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Evidence from UK bank bailouts

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Do CDS spreads reflect default risks? Evidence from UK bank bailouts

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Abstract

CDS spreads are generally considered to reflect the credit risks of their reference entities. However, CDS spreads of the major UK banks remained relatively stable in response to the recent credit crisis. We suggest that this can be explained by changes in loss given default (LGD). To obtain the result we first derive the probabilities of default from stock option prices and then determine the LGD consistent with actual CDS spreads. Our results reveal a significant decrease in the LGD of bailed out banks over the observed period in contrast to banks which were not bailed out and non-financial companies.

Keywords: Credit default swap (CDS), Loss given default (LGD), Volatility surface

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1 Introduction

Credit default swaps (CDS) are one of the most important innovations in financial markets in the last two decades. CDS provide an insurance against the loss by a company's

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(reference entity's) default, and the rate of the yearly payment for this contract is known as the CDS spread. It may be thus expected that the CDS spread reflects the soundness/riskiness of the reference entity. In fact, financial market practitioners use CDS spreads as a riskiness measure (see e.g. Ferguson, 2008). Hull et al. (2004) provide some evidence of the positive relationship between CDS spreads and credit ratings. More recent studies confirm that CDS spreads can be well explained by the variables that are commonly used as determinants of the credit risk (see Ericsson et al., 2009), and by individual firms' volatility and jump risks (see Zhang et al., 2009), however they do not address directly the question that how well CDS spreads reflect credit risks. Although the link between the CDS spread and the credit risk of the reference entity seems quite intuitive and plausible, there is a certain gap in the literatures on this topic. This paper contributes to covering this gap. In particular, we study how the credit risk of major UK banks is reflected in their CDS spreads during 2008-09. Our focus on this time period is motivated mainly by explicit defaults and government bailouts that took place in the UK's banking industry during the recent financial crisis (table 1).

We start with the observation that during 2008-09 there is no significant difference between the CDS spreads of the major UK banks and those of non-financial companies (see figure 1). Neither is there a difference in the spreads between the government "bailed out" banks (RBS and LLOYDS) and those of the "non bailed out" banks (HSBC and Barclays). This contrasts with the US data which show a significant difference (about ten times!) in CDS spreads for banks and non-financials (see figure 2). The only exception to this is a short term turmoil between the Lehman Brothers bankruptcy and the announce of the capital injection by the UK government - during this short period in September 2008 UK banks showed some temporary increase in CDS spreads similarly to their US peers.

However, stock prices reveal that the risk of UK banks significantly increased, as perceived by the market: during the 9 months from July 2008 stock prices decreased by a factor 2 for HBSC, by factor 4 for Barclays, and dropped close to zero for Lloyds and RBS (see figure 3). Our second observation is that the above UK banks were far from riskless. We therefore question how much of this risk is reflected in CDS spreads. In fact both RBS and LLOYDS required UK government public money and availed themselves of the governments asset protection scheme (for details on UK bank bailout measures see e.g. Bank of England, 2009; Hall, 2009, and table 1). [1]

We focus on the role of the loss given default (LGD) in the analysis of CDS pricing mechanism to consider the above problem. Brigo and Mercurio (2006) indicate that the CDS spread is approximately the product of probability of default (PD) and LGD. In the pricing of corporate debt based securitized products, market participants often calculate the credit risk assuming that the LGD is fixed at about 60 % [2]. It follows that with a constant LGD the value of CDS spread reflects the credit risk of the reference entity (at least as measured by the probability of default), which is inconsistent with our observations above. In this paper we show that a model of consistently priced CDS implies that LGD varies in time, which should be taken into account when determining the credit risk based on the CDS spreads.

The LGD inferred from CDS spreads is not, in general, the same as that inferred from the prices of loans and bonds. The debt of reference entities, especially loans, is not always tradable because of illiquidity. Therefore, different types of debt assets which sellers receive in the physical settlement of a CDS would imply different LGD as inferred from the respective CDS spreads[3]. On the other hand recent CDS contracts are designed so that CDS dealers determine the LGD through an auction in the cash settlement to

[1]US Department of Treasury decided to inject capital to all the major US financial institutions in October 2008. However, JP Morgan Chase repaid the government in June 2009. Therefore we treat JP Morgan Chase as a non bailed out institution in figure 2.

[2]Standard & Poors (2006) report and Fitch Ratings (2010) report show that the rating agencies evaluate CDS and give credit ratings to cash and synthetic CDO under the assumption that the LGD of most of the investment grade corporate bonds and CDS is about 60%.

[3]There are three types of settlement in the CDS market. In physical settlement, CDS buyers deliver the debt of the reference entity (typically a loan or bond) and receive the same amount of money as the notional amount of the CDS. In a cash settlement, the sellers pay the difference between the market value of the debt and the notional amount of the CDS to the buyers. If the market value of the debt is not uniquely available, the dealers of the CDS will determine the market value in the auction. Finally, in a fixed amount settlement, the sellers pays a fixed amount of money to the buyers regardless of loss due to the credit event. British Bankers Association (2006) showed that the market share of CDSs which are designed to settle in cash settlement has increased from 13% to 26% in 2006 with the growth of CDS although the share of physical settlement is still 73%.

reduce the settlement cost[4]. The LGD of CDS will be determined typically in a few weeks following the credit event, whilst it often takes months (or even years) to determine the LGD of loans and corporate bonds in the legal bankruptcy processes. Therefore, the LGD of CDS can diverge from those of loans and bonds, particularly under illiquid market conditions where it is difficult to construct an arbitrage position. As a result, the difference can sometimes become large if the market participants are particularly risk averse and unwilling to hold the reference asset until the legal determination of the LGD. In fact, Helwege et al. (2009) show that the LGD of Lehman Brothers was about 91% in October 2008 whilst the bond price was 13% of the face value on the day prior to the auction (approximately 30 % difference between the debt price and the result of the auction). However, other auction results in 2008 were generally consistent with the bond prices.

This motivates us to use a joint estimation approach, in which we derive the PD from the listed stock options of the reference entities and then use this PD to calculate the implied LGD from CDS spreads. There are three general approaches in the area of joint estimation of credit risk. The first approach, typified by Jarrow (2001), develops a joint estimation framework using stock prices. This methodology determines the probability of default as the probability that the stock price collapses to zero. However, since the stock price is a univariate time series, it is not possible to estimate the PD daily, but in some band (typically one month). This method also has the disadvantage of not being too responsive to changes in market conditions. The second approach, employed by Baba and Ueno (2006), Schlafer and Uhrig-Homburg (2009) and Norden and Weber (2010), estimates the PD from the relationship between senior CDS and subordinate CDS. However, this approach, although intuitive and straightforward to implement, is limited to those companies which have a traded market in both senior and subordinate CDS. The third approach which follows Linetsky (2006) and Carr and Wu (2010), develops a framework to estimate the PD implied in stock options. This methodology has the advantage that it can be employed to estimate the daily PD because option prices provide

[4]The International Swap and Derivatives Association (2009) suggests that all the existing CDS contracts should be transferred to cash settlement to improve efficiency and transparency in the CDS settlement process.

wide cross section data for a large number of companies. It also has the advantage of being responsive to changes in market conditions. We use this approach to estimate the PD in this paper.

For the estimation of the unbiased PD we modify the Black and Scholes (1973) option pricing model. In particular, (1) we relax the assumption that the underlying stock price follows a geometric Brownian motion (no default and no jump) and allow for the possibility of default (the price can be zero), (2) we allow the interest rate to be stochastic and use the (non-flat) term structure of the compound interbank funding rate as the closest proxy for the instantaneous rate in continuous-time models, and (3) the volatility of the underlying stock price is stochastic and modeled with the singular perturbation method. A similar approach has been used by Bayraktar and Yang (2010), however we allow for greater flexibility in modeling the term structure of the interest rates by adopting the Hull and White (1990, 1993) extension of the Vasicek (1977) one factor interest rate model. This captures the high instability in market conditions experienced during the financial crisis.

In order to infer the LGD from CDS we derive a CDS pricing model which takes into account counterparty risk. Duffie et al. (2005, 2007) show the impact of intermediary activity on the price of OTC (Over the Counter) securities in contrast to listed securities. Moreover, Taylor and Williams (2009) and Wu (2008) agree that counterparty risk influenced the short term interest rate in money market during the recent financial crisis while they don't agree with the effect of the Federal Reserve's liquidity supply to ease the turmoil of financial markets. These theoretical and empirical studies imply the importance of counterparty risk in CDS pricing.

Finally, we calibrate the model with the actual data and the PD inferred from stock option prices in order to derive the LGD estimate. Our results suggest that the above puzzle of CDS not being able to reflect the default risk of the reference entity can be resolved if one takes into account the variability of LGD. In particular we show that the LGD of the bailed out UK banks decreased dramatically in response to market events, especially after the Lehman Brothers bankruptcy, counteracting the substantial rise in their implied PD. This interplay between the PD and LGD provides a clear explanation of the behavior of their credit spreads during this period of unprecedented turbulence in the markets. In addition we demonstrate that the LGD of banks and non financial sector companies are negatively correlated, whilst the CDS spreads are positively correlated. These findings on the LGD of the non financial sector companies are consistent with Altman et al. (2005) and Zhang (2009) who show the influence of demand and supply of distressed assets, like defaulted bonds, and business cycles on the LGD of loans.

The rest of the paper is organized as follows. In section 2, we introduce the option pricing model under credit risk and derive the approximation formula to estimate the PD implied in listed stock options using the perturbation method. In section 3 we develop the framework to calculate the LGD implied in CDS using the implied PD. In section 4 we consider the calibration method to derive the implied PD and LGD. In section 5 we discuss the results of the estimation and the relationship with other studies, which consider LGD and prices of defaulted bonds. Finally we provide our conclusions in section 6.

2 Listed Stock Option Pricing Model under Credit Risk

The first step in this paper is the estimation of credit risk (PD) implied from listed stock option prices. The original option pricing model (Black-Scholes formula) is derived under the following simplified assumptions; (1) the underlying stock price S_t follows a geometric Brownian motion, (2) the volatility of the stock price σ_t is constant at the current level σ_0 and (3) instantaneous risk free interest rate, $r_t\,(= r(t,t))$ is constant at the current level r_0 and the forward rate between t and t_1, $r(t,t_1)$ is flat:

$$dS_t = S_t\left(\mu_t dt + \sigma_t dW_t\right) \tag{1}$$

$$\sigma_t = \sigma_0 \tag{2}$$

$$r_T = r_t \quad for\ \forall T \geq t \quad and \quad r(t,t_1) = r_t \quad for\ \forall t_1 \geq t \tag{3}$$

We follow the approach of Carr and Wu (2010) and Bayraktar and Yang (2010) in relaxing the above three assumptions. In the first assumption, the stock price is continuous and strictly positive, which implies that the issuer of stock never defaults. Clearly, if the stock issuer defaults, then the stock price will collapse to zero[5]. To capture the impact of credit risk on the stock price and its option prices, the underlying stock price process is extended to the credit risk embedded process using a Cox process, as suggested by Lando (1998). The second assumption (2) is relaxed by introducing additional stochastic processes into the option pricing model to describe the volatility fluctuation, as, for example, in Heston (1993).

To deal with the third assumption, Carr and Wu (2010) and Bayraktar and Yang (2010) model the term structure of interest rate using the Vasicek model. However, this model fails to adequately reproduce the significant curvature experienced in the term structure during the crisis experienced in 2008-2009. To overcome this shortcoming, we employ the Hull and White (1993) term structure model to fit the twisted term structure in 2008-2009. This model fits the data during this period completely and has the advantage (like the Vasicek model) of analytical tractability.

2.1 The Stochastic Processes of the Underlying Stock Price and Interest Rate

In this section we describe the modification to the standard Black-Scholes assumptions. First we introduce the two dimensional Cox process (Poisson Processes of which intensity, $\lambda_t^{(1)}$ and $\lambda_t^{(2)}$, are time varying)

$$\begin{pmatrix} \tilde{N}_t^{(1)} \\ \tilde{N}_t^{(2)} \end{pmatrix} = \begin{pmatrix} N_t^{(1)} \left(\int_0^t \lambda_s^{(1)} ds \right) \\ N_t^{(2)} \left(\int_0^t \lambda_s^{(2)} ds \right) \end{pmatrix} \tag{4}$$

[5]It is important to distinguish between "close to default" and "defaulted" firms. Stock prices of companies which are at the brink of default are still positive. In fact the stock prices of most financial institutions bailed out by governments did not fall to zero. As Dixit and Pindyck (1993) discussed, within the real option framework, even if investors recognize that the net asset value turns negative, stock prices can remain positive since investors can bet the (small) probability that the company would recover to financial soundness and profitability under the limited liability of shareholders. However, the stock will become worthless when the company decides to shut down the business under the negative net asset value (default). We measure the likelihood of this latter event and therefore assume zero recovery at default.

and six correlated Brownian motions with the following correlation structure,

$$E[W_t^{(0)},\, W_t^{(i)}] = \rho_i t \ ,\ E[W_t^{(i)},\, W_t^{(j)}] = \rho_{ij} t \ \ i,j \in \{1,2,3,4,5\},\ \ t \geq 0. \tag{5}$$

To model the time of credit event τ_1 and the dynamics of the underlying stock price, we employ,

$$\tilde{N}_t^{(1)} = \begin{cases} 0 & \tau_1 > t \\ 1 & \tau_1 \leq t \end{cases}$$

where the intensity of the Cox process, $\lambda_t^{(1)}$, is defined

$$\begin{aligned} \lambda_t^{(1)} &= f(Y_t, Z_t), & (6)\\ dY_t &= \frac{1}{\epsilon}(m - Y_t)dt + \frac{\upsilon\sqrt{2}}{\sqrt{\epsilon}} dW_t^{(2)} & (7)\\ dZ_t &= \delta c(Z_t)dt + g(Z_t)dW_t^{(3)} & (8) \end{aligned}$$

$f(Y_t, Z_t)$, $c(Z_t)$ and $g(Z_t)$ are respectively smooth and bounded functions of Z_t and Y_t. The parameters δ and ϵ control the velocity of the processes which are characterized by m, ν, $c(Z_t)$ and $g(Z_t)$. Thus the probability that "$\tau_1 \leq T$ at t" is

$$\begin{aligned} P(\tau_1 \leq T |\, \tau_1 > t) &= E\left[\tilde{N}_T^{(1)} \middle|\, \mathcal{G}_t \right] \\ &= E\left[\int_t^T \lambda_s^{(1)} \exp\left(- \int_0^s \lambda_u^{(1)} du \right) ds \right] & (9) \end{aligned}$$

We denote the credit event indicator process by $I_t = 1_{(\tau_1 \leq t)}$ $(t \geq 0)$ and define $\mathcal{I}_t$ as the filtration generated by I_t. Under this setting, the defaultable stock price process is defined as,

$$d\bar{S}_t = \bar{S}_t \left(r_t dt + \sigma(\tilde{Y}_t) dW_t^{(0)} - d\left(N_t - \int_0^{t\vee\tau} \lambda_u^{(1)} du \right) \right),\ \ \bar{S}_0 = x\,. \tag{10}$$

The credit event indictor of (10) is the jump diffusion factor, where the jump magnitude is fixed at unity. This is the modification of assumption (1).

