How Noisy should a Noisy Signal be: A Model of Bank Runs

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Abstract

In the literature on bank runs where depositors decide whether to withdraw early from the bank or not based on the noisy signals they receive about the future returns, a unique equilibrium is established with a threshold level below which depositor would withdraw. However, these papers assume precise information. In reality noise levels need not be very small. The information that is available to the depositors can be endogenised. This paper finds that to either minimise the probability of a bank-run or maximise the expected utility of the depositors, there should be high transparency of the banks' long term returns.

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1 Introduction

The literature on bank runs has gone through various developments over the last two decades. We have the Diamond and Dybvig (1983) model which says that bank runs are caused by self fulfilling beliefs of depositors. This model has two equilibria: the bank run equilibrium and the no bank run equilibrium. A crucial break through was made to establish unique equilibrium using the global game framework by Morris and Shin (1998) in a model of currency attacks. This paved the way to many models on bank runs looking at different issues, establishing a unique equilibrium where agents receive noisy signals about the fundamental and decide whether to withdraw or not because of self fulfilling beliefs.[1] The fundamental gives information about the long term earning potential of the bank and the proportion of early withdrawals, which in turn gives information about the long term returns of the agents.

The noise reveals how informative the signal is about the true value of the fundamental. Lower the noise, higher the level of informativeness and transparency of the signal. Upto now these models on bank runs assume precise information with the range of noise close to zero. In reality it is possible to choose the amount of information that is made available to the agents. Erland (2005) does an empirical analysis of bank runs and finds that transparency increases financial stability.

This paper attempts to find the optimal level of transparency of the banks'

[1] Morris and Shin(2002), Goldstein and Pauzner (2005 a, b), G. Selvaretnam (2005).

future return that should be chosen for financial stability and depositor welfare. There are papers which have modelled transparency using the global game set up for other issues, where self fulfilling beliefs play a role in the agents' decisions. Heinemann and Illing (2002) analyse how transparency affects the probability of speculative attacks on currency and find that the likelihood of attacks reduces with transparency. In a similar vein, Englmaier and Reisinger (2004) build a model analysing the decision to invest and find that transparency increases the chances of industrialisation.

The model has a continuum of agents who are endowed with one unit which is invested in a bank which operates in a competitive environment. A known fraction of the depositors are impatient, who are hit by a liquidity shock and have to withdraw early. The bank keeps reserves just sufficient to meet the demand of the impatient agents and invests the balance in a long-term project. If patient agents who are not hit by the liquidity shock want to withdraw early, the bank can get a loan which has to be repaid with an interest. Early withdrawers receive returns of one unit. The choice variables in this model are the noise level and the interest rate on the loan that is taken to meet the demand of early withdrawals.

First we look at the case where the decision regarding both these variables are in the hands of the authorities in charge of the financial system in the country. Hence the objective is to minimise the probability of bank runs. They can impose rules about how much information should be divulged to the depositors. We solve for the noise level that minimises the probability of bank runs. The next section determines the level of noise that maximises the expected utilty of the depositors, which is an obvious objective of banks which operate in a competitive market.

It is found that for both objectives, noise should be very small (i.e. high level of transparency). If the noise is very large, it means that the private information is of no value and therefore the depositors don't act on it. If there is to be information, it is better to make it as transparent as possible. It is also found that the bank should have access to loans at minimum cost (i.e. no interest) to meet the demand of patient agents who decide to withdraw because of self-fulfilling beliefs. This supports the concept of deposit insurance which is advocated by Sachs (1998), Radelet and Sachs (1998).

The rest of the paper is organised as follows: In the next section, the model is set up, followed by section 3 where the unique equilibrium is established. Section 4 looks at the results when the probability of bank runs is minimised while section 5 looks at the results when the expected utility of the agents is maximised and section 6 concludes.

2 The Model

There are three periods (t_0, t_1, t_2) and a continuum [0,1] of agents who are the depositors, and one bank operating in a competitive environment. Each agent is endowed with one unit at the beginning of t_0 which is invested in the bank. Consumption happens only in periods t_1 and t_2. All agents are identical and risk averse, and each agent's utility funcion is strictly concave, increasing and twice continuously differentiable.

