Cost effectiveness of R&D and the robustness of Strategic Trade Policy*

Praveen Kujal
Universidad Carlos III de Madrid

Juan Ruiz†
University of Essex and Univ. Carlos III de Madrid

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Abstract

This paper analyzes the incentives for governments to impose export subsidies when firms invest in a cost saving technology before market competition. Governments first impose an export subsidy or a tax. After observing export policy, firms invest in cost reducing R&D and subsequently compete in the market. Governments subsidize exports under Cournot competition. Under Bertrand competition, export subsidies are positive whenever R&D is sufficiently cost-effective at reducing marginal costs, and negative otherwise. The trade policy reversal found in models without endogenous sunk costs disappears if R&D is sufficiently cost-effective. Output subsidies are more robust than implied by the recent literature.

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†Corresponding author. Address: University of Essex, Department of Economics, Wivenhoe Park, Colchester, Essex, CO4 3SQ, United Kingdom. Tel.: +44 (0) 1206 872394, Fax: +44 (0) 1206 872724, e-mail: jruiz@alum.bu.edu.

1 Introduction

Since Eaton and Grossman (1986), one of the major criticisms of the strategic trade literature has been its non-robustness to the mode of market competition. If trade policy is sensitive to the choice of strategic variable by firms and governments are uncertain about the mode of competition then strategic trade policy can be more harmful than beneficial. In this paper, we analyze export subsidies when firms invest in cost-reducing R&D before the market competition stage. Governments choose export subsidies first. After observing governments' choice, firms invest in R&D and then compete in a third market (in prices or quantities). We find that for sufficiently cost effective R&D[1] governments subsidize exports independently of the mode of competition. This suggests that export subsidies are more robust to the type of the market competition than implied by the recent literature.

Several authors have studied the robustness of strategic trade policy using two kinds of models. In the first kind, in a two-stage game, governments first commit to output subsidies and then firms compete in the market. Using this approach Brander and Spencer (1985) show that the optimal trade policy is an export subsidy under Cournot competition. Eaton and Grossman (1986), however, show that the optimal strategic trade policy reverses to an export tax if firms compete in prices.[2] This policy reversal highlights the lack of robustness of strategic trade policy when governments are uncertain about the mode of competition.

In the second kind of models, actions are chosen in a three-stage game: governments first commit to a policy, firms then invest in R&D and later compete in the market. In such models, investing in a strategic variable before the market competition stage captures entry barriers, a feature that is fundamental to oligopolistic market structures (see Sutton, 1991). A further appeal of these models is that they capture firm commitment to a strategic variable before the competition stage (Grossman, 1988). If firms can make sunk investments before the market competition stage then governments have two instruments at their disposal: output and R&D subsidies. If governments use only R&D policy Bagwell and Staiger (1994) show that governments subsidize R&D under both Cournot and Bertrand Competition.[3] Based on this, Brander (1995) suggests that R&D subsidies seem more robust than output subsidies. Neary and Leahy (2000), however, dispute Brander's claim.[4] They show that when governments use two instruments (an output and a R&D subsidy at the same time) then both instruments are not robust to the nature of market competition.[5]

This paper adds another argument against the claim that R&D subsidies are more robust than output subsidies. If governments only subsidize exports and firms invest in R&D (before competing in the market), we show that the optimal trade policy is an export subsidy under both Cournot and Bertrand competition, provided R&D is sufficiently cost-effective. This means that output policy is more robust than previously

[1] We refer to the cost-effectiveness of R&D as the effect of R&D on marginal costs relative to the cost of investing in R&D.

[2] The reversal in the optimal export policy is explained by the fact that outputs are strategic substitutes and prices are strategic complements. See Brander (1995) for a discussion on this.

[3] Spencer and Brander (1983) had shown the optimality of R&D subsidies under Cournot competition. Bagwell and Staiger (1994) develop a model where the effect of R&D investment is stochastic. In the case where R&D reduces the mean but does not affect the variance of costs (the closest case to deterministic R&D), they find that R&D should be subsidized under both Cournot and Bertrand competition. Maggi (1996) finds a similar result in a model where firms invest in capacities (instead of R&D) before the competition stage. The optimal policy in his model is to subsidize capacities.

[4] See Neary and Leahy (2000), page 505.

[5] Neary and Leahy (2000) show that under Cournot competition governments subsidize exports and tax R&D, a result found in Spencer and Brander (1983). However, under Bertrand competition, governments will tax exports and subsidize R&D. The intuition is that governments use export policy to shift profits from foreign firms (as in models without R&D) and use R&D policy to correct the distortion on R&D generated by the strategic behavior of firms. Therefore, if governments use two instruments, strategic policy in the presence of R&D is no longer robust to changes in the mode of competition.

considered by the literature. This is true especially in industries where the marginal cost of R&D is not too high relative to its effect on process innovation.

The papers closest to ours are Spencer and Brander (1983) and Neary and Leahy (2000). Spencer and Brander (1983) show that governments impose an output subsidy under Cournot competition when firms can invest in R&D before competing in the market. They analyze two cases that are different to ours. First, they show the optimality of output subsidies if they are set by governments *after* firms decide their R&D investment. Second, they show that output subsidies are optimal if they are *set jointly* with R&D subsidies before R&D is chosen by firms. In the first part of our paper, we extend their results to the case when R&D subsidies are not available and the government chooses output subsidies before firms invest in R&D.

In a numerical simulation, Neary and Leahy (2000) show that if governments only use output subsidies then the Eaton and Grossman trade policy reversal from Cournot to Bertrand competition is still observed when firms invest in R&D before the market competition stage. In this paper, we show that their result holds only when R&D is relatively ineffective at reducing marginal costs. Our result becomes clear once one realizes that the effect of R&D on profits depends on the level of output. Due to output expansion, an export subsidy increases the ability of domestic R&D to shift profits from the foreign firm. Output expansion, due to the output subsidy, occurs under both Cournot and Bertrand competition. Therefore, only looking at R&D, governments have the incentive to subsidize exports both under price and quantity competition.

The sign of the optimal policy depends upon the net effect of the export subsidy on the R&D and the market competition stage. In a model without R&D, the sign of the strategic trade policy depends on the strategic complementarity or substitutability of the variables chosen by firms in the market competition stage. Under R&D and Cournot competition, a unilateral export subsidy increases welfare both through its effect on R&D and on output. This means that governments want to subsidize exports (Spencer and Brander, 1983). Under Bertrand competition, however, the two effects have the opposite sign. If R&D is sufficiently cost effective then R&D will be relatively elastic with respect to an export subsidy. This high elasticity of R&D will make the effect of the output subsidy on the R&D stage stronger than the effect on the price competition stage. In this case, governments subsidize output under Bertrand competition. Conversely, if R&D is not sufficiently cost-effective then the effect of an output subsidy on the price competition stage dominates the effect on the R&D stage and the optimal policy under Bertrand competition is an output tax.

We use the standard third country model of strategic trade as in Spencer and Brander (1983). Two firms, one located in each country, produce a differentiated good which is exported to a third country. There is no domestic consumption and welfare is measured as producer surplus (profits) net of subsidy costs.[6] In a three stage game of complete information, the domestic government first sets an output subsidy s^1. This is followed by both firms simultaneously deciding their investment in cost-reducing R&D (Δ^i and Δ^j). In the third stage, firms compete in the product market simultaneously choosing quantities, or prices. We also assume that governments commit to an export subsidy while firms commit to their investment in R&D.

We proceed as follows: in section 2 we analyze output subsidies under Cournot competition. In section 3 we perform the same analysis under Bertrand competition. Section 4 presents a numerical simulation that highlights the effect of the convexity of the cost of R&D on the optimal trade policy. Section 5 concludes.

[6] Public funds may have an opportunity cost bigger than one (as in Neary [1994]). We abstract from this issue in this analysis.

2 Cournot Competition

In the first stage of the game, government 1 chooses an export subsidy. Then firms choose R&D investment. Output is chosen in the third stage of the game. R&D investment generates a process innovation of size Δ^i (by firm i), imposing a monetary cost of $\phi(\Delta^i)$ upon the firm. The monetary cost is increasing and convex in the extent of process innovation and reduces total and marginal costs of production. Denoting firms by superscripts and derivatives by subscripts these assumptions translate into:

$$C^i_\Delta = \frac{\partial C^i(x^i,\Delta^i)}{\partial \Delta^i} \leq 0, \qquad C^i_{\Delta\Delta} = \frac{\partial^2 C^i(x^i,\Delta^i)}{\partial \left(\Delta^i\right)^2} \geq 0, \qquad C^i_{x\Delta} = \frac{\partial^2 C^i(x^i,\Delta^i)}{\partial \Delta^i \partial x^i} \leq 0 \tag{1}$$

$$\phi^i_i(\Delta^i) > 0, \qquad \phi^i_{ii}(\Delta^i) > 0 \tag{2}$$

The choice of R&D investment is irreversible and simultaneous for both firms. We assume that goods are imperfect substitutes and that the own-price effect dominates the cross-price effect:[7]

$$\frac{\partial p^i(x^i,x^j)}{\partial x^i} < \frac{\partial p^i(x^i,x^j)}{\partial x^j} < 0 \tag{3}$$

The following assumptions concern the behavior of revenues $R^i(x^i,x^j) = x^i p^i(x^i,x^j)$:

$$R^i_{ii}(x^i,x^j) = x^i \frac{\partial^2 p^i(x^i,x^j)}{\partial \left(x^i\right)^2} + 2\frac{\partial p^i(x^i,x^j)}{\partial x^i} < 0 \tag{4}$$

$$R^i_{jj}(x^i,x^j) = x^i \frac{\partial^2 p^i(x^i,x^j)}{\partial \left(x^j\right)^2} \geq 0 \tag{5}$$

$$R^i_{ij}(x^i,x^j) = x^i \frac{\partial p^i(x^i,x^j)}{\partial x^i \partial x^j} + \frac{\partial p^i(x^i,x^j)}{\partial x^j} < 0 \tag{6}$$

Assumption (4) states that the revenue is concave in own quantity, and is satisfied by demand functions that are not too convex. Assumption (5) states that revenue decreases (at a decreasing rate) with an increase in the other firm's output. This is true in particular for linear demands. Lastly, (6) states that an increase in sales of one good decreases marginal revenue of the other (again satisfied in the case of a linear demand).

Suppose that government 1 subsidizes exports giving a per-unit output subsidy, s^1, to its domestic firm. The profit function of firm 1 and firm 2 can then be written as,

$$\bar{\Pi}^1(x^1,x^2,\Delta^1,s^1) = R^1(x^1,x^2) - C^1(x^1,\Delta^1) - \phi(\Delta^1) + s^1x^1 = \Pi^1(x^1,x^2,\Delta^1) + s^1x^1 \tag{7}$$

$$\bar{\Pi}^2(x^1,x^2,\Delta^2) = \Pi^2(x^1,x^2,\Delta^2) = R^2(x^1,x^2) - C^2(x^2,\Delta^2) - \phi(\Delta^2) \tag{8}$$

The net domestic benefit of country 1 is simply the profit of the domestic firm minus the cost of the subsidy,

$$\bar{B}^1(s^1) = \bar{\Pi}^1(x^1,x^2,\Delta^1,s^1) - s^1x^1 = \Pi^1(x^1,x^2,\Delta^1)$$

2.1 Final Stage: Quantity Competition

In the final stage, firms choose output, x^i, to maximize profits, $\bar{\Pi}^i(x^1,x^2,\Delta^i,s^1)$. The first order condition for the two firms gives us the following expressions:

$$\bar{\Pi}^1_1 = R^1_1(x^1,x^2) - C^1_x(x^1,\Delta^1) + s^1 = 0 \tag{9}$$

[7] Strictly speaking, the condition for the own price effect to dominate the cross price effect is $\left(\frac{\partial p^i(x^i,x^j)}{\partial x^j}\right)^2 < \left(\frac{\partial p^i(x^i,x^j)}{\partial x^i}\right)^2$. In this paper we restrict our attention to the case of imperfect substitutes, that is $\frac{\partial p^i(x^i,x^j)}{\partial x^j} < 0$.

$$\bar{\Pi}_2^2 = R_2^2(x^1, x^2) - C_x^2(x^2, \Delta^2) = 0 \tag{10}$$

with second order condition:[8]

$$\bar{\Pi}_{ii}^i = R_{ii}^i(x^i, x^j) - C_{xx}^i(x^i, \Delta^i) < 0 \tag{11}$$

We assume that the second order condition is always satisfied.[9] Note that assumption (6) implies that quantities are strategic substitutes, and therefore output reaction functions are negatively sloped.

For later use we need to assume that the own effect of output on marginal profit is stronger (greater in absolute value) than the cross effect, that is, $\bar{\Pi}_{ii}^i < \bar{\Pi}_{ij}^i$. This then implies that:

$$\bar{\Pi}_{11}^1 \bar{\Pi}_{22}^2 - \bar{\Pi}_{12}^1 \bar{\Pi}_{12}^2 > 0 \tag{12}$$

The solution of the two equations in (9) gives us equilibrium outputs (as a function of R&D levels chosen in the second stage and the output subsidy chosen by government 1 in the first stage):

$$x^i = \bar{q}^i(\Delta^i, \Delta^j, s^1) \tag{13}$$

Totally differentiating the two first order conditions (9) and (10) we obtain the effect of R&D on output (keeping the output subsidy constant):[10]

$$\bar{q}_{\Delta^i}^i(\Delta^i, \Delta^j, s^1) = \frac{\mathrm{d}x^i}{\mathrm{d}\Delta^i} = \frac{\bar{\Pi}_{jj}^j C_{x\Delta}^i}{\bar{\Pi}_{ii}^i \bar{\Pi}_{jj}^j - R_{ij}^j R_{ij}^i} = \frac{\bar{\Pi}_{jj}^j C_{x\Delta}^i}{\bar{\Pi}_{ii}^i \bar{\Pi}_{jj}^j - \bar{\Pi}_{ij}^j \bar{\Pi}_{ij}^i} > 0 \tag{14}$$

$$\bar{q}_{\Delta^j}^i(\Delta^i, \Delta^j, s^1) = \frac{\mathrm{d}x^i}{\mathrm{d}\Delta^j} = \frac{-R_{ij}^i C_{x\Delta}^j}{\bar{\Pi}_{ii}^i \bar{\Pi}_{jj}^j - R_{ij}^j R_{ij}^i} = \frac{-\bar{\Pi}_{ij}^i C_{x\Delta}^j}{\bar{\Pi}_{ii}^i \bar{\Pi}_{jj}^j - \bar{\Pi}_{ij}^j \bar{\Pi}_{ij}^i} < 0 \tag{15}$$

where the inequalities come from (1), (6) and (11). The intuition is straightforward: an increase in R&D expenditure reduces the marginal cost of production and thus shifts out the reaction curve of firm i. Given that reaction functions are downward sloping, this implies that firm i produces more output while firm j produces less. The effect of the subsidy (s^1) on output is also determined by the effect the output subsidy has on R&D of both firms. Keeping R&D levels Δ^1 and Δ^2 fixed, the *partial* effects are,

$$\bar{q}_{s^1}^1(\Delta^1, \Delta^2, s^1)\big|_{\Delta^1, \Delta^2 \text{ constant}} = \frac{-\bar{\Pi}_{22}^2}{\bar{\Pi}_{11}^1 \bar{\Pi}_{22}^2 - R_{12}^2 R_{12}^1} > 0 \tag{16}$$

$$\bar{q}_{s^1}^2(\Delta^1, \Delta^2, s^1)\big|_{\Delta^1, \Delta^2 \text{ constant}} = \frac{R_{12}^2}{\bar{\Pi}_{11}^1 \bar{\Pi}_{22}^2 - R_{12}^2 R_{12}^1} = \frac{\bar{\Pi}_{12}^2}{\bar{\Pi}_{11}^1 \bar{\Pi}_{22}^2 - \bar{\Pi}_{12}^2 \bar{\Pi}_{12}^1} < 0 \tag{17}$$

The partial effects state that own output is increasing in own (subsidy) and decreasing in the other subsidy. However, R&D levels are influenced by the choice of output subsidies. Therefore, the *total* effect of a change in s^1 should take this into account. (Expressions for $\bar{q}_{s^1}^1$ and $\bar{q}_{s^1}^2$ above would be relevant if output subsidies are chosen *after* R&D levels are set.)

[8] Note that $\bar{\Pi}_{ii}^i = \Pi_{ii}^i$ and $\bar{\Pi}_{ij}^i = \Pi_{ij}^i$ are the same as under free trade.

