EX-ANTE PRICE COMMITMENT WITH RENEGOTIATION IN A DYNAMIC MARKET

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Abstract. This paper studies the *endogenous* determination of the price formation procedure in markets characterized by *match-specific* heterogeneity; such heterogeneity captures, for example, markets in which sellers own differentiated commodities and buyers have heterogeneous preferences. Specifically, we study a dynamic, stochastic model of a market in which, in each time period, agents on one side (e.g., sellers) strategically choose whether or not to "post", or commit themselves to, *incomplete* price contracts *before* they encounter agents of the opposite type. After a pair of agents of the opposite types have encountered each other, their match-specific values from trading with each other are realised. If no price contract was posted, then the terms of trade (and whether or not it occurs) are determined by bilateral negotiations. Otherwise, depending upon the agents' match-specific trading values and equilibrium continuation payoffs, trade occurs (if it does) either on the terms specified in the posted contract or at a renegotiated price (when renegotiation of the posted, incomplete price contract is *mutually beneficial*). We study the Markov subgame perfect equilibria of this market game, and address a variety of issues such as the impact of market frictions on the equilibrium proportion of trades that occur at a price specified in the *ex-ante* posted contract rather than at a price determined by *ex-post* bargaining.

1. Introduction

The *procedure* (or mechanism) of price determination varies not only across markets but often also within the same market. For example, in housing markets, trade sometimes occurs at prices posted by sellers and at other times at prices determined by bilateral negotiations; some houses are even sold at auctions. In labour markets, firms often post

Date: June 14, 2003.

Key words and phrases. match-specific heterogeneity, *ex-ante* incomplete price contracts, mutually beneficial renegotiation, *ex-post* bargaining, large decentralized markets, Markov subgame perfect equilibrium.

We greatly appreciate helpful comments from participants in a seminar at Essex, and from V. Bhaskar and Melvyn Coles.

wages, and, depending upon the nature of the "match" between a firm and a worker, employment may occur either at the posted wage or at a renegotiated wage (when renegotiation is mutually beneficial — perhaps because the posted wage is too low while the worker has turned out to be a good match for the firm). What factors determine the procedure of price formation in any particular market? Under what circumstances can two or more pricing mechanisms *co-exist* in the same market? What role do market frictions play in determining the pricing mechanism?

This paper aims to address these and other issues in the context of frictional markets with *match-specific* heterogeneity. Such heterogeneity is meant to capture, for example, markets in which sellers own differentiated commodities and buyers have heterogeneous preferences. When embedded in the context of endogenous price determination, it leads us to develop and explore a model that is quite different from the other models in the relatively small literature that studies the endogenous determination of the pricing mechanism.

The three main price formation procedures that are typically observed in real-life, and that have received the most attention from economic theorists are auctions, bargaining and price posting. A common feature of the enormous literature on models of decentralized markets, however, is that it takes the price formation procedure as *exogenously* given. Following Vickrey (1961) there is a large literature on models in which prices are determined via auctions; while following Diamond (1981), Mortensen (1982), and, Rubinstein and Wolinsky (1985) there is vast literature on models in which prices are determined by bilateral negotiations; and furthermore, following Diamond (1971) there is a literature on models in which prices are determined by price posting.

In models that allow for a price posting mechanism there is a potential, exogenously built-in *ex-post* inefficiency that arises from the fact that when a pair of agents meet they have to either trade at some convex combination of the two posted prices or not trade at all.[1] This implies that in an environment characterized by match-specific heterogeneity, it is possible that trade may not occur (since it might not be individually rational for at least one of the two agents to trade at such a price) although it might be mutually beneficial for the agents to trade (but at some other price). Thus, *ex-post* renegotiation of the terms of trade can be mutually beneficial. This is not surprising, since the

[1]This point also, it may be noted, applies to models in the literature following Diamond (1971) in which prices are *exogenously* assumed to be determined via a price posting mechanism — for two recent models in this literature, see Burdett and Mortensen (1998) and Masters (1999).

posted price is an incomplete (or, to be precise, non-comprehensive) contract — in that the posted price is not conditioned on the potential match-specific realisations of the agents' respective values from trading with each other. A key novel feature of our market model is that we allow for such mutually beneficial renegotiation to take place. This price formation procedure may be called the *contract posting cum renegotiation mechanism*.[2]

Another novel feature of our model is that we allow one side of the market (e.g., sellers) to choose whether to determine the terms of trade *ex-post* via a bilateral bargaining process, or to determine the terms of trade via the contract posting cum renegotiation mechanism. As indicated above, a main aim of this paper is to *endogenously* determine the pricing mechanism as part of the *market equilibrium*.

As mentioned above, there is a relatively small literature that studies the endogenous determination of the pricing mechanism. Specifically, this literature studies market models that allow for two of the three potential pricing mechanisms mentioned above. For example, while Wang (1993), and, Bulow and Klemperer (1996) study models in which the allowable pricing mechanisms are auctions and bargaining, Peters (1991), Bester (1993), Wang (1995) and, Ellingsen and Rosen (2003) — like us — allow for price posting and bargaining. They differ from ours in that they view these as distinct mechanisms to which participants have to commit ex ante. Under our contract-posting-cum-renegotiation-mechanism, whether the good is sold at a bargained or a posted price will depend on the realized value of trade to the participants.

In the next section we lay down the model, specify the equilibrium concept, and, as an instructive benchmark, characterize the unique equilibrium in the case when the posted price contracts are comprehensive — that is, the posted price is conditioned on the set of admissible pairs of match-specific trading values. Thereafter, the analysis concerns the set of equilibria under the more plausible case when the posted price contracts are incomplete.

In section 3 we derive some *general* results concerning the characteristics of an arbitrary market equilibrium. In particular, we shall show that in any market equilibrium, the pricing mechanism will be the contract posting cum renegotiation mechanism. Several results concerning the impact of market frictions will also be derived here. One key insight

[2]Interestingly, over fifteen years ago, Hart and Moore (1988) established the crucial role of mutually beneficial renegotiation in the context of an incomplete *bilateral* contracting model — for recent surveys of that literature, see Hart (1995) and Tirole (1999).

obtained is that when the matching rates of the two sides of the market are unequal, then aggregate market welfare would be maximised either when the agents with the relatively higher matching rate post incomplete price contracts or when the agents with the relatively lower matching rate post comprehensive price contracts. In particular, the posting of comprehensive price contracts by agents on the short side of the market adversely affects aggregate market welfare.

The issue of the existence of market equilibrium is addressed in section 4. Then, in sections 5 and 6 we derive — under various, alternative additional assumptions — some further results concerning the properties of a market equilibrium such as the impact of market frictions on the equilibrium proportion of trades that occur at the price specified in the posted incomplete contract. A main insight obtained here is that trade in markets with small frictions is likely to occur at negotiated prices, while in markets with large frictions it is more likely to occur at posted prices. An implication of this result — which appears to be consistent with real-life retail markets — is that in retail markets in which buyers search intensively (such as in housing markets) trade is more likely to occur at negotiated prices, while in retail markets in which their intensity of search is negligible (such as in the market for eggs) trade is more likely to occur at prices posted by the sellers. Section 7 summarizes, and discusses some of our key modelling assumptions. We relegate almost all of our technical arguments (formal proofs) to the Appendix.

2. The Model

The market considered in the model operates over an infinite number of discrete points in time with two types of agents, namely, "buyers" and "sellers", who are respectively denoted by type b and type s; there are a large number (formally, a continuum) of each type of agent. The market is in a steady state; that is, the numbers of buyers and sellers in the market are constant over time.[3]

An important feature of this market is the existence of *match-specific, payoff-relevant heterogeneity*: the value to an agent from trading with an agent of the opposite type depends on the nature of their specific match. Agents of the opposite types encounter each other through

[3]As is noted in Osborne and Rubinstein (1990, Part 2), such a steady state assumption may be interpreted as an approximation for the case in which the numbers of buyers and sellers are roughly constant, with any fluctuations being small enough to be ignored by the agents.

a random, pairwise matching process.[4] After they meet, their match-specific values are realised. The buyer's and the seller's values v_b and v_s from trading *with each other* are randomly (and independently) drawn from the distributions F_b and F_s respectively. Thus, if this pair of agents agree to form a match, and trade at price p, then the buyer's and the seller's payoffs are respectively $v_b - p$ and $p + v_s$. We assume that F_k $(k = b, s)$ has a bounded support, denoted by Σ_k; the infimum and the supremum of Σ_k are respectively denoted by $\underline{v}_k$ and $\bar{v}_k$. For notational convenience, we denote the Cartesian product of the supports, $\Sigma_b \times \Sigma_s$, by Σ. It will be assumed that $\bar{v}_b + \bar{v}_s > 0$; for otherwise no gains to trade will exist between any pair of agents.

Unlike the vast majority of models of decentralized markets, in our model each agent on one side of the market (either the sellers or the buyers) will have the *option* (in each time period) to post a price contract *before* encountering an agent of the opposite type (and hence, before the realisation of any match-specific values). If that option is not exercised by an agent, then the terms at which he trades are determined *ex-post* — after he encounters an agent of the opposite type and their match-specific values are realised — via a bilateral bargaining process. This choice is formally equivalent to posting an extreme price at which trade cannot be individually rational for any realized valuations of the good.

For a number of reasons discussed at great length in the *Theory of Incomplete Contracts* — see, for example, Hart (1995) and Tirole (1999) — the posted price contracts may necessarily be incomplete (or, more precisely, non-comprehensive); that is, the posted price may not be conditioned on the set Σ of pairs of match-specific values. For example, although the realised values v_b and v_s will be assumed to be observable by both of the agents, it might be too costly for a third party (such as the courts) to verify the realised values; as such a posted price contract in which the price is contingent on the (*ex-post*) realised values of v_b and/or v_s would be too costly to enforce. Our analysis will therefore centre on the (plausible) case in which the posted price contracts are incomplete, which gives rise to another key novel feature of our model, to which we now turn.

[4] There is a growing literature on *directed* search with posted prices (e.g. Moen 1997). In such models, buyers, for example see all the prices at the same time but which seller they approach is private information. Sellers have only one good to sell each period so failure of buyers to coordinate means that some potential trades are not realized. If agents are ex ante homogeneous (as in our model), symmetric equilibria are characterized by random matching. This suggests that, our results should be robust to allowing for directed rather than random search.

Incompleteness of the posted price contract gives rise to the possibility that for some realisations of the match-specific values there will be room for *mutually beneficial* renegotiation of the price specified in the posted contract. That is, although it will be mutually beneficial for the agents who have encountered each other to form a match and trade (given their match-specific realised values and given their equilibrium payoffs from not trading), it would not be individually rational for at least one of the agents to trade at the price specified in the posted contract. In order to give agents the opportunity to exploit any potential gains to trade that are realised *ex-post* (after they encounter each other), we allow them the option to engage in mutually beneficial renegotiation of the price specified in the posted contract.

The sequence of events that occur at each point in time t, where $t = 0, \Delta, 2\Delta, \ldots$, with $\Delta > 0$ (but small), may be conveniently described by the following five-stage process.

• **Stage 1: Post a Price Contract?** Each agent on one (and only one) side of the market — sellers, for example — simultaneously posts a price contract.[5] If such a contract is comprehensive, then the posted price contract is a function that specifies a price for each pair $(v_b, v_s) \in \Sigma$. However, as has been argued above, such a contract will typically be incomplete. In particular, the posted price contract will specify one real number; this number denotes a price which is independent of the match-specific pairs of agents' values.[6] We denote the type of agents who have this option to post price contracts by i, where $i = b$ or $i = s$; the other type of agents is denoted by j ($j \neq i$).