A fraction λ of the agents are hit by a liquidity shock in t_1, which requires them to definitely withdraw early. They are referred to as the impatient agents and those who are not hit by the liquidity shock are called the patient agents. If the agents decide to withdraw in t_1 they will receive an early return of 1. The depositors might also want to withdraw early because they believe that if they don't, they might end up losing their investment in the bank because sufficiently large number of agents decide to withdraw early.

The bank keeps λ as reserves to meet the demand of impatient agents and invests the money that is deposited in a long term project. Each unit that has been invested till t_2 realises a random return θ.

If the patient agents want to withdraw early in t_1, the bank borrows from an outside party what is needed. This loan has to be settled with interest, so that each unit that is borrowed will have to be repaid with $L\,(\geq 1)$. Because the loan will be from an institution which has the welfare of the financial system in consideration, it is assumed that the bank will receive all the money that is needed.

Those who do not withdraw in t_1 will have to share equally what is left of the earnings from the long term project after the loan is settled (if there is anything left to share) in t_2.

We assume that the economic fundamental θ is uncertain and is drawn from a uniform distribution on $[\underline{\theta}, \overline{\theta}]$ where $\underline{\theta} = 0$ and $\overline{\theta}$ very large. In t_1 each agent i observes a noisy signal $\theta_i = \theta + \epsilon_i$ of the economic fundamental θ. The noise ϵ_i is uniformly and independently distributed among the depositors with support $(-e, +e)$.

Once the agents observe the signal, they will decide whether to withdraw in t_1 or wait till t_2. This decision is based on their beliefs about θ and the number of agents who would withdraw in t_1. A threshold θ^* can be established where, a player withdraws if and only if he observes a θ_i less than θ^* in t_1. Lower the θ^*, lower the probability of a bank run.

The crucial point of this paper is that the range of the noise level, $e,$ is a choice variable that can be chosen by the bank or the regulators. Lower the noise, e, higher the transparency or information available to the agents about their long term return.

Keeping with this strand of literature it is assumed that the fundamental θ has an upper dominant region and lower dominant region so that a unique equilibrium can be established. If θ was such that no patient agent withdraws, the return to the agent by not withdrawing is θ. If θ is sufficiently low such that $u(\theta) < u(1)$, it is better to withdraw early even if no other agent withdraws. If player i's signal is $\theta_i < 1 - e$, he will definitely withdraw.

On the other hand there could be a range of θ which is so high that even if everyone else withdraws it is better for an agent not to withdraw. If everyone else withdraws, by waiting he will receive $u(\theta - L)$. If $u(\theta - L) > u(1)$ the agent is better off by not withdrawing. which means that if the signal he receives is $\theta_i > 1 + L + e$ he will definitely not withdraw. When computing θ^* we only consider the range $[\theta^* - e, \theta^* + e]$ and assume that the dominant regions are extreme enough.

2.1 Threshold level θ^*

We can compute a threshold level of θ^* where if any player observes a value less than θ^* in t_1 he will withdraw. Once the economic fundamental θ is realised, each player i receives a signal $\theta_i = \theta + \epsilon_i$. The strategy for player i can be mapped out as;

$$s_i : \left[\underset{\sim}{\theta} - e, \widetilde{\theta} + e\right] \rightarrow \{\text{withdraw, not withdraw}\},$$

We consider threshold strategies and set out the conditions for θ^* to be a symmetric equilibrium. The threshold strategy for each player would be

$$\begin{aligned} s_i &= \text{withdraw} \quad \text{if } \theta_i < \theta_i^* \\ &= \text{not withdraw} \quad \text{if } \theta_i > \theta_i^*. \end{aligned}$$

Symmetric threshold strategy would mean $\theta_i^* = \theta^*$ for every player i.

If $\theta_i > \theta^*$, agent i believes that the bank's investment is doing sufficiently well and large enough proportion of the depositors believe the same and would not withdraw. Whereas if $\theta_i < \theta^*$, agent i believes that sufficiently high proportion of depositors believe (as he) that they should withdraw because they feel that they would lose out if they waited.