[9] This will be satisfied if marginal costs are increasing or do not decrease faster than marginal revenue.

[10] Full details of the derivation of most mathematical expressions in this paper can be found in Kujal and Ruiz (2003).

2.2 R&D investment

In the R&D (i.e. second) stage, we can rewrite the profit of a firm as a function of R&D and output subsidies: $\bar{\pi}^i(\Delta^i, \Delta^j, s^1) = \bar{\Pi}^i(\bar{q}^i(\Delta_i, \Delta_j, s^1), \bar{q}^j(\Delta_i, \Delta_j, s^1), \Delta^i, s^1) = R^i(\bar{q}^i, \bar{q}^j) - C^i(\bar{q}^i, \Delta^i) - \phi^i(\Delta^i) + s^1\bar{q}^i$. The first order condition for a Nash equilibrium in the choice of R&D is given by the same first order condition as in the case of free trade:

$$\bar{\pi}^i_{\Delta^i}(\Delta^i, \Delta^j, s^1) = R^i_j(x^i, x^j)\bar{q}^j_{\Delta^i}(\Delta^i, \Delta^j, s^1) - C^i_{\Delta}(x^i, \Delta^i) - \phi^i_i(\Delta^i) = 0 \tag{18}$$

With the second order condition,

$$\bar{\pi}^i_{\Delta^i\Delta^i}(\Delta^i, \Delta^j, s^1) = R^i_j\bar{q}^j_{\Delta^i\Delta^i} + \bar{q}^j_{\Delta^i}\frac{\mathrm{d}R^i_j(x^i, x^j)}{\mathrm{d}\Delta^i} - C^i_{x\Delta}\bar{q}^i_{\Delta^i} - C^i_{\Delta\Delta} - \phi^i_{ii} < 0. \tag{19}$$

Where, $\frac{\mathrm{d}R^i_j(x^i,x^j)}{\mathrm{d}\Delta^i} = R^i_{ij}(x^i, x^j)\bar{q}^i_{\Delta^i} + R^i_{jj}(x^i, x^j)\bar{q}^j_{\Delta^i} < 0$ (by (5), (6), (14) and (15)) and $\bar{q}^j_{\Delta^i}\frac{\mathrm{d}R^i_j(x^i,x^j)}{\mathrm{d}\Delta^i} - C^i_{x\Delta}\bar{q}^i_{\Delta^i} > 0$.[11]

We now assume a condition similar to (12). It refers to the effect of R&D on profits. Again, assuming that own effect of R&D on marginal profits is stronger (bigger in absolute value) than the cross effect (i.e. $\bar{\pi}^i_{\Delta^i\Delta^i} < \bar{\pi}^i_{\Delta^i\Delta^j}$) we get

$$\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^j_{\Delta^j\Delta^j} - \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^j_{\Delta^i\Delta^j} > 0. \tag{20}$$

Note that using (1), (3), (5), (6), (14), (15) and the assumption that marginal costs are constant with respect to output (so that $\bar{q}^j_{\Delta^i\Delta^j} = 0$), gives us

$$\pi^i_{ij} = \bar{q}^j_{\Delta^i}\left(R^i_{ij}(x^i, x^j)\bar{q}^i_{\Delta^j} + R^i_{jj}(x^i, x^j)\bar{q}^j_{\Delta^j}\right) - C^i_{x\Delta}\bar{q}^i_{\Delta^j} < 0. \tag{21}$$

Thus, R&D expenditures are *strategic substitutes* and R&D reaction functions are negatively sloped.

To understand (21), notice that firm i sets its R&D Δ^i to satisfy (18). An infinitesimal increase in Δ^i increases profits for firm i since the total cost of production is reduced. Further, the quantity produced by firm j in the last stage also declines which, in turn, increases the revenues of firm i.[12] This increase in revenues has to be compared with the cost of increasing R&D $\phi^i_i(\Delta^i)$.

Consider now an increase in R&D by firm j (Δ_j). An increase in R&D by firm j increases its own quantity and reduces the quantity of firm i. The most important effect is the reduction in x^i (for linear demands the effect on x^j vanishes), since a lower output implies that own R&D (Δ^i) is less effective at increasing profits. Since the marginal cost of R&D for firm 1 does not change, this implies that the optimal level of R&D for firm 1 has to be lower after an increase in Δ_j. Hence $\frac{\mathrm{d}\Delta^i}{\mathrm{d}\Delta^j} < 0$.

2.3 Output subsidies

In order to see the effect of output subsidies on R&D investment, we totally differentiate the two first order conditions for the R&D stage. The following proposition states the effect of an output subsidy on the equilibrium R&D of both firms.

[11] Note that even if we assume that marginal costs are constant with respect to output and linear with respect to R&D, (i.e. $\bar{q}^j_{\Delta^i\Delta^i} = 0$) we still need to ensure that $C^i_{\Delta\Delta} + \phi^i_{ii}$ is big enough for (19) to hold. This implies that as R&D increases its cost-effectiveness has to decline fast enough.

[12] Because of the envelope theorem, the effect of an infinitesimal change on firm i's R&D on profits through its effect on the quantity produced by firm i can be ignored

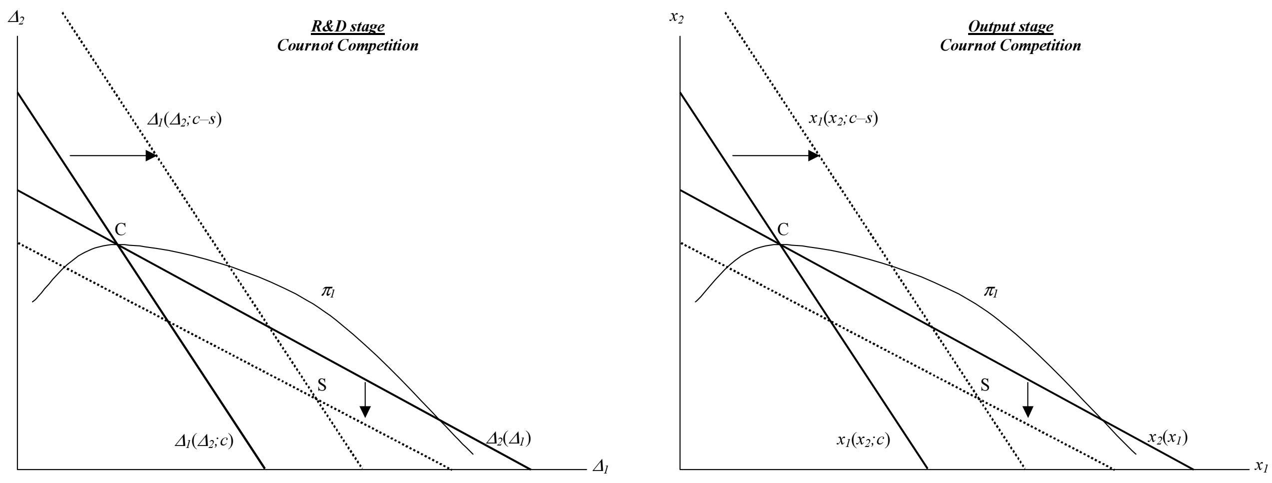

Figure 1: Cournot competition: Effect of an output subsidy s imposed by government 1.

Proposition 1 *An output subsidy by the domestic government increases the equilibrium level of R&D chosen by the domestic firm and reduces the R&D level chosen by the foreign firm. That is,*

$$\frac{d\Delta^1}{ds^1} > 0 \tag{22}$$

$$\frac{d\Delta^2}{ds^1} < 0. \tag{23}$$

Proof. See appendix. ■

Proposition 1 states that an increase in the subsidy s^1 shifts the reaction function of both firms in R&D space. The reaction function of firm 1 shifts outwards while the reaction function of firm 2 shifts inwards. This is illustrated in the left half of figure 1. An output subsidy s^1 moves the equilibrium in the R&D space from point C (free trade) to point S. This implies that for a small increment in its output subsidy, firm 1 will be inside its isoprofit contour (π_1) passing through the free trade equilibrium point C. This analysis, however, does not take into account the effect of output subsidies in output space (i.e. in the third stage). The effect in the output competition stage is illustrated on the right side of figure 1. Notice that an output subsidy s^1, imposed by government 1, increases domestic R&D and lowers foreign R&D (as seen in the left half of figure 1). This reduces domestic marginal costs beyond the direct effect of the subsidy and increases foreign marginal costs. In output space, this means that the domestic output reaction function shifts out and the foreign reaction function shifts in. The resulting equilibrium is at point S, which is inside the isoprofit contour (π_1) that passes through the free trade equilibrium at point C. Therefore, an output subsidy increases welfare for the domestic country both through R&D and output.

To obtain the optimal output subsidy these two effects need to be included. The net benefit of government 1 is $\bar{B}^1(s^1) = \bar{\pi}^1(\Delta^1, \Delta^2, s^1) - s^1 x^1$. Differentiating $\bar{B}^1(s^1)$ with respect to s^1 we obtain,

$$\frac{\partial \bar{B}^1}{\partial s^1} = \bar{\pi}^1_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \bar{\pi}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\pi}^1_{s^1} - x^1 - s^1\frac{\mathrm{d}\bar{q}^1}{\mathrm{d}s^1}. \tag{24}$$

Recall that, $\bar{\pi}^1_{s^1} = R^1_2(x^1, x^2)\bar{q}^2_{s^1}(\Delta^1, \Delta^2, s^1) + x^1$ and $\bar{\pi}^1_{\Delta^1} = 0$ from the R&D stage. Further,

$$\frac{\mathrm{d}\bar{q}^1}{\mathrm{d}s^1} = \bar{q}^1_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \bar{q}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{q}^1_{s^1} > 0. \tag{25}$$

This last inequality simply states that the total effect of an output subsidy on equilibrium output is positive, i.e. an output subsidy makes a firm in that country more competitive in the output stage ($\bar{q}^1_{s^1} > 0$). Further, an output subsidy reduces foreign R&D while increasing domestic R&D in the second stage. This in turn benefits domestic production, i.e. $\bar{q}^1_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \bar{q}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} > 0$. Given this, $\frac{\partial \bar{B}^1}{\partial s^1}$ simplifies to

$$\frac{\partial \bar{B}^1}{\partial s^1} = \bar{\pi}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + R^1_2\bar{q}^2_{s^1} - s^1\frac{\mathrm{d}\bar{q}^1}{\mathrm{d}s^1} = R^1_2\left(\bar{q}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{q}^2_{s^1}\right) - s^1\frac{\mathrm{d}\bar{q}^1}{\mathrm{d}s^1}. \tag{26}$$

The first term reflects the effect of the output subsidy on domestic benefit in the second (R&D) stage.[13] The output subsidy reduces foreign R&D (Δ^2) resulting in an increase in domestic profits. As a result, the effect of a subsidy s^1 on benefits in the second stage is positive. The second term captures what happens in the third (output) stage: an increase in the subsidy s^1 reduces the quantity produced by the foreign firm resulting in an increase in domestic revenues (and profits). The third term reflects the increased subsidy expenditure brought about by an increased production for the domestic firm $s^1\frac{\mathrm{d}\bar{q}^1}{\mathrm{d}s^1}$. The sign of the expression $\frac{\partial \bar{B}^1}{\partial s^1}$ is determined by the net of the three effects pointed out above. Notice that, starting from a subsidy s^1 equal to zero, an infinitesimal increase in the output subsidy increases domestic benefit for the subsidizing country as both the output effect $\left(\bar{q}^2_{s^1}\right)$ and the R&D effect $\left(\bar{q}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}\right)$ move in the same direction.

$$\left.\frac{\partial \bar{B}^1}{\partial s^1}\right|_{s^1=0} = \bar{\pi}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + R^1_2\bar{q}^2_{s^1} > 0 \tag{27}$$

To obtain the precise expression for the optimal output subsidy we set $\frac{\partial \bar{B}^1}{\partial s^1} = 0$:

Proposition 2 *When firms compete à la Cournot, the optimal output subsidy s^{1*} is positive:*

$$s^{1*} = \frac{\bar{\pi}^1_{\Delta^2}\frac{d\Delta^2}{ds^1} + R^1_2\bar{q}^2_{s^1}}{\frac{d\bar{q}^1}{ds^1}} = \frac{R^1_2\left(\bar{q}^2_{\Delta^2}\frac{d\Delta^2}{ds^1} + \bar{q}^2_{s^1}\right)}{\bar{q}^1_{\Delta^1}\frac{d\Delta^1}{ds^1} + \bar{q}^1_{\Delta^2}\frac{d\Delta^2}{ds^1} + \bar{q}^1_{s^1}} > 0 \tag{28}$$

Proof. Immediate from (26) ■

This proposition extends the results in Spencer and Brander (1983). They analyze the case when an output subsidy is set *after* firms invest in R&D and before they choose output. They find that the optimal output subsidy is positive. They also analyze the case of subsidies to R&D *and* output before the R&D stage, finding that output subsidies are also positive. Here we have shown that output subsidies are also positive under Cournot competition if subsidies are set *before* R&D investment.

Note that the separation into two effects related to each of the two stages in which firms play will be useful to characterize the solution in the case of Bertrand competition.

3 Bertrand Competition

Consider again a three–stage game. In stage 1, government 1 imposes an output subsidy s^1. In the second stage, firms simultaneously choose R&D. In the last stage, firms compete in prices. We assume that goods are imperfect substitutes and that the own-price effect dominates the cross-price effect,

$$\frac{\partial x^i(p^i,p^j)}{\partial p^i} < 0 < \frac{\partial x^i(p^i,p^j)}{\partial p^j} \tag{29}$$

$$\left|\frac{\partial x^i(p^i,p^j)}{\partial p^i}\right| > \left|\frac{\partial x^i(p^i,p^j)}{\partial p^j}\right|. \tag{30}$$

[13] Since firm 1 is choosing R&D, Δ^1, to maximize profits then an infinitesimal output subsidy s^1will not affect benefits.

Using previous notation, revenues and costs can be written as, $\hat{R}^i(p^i,p^j) = x^i(p^i,p^j)\cdot p^i = R^i(x^i(p^i,p^j),x^j(p^i,p^j))$ and $\hat{C}^i(p^i,p^j,\Delta^i) = C^i(x^i(p^i,p^j),\Delta^i)$, respectively. Revenues are assumed to satisfy the following properties:

$$\hat{R}^i_{ii}(p^i,p^j) = p^i\frac{\partial^2 x^i(p^i,p^j)}{\partial\left(p^i\right)^2} + 2\frac{\partial x^i(p^i,p^j)}{\partial p^i} < 0 \tag{31}$$

$$\hat{R}^i_{jj}(p^i,p^j) = p^i\frac{\partial^2 x^i(p^i,p^j)}{\partial\left(p^j\right)^2} \geq 0 \tag{32}$$

$$\hat{R}^i_{ij}(p^i,p^j) = p^i\frac{\partial x^i(p^i,p^j)}{\partial p^i\partial p^j} + \frac{\partial x^i(p^i,p^j)}{\partial p^j} > 0 \tag{33}$$

Assumption (31) states that revenue is concave in its own price, a property which is satisfied by demand functions that are not too convex. Assumption (32) is the standard case where revenue is increasing, at a non-decreasing rate, in the other firm's price. This property, in particular, is satisfied by linear demand functions. Lastly, (33) states that an increase in the price of one good increases marginal revenue for the other firm. This is again satisfied in the case of linear demand.

