Effects of the Exchange-Rate Regime on Trade under Monetary Uncertainty: The Role of Price Setting

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Abstract

In a baseline stochastic new open-economy macroeconomics (NOEM) model which parallels alternative invoicing conventions, namely consumer's currency pricing (CCP) vs. producer's currency pricing (PCP), we revisit the question whether the exchange-rate regime matters for trade. We show analytically that under full symmetry, only money shocks and separable but otherwise very general utility, it is irrelevant in affecting expected trade-to-output ratios. A peg-float comparison is nevertheless meaningful under PCP, although not CCP, in terms of volatility of national trade shares: by shutting down the pass-through and expenditure-switching channel, a peg then stabilizes equilibrium trade-to-GDP at its expected level.

JEL Classification: F10, F33, F41.

Keywords: alternative price setting, international trade, exchange-rate regimes, stochastic NOEM models.

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1 Introduction

The present paper belongs to the rapidly growing new open-economy macroeconomics (NOEM) literature.[1] Our objective is to revisit, within this sticky-price optimizing approach and explicitly accounting for monetary uncertainty in general equilibrium, the classic subject of exchange rate and trade determination. In particular, we here reconsider in a fully-symmetric NOEM context and under *alternative* price-setting conventions the question whether the exchange-rate regime matters for international trade. Comparing *consumer's* currency pricing (CCP) vs. *producer's* currency pricing (PCP), we are able to answer in what sense this is the case. In a self-contained theoretical analysis that explicitly parallels a CCP to a PCP model version, we derive from first (micro-)principles important (macro-)outcomes. Some of them are really novel while the positive and normative implications of other have been debated for long, but largely within ad-hoc frameworks in the Mundell-Fleming-Dornbusch tradition.

More precisely, this paper builds on the stochastic representative agent set-up under CCP proposed in Bacchetta and van Wincoop (1998, 2000 a). As noted by these authors, their "benchmark monetary model" – together with the similar ones developed in Obstfeld and Rogoff (1998, 2000) under PCP, we would add – is intended as a starting point in modern research on monetary policy in open economies. Its main contribution, which we pursue here as well, is to recast traditional welfare comparisons of exchange-rate arrangements in a general equilibrium setting that explicitly considers the role of macroeconomic uncertainty. We thus explore Bacchetta and van Wincoop's (2000 a) single-period benchmark also under PCP focusing our attention on trade prices and flows. In essence, we compare the equilibrium outcomes of such a richer model under the polar invoicing practices in cross-border transactions[2] it nests, namely CCP vs. PCP. Theoretical analysis of these extremes allows us to draw some clear-cut, mostly qualitative conclusions on the effects of the exchange-rate regime – modelled simply as float vs. peg – on relative prices and key trade measures as well as on the underlying consumption and labor/leisure choices.

Our principal import is to demonstrate that price-setting assumptions, fundamental in any open-economy model with nominal rigidity, affect in a crucial way optimal consumption allocations under (even only) *monetary* uncertainty and, consequently, any microfounded analysis of international trade. In a preview of our results we can state that irrespective of the invoicing assumed, the exchange-rate regime does not matter for the *expected level* of trade-to-output ratios by country, which is always 1 given symmetry and frictionless trading. Yet under PCP, but not CCP, it matters for the *volatility* of national trade shares. A peg would thus stabilize, under PCP, the equilibrium trade share in each

[1] As defined and exhaustively classified in the recent survey by Lane (2001). A narrower and more technical summary of the basic NOEM methodology is also provided in Sarno (2001).

[2] Friberg (1998) points out to the fact that the currency of *price setting*, the currency of *invoicing* and the currency of *payment*, although theoretically corresponding to three distinct stages of a typical international trade transaction and hence potentially different, practically coincide "with few known exceptions". Therefore we further down use "invoicing" and "price setting" interchangeably (without talking at all about the "currency of payment").

country across states of nature at its expected level. This latter level coincides with the one under CCP, which is the same ex-ante as ex-post. We identify the difference in the effects of the exchange-rate regime on equilibrium trade flows as originating in the particular currency denomination of transactions, hence, the implied *exchange-rate pass-through*, and, ultimately, *expenditure switching*. In our symmetric framework, this major channel of international spillover of monetary shocks is absent under CCP and float. As to the PCP model version, a peg effectively shuts it down, by equalizing at the neutral unitary level the relative price of foreign goods in terms of domestic analogues households in both countries face ex-post.

We would not survey here the voluminous literature, classic as well as modern, on the subject we are interested in. We briefly discuss instead only those lines of relevant research that have strongly influenced our motivation for the paper as well as our modelling strategy. In doing so, we also highlight in the next two subsections two essential features of our set-up which would have important implications in any open-economy model with price rigidity.

1.1 Monetary Uncertainty in General Equilibrium

Monetary uncertainty generating exchange-rate risk is inherent in issues related to international trade, welfare and macroeconomic policy in which risk-averse agents are involved. To be properly studied, such issues have therefore to be cast in general equilibrium frameworks that are *explicitly stochastic*.[3] That is why we have purposefully chosen to follow a recent approach in NOEM theoretical modelling, introduced by Obstfeld and Rogoff (1998, 2000) and Bacchetta and van Wincoop (1998, 2000 a). It extends the deterministic "redux" exchange rate model of Obstfeld and Rogoff (1995, 1996: Chapter 10) and its variations in Corsetti and Pesenti (1997, 2001 a, b, 2002). To our knowledge, the "redux" model was the first microfounded open-economy general-equilibrium framework with rigid prices and monopolistic competition designed to explain exchange-rate dynamics. Traditional research on exchange rates and trade was either general-equilibrium but flexible-price,[4] or sticky-price but ad-hoc.[5] If the impact of uncertainty on exchange rates and, hence, trade and consumption flows was at all considered, analysis was restricted to partial-equilibrium models, as duly pointed out in Bacchetta and van Wincoop (2000 a).[6]

To allow for analytical solutions, the explicitly stochastic NOEM literature has been technically implemented under simplifying assumptions. *Log-normal* processes for shocks and, consequently, for the endogenous variables as well as rather *restrictive* specifications of utility are usually imposed, e.g. in Obstfeld

[3] Earlier models usually considered impulse responses to just a *single (one-time) shock* in an otherwise completely deterministic setting. Accordingly, although sometimes named "stochastic", they are essentially not.

[4] E.g. Helpman and Razin (1979, 1982, 1984), Helpman (1981) and Lucas (1982).

[5] Here one could enumerate papers in the Mundell-Fleming-Dornbusch tradition of the 1960s and 1970s.

[6] See the references cited in their footnote 7, p. 1096. Good surveys can be found in Côté (1994) and in Glick and Wihlborg (1997).

and Rogoff (2000). Often, it is also assumed that the Law of One Price (LOP) and, hence, Purchasing Power Parity (PPP) hold[7] so that the real exchange rate (RER) is constant. To benefit from the insights provided by an analytical solution, we likewise limit our set-up in this initial study to a single period with only monetary uncertainty, as in Bacchetta and van Wincoop's (2000 a) benchmark. Yet in a pursuit of greater generality of our conclusions, we do not restrict attention to neither a CCP nor a PCP-LOP-PPP model version only. Furthermore, we need not specialize, for our purposes here, to a log-normal distribution of disturbances or to a particular class of utility. With respect to the stochastic processes, it proves sufficient to invoke no more restrictions than a jointly *symmetric* distribution for the national money stock growth rates. As to the utility function, we essentially assume that it is well-behaved and separable. These features make our analysis less restrictive than related earlier work, with a few exceptions we know about such as Bacchetta and van Wincoop (1998, 2000 a). The latter two authors do not, however, explicitly consider PCP and its pass-through and expenditure switching implications as well as the effects of the exchange-rate regime on international relative prices.

1.2 Alternative Price Setting in Open Economies

Another important development in NOEM research has been to incorporate considerations of the earlier international trade literature, such as Helpman and Razin (1984) to mention an outstanding example, regarding alternative price setting. Contributions in this particular direction have been due to Betts and Devereux (1996, 2000), Bacchetta and van Wincoop (1998, 2000 a, b, 2001), Devereux and Engel (1998, 1999, 2000), Devereux (2000) and Engel (2000). Extending the original Obstfeld-Rogoff – Corsetti-Pesenti framework of non-segmented markets, these authors introduced international market segmentation in the goods market and what they usually call *pricing-to-market* (PTM)[8] behavior of monopolistically competitive firms, engaging at the same time in microfounded welfare comparisons of exchange-rate regimes. PTM is often alternatively denoted local currency pricing (LCP),[9] but to avoid ambiguity we would rather use a terminology that is hopefully more precise in our context: *producer*'s currency pricing (PCP) and *consumer*'s currency pricing (CCP).[10]

We already noted in what our analysis differs from, or rather extends and complements, the one in Bacchetta and van Wincoop (1998, 2000 a). As to the remaining NOEM literature cited in the preceding paragraph, our study is justified at least in the following three aspects. First, we examine the effects of the exchange-rate regime on *trade* prices and quantities (no matter that our

[7] Obstfeld and Rogoff (1995, 1996: Chapter 10, 1998, 2000) and Devereux (2000), among others.

[8] A term coined by Krugman (1987).

[9] A coinage due to Devereux (1997) to refer to the *special* case of PTM where prices are always set in the currency of the destination market.

[10] Since we do not explicitly distinguish an intermediary import/export sector in the two-country economy we study, as Tille (2000) has first done within NOEM, CCP and PCP are equivalent here to, respectively, *importer*'s (buyer's) and *exporter*'s (seller's) currency pricing.

equilibrium allocations, including imports and exports, have also been the result of underlying optimal consumption/leisure choices), whereas attention in all quoted papers is focused on welfare issues. Second, and as a consequence of not undertaking welfare analysis, we are able to allow for a *more general* utility specification, while the other authors use quite restrictive utility subclasses, perhaps narrowing the scope of validity of their findings. Third, under uncertainty and in cash-in-advance (CiA) sticky-price frameworks – as emphasized in the insightful methodological books by Magill and Quinzii (1996) and Walsh (1998), among others – the assumed timing of decisions and price-setting behavior are crucial to model outcomes. Recognizing these facts and, more importantly, studying their interaction in a symmetric *open-economy* context that makes an explicit parallel between CCP and PCP invoicing is another novel feature of our approach.

The paper is further down organized as follows. Section 2 outlines the stochastic NOEM model of exchange rate and trade determination we employ and highlights the differences in its initial assumptions under CCP vs. PCP. The third section studies, under *float* and full symmetry, the role alternative price setting plays in agents' optimization and in deriving key equilibrium relationships. Section 4 then focuses on the effects of the exchange-rate regime on international relative prices and trade flows, by discussing if and how a *peg* would change the float allocations of the preceding section. Section 5 concludes and *Appendix A* contains the proofs of propositions.

2 A Simple Stochastic NOEM Model of Trade

The present section serves to introduce the model we study. We first describe the *basic* set-up that underlies both our model versions. The essential differences between the *CCP vs. PCP* cases, originating in the relevant currency denomination assumptions and reflected in our invoicing-specific notation, are highlighted next.

2.1 Basic Set-Up

The artificial economy we analyze is made up of two countries, *H(ome)* and *F(oreign)*, assumed of equal size. A continuum of differentiated *brands* belonging to the *same* good *type* is available for consumption. These highly substitutable brands are indexed by i if made in H and by i^* if made in F. Each such brand is produced and sold by a *single* monopolistically competitive firm, also indexed by i in H and i^* in F. Firms in Home are *uniformly* distributed on the *unit* interval $[0, 1]$. Likewise, firms in Foreign produce on $(1, 2]$.

To obtain (short-run) money non-neutrality, we assume sticky prices motivated by menu costs.[11] Moreover, monopolistic competition enables each firm

[11] As first suggested by Mankiw (1985). To recall a classic result in Lucas (1982), with perfectly flexible prices the exchange-rate regime does not matter, even under uncertainty, for optimal real allocations. As to the locus of rigidity, some authors prefer to model sticky

to optimally choose the price(s) at which it sells its product. Prices are *set in advance*, i.e. in our *ex-ante* state 0 (*before* monetary uncertainty has been resolved), and remain *valid for just one period*, i.e. for the *ex-post* state $s \in S$ we consider (*after* shocks in H and F have been observed).[12] Preannounced prices result, in turn, in demand-determined output, on an individual-firm as well as on an aggregate level.[13] In such a (New-)Keynesian situation, technology shocks do not influence production possibilities and output quantities sold.[14]

The two model versions, under CCP vs. PCP, we compare have imposed a specific notation we now summarize. All our quantity variables are denoted by lowercase Latin letters. These quantities can be indexed by up to two subscripts and up to two superscripts. A first *sub*script H or F indicates the origin of the respective variable at the national-economy level, i.e. the country where a particular good i or i^* (first *sub*scripts again but at the individual-firm level) has been produced. Following the tradition, we use an asterisk ($*$) as a first *super*script to denote that a particular quantity variable has been consumed in Foreign. The second *sub*script, 0 for ex-ante quantities and s for ex-post quantities, indexes the state of nature whereas the second *super*script, C (for CCP) or P (for PCP), indicates the assumed price setting. The same notational rules apply to the (money) prices or nominal variables that correspond to all respective quantities in our model, the only difference being that these are denoted by uppercase Latin letters. Greek letters, in turn, designate model parameters and shocks.