Proposition 1 *There exists a threshold level of the fundamental θ^* such that a patient agent will withdraw if and only if he observes a signal less than θ^*.*

Proof. Let the agents withdraw early if they receive a signal less than $\widehat{\theta}$.

Player i who observes signal θ_i has a posterior distribution of θ that is given by $y\,(=\theta/\theta_i)$. We know that y is then uniformly distributed on $[\theta_i - e, \theta_i + e]$ where each of the points is realised with equal probability $\frac{1}{2e}$. In turn he will believe that each of the point $y \in [\theta_i - e, \theta_i + e]$ would have given out signals to the other agents $(y - e, y + e)$ meaning the proportion of patient agents who he believes would withdraw would be a distribution given by $\widetilde{\omega}\,(y) \in [0, 1]$:

$$\begin{aligned}
y \leq \widehat{\theta} - e, &\qquad \widetilde{\omega} = 1. \\
\widehat{\theta} - e < y < \widehat{\theta} + e, &\quad \widetilde{\omega} = \frac{\widehat{\theta} - (y - e)}{2e}. \\
y \geq \widehat{\theta} + e, &\qquad \widetilde{\omega} = 0.
\end{aligned}$$

If a patient agent who observes $\theta_i = \theta^*$ withdraws in t_1 he will definitely receive one unit.

If the agent waits till t_2 will receive either nothing or,

$$\frac{\lambda - \lambda + (1 - \lambda)\, y - (1 - \lambda)\, \widetilde{\omega} L}{(1 - \lambda)\,(1 - \widetilde{\omega})} = \left(\frac{y - \widetilde{\omega} L}{1 - \widetilde{\omega}} \right).$$

The agent will never receive a negative return.

If $y < \widetilde{\omega} L \left(\text{i.e. } y < \frac{(\widehat{\theta}+e)L}{L+2e} \right)$, he believes he will receive nothing in the last period.

If $y > \widetilde{\omega} L \left(\text{i.e. } y > \frac{(\widehat{\theta}+e)L}{L+2e} \right)$, he believes he will receive $\frac{y-\widetilde{\omega}L}{1-\widetilde{\omega}}$ in the last period.

The difference in expected utility from withdrawing and not withdrawing, given signal θ_i is given by:

$$g\left(\theta_i, \widehat{\theta}\right) = EU(withdraw/_{\theta_i}) - EU(not\ withdraw/_{\theta_i}).$$

$$g\left(\theta_i\right) = u\left(1\right) - \int_{\frac{(\widehat{\theta}+e)L}{L+2e}}^{\theta_i+e} \frac{1}{2e} u\left(\frac{y - \widetilde{\omega}\left(y\right) * L}{1 - \widetilde{\omega}\left(y\right)}\right) dy. \tag{1}$$

$g : R \rightarrow R.$

It is clear that $\underset{\theta_i \rightarrow \underline{\theta}}{\text{Lim}} g > 0$ and $\underset{\theta_i \rightarrow \overline{\theta}}{\text{Lim}}\ g < 0$.

Over the range of the integral, $u(.)$ is non negative.

$$\frac{dg}{d\theta_i} = -\frac{1}{2e} u\left(\frac{\theta_i + e - \frac{\widehat{\theta}-\theta_i}{2e} * L}{1 - \frac{\widehat{\theta}-\theta_i}{2e}}\right). \tag{2}$$

Because $g\left(.\right)$ is continuous in θ_i and decreasing we can conclude that there exists a unique point where $g\left(\theta^*\right) = 0$, so that the agent who receives a signal $\theta_i = \theta^*$ will be indifferent between withdrawing and not withdrawing early. ■

3 Minimising the Probability of Bank Runs

The authorities who are interested in the financial stability of the country would be concerned about minimising the probability of bank runs. If they can determine the level of noise and the interest that has to be paid to the lenders, it is reasonable to assume that they would choose noise, e and loan repayment, L to minimise θ^*. This however, may or may not be in the best interest of the depositors. We look at the optimal choices which maximise the expected utility of the depositors in the next section.