We make the following assumptions about costs (which are equivalent to (1) and (2) in the Cournot case):

$$\begin{aligned}
\hat{C}^i_\Delta &= \frac{\partial\hat{C}^i(p^i,p^j,\Delta^i)}{\partial\Delta^i} \leq 0, \qquad \hat{C}^i_{\Delta\Delta} = \frac{\partial^2\hat{C}^i(p^i,p^j,\Delta^i)}{\partial\left(\Delta^i\right)^2} \geq 0, \\
\hat{C}^i_{p^i\Delta} &= \frac{\partial^2 C^i(x^i,\Delta^i)}{\partial\Delta^i\partial x^i}\frac{\partial x^i(p^i,p^j)}{\partial p^i} > 0, \qquad \hat{C}^i_{p^j\Delta} = \frac{\partial^2 C^i(x^i,\Delta^i)}{\partial\Delta^i\partial x^i}\frac{\partial x^i(p^i,p^j)}{\partial p^j} < 0 \\
\phi^i_i(\Delta^i) &> 0, \qquad \phi^i_{ii}(\Delta^i) > 0
\end{aligned} \tag{34}$$

$$\begin{aligned}
\hat{C}^i_{p^i} &= \frac{\partial\hat{C}^i(p^i,p^j,\Delta^i)}{\partial p^i} = \frac{\partial C^i(x^i,\Delta^i)}{\partial x^i}\frac{\partial x^i(p^i,p^j)}{\partial p^i} < 0 \\
\hat{C}^i_{p^ip^i} &= \frac{\partial^2\hat{C}^i(p^i,p^j,\Delta^i)}{\left(\partial p^i\right)^2} = \frac{\partial C^i(x^i,\Delta^i)}{\partial x^i}\frac{\partial^2 x^i(p^i,p^j)}{\partial\left(p^i\right)^2} + \frac{\partial^2 C^i(x^i,\Delta^i)}{\partial\left(x^i\right)^2}\left(\frac{\partial x^i(p^i,p^j)}{\partial p^i}\right)^2 \geq 0 \\
\hat{C}^i_{p^ip^j} &= \frac{\partial^2\hat{C}^i(p^i,p^j,\Delta^i)}{\partial p^i\partial p^j} = \frac{\partial C^i(x^i,\Delta^i)}{\partial x^i}\frac{\partial^2 x^i(p^i,p^j)}{\partial p^i\partial p^j} + \frac{\partial^2 C^i(x^i,\Delta^i)}{\partial\left(x^i\right)^2}\frac{\partial x^i(p^i,p^j)}{\partial p^i}\frac{\partial x^i(p^i,p^j)}{\partial p^j} \leq 0.
\end{aligned} \tag{35}$$

The profit function of firm 1 and firm 2 can now be written as:

$$\bar{\Pi}^1(p^1,p^2,\Delta^1,s^1) = \hat{R}^1(p^1,p^2) - \hat{C}^1(p^1,p^2,\Delta^1) - \phi(\Delta^1) + s^1\cdot x^1(p^1,p^2) \tag{36}$$

$$= \Pi^1(p^1,p^2,\Delta^1) + s^1\cdot x^1(p^1,p^2) \tag{37}$$

$$\bar{\Pi}^2(p^1,p^2,\Delta^2) = \Pi^2(p^1,p^2,\Delta^2) = \hat{R}^2(p^1,p^2) - \hat{C}^2(p^1,p^2,\Delta^2) - \phi(\Delta^2). \tag{38}$$

The net domestic benefit of country 1 is simply the profit of the domestic firm minus the cost of the subsidy:

$$\bar{B}^1(s^1) = \bar{\Pi}^1(p^1,p^2,\Delta^1,s^1) - s^1\cdot x^1(p^1,p^2) = \Pi^1(p^1,p^2,\Delta^1).$$

3.1 Last Stage: Price Competition

In the first stage, firms maximize $\bar{\Pi}^1(p^1,p^2,\Delta^1,s^1)$ and $\bar{\Pi}^2(p^1,p^2,\Delta^2)$ choosing the price p^1 and p^2, respectively. The first order conditions to this problem are:

$$\bar{\Pi}^1_1 = \hat{R}^1_1(p^1,p^2) - \hat{C}^1_{p^1}(p^1,p^2,\Delta^1) + s^1\frac{\partial x^1}{\partial p^1} = 0 \tag{39}$$

$$\bar{\Pi}^2_2 = \hat{R}^2_2(p^1,p^2) - \hat{C}^2_{p^2}(p^1,p^2,\Delta^2) = 0 \tag{40}$$

with the second order conditions:[14]

$$\bar{\Pi}^1_{11} = \hat{R}^1_{11}(p^1, p^2) - \hat{C}^1_{p^1p^1}(p^1, p^j, \Delta^1) + s^1 \frac{\partial^2 x^1}{\partial \left(p^1\right)^2} < 0 \tag{41}$$

$$\bar{\Pi}^2_{22} = \hat{R}^2_{22}(p^1, p^2) - \hat{C}^2_{p^2p^2}(p^1, p^2, \Delta^2) < 0$$

We assume that the second order conditions are satisfied.

For later use we need to assume that the own effect of output on marginal profits is stronger (bigger in absolute value) than the cross effect, that is $\left|\bar{\Pi}^i_{ii}\right| > \left|\bar{\Pi}^i_{ij}\right|$. This implies that

$$\bar{\Pi}^1_{11}\bar{\Pi}^2_{22} - \bar{\Pi}^1_{12}\bar{\Pi}^2_{12} > 0 \tag{42}$$

Note that assumptions (33) and (35) imply that the cross-partial derivative of profits is positive ($\bar{\Pi}^i_{ij} > 0$) for country 2. That is also the case for country 1 as long as $\frac{\partial x^i(p^i,p^j)}{\partial p^i \partial p^j}$ is not too big, which we assume. In that case, prices are strategic complements and price reaction functions are positively sloped. That is, along a price reaction function,

$$\frac{\mathrm{d}p^i}{\mathrm{d}p^j} = -\frac{\bar{\Pi}^i_{ij}}{\bar{\Pi}^i_{ii}} > 0 \tag{43}$$

This is a standard result for Bertrand games with differentiated products.

The solution to the two equations (39) and (40) gives us prices as a function of the R&D levels of both firms (chosen in the previous stage) and output subsidy s^1,

$$p^i = \bar{\psi}^i(\Delta^i, \Delta^j, s^1) \tag{44}$$

To see the effect of R&D investment and subsidies on prices, we differentiate the two first order conditions given in (39) and (40). We obtain

$$\bar{\psi}^i_{\Delta^i}(\Delta^i, \Delta^j, s^1) = \frac{\mathrm{d}p^i}{\mathrm{d}\Delta^i} = \frac{\bar{\Pi}^j_{jj}\hat{C}^i_{p^i\Delta}}{\bar{\Pi}^i_{ii}\bar{\Pi}^j_{jj} - \bar{\Pi}^j_{ij}\bar{\Pi}^i_{ij}} < 0 \tag{45}$$

$$\bar{\psi}^i_{\Delta^j}(\Delta^i, \Delta^j, s^1) = \frac{\mathrm{d}p^i}{\mathrm{d}\Delta^j} = \frac{-\bar{\Pi}^i_{ij}\hat{C}^j_{p^j\Delta}}{\bar{\Pi}^i_{ii}\bar{\Pi}^j_{jj} - \bar{\Pi}^j_{ij}\bar{\Pi}^i_{ij}} < 0 \tag{46}$$

where the inequalities come from (33), (34), (35) and (41). The expressions above state that prices are decreasing both in domestic and foreign R&D. An increase in R&D expenditure reduces the marginal cost of production shifting the reaction curve of firm i downwards. Given that prices are strategic complements, this implies that both firm i and firm j charge a lower price.

Given that the output subsidy is chosen before firms decide on their R&D, the effect of the subsidy on prices has to take into account how it affects the choice of R&D by both firms. The partial effects, keeping R&D levels (Δ^1 and Δ^2) constant, are:

$$\left.\bar{\psi}^1_{s^1}(\Delta^1, \Delta^2, s^1)\right|_{\Delta^1,\Delta^2 \text{ constant}} = \frac{-\bar{\Pi}^2_{22}\left(\frac{\partial x^1}{\partial p^1}\right)}{\bar{\Pi}^1_{11}\bar{\Pi}^2_{22} - \bar{\Pi}^2_{12}\bar{\Pi}^1_{12}} < 0 \tag{47}$$

$$\left.\bar{\psi}^2_{s^1}(\Delta^1, \Delta^2, s^1)\right|_{\Delta^1,\Delta^2 \text{ constant}} = \frac{\bar{\Pi}^2_{12}\left(\frac{\partial x^1}{\partial p^1}\right)}{\bar{\Pi}^1_{11}\bar{\Pi}^2_{22} - \bar{\Pi}^2_{12}\bar{\Pi}^1_{12}} < 0. \tag{48}$$

[14] Note that, for linear demands, $\bar{\Pi}^i_{ii} = \Pi^i_{ii}$ and $\bar{\Pi}^i_{ij} = \Pi^i_{ij}$ are the same as in the case of free trade since $\frac{\partial^2 x^1}{\partial (p^1)^2} = 0$.

Notice that we assume that R&D levels are kept constant, while in fact they are influenced by the choice of output subsidies. The total effect of a change in s^1, therefore, has to also take this into account.[15]

In order to obtain the effect of imposing an output subsidy (before R&D takes place), we turn now to the R&D stage.

3.2 R&D investment

Rewrite the profit of the firm as a function of R&D and output subsidies:

$$\begin{aligned} \bar{\pi}^i(\Delta^i, \Delta^j, s^1) &= \bar{\Pi}^i(\bar{\psi}^i(\Delta_i, \Delta_j, s^1), \bar{\psi}^j(\Delta_i, \Delta_j, s^1), \Delta^i, s^1) \\ &= \hat{R}^i(\bar{\psi}^i, \bar{\psi}^j) - \hat{C}^i(\bar{\psi}^i, \bar{\psi}^j, \Delta^i) - \phi^i(\Delta^i) + s^1 \cdot x^i(\bar{\psi}^i, \bar{\psi}^j) \end{aligned} \tag{49}$$

The first order conditions for a Nash equilibrium in the choice of R&D are,

$$\bar{\pi}^1_{\Delta^1}(\Delta^1, \Delta^2, s^1) = \left[\hat{R}^1_2(p^1, p^2) - \hat{C}^1_{p^2}(p^1, p^2, \Delta^1) + s^1\left(\frac{\partial x^1}{\partial p^2}\right)\right]\psi^2_{\Delta^1}(\Delta^1, \Delta^2, s^1) - \hat{C}^1_{\Delta}(p^1, p^2, \Delta^1) - \phi^1_1(\Delta^1) = 0 \tag{50}$$

$$\bar{\pi}^2_{\Delta^2}(\Delta^1, \Delta^2, s^1) = \left[\hat{R}^2_1(p^1, p^2) - \hat{C}^2_{p^1}(p^1, p^2, \Delta^2)\right]\psi^1_{\Delta^2}(\Delta^2, \Delta^1, s^1) - \hat{C}^2_{\Delta}(p^1, p^2, \Delta^2) - \phi^2_1(\Delta^2) = 0. \tag{51}$$

With the second order conditions:[16]

$$\begin{aligned} \bar{\pi}^1_{\Delta^1\Delta^1} &= \left(\hat{R}^1_2 - \hat{C}^1_{p^2} + s^1\left(\frac{\partial x^2}{\partial p^1}\right)\right)\bar{\psi}^2_{\Delta^1\Delta^1} + \bar{\psi}^2_{\Delta^1}\left(\frac{\mathrm{d}\hat{R}^1_2(p^2, p^1)}{\mathrm{d}\Delta^1} - \frac{\mathrm{d}\hat{C}^1_{p^2}(p^2, p^1, \Delta^1)}{\mathrm{d}\Delta^1} + s^1\frac{\mathrm{d}\left(\frac{\partial x^1}{\partial p^2}\right)}{\mathrm{d}\Delta^1}\right) \\ &\quad -\hat{C}^1_{p^1\Delta}\bar{\psi}^1_{\Delta^1} - \hat{C}^1_{p^2\Delta}\bar{\psi}^2_{\Delta^1} - \hat{C}^1_{\Delta\Delta} - \phi^1_{11} < 0 \end{aligned} \tag{52}$$

$$\bar{\pi}^2_{\Delta^2\Delta^2} = \left(\hat{R}^2_1 - \hat{C}^2_{p^1}\right)\bar{\psi}^1_{\Delta^2\Delta^2} + \bar{\psi}^1_{\Delta^2}\left(\frac{\mathrm{d}\hat{R}^2_1(p^1, p^2)}{\mathrm{d}\Delta^2} - \frac{\mathrm{d}\hat{C}^2_{p^1}(p^1, p^2, \Delta^2)}{\mathrm{d}\Delta^2}\right) - \hat{C}^2_{p^2\Delta}\bar{\psi}^2_{\Delta^2} - \hat{C}^2_{p^1\Delta}\bar{\psi}^1_{\Delta^2} - \hat{C}^2_{\Delta\Delta} - \phi^2_{11} < 0 \tag{53}$$

We assume that the own effect of R&D on marginal profits is stronger (bigger in absolute value) than the cross effect, that is, $\bar{\pi}^i_{\Delta^i\Delta^i} < \bar{\pi}^i_{\Delta^i\Delta^j}$. This implies that,

$$\bar{\pi}^1_{\Delta^1\Delta^1}\bar{\pi}^2_{\Delta^2\Delta^2} - \bar{\pi}^1_{\Delta^1\Delta^2}\bar{\pi}^2_{\Delta^1\Delta^2} > 0 \tag{54}$$

The cross partial derivative $\bar{\pi}^i_{\Delta^i\Delta^j}$ is, in general, difficult to sign. However, for the usual case of linear demand and constant marginal costs, the following proposition establishes that R&D expenditures are strategic substitutes even if firms compete in prices.

Proposition 3 *Under Bertrand competition, R&D expenditures are strategic substitutes for the case of linear demand and constant marginal costs:*

$$\bar{\pi}^i_{\Delta^i\Delta^j} = \bar{\psi}^j_{\Delta^i}\bar{\psi}^i_{\Delta^j}\hat{R}^i_{ij}(p^i, p^j) - \hat{C}^i_{p^i\Delta}\bar{\psi}^i_{\Delta^j} - \hat{C}^i_{p^j\Delta}\bar{\psi}^j_{\Delta^j} < 0 \tag{55}$$

[15] Expressions for $\bar{\psi}^1_{s^1}$ and $\bar{\psi}^2_{s^1}$ (in (47) and (48)) would be relevant if output subsidies are chosen after R&D levels are set.

[16] Notice that $\frac{\mathrm{d}\hat{R}^i_j(p^i,p^j)}{\mathrm{d}\Delta^i} = \hat{R}^i_{ij}(p^i,p^j)\bar{\psi}^i_{\Delta^i} + \hat{R}^i_{jj}(p^i,p^j)\bar{\psi}^j_{\Delta^i} < 0$ (by (32), (33), (45) and (46)) and $\frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta_i)}{\mathrm{d}\Delta^i} = \hat{C}^i_{p^ip^j}(p^i,p^j,\Delta^i)\bar{\psi}^i_{\Delta^i} + \hat{C}^i_{p^jp^j}(p^i,p^j,\Delta^i)\bar{\psi}^j_{\Delta^i} + \hat{C}^i_{p^j\Delta}(p^i,p^j,\Delta^i)$. In general, $\frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta_i)}{\mathrm{d}\Delta^i}$ is hard to sign. However, in the case of linear demand it is equal to $\hat{C}^i_{p^j\Delta}(p^i,p^j,\Delta^i)$, which is negative. For the case of linear demand we also have that $\frac{\mathrm{d}\left(\frac{\partial x^1}{\partial p^2}\right)}{\mathrm{d}\Delta^1} = 0$. Assuming also that marginal costs are constant with respect to output and linear with respect to R&D (i.e. $\bar{\psi}^j_{\Delta^i\Delta^i} = 0$), we get $\bar{\pi}^i_{\Delta^i\Delta^i}(\Delta^i,\Delta^j,s^1) = \bar{\psi}^j_{\Delta^i}\frac{\mathrm{d}\hat{R}^i_j(p^i,p^j)}{\mathrm{d}\Delta^i} - \hat{C}^i_{p^i\Delta}\bar{\psi}^i_{\Delta^i} - 2\hat{C}^i_{p^j\Delta}\bar{\psi}^j_{\Delta^i} - \hat{C}^i_{\Delta\Delta} - \phi^i_{ii}$. This expression can only be negative (for (52) to hold) if $2\hat{C}^i_{p^j\Delta}\bar{\psi}^j_{\Delta^i} + \hat{C}^i_{\Delta\Delta} + \phi^i_{ii}$ is big enough. This is equivalent to saying that as R&D increases, its cost-effectiveness has to decline fast enough, a condition similar to the Cournot case.

Proof. See Appendix. ■

Proposition 3 states that an increase in R&D by firm 2 reduces the marginal profitability of R&D by firm 1. To see how this occurs, notice that firm 1 sets its R&D, Δ^1, to satisfy (50). An infinitesimal increase in Δ^1 has two opposing effects on firm 1's profits. First, profits increase due to the reduction in total costs $\hat{C}^1$. On the other hand the decrease in p^2 (due to increased R&D, Δ^1) decreases firm revenues.[17] The first order condition (50) shows this trade off against the increase in the cost of R&D , $\phi_1^1(\Delta^1)$.