Governments and Shocks In each country, there is a government whose only (passive) role is to proportionally transfer cash denominated in national currency to all domestic households in a random way.[15] Seigniorage is then repaid in a lump-sum fashion, as is standard in the related literature. We interpret such a money supply behavior, equivalent in our context to a *flexible* exchange-rate system, as *exogenous* "monetary policy" and model it in terms of stochastic money stock growth rates. Moreover, we restrict it to be *jointly symmetric*, in the sense we explain next.

For $\forall s \in S$, μ_s and μ_s^* are, respectively, H-money stock and F-money stock *net* rates of growth, having the same means and variances. For the sake of symmetry, *ex-ante* (state 0) national money holdings of the representative house-

(nominal or real) wages, following Taylor (1979) and the earlier Keynesian tradition, while others give preference to sticky prices, following Rotemberg (1982) and Calvo (1983), and as Kimball (1995) has notably insisted. In essence, the two approaches are not so different and – within NOEM – often imply each other, as Hau (2000) and Obstfeld and Rogoff (2000) have recently argued.

[12] Since our focus is not on inflation dynamics (in general) or inflation persistence (in particular), the static stochastic framework we borrow from Bacchetta and van Wincoop (2000 a) and the related NOEM research seems not too constraining.

[13] For this to be realistic, we note that our subsequent analysis applies only to money growth disturbances of a sufficiently *small* magnitude.

[14] That is why we abstract, in this paper, from also modelling productivity disturbances. Even if explicitly accounted for, they will not change much in the present single-period setting.

[15] One could argue that monetary authorities are ultimately unable to perfectly control the money *supply* or precisely estimate the *demand* for money in order to always equilibrate them.

holds in Home and Foreign are assumed *identical* in terms of units of each country's currency:[16] $M_0 = M_0^*$. The *ex-post* (state s) cash balances, i.e. the domestic-currency budgets with which Home and Foreign households dispose for transactions purposes in any realized state of nature $s \in S$, are then respectively given by $M_s \equiv M_0 + \mu_s M_0 = (1 + \mu_s) M_0$ and $M_s^* \equiv M_0^* + \mu_s^* M_0^* = (1 + \mu_s^*) M_0^*$.

The only difference between float vs. peg in terms of the (conditional) joint distribution (up to second moments, inclusive) of national money *growth* shocks (μ_s, μ_s^*) and, hence, of the resulting ex-post money *stocks* (M_s, M_s^*) thus arises from their *covariance* terms. It is imposed by the definition itself of a fixed vs. flexible exchange-rate regime: under (pure) *float*, the (conditional) correlation of national money stocks is 0; under (credible) *peg*, this (conditional) correlation is 1. In essence, our fixed exchange-rate version is thus isomorphic to a model where a monetary union or a single currency area is hit by just one, common money shock.

Timing of Events In the single period we analyze, decisions are made in two stages, ex-ante and ex-post. These stages are defined – and distinguished as the *ex-ante* state 0 and the *ex-post* state s – by the moment of the resolution of monetary uncertainty.

Ex-Ante Behavior Only *firms* optimize ex-ante, solving a stochastic optimization problem. Before knowing the particular state of the world that will materialize but having common views on the joint distribution of the symmetric monetary shocks, they preannounce prices. Due to (prohibitive) menu costs, they cannot change ex-post these optimally prefixed prices.

Ex-Post Behavior After observing the state of the world, *firms* employ labor to produce goods. Output, hence, labor input and, ultimately, leisure hours are simply determined in any realized state of nature by the optimal consumption demand for the respective differentiated product each one of the firms produces. *Households*, contrary to firms, optimize only ex-post. After receiving their random cash, they allocate total money balances across the differentiated goods which make up the real consumption composite. Because of demand-determined output and labor input, households are thus not free to adjust their labor/leisure trade-off once a given state of nature has materialized.

Households In each country, H and F, there is a continuum of *identical* households. The population in each of these economies is assumed constant and is normalized to 1. The representative household (in H as well as in F) likes diversity and *consumes all brands* on the interval $[0, 2]$. It also supplies labor, earning the equilibrium wage, and owns an equal proportion of domestic firms, receiving their profits (in the form of dividends).

[16] At an initial *equilibrium* exchange rate $S_0 = 1$, as will be discussed later.

The representative household in Home[17] maximizes its *ex-post* (state s) utility:

$$\underset{c_s, l_s}{Max} \quad u(c_s, l_s), \quad \forall s \in S. \tag{1}$$

Our utility function is assumed to be *well-behaved* (i.e. to exist, be continuous, twice differentiable and concave) and *separable.* l_s is (hours of) leisure and c_s is a *constant* elasticity of substitution (CES) *real* consumption index defined in the standard way by the following Dixit-Stiglitz (1977) aggregator:[18]

$$c_s \equiv \left[\left(\tfrac{1}{2}\right)^{\frac{1}{\varphi}} (c_{H,s})^{\frac{\varphi-1}{\varphi}} + \left(\tfrac{1}{2}\right)^{\frac{1}{\varphi}} (c_{F,s})^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}}, \quad \forall s \in S, \tag{2}$$

with

$$c_{H,s} \equiv \left(\int_0^1 c_{i,s}^{\frac{\varphi-1}{\varphi}} \, di \right)^{\frac{\varphi}{\varphi-1}} \quad \text{and} \quad c_{F,s} \equiv \left(\int_1^2 c_{i^*,s}^{\frac{\varphi-1}{\varphi}} \, di^* \right)^{\frac{\varphi}{\varphi-1}}.$$

Similarly to Bacchetta and van Wincoop (2000 a) and most NOEM set-ups, we assume that $\varphi = \varphi^* > 1$. Thus φ in the above formulas is the elasticity of substitution in demand between *any* two brands, no matter where they are produced. $c_{i,s}$ is the consumption by the Home representative household of brand i produced by a Home firm i and $c_{i^*,s}$ is its consumption of brand i^* produced by a Foreign firm i^*. $c_{H,s}$ is an index of the consumption by the Home household of all brands produced in Home and $c_{F,s}$ is an analogous index for all brands produced in Foreign. Textbook derivations in this well-known Dixit-Stiglitz (1977) setting[19] give the allocation of consumption across brands:

$$\begin{aligned} c_{H,s} &= \frac{1}{2}\left(\frac{P_{H,s}}{P_s}\right)^{-\varphi} c_s, \quad c_{F,s} = \frac{1}{2}\left(\frac{P_{F,s}}{P_s}\right)^{-\varphi} c_s; \\ c_{i,s} &= \left(\frac{P_{i,s}}{P_{H,s}}\right)^{-\varphi} c_{H,s} = \frac{1}{2}\left[\frac{P_{i,s}}{P_s}\right]^{-\varphi} c_s, \\ c_{i^*,s} &= \left(\frac{P_{i,s}}{P_{F,s}}\right)^{-\varphi} c_{F,s} = \frac{1}{2}\left(\frac{P_{i^*,s}}{P_s}\right)^{-\varphi} c_s. \end{aligned}$$

$P_{i,s}$ is the price (in Home currency) paid by the Home household for one unit of a brand i produced by a Home firm i and $P_{i^*,s}$ is the price (in Home currency) paid by the Home household for one unit of a brand i^* produced by a Foreign firm i^*. $P_{H,s}$ is the price index (in Home currency) paid by the Home household across all Home produced brands and $P_{F,s}$ is the price index (in Home currency) paid by the Home household across all Foreign produced brands. P_s is the price

[17] The notation in which the model is further on set out generally refers to Home, but for Foreign symmetric relationships hold (unless otherwise stated).

[18] Accordingly, the representative household in Home (and, analogously, in Foreign) minimizes the cost of buying a unit of real consumption.

[19] See, for instance, Obstfeld and Rogoff (1996).

index (in Home currency) across all brands consumed by the Home household. These price indexes too are defined in the usual way:[20]

$$\begin{aligned} P_s &= \left(\frac{1}{2}P_{H,s}^{1-\varphi} + \frac{1}{2}P_{F,s}^{1-\varphi}\right)^{\frac{1}{1-\varphi}} \quad \text{with} \\ P_{H,s} &= \left(\int_0^1 P_{i,s}^{1-\varphi} di\right)^{\frac{1}{1-\varphi}} \quad \text{and} \quad P_{F,s} = \left(\int_1^2 P_{i^*,s}^{1-\varphi} di^*\right)^{\frac{1}{1-\varphi}}. \end{aligned}$$

Respective symmetric expressions hold, of course, for Foreign. Note that, up to this point, all of the indexes are written down as independent of the underlying price setting. Their particular variants under CCP vs. PCP invoicing, modified by the appropriate notation, are discussed in the next section.

In this representative agent economy, the aggregate constraints on (per-) household behavior coincide with those of the identical households. They are standard in NOEM but, for completeness, we briefly present them below.

Time Endowment Constraint The endowment of hours to the representative household (in Home) is normalized to 1 in each state,

$$l_s + n_s \equiv 1, \quad \forall s \in S, \tag{3}$$

so that $n_s \equiv 1 - l_s$ is (Home) household's (hours of) labor (supply).

Cash-in-Advance (CiA) Constraint Households need to carry cash before going to the goods market.[21] Moreover, we restrict them to hold and receive from their monetary authority only *domestic* currency. Thus (for Home)

$$\underbrace{c_s P_s}_{H \text{ national } \textit{expenditure} \text{ (in } H \text{ currency)}} \leq \underbrace{M_s}_{\text{available } \textit{cash} \text{ in } H \text{ (in } H \text{ currency)}}, \quad \forall s \in S. \tag{4}$$

National Money Market Equilibrium Since CiA constraints are *binding*[22] and there is no investment and government spending in the model, the nominal value of national output sold (for consumption) is equal to the total stock of money in each of the countries. For Home:

$$Y_s = M_s, \quad \forall s \in S. \tag{5}$$

[20] To represent the minimal expenditure required for the purchase of one unit of the corresponding basket.

[21] The alternative would be to introduce money and, hence, the nominal exchange rate whose determination and regimes we wish to analyze, via a *money-in-the-utility* (MiU) function, also common in monetary general-equilibrium models. Our modelling choice here is anyway not crucial, since Feenstra (1986) has demonstrated the equivalence of these two approaches.

[22] For at least two reasons in our present set-up: (i) this is implied by the *concavity* of utility we assumed; (ii) it is also the optimal strategy for the representative household when no future (i.e. no dynamics) is allowed for, as in the *one-period* stochastic framework analyzed here. The binding CiA implies, in turn, a *unitary* velocity of (quantity theory) money demand (5), which is, certainly, another limitation but one that is common to similar CiA settings.

National Income Identity With a nominal wage rate of W_s and total hours of work amounting to $1 - l_s$, the nominal labor income of the (Home) representative household is given by $W_s(1 - l_s)$. Nominal dividends from firm profits earned by this household are denoted by Π_s. In equilibrium, all income from the activity of firms is distributed to domestic households who are their ultimate owners, as will be assumed (but this happens only at the end of the one-period framework we consider):[23]

$$\underbrace{\underbrace{W_s(1-l_s)}_{\text{labor income}} + \underbrace{\Pi_s}_{\text{ownership income}}}_{H \text{ national (factor) } \textit{income} \text{ (in } H \text{ currency)}} \equiv \underbrace{Y_s}_{H \text{ national } \textit{output} \text{ (in } H \text{ currency)}}, \quad \forall s \in S. \tag{6}$$

First-Order Conditions The following "compact" FONC can be derived in a familiar way from the above-described constrained optimization problem for the H representative household:

$$W_s = \frac{u_{l,s}}{u_{c,s}} P_s, \quad \forall s \in S. \tag{7}$$

$u_{l,s}$ and $u_{c,s}$ in (7) are the marginal utilities of leisure and consumption, respectively, in the realized state s. The real wage rate is thus equal, in equilibrium, to the ratio of these marginal utilities.

Firms Unlike the NOEM alternative of "yeoman-farmers", firms exist in themselves in our model and effect production. A usual restriction in similar settings we impose at this stage too is that firms are owned by *domestic* households only. In the present study we also abstract from an international stock market, as well as of risk-sharing issues in general. As noted, product differentiation makes firms monopolistically competitive. We focus here on the case where differentiated brands belong to the *same* type of a homogeneous good produced in both countries with identical technology common to all firms.[24] Just one factor, labor, available in fixed quantities in both economies, is used as input. For Home:

$$y_s = n_s = 1 - l_s. \tag{8}$$

Such a production function does seem simplistic, but is actually sufficient for the purposes of our sticky-price single-period analysis here. The reason is that, given the (New-)Keynesian set-up we described, it is household demand and not productivity that ultimately determines output.

[23] Factor income is thus not used further on, to buy consumption goods and to lend or borrow, with no future modelled.

[24] In Mihailov (2003 a) we allow for national good types that *differ* in the sense of being less substitutable than brands.

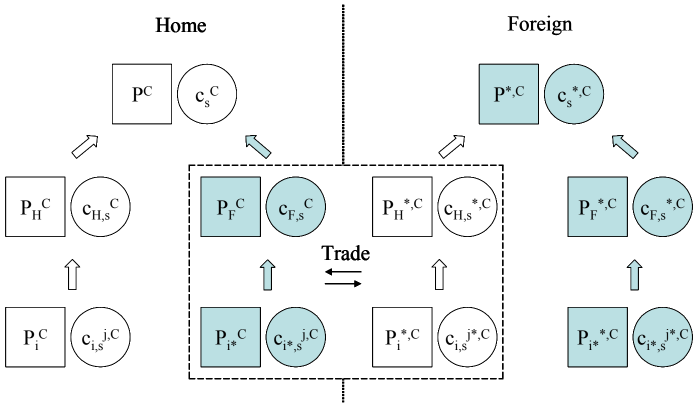

Figure 1: Notation on Price and Quantity Aggregation under CCP

2.2 CCP vs. PCP Version

As already mentioned, the combination of timing and nominal rigidity assumptions plays an important role in similar stochastic CiA models. In our case, it affects in a crucial way the nature of optimization under CCP vs. PCP. More precisely, the exchange rate does not matter for households decision problem under CCP but becomes a key consideration under PCP. The reason is in the particular currency denomination implied by our alternative invoicing assumptions.