Proposition 2 below is central to this paper which says that transparency reduces the probability of bank runs. This is supported by the empirical analysis in Erland (2005).

Proposition 2 *The probability of a bank-run is minimised when the noise level e is at a minimum.*

Proof. The agent is indifferent between withdrawing and not withdrawing when he receives a signal $\theta_i = \theta^*$.

We use equation 1 which gives the indifference condition:

$$h\left(\theta^*\left(e, L\right)\right) = 1 - \frac{1}{2e}\int_{\frac{(\theta^*+e)L}{L+2e}}^{\theta^*+e} u\left(\frac{y - \widetilde{\omega}\left(y\right) * L}{1 - \widetilde{\omega}\left(x\right)}\right) dy. \tag{3}$$

Keep in mind that $\widetilde{\omega} = 0$ when $\theta = \theta^* + e$; $\widetilde{\omega} = 1$ when $\theta = \theta^* - e$ and $\widetilde{\omega} = \frac{\theta^*+e}{2e+L}$ when $\theta = \frac{(\theta^*+e)L}{2e+L}$.

$$\frac{\partial h}{\partial e} = -\frac{2e * u\left(\theta^* + e\right) - 2\int_{\frac{(\theta^*+e)L}{L+2e}}^{\theta^*+e} u\left(\frac{y-\widetilde{\omega}(y)*L}{1-\widetilde{\omega}(y)}\right) dy}{4e^2}. \tag{4}$$

When $y = \theta^* + e$, $u\left(\frac{y-\widetilde{\omega}(y)*L}{1-\widetilde{\omega}(y)}\right) = u\left(\theta^* + e\right)$.

Therefore we can say that $2e * u(\theta^* + e) < 2 \int_{\frac{(\theta^*+e)L}{L+2e}}^{\theta^*+e} u\left(\frac{y-\widetilde{\omega}(y)*L}{1-\widetilde{\omega}(y)}\right) dy$ as long as e is sufficiently small. This means,

$$\frac{\partial h}{\partial e} > 0.$$

$$\frac{\partial h}{\partial \theta^*} = -\frac{1}{2e} u\left(\frac{\theta^* + e - 0 * L}{1 - \widetilde{\omega}(\theta^* + e)}\right) * 1 + \frac{1}{2e} u\left(\frac{\frac{(\theta^*+e)L}{L+2e} - \frac{\theta^* - \frac{(\theta^*+e)L}{L+2e} + e}{2e} * L}{1 - \frac{\theta^* - \frac{(\theta^*+e)L}{L+2e} + e}{2e}}\right) * \frac{L}{L+2e}. \tag{5}$$

This reduces to,

$$\frac{\partial h}{\partial \theta^*} = -\frac{1}{2e} u(\theta^* + e) < 0. \tag{6}$$

Therefore when e is small enough

$$\frac{d\theta^*}{de} > 0.$$

Therefore to minimise θ^*, noise level e should be very low. When there is nearly full transparency ($e \to 0$), the probability of bank run is minimised. ■

If noise is very small, the signal each agent receives is very close to the true value. However, if the authorities are unable to have much transparency, should we have large noise? If the noise is very large, depositors can't learn anything from their private signals and therefore would not consider it when making decisions. Because it is difficult to prevent some information floating around, this model recommends that there should be very clear transparence of information about the bank's long term return (i.e. very small level of noise) if probability of bank runs is to be minimised.

Proposition 3 below says that a bank run is minimised when $L = 1$. This means that the banks should have interest-free loans to meet the demand of self fulfilling withdrawals. This result is quite obvious and trivial. Lower the amount that should be repaid to the lender, higher the returns for those who wait. Therefore to minimise the probability of a bank run, L should be as less as possible. In this case, it will be $L = 1$ so that the banks should have access to interest-free loans to meet the demand of self-fulfilling withdrawals.

