The Price Augmented Risk Premium, theory and applicaton.

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Abstract

This note proposes the Price Augmented Risk Premium (PARP), a decomposition of the multivariage risk premium associated with price and income uncertainty. The PARP is used to measure the likely impact on welfare arising from price fluctuations experienced by UK households over the period 1963-97.

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Contents

1 Introduction.

Ever since the seminal work of Von Neumann and Morgenstern (1944), much of the literature on consumer welfare under uncertainty has focused on uncertainty in income. However, consumers also face uncertainty in prices and this raises the possibility that uncertainty may increase expected welfare. This is because relative price movements may create additional opportunities for consumers to substitute between goods. The aim of this paper is to extend the analysis of consumer utility to include price uncertainty as well as income uncertainty. In order to do this the Price Augmented Risk Premium (PARP), a decomposition of the scalar multivariate risk premium, is proposed and estimated.

In much of the research, the multivariate risk premium has been defined as a vector of risk premia the economic agent is prepared to pay to avoid the uncertainty associated with each stochastic variable. This definition can be found in Pratt (1964) where it is called "risk aversion in the small", in Kihlstrom and Mirman (1974) where it is called the "directional risk premium" and in Duncan (1977) where it is called the "approximate risk premium vector". To the best of my knowledge, the first formal definition of a multivariate risk premium as a scalar is to be found in Karni (1979). Karni calls this the "risk premium of *local risk aversion*" which has the same specification as the premium defined in equation (1) below. Karni's contribution is to state "the restrictions that must be imposed on ... two utility functions in order that one requires a higher risk premium than another for every small multivariate risk."[1]

The PARP proposed in equation (6) is a decomposition of Karni's risk premium into components that are and are not affected by monotonic transformations of the utility function. Components affected by monotonic transformations of the utility function are associated with much studied attitudes to income risk aversion. Components not affected by monotonic transformation of the utility function depend only on ordinal aspects of preferences which can be estimated using standard demand systems.

In section 2 the theory underlying the scalar multivariate risk premium and its decomposition into the PARP are outlined. In section 3 the PARP is estimated using data on UK households. Section 4 concludes.

2 The Theory.

Presented in the next two sub-sections are the Scalar Multivariate Risk Premium and its reformulation into the Price Augmented Risk Premium. Equation (6) illustates how the multivariate risk premium nests the risk premium based on the Arrow-Pratt measure of Absolute Risk Aversion.

2.1 The Scalar Multivariate Risk Premium.

This sub-section recalls the multivariate risk premium developed by Karni and reported in equation (3.1) of his 1979 paper. Any risk premium is generally defined as the difference between the expected value of a lottery and the certainty equivalent of that lottery.[2] If an individual is facing a fair lottery, fair insofar as the expected payoff is

[1] Karni (1979), page 1391.

[2] See Kreps (1990), page 82, for this definition of the risk premium.

zero, the same risk premium can equivalently be defined as the maximum the individual is prepared to pay to avoid the fair lottery. The scalar multivariate risk premium presented here is defined as the maximum the individual is prepared to pay in order to avoid a fair lottery in price and income uncertainty. Thus, this premium is the monetary value the consumer puts on removing all variances and covariances in prices and income. The lottery in this case is fair, by construct, because the expected payoff is taken at the mean expected values of the stochastic variables using second order Taylor series approximations. This premium is therefore not the exact risk premium[3] but rather a risk premium for localised (small) uncertainty.

Proposition 1 *Define the multivariate risk premium by equating expected utility under uncertainty to utility under no uncertainty once the risk premium* π *has been paid,* $E[V(\Psi)] = V(E[\Psi] - \Pi)$*. This multivariate risk premium is approximated by,*

$$\begin{aligned} \pi \quad &\approx \quad \frac{1}{2}\sum_{i=1}^{n} \frac{-\partial^2 V/\partial p_i \partial p_i}{\partial V/\partial m}\sigma_{p_i p_i} + \sum_{i=1}^{n}\sum_{j\neq i}^{n} \frac{-\partial^2 V/\partial p_i \partial p_j}{\partial V/\partial m}\sigma_{p_i p_j} \\ &\quad + \sum_{i=1}^{n} \frac{-\partial^2 V/\partial p_i \partial m}{\partial V/\partial m}\sigma_{p_i m} + \frac{1}{2}\frac{-\partial^2 V/(\partial m)^2}{\partial V/\partial m}\sigma_{mm}. \end{aligned} \tag{1}$$

where all derivatives are evaluated at mean values, $E[.]$ *is the expectations operator,* $V(.)$ *represents the indirect utility function,* m_t *represents total expenditure at time* t*,* σ*'s represent the covariances* $\sigma_{xy} = \mathrm{cov}(x, y)$*, the* p*'s represent prices,*

$$\Pi = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \pi \end{bmatrix}, \qquad \Psi = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \\ m \end{bmatrix}, \qquad \bar{\Psi} = \begin{bmatrix} \bar{p}_1 \\ \bar{p}_2 \\ \vdots \\ \bar{p}_n \\ \bar{m} \end{bmatrix},$$

and $\bar{\Psi}$ *represents the mean values of* Ψ*.*

Proof. The PARP is defined by equating expected utility under uncertainty to utility under no uncertainty once the risk premium has been paid and prices and incomes have been fixed at their expected mean values. This is expressed by,

$$E[V(\Psi)] = V(E[\Psi] - \Pi) \tag{2}$$

where $E[.]$ is the expectations operator. Taking Taylor Series approximations of the left and right hand sides of equation (2) gives equations (3) and (4) respectively.

$$\begin{aligned} E[V(\Psi)] \quad &\approx \quad E\left[V(\Psi) + V_\Psi(\Psi - \bar{\Psi}) + \tfrac{1}{2}(\Psi - \bar{\Psi})' \tfrac{\partial^2 V}{(\partial \Psi)^2}(\Psi - \bar{\Psi})\right] \\ &\approx \quad V(\bar{\Psi}) + \tfrac{1}{2}E\left[(\Psi - \bar{\Psi})' \tfrac{\partial^2 V}{(\partial \Psi)^2}(\Psi - \bar{\Psi})\right] \\ &\approx \quad V(\bar{\Psi}) + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} \frac{\partial^2 V}{\partial p_i \partial p_j}\sigma_{p_i p_j} \end{aligned} \tag{3}$$

[3]Copeland and Weston (1983), pages 85-90.

$$
\begin{aligned}
V(E[\Psi]-\Pi) &\approx V(E[\Psi]) - \pi(\partial V/\partial \Psi) \\
&\approx V(\bar{\Psi}) - \pi(\partial V/\partial m) \qquad (4)
\end{aligned}
$$

Substituting (4) and (3) into (2) gives the approximation,

$$
\pi \approx \frac{-1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\partial^2 V/\partial p_i \partial p_j}{\partial V/\partial m} \sigma_{p_i p_j} \qquad (5)
$$

for the PARP. Substituting m for p_m and re-arranging gives equation (1). **end proof.**

Equation (1) is a generalisation of the standard measure of the income risk premium, $\pi = \frac{1}{2}\left(\frac{-\partial^2 V/(\partial m)^2}{\partial V/\partial m}\right)\sigma_{mm}$, where the term in brackets is the Arrow-Pratt measure of Absolute Risk Aversion (ARA) and the coefficient of Relative Risk Aversion (RRA) is ARA/m. The multivariate risk premium cannot be signed *a priori* because though quasi-convexity of the indirect utility function with respect to m is sufficient to determine that the Arrow-Pratt measure of Absolute Risk Aversion is positive, the other derivatives in equation (1) cannot be signed.

2.2 The Price Augmented Risk Premium.

While succinct, the formulation in (1) is not transparently related to the substitution responses that presumably shape attitudes to price uncertainty and their interaction with income risk aversion. The proposed PARP provides a decomposition of the scalar multivariate risk premium into components that relate to income effects, substitution effects and uncompensated price effects.

Proposition 2 *The multivariate risk premium (1) may be decomposed into the PARP,*

$$
\pi = \pi_{ie} + \pi_{se} + \pi_{ue}, \qquad (6)
$$

$$
\begin{aligned}
\pi_{ie} &= \frac{1}{2}\left(\frac{RRA}{\bar{m}}\right)\left(\sigma_{mm} + \sum_{i=1}^{n}\sum_{j=1}^{n} \bar{x}_i \bar{x}_j \sigma_{p_i p_j} - 2\sum_{i=1}^{n} \bar{x}_i \sigma_{p_i m}\right) \\
\pi_{se} &= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} e_{ij}^{h}\left(\frac{\bar{x}_i}{\bar{p}_j}\right)\sigma_{p_i p_j} \\
\pi_{ue} &= \sum_{i=1}^{n} e_i^{m}\left(\frac{\bar{x}_i}{\bar{m}_t}\right)\left(\sigma_{p_i m} + \sum_{j=1}^{n} \bar{x}_j \sigma_{p_i p_j}\right)
\end{aligned}
$$

where the x's represent the demands for the goods and the $\bar{x}$'s represent the mean values for those demands, e_{ij}^h's represent the Hicksian (compensated) price elasticities of demand and e_i^m's represent the income elasticities of demand.

Proof. Differentiating Roy's identity $\partial V/\partial p_i = -(\partial V/\partial m)x_i(p,m)$, with respect to m and p gives the following three expressions:

$$
\begin{aligned}
A(p_i,m) &= -A(m,m)\bar{x}_i + \partial x_i/\partial m \\
A(p_j,m) &= -A(m,m)\bar{x}_j + \partial x_j/\partial m \\
A(p_i,p_j) &= -A(m,p_j)\bar{x}_i + \partial x_i/\partial p_j
\end{aligned}
$$

where $A(x,y) = \frac{-\partial^2 V/\partial x \partial y}{\partial V/\partial m}$. Substituting $A(p_j, m)$ into $A(p_i, p_j)$ gives,

$$A(p_i, p_j) = -A(m,m)\bar{x}_i\bar{x}_j - \frac{\partial x_j}{\partial m}\bar{x}_i + \frac{\partial x_i}{\partial p_j} \tag{7}$$

Substituting Shephard's Lemma $\partial x_i/\partial p_j = (\partial x_i^u/\partial p_j) - (\partial x_i/\partial m)\bar{x}_j$, into (7) gives

$$A(p_i, p_j) = -A(m,m)\bar{x}_i\bar{x}_j + \frac{\partial x_i^u}{\partial p_j} - \frac{\partial x_j}{\partial m}\bar{x}_i - \frac{\partial x_i}{\partial m}\bar{x}_j \tag{8}$$

when $i = j$ equation (8) simplifies to

$$A(p_i, p_i) = -A(m,m)\bar{x}_i^2 + \frac{\partial x_i^u}{\partial p_i} - 2\frac{\partial x_i}{\partial m}\bar{x}_i \tag{9}$$

Substituting $A(p_j, m)$, $A(p_i, p_j)$ and $A(p_i, p_i)$ into equation (1) and rearranging gives equation (6). **end proof.**

The first component in equation (6) represents attitudes to risk aversion, the second and third components represent attitudes to ordinal preferences. The first component π_{ie} contains the terms relating to risk aversion in real budget fluctuations about the mean bundle. Note that in this first component all the terms are pre-multiplied by $\frac{RRA}{\bar{m}} = \frac{-\partial^2 V/(\partial m)^2}{\partial V/\partial m}$, the standard Arrow-Pratt measure of ARA. Note also that, the income variance term σ_{mm} appears only in this first component and is unambiguously detrimental to consumer welfare, if risk aversion in income is assumed. The second component π_{se} represents the substitution opportunities arising from relative price movements, this contains all the terms relating to compensated demands for the goods. The third component π_{ue} represents the (residual) uncompensated price effects.