• **Stage 2: Random Pairwise Encounters.** Each seller meets a buyer with probability $\lambda_s\Delta < 1$, and with the complement probability $1 - \lambda_s\Delta$ the seller meets no one in this time period, where $\lambda_s > 0$ is a seller's meeting rate.[7] Similarly, each buyer meets a seller with probability $\lambda_b\Delta < 1$, and with the complement probability $1 - \lambda_b\Delta$ the buyer meets no one in this time period, where $\lambda_b > 0$ is a buyer's meeting rate.[8]

[5]The consequences of allowing both sides of the market to post price contracts is briefly discussed in the concluding section.

[6]It should be noted that posting a (constant) sufficiently high price or a sufficiently low price is formally equivalent to not posting any price contract; the terms of trade are (in such circumstances) always determined (at stage 4 below) through a bilateral bargaining process.

[7]That is, each seller meets a buyer according to a Poisson process with parameter λ_s. Similarly, each buyer meets a seller according to an independent Poisson process with parameter λ_b.

[8]The price contracts posted at stage 1 have no influence on the meeting process at stage 2. Indeed, the set of equilibria of our model are identical to the set of

• **Stage 3: Realisation of Match-Specific Values.** After a pair of agents of the opposite types encounter each other, their match-specific values are realised. As mentioned above, we model this in the following (standard) manner. The buyer's and the seller's values v_b and v_s respectively from trading with each other are randomly (and independently) drawn from the distributions F_b and F_s respectively. It is assumed that these values become common knowledge amongst them.[9]

• **Stage 4: Renegotiation?** The two agents now have the opportunity to decide whether or not to renegotiate the price specified in the posted contract. They will renegotiate if and only if both agree to do so — the decision to renegotiate is made simultaneously. If at least one agent refuses to renegotiate, then the process moves to stage 5. However, if both agents choose to renegotiate, then they engage in the following bargaining process. With equal probability, Mother Nature picks either agent to make an offer of a (new) price, which the other agent can accept or reject. In either case the process then moves to stage 5 (with a *new* agreed price, or the price specified in the posted contract).[10]

• **Stage 5: Trade?** The two agents simultaneously decide whether or not to form a match and trade.

Any agent who does not trade at time t waits until time $t + \Delta$ when the five-stage process recurs. Furthermore, any pair of agents who trade

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equilibria of the alternative model in which the contract posting process occurs after the (random) meeting process. What is important is that contract posting occurs before stage 3. The posted price contract is a legally enforceable contract. If the agents agree to match, and trade occurs, then either agent can, *if he wishes to*, enforce trade at the price specified in the posted contract. Neither party, however, can enforce trade — which is voluntary (i.e., following Hart and Moore (1988) amongst others, the posted contracts are *at-will* contracts).

[9]This is restrictive, but we focus here on other aspects of a problem that is already rather rich. The role of this assumption — and others — is discussed in the concluding section.

[10]Although unnecessarily complex to establish, all the results obtained in this paper would also hold if, instead, we adopt the more plausible (Rubinsteinian) infinite-horizon, alternating-offers process in which (i) the outcome associated with perpetual disagreement (or impasse) is that trade (if it occurs) has to occur at the posted price, and (ii) the players' respective "costs" from rejecting offers (as captured, for example, by their respective discount rates) are identical. This follows from the result (established, for example, in Muthoo (1999, p.189)) that the unique subgame perfect equilibrium of such a Rubinsteinian bargaining game is identical to the unique subgame perfect equilibrium of our simple, randomly determined proposer version of the "take-it-or-leave-it-offer" bargaining game in which each party has an *equal* chance of being the proposer.

at time t exit the market. The payoffs to an agent are as follows. If he never trades — and thus stays in the market forever — then his payoff is zero. But if he trades T time units after entering the market and obtains z units of money, then his payoff is $z \exp(-rT)$, where $r > 0$ denote's the agents' common discount rate.[11]

We should emphasize that the rules of renegotiation and trade determination defined in stages 4 and 5 have been chosen to capture in a simple, but rigorous manner the notion that the agents should have the opportunity to tear-up the posted price contract and write a new contract in order to consummate any *ex-post* realised gains to trade. In particular, the objective is to capture the following plausible, specific notions. Trade should be allowed to occur between a pair of agents when it is mutually beneficial for them to trade (given their *ex-post* realised values from trading with each other, and given their *equilibrium* payoffs from not trading with each other). Furthermore, if it is individually rational for a pair of agents to trade at the price specified in the posted contract then trade occurs at that price. On the other hand, if it is not individually rational for some agent to trade at the price specified in the posted contract, but trade is mutually beneficial, then the agents should be allowed to tear-up the posted contract and bargain over a new set of terms of trade, and trade at a renegotiated price. One could certainly write alternative, perhaps more plausible rules of renegotiation and trade determination that would also capture these ideas.

2.1. **The Equilibrium Concept.** The model that we have described above is, in effect, a stochastic, dynamic game with a continuum of players. At any point in time, a continuum of agents (of type i) engage in a simultaneous-move, contract posting process (as defined in stage 1). Subsequently, Nature determines a set of buyer-seller pairs, where each such pair engages in a dynamic, strategic process (as defined in stages 4 and 5). First, each pair simultaneously decide whether or not to renegotiate the terms of trade specified in the posted contract. Conditional on the outcome of that process, they may then engage in a price renegotiation process. Finally, they then simultaneously decide whether or not to trade.

We shall analyze the symmetric subgame perfect equilibria in Markov pure strategies of this stochastic, dynamic game. In addition, we rule out the use of weakly dominated strategies. A pure strategy for an

[11]Without loss generality, but in order to simplify the analysis, it is assumed that the payoff to an agent who trades at time t is realised at the end of the time period (i.e., at time $t + \Delta$).

agent of either type (buyer or seller) is a complete plan of action, for each and every eventuality the agent may find himself after entering the market, and it can depend on the personal history of the agent in arbitrary and complicated ways. A Markov (i.e., payoff-relevant) pure strategy, on the other hand, has the property that the action of an agent at any point in time t is independent of the history up until the end of time period $t - \Delta$, but can be conditioned on events that occur at time t. A Markov pure strategy for an agent of type k ($k = b, s$) is denoted by σ_k.

Given the symmetric nature of our model, we shall characterize those subgame perfect equilibria in which all buyers adopt the same Markov pure strategy and all sellers adopt the same Markov pure strategy. A symmetric (pure-strategy) Markov subgame perfect equilibrium is a pair (σ_b, σ_s) of pure Markov strategies such that for an arbitrary agent his equilibrium strategy (σ_b or σ_s, depending on whether the agent is a buyer or a seller) is optimal at any point in the market game given that all other buyers employ σ_b and all other sellers employ σ_s.[12] For convenience, we call a symmetric (pure-strategy) Markov subgame perfect equilibrium (in which no agent employs a weakly dominated strategy) a *Market Equilibrium* (ME, for short).

For any ME $\sigma = (\sigma_b, \sigma_s)$, let V_b and V_s denote the associated equilibrium expected payoffs to a buyer and a seller respectively at the beginning of any time period t. The sum of these equilibrium payoffs will capture some important properties of the ME, and therefore, for notational convenience, we denote this sum, $V_b + V_s$, by R.

2.2. **Equilibrium when Contracts are Comprehensive.** As a useful benchmark to the main case in which the posted price contracts are incomplete, in this section we characterize the unique ME in the case when the posted price contracts are comprehensive; that is, the posted price is a function of the match-specific pair (v_b, v_s) of trading values. Let $p_i^C(.)$ denote such a comprehensive price contract posted by a type i agent. It is first straightforward to show that in any ME the comprehensive posted price contract has the property that the type i agent extracts all of the surplus from any match.[13] Hence, it immediately

[12]It should be noted that by "optimal" we mean that he cannot profitably deviate to an alternative strategy, Markovian or *non-Markovian*.

[13]That is, in any ME in the case when the posted price contract is comprehensive, $p_b^C(v_b, v_s) = V_s - v_s$ and $p_s^C(v_b, v_s) = v_b - V_b$ — a formal proof of this result is available upon request. This property makes much intuitive sense, since, by being able to condition the posted price on the match-specific pair of trading values, the type i agent can design the comprehensive price contract in such a way that for

follows that in any ME, $V_j = 0$. Consequently, the Bellman equation for V_i is:

$$rV_i = \lambda_i \iint_{v_i+v_j \geq V_i} [v_i + v_j - V_i] dF_i dF_j. \tag{1}$$

Since the right-hand side of (1) is decreasing in V_i, it follows that there exists a unique solution to (1) in V_i. Hence, we have established the following result:

Lemma 1 (Comprehensive Posted Contracts). *Fix i, where $i = b$ or $i = s$. In the case in which the posted price contracts are comprehensive, there exists a unique ME. In equilibrium, the comprehensive posted price contract is*

$$p_i^C(v_b, v_s) = \begin{cases} -v_s & \text{if } i = b \\ v_b & \text{if } i = s. \end{cases}$$

The equilibrium payoff to a type i agent is V_i^C, where V_i^C is the unique solution to (1), and the equilibrium payoff to a type j agent is $V_j^C = 0$. In equilibrium, trade between a buyer and a seller occurs if and only if trade is mutually beneficial — that is, their match-specific pair of values is such that $v_i + v_j \geq V_i^C$. Furthermore, trade always occurs at the price specified in the equilibrium posted comprehensive price contract.

Notice that Lemma 1 is a statement of the Diamond Paradox for this more general environment (cf. Diamond, 1971). It will, however, be shown (in the next section) that the Diamond Paradox does *not* hold when the posted price contracts are incomplete, provided that F_j is non-degenerate.[14]

3. Characterization of Market Equilibria

We now begin our study of the main case in which the posted price contracts are incomplete. In this section we derive several results concerning the characteristics of an arbitrary ME. These results are general to the extent that they are valid for any pair of distributions F_b and F_s

any realisation $v_j \in \Sigma_j$, the type j agent is left indifferent between trading and not trading at the appropriate price specified in such a contract.

[14]Not surprisingly, if F_j is degenerate, then the unique ME when posted contracts are incomplete is (effectively) identical to the unique ME when posted contracts are comprehensive. After all, if there is no variation in the type j agents' trading value, then it does not matter to the type i agents whether they can post comprehensive or incomplete price contracts.

with bounded supports. However, in view of the result stated above, from now on we shall assume that the distribution F_j is non-degenerate (i.e., $\bar{v}_j > \underline{v}_j$).[15]

3.1. **Preliminaries.** The lemma stated below describes the circumstances under which in any ME a pair of agents will and will not renegotiate, will and will not form a match and trade, and, if they match, the terms of trade. The formal proof of Lemma 2 is straightforward, and follows from standard arguments; as such it is omitted.

Lemma 2. *Fix an arbitrary ME, and consider an arbitrary pair of agents who have met (at any time t) such that at stage 1 the price specified in the posted incomplete contract is $p \in \Re$ and such that the realised match-specific pair of values is $(v_b, v_s) \in \Sigma$. Then, in equilibrium:*
(a) if $v_b + v_s < V_b + V_s$, then the agents do not trade,
(b) if $v_b - p \geq V_b$ and $p + v_s \geq V_s$, then trade occurs at the price p specified in the posted contract, and
(c) if $v_b + v_s \geq V_b + V_s$ and either (i) $v_b - p < V_b$, or (ii) $p + v_s < V_s$, then the agents renegotiate, and, trade at price $V_s - v_s$ with probability one-half and at price $v_b - V_b$ with probability one-half.

Notice that Lemma 2(c) implies that if the "posted price" (i.e., the price specified in the posted contract) is either arbitrarily high or arbitrarily low, then, for any realisation of the pair $(v_b, v_s) \in \Sigma$, the terms of trade are determined by the bargaining process; the posted price has no influence whatsoever on the terms of trade.[16] Figure 1, which divides the (v_b, v_s) space according to the four possible equilibrium outcomes described in Lemma 2, may be useful when understanding the Bellman equations for the players' equilibrium expected payoffs, to which we now turn.