This supports the concept of deposit insurance shown in Allen and Gale (1998), Sachs (1998), Radelet and Sachs (1998) where there is a lender of last resort, which would prevent self fulfilling bank runs. However, there are other papers, Cooper and Ross (2002), Schwartz (1998), Calomoris (1998), which warn that having deposit insurance schemes or lender of last resort would give rise to moral hazard where the banks would be irresponsible in their investment. So, to have the advantage of the interest free loan, the banks need to be regulated so that they keep reserves that are at least sufficient for the impatient withdrawers.

Proposition 3 *The probability of a bank run is minimised when* $L = 1$*.*

Proof. We use the indifference condition in equation 3.

$$\frac{d\theta^*}{dL} = -\frac{\frac{\partial h}{\partial L}}{\frac{\partial h}{\partial \theta^*}}.$$

We know that $\frac{\partial h}{\partial \theta^*} < 0$.

$$\frac{\partial h}{\partial L} = +u\left(\frac{\frac{(\theta^*+e)L}{L+2e} - \frac{\theta^* - \frac{(\theta^*+e)L}{L+2e} + e}{2e} * L}{1 - \frac{\theta^* - \frac{(\theta^*+e)L}{L+2e} + e}{2e}}\right) * \frac{2e\left(\theta^* + e\right)}{\left(L + 2e\right)^2} - \int_{\frac{(\theta^*+e)L}{L+2e}}^{\theta^*+e} u'\left(.\right)\left(-\frac{\widetilde{\omega}\left(y\right)}{1 - \widetilde{\omega}\left(y\right)}\right) dy. \tag{7}$$

This reduces to,

$$\frac{\partial h}{\partial L} = -\int_{\frac{(\theta^*+e)L}{L+2e}}^{\theta^*+e} u'\left(\frac{y - \widetilde{\omega}\left(y\right) * L}{1 - \widetilde{\omega}\left(y\right)}\right)\left(-\frac{\widetilde{\omega}\left(y\right)}{1 - \widetilde{\omega}\left(y\right)}\right) dy > 0. \tag{8}$$

Therefore $\frac{d\theta^*}{dL} > 0$. ■

4 Maximising the Expected Utility of Agents

In this section we find the results when the objective is to maximise the expected utility of the depositors. The bank would want to do that because it operates in a competitive market. The expected utility of an agent is given by equation 9.

$$EU = \frac{1}{\overline{\theta} - \underline{\theta}}\left(\begin{array}{c} \lambda u\left(1\right) + \\ \left(1-\lambda\right)\left(\begin{array}{c} \int_{\underline{\theta}}^{\theta^*-e} u\left(1\right) d\theta + \int_{\theta^*+e}^{\overline{\theta}} u\left(\frac{\theta}{1-\lambda}\right) d\theta \\ + \int_{\theta^*-e}^{\theta^*+e}\left(\omega\left(\theta\right) * u\left(1\right)\right) d\theta + \\ \left(\begin{array}{c} \int_{\theta^*-e}^{\frac{(\theta^*+e)L}{2e+L}}\left(1 - \omega\left(\theta\right)\right) * 0 d\theta \\ + \int_{\frac{(\theta^*+e)L}{2e+L}}^{\theta^*+e}\left(1 - \omega\left(\theta\right)\right) * u\left(\frac{\theta - \omega(\theta) * L}{(1-\lambda)1 - \omega(\theta)}\right) d\theta \end{array}\right) \end{array}\right) \end{array}\right). \tag{9}$$

The probability of θ being at any point is *ex ante* $\frac{1}{\bar{\theta}-\underline{\theta}}$. With λ probability the agent can be hit by the liquidity shock and have to withdraw early. In which case, he gets $u(1)$.

With $(1-\lambda)$ probability he will not be hit by the liquidity shock. The first term is when $\theta < \theta^* - e$, so that all the depositors would withdraw early in t_1 and receive early return of 1 for sure. The second term is when $\theta > \theta^* + e$, so that all the patient depositors would wait till t_2 and therefore the entire earning, θ, will be distributed among the patient depositors.