2.3 Consistency Checks.

This sub-section two consistency checks are applied to the PARP. The first is to check that the PARP does not violate the assumption of homogeneity in the demand system and the second is to identifying a form of price variance that consumers find desirable irrespective of preferences.

The first consistency check is that the property of homogeneity of degree (HOD) zero in the demand system extends to the PARP. For example, HOD zero in the demand system implies that if income and all prices double the consumer is no better and no worse off. This property extends to the PARP insofar as when all prices and income fluctuate by the same proportion the PARP equals zero implying that these fluctuations have no impact on consumer welfare. For a formal proof see appendix A.

The second consistency check involves identifying a form of price uncertainty that unambiguously increases welfare (so long a goods are not perfect complements). A desirable form of price uncertainty, limited to the two good case, is identified and is found to be desirable regardless of the choice of utility function. On an intuitive level, this form of uncertainty can be thought of as a "real income preserving spread" process, i.e. price fluctuations may cause the budget constraint to fluctuate but the bundle chosen under the non-stochastic price equivalent can always be afforded. For a formal proof see appendix B. This result raises the possibility that other forms of price and income uncertainty may be welfare enhancing.

3 An Application.

The PARP is estimated for a "representative" UK household using equation (6), information on consumers' expenditure (see sub-section 3.1) and information on consumers' preferences (see sub-section 3.2). The household is representative insofar as mean expenditure for the household is derived by dividing aggregate UK consumer expenditure by estimates of the number of UK households. Estimates of the PARP for the representative household and for households representative of the expenditure quintiles are reported in sub-section 3.3.

3.1 The Data on Consumers' Expenditure.

The data[4] are seasonally unadjusted and refer to the consumption of non-durable goods in the UK over the period 1963q1-1997q4. Summary statistics on product categories, expenditure shares and total expenditures are given in Table 1. The highest mean expenditure share is for product category 4.1 (Electricity, Gas, Coal & Coke) with mean($w_{4.1}$) = 0.092. The second highest is for product category 2.1 (Beer) with mean($w_{2.1}$) = 0.091. From the same table one can see that the highest mean expenditure is for product category 2.1, mean($x_{2.1}$) = £171.06. Total expenditure is based on total expenditure at 1990 prices divided by the number of household in the UK.[5] Summary statistics on total expenditure by the "representative" household are given in the last row of Table 1, from this one can see that mean total expenditure on non-durables, per quarter, for the representative household is £1493.45.

For brevity, statistics for prices and covariances are not reported. Prices have been defined by dividing the implicit deflator for each commodity group by the implicit deflator for total expenditure. Covariances between prices and total expenditure are based on the whole sample period, this implies that the covariance structure for the sample period is representative of the expected covariance structure for each sub-period.

3.2 The Estimated Preference Structure.

The preference structure over goods for the representative household is derived from elasticities estimated using an AIDS and by defining a coefficient of RRA. The formulation of the AIDS and derivation of the elasticities is given in appendix C. The coefficient of RRA is set equal to one, the reasons for choosing this calibration are elaborated upon in appendix D. Estimates of the elasticities based on parameter estimates of an AIDS restricted for both Homogeneity and Symmetry are reported in Table 2. The estimated preference structure is that for the "representative" household. Summary statistics on the compensated cross-price elasticities are not reported as these represent an additional 38*38-38 = 1406 time series. Parameter estimates of the AIDS are reported in appendix E.

All the elasticities seem reasonable with the exception of the compensated own elasticities (e^h_{ii}) for product categories 2.2, 7.2 and 8.1. In these categories maximum

[4]The data are taken from the Office for National Statistics publication "Consumer Trends", this in turn, is based on data from the UK Family Expenditure Surveys.

[5]The number of households is based on population estimates from "Monthly Digest of Statistics", table 2.1 and from estimates of household size from the "General Household Surveys". This gives mid-year estimates and values for each quarter are generated by interpolation.

i	Product group:	w_{it} mean	w_{it} min	w_{it} max	$£x_{it}$ mean	$£x_{it}$ min	$£x_{it}$ max
1.1	Bread, Cakes, Biscuits & Cereals.	0.043	0.034	0.055	63.37	51.10	72.98
1.2	Meat and Bacon.	0.080	0.060	0.109	114.56	81.97	145.00
1.3	Fish.	0.012	0.009	0.019	19.45	12.11	29.83
1.4	Milk, Cheese & Eggs.	0.047	0.026	0.059	69.63	48.76	98.49
1.5	Oils and Fats.	0.010	0.005	0.013	11.58	8.59	14.73
1.6	Fruit.	0.018	0.013	0.023	25.33	15.59	41.49
1.7	Potatoes.	0.013	0.008	0.016	19.61	3.82	34.16
1.8	Vegetables.	0.022	0.016	0.033	31.09	15.08	64.33
1.9	Sugar.	0.005	0.001	0.010	7.09	2.19	14.79
1.10	Confectionery.	0.025	0.020	0.033	36.44	25.69	47.92
1.11	Tea, Coffee & Cocoa.	0.011	0.006	0.015	16.00	8.38	23.85
1.12	Soft Drinks.	0.014	0.005	0.028	21.18	4.57	53.69
1.13	Other Foodstuffs.	0.010	0.008	0.014	13.82	9.31	20.61
2.1	Beer.	0.091	0.057	0.129	171.06	85.68	266.98
2.2	Spirits.	0.031	0.013	0.057	45.96	12.99	104.40
2.3	Wine.	0.020	0.006	0.035	28.91	5.81	63.47
2.4	Cider & Perry.	0.004	0.002	0.008	7.40	3.07	12.88
2.5	Tobacco.	0.086	0.038	0.136	146.81	47.97	250.98
3.1	Men's & boys' Clothing.	0.034	0.026	0.051	46.75	22.95	110.75
3.2	Women's & girls' Clothing.	0.062	0.035	0.116	83.38	21.76	259.16
3.3	Footwear.	0.023	0.015	0.030	32.09	13.26	65.37
4.1	Electricity, Gas, Coal & Coke.	0.092	0.057	0.148	147.57	75.99	228.35
4.2	Petrol and Engine Oil.	0.056	0.022	0.073	82.23	29.23	118.78
5.1	Cleaning.	0.013	0.009	0.015	19.01	15.41	22.83
5.2	DIY goods.	0.024	0.011	0.037	36.50	13.95	61.25
5.3	Textiles & soft furnishings.	0.013	0.008	0.019	17.49	7.57	42.86
5.4	Hardware.	0.019	0.014	0.022	29.40	17.19	38.77
6.1	Books.	0.009	0.006	0.014	20.25	9.01	38.62
6.2	Newpapers.	0.021	0.010	0.035	45.39	17.86	110.43
6.3	Magazines.	0.008	0.005	0.012	16.78	8.05	34.46
7.1	Spectacles.	0.003	0.002	0.007	5.09	1.74	13.39
7.2	Medication.	0.008	0.006	0.012	13.57	8.39	19.04
7.3	Toiletries.	0.024	0.015	0.036	36.60	17.89	60.28
8.1	Sporting goods & toys.	0.015	0.004	0.033	21.83	3.67	72.35
8.2	Jewelry.	0.013	0.007	0.023	19.38	9.48	42.98
8.3	Records.	0.008	0.002	0.029	10.71	1.22	53.42
8.4	Photographic equipment.	0.003	0.001	0.006	4.10	0.52	10.69
8.5	Pets.	0.012	0.008	0.017	16.14	10.09	25.75
	Total	1.000	1.000	1.000	1493.45	1272.68	1915.79

Table 1: Summary Statistics for Weights and Expenditures. w_{it} is the expenditure share and $£x_{it}$ is the expenditure at 1990 prices on product group i at time t.

i	Product group:	$e^h_{(ii)t}$ mean	$e^h_{(ii)t}$ min	$e^h_{(ii)t}$ max	e^m_{it} mean	e^m_{it} min	e^m_{it} max
1.1	Bread, Cakes, Biscuits & Cereals.	-1.08	-1.12	-1.05	0.33	0.15	0.48
1.2	Meat & Bacon.	-1.46	-1.64	-1.28	0.44	0.26	0.60
1.3	Fish.	-1.85	-2.15	-1.51	0.03	-0.32	0.40
1.4	Milk, Cheese & Eggs.	-1.40	-1.72	-1.28	0.22	-0.32	0.41
1.5	Oils & Fats.	-0.52	-0.68	-0.16	0.10	-0.60	0.40
1.6	Fruit.	-1.68	-1.92	-1.50	0.16	-0.13	0.37
1.7	Potatoes.	-1.21	-1.36	-1.15	0.47	0.11	0.59
1.8	Vegetables.	-1.30	-1.42	-1.18	-0.01	-0.36	0.33
1.9	Sugar.	-1.30	-1.76	-1.10	1.16	1.06	1.41
1.10	Confectionery.	-0.75	-0.80	-0.71	1.51	1.39	1.63
1.11	Tea, Coffee & Cocoa.	-0.75	-0.82	-0.57	0.17	-0.48	0.43
1.12	Soft Drinks.	-1.27	-1.62	-1.08	0.25	-0.67	0.71
1.13	Other Foodstuffs.	-1.57	-1.74	-1.41	-0.29	-0.67	0.05
2.1	Beer.	-1.00	-1.08	-0.93	0.74	0.59	0.82
2.2	Spirits.	0.90	0.03	3.02	4.19	2.62	7.96
2.3	Wine.	-1.94	-3.83	-1.42	3.76	2.34	9.01
2.4	Cider & Perry.	-1.35	-1.80	-1.15	0.94	0.85	0.97
2.5	Tobacco.	-0.68	-0.74	-0.46	0.13	-0.77	0.51
3.1	Men's & Boys' Clothing.	-2.12	-2.47	-1.69	2.44	1.94	2.85
3.2	Women's & Girls' Clothing.	-0.41	-0.61	-0.14	2.46	1.72	3.38
3.3	Footwear.	-2.49	-3.28	-2.09	1.73	1.54	2.11
4.1	Electricity, Gas, Coal & Coke.	-0.77	-0.78	-0.73	0.53	0.28	0.72
4.2	Petrol & Engine Oil.	-0.79	-0.81	-0.61	0.18	-1.00	0.40
5.1	Cleaning.	-0.85	-0.87	-0.79	-0.48	-0.98	-0.18
5.2	DIY goods.	-1.23	-1.49	-1.11	0.05	-0.89	0.45
5.3	Textiles & soft furnishings.	-4.04	-5.51	-2.97	0.58	0.38	0.73
5.4	Hardware.	-0.75	-0.79	-0.68	1.66	1.55	1.89
6.1	Books.	-1.51	-1.78	-1.30	3.88	2.77	5.32
6.2	Newpapers.	-1.30	-1.57	-1.14	0.43	-0.04	0.69
6.3	Magazines.	-1.40	-1.58	-1.24	0.29	-0.01	0.57
7.1	Spectacles.	-1.84	-2.51	-1.31	-0.59	-1.83	0.40
7.2	Medication.	-0.26	-0.48	0.03	0.36	0.11	0.55
7.3	Toiletries.	-1.26	-1.42	-1.15	1.40	1.26	1.61
8.1	Sporting goods & toys.	1.01	-0.23	4.29	4.09	2.12	9.23
8.2	Jewelry.	-0.66	-0.79	-0.49	4.32	2.74	6.36
8.3	Records.	-0.85	-0.94	-0.63	8.83	2.24	24.81
8.4	Photographic equipment.	-1.71	-3.01	-1.26	-0.05	-1.97	0.61
8.5	Pets.	-1.49	-1.67	-1.31	-0.18	-0.59	0.23

Table 2: Summary statistics for Price and Income Elasticities. $e^h_{(ii)t}$ is the compensated own price elasticity and e^m_{it} is the income elaticity for product group i at time t.

