Openness and Inflation*

Dudley Cooke†

University of Essex

Abstract: A general equilibrium model of a small open economy is developed to analyze the optimal rate of inflation under discretion. Once agents' welfare is the sole policy objective it is possible to show that openness and inflation no longer have a simple inverse relationship. A greater degree of openness may lead the policy maker to want to exploit the short-run Phillips curve more aggressively, even if involves a smaller short-run benefit, because changes in export demand affect the terms of trade. Inflation can then be higher in a more open economy.

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†Department of Economics, University of Essex. Wivenhoe Park, Colchester CO4 3SQ, Essex, UK. Email: dkcooke@essex.ac.uk

1. Introduction

Will inflation rise or fall as an economy becomes more open? One way to approach this question is through the time inconsistency problem of monetary policy.[1] Typically, the answer is that inflation is lower in an open economy because deteriorations in the terms of trade increase the costs of surprise monetary expansions. As an economy becomes more open, the terms of trade effect becomes stronger, the associated costs greater, and inflation lower.[2] However, this result is generated using a postulated loss function. I develop a general equilibrium model of a small open economy to analyze the optimal rate of inflation under discretion. When the maximization of agents' welfare is the sole policy objective it is possible to provide a simple explanation of why inflation may rise or fall as an economy becomes more open.

Determining the rate of inflation under discretion in a general equilibrium setting rests on the introduction of the costs and benefits of expansionary monetary policy. Previous work studying the welfare implications of monetary policy in open economies, such as Obstfeld and Rogoff (1995), Corsetti and Pesenti (2001) and Benigno (2002), does not incorporate an explicit welfare cost of current inflation. As a result it is not possible to generate a time-consistent (discretionary) equilibrium.[3] One contribution of this paper is to introduce such a welfare cost, in a relatively simple way, and so bring about the existence of a time-consistent equilibrium. This makes it possible to study the relationship between openness and inflation in detail.

In the analysis presented below, a direct cost to current inflation is introduced by assuming beginning-of-period real money balances directly enter the utility function. Households are then required to make a money holding decision before production and consumption decisions.[4] Inflation is costly because it reduces purchasing power. The welfare benefits

[1]For example, see Rogoff's (1985) Mundell-Fleming extension of Barro and Gordon's (1983) analysis.

[2]Romer (1993) and Lane (1997) provide a starting point for the empirical analysis of this relationship. Both of these studies suppose it is the inability of the government to commit to a policy action that determines the rate of inflation.

[3]Benigno (2002) can, in fact, generate a time-consistent equilibrium as a special case. The result relies on some specific values for the structural parameters of the model.

[4]As Nicolini (1998) emphasizes the beginning of period specification is consistent with the worker-

of inflation occur through increases in an inefficiently low level of output. This is captured by supposing that labor is supplied in a monopolistically competitive market and that there are one period nominal wage rigidities. The costs and benefits of surprise inflation are introduced in this way for two reasons. First, even in an open economy, it allows the derivation of an explicit expression for inflation without linear approximations. Second, without sacrificing internal consistency, the assumptions help capture the type of results found using reduced-form models, allowing for some interesting comparisons.

The results show that openness alters inflation via two main mechanisms. First, there is a standard effect which operates through the Phillips curve; when an economy is more open the slope of the Phillips curve is steeper, increasing the inflation cost and reducing the output gain from a surprise monetary expansion. There is a second effect present, however, because export demand interacts with openness and this affects the terms of trade. For a given degree of openness, an increase in export demand improves the terms of trade. This creates a larger incentive for the government to inflate because more favorable terms of trade imply a one unit increase in output leads to a bigger increase in consumption and a larger utility gain. As a result, export demand also influences the openness-inflation relationship. If export demand is sufficiently high, a greater degree of openness may lead the policy maker to exploit the short-run Phillips curve, even if it involves a smaller short-run benefit from doing so. In a rational expectations equilibrium, inflation can then be higher in a more open economy, in contrast to the standard result.

There is a related literature that focuses on discretionary monetary policy in a dynamic general equilibrium setting. Perhaps the most well known approach is Woodford's (2003) cashless linear-quadratic analysis, in which a cost to inflation is introduced through the use of a staggered pricing structure similar to Calvo (1983). Because only a fraction of firms are allowed to re-optimize their price in response to a shock, the costs of expansionary policy (or rather positive levels of inflation) are associated with relative price distortions across goods.[5] This approach has been extended to a two country framework by Clarida

shopper argument used to motivate both CIA and MIU models. The specification used here can also be thought of as implying a precautionary demand for money. This approach has been adopted in Danthine and Donaldson (1986), Neiss (1999) and most recently Perrson *et al.* (2006).

[5]This is also discussed at length in Clarida *et al.* (1999). King and Wolman (2004) provide a critique

et al. (1999, 2002), Benigno and Benigno (2003) and Pappa (2004) to incorporate terms of trade effects. In all of these papers the benefits of expansionary monetary policy occur because of monopolistic competition and nominal rigidities.