Under *CCP*, households pay for imports as well as for home-produced goods directly in their national currency. The equilibrium exchange rate, although observed (calculated implicitly) at the moment of the realization of the national money growth shocks, does not play a role in consumer optimization. It only matters for firms, as their profits from exports denominated in foreign currency are converted back into domestic currency. In short, the ex-post exchange rate does not affect households optimizing behavior and, hence, trade allocations. Under *PCP*, households use part of their domestic-currency money balances in the realized state of nature to buy, at the equilibrium exchange rate, the foreign currency needed for imports. In short, the ex-post exchange rate now influences the optimizing behavior of households and, hence, trade allocations.

For a schematic representation of prices, quantities and their (definitional) interrelations as well as of the general structure of our CCP vs. PCP model versions, compare the respective elements and blocks in figures 1 vs. 2. Additional explanatory comments follow suit.

3 The Role of Price Setting

The model and notation we have introduced thus far enables us to draw, in the present section, an explicit parallel between the essential differences in the opti-

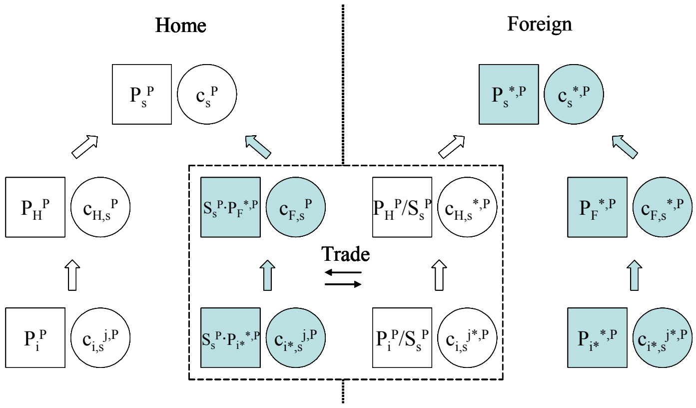

Figure 2: Notation on Price and Quantity Aggregation under PCP

mizing behavior and the resulting consumption demand and monopolistic pricing functions across alternative price setting. On that basis, a formal definition of equilibrium, in the context of our two model versions, is provided. Under *float* and *symmetry*, we then derive CCP vs. PCP results for the exchange-rate level, international relative prices, cross-country consumption/leisure allocations and, most importantly, some key measures of trade flows. The underlying algebra is systematized in more detail in *Appendix A*.

3.1 Optimization and Equilibrium

Consumption Demands and Price Levels The consumption aggregator (2) is only at first sight identical across our alternative price-setting assumptions. The reason is that its components, $c_{H,s}$ and $c_{F,s}$, although seemingly the same, are in fact optimally defined by different expressions under CCP vs. PCP.[25] They originate in some initial, pricing and quantity invoicing-specific conventions but, as the optimization proceeds on and is nationally aggregated, these differences also feed into the resulting analytical outcomes.

Standard derivations à la Dixit-Stiglitz (1977) under CCP vs. PCP result in Home optimal demands for H- (equation (9) below) and F-produced (10) brands and the respective price indices at the domestic absorption (11), import demand (12) and consumer (13) levels as follows:

$$c_{H,s}^{C} = \frac{1}{2}\left(\frac{P_H^C}{P^C}\right)^{-\varphi}\frac{M_s}{P^C} \quad \text{vs.} \quad c_{H,s}^{P} = \frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\varphi}\frac{M_s}{P_s^P}; \tag{9}$$

[25] That is what imposed the more complicated notation we employ furtehr down in the paper, e.g. $c_{H,s}^C$ and $c_{H,s}^C$ vs. $c_{H,s}^P$ and $c_{F,s}^P$, as well as the need to discuss in parallel the key CCP vs. PCP model version differences.

$$c^C_{F,s} = \frac{1}{2}\left(\frac{P^C_F}{P^C}\right)^{-\varphi}\frac{M_s}{P^C} \quad \text{vs.} \quad c^P_{F,s} = \frac{1}{2}\left(\frac{\overbrace{S^P_s P^{*,P}_F}^{\equiv P^P_{F,s}}}{P^P_s}\right)^{-\varphi}\frac{M_s}{P^P_s}; \tag{10}$$

with

$$P^C_H \equiv \left[\int_0^1 \left(P^C_i\right)^{1-\varphi} di\right]^{\frac{1}{1-\varphi}} \quad \text{vs.} \quad P^P_H \equiv \left[\int_0^1 \left(P^P_i\right)^{1-\varphi} di\right]^{\frac{1}{1-\varphi}}; \tag{11}$$

$$P^C_F \equiv \left[\int_1^2 \left(P^C_{i^*}\right)^{1-\varphi} di^*\right]^{\frac{1}{1-\varphi}} \quad \text{vs.} \quad \underbrace{S^P_s P^{*,P}_F}_{\equiv P^P_{F,s}} \equiv \left[\int_1^2 \underbrace{\left(S^P_s P^{*,P}_{i^*}\right)}_{\equiv P^P_{i^*,s}}^{1-\varphi} di^*\right]^{\frac{1}{1-\varphi}}; \tag{12}$$

$$P^C \equiv \left[\frac{1}{2}\left(P^C_H\right)^{1-\varphi} + \frac{1}{2}\left(P^C_F\right)^{1-\varphi}\right]^{\frac{1}{1-\varphi}} \quad \text{vs.} \tag{13}$$

$$P^P_s \equiv \left[\frac{1}{2}\left(P^P_H\right)^{1-\varphi} + \frac{1}{2}\underbrace{\left(S^P_s P^{*,P}_F\right)}_{\equiv P^P_{F,s}}^{1-\varphi}\right]^{\frac{1}{1-\varphi}}.$$

Clearly, under PCP the exchange-rate pass-through to import prices is *unitary*, while under CCP it is *prefixed* (cf. the CCP vs. PCP expression in (12)). For the same reason, the CPI is constant under CCP, P^C, but state-dependent under PCP, P^P_s (cf. equations (13)). This causes demands for even domestically-produced brands, at first sight identical, to be actually different across our alternative price-setting assumptions (cf. the CCP vs. PCP expression in (9)).

Output Prices Similarly to the consumption aggregator (2), the expected market value of real profits[26] which a Home firm $i \in [0,1]$ maximizes is seemingly the same, but is nevertheless differently defined under CCP vs. PCP:

$$\underset{P^C_i, P^{*,C}_i}{Max} E_0 \left[\frac{u_{c,s}}{P^C}\underbrace{\left(P^C_i c^C_{i,s} + S^C_s P^{*,C}_i c^{*,C}_{i,s} - W^C_s c^C_{i,s} - W^C_s c^{*,C}_{i,s}\right)}_{\equiv \Pi^C_{i,s}}\right], s \in S \tag{14}$$

[26]Note that the relevant weights for the states of nature in the formulas we introduce are related to the marginal utility of consumtion of the representative shareholder, $u_{c,s}$.

$$\text{vs. } \underset{P_i^P}{Max} E_0 \left[\frac{u_{c,s}}{P_s^P} \underbrace{\left(P_i^P c_{i,s}^P + P_i^P c_{i,s}^{*,P} - W_s^P c_{i,s}^P - W_s^P c_{i,s}^{*,P} \right)}_{\equiv \Pi_{i,s}^P} \right], s \in S. \tag{15}$$

Under CCP this firm i – which in our setting is also the Home *representative* firm – presets *two* prices, one in national currency and the other in foreign currency. Under PCP just *one* price, in national currency, is prefixed. Using the respective first order conditions, CCP vs. PCP optimal prices of the Home representative firm (relevant for consumer households in the domestic and foreign market) are thus:

$$P_i^C = P_H^C = \frac{\varphi}{\varphi - 1} \frac{E_0 \left[u_{c,s} W_s^C M_s \right]}{E_0 \left[u_{c,s} M_s \right]} \text{ vs.} \tag{16}$$

$$P_i^P = P_H^P = \frac{\varphi}{\varphi - 1} \frac{E_0 \left[\frac{u_{c,s}}{P_s^P} W_s^P \left(P_s^P \right)^{\varphi - 1} \left(M_s + S_s^P M_s^* \right) \right]}{E_0 \left[\frac{u_{c,s}}{P_s^P} \left(P_s^P \right)^{\varphi - 1} \left(M_s + S_s^P M_s^* \right) \right]}; \tag{17}$$

$$P_i^{*,C} = P_H^{*,C} = \frac{\varphi}{\varphi - 1} \frac{E_0 \left[u_{c,s} W_s^C M_s^* \right]}{E_0 \left[u_{c,s} S_s^C M_s^* \right]} \text{ vs.} \tag{18}$$

$$\underbrace{P_{H,s}^{*,P} \equiv \frac{P_H^P}{S_s^P}}_{\text{LOP}} \quad \Rightarrow \quad \underbrace{P_s^{*,P} = \frac{P_s^P}{S_s^P}}_{\text{PPP}}. \tag{19}$$

As evident from (19), the price at which Home representative firm's product sells in Foreign under PCP, $P_{H,s}^{*,P}$, depends on the exchange-rate level that has materialized ex-post, S_s^P. In fact, it is LOP applied to the homogeneous good type (differentiated across monopolistically produced brands) in the present context that underlies the above PCP Foreign import price definition. Moreover as we noted earlier, the price which is preset in the currency of the seller (Home, in the case we comment here) under PCP, P_H^P, becomes state-dependent when converted – via the observed exchange rate, S_s^P – in the currency of the buyer, $P_{H,s}^{*,P}$.

To sum up, the difference between our invoicing conventions boils down to the following: under CCP, CPIs are fixed across states (P^C and $P^{*,C}$); by contrast, under PCP the price of imported goods moves with the exchange rate, hence so do CPIs (P_s^P and $P_s^{*,P}$). This is the major *channel* of monetary shocks transmission – with polar effect via pass-through on optimal expenditure switching – along which we distinguish and interpret the model versions under our alternative price setting.

Definition of Equilibrium We now formally define an equilibrium concept that corresponds to the described sequential optimization.

Definition 1 *In the context of the model versions we presented, an* equilibrium *is a set of quantities and prices, such that:*

1. ***[Ex-Ante Conditions]*** before *the resolution of monetary uncertainty but given commonly held views about the joint symmetric distribution of money growth shocks* (μ_s, μ_s^*)*;*

   (a) *[Firms Stochastic Optimization] given their technology constraint and the* expected *quantities demanded in the* goods *market,* $\{E_0\left[c_{H,s}^C\right],\ E_0\left[c_{H,s}^{*,C}\right],\ E_0\left[c_{F,s}^{*,C}\right],\ E_0\left[c_{F,s}^C\right]\}$ *under CCP or* $\{E_0\left[c_{H,s}^P\right],\ E_0\left[c_{H,s}^{*,P}\right],\ E_0\left[c_{F,s}^{*,P}\right],\ E_0\left[c_{F,s}^P\right]\}$ *under PCP, the prices,* $\{P_H^C,\ P_H^{*,C},\ P_F^{*,C},\ P_F^C\}$ *under CCP or* $\left\{P_H^P,\ P_F^{*,P}\right\}$ *under PCP, that are optimally preset ex-ante (i.e. in state* 0*) and bindingly posted to consumer households for transactions ex-post (in state* s *for* $\forall s \in S$*) solve the profit maximization problem of the representative producer firm in* Home *as well as in* Foreign*;*

2. ***[Ex-Post Conditions]*** following *the resolution of monetary uncertainty and in any state of nature* $s \in S$ *that has materialized;*

   (a) *[Households Labor-Leisure Trade-Off] given its constraints and the posted prices,* $\{P_H^C,\ P_H^{*,C},\ P_F^{*,C},\ P_F^C\}$ *under CCP or* $\left\{P_H^P,\ P_F^{*,P}\right\}$ *under PCP, the representative consumer household in* Home *as well as in* Foreign *spends up all available cash on its* total *real consumption* $\{c_s, c_s^*\}$*; hours of work (employment)* $\{1 - l_s\,,\ 1 - l_s^*\}$ *are supplied by households until firms demand labor to equilibrate ex-post consumption demand for their differentiated products at the resulting equilibrium real wage rates* $\left\{\dfrac{W_s}{P_s},\ \dfrac{W_s^*}{P_s^*}\right\}$*;*

   (b) *[Households Consumer Basket Allocation] given the posted prices,* $\{P_H^C,\ P_H^{*,C},\ P_F^{*,C},\ P_F^C\}$ *under CCP or* $\left\{P_H^P,\ P_F^{*,P}\right\}$ *under PCP, the consumption quantities* $\{c_{H,s}^C,\ c_{H,s}^{*,C},\ c_{F,s}^{*,C},\ c_{F,s}^C\}$ *under CCP or* $\{c_{H,s}^P,\ c_{H,s}^{*,P},\ c_{F,s}^{*,P},\ c_{F,s}^P\}$ *under PCP solve the cost minimization problem à la Dixit-Stiglitz (1977) of the representative consumer household in* Home *as well as in* Foreign*;*

