A Semantic Method

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We may wonder about the status of logical accounts of the meaning of language.

- When does a particular proposal count as a *semantic* theory?
- How do we judge a theory to be “correct”?
- What criteria can we use to decide whether one theory is “better” than another?
- Here we will seek to defend what might be described as a “descriptivist” approach.
A traditional account of what constitutes a semantic theory is to provide a systematic translation of linguistic constructs into an appropriate formalism that captures the salient aspects of behaviour.

In the case of indicative sentences, this could be a translation of sentences into form in which the *truth conditions* of the translated sentences, and the relationships between then, accords with intuitions about the original sentences.
Some questions

- What are the “salient aspects of behaviour”, and in what sense should they be “captured”? 
- What counts as an appropriate formalism, or interpretation?
What is the salient behaviour that we are trying to capture, model or explain: \textit{what is the data}\?

Often the data is messy, with confounding aspects of behaviour.

If we wish to capture a particular aspect of behaviour, there is a question as to what are the most natural lines of division.

It can sometimes be unclear how to factorise the behaviour of a given example into these different aspects.

There may also be questions as to whether it is right to seek to factorise behaviour in this way, or whether a more holistic approach is required.
Simplifying the data

- It is often traditional to use toy examples and scenarios.
- This might be considered a weakness.
- But it could be justified by appeal to natural science: it is conventional to make simplifying assumptions, and capture the behaviour of simplified systems.
- Again we may question the impact of such simplifications, and whether the categories of phenomena are in any sense “natural”, and independent.
- (These are perhaps arguments that semanticists need to be aware of the linguistic data, and perhaps consider cross-linguistic data as a guard against over-generalising from one language.)
In addition to the question of the data, there is also the issue of what kinds of system are assumed appropriate as vehicles for expressing semantic behaviour.

What criteria should be used to determine that one target formalisation (logic or theory) is more appropriate than another?
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Set theory and possible worlds

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Implicitly, many accounts of formal semantics attribute a foundational status to \textit{set theory} (and set-theoretic characterisations of possible worlds in particular).

The goal of a semantic theory is then to find a translation of the phenomena of interest into a such a set-theoretic model (perhaps by way of a logic that is then interpreted by a set-theoretic model).

Such theories may be deemed to have “explanatory” or “predictive” power if a mapping can found into expressions of set-theory that have the appropriate behaviour by virtue of the rules of set-theory.
A classic example of this approach is due to Montague (1973, 1974).

Montague translated a toy fragment of English into a logical representation (IL).

This representation was then interpreted in set theory.

*But* Montague viewed the set theory as the “real” semantics:

- the translations were set up so that the logical representation could be eliminated.
Example: Montague’s Intensionality

- Intensionality can be dealt with using possible worlds.
- These are given using a set-theoretic characterisation
  - A possible world is a [consistent] set of propositions [which are true in that world].
  - Alternative: a proposition is a set of worlds [in which that proposition is true].
- Propositions that have the same truth value in the current world can still be distinguished if their truth values vary at other worlds.
- This can be exploited to model the epistemic modalities.
We need a way of formalising talk about pluralities of individuals.

One approach is to use set theory for plural entities (e.g. Landman)

- “boys” = \{a, b, c\}
- “john and mary” = \{j\} \cup \{m\}

Predication is then of sets of entities.

Singular entities are singleton sets.
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Example: Questions and Answers

Although traditionally used for indicatives, set-theoretic possible worlds have been proposed for other kinds of utterances and sentential forms, such as questions and their answers.

- Questions represent a partition of worlds
  - E.g. yes/no questions partition the world into two sets
- Each set in the partition corresponds to a different possible answer
  - An answer indicates a partition.
  - A correct answer indicates in which partition the current world is located.
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Set theory and possible worlds
Some questions and issues

Why set theory?

- As evident in common practice, and its justification, set theory has a *de facto* and *de jure* foundational status.

- Why are set-theoretic interpretations given primacy over logical, or other formal interpretations?
  - One justification is that existence of a translation of a particular feature into some set-theoretic construct which mimics the desired behaviour provide some form of explanation that goes beyond “mere” description.
  - Set theory is sometimes seen as playing a foundational role that sets it apart form other kinds of formalism (and notation).

- But set-theoretic interpretations are not without problems . . .
Specific Issues

**Intensionality** interpretation as sets gives the wrong results.
- Also, possible worlds do not appear as such (reduced to sets).

**Plurals** what does \{\{j\}, \{m\}\} mean?
- Could be considered for controlling distributive inferences (Landman).
- But shown to be inappropriate (Schwarzschild).