Fix an arbitrary ME, and consider an arbitrary agent of type i (at any time t) who has posted an arbitrary price contract $p \in \Re$. Using Lemma 2, we can now write his equilibrium expected payoff at the beginning of stage 2, before the random meeting process occurs; we denote it

[15] We should emphasize that the results derived in this section are valid whether F_i is degenerate or non-degenerate.

[16] Indeed, as mentioned above (in section 2), posting such a price contract is formally equivalent to not posting any price contract. The type i agent has, effectively, chosen not to post a price contract, but has chosen to determine the terms of trade *ex-post* (after he encounters an agent of the opposite type) via a bilateral bargaining process.

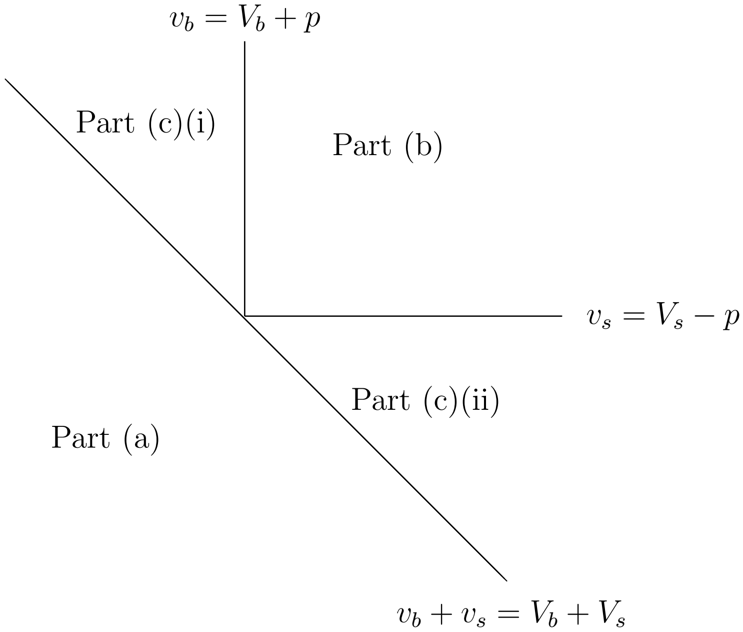

Figure 1. An illustration of Lemma 2.

by $\widehat{Z}_i(p, V_b, V_s)$. However, it is first convenient to introduce some set notation that partitions the (v_b, v_s) space appropriately. Define:

$$\Omega_N = \{(v_b, v_s) \in \Sigma : v_b + v_s < V_b + V_s\}$$
$$\Omega_P(p) = \{(v_b, v_s) \in \Sigma : v_b \geq V_b + p \text{ and } v_s \geq V_s - p\}$$
$$\Omega_R(p) = \{(v_b, v_s) \in \Sigma : v_b + v_s \geq V_b + V_s \text{ and either } v_b < V_b + p \text{ or } v_s < V_s - p\}.$$

Hence, after some simplification,

$$\widehat{Z}_i(p, V_b, V_s) = \frac{V_i}{1 + r\Delta} + \frac{\lambda_i \Delta Z_i(p, V_b, V_s)}{1 + r\Delta},$$

where

$$Z_i(p, V_b, V_s) = \iint_{(v_b, v_s) \in \Omega_P(p)} (v_i - \mathcal{I}p - V_i) dF_b dF_s + \iint_{(v_b, v_s) \in \Omega_R(p)} \left(\frac{v_b + v_s - V_b - V_s}{2} \right) dF_b dF_s, \tag{2}$$

with $$\mathcal{I} = \begin{cases} 1 & \text{if } i = b \\ -1 & \text{if } i = s. \end{cases}$$

Letting p_i^* denote the equilibrium posted price, it follows (by definition) that $V_i = \widehat{Z}_i(p_i^*, V_b, V_s)$. Hence, it follows that V_i satisfies the following Bellman equation:

$$rV_i = \lambda_i \Bigg[\iint_{(v_b,v_s)\in\Omega_P(p_i^*)} (v_i - \mathcal{I}p_i^* - V_i)dF_b dF_s + \iint_{(v_b,v_s)\in\Omega_R(p_i^*)} \left(\frac{v_b + v_s - V_b - V_s}{2}\right) dF_b dF_s \Bigg]. \tag{3}$$

Furthermore, optimality requires that

$$p_i^* = \arg\max_p Z_i(p, V_b, V_s). \tag{4}$$

Finally, using similar arguments to those used above, it can be shown that V_j (where $j \neq i$) satisfies the following Bellman equation:

$$rV_j = \lambda_j \Bigg[\iint_{(v_b,v_s)\in\Omega_P(p_i^*)} (v_j + \mathcal{J}p_i^* - V_j)dF_b dF_s + \iint_{(v_b,v_s)\in\Omega_R(p_i^*)} \left(\frac{v_b + v_s - V_b - V_s}{2}\right) dF_b dF_s \Bigg], \tag{5}$$

with
$$\mathcal{J} = \begin{cases} 1 & \text{if } j = s \\ -1 & \text{if } j = b. \end{cases}$$

We have thus established the following characterization of the set of Market Equilibria.

Proposition 1 (Characterization of Market Equilibria). *Fix i and j, where $i, j = b, s$ with $i \neq j$. For any solution (V_i, V_j, p_i^*) to (3) – (5) there exists a unique ME in which the equilibrium posted price contract is p_i^* and equilibrium expected payoffs to any agent of type i and any agent of type j are respectively V_i and V_j. There exist no other ME.*

3.2. **Equilibrium Pricing Mechanism.** Recall that type i agents choose the mechanism through which the terms of trade are determined. In particular, a ME can be one of the following two types. A ME in which the posted price is arbitrarily high (or arbitrarily low) has the property that the terms of trade are *always* (for any possible pair of match-specific values) determined via *ex-post* bargaining. On the other hand, a ME in which the posted price is, what may be termed, "serious" (in the sense that it is neither too low nor too high) has the property that at least some trades are executed at the *ex-ante* posted price while others at an *ex-post* renegotiated price. In this subsection, we establish that *any* ME is of the latter type.

It is useful to first introduce the concept of the agents' "reservation" values. In any ME, R_b and R_s — where $R_b = V_b + p_i^*$ and $R_s = V_s - p_i^*$ — may be respectively interpreted as a buyer's and a seller's *reservation* values. A buyer would like to trade with a seller at the equilibrium posted price p_i^* if and only if his realised value from trading $v_b \geq R_b$. Similarly, a seller would like to trade with a buyer at the equilibrium posted price p_i^* if and only if his realised value from trading $v_s \geq R_s$. Indeed, trade occurs at the equilibrium posted price p_i^* if and only if $v_b \geq R_b$ and $v_s \geq R_s$. Otherwise trade occurs at a renegotiated price or trade does not occur, depending on whether $v_b + v_s \geq V_b + V_s$ or $v_b + v_s < V_b + V_s$.

Let Θ denote the set of ME. It is instructive to classify the set of ME into the following subsets.[17]

$$\Theta_B = \{(\sigma_b, \sigma_s) \in \Theta : \text{ either } \bar{v}_b < R_b \text{ or } \bar{v}_s < R_s \text{ (but not both)}\}$$
$$\Theta_{PR} = \{(\sigma_b, \sigma_s) \in \Theta : \bar{v}_b \geq R_b \text{ and } \bar{v}_s \geq R_s\}.$$

For any ME $\sigma \in \Theta_B$, trade never occurs at the equilibrium posted price — either because it is too high ($\bar{v}_b < R_b$) or because it is too low ($\bar{v}_s < R_s$); it occurs at a negotiated price when gains to trade exist or not at all when gains to trade do not exist. Proposition 2 below implies that such a ME does not exist. This means that there does not exist a ME in which the agents of type i post an arbitrarily high price (or an arbitrarily low price) such that for all possible realisations $(v_b, v_s) \in \Sigma$, the terms of trade are determined *ex-post* (after a pair of agents of the opposite types meet each other) via bilateral bargaining. Proposition 2 states that $\Theta = \Theta_{PR}$. Thus, in any ME the agents of type i will post a price that is neither too high nor too low such that trade occurs at it for at least some realisations of the match-specific values; for other realisations it occurs at a renegotiated price or not at all (depending on whether or not gains to trade exist).

Proposition 2 (Equilibrium Pricing Mechanism)**.** *Fix i, where $i = b$ or $i = s$. If F_j is non-degenerate (i.e, $\bar{v}_j > \underline{v}_j$), then in any ME $\sigma \in \Theta$, $\bar{v}_b > R_b$ and $\bar{v}_s > R_s$, where $R_b = V_b + p_i^*$ and $R_s = V_s - p_i^*$.*

Proof. In the Appendix. □

[17] Note that since $\bar{v}_b + \bar{v}_s > 0$ and (in any ME) $\bar{v}_b + \bar{v}_s \geq V_b + V_s$, there cannot exist a ME in which $\bar{v}_b < R_b$ and $\bar{v}_s < R_s$.

To capture the essence of the proof, suppose that $i = b$, and, contrary to Proposition 2, that all buyers (such as firms in a labour market) are posting an arbitrarily low price (wage) which is unacceptable to all sellers (workers). In that case a firm and a worker split equally the match surplus (when gains to trade exist) via bargaining. Now suppose a firm unilaterally deviates and posts a wage equal to $V_s - \bar{v}_s + \epsilon$, where $\epsilon > 0$ but sufficiently small. It follows that there exist realisations of v_s (namely, $v_s \in [\bar{v}_s - \epsilon, \bar{v}_s] \bigcap \Sigma_s$) such that workers with such realisations would (like the firm) be willing to trade at such a wage. Since such a wage forces the worker down to (almost) his continuation payoff V_s, the firm would (for such realisations) now get *all* the surplus — rather than have to split it with the worker. The deviation is, therefore, profitable for the firm.[18]

At first blush, this result may seem trivial. After all, by allowing one side of the market to post price contracts (albeit incomplete) is like granting them market power which they are bound to use. To see why this result is interesting, consider once more Figure 1. For realisations of (v_b, v_s) in the regions marked Part (c), the agents bargain and any match surplus is divided equally. Within the region marked Part (b), trade occurs at the posted price and the division of the surplus depends on the actual realisation of the match-specific values v_b and v_s. In particular, given a posted price p, realisations of (v_b, v_s) toward the south-east of this region generates trade such that a seller would *ex-post* regret having posted that price — he would receive less than half the match surplus.[19] The price posting decision amounts to picking a location for the south-west corner of the Part (b) region on the $v_b + v_s = V_b + V_s$ line. The type i agent would like to minimize the probability of outcomes that he would (*ex-post*) regret *vis-a-vis* those which he would welcome. Proposition 2 implies that he can always find a (serious) price at which the benefits from posting it outweigh the opportunity cost of not bargaining.

Proposition 2 implies that in any ME, the equilibrium *proportion* of trades which occur at the posted price (rather than at a renegotiated price) is strictly positive, but (in general) it will be strictly less

[18]Thus, the contract posting cum renegotiation mechanism will be preferred by the firms over the *ex-post* bargaining mechanism, precisely because it allows them to extract a greater amount of surplus from *some* workers without affecting the surplus that they obtain from the others. Without the possibility of mutually beneficial renegotiation, this result may not hold.

[19]The converse is true for a buyer. Symmetrically, at that price, realisations to the north-west of the Part (b) region would lead to trade at which the buyer receives less than half the match surplus.

than one. It is therefore interesting to study how various parameters (such as those which capture market frictions) affect this equilibrium proportion. Such an analysis may, in particular, shed some light on the question of why in some markets trade typically occurs at posted prices (such as in some retail markets), while in other markets it typically occurs at negotiated prices (such as in bazaars and some labour markets). We shall address this issue in sections 5 and 6.

