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Abstract

Ambiguous private information leads to informational inefficiency of market prices in rational expectations equilibrium. This inefficiency implies lower asset prices as uninformed traders require a premium to hold assets. This premium is increasing in the riskiness of the asset and leads to excess volatility, price swings, and abrupt volatility and illiquidity variation across informational efficiency regimes. Public information affects the informational efficiency of price and can also lead to abrupt changes in volatility and illiquidity.

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1 Introduction

Along with their role in rationing assets, market prices aggregate and convey information. In traditional models where asset prices convey information in equilibrium, prices always react to and reveal information.[1] However, growing empirical research indicates that prices react to news in differing ways depending on the state of the economy which may make information transmission difficult.[2] This is not surprising given that the information received by traders and potentially transmitted through market prices can vary widely across sources, asset classes, and time. This paper investigates the ability of market prices to transmit information that is perceived to be ambiguous and shows that the reaction of market prices to news can be very different than in traditional asset market models.

Unlike unambiguous information, market prices will not reveal a range of ambiguous private information in REE. This leads to (i) lower asset prices due to an unrevealed information premium which increases with fundamental risk and (ii) excess volatility and high illiquidity as prices change discontinuously relative to fundamentals across informational efficiency regimes. Moreover, price volatility varies across informational efficiency regimes and the arrival of public information can lead to excess volatility and high illiquidity by affecting the informational efficiency of prices.[3]

These results stem from two facts. The first is that informed traders who receive ambiguous private information about an asset will trade off their asset holdings unless they are compensated by an ambiguity premium. Moreover, they will do so at the same price for a range of information, a property we term portfolio inertia in information. Uninformed traders who take positive positions in the asset will then require an unrevealed information premium in addition to a market risk premium which compensates them for fundamental risk and reduction in asset holders. The information premium is higher for riskier assets. Information is not revealed in REE when the latter premia are lower in aggregate than the ambiguity premium.

The second fact is that price changes discontinuously relative to fundamentals across informational efficiency regimes. Informational inefficiency implies uninformed traders' beliefs are based on a set of possible information compared to exact information under informa-

[1]For instance, Grossman (1981), Radner (1979) *inter alia*.

[2]See Andersen, Bollerslev, Diebold, and Vega (2005), Faust, Rogers, Wang, and Wright (2007), and others.

[3]Differing volatility across regimes is suggestive of time-varying volatility in dynamic models, see Andersen, Bollerslev, and Diebold (2009) for an introduction to the extensive literature on time-varying volatility.

tional efficiency and so beliefs will differ discontinuously across regimes. Since these beliefs drive asset prices in equilibrium, the latter are discontinuous relative to fundamentals across regimes. This discontinuity implies excess volatility and high illiquidity measured by price impact across regimes. Moreover public information affects the range of unrevealed information and can yield discontinuous price changes and anomalous price behavior such as a price fall under good public news.

We extend the standard REE model where market prices aggregate and communicate information (see Radner (1979) and Grossman (1976) among others). Informed traders receive ambiguous private information, i.e. they perceive a set of probability distributions over the underlying fundamentals rather than a single distribution. These traders are ambiguity averse as per the Gilboa and Schmeidler (1989) multiple priors representation, which captures the degree of confidence decision-makers have in probabilistic assessments based on the quality of information, unlike the Savage (1954) decision-making model (Gilboa and Marinacci (2012)).[4] Uninformed traders can be ambiguity-neutral or averse.

The key portfolio inertia in information property is a consequence of the non-smooth multiple priors representation. It is distinct from the portfolio inertia in prices property identified by Dow and da Costa Werlang (1992b), but related since both follow from non-smoothness of the representation. Incorporating this non-smooth decision-making model has provided a number of insights in studying financial markets (Epstein and Schneider 2010).[5] Smooth preference representations such as Klibanoff, Marinacci, and Mukerji (2005), Maccheroni, Marinacci, and Rustichini (2006), and Hansen and Sargent (2007) do not yield inertia in information and so will not generate the informational inefficiency we study here. Experimental evidence in Ahn, Choi, Kariv, and Gale (2011), Asparouhova, Bossaerts, Eguia, and Zame (2012), and Bossaerts, Ghirardato, Guarneschelli, and Zame (2010) provides persuasive support of non-smooth models of ambiguity aversion in financial markets.

Information non-revelation under ambiguity differs from informational inefficiency due to noise-, endowment-, or taste-shock mechanisms which introduce additional exogenous randomness in price to impede information revelation.[6] Our analysis suggests that the ambiguity-based and noise-based mechanisms provide differing, but complementary means of studying financial markets. There is a growing literature studying informational efficiency

[4]See also Ellsberg (1961), Keynes (1921) and Knight (1921).

[5]Much of these are developed with representative agent or homogeneous information frameworks. Chapman and Polkovnichenko (2009) show that ignoring underlying heterogeneity can significantly change estimates for the equity premium and risk free rate.

[6]See Dow and Gorton (2008) for a recent discussion of these

and ambiguity averse traders including Tallon (1998), Caskey (2008), Ozsoylev and Werner (2011), and Mele and Sangiorgi (2011) which use the noise trader mechanism for informational inefficiency and Easley, O'Hara, and Yang (2011) and Condie and Ganguli (2011a) which don't.[7] The latter demonstrates that the informational inefficiency studied here has the desirable property of being robust in the context of general financial market economies.

The paper proceeds as follows. We first develop the financial market model in section 2. Section 3 describes the conditions for and nature of non revelation of information. Section 4 elaborates the pricing implications of partial revelation. Section 5 discusses and extends the model, including comparison to noise-based partial revelation and incorporating non-tradeable labor income, multiple ambiguous signals, and ambiguity averse uninformed traders. Section 6 concludes.

2 A model of ambiguous private information

The model is populated by two types of investors, denoted by $n \in \{I, U\}$. I-investors receive a private signal and are referred to as informed investors whereas U-investors are uninformed. All investors live for 3 periods and trade assets in the market. Time is indexed by $t = 0, 1, 2$. Investors observe information and trade at $t = 1$. All uncertainty is resolved and consumption occurs at $t = 2$.

Two assets are traded in the market. The first asset is a risk-free bond whose payoff is denoted V_f.[8] This asset is in perfectly elastic supply. The second asset, called the stock, has an uncertain terminal value denoted by V. It is assumed to be in unit net supply. At time 0, type n-investors are endowed in aggregate with a fraction $x_0^n > 0$ of the uncertain asset and 0 units of the bond. Trade occurs in period 1 with the resolution of uncertainty occurring in period 2.

We assume that the log stock payoff $\ln V$, denoted v henceforth, is normally distributed with mean μ and variance σ^2. In period 0, all investors have identical information about the expected payoff of the uncertain asset. Both types of investors believe that v is normally distributed with variance σ^2. Both types are uncertain about the mean of v and their beliefs over μ are given by a normal distribution that has mean μ_0 and precision ρ_0. However, the two types of traders differ in their reciept and perception of information.

[7] de Castro, Pesce, and Yannelis (2010) introduce and prove existence, incentive compatibility, and Pareto efficiency of a separate equilibrium concept they call 'maximin rational expectations equilibrium'.

[8] It would perhaps be more appropriate to use the term 'uncertainty-free' to describe this asset in our setting, but we stay with the usual terminology.

At $t = 1$, I-investors receive a private signal

$$s = \mu + \epsilon \tag{1}$$

that conveys information about μ, where ϵ is a stochastic error term.[9] The signal is interpreted differently by the informed I-investors and the uninformed U-investors, if the latter observe it. This differential interpretation is related to the signal error term ϵ.

Both types of investor agree that the signal error ϵ is distributed normally with precision ρ_ϵ but have differing assessments of the mean μ_ϵ of the error term. I-investors believe the information may be biased but are unsure about the direction of this bias. I-investors' lack of confidence about the signal bias is modeled as *ambiguity in the signal* in the sense that they know only that $\mu_\epsilon \in [-\delta, \delta]$ where $\delta > 0$. The size of this interval captures the I-investors degree of confidence in the information. We denote I-investors' assessment of the mean by μ_ϵ^I. In this structure I-investors use a set of likelihoods, indexed by $\mu_\epsilon^I \in [-\delta, \delta]$, in updating their beliefs, which we discuss in section 2.1.

I-investors may doubt the unbiasedness of a signal because of concerns about the signal source, because the information is intangible in the sense of Daniel and Titman (2006), or because the relationship between the signal and the stock is ambiguous, for example, receiving ambiguous private information about a non-traded asset like labor income, whose payoff is correlated with that of the stock (see section 5.2) among other possibilities. See also the discussions in Epstein and Schneider (2008) and Illeditsch (2011).[10]

On the other hand, for simplicity, we assume U-investors believe the signal is unbiased, i.e. their assessment of the mean $\mu_\epsilon^U = 0$.[11] This structure implies that the informational inefficiency derives from the ambiguity in information perceived by the ambiguity-averse recipients and not the uninformed investors. That is, it is not the uninformed investors' inability to interpret information which drives informational inefficiency. This model can be extended easily to allow for other investors who receive private signals that are not perceived to be ambiguous. However, such signals will be revealed in equilibrium and as such, the results of the model would not change. We choose to interpret the priors of traders in this model as having already incorporated all private signals that have been revealed.

[9] A similar signal structure without ambiguity appears in Peress (2009) relating to the analysis of Peress (2004).

[10] These papers model ambiguity through an interval of signal variances.

[11] This assumption is relaxed in section 5.4 at the cost of some notational simplicity, but without much additional insight into partial revelation.

2.1 Decision making

Ambiguous information is processed and incorporated using an updating rule developed in Epstein and Schneider (2007) and Epstein and Schneider (2008). This rule reduces to Bayes' rule when the infomation is unambiguous.[12]

Investors' prior over μ has mean μ_0 and precision ρ_0. Bayesian updating implies that given μ_0 and $\mu_\epsilon \in [-\delta, \delta]$, μ conditional on the signal s is normally distributed with mean

$$\mu|s = \frac{\rho_0\mu_0 + \rho_\epsilon (s + \mu_\epsilon)}{\rho_0 + \rho_\epsilon} \tag{3}$$

and precision $\rho_0 + \rho_\epsilon$. Therefore, the set of updated beliefs of an I-investor about μ is the set of normal distributions with precision $\rho_0 + \rho_\epsilon$ and means

$$[\underline{\mu}^I|s, \overline{\mu}^I|s] = \left[\frac{\rho_0\mu_0 + \rho_\epsilon(s-\delta)}{\rho_0 + \rho_\epsilon}, \frac{\rho_0\mu_0 + \rho_\epsilon(s+\delta)}{\rho_0 + \rho_\epsilon}\right]. \tag{4}$$

Investors' von Neumann-Morgenstern utility u is in the constant relative risk aversion (CRRA) class with common CRRA coefficient γ, i.e.

$$u(W_2) = \frac{W_2^{1-\gamma}}{1-\gamma} \tag{5}$$

for $\gamma \neq 1$, where terminal wealth $W_2 = \theta^n R + (1-\theta^n)R_f$, R and R_f are the gross returns on the stock and bond respectively and θ^n is the fraction of wealth invested in the stock. For $\gamma = 1$, $u(W_2) = \ln W_2$.[13]

Ambiguity averse investors make decisions using the Gilboa and Schmeidler (1989) representation.[14] Denoting by M^n the set of distributions representing investor n's beliefs given

[12] Epstein and Schneider (2007) consider the set of possible Bayes updates that arise from a set of likelihoods and a prior. If a decision maker has a prior and the set of likelihoods is $\{L(s|\cdot)\}_{L\in\mathcal{L}}$ for some index set $\mathcal{L}$, then the set of updated beliefs about event B is given by

$$\{Pr(B|s)\} = \left\{ \frac{Pr(B)L(s|B)}{\int L(s|B)dB} \middle| L \in \mathcal{L} \right\}. \tag{2}$$

Investors make decisions only once after receiving information, so issues of dynamic inconsistency do not arise, but intertemporal decision making would be dynamically consistent with this updating rule and our assumptions.

[13] For expositional purposes, we will generally use (5) for risk preferences of investors and explicitly indicate when log utility is used.

[14] The ambiguity aversion of investors in this representation can be formalised using the analysis of Gajdos, Hayashi, Tallon, and Vergnaud (2008).

his information, the utility from portfolio θ^n is

$$\mathbf{U}^n(\theta^n) = \min_{m \in M^n} \mathbb{E}_m[u(W_2(\theta^n))] = \min_{m \in M^n} \mathbb{E}_m \left(\frac{W_2(\theta^n)^{1-\gamma}}{1-\gamma} \right), \tag{6}$$

This includes the case of Savage (1954) expected utility U-investors who do not perceive any ambiguity and so M^U is a single probability distribution.