Variable		mean	min	max
$\pi_{ie,t}$	(income effect)	£7.62	£4.51	£18.55
$\pi_{se,t}$	(substitution effect)	£-13.08	£-25.58	£-4.71
$\pi_{ue,t}$	(uncompensated effect)	£-1.33	£-5.33	£13.54
π_t	(PARP)	£-6.78	£-20.73	£18.78
m_t	(total expenditure)	£1493.45	£1272.68	£1915.79
$\frac{100\pi_t}{m_t}$		-0.48%	-1.44%	0.94%

Table 3: Summary statistics on the PARP for the representative household, 1963q1-1997q4.

values suggest that for at least some observations these compensated own elasticities are positive. These discrepancies in the elasticities may arise for a number of reasons. These reasons include aggregation over products, aggregation over households, measurement errors, and failure to model the non-stationary nature of the data.[6]

Estimates of the income elasticities (e_i^m) suggest that most goods are necessities. Product categories classified as inferior by their mean income elasticities are 1.8, 1.13, 5.1, 7.1, 8.4 and 8.5. The product categories classified as superior by their mean income elasticities are 2.2, 2.3, 3.1, 3.2, 3.3, 5.4, 6.1, 7.3, 8.1, 8.2 and 8.3. Some of these elasticities may seem dubious in light of the products represented by the categories, these discrepancies are likely have arisen for the reasons outlined above.

3.3 Estimates of the PARP.

The PARP for the representative UK household, in each quarter, is estimated using the expenditures given by the data, the preference structure estimated using the AIDS and by setting the coefficient of RRA to one. Summary statistics on values of the components of the PARP are reported in table 3. The minimum value for the component relating to income effects shows it to be always positive, this represents the amount the representative household would be willing to pay in order to avoid these pure income (expenditure) fluctuations. The maximum value for the component relating to substitution effects show it to be always negative, this represents the amount the representative household would have to be compensated to have these substitution opportunities removed. The component relating to the uncompensated price effects takes on both negative and positive values, suggesting that the net effects acting through the income elasticities can be either welfare enhancing or reducing. The overall value of the PARP takes on both negative and positive values but the mean value is negative, this suggest that on average the price and expenditure fluctuations are welfare enhancing for the representative household. This welfare gain represents roughly £6.78 per quarter at 1990 prices. As a proportion of total expenditure this gain is small, representing just 0.48% of total expenditure on non-durables.

[6]Failure to model the non-stationarity of the data is the greatest shortcoming. The data exhibits non-stationarity with the presence of unit roots and seasonal roots. Dealing with non-stationary data that contains seasonal components is non-trivial and is therefore left as a future extension.

Variable	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
mean(π_{ie})	£18.71	£10.43	£7.87	£6.82	£6.10
mean(π_{se})	£-5.73	£-9.48	£-12.59	£-15.19	£-20.05
mean(π_{ue})	£-3.69	£-2.58	£-1.51	£-0.50	£1.64
mean(π)	£9.28	£-1.63	£-6.23	£-8.88	£-12.31
mean(m)	£507.77	£970.74	£1418.78	£1836.95	£2733.02
mean($\frac{100\pi}{m}$)	1.85%	-0.19%	-0.47%	-0.52%	-0.48%

Table 4: Mean values over the time interval by expenditure quintiles, 1963q1-1997q4.

The same values are also used to estimate the PARP for households "representative" of five expenditure quintiles. Given one cannot use the data to estimate the preference structure for households at different expenditure levels, this preference structure is generated by assuming the underlying parameters estimated by the AIDS hold for all households. Expenditures, demands and elasticities are generated for households representative of each quintile by altering the value of total expenditure according to total expenditure statistics from the UK Family Expenditure Survey. These statistics suggest that expenditures in the five quintiles represent 34%, 65%, 95%, 123% and 183% respectively of expenditure on non-durables in the mean representative household. Summary statistics for the PARP and its components in each expenditure quintile are reported in table 4, these suggest that the greater the level of expenditure the larger the gains arising from price uncertainty. These gains increase as a proportion of total expenditure on non-durables up to the fourth quintile and then fall slightly for the fifth quintile. The only households to experience a net loss from the price fluctuations are those in the lowest expenditure quintile.

4 Conclusion.

The empirical results suggest the overall value of the PARP (π) is positive for households in the lowest expenditure quintile suggesting price uncertainty is welfare reducing for this group, and negative for the other quintiles suggesting price uncertainty is welfare enhancing for these groups. For all household types and in all time periods the component relating to income effects π_{ie} is positive, suggesting pure income fluctuations are always welfare reducing. The component relating to substitution effects π_{se} is always negative, suggesting substitution opportunities are welfare enhancing. The component relating to uncompensated price effects π_{ue} varies in sign and its magnitude is relatively small.

As we have seen, uncertainty in prices of non-durable goods may be desirable or undesirable, according to the structure of the uncertainty and the form of preferences. This in itself is not a new idea. What is new is the decomposition with which it is presented. This is the familiar framework of the Arrow-Pratt measure of absolute risk aversion, expanded into price space. Unfortunately, this analysis may be unrealistic insofar as it allows the consumer too much freedom in his decision-making. The consumer in these examples is not affected by liquidity constraints, nor by the need to purchase consumer durables. In practice it is probably factors such as liquidity constraints, the need to purchase durables and the cost of acquiring information in an uncertain environment that generate the greatest costs in an uncertain environment. A more complete analysis would add to these other factors to the PARP.

Appendix.

A Homogeneity in the Demand System.

Conjecture 1 *If all prices and income vary in a collinear manner, then the risk premium associated with avoiding this form of uncertainty is equal to zero, i.e. if* $\Psi = \lambda\bar{\Psi}$*, where* λ *is a scalar and* Ψ*,* $\bar{\Psi}$ *are the vectors defined on page 4, then* $\pi = 0$*. In this special case, where the consumer faces stochastic income and prices but can observe income and prices at the point of purchase, a proportionate change in income and all prices will leave the consumer's utility unchanged.*

Proof. The indirect utility function is homogenous of degree zero implying[7] $V(\lambda\Psi) = \lambda^0 V(\Psi)$, for all λ, this can be expressed as $0 = \frac{\partial V}{\partial m}m + \sum_{i=1}^{n}\frac{\partial V}{\partial p_i}p_i$. By substituting in Roy's identity $(-\partial V/\partial p)/(\partial V/\partial m) = x_i$, this can be re-written as,

$$0 = \frac{\partial V}{\partial m}\left(m - \sum_{i=1}^{n} x_i p_i\right) \tag{10}$$

Homogeneity of degree zero in the indirect utility function also implies $\partial V/\partial\Psi$ is homogenous of degree minus one, implying[8] $\lambda(\partial V/\partial\Psi)(\lambda\Psi) = \partial V/\partial\Psi$. Differentiating with respect to λ gives the following two expressions:

$$\frac{\partial V}{\partial p_i} + \sum_{j=1}^{n}\frac{\partial^2 V}{\partial p_i \partial p_j}p_j + \frac{\partial^2 V}{\partial p_i \partial m}p_i = 0 \tag{11}$$

$$\frac{\partial V}{\partial m} + \frac{\partial^2 V}{(\partial m)^2}m + \sum_{j=1}^{n}\frac{\partial^2 V}{\partial m \partial p_j}p_j = 0 \tag{12}$$

The assumptions on the price variations imply:

$$\sigma_{p_i p_j} = p_i p_j \lambda^2, \quad \sigma_{p_i m} = p_i m \lambda^2, \quad \sigma_{mm} = m^2\lambda^2, \quad \forall i, j = 1...n \tag{13}$$

Substituting in equations (13) into equation (1) and rearranging gives:

$$\pi = \frac{-\lambda^2}{2\frac{\partial V}{\partial m}}\left(\sum_{i=1}^{n}\left(p_i\left(\frac{\partial^2 V}{\partial p_i \partial m}p_i + \sum_{j=1}^{n}\frac{\partial^2 V}{\partial p_i \partial p_j}p_j\right)\right) + m\left(\frac{\partial^2 V}{(\partial m)^2}m + \sum_{i=1}^{n}\frac{\partial^2 V}{\partial p_i \partial m}p_i\right)\right) \tag{14}$$

Substituting in equations (11) and (12) into the above expression, and then substituting in equation (10) gives: $\pi = \frac{1}{2}\frac{\lambda^2}{\frac{\partial V}{\partial m}}\left(\frac{\partial V}{\partial m}m + \sum_{i=1}^{n}\frac{\partial V}{\partial p_i}p_i\right) = 0$. Thus the risk premium for covariances that keep relative prices and income constant is equal to zero. **end proof.**

[7] see Dixit and Norman (1988), pages 312-313. Varian (1992), pages 17-18.

[8] Ibid.

B A Desirable Form of Price Uncertainty.

Proposition 3 *Consider a consumer who consumes just two goods. If income $\bar{m}$ is constant but prices fluctuate such that the following equality always holds: $p_1\bar{x}_1 + p_2\bar{x}_2 = \bar{m}$, where $\bar{x}_1$ and $\bar{x}_2$ are fixed at levels optimal under the zero price variance case. Then, the expected risk premium is negative $\pi < 0$ and the consumer is better off than under the certainty price equivalent.*

Proof Assume there are price fluctuations where the following equality always holds: $p_1\bar{x}_1 + p_2\bar{x}_2 = \bar{m}$, this satisfies $p_1\bar{x}_1 = -p_2\bar{x}_2 + \bar{m}$, which can be re-written as:

$$\mathrm{var}(p_1\bar{x}_1) = \mathrm{var}(p_2\bar{x}_2) \tag{15}$$

and also satisfies $\mathrm{var}(p_1\bar{x}_1 + p_2\bar{x}_2) = 0$, which can be re-written as:

$$\mathrm{var}(p_1\bar{x}_1) + \mathrm{var}(p_2\bar{x}_2) + 2\mathrm{cov}(p_1\bar{x}_1, p_2\bar{x}_2) = 0 \tag{16}$$

Substituting equation (15) into equation (16) gives:

$$\begin{aligned} 2\mathrm{var}(p_1\bar{x}_1) + 2\mathrm{cov}(p_1\bar{x}_1, p_2\bar{x}_2) &= 0 \\ \mathrm{var}(p_1\bar{x}_1) = \mathrm{var}(p_2\bar{x}_2) &= -\mathrm{cov}(p_1\bar{x}_1, p_2\bar{x}_2) \\ \bar{x}_1^2\sigma_{p_1p_1} = \bar{x}_2^2\sigma_{p_2p_2} &= -\bar{x}_1\bar{x}_2\sigma_{p_1p_2} \end{aligned} \tag{17}$$