A second approach, more related to this paper, is taken by Albanesi *et al.* (2003). In this case, a cost to current inflation is introduced by assuming that households use previously accumulated cash to buy a subset of goods consistent with Svensson's (1985) timing change. Higher realized inflation then forces households to substitute towards non-cash goods, which lowers welfare. However, inflation also raises output because some prices are fixed. This reduces the monopoly distortion and raises welfare, generating a policy trade-off. Arseneau (2004) analyzes a two country model with a cash-in-advance constraint. Following Ireland (1997), he assumes there is an upper bound to the rate of money growth to generate a discretionary outcome. Interestingly, both Albanesi *et al.* and Arseneau show that there can be multiple discretionary Markov equilibria.[6] This paper shows that there is either an interior solution on no solution, depending on an interplay between openness and the monopoly distortion.

The remainder of the paper is organized as follows. In section two I describe the model economy. In section three I solve the model for an arbitrary rate of money growth. Section four computes the discretionary rate of inflation and looks in detail at it's relationship with openness. Section five concludes.

2. Model Economy

There are two economies - domestic and foreign. Both consist of a continuum of $j \in [0, 1]$ households which supply a differentiated labor type, hold real money balances and nominal bonds and consume domestic and foreign goods. Firms produce a single specialized output using labor as the input. The government controls the supply of money through lump-sum transfers. The foreign economy is assumed to be large relative to the domestic economy implying that the domestic economy takes financial conditions in the foreign economy as given and that domestic exports form a negligible component of the foreign

of the LQ approach in the context of discretionary monetary policy making.

[6] Multiple discretionary equilibria also arise in Armenter and Bodenstein's (2005) analysis of a model with financially constrained firms.

economy's consumption basket. Consumption, output and the nominal price of the domestic output are denoted with h-subscripts and for foreign consumption, output and prices f is used. Asterisks denote foreign economy variables.

2.1. Firms

Firms maximize profits, $\vartheta_t(j) = P_{h,t}y_t - \int_0^1 w_t(j)l_t(j)dj$, choosing amongst differentiated labor types, $l_t(j)$, subject to a constant elasticity of substitution production function,

$$y_t = \left(\int_0^1 l_t(j)^{(\sigma-1)/\sigma}dj\right)^{\sigma/\alpha(\sigma-1)} \tag{1}$$

where $w_t(j)$ is the jth individuals nominal wage, $P_{h,t}$ is the GDP deflator, y_t domestic output, $\alpha > 1$ measures the returns to scale in production, and $\sigma > 1$ measures the elasticity of input substitution. Conditional labor demand is,

$$l_t(j) = (w_t(j)/w_t)^{-\sigma}\, y_t^{\alpha} \tag{2}$$

where $w_t = \left(\int_0^1 w_t(j)^{1-\sigma}dj\right)^{1/(1-\sigma)}$ is the wage index. Since all households set the same wage in equilibrium, final labor demand is given by,

$$y_t = (\alpha w_t/P_{h,t})^{1/(1-\alpha)} \tag{3}$$

In an open economy, output depends on the GDP deflator. It is possible to set $\alpha = 1$, but with predetermined nominal wages, the GDP deflator will also be predetermined such that the domestic component of the inflation rate will be independent of monetary surprises.[7]

2.2. Households

Suppressing the j index, household utility is,

$$U_0 = \sum_{t=0}^{\infty}\beta^t\left[\ln C_t + \upsilon\left(m_t\right) - \phi l_t^{\kappa}/\kappa\right] \tag{4}$$

[7] In a closed economy $\alpha \neq 1$ is a necessary assumption for the overall price level not to be predetermined when there is nominal wage rigidity. In an open economy the overall price may vary through changes in the exchange rate.

where consumption, C_t, is Cobb-Douglas index of domestic and foreign goods; $C_{h,t}$ and $C_{f,t}$ respectively.

$C_t \equiv C_{h,t}^n C_{f,t}^{1-n} / n^n (1-n)^{1-n}$

The $\upsilon(m_t)$ term captures the liquidity services of real money balances, $m_t \equiv M_t/P_t$, where P_t is the consumer price index (CPI), and is assumed to be strictly increasing and strictly concave. Finally, l_t is labor supply. The parameter n is a measure of the degree of openness, $\beta \in (0,1)$ is the discount factor, $\phi > 0$ is a weight attached to the disutility of labor and $1/(1-\kappa)$ is the elasticity of labor supply, with $\kappa > 1$.

Households maximize utility choosing a sequence of nominal bond and domestic nominal money holdings, consumption, and a desired wage rate, subject to the sequence of constraints,

$$B_t + M_{t+1} = \vartheta_t + w_t l_t - P_{h,t} C_{h,t} - P_{f,t} C_{f,t} + T_t + B_{t-1}(1 + i_{t-1}) + M_t \tag{5}$$

and conditional labor demand, where B_t are bonds denominated in domestic currency which pay a nominal net rate of interest i_t and T_t are lump-sum transfers.