Utility $\mathbf{U}^n$ is everywhere differentiable except when the terminal wealth from portfolio holdings is not uncertain, i.e. when the investor trades away his holdings of the stock and holds only the risk-free asset. This non-differentiability is key for the partial revelation equilibria.[15]

2.2 Market prices and rational expectations equilibria

Trade in the assets occurs in period 1. A price function $\mathbf{P}$ maps signal values s to prices, i.e. $\mathbf{P}(s) = (P(s), R_f(s))$, where $P(\cdot)$ denotes the stock price and $R = V/P$. Information is revealed through prices when the function $\mathbf{P}$ is invertible. When this occurs for all signals, U-investors correctly infer each signal by observing the market prices and the price function $\mathbf{P}$ is said to be *fully-revealing*.

When the function is not invertible, the market prices will not reveal all information and the function is said to be *partially revealing*. When prices are partially revealing, multiple signal values may be consistent with the observed market prices (P, R_f) and U-investors know only that some signal from the set $\mathbf{P}^{-1}(P, R_f)$ was observed by I-investors.

Initial wealth for n-investors at price P is $W_0^n = x_0^n P$. Thus, the market clearing condition for the stock is

$$\frac{\theta^I}{P} W_0^I + \frac{\theta^U}{P} W_0^u = 1 \tag{7}$$

The rational expectations equilibrium (REE) concept requires that individuals behave optimally given the information that they have and that they make use of all available information.

Definition 1. A *rational expectations equilibrium* is a set of portfolio weights $\{\theta^I(s), \theta^U(s)\}$ and a price function $\mathbf{P}$, which specifies prices $P(s)$ and $R_f(s)$ for each signal s, such that the following hold almost surely.

[15]Though we will not explore this here, other portfolio positions where utility is non-differentiable could be used for studying the kind of partial revelation we present here.

1. Each I-investor has information s and chooses a portfolio $\theta^I(s)$ that satisfies

$$\theta^I(s) \in \arg\max \mathbf{U}^I(\theta|s) \tag{8}$$

2. Each U-investor has information $\mathbf{P}^{-1}(P(s), R_f(s))$ and chooses a portfolio $\theta^U(s)$ that satisfies

$$\theta^U(s) \in \arg\max \mathbf{U}^U(\theta|\mathbf{P}^{-1}(P(s), R_f(s))) \tag{9}$$

3. The market clearing condition (7) holds.

Given this definition, an REE is said to be *fully revealing* when the equilibrium price function is fully revealing and it is said to be *partially revealing* otherwise. In the above definition, we specify I-investors' information as the private signal s since the price does not convey any additional information to them.

2.3 Investor demand and inertia

We solve for investor demand by adapting the standard method for approximating asset returns given the lognormality assumption on the payoff distribution (for example, see Campbell and Viciera (2002)).[16] Throughout, lowercase letters represent the natural log of model variables. Since we can work with relative prices, we normalize $R_f = 1$, i.e. $\ln R_f \equiv r_f = 0$ hereafter and work with the log stock price $p \equiv \ln P$, in analysing REE. Investor demand is given the following result.

Proposition 1. *The optimal portfolio under beliefs* $[\underline{\mu}^n|s, \overline{\mu}^n|s]$ *is given by*

$$\theta^n(p) = \begin{cases} \frac{1}{\gamma\sigma^2}\left(\underline{\mu}^n|s + \frac{1}{2}\sigma^2 - p\right) & \underline{\mu}^n|s + \frac{1}{2}\sigma^2 > p \\ 0 & \underline{\mu}^n|s + \frac{1}{2}\sigma^2 \le p \le \overline{\mu}^n|s + \frac{1}{2}\sigma^2 \\ \frac{1}{\gamma\sigma^2}\left(\overline{\mu}^n|s + \frac{1}{2}\sigma^2 - p\right) & \overline{\mu}^n|s + \frac{1}{2}\sigma^2 < p \end{cases} \tag{10}$$

In the above expression, note that the case of $\underline{\mu}^n|s + \frac{1}{2}\sigma^2 \le p \le \overline{\mu}^n|s + \frac{1}{2}\sigma^2$ corresponds to a situation where n-investors trade from their non-zero initial stock position to a zero position in the stock. Thus, this demand is not a no-trade position.

[16] The approximation becomes exact as the discrete time interval shrinks to zero. Details are provided in section A.1.

I-investors require an *ambiguity premium* whenever they do not trade away their stock holding to a zero position. This premium is in addition to the usual risk premium observed in these models. I-investors require a reduction (respectively, an increase) of $\delta\rho_\epsilon/(\rho_0+\rho_\epsilon)$ in the stock price when they are long (respectively, short) in the stock given their effective belief $\underline{\mu}^I|s$ (respectively, $\overline{\mu}^I|s$). Whenever the price does not incorporate this ambiguity premium, they trade away their stock holding to a zero position.

I-investors' demand also exhibits two interesting and complementary facts. The first is that for any given signal value s, there exists a range of prices for which it is optimal for I-investors to trade away their stock holdings to a zero position ($\theta^I = 0$). This corresponds to portfolio inertia in prices at the risk-free portfolio first noted by Dow and da Costa Werlang (1992b).

The second fact is that for a given price p, I-investors will find it optimal to trade to a zero position under distinct signals s, s' when $p + \frac{1}{2}\sigma^2 \in [\underline{\mu}^I|s, \overline{\mu}^I|s] \cap [\underline{\mu}^I|s', \overline{\mu}^I|s']$ That is, at $\theta^I = 0$, there is *portfolio inertia with respect to information.* We show below that this inertia leads to the existence of partially revealing REE.[17] Whether or not the price incorporates the ambiguity premium of $\delta\rho_\epsilon/(\rho_0+\rho_\epsilon)$ plays an important role in informational inefficiency since it determines whether the inertia position is optimal. Finally, note also that this inertia does not appear in smooth models of preferences and so these models will not display the partial revelation property we study here.

3 Equilibrium partial revelation

3.1 The necessity of inertia for partial revelation

Non-revelation of signals s and s' requires that $p(s) = p(s') = p$. If I-investors find it optimal to not trade away their stock holdings then the equilibrium price will be monotone in the signal and hence revealing, as the next proposition proves.

Proposition 2. *If markets clear with $\theta^I(s) \neq 0$ then signal s is revealed in any rational expectations equilibrium price.*

Thus, the existence of partial revelation requires that for a given price there is a range of signals for which I-investors wish to trade to a zero position in the stock. Then the

[17] Condie and Ganguli (2011a) use this property in the context of general financial market exchange economies to establish robust existence of partially revealing REE.

market-clearing price p satisfies

$$p = \mu_{PR}^U + \frac{1}{2}\sigma^2 - \frac{\gamma\sigma^2}{x_0^U}. \tag{11}$$

where μ_{PR}^U denotes U-investors' updated beliefs about the mean of μ given the available price information. We refer to the term $\frac{\gamma\sigma^2}{x_0^U}$ as the *market risk premium.*

3.2 The price function and uninformed investor beliefs

The above requirements of optimality and market-clearing are related and complicated by the fact that U-investors infer information from the prevailing price. This inference potentially leads to changes in the beliefs of U-investors which leads to changes in the market prices. Thus, the equilibrium prices and beliefs of U-investors must be solved for simultaneously. The solution to this problem is a set of signals that are not revealed in REE and beliefs for U-investors that are consistent with the knowledge that a signal in the set of unrevealed signals has been received.

First, we characterise in terms of U-investor and I-investor beliefs, the range of signals for which I-investors will trade away their stockholding to U-investors at a given price p.

Lemma 1. *The range of signals for which markets clear with I-investors trading away their stockholding when U-investor beliefs are μ_{PR}^U is expressed implicitly as*

$$\left[\mu_{PR}^U + \delta + \frac{\rho_0}{\rho_\epsilon}(\mu_{PR}^U - \mu_0) - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon}\frac{\gamma\sigma^2}{x_0^U}, \mu_{PR}^U - \delta + \frac{\rho_0}{\rho_\epsilon}(\mu_{PR}^U - \mu_0) - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon}\frac{\gamma\sigma^2}{x_0^U}\right] \tag{12}$$

Let a denote the lower bound of this interval and b denote the upper bound. Then when the signal is not revealed, U-investors know only that the signal observed by I-investors lies in the interval $[a, b]$. U-investor beliefs μ_{PR}^U are constant over the interval $[a, b]$ and denoted $\mu_{PR}^U|[a, b]$. For signals $s \notin [a, b]$, (3) implies beliefs $\mu_{PR}^U|s$ are monotone and linear in s .

The interval bounds a and b are determined endogenously by the implicit expression in (12) since $\mu_{PR}^U|[a, b]$ depends on the values of these bounds. The next result characterises updated U-investor beliefs based on the knowledge that the signal lies in $[a, b]$. ϕ and Φ denote the standard normal density and distribution functions respectively.

Lemma 2. *U-investors' updated belief $\mu_{PR}^U|[a, b]$ about the mean log stock payoff conditional* *on knowing that the signal lies in an interval* $[a, b]$ *is*

$$\mu_{PR}^U|[a, b] = \mathbb{E}\left[\mu|a \leq s \leq b\right] = \mu_0 + \frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon}\Delta(a, b). \tag{13}$$

where

$$\Delta(a, b) = \frac{\phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(a - \mu_0)\right) - \phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(b - \mu_0)\right)}{\Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(b - \mu_0)\right) - \Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(a - \mu_0)\right)}\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0 + \rho_\epsilon}} \tag{14}$$

The term

$$\frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon}\Delta(a, b) \tag{15}$$

represents the change in U-investor beliefs when they know only that I-investors received a signal that is not revealed by price, i.e. $s \in [a, b]$.[18] The equilibrium price function must be consistent with this inference by U-investors, which implies that any partially-revealing price function p_{PR} satisfies

$$p_{PR}(s) = \begin{cases} \mu_0 + \frac{1}{2}\sigma^2 + \frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}\Delta(a, b) - \frac{\gamma\sigma^2}{x_0^U} & \text{if } s \in [a, b] \\ \mu_0 + \frac{1}{2}\sigma^2 + \frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}(s - \mu_0) - \frac{\gamma\sigma^2}{x_0^U} & \text{if } s < a \text{ or } s > b \end{cases} \tag{16}$$

This function is non-linear in signals and exhibits discontinuities at signal values a and b since U-traders' updated beliefs, $\mu_{PR}^U|[a, b]$, lie strictly between the updated belief based on the signal value a, $\mu_{PR}^U|a$, and the updated belief based on the signal value b, $\mu_{PR}^U|b$.

Combining (13) and (12) shows that the existence of an interval $[a, b]$ of unrevealed signals and hence the existence of partially revealing REE follows from the existence of solutions a and b to

$$\mu_0 + \Delta(a, b) + \delta - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon}\frac{\gamma\sigma^2}{x_0^U} = b \tag{17}$$

$$\mu_0 + \Delta(a, b) - \delta - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon}\frac{\gamma\sigma^2}{x_0^U} = a \tag{18}$$

Subtracting the left-hand sides of equations (17) and (18) implies that if the interval $[a, b]$ of unrevealed signals exists then it has length 2δ. Using this information and rearranging

[18]U-investors are infering all available information from the non-revelation of the signal.

equation (17) reduces the problem to finding a solution to

$$\frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon}\left(b - \mu_0 - \Delta(b - 2\delta, b)\right) + \left(\frac{\gamma\sigma^2}{x_0^U} - \frac{\delta\rho_\epsilon}{\rho_0 + \rho_\epsilon}\right) = 0 \tag{19}$$

A solution to (19) always exists when the ambiguity premium required by I-investors is at least as large as the market risk premium required by U-investors as we show next. Generally, there is not an analytical solution for the interval bounds a and b, though they can be found numerically.