Consider now an infinitesimal increase in R&D by firm 2. This reduces both p^1 and p^2. However, the fall in own price (p^2) is greater than the price decline for the rival.[18] A bigger price increase for firm 1 means that it now sells less. Lower output reduces the effectiveness of Δ^1 in reducing total costs for firm 1. This is captured by the last two terms of (55). The first term captures the effect of an increase in Δ^2 on the marginal effect of Δ^1 on firm 1's revenue. The fall in quantity (x^1), associated with an increase in Δ^2, makes the revenue loss of an increase in Δ^1 less important. This accounts for $\bar{\psi}^j_{\Delta^i}\bar{\psi}^i_{\Delta^j}\hat{R}^i_{ij}(p^i,p^j)$ being positive.

Note that the (direct) effect on costs dominates the (indirect) effect on revenue (as shown in the proof of proposition 3). The positive effect of investing in R&D for firm 1 weakens due to an increase in Δ^2. Since the marginal cost of R&D $\phi_1^1(\Delta^1)$ is unaffected by a change in Δ^2, an increase in foreign R&D (Δ^2) makes own R&D less attractive. Therefore, firm 1 optimally invests less in R&D in response to an increase in Δ^1, implying that $\bar{\pi}^i_{\Delta^i\Delta^j} < 0$.

A corollary of the previous proposition is that the slope of firm i's R&D reaction function is negative. Note that R&D reaction functions are negatively sloped (i.e. strategic substitutes) both under Cournot and Bertrand competition because the main effect of R&D comes through total costs. In both cases an increase in R&D by firm 2 reduces firm 1's output thereby decreasing the capacity of Δ^1 to reduce firm 1's total costs. Under Cournot competition, the effect on marginal revenue adds to this effect on costs. With Bertrand competition, the effect on marginal revenue dampens (but does not dominate) the effect on costs (as shown in proposition 3).

The next section describes the effect of output subsidies on R&D and price choices, under Bertrand competition.

3.3 Output Subsidies

In order to see the effect of output subsidies on R&D investment, we totally differentiate the two first order conditions given by (50) and (51). Unfortunately, no clear-cut solutions exist when we depart from the case of linear demand and constant marginal costs. However, as this is the standard case analyzed in much of the literature on the subject, we concentrate our analysis of output subsidies on this scenario.

The next proposition describes the effect of an output subsidy on the equilibrium R&D chosen by firms.

Proposition 4 *Under Bertrand competition, an output subsidy by the domestic government increases R&D of the domestic firm, and reduces R&D of the foreign firm:*

$$\frac{d\Delta^1}{ds^1} > 0 \tag{56}$$

$$\frac{d\Delta^2}{ds^1} < 0. \tag{57}$$

[17] From the envelope theorem we can ignore the effect on firm 1's price on its profits.

[18] This can be easily seen comparing $\bar{\psi}^i_{\Delta^i}$ and $\bar{\psi}^i_{\Delta^j}$ on (45) and (46), and recalling that own effects dominate cross effects in the price stage.

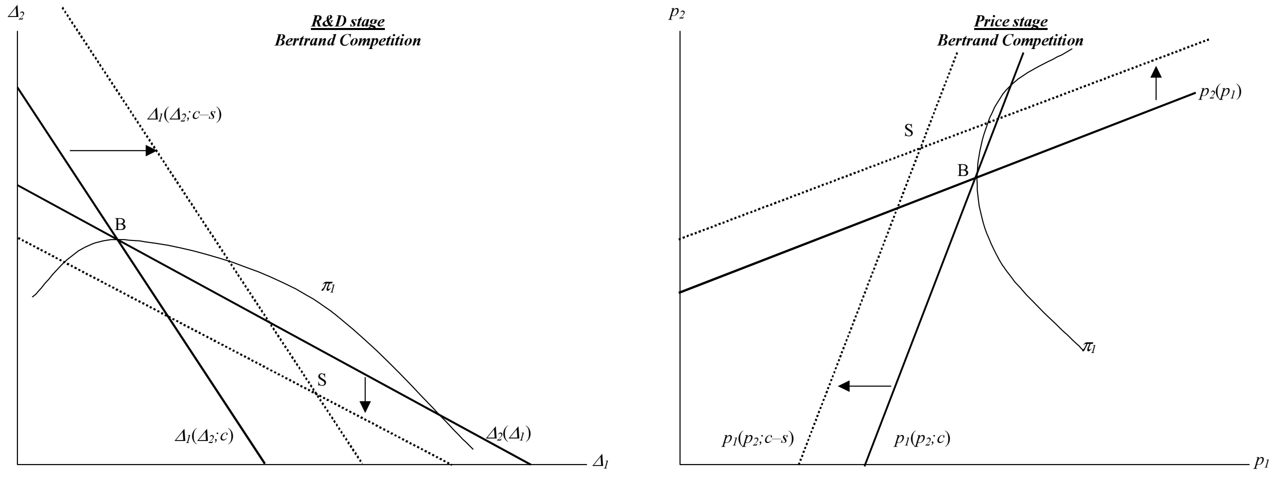

Figure 2: Bertrand Competition: Effect of an output subsidy s imposed by government 1.

Proof. See appendix ■

The intuition for this proposition is straightforward once we consider how R&D influences profits. Recall (from the discussion of proposition 3) that the incentives to invest in R&D decrease if output declines: the beneficial effects of cost reduction are smaller if output is lower. Consider now an increase in the output subsidy s^1. The output subsidy results in a reduction in the price of both goods. However, p^1 declines by a greater amount than p^2. As a result, output of firm 1 increases while output of firm 2 decreases. The output expansion creates an even greater incentive for firm 1 to invest in R&D (shifts its R&D reaction function out). The effect on firm 2 is just the contrary: the incentives for firm 2 to invest in R&D decline (firm 2's R&D reaction function shifts in) due to the output subsidy, s^1.

This is the same type of effect as was observed under Cournot competition. An increase in the output subsidy increases quantity produced thereby positively affecting the incentives to invest in R&D for the home firm. In both cases the foreign firm reduces its R&D due to decreased foreign production. As one would expect, an output subsidy imposed by the domestic government affects domestic R&D more than foreign R&D. This result, formalized in the next corollary, is used later to determine the sign of the optimal output subsidy.

Corollary 5 *Under Bertrand competition, the effect of an output subsidy on own R&D expenditures is stronger than on foreign R&D expenditures:*

$$\left|\frac{d\Delta^1}{ds^1}\right| > \left|\frac{d\Delta^2}{ds^1}\right| \tag{58}$$

Proof. See appendix ■

We can conduct a graphical analysis similar to the Cournot case. As with quantity competition, an increase in output subsidy (s^1) shifts the R&D reaction function of firm 1 out and that of firm 2 in (left half of figure 2) This means that the equilibrium in R&D space moves from B (free trade) to S. For a small output subsidy, this leaves firm 1 inside its isoprofit contour (π_1) that passes through the free trade point B : just looking at the R&D stage an output subsidy increases welfare for the domestic country. However, as in the Cournot case, we have to also take into account the effect of the subsidy in the price competition stage. This is illustrated in the right half of figure 2. As in the case of Cournot competition, an output subsidy increases domestic and reduces foreign R&D, reducing domestic marginal costs beyond the direct effect of the subsidy and increasing foreign marginal costs. This means that the domestic price reaction function shifts in and the foreign price reaction function shifts out, moving the equilibrium from B to S. From corollary 5 we know that even if we only take into account the effect of R&D on the price stage, the reaction function of firm 1 will shift more that the reaction function of firm 2. This leaves point S outside the isoprofit contour π_1 passing through point B in the price space. Therefore an output subsidy reduces welfare for the home government in the price stage. The *net effect* on the two stages determines whether an output subsidy increases or reduces welfare.

Formally, define the net domestic benefit of government 1 as $\bar{B}^1(s^1) = \bar{\pi}^1(\Delta^1, \Delta^2, s^1) - s^1 x^1(\bar{\psi}^1, \bar{\psi}^2)$. Taking the derivative of $\bar{B}^1(s^1)$ with respect to s^1:

$$\frac{\partial \bar{B}^1}{\partial s^1} = \bar{\pi}^1_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \bar{\pi}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\pi}^1_{s^1} - x^1 - s^1\frac{\partial x^1}{\partial p^1}\frac{\mathrm{d}\bar{\psi}^1}{\mathrm{d}s^1} - s^1\frac{\partial x^1}{\partial p^2}\frac{\mathrm{d}\bar{\psi}^2}{\mathrm{d}s^1} \tag{59}$$

which can be rewritten as (see appendix):

$$\frac{\partial \bar{B}^1}{\partial s^1} = m^1\left(\frac{\partial x^1}{\partial p^2}\right)\bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + m^1\left(\frac{\partial x^1}{\partial p^2}\right)\bar{\psi}^2_{s^1} - s^1\left[\frac{\partial x^1}{\partial p^1}\frac{\mathrm{d}\bar{\psi}^1}{\mathrm{d}s^1} + \frac{\partial x^1}{\partial p^2}\frac{\mathrm{d}\bar{\psi}^2}{\mathrm{d}s^1}\right] \tag{60}$$

where $m^1 \equiv p^1 - \frac{\partial C^1}{\partial x^1} + s^1 > 0$ is the gross benefit per unit sold, including the output subsidy. Note that the terms $\frac{\mathrm{d}\bar{\psi}^i}{\mathrm{d}s^1}$ capture the *total* effect of the output subsidy on prices. They take into account that the subsidy also affects the choice of R&D by both firms in the second stage (and these, in turn, affect prices).

The first term on the right hand side of (60) shows the effect of the output subsidy on domestic benefit in the *second stage* (R&D investment). A domestic output subsidy reduces foreign R&D investment $\left(\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} < 0\right)$, which in turn increases the foreign price p^2. The increase in p^2 increases domestic output x^1 and hence firm 1's profits. Notice that due to the envelope theorem, the effect of an infinitesimal increase in the subsidy s^1 on domestic benefit $\bar{B}^1$ (through domestic R&D) can be ignored.

The second term in (60) captures the effect of an output subsidy on domestic benefit in the *third stage* (price competition stage). A domestic output subsidy reduces the foreign price in the price competition stage ($\bar{\psi}^2_{s^1} < 0$). The reduction in the foreign price p^2 reduces domestic output and profits. Again the envelope theorem allows us to ignore the effect of the output subsidy on domestic benefits through the domestic price p^1.

Notice that, starting from a subsidy s^1 equal to zero, an infinitesimal increase in the subsidy increases domestic benefits if and only if the *R&D stage effect* $\left(\bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}\right)$ is stronger than the *price stage effect,* $\left(\bar{\psi}^2_{s^1}\right)$.

$$\left.\frac{\partial \bar{B}^1}{\partial s^1}\right|_{s^1=0} = \left[p^1 - \frac{\partial C^1}{\partial x^1}\right]\left(\frac{\partial x^1}{\partial p^2}\right)\left(\bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\psi}^2_{s^1}\right) \tag{61}$$

The third term in (60) captures the increase in the subsidy bill brought about by an increase in domestic output. It includes the direct effect of the subsidy in the price competition stage as well as the R&D stage effect and price stage effect. To obtain the expression for the optimal output subsidy we need to solve

$$\frac{\partial \bar{B}^1}{\partial s^1} = 0 \tag{62}$$

with the second order condition

$$\frac{\partial^2 \bar{B}^1}{(\partial s^1)^2} < 0. \tag{63}$$

Solving (62), the precise expression for the optimal output subsidy is obtained:

$$s^{1*} = m^1 \left(\frac{\partial x^1}{\partial p^2} \right) \frac{\bar{\psi}^2_{\Delta^2} \frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\psi}^2_{s^1}}{\frac{\partial x^1}{\partial p^1} \frac{\mathrm{d}\bar{\psi}^1}{\mathrm{d}s^1} + \frac{\partial x^1}{\partial p^2} \frac{\mathrm{d}\bar{\psi}^2}{\mathrm{d}s^1}} \tag{64}$$

where $m^1 = p^1 - \frac{\partial C^1}{\partial x^1} + s^{1*}$ as before. The denominator in (64) is positive,[19] and thus the sign of the optimal subsidy depends on whether the effect on the R&D stage or on the price stage dominates in the numerator of (64). Define $\theta = -\frac{\hat{C}^i_{p^i\Delta}}{\frac{\partial x^i}{\partial p^i}} = -\frac{\partial^2 C^i(x^i,\Delta^i)}{\partial \Delta^i \partial x^i}$ as the effectiveness of R&D at reducing marginal costs of production. As we will see, the sign of the optimal output subsidy is ambiguous and depends on the cost of R&D (ϕ^1_{11}) relative to the effectiveness of R&D (θ). Notice from (45) that $\bar{\psi}^i_{\Delta^i}$ is independent of ϕ^1_{11}. Therefore, $\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}$ is the only term in the numerator of (64) that depends on ϕ^1_{11}. The following lemma helps to understand the role of the cost of R&D on the elasticity of R&D to output subsidies.

Lemma 6 *The influence of output subsidies on R&D decreases as the marginal cost of R&D increases. Specifically,*

$$\frac{\partial \left| \frac{d\Delta^1}{ds^1} \right|}{\partial \phi^1_{11}} < 0 \tag{65}$$

$$\frac{\partial \left| \frac{d\Delta^2}{ds^1} \right|}{\partial \phi^1_{11}} < 0. \tag{66}$$

Proof. See Appendix ∎

An increase in ϕ^1_{11} makes R&D investment more convex. As a result, R&D is less elastic to an output subsidy, and therefore the *R&D stage* effect of an output subsidy in (64) is weaker. Whenever the R&D stage effect is weak, the optimal output subsidy is influenced more by the *price stage* effect and should be optimally set below zero (an output tax).

The domestic government only takes into account the effect of an output subsidy on price competition when the effect of an output subsidy on foreign R&D is smaller (ϕ^1_{11} becomes higher). Contrarily, the government only takes into account the effect of the output subsidy on the R&D stage when ϕ^1_{11} is small enough. The following proposition formalizes this result, showing that we could have an output subsidy or a tax depending on the convexity of the cost of investment in R&D, i.e. ϕ^i_{11}.[20]

Proposition 7 *Under Bertrand competition, the optimal output subsidy s^{1*} can be positive or negative, depending on the convexity of the cost of R&D (ϕ^i_{11}). The optimal output subsidy is positive (an output subsidy) when the cost of additional investment in R&D is sufficiently low (low ϕ^i_{11}), and negative (an output tax) when ϕ^i_{11} is sufficiently high. Specifically,*

$$\exists \bar{\phi} < \infty \text{ such that if } \phi^i_{11} > \bar{\phi} \text{ then } s^{1*} < 0$$

$$\exists \underline{\phi} > \theta \bar{\pi}^1_{\Delta^1 s^1} - \pi^i_{\Delta_i \Delta_j} \text{ such that if } \phi^i_{11} < \underline{\phi} \text{ then } s^{1*} > 0.$$

[19] See the proof of proposition 7.

[20] Notice, however, that ϕ^i_{11} is bounded below by the stability condition (54) and therefore cannot take values below $\theta\bar{\pi}^1_{\Delta^1 s^1} - \pi^i_{\Delta_i \Delta_j}$. See the proof of lemma 6.

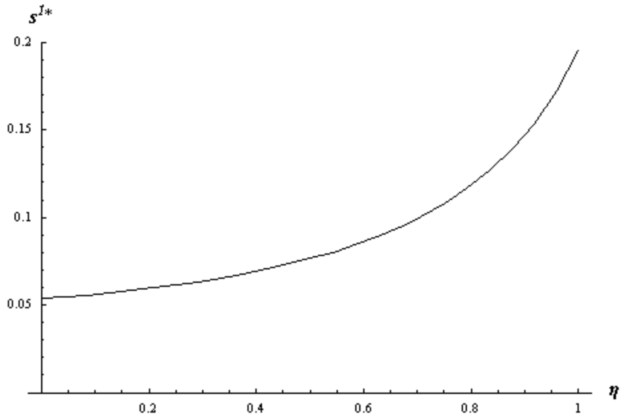

Figure 3: Cournot: Optimal output subsidy $\left(s^{1*}\right)$ as a function of the cost-effectiveness of R&D $\left(\eta = \frac{\theta^2}{b\phi}\right)$. (for $a - c = 1$, $\gamma = 0.5$)

Proof. See Appendix ■

As ϕ_{11}^i increases, the cost of investing in R&D becomes more convex. A steeper R&D cost function makes R&D less elastic with respect to an output subsidy. This reduces the effect of the subsidy on the foreign firm's R&D reaction function, leaving the effect on the foreign firm price reaction function unaffected. This implies that the domestic government has an incentive to reduce the output subsidy, or even tax output, as in the standard Bertrand game without R&D investment.