Any indifference curve of the indirect utility function may either be expressed as function of prices at constant level of utility as expressed, say, by: $V(p_1, p_2) = \bar{u}$, or the same indifference curve can be expressed by one price as a function of the other price and constant utility: $p_1 = f(p_2, \bar{u})$. Substituting the latter into the former gives: $V(f(p_2, \bar{u}), p_2) = \bar{u}$. Differentiating this with respect to p_2 provides a definition for the slope of the indifference curve $\frac{\partial V}{\partial p_1}\frac{\partial f}{\partial p_2} + \frac{\partial V}{\partial p_2} = 0$, which implies $\frac{\partial f}{\partial p_2} = -\frac{\partial V}{\partial p_2}/\frac{\partial V}{\partial p_1}$. Equating this with Roy's Identity provides the ratio of demands for the two goods at any given level of utility:

$$\frac{\partial f}{\partial p_2} = \left(\left. \frac{-\partial V/\partial p_2}{\partial V/\partial p_1} \right|_{\bar{u}} \right) = -\left(\left. \frac{-(\partial V/\partial p_2)/(\partial V/\partial m)}{-(\partial V/\partial p_1)/(\partial V/\partial m)} \right|_{\bar{u}} \right) = -\frac{x_2}{x_1} \tag{18}$$

The strict quasi-convexity condition for the indifference curve can therefore be expressed on (18) by the condition

$$\frac{\partial}{\partial p_2}\left(\left. \frac{-\partial V/\partial p_2}{\partial V/\partial p_1} \right|_{\bar{u}} \right) > 0 \tag{19}$$

Carrying out the differentiation, substituting in (18), substituting in Roy's identity and substituting in (17) gives

$$\begin{aligned} \frac{\partial \left(\left. \frac{-\partial V/\partial p_2}{\partial V/\partial p_1} \right|_{\bar{u}} \right)}{\partial p_2} &= \frac{1}{\left(\frac{\partial V}{\partial p_1}\right)^2} \left(\frac{-\partial V}{\partial p_2} \left(\frac{\partial V^2}{\partial p_1 \partial p_1}\frac{\partial f}{\partial p_2} + \frac{\partial V^2}{\partial p_1 \partial p_2} \right) + \frac{\partial V}{\partial p_1} \left(\frac{\partial V^2}{\partial p_1 \partial p_2}\frac{\partial f}{\partial p_2} + \frac{\partial V^2}{\partial p_1 \partial p_2} \right) \right) \\ &= \frac{\bar{x}_2 \left(\frac{\partial V}{\partial m}\right)}{\left(\frac{\partial V}{\partial p_1}\right)^2} \left(\frac{\partial V^2}{(\partial p_1)^2}\sigma_{p_1p_1} + \frac{\partial V^2}{(\partial p_2)^2}\sigma_{p_2p_2} + 2\frac{\partial V^2}{\partial p_1 \partial p_2}\sigma_{p_1p_2} \right) \end{aligned} \tag{20}$$

Applying property (19) to equation (20) implies inequality (21) holds.

$$\frac{\partial V^2}{(\partial p_1)^2}\sigma_{p_1p_1} + \frac{\partial V^2}{(\partial p_2)^2}\sigma_{p_2p_2} + 2\frac{\partial V^2}{\partial p_1 \partial p_2}\sigma_{p_1p_2} > 0 \tag{21}$$

Imposing $\mathrm{var}(\bar{m}) = 0$ and the inequality (21) on equation (1) defines a negative risk premium: $\pi = \left(\frac{1}{2}\frac{\partial V^2}{(\partial p_1)^2}\sigma_{p_1p_1} + \frac{1}{2}\frac{\partial V^2}{(\partial p_2)^2}\sigma_{p_2p_2} + \frac{\partial V^2}{\partial p_1\partial p_2}\sigma_{p_1p_2}\right) \Big/ \frac{-\partial V}{\partial m} < 0$. **end proof.**

C The Almost Ideal Demand System.

In this sub-section the equations that form the theoretical basis of the AIDS and of the preference structure for the representative household are presented. The logarithmic AIDS cost function taken from Deaton and Muellbauer (1980), is given in equation (22) below which can be expressed more compactly in terms of direct utility and the functions $a\{p\}$ and $b\{p\}$,

$$\ln c\{u, p\} = a\{p\} + ub\{p\} \tag{22}$$

where,

$$a\{p\} = \alpha_0 + \textstyle\sum_{k=1}^{n} \alpha_k \ln p_k + \frac{1}{2}\sum_{k=1}^{n}\sum_{j=1}^{n} \gamma_{kj}^* \ln p_k \ln p_j \tag{23}$$

$$b\{p\} = \beta_0 \textstyle\prod_{k=1}^{n} p_k^{\beta_k}. \tag{24}$$

the α's β's and γ's are parameters, m is nominal income, u is direct utility and the p's are prices for each commodity category. Taking the exponential of equation (22) gives the expenditure function in non logarithmic terms,

$$c\{u, p\} = \exp\left(a\{p\} + ub\{p\}\right) \tag{25}$$

Inverting the AIDS expenditure function gives the indirect utility function, $V\{m, p\}$, represented by equation (26).

$$V\{m, p\} = \frac{\ln m - a\{p\}}{b\{p\}} \tag{26}$$

In order to estimate the demand system, expressions for the budget shares for each commodity category are defined. This is done by differentiating equation (22) with respect to $\ln p_i$ and substituting in $u\beta_0 \prod_{k=1}^{n} p_k^{\beta_k} = \ln(M/P)$, where $\ln P = \alpha_0 + \sum_{k=1}^{n} \alpha_k \ln p_k + \frac{1}{2}\sum_{k=1}^{n}\sum_{j=1}^{n} \gamma_{kj}^* \ln p_k \ln p_j$. This gives equation (27) which is equation (8) in Deaton and Muellbauer.

$$w_i = + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln(m/P)\alpha_i \tag{27}$$

where $w_i = p_i q_i / c\{u, p\}$ is the budget share for good i and $\gamma_{ij} = \frac{1}{2}(\gamma_{ij}^* + \gamma_{ij}^*)$. Note that the expression for γ_{ij} is not the condition of symmetry described below, it is a condition satisfied by construct in the system, see Deaton and Muellbauer (1980) page 313. Equation (27) represents the system of equations that are generally estimated in these kinds of models. To facilitate estimation, most researchers replace the exact price index $\ln P = \alpha_0 + \sum_{k=1}^{n} \alpha_k \ln p_k + \frac{1}{2}\sum_{k=1}^{n}\sum_{j=1}^{n} \gamma_{kj}^* \ln p_k \ln p_j$. with the Stone Price

Index Approximation $\ln P = \sum_{k=1}^{n} w_k \ln p_k$. Indeed, this substitution is used in the original work by Deaton and Muellbauer.

To be consistent with the theory of consumer demand the demand system should satisfy the restrictions of additivity, homogeneity and symmetry. Additivity simply requires that the sum of all individual expenditures equals total expenditure. Homogeneity is the requirement that all demand functions in the system should be homogenous of degree zero, such that if all prices and total expenditure change by the same multiplicative term then demands do not change. Symmetry requires that the compensated cross price derivatives between two goods should be equal, for example, the compensated cross price derivative of potatoes with respect to a change in the price of fish should equal the compensated cross price derivative of fish with respect to a change in the price of potatoes. These imply the following restrictions on the AIDS model:

$$\begin{array}{rl} \text{Additivity:} & \sum_{i=1}^{n} \alpha_i = 1 \text{ and } \sum_{i=1}^{n} \beta_i = \sum_{i=1}^{n} \gamma_{ij} = 0 \\ \text{Homogeneity:} & \sum_{j=1}^{n} \gamma_{ij} = 0 \\ \text{Symmetry:} & \gamma_{ij} = \gamma_{ji}, \qquad \forall \quad i, j. \end{array}$$

Additivity is satisfied by construct in the dataset, homogeneity and symmetry must be imposed and tested for.

The demands and the preference structure for the representative household can be derived from the structure of the AIDS by the transformations presented below. Differentiating the expenditure function, given by equation (25), with respect to price gives the Hicksian (compensated) demands.

$$x_i^h = \frac{\partial c\{u, p\}}{\partial p_i} = \left(\frac{\partial a\{p\}}{\partial p_i} + \frac{\partial b\{p\}}{\partial p_i} \right) \exp\left(a\{p\} + b\{p\}\right) \tag{28}$$

where the partial derivatives (29) and (30) are given by differentiating (23) and (24),

$$\frac{\partial a\{p\}}{\partial p_i} = \frac{\alpha_i}{p_i} + \frac{1}{p_i} \sum_{i=1}^{n} \frac{\gamma_{ij}^* + \gamma_{ji}^*}{2} \ln p_i = \frac{\alpha_i + \sum_{i=1}^{n} \gamma_{ij} \ln p_i}{p_i} \tag{29}$$

$$\frac{\partial b\{p\}}{\partial p_i} = \frac{\beta_0 \prod_{k=1}^{n} p_k^{\beta_k}}{p_i} = \frac{b\{p\}}{p_i} \tag{30}$$

By substitution, equation (28) can be re-written more compactly as: $x_i^h = \frac{w_i}{p_i} c\{u, p\}$, this serves as a consistency check on this equation. The compensated price elasticities are derived by differentiating the Hicksian demands with respect to prices and substituting in $u\beta_0 \prod_{k=1}^{n} p_k^{\beta_k} = \ln(m/P)$,

$$e_{ij}^h = \frac{\partial x_i^h}{\partial p_j} \frac{p_j}{x_i^h} = \left(w_i w_j + \gamma_{ij} + \beta_i \beta_j \ln\left(\frac{m}{P}\right) \right) \frac{1}{w_i} - \delta_{ij} \tag{31}$$

where $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$.

The Marshallian demands are derived using Roy's Identity $x_i = \frac{-\partial V/\partial p_i}{\partial V/\partial m}$. Differentiating the indirect utility function (26) produces $\partial V(p, m)/\partial m_t = 1/m_t b\{p\}$ and $\partial V(p, m)/\partial p_{t,i} = \left(\alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_i + \beta_i \left(\ln m - a\{p\} \right) \right) / \left(-p_i b\{p\} \right)$. The Marshallian demand for good i is given by the ratio of these two derivatives,

$$x_{t,i} = \frac{-\partial V/\partial p_{i,t}}{\partial V/m_t} = \frac{m}{p_i} \left(\alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_i + \beta_i \ln\left(\frac{m}{P}\right) \right) \tag{32}$$

As a consistency check, see that equation (32) can be expressed more compactly as $x_{t,i} = (w_{t,i}/p_{t,i})m_t$, which is clearly correct.

The income elasticity of demand is given by differentiating the Marshallian demand with respect to income:

$$e_{it}^{m} = \frac{\partial x_{it}}{\partial m_t}\frac{m_t}{x_{it}} = 1 + \beta_i/w_{it} \tag{33}$$

Equation (33) corresponds to the formulation for income elasticity reported in Anderson and Blundell (1983), table I.

D Relative Risk Aversion.