Secondly, the stochastic volatility function is defined as a smooth and bounded function of $\tilde{Y}_t$. The stochastic process, $\tilde{Y}_t$, follows a mean reverting process,

$$d\tilde{Y}_t = \left(\frac{1}{\epsilon} \left(\tilde{m} - \tilde{Y}_t \right) + \frac{\tilde{\upsilon}\sqrt{2}}{\sqrt{\epsilon}} \Lambda(\tilde{Y}_t) \right) dt + \frac{\tilde{\upsilon}\sqrt{2}}{\sqrt{\epsilon}} dW_t^{(4)}\,. \tag{11}$$

This is introduced to modify assumption (2). The parameter $1/\epsilon$ is the rate of mean reversion of the process while ϵ also corresponds to the time scale of the process.

While the stock price drops to zero at the moment of the credit event (default) and thereafter stays at zero, the pre-default stock price dynamics has continuous paths.

$$dS_t = S_t\left((r_t + \lambda_t^{(1)})\,dt + \sigma(\tilde{Y}_t)dW_t^{(0)}\right),\ S_0 = x \ and\ \lambda_0^{(1)} = \lambda^{(1)}\,. \tag{12}$$

Finally the risk free interest rate, r_t is assumed to follow Hull and White (1993) model

$$dr_t = (\alpha_t - \beta r_t)dt + \eta\, dW_t^{(1)}, \quad r_0 = r\,. \tag{13}$$

where

$$\alpha_t = \frac{\partial f^M(0,t)}{\partial t} + \beta f^M(0,t) + \frac{\eta^2}{2\beta}(1 - \exp(-2\beta t)) \tag{14}$$

and $f^M(t,T)$ is the market forward rate from time t to time T. This is the modification of (3).

The modifications to the assumptions of Black-Scholes (1)-(3) are now completed. This theoretical framework is now employed to derive the modified equations for both European call and put options.

2.2 Option Pricing under Credit Risk

In the framework described above, it is possible to construct the enlargement filtration $\mathcal{G}_t$ of the filtration $\mathcal{F}_t$ generated by the vector of Brownian motions and credit indicator filtration $\mathcal{I}_t$ ($\mathcal{G}_t = \mathcal{I}_t \vee \mathcal{F}_t$). We can calculate the price of the defaultable contingent claim $P(t,T)$ as the conditional expectation of the pay off, $h(\bar{S}_T)(= P(T,T))$ by proposition 5.1.1 in Bielecki and Rutkowski (2002):

$$\begin{aligned} P(t,T) &= E\left[\exp\left(-\int_t^T r_s ds\right) h(\bar{S}_T) 1_{(\tau_1 > T)} \middle| \mathcal{G}_t\right] \\ &= 1_{(\tau_1 > t)} E\left[\exp\left(-\int_t^T \left(r_s + \lambda_s^{(1)}\right) ds\right) h(S_T) \middle| \mathcal{F}_t\right]\,. \end{aligned} \tag{15}$$

Duffie and Singleton (1999) obtain the equivalent result for defaultable bonds.

Equation (15) provides some useful special cases. First, if $h(\bar{S}_T) \equiv 1$, then $P(t,T)$ is a (non-defaultable) risk free discount bond price, $B^0(t,T)$.

$$B^0(t,T) = E\left[\exp\left(-\int_t^T r_s ds\right) \middle| \mathcal{F}_t\right] \tag{16}$$

In the setting described by (13), it is possible to obtain the explicit solution of (16)

$$B^0(t,T) = \exp(a(t,T) - b(t,T)r_t)$$

where

$$\begin{aligned} b(t,T) &= \frac{1-\exp(\beta(T-t))}{\beta} \\ a(t,T) &= \frac{f^M(0,T)}{f^M(0,t)}\left(b(t,T)f^M(0,t) - \frac{\eta^2}{4\beta}(1-\exp(-2\beta t))b(t,T)^2\right). \end{aligned} \tag{17}$$

If $h(\bar{S}_T) \equiv 1_{(\tau_1 > T)} + 1_{(\tau_1 \le T)}(1-l_1)P(t,\tau_1 -)$ and l_1 is the rate of loss at default, then $P(t,T)$ is a defaultable discount bond price, $B^c(t,T)$.

$$\begin{aligned} B^c(t,T) &= E\left[\exp\left(-\int_t^T r_s ds\right)(1-1_{(\tau_1\le T)}l_1)\Big|\mathcal{G}_t\right] \\ &= E\left[\exp\left(-\int_t^T \left(r_s + l_1\lambda_s^{(1)}\right)ds\right)\Big|\mathcal{F}_t\right]. \end{aligned} \tag{18}$$

Finally, if $h(\bar{S}_T) \equiv (S_T - K)^+$, $P(t,T)$ represents the price of a European call option with strike price K ($K > 0$) which reduces to

$$\begin{aligned} Call(t,T) &= E\left[\exp\left(-\int_t^T \left(r_s+\lambda_s^{(1)}\right)ds\right)(X_T-K)^+\Big|\mathcal{F}_t\right] \\ &= xN(d_1) - K\,E\left[\exp\left(-\int_t^T \left(r_s+\lambda_s^{(1)}\right)ds\right)\Big|\mathcal{F}_t\right]N(d_2), \end{aligned} \tag{19}$$

where $N()$ is the standard normal distribution function and

$$\begin{aligned} d_1 &= \frac{\log\left(\frac{x}{kB_0^c(t,T)}\right)+\frac{1}{2}\sigma(t,T)}{\sqrt{\sigma(t,T)}}, d_2 = d_1 - \sqrt{\sigma(t,T)}. \\ \sigma(t,T) &= E\left[\int_t^T \sigma\left(\tilde{Y}_t\right)^2 dt\Big|\mathcal{F}_t\right] \\ B_0^c(t,T) &= E\left[\exp\left(-\int_t^T \left(r_s+\lambda_s^{(1)}\right)ds\right)\Big|\mathcal{F}_t\right]. \end{aligned} \tag{20}$$

The price of put option is obtained by using put-call parity,

$$\begin{aligned} Put(t,T) &= -x\,N(-d_1) + K\,E\left[N(-d_2)\exp\left(-\int_t^T \left(r_s+\lambda_s^{(1)}\right)ds\right)\Big|\mathcal{F}_t\right] \\ &\quad +K\,E\left[\exp\left(-\int_t^T r_s\,ds\right) - \exp\left(-\int_t^T (r_s+\lambda_s^{(1)})ds\right)\Big|\mathcal{F}_t\right]. \end{aligned} \tag{21}$$

To illustrate in a simple way the qualitative effect of credit risk on the option prices; keeping the volatility of the underlying asset and interest rate constant as in (2) and (3), the impact of credit risk on the price of a call option is equivalent to that of a rise in interest rate. In the above condition, equation (19) is equivalent to the original Black-Scholes formula with the non defaultable interest rate r_s replaced by $r_s + \lambda_s^{(1)}$. On the other hand, the impact of credit risk on put options is not as straightforward. As derived in (21), the last term is positive if $\lambda_s^{(1)}$ is positive. Then, particularly for the out of money put option, this model produces a higher premium than the standard Black-Scholes model because the impact of the last term becomes relatively larger as the first two terms becomes smaller. From an economic point of view, this represents an insurance premium to cover the possibility of default.

2.3 Modeling the Volatility Surface of Stock Options

In this section we model the volatility surface of the listed stock options using the singular perturbation method. As Fouque et al. (2000, 2003) demonstrate the singular perturbation method provides an accurate approximation of option prices without the need to specify the individual parameters of the stochastic processes in section 2.1.

The perturbation method is divided into two steps. First, we formulate the model as the modification of the simple model, which is analytically solved. It is possible to derive the analytical solution of the option price, P_0 when the volatility of underlying stock price process $\sigma(\tilde{Y}_t)$ and the intensity of the hazard rate $\lambda_t^{(1)}$ are fixed at $\bar{\sigma}_1^2$ and $\bar{\lambda}^{(1)}$ respectively. Denote $Call_0$ if P_0 is the price of a call option,

$$Call_0(t,T) = xN(d_1) - KE\left[\exp\left(-\int_t^T \left(r_s + \bar{\lambda}^{(1)}\right) ds\right) \bigg| \mathcal{F}_t\right] N(d_2)\,, \tag{22}$$

where

$$\begin{aligned} d_1 &= \frac{\log\left(\frac{x}{k\bar{B}_0^c(t,T)}\right) + \frac{1}{2}\bar{\sigma}(t,T)}{\sqrt{\bar{\sigma}(t,T)}}, d_2 = d_1 - \sqrt{\bar{\sigma}(t,T)}\,. \\ \bar{\sigma}(t,T) &= \bar{\sigma}_1^2(T-t) + \eta^2\int_t^T b^2(s,T)ds + 2\eta\bar{\rho}\bar{\sigma}_1\int_t^T b(s,T)ds \\ \bar{B}_0^c(t,T) &= E\left[\exp\left(-\int_t^T \left(r_s + \bar{\lambda}^{(1)}\right) ds\right) \bigg| \mathcal{F}_t\right]\,. \end{aligned} \tag{23}$$

Similarly the price of a put option Put_0 is

$$\begin{aligned} Put_0(t,T) &= -x\,N(-d_1) + K\,E\left[N(-d_2)\exp\left(-\int_t^T \left(r_s + \bar{\lambda}^{(1)}\right) ds\right)\Big|\mathcal{F}_t\right] \\ &\quad + K\,E\left[\exp\left(-\int_t^T r_s\ ds\right) - \exp\left(-\int_t^T (r_s + \bar{\lambda}^{(1)}) ds\right)\Big|\mathcal{F}_t\right]. \quad (24) \end{aligned}$$

The second step is to calculate the approximation of (19) and (21) using the asymptotics of (22) and (24) to correct the error between the original model and simplified model. We calculate the model option price $\tilde{P}_{\epsilon,\delta}$ using the first order asymptotics on $\sqrt{\epsilon}$ and $\sqrt{\delta}$ to approximate the actual option price P,

$$\tilde{P}_{\epsilon,\delta} = P_0 + \sqrt{\epsilon}P_{1,0} + \sqrt{\delta}P_{0,1}\,. \quad (25)$$

Fouque et al. (2003) prove the validity of the approximation, in particular that there exists a constant C such that $|P - \tilde{P}_{\epsilon,\delta}| \leq C$ when the payoff function $h(S_T)$ is smooth, and $|P - \tilde{P}_{\epsilon,\delta}| \leq C\left(\epsilon \log\epsilon + \delta + \sqrt{\epsilon\delta}\right)$ when $h(S_T)$ is the payoff of a call or put option. The model price of stock option, $\tilde{P}_{\epsilon,\delta}$ is expressed explicitly (See Appendix A for the derivation),

$$\begin{aligned} \tilde{P}_{\epsilon,\delta}(T,K,\bar{\lambda}^{(1)}(z),V) &= P_0(T,K,\bar{\lambda}^{(1)}(z)) + V_1^\epsilon g_1(T,K,\bar{\lambda}^{(1)}(z)) + V_2^\epsilon g_2(T,K,\bar{\lambda}^{(1)}(z)) \\ &+ V_3^\epsilon g_3(T,K,\bar{\lambda}^{(1)}(z)) + V_4^\epsilon g_4(T,K,\bar{\lambda}^{(1)}(z)) + V_5^\epsilon g_5(T,K,\bar{\lambda}^{(1)}(z)) \\ &+ V_6^\epsilon g_6(T,K,\bar{\lambda}^{(1)}(z)) + V_7^\epsilon g_7(T,K,\bar{\lambda}^{(1)}(z)) + V_1^\delta g_8(T,K,\bar{\lambda}^{(1)}(z)) \\ &+ V_2^\delta g_9(T,K,\bar{\lambda}^{(1)}(z))\,. \quad (26) \end{aligned}$$

As shown in Appendix A, all coefficients $V\ (= (V_1^\epsilon, V_2^\epsilon, V_3^\epsilon, V_4^\epsilon, V_5^\epsilon, V_6^\epsilon, V_7^\epsilon, V_1^\delta, V_2^\delta))$ in (26) are the products of the parameters of (6)-(13). It follows that it is possible to estimate the option price under credit risk without specification of the individual parameters of the stochastic processes, but rather the coefficients, V in the above equation (see Bayraktar and Yang (2010) for detail). This is important as it allows a calibration of the model which minimizes model risk.

In Appendix A we show that equation (26) approximates $\tilde{P}_{\epsilon,\delta}$ using the Greeks (risk sensitivities) of the analytical solution P_0. Moreover, equation (26) is independent of two state variables, Y_t and $\tilde{Y}_t$ due to averaging. It depends only on the level of Z_t through $\bar{\lambda}^{(1)}(z)$. As P_0 is a function of x and $\bar{\lambda}^{(1)}(z)$, we can calculate the model free approximation, $\tilde{P}_{\epsilon,\delta}$, on Y_t and $\tilde{Y}_t$ except the mean reversion defined in (7) and (11).

3 Valuation of CDS using the PD implied in the Listed Stock Options

The second step in this paper is to calculate the LGD of CDS which is consistent with the PD implied in the stock options. We introduce counterparty risk into the CDS pricing model for this calculation. The loss by counterparty risk arises when the protection seller cannot compensate the loss from default of the reference entity if the seller and reference entity default simultaneously.

3.1 CDS Pricing without counterparty risk

Bluhm et al. (2002) show that it is possible to determine the fair (no arbitrage) CDS spread as the spread to balance the expected cash flow of the protection seller and that of the protection buyer in the frictionless market. That is the expected value of the protection buyer's cash flow (premium leg), C_F^{buy}, equals that of the protection seller's cash flow (default leg), C_F^{sell}, under the fair CDS spread.

Suppose that $CDS_F(t,T)$ is the CDS spread, where the contract starts at t and matures at T, and T_m is the time of the premium payment ($m = 1, \cdots, M$ and $T_M = T$), the expected payment of the CDS premium, $C_F^{buy}(t,T)$ is

$$\begin{aligned} C_F^{buy}(t,T) &= E\left[\sum_{m=1}^{M} \exp\left(-\int_t^{T_m} r_s ds\right) 1_{(\tau_1 > T_m)} CDS_F(t,T) \Big| \mathcal{G}_t\right] \\ &= CDS_F(t,T) 1_{(t<\tau_1)} \sum_{m=1}^{M} E\left[\exp\left(-\int_t^{T_m} (r_s + \lambda_s^{(1)}) ds\right) \Big| \mathcal{F}_t\right] \end{aligned} \quad (27)$$

and the expected payment triggered by the credit event, $C_F^{sell}(t,T)$ is

$$\begin{aligned} C_F^{sell}(t,T) &= E\left[\exp\left(-\int_t^{\tau_1} r_s ds\right) 1_{(\tau_1 \le T_M)} l \Big| \mathcal{G}_t\right] \\ &= 1_{(t<\tau_1)} E\left[l \int_t^T \exp\left(-\int_t^u (r_s + \lambda_s^{(1)}) ds\right) \lambda_u^{(1)} du \Big| \mathcal{F}_t\right] \end{aligned} \quad (28)$$

Therefore, when $C_F^{buy}(t,T)$ equals $C_F^{sell}(t,T)$, the no arbitrage CDS spread is

$$CDS_F(t,T) = 1_{(t<\tau_1)} \frac{E\left[l \int_t^T \exp\left(-\int_t^u (r_s + \lambda_s^{(1)}) ds\right) \lambda_u^{(1)} du \Big| \mathcal{F}_t\right]}{\sum_{m=1}^{M} E\left[\exp\left(-\int_t^{T_m} (r_s + \lambda_s^{(1)}) ds\right) \Big| \mathcal{F}_t\right]}. \quad (29)$$

Equation (29) determines the CDS spreads using only three parameters, PD, LGD and risk free interest rate, and assumes no dependence between these risk factors, a reasonable assumption as pointed out by Jankowitsch et al. (2008). Equation (29) is designated in our analysis as the benchmark model given its widespread use in empirical studies (Huang et al., 2009 and Das et al., 2009). However, equation (29) is not a complete model of CDS pricing, in particular Duffie (1999) shows that it is valid only if the contract is free from counterparty risk.

