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Market Efficiency and Coalition Structures

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Abstract

We consider a three-stage game in which symmetric firms decide whether to invest in a cost-reducing technology, then they have the possibility to merge (forming coalitions), and eventually, in the third stage, a Cournot oligopoly game is played by the resulting firms (coalitions). We show that, contrary to the existing literature, the monopoly market structure may fail to form even when the number of initial firms is just three. We then introduce a *weighted* sharing rule and show that a situation in which all firms acquire the cost-reducing asset cannot be sustained as a Subgame Perfect Equilibrium.

JEL Classification: C71, C78, D43
Key words: *investment, coalition formation, oligopoly market structure.*

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1 Introduction

1.1 Background

The study of coalition of players goes back to the first book on game theory: the seminal contribution of von Neumann and Morgenstern (1953). After this seminal work, the research on coalition was mainly characterized by models studying coalitions in "characteristic form". These models postulates the existence of the grand coalition, assuming away any issue concerning the process of coalition formation. The focus was on how the value generated by the coalition would be divided among the members. Interestingly, it is only recently that economic theory has begun to pay the deserved attention to the problem of coalition formation. A recent stream of research has the merit of explicitly considering the formation of coalitions and the externalities that arise among them. The scope of application of this theoretical breakthrough spans over all fields of economics. Any association of economic agents can be seen as a coalition, from political parties and federation of states, to firms and cartels.

The theory of coalitions in Oligopolies, was one of the first application proposed[1]. In this case the players of the game are firms which can form coalitions in order to compete in the market. In practice, these coalitions can be thought of as partnerships, cartels or horizontal merger. The economic theory on horizontal mergers considers two main reasons for merging: reducing competition in the market, and exploiting synergies. The latter is characterized by the presence of economies of scale in some aspects of the

[1] See as a reference Brown and Chiang [2003].

production process. For instance, through merging it is possible to eliminate duplications of fixed costs and increase specialization. Most of the models take the synergy among firms as an exogenous factor, focusing on the effects of concentration on the equilibrium social welfare. If no synergies come from concentration, the effect on social welfare is clearly negative – monopoly reduces social welfare. In case of synergies, however, the answer is not so simple. The benefits from synergies could compensate (or offset) the negative impact of concentration.

1.2 Our contribution

We propose a model of mergers and acquisitions in which the synergy resulting from concentration is endogenous. Each firm is ex-ante identical, they have the possibility to invest and modify their level of efficiency. As a consequence the decision to merge would depend on this initial investment. The investment consists on the acquisition of a particular *asset* which reduces the production costs of the firm (and of the firm resulting from the merger). This, to our knowledge, is the first work which directly addresses the *investment* issue in a coalition model[2]. The other important contribution is on the theory of coalition formation. We propose a sharing rule that accounts for the possible asymmetry among firms in the second stage[3]. In fact, with

[2]Hart and Moore [1990] consider the effect of asset ownership on the formation of a coalition of players, but they postulates the formation of the grand coalition, using the Shapley value as sharing rule. Espinosa and Macho-Stadler [2003], consider a model in which coalitions form in the first stage and then members decide the effort to put in cost reduction. They introduce the problem of moral hazard. However, their benchmark case – perfect information – is similar to our model but with a reverse order: in our model players commit themselves to a particular "effort" before coalitions are formed.

[3]To our knowledge few papers consider coalitions with asymmetric players, and they all restrict the analysis to particular types of coalition structures. Many of these papers deal

symmetric players, Ray and Vohra [1999] show that the "equal" sharing rule results as the unique equilibrium of a sequential coalition formation game.

We consider a three-stage game in which ex-ante symmetric firms decide whether to invest (acquire the asset), then, in the second stage, they can form coalitions[4] (e.g. merging), and eventually, in the third stage, a Cournot oligopoly game is played among the resulting firms (coalitions). In order to characterize the coalition formation game we rely on the concept of stability introduced by Ray and Vohra [1997], in which a coalition structure is stable if it can be supported by an *equilibrium binding agreement* (EBA) strategy profile. We deal with the cooperative behaviour inside a coalition by assuming an exogenous sharing rule. In this case, the third stage can be represented by a *valuation*, a function which maps coalition structures into vectors of individual payoffs.

We characterize the impact of investing in a cost-saving asset on the merging strategy of each firm, and hence on the resulting market structure. We found that two main forces drive the merging strategy of firms: a "Stigler" effect and a "synergy" effect. The former, maintained by Stigler [1950], accounts for the incentive of each firm not to participate in a merger. Basically, Stigler claims that a firm which do not participate in a merger may enjoy a positive externality because of the reduction in the number of firms in the market (lower competition). The latter, considered in our model, accounts for the possibility that firms reduce their costs by merging, i.e. the resulting

with international agreements, either on environmental or trade issues, and all of them consider games in which only one non-degenerate coalition (a coalition with two or more members) can form. See for a reference Carraro [1997], and Yi [1997].

[4]That represents any type of agreement in which firms in the coalition set quantity cooperatively, and share their production technology.

firm would be more efficient. Therefore, the two forces work in different directions. We show that the acquisition of the cost-saving asset influences the market structure in a counterintuitive way. With three firms and no asset, monopoly (the grand coalition) is always a stable market structure, while if all firms acquire the asset the monopoly structure is no more stable, either firms do not merge or only two of them merge. This is in contrast with both Bloch [1996] and Ray and Vohra [1997], where a similar game with three firms always leads to the grand coalition.

The main result of the model is that, when the investment decision is endogenous, a situation in which two or more firms acquire the cost-saving asset is never a (Subgame) Perfect Equilibrium of this game[5]. Indeed, under some conditions only one firm invests and the resulting market structure is a monopoly (the three firms merge). This counterintuitive result seems to suggest that, the asset is not always a *plus* for a firm, it can be also a burden. We will discuss this issue in more detail in the next sections.

The results of our model have also an important impact in terms of economic policy. The equilibrium in which only one firm invests holds even if the cost of the investment tends to zero. Therefore a government subsidy aiming at increasing the efficiency of firms in the market has no impact. We leave for future research the analysis of whether an anti-trust policy which blocks the grand coalition may improve the incentive to invest of each firm.

[5]Technically we do not have a subgame perfect equilibrium, because in the second stage we do not have a Nash equilibrium. However, the sequential rationality feature of the SPE is maintained in our equilibrium. In the first stage the Nash equilibrium is computed taking into account the equilibrium outcome on the other stage games.

1.3 Alternative applications

The theoretical model we propose can be applied to any environment in which players make an action prior to the decision to form coalitions. For instance, we can consider individual workers, they can decide whether to enter a specific market alone or to form a coalition with other agents. We have in mind lawyers, consultants, state agents, all types of workers that can compete in the market alone or through a partnership. In this scenario, the investment decision can be thought of as human capital – the acquisition of a certain skill by the worker. The main questions to address would be: how the stability of a partnership is influenced by the possibility of acquiring human capital.

Another possible application is the analysis of bidding rings. In this case, bidders can make an investment which alters their payoff from the auctioned object, influencing the stability of the ring.

1.4 The structure of the paper

We start by considering a model in which there are only three firms with a binary investment decision: either acquire the asset or not. In order to compare our results with the existing literature, we present a reduced form[6] of the model in which all firms have acquired the asset and the "equal" sharing rule is imposed; then, we consider the full model introducing a more appropriate sharing rule, that we call *weighted*. We investigate how the acquisition of the cost-saving technology modifies the incentives to merge. We, then, look for a Subgame Perfect Equilibrium of this sequential game,

[6] It is a reduced form because the investment decision is exogenously given.

and draw conclusions in terms of policy and social welfare. A final section conclude mentioning the limitations of our approach and the scope for future research.

2 Set up of the model

Consider $N = 3$ identical firms. At date 1, each firm chooses an action x_i for $i = 1, 2, 3$, so that $x \in X$ represents the vector of all actions, and X is the set of available strategies[7]. At date 2, firms can merge (forming coalitions). To avoid confusion we refer to the set of original firms as *firms* and to the firms resulting from the merging process as *coalitions*[8]. The structure of the market is represented by the set of coalition structures, $\mathcal{P}$, where each coalition is denoted by s and characterized by the number of members m_s, so that a coalition with $m_s = 1$ is a firm that did not merge. The action chosen by the firms participating in the merger is $x_s \subseteq x$, i.e. x_s represents the vector of actions chosen by the subset of firms that merged creating a new firm (coalition s). Notice that x_s is a vector, while x_i is a scalar. At date 3, the resulting coalitions will compete à la Cournot, and the profits are shared among members of the same coalition according to the valuation $v : \mathcal{P} \rightarrow \mathbb{R}^n$.

The cost function of each coalition is characterized by the following marginal cost:

$$c_s = C_0 - f(m_s, x_s) \tag{1}$$

[7]Since players are symmetric at this stage $X = X_i \, \forall i$.