   (c) *[Goods Market Clearing] all quantities under CCP or PCP satisfy the feasibility conditions for each* differentiated brand *so that all product-brand markets – and, hence, the* international *product-type market as a whole – clear;*

   (d) *[Forex Market Clearing] the* international *forex market clears as well.*

3.2 Equilibrium Nominal Exchange Rate

The simple symmetric structure of the model we analyze allows an explicit derivation of the equilibrium nominal exchange rate (NER).[27] It solves the international *forex market clearing* condition which states that excess supply of each of the two currencies (expressed in the same monetary unit[28]) is zero in any state of nature $s \in S$:[29]

$$\underbrace{P_F^C c_{F,s}^C}_{F \text{ export revenues} \Leftrightarrow HC \text{ supply}} - S_s^C \cdot \underbrace{P_H^{*,C} c_{H,s}^{*,C}}_{H \text{ export revenues} \Leftrightarrow HC \text{ demand}} = 0 \quad (20)$$

$$\text{vs.} \quad \underbrace{S_s^P \cdot P_F^{*,P} c_{F,s}^P}_{H \text{ import demand} \Leftrightarrow HC \text{ supply}} - \underbrace{P_H^P c_{H,s}^{*,P}}_{F \text{ import demand} \Leftrightarrow HC \text{ demand}} = 0. \quad (21)$$

Substituting for optimal demands above as well as for the ideal H and F CPI definitions further on in the algebraic manipulation derives the following *general* expressions for the equilibrium NER under CCP vs. PCP:

$$S_s^C = \frac{1 + \left(\frac{P_F^{*,C}}{P_H^{*,C}} \right)^{1-\varphi}}{1 + \left(\frac{P_H^C}{P_F^C} \right)^{1-\varphi}} \frac{M_s}{M_s^*} \quad \text{vs.} \quad S_s^P = \frac{1 + \left(\frac{S_s^P P_F^*}{P_H^P} \right)^{1-\varphi}}{1 + \left(\frac{\frac{P_H}{S_s^P}}{P_F^{*,P}} \right)^{1-\varphi}} \frac{M_s}{M_s^*}. \quad (22)$$

Equilibrium NER under Full Symmetry Under *full symmetry*, i.e. with $P_H^C = P_F^{*,C}$, $P_F^C = P_H^{*,C}$, $P^C = P^{*,C}$ under CCP vs. $P_H^P = P_F^{*,P}$, $P_{F,s}^P \equiv S_s^P P_F^{*,P}$, $P_{H,s}^{*,P} \equiv \frac{P_H^P}{S_s^P}$, $P_s^P = S_s^P P_s^{*,P}$ under PCP, the above expressions simplify to

$$S_s^C = \frac{M_s}{M_s^*} \text{ vs. } S_s^P = \left(\frac{M_s}{M_s^*} \right)^{\frac{1}{\varphi}}. \quad (23)$$

The equilibrium exchange rate (23) under CCP vs. PCP only differs in including or not the key model parameter, $\varphi = \varphi^* > 1$. This result implies that, in equilibrium, the NER should be *less volatile* under PCP than under CCP,[30] just because of substitutions via imports/exports induced by the pass-through effect under PCP. In both cases, however, the equilibrium exchange rate

[27] The nominal exchange rate is defined in the usual way as the *Home*-currency price of *Foreign* money.

[28] Taking the currency of H as the common unit of account below.

[29] Note as well that because of symmetry this condition also imposes, in effect, *balanced trade* for both economies no matter the particular state that has materialized.

[30] A point first made by Betts and Devereux (1996). It is also evident that, for a given *symmetric* distribution of money growth shocks, NER volatility will thus be lower under PCP by a magnitude depending directly on the particular value of *consumption demand substitutability*, φ, or, alternatively, the degree of *monopolistic competition*, $\frac{\varphi}{\varphi-1}$.

is a function of *fundamentals*, namely the money stocks in Home and Foreign. The *more general* formula (22) does *not* impose full symmetry in order to apply simplifying substitutions relying on *PPP*, $P_s^P = S_s^P P_s^{*,P}$ under PCP or even stronger equations such as, in our CCP case, $P^C = P^{*,C}$. The benefit from looking at (22) is that this formula makes evident another principal difference between the price-setting assumptions we study here. In general, the equilibrium exchange rate in a sticky-price model of trade will depend not only on relative money stocks but also on relative price levels resulting from aggregation of the optimally prefixed prices of domestic and foreign (highly substitutable) brands. This is true for both the cases of CCP and PCP, but the difference is, again, that in our PCP version import prices are state-dependent, and hence sensitive to (or affected by) the ex-post exchange rate, whereas this is not so under CCP.[31]

Optimal Firm Prices under Full Symmetry Using (7) and its equivalent for Foreign as well as (23) under CCP and PCP to substitute for the endogenous variables W_s, W_s^* and S_s in (16) through (19), the optimal firm prices derived earlier can now be fully determined. The final model solutions for prices in terms of exogenous variables and parameters only are thus:

$$P_i^C = P_H^C = \frac{\varphi}{\varphi - 1} P^C \frac{E_0\left[u_{l,s} M_s\right]}{E_0\left[u_{c,s} M_s\right]} \text{ vs.}$$

$$P_i^P = P_H^P = \frac{\varphi}{\varphi - 1} \frac{E_0\left[u_{l,s}\left(P_s^P\right)^{\varphi-1}\left(M_s + M_s^{\frac{1}{\varphi}} M_s^{*\frac{\varphi-1}{\varphi}}\right)\right]}{E_0\left[\frac{u_{c,s}}{P_s^P}\left(P_s^P\right)^{\varphi-1}\left(M_s + M_s^{\frac{1}{\varphi}} M_s^{*\frac{\varphi-1}{\varphi}}\right)\right]};$$

$$P_i^{*,C} = P_H^{*,C} = \frac{\varphi}{\varphi - 1} P^{*,C} \frac{E_0\left[u_{l,s} M_s^*\right]}{E_0\left[u_{c,s} M_s\right]} \text{ vs.}$$

$$\underbrace{P_{H,s}^{*,P} \equiv \frac{P_H^P}{\left(\frac{M_s}{M_s^*}\right)^{\frac{1}{\varphi}}}}_{\text{LOP}} \quad \Rightarrow \quad \underbrace{P_s^{*,P} = \frac{P_s^P}{\left(\frac{M_s}{M_s^*}\right)^{\frac{1}{\varphi}}}}_{\text{PPP}}.$$

As in Bacchetta and van Wincoop (2000 a), it is easily seen that under *CCP* the prices set by the Home representative firm domestically, P_H^C, and abroad, $P_H^{*,C}$, will be the same only if $E_0\left[u_{l,s} M_s\right] = E_0\left[u_{l,s} M_s^*\right]$. This will always be true under peg, since then M_s^* can be substituted by M_s everywhere in the formulas up to here, but not generally under float. Bacchetta and van Wincoop (2000 a) formally prove, in their Lemma 1 and related Proposition 1, that $E_0\left[u_{l,s} M_s\right] = E_0\left[u_{l,s} M_s^*\right]$ and, hence, $P_H^C = P_H^{*,C}$ is true *only* when

[31] Another parameter that will also, in principle, determine the equilibrium exchange rate in this type of NOEM set-ups could be a nationally-specific elasticity of substitution in consumption, $\varphi \neq \varphi^*$ (or, equivalently, a nationally-specific degree of product market monopolization, $\frac{\varphi}{\varphi-1} \neq \frac{\varphi^*}{\varphi^*-1}$).

utility is *separable* in consumption and leisure. To be able to continue now with our focus, in this initial study, on the fully-symmetric case, in section 2 we purposefully assumed this less general case which is nevertheless widespread when it comes to modelling preferences. With separable utility under CCP and float, the prices optimally *preset* domestically and abroad will therefore be the *same*, due to symmetry, so that $P_H^C = P_H^{*,C} = P_F^C = P_F^{*,C}$.

It is also clear from the respective formula above for Home and the corresponding one for Foreign that under *PCP* and float, when just one price is optimally prefixed in each country, in the domestic currency, the two preannounced prices will have the same level, $P_H^P = P_F^{*,P}$, given symmetry and separability again. Yet the respective ex-post PCP prices in the foreign currency, $P_{H,s}^{*,P}$ and $P_{F,s}^P$, will in general not be equal to those preset domestically, as we stressed earlier. Observe, however, for future use that under PCP and *peg* the domestic-currency prices of home and foreign substitutes faced by consumers in a given country will be the same for any $s \in S$, so that $P_H^P = P_H^{*,P} = P_F^P = P_F^{*,P}$ (ex-post as well as ex-ante).

A final set of key equations in the model provides, under full symmetry, straightforward expressions for some traditional characteristics of international trade. In addition to the trade share in output *by country* considered in Bacchetta and van Wincoop (2000 a) under CCP, in our present extension we also discuss two other aspects, missing in their study and central to understanding the CCP vs. PCP outcomes of our analysis. These aspects concern *international relative prices* and the share of *world* trade in *world* output.

3.3 Equilibrium Relative Prices

Relative Price of Foreign to Domestic Goods We saw that under CCP with jointly symmetric money shocks and separable preferences, all prices are optimally prefixed in the currency of the buyer at the *same* level: $P_H^C = P_H^{*,C} = P_F^C = P_F^{*,C}$. As a consequence, the relative price of foreign-produced goods in terms of domestically-produced ones in both countries is predetermined at 1:[32]

$$p_H^C \equiv \frac{P_F^C}{P_H^C} = 1 = \frac{P_H^{*,C}}{P_F^{*,C}} \equiv p_F^{*,C} \text{ for } \forall s \in S. \tag{24}$$

Under PCP, the prices which firms preannounce in their *domestic* currency have likewise the *same* level across countries, $P_H^P = P_F^{*,P}$. However, the corresponding *foreign*-currency prices obtained via LOP, $P_{H,s}^{*,P}$ and $P_{F,s}^P$, can remain equal to the domestic-currency ones only if some low-probability state of *relative* monetary equilibrium, $s_e \in S_e \subset S$, occurs. In general, the resulting relative prices of foreign-produced goods in terms of domestically-produced ones under PCP are *reciprocal* across countries and reflect directly the ex-post nominal exchange rate:

[32] In such a way, any effects of the ex-post values of these key international relative prices on consumer behavior are precluded under CCP.

$$p_{H,s}^{P} \equiv \frac{\overbrace{S_s^P P_F^{*,P}}^{\equiv P_{F,s}^{P}}}{P_H^P} = S_s^P = \left(\frac{\overbrace{\frac{P_H^P}{S_s^P}}^{\equiv P_{H,s}^{*,P}}}{P_F^{*,P}} \right)^{-1} \equiv \left(p_{F,s}^{*,P} \right)^{-1} \neq 1 \text{ unless } s_e. \tag{25}$$

Terms of Trade In our symmetric set-up, the terms of trade (ToT) are *inversely* defined – across countries for the same invoicing convention as well as across price setting assumptions for each of the countries with respect to the nominal exchange rate. Our *CCP* model version thus implies a *negative* relationship between the NER and the ToT: a nominal depreciation *improves* the terms of trade. Just the opposite effect is, however, predicted by our *PCP* model version: the relationship between the NER and the ToT is *positive*, so that a nominal depreciation *weakens* the terms of trade and induces, in turn, *expenditure switching*, an international spillover channel largely debated in the Mundell-Fleming-Dornbusch tradition:

$$(ToT)_{H,s}^{C} \equiv \frac{\overbrace{P_F^C}^{\equiv P_H^{Im,C}}}{\underbrace{S_s^C P_H^{*,C}}_{P_H^{Ex,C}}} = \frac{1}{S_s^C} = \left(\frac{\overbrace{P_H^{*,C}}^{\equiv P_F^{Im,C}}}{\underbrace{\frac{P_F^C}{S_s^C}}_{\equiv P_{F,s}^{Ex,C}}} \right)^{-1} \equiv \left[(ToT)_{F,s}^{*,C} \right]^{-1} \neq 1 \text{ unless } s_e \text{ vs.} \tag{26}$$

$$(ToT)_{H,s}^{P} \equiv \frac{\overbrace{P_{F,s}^P}^{\equiv P_{H,s}^{Im,P}}}{\underbrace{P_H^P}_{\equiv P_{H,s}^{Ex,P}}} = \frac{S_s^P P_F^{*,P}}{P_H^P} = S_s^P = \left(\frac{\overbrace{\frac{P_H^P}{S_s^P}}^{\equiv P_{F,s}^{Im,P}}}{\underbrace{P_F^*}_{\equiv P_{F,s}^{Ex,P}}} \right)^{-1} \equiv \left[(ToT)_{F,s}^{*,P} \right]^{-1} \neq 1 \text{ unless } s_e. \tag{27}$$

This latter result, which we have explicitly derived from microfoundations, is in line with findings in other recent NOEM papers, in particular with the Obstfeld-Rogoff (2000) correlation approach of checking for pricing-to-market in macrodata.[33]

Real Exchange Rate In compliance with the PPP literature, our symmetric *PCP* model results in a microfounded real exchange rate (RER) that is *constant* (across states of nature) at 1 in equilibrium:

[33] Our theoretical point here is the subject of related empirical work in Mihailov (2003 b).

$$(RER)_H^P \equiv \frac{S_s^P P_s^{*,P}}{P_s^P} = \frac{P_s^P}{P_s^P} = 1 = \frac{P_s^{*,P}}{P_s^{*,P}} = \frac{\frac{P_s^P}{S_s^P}}{P_s^{*,P}} \equiv (RER)_F^{*,P} \text{ for } \forall s \in S. \tag{28}$$

On the other hand, our *CCP* version leads to a parallel equilibrium outcome of a RER that *moves one-to-one* with the NER (across states of nature), as consistent with the higher RER volatility implied by PTM-based models:

$$(RER)_{H,s}^C \equiv \frac{S_s^C P^{*,C}}{P^C} = S_s^C = \left(\frac{\frac{P^C}{S_s^C}}{P^{*,C}} \right)^{-1} \equiv \left[(RER)_{F,s}^{*,C} \right]^{-1} \neq 1 \text{ unless } s_e. \tag{29}$$

3.4 Equilibrium Consumption and Leisure across Countries

To better understand the implications of the *microfounded* general-equilibrium framework we study for CCP vs. PCP trade flows, we now have to first consider its outcomes across price setting in terms of the ingredients of the utility function, namely consumption and leisure. Our essential points are summarized in the *propositions* we state in their logical order throughout the present subsection. Proofs, based largely on earlier definitions and derivations, are provided in *Appendix A* whereas interpretations follow further down in the main text.