Proposition 3. *1. An interval $[a, b]$ of unrevealed signals exists if and only if the market risk premium $\frac{\gamma\sigma^2}{x_0^U}$ is less than or equal to the ambiguity premium $\frac{\delta\rho_\epsilon}{\rho_0+\rho_\epsilon}$.*

2. The size of the set of unrevealed signals is 2δ.

3. The unique partially revealing price function takes the form given in equation (16) and is discontinuous at a and b.

4. Trade volume is x_0^I for all s.

4 Pricing implications of partial revelation

4.1 Premia and the informativeness of prices

The partially-revealing price function p_{PR} in (16) can be decomposed into portions determined by ex-ante beliefs $(\mu_0 + \frac{1}{2}\sigma^2)$ and by two premia. The first premium is the market risk premium $\frac{\gamma\sigma^2}{x_0^U}$, henceforth market risk, and reflects the premium required by U-investors in order for them to be willing to hold all of the stock. Market risk comprises compensation for the fundamental risk $\gamma\sigma^2$ given investors' risk aversion and the stock payoff volatility and compensation $\frac{1}{x_0^U}$, for the reduction in stockholders that occurs when I-investors trade away their asset holdings. If x_0^U is small, U-investors are required to purchase a large fraction of the total asset stock from I-investors when the latter wish to trade to a zero position. This is an increasingly risky portfolio and U-investors require an increasing amount of compensation to take on this additional risk. However, if x_0^U is large, then U-investors own most of the market and taking on the remainder of the assets does not greatly increase the compensation required by U-investors.

The second premium

$$-\frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon}\Delta(a, b) \tag{20}$$

is novel to this paper and arises only when private ambiguous information is not revealed in market prices. We refer to it as the *unrevealed information premium.* $\Delta(a, b)$ is always negative and paramaterizes the premium that U-traders require when they know that there is information in the market that they haven't observed. The relation between these two premia is given in the next result.

Proposition 4. *1. If market risk is zero, $b = \mu_0 + \delta$ and the unrevealed information premium is zero.*

2. The unrevealed information premium is increasing in the market risk premium and is positive whenever the market risk premium is positive.

3. If the signal is not revealed in equilibrium then the U-investors ascribe it to bad news, i.e. the expected value of the signal is less than μ_0.

When the market risk premium is zero, investors are risk-neutral for instance, then there is no information available from the fact that the signal has not been revealed in equilibrium. Since $[\mu_0 - \delta, \mu_0 + \delta]$ is centered around μ_0, the symmetry of the normal distribution about its mean implies that the updated beliefs of U-investors will be μ_0, i.e. the beliefs of U-investors don't change upon learning that the signal has not been revealed in equilibrium.

The unrevealed information premium is increasing in the market risk premium. As the market risk premium increases, the incentive to hold the risky asset increases ceteris paribus. Therefore, when market risk is high *and* I-investors trade away their stockholding, it is because the signal that they have received is relatively bad. Since market risk is usually non-zero for traded assets this asymmetry implies that non-revealed information tends to be interpreted as bad news on average. Thus, the unrevealed information premium will always be positive.

Figure 1(b) illustrates these features of equilibrium. The set $[a, b]$ of unrevealed signal values moves to the left and the price in the partial-revelation region declines as market risk increases due to the increase in the unrevealed information premium. When market risk is zero or moderate, both moderately good news ($s > \mu_0$) and bad news ($s < \mu_0$) are not revealed.

The first statement in Proposition 3 as well as statement 2 in Proposition 4 highlight the role that the relative market share of those who receive the ambiguous signal plays in the revelation of information. From these two results follows the intuitive outcome that if the market share of those who have received the private signal is large enough, the signal will be revealed in equilibrium, regardless of its value. That is, if enough traders in the market know the information, it will be revealed in equilibrium. As the market share of those who are privately informed increases, so does the market risk premium. U-investors are required to hold an increasingly large portion of the market and must be compensated to do so. As this market risk premium increases, the incentives for I-investors to hold the risky asset and hence reveal their signal increase.

Next we note that the informativeness of prices depends fundamentally on the amount of ambiguity in the signal.

Proposition 5. *The size of the set of signals that are obscured in equilibrium is strictly increasing in the amount of ambiguity in the signal.*

As ambiguity (measured by δ), increases so does the set of signals that don't get revealed in equilibrium. Market environments that are characterized by large amounts of ambiguous information will also tend to have prices that are less informative. Furthermore, in this model the market mechanism does not impose any additional ambiguity. Thus, the size of the unrevealed region of prices corresponds exactly to the amount of ambiguity that I-investors perceive in the signal.

4.2 Price volatility and jumps

The comparitive statics of this model indicate that the revelation and non-revelation of privately observed signals have implications for the volatility of equilibrium stock prices. The first implication is that the discontinuous nature of the price function is suggestive of *crashes, jumps, and excess price volatility.* Market prices display excess volatility in the neighborhood of the points of discontinuity of the price function as moving between information regimes with small changes in the signal leads to a disproportionately large change in prices. For example, consider an economy for which information is not revealed, but for which the signal is close to a in figure 1(a). In this case, if the news that arrives becomes just slightly worse, then it is revealed through a large, discontinuous fall in prices. This is true for all unrevealed signals that are worse news than the average of the unrevealed signals. On the other hand, this price swing will be positive if the revealed signal is better than the average unrevealed signal, and hence is above b. Similarly, a change from revelation to non revelation of signals is accompanied by a discontinuous move in price as U-investors' information changes from an exact signal value to a set of possible signal values. Figure 2 depicts this excess volatility for signals close to b.

The second implication centers around the price's sensitivity to received signals within the two revelation regimes. Movements in price are the mechanism by which prices convey information. Since price does not respond to changes in the signal for unrevealed signals ($s \in [a, b]$), price volatility is lower when information is not revealed. Since price is sensitive to changes in the signal, volatility is higher for revealed signals ($s < a$ or $s > b$). This result suggests caution in the face of financial market policy options that might unduly limit market volatility, whether this is the goal of the policy or not. Periods of higher price volatility are not necessarily bad if prices are successfully incorporating new information and transmitting that information to market participants. Likewise, for these same reasons, periods of lower market volatility are not necessarily desirable.

We can summarize the above analysis as follows.

Proposition 6. *1. Transitions into and out of signal revelation are marked by excess price volatility.*

2. Price volatility conditional on revelation of information is strictly positive and is zero conditional on non-revelation of signals.

Taken together, these two results on price volatility with ambiguous information imply that the transition out of periods of low market volatility can be hectic. If the market changes in such a way that information is revealed, this happens concurrently with a large price swing, followed by a period of relatively high market volatility. Thus, when information becomes revealed in market prices an initial increase in market volatility is likely to be followed by higher expected volatility in the future. On the other hand, when moving into periods of non-revelation, the initial excess volatility is followed by a period of relative tranquility in asset prices.

These mechanisms and results are different from those in other papers. Illeditsch (2011) shows that if traders are risk-averse, there is a discontinuity in price when a public signal confirms the prior mean, implying excess volatility and volatility variation, while we establish excess volatility and variation even with risk-neutrality. Dow and da Costa Werlang (1992a) has an example of excess volatility due to a violation of the standard Bayesian variance decomposition formula under ambiguity, which is not required here. Mandler (2012) shows excess volatility relative to a stochastic technology parameter in a sequential production economy and Mele and Sangiorgi (2011) find price swings under ambiguity with costly information and noise-based partial revelation.

4.3 Price impact and liquidity

Revelation and non-revelation of information also have implications for the price impact of trade under asymmetric information. The price impact of trade can be used as a measure of market illiquidity in the presence of adverse selection due to differentially informed investors, see for example Brennan and Subrahmanyam (1996) and Vayanos and Wang (2012)).[19] Investors are price takers in this model, so price impact measures directly the effect of asymmetric information and does not include any effect of strategic behavior by investors. It is closely related to Kyle's lambda if investors are price takers in Kyle (1985) and there are no noise traders.[20]

Under partial revelation, trade volume is $x_0^I > 0$ for all signal values since I-investors sell their stockholding to U-investors when the ambiguity premium exceeds market risk. Comparing the partial revelation price for distinct signals s and s' provides a measure of the price impact purely due to asymmetric information. This price impact differs depending on whether signals are revealed or not. For distinct signals $s, s' \in [a, b]$, neither of which are revealed in equilibrium, price impact is zero since the same price $p_{PR}(s) = p_{PR}(s')$ prevails under both signals. Thus, the lack of informational efficiency in price coincides with a very liquid market.

For revealed signals $s \notin [a, b]$ or $s' \notin [a, b]$, the price impact is non-zero since the price $p_{PR}(\cdot)$ changes with the signal value. If $s, s' \notin [a, b]$ then price impact is positive, but market illiquidity is relatively low in the sense that the price changes $|p_{PR}(s) - p_{PR}(s')|$, as a function of information changes $|s - s'|$, are continuous. However, if one signal is revealed and the other is not, then the price impact is discontinuously large relative to the change $|s - s'|$, as U-investors now require the unrevealed information premium for the same trade x_0^I. So, a change in the informational efficiency of price coincides with a jump in illiquidity and discrete fall in price if for instance the information changes from $s \in [a, b]$ to $s' < a$ or from $s > b$ to $s' \in [a, b]$.

Let $\lambda(s, s') \equiv |p_{PR}(s) - p_{PR}(s')|$ denote the price impact of trade by I-investors due to a

[19]The notion of illiquidity due to adverse selection can be traced back at least to Bagehot (1971). The papers above discuss liquidity under noise-based partial revelation.

[20]Pasquariello (2012) develops a noise-based Kyle (1985) model where price-taking speculators have prospect theory preferences.

change in the private signal from s to s'. Then,

$$\lambda(s,s') = \begin{cases} 0 & \text{if } s \in [a,b] \text{ and } s' \in [a,b] \\ \frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}|s-s'| & \text{if } s \notin [a,b] \text{ and } s' \notin [a,b] \\ \frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}|\mu_0+\Delta(a,b)-s'| & \text{if } s \in [a,b] \text{ and } s' \notin [a,b]. \end{cases} \tag{21}$$

This discussion can be summarized as follows.

Proposition 7. *The price impact of trade due to a change in information is (i)* 0 *if neither signal is revealed, (ii) positive if either of the signals is revealed, and (iii) discontinuously large if one signal is not revealed while the other is revealed. Discontinuous changes in price impact can coincide with discontinuous falls in price.*

Figure 3 depicts the price impact $\lambda(s,s')$ for the case where $s \in [a,b]$ and the case that $s \notin [a,b]$ as a function of s'. There are discontinuities at $s' = a$ and $s' = b$ as indicated by the discussion above. The price impact is positive, except for $s, s' \in [a,b]$ and for $s' = s$, and continuous otherwise. Pasquariello (2012) documents empirical evidence for variation in price impact and liquidity measures of stocks, while Vayanos and Wang (2012) also document the evidence on the positive relation between illiquidity and returns and on large cross-sectional variation in the Kyle (1985) lambda measure.

The above discussion suggests that liquid markets may be performing poorly in aggregating and communicating information, with the consequence that uninformed investors do not obtain compensation for the adverse selection risk they face. On the other hand, a positive price impact may just reflect the market's informational efficiency. In particular, a jump in illiquidity may in fact be a consequence of the market moving into a regime of informational efficiency from inefficiency.

4.4 Public information

Public information affects both I- and U-investors by reducing the disparity in their beliefs, which affects prices and informational efficiency. Suppose investors observe *a public signal*

$$\zeta = \mu + \epsilon_\zeta, \tag{22}$$

where ϵ_ζ is normally distributed with mean 0 and precision ρ_ζ. Since public information is observed by all investors we assume for simplicity that it is unambiguous. Let $\hat{\rho}_0 = \rho_0 + \rho_\zeta$

and $\hat{\mu}_0 = (\rho_0\mu_0 + \rho_\zeta\zeta)/\hat{\rho}_0$.

I-investors' beliefs about μ are now given by the set of distributions with precision $\hat{\rho} + \rho_\epsilon$ and means

$$\left[\underline{\mu}^I|(s,\zeta), \overline{\mu}^I|(s,\zeta)\right] = \left[\frac{\hat{\rho}_0\hat{\mu}_0 + \rho_\epsilon\,(s-\delta)}{\hat{\rho}_0 + \rho_\epsilon}, \frac{\hat{\rho}_0\hat{\mu}_0 + \rho_\epsilon\,(s+\delta)}{\hat{\rho}_0 + \rho_\epsilon}\right]. \tag{23}$$

Reasoning similar to that for equation (12) indicates that with public information, a range of unrevealed information will exist when market risk doesn't exceed the ambiguity premium, which is now $\frac{\rho_\epsilon\delta}{\hat{\rho}_0+\rho_\epsilon}$, but the range will be distinct from that without public information. Denote the range of unrevealed private information by $[a_\zeta, b_\zeta]$.

