A Semantic Method

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Outline

Topic

Introduction

We may wonder about the status of logical accounts of the meaning of language.

- When does a particular proposal count as a *semantic* theory?
- How do we judge a theory to be "correct"?
- What criteria can we use to decide whether one theory is "better" than another?
- Here we will seek to defend what might be described as a "descriptivist" approach.

Traditional formal semantics

- A traditional account of what constitutes a semantic theory is to provide a systematic translation of linguistic constructs into an appropriate formalism that captures the salient aspects of behaviour.
- In the case of indicative sentences, this could be a translation of sentences into form in which the *truth conditions* of the translated sentences, and the relationships between then, accords with intuitions about the original sentences.

Some questions

- What are the “salient aspects of behaviour”, and in what sense should they be “captured”?
- What counts as an appropriate formalism, or interpretation?.

The data

- What is the salient behaviour that we are trying to capture, model or explain: *what is the data?*
- Often the data is messy, with confounding aspects of behaviour.
- If we wish to capture a particular aspect of behaviour, there is a question as to what are the most natural lines of division.
- It can sometimes be unclear how to factorise the behaviour of a given example into these different aspects.
- There may also be questions as to whether it is right to seek to factorise behaviour in this way, or whether a more holistic approach is required.

Simplifying the data

- It is often traditional to use toy examples and scenarios.
- This might be considered a weakness.
- But it could be justified by appeal to natural science: it is conventional to make simplifying assumptions, and capture the behaviour of simplified systems.
- Again we may question the impact of such simplifications, and whether the categories of phenomena are in any sense "natural", and independent.
- (These are perhaps arguments that semanticists need to be aware of the linguistic data, *and* perhaps consider cross-linguistic data as a guard against over-generalising from one language.)

The interpretation

- In addition to the question of the data, there is also the issue of what kinds of system are assumed appropriate as vehicles for expressing semantic behaviour.
- What criteria should be used to determine that one target formalisation (logic or theory) is more appropriate than another?

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Conventional formal semantics

- Implicitly, many accounts of formal semantics attribute a foundational status to **set theory** (and set-theoretic characterisations of possible worlds in particular).
- The goal of a semantic theory is then to find a translation of the phenomena of interest into a such a set-theoretic model (perhaps by way of a logic that is then interpreted by a set-theoretic model).
- Such theories may be deemed to have "explanatory" or "predictive" power if a mapping can found into expressions of set-theory that have the appropriate behaviour by virtue of the rules of set-theory.

Example: The Montagovian approach

- A classic example of this approach is due to Montague (1973, 1974).
- Montague translated a toy fragment of English into a logical representation (IL).
- This representation was then interpreted in set theory.
- *But* Montague viewed the set theory as the "real" semantics:
  - the translations were set up so that the logical representation could be eliminated.

Example: Montague's Intensionality

- Intensionality can be dealt with using possible worlds.
- These are given using a set-theoretic characterisation
  - A possible world is a [consistent] set of propositions [which are true in that world].
  - Alternative: a proposition is a set of worlds [in which that proposition is true].
- Propositions that have the same truth value in the *current world* can still be distinguished if their truth values vary at other worlds.
- This can be exploited to model the epistemic modalities.

Example: Plurals

- We need a way of formalising talk about pluralities of individuals.
- One approach is to use set theory for plural entities (e.g. Landman)
  - "boys" $= \{a, b, c\}$
  - "john and mary" $= \{j\} \cup \{m\}$
- Predication is then of sets of entities.
- Singular entities are singleton sets.

Example: Questions and Answers

Although traditionally used for indicatives, set-theoretic possible worlds have been proposed for other kinds of utterances and sentential forms, such as questions and their answers.

- Questions represent a partition of worlds
  - E.g. yes/no questions partition the world into two sets
- Each set in the partition corresponds to a different possible answer
  - An answer indicates a partition.
  - A correct answer indicates in which partition the current world is located.

Why set theory?

- As evident in common practice, and its justification, set theory has a *de facto* and *de jure* foundational status.
- Why are set-theoretic interpretations given primacy over logical, or other formal interpretations?
  - One justification is that existence of a translation of a particular feature into some set-theoretic construct which mimics the desired behaviour provide some form of explanation that goes beyond "mere" description.
  - Set theory is sometimes seen as playing a foundational role that sets it apart form other kinds of formalism (and notation).
- But set-theoretic interpretations are not without problems . . .

Specific Issues

Intensionality interpretation as sets gives the wrong results.

- Also, possible worlds do not appear as such (reduced to sets).

Plurals what does $\{\{j\},\{m\}\}$ mean?

- Could be considered for controlling distributive inferences (Landman).
- But shown to be inappropriate (Schwarzschild).

Questions and answers set-theoretic PW model has oddities.

