INVITED COMMENTARY

Using Modeling to Understand How Athletes in Different Disciplines Solve the Same Problem: Swimming Versus Running Versus Speed Skating

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Every new competitive season offers excellent examples of human locomotor abilities, regardless of the sport. As a natural consequence of competitions, world records are broken every now and then. World record races not only offer spectators the pleasure of watching very talented and highly trained athletes performing muscular tasks with remarkable skill, but also represent natural models of the ultimate expression of human integrated muscle biology, through strength, speed, or endurance performances.[1,2] Given that humans may be approaching our species limit for muscular power output,[2,3] interest in how athletes improve on world records has led to interest in the strategy of how limited energetic resources are best expended over a race.[4,5] World record performances may also shed light on how athletes in different events solve exactly the same problem—minimizing the time required to reach the finish line. We have previously applied mathematical modeling to the understanding of world record performances in terms of improvements in facilities/equipment and improvements in the athletes' physical capacities.[6] In this commentary, we attempt to demonstrate that differences in world record performances in various sports can be explained using a very simple modeling process.

Pacing strategy is largely related to the duration of a competitive event, being much more "all out" in shorter events and typically more "faster-slower-faster" as event duration increases.[7,8] However, pacing strategy may also vary with the nature of the event. For example, we observe that although the world record times for 200 m swimming, 800 m running, and 1500 m speed skating are almost identical (102.00 s, 101.01 s, and 101.04 s, respectively)—implying very similar net energetic requirements—the velocity patterns in which the events are contested are strikingly different. Swimmers traditionally display very evenly paced races, runners

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accelerate quickly and then decelerate slowly over their race, and speed skaters display a very high early velocity with a comparatively large slowdown (Figure 1, left panel). The trend lines in Figure 1 (left) represent the pacing strategy of athletes during world record performances and appear to have slopes of –0.0035, –0.0088, and –0.0225 m·s⁻² for swimming, running, and speed skating, respectively. In 800 m running, there is evidence of a more complex pacing strategy, with a spurt at the end of a generally slowing velocity profile. In 1500 m speed skating, there are small oscillations of the velocity profile associated with skating the curves vs the straights, and in swimming there is evidence of velocity variations associated with the start and push-off on the turns vs free water swimming. This raises the question of why these races, which require essentially equal time and thus net energetic requirement, are contested with such different global velocity patterns.

The factors that dictate the pacing pattern in a given race are (a) the difficulty of accelerating at the beginning of the race, (b) the magnitude of slowdown resulting from the loss of power output due to fatigue, (c) the power losses to the environment, and (d) the amount of essentially wasted kinetic energy at the end of the race. In any competitive event, the adopted pacing strategy presumably represents an optimal solution of the variation of these factors, as well as the energy-producing ability and skill (eg, propulsive efficiency) of the athlete. During actual races, the optimal solution can easily be overridden by race tactics, but in world record events it seems reasonable to suggest that the energetically optimal race strategy and competitive race tactics are likely to be extremely similar, if not exactly the same.
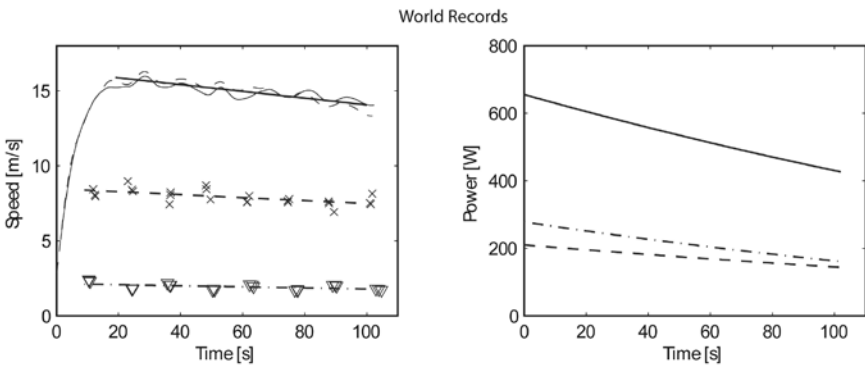