The following section performs a numerical exercise to highlight the results of price and quantity competition.

4 A Numerical Example

In this example,[21] we consider linear demands and constant marginal costs with respect to output. In particular, assume that the inverse demand for good i is given by:

$$p^i = a - b(x^i + \gamma x^j). \tag{67}$$

With $0 < \gamma < 1$. Cost functions are linear in output,

$$C(x^i, \Delta^i) = \left(c - \theta\Delta^i\right) x^i \tag{68}$$

and the monetary cost of Δ^i units of R&D is quadratic:

$$\phi(\Delta^i) = \phi\frac{\left(\Delta^i\right)^2}{2}. \tag{69}$$

The optimal output subsidy is always positive under Cournot competition, as both the R&D stage effect ($\bar{q}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}$) and the price stage effect ($\bar{q}^2_{s^1}$) have the same sign (see proposition 2). R&D becomes more elastic with respect to the output subsidy as the cost of R&D becomes flatter (i.e. ϕ_{11}^i falls). In this case the government has greater incentives to subsidize output thereby reducing foreign R&D. Figure 3 shows the optimal subsidy as a function of the cost-effectiveness of R&D (defined as $\eta = \frac{\theta^2}{\phi b}$). The optimal subsidy is increasing in η.

[21] The mathematica code used to generate the numerical results is available from the authors upon request.

Cournot Competition: numerical simulation			
Product differentiation	γ	0.5	0.5
Cost-effectiveness of R&D	$\eta = \frac{\theta^2}{\phi b}$	0.3	0.7
Price firm 1	p^1	$0.2765a + 0.7235c$	$0.0689a + 0.9311c$
Price firm 2	p^2	$0.3035a + 0.6965c$	$0.1349a + 0.8651c$
Output firm 1	x^1	$0.5004\left(\frac{a-c}{b}\right)$	$0.6648\left(\frac{a-c}{b}\right)$
Output firm 2	x^2	$0.4463\left(\frac{a-c}{b}\right)$	$0.5328\left(\frac{a-c}{b}\right)$
R&D firm 1	Δ^1	$0.1601\left(\frac{a-c}{\theta}\right)$	$0.4964\left(\frac{a-c}{\theta}\right)$
R&D firm 2	Δ^2	$0.1428\left(\frac{a-c}{\theta}\right)$	$0.3977\left(\frac{a-c}{\theta}\right)$
Unit profit firm 1	$m^1 = p^1 - c + s^1$	$0.3403\,(a-c)$	$0.1684\,(a-c)$
Unit profit firm 2	$m^2 = p^2 - c$	$0.3035\,(a-c)$	$0.1349\,(a-c)$
Total profits firm 1	π^1	$0.2076\frac{(a-c)^2}{b}$	$0.2659\frac{(a-c)^2}{b}$
Total profits firm 2	π^2	$0.1652\frac{(a-c)^2}{b}$	$0.1707\frac{(a-c)^2}{b}$
Benefits country 1	B^1	$0.1757\frac{(a-c)^2}{b}$	$0.1998\frac{(a-c)^2}{b}$
Benefits country 2	B^2	$0.1652\frac{(a-c)^2}{b}$	$0.1707\frac{(a-c)^2}{b}$
Optimal output subsidy	s^{1*}	$\mathbf{0.0638}\,(a-c)$	$\mathbf{0.0995}\,(a-c)$
Government's SOC	$\frac{\partial^2 B^1}{(\partial s^{1*})^2}$	$-\frac{0.5987}{b}$	$-\frac{0.8141}{b}$

Table 1: Numerical simulation under Cournot Competition in the third stage

The case of Bertrand competition is slightly more complicated. We have to satisfy (63), the second order condition of the government maximization problem. As we expected, the optimal subsidy also depends on the cost–effectiveness of R&D (η). Figure 4 shows the optimal output subsidy, which is increasing in η (decreasing in ϕ_{11}^i). Note that as the R&D effect becomes stronger (η increases) the government reverses its policy from an output tax to an output subsidy.[22] Note also that, interestingly, there is a set of parameter values for which free trade ($s^{1*} = 0$) is an equilibrium in the Bertrand case, even in the presence of imperfect competition.

Tables 1 and 2 present numerical results for $\gamma = 0.3$ and two different values of η (0.3 and 0.7). Notice that all relevant quantities are positive and that the second order condition for the government's maximization problem is satisfied. Table 2 shows that, depending on the cost-effectiveness of R&D (η), there could be a policy reversal under Bertrand competition.[23]

[22] For the Bertrand example in this section, (figure 4), the stability condition (54) translates into

$$\eta < \frac{(1-\gamma^2)(4-\gamma^2)^2}{2(2-\gamma^2)(2+\gamma-\gamma^2)}$$

For the value in the numerical example ($\gamma = 0.5$), we require $\eta < 1.33929$ to satisfy that condition.

[23] The numerical simulations presented in section 3 of Neary and Leahy (2000) assume, for the Cournot case, a set of parameters, which with our notation, imply $b = \gamma = \theta = a - c = 1$ and $\eta = \frac{1}{\phi} = 0.2$. For that set of parameters we obtain an optimal subsidy $s^{1*} = 0.3089$, which roughly corresponds to what they refer to as the second–best optimal output subsidy. This is represented by the intersection of the flatter line with the vertical axis in their figure 3.

For the Bertrand simulation, they use a set of parameters $b = \theta = a - c = 1$ and $\eta = \frac{1}{\phi} = 0.4$, with inverse demands

$$x^i = a - b(p^i - p^j)$$

which means that cross price effects are as strong as own price effects. Therefore we cannot compare directly with their results. They find that the optimal output subsidy is negative (point C in their figure 4). If we set $\gamma = 0.5$ with their other parameters, in our simulation we obtain a negative output subsidy (i.e. a tax) equal to $s^{1*} = -0.0224$. We only need to have a cost–effectiveness of R&D beyond 0.6 to obtain a positive output subsidy, as shown in figure 4.

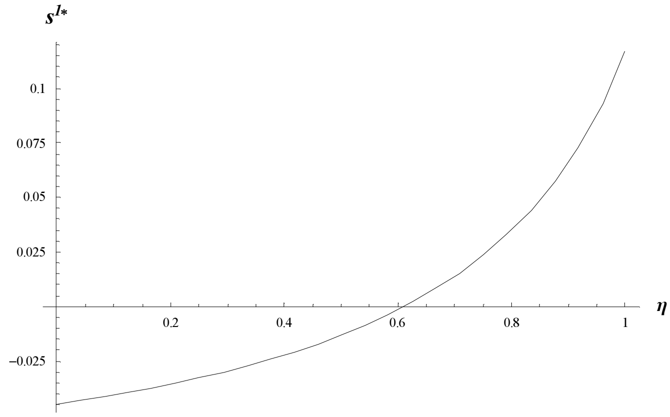

Figure 4: Bertrand: Optimal output subsidy $\left(s^{1*}\right)$ as a function of the cost-effectiveness of R&D $\left(\eta = \frac{\theta^2}{b\phi}\right)$. (for $a - c = 1$, $\gamma = 0.5$)

Bertrand Competition: numerical simulations			
Product differentiation	γ	0.5	0.5
Cost-effectiveness of R&D	$\eta = \frac{\theta^2}{\phi b}$	0.3	0.7
Price firm 1	p^1	$0.2575a + 0.7425c$	$0.0483a + 0.9517c$
Price firm 2	p^2	$0.2422a + 0.7578c$	$0.0598a + 0.9401c$
Output firm 1	x^1	$0.4848\left(\frac{a-c}{b}\right)$	$0.6422\left(\frac{a-c}{b}\right)$
Output firm 2	x^2	$0.5154\left(\frac{a-c}{b}\right)$	$0.6191\left(\frac{a-c}{b}\right)$
R&D firm 1	Δ^1	$0.1357\left(\frac{a-c}{\theta}\right)$	$0.4196\left(\frac{a-c}{\theta}\right)$
R&D firm 2	Δ^2	$0.1443\left(\frac{a-c}{\theta}\right)$	$0.4044\left(\frac{a-c}{\theta}\right)$
Unit profit firm 1	$m^1 = p^1 - c + s^1$	$0.2278\,(a-c)$	$0.0621\,(a-c)$
Unit profit firm 2	$m^2 = p^2 - c$	$0.2422\,(a-c)$	$0.0598\,(a-c)$
Total profits firm 1	π^1	$0.1455\frac{(a-c)^2}{b}$	$0.1836\frac{(a-c)^2}{b}$
Total profits firm 2	π^2	$0.1645\frac{(a-c)^2}{b}$	$0.1706\frac{(a-c)^2}{b}$
Benefits country 1	B^1	$0.1599\frac{(a-c)^2}{b}$	$0.1747\frac{(a-c)^2}{b}$
Benefits country 2	B^2	$0.1645\frac{(a-c)^2}{b}$	$0.1706\frac{(a-c)^2}{b}$
Optimal output subsidy	s^{1*}	$\mathbf{-0.0297}\,(a-c)$	$\mathbf{0.0138}\,(a-c)$
Government's SOC	$\frac{\partial^2 B^1}{(\partial s^{1*})^2}$	$-\frac{0.8055}{b}$	$-\frac{1.1226}{b}$

Table 2: Numerical simulation under Bertrand Competition in the third stage

5 Conclusions

This paper shows that for sufficiently cost effective R&D the trade policy reversal in Eaton and Grossman (1986) is not observed. Our result suggests that output subsidies are more robust than otherwise implied by the literature on strategic trade. If exporting industries make long run investments before competing in the market then governments have a case for using output subsidies even if they are uncertain about the mode of competition in the market.

We show that a necessary condition for output subsidies to be robust is that R&D be sufficiently cost effective. If the cost of R&D is too convex then R&D expenditures will be relatively inelastic to the export subsidy. In this case, the effect of an export subsidy on R&D will be negligible and will thus be arbitrarily close to the case when there is no R&D investment (Brander and Spencer (1985), Eaton and Grossman (1986)). If R&D costs are not too convex then R&D is responsive to an output subsidy. In this case, the effect of the output subsidy on the R&D stage reinforces the effect of the output subsidy on the market competition stage under Cournot competition, and dominates it under Bertrand competition. Thus, regardless of the mode of competition, the optimal policy is an output subsidy if R&D is sufficiently cost-effective.

Our condition on the curvature of the cost of R&D is reminiscent of Maggi (1996). In his model, firms invest in capacity and then compete in prices in the product market. Maggi shows that going from Cournot to Bertrand competition the optimal policy changes from an output subsidy to a tax. The key parameter is his model is the convexity of the cost function. A more convex cost function (i.e. steeper marginal cost) results in firm behavior closer to price competition. The optimal trade policy in this case is an output tax. Contrarily, a flatter marginal cost implies that the optimal policy is an output subsidy. In contrast to Maggi (1996), in our model marginal costs are constant. Under Bertrand competition, whether the optimal policy is an output subsidy or a tax, depends on the convexity of the cost of R&D . Under Cournot competition, the optimal trade policy is always an output subsidy.

Appendix

A Proof of Proposition 1

Differentiate totally the two first order conditions given by (18) to get:

$$\frac{\mathrm{d}\Delta^i}{\mathrm{d}s^1} = \frac{-\bar{\pi}^j_{\Delta^j\Delta^j}\bar{\pi}^i_{\Delta^i s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^j_{\Delta^j s^1}}{\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^j_{\Delta^j\Delta^j} - \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^j_{\Delta^i\Delta^j}} \tag{70}$$

In order to obtain the value of the expressions in (70) we first need to sign the *total* effect of subsidies on marginal revenues (including the effect on the last stage (quantity competition). We therefore have

$$\frac{\mathrm{d}R^1_2(x^1,x^2)}{\mathrm{d}s^1} = R^1_{12}(x^1,x^2)\bar{q}^1_{s^1} + R^1_{22}(x^1,x^2)\bar{q}^2_{s^1} < 0 \tag{71}$$

$$\frac{\mathrm{d}R^2_1(x^1,x^2)}{\mathrm{d}s^1} = R^2_{11}(x^1,x^2)\bar{q}^1_{s^1} + R^2_{12}(x^1,x^2)\bar{q}^2_{s^1} > 0 \tag{72}$$

by (5), (6), (16) and (17). Using these signs we can now turn to the elements in (70)

$$\bar{\pi}^i_{\Delta^i\Delta^i} = R^i_j\bar{q}^j_{\Delta^i\Delta^i} + \bar{q}^j_{\Delta^i}\frac{\mathrm{d}R^i_j(x^i,x^j)}{\mathrm{d}\Delta^i} - C^i_{x\Delta}\bar{q}^i_{\Delta^i} - C^i_{\Delta\Delta} - \phi^i_{ii} < 0 \tag{73}$$

$$\bar{\pi}^i_{\Delta^i\Delta^j} = R^i_j\bar{q}^j_{\Delta^i\Delta^j} + \bar{q}^j_{\Delta^i}\frac{\mathrm{d}R^i_j(x^i,x^j)}{\mathrm{d}\Delta^j} - C^i_{x\Delta}\bar{q}^i_{\Delta^j} < 0 \tag{74}$$

$$\bar{\pi}^1_{\Delta^1 s^1} = R^1_2\bar{q}^2_{\Delta^1 s^1} + \bar{q}^2_{\Delta^1}\frac{\mathrm{d}R^1_2(x^1,x^2)}{\mathrm{d}s^1} - C^1_{x\Delta}\bar{q}^1_{s^1} > 0 \tag{75}$$

$$\bar{\pi}^2_{\Delta^2 s^1} = R^2_1\bar{q}^1_{\Delta^2 s^1} + \bar{q}^1_{\Delta^2}\frac{\mathrm{d}R^2_1(x^1,x^2)}{\mathrm{d}s^1} - C^2_{x\Delta}\bar{q}^2_{s^1} < 0 \tag{76}$$

where the first inequality is the second order condition of the maximization in the R&D stage, the second inequality repeats (21), and the last two inequalities are derived from (71), (72), (16), (17), (15) and noting that for linear demand and constant marginal costs, $\bar{q}^i_{\Delta^j}$ is independent of s^1. Therefore

$$\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} = \frac{-\bar{\pi}^2_{\Delta^2\Delta^2}\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^1_{\Delta^1\Delta^2}\bar{\pi}^2_{\Delta^2 s^1}}{\bar{\pi}^1_{\Delta^1\Delta^1}\bar{\pi}^2_{\Delta^2\Delta^2} - \bar{\pi}^1_{\Delta^1\Delta^2}\bar{\pi}^2_{\Delta^1\Delta^2}} > 0 \tag{77}$$

$$\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} = \frac{-\bar{\pi}^1_{\Delta^1\Delta^1}\bar{\pi}^2_{\Delta^2 s^1} + \bar{\pi}^2_{\Delta^2\Delta^1}\bar{\pi}^1_{\Delta^1 s^1}}{\bar{\pi}^2_{\Delta^2\Delta^2}\bar{\pi}^1_{\Delta^1\Delta^1} - \bar{\pi}^2_{\Delta^2\Delta^1}\bar{\pi}^1_{\Delta^2\Delta^1}} < 0 \tag{78}$$

■

B Proof of Proposition 3.