The parameter estimates of the AIDS cannot be used to "pin" down consumer's attitudes to risk with respect to income effects. This is because the indirect utility function, specified by equation (26), depends on cardinal properties of preferences. This means that the direct utility function can undergo any monotonic transformation and still be a valid representation of preferences. For example, equation (22) can be re-written as $\ln c\{u,p\} = a\{p\} + f\{u\}b\{p\}$ where $df\{u\}/du > 0$. This also implies that a valid transformation for equation (26) is $V\{p,m\} = f^{-1}\left\{\frac{\ln m - a\{p\}}{b\{p\}}\right\}$. In order to compute general case for the Arrow-Pratt measures of ARA and RRA, one needs the derivatives of the indirect utility function, subject to the transformation $f\{.\}$, these give,

$$\begin{aligned}\frac{dV(p,m)}{dm_t} &= \frac{\partial f^{-1}\left\{\frac{\ln m - a\{p\}}{b\{p\}}\right\}}{\partial m_t}\frac{1}{m_t b\{p\}} \\ \frac{d^2V(p,m)}{(dm_t)^2} &= \frac{\partial^2 f^{-1}\left\{\frac{\ln m - a\{p\}}{b\{p\}}\right\}}{(\partial m_t)^2}\frac{1}{m_t^2 b\{p\}^2} - \frac{\partial f^{-1}\left\{\frac{\ln m - a\{p\}}{b\{p\}}\right\}}{\partial m_t}\frac{1}{m_t^2 b\{p\}}\end{aligned}$$

The ratio of these two partial derivatives gives the Arrow-Pratt measures of absolute risk aversion (ARA) and relative risk aversion (RRA) for the AIDS,

$$ARA = \frac{-d^2V/dm_t^2}{dV/dm_t} = \left(1 - \frac{\partial^2 f^{-1}\left\{\frac{\ln m - a\{p\}}{b\{p\}}\right\}}{(\partial m_t)^2} \div \frac{\partial f^{-1}\left\{\frac{\ln m - a\{p\}}{b\{p\}}\right\}}{\partial m_t} b\{p\}\right)\frac{1}{m_t} \tag{34}$$

$$RRA = 1 - \frac{\partial^2 f^{-1}\left\{\frac{\ln m - a\{p\}}{b\{p\}}\right\}}{(\partial m_t)^2} \div \frac{\partial f^{-1}\left\{\frac{\ln m - a\{p\}}{b\{p\}}\right\}}{\partial m_t} b\{p\} \tag{35}$$

Given these difficulties, the simple strategy of calibrating the model so that $RRA = 1$ is used. A constant coefficient of RRA implies that agents' risk aversion is proportional to their wealth and much of the literature suggest a range of $\frac{1}{2} < RRA < 2$, see Siegel and Hoban (1982), Choi and Menezes (1985), Szpiro (1986) and Gollier et al. (1997) for evidence of this. Calibrating the coefficient of RRA to equal one seems a reasonable compromise. Different values of RRA simply alter the value of the component π_{ie} in equation (6) by a scalar multiplicative term.

E Parameter Estimates for the AIDS.

Parameter estimates for equation (27) are repored in Table C, these are the values for the α's, β's and γ's. These parameter estimates are used to generate the preference structure of the representative household reported in Table (2). To avoid the problem of collinearity between the equations, the system is estimated using just the first 37 equations. Parameters for the 38^{th} equation are calculated using the property of additivity reported in Appendix C.

The restrictions of Homogeneity and Symmetry are rejected both individually and jointly. Rejection of the the joint hypothesis of homogeneity and symmetry is strong with $F(703, 3700) = 9.87$ for the test. Despite this, these restrictions are imposed (using parameter estimates of the unrestricted AIDS did not substantially alter the final estimates of the PARP). Rejection of the hypotheses of homogeneity and symmetry may arise for any number of reasons, such as measurement error, aggregation over households or aggregation over products. However, in this case the most likely reason for rejection arises from the non-stationary nature of the data. Most estimates of aggregate AIDS's typically are for systems with just 4 or 5 product categories (rather than the 38 used here) and use seasonally adjusted data. Using seasonally adjusted data permits the use of the Fully Modified Least Squares Estimator of Phillips and Hansen (1990) to remove the bias arising from non-stationarity. However, seasonal adjustment can affect parameter estimates. Attempts at estimating this demand system using seasonally adjusted data lead to parameter estimates that often implied the existence of positive compensated own price elasticities.

