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Abstract

Holmstrom (1982) argues that a principal is required to restrain moral hazard in a team: wasting output in a certain state is required to enforce efficient effort, and the principal is a commitment device for such enforcement. Under competition in commodity and team-formation markets, I extend his model *à la* Prescott and Townsend (1984) to show that competitive contracts can exploit the futures market to transfer output across states instead of wasting it. Thus, the futures market replaces the role of principals. An important feature of such contracts is exclusiveness: private access to the the futures market by team members is not allowed. The duality of linear programming is exploited to characterize a market environment and the contractual agreements for efficiency.

JEL Classification: D53, D71, D86, L22

Keywords: general equilibrium, team, contract theory, futures market, duality of linear programming

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1 Introduction

Holmstrom (1982) argues that a principal is required to implement efficient effort in a team facing a moral hazard problem. Given budget balance, the incentive compatibility constraint for efficient effort may not be satisfied. Therefore, waste penalty at the low output state is required to widen the gap between consumption in the high output and the low output states. To enforce the waste, a principal is brought into the team as a recipient of the waste; hence, the principal is referred to as a "budget breaker."

The main questions of the current paper are: can such output-wasting contracts survive competition in commodity and team-formation markets? If not, what kinds of characteristics do the surviving contracts have? To answer the questions, Holmstrom's model is extended *à la* Prescott and Townsend (1984). A finite number of individuals form a team in order to produce stochastic output. Output is observed, but effort is not; hence, teams have to deal with the classic moral hazard problem. I characterize the efficient assignment of individuals to teams and the efficient allocation of commodities to individuals, then I describe the markets and the contracts that are required for efficiency.

Since teams are not divisible, randomized matching improves efficiency.[1] A lottery market is required to implement such randomization. For no arbitrage condition in the lottery market, contract arbitrageurs are derived from the model. They design contracts, and trade them with individuals for profit. With free entry, contract arbitrageurs will end up with zero profit in equilibrium.

In the aforementioned economy, I find that Holmstrom's contract is no longer optimal. Trade of teams' uncertain future output (or, equivalently, the futures market trade[2]) makes it possible to widen the gap between consumption, for example, at high and low output states without breaking the budget balance. Instead of being wasted, the output is transferred to another state where more reward to the members is required; hence, I show a positive efficiency implication of the futures market. The mechanism of the futures market trade and that of Holmstrom are similar: both

[1] For example, Rogerson (1988) shows that randomized assignment of labor can improve efficiency due to the indivisibility of labor.

[2] "Contingent claims trade" commonly refers to trade of contingencies due to exogenous shock. Because the uncertainty of team's output depends on members' effort, the term "futures market" is used instead.

widen the consumption gap between the states. However, the organizational implications are quite different: no principal is required for the commitment in the current framework since there is no waste. Another characteristic of efficient contracts is exclusiveness: members of a team are not allowed to have private access to the futures market. It is known in the literature that private access to a futures market is inefficient.[3] But, the novelty here is to derive the exclusiveness from the model.

I analyze the problem using the duality of linear programming. This dual approach is based on a line of works: Makowski and Ostroy (1996, 2003) and Rahman (2005). I extend their methodology by dropping the assumption of quasi-linear utilities. I do this by finding a correct weight for each individual in the planner's problem, so that transferable utility can be interpreted as non-transferable utility.[4]

Prescott and Townsend (2006) look at an economic environment that is similar to the present paper. However, there are several differences. Firstly, my paper focuses on the characterization of contracts rather than the types of firms in equilibrium. Secondly, the derivation of the economic environment through the duality of linear programming reveals the intuition hidden behind the fixed point theorem. The obtained intuition makes it easy to prove the exclusiveness of contracts and the existence of equilibrium. Finally, I detail a model with heterogeneous individuals.

Other researchers looked at similar economies. Cole and Prescott (1997) show how the classical results from the competitive analysis of convex finite-agent economies can be reinterpreted to apply to a team model with a lottery market. Ellickson, Grodal, Scotchmer, and Zame (1999, 2001) examine both continuum and finite club economies without a lottery trade. Zame (2006) extends the club economy to allow for information asymmetry without the exclusiveness. Jerez (2003, 2005) identifies welfare effects associated with incentives, and extends the results into a general equilibrium model.