Figure 1 — Left panel: Individual velocity–distance data in the three best all-time men's performances in 200 m freestyle swimming, 800 m running, and two 1500 m men's speed skating performances that improved the world record at the Salt Lake City Olympics in 2002, with the trend lines representing the pacing strategy (solid line, speed skating; broken line, running; and dotted line, swimming). The swimming data is demonstrating the velocity gains associated with the start and the push-off at each turn relative to the slower velocity during the free water swimming segments. Note that in the running data, despite a general tendency to decelerate throughout the race after an initial acceleration, there is evidence of a spurt during the terminal portion of the race. The oscillations in the speed skating data are associated with skating the curved and straight sections of the track. Right panel: Mechanical power associated with losses to friction (solid line, speed skating; broken line, running; and dotted line, swimming).

To address the question of why swimming, running, and speed skating races of the same duration are contested with different pacing patterns, we made a simple model. We can describe the velocity pattern for every race in a simple manner: $v = v_{initial} - bt$, where b is average acceleration (ie, slowdown in these comparatively short races) and t is time (Figure 2, left). The kinetic energy at the finish line equals $(E_{kin, final}) = \frac{1}{2}mv_{final}^2$. The drag through the medium in which the athlete moves (water, air) equals $F_{drag} = \frac{1}{2}(\rho C_d A v^2)$, where ρ is the density, C_d is the drag coefficient, and A is the frontal area. Together, ρ, C_d, and A make up the drag constant: $k = \frac{1}{2}\rho C_d A$. For running,[9] k equals 0.345 kg·m^{-1}; for speed skating,[10] k equals 0.14 kg·m^{-1}; and for swimming,[11] k equals 28.9 kg·m^{-1}. In a non-moving medium, the power lost to drag equals $P_{drag} = F_{drag}v = kv^3$. Substitution gives $P_{drag} = k(v_{initial} - bt)^3$ for the instantaneous mechanical power loss to drag. The calculated P_{drag} for speed skating, running, and swimming are shown in Figure 1 (right). The total amount of external energy spent during the race (E_{lost}) is thus equal to the total energy lost to drag and $E_{kin, final}$:

$$E_{lost} = \int_0^{t_{final}} P_{drag} + E_{kin, final}$$

It appears unlikely that there are great differences in the power-producing capabilities within any group of athletes capable of making it to an Olympic final. Given a limited power-producing ability, an athlete needs to minimize E_{lost}. Because of the nonlinear dependence of P_{drag} on velocity and the dependence of $E_{kin, final}$ on v_{final}, the velocity profile (slope b in $v = v_{initial} - bt$) affects E_{lost}. The effect of velocity profile (slope b) on E_{lost} has been modeled, and the results show clear optima for slope b for all three events (Figure 2, right). The theoretically optimal race profiles (ie, the slowdowns that minimize E_{lost} according to our model) are shown in Figure 3 (left), together with the world record velocity data. There is a remarkable similarity between the modeled race profiles and the velocity profiles of the athletes setting world records, although the associated power losses to drag (Figure 3, right) show some notable differences with the calculated power for the world records (Figure 1, right).
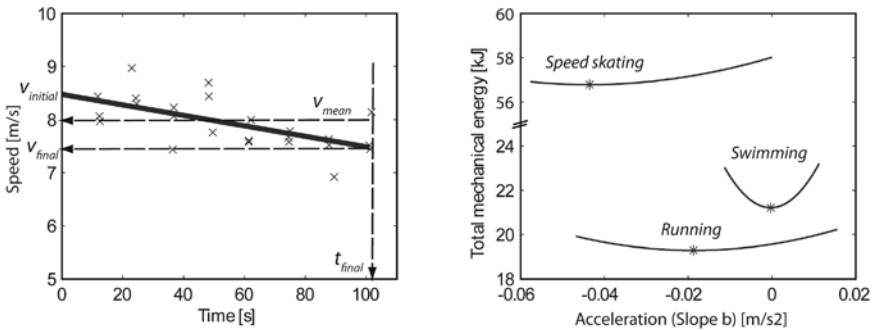