**Questions and answers** set-theoretic PW model has oddities.
- To produce true answers you need to know which world you are in.
- So why would you ask questions?
- Issue of computational tractability (Bos & Gabsdil)
Arguments from Benacerraf

- Number theory can be derived using an appropriate “encoding” of numbers as sets,
  - e.g. $1 = \{\{\}\}$, $2 = \{\{\}\}, \{\{\}\} \ldots$
- But there may be different encodings
  - e.g. $1 = \{\{\}\}$, $2 = \{\{\}\}, \{\}\} \ldots$
- These encodings may vary in their behaviour, both from each other, and from the common understanding of what numbers are.
  - e.g. is $2 \in 3$?
- These issues are used to justify the view that numbers do not refer to some specific concrete realisation, but instead are structural things in themselves.
  - That structure may be manifest in many other systems [whose behaviours go beyond that of numbers].
Arguments from Dummett

- Jumping straight to set-theory leads to metaphysical questions about meaning and language being overlooked.
  - It presupposes that the ontology of language is that of sets.
  - All other metaphysical options and ontological choices are ignored.

(It seems these particular arguments are independent of Dummett’s case for constructivism.)
Similar arguments can be applied to set-theoretic semantic theories.

- Formal semantics should focus on determining appropriate “structural” characterisations of behaviour, independent of a specific set-theoretic interpretation.
- This also allows ontological issues to be treated more seriously.

(Cf. Feferman’s notions of adequacy and faithfulness.)
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Less reductive alternatives

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The alternative to a set-theoretic approach (of mapping phenomena more-or-less directly into set-theoretic constructs) is to

- help ourselves to “new” primitives and ontological categories, and
- devise logical rules and axioms that capture the appropriate inferential behaviour (as in Turner 1992) in terms of those primitives.

Even if we don’t think meaning (of language) should inform our metaphysics, we should at least allow ontological considerations to inform our analysis of meaning.

(A set-theoretic model then can be used to demonstrate a degree of formal hygiene, rather than being a primary objective.)
Examples: Alternatives to Montague

- There are a number of theories that consider “independent” formalisations of behaviour
  - *Property Theory* (e.g. Chierchia and Turner, 1988; Bealer 1982).
  - *Situation Theory* (e.g. Barwise 1987)
- By avoiding sets, they avoid the need to work around implicitly extensional behaviour.
- *(Proof-theoretic NL semantics may also count as a general alternative, e.g. Francez & Dyckhoff, 2007.)*
Examples: Plurals

- Appropriate structural axioms can be formulated, akin to lattice theory, or merology (Link, Schwarzschild etc.).
  - These can be axiomatised independently of any particular set-theoretic interpretation.
  - Particular set-theoretic structures may exemplify lattices, but that does not mean that there is no independent notion of a lattice.

(An additional, separate, methodological issue arises in that mathematicians may use the language of set theory to formulate the notion of a lattice.)
Alternative theories exist (e.g. Ginzburg & Sag) that do not rely on an overt reduction to possible worlds.

- Analysis with situations or type theory (e.g. using dependent record types).
- Other approaches may be possible (e.g. taking questions to be a new basic category, as with propositions in Property Theory).

(Questions about reduction to abstraction.)
The argument is not that these accounts provide the best or the most *comprehensive* analysis of the phenomena in question.

But their flaws in adequacy are no different in kind from those set-theoretic accounts that tend to model idealised versions of the phenomena in question.

Using set theory, or not, does not avoid the hard problems in semantics.

But using set theory may lead to a failure in faithfulness (cf. Feferman).
This approach might be criticised as mere “descriptivism”:
- it just characterises the data in some formalism;
- it lacks any predictive or explanatory power.

Some may argue that these faults do not arise in set-theoretic semantics.
Issues with justification of set theory

Why should set theory be seen to be predictive or explanatory?

- Given that ZF set theory is the most powerful theory, what *explanatory* power is there in showing that there is a mapping into it?

There *is* a constructive element here: a mapping from language into set theory has to be provided.

- But it could be argued that the *mapping* itself is (merely) a proxy description.
- The relevant intended behaviour is not explicit in the set-theory by itself.

*(And ontological/metaphysical questions do not apppear to be considered very seriously.)*
Even if we deny set theory a foundational role in semantics, there are still foundational questions.

We can consider which *foundational framework* (cf. Feferman 1992) is most appropriate, or required.

For example:

- Finitary v. Infinitary
- Uncountable v. Countable
- Impredicative v. Predicative
- Non-constructive v. Constructive

And we can take into account other, metaphysical issues.
We argue that the role of formal theory is to provide an adequate and faithful presentation of observed behaviour.

Reductive set-theoretic analyses of semantics are open to a version of Benacerraf’s and Dummett’s criticisms.

Reductions to set theory perhaps make it too easy to avoid, or fail to take account of, questions of ontology and formal power.

Any remaining claims about the inadequacies of “descriptivist” accounts compared to set-theoretic reductions must rely on criteria and assumptions that lie outside the domain of formal semantics as such.