Proposition 8. *1. An interval of unrevealed signals $[a_\zeta, b_\zeta]$ exists if and only if $\frac{\gamma\sigma^2}{x_0^U} \leq \frac{\rho_\epsilon\delta}{\hat{\rho}_0+\rho_\epsilon}$.*

2. The size of the set of unrevealed signals is 2δ.

3. The partially revealing price function with public information $p_{PR,\zeta}$ is given by

$$p_{PR,\zeta}(s) = \begin{cases} \hat{\mu}_0 + \frac{1}{2}\sigma^2 + \frac{\rho_\epsilon}{\hat{\rho}_0+\rho_\epsilon}\Delta(a_\zeta, b_\zeta) - \frac{\gamma\sigma^2}{x_0^U} & \text{if } s \in [a_\zeta, b_\zeta] \\ \hat{\mu}_0 + \frac{1}{2}\sigma^2 + \frac{\rho_\epsilon}{\hat{\rho}_0+\rho_\epsilon}(s - \hat{\mu}_0) - \frac{\gamma\sigma^2}{x_0^U} & \text{if } s < a_\zeta \text{ or } s > b_\zeta \end{cases} \tag{24}$$

and is discontinous at a_ζ and b_ζ.

Figure 4 illustrates the effect of public information, even when it does not convey any news, i.e. simply confirms the prior mean, $\zeta = \mu_0$. The dotted lines depict the price function with no public signal, while the dashed lines depict the price function for public signal $\zeta = \mu_0$. With the public signal, the set of unrevealed private signals comprises worse signal values relative to without the public signal, i.e. $b_\zeta < b$. The stock price corresponding to signal $s < \mu_0$ is higher when $s \in [a, b]$ (the flat portion of the dotted line), but $s > b_\zeta$. Similarly, the stock price corresponding to signal $s < a < \mu_0$ is higher when $s \in [a_\zeta, b_\zeta]$.

The change in the range of unrevealed information implies price-related phenomena that might otherwise be considered anomalous. Typically, one would expect that the stock price will rise following a public signal that conveys good news ($\zeta > \mu_0$). However, the price falls if previously unrevealed private information is now revealed and swamps the public news. In the left panel of Figure 5 if there is no public signal (dotted line) private information s_1 is not revealed by equilibrium price. However, public good news ($\zeta > \mu_0$) means signal s_1 is revealed by equilibrium price (chain-dotted line) and the bad news ($s_1 < \mu_0$) in the private information then swamps the good news in the public signal meaning a lower stock price.

Similarly, in the right panel, good news private information ($s_2 > \mu_0$) is revealed under bad public news ($\zeta < \mu_0$) by price (chain-dotted line) and the stock price is higher than without public information (black solid line).

These examples also demonstrate that the stock price need not move continuously in the public signal. For any two public signals, there will be a private signal sufficiently close to the boundary a (or b), for which the market will be in different informational efficiency regimes under the two public signals. Hence, public information can also cause excess volatility when the private signal is revealed under one public signal and not revealed under another. Finally, for similar reasons price impact of trade can differ discontinuously with public information if the information regimes are different before and after the public signal.

Proposition 9. *In an economy with ambiguous private information and public signals*

1. *the stock price can fall when public information is good and rise when it is bad,*

2. *the stock price displays excess volatility with public information if $s \in [a, b]$ and $s \notin [a_\zeta, b_\zeta]$ or $s \notin [a, b]$ and $s \in [a_\zeta, b_\zeta]$, and*

3. *price impact $\lambda(s, s')$ increases discontinuously if $s' \in [a, b]$ and $s' \notin [a_\zeta, b_\zeta]$ given $s \in [a, b]$.*

5 Model discussion and extensions

5.1 Noise-based partial revelation and full revelation

The predominant approach to partial revelation introduces an exogenous source of stochastic variation in price such as noise traders, endowment shocks or taste shocks. We refer to this approach as the noise-based approach for brevity (see also Dow and Gorton (2008)). This added variation implies changes in price are not due solely to changes in information, meaning price is not invertible as a function of private information and is therefore partially revealing.

The price function in the commonly-used Grossman and Stiglitz (1980) and related noise-based frameworks is linear due to the assumption of normal distributions, CARA utility, no wealth constraints, and unambiguous beliefs. Different distributional or utility assumptions (Mailath and Sandroni (2003), Barlevy and Veronesi (2003), Breon-Drish (2012)) or wealth constraints (Yuan (2005)) can yield non-linear price functions, while exogenous portfolio insurance or hedging demand can yield a discontinuous price function (Gennotte and Leland

(1990)).[21] REE models with ambiguity averse traders and noise traders such as Ozsoylev and Werner (2011) and Mele and Sangiorgi (2011) and without noise traders such as Easley, O'Hara, and Yang (2011) feature continuous price functions.[22]

Under the noise-based approach, partial revelation typically does not have volatility implications beyond what noise adds in a manner qualitatively similar to how noise alters volatility in a symmetric information setting. Another distinguishing feature is that in common noise-based CARA-normal models information on trading volume makes partially revealing prices fully revealing (Blume, Easley, and O'Hara (1994) and Schneider (2009)). This is not true in the present framework since trade is constant at x_0^I for all signals.

The noise-based approach is also used to rule out the existence of fully revealing REE and provide a resolution to the Grossman and Stiglitz (1980) paradox of costly information acquisition. Partial revelation under ambiguity does not rule out full revelation REE; indeed one always exists as noted below.[23] The present model also does not analyse costly information acquistion. It is not clear that all information used in financial markets involves a direct cost, such as information from a non-traded asset like labor income, whose payoff is correlated with, and hence informative about, the stock payoff (see section 5.2). Moreover, Bernardo and Judd (2000), Muendler (2007), and Krebs (2007) indicate that the co-existence of informationally efficient prices and costly information is not paradoxical outside of the widely-used CARA-normal models in the noise-based approach.

Proposition 10. *A fully revealing REE always exists with ambiguous information.*

Partial revelation under ambiguity involves a range of signal values not being revealed and yields a non-linear discontinuous price function and discontinuous variation in price volatility and price impact as noted previously. Overall, the differences in partial revelation due to ambiguous information and noise-based partial revelation suggest that in principle, these approaches may provide differing testable implications and be useful in complementary ways for studying financial markets.

[21] Price is discontinuous at a point in the noise variable in Barlevy and Veronesi (2003).

[22] In Ozsoylev and Werner (2011) ambiguity-averse traders do not receive any private signals, while in Mele and Sangiorgi (2011) private information eliminates ex-ante ambiguity. In Easley, O'Hara, and Yang (2011), ambiguity-averse, uninformed 'simple' traders are ambiguous about the trading strategy of 'opaque' traders which yields a price function which is not fully informative for the 'simple' traders.

[23] The analyses of Radner (1979), Grossman (1981), and Condie and Ganguli (2011b) also suggest that a full revelation REE will exist.

5.2 Non-tradeable labor income

The information structure used thus far can be reinterpreted as one in which investors receive information about a non-tradeable asset such as labor income whose payoff is correlated with the stock. Investors can use the stock to hedge against their labor income fluctuations and in turn use information about labor income to update their information about the stock payoff. Benzoni, Dufrense, and Goldstein (2007) and references therein provide a perspective on how labor income risk can affect investor choices. Related formulations of hedging motives are also commonly used in the noise-based REE literature.[24]

Suppose I-investors have non-tradeable labor income that provides a return R_l on initial wealth and $r_l \equiv \ln R_l$ and μ are jointly normally distributed with means (μ_l, μ_0), precisions (ρ_l, ρ_0) and covariance $\eta \neq 0$. I-investors have private information $s = r_l + \epsilon$ about labor income, where ϵ is independent of r_l and normally distributed with precision ρ_ϵ so the signal variance is $\sigma_s^2 = \rho_l^{-1} + \rho_\epsilon^{-1}$. First, suppose there is there is no ambiguity in this signal and it is unbiased (i.e., $\mathbb{E}\epsilon = 0$). Joint normality of s, r_l, and μ implies that the covariance of s and μ is η and the updated distribution of μ given the observation of the private signal s is normal with mean

$$\mu_0 + \frac{\eta}{\sigma_s^2}(s - \mu_l). \tag{25}$$

If the mean of the signal (or equivalently, of ϵ) is ambiguous and indexed by $[-\delta, \delta]$, then the updated beliefs are given by a set of normal distributions with means

$$[\underline{\mu}^I|s, \overline{\mu}^I|s] = \left[\mu_0 + \frac{\eta}{\sigma_s^2}(s - \delta - \mu_l), \mu_0 + \frac{\eta}{\sigma_s^2}(s + \delta - \mu_l)\right]. \tag{26}$$

Terminal wealth is given by

$$W_2 = W_0(\theta R + (1 - \theta)R_f + R_l). \tag{27}$$

Using an approximation of payoffs and assuming $r_f = 0$ as in section 2.3, we have the following result on investor demand.

Lemma 3. *The optimal portfolio weight on the stock for investor n who observes information*

[24] For example Biais, Bossaerts, and Spatt (2010), Schneider (2009), Goldstein and Guembel (2008), Watanabe (2008), and the references therein where the hedging motive is closely tied to the noise which prevents prices from revealing information.

about non-tradable labor income that is correlated with the mean stock payoff is given by

$$\theta^n(s) = \begin{cases} \frac{\underline{\mu}^n|s+\frac{1}{2}\sigma^2-p}{\gamma\sigma^2} & \textit{if } \underline{\mu}^n|s+\frac{1}{2}\sigma^2-p>0 \\ 0 & \textit{if } \underline{\mu}^n|s \leq p-\frac{1}{2}\sigma^2 \leq \overline{\mu}^n|s \\ \frac{\overline{\mu}^n|s+\frac{1}{2}\sigma^2-p}{\gamma\sigma^2} & \textit{if } \overline{\mu}^n|s+\frac{1}{2}\sigma^2-p<0. \end{cases} \tag{28}$$

This demand is similar to that in the case when signals are directly related to the asset and the rest of the analysis conducted previously will then follow similarly.

5.3 Multiple signals observed by I-investors

We now consider the scenario where I-investors may observe a sequence of identically and independently drawn signals before trading occurs in the framework of Section 2. We show that the partial revelation developed in this paper does not dissappear with such repeated observation of information by I-investors. For simplicity, we assume that I-investors perceive the same ambiguity $[-\delta, \delta]$ in all of the signals and U-investors consider them unbiased.

Suppose I-investors observe a sequence denoted by $(s_1, s_2, \ldots, s_K)$ of $K \geq 1$ signals $s_k = \mu + \epsilon_k$. Their updated beliefs about μ are represented by the set of normal distributions with precision $\rho_0 + K\rho_\epsilon$ and means $[\underline{\mu}^I|(s_1, \ldots, s_K), \overline{\mu}^I|(s_1, \ldots, s_K)]$, where

$$\underline{\mu}^I|(s_1, \ldots, s_K) = \frac{\rho_0\mu_0 + \rho_\epsilon \sum_{k=1}^K (s_k - \delta)}{\rho_0 + K\rho_\epsilon} \text{ and } \overline{\mu}^I|(s_1, \ldots, s_K) = \frac{\rho_0\mu_0 + \rho_\epsilon \sum_{k=1}^K (s_k + \delta)}{\rho_0 + K\rho_\epsilon}. \tag{29}$$

The length of the interval is given by $2K\delta/(\rho_0 + K\rho_\epsilon)$ which approaches $2\delta/(\rho_0 + \rho_\epsilon)$ as $K \to \infty$. This implies that repeated observation of ambiguous information does not eliminate the ambiguity perceived by I-investors.[25]

The ambiguity premium now required by I-investors to hold the asset is $\frac{K\delta\rho_\epsilon}{\rho_0+K\rho_\epsilon}$ which is increasing in K. Thus, the condition for partial revelation, $\frac{\gamma\sigma^2}{x_0^U} \leq \frac{K\delta\rho_\epsilon}{\rho_0+K\rho_\epsilon}$, becomes easier to satisfy as the number of signals increases. When U-investors hold all the stock, the market-clearing price satisfies $p = \mu^U_{PR}(s_1, \ldots, s_K) - \frac{\gamma\sigma^2}{x_0^U} + \frac{1}{2}\sigma^2$, where $\mu^U_{PR}(s_1, \ldots, s_K)$ denotes the expected value of μ under U-investors' updated beliefs.

When I-investors observe K signals before trading, information is revealed or not revealed in the form of $\sum_{k=1}^K s_k$. Reasoning similar to section 3.2 shows that a range denoted $[a_K, b_K]$ of values of $\sum_{k=1}^K s_k$ is not revealed. This leads to the following analogue of Proposition 3.