[8]In the second stage of the model, even a firm which did not merge, it is referred to as a *coalition* of one member.

where $f(m_s, x_s) : \mathbb{R}^{(1+m_s)} \rightarrow \mathbb{R}$, represents the effect of each firm on coalition s marginal cost; this effect depends on the number of members, m_s, and on the action chosen by each of them in the first period x_s. In other words, the cost function of the new firm is characterized by a common cost factor C_0 and the access to the cost-saving assets of the merged firms. The merger produces a synergy only if some of the firms have invested. This synergy created can be specified in different ways. For instance, we could assume that the asset each firm owns is productive only if shared with other firms,

$$c_s = C_0 - \left[\sum_{i \in s} x_i\right]^{\alpha}$$

with $\alpha > 1$ for a convex cost-saving function and $0 \geq \alpha \geq 1$ for a concave-cost saving function. This assumption would provide a strong incentive to merge for firms that owns an asset. Alternatively, following Brown and Chiang [2003], we can model the marginal cost of each coalition as:

$$c_s = C_0 - \alpha \sum_{i \in s} x_i - \frac{\beta}{2} \left(\sum_{i \in s} x_i\right)^2$$

where α represents the first order effect and β the second order effect of the investment. With $\beta = 0$ we obtain a linear cost-saving function. In reality all we need is to model some kind of synergy resulting from the merger of firms in case they acquire the asset. For this reason, we simply consider a linear synergy among firms.

Main assumptions of the model:

Assumption A *Marginal Cost.* The action taken at date 1, contributes to the marginal cost of each coalition in the following way

$$c_s = C_0 - \alpha \sum_{i \in s} x_i \qquad \text{for all } s \in \mathcal{P}_k \quad \text{and} \quad \mathcal{P}_k \in \mathcal{P} \tag{2}$$

where c_s represents the marginal cost of coalition s, which is given by a fix and common factor C_0, linearly reduced by the investment of firms in the coalition. The parameter α accounts for the degree of *synergy* among coalition members[9], we assume $\alpha > 0$. This assures a positive synergy effect when "efficient" firms merge (an efficient firm is a player of the game with action $x = 1$). The firms' investment linearly affects the cost function of the firm ($\frac{\partial c_s}{\partial x_i} = -\alpha$ for all i); the cost function captures the efficiency gains from pooling efficient firms, if no efficient firms participate in the merger the resulting coalition's cost is the same as the cost of each merging firm.

Assumption B *Identical Action Cost.* The cost of taking action x_i is the same for any i, and equal to ϕ; this reflects the ex-ante symmetry of all firms.

Assumption C *Binary investment choice* We restrict the strategy set of each firm, at time 1, to $X = \{0, 1\}$. As a consequence the cost of action $x = 1$ is fixed and equal to $\phi \in \mathbb{R}^+$. In other words, there is only one type of asset firms can acquire. Each firm cannot acquire more than one asset. And finally, there is no competition for the assets, i.e. they are always available assets.

[9] See for instance Perry and Porter [1985], for an analysis of the synergy influencing horizontal mergers among firms. In their model, however, the decision to acquire the cost reducing asset is *not* endogenous.

In this section we characterize the last two stages of the game. Firstly, we compute the Cournot Oligopoly equilibrium with many coalitions and (possibly) different marginal costs. Secondly, we characterize the coalition formation game applying Ray and Vohra [1997]'s *Equilibrium Binding Agreements* stability concept.

2.1 Cournot Equilibrium

We assume that each coalition produces the same good, and they face the same linear demand $P = a - bQ$, where P is the market price and Q the total output of the industry; the parameters of the demand function are both positive, $a, b > 0$.

The Cournot equilibrium is characterized by the following conditions, where S represents the number of coalitions in a specific market structure:

Industry Output	$Q = \frac{(a-\bar{c})S}{b(S+1)}$	$\frac{dQ}{dS} > 0$
Market Price	$P = \frac{a+S\bar{c}}{S+1}$	$\frac{dP}{dS} < 0$
Firm Output	$q_s = \frac{a+\sum_{j\neq s} c_j - Sc_s}{(S+1)b}$	$\frac{dq_s}{dS} < 0$
Firm Profit	$\pi_s = \frac{1}{b}\left[\frac{a+\sum_{j\neq s} c_j - Sc_s}{(S+1)b}\right]^2$	$\frac{\pi_s}{dS} < 0$

We are interested in the last expression, which represents the worth (profit) of coalition s. Clearly, this worth depends not only on the marginal cost of coalition s, but also on the marginal cost of the other coalitions $j \neq s$, and hence it ultimately depends on the market structure and the action chosen by all firms $i \in N$.

2.2 Coalition Formation

The coalition formation game is characterized by two critical features: (i) how firms influence the cost of the coalition and (ii) the sharing rule. In general, we should consider also the way in which coalitions interact, but in our model this is pinned down by the assumption of Cournot competition.

We decided to use a *cooperative* concept of stability: the equilibrium binding agreement (EBA) introduced by Ray and Vohra [1997]. A coalition structure is stable if it can be supported by an EBA. Equilibrium Binding Agreements are strategy profiles that are immune to coalition deviations which (i) are consistent (no further deviation) and (ii) can be supported by the external players, i.e. the deviation is profitable considering the optimal reply of the remaining players. One important feature of this concept is that coalitions can only break up in smaller coalitions as a consequence of a deviation[10]. Note that, when a player decides whether to deviate he will consider all the possible reactions of both, the subset of deviating players and the players who have not deviated, until a stable coalition structure is reached. An synthetic formal definition of this concept is provided by Yi and Shin [1995]:

> "a nondegenerate coalition structure $\mathcal{P} = \{n_1, n_2, \ldots, n_m\}$ is stable under the Equilibrium Binding Agreement rule if and only if there do not exist $\mathcal{P}^1 = \{n_1^1, n_2^1, \ldots, n_{m(1)}^1\}$, $\mathcal{P}^2 = \{n_1^2, n_2^2, \ldots, n_{m(2)}^2\}$, ..., $\mathcal{P}^R = \{n_1^R, n_2^R, \ldots, n_{m(R)}^R\}$ such that
>
> (1) $\mathcal{P}^1 = \mathcal{P}$ and $\mathcal{P}^{r+1} = \mathcal{P}^r \setminus \{n_{i(r)}^r\} \bigcup \{\hat{n}_{i(r)}^r, n_{i(r)}^r - \hat{n}_{i(r)}^r\}$, for

[10] As stated by Muthoo [2004], it is implicit the idea that breaking up a coalition is costless comparing to forming a new coalition.

some $i(r) = 1, \ldots, m(r)$ and for all $r = 1, \ldots, R-1$;

(2) $\mathcal{P}^r$ is stable but $\mathcal{P}^2, \mathcal{P}^3, \ldots, \mathcal{P}^{R-1}$ are not; and

(3) $\hat{n}^1_{i(1)}$ leading perpetrators are better off under the final coalition structure $\mathcal{P}^R$ than under the original coalition structure $\mathcal{P} = \mathcal{P}^1$.

The application of this concept usually produces more than one stable coalition structure. In order to compute the equilibrium of the game, we assume the most *concentrated*[11] *stable* coalition structure as the equilibrium of the coalition formation stage.

In games with transferable utility, the stability of a coalition structure ultimately depends on the sharing rule adopted. Assuming an exogenous sharing rule allows to represent the gain from cooperation inside a coalition as a *valuation* which maps coalition structures into vectors of individual payoffs[12] Ray and Vohra [1999] show that, in an infinite-horizon model of coalition formation with *symmetric* players, the "equal" sharing rule results as the outcome of any equilibrium without delay. In fact, in our model players (firms) are symmetric ex-ante (prior of the action choice) but they can be asymmetric ex-post. We acknowledge the importance of an appropriate sharing rule, and we postpone a detailed discussion to section four. The assumption of an equal sharing rule does not seem to be appropriate in our

[11]We use the Yi and Shin [1995]'s concept of concentration as reported in Bloch [2003]: "A coalition structure $\mathcal{P} = \{n_1, n_2, \ldots, n_m\}$ is a *concentration* of a coalition structure $\mathcal{P}' = \{n'_1, n'_2, \ldots, n'_m\}$ if there exist two subcoalition structures, D and D' of $\mathcal{P}$ and $\mathcal{P}'$, $D = \{k_1, k_2, \ldots, k_s\}$ with $k_1 \geq k_2 \geq \ldots \geq k_s$ and $D' = \{k'_1, k'_2, \ldots, k'_{s+t}\}$ with $k'_1 \geq k'_2 \geq \ldots \geq k'_{s+t}$ such that (i) $\mathcal{P} \setminus D = \mathcal{P}' \setminus D'$ and (ii) $\sum_{i=1}^{j} k_i > \sum_{i=1}^{j} k'_i$ for all $j = 1, 2, \ldots, s$ ".