Proposition 1 *(Relative Consumption)* Relative *real consumption is ultimately determined by the* relative *money stock,* no matter *the particular* price setting *assumed.*

To put it differently, Proposition 1 establishes that it is national money shocks and, consequently, relative money stocks (or relative wealth in our simple NOEM framework) that really matter – via demand and trade – for ex-post real consumption differences across the ex-ante symmetric countries, irrespective of the invoicing convention. Note, however, that under CCP but not PCP the relative money stock is also the equilibrium nominal exchange rate and that under PCP but not CCP the elasticity of consumption demand, $\varphi > 1$, mitigates[34] the effect of relative monetary disequilibria. More importantly, there is another, principal difference between our price-setting assumptions which results from the fact that CCP prevents substitution across borders, and hence expenditure switching, while under PCP such substitution is optimal, as we show next.

Proposition 2 *(Consumption Switching) Under* CCP *the optimal split-up of real consumption between demand for domestic and foreign goods is always* 50 : 50 *whereas under* PCP *it is ultimately determined by the* relative *money stock.*

[34] Compared to the CCP case.

Proposition 2 is of major importance for understanding our equilibrium trade share outcomes across price-setting assumptions to be discussed in more detail later on. It implies that in the *CCP* model version a monetary expansion – coordinated under peg or unilateral under float – does *not* induce any *bias* in goods consumption. In the *PCP* case, by contrast, a monetary expansion in one of the countries results – by depreciating (appreciating, for the other country) the equilibrium exchange rate, making imports more expensive (cheaper) and inducing substitution away from (into) them – in a *bias in both countries* favoring consumption of the goods produced in the *expansionary* country.

Proposition 3 *(Relative Leisure) Under* CCP *equilibrium output, employment and leisure (but not consumption) are always equal across countries whereas under* PCP *output, employment and leisure (as well as consumption) are ultimately determined by the* relative *money stock and are thus not generally equal across nations.*

The basic intuition behind Proposition 3 is that under CCP – when there is optimally no consumption switching away from the preferred 50 : 50 split-up – the two countries always produce the *same* real quantities of *output*, no matter the particular state of nature that has occurred. Because of the identical technologies, the two countries furthermore employ the *same* amount of *labor*, i.e. employment is the same as well. Therefore, the hours of *leisure* the representative household in Home and in Foreign enjoys – residually, due to the demand-determined output and, hence, labor input – under CCP are always the *same* too. By contrast, under PCP – when consumers switch to the cheaper product due to the now operating pass-through channel – the two countries do *not* produce the *same* real quantities of *output*, unless some state of nature of relative monetary equilibrium has materialized. Due to the identical technologies again, the two countries do *not* employ the *same* amount of *labor*. Consequently, the hours of *leisure* the representative household in Home and in Foreign enjoys under PCP are generally *not* the *same* either.

To provide certain parallels between the present set-up and the preceding related literature,[35] we finally consider the traditional example of the impact of a one-time money supply shock. Since the model here is explicitly stochastic, we shall rather be talking about *relative* monetary expansion or *relative* money stock disequilibrium. In order not to violate the credibility of our sticky-price environment, we more precisely analyze ex-post allocations in response to money stock growth shocks of a *small* magnitude occurring after the initial symmetric equilibrium.

Proposition 4 *(Impact of Monetary Expansion) In our CCP model version, any* relative *monetary disequilibrium under float increases the* ex-post *utility of the residents of the expansionary country* relative to *the* ex-post *utility of the residents of the contractionary country. Interestingly, PCP* inverses *this beggar*-thy-neighbor *conclusion into a beggar*-thyself *one.*

[35] E.g. Obstfeld-Rogoff (1995) and the numerous papers building on it.

The logic underlying Proposition 4 is that in our *CCP* model version households in both economies enjoy equal amount of leisure in any state that has materialized, but at the same time those in the expansionary economy consume more *relative to* their neighbors in the contractionary economy. So overall, ex-post utility is higher in the expansionary country, a result reminiscent of (but not identical to) "beggar-*thy-neighbor*" policies debated in the Mundell-Fleming-Dornbusch tradition. Under *PCP*, by contrast, the gain of the Home representative household in consumption relative to the Foreign one is lower than its simultaneous relative loss in leisure (when consumption and leisure are *equally* valued, as we assume for our purposes here). Under PCP, therefore, Home residents are worse-off than Foreign ones in *cross-country* ex-post utility terms following a Home *relative* monetary expansion, a finding similar (but not equivalent) to classic and more recent "beggar-*thyself*" reasoning.

3.5 Equilibrium Trade Flows

Trade Shares by Country Under CCP vs. PCP, Home[36] equilibrium (ex-post) *foreign trade* / GDP ratio in each state of nature $s \in S$ is defined by

$$(ft)^C_{H,s} \equiv \frac{(Ex)^C_{H,s} + (Im)^C_{H,s}}{(DA)^C_{H,s} + (Ex)^C_{H,s}} = \frac{S^C_s \cdot P^{*,C}_H \cdot c^{*,C}_{H,s} + P^C_F \cdot c^C_{F,s}}{P^C_H \cdot c^C_{H,s} + S^C_s \cdot P^{*,C}_H \cdot c^{*,C}_{H,s}} \text{ vs.} \tag{30}$$

$$(ft)^P_{H,s} \equiv \frac{(Ex)^P_{H,s} + (Im)^P_{H,s}}{(DA)^P_{H,s} + (Ex)^P_{H,s}} = \frac{P^P_H \cdot c^{*,P}_{H,s} + \overbrace{S^P_s \cdot P^{*,P}_F}^{\equiv P^P_{F,s}} \cdot c^P_{F,s}}{P^P_H \cdot c^P_{H,s} + P^P_H \cdot c^{*,P}_{H,s}}, \tag{31}$$

where $(Ex)^C_{H,s}$ denotes Home exports, $(Im)^C_{H,s}$ Home imports and $(DA)^C_{H,s}$ Home domestic absorption, all these three Home-currency *values* (prices multiplied by quantities) under CCP and in any state $s \in S$ that has materialized. $(Ex)^P_{H,s}$, $(Im)^P_{H,s}$ and $(DA)^P_{H,s}$ are, of course, the respective PCP values.

Substitutions for optimal demands and use of the Home ideal CPI definition derive – under *full symmetry* and *separable preferences* – the CCP vs. PCP *trade share curve* for Home:

$$(ft)^C_H = \frac{2}{\left(\frac{P^C_H}{P^{*,C}_H}\right)^{1-\varphi} + 1} = \frac{2}{\left(\frac{E_0[u_{l,s}M_s]}{E_0[u_{l,s}M^*_s]}\right)^{1-\varphi} + 1} = const = 1 \text{ vs.} \tag{32}$$

$$(ft)^P_{H,s} = \frac{2}{\left(\frac{P^P_s}{P^{*,P}_s}\right)^{\varphi-1} + 1} = \frac{2}{(S^P_s)^{\varphi-1} + 1} = \frac{2}{\left(\frac{M_s}{M^*_s}\right)^{\frac{\varphi-1}{\varphi}} + 1} \neq 1 \text{ unless } s_e. \tag{33}$$

[36] For Foreign, the corresponding expressions are symmetric.

The corresponding *trade share curve* for *Foreign* is symmetrically given by

$$(ft)_F^C = \frac{2}{\left(\frac{P_F^{*,C}}{P_H^C}\right)^{1-\varphi}+1} = \frac{2}{\left(\frac{E_0\left[u_{l,s}^* M_s^*\right]}{E_0\left[u_{l,s}^* M_s\right]}\right)^{1-\varphi}+1} = const = 1 = (ft)_H^C \text{ vs.} \tag{34}$$

$$(ft)_{F,s}^P = \frac{2}{\left(\frac{P_s^{*,P}}{P_s^P}\right)^{\varphi-1}+1} = \frac{2}{\left(\frac{1}{S_s^P}\right)^{\varphi-1}+1} = \frac{2}{\left(\frac{M_s^*}{M_s}\right)^{\frac{\varphi-1}{\varphi}}+1} \neq 1 \neq (ft)_{H,s}^P \text{ unless } s_e. \tag{35}$$

These two pairs of equations compare directly the impact of our alternative price-setting assumptions on trade, measured relative to output.[37] Under CCP, (32) and (34) show that the *equilibrium* trade share is constant at 1 in each country and in any state of nature that has materialized. Under PCP, by contrast, this is not generally the case: as clear from (33) and (35), national trade-to-output ratios now both become state-dependent, i.e. volatile, unless some low-probability state s_e of relative monetary equilibrium occurs.

To see the intuition behind, assume a Home *relative* monetary expansion and compare the numerator and denominator in (30) under float. Under CCP, *no substitution* occurs between domestic and foreign brands of the same product type we model here, due to the preset *buyer*'s currency prices and the resulting foreign/domestic relative price *equality* across countries in (24). That is why the additional (or excessive, with respect to Foreign) Home cash in the observed state of nature $s_H \in S_H \subset S$ splits up evenly $(50:50)$ into a domestic demand increase and an import demand increase:[38] $(DA)_{H,s_H}^C \uparrow = (Im)_{H,s_H}^C \uparrow$. Thus, the denominator in (30) changes by the same amount as the numerator, and the trade/output ratio remains constant (across states).

Under PCP, by contrast, prices are prefixed in the currency of the *seller*. Therefore, the observed nominal exchange rate affects import prices, and hence consumer price levels, thus partly "flexibilizing" our otherwise fix-price model. The ex-post NER feeds on into the foreign/domestic relative price *reciprocity* across countries highlighted in (25). This key relative price is now state-dependent and, in turn, influences itself optimal consumer decisions on cross-border *substitution*[39] in demand. Home import demand falls as more expensive imports resulting from the depreciated exchange rate (relative to its ex-ante equilibrium of 1) are substituted away and into domestic analogues so that domestic demand rises, as well as Home exports, for the same (or rather symmetric) reason applied to Foreign importing households:[40] $(Im)_{H,s_H}^P \downarrow = (Ex)_{H,s_H}^P \uparrow = (DA)_{H,s_H}^P \uparrow$.

[37] The first equality in the formulas expresses the trade/GDP ratio as a function of *price levels*. The last equality is, in turn, the reduced-form version which expresses trade relative to output as a function of the *exogenous variables* only.

[38] As formally shown in Proposition 2.

[39] Whose *degree* depends on the particular value of the key *elasticity* parameter $\varphi = \varphi^* > 1$.

[40] According to Proposition 2, again.

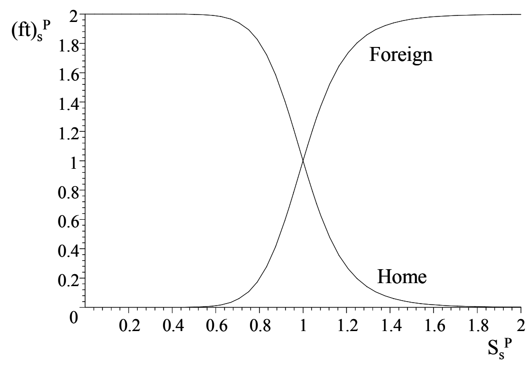

Figure 3: PCP Trade Share Curves under "Usual" Monopolistic Competition (for a markup of 10%, i.e. $\varphi = 11$)

Thus, the denominator in (31) goes up whereas the numerator stays flat, as rising exports and falling imports *compensate exactly* each other *in value.* The equilibrium trade share in Home is consequently less than its CCP magnitude of 1, and the trade share in Foreign is more than 1, following a Home *relative* monetary expansion.

To illustrate the interpretation suggested above, we present in Figure 3 the *PCP trade share curves* for Home, equation (33), and for Foreign, equation (35), according to a baseline computation we have performed setting $\varphi = 11$. This latter value of the elasticity of substitution in consumption demand is consistent with a markup $\frac{\varphi}{\varphi-1}$ of 10%, a largely consensual estimate in empirical studies. For completeness, we have also studied the cases of a very elastic demand, $\varphi = 101$, which corresponds to a tiny markup of only 1% as in Figure 4 and of almost inelastic demand, $\varphi = 2$, corresponding to a huge markup of 100% as in Figure 5. The graphs show the trade share in output $(ft)_s^P$ (on the vertical axis) under PCP, float and full symmetry as a function of the equilibrium nominal exchange rate S_s^P or, ultimately, the underlying relative money stock $\frac{M_s}{M_s^*}$ (on the horizontal axis).