	$w_{1.1}$	$w_{1.2}$	$w_{1.3}$	$w_{1.4}$	$w_{1.5}$	$w_{1.6}$	$w_{1.7}$	$w_{1.8}$	$w_{1.9}$	$w_{1.10}$	$w_{1.11}$	$w_{1.12}$	$w_{1.13}$	$w_{2.1}$	$w_{2.2}$	$w_{2.3}$	$w_{2.4}$	$w_{2.5}$	$w_{3.1}$
	Dependent variable: Expenditure share (w_i).																		
	Parameter estimates, (γ_{ij}'s, β_i's and α_i's).																		
$\ln p_{1.1}$	-0.021	0.003	-0.010	-0.003	0.002	0.003	-0.000	0.007	-0.001	-0.011	-0.000	0.002	-0.000	-0.012	-0.002	0.008	-0.005	0.010	-0.023
$\ln p_{1.2}$	0.003	-0.065	-0.000	0.005	-0.003	0.005	-0.002	0.008	-0.005	-0.006	-0.003	0.013	0.006	-0.033	0.007	0.025	-0.000	0.001	-0.002
$\ln p_{1.3}$	-0.010	-0.000	-0.014	-0.007	0.001	0.001	-0.001	-0.002	0.000	-0.002	-0.001	-0.001	-0.003	0.004	0.003	0.010	-0.000	0.000	0.012
$\ln p_{1.4}$	-0.003	0.005	-0.007	-0.012	0.004	0.002	0.000	0.005	0.001	-0.003	-0.002	0.003	-0.011	-0.014	0.031	0.008	-0.004	-0.009	-0.010
$\ln p_{1.5}$	0.002	-0.003	0.001	0.004	-0.001	0.001	0.000	0.000	0.001	0.001	-0.001	0.002	-0.002	-0.002	-0.000	0.001	-0.000	0.001	-0.008
$\ln p_{1.6}$	0.003	0.005	0.001	0.002	0.001	-0.011	-0.002	0.007	-0.001	-0.003	0.000	0.004	0.000	-0.003	0.015	-0.005	-0.001	0.004	-0.006
$\ln p_{1.7}$	-0.000	-0.002	-0.001	0.000	0.000	-0.002	-0.004	0.003	-0.000	0.001	0.000	0.001	-0.000	0.003	-0.000	0.001	-0.001	0.004	0.001
$\ln p_{1.8}$	0.007	0.008	-0.002	0.005	0.000	0.007	0.003	-0.005	0.000	-0.013	0.001	0.009	0.001	0.005	0.003	-0.007	0.000	0.010	-0.013
$\ln p_{1.9}$	-0.001	-0.005	0.000	0.001	0.001	-0.001	-0.000	0.000	-0.002	0.000	-0.000	0.001	0.001	-0.002	0.002	0.003	0.000	-0.001	-0.004
$\ln p_{1.10}$	-0.011	-0.006	-0.002	-0.003	0.001	-0.003	0.001	-0.013	0.000	0.012	0.001	-0.003	-0.005	0.012	0.022	0.003	-0.001	-0.013	0.014
$\ln p_{1.11}$	-0.000	-0.003	-0.001	-0.002	-0.001	0.000	0.000	0.001	-0.000	0.001	-0.004	0.001	-0.001	0.002	-0.001	0.002	-0.001	-0.001	-0.002
$\ln p_{1.12}$	0.002	0.013	-0.001	0.003	0.002	0.004	0.001	0.009	0.001	-0.003	0.001	-0.005	0.001	0.012	0.003	0.001	-0.004	0.004	-0.011
$\ln p_{1.13}$	-0.000	0.006	-0.003	-0.011	-0.002	0.000	-0.000	0.001	0.001	-0.005	-0.001	0.001	-0.009	-0.006	0.012	-0.000	0.000	0.002	0.008
$\ln p_{2.1}$	-0.012	-0.033	0.004	-0.014	-0.002	-0.003	0.003	0.005	-0.002	0.012	0.002	0.012	-0.006	-0.017	0.039	0.016	-0.006	-0.070	0.002
$\ln p_{2.2}$	-0.002	0.007	0.003	0.031	-0.000	0.015	-0.000	0.003	0.002	0.022	-0.001	0.003	0.012	0.039	-0.124	-0.018	0.007	0.021	0.003
$\ln p_{2.3}$	0.008	0.025	0.010	0.008	0.001	-0.005	0.001	-0.007	0.003	0.003	0.002	0.001	-0.000	0.016	-0.018	-0.034	-0.001	0.010	-0.004
$\ln p_{2.4}$	-0.005	-0.000	-0.000	-0.004	-0.000	-0.001	-0.001	0.000	0.000	-0.001	-0.001	-0.004	0.000	-0.006	0.007	-0.001	-0.001	-0.000	0.002
$\ln p_{2.5}$	0.010	0.001	0.000	-0.009	0.001	0.004	0.004	0.010	-0.001	-0.013	-0.001	0.004	0.002	-0.070	0.021	0.010	-0.000	0.008	0.007
$\ln p_{3.1}$	-0.023	-0.002	0.012	-0.010	-0.008	-0.006	0.001	-0.013	-0.004	0.014	-0.002	-0.011	0.008	0.002	0.003	-0.004	0.002	0.007	0.038
$\ln p_{3.2}$	0.017	0.018	0.017	0.031	0.009	-0.005	0.004	-0.006	0.007	0.014	0.015	0.001	0.004	0.009	-0.009	-0.045	0.001	0.025	-0.001
$\ln p_{3.3}$	0.013	0.003	0.001	0.006	0.002	0.001	-0.002	0.000	0.001	-0.009	0.001	-0.002	-0.001	-0.009	-0.004	-0.009	0.000	0.012	-0.021
$\ln p_{4.1}$	-0.013	0.003	-0.010	-0.006	0.002	-0.002	0.002	-0.011	0.001	0.004	0.001	-0.009	-0.005	-0.034	0.050	0.026	0.003	-0.009	0.007
$\ln p_{4.2}$	0.001	0.009	-0.002	-0.003	-0.000	0.002	0.001	0.002	-0.000	-0.003	-0.002	-0.002	-0.003	-0.001	0.006	0.007	0.001	-0.011	-0.002
$\ln p_{5.1}$	0.004	0.001	-0.005	-0.003	-0.001	0.001	-0.001	0.001	-0.001	-0.005	-0.002	0.001	0.000	0.000	-0.000	0.001	-0.001	-0.000	-0.018
$\ln p_{5.2}$	0.006	-0.007	0.006	-0.004	-0.005	0.001	0.001	0.003	0.002	-0.007	-0.002	-0.001	-0.003	-0.009	-0.002	0.003	0.004	0.020	0.026
$\ln p_{5.3}$	0.010	0.003	-0.003	0.005	-0.001	0.003	-0.001	0.001	-0.001	0.007	-0.004	-0.002	0.001	0.013	-0.011	0.003	-0.003	-0.002	-0.013
$\ln p_{5.4}$	0.009	0.010	0.003	-0.014	0.000	-0.000	-0.001	-0.003	0.002	0.004	-0.000	-0.002	-0.003	0.013	-0.005	0.013	0.000	-0.004	-0.000
$\ln p_{6.1}$	-0.007	-0.008	0.003	-0.005	-0.003	-0.004	-0.000	-0.009	-0.001	0.012	0.000	-0.003	-0.003	0.009	-0.002	0.004	0.001	-0.010	0.026
$\ln p_{6.2}$	-0.010	-0.014	-0.007	-0.003	-0.001	0.002	-0.001	0.003	-0.002	-0.006	-0.003	-0.002	-0.000	0.015	0.003	0.005	-0.002	-0.006	0.003
$\ln p_{6.3}$	0.002	-0.003	-0.001	0.002	-0.001	-0.000	-0.000	0.000	-0.000	-0.000	-0.002	0.001	0.001	0.004	0.001	-0.002	-0.001	-0.002	-0.007
$\ln p_{7.1}$	0.004	0.002	-0.002	-0.001	-0.001	0.000	0.000	0.001	-0.000	-0.001	-0.001	-0.001	0.001	0.001	-0.002	-0.000	-0.000	0.003	0.001
$\ln p_{7.2}$	0.007	0.004	0.000	0.000	-0.001	0.001	-0.001	-0.000	-0.001	-0.002	-0.001	-0.001	0.001	0.000	0.003	-0.007	0.001	-0.001	-0.011
$\ln p_{7.3}$	0.004	0.003	0.005	-0.007	-0.002	-0.001	-0.001	0.000	0.000	-0.008	0.001	-0.004	0.005	0.002	-0.002	-0.008	0.004	0.002	0.007
$\ln p_{8.1}$	0.008	0.009	0.006	0.011	0.000	-0.003	0.001	-0.003	0.001	0.002	0.004	-0.000	0.005	0.003	-0.004	-0.010	-0.000	0.004	-0.011
$\ln p_{8.2}$	-0.001	0.004	0.003	-0.002	0.001	-0.006	-0.002	-0.002	0.000	-0.000	0.002	-0.001	-0.002	0.011	-0.016	-0.002	-0.001	0.001	0.005
$\ln p_{8.3}$	0.010	0.004	-0.004	0.017	0.003	-0.001	0.001	0.006	0.001	0.000	0.004	0.001	0.001	0.038	-0.026	-0.003	0.001	0.002	-0.014
$\ln p_{8.4}$	-0.001	0.002	0.002	-0.001	0.001	0.001	-0.001	0.000	-0.002	-0.004	-0.001	0.001	0.002	-0.003	0.001	-0.002	-0.002	0.002	-0.001
$\ln p_{8.5}$	-0.008	0.001	-0.002	-0.008	-0.000	-0.000	-0.001	-0.001	-0.000	-0.007	-0.001	-0.007	0.003	0.008	-0.003	0.009	0.009	-0.013	0.016
$\ln \frac{m}{P}$	-0.018	-0.018	-0.011	-0.024	-0.002	-0.001	-0.001	-0.002	-0.001	-0.000	-0.005	-0.009	-0.009	-0.018	0.042	0.031	-0.003	-0.011	0.018
const.	0.169	0.205	0.092	0.215	0.025	0.025	0.020	0.039	0.011	0.027	0.047	0.087	0.079	0.216	-0.280	-0.207	0.025	0.142	-0.094
	t-statistics																		
$\ln p_{1.1}$	-2.500	0.380	-1.706	-0.310	0.800	0.434	-0.240	0.129	-0.268	-2.198	-0.041	0.381	-0.095	-2.115	-0.139	11.452	-0.433	0.125	-0.437
$\ln p_{1.2}$	0.134	-35.453	-0.027	2.068	-0.098	2.515	-0.569	0.716	-2.302	-1.372	-3.104	1.839	0.832	-3.015	0.672	4.682	-0.112	0.030	-0.101
$\ln p_{1.3}$	-1.403	-0.030	-11.348	-0.610	0.028	0.033	-0.508	-1.819	0.033	-1.563	-0.394	-0.140	-0.143	1.909	0.166	6.367	-0.002	0.008	0.253
$\ln p_{1.4}$	-0.190	0.802	-0.775	-3.009	0.564	0.337	0.017	1.161	0.219	-3.833	-0.426	1.611	-1.773	-1.264	15.121	1.146	-0.115	-0.150	-0.477
$\ln p_{1.5}$	0.733	-0.242	0.234	0.100	-0.608	0.090	0.015	0.075	0.506	0.754	-0.112	0.348	-0.127	-0.317	-0.003	0.539	-0.028	0.048	-0.031
$\ln p_{1.6}$	0.230	3.710	0.046	0.072	0.082	-1.665	-6.015	2.114	-0.769	-0.835	0.017	0.266	0.136	-0.213	3.386	-0.200	-0.429	0.069	-0.110
$\ln p_{1.7}$	-0.038	-0.190	-0.181	0.025	0.008	-0.026	-2.474	0.989	-0.654	0.109	0.085	0.117	-0.017	2.178	-0.015	0.025	-0.038	0.148	0.251
$\ln p_{1.8}$	0.393	2.798	-0.042	1.911	0.012	0.409	4.061	-1.880	0.519	-1.563	0.376	1.059	0.121	0.391	0.374	-0.985	0.022	0.035	-0.620
$\ln p_{1.9}$	-0.632	-0.412	0.003	0.027	0.196	-0.471	-0.300	0.530	-0.853	0.052	-0.024	0.329	0.048	-0.602	0.026	1.786	0.005	-0.018	-0.336
$\ln p_{1.10}$	-0.746	-1.312	-0.199	-0.276	0.016	-0.444	0.732	-29.667	0.125	5.268	0.254	-0.287	-2.699	0.760	0.373	0.180	-0.134	-2.355	0.968
$\ln p_{1.11}$	-0.005	-0.073	-0.344	-0.090	-0.044	0.033	0.178	1.263	-0.045	0.252	-0.753	0.108	-0.080	0.352	-0.056	0.261	-0.005	-0.041	-0.126
$\ln p_{1.12}$	0.102	0.353	-0.041	0.314	1.255	0.940	1.816	4.204	0.100	-0.313	0.634	-0.507	0.195	0.216	0.669	0.104	-0.180	0.331	-0.897
$\ln p_{1.13}$	-0.047	0.480	-0.293	-0.099	-0.232	0.074	-1.443	0.260	0.491	-1.999	-0.191	0.578	-0.539	-0.126	0.267	-0.066	0.020	0.127	1.873
$\ln p_{2.1}$	-0.170	-10.584	0.183	-0.615	-0.746	-0.772	11.563	0.892	-0.537	2.684	0.523	1.338	-0.970	-1.120	2.117	0.229	-0.585	-4.396	0.797
$\ln p_{2.2}$	-0.037	0.245	0.278	14.043	-0.012	11.053	-0.434	0.462	0.170	24.521	-0.179	1.282	0.191	10.197	-3.184	-1.219	1.392	1.561	0.666
$\ln p_{2.3}$	0.429	2.178	0.083	0.823	0.307	-7.935	0.417	-4.911	0.979	0.614	2.323	0.115	-0.009	0.481	-1.007	-24.547	-0.166	2.107	-0.342
$\ln p_{2.4}$	-1.010	-0.017	-0.003	-0.738	-0.087	-0.866	-0.563	0.099	0.017	-0.425	-0.110	-0.995	0.003	-0.411	0.034	-0.172	-0.125	-0.198	0.128
$\ln p_{2.5}$	0.232	0.072	0.024	-1.340	0.914	1.131	1.978	1.359	-0.831	-2.340	-0.642	0.089	0.492	-2.342	0.491	3.224	-0.087	1.672	0.296
$\ln p_{3.1}$	-1.316	-0.018	1.115	-1.569	-11.901	-0.969	2.641	-6.862	-0.565	22.256	-0.245	-0.304	0.219	0.123	0.727	-0.833	1.194	0.617	1.436
$\ln p_{3.2}$	0.465	0.670	3.139	5.665	6.655	-0.722	1.692	-1.526	1.668	3.020	6.186	0.086	0.276	0.061	-0.477	-11.019	1.045	1.250	-0.170
$\ln p_{3.3}$	0.729	1.321	0.173	2.900	0.671	0.149	-0.228	0.463	0.074	-6.424	0.026	-0.663	-0.033	-0.281	-0.442	-2.735	0.228	0.459	-2.692
$\ln p_{4.1}$	-0.068	0.308	-1.385	-6.129	0.361	-0.880	1.113	-2.746	1.139	0.681	0.070	-0.338	-0.356	-11.308	4.172	21.139	0.541	-0.302	0.306
$\ln p_{4.2}$	0.028	1.676	-0.330	-1.332	-0.061	2.860	0.137	0.773	-0.018	-1.239	-0.217	-0.216	-0.017	-0.101	0.537	11.903	0.103	-1.472	-0.334
$\ln p_{5.1}$	1.086	0.170	-2.282	-0.678	-0.067	0.280	-1.077	0.302	-0.705	-0.214	-1.083	0.048	0.014	0.024	-0.005	1.139	-0.091	-0.043	-1.387
$\ln p_{5.2}$	0.355	-0.923	5.351	-0.456	-2.497	1.785	0.201	5.391	0.268	-0.380	-0.133	-0.094	-0.966	-0.960	-0.424	1.107	0.305	0.604	10.365
$\ln p_{5.3}$	1.154	0.440	-1.428	0.432	-0.422	2.866	-0.319	0.346	-0.480	1.096	-0.655	-0.017	0.099	1.510	-6.136	0.476	-0.939	-0.260	-0.988
$\ln p_{5.4}$	0.737	4.615	0.676	-1.135	0.044	-0.782	-0.214	-2.516	0.062	2.588	-0.020	-0.061	-0.420	1.712	-1.482	1.830	0.012	-0.235	-0.020
$\ln p_{6.1}$	-0.650	-7.004	0.339	-1.686	-1.189	-2.946	-0.654	-1.707	-0.054	0.920	0.016	-1.401	-0.253	3.152	-0.280	0.455	0.037	-2.755	1.697
$\ln p_{6.2}$	-1.035	-6.300	-0.577	-0.822	-0.100	1.961	-0.214	1.521	-0.188	-1.007	-0.047	-0.175	-0.003	11.389	0.219	2.351	-0.167	-0.357	1.796
$\ln p_{6.3}$	0.708	-0.508	-0.052	0.195	-0.952	-0.010	-0.263	0.012	-0.124	-0.034	-0.122	0.235	0.126	1.418	0.056	-0.421	-0.029	-0.127	-0.533
$\ln p_{7.1}$	2.493	0.249	-0.459	-0.461	-0.182	0.551	0.028	0.037	-0.000	-0.128	-0.533	-0.196	0.181	0.199	-0.076	-0.009	-0.092	0.126	0.231
$\ln p_{7.2}$	1.969	0.327	0.013	0.088	-0.271	0.610	-0.430	-0.012	-0.080	-0.034	-0.092	-0.182	0.918	0.009	0.473	-1.805	0.040	-0.217	-0.586
$\ln p_{7.3}$	0.457	0.247	0.347	-6.849	-0.294	-3.880	-0.028	0.025	0.008	-0.603	0.454	-0.601	1.738	0.135	-0.769	-1.013	0.304	0.091	1.232
$\ln p_{8.1}$	0.518	2.720	1.394	1.920	0.538	-1.551	0.036	-0.273	0.169	2.038	1.045	-0.108	0.682	0.156	-0.406	-6.003	-0.001	0.807	-0.162
$\ln p_{8.2}$	-0.045	0.781	0.310	-0.501	0.141	-10.797	-0.447	-0.483	0.003	-0.006	0.463	-1.210	-0.129	2.603	-5.858	-0.278	-0.298	0.037	0.103
$\ln p_{8.3}$	0.451	0.250	-2.289	2.605	1.552	-0.165	0.476	0.564	0.044	0.148	1.015	0.362	0.070	4.352	-5.137	-0.621	0.064	0.294	-0.779
$\ln p_{8.4}$	-0.164	0.519	0.239	-1.486	0.126	0.275	-0.093	0.006	-0.998	-1.138	-0.922	0.131	0.130	-0.099	0.728	-0.238	-0.277	0.023	-0.325
$\ln p_{8.5}$	-1.501	0.083	-0.342	-1.473	-0.135	-0.152	-0.281	-0.024	-0.043	-2.037	-0.950	-0.810	0.762	1.025	-0.536	7.783	0.319	-0.192	0.426
$\ln \frac{m}{P}$	-1.034	-10.762	-1.100	-14.689	-0.084	-2.320	-0.100	-0.182	-0.302	-0.162	-4.084	-0.754	-1.373	-1.248	11.938	3.735	-0.295	-0.450	1.105
const.	35.579	23.081	79.391	28.350	1.117	6.826	4.424	37.652	2.080	25.350	15.592	6.652	3.827	74.406	-44.897	-85.641	0.244	23.423	-2.767
R_i^2	0.934	0.925	0.827	0.961	0.978	0.445	0.419	0.720	0.989	0.577	0.948	0.874	0.670	0.683	0.477	0.900	0.820	0.964	0.842
$\hat{\sigma}_i$	0.001	0.004	0.001	0.002	0.000	0.002	0.001	0.002	0.000	0.002	0.001	0.003	0.001	0.010	0.008	0.003	0.001	0.006	0.003

Almost Ideal Demand System estimated by Ordinary Least Squares, restricted for homogeneity (37 restrictions) and symmetry (666 restrictions). Restrictions rejected individually and jointly.