The optimal allocation of expenditure between domestic and imported goods is,

$$C_{h,t} = n P_t C_t / P_{h,t} \quad \text{and} \quad C_{f,t} = (1-n) P_t C_t / P_{f,t} \tag{6}$$

where $P_t \equiv P_{h,t}^n P_{f,t}^{1-n}$, and total consumption expenditures by domestic households are therefore $P_{h,t} C_{h,t} + P_{f,t} C_{f,t} = P_t C_t$. The following conditions also hold for all $t \geq 0$,

$$P_{t+1} C_{t+1} = P_t C_t \beta (1 + i_t) \tag{7}$$

$$w_t = \sigma \phi P_t C_t / (\sigma - 1) l_t^{1-\kappa} \tag{8}$$

$$\upsilon\prime(m_{t+1}) = C_{t+1} / i_t \tag{9}$$

where $\upsilon\prime(m_{t+1})$ is the derivative of $\upsilon(m_{t+1})$. Equation (7) is the standard consumption Euler equation. Equation (8) describes labor supply and includes the monopoly markup, $\sigma/(\sigma-1)$, and the CPI. Once a rigid nominal wage is assumed labor is demand determined in the short-run because agents are always willing to supply more labor given the existence of monopoly profits. Equation (9) expresses the demand for money. This expression is non-standard and reflects the assumption that money holdings are effectively chosen in period $t-1$, and so M_t is predetermined in period t. Expansionary policy raises the price level and this implies a utility cost in terms of forgone real balances.

The foreign economy is identical. But as it becomes infinitely large relative to the domestic economy the proportion of domestic goods in it's consumption basket diminishes. That is, $n^* \to 0$, so that $C_t^* \cong C_{f,t}^*$ and $P_t^* \cong P_{f,t}^*$, which is normalized to unity as it is exogenous. However, this does not mean export demand is zero since C_t^* itself is large. Defining the inverse terms of trade as $\rho_t \equiv P_{f,t}/P_{h,t}$, as the law of one price holds for the domestic good (i.e. $P_{h,t} = s_t P_{h,t}^*$), foreign consumption of domestic production is given by,

$$C_{h,t}^* = g_t^* \rho_t \tag{10}$$

The composite variable $g_t^* \equiv n^* C_t^*$ is a measure of export demand and is exogenous from the viewpoint of the domestic economy.[8]

2.3. Government

The government's budget constraint is given by the following.

$$T_t = M_{t+1} - M_t \tag{11}$$

Now all the constraints have been introduced it is clear how the beginning-of-period real money balances assumption works. Because period t nominal money holdings are predetermined, a monetary expansion in period t is not an increase in M_t, rather M_{t+1} increases, and this produces the increase in P_t. This explains how surprise inflation in period t unambiguously reduces current real money balances.

2.4. Equilibrium

[8]PPP does not hold since there is a form of consumption home bias, that is, $n \neq n^*$. Defining the consumption based real exchange rate as $q_t = s_t/P_t$, it is clear that $q_t = \rho_t^n$, so the less open the domestic economy, a given deterioration in the terms trade implies a greater real depreciation in the domestic currency.

The real side of the economy can be described by a simple supply and demand system. As perfect foresight is assumed, the labor market is in equilibrium for periods $t \geq 1$, despite the nominal wage rigidity. From (3) and (8) the supply of goods takes the form,

$$y_t = \left[\alpha\phi\sigma C_t \rho_t^{1-n} / (\sigma - 1)\right]^{1/(1-\kappa\alpha)} \tag{12}$$

and domestic output depends on consumption and the terms of trade.

The second condition that describes the real side of the economy is a goods market equilibrium condition. This is derived by combining the resource constraint, $y_t = C_{h,t} + C_{h,t}^*$, with the CPI and the conditions that describe the demand for domestic goods. From here on, the foreign economy is assumed to be in a zero inflation steady state such that $C_t^* = C^*$ for all t. This implies export demand is constant, i.e. $g_t^* = g^*$. In this case, goods market equilibrium is given by,

$$y_t = n\rho_t^{1-n} C_t + g^* \rho_t \tag{13}$$

for $t \geq 0$. Now an increase in g^* can also be viewed as an output shifter or an alternative openness parameter.

Finally, the national budget constraint can be written in the following way.

$$B_t - B_{t-1}(1 + i_{t-1}) = P_{h,t} y_t - P_t C_t \tag{14}$$

As usual, the end of period bond level is equal to domestic output minus the rate of absorption plus interest from claims on bonds.

3. Model Solution

To solve the model I show that the current account is zero for an arbitrary rate of money growth, μ_t. The logic is as follows. To understand how the current account behaves it first is necessary to pin-down the nominal exchange rate. Because it's evolution is governed by a UIP condition, a characterization of the domestic nominal interest rate is required. However, real money balances do not enter utility in a logarithmic manner, so the behavior of the nominal interest rate depends on the real interest rate and this, in turn, is affected by the rigid money wage.

3.1. Exchange Rate and Current Account

In periods $t \geq 1$ market clearing requires both (12) and (13) to hold. These conditions show that output is an implicit function of consumption and the terms of trade, such that, $\rho_t = \rho(C_t)$ and $y_t = y(C_t)$. Once we account for these implicit functions, combining the consumption Euler and real UIP conditions produces a self-contained first-order linear difference equation in the composite variable $C_t/\rho(C_t)$.

$$C_{t+1}/\rho(C_{t+1}) = C_t/\rho(C_t) \tag{15}$$

where $\beta = \beta^*$ is assumed to rule out the domestic economy becoming large over time. Equation (15) implies $C_t = C_{t+1}$ for $t \geq 1$ so that the consumption profile of the domestic economy is flat. This has the further implication that $\rho_t = \rho_{t+1}$, for $t \geq 1$ and since $y_t = y(C_t)$, that $y_t = y_{t+1}$ for $t \geq 1$. I denote these solutions C, y, ρ. As a result, the domestic real interest rate must be at its steady state value, $1 + r_t = 1/\beta$. This argument does not tie down the initial period real interest rate, r_0, since this depends on the rigidity in the labor market and under such conditions (15) fails to hold. But given the timing assumptions over nominal money holdings it is not necessary to determine the initial level of the real interest rate to solve the model.