[12]See Bloch [1996] for a discussion on the sensibility of this assumption.

case, nevertheless we start by assuming the equal sharing rule in order to compare our model with the results in Ray and Vohra [1997] and Bloch [1996]; furthermore, there might be some institutional scenarios in which such a rule is imposed. For instance, when "fairness" is the main concern; or we can think of a situation in which players decide, prior to any action, to commit to a particular sharing rule, and since they are symmetric the equal sharing rule seems to be a plausible choice.

3 A reduced form model: symmetry and Equal Sharing Rule

In this section we consider a reduced form of the model, in which the investment decision is exogenously given. In particular, we assume that all firms have acquired the asset. This assumption makes all firms identical in the coalition game, so that the assumption of an equal sharing rule is justifiable. Hence we can compare the results we obtain with the existing literature on coalition market structure.

We consider a coalition game in which an equal sharing rule is exogenously imposed. We call this game $\hat{\Gamma}$.

Assumption D' *Equal Sharing Rule* The worth of each coalition is shared in equal parts among its members.

We characterized in the previous section the Cournot game that coalitions play in the third stage. Now we characterize the stable coalition structure given the investment decision of each firm at date 1.

In order to contrast our results with the existing literature on oligopoly and coalitions we consider the symmetric case $x_1 = x_2 = x_3 = 1$, in which each firm acquires the asset. Most of the models of coalition formation in oligopoly assume symmetric firms, and no synergy in merging (i.e. the cost of the resulting firm is the same as the cost of the merging firms). Here we want to introduce a certain degree of synergy, by assuming that all firms have acquired the cost reducing investment, so that the resulting firm has a lower cost than the merging firms. In the following we assume $\alpha = 1$, this is without loss of generality, because our aim is to compare our model with models in which $\alpha = 0$.

Coalition Structure		Cost Function	PerCapita Profit	Tot Profit
$\mathcal{P}_1$	$\{1,2,3\}$	$c = C_0 - 3$	$\bar{\pi} = \frac{(A+3)^2}{12b}$	$\Pi = \frac{(A+3)^2}{4b}$
$\mathcal{P}_2$	$\{1,2\}\{3\}$	$c_1 = C_0 - 2$ $c_2 = C_0 - 1$	$\pi'' = \frac{(A+3)^2}{18b}$ $\pi' = \frac{A^2}{9b}$	$\Pi = \frac{2A^2+6A+9}{9b}$
$\mathcal{P}_3$	$\{1\}\{2\}\{3\}$	$c^* = C_0 - 1$	$\pi^* = \frac{(A+1)^2}{16b}$	$\Pi = \frac{3(A+1)^2}{16b}$

Let $A = (a - C_0)$ be the difference between the demand size and the cost structure of the industry. As we will see this relation plays an important role for the stability of a coalition structure. Since firms are symmetric we did not report the case of coalitions of firm 2 with firm 3 and firm 1 with firm 3, these two coalition strucutres would be the same as the coalition structure $\mathcal{P}_2$. In the above table, $\bar{\pi}$ represents the per capita worth of the grand coalition, while π^* is the per capita worth of the singleton coalitions; π'' is the per capita worth of a coalition with two firms, while π' is the per capita worth of a coalition with only one firm.

Lemma 1 *In case of bilateral deviation from the grand coalition the per* *capita payoff they get as a result of the deviation, is always lower than their payoff in the grand coalition, i.e.* $\pi'' < \pi$.

Proof. The proof of this and the following two lemmas, is straightforward, we just need to compare the per-capita payoff in different coalitions and check which one prevails. ■

Lemma 2 *The payoff of a single firm deviating from the grand coalition, in case the other firms remain in the original coalition, is greater than his/her payoff in the grand coalition if and only if* $A \geq 19.39$.

Note that in Ray and Vohra [1997] this lemma is always satisfied, does not depend on the value of A. This point will be clarified later.

Lemma 3 *The singleton profit is lower than the per capita profit of any deviation from the grand coalition, i.e.* $\pi^* \leq \pi''$, *if and only if* $A \leq 31.97$.

In this case, firms who are left in a two member coalition have no incentive to separate only if $A > 31.97$.

Now, considering the three lemmas we can get the following proposition:

Proposition 1 *In the coalition game* $\hat{\Gamma}(x = 1)$ *with demand* $P = a - bQ$ *and marginal cost* $c_s = C_0 - \sum_{i \in s} x_i$, *for* $A \equiv (a - C_0)$ *such that* $19.38 \leq A \leq 31.97$ *the grand coalition is blocked by the stable coalition structure* $\{i, j\}\{k\}$.

Proof. The structure of the proof relies on 4 separate remarks. Firstly, the singleton payoff is never greater than the per capita grand coalition payoff. Secondly, the per capita payoff of the two-firm coalition in $\mathcal{P}_2$ is always lower than the per capita payoff of the grand coalition. Given this two remarks, we need to check only for incentives to unilaterally deviate from the grand coalition. Thirdly, the incentive to unilaterally deviate of each firm depends on how the other two firms will react. Finally, in coalition structure $\mathcal{P}_2$ members of the two-firm coalition will stick together if $A \leq 31.97$. By simple comparison of per capita payoff we have that unilateral deviation is profitable only for $19.39 \leq A \leq 31.97$. If $A > 31.97$ one firm would like to deviate, but he knows that in this case the other two firms will split and all of them will end up in the singleton coalition structure which gives a lower payoff than $\mathcal{P}_1$. When $A < 11.39$ there is no incentive to unilaterally deviate. Even if the other two firms will stick together, the payoff of the lonely firm in coalition $\mathcal{P}_2$ is lower than in the grand coalition. ■

In contrast to Ray and Vohra [1997], the grand coalition is not stable. The intuition for this result relies on the effect each firm has on the marginal cost of the coalition, and, hence, on the share of profits that the coalition can capture. In Ray and Vohra [1997] as well as in Bloch [1996], what matters is only the number of coalitions not their size (the number of firms in each coalition). Notice that the game $\hat{\Gamma}(x = 1)$ is analogous to the coalition game in Ray and Vohra [1997]: it is a Transferable Utility symmetric game with positive externalities. The difference is in the marginal cost which is not fixed across coalitions but depends on the number of members.

We can identify two effects driving the incentive to join a coalition:

▷ **competition externality (Stigler effect)** firms which do not participate in a merger (coalition) can benefit more than firms involved in the merger[13]. It depends on the change in the number of coalitions *competing* in the market;

▷ **efficiency effect (synergy effect)** a firm leaving a coalition modifies the worth of all coalitions because of his/her impact on the marginal cost of the coalition he leaves and the coalition he joins (this effect is present even if the number of coalitions in the market does not change)

The first effect is present in all models of coalitions with positive externalities, and, in case of Cournot competition, it gives an incentive for a firm to exit a coalition. The second effect is driven by the possibility of a firm to reduce the marginal cost of a coalition. In this case, deviating to form a singleton might not be convenient, because the deviating firm needs to weight the benefit of not sharing the coalition worth, with a lower share of the total worth, as a result of a higher marginal cost. Which one of the two effects prevails depends on the size of demand and the underling cost structure of the industry, C_0. Here comes the importance of A for the equilibrium structure[14] . We showed that the grand coalition is stable only for a certain range of values of A, while in Ray and Vohra [1997] the grand coalition, with only three firms, is always stable. The parameter A can be interpreted as a proxy of the importance of the cost structure relative to the demand size in the formation of the per capita profit. In fact, when A is large, meaning that the cost is a negligible part of the profits, the competition effects prevails –

[13] This concept was proposed by Stigler [1950].

[14] Also Espinosa and Macho-Stadler [2003] in a model of coalition formation with moral hazard highlight the role of the ratio between demand size and structure of costs. Their model, indeed, is similar to ours in the fact that partners of the coalition simultaneously decide the effort in reducing the marginal cost of the coalition. Therefore, the per capita payoff does not depend just on the number of coalitions in the market.

a firm is willing to have a higher marginal cost and exploit the high demand on his own. When A is small, costs are an important component of firms' profit and the efficiency effects prevail, so that firms are not willing to incur higher costs by deviating. So, for small values of A, the grand coalition is stable because the efficiency effect prevails, for large values of A the grand coalition is stable as well because the competition effect is so strong that when one firm deviates the coalition disintegrates in singletons. However, for intermediate values of A, the two forces balance and the grand coalition is not stable. To sum up, the main difference with models of cartel formation is the introduction of an element of synergy among firm's members which makes it more difficult to leave the grand coalition.