Table C: AIDS estimated by OLS.

$w_{3.2}$	$w_{3.3}$	$w_{4.1}$	$w_{4.2}$	$w_{5.1}$	$w_{5.2}$	$w_{5.3}$	$w_{5.4}$	$w_{6.1}$	$w_{6.2}$	$w_{6.3}$	$w_{7.1}$	$w_{7.2}$	$w_{7.3}$	$w_{8.1}$	$w_{8.2}$	$w_{8.3}$	$w_{8.4}$	$w_{8.5}$	
Dependent variable: Expenditure share (w_i).																			
Parameter estimates, (γ_{ij}'s, β_i's and α_i's).																			
0.017	0.013	-0.013	0.001	0.004	0.006	0.010	0.009	-0.007	-0.010	0.002	0.004	0.007	0.004	0.008	-0.001	0.010	-0.001	-0.008	$\ln p_{1.1}$
0.018	0.003	0.003	0.009	0.001	-0.007	0.003	0.010	-0.008	-0.014	-0.003	0.002	0.004	0.003	0.009	0.004	0.004	0.002	0.001	$\ln p_{1.2}$
0.017	0.001	-0.010	-0.002	-0.005	0.006	-0.003	0.003	0.003	-0.007	-0.001	-0.002	0.000	0.005	0.006	0.003	-0.004	0.002	-0.002	$\ln p_{1.3}$
0.031	0.006	-0.006	-0.003	-0.003	-0.004	0.005	-0.014	-0.005	-0.003	0.002	-0.001	0.000	-0.007	0.011	-0.002	0.017	-0.001	-0.008	$\ln p_{1.4}$
0.009	0.002	0.002	-0.000	-0.001	-0.005	-0.001	0.000	-0.003	-0.001	-0.001	-0.001	-0.001	-0.002	0.000	0.001	0.003	0.001	-0.000	$\ln p_{1.5}$
-0.005	0.001	-0.002	0.002	0.001	0.001	0.003	-0.000	-0.004	0.002	-0.000	0.000	0.001	-0.001	-0.003	-0.006	-0.001	0.001	-0.000	$\ln p_{1.6}$
0.004	-0.002	0.002	0.001	-0.001	0.001	-0.001	-0.001	-0.000	-0.001	-0.000	0.000	-0.001	-0.001	0.001	-0.002	0.001	-0.001	-0.001	$\ln p_{1.7}$
-0.006	0.000	-0.011	0.002	0.001	0.003	0.001	-0.003	-0.009	0.003	0.000	0.001	-0.000	0.000	-0.003	-0.002	0.006	0.000	-0.001	$\ln p_{1.8}$
0.007	0.001	0.001	-0.000	-0.001	0.002	-0.001	0.002	-0.001	-0.002	-0.000	-0.000	-0.001	0.000	0.001	0.000	0.001	-0.002	-0.000	$\ln p_{1.9}$
0.014	-0.009	0.004	-0.003	-0.005	-0.007	0.007	0.004	0.012	-0.006	-0.000	-0.001	-0.002	-0.008	0.002	-0.000	0.000	-0.004	-0.007	$\ln p_{1.10}$
0.015	0.001	0.001	-0.002	-0.002	-0.002	-0.004	-0.000	0.000	-0.003	-0.002	-0.001	-0.001	0.001	0.004	0.002	0.004	-0.001	-0.001	$\ln p_{1.11}$
0.001	-0.002	-0.009	-0.002	0.001	-0.001	-0.002	-0.002	-0.003	-0.002	0.001	-0.001	-0.001	-0.004	-0.000	-0.001	0.001	0.001	-0.007	$\ln p_{1.12}$
0.004	-0.001	-0.005	-0.003	0.000	-0.003	0.001	-0.003	-0.003	-0.000	0.001	0.001	0.001	0.005	0.005	-0.002	0.001	0.002	0.003	$\ln p_{1.13}$
0.009	-0.009	-0.034	-0.001	0.000	-0.009	0.013	0.013	0.009	0.015	0.004	0.001	0.000	0.002	0.003	0.011	0.038	-0.003	0.008	$\ln p_{2.1}$
-0.009	-0.004	0.050	0.006	-0.000	-0.002	-0.011	-0.005	-0.002	0.003	0.001	-0.002	0.003	-0.002	-0.004	-0.016	-0.026	0.001	-0.003	$\ln p_{2.2}$
-0.045	-0.009	0.026	0.007	0.001	0.003	0.003	0.013	0.004	0.005	-0.002	-0.000	-0.007	-0.008	-0.010	-0.002	-0.003	-0.002	0.009	$\ln p_{2.3}$
0.001	0.000	0.003	0.001	-0.001	0.004	-0.003	0.000	0.001	-0.002	-0.001	-0.000	0.001	0.004	-0.000	-0.001	0.001	-0.002	0.009	$\ln p_{2.4}$
0.025	0.012	-0.009	-0.011	-0.000	0.020	-0.002	-0.004	-0.010	-0.006	-0.002	0.003	-0.001	0.002	0.004	0.001	0.002	0.002	-0.013	$\ln p_{2.5}$
-0.001	-0.021	0.007	-0.002	-0.018	0.026	-0.013	-0.000	0.026	0.003	-0.007	0.001	-0.011	0.007	-0.011	0.005	-0.014	-0.001	0.016	$\ln p_{3.1}$
-0.153	0.003	0.030	0.013	0.025	-0.005	0.028	-0.003	-0.005	0.038	0.012	0.002	-0.005	-0.025	-0.031	0.001	-0.014	-0.006	-0.012	$\ln p_{3.2}$
0.003	-0.008	0.006	0.002	0.005	0.007	-0.004	-0.003	-0.003	0.004	0.002	-0.000	-0.002	-0.009	0.002	0.002	0.003	0.001	0.008	$\ln p_{3.3}$
0.030	0.006	-0.017	-0.003	-0.006	-0.015	-0.011	0.003	0.006	-0.006	-0.003	-0.004	0.001	-0.001	0.008	0.010	-0.001	-0.003	0.002	$\ln p_{4.1}$
0.013	0.002	-0.003	-0.017	-0.000	-0.009	-0.001	-0.009	0.001	0.002	0.001	-0.001	0.001	0.003	0.008	-0.001	0.003	0.001	0.010	$\ln p_{4.2}$
0.025	0.005	-0.006	-0.000	-0.004	0.002	0.003	-0.003	-0.000	-0.008	-0.000	0.001	0.007	0.004	0.007	-0.001	0.001	0.001	-0.006	$\ln p_{5.1}$
-0.005	0.007	-0.015	-0.009	0.002	-0.029	0.003	-0.011	0.009	0.009	-0.000	-0.001	0.003	-0.005	-0.000	0.008	0.001	0.006	-0.004	$\ln p_{5.2}$
0.028	-0.004	-0.011	-0.001	0.003	0.003	-0.027	-0.002	-0.003	-0.003	-0.001	-0.001	-0.002	0.007	0.008	0.001	0.002	-0.000	-0.001	$\ln p_{5.3}$
-0.003	-0.003	0.003	-0.009	-0.003	-0.011	-0.002	-0.004	0.007	-0.005	-0.005	-0.001	-0.002	0.018	0.004	0.001	-0.001	-0.001	-0.005	$\ln p_{5.4}$
-0.005	-0.003	0.006	0.001	-0.000	0.009	-0.003	0.007	0.002	0.002	-0.002	-0.002	-0.001	0.002	-0.000	0.001	-0.008	-0.002	-0.004	$\ln p_{6.1}$
0.038	0.004	-0.006	0.002	-0.008	0.009	-0.003	-0.005	0.002	-0.014	-0.004	-0.003	0.001	0.005	0.008	0.002	0.005	-0.002	-0.005	$\ln p_{6.2}$
0.012	0.002	-0.003	0.001	-0.000	-0.000	-0.001	-0.005	-0.002	-0.004	-0.002	0.001	-0.001	0.007	0.003	0.000	0.002	-0.001	-0.002	$\ln p_{6.3}$
0.002	-0.000	-0.004	-0.001	0.001	-0.001	-0.001	-0.001	-0.002	-0.003	0.001	-0.001	0.002	0.004	-0.001	-0.000	-0.000	0.001	0.000	$\ln p_{7.1}$
-0.005	-0.002	0.001	0.001	0.007	0.003	-0.002	-0.002	-0.001	0.001	-0.001	0.002	-0.002	0.002	-0.003	0.001	0.000	0.001	0.007	$\ln p_{7.2}$
-0.025	-0.009	-0.001	0.003	0.004	-0.005	0.007	0.018	0.002	0.005	0.007	0.004	0.002	-0.001	-0.004	-0.004	-0.010	0.003	-0.000	$\ln p_{7.3}$
-0.031	0.002	0.008	0.008	0.007	-0.000	0.008	0.004	-0.000	0.008	0.003	-0.001	-0.003	-0.004	-0.026	-0.000	-0.011	0.003	0.002	$\ln p_{8.1}$
0.001	0.002	0.010	-0.001	-0.001	0.008	0.001	0.001	0.001	0.002	0.000	-0.000	0.001	-0.004	-0.000	-0.012	-0.005	0.001	0.004	$\ln p_{8.2}$
-0.014	0.003	-0.001	0.003	0.001	0.001	0.002	-0.001	-0.008	0.005	0.002	-0.000	0.000	-0.010	-0.011	-0.005	-0.012	0.001	0.004	$\ln p_{8.3}$
-0.006	0.001	-0.003	0.001	0.001	0.006	-0.000	-0.001	-0.002	-0.002	-0.001	0.001	0.001	0.003	0.003	0.001	0.001	0.000	0.001	$\ln p_{8.4}$
-0.012	0.008	0.002	0.010	-0.006	-0.004	-0.001	-0.005	-0.004	-0.005	-0.002	0.000	0.007	-0.000	0.002	0.004	0.004	0.001	0.009	$\ln p_{8.5}$
0.079	0.004	-0.022	-0.025	-0.013	-0.009	-0.007	0.001	0.008	-0.015	-0.005	-0.003	0.004	0.013	0.020	0.005	0.016	-0.003	-0.005	$\ln \frac{m}{P}$
-0.507	-0.004	0.251	0.248	0.110	0.096	0.068	0.013	-0.053	0.124	0.041	0.027	-0.017	-0.064	-0.126	-0.021	-0.106	0.027	0.053	const.
t-statistics																			
1.120	6.083	-3.868	0.427	0.516	0.339	0.886	5.922	-0.759	-6.568	0.042	0.710	0.244	0.282	1.581	-0.186	4.429	-0.143		$\ln p_{1.1}$
0.101	0.352	1.057	7.326	0.090	-1.483	0.863	1.389	-6.813	-2.052	-0.060	0.046	0.320	2.699	1.478	2.412	0.785	0.347		$\ln p_{1.2}$
0.453	0.266	-3.480	-0.721	-0.256	0.467	-0.504	0.682	0.362	-2.895	-0.043	-0.070	0.000	0.914	0.946	3.061	-0.414	1.188		$\ln p_{1.3}$
8.969	0.928	-5.964	-0.427	-0.157	-0.107	3.994	-1.590	-2.176	-0.106	0.496	-0.028	0.016	-2.583	2.029	-1.088	1.429	-0.057		$\ln p_{1.4}$
0.560	0.346	4.075	-0.040	-0.104	-0.501	-0.227	0.444	-0.304	-0.028	-0.031	-0.050	-0.366	-0.519	0.222	0.203	0.199	0.016		$\ln p_{1.5}$
-0.702	0.244	-1.822	0.141	0.085	0.057	0.820	-0.025	-0.959	0.240	-0.001	0.000	0.060	-0.378	-2.834	-0.863	-0.307	0.077		$\ln p_{1.6}$
0.367	-0.857	0.902	0.032	-0.036	0.199	-0.113	-0.448	-0.009	-0.497	-0.010	0.003	-0.121	-0.174	0.282	-0.245	0.229	-0.032		$\ln p_{1.7}$
-0.566	0.387	-2.518	0.499	0.221	0.144	1.613	-0.282	-0.288	0.184	0.016	0.120	-0.007	0.029	-0.692	-0.223	0.693	0.005		$\ln p_{1.8}$
0.845	0.299	0.179	-0.014	-0.110	0.174	-0.225	0.574	-0.123	-0.259	-0.002	-0.000	-0.076	0.227	0.161	0.114	0.341	-0.050		$\ln p_{1.9}$
4.565	-1.952	0.693	-0.104	-2.028	-0.285	3.697	0.108	4.531	-0.432	-0.013	-0.067	-0.299	-6.764	0.233	-0.006	0.091	-0.182		$\ln p_{1.10}$
10.035	0.089	0.931	-0.216	-0.155	-0.754	-0.500	-0.010	0.004	-0.554	-0.592	-0.053	-0.230	0.529	0.379	0.119	3.952	-0.029		$\ln p_{1.11}$
0.331	-0.193	-1.379	-0.174	0.133	-0.049	-0.694	-0.153	-0.331	-0.028	0.090	-0.103	-1.023	-0.735	-0.078	-0.279	0.170	0.143		$\ln p_{1.12}$
0.569	-0.095	-0.265	-1.046	0.034	-0.540	0.048	-1.248	-0.118	-0.002	0.148	0.047	0.514	0.822	1.152	-0.186	0.372	0.068		$\ln p_{1.13}$
0.706	-3.133	-6.142	-0.098	0.096	-0.318	0.558	0.583	0.877	11.283	0.420	0.313	0.016	0.294	0.203	6.336	7.086	-0.269		$\ln p_{2.1}$
-0.496	-1.589	4.667	0.660	-0.004	-0.162	-1.280	-0.585	-0.022	0.544	0.123	-0.818	0.242	-1.359	-1.071	-1.745	-41.761	0.016		$\ln p_{2.2}$
-2.379	-1.241	12.358	0.437	0.396	0.033	1.212	0.665	0.153	1.521	-0.269	-0.031	-0.525	-1.723	-1.381	-0.379	-0.734	-0.147		$\ln p_{2.3}$
0.207	0.211	0.229	0.439	-0.058	0.047	-0.155	0.017	0.627	-0.423	-0.212	-0.036	0.051	0.290	-0.023	-0.079	0.985	-0.010		$\ln p_{2.4}$
5.582	3.006	-1.200	-0.761	-0.080	0.729	-0.310	-0.042	-0.982	-1.639	-1.157	0.153	-0.137	0.417	0.601	0.749	0.392	0.016		$\ln p_{2.5}$
-0.081	-25.243	0.552	-0.531	-0.285	3.890	-0.799	-0.009	5.387	0.978	-2.559	0.038	-3.761	0.962	-2.189	0.574	-6.427	-0.031		$\ln p_{3.1}$
39.011	0.775	19.732	0.676	0.510	-0.088	3.799	-1.295	-0.759	31.643	1.803	0.063	-0.547	-15.873	-3.662	0.250	-0.617	-0.553		$\ln p_{3.2}$
0.434	-2.950	0.535	0.244	0.285	0.285	-0.043	-0.288	-0.466	6.333	0.165	-0.014	-0.943	-1.098	1.747	0.151	0.160	0.012		$\ln p_{3.3}$
19.914	1.211	-5.213	-0.041	-1.445	-0.283	-0.622	0.658	1.133	-4.980	-0.174	-0.560	0.300	-0.145	1.107	2.318	-0.128	-0.070		$\ln p_{4.1}$
1.688	2.759	-0.188	-0.307	-0.006	-0.395	-0.885	-1.587	0.462	0.534	0.032	-0.073	1.090	0.266	3.666	-0.027	2.178	0.006		$\ln p_{4.2}$
4.695	1.101	-1.176	-0.011	-0.245	0.007	0.346	-0.522	-0.406	-1.557	-0.035	0.177	1.375	3.787	0.675	-0.021	0.083	0.018		$\ln p_{5.1}$
-0.568	5.683	-0.268	-1.993	0.062	-0.511	0.850	-2.297	4.467	1.330	-0.026	-0.102	0.799	-0.598	-0.101	0.695	0.106	0.013		$\ln p_{5.2}$
26.316	-0.615	-0.252	-0.035	0.175	0.591	-5.423	-0.866	-0.580	-0.429	-0.039	-0.515	-0.332	3.042	0.233	0.227	0.129	-0.005		$\ln p_{5.3}$
-0.320	-1.309	0.195	-0.541	-0.018	-0.460	-0.313	-4.704	0.885	-2.927	-0.590	-0.125	-2.471	1.645	0.138	0.038	-0.186	-0.071		$\ln p_{5.4}$
-2.175	-0.142	1.629	0.024	-0.011	0.734	-0.664	4.088	0.186	0.360	-0.091	-0.228	-0.228	0.548	-0.019	0.087	-0.137	-0.052		$\ln p_{6.1}$
3.549	0.226	-0.185	0.091	-2.406	0.555	-2.149	-1.293	0.205	-0.678	-1.282	-0.230	0.529	0.135	3.522	0.084	0.401	-0.097		$\ln p_{6.2}$
3.290	0.339	-0.197	0.003	-0.010	-0.017	-1.716	-0.698	-0.513	-0.743	-0.107	0.865	-0.149	0.233	0.146	0.000	1.448	-0.040		$\ln p_{6.3}$
0.042	-0.062	-0.123	-0.036	0.130	-0.082	-0.717	-0.136	-0.556	-0.259	0.108	-0.073	0.875	0.405	-0.119	-0.003	-0.089	0.052		$\ln p_{7.1}$
-0.155	-0.181	0.105	0.268	0.658	0.539	-0.528	-0.149	-0.122	0.329	-0.047	0.614	-0.086	0.767	-0.136	0.039	0.081	0.047		$\ln p_{7.2}$
-2.352	-1.687	-0.005	0.147	0.429	-2.099	1.141	6.608	0.797	0.462	3.011	0.295	0.111	-0.034	-0.461	-1.573	-2.610	0.350		$\ln p_{7.3}$
11.798	0.156	0.261	0.911	0.814	-0.069	0.999	0.796	-0.037	1.078	0.172	-0.209	-0.393	-0.398	-0.277	-0.041	-3.109	0.777		$\ln p_{8.1}$
0.029	0.367	3.287	-0.104	-0.248	0.731	0.075	0.068	0.979	0.142	0.001	-0.007	0.603	-0.179	-0.022	-2.206	-1.592	0.181		$\ln p_{8.2}$
-1.471	0.050	-0.059	0.304	0.775	0.030	0.673	-0.246	-1.665	3.232	0.073	-0.012	0.016	-1.005	-5.901	-0.667	-10.645	0.068		$\ln p_{8.3}$
-0.311	0.101	-0.447	0.061	0.240	0.222	-0.111	-0.080	-0.619	-0.166	-0.133	0.095	0.179	0.026	0.357	0.211	2.452	0.014		$\ln p_{8.4}$
-1.249	7.526	0.230	2.881	-0.835	-0.140	-0.095	-3.405	-0.599	-1.548	-0.019	0.109	0.528	-0.011	0.420	0.663	3.634	0.027		$\ln p_{8.5}$
0.757	0.724	-2.589	-14.984	-1.050	-1.163	-1.920	0.127	12.040	-0.921	-0.068	-0.095	0.629	6.023	3.487	2.267	6.149	-0.063		$\ln \frac{m}{P}$
23.297	-1.683	34.766	72.978	6.682	26.206	9.410	2.353	-10.240	23.055	1.737	2.016	-0.274	-6.623	-22.613	-20.177	-23.024	2.128		const.
0.940	0.593	-0.016	0.800	0.900	0.921	0.849	0.507	0.529	0.966	0.940	0.982	0.923	0.966	0.937	0.495	0.798	0.744		R_i^2
0.005	0.002	0.028	0.006	0.001	0.003	0.001	0.002	0.002	0.001	0.000	0.000	0.001	0.001	0.002	0.003	0.003	0.001		$\hat{\sigma}_i$