Figure 2 — Left panel: Simple model in which the velocity profile of the race is described with $v = v_{initial} - bt$ (solid line). The velocity data obtained from the 200 m running races (left panel) were used as an example. The term $v_{initial}$ is the velocity at the beginning of the race; v_{final}, the velocity at the finish; b, average acceleration; and t, time. Right panel: Solution of the simple model demonstrating the optimal slowdown of the athlete (b in $v = v_{initial} - bt$) to achieve the shortest race time.
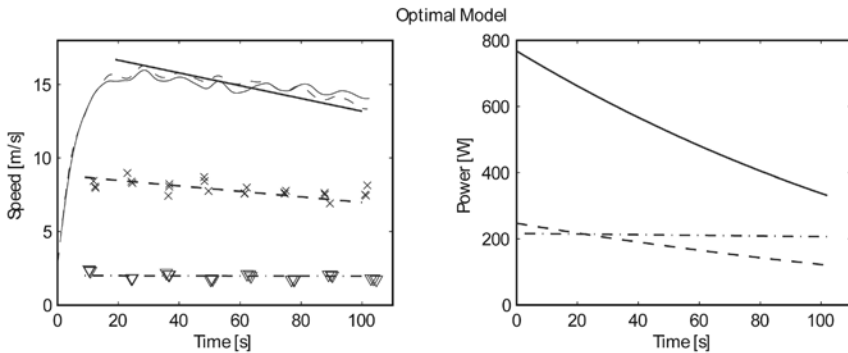
Figure 3 — Left panel: Modeled optimal pacing strategy in 200 m swimming, 800 m running, and 1500 m speed skating based on minimization of E_{lost}, with the performances as described in Figure 1 (solid line, speed skating; broken line, running; and dotted line, swimming). Right panel: Mechanical power associated with losses to friction as calculated for the optimal velocity profiles (solid line, speed skating; broken line, running; and dotted line, swimming).

Our simple model shows that for 200 m swimming, the optimal solution is an almost even pace ($b = -0.0003$ m·s^{-2}; Figure 2, right), with a $v_{initial}$ very close to average velocity (v_{mean}) and ideal 50 m splits of 24.9%, 25.0%, 25.1%, and 25.2% of t_{final}. For 800 m running, a higher relative $v_{initial}$ (112% of v_{mean}, corresponding to $b = -0.0187$ m·s^{-2}; Figure 2, right) is optimal. The model predicts the shortest race time when the first and second laps are 47.0% and 53.0% of t_{final}.

In speed skating, because of the relatively low value for k and the very high value for kinetic energy at the finish, the optimal $v_{initial}$ is higher than in swimming and running (115% of v_{mean}, corresponding to $b = -0.0435$ m·s^{-2}; Figure 2, right). It can be predicted that at a low-altitude oval rink (and, hence, relatively high k), such as that in Vancouver (site of the 2010 Olympics), the optimal $v_{initial}$ would be smaller than that at a high-altitude oval (eg, Calgary and Salt Lake City, sites of the 1988 and 2002 Olympics). However, evidence from both the Torino 2006 Olympics (also a low-altitude oval) and the Vancouver 2010 Olympics (with the rink exactly at sea level) still indicates a very high $v_{initial}$ (optimal for Vancouver: 114% of v_{mean}, corresponding to $b = -0.0375$ m·s^{-2}; realized in Vancouver: 112% of v_{mean}, corresponding to $b = -0.0300$ m·s^{-2}).