The nominal interest rate for periods $t \geq 0$ can now be determined by transforming the money demand function into a difference equation in real money balances. The resulting expression also involves the endogenous real variables r_{t+1} and C_{t+1}. But since $1+r_{t+1} = 1/\beta$ and $C_{t+1} = C$ for all t the evolution of real money balances are not affected. This leads to the following condition,

$$m_{t+2}\left[1 + \upsilon'(m_{t+2})\, C\right] = m_{t+1}/\beta\mu_{t+1} \tag{16}$$

where $\mu_{t+1} \equiv M_{t+1}/M_{t+2}$ is defined as the inverse growth rate of the money stock. As m_{t+1} is non-predetermined, when money growth is constant at $\mu_t = \mu$, we need a saddle path condition to hold such that that real balances jump to their steady state value for periods $t \geq 1$. Differentiating (16) and evaluating this expression at the steady state,

$$\frac{dm_{t+2}}{dm_{t+1}} = \frac{1+i}{1 + i\left[1 + m\upsilon''(m)/\upsilon'(m)\right]}$$

This condition shows that following an unanticipated permanent change in the money supply the nominal interest rate must jump immediately to its steady state value, $1 + i_t = 1/\beta\mu$ for all t.[9] It is now apparent that the timing of households decisions over money balances implies the fixed money wage in period t does not affect the behavior of the nominal interest rate in period t. This rules out any exchange rate dynamics and determines the evolution of the exchange rate as $s_{t+1}/s_t = \mu$ for all t, but not the initial value.

To solve for s_0 explicitly the national intertemporal budget constraint is used. This is derived by solving (14) forward.

$$(1+i_{-1})\,B_{-1} = -\sum_{t=0}^{\infty}\left[s_t g^* - P_t C_t\,(1-n)\right]/\left[(1+i_0)\ldots(1+i_{t-1})\right] \tag{17}$$

where $[(1+i_0)\ldots(1+i_{t-1})] \equiv 1$ when $t = 0$. Ponzi games are ruled out so that the following condition also holds, $\lim_{t\to\infty} B_t/\left[(1+i_{t-1})\ldots(1+i_0)\right] = 0$. Setting the initial level of debt equal to zero, noting $1 + i_t = 1/\beta\mu$ holds in all periods, using the nominal consumption Euler equation (7), the nominal UIP condition, and accounting for initial conditions as $s_t = (1/\mu)^t s_0$ and $P_t C_t = (1/\mu)^t P_0 C_0$, it is possible to rewrite the right-hand side of (17) as $\beta\mu\left[P_0 C_0\,(n-1) + g^* s_0\right]/(1-\beta)$, because $\sum_{t=0}^{\infty}\beta^{t+1} = \beta/(1-\beta)$. This implies there is an explicit expression for the initial value of the nominal exchange rate.

$$s_0 = P_0 C_0 (1-n)/g^* \tag{18}$$

This condition holds for all periods, not just the current period. The time-invariance of (18) depends not only on the simplifying assumptions over consumer preferences (in particular, the Cobb-Douglas consumption index and unit intertemporal elasticity of substitution of total consumption) but it also requires export demand to be constant. Given the solution for the nominal exchange rate it is also possible to solve for the current account. Substituting the real version of (18) into the goods market equilibrium condition, (13), a zero current account condition holds in all periods. That is,

$$P_{h,t} y_t = P_t C_t \tag{19}$$

[9] Also see Obstfeld and Rogoff (1983) for a related analysis.

If the nominal interest rate is constant and at its steady state in all periods, and in the initial period the bond stock is assumed to be zero, in all future periods the bond stock will remain at zero. Thus a zero current account in the initial period implies a zero current account in all periods, and following a change in the money supply the economy does not run a current account imbalance. This result is central in obtaining a closed form solution to the model.[10]

3.2. Natural Rate of Output and Social Planning Problem

As there is a zero current account it is possible to derive an explicit solution for the natural rate of output, $y_t = \overline{y}\ \forall t \geq 1$, by appealing to the labor market equilibrium condition, equation (12).

Remark 1 *The natural rate of output is below the competitive level and depends on preferences and technology,* $\overline{y} = [(\sigma - 1)/\sigma\phi\alpha]^{1/\kappa\alpha}$. *The natural rate of output is independent of the openness parameter and export demand.*

The labor market becomes more competitive as $\sigma \to \infty$ ($\overline{y} \to (1/\phi\alpha)^{1/\kappa\alpha} \equiv y^{pc}$) and thus it might appear that the presence of monopolistic competition gives the government an incentive to increase output by making a larger than expected money transfer.[11] But this decision must depend on the optimal level of output. Since households have access to two traded goods it does not necessarily follow that the perfect competition and optimal levels of output coincide.

To examine this idea in more detail I setup the social planning problem for the economy. The social planners' constrained optimization problem (dropping t subscripts) is to choose C_h, C_f and l to maximize,

$$U = \ln\left[C_h^n C_f^{1-n}/n^n(1-n)^{1-n}\right] - \phi l^\kappa/\kappa \tag{20}$$

[10] Here the preference restrictions on the model required to derive this solution are somewhat more stringent than in Corsetti and Pesenti (2001) because PPP does not hold.