[25] See also Epstein and Schneider (2007).

Proposition 11.

1. *An interval* $[a_K, b_K]$ *of unrevealed signals exists if and only if* $\frac{K\rho_\epsilon\delta}{\rho_0+K\rho_\epsilon} \leq \frac{\gamma\sigma^2}{x_0^U}$.

2. *The length of the interval* $[a_K, b_K]$ *of unrevealed signals is* 2δ.

3. *The partially revealing price function* p_{PR} *satisfies*

$$p_{PR}(s_1,\ldots,s_K) = \begin{cases} \mu_0 + \frac{1}{2}\sigma^2 + \frac{\rho_\epsilon}{\rho_0+K\rho_\epsilon} K\Delta_K(a_K,b_K) - \frac{\gamma}{x_0^U}\sigma^2 & \text{if } \sum_{k=1}^K s_k \in [a_K,b_K] \\ \mu_0 + \frac{1}{2}\sigma^2 + \frac{\rho_\epsilon}{\rho_0+K\rho_\epsilon}\left(\sum_{k=1}^K (s_k-\mu_0)\right) - \frac{\gamma}{x_0^U}\sigma^2 & \text{if } \sum_{k=1}^K s_k \notin [a_K,b_K]\,. \end{cases} \tag{30}$$

with discontinuities at a_K *and* b_K.

5.4 Ambiguity averse U-investors

We extend the model to allow U-investors to also perceive ambiguity in the signals if they observe it through price while retaining the structure outlined in section 2. U-investors also consider a range $[-\delta^U, \delta^U]$ of possible values for the mean μ_ϵ^U of the error term ϵ, where $\delta^U > 0$.

We assume that $\delta^U < \delta$, so that $[-\delta^U, \delta^U]$ is a strict subset of $[-\delta, \delta]$. This assumption is needed for market clearing to be consistent with I-investors trading away all their stock-holding. Since U-investors now require an uninformed ambiguity premium $\frac{\delta^U\rho_\epsilon}{\rho_0+\rho_\epsilon}$ also to hold the stock, partial revelation will require that the total premium required by U-investors to hold all the stock not exceed I-investors' ambiguity premium for holding the stock.

As in section 3, under the conditions stated above, there will be an interval $[a, b]$ of signals which aren't revealed. Define

$$\underline{\Delta}(a,b) = \min_{d\in[-\delta^U,\delta^U]} \frac{\phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(a-\mu_0-d)\right) - \phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(b-\mu_0-d)\right)}{\Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(b-\mu_0-d)\right) - \Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(a-\mu_0-d)\right)} \sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}. \tag{31}$$

Proposition 12.

1. *An interval* $[a, b]$ *of unrevealed signals if and only if* $\frac{\rho_\epsilon}{\rho_0+\rho_\epsilon} \leq \frac{\gamma\sigma^2}{x_0^U} + \frac{\rho_\epsilon\delta^U}{\rho_0+\rho_\epsilon}$.

2. *The length of the interval* $[a, b]$ *of unrevealed signals is* 2δ.

3. *The partially revealing price function p_{PR} satisfies*

$$p_{PR}(s) = \begin{cases} \mu_0 + \frac{1}{2}\sigma^2 + \frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}\underline{\Delta}(a,b) - \frac{\delta^U \rho_\epsilon}{\rho_0+\rho_\epsilon} - \frac{\gamma}{x_0^U}\sigma^2 & \textit{if } s \in [a,b] \\ \mu_0 + \frac{1}{2}\sigma^2 + \frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}(s-\mu_0) - \frac{\delta^U \rho_\epsilon}{\rho_0+\rho_\epsilon} - \frac{\gamma}{x_0^U}\sigma^2 & \textit{if } s < a \textit{ or } s > b \end{cases} \quad (32)$$

and is discontinuous at a and b.

Using these results, we can then obtain qualitatively similar results to those in section 4.

6 Concluding remarks

In this paper, we show that partially revealing REE arise and affect market variables when ambiguous private information is received by investors who exhibit inertia with respect to information under the Gilboa and Schmeidler (1989) model of decision-making.

Partial revelation of ambiguous information leads to a decline in stock price. Changes in informational efficiency lead to price swings, excess price volatility, and volatility variation. Moreover, markets are more liquid with informationally inefficient prices and illiquidity can spike with small changes in fundamentals. Public information directly affects the informational efficiency of prices and can lead to excess volatility, illiquidity jumps, and price changes in the opposite direction to public news. This informational inefficiency persists with the repeated observation of information by informed traders or when uninformed traders are also ambiguity-averse. These predictions provide insight into possible causes of time-varying volatility that are implicit to the market mechanism as opposed to being driven by asset fundamentals. The model also suggests that volatility is an important by-product of informative prices and that regulatory actions designed to limit volatility should be taken with care.

We have focused on a single type of informed investor and a single uncertain asset in order to highlight the information transmission role of prices. Future areas for research would allow for multiple types of informed investors who receive different information, thus enabling the study of information aggregation and transmission as well as the study of multiple traded assets. In such models, non-revelation of information will require conditions similar to those in (19).

A Appendix

A.1 Proofs for Section 2

An approximate solution for investor demand. To solve for investor demand, we approximate the portfolio return by a lognormal random variable adapting the method of Campbell and Viciera (2002).[26] Terminal wealth is given by

$$W_2 = W_0(\theta R + (1-\theta)R_f). \tag{33}$$

Expressing the returns in logs and using a second-order Taylor series approximation around $r - r_f = 0$ gives

$$\ln \theta R + (1-\theta)R_f = r_f + \ln(1 + \theta\left(e^{r-r_f} - 1\right)) \approx \theta\left(r - r_f\right) + \frac{1}{2}\theta(r - r_f)^2. \tag{34}$$

Replacing the second-order term with its unconditional expectation yields

$$\theta\left(r - r_f\right) + \frac{1}{2}\theta(1-\theta)\sigma^2 \tag{35}$$

as our lognormal approximation of market returns.

If W_2 is lognormally distributed then the solution to the individual's optimization problem is equivalent to the solution to

$$\max_{\theta} \min_{m \in M} \ln \mathbb{E}_m \left[\frac{(W_2)^{1-\gamma}}{1-\gamma}\right]. \tag{36}$$

By the lognormality of W_2,

$$\begin{aligned} \ln \mathbb{E}_m \left[(W_2)^{1-\gamma}\right] &= \mathbb{E}_m \ln(W_2)^{1-\gamma} + \frac{1}{2}Var \ln(W_2)^{1-\gamma} \\ &= (1-\gamma)\mathbb{E}_m[w_0 + \ln(\theta R + (1-\theta)R_f)] + \frac{1}{2}(1-\gamma)^2(\theta)^2\sigma^2. \end{aligned} \tag{37}$$

Since $\mathbb{E}_m(w_0)$ and r_f are non-stochastic and $1-\gamma$ is a scale factor, solving the optimization problem is equivalent to solving

$$\max_{\theta} \min_{m \in M} \mathbb{E}_m \theta\left(r - r_f\right) + \frac{\theta(1-\theta)}{2}\sigma^2 + \frac{(1-\gamma)\theta^2}{2}\sigma^2. \tag{38}$$

[26]We show the derivation for $\gamma \neq 1$, the derivation for log utility ($\gamma = 1$) is similar.

using the approximation in (35).

Proof of Proposition 1. Using (38), our normalization of $r_f = 0$ and the expressions for the updated beliefs in (4), the first order conditions for n-investors are

$$\begin{array}{ll} 0 = \min_{m\in[\underline{\mu}^n|s,\overline{\mu}^n|s]} \mathbb{E}_m r + \frac{1}{2}\sigma^2 - \gamma\theta\sigma^2 & \text{if } \theta > 0 \\ 0 \in \left\{\mathbb{E}_m r + \frac{1}{2}\sigma^2 : m \in [\underline{\mu}^n|s, \overline{\mu}^n|s]\right\} & \text{if } \theta = 0 \\ 0 = \max_{m\in[\underline{\mu}^n|s,\overline{\mu}^n|s]} \mathbb{E}_m r + \frac{1}{2}\sigma^2 - \gamma\theta\sigma^2 & \text{if } \theta < 0. \end{array} \tag{39}$$

Therefore, n-investors trade off their stockholding if and only if

$$\begin{aligned} \min_{m\in[\underline{\mu}^n|s,\overline{\mu}^n|s]} \mathbb{E}_m v + \frac{1}{2}\sigma^2 - p \leq 0 \leq \max_{m\in[\underline{\mu}^n|s,\overline{\mu}^n|s]} \mathbb{E}_m v + \frac{1}{2}\sigma^2 - p \\ \Leftrightarrow \underline{\mu}^n + \frac{1}{2}\sigma^2 \leq p \leq \overline{\mu}^n + \frac{1}{2}\sigma^2. \end{aligned} \tag{40}$$

□

A.2 Proofs for Section 3

Proof of Proposition 2. Suppose markets clear with $\theta^I(s) = \frac{1}{\gamma\sigma^2}\left(\mu^I|s + \frac{1}{2}\sigma^2 - p\right) > 0$. Then the market clearing price is

$$p = x_0^I\left(\frac{\rho_0\mu_0 + \rho_\epsilon}{\rho_0 + \rho_\epsilon}(s-\delta)\right) + x_0^U\mu^U - \gamma\sigma^2 + \frac{1}{2}\sigma^2, \tag{41}$$

where μ^U denotes the mean of μ under U-investors' beliefs. If this price were to be non-revealing then μ^U must be a decreasing function of the signal s, which cannot occur, either in partial or full revelation. Thus, the signal must be revealed.

When the signal s is revealed by the price, U-investors' updated belief $\mu^U_{PR}|s$ about the mean of μ is $\frac{\rho_0\mu_0+\rho_\epsilon s}{\rho_0+\rho_\epsilon}$. With this belief, the market clearing price satisfies

$$p = \left(\frac{\rho_0\mu_0 + \rho_\epsilon}{\rho_0 + \rho_\epsilon}(s)\right) - x_0^I\frac{\delta\rho_\epsilon}{\rho_0 + \rho_\epsilon} + \gamma\sigma^2 + \frac{1}{2}\sigma^2 \tag{42}$$

and again this price is consistent with revelation of s to U-investors and $\theta^I(s) > 0$. Hence with $\theta^I(s) > 0$, the market clearing price reveals the signal to U-investors. Similar arguments show that if $\theta^I(s) < 0$, then price reveals the signal s. □

Proof of Lemma 1. The demand function of I-investors given in (1), together with the definition of $\mu^I|s$ in (4) imply that the range of signals for which I-investors will trade away their share holdings at the price p is

$$-\frac{\rho_0}{\rho_\epsilon}\mu_0 + \delta - \frac{\rho_0+\rho_\epsilon}{\rho_\epsilon}\left(\frac{1}{2}\sigma^2 - p\right) \geq s \geq -\frac{\rho_0}{\rho_\epsilon}\mu_0 - \delta - \frac{\rho_0+\rho_\epsilon}{\rho_\epsilon}\left(\frac{1}{2}\sigma^2 - p\right). \tag{43}$$

Using the price expression given in equation (11) we obtain (12). □

Proof of Lemma 2. When U-investors are not able to infer the signal, their belief about the mean of μ is obtained by using the updated beliefs conditional on the knowledge that the signal is in an interval $[a, b]$. This conditional expected value is

$$\begin{aligned}\mathbb{E}\left[\mu | a \leq s \leq b\right] &= \mathbb{E}\left[\mu^U_{PR|s} | a \leq s \leq b\right] \\ &= \frac{1}{\rho_0+\rho_\epsilon}\left(\rho_0\mu_0 + \rho_\epsilon \mathbb{E}[s|a \leq s \leq b]\right)\end{aligned} \tag{44}$$

Since U-investors believe s is normally distributed with mean μ_0 and variance $\rho_0^{-1} + \rho_\epsilon^{-1}$, using the properties of the truncated normal distribution (see e.g. Johnson and Kotz (1970)),

$$\mathbb{E}\left[s | a \leq s \leq b\right] = \mu_0 + \Delta(a, b) \tag{45}$$

where

$$\Delta(a,b) = \frac{\phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(a-\mu_0)\right) - \phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(b-\mu_0)\right)}{\Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(b-\mu_0)\right) - \Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(a-\mu_0)\right)}\sqrt{\frac{\rho_0+\rho_\epsilon}{\rho_0\rho_\epsilon}}, \tag{46}$$

where ϕ and Φ denote the standard normal density and distribution functions respectively. Hence,

$$\mu^U_{PR}|[a,b] = \mu_0 + \frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}\Delta(a,b). \tag{47}$$

□

The next lemma collects several facts that are useful in proving subsequent results.