When θ is between $\theta^* - e$ and $\theta^* + e$ we have a partial run where the agent might have to either run or not run. There is $\omega(\theta)$ probability that an agent would run and $(1-\omega(\theta))$ probability that he would not run. If he does run he will receive utility of 1 unit. But if he does not run, he could either receive nothing (if too many depositors had withdrawn early) or $u\left(\frac{\theta-\omega L}{(1-\omega)(1-\lambda)}\right)$.

The proportion of agents who run can be looked at in three categories.

$\omega = 0$ if $\theta > \theta^* + e$. When θ is big enough no patient agent will run.

$\omega = 1$ if $\theta < \theta^* - e$ When θ is low enough everyone will run.

$\omega = \frac{\theta^* - \theta + e}{2e}$ if $\theta^* - e \leq \theta \leq \theta^* + e$. This is when there will be a partial run.

Now we come to the other key result in the paper. According to proposition 4 the expected utility of the agents is maximised if there is high transparency of information.

Proposition 4 *To maximise the expected utility of the depositors, the level of noise e should be minimised.*

Proof. The expected utility of an agent given in equation 9 can be rearranged as follows:

$$EU = \frac{\lambda}{\overline{\theta}-\underline{\theta}} u(1) + \frac{1-\lambda}{\overline{\theta}-\underline{\theta}} \begin{pmatrix} {}_{\underline{\theta}}\int^{\theta^*-e} u(1)\, d\theta + {}_{\theta^*+e}\int^{\overline{\theta}} u\left(\frac{\theta}{1-\lambda}\right) d\theta \\ + {}_{\theta^*-e}\int^{\theta^*+e} \omega(\theta)\, u(1)\, d\theta \\ + {}_{\frac{(\theta^*+e)L}{2e+L}}\int^{\theta^*+e} u\left(\frac{\theta-\omega(\theta)*L}{(1-\lambda)(1-\omega(\theta))}\right) d\theta + \\ - {}_{\frac{(\theta^*+e)L}{2e+L}}\int^{\theta^*+e} \omega(\theta)\, u\left(\frac{\theta-\omega(\theta)*L}{(1-\lambda)(1-\omega(\theta))}\right) d\theta \end{pmatrix}. \tag{10}$$

Detailed workings of $\frac{dEU}{de}$ is in the appendix.

$$\frac{dEU}{de} = \frac{1}{\overline{\theta}-\underline{\theta}} \left(\int_{\frac{(\theta^*+e)L}{2e+L}}^{\theta^*+e} \left(\frac{\theta-\theta^*}{2e^2}\right) \begin{pmatrix} \frac{u'(.)(\theta-L)}{(1-\lambda)(1-\omega(\theta))} \\ -u\left(\frac{\theta-\omega(\theta)*L}{(1-\lambda)(1-\omega(\theta))}\right) \end{pmatrix} d\theta \right). \tag{11}$$

Because $u(c) > cu\prime(.)$,

$$u\left(\frac{\theta-\omega(\theta)*L}{(1-\lambda)(1-\omega(\theta))}\right) > \left(\frac{\theta-\omega(\theta)*L}{(1-\lambda)(1-\omega(\theta))}\right) u'\left(\frac{\theta-\omega(\theta)*L}{(1-\lambda)(1-\omega(\theta))}\right).$$

Therefore,

$$u\left(\frac{\theta-\omega\left(\theta\right)*L}{\left(1-\lambda\right)\left(1-\omega\left(\theta\right)\right)}\right)>\left(\frac{\theta-L}{\left(1-\lambda\right)\left(1-\omega\left(\theta\right)\right)}\right)u'\left(\frac{\theta-\omega\left(\theta\right)*L}{\left(1-\lambda\right)\left(1-\omega\left(\theta\right)\right)}\right).$$

It is clear that $\frac{dEU}{de}<0$.

Therefore to maximise EU, noise e should be at a minimum. ■

When the agents have good information about the long term return, they can take an informed decision in t_1 whether to withdraw early or not. Partial runs are minimised. We have also seen that this would minimise the probability of bank runs as well which also contributes to the increase in expected utility.