[11] In ad-hoc formulations, it is this feature that, in effect, creates the inflation bias.

such that,

$$
\begin{aligned}
y &= l^{1/\alpha} && (21)\\
y &= C_h + C_h^* && (22)\\
C_h^* &= \rho C_f && (23)\\
C_h^* &= \rho g^* && (24)
\end{aligned}
$$

Equation (20) is the utility function without the real money balances term (i.e. ignoring monetary frictions) and with the consumption sub-index replacing C, (21) is the production function, (22) is the resource constraint, (23) is a balanced trade condition and (24) is the export demand function. From (23) and (24), for any arbitrarily given value of g^* and C_f the only possible solutions for ρ and C_h^* are for them both to equal zero. Relaxing the assumption that C_f is given, $C_f = g^*$, but in this case (23) and (24) no longer uniquely determine ρ and C_h^* as functions of g^* and C_f. Proceeding with the idea that C_f is pegged to g^* it is possible to substitute foreign goods consumption, C_f, directly into the utility function and drop the final constraint (24). By imposing $C_h^* = \rho = 0$ the problem is reduced still further by the elimination of (24) and then C_h^* from the resource constraint. Substituting the remaining constraint into the maximand reduces (20)-(24) to a simple unconstrained problem of choosing C_h. The first-order condition is,

$$n - \phi\alpha C_h^{\alpha\kappa} = 0$$

Re-substituting the transformed resource constraint (i.e. $y = C_h$) into the first order condition gives the desired result.

Remark 2 *The solution to the social planners' problem depends on exogenous parameters and in particular on the degree of openness, n. The optimal output level is given by, $\widetilde{y} = (n/\phi\alpha)^{1/\kappa\alpha}$. The optimal output level does not depend on export demand.*

There are two important points concerning this result. First, although $\rho = 0$ is a special type of result which derives from the specific assumption made over consumer preferences, there is an intuitive explanation. Foreign consumption of the domestic good (i.e. exports) and the terms of trade do not affect domestic utility directly, and since negative values are excluded, from the viewpoint of the social planner, their optimal values are zero. The fact that it is feasible to drive these values to zero is a consequence of the particular unit-elastic demand functions implied by the model. In this case, domestic labor goes entirely into raising C_h as opposed to C_h^*, because only domestic consumption raises welfare. Under these conditions the social planning outcome is a corner solution.[12]

The second point turns out to be important when considering optimal monetary policy. In the open economy, when the household has a choice over two goods, the social planning problem and the perfectly competitive market outcome diverge. In a closed economy these values would be the same because taking $n \to 1$ in the planning outcome or taking $\sigma \to \infty$ in the market outcome both imply $y = (1/\phi\alpha)^{1/\kappa\alpha}$. The reason for this divergence follows similar lines to an optimal tariff argument. If a country is large enough in world markets it can impose a tariff on imports to alter the terms of trade, which increases welfare. The tariff reduces the overall volume of trade in the world and generates production and consumption costs, but by improving the terms of trade a moderate tariff can produce benefits that outweigh these costs. In trade theory the small open economy assumption usually implies that both the domestic and foreign prices of goods are taken as given by the domestic economy, but the Mundell-Fleming type assumptions employed here imply that the domestic economy has power over the domestic price level because it exports a specialized output. In a decentralized economy no individual private agent is able to affect the terms of trade despite monopolistic power over the wage rate, but the social planner does have this ability. The social planner therefore effectively coordinates the behavior of all private agents to the detriment of the foreign economy, increasing welfare by improving the terms of trade.[13]

4. Optimal Monetary Policy

Having solved the model for an arbitrary rate of money growth I now consider the optimal rate of money growth under discretion. I suppose the government and households play a one-shot game so there is no history dependence in the government's decision process.

[12] This result has a natural analogy with the profit maximizing problem of a monopolist that faces by a constant elasticity demand curve where the elasticity is less than one. In that case, it is optimal to set the price to infinity and the quantity sold to zero.

[13] Although the optimal tariff argument may seem a little strange in this context it is an application of a beggar-thy-neighbor argument stressed in the analysis of Tille (2001).

In this case, the government's choice over money growth is equivalent to a choice over real balances and the resulting equilibrium is a Markov equilibrium. The key results examined here are; how the rate of inflation varies with (i) export demand, (ii) openness and (iii) how the openness-inflation relationship depends on export demand.

4.1. Government's Optimization Problem

The governments constrained optimization problem (dropping t subscripts) is to choose m to maximize,

$$U = \ln C + \upsilon(m) - \phi l^{\kappa}/\kappa \tag{25}$$

such that,

$$\begin{aligned} C &= [(1-n)/g^*]^{n-1} y^n & (26)\\ y &= l^{1/\alpha} & (27)\\ l &= (m/m^e)^{\alpha/(n-\alpha)} \bar{l} & (28) \end{aligned}$$

with m^e given. Equation (26) is the zero current account condition, that is (19), combined with the solution for the exchange rate, (18). Equation (27) is the production function.