The result stated in the following corollary is a straightforward, immediate consequence of Proposition 2. We therefore omit its formal proof, and, instead provide some intuition for it after its statement.

Corollary 1. *Fix i, where $i = b$ or $i = s$. If F_j is non-degenerate (i.e., $\bar{v}_j > \underline{v}_j$), then in any ME, $V_j > 0$.*

Corollary 1 implies that, unlike in the Diamond Paradox (and unlike in the case of comprehensive contracting discussed above in section 2.2), the type i agents do not receive all the match surplus. The intuition behind this result comes partly from the observation that since price contracts are incomplete, renegotiation of the price specified in the equilibrium posted contract may occur for some (*ex-post*) realisations of the match-specific values.

3.3. **Role of Market Frictions.** We now derive some results concerning the role of the main parameters (namely, r, λ_b and λ_s) on various aspects of an arbitrary ME. Our first result concerns the role of the matching rates on *aggregate* market welfare. We define aggregate market welfare to be the sum of the payoffs of *all* the agents in the market (in a steady state). That is, aggregate market welfare is $W = V_b N_b + V_s N_s$, where N_b and N_s are respectively the (steady state) measures of buyers and sellers in the market. Since agents meet in pairs, it must be the case that $\lambda_b N_b = \lambda_s N_s$. Hence, after normalizing the measures so that $N_b + N_s = 1$, it follows that aggregate market welfare

$$W = \frac{\lambda_s V_b + \lambda_b V_s}{\lambda_b + \lambda_s}. \tag{6}$$

We now state our result concerning the role of the matching rates on aggregate market welfare in the following proposition; it will be discussed after its formal statement.

Proposition 3 (Matching Rates and Market Welfare)**.** *Assume that F_j is non-degenerate (i.e., $\bar{v}_j > \underline{v}_j$); and, for each $i = b, s$, let W_i^C denote the aggregate market welfare in the unique ME when the type i*

agents post comprehensive price contracts, and, let W_i^I *denote the aggregate market welfare in an arbitrary ME when the type i agents post incomplete price contracts.*
(a) If $\lambda_b = \lambda_s$*, then* $W_b^C = W_b^I = W_s^C = W_s^I$*.*
(b) If $\lambda_b < \lambda_s$*, then* $W_b^C > W_b^I$ *and* $W_s^I > W_s^C$*.*

Proof. See Appendix. □

Thus, interestingly, when the matching rates are identical, aggregate market welfare is the same whether the posted price contracts are incomplete or comprehensive. Not surprisingly, though, it does not matter which side of market gets to post the price contracts. It should be noted, however, that the distribution of welfare between the two sides of the market will crucially depend on whether contracts are incomplete or comprehensive (cf. Lemma 1 and Corollary 1).

Another important result contained in Proposition 3 is that when matching rates are not identical, the posting of comprehensive price contracts by agents with the higher matching rate adversely affects aggregate market welfare. The insight contained here is that when one side of the market has too much bargaining power — by not only being able to post comprehensive price contracts but also by having a relatively higher matching rate — aggregate market welfare and market efficiency is compromised. By making such agents post incomplete price contracts (with the crucial option to engage in mutually beneficial renegotation), on the other hand, gives some bargaining power to the other side of the market which has a lower matching rate.

A more general insight that one may extract from Proposition 3 is that the distribution of bargaining power amongst market traders has efficiency consequences. In particular, a social planner with an objective to maximise aggregate market welfare would not allow agents on the *short*-side of the market (i.e., agents who have the relatively higher matching rate) to post comprehensive price contracts. It may be worth noting that this insight suggests *a* potential explanation for the existence of incomplete contracts — namely, that such contracts help distribute bargaining power more evenly amongst the concerned parties.

It should be noted that Proposition 3 implies that aggregate market welfare would be maximised *either* (i) when the agents with the relatively higher matching rate post incomplete price contracts *or* (ii) when the agents with the relatively lower matching rate post comprehensive price contracts. We have not been able to establish whether it is the former or the latter that generates the higher aggregate market welfare.

The next proposition concerns the role of the matching rates on *each* agent's equilibrium payoff.

Proposition 4 (Matching Rates and Equilibrium Payoffs). *Fix i, where $i = b$ or $i = s$. If F_j is non-degenerate, then in any ME, if $\lambda_i \geq \lambda_j$ then $V_i > V_j$, and, if $\lambda_i < \lambda_j$ and the difference $\lambda_j - \lambda_i$ is not too large then (also) $V_i > V_j$.*

Proof. See Appendix. □

The results stated in this proposition follow since a type i agent extracts a relatively greater amount of surplus from *some* type j agents (i.e., for some realisations of v_j) by trading at the equilibrium posted price. Not surprisingly, the ability to post an incomplete price contract (even with the possibility of mutually beneficial renegotiation) gives a type i agent a relatively greater equilibrium payoff. And, this is valid even when there are far more agents of that type than agents of the opposite type in the market (which is captured by having $\lambda_j > \lambda_i$).

When the discount rate is arbitrarily small (close to zero), the market contains negligible frictions. The result contained in the following proposition addresses, in particular, this limiting case of negligible market frictions.

Proposition 5 (Discount Rate and Sum of the Equilibrium Payoffs). *Fix i, where $i = b$ or $i = s$. If F_j is non-degenerate, then in any ME, the sum R of the equilibrium payoffs to a pair of agents of the opposite types is strictly decreasing in r. Furthermore, $R \to \bar{v}_b + \bar{v}_s$ as $r \to 0$, and, $R \to 0$ as $r \to \infty$.*

Proof. In the Appendix. □

As would be expected, aggregate market welfare increases as the degree of market frictions decreases. In particular, Proposition 5 implies that when market frictions are negligible, in any ME, trade occurs between a buyer and a seller if and only if the realised match-specific pair of values (v_b, v_s) are arbitrarily close to the pair $(\bar{v}_b, \bar{v}_s)$. This makes intuitive sense; when frictions are negligible, the cost to each agent of locating the almost perfect match is negligible — and therefore, each agent waits to match with an agent of the opposite type who will generate almost maximal value for him.

At this level of generality, we have not been able to derive the limiting (as $r \to 0$) equilibrium payoffs to a type i agent and a type j agent. However, the specific case studied in section 5 below suggests that in general, when market frictions are negligible and the matching rates are identical, a type i agent will obtain a relatively greater equilibrium payoff. Thus, even under almost frictionless conditions, the ability to post price contracts (subject to mutually beneficial renegotiation) may confer a strategic advantage. This observation suggests that when market frictions are negligible, the ME outcome may *not* approximate the "competitive" equilibrium outcome of our market.

4. Existence of Market Equilibria

It has proved difficult to establish the existence of a ME without imposing some additional restrictions on the distributions F_i and F_j. We have established three existence results, two of which are fairly general.

Our first existence result (Proposition 6(a) below) assumes that F_i and F_j are continuous. However, this result requires assuming that a type i agent can randomize over the choice of his posted price contract. It has not been possible to establish existence of a ME (in this, general case) without allowing for such mixed strategies.

Our second existence result (Proposition 6(b) below) assumes that F_i is degenerate; this assumption considerably simplifies the equations that characterize a ME, but without any loss in the main strategic considerations that underlie our market model. In addition, this existence result requires some appropriate restrictions on F_j, which, in fact, suffice for the existence of a unique ME. This ME is in pure strategies. The assumption that F_i is degenerate — that is, $\bar{v}_i = \underline{v}_i$ — means that the value to a type i agent from trading is the same across all type j agents; we denote this single value by v_i^*.[20]

Our third, and final existence result (Proposition 6(c) below) is for the case when F_i and F_j are uniformly distributed.

Proposition 6 (Existence of Market Equilibria)**.** *Fix i, where $i = b$ or $i = s$.*
(a) If F_i and F_j are continuous, then there exists a (mixed-strategy) ME.
(b) If F_j is differentiable, $1 - F_j$ is log-concave, and F_i is degenerate

[20]It may be noted that the assumption that F_i is degenerate captures many kinds of markets such as some retail markets (or bazaars) in which sellers post prices, and, each seller does not care as to which particular buyer she trades with.

(i.e., $\bar{v}_i = \underline{v}_i$), then there exists a unique (pure-strategy) ME.
(c) If both F_i and F_j are uniformly distributed, then there exists a unique (pure-strategy) ME.

Proof. The proofs of parts (a) and (b) are in the Appendix. The proof of part (c) is straightforward (although algebraically messy), and hence it is omitted.[21] □

For part (b) of this proposition, it may be noted that while differentiability of F_j is assumed to simplify the arguments (and could be dispensed with), log-concavity of $1 - F_j$ is crucial to our argument, and, it may be noted, it is an assumption often used for the existence of a pure-strategy equilibrium. In the context of our model under the additional assumption that F_i is degenerate, it also suffices for establishing the uniqueness of equilibrium.

5. An Example: The Case of Uniform Distributions

In order to obtain some additional insights concerning the properties of the Market Equilibria, over and above those obtained in section 3 above, we now study the unique ME when both F_i and F_j are uniformly distributed.[22] Although the results discussed here are specific to this case, the intuition behind them suggest that some (but certainly not all) of these results may actually hold more generally. There are several issues of interest that we wish to obtain some insight into, but which we were unable to do in the general context studied in section 3 above. In particular, the issue of the impact of market frictions on (i) the equilibrium proportion of trades that occur at the posted price, and (ii) the equilibrium payoffs to type i and type j agents.

In this section (only) we assume, without loss of generality, that $i = b$. That is, it is the buyers who get to post the incomplete price contracts. This assumption fits, for example, labour markets in which firms (the buyers) post wages (prices).

Although it is possible to obtain a closed-form, analytical characterization of the unique ME for any set of parameters, we have done so only when the discount rate is sufficiently small. In that case we obtain several insightful results, which we report in the following proposition.[23] They will be discussed after we state this proposition *and* after we report the results of a simulation aimed to obtain insights into the properties of the unique ME for larger values of the discount rate.

[21] It is available upon request.
[22] Recall that Proposition 6(b) states that in this case, there exists a unique ME.
[23] A proof of this proposition is available upon request.

Proposition 7 (The Case of Uniform Distributions)**.** *If $i = b$, and, both F_b and F_s are uniformly distributed, then the unique ME possesses, in particular, the following properties. There exists an $\bar{r}$ such that over the interval $(0, \bar{r})$, a change in the discount rate r has in general an ambiguous effect on the equilibrium posted price. However, if $\lambda_b = \lambda_s$, then the equilibrium posted price is strictly decreasing in r; but if λ_s is sufficiently larger than λ_b, then the equilibrium posted price is strictly increasing in r. Moreover, the equilibrium proportion of trades which occur at the posted price — we denote this proportion by τ — equals 1/3 for any $r \in (0, \bar{r})$. For any $r > \bar{r}$, τ is strictly increasing in r.*

Before we discuss the results stated in this proposition, we first report the results of a simulation that reveals, in particular, a few other interesting properties of the unique ME for larger values of the discount rate, but when the matching rates are identical.[24] The results of the simulation in question are stated in Table 1. As is evident, the results of this simulation are consistent with the appropriate, analytically derived results stated in Proposition 7 above. There are, however, some other revealing results in this simulation that we have not been able to establish analytically. For example, as can be seen from Table 1, the difference $V_b - V_s$ does *not* vanish as r becomes negligible. This result is particularly interesting as it implies (and this was mentioned in our discussion above following Proposition 5) that our ME outcome does *not* approximate the "competitive" equilibrium outcome even as market frictions become negligible.[25] The message here is that even under frictionless conditions, the option to post price contracts (albeit incomplete, and, given the possibility to engage in mutually beneficial renegotiation) confers a strategic advantage.