Lemma 4. *Suppose s is normally distributed with mean μ_0 and variance $\sigma_s^2 \equiv \rho_0^{-1} + \rho_\epsilon^{-1}$.*

Let $f(s)$ denote the pdf of s. Then

$$
\begin{aligned}
&\textit{1. } \frac{\partial}{\partial b}\int_{b-2\delta}^{b} sf(s)ds = bf(b) - (b-2\delta)f(b-2\delta) \\
&\textit{2. } \frac{\partial}{\partial b}\int_{b-2\delta}^{b} f(s)ds = f(b) - f(b-2\delta) \\
&\textit{3. } f(b-2\delta) = f(b)e^{-\frac{2\delta(\delta+\mu_0)}{\sigma_s^2}} e^{\frac{2\delta b}{\sigma_s^2}} \\
&\textit{4. } bf(b) - (b-2\delta)f(b-2\delta) = \int_{b-2\delta}^{b} f(s)ds - \int_{b-2\delta}^{b} s\left(\frac{s-\mu_0}{\sigma_s^2}\right)f(s)ds \\
&\textit{5. } f(b-2\delta) - f(b) = \frac{1}{\sigma_s^2}\int_{b-2\delta}^{b} sf(s)ds - \frac{\mu_0}{\sigma_s^2}\int_{b-2\delta}^{b} f(s)ds \\
&\textit{6. } 0 < \frac{\partial \mathbb{E}[s|b-2\delta \le s \le b]}{\partial b} < 1 \textit{ for all } -\infty < b < \infty
\end{aligned}
\tag{48}
$$

Proof. The first two results follow from Leibniz's rule and the third by rearranging terms in $f(b-2\delta)$. The fourth follows from integrating $\int_{b-2\delta}^{b} f(s)ds$ by parts where $u = f(s)$ and $dv = ds$. The fifth follows from observing that

$$f'(s) = -\left(\frac{s-\mu}{\sigma_s^2}\right)f(s) \tag{49}$$

and integrating both sides of equation (49) over the region $[b-2\delta, b]$.

To show

$$0 < \frac{\partial \mathbb{E}[s|b-2\delta \le s \le b]}{\partial b} < 1 \tag{50}$$

calculate

$$
\begin{aligned}
\frac{\partial E[s|b-2\delta \leq s \leq b]}{\partial b} =& \frac{\partial}{\partial b} \frac{\int_{b-2\delta}^{b} sf(s)ds}{\int_{b-2\delta}^{b} f(s)ds} \\
=& \frac{\left(\int_{b-2\delta}^{b} f(s)ds\right)(bf(b)-(b-2\delta)f(b-2\delta)) - \left(\int_{b-2\delta}^{b} sf(s)ds\right)(f(b)-f(b-2\delta))}{\left(\int_{b-2\delta}^{b} f(s)ds\right)^2} \\
=& \frac{\left(\int_{b-2\delta}^{b} f(s)ds\right)(bf(b)-(b-2\delta)f(b-2\delta))}{\left(\int_{b-2\delta}^{b} f(s)ds\right)^2} \\
&+ \frac{\left(\int_{b-2\delta}^{b} sf(s)ds\right)\left(\frac{1}{\sigma_s^2}\int_{b-2\delta}^{b} sf(s)ds - \frac{\mu_0}{\sigma_s^2}\int_{b-2\delta} bf(s)ds\right)}{\left(\int_{b-2\delta}^{b} f(s)ds\right)^2} \\
=& \frac{\left(\int_{b-2\delta}^{b} f(s)ds\right)\left(\int_{b-2\delta}^{b} f(s)ds - \int_{b-2\delta}^{b} s\left(\frac{s-\mu_0}{\sigma_s^2}\right)f(s)ds\right)}{\left(\int_{b-2\delta}^{b} f(s)ds\right)^2} \\
&+ \frac{\left(\int_{b-2\delta}^{b} sf(s)ds\right)\left(\frac{1}{\sigma_s^2}\int_{b-2\delta}^{b} sf(s)ds - \frac{\mu_0}{\sigma_s^2}\int_{b-2\delta}^{b} f(s)ds\right)}{\left(\int_{b-2\delta}^{b} f(s)ds\right)^2} \\
=& 1 - \frac{1}{\sigma_s^2}\frac{\int_{b-2\delta}^{b} s^2 f(s)ds}{\int_{b-2\delta}^{b} f(s)ds} + \frac{\mu_0}{\sigma_s^2}\frac{\int_{b-2\delta}^{b} sf(s)ds}{\int_{b-2\delta}^{b} f(s)ds} \\
&- \frac{1}{\sigma_s^2}\left[\frac{\int_{b-2\delta}^{b} sf(s)ds}{\int_{b-2\delta}^{b} f(s)ds}\right]^2 - \frac{\mu_0}{\sigma_s^2}\frac{\int_{b-2\delta}^{b} sf(s)ds}{\int_{b-2\delta}^{b} f(s)ds} \\
=& 1 - \frac{1}{\sigma_s^2}E[s^2|b-2\delta \leq s \leq b] + \frac{1}{\sigma_s^2}E[s|b-2\delta \leq s \leq b]^2 \\
=& 1 - \frac{1}{\sigma_s^2}Var(s|b-2\delta \leq s \leq b)
\end{aligned} \tag{51}
$$

The third and fourth equalities follow from facts 5 and 4, respectively and the rest are simplifications. The result follows since

$$0 < Var(s|b-2\delta \leq s \leq b) < \sigma_s^2 \equiv Var(s). \tag{52}$$

□

Proof of Proposition 3. Statements 2 and 3 are proved in the body of the text. For statement 1, we first prove that if the interval $[a,b]$ exists then $\frac{\gamma\sigma^2}{x_0^U} < \frac{\delta\rho_\epsilon}{\rho_0+\rho_\epsilon}$. Define $h(\cdot)$ as

$$h(b) = b - \mu_0 - \Delta(b-2\delta, b) = b - \mathbb{E}[s|s \in [b-2\delta, b]]. \tag{53}$$

If $\frac{\gamma\sigma^2}{x_0^U} > \frac{\delta\rho_\epsilon}{\rho_0+\rho_\epsilon}$ then (19) requires $h(b) < 0 \Leftrightarrow b < E[s|b-2\delta \leq s \leq b]$, which is a contradiction.

Rewrite equation (19) as

$$h(b) = \delta - \frac{\rho_0+\rho_\epsilon}{\rho_\epsilon}\frac{\gamma\sigma^2}{x_0^U}. \tag{54}$$

We prove sufficiency by showing that $h(\cdot)$ takes all values in $[0,\delta]$. When $\frac{\gamma\sigma^2}{x_0^U} = 0$, given the symmetry of a normal pdf around its mean, $b_0 = \mu_0+\delta$ solves (19) since $\Delta(\mu_0-\delta, \mu_0+\delta) = 0$. Moreover, $h(b_0) = \delta$.

Since for any $-\infty < b < \infty$, $h'(b)$ exists and from (50) $0 < h'(b) < 1$, it suffices to show that

$$\lim_{b\to-\infty} h(b) = 0. \tag{55}$$

Using L'Hopital's rule, 1-3 in Lemma 4 and rearranging terms,

$$\begin{aligned}
\lim_{b\to-\infty} \mathbb{E}\left[s|s \in [b-2\delta, b]\right] &= \lim_{b\to-\infty} \frac{\int_{b-2\delta}^{b} sf(s)ds}{\int_{b-2\delta}^{b} f(s)ds} \\
&= \lim_{b\to-\infty} \frac{\frac{\partial}{\partial b}\int_{b-2\delta}^{b} sf(s)ds}{\frac{\partial}{\partial b}\int_{b-2\delta}^{b} f(s)ds} \\
&= \lim_{b\to-\infty} \frac{bf(b)-(b-2\delta)f(b-2\delta)}{f(b)-f(b-2\delta)} \\
&= \lim_{b\to-\infty} \frac{b(f(b)-f(b-2\delta))+2\delta f(b-2\delta)}{f(b)-f(b-2\delta)} \\
&= \lim_{b\to-\infty} b + 2\delta\frac{f(b-2\delta)}{f(b)-f(b-2\delta)} \\
&= \lim_{b\to-\infty} b + 2\delta\frac{f(b)e^{-\frac{2\delta(\delta+\mu_0)}{\sigma_s^2}}e^{\frac{2\delta b}{\sigma_s^2}}}{f(b)\left(1-e^{-\frac{2\delta(\delta+\mu_0)}{\sigma_s^2}}e^{\frac{2\delta b}{\sigma_s^2}}\right)} \\
&= \lim_{b\to-\infty} b + 2\delta\frac{e^{-\frac{2\delta(\delta+\mu_0)}{\sigma^2}}e^{\frac{2\delta b}{\sigma_s^2}}}{1-e^{-\frac{2\delta(\delta+\mu_0)}{\sigma_s^2}}e^{\frac{2\delta b}{\sigma_s^2}}},
\end{aligned} \tag{56}$$

which yields

$$
\begin{aligned}
\lim_{b\to-\infty} h(b) &= \lim_{b\to-\infty} b - E\left[s|s\in[b-2\delta,b]\right] \\
&= \lim_{b\to-\infty} b - b - 2\delta\frac{e^{-\frac{2\delta(\delta+\mu)}{\sigma^2}}e^{\frac{2\delta b}{\sigma^2}}}{1-e^{-\frac{2\delta(\delta+\mu)}{\sigma^2}}e^{\frac{2\delta b}{\sigma^2}}} \\
&= \lim_{b\to-\infty} -2\delta\frac{e^{-\frac{2\delta(\delta+\mu)}{\sigma^2}}e^{\frac{2\delta b}{\sigma^2}}}{1-e^{-\frac{2\delta(\delta+\mu)}{\sigma^2}}e^{\frac{2\delta b}{\sigma^2}}} \\
&= 0.
\end{aligned}
\tag{57}
$$

□

A.3 Proofs for section 4

Proof of Proposition 4. Result 1 is proved in the proof to Proposition 3 and result 3 is a corollary of result 2, which we prove next. Using the sixth result in Lemma 4 and the fact that $\mathbb{E}[s|b-2\delta\leq s\leq b]=\mu_0+\Delta(b-2\delta,\delta)$ yields $0<\partial\Delta(b-2\delta,\delta)/\partial b<1$. Applying the implicit function theorem to equation (19) then yields

$$
\frac{\partial\Delta(b-2\delta,\delta)}{\partial\frac{\gamma\sigma^2}{x_0^U}} = -\frac{1}{\frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}\left(1-\frac{\partial\Delta(b-2\delta,\delta)}{\partial b}\right)} < 0 \tag{58}
$$

□

Proof of Proposition 5. This follows from statement 2 in Proposition 3. □

Proof of Proposition 6. Result 1 follows from the discontinuity of the price function defined in equation (16). Formally, $\lim_{s\uparrow a} p_{PR}(s)=p_a^+<p_a^-=\lim_{s\downarrow a}$, $Var(p_{PR}|a-\varepsilon<s<\varepsilon)>Var(p_{PR}|a-\varepsilon<s<\varepsilon)$ for small $\varepsilon>0$. A similar argument applies at b.