Section 2 provides two motivating examples that illustrate a few of the results. Section 3 builds

[3]For example, see Fisher (1992), Tommasi and Weinschelbaum (2004), Rothschild and Stiglitz (1976), Prescott and Townsend (1984), and Bisin and Gottardi (2006).

[4]This technique is based on Shapley (1969), but the added market interpretation complicates the proof. Negishi (1960) developed a similar idea.

a model. Section 4 characterizes contracts. My conclusion summarizes the results.

2 Examples

One of the main points in Holmstrom (1982) is that a principal is required to restrain moral hazard problem in a team. The following example summarizes his argument.

Example 1. [Holmstrom (1982)] Team T consisting of two identical individuals (1 and 2) can have one of two possible outcomes, $q_L = 1$ or $q_H = 2$. The realization probability of $q \in \{q_H, q_L\}$ is determined by effort of individuals, i.e. the probability is denoted by $\varphi_T(q; e_T)$ where $e_T := (e_1, e_2)$ represents the vector of individuals' effort $e_i \in \{e_H, e_L\}$: individuals can choose high effort or low effort. Let the probabilities to be $\varphi_T(q_H; e_H, e_H) = 0.7$, $\varphi_T(q_H; e_H, e_L) = \varphi_T(q_H; e_L, e_H) = 0.6$, $\varphi_T(q_H; e_L, e_L) = 0.5$. The utility function of each individual i in terms of consumption and effort is $z_i - C(e_i)$, where $C(e_H) = 0.06$ and $C(e_L) = 0$. Expected utility is

$$\sum_{q \in \{q_L, q_H\}} z_i(q)\varphi_T(q; e_T) - C_i(e_i)$$

where $z_i(q)$ is consumption at outcome $q \in \{q_L, q_H\}$.

(e_H, e_H) is the most efficient assignment of effort.[5] Effort is not observable or not contractible; therefore, incentive compatibility constraint for (e_H, e_H) is

$$\sum_{q \in \{q_L, q_H\}} z_i(q)\varphi_T(q; (e_H, e_H)) - C(e_H) \geq \sum_{q \in \{q_L, q_H\}} z_i(q)\varphi_T(q; (e_L, e_H)) - C(e_L).$$

However, split-the-output contract ($z_i(q) = q/2$) does not satisfy the incentive compatibility constraint, i.e.

$$\sum_{q \in \{q_L, q_H\}} \frac{q}{2}\varphi_T(q; (e_H, e_H)) - C(e_H) < \sum_{q \in \{q_L, q_H\}} \frac{q}{2}\varphi_T(q; (e_L, e_H)) - C(e_L).$$

[5]For (e_H, e_H) (or (e_H, e_L)), the cost of effort, $2 \times 0.06 = 0.12$ (or 0.06), is smaller than the benefit of the more efficient probability, $(\varphi_T(q_H; e_H, e_H) - \varphi_T(q_H; e_L, e_L))(q_H - q_L) = 0.2$ (or $(\varphi_T(q_H; e_H, e_H) - \varphi_T(q_H; e_H, e_L))(q_H - q_L) = 0.1$). Therefore, the benefit of implementing (e_H, e_H) (or (e_H, e_L)) over (e_L, e_L) is 0.08 (or 0.04).

In other words, gap between $q_H/2$ and $q_L/2$ is not sufficiently large to implement (e_H, e_H). In fact, it is easily verified that (e_L, e_L) is the only implementable effort with the split-the-output contract[6].

However, by wasting output at state q_L, efficient effort (e_H, e_H) can be implemented, i.e.

$$\sum_{q\in\{q_L,q_H\}} z(q)\varphi_T(q;(e_H,e_H)) - C(e_H) \geq \sum_{q\in\{q_L,q_H\}} z(q)\varphi_T(q;(e_L,e_H)) - C(e_L)$$

when $z(q_H) = q_H/2$, $z(q_L) = (q_L - W)/2$, and $W \geq 0.2$. W represents the waste at state q_L.