Note that,

$$\begin{aligned}\bar{\pi}^1_{\Delta^1\Delta^2} \quad = \quad & \left(\hat{R}^1_2 - \hat{C}^1_{p^2} + s^1\left(\frac{\partial x^1}{\partial p^2}\right)\right)\bar{\psi}^2_{\Delta^1\Delta^2} + \bar{\psi}^2_{\Delta^1}\left(\frac{\mathrm{d}\hat{R}^1_2(p^1,p^2)}{\mathrm{d}\Delta^2} - \frac{\mathrm{d}\hat{C}^1_{p^2}(p^1,p^2,\Delta^1)}{\mathrm{d}\Delta^2} + s^1\frac{\mathrm{d}\left(\frac{\partial x^1}{\partial p^2}\right)}{\mathrm{d}\Delta^2}\right) \\ & -\hat{C}^1_{p^1\Delta}\bar{\psi}^1_{\Delta^2} - \hat{C}^1_{p^2\Delta}\bar{\psi}^2_{\Delta^2}\end{aligned} \tag{79}$$

$$\bar{\pi}^2_{\Delta^2\Delta^1} = \left(\hat{R}^2_1 - \hat{C}^2_{p^1}\right)\bar{\psi}^1_{\Delta^2\Delta^1} + \bar{\psi}^1_{\Delta^2}\left(\frac{\mathrm{d}\hat{R}^2_1(p^1,p^2)}{\mathrm{d}\Delta^1} - \frac{\mathrm{d}\hat{C}^2_{p^1}(p^1,p^2,\Delta^2)}{\mathrm{d}\Delta^1}\right) - \hat{C}^2_{p^2\Delta}\bar{\psi}^2_{\Delta^1} - \hat{C}^2_{p^1\Delta}\bar{\psi}^1_{\Delta^1} \tag{80}$$

Here, $\frac{\mathrm{d}\hat{R}^i_j(p^i,p^j)}{\mathrm{d}\Delta^j} = \hat{R}^i_{ij}(p^i,p^j)\bar{\psi}^i_{\Delta^j} + \hat{R}^i_{jj}(p^i,p^j)\bar{\psi}^j_{\Delta^j} < 0$ (from (32), (33), (45) and (46)) and $\frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta^i)}{\mathrm{d}\Delta^j} = \hat{C}^i_{p^ip^j}(p^i,p^j,\Delta^i)\bar{\psi}^i_{\Delta^j} + \hat{C}^i_{p^jp^j}(p^i,p^j,\Delta^i)\bar{\psi}^j_{\Delta^j}$. Both the second order condition (52) and the stability condition (54) impose bounds on ϕ^i_{ii} (this is discussed in the determination of the optimal subsidy).

Note that, under the assumption that marginal costs are constant, we have $\bar{\psi}^j_{\Delta^i\Delta^j} = 0$. If demand is linear then $\hat{R}^i_{jj} = 0$ and the slope of the demand function is not influenced by R&D. Formally:

$$\frac{\mathrm{d}\left(\frac{\partial x^i}{\partial p^i}\right)}{\mathrm{d}\Delta^j} = \frac{\partial^2 x^i(p^i,p^j)}{\partial\left(p^i\right)^2}\bar{\psi}^i_{\Delta^j} + \frac{\partial^2 x^i(p^i,p^j)}{\partial p^i \partial p^j}\bar{\psi}^j_{\Delta^j} = 0 \tag{81}$$

Both linearity of demand and constant marginal costs together imply $\hat{C}^i_{p^ip^j} = \hat{C}^i_{p^jp^j} = \frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta^i)}{\mathrm{d}\Delta^j} = 0$. Then, we can simplify both expressions $\bar{\pi}^i_{\Delta^i\Delta^j}$ to:

$$\begin{aligned}
\bar{\pi}^i_{\Delta^i\Delta^j} &= \bar{\psi}^j_{\Delta^i}\bar{\psi}^i_{\Delta^j}\hat{R}^i_{ij}(p^i,p^j) - \hat{C}^i_{p^i\Delta}\bar{\psi}^i_{\Delta^j} - \hat{C}^i_{p^j\Delta}\bar{\psi}^j_{\Delta^j} \\
&= \frac{-\bar{\Pi}^j_{ij}\hat{C}^i_{p^i\Delta}}{\bar{\Pi}^i_{ii}\bar{\Pi}^j_{jj} - \bar{\Pi}^j_{ij}\bar{\Pi}^i_{ij}}\frac{-\bar{\Pi}^i_{ij}\hat{C}^j_{p^j\Delta}}{\bar{\Pi}^i_{ii}\bar{\Pi}^j_{jj} - \bar{\Pi}^j_{ij}\bar{\Pi}^i_{ij}}\hat{R}^i_{ij} - \hat{C}^i_{p^i\Delta}\frac{-\bar{\Pi}^i_{ij}\hat{C}^j_{p^j\Delta}}{\bar{\Pi}^i_{ii}\bar{\Pi}^j_{jj} - \bar{\Pi}^j_{ij}\bar{\Pi}^i_{ij}} - \hat{C}^i_{p^j\Delta}\frac{\bar{\Pi}^i_{ii}\hat{C}^j_{p^j\Delta}}{\bar{\Pi}^i_{ii}\bar{\Pi}^j_{jj} - \bar{\Pi}^j_{ij}\bar{\Pi}^i_{ij}} \\
&= \frac{\bar{\Pi}^j_{ij}\hat{C}^i_{p^i\Delta}\bar{\Pi}^i_{ij}\hat{C}^j_{p^j\Delta}\hat{R}^i_{ij} + \hat{C}^i_{p^i\Delta}\bar{\Pi}^i_{ij}\hat{C}^j_{p^j\Delta}\left(\bar{\Pi}^i_{ii}\bar{\Pi}^j_{jj} - \bar{\Pi}^j_{ij}\bar{\Pi}^i_{ij}\right) - \hat{C}^i_{p^j\Delta}\Pi^i_{ii}\hat{C}^j_{p^j\Delta}\left(\bar{\Pi}^i_{ii}\bar{\Pi}^j_{jj} - \bar{\Pi}^j_{ij}\bar{\Pi}^i_{ij}\right)}{\left(\bar{\Pi}^i_{ii}\bar{\Pi}^j_{jj} - \bar{\Pi}^j_{ij}\bar{\Pi}^i_{ij}\right)^2}
\end{aligned} \tag{82}$$

Recall that, for linear demands, $\bar{\Pi}^i_{ij} = \hat{R}^i_{ij}$. Notice also that in the case of linear demands, $\bar{\Pi}^i$ is quadratic and all second derivatives of $\bar{\Pi}^i(p^1,p^2)$ with respect to prices are thus constant. Therefore, $\bar{\Pi}^i_{ii} = \bar{\Pi}^j_{jj}$ and $\bar{\Pi}^j_{ij} = \bar{\Pi}^i_{ij}$. If we also have constant marginal costs, all second derivatives of $\hat{C}^i(p^1,p^2,\Delta^i)$ are constant. Therefore $\hat{C}^i_{p^i\Delta} = \hat{C}^j_{p^j\Delta}$. Remember also that $\left|\bar{\Pi}^i_{ii}\right| > \left|\bar{\Pi}^i_{ij}\right|$, $\left|\hat{C}^i_{p^i\Delta}\right| > \left|\hat{C}^i_{p^j\Delta}\right|$ and $\left|\frac{\partial x^i}{\partial p^i}\right| > \left|\frac{\partial x^i}{\partial p^j}\right|$. All this implies that

$$\begin{aligned}
\bar{\pi}^i_{\Delta^i\Delta^j} &= \frac{\left(\bar{\Pi}^i_{ij}\right)^3\left(\hat{C}^i_{p^i\Delta}\right)^2 + \left(\hat{C}^i_{p^i\Delta}\right)^2\bar{\Pi}^i_{ij}\left(\bar{\Pi}^i_{ii}\right)^2 - \left(\hat{C}^i_{p^i\Delta}\right)^2\left(\bar{\Pi}^i_{ij}\right)^3 - \hat{C}^i_{p^j\Delta}\left(\bar{\Pi}^i_{ii}\right)^3\hat{C}^i_{p^i\Delta} + \hat{C}^i_{p^j\Delta}\bar{\Pi}^i_{ii}\hat{C}^i_{p^i\Delta}\left(\bar{\Pi}^i_{ij}\right)^2}{\left(\left(\bar{\Pi}^i_{ii}\right)^2 - \left(\bar{\Pi}^i_{ij}\right)^2\right)^2} \\
&= \frac{\left(\hat{C}^i_{p^i\Delta}\right)^2\bar{\Pi}^i_{ij}\left(\bar{\Pi}^i_{ii}\right)^2 - \hat{C}^i_{p^j\Delta}\left(\bar{\Pi}^i_{ii}\right)^3\hat{C}^i_{p^i\Delta} + \hat{C}^i_{p^j\Delta}\bar{\Pi}^i_{ii}\hat{C}^i_{p^i\Delta}\left(\bar{\Pi}^i_{ij}\right)^2}{\left(\left(\bar{\Pi}^i_{ii}\right)^2 - \left(\bar{\Pi}^i_{ij}\right)^2\right)^2} \\
&= \frac{\left(\hat{C}^i_{p^i\Delta}\right)^2\left(\bar{\Pi}^i_{ii}\right)^3}{\left(\left(\bar{\Pi}^i_{ii}\right)^2 - \left(\bar{\Pi}^i_{ij}\right)^2\right)^2}\left[\frac{\bar{\Pi}^i_{ij}}{\bar{\Pi}^i_{ii}} - \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}} + \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}}\left(\frac{\bar{\Pi}^i_{ij}}{\bar{\Pi}^i_{ii}}\right)^2\right] \\
&= \frac{\left(\hat{C}^i_{p^i\Delta}\right)^2\left(\bar{\Pi}^i_{ii}\right)^3}{\left(\left(\bar{\Pi}^i_{ii}\right)^2 - \left(\bar{\Pi}^i_{ij}\right)^2\right)^2}\left[\frac{1}{2}\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}} - \frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}} + \frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}\left(\frac{1}{2}\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}\right)^2\right] = \frac{\left(\hat{C}^i_{p^i\Delta}\right)^2\left(\bar{\Pi}^i_{ii}\right)^3}{\left(\left(\bar{\Pi}^i_{ii}\right)^2 - \left(\bar{\Pi}^i_{ij}\right)^2\right)^2}\left[\frac{1}{2}\gamma - \frac{1}{4}\gamma^3\right] < 0
\end{aligned} \tag{83}$$

since $\gamma = -\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}$ is between zero and one and $\bar{\Pi}^i_{ii} < 0$. Notice that γ measures the degree of product differentiation and is bounded between 0 (independent goods) and 1 (perfect substitutes). ■

C Proof of Proposition 4

For later use we need to compute $\bar{\psi}^i_{\Delta^i\Delta^j}$ and $\bar{\psi}^i_{\Delta^j\Delta^j}$. Differentiating (46) we obtain

$$\bar{\psi}^i_{\Delta^i\Delta^j} = \frac{\bar{\Pi}^i_{ij}\hat{C}^j_{p^j\Delta}\hat{C}^i_{p^ip^i\Delta}\bar{\Pi}^j_{jj}}{\left(\bar{\Pi}^i_{ii}\bar{\Pi}^j_{jj}-\bar{\Pi}^j_{ij}\bar{\Pi}^i_{ij}\right)^2} \tag{84}$$

$$\bar{\psi}^i_{\Delta^j\Delta^j} = \frac{\left(-\hat{R}^i_{ij}\hat{C}^j_{p^j\Delta\Delta}\right)\left(\bar{\Pi}^i_{ii}\bar{\Pi}^j_{jj}-\bar{\Pi}^i_{ij}\bar{\Pi}^j_{ij}\right)+\bar{\Pi}^i_{ij}\hat{C}^j_{p^j\Delta}\hat{C}^j_{p^jp^j\Delta}\bar{\Pi}^i_{ii}}{\left(\bar{\Pi}^i_{ii}\bar{\Pi}^j_{jj}-\bar{\Pi}^i_{ij}\bar{\Pi}^j_{ij}\right)^2} \tag{85}$$

Note that $\bar{\psi}^i_{\Delta^i\Delta^j}$ is zero for constant marginal costs with respect to output and $\bar{\psi}^i_{\Delta^j\Delta^j}$ is zero for marginal costs that are constant with respect to output and linear with respect to R&D.

For the first part of the proof, we will follow similar steps as the proof of proposition 1. We start by differentiating totally the two first order conditions given by (50) and (51). Using Cramer's rule:

$$\frac{\mathrm{d}\Delta^i}{\mathrm{d}s^1} = \frac{-\bar{\pi}^j_{\Delta^j\Delta^j}\bar{\pi}^i_{\Delta^is^1}+\bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^j_{\Delta^js^1}}{\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^j_{\Delta^j\Delta^j}-\bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^j_{\Delta^i\Delta^j}} \tag{86}$$

To obtain the value of expressions in (70) we need to obtain the *total* effect of subsidies on marginal revenues (including the effect on the last (price competition) stage). We have

$$\frac{\mathrm{d}\hat{R}^i_j(p^i,p^j)}{\mathrm{d}\Delta^i} = \hat{R}^i_{ij}(p^i,p^j)\bar{\psi}^i_{\Delta^i}+\hat{R}^i_{jj}(p^i,p^j)\bar{\psi}^j_{\Delta^i} < 0 \tag{87}$$

$$\frac{\mathrm{d}\hat{R}^i_j(p^i,p^j)}{\mathrm{d}\Delta^j} = \hat{R}^i_{ij}(p^i,p^j)\bar{\psi}^i_{\Delta^j}+\hat{R}^i_{jj}(p^i,p^j)\bar{\psi}^j_{\Delta^j} < 0 \tag{88}$$

$$\frac{\mathrm{d}\hat{R}^i_j(p^i,p^j)}{\mathrm{d}s^1} = \hat{R}^i_{ij}(p^i,p^j)\bar{\psi}^i_{s^1}+\hat{R}^i_{jj}(p^i,p^j)\bar{\psi}^j_{s^1} < 0 \tag{89}$$

where the inequalities are obtained from (32), (33), (45), (46), (47) and (48).

Turn next to the *total* effect of R&D on marginal costs:

$$\frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta^i)}{\mathrm{d}\Delta^i} = \hat{C}^i_{p^ip^j}(p^i,p^j,\Delta^i)\bar{\psi}^i_{\Delta^i}+\hat{C}^i_{p^jp^j}(p^i,p^j,\Delta^i)\bar{\psi}^j_{\Delta^i}+\hat{C}^i_{p^j\Delta}(p^i,p^j,\Delta^i) < 0 \tag{90}$$

$$\frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta^i)}{\mathrm{d}\Delta^j} = \hat{C}^i_{p^ip^j}(p^i,p^j,\Delta^i)\bar{\psi}^i_{\Delta^j}+\hat{C}^i_{p^jp^j}(p^i,p^j,\Delta^i)\bar{\psi}^j_{\Delta^j} = 0 \tag{91}$$

$$\frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta^i)}{\mathrm{d}s^1} = \hat{C}^i_{p^ip^j}(p^i,p^j,\Delta^i)\bar{\psi}^i_{s^1}+\hat{C}^i_{p^jp^j}(p^i,p^j,\Delta^i)\bar{\psi}^j_{s^1} = 0 \tag{92}$$

where the inequalities are derived from (34), (35), (45), (46), (47) and (48). We also assume linear demand and constant marginal cost with respect to output (so that $\hat{C}^i_{p^jp^j} = \hat{C}^i_{p^ip^j} = 0$). Finally, note that for linear demands, the slope of the demand function is not influenced by R&D (equation 81)