For result 2 consider the variance of p_{PR} conditional on the signal being revealed or not revealed. When $s\in[a,b]$, price is constant at

$$
p_{PR}(s)=\mu_0+\frac{1}{2}\sigma^2+\frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}\Delta(a,b)-\frac{\gamma}{x_0^U}\sigma^2. \tag{59}
$$

and conditional price volatility is

$$
Var\left[p_{PR}|s\in[a,b]\right]=0. \tag{60}
$$

When $s < a$ or $s > b$ then price is

$$p_{PR}(s) = \mu_0 + \frac{1}{2}\sigma^2 + \frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon}(s - \mu_0) - \frac{\gamma}{x_0^U}\sigma^2. \tag{61}$$

and conditional price volatility is

$$\begin{aligned} Var(p_{PR}|s < a \text{ or } s > b) &= \left(\frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon}\right)^2 Var(s|s < a \text{ or } s > b) \\ &= \left(\frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon}\right)^2 \Big(\hat{F}(a)Var(s|s < a) + \hat{F}(b)Var(s|s > b)\Big) > 0 \end{aligned} \tag{62}$$

where $\hat{F}(a) = \frac{F(a)}{F(a)+1-F(b)}$ and $\hat{F}(b) = \frac{1-F(b)}{F(a)+1-F(b)}$ with $F(\cdot)$ denoting the cdf of s and, with $\rho_s = \frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}$,

$$Var(s|s < a) = \frac{1}{\rho_s}\left[1 - \frac{\phi\left(\sqrt{\rho_s}\,(\mu_0 - a)\right)}{1 - \Phi\left(\sqrt{\rho_s}\,(\mu_0 - a)\right)}\left(\frac{\phi\left(\sqrt{\rho_s}\,(\mu_0 - a)\right)}{1 - \Phi\left(\sqrt{\rho_s}\,(\mu_0 - a)\right)} - \sqrt{\rho_s}\,(\mu_0 - a)\right)\right] \tag{63}$$

and

$$Var(s|s > b) = \frac{1}{\rho_s}\left[1 - \frac{\phi\left(\sqrt{\rho_s}\,(b - \mu_0)\right)}{1 - \Phi\left(\sqrt{\rho_s}\,(b - \mu_0)\right)}\left(\frac{\phi\left(\sqrt{\rho_s}\,(b - \mu_0)\right)}{1 - \Phi\left(\sqrt{\rho_s}\,(b - \mu_0)\right)} - \sqrt{\rho_s}\,(b - \mu_0)\right)\right]. \tag{64}$$

□

Proof of Proposition 7. These statements follow from the definition of $\lambda(s, s')$ in (21). The discontinuity in signals follows from the discontinuity of price in signals. □

Proof of Propositions 8 and 9. We demonstrate how a public signal changes the range of unrevealed signals. The results then follow by reasoning similar to the case of no public information and by comparing the values of the respective price, price volatility, and price impact variables to the case of no public information.

Proceeding along the same lines as in section 3 shows that U-investor beliefs are given by

$$\mu_{PR}^U|([a_\zeta, b_\zeta], \zeta) = \frac{\hat{\mu}_0 + \rho_\epsilon(\mu_0 + \Delta(a_\zeta, b_\zeta))}{\hat{\rho}_0 + \rho_\epsilon} \tag{65}$$

if $s \in [a_\zeta, b_\zeta]$ where

$$\Delta(a_\zeta, b_\zeta) = \frac{\phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(a_\zeta - \mu_0)\right) - \phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(b_\zeta - \mu_0)\right)}{\Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(b_\zeta - \mu_0)\right) - \Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(a_\zeta - \mu_0)\right)}\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}, \tag{66}$$

the length of the interval of unrevealed signals is 2δ, and the existence of the interval follows from the existence of a solution to

$$\frac{\rho_\epsilon}{\hat{\rho}_0 + \rho_\epsilon}\left(b_\zeta - \hat{\mu}_0 - \Delta(b_\zeta - 2\delta, b_\zeta)\right) + \frac{\gamma\sigma^2}{x_0^U} - \frac{\delta\rho_\epsilon}{\hat{\rho}_0 + \rho_\zeta + \rho_\epsilon} = 0. \tag{67}$$

The proof then follows reasoning to that for Proposition 3, subject to a simple change of notation.

The proofs of 1-3 in Proposition 9 follow from the discussion in the main text and reasoning similar to that for the proofs of Proposition 6 and Proposition 7. □

A.4 Proofs for Section 5.

Proof of Proposition 10. If $\frac{\gamma\sigma^2}{x_0^U} > \frac{\rho_\epsilon\delta}{\rho_0+\rho_\epsilon}$ the price function

$$p_{FR}(s) = \frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} + \frac{1}{2}\sigma^2 - \gamma\sigma^2 - x_0^I\frac{\delta\rho_\epsilon}{\rho_0 + \rho_\epsilon} \tag{68}$$

is a fully-revealing REE price function and if $\frac{\gamma\sigma^2}{x_0^U} \leq \frac{\rho_\epsilon\delta}{\rho_0+\rho_\epsilon}$ then

$$p_{FR}(s) = \frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} + \frac{1}{2}\sigma^2 - \frac{\gamma\sigma^2}{x_0^U}. \tag{69}$$

This is also a fully revealing REE price function. □

Proof of Lemma 3. Terminal wealth is $W_2 = W_0(\theta R + (1-\theta)R_f + R_l)$ and, as before, we provide a lognormally distributed approximation to the return on initial wealth using a second order Taylor series.

Arguments similar to those prior to the proof of Proposition 1 yield

$$\ln(\theta R + (1-\theta)R_f + R_l)/R_f \approx \theta(r - r_f) + \frac{R_l}{R_f} + \frac{\theta - \theta^2}{2}\sigma^2 + \frac{1}{2}\sigma_l^2 \tag{70}$$

by using the appropriate derivatives evaluated at $(r - r_f, R_l/R_f) = (0, 0)$ and replacing the second order terms with their expectations.

Using $Var(r_w) = \theta^2\sigma^2 + \sigma_l^2$ and reasoning similarly as before, n-investors' objective function is

$$\max_{\theta} \min_{m \in M^n} \mathbb{E}_m \left[\theta(r - r_f) + \frac{R_l}{R_f} + \frac{\theta - \theta^2}{2}\sigma^2 + \frac{1}{2}\sigma_l^2 - r_f + \frac{1-\gamma}{2}\left[\theta^2\sigma^2 + \sigma_l^2\right] \right] \tag{71}$$

which simplifies to

$$\max_{\theta} \min_{m \in M^n} \mathbb{E}_m \left[\theta(r - r_f) + \frac{\theta - \theta^2}{2}\sigma^2 + \frac{1-\gamma}{2}\theta^2\sigma^2 \right] \tag{72}$$

since r_f, $\frac{1}{2}\sigma_l^2$, $\mathbb{E}_m\left(\frac{R_l}{R_f}\right)$, and $\frac{1-\gamma}{2}\sigma_l^2$ do not affect the optimal choice of θ.

The first order conditions are

$$\begin{array}{ll} 0 = \min_{m \in M^n} \mathbb{E}_m \left[(r - r_f) + \frac{1}{2}\sigma^2 - \theta\sigma^2 + (1-\gamma)\,\theta\sigma^2\right] & \text{if } \theta > 0 \\ 0 \in \left\{\mathbb{E}_m \left[(r - r_f) + \frac{1}{2}\sigma^2 - \theta\sigma^2 + (1-\gamma)\,\theta\sigma^2\right] : m \in M^n\right\} & \text{if } \theta = 0 \\ 0 = \max_{m \in M^n} \mathbb{E}_m \left[(r - r_f) + \frac{1}{2}\sigma^2 - \theta\sigma^2 + (1-\gamma)\,\theta\sigma^2\right] & \text{if } \theta < 0 \end{array} \tag{73}$$

which implies a demand of

$$\theta^n(M^n) = \begin{cases} \frac{\underline{\mu}^n - r_f + \frac{1}{2}\sigma^2 - p}{\gamma\sigma^2} & \text{if } \underline{\mu}^n - r_f + \frac{1}{2}\sigma^2 - p > 0 \\ 0 & \text{if } \underline{\mu}^n - r_f \le p - \frac{1}{2}\sigma^2 \le \overline{\mu}^n - r_f \\ \frac{\overline{\mu}^n - r_f + \frac{1}{2}\sigma^2 - p}{\gamma\sigma^2} & \text{if } \overline{\mu}^n - r_f + \frac{1}{2}\sigma^2 - p < 0 \end{cases} \tag{74}$$

using $[\underline{\mu}^n, \overline{\mu}^n]$ to denote the corresponding interval of means. Normalizing $r_f = 0$ and using $[\underline{\mu}^n|s, \overline{\mu}^n|s]$ provides the result. □

Proof of Proposition 11. The proof is similar that of Proposition 3 by substituting ρ_ϵ with $K\rho_\epsilon$ and s by $\sum_{k=1}^{K} s_k$. □

Proof of Proposition 12. The market clearing price when I-investors trade away their stockholdings to U-investors is

$$p = \underline{\mu}_{PR}^U + \frac{1}{2}\sigma^2 - \frac{\gamma\sigma^2}{x_0^U}. \tag{75}$$

where $\underline{\mu}_{PR}^U$ is the lower bound of the interval of means under U-investor updated beliefs since

$\theta^U > 0$.

When U-investors only know that the signal is in the range $[a, b]$,

$$
\begin{aligned}
\underline{\mu}^U|[a,b] &= \min_{d\in[-\delta^U,\delta^U]} \mathbb{E}_d \left[\min_{d'\in[-\delta^U,\delta^U]} \mathbb{E}_{d'}[\mu|s] \middle| [a,b] \right] \\
&= \min_{d\in[-\delta^U,\delta^U]} \mathbb{E}_d \left[\frac{\rho_0\mu_0 + \rho_\epsilon(s-\delta^U)}{\rho_0+\rho_\epsilon} \middle| s \in [a,b] \right] \\
&= \frac{\rho_0}{\rho_0+\rho_\epsilon}\mu_0 - \frac{\rho_\epsilon\delta^U}{\rho_0+\rho_\epsilon} + \frac{\rho_\epsilon}{\rho_0+\rho_\epsilon} \min_{d\in[-\delta^U,\delta^U]} \mathbb{E}_d[s|s\in[a,b]]
\end{aligned} \tag{76}
$$

The set of distributions of s comprises normal distributions with variance $\rho_0^{-1}+\rho_\epsilon^{-1}$ and means $[\mu_0 - \delta^U, \mu_0 + \delta^U]$. From Lemma 2, for each $d \in [-\delta^U, \delta^U]$, $\mathbb{E}_d[s|s \in [a,b]] = \mu_0 + \Delta_d(a,b)$ where

$$
\Delta_d(a,b) = \frac{\phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(a-\mu_0-d)\right) - \phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(b-\mu_0-d)\right)}{\Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(b-\mu_0-d)\right) - \Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(a-\mu_0-d)\right)} \sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}} \tag{77}
$$

Using $\underline{\Delta}(a,b)$ as defined in (31), the market clearing condition above implies a price of

$$
p = \mu_0 + \frac{1}{2}\sigma^2 + \frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}\underline{\Delta}(b, b-2\delta) - \frac{\rho_\epsilon\delta^U}{\rho_0+\rho_\epsilon} - \frac{\gamma}{x_0^U}\sigma^2. \tag{78}
$$

Plugging this price into the conditions necessary for partial revelation as in Section 3, the existence of a range $[a,b]$ of unrevealed signals follows from the existence of a solution to

$$
\frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}\left(b-\mu_0-\underline{\Delta}(b-2\delta,b)\right) + \left(\frac{\gamma\sigma^2}{x_0^U} + \frac{\rho_\epsilon\delta^U}{\rho_0+\rho_\epsilon} - \frac{\rho_\epsilon\delta}{\rho_0+\rho_\epsilon}\right) = 0. \tag{79}
$$