A key intuition that underlies all of these results runs as follows. When the matching rates are identical, then the only asymmetry between a firm (a buyer) and a worker (a seller) is that the firm has the option to post a price contract, which works to its advantage; in particular, the *greater* the degree of market frictions (i.e., the higher the value of r) the *bigger* is that advantage. This is because the worker's "outside option" — which is to wait and find an alternative trading partner — is less attractive the greater the degree of market frictions. However, when a worker's matching rate is sufficiently large relative to

[24]The model was simulated, with most of the results to be shortly discussed captured also in the several other simulations that we conducted but which we shall not report here.

[25]In a "competitive" equilibrium outcome of our market, all agents would earn the same expected payoff.

r	R	p_b	V_b	V_s	τ
.001	14.2	4.22	9.15	5.05	0.33
0.01	13.3	3.93	8.71	4.59	0.41
0.1	10.9	2.45	8.22	2.68	0.78
0.2	9.69	1.87	7.82	1.87	0.9
0.3	8.86	1.47	7.39	1.47	0.91
0.4	8.21	1.22	6.99	1.22	0.97
0.5	7.67	1.00	6.67	1	1

Table 1. Results of a simulation: Equilibrium when F_b and F_s are uniformly distributed, $\lambda_b = \lambda_s = 1$, $\bar{v}_s = 1$, $\underline{v}_s = 0$, $\bar{v}_b = 14$ and $\underline{v}_b = 8$.

a firm's matching rate, the worker's outside option is relatively more attractive than the firm's outside option, and this works to the worker's advantage. Hence, for example, when λ_s is sufficiently larger than λ_b, the equilibrium posted price is strictly increasing in r.

As can be seen from Table 1, when the matching rates are identical, both V_b and V_s are strictly decreasing in r. Furthermore, the ratio V_b/V_s increases with the discount rate. This result indicates that the *relative* advantage that type i agents have over type j agents increases with the degree of market frictions.

Although the result established in Proposition 7 that τ, the equilibrium proportion of trades at the posted price, is constant for small values of r may not be robust to other specifications of the distributions, the result that τ is (in general) increasing in r will be shown (in the next section) to hold under a more general class of distributions. Thus, a key insight from our model may be put as follows: *trade in markets with small frictions is likely to occur at negotiated prices, while in markets with large frictions it is more likely to occur at posted prices.* Let us provide some intuition behind this insight.

To illustrate the intuition in a fairly transparent manner, let us consider the two extreme cases of very large and very small degrees of market friction. In the former case, waiting to find an alternative trading partner is quite costly, and thus, the agents' outside options are pretty unattractive. This immediately implies that trade at the posted price will tend to be individually rational for any pair of agents. In the latter case, on the other hand, the reverse holds: the outside options of both agents are pretty attractive, and thus, trade at the posted price will tend *not* to be individually rational for at least one them. Hence, trade is more likely to occur at an *ex-post* negotiated price.

In most retails markets in the rich, OECD countries, trade typically always occurs at posted prices. On the other hand, in retail markets in the relatively poorer, developing countries (such as India), buyers typically trade at an *ex-post* negotiated price (and not at the sellers' posted prices). We argue that this *casual* observation can be potentially explained by differences in the degree of market frictions — and appealing to the insight obtained above — since the value of time (or subjective discount rate) of buyers in the rich, OECD countries is typically larger than that of buyers in the poorer, developing countries.[26]

6. No Variation in Type $\boldsymbol{i}$ Agents' Value

In this section we study the unique ME under the hypothesis of Proposition 6(b). We should emphasize that the assumption that F_i is degenerate considerably simplifies the equations which characterize a ME, without any loss in the main strategic considerations that underlie our market model (since, in particular, we continue to assume that F_j is almost arbitrary, but non-degenerate). The assumption that F_i is degenerate, it may be noted, captures many kinds of markets such as some retail markets (or bazaars) in which sellers post prices, and, each seller does not care as to which particular buyer she trades with.

Our main objective here is, in particular, to explore the robustness or otherwise of the results obtained above (in the context of uniform distributions) concerning the role of the main parameters on the equilibrium proportion of trades which occur at the posted price. Since the (general) results obtained in section 3 are valid when F_i is degenerate, there is no point reproducing such results here when F_i is explicitly assumed to be degenerate. We shall only derive results on key issues that proved difficult to obtain in the general case (without the added assumption of F_i being degenerate).

6.1. Equilibrium Trade.

Here we examine how the parameters affect the equilibrium probability of trade between an arbitrary pair of agents.

[26] We are not suggesting for a moment that in *all* matters and *all* spheres of life, individuals in the rich, OECD countries are more impatient than individuals in the poorer, developing countries. In some matters, the reverse is the case. However, when acting as buyers in retail markets — such as when buying food or consumer durables — the richer individual is relatively more impatient to buy and leave the market as he has relatively less time at hand for shopping. In contrast, when acting as workers in labour markets, individuals in the rich, OECD countries are far less impatient to trade; as such posted wages are often renegotiated.

This probability is, of course,

$$\gamma = 1 - F_j(R - v_i^*),$$

where $R = V_i + V_j$. For simplicity of calculation, we shall assume that $\lambda_b = \lambda_s = \lambda$; and denote r/λ by $\widehat{r}$. We obtain the following result:

Lemma 3 (Equilibrium Probability of Trade)**.** *Fix i, where $i = b$ or $i = s$. Assume that F_j is differentiable, $1 - F_j$ is log-concave, F_i is degenerate ($\bar{v}_i = \underline{v}_i = v_i^*$), and, $\lambda_b = \lambda_s = \lambda$. For any parameter values such that $R > v_i^* + \underline{v}_j$ (where $R = V_i + V_j$), the equilibrium probability of trade γ is strictly increasing in $\widehat{r}$ (where $\widehat{r} = r/\lambda$), and it is also strictly increasing in v_i^*.*

Proof. In the Appendix. □

Thus, perhaps not surprisingly, the equilibrium probability of trade between an arbitrary pair of agents increases with the degree of market frictions and with the expected total value of a match.[27] An implication of this result is that a *proportional* mean preserving spread of F_j generates a reduced rate of trading.

Proposition 5 implies that R is strictly decreasing in $\widehat{r}$, with $R \to 0$ as $\widehat{r} \to \infty$, and $R \to v_i^* + \bar{v}_j$ as $\widehat{r} \to 0$. These results imply that the comparative-static results stated in Lemma 3 are valid for *any* parameter values when $v_i^* + \underline{v}_j \leq 0$. But if $v_i^* + \underline{v}_j > 0$, then, when $\widehat{r}$ is sufficiently large, $R < v_i^* + \underline{v}_j$ (i.e., $R < v_i^* + v_j$ for *any* $v_j \in \Sigma_j$). Hence, when $\widehat{r}$ is sufficiently large, the equilibrium probability of trade equals one. Indeed, this makes intuitive sense: when the degree of market frictions is sufficiently large, the equilibrium payoffs to any pair of agents from not trading (and thus, waiting to find an alternative trading partner) will be so small that they will *always* (for any $v_j \in \Sigma_j$) find it mutually beneficial to trade with each other.

6.2. **Equilibrium Proportion of Trades at the Posted Price.** We now explore how the parameters affect the equilibrium proportion of trades which occur at the posted price (rather than at a renegotiated price). Let τ denote this proportion; thus, $1 - \tau$ denotes the proportion of trades that occur at a renegotiated price. In the unique ME, trade occurs at the posted price for any $v_j \in [R_j, \bar{v}_j]$, and at a renegotiated price for any $v_j \in [R - v_i^*, R_j]$, where R_j is a type j agent's equilibrium

[27]It may be noted that in terms of the pattern of trade, changes in v_i^* are identical to changes in the expected value of v_j.

reservation value and $R = V_i + V_j$. Furthermore, for any $v_j \in [\underline{v}_j, R - v_i^*]$ trade does not occur (since it is not mutually beneficial to do so). Thus,

$$\tau = \frac{1 - F_j(R_j)}{1 - F_j(R - v_i^*)} = \frac{1 - F_j(R_j)}{\gamma}. \tag{7}$$

We obtain the following result:

Proposition 8 (Equilibrium Trade at the Posted Price)**.** *Fix i, where $i = b$ or $i = s$. Assume that F_j is differentiable, $1 - F_j$ is log-concave, F_i is degenerate ($\bar{v}_i = \underline{v}_i = v_i^*$), and, $\lambda_b = \lambda_s = \lambda$.*
(a) The derivative of τ with respect to $\widehat{r}$ is in general ambiguous, where $\widehat{r} = r/\lambda$. If, however, $v_i^* + \underline{v}_j > 0$, then there exists a $\widehat{r}^*$ and $\widehat{r}^{**}$ where $\widehat{r}^* > \widehat{r}^{**} > 0$ such that over the interval $(\widehat{r}^{**}, \widehat{r}^*)$, τ is strictly increasing in $\widehat{r}$, and, $\tau = 1$ for all $\widehat{r} > \widehat{r}^*$.
(b) The derivative of τ with respect to v_i^* is in general ambiguous. However, there exists a $\widehat{v}_i^*$ and $\tilde{v}_i^*$ where $\widehat{v}_i^* > \tilde{v}_i^* > 0$ such that over the interval $(\tilde{v}_i^*, \widehat{v}_i^*)$, τ is strictly increasing in v_i^*, and, $\tau = 1$ for all $v_i^* > \widehat{v}_i^*$.

Proof. In the Appendix. □

The intuition for why the comparative-static results reported in the proposition above are in general ambiguous is because changes in the appropriate parameters have the same qualitative effect on both the denominator of the RHS of (7) — which is the equilibrium probability of trade — and on the numerator of the RHS of (7) — which is the equilibrium probability of trade at the posted price. We now discuss the other results contained in Proposition 8.

An implication of the result in part (a) of the proposition is — as, indeed, we also discovered in the context of the case of uniform distributions in section 5 above — that trade in markets with relatively small frictions is more likely to occur at negotiated prices, while in markets with relatively large frictions it is more likely to occur at posted prices. This result makes much sense, and the intuition for it was provided in section 5. On the other hand, the appropriate result in part (b) of the proposition is at first blush seemingly inconsistent with real-life markets. The result implies that trade in retail markets for expensive items (such as cars and houses) should be more likely to occur at posted prices, while in retail markets for relatively cheap items (such as foodstuffs) trade is more likely to occur at negotiated prices.

As we now explain in the context of retail markets, the reason for why our model generates this counter-intuitive relationship between the equilibrium proportion of trades at the posted price and the expected total value of a match is because in our model the matching rate (or equivalently the intensity of search) is *exogenously* given.

Notice that an implication of the result (contained in Proposition 8(a)) is that (when the matching rate is sufficiently small) the equilibrium proportion of trades at the posted price is strictly decreasing in the matching rate. Since the matching rate is determined by the intensity with which buyers search for sellers, this result implies that the more intensively buyers engage in search the more likely it is that trade occurs at negotiated prices. But casual observation suggests that in real-life, retail market buyers tend to search intensively when buying an expensive item (such as a car or a house) and not when buying cheap items (such as foodstuffs). This is because it is hardly more costly to find another auto dealership than it is to find another supermarket. Accordingly, a simple extension of the model (which goes beyond the scope of the current paper) would be to endogenize search intensity. With search costs fixed, agents will search more intensively for larger ticket items inducing a higher proportion of trades at bargained prices.

We now draw attention to another implication of the appropriate result contained in Proposition 8(b). In the context of labour markets, suppose that firms are posting wages, and that the variance of the distribution of a worker's match-specific value decreases. Since a decrease in the variance of F_j is formally equivalent to an increase in v_i^*,[28] it follows immediately from Proposition 8(b) that as the variance of F_j decreases, the equilibrium proportion of trades at the posted price will increase. This makes intuitive sense. That is partly because as the variability in v_j decreases, the likelihood that trade at the posted wage is individually rational for both parties increases. And partly because as the variability in v_j decreases, after having encountered some firm, a worker's incentive to search for an alternative firm decreases, since the likelihood of her obtaining a better match-specific value has decreased. In the context of retail market, this result suggests that when comparing 2 items of similar expected value, such as painting and computer, the item over which the idiosyncratic component of value has greater variance is more likely to be sold at a bargained price (*i.e.* the painting).