- To produce true answers you need to know which world you are in.
- So why would you ask questions?
- Issue of computational tractability (Bos & Gabsdil)

Arguments from Benacerraf

- Number theory can be derived using an appropriate "encoding" of numbers as sets,
  - e.g. $1 = \{\{\}\}$, $2 = \{\{\{\}\}\} \ldots$
- But there may be different encodings
  - e.g. $1 = \{\{\}\}$, $2 = \{\{\{\}\}, \{\}\} \ldots$
- These encodings may vary in their behaviour, both from each other, and from the common understanding of what numbers are.
  - e.g. is $2 \in 3$?
- These issues are used to justify the view that numbers do not refer to some specific concrete realisation, but instead are structural things in themselves.
  - That structure may be manifest in many other systems [whose behaviours go beyond that of numbers].

Arguments from Dummett

- Jumping straight to set-theory leads to metaphysical questions about meaning and language being overlooked.
  - It presupposes that the ontology of language is that of sets.
  - All other metaphysical options and ontological choices are ignored.
- (*It seems these particular arguments are independent of Dummett's case for constructivism.*)

Application to Semantics

- Similar arguments can be applied to set-theoretic semantic theories.
  - Formal semantics should focus on determining appropriate "structural" characterisations of behaviour, *independent* of a specific set-theoretic interpretation.
  - This also allows ontological issues to be treated more seriously.
- (*Cf. Feferman's notions of* adequacy *and* faithfulness.)

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Alternative Approaches

- The alternative to a set-theoretic approach (of mapping phenomena more-or-less directly into set-theoretic constructs) is to
  - help ourselves to "new" primitives and ontological categories, and
  - devise logical rules and axioms that capture the appropriate inferential behaviour (as in Turner 1992) in terms of those primitives.
- Even if we don't think meaning (of language) should inform our metaphysics, we should at least allow ontological considerations to inform our analysis of meaning.
- (*A set-theoretic model then can be used to demonstrate a degree of formal hygiene, rather than being a primary objective.*)

Examples: Alternatives to Montague

- There are a number of theories that consider "independent" formalisations of behaviour
  - *Property Theory*
    (e.g. Chierchia and Turner, 1988; Bealer 1982).
  - *Situation Theory*
    (e.g. Barwise 1987)
- By avoiding sets, they avoid the need to work around implicitly extensional behaviour.
- (*Proof-theoretic NL semantics may also count as a general alternative, e.g. Francez & Dyckhoff, 2007.*)

Examples: Plurals

- Appropriate structural axioms can be formulated, akin to lattice theory, or merology (Link, Schwarzschild etc.)
  - These can be axiomatised independently of any particular set-theoretic interpretation.
  - Particular set-theoretic structures may *exemplify* lattices, but that does not mean that there is no independent notion of a lattice.
- (*An additional, separate, methodological issue arises in that mathematicians may use the language of set theory to formulate the notion of a lattice.*)

Examples: Questions and Answers

- Alternative theories exist (e.g. Ginzburg & Sag) that do not rely on an overt reduction to possible worlds.
  - Analysis with situations or type theory (e.g. using dependent record types).
  - Other approaches may be possible (e.g. taking *questions* to be a new basic category, as with *propositions* in Property Theory).
- (*Questions about reduction to abstraction.*)

Examples: Comment

- The argument is not that these accounts provide the best or the most *comprehensive* analysis of the phenomena in question.
- But their flaws in adequacy are no different in kind from those set-theoretic accounts that tend to model idealised versions of the phenomena in question.
- Using set theory, or not, does not avoid the hard problems in semantics.
- But using set theory may lead to a failure in faithfulness (cf. Feferman).

Criticisms of this alternative approach

- This approach might be criticised as mere "descriptivism":
  - it just characterises the data in some formalism;
  - it lacks any predictive or explanatory power.
- Some may argue that these faults do not arise in set-theoretic semantics.

Issues with justification of set theory

- Why should set theory be seen to be predictive or explanatory?
  - Given that ZF set theory is the most powerful theory, what *explanatory* power is there in showing that there is a mapping into it?
- There *is* a constructive element here: a mapping from language into set theory has to be provided.
  - But it could be argued that the *mapping* itself is (merely) a proxy description.
  - The relevant intended behaviour is not explicit in the set-theory by itself.
- (*And ontological/metaphysical questions do not apppear to be considered very seriously.*)

Questions of Foundations

- Even if we deny set theory a foundational role in semantics, there are still foundational questions.
- We can consider which *foundational framework* (cf. Feferman 1992) is most appropriate, or required.
- For example:
  - Finitary v. Infinitary
  - Uncountable v. Countable
  - Impredicative v. Predicative
  - Non-constructive v. Constructive
- And we can take into account other, metaphysical issues.

Conclusion

- We argue that the role of formal theory is to provide an adequate and faithful presentation of observed behaviour.
- Reductive set-theoretic analyses of semantics are open to a version of Benacerraf's and Dummett's criticisms.
- Reductions to set theory perhaps make it too easy to avoid, or fail to take account of, questions of ontology and formal power.
- Any remaining claims about the inadequacies of "descriptivist" accounts compared to set-theoretic reductions must rely on criteria and assumptions that lie outside the domain of formal semantics as such.