The proof then follows similarly to that of Proposition 3. □

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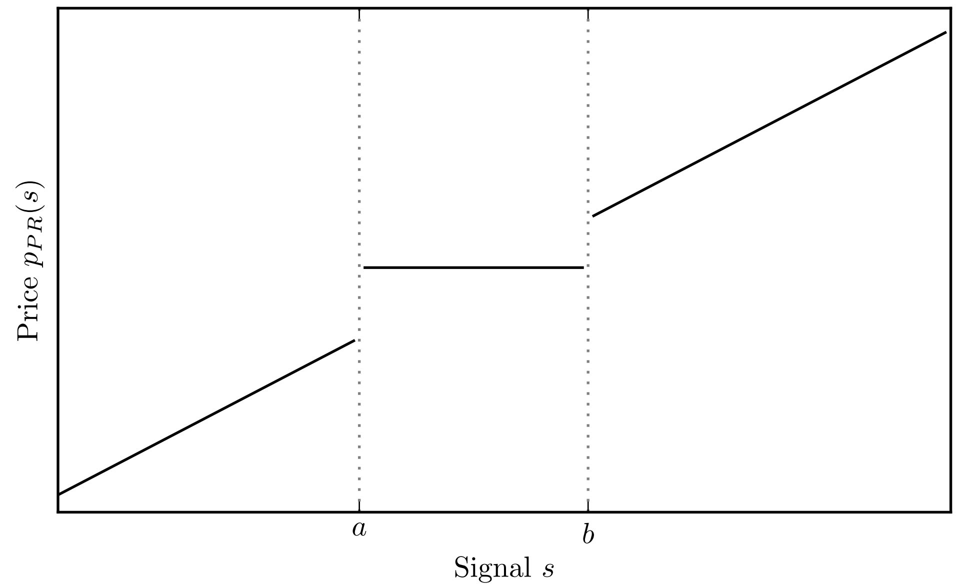
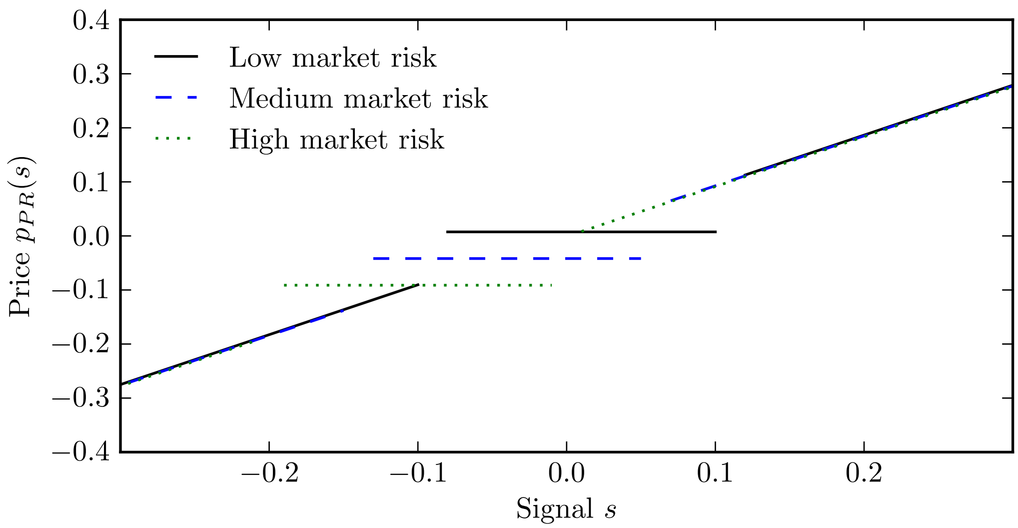


(a) Partial revelation price function p_{PR}



(b) For differing levels of market risk $\left(\frac{\gamma\sigma^2}{x_0^U}\right)$

Figure 1: Equilibrium price function

The parameter values for all plots in the paper are $\rho_0 = 10$, $\rho_\epsilon = 120$, $\sigma = 0.05448$, $\mu_0 = 0.006$ and $\delta = 0.10$. The values for μ_0 and σ were taken from the average monthly equity premium on the CRSP value-weighted market index (using the Fama risk-free rates available from CRSP) from January 1925 to December 2011. For each market risk scenario, $x_0^U = 0.99$. Low, medium, and high market risk correspond to $\gamma = 0$ (risk-neutral), $\gamma = \frac{1}{2}$ and $\gamma = 1$ (log utility) respectively.

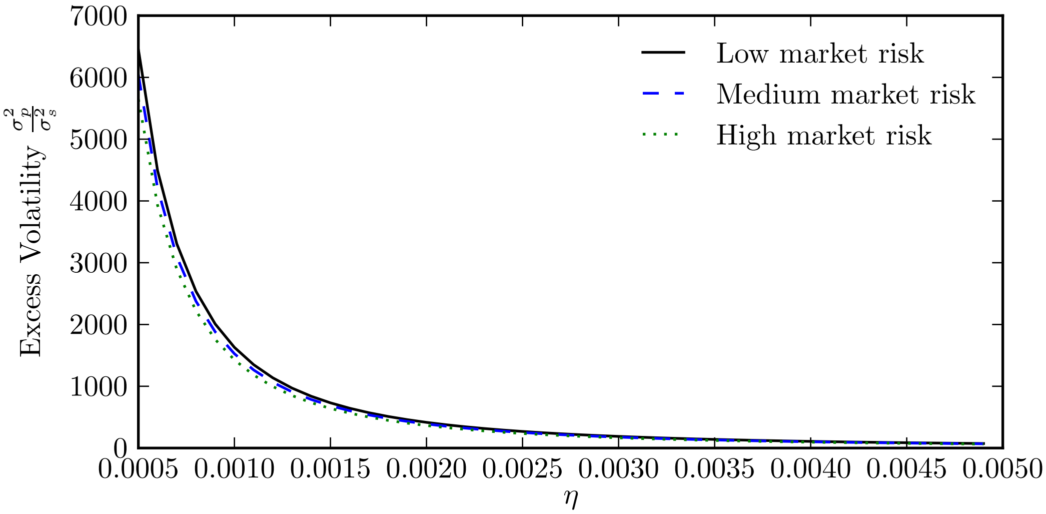


Figure 2: Excess volatility of price with $\sigma_p^2 = \text{Var}(p_{PR}|b - \eta < s < b + \eta)$, $\sigma_s^2 = \text{Var}(s|b - \eta < s < b + \eta)$ for small $\eta > 0$. Parameter values are as in Figure 1(b).

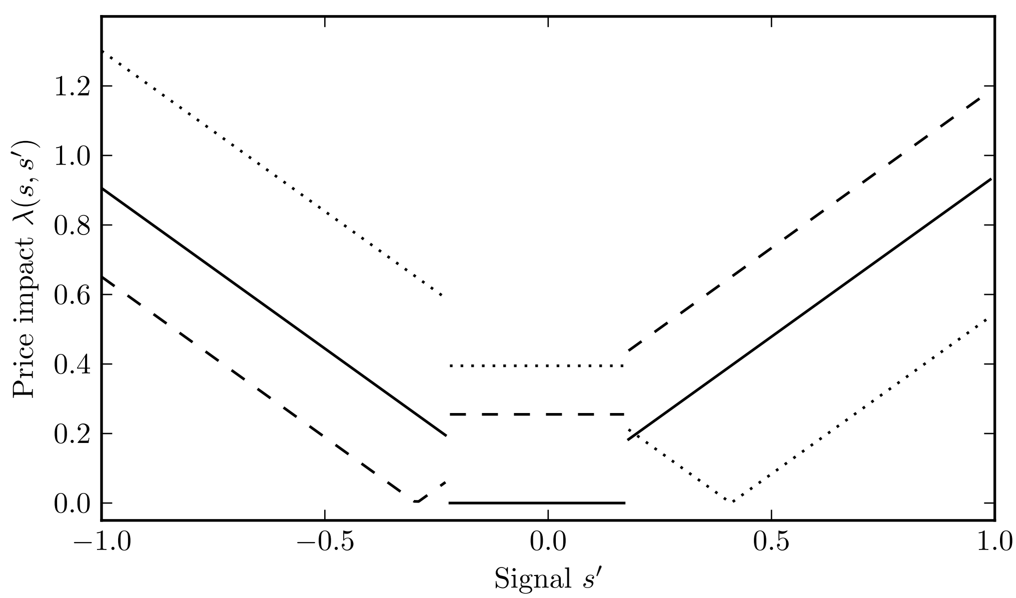


Figure 3: Price impact $\lambda(s, s')$ as a function of s' for a given s: (i) solid lines for $s \in [a, b]$, (ii) dashed lines for $s < a$, and (iii) dotted lines for $s > b$. Price impact $\lambda(s, s') = 0$ when $s = s'$. The plot is for $\gamma = 1$ (log utility) investors. Other parameter values are the same as in Figure 1(b).

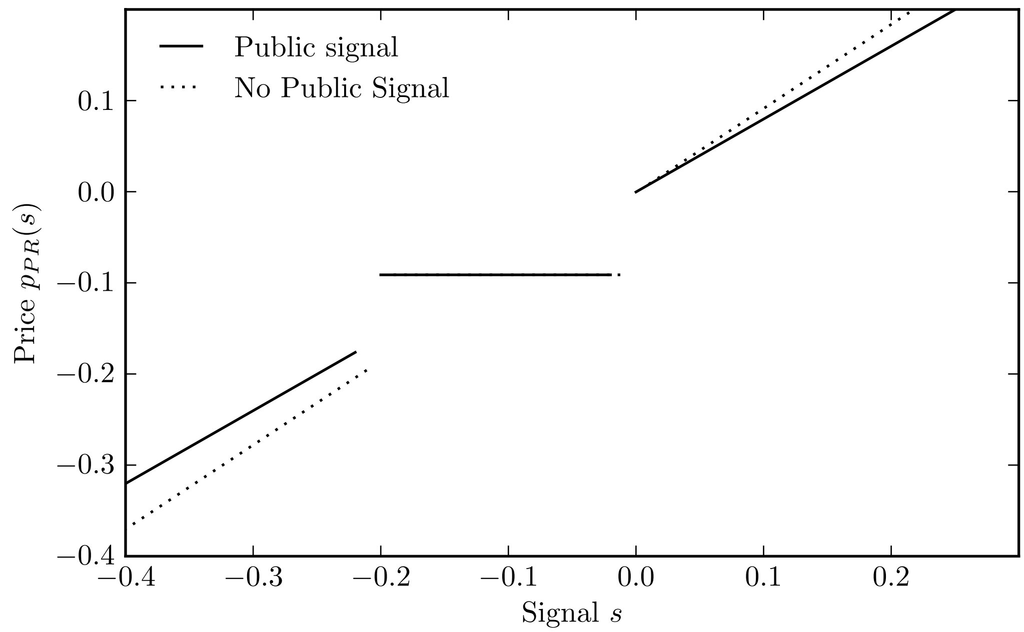


Figure 4: Effect of a public signal ζ that confirms the mean ($\zeta = \mu_0$). The plot is for $\gamma = 1$ investors, parameters are the same as those of Figure 1(b), and the public signal has precision $\rho_\zeta = 20$.

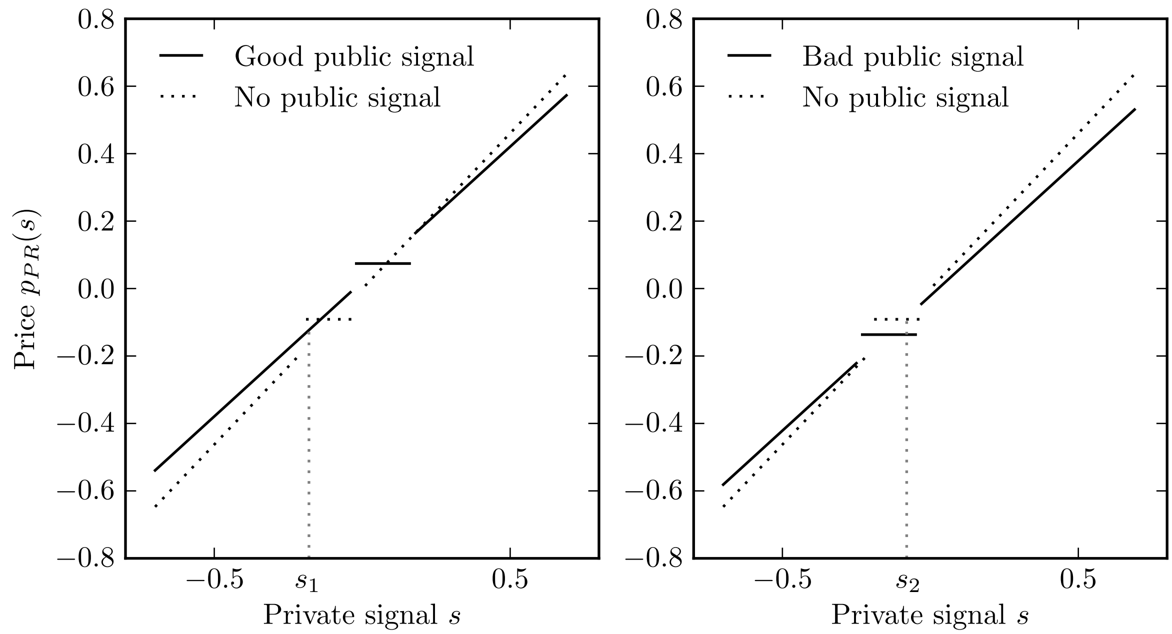


Figure 5: Effect of a good ($\zeta > \mu_0$) and bad ($\zeta < \mu_0$) public signal on the equilibrium price function. The good public signal is $s = \mu_0 + \frac{1}{2}\rho_0^{-0.5}$ while the bad signal is $s = \mu_0 - \frac{1}{2}\rho_0^{-0.5}$ The plot is for $\gamma = 1$ (log utility) investors. Parameters are the same as those of Figure 1(b) and the public signal has precision $\rho_\zeta = 20$.