[28]This is because an increase in v_i^* is equivalent to an increase in the expected value of v_j, which, in turn, is equivalent (after normalization) to keeping the expected value of v_j unchanged but decreasing the variance of v_j.

7. Summary and Concluding Remarks

The two most fundamental results obtained in this paper — concerning markets characterized by match-specific heterogeneity — are as follows:

• In such markets, some trades are executed at posted prices, while others at negotiated prices — with the exact proportions depending on the fundamentals such as the degree of market frictions. In particular, trade in markets with small frictions is likely to occur at negotiated prices, while in markets with large frictions it is more likely to occur at posted prices. This implies, for example, that in retail markets in which buyers search intensively (such as in housing markets) trade is more likely to occur at negotiated prices, while in retail markets in which their intensity of search is negligible (such as in the food market) trade is more likely to occur at posted prices.

• In general (when the numbers of sellers and buyers are unequal, or equivalently when the matching rates are unequal) the posting of comprehensive price contracts by agents on the short-side of the market adversely affects aggregate market welfare; a social planner, for example, would not allow agents on the short-side of the market to post comprehensive price contracts. To put it differently, when one side of the market has too much bargaining power — by not only being able to post comprehensive price contracts but also by having a relatively higher matching rate — aggregate market welfare is compromised. Aggregate market welfare is maximised either when the agents with the relatively higher matching rate post incomplete price contracts or when the agents with the relatively lower matching rate post comprehensive price contracts. Thus, in order to promote aggregate market welfare it is important that bargaining power, broadly interpreted, is evenly distributed between the two sides of the market.

Another point that deserves some emphasis is:

• When contracts are incomplete (as they typically are), the Diamond paradox should not be of concern in such markets (provided, of course, that the heterogeneity is not degenerate). However, with regard to the other side of the same coin, such markets will not be Walrasian even in the limit as market frictions vanish; the price-posting agents will always retain some positive degree of market power.

Although our market model is the first to combine in a single framework (of endogenous price determination) the triple features of (i) match-specific heterogeneity, (ii) the option to post incomplete price contracts, and (iii) the option to engage in mutually beneficial renegotiation, the model contains some restrictive, simplifying assumptions. We conclude by considering some of them.

One such is the assumption that when a pair agents of the opposite types encounter each other, their match-specific trading values become common knowledge between them. This assumption ought to be relaxed in future research, since it is far more plausible that after a pair of agents encounter each other, the realisation of type k agent's $(k = b, s)$ match-specfic value v_k is his private information. A potential way to extend our model in order to incorporate the consequences of such private information would be to introduce another stage to the game that takes place after stage 3 and before stage 4. In that (new) stage, stage 3.5 say, the matched agents simultaneously announce their trading values. Of course, each agent may have an incentive to lie about his trading value. For example, suppose that a buyer's true realised value is sufficiently high such that it is individually rational for him to trade at the posted price but he would then get less than half of the match surplus if trade occurred at the posted price. In that circumstance, he would have an incentive to announce a relatively lower value in order to induce renegotiation of the posted price (and thus obtain half the match surplus). The analysis of such an extended model would not be trivial, but it would be worth pursuing in order to address the robustness or otherwise of the main results obtained in this paper.

Second, although the assumption that only one side of the market has the option to post price contracts may have merit from an applicability point of view — since it is consistent with several real-life markets — it is interesting and important from a theoretical perspective to give both sides of the market this option; and thus, to endogenously determine (as part of a market equilibrium) the conditions and circumstances under which only one side exercises such an option. This extended model would provide much insight and understanding about the workings of real-life markets where only one side posts incomplete price contracts. When extending our model to capture this feature, it may be assumed, for example, that when both types of agents post prices, p_b and p_s, then, when deciding whether or not to renegotiate, and whether or not to trade, the "default" price is $p = (p_b + p_s)/2$ if $p_b \geq p_s$. But if $p_b < p_s$ then there is no default price — which means that in that case, trade can only occur at an *ex-post* negotiated price.

Third, it would be interesting to extend our model by allowing the trading value to be partly match-specific, but also partly endogenous — as a function of some investment decision. That is, the value v_k to a type k agent from trading with some particular type m agent ($m \neq k$) is a function $f_k(\theta_k, I_k)$, where θ_k is the match-specific component (randomly realised after encountering this particular agent), while I_k is his investment level made upfront before encountering any agent. Such an extended model could also be interpreted from the perspective of the incomplete *bilateral* contracting literature: unlike in that literature, in such a model the parties' "outside options" would now be endogenously determined as part of a market equilibrium.

Fourth, the assumption that the intensities with which agents search is exogenous should be relaxed. As we informally discussed in section 6.2 above, such an extended model is necessary in order to obtain the plausible relationship between the equilibrium proportion of trades that occur at posted prices (*vis-a-vis* negotiated prices) and the expected total value of a match. This is because as the expected total value of a match increases, it is intuitive that agents in real-life markets increase their intensity of search — which (using the results obtained in this paper) would imply an increase in the likelihood of trade occuring at negotiated prices (rather than at posted prices).

Appendix

Proof of Proposition 2. Fix i, where $i = b$ or $i = s$, and fix an arbitrary ME $\sigma \in \Theta$. We first establish, by contradiction, that $\bar{v}_b + \bar{v}_s > V_b + V_s$. Thus, suppose that $\bar{v}_b + \bar{v}_s \leq V_b + V_s$. This would imply that $\Omega_N = \Sigma$ and $\Omega_P(p_i^*) = \Omega_R(p_i^*) = \emptyset$. Hence, it follows from (3) and (5) that $V_b = V_s = 0$. This implies that $\bar{v}_b + \bar{v}_s \leq 0$, which is a contradiction. We note that $\bar{v}_b + \bar{v}_s > R$ implies that $V_s - \bar{v}_s < \bar{v}_b - V_b$, where $R = V_b + V_s$. Define, for each $\epsilon \in [0, \bar{v}_b + \bar{v}_s - R]$,

$$p_i(\epsilon) = \begin{cases} V_s - \bar{v}_s + \epsilon & \text{if } i = b \\ \bar{v}_b - V_b - \epsilon & \text{if } i = s. \end{cases}$$

The following Claim implies that $V_s - \bar{v}_s < p_i^* < \bar{v}_b - V_b$, which, in turn (and as required), implies Proposition 2.

Claim A.1. *There exists an $\epsilon \in (0, \bar{v}_b + \bar{v}_s - R)$ such that $Z_i(p_i(\epsilon), V_b, V_s) > Z_i(p, V_b, V_s)$ for any $p \geq \bar{v}_b - V_b$ and for any $p \leq V_s - \bar{v}_s$.*

Proof of Claim A.1. If p is such that either $p \geq \bar{v}_b - V_b$ or $p \leq V_s - \bar{v}_s$, then $\Omega_P(p) = \emptyset$, which (using (2)) implies that for any such p,

$$Z_i(p, V_b, V_s) = \int_{v_i = R - \bar{v}_j}^{\bar{v}_i} \int_{v_j = R - v_i}^{\bar{v}_j} \left(\frac{v_b + v_s - R}{2} \right) dF_b dF_s. \tag{8}$$

Notice that for any such p, $Z_i(p, V_b, V_s)$ is independent of p — since for any such p the terms of trade are determined (for any possible realisation of v_b

and v_s) *ex-post* via bilateral bargaining; the posted price p is redundant. Furthermore, using (2), we obtain that

$$Z_i(p_i(\epsilon), V_b, V_s) = \int_{v_i=R-\bar{v}_j+\epsilon}^{\bar{v}_i} \int_{v_j=\bar{v}_j-\epsilon}^{\bar{v}_j} (v_i+\bar{v}_j-R-\epsilon) dF_b dF_s + C + D, \text{ where}$$

$$C = \int_{v_i=R-\bar{v}_j}^{v_i=R-\bar{v}_j+\epsilon} \int_{v_j=R-v_i}^{\bar{v}_j} \left(\frac{v_b+v_s-R}{2} \right) dF_b dF_s \quad \text{and}$$

$$D = \int_{v_i=R-\bar{v}_j+\epsilon}^{\bar{v}_i} \int_{v_j=R-v_i}^{\bar{v}_j-\epsilon} \left(\frac{v_b+v_s-R}{2} \right) dF_b dF_s.$$

For notational convenience, define $G_i(\epsilon) = 2[Z_i(p_i(\epsilon), V_b, V_s) - Z_i(p, V_b, V_s)]$. Since the right-hand side of (8) equals

$$\int_{v_i=R-\bar{v}_j+\epsilon}^{\bar{v}_i} \int_{v_j=\bar{v}_j-\epsilon}^{\bar{v}_j} \left(\frac{v_b+v_s-R}{2} \right) dF_b dF_s + C + D, \qquad \text{it follows that}$$

$$G_i(\epsilon) = \int_{v_i=R-\bar{v}_j+\epsilon}^{\bar{v}_i} \int_{v_j=\bar{v}_j-\epsilon}^{\bar{v}_j} \left[(v_i+\bar{v}_j-R-\epsilon) - [v_j - (\bar{v}_j-\epsilon)] \right] dF_b dF_s.$$

Thus, for any $\epsilon \in [0, \bar{v}_b + \bar{v}_s - R]$,

$$G_i(\epsilon) = -\Psi_j'(\bar{v}_j - \epsilon)\Psi_i(R - \bar{v}_j + \epsilon) + \Psi_j(\bar{v}_j - \epsilon)\Psi_i'(R - \bar{v}_j + \epsilon),$$

where for each $x = b, s$ and for any $z \in \Re$,

$$\Psi_x(z) = \int_{v_x=z}^{v_x=\bar{v}_x} [1 - F_x(v_x)] dv_x = \int_{v_x=z}^{v_x=\bar{v}_x} (v_x - z) dF_x. \tag{9}$$

It may be noted that $\Psi_x'(z) = -[1 - F_x(z)]$.

Since $\Psi_x'(z) = -[1 - F_x(z)]$, and for any $\epsilon > 0$, $\Psi_j(\bar{v}_j - \epsilon) < \epsilon[1 - F_j(\bar{v}_j - \epsilon)]$, it follows that for any $\epsilon \in (0, \bar{v}_b + \bar{v}_s - R]$,

$$G_i(\epsilon) > [1 - F_j(\bar{v}_j - \epsilon)][\Psi_i(R - \bar{v}_j + \epsilon) + \epsilon\Psi_i'(R - \bar{v}_j + \epsilon)].$$

We now show that there exists an $\bar{\epsilon}_i$, where $0 < \bar{\epsilon}_i < \bar{v}_b + \bar{v}_s - R$, such that for any $\epsilon \in (0, \bar{\epsilon}_i)$ the right-hand side of the above inequality is strictly positive — which implies that $G_i(\epsilon) > 0$, as required.

Let $h_i(\epsilon) = \Psi_i(R - \bar{v}_j + \epsilon) + \epsilon\Psi_i'(R - \bar{v}_j + \epsilon)$. Since (by definition) F_i is right-continuous, it follows that h_i is right-continuous at $\epsilon = 0$. Now notice (since $\bar{v}_b + \bar{v}_s > R$) that $h_i(0) > 0$. It thus follows that there exists an $\bar{\epsilon}_i$ (where $0 < \bar{\epsilon}_i < \bar{v}_b + \bar{v}_s - R$) such that for any $\epsilon \in (0, \bar{\epsilon}_i)$, $h_i(\epsilon) > 0$. The desired result follows immediately, since for any $\epsilon > 0$, $1 - F_j(\bar{v}_j - \epsilon) > 0$.