A comparison among the reported three cases shows that the *degree* of substitutability $\varphi > 1$ across the individualized brands that nations exchange within the same type of good under PCP trade – or, alternatively, the *degree* of imperfect competition identified by the monopolistic markup $\frac{\varphi}{\varphi-1} > 1$ charged over price – matters a lot in related analyses. In particular, PCP trade share curves are much flatter and more curved in the vicinity of 1 under *low* substitutability and *highly monopolized* world market structure relative to the "normal" situation ($\varphi = 11$). By contrast, these same curves are almost vertical and straight in the *near* vicinity of 1 with *high* substitutability and competition *close to perfect.*

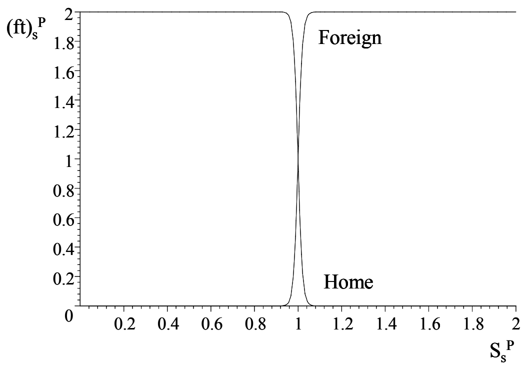

Figure 4: PCP Trade Share Curves under Near-Perfect Competition (for a markup of 1%, i.e. $\varphi = 101$)

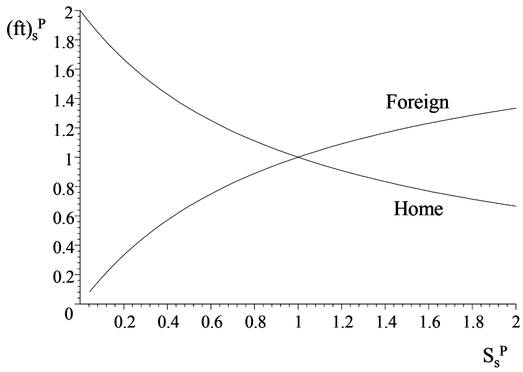

Figure 5: PCP Trade Share Curves under High Monopolistic Competition (for a markup of 100%, i.e. $\varphi = 2$)

World Trade Share

Proposition 5 *(World Trade Share) For the world economy as a whole, the trade-to-output ratio is* constant *at* 1 *in any state of nature* $s \in S$*, due to symmetry and* no matter *the particular price setting assumed.*

Under *CCP*, *Home* nominal trade is always equal to *Home* nominal output so that the *Home* trade share in output is constant at 1, irrespective of the state of nature that has materialized. The same is true for *Foreign*, and as a consequence the (equally-weighted) world trade-to-GDP ratio is also 1 for $\forall s \in S$.[41] Under *PCP*, by contrast, *Home* and *Foreign* trade shares in output are state-dependent and *not* equal to each other and to 1 unless relative monetary equilibrium occurs ($s_e \subset S$). However, as can also be verified by looking at figures 3, 4, and 5 the *Home* and *Foreign* trade share curves are *complementary* in the sense that at each point they sum to 2, so that world trade equals world output for $\forall s \in S$.

4 Effects of the Exchange-Rate Regime

Making further use of the equilibrium solutions under float we characterized thus far, the present section summarizes the implications of a peg, and therefore of the alternative exchange-rate regimes we study here, for international trade prices and flows. Our regime comparisons discussed below are made along two dimensions, namely with respect to *ex-post (equilibrium)* and *ex-ante (expected)* trade measures. The reason is that when evaluating float vs. peg under (monetary) uncertainty it is the expected levels of the relevant variables, i.e. integrated over the entire distribution of shocks, that can be meaningfully compared, the *ex-post* ones being stochastic, i.e. state-specific. We saw, however, that our equilibrium model outcomes concerning, in particular, the share of nominal trade in nominal output by country were not necessarily state-dependent, and whether they were or not depended on the currency of invoicing assumed. Moreover, the *equilibrium* solutions are a necessary first *step* in deriving the *expected* ones. That is why we also retain in what follows the ex-post dimension of our analysis.

4.1 Comparative Synthesis of Equilibrium Results

Table 1 captures in a synthetic form the effects we evoked in our propositions up to now. It compares the *equilibrium* model outcomes under a *flexible* exchange-rate regime across the alternative price-setting conventions studied.

Similarly, Table 2 provides a compact account of our CCP vs. PCP *equilibrium* findings under a *fixed* exchange-rate regime, i.e. with $M_s \equiv M_s^*$ for

[41] This latter equality does not, however, also mean that real consumption is equal in the two countries, which will be true only under equal money growth rates in a given state of nature s_e (recall Proposition 1).

	CCP	PCP
NER	$S_s^c = \frac{M_s}{M_s^*} \neq 1$ unless s_e	$S_s^P = \left(\frac{M_s}{M_s^*}\right)^{\frac{1}{\varphi}} \neq 1$ unless s_e
relative prices		
foreign/home	$p_H^C = p_F^{*,C} = 1$	$\overbrace{p_{H,s}^P = \left(p_{F,s}^{*,P}\right)^{-1} = S_s^P =}^{\neq 1 \text{ unless } s_e}$
ToT	$(ToT)_{H,s}^C = \frac{1}{(ToT)_{F,s}^{*,C}} = \frac{1}{S_s^C} =$	$\overbrace{= (ToT)_{H,s}^P = \frac{1}{(ToT)_{F,s}^{*,P}}}^{\neq 1 \text{ unless } s_e}$
RER	$= \frac{1}{(RER)_{H,s}^C} = (RER)_{F,s}^{*,C} \neq 1$ unless s_e	$(RER)_H^P = (RER)_F^{*,P} = 1$
consumption		
relative	$c_s^C \neq c_s^{*,C}$ unless s_e	$c_s^P \neq c_s^{*,P}$ unless s_e
split-up	$\frac{c_{H,s}^C}{c_{F,s}^C} = \frac{c_{F,s}^{*,C}}{c_{H,s}^{*,C}} = 1, \forall s$	$1 \neq \frac{c_{H,s}^P}{c_{F,s}^P} \neq \frac{c_{F,s}^{*,P}}{c_{H,s}^{*,P}} \neq 1$ unless s_e
aggregates	$c_{H,s}^C = c_{F,s}^C \neq c_{H,s}^{*,C} = c_{F,s}^{*,C}, \forall s$	$c_{H,s}^P \neq c_{F,s}^P \neq c_{H,s}^{*,P} \neq c_{F,s}^{*,P}$ unless s_e
labor/leisure		
employment	$n_s^C = n_s^{*,C}, \forall s$	$n_s^P \neq n_s^{*,P}$ unless s_e
leisure	$l_s^C = l_s^{*,C}, \forall s$	$l_s^P \neq l_s^{*,P}$ unless s_e
trade-to-output		
by country	$(ft)_H^C = (ft)_F^{*,C} = 1$	$\overbrace{(ft)_{H,s}^P}^{\neq 1} \neq \overbrace{(ft)_{F,s}^{*,P}}^{\neq 1}$ unless s_e
world	$\frac{1}{2}(ft)_H^C + \frac{1}{2}(ft)_F^{*,C} = 1$	$\frac{1}{2}(ft)_{H,s}^P + \frac{1}{2}(ft)_{F,s}^{*,P} = 1, \forall s$

Table 1: Equilibrium Results under Float

$\forall s \in S$. It helps clarify in an explicit manner the parallels and divergencies with regard to the corresponding float results in Table 1.

	CCP	PCP
NER	$S_s^C = \frac{M_s}{M_s} = 1, \forall s$	$S_s^P = \left(\frac{M_s}{M_s}\right)^{\frac{1}{\varphi}} = 1, \forall s$
relative prices		
foreign/home	*same* as under float	$p_H^P = p_F^{*,P} = S^P =$
ToT	$(ToT)_H^C = (ToT)_F^C = S^C =$	$= (ToT)_H^P = (ToT)_F^P = 1$
RER	$= (RER)_H^C = (RER)_F^C = 1$	*same* as under float
consumption		
relative	$c_s^C = c_s^{*,C}, \forall s$	$c_s^P = c_s^{*,P}, \forall s$
split-up	*same* as under float	$\frac{c_{H,s}^P}{c_{F,s}^P} = \frac{c_{F,s}^{*,P}}{c_{H,s}^{*,P}} = 1, \forall s$
aggregates	$c_{H,s}^C = c_{F,s}^C = c_{H,s}^{*,C} = c_{F,s}^{*,C}, \forall s$	$c_{H,s}^P = c_{F,s}^P = c_{H,s}^{*,P} = c_{F,s}^{*,P}, \forall s$
labor/leisure		
employment	*same* as under float	$n_s^P = n_s^{*,P}, \forall s$
leisure	*same* as under float	$l_s^P = l_s^{*,P}, \forall s$
trade-to-output		
by country	*same* as under float	$(ft)_H^P = (ft)_F^P = 1$
world	*same* as under float	*same* as under float

Table 2: Equilibrium Results under Peg

On the basis of these two comparative tables, we next discuss the impact of alternative exchange rate-regimes on trade prices and flows, given CCP or PCP.

4.2 Relative Prices under Peg

As far as the key international prices are concerned, a peg makes a difference with respect to a float in that it ensures all three relative prices we considered – the foreign/domestic output price, the ToT and the RER – to be equal to 1, i.e. to the *fixed* NER (cf. tables 1 and 2) not only ex-ante (in expectation) but also ex-post (in equilibrium) in any realized state. Consequently, Home as well as Foreign agents perceive these prices in the *same neutral* way which does not induce substitutions in consumption via pass-through and expenditure switching. Under float and *CCP* (see Table 1), this is not generally the case for the ToT and the RER, no matter that the relative price of foreign-produced goods in terms of domestic goods is always predetermined at 1 (so that the expenditure-switching channel is inoperative). Under float and *PCP* (see again Table 1), it is not generally the case for this latter relative price (so that now NER pass-through induces optimal expenditure switching) and for the ToT, no matter that the equilibrium RER is always 1, due to PPP.

4.3 Expected Trade Flows

Proposition 6 *(Expected Trade Share) Under full symmetry and separable preferences, the* expected *trade-to-output ratio in each of the countries is always* 1, no matter *the particular price setting and exchange-rate regime modelled.*

Under CCP, equations (32) and (34) we derived earlier showed that the value of trade is equal to the value of output, irrespective of the particular state of nature that has materialized. To put it differently, both trade and output do vary in *value* across states, but under CCP when there is no consumption switching this variation is in the same direction and proportion so that their *ratio* always remains constant, at 1 under full symmetry and separable preferences. Therefore, *expected* trade-to-output is 1 under CCP, given the above assumptions:

$$E_0\left[(ft)_H^C\right] = E_0\left[(ft)_F^C\right] = E_0\left[1\right] = 1, \quad s \in S. \tag{36}$$

Taking expectations from the equilibrium trade share formulas, (33) and (35), under PCP with float and *full symmetry* is shown in *Appendix A* to derive the same result:

$$E_0\left[(ft)_{H,s}^P\right] = 1 = E_0\left[(ft)_{F,s}^P\right], \quad s \in S. \tag{37}$$

We thus conclude that *expected* trade-to-output is 1 under PCP too.

To sum-up, our alternative assumptions on invoicing and monetary arrangements are neutral to *expected* trade shares, the *relevant* measure to compare them under uncertainty, as in our framework.[42]

However, there is one essential way, valid only under PCP, in which the exchange-rate regime does matter for trade in our set-up. It is that a peg eliminates – by preventing any exchange-rate pass-through on relative prices and, hence, by shutting down the expenditure switching channel – the volatility of trade in terms of output across states of nature. Comparing the trade share formulas (33) and (35) makes it easy to see that a peg under PCP *restores* in any $s \in S$ the ex-post equality, typical under CCP with float, between nominal trade and nominal output in each of the countries. This interesting parallel is highlighted next.

Corollary 1 *(Trade-Output Equalization under PCP with Peg) A fixed exchange-rate regime, by maintaining* relative *money stock equilibrium in* any *state of nature, guarantees under PCP – via the optimal consumption split-up channel – equilibrium trade to be equal to output in both countries modelled.*

Proof. Follows directly from the proofs of propositions 2 and 4. ■

Note that trade-output equalization obtains always under CCP even with float,[43] so a peg is in that case not needed to bring about such a result.

[42] Under full symmetry and separable preferences.
[43] Given full symmetry and separable preferences.

Given the *equal* preference for Home and Foreign product brands in the simple stochastic model of international trade we analyzed, there may thus be some role for a peg in eliminating ex-post consumption switching under float and, consequently, stabilizing the equilibrium trade-to-output in both countries. But rough calculations similar to those in the proof of Proposition 4 (Impact of Monetary Expansion) have indicated that there is a cost to such stabilization in terms of some loss (not a big one, it is true) of *world* consumption relative to a PCP with float. Since (slightly) reduced real world consumption implies, in this framework, (slightly) increased world leisure, a deeper welfare analysis of the set-up we considered requires an explicit specification of the utility function, which we preferred to keep general for our purposes here, and thus goes beyond the scope of the present study.