3.2 CDS Pricing with counterparty risk

In this section we extend the benchmark CDS pricing model described above, by explicitly introducing counterparty risk. Although some articles already model the counterparty risk in CDS pricing (Hull and White, 2001, Leung and Kwok, 2005 and Crépey et al., 2010), these models are not tested empirically. This is mainly because the individual transaction data of CDS are not generally available. However, the implementation of counterparty risk in CDS pricing is not completely impossible if the data is not available. We focus on not the counterparty risk in individual contracts but the "average" counterparty risk implied in the daily final prices of CDS. As we show in section 4, we take the TED spread[6] as a proxy for the average counterparty risk of the financial institutions, which are the dealers of CDS contracts.

To analyze the counterparty risk in CDS, we define the time of default of the counterparty using the second component of (4). First we denote stopping time τ_2 as the time of default of counter party in CDS contract. Then its intensity process is defined as

$$\lambda_t^{(2)} = \tilde{f}\left(\tilde{Z}_t\right) \tag{30}$$

$$d\tilde{Z}_t = \tilde{c}(\tilde{Z}_t)dt + \tilde{g}(\tilde{Z}_t)dW_t^{(5)} . \tag{31}$$

In contrast to $\lambda_t^{(1)}$, $\lambda_t^{(2)}$ depend only on $\tilde{Z}_t$. As we model the average counterparty risk, we do not consider the influence of the stock price on the credit risk of the counterparty.

[6] TED spread is the abbreviation of ***T***resury and ***E***uro ***D***ollar spread. It is originally defined as the spread between 3 months US Treasury bill and 3 months euro dollar rate. Now it is generally used to denote the spread between interbank interest rate and government bond yield. In fact Taylor and Williams (2009) defined the spread between OIS(over night index swap) of FF (Federal Fund) rate and 3 month LIBOR as TED spread.

There are three types of approaches to study the impact of the counterparty risk on CDS spread. First Hull and White (2001) consider three cases, default of the reference entity, that of the protection seller and no default. They focus on the credit event of the reference entity and the cancelation of the contract on the seller's default. Hull and White (2001) do not, however, model the simultaneous default of the reference entity and the counterparty explicitly. Second, Leung and Kwok (2005) analyze not only the impact of the default of the seller but also that of the buyer. Finally, Crépey et al. (2010) analyze not only the impact of the seller's default but also the impact of the simultaneous default of the reference entity and the seller.

Leung and Kwok (2005) show that the buyer's default does not influence the CDS spread if the buyer's default is not highly correlated with the sellers and the reference entity. This allows us to focus on the default of the reference entity and that of the seller, as well as their simultaneous default, omitting the buyer's default possibility.

There are four possible types of pay off in the CDS contract with the counterparty risk of protection sellers.

1. The reference entity defaults earlier than the expiration of the contract and the default of the counterparty ($\tau_1 < T$, $\tau_1 < \tau_2$). The payment of the premium will terminate at τ_1 and the counterparty will compensate the loss by the reference entity's default, l_1 of the notional amount.

2. The counterparty defaults earlier than the expiration of the contract and the default of the reference entity ($\tau_2 < T$, $\tau_2 < \tau_1$). The contract will be cancelled at τ_2. It is normally necessary to settle the net value of the contract, which is equal to the reconstruction cost of a new contract on the same reference entity, in the cancellation. However, the protection seller can settle some proportion of the net value, $1 - l_2$.

3. Neither the reference entity nor the counterparty defaults earlier than the expiration of the contract ($T < \tau_1$, $T < \tau_2$). The protection buyer pays the premium until the expiration of the contract.

4. Both the reference entity and the counterparty default simultaneously before the expiration of the CDS contract ($\tau_1 = \tau_2 < T$). The payment of the premium will

stop at τ_2 and the protection buyer can receive a certain proportion of loss by the default of the reference entity, $l_1(1-l_2)$.

We model cases 1, 3 and 4. The loss in case 2 is negligible as the net value of the contract is regularly cleared to minimize the uncovered exposure between the buyer and the seller[7]. Case 4 is important for the valuation of CDS because the buyer fails to recover the loss. The simultaneous default is seemingly rare. However, it is interpreted as the default of the counterparty prior to the CDS settlement arising from the reference entity's default.

In this framework, the premium leg incorporating counterparty risk, $C_C^{buy}(t,T)$ is

$$\begin{aligned} C_C^{buy}(t,T) &= E\left[\sum_{m=1}^{M}\exp\left(-\int_t^{T_m} r_s ds\right) 1_{(\min(\tau_1,\tau_2)>T_m)} CDS_C(t,T) \middle| \mathcal{G}_t\right] \\ &= CDS_C(t,T)\sum_{m=1}^{M} E\left[\exp\left(-\int_t^{T_m}\left(r_s+\lambda_s^{(1)}+\lambda_s^{(2)}\right) ds\right)\middle| \mathcal{F}_t\right] \qquad (32) \end{aligned}$$

The default leg incorporating counterparty risk, $C_C^{sell}(t,T)$ is

$$\begin{aligned} C_C^{sell}(t,T) &= E\left[\exp\left(-\int_t^{\tau_1\vee\tau_2} r_s ds\right)\left(1_{(\tau_1\le T,\,\tau_1<\tau_2)} l_1 + 1_{(\tau_1=\tau_2,\,\tau_1<T)} l_1(1-l_2)\right)\middle| \mathcal{G}_t\right] \\ &= E\left[\exp\left(-\int_t^{\tau_1\vee\tau_2} r_s ds\right)\left(1_{(\tau_1\le T,\,\tau_1<\tau_2)} l_1 + 1_{(\tau_1-\tau_2<\gamma,\,\tau_1<T)} l_1(1-l_2)\right)\middle| \mathcal{G}_t\right] \\ &= E\left[l_1\int_t^T \exp\left(-\int_t^s r_u du\right) d\left(F_1(s)(1-F_2(s))\right)\middle| \mathcal{F}_t\right] \\ &+ E\left[\int_t^T \exp\left(-\int_t^s r_u du\right) l_1(1-l_2) d((F(s)-F(s-\gamma))\left(1-F(s-\gamma)\right))\middle| \mathcal{F}_t\right] \\ &= E\left[l_1\int_t^T \exp\left(-\int_t^s r_u du\right)\Big(S_1(s)+S_2(s)+S_3(s)\Big) ds\middle| \mathcal{F}_t\right] \\ &+ E\left[l_1(1-l_2)\int_t^T \exp\left(-\int_t^s r_u du\right)\Big(S_4(s)+S_5(s)+S_6(s)\Big) ds\middle| \mathcal{F}_t\right] \qquad (33) \end{aligned}$$

where $F(t)$ is the joint probability distribution function that τ_1 and τ_2 are less than t, $F_1(t)$ and $F_2(t)$ are the marginal distribution function of τ_1 and τ_2, γ is the time between

[7]This is a reasonable approximation as the majority of the CDS are contracts between financial institutions. In fact Sorkin (2009) shows that AIG requested government support because AIG had to fund money not for the settlement of the credit derivatives but for the additional collateral of the credit derivatives.

a credit event and the auction of the CDS (on average two weeks), and

$$\begin{aligned}
S_1(s) &= \lambda_s^1 \exp\left(-\int_0^s \lambda_u^{(1)} du\right), && (34)\\
S_2(s) &= \lambda_s^1 \exp\left(-\int_0^s \lambda_u^{(1)} du\right) \int_t^s \lambda_u^{(2)} \exp\left(-\int_0^u \lambda_q^{(2)} dq\right) du\,, && (35)\\
S_3(s) &= \lambda_s^{(2)} \exp\left(-\int_0^s \lambda_u^{(2)} du\right) \int_t^s \lambda_u^{(1)} \exp\left(-\int_0^u \lambda_q^{(1)} dq\right) du\,, && (36)\\
S_4(s) &= \left(\lambda_s^{(1)} + \lambda_s^{(2)}\right)\left(\exp\left(-\int_{s-\gamma}^s \left(\lambda_u^{(1)} + \lambda_u^{(2)}\right) du\right)\right), && (37)\\
S_5(s) &= \left(\lambda_s^{(1)} + \lambda_s^{(2)}\right)\left(\exp\left(-\int_{s-\gamma}^s \left(\lambda_u^{(1)} + \lambda_u^{(2)}\right) du\right)\right)\\
&\quad \times \left(\int_t^{s-\gamma} \left(\lambda_u^{(1)} + \lambda_u^{(2)}\right) \exp\left(-\int_0^u \left(\lambda_q^{(1)} + \lambda_q^{(2)}\right) dq\right) du\right), && (38)\\
S_6(s) &= \left(\left(\lambda_s^{(1)} + \lambda_s^{(2)}\right) \exp\left(-\int_0^s \left(\lambda_u^{(1)} + \lambda_u^{(2)}\right) du\right)\right)\\
&\quad \times \left(\int_t^s \left(\lambda_u^{(1)} + \lambda_u^{(2)}\right) \exp\left(-\int_{u-\gamma}^u \left(\lambda_q^{(1)} + \lambda_q^{(2)}\right) dq\right) du\right). && (39)
\end{aligned}$$

Finally the fair CDS spread under counterparty risk is

$$CDS_C(t,T) = \frac{C_C^{sell}(t,T)}{\sum_{m=1}^{M} E\left[\exp\left(-\int_t^{T_m} (r_s + \lambda_s^{(1)} + \lambda_s^{(2)})\, ds\right) \middle| \mathcal{F}_t\right]} \qquad (40)$$

where l_1 and l_2, are respectively the LGD of the reference entity and the counterparty which appear in (33) . We omit the accrued premium on default following Brigo and Chourdakis (2009). Equation (40) implies that higher counterparty risk decreases the CDS spread. This intuitively means that a guarantee by a less credible entity is less valuable and reliable.

4 Data and Estimation Method

We now have the theoretical framework to analyze the cause of the relative stability in the UK bank CDS spread shown in figure 1. As discussed above it is conceptually possible to divide the CDS spread into PD and LGD. These risk factors are interpreted as the "likelihood" of default and the "depth" of the loss on default respectively. Our model can identify the impact of the individual risk factors. Therefore, even if the CDS spreads of two companies are close, the credit risk implications may be very different.

4.1 Data

The sample period, July 2008-December 2009, contains Lehamn Brothers' bankruptcy, the subsequent financial crisis and the bank bailouts by the UK government. The credit derivatives markets have been exposed to the most challenging and volatile conditions during this period.

We calibrate our stock option pricing model and CDS model to actual data of four major UK banks (HSBC Holdings, Barclays Bank, Lloyds Banking Group and Royal Bank of Scotland Group) and three major UK non financial sector companies (BP, BT and Tesco) over the period of 1 July 2008-31 December 2009. The sources of data we use are:

- Daily stock price data are obtained from Datastream.
- Daily listed stock option price data are also obtained from Datastream. The data contains options of all expiration dates and strike prices which were traded on Euroclear over the sample period.
- The term structure of the funding rate is derived from 3 months LIBOR and interest rate futures traded in Euroclear.
- Daily CDS spreads are obtained from Datastream.

We estimate the PD over one year horizon using the stock option data. However, options with a maturity of exactly one year are not always available as Euroclear operates a three month expiring cycle. Hence we employ call and put options with maturities remaining from 6 months to 18 months. Therefore, we effectively estimate the average of the PD between 6 months and 18 months. The listed options are American style and we convert the prices into European style by extracting the early exercise premium following the methodology of Bayraktar and Yang (2010) and Carr and Wu (2010).

We calculate the implied LGD for the one year CDS using the PD obtained from stock options as the maturity of the CDS is consistent with the maturity of the listed stock options. Moreover, as figure 5 demonstrates, the term structures of CDS of the UK banks and UK companies were upward sloping in the third quarter of 2008 and the second half of 2009 whilst they were downward sloping (inverted) in the fourth quarter of 2008 and the first half of 2009. The inverted nature of term structure during the peak of the financial crisis illustrates that the one year CDS spread is a more sensitive measure. This implies that one year CDS can reflect the credit risk more sensitively. We focus on the CDS of senior debts to compare the impacts of the financial crisis and the government's bailout on the credit risk of the banks to those to the non financial companies in the same seniority.

4.1.1 The Term Structure of the Funding Rate

As discussed in section 2.1, the risk free interest rate is modeled as in Hull and White (1993). Moreover, we model the term structure of the compound interbank funding rate and not the term structure of spot rates such as government bond yield or swap rate. The reason for this is that the term of the funding rate in the replicating portfolio of listed derivatives is short regardless of the maturity of the derivative due to daily settlement (see Black (1976)). That is the value of the listed derivatives is zero after the daily settlement[8] although the price of these derivatives is not zero. Under these settlement conditions, the funding rate is the continuous roll of short term interest rate from current date t to maturity date T.

To construct the term structure of the compound interest rate, we employ the prices of interest rate futures instead of the spot rate. In the interest futures market, the futures of 3 month LIBOR are available in 3 month interval until December 2015 (as at December 2009). Under the expectation hypothesis of the term structure one would expect that the implied forward rates generated by the spot rate are equivalent to the term structure of the compound short rate, i.e. the forward rate, $r(t, s, T)$ at time t between s and T ($s \geq t$) would be equal to the compound short rate,

$$\int_t^T r_s \, ds = r(t, t, T)(= r(t, T)). \tag{41}$$

However, as Campbell et al. (1997) surveyed the expectation hypothesis (41) rarely satisfies actual data. As a result, the implied forward rate cannot be used as the funding rate process in listed option pricing. In contrast, the term structure based on interest

[8] Black (1976) pointed out the funding term is 1day. However, the settlement is not always executed every business day in practice. If the net value of the derivatives are less than margin call rate (usually 5% or 10%), it is not cleared to save the cost of settlement.

futures is not only theoretically consistent with listed option pricing models and can be used to determine the funding rate in listed option pricing but also it is a relatively unbiased estimator of the expectation of the future term structure.

4.2 The Estimation Methodology

The estimation of the parameters involves three steps. First we calculate the first term of (26). Secondly, we calibrate the volatility surface and estimate the implied PD. Finally, we calculate the implied LGD from CDS spreads based on the implied PDs obtained from the second step.

In the first step, we estimate the mean value of the volatility of the underlying stock, $\bar{\sigma}_1$, and the funding rate, $\bar{\sigma}_0$ using the past 20 business days (approximately 1 month) data. Next we calibrate the term structure model (13) to the actual term structure of the funding interest rate. As shown in Brigo and Mercurio (2006), the Hull and White (1993) model can always fit the currently observed term structure regardless of the parameters. Therefore, we calibrate the model not to the spot rate curve but to the forward rate curve. Then we can calculate the first term of (26)

In the second step, the parameters of (26) are calibrated with actual daily data. There are 10 unknown parameters, (V_1^ϵ, V_2^ϵ, V_3^ϵ, V_4^ϵ, V_5^ϵ, V_6^ϵ, V_7^ϵ, V_1^δ, V_2^δ, $\bar{\lambda}^{(1)}$) to estimate in equation (26). In this procedure we minimize the weighted mean squared error to avoid the bias caused by the difference of the number of options, $N_{i,t}$, at individual maturities.