Proof of Proposition 3. For each $i = b, s$, let R_i^C denote the sum of the equilibrium payoffs to a pair of agents of the opposite types in the unique ME when the type i agents post comprehensive price contracts, and, let R_i^I denote the sum of the equilibrium payoffs to a pair of agents of the opposite types in an arbitrary ME when the type i agents post incomplete price contracts.

First suppose that $\lambda_b = \lambda_s$. It follows from Claim A.2 below that $R_b^I = R_b^C$ and $R_s^I = R_s^C$. Part (a) of the proposition now follows immediately since (by Lemma 1) $R_b^C = R_s^C$, and since (when the matching rates are identical) aggregate market welfare W equals one-half of the sum of the payoffs to a pair of agents of the opposite types.

Now we establish part (b) of the proposition. Thus, now suppose that $\lambda_b < \lambda_s$. It follows from Claim A.2 that $R_b^I > R_b^C$ and $R_s^I < R_s^C$. Furthermore, it follows from (6) that

$$W_b^C = \frac{\lambda_s}{\lambda_b + \lambda_s} R_b^C \quad \text{and} \quad W_b^I = \frac{\lambda_s}{\lambda_b + \lambda_s} \Big[V_b^I + \Big(\frac{\lambda_b}{\lambda_s}\Big) V_s^I \Big].$$

Now Lemma 1 implies that

$$R_b^C = \frac{\lambda_b}{r} \iint\limits_{v_b + v_s \geq R_b^C} [v_b + v_s - R_b^C] dF_b dF_s,$$

and equations 3 and 5 imply that

$$V_b^I + \Big(\frac{\lambda_b}{\lambda_s}\Big) V_s^I = \frac{\lambda_b}{r} \iint\limits_{v_b + v_s \geq R_b^I} [v_b + v_s - R_b^I] dF_b dF_s.$$

Hence, since the integral

$$\iint\limits_{v_b + v_s \geq R} [v_b + v_s - R] dF_b dF_s$$

is strictly decreasing in R, and since Claim A.2 below implies that $R_b^I > R_b^C$, it therefore follows that $W_b^I < W_b^C$. By a symmetric argument, since Claim A.2 implies that $R_s^I < R_s^C$, it follows that $W_s^I > W_s^C$.

Claim A.2. *Fix i, where $i = b, s$. If F_j is non-degenerate, then in any ME*

$$R_i^I \gtreqless R_i^C \quad \text{if} \quad \lambda_i \lesseqgtr \lambda_j.$$

Proof of Claim A.2. It follows from equations 3 and 5 that

$$\frac{rV_i}{\lambda_i} + \frac{rV_j}{\lambda_j} = \iint\limits_{v_b + v_s \geq R} [v_b + v_s - R] dF_b dF_s,$$

where $R = V_b + V_s$. Now define $k = \lambda_j / \lambda_i$. After substituting for λ_j (using this latter expression) in the above equation, and re-arranging, we obtain that

$$\frac{r(1-k)V_j}{k} = \xi(R), \tag{10}$$

where

$$\xi(R) = \lambda_i \iint\limits_{v_b+v_s \geq R} [v_b + v_s - R] dF_b dF_s - rR.$$

Proposition 1, of course, implies that in any ME, V_j and R must satisfy (10). Now notice that it follows from Lemma 1 that R_i^C is the unique value of R such that $\xi(R) = 0$. Furthermore, note that since the first term of $\xi(.)$ is strictly decreasing in R, it follows that $\xi(.)$ is strictly decreasing in R. Given these properties of ξ, it is now trivial to establish Claim A.2. First note that if $k = 1$ then it follows from (10) that in any ME, $\xi(R) = 0$; hence, $R_i^I = R_i^C$. Secondly, note that if $k < 1$ then — since in any ME, $V_j > 0$ (cf. Corollary 1) — it follows from (10) that in any ME, $\xi(R) > 0$; hence, $R_i^I < R_i^C$. And finally, by a similar argument, if $k > 1$, then in any ME, $R_i^I > R_i^C$.

Proof of Proposition 4. In order to establish this proposition, we first need to derive Claim A.3 (stated below) that provides an alternative characterization of the set of ME. It then follows by subtracting (12) from (11) — stated below in Claim A.3 — that the difference between the equilibrium payoffs (in any ME)

$$V_i - V_j = \frac{(\lambda_i - \lambda_j)R}{\lambda_b + \lambda_s} + \frac{2\lambda_b \lambda_s \widehat{G}_i(R)}{r(\lambda_b + \lambda_s)},$$

where R and $\widehat{G}_i(R)$ are defined below in Claim A.3. Proposition 4 is now an immediate consequence of the observation that $\widehat{G}_i(R) > 0$ and $R > 0$.

Claim A.3. *Fix i and j, where $i, j = b, s$ with $i \neq j$. (V_i, V_j, p_i^*) defines a ME if and only if*

$$V_i = \frac{\lambda_i R}{\lambda_b + \lambda_s} + \frac{\lambda_b \lambda_s \widehat{G}_i(R)}{r(\lambda_b + \lambda_s)} \tag{11}$$

$$V_j = \frac{\lambda_j R}{\lambda_b + \lambda_s} - \frac{\lambda_b \lambda_s \widehat{G}_i(R)}{r(\lambda_b + \lambda_s)} \tag{12}$$

$$p_i^* = \begin{cases} V_s - \bar{v}_s + \epsilon_b^* & \textit{if } i = b \\ \bar{v}_b - V_b - \epsilon_s^* & \textit{if } i = s, \end{cases}$$

where R is a fixed point of $\widehat{\Gamma}_i$, $\epsilon_i^* \in \Phi_i(R)$ such that $R = \Gamma_i(R, \epsilon_i^*)$, and

$$\widehat{G}_i(R) = \max_{\epsilon \in [0, \bar{q}+\bar{e}-R]} G_i(\epsilon; R), \quad \textit{with}$$

$$\Phi_i(R) = \{\epsilon_i^* : G_i(\epsilon_i^*; R) \geq G_i(\epsilon; R) \ \ \forall \epsilon \in [0, \bar{v}_b + \bar{v}_s - R]\}, \quad \text{where}$$

$$G_i(\epsilon; R) = -\Psi_j'(\bar{v}_j - \epsilon)\Psi_i(R - \bar{v}_j + \epsilon) + \Psi_j(\bar{v}_j - \epsilon)\Psi_i'(R - \bar{v}_j + \epsilon) \tag{13}$$

(14)

$$\Gamma_i(R,\epsilon_i^*) = \frac{\lambda_b+\lambda_s}{2r}\left[\int_{v_i=R-\bar{v}_j+\epsilon_i^*}^{\bar{v}_i}\Psi_j(R-v_i)dF_i+\int_{v_j=\bar{v}_j-\epsilon_i^*}^{\bar{v}_j}\Psi_i(R-v_j)dF_j\right]+ \frac{\lambda_i\Psi_j(\bar{v}_j-\epsilon_i^*)\Psi_i'(R-\bar{v}_j+\epsilon_i^*)}{r}+\frac{\lambda_j\Psi_j'(\bar{v}_j-\epsilon_i^*)\Psi_i(R-\bar{v}_j+\epsilon_i^*)}{r}$$

Furthermore, for each $R\in[0,\bar{v}_b+\bar{v}_s]$, $\widehat{\Gamma}_i(R)=\{\Gamma_i(R,\epsilon_i^*):\epsilon_i^*\in\Phi_i(R)\}$.

Proof of Claim A.3. It is easy to show that equations 3 and 5 can be respectively rewritten as follows:[29]

(15)

$$\frac{2rV_i}{\lambda_i}=\int_{v_i=R_i}^{\bar{v}_i}\Psi_j(R-v_i)dF_i+\int_{v_j=R_j}^{\bar{v}_j}\Psi_i(R-v_j)dF_j+2\Psi_j(R_j)\Psi_i'(R_i)$$

(16)

$$\frac{2rV_j}{\lambda_j}=\int_{v_i=R_i}^{\bar{v}_i}\Psi_j(R-v_i)dF_i+\int_{v_j=R_j}^{\bar{v}_j}\Psi_i(R-v_j)dF_j+2\Psi_i(R_i)\Psi_j'(R_j),$$

where $R_b=V_b+p_i^*$, $R_s=V_s-p_i^*$, $R=R_b+R_s$ and, for each $x=b,s$ and for any $z\in\Re$, $\Psi_x(z)$ is define in (9). It follows immediately from the arguments in the proof of Proposition 2 above that in any ME, the equilibrium posted price p_i^* is as follows:

$$p_i^*=\begin{cases}V_s-\bar{v}_s+\epsilon_b^* & \text{if } i=b\\ \bar{v}_b-V_b-\epsilon_s^* & \text{if } i=s,\end{cases}$$

for some $\epsilon_i^*\in\Phi_i(R)$, where $\Phi_i(R)$ is defined in Claim A.3. It thus follows (from Proposition 1) that a ME can be characterized by a triple (V_b,V_s,ϵ_i^*) which satisfies equations 15 and 16 *with* $R_j=\bar{v}_j-\epsilon_i^*$ and $R_i=R-\bar{v}_j+\epsilon_i^*$, and such that $\epsilon_i^*\in\Phi_i(R)$. Substituting for R_i and R_j in (15) and (16) using these latter expressions, it follows (by adding (15) and (16)) that $R=\Gamma_i(R,\epsilon_i^*)$, where $\Gamma_i(R,\epsilon_i^*)$ is defined in Claim A.3. Hence, (V_b,V_s,p_i^*) defines a ME *only if* $R(=V_b+V_s)$ is a fixed point of $\widehat{\Gamma}_i$, where the correspondence $\widehat{\Gamma}_i$ is defined in Claim A.3. This, then, establishes Claim A.3.

Proof of Proposition 5. Fix an arbitrary ME. In order to emphasize the dependence of the sum of the equilibrium payoffs to a pair of agents of the opposite types on the discount rate, we write it as $R(r)$. Now consider two arbitrary values of the discount rate, $r_H>r_L$. We need to show that $R(r_L)>R(r_H)$. We argue by contradiction. Thus, suppose, to the contrary, that $R(r_L)\le R(r_H)$. It follows from equation (10) — since for any R, $\xi(R;r)$ is strictly decreasing in r — that therefore $r_HV_j(r_H)<r_LV_j(r_L)$. But this implies that $V_j(r_H)<V_j(r_L)$, which, in turn, implies that $V_i(r_H)<V_i(r_L)$. Hence, it follows that $R(r_H)<R(r_L)$, which contradicts our supposition.

[29]A proof is available upon request.

We now show that $R \to \bar{v}_b + \bar{v}_s$ as $r \to 0$. In the limit as $r \to 0$, the LHS of (10) converges to zero, and hence, the RHS must also converge to zero. This implies that in the limit as $r \to 0$,

$$\iint\limits_{v_b+v_s \geq R} [v_b + v_s - R] dF_b dF_s$$

must converge to zero. Hence, R must converge to $\bar{v}_b + \bar{v}_s$.

We now show that $R \to 0$ as $r \to \infty$. After dividing equation 10 by r, it follows that in the limit as $r \to \infty$, it must be the case that $V_j/k + V_i \to 0$. Hence, $R \to 0$.

Proof of Proposition 6

Proof of Part (a)

Here we establish the existence of a ME by allowing agents of type i to randomize over their choice of the posted price. This existence result only requires that both F_b and F_s are continuous, which allows us to use the *Theorem of the Maximum*, from which it follows that $\Phi_i(.)$ is compact valued and upper hemi-continuous (uhc), where Φ_i is defined in Claim A.3 above.