Equation (28) is the Phillips curve for the economy and is derived by introducing expectations into the labor demand equation.[14] The Phillips curve is defined in terms of expected real money balances, $m_t^e \equiv M_t/P_t^e$, not inflation. However, choosing a value of m_t^e is equivalent to choosing the inverse of the (fixed) money wage, w_t, because expected real balances are simply the expected price level normalized by the beginning of period (and therefore predetermined) nominal money supply. Deciding on w_t requires a forecast of the CPI so in this model m_t and m_t^e play the role of actual and expected inflation. Lower n, which is an increase in the degree of openness, reduces $1/(n-\alpha)$ in absolute value and thus a given reduction in m_t has a smaller effect on domestic output. This type of relationship is stressed in previous studies because, given the wage rigidity, the terms of trade affect the slope of the Phillips curve.

[14]The economy's Phillips curve is therefore similar to the new-classical Phillips curve, as only surprise inflation matters. The New Keynesian Phillips curve, which can be obtained when there is a staggered pricing structure, emphasizes the role of expected future inflation.

The solution to the governments problem describes a reaction function, which makes real balances an implicit function of expected real balances. Writing this out with real money balances on the left-hand side,

$$m\left[\frac{\phi\alpha}{n-\alpha}\widetilde{y}^{\alpha\kappa}+m\upsilon\prime(m)\right]^{(\alpha-n)/\alpha\kappa}=m^{e}\left[\frac{\phi\alpha}{n-\alpha}\overline{y}^{\alpha\kappa}\right]^{(\alpha-n)/\alpha\kappa} \tag{29}$$

Equation (29) also makes use of the definitions of the natural rate of output and the social planners' output level, given in the previous section. Solving for equilibrium real balances requires the households reaction function, which is given by the long-run Phillips curve. To obtain the equilibrium level of real balances I set $m=m^{e}=m^{d}$, which makes equilibrium real balances a function of the optimal and monopolistic levels of output.

$$m^{d}\upsilon\prime\left(m^{d}\right)=\frac{\phi\alpha}{n-\alpha}\left(\overline{y}^{\alpha\kappa}-\widetilde{y}^{\alpha\kappa}\right) \tag{30}$$

It is now clear that if the natural rate of output exceeds the optimal level, then the level of real money balances that obtains in equilibrium is undefined. The condition for a well defined equilibrium ($\widetilde{y}>\overline{y}$) can be expressed as $n>(\sigma-1)/\sigma\equiv\delta\in(0,1)$. Thus, because the optimal level of output is determined by the structural parameters of the model it cannot be guaranteed the openness-monopoly distortion ratio is greater than one and that the condition holds. In a closed economy, $n\rightarrow1$, and the socially optimal level of output is always greater than the monopolistic natural rate, providing a well defined level of equilibrium real balances.

To further understand the properties of the equilibrium I derive the following condition from the government's reaction function.

$$\frac{\partial m}{\partial y}\left(\frac{y}{m}\right)=\frac{\phi\kappa\alpha^{2}y^{\alpha\kappa}/\left(n-\alpha\right)}{\left[\upsilon\prime(m)+m\upsilon\prime\prime(m)\right]m}$$

As $\alpha>n$, $\upsilon(m)$ must be such that $\upsilon\prime(m)+m\upsilon\prime\prime(m)<0$, because, only in this case, will the marginal cost to inflating to be increasing in the discretionary rate of inflation. If, for example, $\upsilon(m)$ is logarithmic, the government's reaction function will be vertical in $(-\ln m,\ln y)$ space, and a rational expectations equilibrium will only occur as a special case. Intuitively, this is because the governments set of indifference curves will be linear displacements of one another, which can be seen by holding utility constant, using the constraint (26), and taking the derivative of (25) with respect to m. However, even if $v\prime(m) + mv\prime\prime(m) < 0$, $\overline{y} > \widetilde{y}$ is still a necessary condition for an equilibrium because otherwise the indifference curves will be negatively sloped in $(-\ln m, \ln y)$ space.[15] These two separate requirements are clear from the following representation of the optimization problem.