5 Concluding Comments

The objective of this paper was to analyze the implications of alternative price setting in evaluating the effects of the exchange-rate regime on international trade. The recent NOEM modelling approach underlying much related research has provided a modern toolkit to revisit this classic but still unresolved issue. To study it within an appropriate framework, we essentially extended Bacchetta and van Wincoop's (2000 a) stochastic "benchmark monetary model" based on consumer's currency pricing (CCP) to a producer's currency pricing (PCP) version as well.

Our analysis confirmed in a broader context that a peg does not necessarily imply a higher trade share in output relative to a float, for any of the two identical countries or currency blocs modelled as well as for the world economy as a whole. With full symmetry, only monetary shocks and separable but otherwise very general utility, the exchange-rate regime does not matter for the *expected level* of trade-to-output ratios across nations, irrespective of the assumed price setting. This important result was explicitly derived from microfoundations and formally proved within our purposefully kept simple analytical framework. We also pointed out that once nominal rigidity is distinguished across open-economy invoicing practices, a comparison of exchange-rate regimes is nevertheless meaningful under *PCP*, although not CCP, in terms of *volatility* of relative prices and, hence, national trade shares. More precisely, the equilibrium trade share *by country* becomes volatile across states of nature under PCP, although it is still constant at 1 for the *world* as a whole, just like in the CCP model version. There is, thus, an effect of a peg under PCP, absent under CCP, in *stabilizing* across states of nature equilibrium trade-to-output in each of the economies at its expected level of 1.

We identified the difference in the impact of exchange-rate regimes on national trade share variability as originating in the currency denomination of transactions and, hence, the *exchange-rate pass-through* implied by our alternative price-setting assumptions. Consequently, the expenditure-switching channel functions well under (full) PCP but not at all under (full) CCP. We showed, in particular, that under *both* CCP and PCP relative real consumption is determined in equilibrium by the relative money stock, although in a different way. We also demonstrated that the optimal split-up of real consumption between demand for domestic and foreign goods is 50 : 50 under *CCP* no matter the state of nature, so any kind of monetary expansion – coordinated under peg or unilateral under float – does not induce switching in consumption. Under *PCP*, this optimal split-up depends instead on the relative money stock in the realized state. Thus, a monetary expansion under float in one of the countries results in a bias in both countries favoring consumption of the goods produced in the expansionary country. Finally, we proved that under *CCP* equilibrium output, employment and, ultimately, leisure (but not consumption) are always the same across countries, whereas under *PCP* they are determined (as well as consumption) by the relative money stock and are therefore not equal between nations unless in the case of relative monetary equilibrium.

A Proofs of Propositions

A.1 Proof of Proposition 1 (Relative Consumption)

Proof.

- Under *CCP* and *full* symmetry with *separable* preferences (recall that in this case $P_H^C = P_F^C = P_H^{*,C} = P_F^{*,C}$ and thus $P^C = P^{*,C}$), *relative* real consumption can be expressed as follows:

$$\frac{c_s^C}{c_s^{*,C}} \equiv \frac{c_{H,s}^C + c_{F,s}^C}{c_{F,s}^{*,C} + c_{H,s}^{*,C}} = \frac{\frac{1}{2}\left(\frac{P_H^C}{P^C}\right)^{-\varphi}\frac{M_s}{P^C} + \frac{1}{2}\left(\frac{P_F^C}{P^C}\right)^{-\varphi}\frac{M_s}{P^C}}{\frac{1}{2}\left(\frac{P_F^{*,C}}{P^{*,C}}\right)^{-\varphi}\frac{M_s^*}{P^{*,C}} + \frac{1}{2}\left(\frac{P_H^{*,C}}{P^{*,C}}\right)^{-\varphi}\frac{M_s^*}{P^{*,C}}} =$$

$$= \frac{\left(\frac{P_H^C}{P^C}\right)^{-\varphi}\frac{M_s}{P^C}}{\left(\frac{P_F^{*,C}}{P^{*,C}}\right)^{-\varphi}\frac{M_s^*}{P^{*,C}}} = \frac{M_s}{M_s^*} = S_s^C \neq 1 \text{ unless } s_e \subset S;$$

- Under *PCP* and *full* symmetry with *separable* preferences (so that $P_H^P = P_F^{*,P}$, $P_{F,s}^P = S_s^P P_F^{*,P}$, $P_{H,s}^{*,P} = \frac{P_H^P}{S_s^P}$ and thus $P_s^P = S_s^P \Leftrightarrow P_s^{*,P} = \frac{P_s^P}{S_s^P}$), analogous reasoning derives *relative* real consumption to be:

$$\frac{c_s^P}{c_s^{*,P}} \equiv \frac{c_{H,s}^P + c_{F,s}^P}{c_{F,s}^{*,P} + c_{H,s}^{*,P}} = \frac{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\varphi}\frac{M_s}{P_s^P} + \frac{1}{2}\left(\frac{P_{F,s}^P}{P_s^P}\right)^{-\varphi}\frac{M_s}{P_s^P}}{\frac{1}{2}\left(\frac{P_F^{*,P}}{P_s^{*,P}}\right)^{-\varphi}\frac{M_s^*}{P_s^{*,P}} + \frac{1}{2}\left(\frac{P_{H,s}^{*,P}}{P_s^{*,P}}\right)^{-\varphi}\frac{M_s^*}{P_s^{*,P}}} =$$

$$= \frac{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\varphi}\frac{M_s}{P_s^P} + \frac{1}{2}\left(\frac{S_s^P P_F^{*,P}}{P_s^P}\right)^{-\varphi}\frac{M_s}{P_s^P}}{\frac{1}{2}\left(\frac{P_F^{*,P}}{\frac{P_s^P}{S_s^P}}\right)^{-\varphi}\frac{M_s^*}{\frac{P_s^P}{S_s^P}} + \frac{1}{2}\left(\frac{\frac{P_H^P}{S_s^P}}{\frac{P_s^P}{S_s^P}}\right)^{-\varphi}\frac{M_s^*}{\frac{P_s^P}{S_s^P}}} =$$

$$= \frac{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\varphi}\frac{M_s}{P_s^P}\left[1 + \left(S_s^P\right)^{-\varphi}\right]}{\frac{1}{2}\left(\frac{P_F^{*,P}}{P_s^P}\right)^{-\varphi}\frac{M_s^*}{P_s^P}\left[\left(S_s^P\right)^{-\varphi}S_s^P + S_s^P\right]} =$$

$$= \frac{M_s}{M_s^*}\frac{\left[1 + \left(S_s^P\right)^{-\varphi}\right]}{\left[\left(S_s^P\right)^{-\varphi} + 1\right]S_s^P} = \left(S_s^P\right)^{\varphi}\frac{1}{S_s^P} =$$

$$= \left(S_s^P\right)^{\varphi - 1} = \left(\frac{M_s}{M_s^*}\right)^{\frac{\varphi-1}{\varphi}} \neq 1 \text{ unless } s_e \subset S.$$

This completes our proof. ■

A.2 Proof of Proposition 2 (Consumption Switching)

Proof.

- Under *CCP* and *full* symmetry with *separable* preferences ($P_H^C = P_F^C = P_H^{*,C} = P_F^{*,C}$ and $P^C = P^{*,C}$), the optimal *split-up* of real consumption between demand of domestic and foreign goods can be expressed as follows:

$$\text{for } \textit{Home}\text{:} \; \frac{c_{H,s}^C}{c_{F,s}^C} = \frac{\frac{1}{2}\left(\frac{P_H^C}{P^C}\right)^{-\varphi}\frac{M_s}{P^C}}{\frac{1}{2}\left(\frac{P_F^C}{P^C}\right)^{-\varphi}\frac{M_s}{P^C}} = 1 \Leftrightarrow c_{H,s}^C = c_{F,s}^C \text{ for } \forall s \in S,$$

$$\text{for } \textit{Foreign}\text{:} \; \frac{c_{F,s}^{*,C}}{c_{H,s}^{*,C}} = \frac{\frac{1}{2}\left(\frac{P_F^{*,C}}{P^{*,C}}\right)^{-\varphi}\frac{M_s^*}{P^{*,C}}}{\frac{1}{2}\left(\frac{P_H^{*,C}}{P^{*,C}}\right)^{-\varphi}\frac{M_s^*}{P^{*,C}}} = 1 \Leftrightarrow c_{F,s}^{*,C} = c_{H,s}^{*,C} \text{ for } \forall s \in S.$$

- Under *PCP* and *full* symmetry with *separable* preferences ($P_H^P = P_F^{*,P}$, $P_{F,s}^P = S_s^P P_F^{*,P}$, $P_{H,s}^{*,P} = \frac{P_H^P}{S_s^P}$ and $P_s^P = S_s^P P_s^{*,P} \Leftrightarrow P_s^{*,P} = \frac{P_s^P}{S_s^P}$), analogous reasoning derives the optimal *split-up* of real consumption to be:

$$\text{for } \textit{Home}\text{:} \; \frac{c_{H,s}^P}{c_{F,s}^P} = \frac{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\varphi}\frac{M_s}{P_s^P}}{\frac{1}{2}\left(\frac{S_s^P P_F^{*,P}}{P_s^P}\right)^{-\varphi}\frac{M_s}{P_s^P}} = \left(S_s^P\right)^{\varphi} = \frac{M_s}{M_s^*} \neq 1 \text{ unless } s_e \subset S,$$

$$\text{for } \textit{Foreign}\text{:} \; \frac{c_{F,s}^{*,C}}{c_{H,s}^{*,C}} = \frac{\frac{1}{2}\left(\frac{P_F^{*,P}}{P_s^{*,P}}\right)^{-\varphi}\frac{M_s^*}{P_s^{*,P}}}{\frac{1}{2}\left(\frac{\frac{P_H^P}{S_s^P}}{P_s^{*,P}}\right)^{-\varphi}\frac{M_s^*}{P_s^{*,P}}} = \left(S_s^P\right)^{-\varphi} = \frac{M_s^*}{M_s} \neq 1 \text{ unless } s_e \subset S.$$

This completes our proof. ■

A.3 Proof of Proposition 3 (Relative Leisure)

Proof.

- Under *CCP* and *full* symmetry with *separable* preferences ($P_H^C = P_F^C = P_H^{*,C} = P_F^{*,C}$ and $P^C = P^{*,C}$), *relative* real output can be expressed as:

$$\frac{y_s^C}{y_s^{*,C}} \equiv \frac{c_{H,s}^C + c_{H,s}^{*,C}}{c_{F,s}^{*,C} + c_{F,s}^C} = \frac{\frac{1}{2}\left(\frac{P_H^C}{P^C}\right)^{-\varphi}\frac{M_s}{P^C} + \frac{1}{2}\left(\frac{P_H^{*,C}}{P^{*,C}}\right)^{-\varphi}\frac{M_s^*}{P^{*,C}}}{\frac{1}{2}\left(\frac{P_F^{*,C}}{P^{*,C}}\right)^{-\varphi}\frac{M_s^*}{P^{*,C}} + \frac{1}{2}\left(\frac{P_F^C}{P^C}\right)^{-\varphi}\frac{M_s}{P^C}} =$$

$$= \frac{\frac{1}{2}\left(\frac{P_H^C}{P^C}\right)^{-\varphi}\frac{1}{P^C}\left(M_s + M_s^*\right)}{\frac{1}{2}\left(\frac{P_F^{*,C}}{P^{*,C}}\right)^{-\varphi}\frac{1}{P^{*,C}}\left(M_s^* + M_s\right)} = 1 \Leftrightarrow y_s^C = y_s^{*,C} \text{ for } \forall s \in S.$$

Consequently:

$$\frac{y_s^C}{y_s^{*,C}} \equiv \frac{n_s^C}{n_s^{*,C}} \equiv \frac{1 - l_s^C}{1 - l_s^{*,C}} = 1 \Rightarrow \frac{l_s^C}{l_s^{*,C}} = 1 \Leftrightarrow l_s^C = l_s^{*,C} \text{ for } \forall s \in S.$$

- Under *PCP* and *full* symmetry with *separable* preferences ($P_H^P = P_F^{*,P}$, $P_{F,s}^P = S_s^P P_F^{*,P}$, $P_{H,s}^{*,P} = \frac{P_H^P}{S_s^P}$ and $P_s^P = S_s^P P_s^{*,P} \Leftrightarrow P_s^{*,P} = \frac{P_s^P}{S_s^P}$), analogous reasoning derives *relative* real output to be:

$$\frac{y_s^P}{y_s^{*,P}} \equiv \frac{c_{H,s}^P + c_{H,s}^{*,P}}{c_{F,s}^{*,P} + c_{F,s}^P} = \frac{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\varphi}\frac{M_s}{P_s^P} + \frac{1}{2}\left(\frac{P_{H,s}^{*,P}}{P_s^{*,P}}\right)^{-\varphi}\frac{M_s^*}{P_s^{*,P}}}{\frac{1}{2}\left(\frac{P_F^{*,P}}{P_s^{*,P}}\right)^{-\varphi}\frac{M_s^*}{P_s^{*,P}} + \frac{1}{2}\left(\frac{P_{F,s}^P}{P_s^P}\right)^{-\varphi}\frac{M_s}{P_s^P}} =$$

$$= \frac{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\varphi}\frac{M_s}{P_s^P} + \frac{1}{2}\left(\frac{\frac{P_H^P}{S_s^P}}{\frac{P_s^P}{S_s^P}}\right)^{-\varphi}\frac{M_s^*}{\frac{P_s^P}{S_s^P}}}{\frac{1}{2}\left(\frac{P_F^{*,P}}{\frac{P_s^P}{S_s^P}}\right)^{-\varphi}\frac{M_s^*}{\frac{P_s^P}{S_s^P}} + \frac{1}{2}\left(\frac{S_s^P P_F^{*,P}}{P_s^P}\right)^{-\varphi}\frac{M_s}{P_s^P}} =$$

$$= \frac{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\varphi}\frac{1}{P_s^P}\left(M_s + S_s^P M_s^*\right)}{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\varphi}\frac{1}{P_s^P}\left[\left(S_s^P\right)^{-\varphi} S_s^P M_s^* + \left(S_s^P\right)^{-\varphi} M_s\right]} =$$

$$= \frac{\left(M_s + S_s^P M_s^*\right)}{\left[S_s^P M_s^* + M_s\right]\left(S_s^P\right)^{-\varphi}} = \left(S_s^P\right)^{\varphi} =$$

$$= \left[\left(\frac{M_s}{M_s^*}\right)^{\frac{1}{\varphi}}\right]^{\varphi} = \frac{M_s}{M_s^*} \neq 1 \text{ unless } s_e \subset S.$$