Note that, for example at $W = 0.2$, the waste strictly improves welfare since

$$\begin{aligned}\varphi_T(q_H; e_H, e_H)\frac{q_H}{2} + \varphi_T(q_L; e_H, e_H)\frac{q_L - W}{2} - C(e_H) = 0.76 \\ > 0.75 = \varphi_T(q_H; e_L, e_L)\frac{q_H}{2} + \varphi_T(q_L; e_L, e_L)\frac{q_L}{2} - C(e_L).\end{aligned}$$

However, once q_L is realized, individuals want to renegotiate over the waste since the waste is not efficient *ex-post*. Seeing this renegotiation possibility, the commitment of the waste is not credible. Holmstrom argues that, if a third party (a principal) receives the waste, the team members and the principal have conflicting interests; the renegotiation possibility is eliminated. Therefore, the efficiency is recovered. ∎

The team of Holmstrom is in an isolated environment where it does not or cannot trade with the outside. Suppose that there are many teams in the economy, and that teams can trade in a futures market of commodity. A different result can be shown as in the following example.

Example 2. There are continuum of identical teams of mass 1. The technology of the teams is identical to that of Example 1. Consider a benevolent planner taking $\epsilon\frac{\varphi_T(q_H;\cdot)}{\varphi_T(q_L;\cdot)}$ amount of resource from the teams with q_L and giving ϵ to the teams with q_H; hence, the effective disposable output becomes $q \in \{q_H + \epsilon, q_L - \epsilon\frac{\varphi_T(q_H;\cdot)}{\varphi_T(q_L;\cdot)}\}$ for each team. The transfer satisfies the resource constraint[7] since

$$q_H\varphi_T(q_H;\cdot) + q_L\varphi_T(q_L;\cdot) = (q_H + \epsilon)\varphi_T(q_H;\cdot) + \left(q_L - \epsilon\frac{\varphi_T(q_H;\cdot)}{\varphi_T(q_L;\cdot)}\right)\varphi_T(q_L;\cdot).$$

[6] (e_H, e_L) can be implemented by an asymmetric output sharing; but, it is still less efficient than (e_H, e_H).

[7] Because of the assumption of continuum and the Law of large number, the expected resource for each team is mathematically identical to the actual resource in the economy.

For large enough ϵ, the incentive compatibility constraint for (e_H, e_H) can be achieved, i.e.

$$\sum_{q\in\{q_L,q_H\}} z(q)\varphi_T(q;(e_H,e_H)) - C(e_H) \geq \sum_{q\in\{q_L,q_H\}} z(q)\varphi_T(q;(e_L,e_H)) - C(e_L)$$

where $z(q_H) = (q_H + \epsilon)/2$ and $z(q_L) = \left(q_L - \epsilon\frac{\varphi_T(q_H;e_H,e_H)}{\varphi_T(q_L;e_H,e_H)}\right)\Big/ 2$. ∎

The transfer of resource can be achieved by the futures market trade of teams.[8] Therefore, Example 2 shows that, if there is a continuum of identical teams, the resource does not have to be wasted. The most crucial point for efficiency[9] is to create income stream of ϵ for state q_H and $-\epsilon\frac{\varphi_T(q_H;e_H,e_H)}{\varphi_T(q_L;e_H,e_H)}$ for state q_L. Considering the vast possibility of the futures market trade in the complex real world, the team would be able to widen the consumption gap by a certain portfolio of financial instruments; hence, the example shows a positive efficiency implication of the futures market. Since there is no waste, there is no need for a principal. The futures market replaces the role of principals as a commitment device; hence, the example shows an organizational implication of the futures market.

Note that individuals are not allowed to trade commodities privately in the futures market. Suppose that individuals could purchase insurance to achieve consumption smoothing. The incentive compatibility constraint would not hold anymore. Private access to the futures market or an insurance destroys the incentive compatibility constraints; hence, the example shows the contractual agreement prohibiting individuals to trade commodities privately in the futures market.

3 A Contractual Team-Formation Problem

Heterogeneous individuals form teams to produce stochastic output. The probability of output is dependent upon the effort of the team members. Since effort is non-observable or non-contractible, there is a moral hazard problem. A team economy is detailed in Section 3.1 along with the definition

[8]Relative price of output at state q_L and state q_H for such futures trade is $\frac{p_L}{p_H} = \frac{\varphi_T(q_L;e_H,e_H)}{\varphi_T(q_H;e_H,e_H)}$.