Sample period is 1963q1 to 1997q4 (140 observations)
$\hat{\sigma}_i$ represents the root mean squared error for each equation.

Table C (continued): AIDS estimated by OLS.

References

Anderson, G. and Blundell, R. (1983). Testing restrictions in a flexible dynamic demand system: An application to consumers' expenditure in Canada. *Review of Economic Studies*, L:397–410.

Arrow, K. (1970). *Essays in the Theory of Risk Bearing*. Norton-Holland, Oxford, 3rd edition.

Choi, E. K. and Menezes, C. F. (1985). On the magnitude of relative risk aversion. *Economics Letters*, 18:125–128.

Copeland, T. E. and Weston, J. F. (1983). *Financial Theory and Corporate Policy*. Addison-Wesley, Reading Massachusetts, 2nd edition.

Deaton, A. and Muellbauer, J. (1980). An almost ideal demand system. *American Economic Review*, 70(3):312–326.

Dixit, A. K. and Norman, V. D. (1988). *The Theory of International Trade: A dual, General Equilibrium Approach*. Welwyn-Nisbett, Cambridge.

Duncan, G. T. (1977). A matrix measure of multivariate risk aversion. *Econometrica*, 45(4):895–903.

Gollier, C., Lindsey, J., and Zeckhauser, R. J. (1997). Investment flexibility and the acceptance of risk. *Journal of Economic Theory*, 76(2):219–241.

Karni, E. (1979). On multivariate risk aversion. *Econometrica*, 47(6):1391–1401.

Kihlstrom, R. E. and Mirman, L. J. (1974). Risk aversion with many commodities. *Journal of Economic Theory*, 8(3):361–388.

Kreps, D. M. (1990). *A Course in Microeconomic Theory*. Harvester-Wheatshelf, New York.

Phillips, P. C. B. and Hansen, B. E. (1990). Statistical inference in instrumental variables regression with i(1) processes. *Review of Economic Studies*, 57(1):99–125.

Pratt, J. W. (1964). Risk aversion in the small and in the large. *Econometrica*, 32(1):122–136.

Siegel, F. W. and Hoban, J. P. (1982). Relative risk aversion revisited. *Review of Economics and Statistics*, 64(3):481–487.

Szpiro, G. G. (1986). Relative risk aversion around the world. *Economics Letters*, 20:19–21.

Varian, H. R. (1992). *Microeconomic Analysis*. Norton, New York, London, 3rd edition.

Von Neumann, J. and Morgenstern, O. (1944). *The Theory of Games and Economic Behaviour*. Princeton University Press, Princeton.