In the appendix we present the whole model[15] with the "equal" sharing rule. We show that for intermediate values of A, the following proposition applies:

Proposition 2 *The Subgame Perfect Equilibrium of the game $\hat{\Gamma}(x)$ is given by the following strategy profile:*

- ▷ *for $\phi < 3.45 \implies$ at date 1, two firms invest (i=1, j=1, k=0), at date 2, the grand coalition {i,j,k} forms;*
- ▷ *for $3.45 < \phi < 3.55 \implies$ there are two equilibria:*
  - *(i) at date 1, only one firm invests (i=0, j=1, k=0), at date 2, the grand coalition ({i,j,k}) forms;*
  - *(ii) at date 1, nobody invests (i=0, j=0, k=0), at date 2, the grand coalition ({i,j,k}) forms;*

[15] We endogenize the investment decision

▷ *for* $\phi > 3.75 \Longrightarrow$ *at date 1, nobody invests (i=0, j=0, k=0), at date 2 the grand coalition (*$\{i,j,k\}$*) forms.*

Notice that the situation in which all firms acquire the asset is never sustained as an equilibrium. We will discuss more in detail this result at the end of the section. Now, we turn to our main model, where the decision to invest is endogenous, and we propose a different sharing rule to take into account the potential asymmetry among firms.

4 The model with a Weighted Sharing Rule

We propose the full model, where the investment decision is endogenous. In this case firms can by asymmetric in the coalition game, it seems more natural to devise a mechanism that assigns to each member a share of the worth as a function of his/her "importance" in the coalition. We start by making some general remarks about sharing the worth of a coalition.

The literature on coalition games in characteristic function[16], considers two main conditions that an *imputation*, i.e. a sharing rule, should satisfy:

- individual rationality;
- group rationality.

Let π_i be the individual payoff of firm i and π^* her payoff by acting non cooperatively, i.e. singleton coalition. Individual rationality implies that a player must be guaranteed a payoff not lower than the payoff he/she can obtain by acting non cooperatively, i.e. $\pi_i \geq \pi^*$. Group rationality

[16]Remember that the characteristic function approach does not allow for externalities among coalitions. See for a good reference Friedman [1989].

guarantees that there is no other agreement that gives all, or some of the players, a higher payoff without decreasing the payoff of somebody else. This is basically a version of Pareto efficiency, $\sum_{i \in N} \pi_i = v(N)$. In our model the application of these two principles are not immediate. As concerns the individual rationality, the payoff a player can obtain, staying alone, depends on the whole coalition structure, while the group rationality seems to be tailored just for the grand coalition.

An important and widely used sharing rule is the Shapley Value. The Shapley value satisfies the following axioms:

(i) Group Rationality $\sum_{i \in N} \phi_i(v) = v(N)$;

(ii) Contribution: if player i adds $v(\{i\})$ in any coalition the he/she should receive $v(\{i\})$;

(iii) Symmetry: if two games differ only for the order of players, then the Shaply value is the same in both games;

(iv) Linearity: given a game $\Gamma = \Gamma_1 + \Gamma_2$ the Shapley value is the sum of the Shapley value in each of the two separate games.

The per capita payoff depends on the contribution a player makes to each coalition of which he can be a member. This is very important when players are asymmetric. However, since the Shapley value assumes the stability of the grand coalition, if we apply it to coalition structures where the grand coalition fails to form the problem is again how to treat the externalities among coalitions.

We propose the following properties a sharing rule should satisfy in coalition games in partition function with asymmetric players:

(a) Identical players should obtain the same share of worth;

(b) Contribution: the share of each player should be a function of his/her "contribution" to the coalition he/she is a member;

(c) Efficiency (Group Rationality), a sharing rule should distribute all the worth of a coalition structure – the difference with the usual group rationality condition is that we require it to be valid within and for each possible coalition structure;

Note that we do not require the individual rationality principle to necessarily apply. This is because we allow for a more general set up in which a player can actually gain from staying alone.

Condition (b) is particularly important because we can have several sharing rules according to the stability concept we adopt. What happens when a player deviates? Can he form another coalition or not? What happens to the players in a coalition after a deviation? Do they remain together or they split? Can they form other coalitions? These are all important question that require a very careful analysis.

We propose the following mechanism, in which the "contribution" of a player is determined according to her *stable* outside option. Before describing the mechanism, we need to define an "ordered" set of coalitions:

Definition 1 (Ordered coalition structure) *Let $\mathcal{C}_k$ be a subset of partitions, where the largest coalition has exactly k members.*

We are now ready to define our sharing rule.

[**Weighted Sharing Rule**] Let s be a coalition in $\mathcal{P}'$, with $|s| = k$ members. Let us order coalition structures so that $\mathcal{P}' \in \mathcal{C}_k$. The contribution of player $j \in s$ is defined as the difference $v(s) - v(s_{-j})$ if s_{-j} is stable, where s_{-j} represents coalition s without player j. If s_{-j} is not stable, define $\hat{\mathcal{P}}$ as the stable coalition structure determined with the players in s_{-j} and $\hat{s}$ the coalitions in partition $\hat{\mathcal{P}}$, then $v(s_{-j}) = \sum_{\hat{s}} v(\hat{s})$. The share of player j is given as the weighted sum of the contributions of all players in s. This procedure is reiterated for any $\mathcal{C}_k$ starting from $\mathcal{C}_2$ up to $\mathcal{C}_n$.

This procedure is based on Ray and Vohra [1997] restriction on the possibility of forming coalitions: coalitions can only split in smaller coalitions. It works also for multiple deviations. The next example helps to clarify this concept. Notice that in case of identical players our sharing rule is equivalent to the "equal" sharing rule.

4.1 Example

Consider three players $i = 1, 2, 3$. We have five possible coalition structures $\mathcal{P}$. We order this coalition structures according to the number of players in the largest coalition in each $\mathcal{P}$. Let $\mathcal{C}_k$ be the set of coalition with identical order. In case of 3 players, we would have

$$\begin{aligned}
\mathcal{C}_3 =& \{1,2,3\} \\
\mathcal{C}_2 =& \{1,2\}\{3\} \\
& \{1,3\}\{2\} \\
& \{2,3\}\{1\} \\
\mathcal{C}_1 =& \{1\}\{2\}\{3\}
\end{aligned}$$

Notice that the number of coalition structure in each $\mathcal{C}_k$ represents the combination of n elements in groups of size k, i.e. $c(n,k)$. So, the number of coalition structures in $\mathcal{C}_2 \equiv c(3,2) = 3$.

Coalitions in $\mathcal{C}_1$ are stable by definition and the worth of a coalition represents also the per capita payoff. Coalition structures in $\mathcal{C}_2$ are characterized by one singleton and one coalition with two players[17]. Let us focus on the division of surplus of the following coalitions:

$$\begin{array}{ll}
\{1,2\} & \{3\} \\
\{1,3\} & \{2\} \\
\{3,2\} & \{1\}
\end{array}$$

Take the first line. The payoff of player 3 is trivially the worth of his singleton coalition, while for {1,2} the marginal "contribution" of player 1 and 2 are

[17] In general, for $n > 3$, $\mathcal{C}_2$ would be characterized by a series of singletons and the two-member coalitions.

w_1 and w_2, respectively,

$$\begin{aligned} w_1 &= v(\{1,2\}) - v(\{2\}) \\ w_2 &= v(\{1,2\}) - v(\{1\}) \end{aligned}$$

These two conditions represent their *stable* outside option, because the singleton coalition is stable by definition. The share of worth for player 1 and 2 would be ω_1 and ω_2,

$$\begin{aligned} \omega_1 &= v(\{1,2\}) \left(\frac{w_1}{w_1 + w_2} \right) \\ \omega_2 &= v(\{1,2\}) \left(\frac{w_2}{w_1 + w_2} \right) \end{aligned}$$

In this way we can compute the per capita payoff of each coalition structure in $\mathcal{C}_2$.

Then we move up to $\mathcal{C}_3$, i.e. the grand coalition {1,2,3}. We repeat the same procedure, being careful to check the stability of the outside option. For instance, when checking player 1 contribution we need to consider the worth of {2,3} if stable, otherwise we consider $v(\{2\}) + v(\{3\})$. Note that when we check the stability of coalition {2,3} we consider the per capita payoff computed in $\mathcal{C}_2$.

4.2 Coalition formation game

Our aim in this section is to investigate the merging behavior of firms as a function of the acquisition of the asset. This would depend on the value of α the marginal cost-saving impact. In order to make the model more tractable we specify a value for $A = 20$, which represents an intermediate level as defined in the previous section. In particular we set the demand function equal to $P = 30 - Q$ and the marginal cost function equal to $c_s = 10 - \alpha \sum_{i \in s} x_i$. In order to have "active" firms in the market we need to impose the condition $\alpha \leq \frac{10}{\sum_{i \in s} x_i}$. Since the sharing rule corresponds to an equal sharing rule in case of identical firms, the distribution of payoffs where either all firms invest or no firm invests is the same as in the case of equal sharing rule. Firstly, we present all possible investment scenarios and we compute the stable coalition structure in each of them, then we investigate the investment decision of each firm in the first stage.