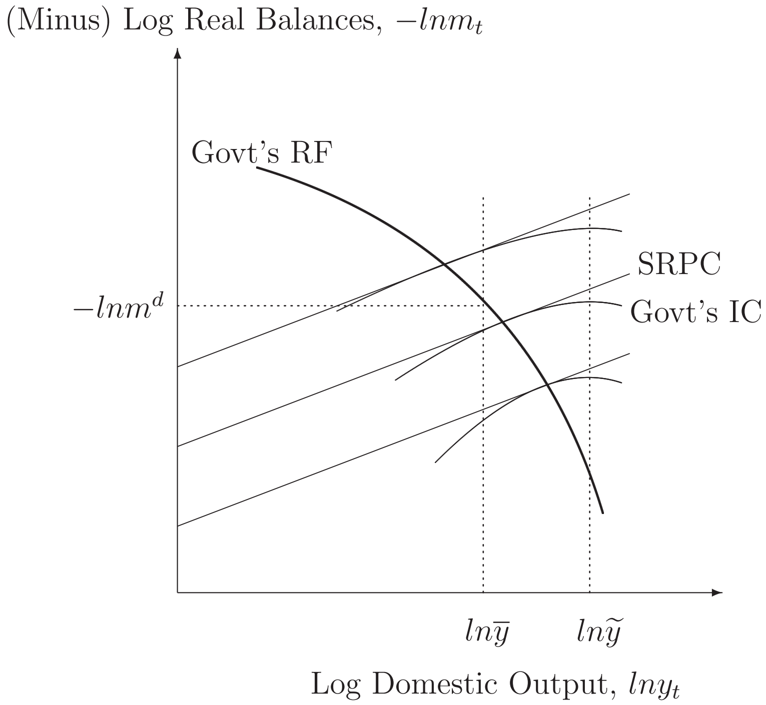

Figure 1: Government's Optimization Problem

The short-run Phillips curve in figure one only allows for tangency points on the positively sloped portion of the government's set of indifference curves, where the natural rate of output is below the socially optimal level. If the natural rate of output is above the social optimum the outcome of the game will not produce a feasible equilibrium. Second, the convex cost of holding nominal money (or rather of inflation) is also crucial for the equilibrium to be well defined. The special case in which the public and government reaction functions are vertical and coincide is similar to Beningo's (2002) analysis of the

[15]It should be clear that as $\alpha > n$ there are a family of positively sloped Phillips Curves in $(-\ln m, \ln y)$ space.

discretionary equilibrium. This case is also consistent with the ad-hoc Barro-Gordon model when the loss function is linear in inflation and quadratic in output.

4.2. Optimal Inflation

The final step in solving for the optimal rate of inflation is to equate equilibrium real balances with steady state real balances. In the steady state, real balances are stationary so $m_t = m$ and inflation is equal to the money growth rate such that $\mu\pi = 1$, where π denotes gross inflation. From the money demand function, this implies $\upsilon\prime(m) = (\pi - \beta)\overline{C}\beta$. Using (30), the discretionary rate of inflation, denoted π^d, can be written in the compact form,

$$\pi^d/\beta = 1 + \underbrace{\upsilon\prime\left(m^d\right)\overline{C}}_{\text{inflation bias}} \tag{31}$$

Inflation is split between the Friedman rule level of inflation and a bias term representing the incentives that drive the government to inflate the economy. The Friedman level of inflation in this economy is $\pi^d = \beta$ and is the outcome when the government has access to a commitment technology. The basic intuition for this result is that there is a wedge generated between the private and social marginal cost of holding money, and when $i > 0$, this generates an inefficiency. If there were no opportunity cost to holding money this inefficiency would disappear, but this requires that inflation equal the inverse of the real interest rate, which is given by β. In turn, the bias term is split between consumption and real money balances. The consumption-real money balances split is important because, as I will argue, the consumption term, which appears from the use of micro-foundations, plays a role in overturning many of the results in the reduced-form literature, whilst the real balances part captures the essence of this approach.

To make progress I adopt the following functional form.

$$\upsilon(m) = \frac{a\epsilon}{\epsilon - 1} m^{(\epsilon-1)/\epsilon}$$

where a is the weight on real balances in utility and ϵ captures the elasticity of substitution between consumption and real money balances. The parameter ϵ also controls the convexity of the inflation cost and as $\epsilon \to 1$ the cost becomes linear such that it is optimal for the government to raise the growth rate of money ever higher. To generate the discretionary equilibrium $v\prime(m) + mv\prime\prime(m) < 0$ requires $\epsilon < 1$.[16] I now rewrite inflation in terms of exogenous parameters only.

$$\pi^d = \beta + a\beta \underbrace{[(n-\delta)/a(\alpha-n)]^{1/(1-\epsilon)}}_{\equiv (m^d)^{-1/\epsilon}} \underbrace{\left[(\delta/\phi\alpha)^{n/\kappa\alpha}\right][(1-n)/g^*]^{n-1}}_{\equiv \overline{C}} \tag{32}$$

This makes clear that the equilibrium rate of inflation is a potentially complicated function of the degree of openness. There are three basic mechanisms through which openness affects inflation. The first term in square brackets represents the effect of changes in real money balances on utility, the second represents leisure and the final term the effect through trade-weighted export demand and the terms of trade.

4.2.1. Export Demand

An important preliminary result relates to the impact of export demand on inflation. It is clear that higher export demand leads to a higher rate of inflation, for a given degree of openness. The intuition for this result is straightforward because export demand only enters equation (32) through it's impact on the terms of trade. Using the solution for the exchange rate and the natural rate of output, export demand can be shown to have a positive relationship with the terms of trade.

$$\rho = (1-n)(\delta/\phi\alpha)^{1/\kappa\alpha}/g^*$$

where ρ is the inverse terms of trade.[17] When export demand rises, the terms of trade are improved, and a one unit increase in output produces a bigger increase in consumption and a larger increase in utility. The temptation to inflate is greater and this increases inflation. This result can only be obtained once agents' welfare is used to make policy decisions. Reduced-form approaches, such as Romer (1993), have not emphasized the significance of export demand on inflation. In the context of rising global inflation the analysis therefore presents a simple explanation of why inflation may rise in countries which exports commodities that are in high demand. Because export demand is exogenous from the

[16] This is also the empirically more reasonable case.

[17] Notice that a change in export demand has a larger effect on the terms of trade the more open the economy.

viewpoint of the government this result also undermines the argument that it is possible to reduce any upward bias in inflation via institutional arrangements. Once there is an exogenous increase in export demand, any inflation already present in the economy will be exacerbated.