Using these inequalities we can now turn to the elements of (86). We will use the fact that, for linear demands, $\bar{\Pi}^i_{ij} = \hat{R}^i_{ij}$ and both are quadratic with constant second derivatives with respect to prices. Therefore, $\bar{\Pi}^i_{ii} = \bar{\Pi}^j_{jj}$ and $\bar{\Pi}^j_{ij} = \bar{\Pi}^i_{ij}$. If we also have constant marginal costs, all second derivatives of $\hat{C}^i(p^1,p^2,\Delta^i)$ are constant. Therefore $\hat{C}^i_{p^i\Delta} = \hat{C}^j_{p^j\Delta}$. For linear demand and constant marginal costs we have $\bar{\psi}^i_{\Delta^j\Delta^j} = \bar{\psi}^i_{\Delta^i\Delta^j} = \frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta^i)}{\mathrm{d}\Delta^j} = \frac{\mathrm{d}\hat{C}^i_{p^j}(p^i,p^j,\Delta^i)}{\mathrm{d}s^1} = \frac{\mathrm{d}\left(\frac{\partial x^i}{\partial p^i}\right)}{\mathrm{d}\Delta^j} = \hat{C}^i_{\Delta\Delta} = 0$. Remember also that $\left|\bar{\Pi}^i_{ii}\right| > \left|\bar{\Pi}^i_{ij}\right|$, $\left|\frac{\partial x^i}{\partial p^i}\right| > \left|\frac{\partial x^i}{\partial p^j}\right|$, and that assumption (30) means $\left|\hat{C}^i_{p^i\Delta}\right| > \left|\hat{C}^i_{p^j\Delta}\right|$. All these imply that

$$
\begin{aligned}
\bar{\pi}^1_{\Delta^1\Delta^1} &= \left(\hat{R}^1_2 - \hat{C}^1_{p^2} + s^1\left(\frac{\partial x^2}{\partial p^1}\right)\right)\bar{\psi}^2_{\Delta^1\Delta^1} + \bar{\psi}^2_{\Delta^1}\left(\frac{\mathrm{d}\hat{R}^1_2(p^2,p^1)}{\mathrm{d}\Delta^1} - \frac{\mathrm{d}\hat{C}^1_{p^2}(p^2,p^1,\Delta^1)}{\mathrm{d}\Delta^1} + s^1\frac{\mathrm{d}\left(\frac{\partial x^1}{\partial p^2}\right)}{\mathrm{d}\Delta^1}\right) \\
&\quad -\hat{C}^1_{p^1\Delta}\bar{\psi}^1_{\Delta^1} - \hat{C}^1_{p^2\Delta}\bar{\psi}^2_{\Delta^1} - \hat{C}^1_{\Delta\Delta} - \phi^1_{11} \\
&= \bar{\psi}^2_{\Delta^1}\left(\hat{R}^1_{12}\bar{\psi}^1_{\Delta^1} - \hat{C}^1_{p^2\Delta}\right) - \hat{C}^1_{p^1\Delta}\bar{\psi}^1_{\Delta^1} - \hat{C}^1_{p^2\Delta}\bar{\psi}^2_{\Delta^1} - \phi^1_{11} \\
&= \frac{-\Pi^2_{21}\hat{C}^1_{p^1\Delta}}{\Pi^2_{22}\Pi^1_{11} - \Pi^1_{21}\Pi^2_{21}}\left(\frac{\hat{R}^1_{12}\Pi^2_{22}\hat{C}^1_{p^1\Delta}}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}} - \hat{C}^1_{p^2\Delta}\right) - \hat{C}^1_{p^1\Delta}\frac{\Pi^2_{22}\hat{C}^1_{p^1\Delta}}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}} - \hat{C}^1_{p^2\Delta}\frac{-\Pi^2_{21}\hat{C}^1_{p^1\Delta}}{\Pi^2_{22}\Pi^1_{11} - \Pi^1_{21}\Pi^2_{21}} - \phi^1_{11} \\
&= \frac{-\Pi^i_{ij}\hat{C}^i_{p^i\Delta}}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2}\left(\Pi^i_{ij}\frac{\Pi^i_{ii}\hat{C}^i_{p^i\Delta}}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2} - \hat{C}^i_{p^j\Delta}\right) - \hat{C}^i_{p^i\Delta}\frac{\Pi^i_{ii}\hat{C}^i_{p^i\Delta}}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2} - \hat{C}^i_{p^j\Delta}\frac{-\Pi^i_{ij}\hat{C}^i_{p^i\Delta}}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2} - \phi^1_{11} \\
&= \frac{\left(\Pi^i_{ii}\right)^3\left(\hat{C}^i_{p^i\Delta}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}} - \left(\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right)^3\frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}} - 1 + \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}}\frac{\Pi^i_{ij}}{\Pi^i_{ii}} - \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}}\left(\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right)^3\right] - \phi^1_{11} \\
&= \frac{\left(\Pi^i_{ii}\right)^3\left(\hat{C}^i_{p^i\Delta}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[\left(\frac{1}{2}\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}\right)\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}} - \left(\frac{1}{2}\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}\right)^3\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}} - 1 + \frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}\left(\frac{1}{2}\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}\right) - \frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}\left(\frac{1}{2}\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}\right)^3\right] - \phi^1_{11} \\
&= \frac{\left(\Pi^i_{ii}\right)^3\left(\hat{C}^i_{p^i\Delta}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[\frac{1}{2}\gamma^2 - \frac{1}{8}\gamma^4 - 1 + \frac{1}{2}\gamma^2 - \frac{1}{8}\gamma^4\right] - \phi^1_{11} \\
&= \frac{\left(\Pi^i_{ii}\right)^3\left(\hat{C}^i_{p^i\Delta}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[\gamma^2 - \frac{1}{4}\gamma^4 - 1\right] - \phi^1_{11} \qquad (93)
\end{aligned}
$$

where $\gamma = -\frac{\frac{\partial x^i}{\partial p^j}}{\frac{\partial x^i}{\partial p^i}}$ measures the degree of product differentiation as in the proof of proposition 3.

$$
\begin{aligned}
\bar{\pi}^2_{\Delta^2\Delta^2} &= \left(\hat{R}^2_1 - \hat{C}^2_{p^1}\right)\bar{\psi}^1_{\Delta^2\Delta^2} + \bar{\psi}^1_{\Delta^2}\left(\frac{\mathrm{d}\hat{R}^2_1(p^1,p^2)}{\mathrm{d}\Delta^2} - \frac{\mathrm{d}\hat{C}^2_{p^1}(p^1,p^2,\Delta^2)}{\mathrm{d}\Delta^2}\right) - \hat{C}^2_{p^2\Delta}\bar{\psi}^2_{\Delta^2} - \hat{C}^2_{p^1\Delta}\bar{\psi}^1_{\Delta^2} - \hat{C}^2_{\Delta\Delta} - \phi^2_{11} \\
&= \bar{\psi}^1_{\Delta^2}\left(\hat{R}^2_{12}\bar{\psi}^2_{\Delta^2} - \hat{C}^2_{p^1\Delta}\right) - \hat{C}^2_{p^2\Delta}\bar{\psi}^2_{\Delta^2} - \hat{C}^2_{p^1\Delta}\bar{\psi}^1_{\Delta^2} - \phi^2_{11} \\
&= \bar{\pi}^1_{\Delta^1\Delta^1} \qquad (94)
\end{aligned}
$$

$$\begin{aligned}
\bar{\pi}^1_{\Delta^1 s^1} &= \left(\hat{R}^1_2 - \hat{C}^1_{p^2} + s^1\left(\frac{\partial x^1}{\partial p^2}\right)\right)\bar{\psi}^2_{\Delta^1 s^1} + \bar{\psi}^2_{\Delta^1}\left(\frac{\mathrm{d}\hat{R}^1_2(p^1,p^2)}{\mathrm{d}s^1} - \frac{\mathrm{d}\hat{C}^1_{p^2}(p^1,p^2,\Delta^1)}{\mathrm{d}s^1} + \left(\frac{\partial x^1}{\partial p^2}\right)\right) \\
&\quad -\hat{C}^1_{p^1\Delta}\bar{\psi}^1_{s^1} - \hat{C}^1_{p^2\Delta}\bar{\psi}^2_{s^1} \\
&= \bar{\psi}^2_{\Delta^1}\left(\hat{R}^1_{12}\bar{\psi}^1_{s^1} + \left(\frac{\partial x^1}{\partial p^2}\right)\right) - \hat{C}^1_{p^1\Delta}\bar{\psi}^1_{s^1} - \hat{C}^1_{p^2\Delta}\bar{\psi}^2_{s^1} \\
&= \frac{-\Pi^2_{21}\hat{C}^1_{p^1\Delta}}{\Pi^2_{22}\Pi^1_{11} - \Pi^1_{21}\Pi^2_{21}}\left(-\frac{\hat{R}^1_{12}\Pi^2_{22}\left(\frac{\partial x^1}{\partial p^1}\right)}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}} + \left(\frac{\partial x^1}{\partial p^2}\right)\right) + \hat{C}^1_{p^1\Delta}\frac{\Pi^2_{22}\left(\frac{\partial x^1}{\partial p^1}\right)}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}} - \hat{C}^1_{p^2\Delta}\frac{\Pi^2_{12}\left(\frac{\partial x^1}{\partial p^1}\right)}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}} \\
&= \frac{\left(\frac{\partial x^1}{\partial p^1}\right)\hat{C}^i_{p^i\Delta}\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[-\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\frac{\frac{\partial x^1}{\partial p^2}}{\frac{\partial x^1}{\partial p^1}} + \frac{\frac{\partial x^1}{\partial p^2}}{\frac{\partial x^1}{\partial p^1}}\left(\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right)^3 + 1 - \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}}\frac{\Pi^i_{ij}}{\Pi^i_{ii}} + \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}}\left(\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right)^3\right] \\
&= \frac{\left(\frac{\partial x^1}{\partial p^1}\right)\hat{C}^i_{p^i\Delta}\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[-\frac{1}{2}\gamma^2 + \frac{1}{8}\gamma^4 + 1 - \frac{1}{2}\gamma^2 + \frac{1}{8}\gamma^4\right] \\
&= \frac{\left(\frac{\partial x^1}{\partial p^1}\right)\hat{C}^i_{p^i\Delta}\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[-\gamma^2 + \frac{1}{4}\gamma^4 + 1\right] > 0
\end{aligned} \tag{95}$$

$$\begin{aligned}
\bar{\pi}^2_{\Delta^2 s^1} &= \left(\hat{R}^2_1 - \hat{C}^2_{p^1}\right)\bar{\psi}^1_{\Delta^2 s^1} + \bar{\psi}^1_{\Delta^2}\left(\frac{\mathrm{d}\hat{R}^2_1(p^1,p^2)}{\mathrm{d}s^1} - \frac{\mathrm{d}\hat{C}^2_{p^1}(p^1,p^2,\Delta^2)}{\mathrm{d}s^1}\right) - \hat{C}^2_{p^2\Delta}\bar{\psi}^2_{s^1} - \hat{C}^2_{p^1\Delta}\bar{\psi}^1_{s^1} \\
&= \bar{\psi}^1_{\Delta^2}\hat{R}^2_{21}(p^1,p^2)\bar{\psi}^2_{s^1} - \hat{C}^2_{p^2\Delta}\bar{\psi}^2_{s^1} - \hat{C}^2_{p^1\Delta}\bar{\psi}^1_{s^1} \\
&= \frac{-\Pi^1_{12}\hat{C}^2_{p^2\Delta}}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}}\left(\hat{R}^2_{21}\frac{\Pi^2_{12}\left(\frac{\partial x^1}{\partial p^1}\right)}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}}\right) - \hat{C}^2_{p^2\Delta}\frac{\Pi^2_{12}\left(\frac{\partial x^1}{\partial p^1}\right)}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}} - \hat{C}^2_{p^1\Delta}\frac{-\Pi^2_{22}\left(\frac{\partial x^1}{\partial p^1}\right)}{\Pi^1_{11}\Pi^2_{22} - \Pi^2_{12}\Pi^1_{12}} \\
&= \frac{-\Pi^i_{ij}\hat{C}^i_{p^i\Delta}}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2}\left(\frac{\left(\Pi^i_{ij}\right)^2\left(\frac{\partial x^1}{\partial p^1}\right)}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2}\right) - \hat{C}^i_{p^i\Delta}\frac{\Pi^i_{ij}\left(\frac{\partial x^1}{\partial p^1}\right)}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2} - \hat{C}^i_{p^j\Delta}\frac{-\Pi^i_{ii}\left(\frac{\partial x^1}{\partial p^1}\right)}{\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2} \\
&= \left(\frac{\partial x^1}{\partial p^1}\right)\frac{-\left(\Pi^i_{ij}\right)^3\hat{C}^i_{p^i\Delta} - \hat{C}^i_{p^i\Delta}\Pi^i_{ij}\left(\Pi^i_{ii}\right)^2 + \hat{C}^i_{p^i\Delta}\left(\Pi^i_{ij}\right)^3 + \hat{C}^i_{p^j\Delta}\left(\Pi^i_{ii}\right)^3 - \hat{C}^i_{p^j\Delta}\Pi^i_{ii}\left(\Pi^i_{ij}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2} \\
&= \frac{\left(\frac{\partial x^1}{\partial p^1}\right)\hat{C}^i_{p^i\Delta}\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[-\frac{\Pi^i_{ij}}{\Pi^i_{ii}} + \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}} - \frac{\hat{C}^i_{p^j\Delta}}{\hat{C}^i_{p^i\Delta}}\left(\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right)^2\right] \\
&= \frac{\left(\frac{\partial x^1}{\partial p^1}\right)\hat{C}^i_{p^i\Delta}\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[-\frac{1}{2}\gamma + \frac{1}{4}\gamma^3\right] < 0
\end{aligned} \tag{96}$$

The second order condition (52) means that $\bar{\pi}^i_{\Delta^i\Delta^i} < 0$, whereas the stability condition (54) implies that $\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 > \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2$. Also, from proposition 3:

$$\bar{\pi}^1_{\Delta^1\Delta^2} = \bar{\pi}^2_{\Delta^2\Delta^1} = \frac{\left(\hat{C}^i_{p^i\Delta}\right)^2\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[\frac{1}{2}\gamma - \frac{1}{4}\gamma^3\right] < 0 \tag{97}$$

All these imply:

$$\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} = \frac{-\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^2_{\Delta^2 s^1}}{\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} > 0 \tag{98}$$

and

$$\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} = \frac{-\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^2_{\Delta^2 s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^1_{\Delta^1 s^1}}{\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} < 0 \tag{99}$$

Which is the statement of the proposition.

From (93), (97), (95) and (96) we can also derive the following relationships, to be used later:

$$\bar{\pi}^1_{\Delta^1 s^1}\left(-\frac{\hat{C}^i_{p^i\Delta}}{\frac{\partial x^1}{\partial p^1}}\right) - \phi^1_{11} = \theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11} = \bar{\pi}^i_{\Delta^i\Delta^i} \tag{100}$$

$$\bar{\pi}^2_{\Delta^2 s^1}\left(-\frac{\hat{C}^i_{p^i\Delta}}{\frac{\partial x^1}{\partial p^1}}\right) = \theta\bar{\pi}^2_{\Delta^2 s^1} = \bar{\pi}^i_{\Delta^i\Delta^j} \tag{101}$$

$$\frac{\bar{\pi}^1_{\Delta^1 s^1}}{\bar{\pi}^i_{\Delta^i\Delta^j}} = \frac{1}{\theta}\frac{\left[\gamma^2 - \frac{1}{4}\gamma^4 - 1\right]}{\left[\frac{1}{2}\gamma - \frac{1}{4}\gamma^3\right]} \tag{102}$$

$$\bar{\pi}^2_{\Delta^2 s^1}\theta\frac{\left[\gamma^2 - \frac{1}{4}\gamma^4 - 1\right]}{\left[\frac{1}{2}\gamma - \frac{1}{4}\gamma^3\right]} - \phi^1_{11} = \bar{\pi}^i_{\Delta^i\Delta^i} \tag{103}$$

■

D Proof of Corollary 5

Note, first, that from (95) and (96), $\left|\bar{\pi}^1_{\Delta^1 s^1}\right| > \left|\bar{\pi}^2_{\Delta^2 s^1}\right|$ for γ between 0 and 1. Also, from (98) and (99):

$$\begin{aligned}
\left|\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}\right| - \left|\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}\right| &= \frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} \\
&= \frac{-\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^2_{\Delta^2 s^1}}{\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} + \frac{-\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^2_{\Delta^2 s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^1_{\Delta^1 s^1}}{\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} \\
&= \frac{-\bar{\pi}^i_{\Delta^i\Delta^i}\left(\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^2_{\Delta^2 s^1}\right) + \bar{\pi}^i_{\Delta^i\Delta^j}\left(\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^2_{\Delta^2 s^1}\right)}{\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} \\
&= \frac{-\left(\bar{\pi}^i_{\Delta^i\Delta^i} - \bar{\pi}^i_{\Delta^i\Delta^j}\right)\left(\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^2_{\Delta^2 s^1}\right)}{\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} \\
&= -\frac{\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^2_{\Delta^2 s^1}}{\bar{\pi}^i_{\Delta^i\Delta^i} + \bar{\pi}^i_{\Delta^i\Delta^j}} > 0 \qquad (104)
\end{aligned}$$

where the inequality comes from the denominator being negative ((52) and proposition 3), $\bar{\pi}^1_{\Delta^1 s^1} > 0$ by (95) and $\left|\bar{\pi}^1_{\Delta^1 s^1}\right| > \left|\bar{\pi}^2_{\Delta^2 s^1}\right|$. ■

E Derivation of equation (60)

Recall that from the first order condition in the R&D stage, $\bar{\pi}^1_{\Delta^1} = 0$, and also,

$$\bar{\pi}^1_{s^1} = \left[\hat{R}^1_2(p^1,p^2) - \hat{C}^1_{p^2}(p^1,p^2,\Delta) + s^1\left(\frac{\partial x^1}{\partial p^2}\right)\right]\bar{\psi}^2_{s^1}(\Delta^1,\Delta^2,s^1) + x^1 = m^1\left(\frac{\partial x^1}{\partial p^2}\right)\bar{\psi}^2_{s^1} + x^1 \tag{105}$$

$$\bar{\pi}^1_{\Delta^2} = \left[\hat{R}^1_2(p^1,p^2) - \hat{C}^1_{p^2}(p^1,p^2,\Delta) + s^1\left(\frac{\partial x^1}{\partial p^2}\right)\right]\bar{\psi}^2_{\Delta^2}(\Delta^1,\Delta^2,s^1) = m^1\left(\frac{\partial x^1}{\partial p^2}\right)\bar{\psi}^2_{\Delta^2} \tag{106}$$

$$\frac{\mathrm{d}\bar{\psi}^1}{\mathrm{d}s^1} = \bar{\psi}^1_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \bar{\psi}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\psi}^1_{s^1} \tag{107}$$

and

$$\frac{\mathrm{d}\bar{\psi}^2}{\mathrm{d}s^1} = \bar{\psi}^2_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} + \bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\psi}^2_{s^1}. \tag{108}$$

where $m^1 \equiv p^1 - \frac{\partial C^1}{\partial x^1} + s^1$. The last two expressions capture the *total* effect of the output subsidy on prices. They take into account that the subsidy also affects the choice of R&D by both firms in the second stage (and these, in turn, affect prices). This effect (through R&D) is reflected in the first two terms of the expression.