All Investment $\Rightarrow x_1 = x_2 = x_3 = 1$

Coalition Structure		Cost Function		PerCapita Profit
$\mathcal{P}_1$	{1,2,3}	$\bar{c} = 10 - 3\alpha$		$\bar{\pi} = \frac{(20+3\alpha)^2}{12}$
$\mathcal{P}_2$	{1,2} {3}	$c_{12} = 10 - 2\alpha$	$c_3 = 10 - \alpha$	$\pi_1 = \pi_2 = \frac{(20+3\alpha)^2}{18}$
				$\pi_3 = \frac{400}{9}$
$\mathcal{P}^*$	{1}{2}{3}	$c^* = 10 - \alpha$		$\pi^* = \frac{(20+\alpha)^2}{16}$

The stability of each coalition structure depends on the value of α, the marginal impact of the asset on the cost function.

Proposition 3 (All Invest) *The grand coalition is blocked by coalition structure* $\mathcal{P}_2$ *in case,* $0.626 \leq \alpha \leq 1.031$*; otherwise the grand coalition is stable.*

For a small impact of the asset on the cost function of the coalition, firms prefer to merge. In other words, it is not convenient for a firm to exit the grand coalition. However, as the impact increases, the grand coalition is blocked by $\mathcal{P}_2$. Two firms find it convenient to merge, and the third one can "free ride" on them. From this point of view the presence of the cost-reducing asset blocks the grand coalition.

When the impact is very large, the grand coalition becomes stable again because the synergy of the assets more than compensate the "Stigler" effect.

No Investment $\Rightarrow x_1 = x_2 = x_3 = 0$

Coalition Structure		Cost Function	PerCapita Profit	
$\mathcal{P}_1$	**{1,2,3}**	$\bar{c} = 10$	$\pi_1 = \pi_2 = \pi_3 = \frac{A^2}{12}$	
$\mathcal{P}_2$	{1,2} {3}	$c_{12} = c_3 = 10$	$\pi_1 = \pi_2 = \frac{A^2}{18}$	$\pi_3 = \frac{A^2}{9}$
$\mathcal{P}^*$	{1}{2}{3}	$c^* = 10$	$\pi^* = \frac{A^2}{16}$	

The grand coalition is always stable. This is in line with the literature on coalition formation and oligopoly with identical firms. Since there is no synergy effect, a merger of two firms is never convenient. Hence, in order to avoid competition (coalition structure $\mathcal{P}^*$), they prefer to merge in a big monopoly firm.

ONE Investor $\Rightarrow x_1 = 1\ x_2 = x_3 = 0$

Coalition Structure		Cost Function	PerCapita Profit		
$\mathcal{P}_1$	{1,2,3}	$\bar{c} = 10 - \alpha$	$\pi_1 = \bar{\upsilon}\bar{\omega}_1$	$\pi_2 = \pi_3 = \bar{\upsilon}\bar{\omega}_2$	
$\mathcal{P}_2$	{1,2} {3}	$c_{12} = 10 - \alpha\ c_3 = 10$	$\pi_1 = \upsilon\omega_1$	$\pi_2 = \upsilon\omega_2$	$\pi_3 = \frac{(20-\alpha)^2}{9}$
$\mathcal{P}_3$	{1} {2,3}	$c_1 = 10 - \alpha\ c_{23} = 10$	$\pi_1 = \frac{(20+2\alpha)^2}{9}$	$\pi_2 = \pi_3 = \frac{(20-\alpha)^2}{18}$	
$\mathcal{P}^*$	{1}{2}{3}	$c_1 = 10 - \alpha\ c_2 = c_3 = 10$	$\pi_1 = \frac{(20+3\alpha)^2}{16}$	$\pi_2 = \pi_3 = \frac{(20-\alpha)^2}{16}$	

Firstly notice that the coalition structure $(\{1,3\}\{2\})$ is analogous in terms of payoffs to coalition structure $\mathcal{P}_2$, hence we have omitted it. In the table the profit of the grand coalition is represented by

$$\bar{v} = \left[\frac{20+\alpha}{2}\right]^2$$

while the profit of the two-firm coalition in $\mathcal{P}_2$ is

$$v = \left[\frac{20+2\alpha}{3}\right]^2$$

The weights associated with the per capita payoff are:

$$\bar{\omega}_1 = \left[\frac{1200+3\alpha^2+200\alpha}{3600-7\alpha^2+280\alpha}\right] \qquad \bar{\omega}_2 = \left[\frac{1200-5\alpha^2+40\alpha}{3600-7\alpha^2+280\alpha}\right]$$

$$\omega_1 = \left[\frac{2800+55\alpha^2+1640\alpha}{5600+92\alpha^2+1840\alpha}\right] \qquad \omega_2 = \left[\frac{2800+97\alpha^2+200\alpha}{5600+92\alpha^2+1840\alpha}\right]$$

Before any consideration on the stability of the coalition structure, notice that the worth of coalition $\{1,2\}$ in $\mathcal{P}_2$ and coalition $\{1\}$ in $\mathcal{P}_3$ are the same, i.e. firm 2 apparently does not bring anything to the new coalition. However, we cannot conclude that her marginal contribution is nil. What firm 2 actually brings, by merging with firm 1, is a reduction of competition in the market, and this is a benefit for firm 1 (and for firm 3 as well). The weighted sharing rule does take into account this aspect.

The second thing to notice is the effect of the Asset on the strategy of firm 1. Let us consider coalition $\{1,2\}$ in $\mathcal{P}_2$. The incentive of firm 1 to deviate increases with α the marginal impact of the asset on the cost function. With $\alpha = 0$, the coalition is stable, while with $\alpha = 1$ this coalition is not stable. This consideration has an effect on the distribution of the worth in the coalition, the relative share of the firm with the asset, in our case firm 1, increases with α.

Proposition 4 (One invests) *The grand coalition is stable for $\alpha \leq 4$, the singleton coalition is the only stable coalition structure for $\alpha > 4$.*

Proof. In the appendix. ■

To understand this point notice that the share of profit that goes to the firm which owns the asset increases with the marginal impact, α. When the marginal impact of the asset is $\alpha < 4$, firm 1 (the firm with the asset) has no incentive to block the grand coalition. In other words, it is not convenient to form a two-firm coalition, firm 1 is willing to merge only in the grand coalition. Conversely, the other firms in the grand coalition have no incentive to deviate, because they know they will end up in a very competitive market, i.e. $\mathcal{P}^*$.

When the impact of the asset is larger, $\alpha > 4$, firm 1 (the owner of the asset) has no interest to merge with anybody, the benefit it would receive in terms of lower competition does not compensate the profit it has to give up after merging. In other words, the Stigler effect prevails because no reduction in costs would result from merging. Hence, what leads firm 1 to merge is to enjoy monopoly power, and only in case of a grand coalition this aspect prevail on the "Stigler" effect.

TWO Investors $\Rightarrow x_1 = 1 \; x_2 = 1 x_3 = 0$

Coalition Structure		Cost Function	PerCapita Profit		
$\mathcal{P}_1$	{1,2,3}	$c = 10 - 2\alpha$	$\pi_1 = \bar{\omega}_1$	$\pi_2 = \bar{\omega}_2$	$\pi_3 = \bar{\omega}_3$
$\mathcal{P}_2$	{1,2} {3}	$c_{12} = 10 - 2\alpha$ $c_3 = 10$	$\pi_1 = \pi_2 = \frac{(20+4\alpha)^2}{18}$		$\pi_3 = \frac{(20-2\alpha)^2}{9}$
$\mathcal{P}_3$	{1,3} {2}	$c_{13} = 10 - \alpha$ $c_2 = 10 - \alpha$	$\pi_1 = \omega_1$	$\pi_3 = \omega_2$	$\pi_2 = \frac{(20+\alpha)^2}{9}$
$\mathcal{P}^*$	{1}{2}{3}	$c_1 = c_2 = 10 - \alpha$ $c_3 = 10$	$\pi_1 = \pi_2 = \frac{(20+2\alpha)^2}{16}$		$\pi_3 = \frac{(20-2\alpha)^2}{16}$

In the table, the value of the payoffs labelled ω are given by the following expressions:

$$\bar{\omega}_1 = \bar{\omega}_2 = \frac{(\alpha+10)^2(\alpha^2+40\alpha+100)}{3\alpha^2+100\alpha+300} \quad \bar{\omega}_3 = \frac{(\alpha+10)^4}{3\alpha^2+100\alpha+300}$$

$$\omega_1 = \frac{(\alpha+2)(\alpha-70)(\alpha+20)^2}{18(\alpha^2-32\alpha-140)} \quad \omega_3 = \frac{(\alpha-10)(\alpha+14)(\alpha+20)^2}{18(\alpha^2-32\alpha-140)}$$

Coalition $\mathcal{P}_3$ stability depends on the value of α. Both firms[18] in coalition $\{1,3\}$ would deviate only for $\alpha \leq 1.43$. We see that also in this case the investment, i.e. the acquisition of the asset, plays a positive role on the stability of a coalition. When the impact of the cost-saving asset is large coalition structure $\mathcal{P}_3$ is stable.