The next proposition is not surprising, which says the less we give the lender, the better for the agents. This shows that an existence of an institution prepared to lend what is necessary to early withdrawers can reduce bank runs and be beneficial to depositors. It would probably may not even have to give out money in t_1 because people will not want to withdraw early any way.

Proposition 5 *Expected utility of the depositors is maximised when the repayable amount to the lender $L=1$.*

Proof. We use the expected utility as given in equation 9 above:

$$
\frac{dEU}{dL} = \frac{1-\lambda}{\overline{\theta}-\underline{\theta}} \begin{pmatrix} \frac{d\theta^*}{dL} - \frac{d\theta^*}{dL} u\left(\frac{\theta^*+e}{1-\lambda}\right) d\theta + 0 - \frac{d\theta^*}{dL} + \frac{d\theta^*}{dL} u\left(\frac{\theta^*+e}{1-\lambda}\right) d\theta - 0 \\ + \frac{(\theta^*+e)L}{2e+L} \int^{\theta^*+e} \frac{d}{dL}\left(\left(1-\omega\left(\theta\right)\right) u\left(\frac{\theta-\omega(\theta)*L}{(1-\lambda)(1-\omega(\theta))}\right)\right) d\theta. \end{pmatrix} \tag{12}
$$

$$
= \frac{1-\lambda}{\overline{\theta}-\underline{\theta}} \left(\frac{(\theta^*+e)L}{2e+L} \int^{\theta^*+e} \frac{d}{dL}\left(\left(1-\omega\left(\theta\right)\right) u\left(\frac{\theta-\omega\left(\theta\right)*L}{\left(1-\lambda\right)\left(1-\omega\left(\theta\right)\right)}\right)\right) d\theta. \right)
$$

The return that remains for late withdrawers, $\left(1-\omega\left(\theta\right)\right) u\left(\frac{\theta-\omega(\theta)*L}{(1-\lambda)(1-\omega(\theta))}\right)$, reduces when the L increases.

Therefore $\frac{dEU}{dL} < 0$ which means, to maximise expected utility, L should be at a minimum. ■

5 Conclusion

This paper has attempted to throw some light about the level of transparency that banks should have in the context of bank runs. When depositors are exposed to noisy signals about the future returns of a bank, the probability of bank runs and their expected returns depend on the level of noise. Most literature in this area of research assume noise to be very small. However in reality the level of noise differs depending on bank policies, bank regulators' policies, education level of agents which affect how much they can interpret the information they receive etc. Higher the noise level, higher the probability of partial runs. If the probability of a partial run is big enough, it should be taken into consideration in the analysis. This model recommends high transparency to maximise the expected utility of the depositors and to minimise the probability of bank runs.

A Appendix

$$EU=\frac{1}{\overline{\theta}-\underline{\theta}}\left(\begin{array}{c}\lambda u\left(1\right)+\\ \left(1-\lambda\right)\left(\begin{array}{c}\int_{\underline{\theta}}^{\theta^{*}-e}u\left(1\right)d\theta+{}_{\theta^{*}+e}\int^{\overline{\theta}}u\left(\frac{\theta}{1-\lambda}\right)d\theta\\ +\int_{\theta^{*}-e}^{\theta^{*}+e}\left(\omega\left(\theta\right)*u\left(1\right)\right)d\theta+\\ \left(\begin{array}{c}\int_{\theta^{*}-e}^{\frac{(\theta^{*}+e)L}{2e+L}}\left(1-\omega\left(\theta\right)\right)*0d\theta\\ \\ +\int_{\frac{(\theta^{*}+e)L}{2e+L}}^{\theta^{*}+e}\left(1-\omega\left(\theta\right)\right)*u\left(\frac{\theta-\omega(\theta)*L}{(1-\lambda)1-\omega(\theta)}\right)d\theta\end{array}\right)\end{array}\right)\end{array}\right).$$

Recall that ω is zero when $\theta=\theta^{*}+e$, and 1 when $\theta=\theta^{*}-e$, and $\frac{(\theta^{*}+e)}{2e+L}$ when $\theta=\frac{(\theta^{*}+e)L}{2e+L}$.