The best three 200 m world record swimming races were set with 50 m splits of 23.5%, 25.4%, 25.8%, and 25.2% of t_{final}. In swimming, k is high whereas $E_{kin, final}$ is small (due to low a v_{mean} of approx. 2 m·s^{-1}). In swimming, the disadvantage of drag in an unevenly paced race is larger than the advantage of a low $E_{kin, final}$. The 200 m world records were swum less evenly paced than the model suggests for optimal performance. The model suggests that even power production to balance the power loss to the environment is optimal (Figure 3, right), but this is probably not realistic for fatiguing events of relatively short duration. In contrast, the 400 m split times of the three best performances in 800 m running were 48.1% and 51.9% of t_{final}. Our model suggests that runners could benefit from starting even faster (Figure 3, right). For speed skating races, our model suggests that further progress in the world record can be achieved with a more all-out starting strategy (eg, a higher $v_{initial}$ and associated higher power output; Figure 3, right). On the other hand, very high muscular power output during the opening segment of an event

carries the risk of developing unacceptably large homeostatic disturbances early in the event, with subsequent loss of both power output and, potentially, propulsive efficiency.[12] Thus, our simple model, based on the trade-off between minimizing the essentially wasted kinetic energy at the end of a race and minimizing energy losses to friction, suggests that world records are attained by the athlete taking a calculated risk that he or she can use early power output to optimize energy losses, while not inducing an "energetic catastrophe."

From a larger perspective, however, the point of this simple exercise is to demonstrate that sport performance can be analyzed using relatively simple models and that athletes can predict ways to improve by optimizing the use of their energetic resources. Finally, it seems probable that in events of different durations (eg, running 1500 m, skating 3000 m, swimming 400 m) a similar solution might be achieved to optimize the energetic and performance balance within the somewhat different "landscape" provided by events of different duration.

References

1. Hill AV. The physiological basis of athletic records. *Sci Monthly,* 1925;21:409–425.
2. Berthelot G, Thibault V, Tafflet M, Escolano S, El Helou N, Jouven X, Hermine O, Toussaint JF. The citius end: world records progression announces the completion of a brief ultra-physiological quest. *PLoS ONE* 2008;3(2):e1552. doi:10.1371/journal. pone.0001552.
3. Seiler KS, de Koning JJ, Foster C. The fall and rise of the gender difference in elite anaerobic performance 1952–2006. *Med Sci Sports Exerc.* 2006;39:534–540.
4. Foster C, Hendrickson K, Peyer K, Reiner B, de Koning JJ, Lucia A, Battista RA, Hettinga FJ, Porcari JP, Wright G. Pattern of developing the performance template. *Br J Sports Med* 2009;43:765–769.
5. Tucker R, Noakes TD (2009) The physiological regulation of pacing strategy during exercise: a critical review. *Br J Sports Med* 2009:43e1. doi:10.1136/bjsm.2009.057562.
6. De Koning JJ. World records: How much athlete? How much technology? *Int J Sports Physiol Perf* 2010;5:262–267.
7. Foster C, de Koning JJ, Hettinga FJ, Lampen J, Dodge C, Bobbert M, Porcari JP. Effect of competitive distance on energy expenditure during simulated competition. *Int J Sports Med* 2004;25:198–204.
8. Tucker R, Lambert MI, Noakes TD. An analysis of pacing strategies during men's world record performances in track athletics. *Int J Sports Physiol Perf.* 2006;1:233–245.
9. Kyle CR, Caiozzo VJ. The effect of athletic clothing aerodynamics upon running speed. *Med Sci Sports Exerc.* 1986;18:509–515.
10. De Koning JJ, Foster C, Lampe J, Hettinga, FJ, Bobbert MF. Experimental evaluation of the power balance model of speed skating. *J Appl Physiol.* 2005;98:227–233.
11. Toussaint HM, Roos PE, Kolmogorov S. The determination of drag in front crawl swimming. *J Biomech.* 2004;37:1655–1663.
12. Hettinga FJ, de Koning JJ, Schmidt LJI, Wind NAC, McIntosh B, Foster C. Optimal pacing strategy: from theoretical modeling to reality in 1500-m speed skating. *Br J Sports Med*. 2009;45:30–35.