Coalition structure $\mathcal{P}_2$ stability follows the same driving forces. In particular, for $\alpha > 0.65$, $\mathcal{P}_2$ is stable, while for $\alpha < 0.65$ is not stable. Notice that as the effect of the asset tends to zero, we go back to the case of no investment, where coalition structure $\mathcal{P}_2$ is analogous to $\mathcal{P}_3$. This is the reason why, for $\alpha \leq 0.65$ both coalition structures are *not* stable.

Let us focus on the stability of the grand coalition. For $\alpha \leq 0.65$ both $\mathcal{P}_2$ and $\mathcal{P}_3$ are not stable. In this case, all three firms have an incentive to

[18] Actually the discriminant value for firm 1 is about 1.42, and for firm 3 is about 1.43.

merge and form a grand coalition. For $0.65 < \alpha \leq 1.43$, coalition structure $\mathcal{P}_2$ is stable, and it blocks the grand coalition. The two firms with the asset find it profitable to merge even without the other firm. In other words, the firm with no asset has an incentive to deviate from the grand coalition because the other two will remain together. It seems that the asset is a burden for these two firms, if they had no asset the grand coalition would form. The last possibility is a very strong impact of the asset. If $\alpha > 1.43$ both coalition $\mathcal{P}_2$ and $\mathcal{P}_3$ are stable, and both can block the grand coalition. Any market structure in which two firms merge is stable, regardless of the asset ownership.

Proposition 5 (two invest) *In case only two firms acquire the asset, the grand coalition is stable for* $\alpha \leq 0.65$*; a market structure in which the two firms with the asset merge, is stable for* $0.65 < \alpha \leq 1.43$*; and finally, a market structure with any two firms merging is stable for* $\alpha > 1.43$*.*

This proposition is clearly driven by the impact of the asset on the cost function. As α increases the possibility to have a monopoly market structure decreases.

4.3 Investment decision

We now consider the first stage of the game, where firms simultaneously decide whether to invest. In the table 1 we present a summary of stable coalition structures in each investment scenario.

For the case of two investments, the first line refers to the fact that a coalition in which *any* two firms merge is stable, while the second line refers

Table 1: Summary of stable coalition structures

Investment decision	Stable Coalition Structure	Condition on α
All invest	{i,j} {k}	$0.626 < \alpha \leq 1.031$
	{i,j,k}	otherwise
No investment	{i,j,k}	for all α
ONE invests $x_i = 1$	{i} {j} {k}	$\alpha > 4$
	{i,j,k}	$\alpha \leq 4$
TWO invest $x_1 = x_2 = 1$	{i,j} {k}	$\alpha > 1.43$
	{1,2} {3}	$0.65 < \alpha \leq 1.43$
	{i,j,k}	$\alpha \leq 0.65$

to the fact that only coalitions in which the two firms who have invested merge is stable. The main difference with the outcome of the "equal" sharing rule, relates to the two asymmetric situations.

Let us compute the SPE for $\alpha = 1$. This is the case in which the asset has a moderate impact on the cost function. Actually the SPE we find is valid for all cases in which $0.65 < \alpha \leq 1.031$.

Given the cost of investment ϕ, we have the following SPE,

Proposition 6 *The Subgame Perfect equilibrium of the game depends on the cost of investment ϕ,*

- ▷ *if $\phi < 7.85$, at date 1 only one firm invests, at date 2 the grand coalition forms;*
- ▷ *if $\phi > 7.85$, at date 1 nobody invests, at date 2 the grand coalition forms.*

Proof. The proof of the SPE relies on showing that the strategy described is a Nash equilibrium in any subgame. We have three subgames. We proceeded by backward induction to compute the equilibrium in the

Cournot game, then the equilibrium in the coalition formation game, and now we should show the equilibrium of the simultaneous investment game. Let us define the profit of each firm as π_i; the stable coalition structure as $\mathcal{P}^*(x)$, which depends on the action chosen by the three firms; and finally, the sharing rule which allows to link the coalition structure to the individual payoff, $v(x)$. The problem faced by each firm i at date 1 is

$$\begin{array}{ll} \max_{x_i} & \pi_i(x_j, x_k, \phi) \\ \text{s.t.} & \pi_i \in \ v : \mathcal{P} * (x) \rightarrow \mathbb{R}^n \end{array}$$

We obtain a system of 3 first order conditions to solve for the three variables x_i, x_j, x_k. This solution would depend on the cost of the investment ϕ. Note that we are assuming that $\alpha = 1$, therefore we have a specific payoff for any equilibrium structure. The equilibrium is given by the optimal strategy of each firm given the strategy of the other two firms. This would be a standard game in strategic form with perfect and complete information, so the proof of our result follows the standard proof of an SPE in such a game. ■

A corollary of this equilibrium is that a firm with an asset is willing to share it with inefficient firms only if a monopoly is created.

The main policy implication of our result is that when the investment decision is endogenous, even for a low cost of the asset, i.e. low ϕ, a situation in which all firms acquire the asset is never sustained as an equilibrium. The reason, as noted before, lies in the instability of the grand coalition, because the threat to go to a singleton competition structure is not credible if they all have acquired the asset. Hence, firms could avoid to acquire the asset in order to keep the credible threat of singleton competition.

It is common wisdom that a certain degree of monopoly power is necessary for a firm to grow and become more efficient. Our model, however, shows that this is not always true. Even if the equilibrium is characterized by a monopoly market structure, only one firm acquires the cost-reducing asset.

We believe[19] that only when the impact of the asset is very large, we may have an equilibrium characterized by monopoly and full efficiency (all firms acquire the asset). When all firms acquire the asset, the grand coalition is stable, i.e. the synergy prevails on the "Stigler" effect. On the other side, when the synergy effect is very small, the grand coalition is still stable, but for a different reason: when one player deviates the other two have no incentive to stick together because of the low synergy effect, and they end up in the singleton coalition; this would prevent each player from deviating.

A comparison between the equilibrium under the "equal" and the "weighted" sharing rule, shows that with the latter the cost level at which investing is not convenient for anybody is higher, from 3.75 to 7.85. However, with the "weighted" sharing rule there is no possibility of having more than one firm to invest. An intuitive argument for this result would be that the "weighted" rule while giving an higher return for the investment, hinders the stability of the grand coalition when two firms invest, so that the higher return can be enjoyed only when one firm invests.

[19]We have not formally shown this point. We leave this for future research.

5 Social Welfare and Policy implications

The welfare theory tells us that the maximum social welfare is achieved in perfect competition. We should, therefore, avoid monopoly structures. However, as shown in Motta [2004], the trade-off between number of firms in a market and welfare is not so clear. From a "static" point of view, the presence of economies of scale may offset the negative impact of monopolies on social welfare. For instance, in a market characterized by high fixed costs, the presence of many small firms does not allow to exploit the economies of scale. From a "dynamic" point of view the possibility to enjoy monopoly power provides the incentives to invest. In our model both issues are considered.

It seems that firms have two ways in which they can increase their monopoly power:

(a) Investing in a cost-saving asset, in order to become more efficient than the other firms, and gain a larger share of the market;

(b) merging with other firms.

These two strategies are not necessarily substitutes. It might be that firms both invest and decide to merge. Our model, however, shows that the two strategies have a certain degree of complementarity. In equilibrium, they all merge but only one firm invests.

In our model, however, we need to take into account the increase in efficiency resulting from the merging behavior of firms. The key point is how much of this efficiency gain is passed on to consumers. Notice, however, that the Subgame Perfect Equilibrium we obtain is characterized by monopoly market structure where at maximum one firm invests. This situation cannot be Pareto superior to a monopoly market structure in which more than one firm invests.

Furthermore, our model seems to suggest that in order to increase social welfare a government policy should be directed towards restrictions on the possibility to merge more than subsidizing investments. The latter policy has no effect in equilibrium, because even if the government reduces the cost of the investment, only one firm invests.

We leave for future research the analysis of the effect on the equilibrium of an anti-trust law that may reduce the possibility to merge. Intuitively, once the grand coalition is blocked by law, other equilibria may emerge in which more than one firm acquires the asset.

6 Concluding comments

We presented a model of mergers and acquisitions in which the investment decision of each firm is endogenous. The result we obtain in this case is that in equilibrium, under some conditions, only one firm invest and they all decide to merge in a monopolistic firm. The results are mainly driven by the interaction of two forces: the Stiglitz effect, and the synergy effect. The interaction of these two forces is captured by the demand parameter A and the marginal impact of the cost reducing investment α. We characterized the equilibrium for an intermediate range of these values, in which there is no force which prevails on the other.