Keep in mind the following:

$$\frac{d\omega}{de}=\frac{\theta-\theta^{*}}{2e^{2}}.$$

$$\frac{d}{de}u\left(\frac{\theta-\omega\left(\theta\right)*L}{\left(1-\lambda\right)\left(1-\omega\left(\theta\right)\right)}\right)=\frac{u'\left(.\right)}{\left(1-\lambda\right)}\left(\frac{\left(1-\omega\right)\left(-L\right)\frac{d\omega}{de}+\left(\theta-\omega L\right)\frac{d\omega}{de}}{\left(1-\omega\right)^{2}}\right)$$

$$=\frac{u'\left(.\right)\left(\frac{\theta-\theta^{*}}{2e^{2}}\right)\left(\theta-L\right)}{\left(1-\lambda\right)\left(1-\omega\right)^{2}}.$$

$$\frac{d}{de}\omega u\left(\frac{\theta-\omega\left(\theta\right)*L}{\left(1-\lambda\right)\left(1-\omega\left(\theta\right)\right)}\right)=u\left(.\right)\frac{\theta-\theta^{*}}{2e^{2}}+\omega\frac{u'\left(.\right)\left(\frac{\theta-\theta^{*}}{2e^{2}}\right)L\left(\theta-1\right)}{\left(1-\lambda\right)\left(1-\omega\left(\theta\right)\right)^{2}}.$$

Therefore

$$\frac{dEU}{de} = \frac{1}{\overline{\theta} - \underline{\theta}} \begin{pmatrix} -1 - u\left(\frac{\theta^*+e}{1-\lambda}\right) + 0 + 1 + {}_{\theta^*-e}\int^{\theta^*+e} \frac{\theta-\theta^*}{2e^2} d\theta + u\left(\frac{\theta^*+e}{(1-\lambda)}\right) - 0 \\ + {}_{\frac{(\theta^*+e)L}{2e+L}}\int^{\theta^*+e} \frac{u'(.)\left(\frac{\theta-\theta^*}{2e^2}\right)(\theta-L)}{(1-\lambda)(1-\omega(\theta))^2} d\theta + 0 + 0 \\ - {}_{\frac{(\theta^*+e)L}{2e+L}}\int^{\theta^*+e} \left(u\left(.\right) \frac{\theta-\theta^*}{2e^2} + \omega \frac{u'(.)\left(\frac{\theta-\theta^*}{2e^2}\right)(\theta-L)}{(1-\lambda)(1-\omega(\theta))^2} \right) d\theta \end{pmatrix}. \quad (13)$$

$$\frac{dEU}{de} = \frac{1}{\overline{\theta} - \underline{\theta}} \begin{pmatrix} {}_{\theta^*-e}\int^{\theta^*+e} \frac{\theta-\theta^*}{2e^2} d\theta \\ + {}_{\frac{(\theta^*+e)L}{2e+L}}\int^{\theta^*+e} \frac{u'(.)\left(\frac{\theta-\theta^*}{2e^2}\right)(\theta-L)}{(1-\lambda)(1-\omega(\theta))^2} d\theta \\ - {}_{\frac{(\theta^*+e)L}{2e+L}}\int^{\theta^*+e} \left(u\left(.\right) \frac{\theta-\theta^*}{2e^2} + \omega \frac{u'(.)\left(\frac{\theta-\theta^*}{2e^2}\right)(\theta-L)}{(1-\lambda)(1-\omega(\theta))^2} \right) d\theta \end{pmatrix}. \quad (14)$$

$$\frac{dEU}{de} = \frac{1}{\overline{\theta} - \underline{\theta}} \left(\int_{\frac{(\theta^*+e)L}{2e+L}}^{\theta^*+e} \left(\frac{\theta-\theta^*}{2e^2}\right) \begin{pmatrix} \frac{u'(.)(\theta-L)}{(1-\lambda)(1-\omega(\theta))} \\ -u\left(\frac{\theta-\omega(\theta)*L}{(1-\lambda)(1-\omega(\theta))}\right) \end{pmatrix} d\theta \right). \quad (15)$$

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