1. First, we estimate parameters $V(= (V_1^\epsilon, V_2^\epsilon, V_3^\epsilon, V_4^\epsilon, V_5^\epsilon, V_6^\epsilon, V_7^\epsilon, V_1^\delta, V_2^\delta))$ with $\bar{\lambda}_{t-1}^{(1)}$ [9]

$$\bar{V} = \arg\min_V \sum_i \sqrt{\sum_j (P(t, T_i, K_{i,j}) - \tilde{P}_{\epsilon,\delta}(T_i, K_{i,j}, \bar{\lambda}_{t-1}^{(1)}, V))^2 / N_{i,t}} \quad (42)$$

   where $N_{i,t}$ is the number of the maturity T_i options traded at t.

[9] The initial value of the probability of default, $\bar{\lambda}_0^1$, is calculated from equation (29) under 60% LGD. To eliminate the dependency on the initial value we ignore the first 20 days results.

2. Second, we estimate PD with the parameters $\bar{V}$ from (42),

$$\bar{\lambda}_t^{(1)} = \arg\min_{\lambda_t^1} \sum_i \sqrt{\sum_j (P(t,T_i,K_{i,j}) - \tilde{P}_{\epsilon,\delta}(T_i,K_{i,j},\lambda_t^{(1)},\bar{V}))^2/N_{i,t}} \tag{43}$$

The third step is the estimation of the LGD implied in CDS by using the implied PD from listed stock option. As a proxy for the "average" counterparty risk we take the UK TED spread, which is an indicator for the funding condition of financial institutions and the degree of risk aversion by investors. The only free parameter in (40), l_1 is calibrated with daily actual CDS spread by the minimization of the error function,

$$\hat{l}_1 = \arg\min_{l_1} \sqrt{(CDS(t,T) - CDS_C(t,T,l_1,\hat{l}_2,\hat{\lambda}_t^{(1)},\hat{\lambda}_t^{(2)}))^2}\,, \tag{44}$$

where

$$\hat{\lambda}_t^{(1)} = \bar{\lambda}_t^{(1)} + CDS^*(t,T) \tag{45}$$

$$r_t^* = r_t - CDS^*(t,T) \tag{46}$$

and $CDS^*(t,T)$ is the CDS spread of the government, $\hat{\lambda}_t^{(2)}$ is calculated from the UK TED spread using (18) with $\hat{l}_2 = 60\%$ and r_t^* is the credit risk adjusted interest rate from the compound interest rate calculated from 3 months interest futures, r_t.

We modify the credit risk embedded in the funding rate r_t using (45) and (46). Our option pricing model with a defaultable stock assumes r_t to be a non-defaultable interest rate. However, the calculated funding rate, r_t obviously contains two kinds of credit risks, the credit risk of the government and that of financial institutions. Thus the estimated value $\bar{\lambda}_t^{(1)}$ in (43) is possibly biased.

We adjust the estimation result by accounting for the first type of credit risk, the credit risk of the government. No government is perfectly free from credit risk. The CDS spread of the UK government, which has been rated AAA, was non-zero even before the financial crisis. This implies that $\bar{\lambda}_t^{(1)}$ estimated in (43) can be lower than the true value $\lambda_t^{(1)}$ if the government bond yield is assumed to be the risk free interest rate. That is the calibration optimizes the level of the sum, $r_t + \bar{\lambda}_t^{(1)}$ regardless of the credit risk embedded in r_t. To modify the bias of the credit risk contained in the funding rate, we estimate the intensity of the Cox process, $\hat{\lambda}_t^{(1)}$ using the estimated value, $\bar{\lambda}_t^{(1)}$ and CDS spread of the

UK government, $CDS^*(t,T)$. Therefore, even if $\bar{\lambda}_t^{(1)}$ is zero, it implies that the credit risk does not influence the stock option prices explicitly and the implied PD of the company is identical to the CDS spread of the government.

However, we do not adjust for the credit risk of financial institutions because the TED spread, the spread between the government bond yield and the interbank interest rate represents frictional costs in the valuation of the credit risk of the reference entity. Clearly the swap rate and LIBOR are not free of credit risk, because they are the interest rates of the contracts between financial institutions. In fact, in any transaction such as the construction of replicating portfolio and risk hedging to hold securities as inventory, LIBOR and swap rate are the possible lowest funding rates for market participants. Therefore, the swap spread does not imply the credit risk of the underlying assets but the friction cost of market makers and dealers.

5 Estimation Results and their Implications

To make clear the implication of the PD implied in stock options ($\hat{\lambda}_t^{(1)}$) on the market valuation of credit risk, we compare it to the benchmark calculation of PD obtained from CDS spreads under assumption of a constant 60% LGD using (29). Under a constant LGD model, movement in the CDS spread is solely due to changes in PD. However, there is no guarantee that the LGD is constant and we do not make this assumption. Next we consider the implication of the LGD implied in the CDS. As we will demonstrate, the implied PD moved in a different direction to that of the CDS spreads, the implied LGD is far from constant and reflects the situation of the individual entity.

5.1 Probability of Default Implied in Listed Stock Options

It is clear from figure 6 and 7 that the implied PD of the UK banks were not much higher than non financial companies in the third quarter of 2008. Although the CDS spreads of the UK banks temporarily spiked following the Lehman Brothers bankruptcy on September 16 2008, the implied PD of the UK banks did not change significantly. As summarized in table 1, European banks such as Fortis and Bradford&Bingley were nationalized, and the US congress rejected TARP (the capital injection and problematic asset purchase plan) at that time. To ease the turmoil, the UK government implemented the capital injection measures and raised the limit of the bank deposit guarantee. As a consequence, the CDS spreads declined but stayed at a higher level than before Lehman's bankruptcy. As the implied PD did not respond significantly to the events in the global financial markets, the movement in the CDS spreads is due more to the general rise in risk aversion during this period and not due to specific company concerns.

In contrast there are significant differences among the implied PD of the UK banks in the fourth quarter of 2008 and the first quarter of 2009. The UK government provided LLOYDS and RBS with public money to strength the capital base at the beginning of October 2008. Therefore, from the middle of October 2008, the implied PD of LLOYDS rose and diverged from that obtained from the benchmark calculation. RBS shows similar behavior from the middle of December 2008. At the peak in March 2009, the implied PD of these banks reached around 70%, while the PD of these banks inferred from the benchmark calculation remained around 3%. In contrast, the implied PD of HSBC remained at almost the same level as that obtained from the benchmark calculation. Finally, the implied PD of Barclays demonstrates behavior between those of the "bailed out" banks and HSBC. The implied PD of Barclays peaked at around 25% at March 2009. To summarize, although the CDS spreads of the UK banks were very similar, the implied PD clearly indicates significant differences in the perceived credit risk of RBS and LLOYDS (and to a lesser extent, Barclays) and that of HSBC.

The timing of the rise as well as the level of the implied PD at the peak indicates the view of investors in the banks. We can interpret the movement of the implied PD as follows.

- The record high losses of LLOYDS in the forth quarter of 2008 is mainly due to the losses incurred by HBOS, which was taken over by LLOYDS. Although the UK government provided LLOYDS with public funds to strengthen the capital structure in October 2008, investors did not recognize it as sufficient to absorb the losses in HBOS. Even after the government announced the introduction of APS and additional capital injection, investors were not confident that LLOYDS could escape from nationalization or bankruptcy. In fact LLOYDS became a partly state owned bank.

- RBS also made record high losses in this period. In contrast to LLOYDS, it was mainly due to the mark down of securitized products and losses in ABN AMRO. Therefore, investors perceived that the bank was in danger of bankruptcy as asset prices, especially securitized product prices, fell rapidly at the end of the fourth quarter of 2008. Investors doubted the sustainability of RBS even under the APS and additional capital injection. RBS also became as partly state owned bank.

- Barclays is a different case. Whilst Barclays already announced the issue of preferred shares to a Qatar government fund in October 2008, investors were not sure that Barclays could absorb the potential loss with the existing capital structure.

Finally it can be seen from figure 6 that for RBS and LLOYDS (and Barclays) there is only a gradual decrease in the implied PD from its peak. In figure 8 it can be seen that the difference between the implied PD and the PD obtained from the benchmark calculation for RBS, LLOYDS and Barclays approaches that of HSBC (and non financial companies) by December 2009. These results imply that the actions of the UK government did not immediately result in the decrease of the implied PD of the banks. In fact the implied PD rose gradually following the first capital injection in October 2008, and rose significantly around the second capital injection in February and March 2009. Investors were unsure if these banks could absorb the potential significant losses even after the new capital was obtained from the government. This contrasts with the CDS spreads of LLOYDS and RBS. After the first capital injection in October 2008, the CDS spreads of RBS and LLOYDS were nearly as high as that of HSBC until the announcement of the Asset Protection Scheme (APS) in the fourth quarter of 2008 (see Hall (2009) for the details). At the time of the second capital injection and the acceptance of the APS the difference between the CDS spread of RBS and LLOYDS and that of HSBC was less than 100bp. These results demonstrate that, not only direct government support, but also the perceived ability of the banks to recover from the significant losses matters to investors and only then does the implied PD recover.

The implied PD of HSBC and the non financial sector companies contrast with those of Barclays, LLOYDS and RBS (see figure 7). Whilst the implied PD of these entities are generally higher than those of the benchmark calculation in the third quarter of 2008 and the second half of 2009, they are approximately equal to those obtained by the benchmark calculations in the fourth quarter of 2008 and the first half of 2009. It implies that additional factors contribute to their CDS spreads. Throughout the sample period of the analysis, their implied PD is greater than or equal to the PD obtained via the benchmark calculation, and the difference varies over time (figure 8). The difference with the benchmark calculation is at a minimum for the non financial companies between the fourth quarter of 2008 and first quarter of 2009. This cyclical movement indicates that the LGD implied in CDS can vary over time even if the reference entities are not subject to external credit support like RBS and LLOYDS.

5.2 Loss Given Default Implied in CDS spread

As discussed previously, the implied PD is used to determine the implied LGD. Figure 9 illustrates that the implied LGD of the UK banks increased temporarily after the Lehman Brothers bankruptcy followed by a rapid decrease (during September 2008). After September 2008 the implied LGD of UK banks (except HSBC) continued to decrease. In particular, the implied LGD reached minimum levels, less than 10% for RBS and LLOYDS, and 10-20% for Barclays after January 2009. The implied LGD of these banks remained at these low levels until the third capital injection to RBS and LLOYDS in the fourth quarter of 2009. The LGD of HSBC and the non financial companies (see figure 10) were negatively correlated with those of Barclays, LLOYDS and RBS, while the CDS spreads were positively correlated with each other during this period. These results are interpreted as follows.

The sharp rise in the implied LGD of the UK banks shortly after the Lehman Brothers bankruptcy implies that the rise in CDS spread in September 2008 was caused not by the specific credit risk in a single name, but rather a shift of investor risk preference due to the turmoil of financial market. In fact this is the period in which the UK TED spread showed significant turmoil (figure 4).

The low implied LGD of LLOYDS and RBS, subsequent to January 2009, is interpreted as the debt investors' expectation that these banks could reorganize their capital structure without loss to the debt investors. As discussed in section 5.1, the government failed to convince the equity investors of the solvency of these banks via the capital injections and the APS. However, the government could obtain the confidence of the debt investors that these banks could avoid bankruptcy and strengthen their capital base regardless of their solvency. This is mainly because the government took over a large proportion of the shares in the second capital injections and set the floor of the loss via ASP. In other words, the government succeeded in reducing the investors' uncertainty on the reorganization of these banks via the strong commitment as the shareholder as well as the regulator.

Debt investors are indifferent as to what extent and what particular scheme the UK government would adopt to reorganize RBS and LLOYDS if they are confident that the banks would not become bankrupt. The nationalization of banks may trigger a credit event. For example the determination committee of ISDA agreed that the nationalizations of Fannie May and Freddie Mac in September 2008 were credit events, however the final settlement of the CDS contracts resulted in 0-8% loss. In contrast, due to the difference in the legal status of the state control, Northern Rock did not trigger a credit event as a result of its control by the Bank of England in February 2008. Moreover, the support by the UK government of LLOYDS and RBS did not trigger a credit event as the UK government did not take over these banks completely after the third capital injection (the government holds 41% of LLOYDS and 70% of RBS after the capital injections in November 2009). This contrast to the bankruptcy of Lehman Brothers, in which the LGD of the CDS settled at 91%. Therefore, debt investors recognized that the risk of RBS and LLOYDS was quite limited and they were confident that the UK government would support both RBS and LLOYDS although they were not certain whether these banks could avoid triggering a credit event.

Finally the rise in the implied LGD of HSBC and the non financial companies, between the fourth quarter of 2008 and mid 2009, is a contributing factor to the rise in their CDS spreads. As the UK government did not introduce the support scheme for non financial companies (analogous to the General Motors bankruptcy scheme instigated by the US government in July 2009), the movement in the implied LGD is independent of the government actions in contrast to the implied LGD of the UK banks. In fact the movements of the implied LGD are consistent with empirical studies on the LGD of bonds and loans. Altman et al. (2005) show that the LGD of corporate bonds and the price of other distressed asset are sensitive to demand and supply. In fact, during the fourth quarter of 2008 and the first half of 2009, most financial asset prices declined. As Duffie et al. (2005, 2007) and Brunnermeier and Pedersen (2009) model, this is not only due to the fall in the expected future cash flow of the assets but also due to the decline in the demand through investor's risk aversion and the shortage in liquidity shown in the record TED spread (figure 4). That is, liquidity risk is still one of the potential risk factor in the LGD of CDS even in the cash settlement although we have not incorporated the impact of liquidity risk in our model. In addition Zhang (2009) points out that the LGD of loans is negatively correlated to business cycles. Therefore, our results imply that the LGD of CDS without specific government support can move consistently with that of loans and bonds. This is evidence that their CDS is, to some extent, consistent with the arbitrage pricing with the underlying bonds or loans.

6 Concluding Remarks

The recent financial crises had a devastating effect on the UK banking system, forcing the UK government's partial ownership of RBS and LLOYDS. However, the one year CDS spread levels for all the major UK banks showed little difference during July 2008-December 2009, either between each other or with non-financial companies. In order to explain this anomaly, we have determined the implied PD from a calibration of the implied volatility of listed stock options, careful to include an appropriate model for the funding term structure. The implied PD is then employed to derive an implied LGD from the quoted CDS spread using a model for CDS spreads that incorporates counterparty risk.

Our result explains the benign levels of the UK bank one year CDS spreads, during a period of intense market turmoil, as being due to the significant interplay between implied PD and LGD. In particular the LGD of RBS and LLOYDS dropped below 10% while their implied PD derived from the listed stock options peaked at around 70% in March 2009. This strongly indicates that the participants in the listed stock option markets saw significant risk in the erosion of existing shareholders equity, whilst debt investors were confident that the government would support any losses to bond holders. Therefore, although UK bank CDS spreads were relatively stable during this period except for the short term turmoil after Lehman Brothers bankruptcy, this does not imply the soundness of the UK banks, but rather low uncertainty on the reorganization scheme during the period of UK government intervention.