The equilibrium we obtained is characterized by a monopoly structure (the three firms merge), while at most one firm invests. It is interesting to note that even if the cost of the investment tends to zero still only one firm invests.

This result seems to suggest that subsidizing firms does not provide any further incentive to invest. More effective would be an anti-trust law which forbids the formation of a monopoly market structure.

The policy implication should be weighted with the restrictions we have imposed in our model, mainly the presence of only three firms with no possibility for new firms to enter.

We leave for future research the extension of the model to more than three firms and the analysis of different way in which the investment affects the efficiency of the coalition.

A Appendix

In this appendix, we present the full model under two different sharing rules: the equal sharing rule and the weighted sharing rule.

A.1 Equal sharing rule: Example 1

We present a game in which the grand coalition does not form when all firms invest. We now consider and example which satisfies proposition 1, i.e. the two effects balance $19.38 \leq A \leq 31.97$. In appendix A.2 we present an example in which the competition effect prevails and, therefore, the grand coalition is stable, as in Ray and Vohra [1997] and Bloch [1996]. Let us consider the following demand and marginal cost functions

$$\begin{aligned} P &= 30 - Q \\ c_s &= 10 - \sum_{i \in s} x_i \end{aligned}$$

where π represents per member payoff, while Π represents total payoff (industry profit). In this example, the marginal effect of the investment is $\alpha = 1$.

Everybody invests $\Rightarrow x_i = 1 \, \forall \, i$

Coalition Structure		Cost Function	PerCapita Profit	Tot Profit
$\mathcal{P}_1$	$\{1,2,3\}$	$c = 7$	$\bar{\pi} = 44.08\bar{3}$	$\Pi = 123.25$
$\mathcal{P}_2$	**{1,2} {3}**	$c_1 = 8$ $c_2 = 9$	$\pi'' = 29.3\bar{8}$ $\pi' = 44.\bar{4}$	$\Pi = 103.\bar{2}$
$\mathcal{P}^*$	$\{1\}\{2\}\{3\}$	$c^* = 9$	$\pi^* = 27.5625$	$\Pi = 82.6875$

The grand coalition fails to form. This is important because the first best is not achieved. Since firms are symmetric it does not matter who is in the coalition of two firms, once one of the three firms decides to deviate the other two cannot do better than stick together. It would be more appropriate to say that the stable coalition structure is any permutation[20] of the structure [{1,2} {3}].

[20] In combinatory calculus, this is actually a *combination* of 3 firms, two at a time, $c(3,2)$.

No investment $\Rightarrow x_i = 0 \, \forall \, i$

Coalition Structure		Cost Function	PerCapita Profit	Tot Profit
$\mathcal{P}_1$	**{1,2,3}**	$c = 10$	$\bar{\pi} = 33.\bar{3}$	$\Pi = 100$
$\mathcal{P}_2$	{1,2} {3}	$c_1 = c_2 = 10$	$\pi'' = 22.\bar{2}$ $\pi' = 44.\bar{4}$	$\Pi = 88.\bar{8}$
$\mathcal{P}^*$	{1}{2}{3}	$c^* = 10$	$\pi^* = 25$	$\Pi = 75$

This is the same as in Ray and Vohra [1997], with three firms the grand coalition is stable.

ONE Investor $\Rightarrow x_1 = 1 \; x_2 = x_3 = 0$

Coalition Structure		Cost Function	PerCapita Profit	Tot Profit
$\mathcal{P}_1$	**{1,2,3}**	$c = 9$	$\bar{\pi} = 36.75$	$\Pi = 110.25$
$\mathcal{P}_2$	{1,2} {3}	$c_1 = 9$ $c_2 = 10$	$\pi'' = 26.\bar{8}$ $\pi' = 40.\bar{1}$	$\Pi = 93.\bar{8}$
$\mathcal{P}_3$	{1} {2,3}	$c_1 = 9$ $c_2 = 9$	$\pi' = 53.\bar{7}$ $\pi'' = 20.0\bar{5}$	$\Pi = 93.\bar{8}$
$\mathcal{P}^*$	{1}{2}{3}	$c_1 = 9$ $c_2 = c_3 = 10$	$\pi_1 = 33.0625$ $\pi_2 = \pi_3 = 22.5625$	$\Pi = 78.1875$

Players are no longer symmetric, so coalition structure $\mathcal{P}_2$ is different than coalition structure $\mathcal{P}_3$. The grand coalition is stable, even if one firm makes a cost-saving investment. That means firm 2 and firm 3 can free ride on firm 1.

TWO Investors $\Rightarrow x_1 = x_2 = 1 \; x_3 = 0$

Coalition Structure		Cost Function	PerCapita Profit	Tot Profit
$\mathcal{P}_1$	**{1,2,3}**	$c = 8$	$\bar{\pi} = 40.\bar{3}$	$\Pi = 121$
$\mathcal{P}_2$	{1,2} {3}	$c_1 = 8$ $c_2 = 10$	$\pi'' = 32$ $\pi' = 36$	$\Pi = 100$
$\mathcal{P}_3$	{1} {2,3}	$c_1 = 9$ $c_2 = 9$	$\pi' = 49$ $\pi'' = 24.5$	$\Pi = 98$
$\mathcal{P}^*$	{1}{2}{3}	$c_1 = c_2 = 9$ $c_3 = 10$	$\pi_1 = \pi_2 = 30.25$ $\pi_3 = 20.25$	$\Pi = 80.75$

Also in this case the grand coalition is stable, and firm 3 can free ride on the other two firms.

In this simple example two things are worth noticing,

1. the grand coalition fails to form when everybody invests,

2. the grand coalition forms even if (and only if) some firms free ride.

The stability of the grand coalition depends on the investment decision and not only on the Cournot competition game. With only three firms we may or may not have a stable grand coalition, depending on the investment decision of firms at stage 1.

We now turn to check the incentives to invest of the three firms given the outcome of the coalition formation game. Remember that the cost of investment is the same for everybody and equal to ϕ. The following table summarizes the payoff of each firm in every possible investment scenario of the game.

Investment decision	Stable Coalition Structure	Individual payoff
All invest	{i,j} {k}	$(29.3\bar{8})$ $(44.\bar{4})$
No investment	{i,j,k}	$33.\bar{3}$
ONE invests $x_i = 1$	{i,j,k}	36.75
TWO invest $x_i = x_j = 1$	{i,j,k}	$40.\bar{3}$

Remember that firms are ex-ante symmetric and that they simultaneously make the investment decision. The **SPE** depends on the level of the investment cost ϕ.

Proposition 7 *The Subgame Perfect Equilibrium of the game $\hat{\Gamma}(x)$ is given by the following strategy profile:*

- ▷ *for $\phi < 3.45 \Longrightarrow$ at date 1, two firms invest (i=1, j=1, k=0), at date 2, the grand coalition $\{i,j,k\}$ forms;*

- ▷ *for $3.45 < \phi < 3.55 \Longrightarrow$ there are two equilibria:*

(i) *at date 1, only one firm invests (i=0, j=1, k=0), at date 2, the grand coalition ({i,j,k}) forms;*

(ii) *at date 1, nobody invests (i=0, j=0, k=0), at date 2, the grand coalition ({i,j,k}) forms;*

▷ *for* $\phi > 3.75 \Longrightarrow$ *at date 1, nobody invests (i=0, j=0, k=0), at date 2 the grand coalition ({i,j,k}) forms.*

Proof. To prove that this is a strategy profile which sustains a SPE, we need to check that in each stage the strategy profile implies a Nash equilibrium of the simultaneous stage game. Note, however, that in the second stage we use the concept of stable coalition structure, instead of the Nash equilibrium. The equilibrium in the second and third stage was showed before. Here we focus on the equilibrium in the first stage given the equilibrium in the other two stages. This is a Nash equilibrium of a complete and perfect information game, so we omit the formal proof. ■

In equilibrium either nobody invests or at least one firm free rides, but the grand coalition always forms. Even if we cannot guarantee the uniqueness of the equilibrium for any level of ϕ, we have a unique SPE for large and small investment cost. Large investment costs intuitively discourage firms to invest.