A type i agent randomizes over two prices — that is, over two values of ϵ_i^*. The value of ϵ_i^* is realized after he encounters an agent of the opposite type but before the match-specific values are realized. A randomization is a triple $(\epsilon_{i1}^*, \epsilon_{i2}^*, \pi)$ where p is the probability with which ϵ_{i1}^* is selected.

From the definition and derivation of $G_i(\epsilon; R)$ — which is provided in the proof of Proposition 2 — it should be clear that the amended problem for the type i agent is now to pick $(\epsilon_{i1}, \epsilon_{i2}, \pi)$ from $[0, \bar{v}_i + \bar{v}_j - R]^2 \times [0, 1]$ to maximize $\pi G_i(\epsilon_{i1}; R) + (1 - \pi) G_i(\epsilon_{i2}; R)$. Hence, the optimal pair ϵ_{i1}^* and ϵ_{i2}^* can be chosen arbitrarily from $\Phi_i(R)$. Given this choice, π^* is arbitrary.

Amending (15) and (16) to allow for randomizations, and then adding them implies that in equilibrium $R \in \Lambda(R)$, where

$$\Lambda(R) = \{\gamma : \gamma = \pi \min\{\widehat{\Gamma}_i(R)\} + (1-\pi) \max\{\widehat{\Gamma}_i(R)\} \quad \text{for some} \quad \pi \in [0,1]\}.$$

Because $\Gamma_i(R, .)$ is continuous, $\widehat{\Gamma}_i(.)$ is compact valued and uhc. Hence, $\Lambda : [0, \bar{v}_i + \bar{v}_j] \to [0, \bar{v}_i + \bar{v}_j]$ is compact valued, convex valued and uhc. An equilibrium is fully characterized as a fixed point of Λ. This is because any $R^* \in \Gamma(R^*)$ identifies a randomization $\{\epsilon_{i1}^*, \epsilon_{i2}^*, \pi^*\}$ such that $\Gamma_i(R^*, \epsilon_{i1}^*) = \min\{\widehat{\Gamma}_i(R^*)\}$ and $\Gamma_i(R^*, \epsilon_{i2}^*) = \max\{\widehat{\Gamma}_i(R^*)\}$ and $\pi = [R^* - \min\{\widehat{\Gamma}(R^*)\}]/[\max\{\widehat{\Gamma}(R^*)\} - \min\{\widehat{\Gamma}(R^*)\}]$, which is consistent with type i agents' optimal behaviour at $R = R^*$. Given this randomization, the threshold value for the sum of the match specific preference shocks, $v_i + v_j$, at which trade occurs is exactly R^*. Existence of a ME is now a consequence of *Kakutani's Fixed Point Theorem.*

Proof of Part (b)

In order to establish this part of the proposition, we shall make use of the characterization of the set of ME given in Claim A.3 above.

It is straightforward to show that when F_i is degenerate, the expression for $G_i(\epsilon; R)$ defined in Claim A.3 reduces to:

$$G_i(\epsilon; R) = -(v_i^* + \bar{v}_j - \epsilon - R)\Psi_j'(\bar{v}_j - \epsilon) - \Psi_j(\bar{v}_j - \epsilon).$$

Hence, if F_j is differentiable then the derivative of G_i with respect to ϵ,

$$G_i'(\epsilon; R) = (v_i^* + \bar{v}_j - \epsilon - R)\Psi_j''(\bar{v}_j - \epsilon) + 2\Psi_j'(\bar{v}_j - \epsilon). \tag{17}$$

It follows from Claim A.1 above that for any $\epsilon_i^* \in \Phi_i(R)$, it must be the case that $0 < \epsilon_i^* < \bar{v}_b + \bar{v}_s - R$. Hence, it follows that $\epsilon_i^* \in \Phi_i(R)$ *only if* $G_i'(\epsilon_i^*; R) = 0$. It thus follows immediately from (17) that $\epsilon_i^* \in \Phi_i(R)$ *only if* ϵ_i^* is a solution to the following equation in ϵ (where $\epsilon \in [0, v_i^* + \bar{v}_j - R]$):

$$v_i^* + \bar{v}_j - \epsilon - R = -\frac{2\Psi_j'(\bar{v}_j - \epsilon)}{\Psi_j''(\bar{v}_j - \epsilon)}. \tag{18}$$

Notice that the left-hand side of (18) is strictly decreasing in ϵ over the closed interval $[0, v_i^* + \bar{v}_j - R]$ — taking a strictly positive value at $\epsilon = 0$, and taking a value of zero when $\epsilon = v_i^* + \bar{v}_j - R$. Over the same closed interval, log-concavity of $1 - F_j$ implies that the right-hand side is increasing from zero to a strictly positive number. Hence, there exists a unique solution in ϵ to (18). It is thus follows that this unique solution constitutes the unique element of $\Phi_i(R)$. Hence, $\widehat{\Gamma}_i$ is a function, and thus, R is a fixed point of $\widehat{\Gamma}_i$ if and only if $R = \widehat{\Gamma}_i(R, \epsilon_i^*(R))$. That is, using (14) — after simplifying it using the assumption that F_i is degenerate — if and only if R satisfies

$$\begin{aligned} 2rR = {} & (\lambda_i + \lambda_j)\Psi_j(R - v_i^*) + \\ & (\lambda_j - \lambda_i)[(v_i^* + \bar{v}_j - \epsilon_i^*(R) - R)\Psi_j'(\bar{v}_j - \epsilon_i^*(R)) + \Psi_j(\bar{v}_j - \epsilon_i^*(R))]. \end{aligned} \tag{19}$$

We now establish that there exists a unique solution in R to the above equation. Hence, it follows from Claim A.3 that there exists a unique ME. The set of feasible values of R is the closed interval $[0, v_i^* + \bar{v}_j]$. The left-hand side of (19) increases over this interval — taking the value of zero when $R = 0$, and taking the value of $2r(v_i^* + \bar{v}_j)$ when $R = v_i^* + \bar{v}_j$. Differentiating the right-hand side of (19) with respect to R yields

$$(\lambda_i + \lambda_j)\Psi_j'(R - v_i^*) - (\lambda_j - \lambda_i)\Bigg[(v_i^* + \bar{v}_j - \epsilon_i^*(R) - R)\epsilon_i^{*\prime}(R)\Psi_j''(\bar{v}_j - \epsilon_i^*(R)) + (2\epsilon_i^{*\prime}(R) + 1)\Psi_j'(\bar{v}_j - \epsilon_i^*(R))\Bigg].$$

Then, using (18) this derivative reduces to

$$(\lambda_i + \lambda_j)\Psi_j'(R - v_i^*) - (\lambda_j - \lambda_i)\Psi_j'(\bar{v}_j - \epsilon_i^*(R)),$$

which is less than or equal to zero — since $0 \leq \epsilon_i^*(R) \leq v_i^* + \bar{v}_j - R$. Finally, note that at $R = v_i^* + \bar{v}_j$, the right-hand side of (19) is zero — since $\epsilon_i^*(v_i^* + \bar{v}_j) = 0$. Hence, since both sides of (19) are continuous in R, there exists a unique solution in R to (19).

Proof of Lemma 3. Under the restriction that $\lambda_i = \lambda_j$, equation 19 reduces to $\widehat{r}R = \Psi_j(R - v_i^*)$. Hence, we obtain that

$$0 < \frac{\partial R}{\partial v_i^*} < 1 \quad \text{and} \quad \frac{\partial R}{\partial \widehat{r}} < 0.$$

The lemma now follows immediately from this results.

Proof of Proposition 8. Fix $x = \widehat{r}, v_i^*$. It follows from (7) that

$$\frac{\partial \tau}{\partial x} = \frac{1}{\gamma^2}\Big[-\gamma F_j'(R_j)\frac{\partial R_j}{\partial x} + [1 - F_j(R_j)]F_j'(R - v_i^*)\frac{\partial (R - v_i^*)}{\partial x}\Big].$$

Equation 18 can be written as:

$$v_i^* + R_j - R + \frac{2\Psi_j'(R_j)}{\Psi_j''(R_j)} = 0.$$

Given the comparative-static results on R derived above in the proof of Lemma 3, and since (by the assumption that $1-F_j$ is log-concave) $\Psi_j'(R_j)/\Psi_j''(R_j)$ is strictly increasing in R_j, it is straightforward to show that therefore R_j is strictly decreasing in x. Hence, it follows that the derivative of τ with respect to x is in general ambiguous.

Now suppose that $v_i^* + \underline{v}_j > 0$. Once again, fix $x = \widehat{r}, v_i^*$. It is straightforward to show from the equation that determines R — namely, $\widehat{r}R = \Psi_j(R - v_i^*)$ — that there exists a value of x — call it $\bar{x}$ — such that for any $x > \bar{x}$, $R - v_i^* \leq \underline{v}_j$. Thus, for any $x > \bar{x}$, the equilibrium probability of trade $\gamma = 1$. Now note that since R_j is strictly decreasing in x (as shown above), the equilibrium probability of trade at the posted price $1 - F_j(R_j)$ is strictly increasing in x. The appropriate results in Proposition 8 (when $v_i^* + \underline{v}_j > 0$) concerning the relationship between τ and x (for values of $x > \bar{x}$) now follow immediately.

References

Bester, Helmut (1993), "Bargaining versus Price Competition in Markets with Quality Uncertainty", *American Economic Review*, **83**, 278-88.

Bulow, Jeremy and Paul Klemperer (1996), "Auctions vs. Negotiations", *American Economic Review*, **86**, 180-94.

Burdett, Ken and Dale Mortensen (1998), "Wage Differentials, Employer Size, and Unemployment" *International Economic Review*, **39**, 257–273.

Diamond, Peter (1971), "A Model of Price Adjustment", *Journal of Economic Theory*, **3**, 156-69.

Diamond, Peter (1981), "Mobility Costs, Frictional Unemployment, and Efficiency", *Journal of Political Economy*, **89**, 798-812.

Ellingsen, Tore and Asa Rosen (2003), "Fixed or Flexible? Wage-Setting in Search Equilibrium", *Economica*, **70**, 233-50.

Hart, Oliver (1995), *Firms, Contracts and Financial Structure*, Oxford: Oxford University Press.

Hart, Oliver and John Moore (1988), "Inomplete Contracts and Renegotiation", *Econometrica*, **56**, 755-786.

Masters, Adrian (1999), "Wage Posting in Two-Sided Search and the Minimum Wage", *International Economic Review*, **40**, 809-26.

Moen, Espen (1997), "Competitive Search Equilibrium," *Journal of Political Economy*, **105**, 358-411.

Mortensen, Dale (1982), "The Matching Process as a Noncooperative Bargaining Game", in McCall, J.J. (ed.), *The Economics of Information and Uncertainty*, Chicago: Chicago University Press.

Muthoo, Abhinay (1999), *Bargaining Theory with Applications*, Cambridge: Cambridge University Press.

Osborne, Martin and Ariel Rubinstein (1990), *Bargaining and Markets*, San Diego: Academic Press.

Peters, Michael (1991), "Ex Ante Price Offers in Matching Games Non-Steady States", *Econometrica*, **59**, 1425-1455.

Rubinstein, Ariel and Asher Wolinsky (1985), "Equilibrium in a Market with Sequential Bargaining" *Econometrica*, **102**, 581-593.

Tirole, Jean (1999), "Incomplete Contracts: Where Do We Stand?", *Econometrica*, **67**, 741-782.

Vickrey, William (1961), "Counterspeculation, Auctions and Competitive Sealed Tenders", *Journal of Finance*, **16**, 8-37.

Wang, Ruqu (1993), "Auctions versus Posted-Price Selling", *American Economic Review*, **83**, 838-51.

Wang, Ruqu (1995), "Bargaining versus Posted Price Selling", *European Economic Review*, **39**, 1747-64.

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