Hence:

$$\frac{y_s^P}{y_s^{*,P}} \equiv \frac{n_s^P}{n_s^{*,P}} \equiv \frac{1 - l_s^P}{1 - l_s^{*,P}} = \frac{M_s}{M_s^*} = \left(S_s^P\right)^{-\varphi} \neq 1 \text{ unless } s_e \subset S \Rightarrow$$

$$\Rightarrow \frac{l_s^C}{l_s^{*,C}} \neq 1 \Leftrightarrow l_s^C \neq l_s^{*,C} \text{ unless } s_e \subset S.$$

This completes our proof. ■

A.4 Proof of Proposition 4 (Impact of Monetary Expansion)

Proof. To evaluate and compare *ex-post* utility *across countries* following a *relative* monetary expansion under float, we need to take account of the simultaneous effects on *relative* real consumption and *relative* leisure. Assume at this point that a higher positive (or a lower negative) money growth has occurred in Home in a given state $s_H \in S_H \subset S$, so that $\frac{M_{s_H}}{M^*_{s_H}} > 1$.[44]

- Under *CCP* and *full* symmetry with *separable* preferences, we then obtain directly from Proposition 1 that $\frac{c^C_{s_H}}{c^{*,C}_{s_H}} > 1 \Leftrightarrow c^C_{s_H} > c^{*,C}_{s_H}$, so the Home representative household consumes more than the Foreign one in this state of nature. From Proposition 3 we also know that *relative* leisure is independent, under CCP, from money stocks, so $\frac{l^C_{s_H}}{l^{*,C}_{s_H}} = 1 \Leftrightarrow l^C_{s_H} = l^{*,C}_{s_H}$. Taking account of both utility index components, namely real consumption and leisure hours, we can conclude that the relative monetary expansion has tilted *relative ex-post* utility in favor of the expansionary country.

- Under *PCP* and *full* symmetry with *separable* preferences, we obtain – again from Proposition 1 – that $\frac{c^P_{s_H}}{c^{*,P}_{s_H}} > 1 \Leftrightarrow c^P_{s_H} > c^{*,P}_{s_H}$, so the Home representative household consumes again more than the Foreign one. From Proposition 3 we can see that, under PCP, $\frac{1-l_{s_H}}{1-l^*_{s_H}} = \frac{M_{s_H}}{M^*_{s_H}} > 1$, so $\frac{l_{s_H}}{l^*_{s_H}} < 1$, with $0 < l_{s_H} < l^*_{s_H} < 1$. We now have to know whether the *relative gain in real consumption* in the expansionary economy is higher or lower than the *relative loss in leisure.* This type of calculation depends *qualitatively* on the magnitude of the *relative* monetary disequilibrium, $\frac{M_{s_H}}{M^*_{s_H}} > 1$, and *quantitatively* on the degree of substitutability in consumption, $\varphi > 1$. For our purposes here, we abstract from unrealistic relative monetary disequilibria[45] and focus on cases that are consistent with our sticky-price set-up. It turns out that under PCP the gain of the Home representative household in consumption relative to the Foreign one is lower than its simultaneous relative loss in leisure, when consumption and leisure are separable and *equally* valued, as we assume for our purposes here.[46] Under PCP, therefore, Home residents are worse-off than Foreign ones in *cross-country* utility terms following a Home *relative* monetary expansion. Let us take as an illustrative example a (realistic) case where $\mu_{s_H} = 4\%$ and

[44] Of course, we would arrive at the same conclusions if we start from a symmetric state of the world characterized by a Foreign *relative* monetary expansion $s_F \in S_F \subset S$, so that $\frac{M_{s_F}}{M^*_{s_F}} < 1$.

[45] Although we have computed such as well, to numerically verify that they do *not* change our conclusions.

[46] And starting from an *initial* symmetric equilibrium with 8 hours of labor and 8 hours of leisure (and 8 hours of sleep), so that $n_0 = n^*_0 = l_0 = l^*_0 = \frac{1}{2}$ if our time endowment (less the "optimum" of sleep) is normalized to 1, as in (3) (and as usual).

$\mu^*_{s_H} = 2\%$ so that $\frac{M_{s_H}}{M^*_{s_H}} = \frac{104}{102} = 1.0196 > 1$. Furthermore, consider (as being close to reality) a monopolistic markup of 10%, and thus a corresponding parameter value of $\varphi = 11$. Using our model to perform such a calculation,[47] we find that the Home representative household consumes 1.0178 times (+1.78 percentage points) more than the Foreign one but also works 1.0200 times more and so has $\frac{1}{1.0200} = 0.9804$ times (-1.96 percentage points) less leisure. Note, however, that given our choice of parameter values above (or parameter regions, more generally) considered as realistic under nominal rigidity, the magnitude of *relative* utility effects measured by the reported *difference* in terms of percentage points ($+1.78 - 1.96 = -0.18$) appears somewhat small to be easily perceptible, and motivating indeed, in the optimizing behavior of the rational agents we model. Another observation to make here is that a *lower* substitutability exacerbates the gap between the relative consumption gain and the relative leisure loss while a *higher* substitutability, by contrast, reduces it.[48] In the limit, when $\varphi \to \infty$ and competition is perfect, one could infer form our numerical examples that the gain in relative consumption following a domestic monetary expansion under float and PCP will be *exactly offset* by the loss in relative leisure, in percentage terms, and ex-post *cross-country* utility will remain unchanged. Thus PCP, and hence *PPP*, with *perfect competition* would act as a risk-sharing device between the two nations we model, a finding that has been pointed out in other NOEM papers as well.

This completes our proof. ■

A.5 Proof of Proposition 5 (World Trade Share)

Proof.

- Under *CCP*, *Home* nominal trade is always equal to *Home* nominal output so that the *Home* trade share in output is constant at 1, irrespective of the state of nature that has materialized. The same is true for *Foreign*, and as a consequence $(ft)^{*,C}_F = 1 = (ft)^C_H$ so that $\frac{1}{2}(ft)^C_H + \frac{1}{2}(ft)^{*,C}_F = 1$, for $\forall s \in S$.

- Under *PCP*, by contrast, *both* these trade-to-output ratios are state-dependent and generally *not* equal to each other and to 1: $1 \neq (ft)^P_{H,s} \neq (ft)^{*,P}_{F,s} \neq 1$ unless $s_e \subset S$. However due to symmetry, the *Home* and *Foreign* trade shares in GDP are *complementary* in the sense that in any state of nature $s \in S$ they sum to 2:

[47] The details are available upon request.

[48] The details of the similar computations we performed with $\varphi = 2$ and $\varphi = 101$ (as well as with other values for the relative monetary disequilibrium) are also available upon request.

$$(ft)^P_{H,s} + (ft)^{*,P}_{F,s} = \frac{2}{\left(S^P_s\right)^{1-\varphi}+1} + \frac{2}{\left(\frac{1}{S^P_s}\right)^{1-\varphi}+1} =$$

$$= \frac{2}{\left(S^P_s\right)^{1-\varphi} + \frac{(S^P_s)^{1-\varphi}}{(S^P_s)^{1-\varphi}}} + \frac{2}{\frac{1}{(S^P_s)^{1-\varphi}} + \frac{(S^P_s)^{1-\varphi}}{(S^P_s)^{1-\varphi}}} = \frac{2}{\frac{(S^P_s)^{2(1-\varphi)}+(S^P_s)^{1-\varphi}}{(S^P_s)^{1-\varphi}}} + \frac{2}{\frac{1+(S^P_s)^{1-\varphi}}{(S^P_s)^{1-\varphi}}} =$$

$$= \frac{2\left(S^P_s\right)^{1-\varphi}}{\left(S^P_s\right)^{1-\varphi}\left[\left(S^P_s\right)^{(1-\varphi)}+1\right]} + \frac{2\left(S^P_s\right)^{1-\varphi}}{1+\left(S^P_s\right)^{(1-\varphi)}} = \frac{2}{1+\left(S^P_s\right)^{(1-\varphi)}} + \frac{2\left(S^P_s\right)^{1-\varphi}}{1+\left(S^P_s\right)^{(1-\varphi)}} =$$

$$= \frac{2+2\left(S^P_s\right)^{1-\varphi}}{1+\left(S^P_s\right)^{(1-\varphi)}} = \frac{2\left[1+\left(S^P_s\right)^{1-\varphi}\right]}{1+\left(S^P_s\right)^{(1-\varphi)}} = 2$$

Thus, (equally-weighted) world trade *equals* world output in any state of nature $s \in S$:

$$\frac{1}{2}(ft)^P_{H,s} + \frac{1}{2}(ft)^{*,P}_{F,s} = 1, \text{ for } \forall s \in S.$$

This completes our proof. ■

A.6 Proof of Proposition 6 (Expected Trade Share)

Proof. [49] Start by recalling our result in (33):

$$(ft)^P_{H,s} = \frac{2}{1+\left(S^P_s\right)^{\varphi-1}} = \frac{2}{1+\left[\frac{M_s}{M^*_s}\right]^{\frac{\varphi-1}{\varphi}}}.$$

Since we have assumed a *jointly symmetric* distribution of money shocks, there are three *kinds* of state of nature: (i) $M_s = M^*_s$ (hence $(ft)^P_{H,s} = 1$), (ii) $M_s > M^*_s$, and (iii) $M_s < M^*_s$. So the expected trade share is:

$$E_0\left[(ft)^P_{H,s}\right] = 1 - \sum_{s,\ \frac{M_s}{M^*_s}>1} \Pr(s) - \sum_{s',\ \frac{M_{s'}}{M^*_{s'}}<1} \Pr(s') + \sum_{s,\ \frac{M_s}{M^*_s}>1} \Pr(s)\frac{2}{1+\left[\frac{M_s}{M^*_s}\right]^{\frac{\varphi-1}{\varphi}}} + \sum_{s',\ \frac{M_{s'}}{M^*_{s'}}<1} \Pr(s')\frac{2}{1+\left[\frac{M_{s'}}{M^*_{s'}}\right]^{\frac{\varphi-1}{\varphi}}},$$

where $1 - \sum_{s,\ \frac{M_s}{M^*_s}>1} \Pr(s) - \sum_{s',\ \frac{M_{s'}}{M^*_{s'}}<1} \Pr(s')$ is the total probability of the states where $M = M^*$. Symmetry: for each state s where $\frac{M_s}{M^*_s} > 1$ there

[49] I am grateful to Cédric Tille for suggesting to compress my analogous, but longer, proof into the elegant form below.

is exactly one mirror state s' where $\frac{M_{s'}}{M^*_{s'}} = \left[\frac{M_s}{M^*_s}\right]^{-1}$, with $\Pr(s) = \Pr(s')$. Therefore we write:

$$\begin{aligned}
E_0\left[(ft)^P_{H,s}\right] &= 1 - 2\sum_{s,\ \frac{M_s}{M^*_s}>1} \Pr(s) + \sum_{s,\ \frac{M_s}{M^*_s}>1} \Pr(s)\left[\frac{2}{1+\left[\frac{M_s}{M^*_s}\right]^{\frac{\varphi-1}{\varphi}}} + \frac{2}{1+\left[\frac{M_s}{M^*_s}\right]^{-\frac{\varphi-1}{\varphi}}}\right] \\
&= 1 - 2\sum_{s,\ \frac{M_s}{M^*_s}>1} \Pr(s) + 2\sum_{s,\ \frac{M_s}{M^*_s}>1} \Pr(s)\left[\frac{1}{1+\left[\frac{M_s}{M^*_s}\right]^{\frac{\varphi-1}{\varphi}}} + \frac{\left[\frac{M_s}{M^*_s}\right]^{\frac{\varphi-1}{\varphi}}}{1+\left[\frac{M_s}{M^*_s}\right]^{\frac{\varphi-1}{\varphi}}}\right] \\
&= 1 - 2\sum_{s,\ \frac{M_s}{M^*_s}>1} \Pr(s) + 2\sum_{s,\ \frac{M_s}{M^*_s}>1} \Pr(s) \\
&= 1
\end{aligned}$$

The same logic applies to the Foreign expected trade-to-output ratio, $E_0\left[(ft)^{P,*}_{F,s}\right]$,for no matter what distribution of (money) shocks provided that it is *jointly symmetric*.

This completes our proof. ■

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