With these expressions we can rewrite $\frac{\partial \bar{B}^1}{\partial s^1}$:

$$\frac{\partial \bar{B}^1}{\partial s^1} = m^1\left(\frac{\partial x^1}{\partial p^2}\right)\bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + m^1\left(\frac{\partial x^1}{\partial p^2}\right)\bar{\psi}^2_{s^1} - s^1\left[\frac{\partial x^1}{\partial p^1}\frac{\mathrm{d}\bar{\psi}^1}{\mathrm{d}s^1} + \frac{\partial x^1}{\partial p^2}\frac{\mathrm{d}\bar{\psi}^2}{\mathrm{d}s^1}\right]$$

This is expression (60) in the main text.

F Proof of Lemma 6

Before proving the statement of the lemma, we need to derive the restrictions on ϕ^i_{11} implied by the second order condition (52) and the stability condition (54).

From the definition of $\bar{\pi}^i_{\Delta^i\Delta^i}$ in (93), in order to satisfy the second order condition $\bar{\pi}^i_{\Delta^i\Delta^i} < 0$ we need to ensure

$$\phi^i_{11} > \frac{\left(\Pi^i_{ii}\right)^3\left(\hat{C}^i_{p^i\Delta}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2 - \left(\Pi^i_{ij}\right)^2\right)^2}\left[\gamma^2 - \frac{1}{4}\gamma^4 - 1\right] = \theta\bar{\pi}^1_{\Delta^1 s^1} > 0 \tag{109}$$

where $\theta = -\frac{\hat{C}^i_{p^i\Delta}}{\frac{\partial x^i}{\partial p^i}} = -\frac{\partial^2 C^i(x^i,\Delta^i)}{\partial\Delta^i\partial x^i}$ measures how fast marginal costs are reduced per unit of R&D.

On the other hand, the stability condition in (54) translates into:

$$\begin{aligned}\frac{\bar{\pi}^i_{\Delta_i\Delta_i}}{\bar{\pi}^i_{\Delta_i\Delta_j}} &= \frac{\frac{\left(\Pi^i_{ii}\right)^3\left(\hat{C}^i_{p^i\Delta}\right)^2}{\left(\left(\Pi^i_{ii}\right)^2-\left(\Pi^i_{ij}\right)^2\right)^2}\left[\gamma^2 - \frac{1}{4}\gamma^4 - 1\right] - \phi^1_{11}}{\frac{\left(\hat{C}^i_{p^i\Delta}\right)^2\left(\Pi^i_{ii}\right)^3}{\left(\left(\Pi^i_{ii}\right)^2-\left(\Pi^i_{ij}\right)^2\right)^2}\left[-\frac{1}{2}\gamma + \gamma - \frac{1}{4}\gamma^3\right]} > 1 \\ &= \frac{\left[\gamma^2 - \frac{1}{4}\gamma^4 - 1\right]}{\left[\frac{1}{2}\gamma - \frac{1}{4}\gamma^3\right]} - \frac{\phi^i_{11}}{\bar{\pi}^i_{\Delta_i\Delta_j}} > 1 \end{aligned} \tag{110}$$

using (102):

$$\begin{aligned}\phi^i_{11} + \bar{\pi}^i_{\Delta_i\Delta_j} &> \bar{\pi}^i_{\Delta_i\Delta_j}\frac{\left[\gamma^2 - \frac{1}{4}\gamma^4 - 1\right]}{\left[\frac{1}{2}\gamma - \frac{1}{4}\gamma^3\right]} \\ \phi^i_{11} &> \theta\bar{\pi}^1_{\Delta^1 s^1} - \bar{\pi}^i_{\Delta_i\Delta_j} > 0\end{aligned} \tag{111}$$

Since $\pi^i_{\Delta_i\Delta_j} < 0$, then only (111) is binding..

From the definition of $\frac{\mathrm{d}\Delta 1}{\mathrm{d}s^1}$ and $\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}$ (98), (99) and the identities (103), (101), (100) and (102) we have

$$\begin{aligned}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} &= \frac{-\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^2_{\Delta^2 s^1}}{\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} = \frac{\left(-\theta\bar{\pi}^1_{\Delta^1 s^1} + \phi^1_{11}\right)\bar{\pi}^1_{\Delta^1 s^1} + \frac{\left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2}{\theta}}{\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} \\ &= \frac{1}{\theta}\frac{\left(-\left(\theta\bar{\pi}^1_{\Delta^1 s^1}\right)^2 + \phi^1_{11}\theta\bar{\pi}^1_{\Delta^1 s^1} + \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)}{\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} > 0\end{aligned} \tag{112}$$

$$\begin{aligned}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} &= \frac{-\bar{\pi}^i_{\Delta^i\Delta^i}\bar{\pi}^2_{\Delta^2 s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^1_{\Delta^1 s^1}}{\left(\bar{\pi}^i_{\Delta^i\Delta^i}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} = \frac{\left(-\theta\bar{\pi}^1_{\Delta^1 s^1} + \phi^1_{11}\right)\frac{\bar{\pi}^i_{\Delta^i\Delta^j}}{\theta} + \bar{\pi}^i_{\Delta^i\Delta^j}\bar{\pi}^1_{\Delta^1 s^1}}{\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} \\ &= \frac{1}{\theta}\frac{\phi^1_{11}\bar{\pi}^i_{\Delta^i\Delta^j}}{\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2} < 0 \end{aligned} \tag{113}$$

Notice that all the terms in the expressions above do not depend on ϕ^1_{11} except, of course ϕ^1_{11}. Taking the derivative with respect to ϕ^1_{11}

$$\begin{aligned}\frac{\partial\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}}{\partial\phi^1_{11}} &= \frac{1}{\theta}\frac{\theta\bar{\pi}^1_{\Delta^1 s^1}\left[\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right] + 2\left(-\left(\theta\bar{\pi}^1_{\Delta^1 s^1}\right)^2 + \phi^1_{11}\theta\bar{\pi}^1_{\Delta^1 s^1} + \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)}{\left(\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)^2} \\ &= \frac{1}{\theta}\frac{-\left(\theta\bar{\pi}^1_{\Delta^1 s^1}\right)^3 + \theta\bar{\pi}^1_{\Delta^1 s^1}\left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2 + 2\phi^1_{11}\left(\theta\bar{\pi}^1_{\Delta^1 s^1}\right)^2 - \left(\phi^1_{11}\right)^2\theta\bar{\pi}^1_{\Delta^1 s^1} - 2\phi^1_{11}\left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2}{\left(\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)^2} \\ &= -\frac{1}{\theta}\frac{\left(\theta\bar{\pi}^1_{\Delta^1 s^1}\right)\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 + \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\left(2\phi^1_{11} - \theta\bar{\pi}^1_{\Delta^1 s^1}\right)}{\left(\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)^2} < 0 \end{aligned} \tag{114}$$

$$\begin{aligned}\frac{\partial\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}}{\partial\phi^1_{11}} &= \frac{1}{\theta}\frac{\bar{\pi}^i_{\Delta^i\Delta^j}\left[\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right] + 2\phi^1_{11}\bar{\pi}^i_{\Delta^i\Delta^j}\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)}{\left(\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)^2} \\ &= \frac{1}{\theta}\frac{\bar{\pi}^i_{\Delta^i\Delta^j}\left[\left(\theta\bar{\pi}^1_{\Delta^1 s^1}\right)^2 - 2\theta\phi^1_{11}\bar{\pi}^1_{\Delta^1 s^1} + \left(\phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right] + 2\phi^1_{11}\bar{\pi}^i_{\Delta^i\Delta^j}\theta\bar{\pi}^1_{\Delta^1 s^1} - 2\phi^1_{11}\bar{\pi}^i_{\Delta^i\Delta^j}\phi^1_{11}}{\left(\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)^2} \\ &= \frac{1}{\theta}\frac{\bar{\pi}^i_{\Delta^i\Delta^j}\left[\left(\theta\bar{\pi}^1_{\Delta^1 s^1}\right)^2 - \left(\phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right]}{\left(\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)^2} \\ &= -\frac{1}{\theta}\frac{\bar{\pi}^i_{\Delta^i\Delta^j}\left[\left(\phi^1_{11}\right)^2 - \left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \bar{\pi}^i_{\Delta^i\Delta^j}\right)\left(\theta\bar{\pi}^1_{\Delta^1 s^1} + \bar{\pi}^i_{\Delta^i\Delta^j}\right)\right]}{\left(\left(\theta\bar{\pi}^1_{\Delta^1 s^1} - \phi^1_{11}\right)^2 - \left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2\right)^2} \\ &> 0 \end{aligned} \tag{115}$$

where the inequalities are derived using (111). Since $\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1} > 0$ and $\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} < 0$, the statement of the proposition follows ■

G Proof of Proposition 7

Rewrite the optimal subsidy as

$$s^{1*} = m^1\left(\frac{\partial x^1}{\partial p^2}\right)\frac{\bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1} + \bar{\psi}^2_{s^1}}{\frac{\partial x^1}{\partial p^1}\frac{\mathrm{d}\bar{\psi}^1}{\mathrm{d}s^1} + \frac{\partial x^1}{\partial p^2}\frac{\mathrm{d}\bar{\psi}^2}{\mathrm{d}s^1}} \tag{116}$$

where $m^1 = p^1 - \frac{\partial C^1}{\partial x^1} + s^1 > 0$ is the gross benefit per unit sold, including the output subsidy. Of course, m^1 has to be positive (otherwise firm 1 would have negative profits).

Turn now to the sign of the denominator in (116). It is positive since

$$
\begin{aligned}
\frac{\partial x^1}{\partial p^1}\frac{\mathrm{d}\bar{\psi}^1}{\mathrm{d}s^1}+\frac{\partial x^1}{\partial p^2}\frac{\mathrm{d}\bar{\psi}^2}{\mathrm{d}s^1} &= \frac{\partial x^1}{\partial p^1}\left(\bar{\psi}^1_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}+\bar{\psi}^1_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\bar{\psi}^1_{s^1}+\left(\frac{\frac{\partial x^1}{\partial p^2}}{\frac{\partial x^1}{\partial p^1}}\right)\left(\bar{\psi}^2_{\Delta^1}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}+\bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\bar{\psi}^2_{s^1}\right)\right)\\
&= \frac{\partial x^1}{\partial p^1}\bar{\psi}^i_{\Delta^i}\left(\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}+\frac{\bar{\psi}^i_{\Delta^j}}{\bar{\psi}^i_{\Delta^i}}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\frac{\bar{\psi}^1_{s^1}}{\bar{\psi}^i_{\Delta^i}}-\gamma\left(\frac{\bar{\psi}^i_{\Delta^j}}{\bar{\psi}^i_{\Delta^i}}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}+\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\frac{\bar{\psi}^2_{s^1}}{\bar{\psi}^i_{\Delta^i}}\right)\right)\\
&= \frac{\partial x^1}{\partial p^1}\bar{\psi}^i_{\Delta^i}\left(\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}-\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}-\frac{\frac{\partial x^i}{\partial p^i}}{\hat{C}^i_{p^i\Delta}}-\gamma\left(-\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}+\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}-\frac{\bar{\psi}^1_{s^1}}{\bar{\psi}^i_{\Delta^i}}\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right)\right)\\
&= \frac{\partial x^1}{\partial p^1}\bar{\psi}^i_{\Delta^i}\left(\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}-\frac{\frac{\partial x^i}{\partial p^j}}{2\frac{\partial x^i}{\partial p^i}}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\frac{1}{\theta}-\gamma\left(-\frac{\frac{\partial x^i}{\partial p^j}}{2\frac{\partial x^i}{\partial p^i}}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}+\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\frac{\gamma}{2\theta}\right)\right)\\
&= \frac{\partial x^1}{\partial p^1}\bar{\psi}^i_{\Delta^i}\left(\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}+\frac{\gamma}{2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\frac{1}{\theta}-\gamma\left(\frac{\gamma}{2}\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}+\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\frac{\gamma}{2\theta}\right)\right)\\
&= \frac{\partial x^1}{\partial p^1}\bar{\psi}^i_{\Delta^i}\left(\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}\left(1-\frac{\gamma^2}{2}\right)-\frac{\gamma}{2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\frac{1}{\theta}\left(1-\frac{\gamma^2}{2}\right)\right)>0 \qquad (117)
\end{aligned}
$$

where the inequality comes from $\frac{\mathrm{d}\Delta^1}{\mathrm{d}s^1}>0>\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}$ (Proposition 4) and $\left(1-\frac{\gamma^2}{2}\right)>\frac{\gamma}{2}>0$ for γ between zero and one.

Therefore the sign of s^{1*} is the same as the sign of $\bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\bar{\psi}^2_{s^1}$.

Using equation (43) we have:

$$\bar{\psi}^2_{s^1}=\bar{\psi}^1_{s^1}\frac{\mathrm{d}p^2}{\mathrm{d}p^1} \qquad (118)$$

Recall that

$$\bar{\psi}^2_{\Delta^2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\bar{\psi}^2_{s^1}=\bar{\psi}^i_{\Delta^i}\left(\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\frac{\bar{\psi}^2_{s^1}}{\bar{\psi}^i_{\Delta^i}}\right)=\bar{\psi}^i_{\Delta^i}\left(\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}-\frac{\bar{\psi}^1_{s^1}}{\bar{\psi}^i_{\Delta^i}}\frac{\Pi^i_{ij}}{\Pi^i_{ii}}\right)=\bar{\psi}^i_{\Delta^i}\left(\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\frac{\gamma}{2\theta}\right) \qquad (119)$$

Since $\bar{\psi}^i_{\Delta^i}$ is independent of ϕ^i_{11}, then a change in ϕ^i_{11} only affects $\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}$ (i.e. the R&D stage effect). From lemma 6, $\frac{\partial\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}}{\partial\phi^1_{11}}>0$ and so $\bar{\psi}^i_{\Delta^i}\left(\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\frac{\gamma}{2\theta}\right)$ in decreasing on ϕ^i_{11}. Left to show is that $\bar{\psi}^i_{\Delta^i}\left(\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\frac{\gamma}{2\theta}\right)$ can be positive or negative for permissible values of ϕ^i_{11}.

From (113) we have

$$\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}=\frac{1}{\theta}\frac{\phi^1_{11}\bar{\pi}^i_{\Delta^i\Delta^j}}{\left(\theta\bar{\pi}^1_{\Delta^1s^1}-\phi^1_{11}\right)^2-\left(\bar{\pi}^i_{\Delta^i\Delta^j}\right)^2}<0 \qquad (120)$$

And so $\lim_{\phi^1_{11}\nearrow\infty}\bar{\psi}^i_{\Delta^i}\left(\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\frac{\gamma}{2\theta}\right)=\bar{\psi}^i_{\Delta^i}\frac{\gamma}{2\theta}<0$ and $\lim_{\phi^1_{11}\searrow\left(\theta\bar{\pi}^1_{\Delta^1s^1}-\pi^i_{\Delta_i\Delta_j}\right)}\bar{\psi}^i_{\Delta^i}\left(\frac{\mathrm{d}\Delta^2}{\mathrm{d}s^1}+\frac{\gamma}{2\theta}\right)=+\infty$. By continuity, the claim of the proposition follows. ■

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