In contrast both the implied PD and LGD of the non-financial sector companies, and the non bail-out banks (HSBC), which could recapitalize without explicit public support, gradually rose in the fourth quarter of 2008 and the first half of 2009, following the Lehman Brothers bankruptcy. This is consistent with empirical studies on the LGD of bonds and loans.

We also run a standard arbitrage-free pricing model of CDS with no counterpary risk for comparison as a benchmark case. Our results confirm that counterparty risk adds to the LGD of the reference entities, however this effect is not large as compared to the benchmark case. Remarkably, the implied LGD remains unstable in time and is often below 60% for most of the companies in our sample.

We end this section by providing an answer to the question posed in the title of the paper. CDS spreads do reflect default risks, provided careful consideration is given to the variability in LGD. The UK banks that were in serious financial difficulties during the crisis and were ultimately bailed out by the UK government (RBS and LLOYDS), had one year CDS spread levels comparable to those of non-financial companies (and financially sound banks) throughout the crisis. Naively assuming a constant LGD would therefore suggest that their PD was close to that of the sound companies: clearly an inappropriate interpretation at the peak of the crisis. In fact, the CDS levels for RBS and LLOYDS masked the dramatic interplay between the substantial increase in their implied PD and the compensating reduction in their LGD. This is the key result of this article.

A The Derivation of Equation (26)

A.1 Feynman - Kac formula for option pricing model

Here we denote the price of contingent claim of defaultable asset $h(x)$ as $P_{\epsilon,\delta}$ to emphasize the parameters, ϵ and δ. From the Feynman-Kac formula (See Karatzas and Shreve (1988) for detail), $P_{\epsilon,\delta}$ is a solution of

$$\begin{aligned} \mathcal{L}^{\epsilon,\delta} P_{\epsilon,\delta}(t,x,r,y,\tilde{y},z) &= 0\,, && (47) \\ P_{\epsilon,\delta}(t,x,r,y,\tilde{y},z) &= h(x)\,, && (48) \end{aligned}$$

where the operator $\mathcal{L}^{\epsilon,\delta}$ is defined as

$$\mathcal{L}^{\epsilon,\delta} = \frac{1}{\epsilon}\mathcal{L}_0 + \frac{1}{\sqrt{\epsilon}}\mathcal{L}_1 + \mathcal{L}_2 + \sqrt{\delta}\mathcal{M}_1 + \delta\mathcal{M}_2 + \frac{\sqrt{\delta}}{\sqrt{\epsilon}}\mathcal{M}_3\,, \tag{49}$$

in which respective operators are listed below,

$$\begin{aligned} \mathcal{L}_0 &= \nu^2\frac{\partial^2}{\partial y^2} + (m-y)\frac{\partial}{\partial y} + \tilde{\nu}^2\frac{\partial^2}{\partial \tilde{y}^2} + (\tilde{m}-\tilde{y})\frac{\partial}{\partial \tilde{y}} + 2\rho_{24}\nu\tilde{\nu}\frac{\partial^2}{\partial y\partial\tilde{y}}, \\ \mathcal{L}_1 &= \rho_2\sigma(\tilde{y})\nu\sqrt{2}x\frac{\partial^2}{\partial x\partial y} + \rho_{12}\eta\nu\sqrt{2}x\frac{\partial^2}{\partial r\partial y} + \rho_4\sigma(\tilde{y})\tilde{\nu}\sqrt{2}x\frac{\partial^2}{\partial x\partial\tilde{y}} \\ &\quad +\rho_{14}\eta\tilde{\nu}\sqrt{2}\frac{\partial^2}{\partial\tilde{y}\partial r} - \Lambda(\tilde{y})\tilde{\nu}\sqrt{2}\frac{\partial}{\partial\tilde{y}}, \\ \mathcal{L}_2 &= \frac{\partial}{\partial t} + \frac{1}{2}\sigma^2(\tilde{y})x^2\frac{\partial^2}{\partial x^2} + (r+f(y,z))x\frac{\partial}{\partial x} + (\alpha_t - \beta r)\frac{\partial}{\partial r} + \sigma(\tilde{y})\eta\rho_1 x\frac{\partial^2}{\partial x\partial r} \\ &\quad +\frac{1}{2}\eta^2\frac{\partial^2}{\partial r^2} - (r+f(y,z))\,, \\ \mathcal{M}_1 &= \sigma(\tilde{y})\rho_3 g(z)x\frac{\partial^2}{\partial x\partial z} + \eta\rho_{13}g(z)\frac{\partial^2}{\partial r\partial z}, \\ \mathcal{M}_2 &= c(z)\frac{\partial}{\partial z} + \frac{1}{2}g^2(z)\frac{\partial^2}{\partial z^2}, \\ \mathcal{M}_3 &= \rho_{23}\nu\sqrt{2}g(z)\frac{\partial^2}{\partial y\partial z} + \rho_{34}\tilde{\nu}\sqrt{2}g(z)\frac{\partial^2}{\partial\tilde{y}\partial z}. \end{aligned}$$

A.2 Asymptotic Expansions

We use an asymptotic expansion for the approximation of $P_{\epsilon,\delta}$ (see Bayraktar and Yang (2010)), as both $\sqrt{\epsilon}$ and $\sqrt{\delta} \to 0$, to derive an expansion of $P_{\epsilon,\delta}$ in powers of $\sqrt{\delta}$ and $\sqrt{\epsilon}$ respectively. First we expand $P_{\epsilon,\delta}$ in powers of $\sqrt{\delta}$

$$P_{\epsilon,\delta} = P_0 + \sqrt{\delta}P_{0,1} + \delta P_{0,2} + \cdots \tag{50}$$

By substituting (50) into (47),

$$\begin{aligned} \mathcal{L}^{\epsilon,\delta}P_{\epsilon,\delta} &= \left(\frac{1}{\epsilon}\mathcal{L}_0 P_{\epsilon,0} + \frac{1}{\sqrt{\epsilon}}\mathcal{L}_1 P_{\epsilon,0} + \mathcal{L}_2 P_{\epsilon,0}\right) \\ &\quad +\sqrt{\delta}\left(\frac{1}{\sqrt{\epsilon}}\mathcal{L}_0 P_{\epsilon,1} + \frac{1}{\sqrt{\epsilon}}\mathcal{L}_1 P_{\epsilon,1} + \mathcal{L}_2 P_{\epsilon,1} + \mathcal{M}_1 P_{\epsilon,0} + \frac{1}{\sqrt{\epsilon}}\mathcal{M}_3 P_{\epsilon,0}\right) \\ &\quad +\left(\sqrt{\delta}\right)^2\left(\mathcal{L}_2 P_{\epsilon,2} + \mathcal{M}_2 P_{\epsilon,0} + \mathcal{M}_2 P_{\epsilon,0} + \frac{1}{\sqrt{\epsilon}}\mathcal{L}_1 P_{\epsilon,2} + \frac{1}{\epsilon}\mathcal{L}_0 P_{\epsilon,2}\right) + \cdots \\ &= 0 \end{aligned} \tag{51}$$

Since $\mathcal{L}^{\epsilon,\delta}P_{\epsilon,\delta}=0$, the first term of (51) is

$$\left(\frac{1}{\epsilon}\mathcal{L}_0+\frac{1}{\sqrt{\epsilon}}\mathcal{L}_1+\mathcal{L}_2\right)P_{\epsilon,0}=0 \tag{52}$$

and given $\sqrt{\delta}\neq 0$, the second term is also zero,

$$\left(\frac{1}{\sqrt{\epsilon}}\mathcal{L}_0+\mathcal{L}_1+\mathcal{L}_2\right)P_{\epsilon,1}=-\left(\mathcal{M}_1+\frac{1}{\sqrt{\epsilon}}\mathcal{M}_3\right)P_{\epsilon,0}\,. \tag{53}$$

Secondly, we expand the solution of (52) and (53) in powers of $\sqrt{\epsilon}$,

$$P_{\epsilon,0} = P_0+\sqrt{\epsilon}P_{1,0}+\left(\sqrt{\epsilon}\right)^2P_{2,0}+\left(\sqrt{\epsilon}\right)^3P_{3,0}+\cdots \tag{54}$$

$$P_{\epsilon,1} = P_{0,1}+\sqrt{\epsilon}P_{1,1}+\left(\sqrt{\epsilon}\right)^2P_{2,1}+\left(\sqrt{\epsilon}\right)^3P_{3,1}+\cdots \tag{55}$$

Inserting (54) into (52), we obtain

$$\begin{aligned}&\left(\sqrt{\epsilon}\right)^{-2}\mathcal{L}_0P_0+\left(\sqrt{\epsilon}\right)^{-1}\left(\mathcal{L}_0P_{1,0}+\mathcal{L}_1P_0\right)+\left(\mathcal{L}_0P_{2,0}+\mathcal{L}_1P_{1,0}+\mathcal{L}_2P_0\right)\\&+\sqrt{\epsilon}\left(\mathcal{L}_0P_{3,0}+\mathcal{L}_1P_{2,0}+\mathcal{L}_2P_{1,0}\right)+\left(\sqrt{\epsilon}\right)^2\left(\mathcal{L}_1P_{3,0}+\mathcal{L}_2P_{2,0}\right)+\cdots\\&=0\,.\end{aligned} \tag{56}$$

Since $\sqrt{\epsilon}\neq 0$, the first term of the equation (56) implies that P_0 is independent of y and $\tilde{y}$. As a result $P_{1,0}$ is also independent of y and $\tilde{y}$ ($\mathcal{L}_0P_{1,0}=0$) from the second term and the independence of P_0 on y and $\tilde{y}$ ($\mathcal{L}_1P_0=0$).

Next we can reduce the third term using the independence of $P_{1,0}$ on y and $\tilde{y}$ ($\mathcal{L}_1P_{1,0}=0$),

$$\mathcal{L}_0P_{2,0}+\mathcal{L}_2P_0=0\,. \tag{57}$$

This is a Poisson equation for $P_{2,0}$ (see Fouque et al. (2000)). The solvability of the equation requires the centering condition,

$$\langle\mathcal{L}_2\rangle P_0=0\,, \tag{58}$$

where $\langle\cdot\rangle$ denotes the average with respect to the invariant distribution of $(Y_t,\tilde{Y}_t)$, which have joint density

$$\Psi\left(y,\tilde{y}\right)=\frac{1}{2\pi\nu\tilde{\nu}}\exp\left(-\frac{1}{2\left(1-\rho_{24}^2\right)}\left[\left(\frac{y-m}{\nu}\right)^2+\left(\frac{\tilde{y}-\tilde{m}}{\tilde{\nu}}\right)^2-2\rho_{24}\frac{\left(y-m\right)\left(\tilde{y}-\tilde{m}\right)}{\nu\tilde{\nu}}\right]\right)\,. \tag{59}$$

Let us denote the averaging of $\mathcal{L}_2$ as

$$\begin{aligned}\langle \mathcal{L}_2 \rangle &= \frac{\partial}{\partial t} + \frac{1}{2}\bar{\sigma}_2(\tilde{y})x^2\frac{\partial^2}{\partial x^2} + (r + \bar{\lambda}^{(1)}(z))x\frac{\partial}{\partial x} + (\bar{\alpha} - \beta r)\frac{\partial}{\partial r} \\ &\quad + \bar{\sigma}_1(\tilde{y})\eta\rho_1 x\frac{\partial^2}{\partial x \partial r} - (r + \bar{\lambda}^{(1)}(z)), \end{aligned} \tag{60}$$

where $\bar{\sigma}_1 = \langle \sigma(\tilde{y}) \rangle$, $\bar{\sigma}_2 = \langle \sigma^2(\tilde{y}) \rangle$, $\bar{\lambda}^{(1)}(z) = \langle f(y,z) \rangle$ and $\bar{\alpha} = \langle \alpha_t \rangle$. Under the terminal condition

$$P_0(T, x, r, z) = h(x), \tag{61}$$

equation (58) defines the leading order term P_0. Using (57) we can also define the solution of the Poisson equation,

$$P_{2,0} = -\mathcal{L}_0^{-1}(\mathcal{L}_2 - \langle \mathcal{L}_2 \rangle)P_0 . \tag{62}$$

Finally the fourth term of (56), term of $\sqrt{\epsilon}$ yields a Poisson equation

$$\mathcal{L}_0 P_{3,0} + \mathcal{L}_1 P_{2,0} + \mathcal{L}_2 P_{1,0} = 0 . \tag{63}$$

For the complete identification of $P_{1,0}$, the solvability of this equation requires

$$\langle \mathcal{L}_2 P_{1,0} \rangle = -\langle \mathcal{L}_1 P_{2,0} \rangle = \left\langle \mathcal{L}_1 \mathcal{L}_0^{-1}(\mathcal{L}_2 - \langle \mathcal{L}_2 \rangle) \right\rangle P_0 \tag{64}$$

under the terminal condition

$$P_{1,0}(T, x, r, z) = 0 . \tag{65}$$

Next, we will express the centoring condition (64) more explicitly. Using (60), we can rewrite $\mathcal{L}_0 P_{2,0}$ as

$$\begin{aligned}(\mathcal{L}_2 - \langle \mathcal{L}_2 \rangle)P_0 &= \frac{1}{2}\left(\sigma^2(\tilde{y}) - \bar{\sigma}_2\right)x^2\frac{\partial^2 P_0}{\partial x^2} + (\sigma(\tilde{y}) - \bar{\sigma}_1)\eta\rho_1 x\frac{\partial^2 P_0}{\partial x \partial r} \\ &\quad + \left(f(y,z) - \bar{\lambda}^{(1)}(z)\right)\left(x\frac{\partial P_0}{\partial x} - P_0\right) + (\alpha_t - \bar{\alpha})\frac{\partial P_0}{\partial r} . \end{aligned} \tag{66}$$

This is identical to equation (3.20) of Bayraktar and Yang (2010) except for the last term. Therefore,

$$\begin{aligned}\mathcal{L}_0^{-1}(\mathcal{L}_2 - \langle \mathcal{L}_2 \rangle)P_0 &= \frac{1}{2}\kappa(y,\tilde{y})x^2\frac{\partial^2 P_0}{\partial x^2} + \psi(y,\tilde{y})\eta\rho_1 x\frac{\partial^2 P_0}{\partial x \partial r} \\ &\quad + \phi(y,\tilde{y},z)\left(x\frac{\partial P_0}{\partial x} - P_0\right) + \varsigma(y,\tilde{y},r)\frac{\partial P_0}{\partial r} \end{aligned} \tag{67}$$

where ψ, κ, ϕ and ς are the solutions to the Poisson equations

$$\begin{aligned}
\mathcal{L}_0\psi(\tilde{y}) &= \sigma(\tilde{y}) - \bar{\sigma}_1, && (68)\\
\mathcal{L}_0\kappa(\tilde{y}) &= \sigma^2(\tilde{y}) - \bar{\sigma}_2, && (69)\\
\mathcal{L}_0\phi(y,z) &= \left(f(y,z) - \bar{\lambda}^{(1)}(z)\right), && (70)\\
\mathcal{L}_0\varsigma\,(y,\tilde{y},r) &= \alpha_t - \bar{\alpha}. && (71)
\end{aligned}$$