Remark 2 *In case everybody invests, the grand coalition fails to form. Only if at least one firm does not invest the grand coalition forms. So, the grand coalition may fail to form even with only 3 firms, while in Ray and Vohra [1997], for the grand coalition to fail a large number of firms is needed,* $n > 4$*.*

A.2 Equal sharing rule: Exercise 2

This exercise shows that, when the demand is quite small compared to the cost structure, the efficiency effect prevails on the competition effect. We consider $A < 19.38$. Let us consider the following demand and marginal cost functions:

$$\begin{aligned} P &= 10 - Q \\ c_s &= 4 - \sum_{i \in s} x_i \end{aligned}$$

Symmetric investment: $x_1 = x_2 = x_3 = 1$

Coalition Structure	MC	Profit per capita	Total profit
{1,2,3}	$c = 1$	$\pi = 6.75$	20.25
{1,2} {3}	$c_1 = 2$ $c_2 = 3$	$\pi_1 = 4.5$ $\pi_2 = 4$	13
{1} {2} {3}	$c_1 = c_2 = c_3 = 3$	$\pi_1 = \pi_2 = \pi_3 = 3.06$	9.18

The stable solution is the grand coalition. In this case, even if firms are symmetric, the *efficiency* effect prevails on the *competition* effect. In contrast with Bloch [1996], not only the number of coalitions matters but also the size of the coalition. Coalitions have both a positive and a negative effect, according to the definition of externality proposed by Yi [1997].

No investment: $x_1 = x_2 = x_3 = 0$ **[Bloch, 1996]**

Coalition Structure	MC	Profit per capita	Total profit
{1,2,3}	$c = 4$	$\pi = 3$	9
{1,2} {3}	$c_1 = 4$ $c_2 = 4$	$\pi_1 = 2$ $\pi_2 = 4$	8
{1} {2} {3}	$c_1 = c_2 = c_3 = 4$	$\pi_1 = \pi_2 = \pi_3 = 2.25$	6.75

The stable coalition is the grand coalition.

Asymmetric investment: $x_1 = 1$ $x_2 = x_3 = 0$

Coalition Structure	MC	Profit per capita
{1,2,3}	$c = 3$	$\pi = 4.08$
{1,2} {3}	$c_1 = 3$ $c_2 = 4$	$\pi_1 = 3.55$ $\pi_2 = 2.77$
{1,3} {2}	$c_1 = 3$ $c_2 = 4$	$\pi_1 = 3.55$ $\pi_2 = 2.77$
{2,3} {1}	$c_1 = 4$ $c_2 = 3$	$\pi_1 = 1.38$ $\pi_2 = 7.11$
{1} {2} {3}	$c_1 = 3$ $c_2 = 4$ $c_3 = 4$	$\pi_1 = 5.0625$ $\pi_2 = \pi_3 = 1.5625$

The stable coalition structure is the singleton solution, this is because firm 1 can create a coalition more efficiently than the other two firms and can therefore exploit the Cournot competition at his favour.

Asymmetric investment: $x_1 = x_2 = 1$ $x_3 = 0$

Coalition Structure	MC	Profit per capita
{1,2,3}	$c = 2$	$\pi = 5.33$
{1,2} {3}	$c_1 = 2$ $c_2 = 4$	$\pi_1 = 5.55$ $\pi_2 = 1.77$
{1,3} {2}	$c_1 = 3$ $c_2 = 3$	$\pi_1 = 2.72$ $\pi_2 = 5.44$
{2,3} {1}	$c_1 = 3$ $c_2 = 3$	$\pi_1 = 2.72$ $\pi_2 = 5.44$
{1} {2} {3}	$c_1 = 3$ $c_2 = 3$ $c_3 = 4$	$\pi_1 = \pi_2 = 4$ $\pi_3 = 1$

The stable coalition structure is characterized by the most efficient firms joining the same coalition. In this case, a firm with an investment prefers to stay with another firm with an investment and than compete with the other firms because they will be much more efficient with respect to the other firms.

In this simple reduced form of the model it emerges that firms are willing to give up monopoly profits if they can form a coalition structure such that the most efficient firms are in the same coalition.

Let us consider the investment decision of each firm at date 1.

Proposition 8 *The outcome of the unique Subgame Nash Equilibrium of game*

Γ^0 *is the grand coalition for any level of* ϕ*, with the SPE characterized by (i)* $\{a_1 = a_2 = a_3 = 1\}$ *if* $\phi \leq 3.75$ *and (ii)* $\{a_1 = a_2 = a_3 = 0\}$ *if* $\phi > 3.75$.

The reason is that firms prefer to invest if their net outcome is higher than what they can get by not investing.

A.3 Weighted sharing rule

We now present the same exercise as in the previous section, using a different sharing rule: the weighted sharing rule. We keep the assumption $\alpha = 1$.

Coalition Structure		Cost Function	PerCapita Profit	Tot Profit
$\mathcal{P}_1$	**{1,2,3}**	$c = 9$	$\pi_1 = 41.18$ $\pi_2 = \pi_3 = 34.54$	$\Pi = 110.25$
$\mathcal{P}_2$	{1,2} {3}	$c_1 = 9$ $c_2 = 10$	$\pi_1 = 32.33$ $\pi_2 = 21.45$ $\pi_3 = 40$	$\Pi = 93.\bar{8}$
$\mathcal{P}_3$	{1} {2,3}	$c_1 = 9$ $c_2 = 9$	$\pi_1 = 53.\bar{7}$ $\pi_2 = \pi_3 = 20.0\bar{5}$	$\Pi = 93.\bar{8}$
$\mathcal{P}^*$	{1}{2}{3}	$c_1 = 9$ $c_2 = c_3 = 10$	$\pi_1 = 33.06$ $\pi_2 = \pi_3 = 22.56$	$\Pi = 78.19$

Notice a slight change in notation: with π_i we denote the payoff of firm i. This change is necessary because members of a coalition may have different payoffs. The main difference with the previous game is the non equal division of the worth in the grand coalition. Since firm 1 invested in the previous stage, she gets more than the other firms. However, the grand coalition is still stable.

TWO Investors $\Rightarrow x_1 = 1$ $x_2 = 1 x_3 = 0$

Coalition Structure		Cost Function	PerCapita Profit	Tot Profit
$\mathcal{P}_1$	{1,2,3}	$c = 10 - 2\alpha$	$\pi_1 = \pi_2 = 43.08$ $\pi_3 = 34.83$	$\Pi = 121$
$\mathcal{P}_2$	**{1,2} {3}**	$c_{12} = 10 - 2\alpha$ $c_3 = 10$	$\pi_1 = \pi_2 = 32$ $\pi_3 = 36$	$\Pi = 100$
$\mathcal{P}_3$	{1,3} {2}	$c_{13} = 10 - \alpha$ $c_2 = 10 - \alpha$	$\pi_1 = 29.66$ $\pi_3 = 19.34$ $\pi_2 = 49$	$\Pi = 98$
$\mathcal{P}^*$	{1}{2}{3}	$c_1 = c_2 = 10 - \alpha$ $c_3 = 10$	$\pi_1 = \pi_2 = 30.25$ $\pi_3 = 20.25$	$\Pi = 80.75$

Two firms have acquired the cost-saving asset. The grand coalition is not stable because the two efficient firms prefer to stay together rather than separate.

This is actually exploited by the inefficient firm that would deviate in the grand coalition. Notice that in case of an "equal" sharing rule, the grand coalition would be stable, because for the inefficient firm it would be no more profitable to deviate.

In this case, we can draw two main conclusions, firms tend to merge with firms which have acquire an asset in order to exploit the synergy. The same synergy, however, makes it profitable for the inefficient firm to deviate from the grand coalition.

Investment decision. We now consider the first stage of the game, where firms simultaneously decide whether to invest. In the next table we present a summary of firms' payoff in each stable coalition structure.

Investment decision	Stable Coalition Structure	Individual payoff
All invest	{i,j} {k}	$(\pi_i = \pi_j = 29.3\bar{8})$ $\pi_k = 44.\bar{4}$
No investment	{i,j,k}	$\pi_1 = \pi_2 = \pi_3 = 33.\bar{3}$
ONE invests $x_i = 1$	{i,j,k}	$\pi_1 = 41.18$ $(\pi_2 = \pi_3 = 34.54)$
TWO invest $x_i = x_j = 1$	{i,j} {k}	$(\pi_1 = \pi_2 = 32)$ $\pi_3 = 36$

The difference with the outcome of the "equal" sharing rule, relates to the two asymmetric situations: one invests and two invest. When only one firm invests the grand coalition is still stable, but there is a different distribution of worth among firms. When two firms invest the grand coalition is no longer stable.

Given the cost of investment ϕ, we have the following SPE,

Proposition 9 *The Subgame Perfect equilibrium of the game* $\Gamma(N = 3, x, \sigma(S))$ *depends on the cost of investment* ϕ*,*

- ▷ *if* $\phi < 7.85$*, at date 1 only one firm invests, at date 2 the grand coalition forms;*
- ▷ *if* $\phi > 7.85$*, at date 1 nobody invests, at date 2 the grand coalition forms.*

A comparison between the equilibrium under the "equal" and the "weighted" sharing rule, shows that with the latter the cost level at which investing is not convenient for anybody is higher, from 3.75 to 7.85. However, with the "weighted" sharing rule there is no possibility of having more than one firm to acquire the cost-saving technology.

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