4.2.2. Openness

The major point I address is whether inflation has an inverse relationship with openness. However, inspecting (32) it is clear that inflation is related to inflation through all three channels stressed above. The reason is that it is not simply the Phillips curve that alters as openness changes. Previously it was shown that the optimal level of output changes with openness, and this is associated with the government's set of indifference curves. A preliminary comment is that the relationship between openness and equilibrium real balances, which is associated with the Phillips curve, is unambiguous. Taking the derivative of (30) with respect to n,

$$\frac{\partial m^d}{\partial n}\frac{1}{m^d} = \epsilon(\alpha-\delta)/(\epsilon-1)(\alpha-n)(n-\delta) \tag{33}$$

The structural parameters of the economy are such that $\alpha > \delta$, $n > \delta$, and $\epsilon < 1$, and so (33) is negative. Relating this result back to inflation, recall from (31), that holding consumption constant, $\partial\pi^d/\partial m^d < 0$.[18] Given this relationship, the effect of openness via real balances is negative; a more open economy has a lower equilibrium rate of inflation. Thus the effect from real balances and the relationship between openness and inflation is similar to that in previous studies because surprise monetary expansions are more costly the more open the economy.

To fully describe the reaction of inflation it is necessary to consider how steady state consumption reacts to changes in openness. Taking the two parts of consumption separately, see again (32), $(\delta/\phi\alpha)^{n/\kappa\alpha}$ is decreasing in n because $\delta/\phi\alpha < 1$, and this counteracts the cost of lower real balances. The second term, $[(1-n)/g^*]^{n-1}$, increases or decreases with openness depending on level of export demand, g^*. We already know that inflation is increasing in export demand because improvements in the terms of trade provide an

[18]Note also that m^d is independent of g^* but that π^d is not because steady state consumption contains an interaction between openness and export demand.

incentive to inflate the economy. However, now it is clear that export demand influences the relationship between openness and inflation. Differentiating (32) with respect to n results in the following expression.

$$\frac{\partial \pi^d}{\partial n} \frac{1}{\pi^d - \beta} = 1 + [(\alpha - \delta) / \Gamma (1 - \epsilon)] + [(1/\kappa\alpha) \ln (\delta/\phi\alpha)] + \ln [(1 - n) / g^*] \tag{34}$$

where $\Gamma \equiv (n - \delta)(\alpha - n) > 0$. It is now possible to see how export demand affects the openness-inflation relationship and to identify the influence of openness on inflation through the three mechanisms which correspond to the three terms in square brackets in equation (34).[19] It is again worth noting that the leisure effect, by itself, suggests openness and inflation have a positive relationship and the real money balance effect suggests there is the standard negative relationship. Taking these two together, however, inflation has an unambiguously negative relation with openness.

Now consider the export demand channel. Recall that n is constrained from below by the monopoly distortion, δ, for the reasons discussed above. When the economy is very closed, $(1 - n)$ is near zero and this restriction becomes less important. In this case, the trade-weighted export demand term becomes more and more negative, given $g^* > (1 - n)$, which must surely hold for high levels of n. If $(1 - n) / g^*$ is sufficiently small, for example, when n is near one and export demand is high, this term dominates the right-hand side of equation (34). In this case, inflation has a positive relation with openness. However, it is also clear that there is a limit to this effect. For a given level of export demand, as n falls, the export demand term rises and may turn positive. A secondary role is played by the real money balances channel because the first term in square brackets is rising in openness for low values of n. Thus it is possible to identify a non-monotonic relationship between openness and inflation which, in this case, works through a tension between the impact of real balances and trade-weighted export demand on utility. If we think of g^* as varying across time, which it surely must do, then this suggests a very tenuous link between openness and inflation in the data.

Initially, this result may appear surprising, but there is also a natural link with the existing theoretical literature. In analyzing the welfare implications of interdependence

[19] Recall that ad-hoc specifications imply $\partial \pi^d / \partial n > 0$.

and monetary policy Corsetti and Pesenti (2001) comment that domestic market failures, i.e. monopoly distortions, need not give rise to an inflation bias when looking at optimal policy because any bias in inflation depends on the relation of openness to other variables. This result essentially derives from the possibility of a beggar-thyself effect of monetary policy when an economy is relatively small via the terms of trade and disutility of leisure. That the analysis in Corsetti and Pesenti (2001) stops short of looking at optimal policy, instead focusing on the role of exogenous changes in the money supply, is less important. The analysis presented there and the three channels identified here both only arise when the representative agents utility function is used as the metric for policy decisions.

5. Conclusion

This paper develops a general equilibrium model of a small open economy to analyze the optimal rate of inflation under discretion. The paper makes two main contributions. First, it is one a few papers that analyzes discretionary monetary policy using a dynamic general equilibrium framework in an open economy. Second, it provides a strong case for the argument that the openness-inflation relationship is dependent on the structure of the economy. To repeat the basic argument. The standard mechanism suggests that inflation depends on openness because surprise monetary expansions affect the terms of trade. The more open the economy, the stronger this effect, and the more costly it is to inflate. Here, however, export demand also affects the terms of trade. This implies that inflation is dependent on export demand and the impact changes in export demand have on inflation depend on the degree of openness. The result is that the openness-inflation relationship depends on the level of export demand. If export demand is high this distorts the terms of trade to such an extent that inflation may rise with openness.

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