Applying the differential operator $\mathcal{L}_1$ to the last expression, we can calculate $P_{1,0}$ explicitly,

$$\begin{aligned}
\left\langle \mathcal{L}_1\mathcal{L}_0^{-1}(\mathcal{L}_2 - \langle\mathcal{L}_2\rangle)\right\rangle P_0 &= \sqrt{2}\left(\rho_2\nu\left\langle\sigma\phi_y\right\rangle(z) - \frac{1}{2}\tilde{\nu}\left\langle\Lambda\kappa_{\tilde{y}}\right\rangle\right)x^2\frac{\partial^2 P_0}{\partial x^2}\\
&+ \sqrt{2}\left(\rho_{12}\eta\nu\left\langle\phi_y\right\rangle(z) + \tilde{\nu}\left\langle\Lambda\varsigma_{\tilde{y}}\right\rangle\right)\frac{\partial}{\partial r}\left(x\frac{\partial P_0}{\partial x} - P_0\right)\\
&+ \frac{\sqrt{2}}{2}\rho_4\tilde{\nu}\left\langle\sigma\kappa_{\tilde{y}}\right\rangle x\frac{\partial}{\partial x}\left(x^2\frac{\partial^2 P_0}{\partial x^2}\right) + \sqrt{2}\rho_1\rho_4\tilde{\nu}\eta\left\langle\sigma\psi_{\tilde{y}}\right\rangle x\frac{\partial}{\partial x}\left(x\frac{\partial^2 P_0}{\partial x\partial r}\right)\\
&+ \frac{\sqrt{2}}{2}\rho_{14}\eta\tilde{\nu}\left\langle\kappa_{\tilde{y}}\right\rangle\frac{\partial}{\partial r}\left(x^2\frac{\partial^2 P_0}{\partial x^2}\right) + \sqrt{2}\rho_1\rho_{14}\eta^2\tilde{\nu}\left\langle\psi_{\tilde{y}}\right\rangle\frac{\partial}{\partial r}\left(x\frac{\partial^2 P_0}{\partial x\partial r}\right)\\
&+ \sqrt{2}\left(\rho_2\nu\left\langle\sigma\varsigma_y\right\rangle + \rho_4\tilde{\nu}\left\langle\sigma\varsigma_{\tilde{y}}\right\rangle - \tilde{\nu}\left\langle\Lambda\psi_{\tilde{y}}\right\rangle - \tilde{\nu}\left\langle\Lambda\varsigma_{\tilde{y}}\right\rangle\right)x\frac{\partial^2 P_0}{\partial x\partial r}\\
&+ \sqrt{2}\left(\rho_{12}\eta\nu\left\langle\varsigma_y\right\rangle + \rho_{14}\eta\tilde{\nu}\left\langle\varsigma_{\tilde{y}}\right\rangle\right)\frac{\partial^2 P_0}{\partial r^2} \qquad (72)
\end{aligned}$$

It is possible to get the explicit expression of P_1^ϵ by inserting (55) into (53). Discussed above, $P_{0,1}$ is independent of y and $\tilde{y}$ and satisfies

$$\langle\mathcal{L}_2\rangle = -\langle\mathcal{M}_1\rangle P_0,\;\; P_{0,1}(T,x,r;z) = 0 \qquad (73)$$

Using the proposition 3.2, 3.3 and Remark 3.1 in Bayraktar and Yang (2010), the first order expansions on $\sqrt{\epsilon}$ and $\sqrt{\delta}$ are derived respectively

$$\begin{aligned}
\sqrt{\epsilon}P_{1,0} &= -(T-t)\left(V_1^\epsilon(z)x^2\frac{\partial^2 P_0}{\partial x^2} + V_2^\epsilon x\frac{\partial}{\partial x}\left(\frac{\partial^2 P_0}{\partial x^2}\right)\right)\\
&+ V_3^\epsilon(z)\left(-x\frac{\partial^2 P_0}{\partial x\partial\alpha} - \frac{\partial P_0}{\partial\alpha}\right) + V_4^\epsilon x^2\frac{\partial^3 P_0}{\partial x^2\partial\alpha} + V_5^\epsilon x\frac{\partial^2 P_0}{\partial\eta\partial x} + V_6^\epsilon x\frac{\partial^2 P_0}{\partial x\partial\alpha}\\
&+ V_7^\epsilon\frac{\partial^2 P_0}{\partial r^2} \qquad (74)\\
\sqrt{\delta}P_{0,1} &= V_1^\delta(z)\frac{(T-t)^2}{2}\left(x^2\frac{\partial^2 P_0}{\partial x^2}\right) + V_2^\delta\left[\left(x\frac{\partial^2 P_0}{\partial\alpha\partial x} - \frac{\partial P_0}{\partial\alpha}\right)\right.\\
&\left. -(T-t)\left(x\frac{\partial^2 P_0}{\partial r\partial x} - \frac{\partial P_0}{\partial r}\right) + \frac{(T-t)^2}{2}\left(x^2\frac{\partial^2 P_0}{\partial x^2} - x\frac{\partial P_0}{\partial x} + P_0\right)\right] \qquad (75)
\end{aligned}$$

in which

$$
\begin{aligned}
V_1^\epsilon(z) &= \sqrt{\epsilon}\left(\rho_2\nu\sqrt{2}\left\langle\sigma\phi_y\right\rangle(z) - \frac{1}{2}\tilde{\nu}\sqrt{2}\left\langle\Lambda\kappa_{\tilde{y}}\right\rangle\right), \; V_2^\epsilon = \frac{\sqrt{2}}{2}\sqrt{\epsilon}\rho_4\tilde{\nu}\left\langle\sigma\kappa_{\tilde{y}}\right\rangle, \\
V_3^\epsilon(z) &= \sqrt{\epsilon}\left(\rho_{12}\eta\nu\sqrt{2}\left\langle\phi_y\right\rangle(z) + \sqrt{2}\tilde{\nu}\left\langle\Lambda\varsigma_{\tilde{y}}\right\rangle\right) \\
V_4^\epsilon &= -\sqrt{\epsilon}\left(\frac{1}{2}\rho_{14}\eta\tilde{\nu}\sqrt{2}\left\langle\kappa_{\tilde{y}}\right\rangle - \rho_4\tilde{\nu}\sqrt{2}\left\langle\sigma\psi_{\tilde{y}}\right\rangle\eta\rho_1 + \rho_{14}\eta\tilde{\nu}\sqrt{2}\left\langle\psi_{\tilde{y}}\right\rangle\bar{\sigma}_1\rho_1^2\right), \\
V_5^\epsilon &= -\sqrt{\epsilon}\left(\rho_{14}\eta\tilde{\nu}\sqrt{2}\left\langle\psi_{\tilde{y}}\right\rangle\rho_1\right) \\
V_6^\epsilon &= \sqrt{\epsilon}(-\sqrt{2}\rho_4\tilde{\nu}\left\langle\sigma\psi_{\tilde{y}}\right\rangle\eta\rho_1 + \sqrt{2}\rho_{14}\eta\tilde{\nu}\left\langle\psi_{\tilde{y}}\right\rangle\bar{\sigma}_1\rho_1^2 \\
&\quad -\sqrt{2}\tilde{\nu}\eta\left\langle\Lambda\psi_{\tilde{y}}\right\rangle\rho_1\rho_2\nu\left\langle\sigma\varsigma_y\right\rangle + \sqrt{2}\rho_4\tilde{\nu}\left\langle\sigma\varsigma_{\tilde{y}}\right\rangle - \sqrt{2}\tilde{\nu}\left\langle\Lambda\varsigma_{\tilde{y}}\right\rangle) \\
V_7^\epsilon &= \sqrt{\epsilon}\left(\sqrt{2}\rho_{12}\eta\nu\left\langle\varsigma_y\right\rangle + \sqrt{2}\rho_{14}\eta\tilde{\nu}\left\langle\varsigma_{\tilde{y}}\right\rangle\right) \\
V_1^\delta(z) &= \sqrt{\delta}\bar{\lambda}^{(1)}(z)\bar{\sigma}_1\rho_3 g(z), \; V_2^\delta(z) = \sqrt{\delta}\bar{\lambda}^{(1)}(z)\eta\rho_{13}g(z).
\end{aligned}
$$

As Fouque et al. (2000) showed, the first order expansions, $P_{1,0}$ and $P_{0,1}$ are independent of the level of Y_t and $\tilde{Y}_t$. Then the first order expansion in ϵ and δ,

$$
\tilde{P}_{\epsilon,\delta} = P_0 + \sqrt{\epsilon}P_{1,0} + \sqrt{\delta}P_{0,1}, \tag{76}
$$

is given by the 10 parameters ($\bar{\lambda}^{(1)}$,$\mathbf{V}^\epsilon$and$\mathbf{V}^\delta$) approximation.

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Table 1: Major Events of the Financial Crisis (2008-2009) in the Financial System

Date	Event	Country
03/14/2008	FRB set the lending facility for Bear Sterns	United States
03/16/2008	JPMorgan Chase took over Bear Sterns	United States
09/06/2008	Freddie Mac & Fannie May were nationalized.	United States
09/15/2008	Lehman Brothers Bankruptcy	United States
09/15/2008	Bank of America decided to take over Meril Lynch.	United States
09/16/2008	FRB set the lending facility for AIG.	United States
09/18/2008	Lloyds announced to take over HBOS.	United Kingdom
09/21/2008	Goldman Sachs & Morgan Stanley became Bank Holding Company (BHC)	United States
09/28/2008	Fortis was nationalized.	Benelux countries
09/29/2008	Bradford&Bingley (B&B) was nationalized.	United Kingdom
09/30/2008	US congress rejected TARP (Troubled Asset Relief Program)	United States
10/03/2008	UK raised the limit of bank deposit guarantee.	United Kingdom
10/12/2008	Capital injection to RBS, Lloyds and HBOS.	United Kingdom
10/14/2008	Capital injection to the US major banks.	United States
10/31/2008	Barclays announced capital raising.	United Kingdom
11/18/2008	Barclays announced the details of the capital raising.(existing shareholders and Qatar)	United Kingdom
11/23/2008	Capital Injection (2nd) and Asset Guarantee (Citi Group)	United States
01/16/2009	Capital Injection (2nd) and Asset Guarantee (Bank of America)	United States

Table 1: Major Events of the Financial Crisis (2008-2009) in the Financial System

Date	Event	Country
01/17/2009	UK announced the asset protection scheme (APS).	United Kingdom
02/17/2009	UK announced to replace the preferred share with common stock (RBS).	United Kingdom
02/26/2009	Capital Injection (2nd) and the APS (RBS)	United Kingdom
03/02/2209	HSBC issued common stocks to shareholders	United Kingdom
03/07/2009	UK announced to replace the preferred share with common share and the ASP (Lloyds)	United Kingdom
03/30/2009	Barclays announced not to adopt the APS.	United Kingdom
11/03/2009	Capital Injection (3rd, RBS & LLOYDS).	United Kingdom

Figure 1: CDS Spread (United Kingdom, 1Year)

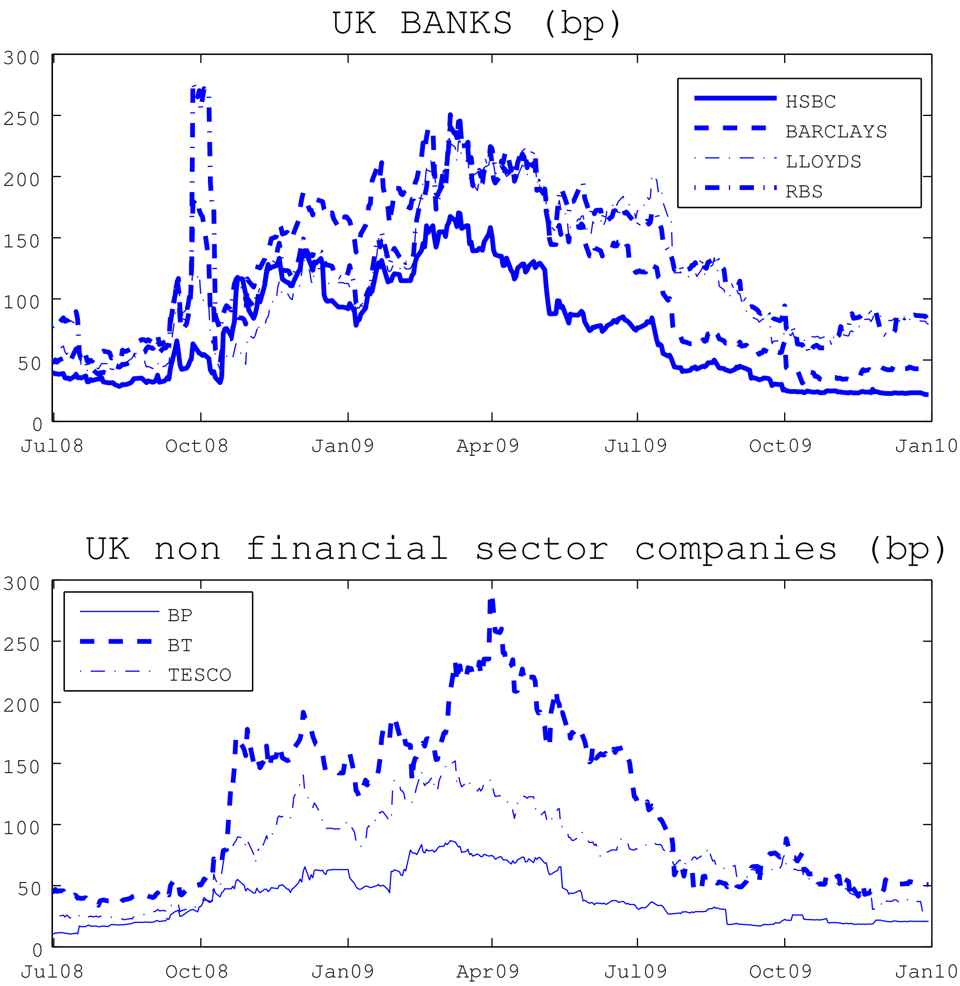
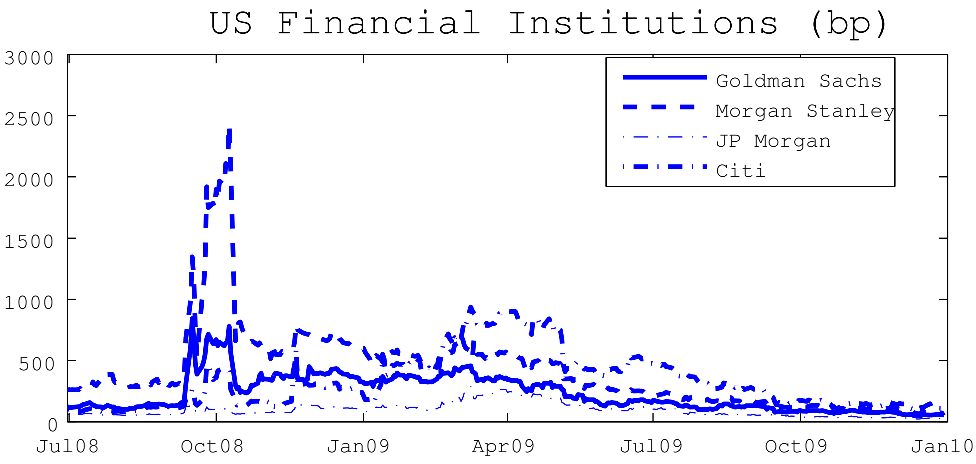
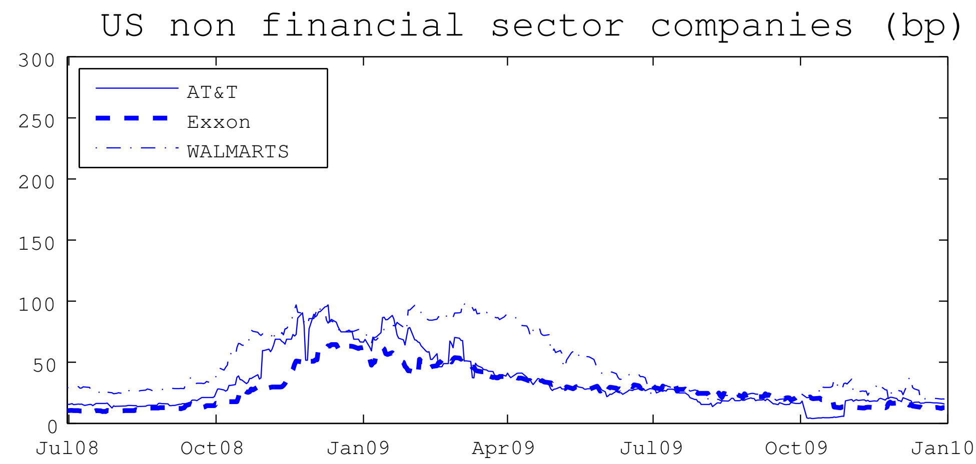


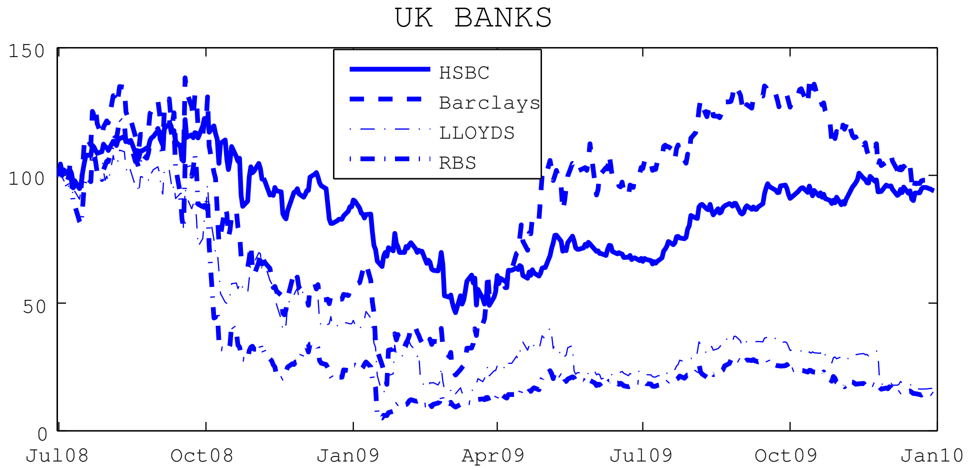
Figure 2: CDS Spread (United States, 1Year)

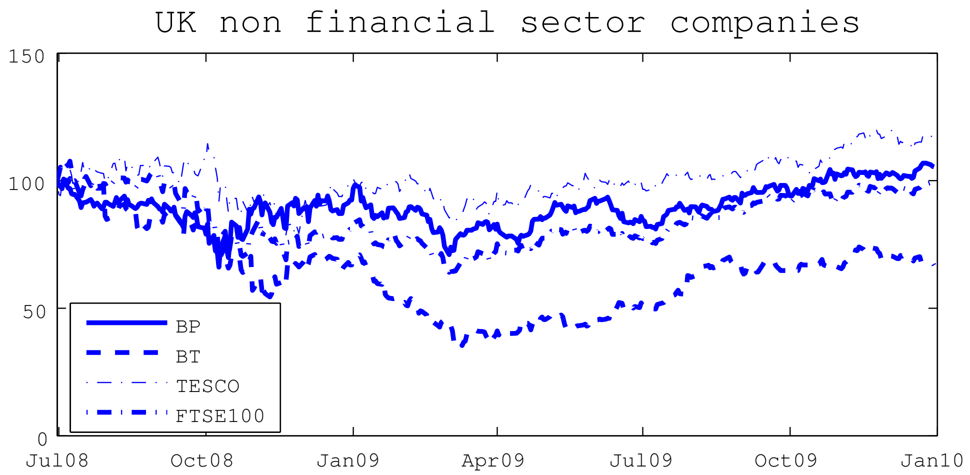




, Note the difference in scale between the two figures.

Figure 3: Stock Prices of the UK banks and companies (%)





, All stock prices are indexed 100 at July 1, 2008.

Figure 4: UK TED spread

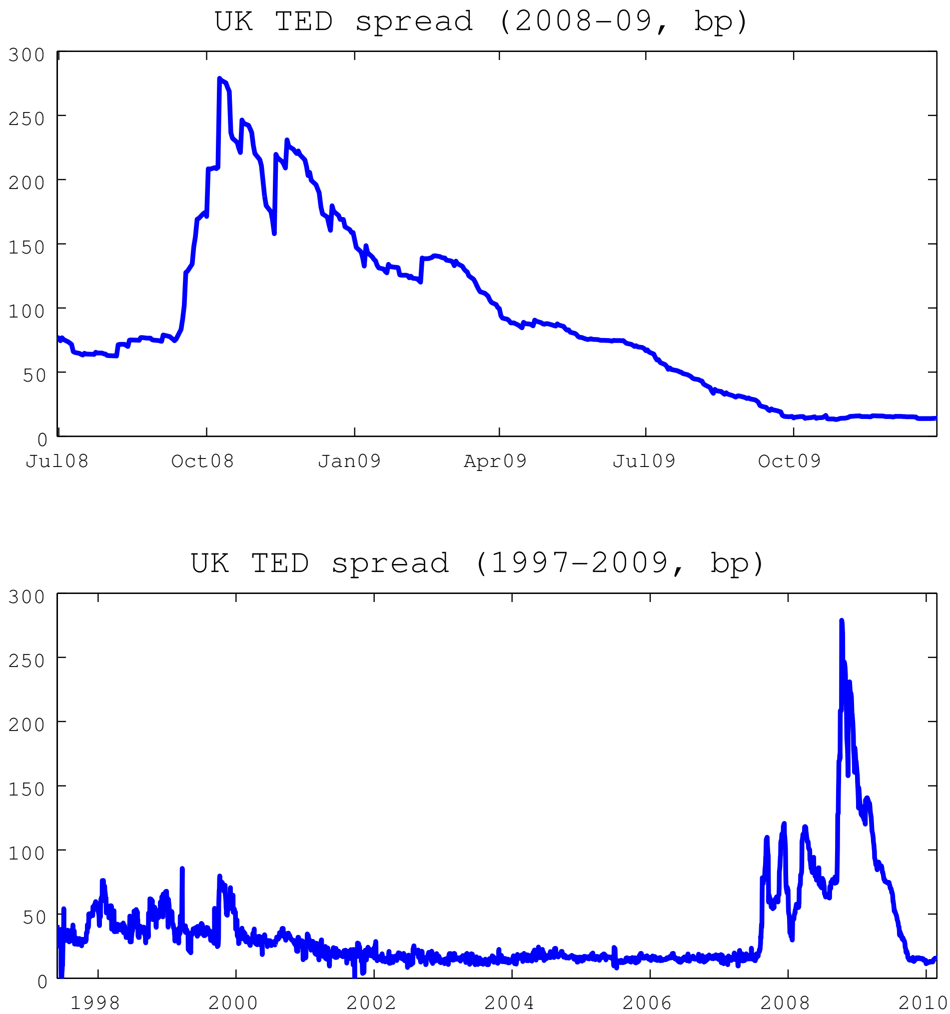
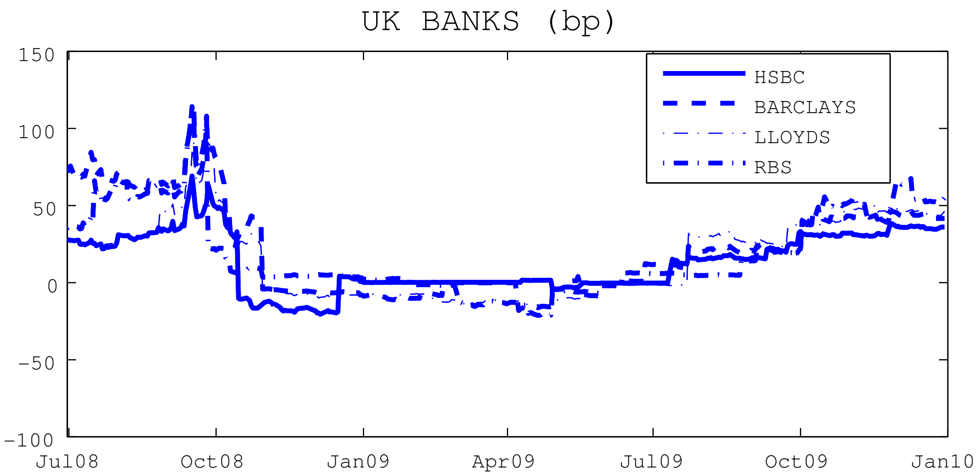


Figure 5: The spread between 5 Year CDS - 1 Year CDS



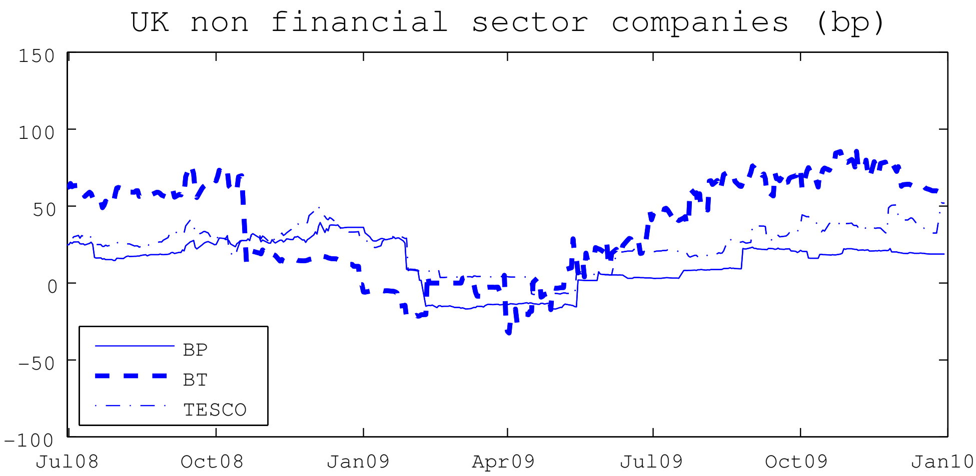
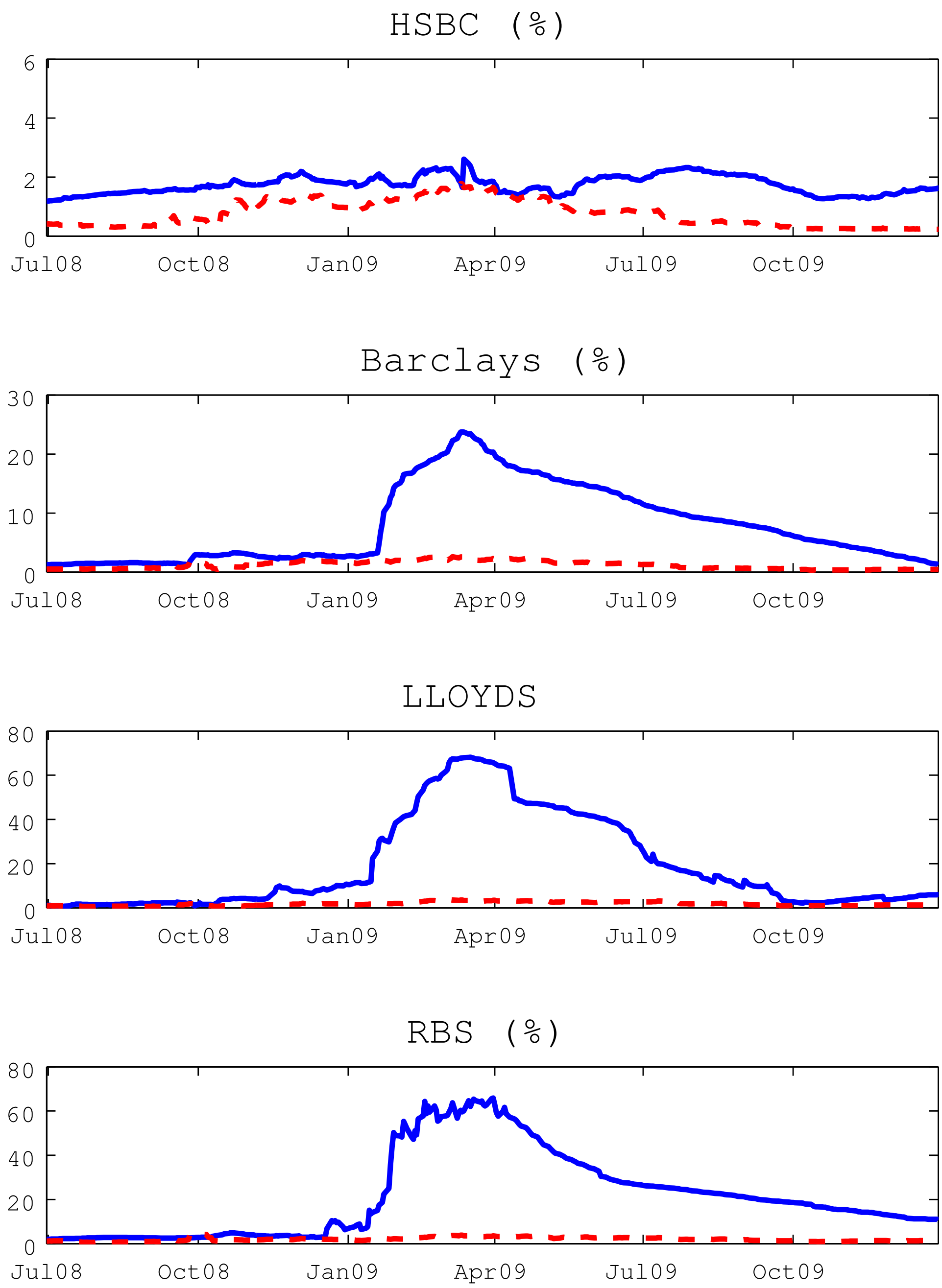
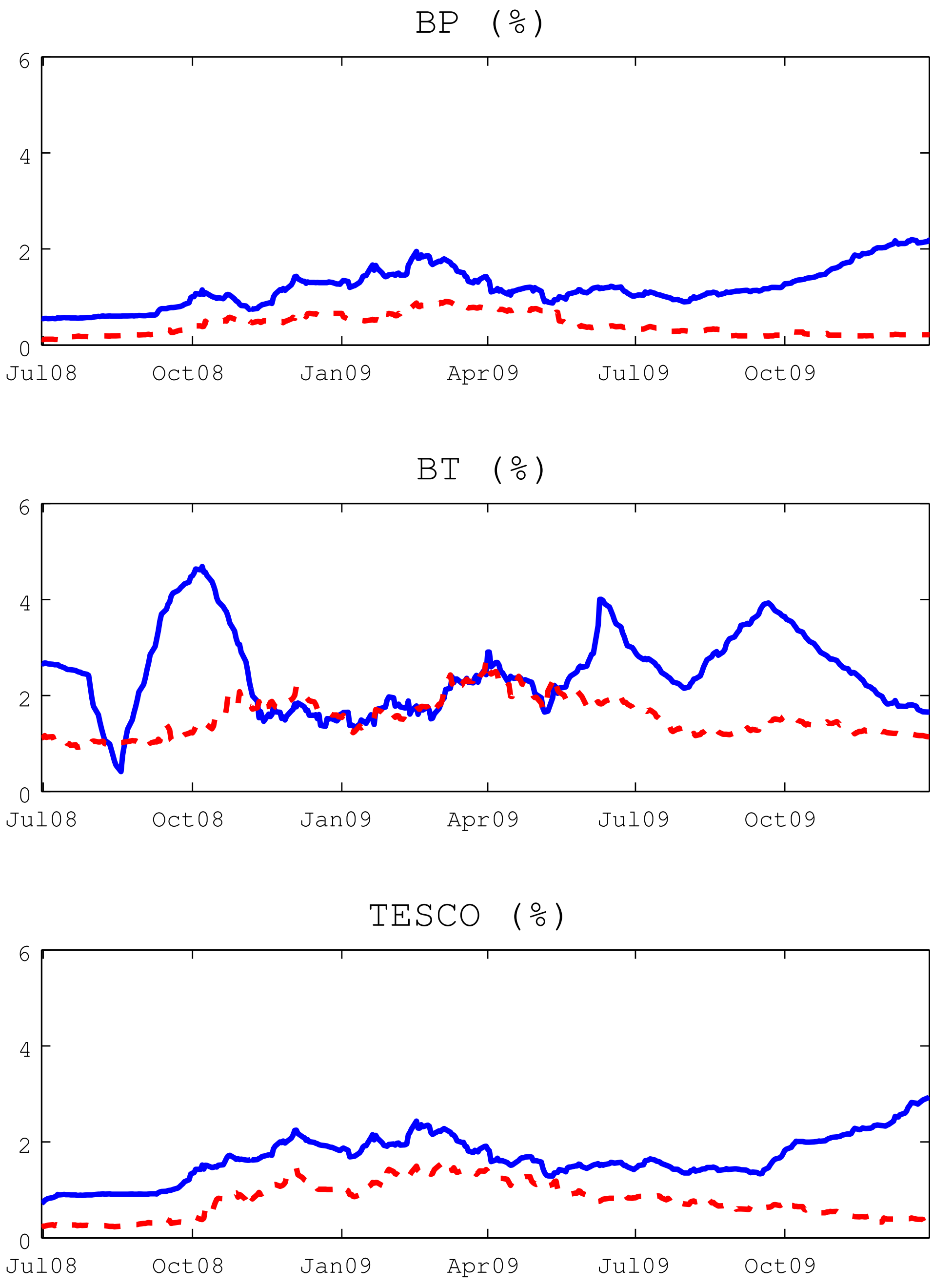


Figure 6: Implied PD of the major UK Banks, (%)



The solid curve depicts the PD implied from listed stock options and the dashed curve shows the PD calculated using the benchmark model (29) assuming a constant 60% LGD.

Figure 7: Implied PD of Non Financial Companies (%)



The solid curve depicts the PD implied from listed stock options and the dashed curve shows the PD calculated using the benchmark model(29) assuming a constant 60% LGD.

Figure 8: Ratio between the implied PD and the PD obtained using the benchmark model (29) assuming a constant 60% LGD

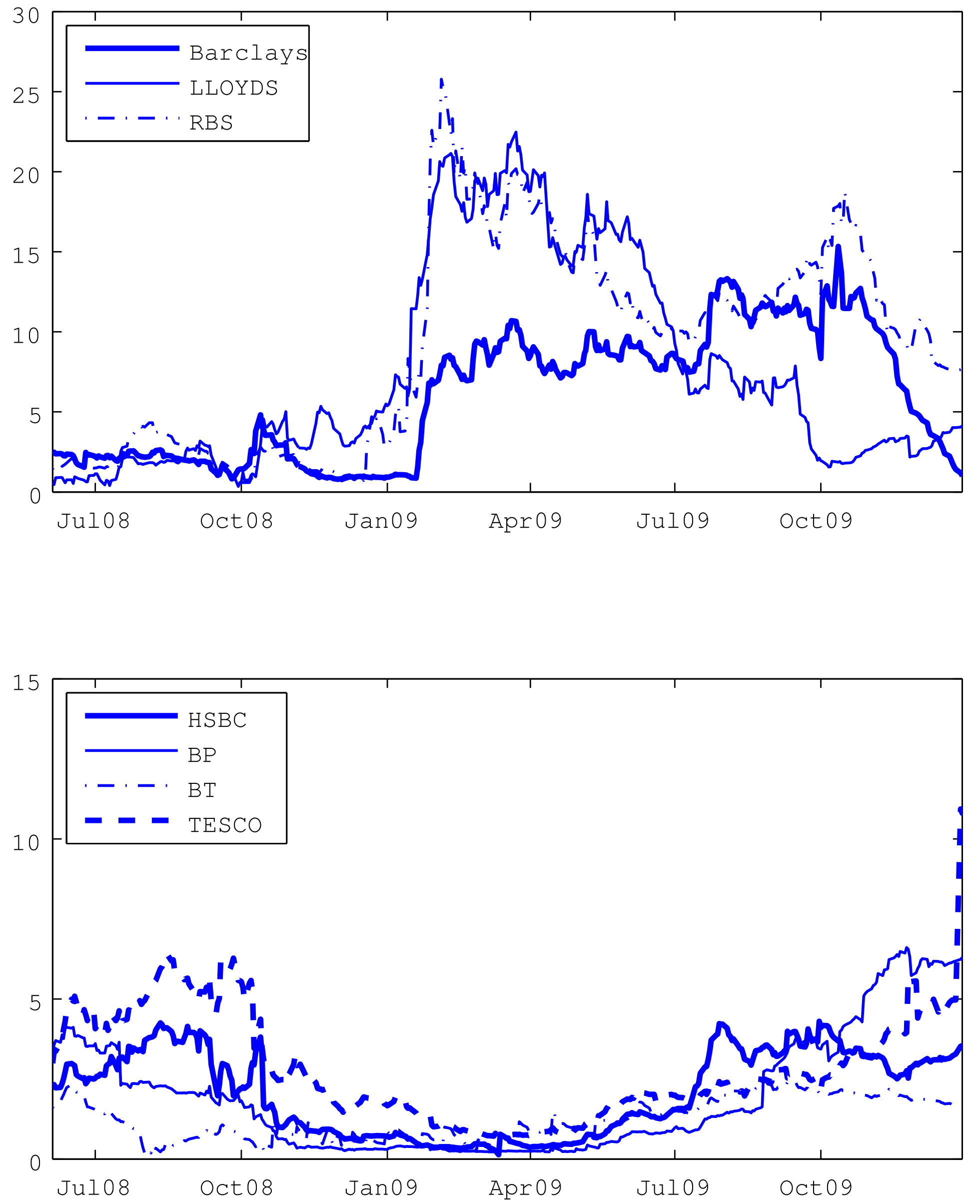
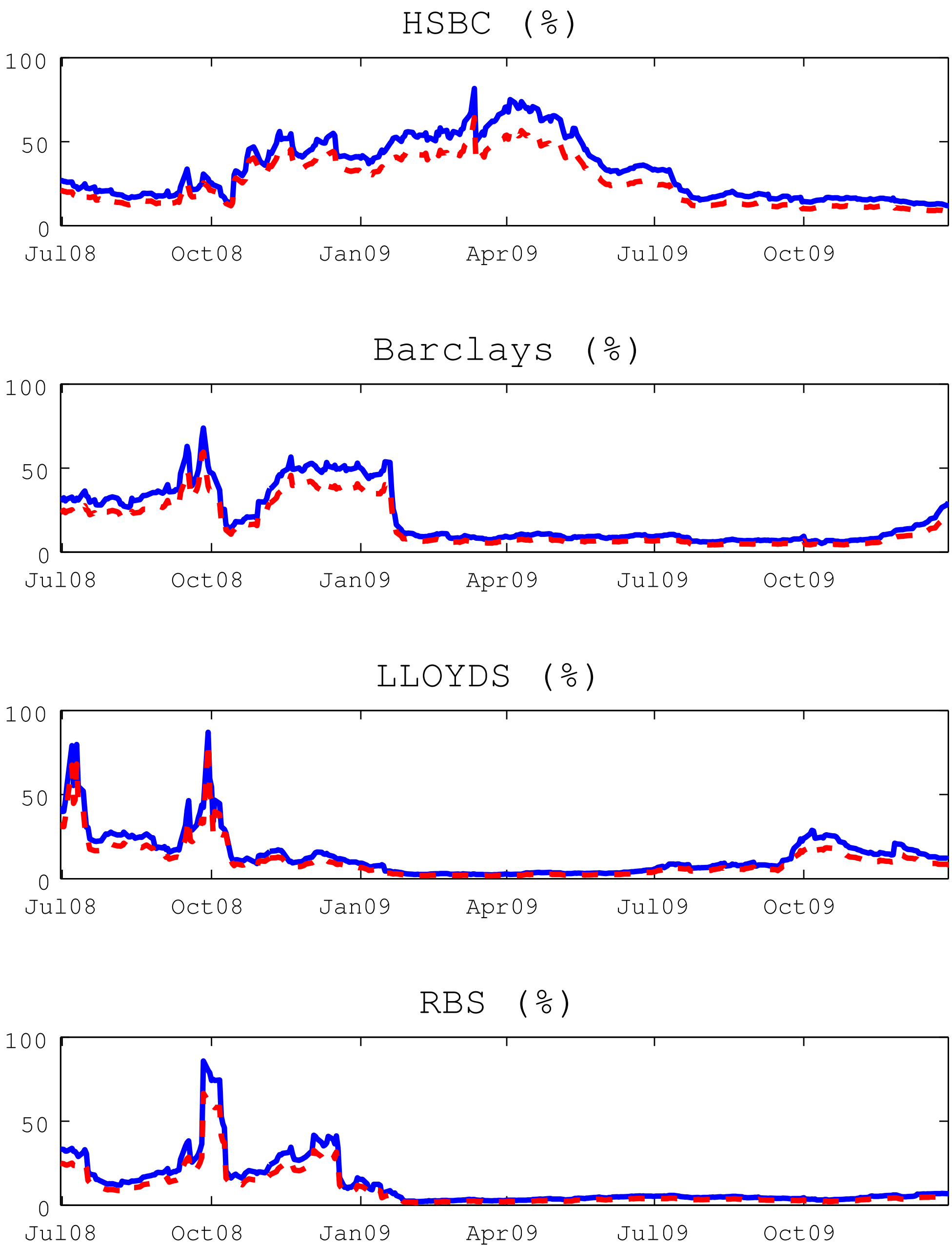
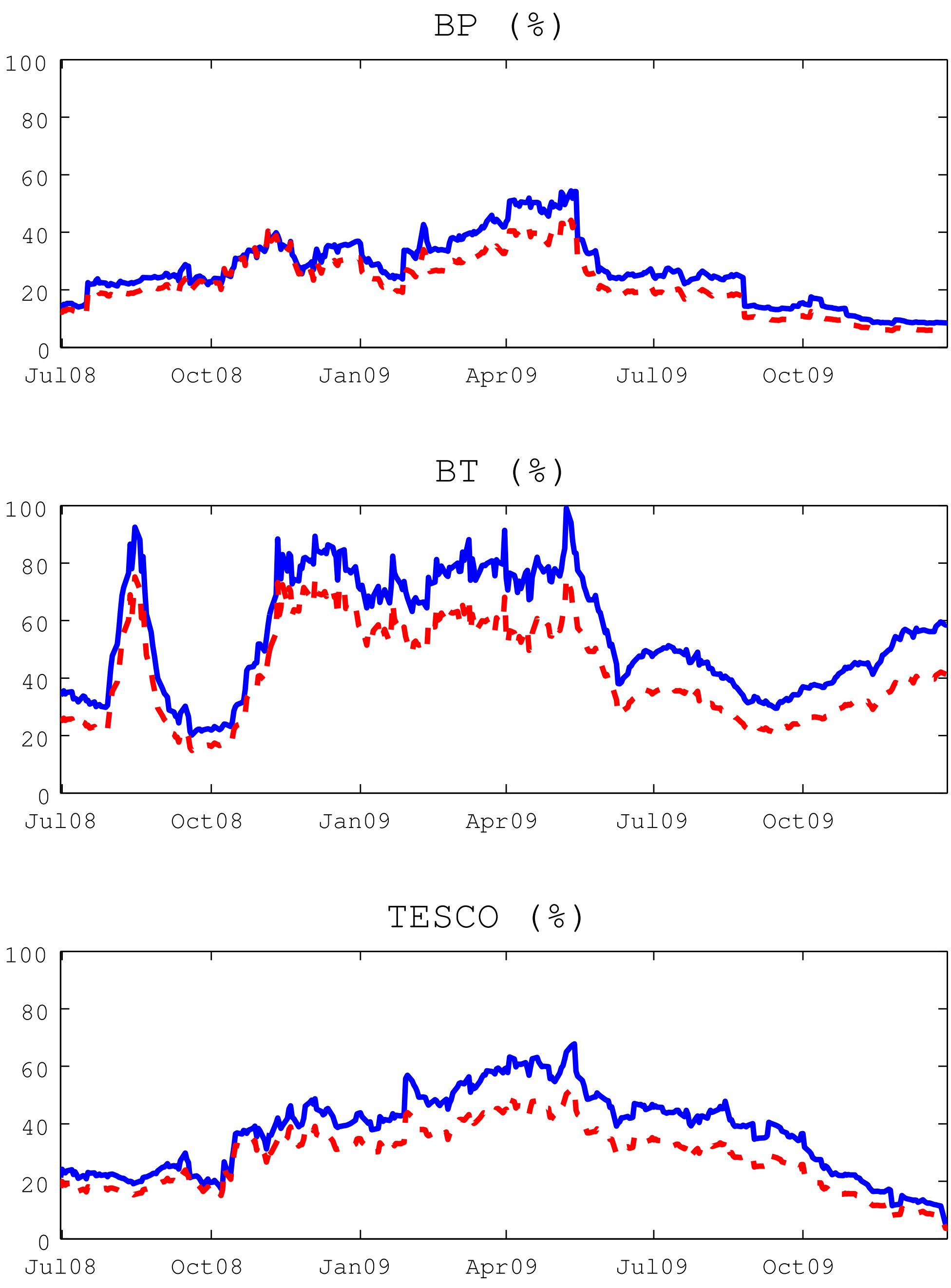


Figure 9: Implied LGD of the major UK Banks (%)



The solid curve shows the LGD implied from the CDS spreads with counterparty risk and the dashed curve is the implied LGD without counterparty risk.

Figure 10: Implied LGD of Non Financial Companies (%)



The solid curve shows the LGD implied from the CDS spreads with counterparty risk and the dashed curve is the implied LGD without counterparty risk.