[9]The utility function was assumed to be linear; hence, the first best is achieved since there is no welfare loss due to the lack of consumption smoothing. If it is assumed to be strictly concave, ϵ should be large enough to make the incentive compatibility constraint binding.

of an efficient assignment of individuals and an efficient allocation of commodities. Characterization of an idealized market environment for efficiency is stated in Section 3.2.

3.1 Description of Team Production and Planner's Problem

Assignments of Team and Effort: There are continuums of individuals with finite types. Each type is of mass 1. A typical type is denoted by $i \in N$, where N denotes the set of types. Let $\Omega = 2^N \setminus \{\emptyset\} = \{T | T \subset N, T \neq \emptyset\}$ to be the set of all the possible teams. When one of each $i \in T$ are matched together, it is said that *team T is formed*.

Effort implemented in team T is denoted by $e_T := (e_i)_{i \in T} \in \mathcal{E}^{|T|}$, where $\mathcal{E}$ is a finite set of possible effort[10]. I write (T, e_T) to designate team T with effort e_T. Individual i is said to be *assigned* to team (T, e_T) when i is in team T implementing effort e_T.

Commodity, Output, Technology: There are L commodities: the commodity space is $\mathbb{R}^L$. For given effort e_T in team T, output q is produced with probability $\varphi_T(q; e_T)$. Support of q is finite, $Q = \{1, \ldots, q, \ldots, Q\} \subset \mathbb{R}^L_+$.[11] In other words, the realization probability of output q depends on team T and effort e_T. So, probability $\varphi_T(q; e_T)$ defines technology of a team. Note that a single person team, $T = \{i\}$, also has a production technology.

Allocation and Utility function: $z_i(q) \in \mathbb{R}^L$ is allocation of commodities to i at output q. Note that some components of $z_i(q)$ can be negative. $z_i := (z_i(q))_{q \in Q} \in \mathbb{R}^{L \times |Q|}$ is allocation profile for all output states. $z_T := (z_i)_{i \in T} \in \mathbb{R}^{L \times |Q| \times |T|}$ is allocation profile for all the members of team T.

Expected utility of i for given (T, e_T, z_T) is

$$\sum_{q \in Q} v_i(z_i(q)) \varphi_T(q; e_T) - C_i(e_i)$$

where $v_i(\cdot)$ is the utility function and $C_i(e_i)$ is dis-utility of making effort e_i.

Probabilities: The planner assigns individuals to teams and allocate commodities to the individuals probabilistically. The probabilistic assignment improves efficiency by convexifying non-convex

[10] It is without loss of generality to assume identical support of effort for all individuals. If individual i cannot choose one $e \in \mathcal{E}$, the assumption of the infinite costs for i to choose e avoids such an assignment.

[11] Output can be in set $\mathbb{R}^L$ for further generalization, where negative components of vector q are inputs.

domain due to the indivisible structure of teams. For example, Rogerson (1988) shows that randomized assignment of labor can improve efficiency due to the indivisibility of labor.

Define $x_i(T, e_T, z_T)$ to be the probability of assigning type i to (T, e_T) and allocating commodities z_T to i.[12] Suppose $x_i(T, e_T, z_T) = x_i(T, e_T, z'_T) = x_i(T, e'_T, z''_T) = x_i(\{i\}, e''_T, z'''_T) = 1/4$. In the probabilistic assignment/allocaion, individual i is assigned to (T, e_T, z_T) with probability 1/4, to (T, e_T, z'_T) with probability 1/4, to (T, e'_T, z''_T) with probability 1/4, or to form one person team $(\{i\}, e''_T)$ with commodities z'''_T with probability 1/4.

Note that the probability is also interpreted as fraction of individuals due to the Law of large numbers and the assumption of mass 1 of each type. However, the assumption is not crucial in analyzing the model: with mass r_i of type i, the probability multiplied by r_i would be the same to the fraction. And, the qualitative characteristics would not change. "Fraction", "mass", and "probability" are used interchangeably from now on. The planner face the following constraint.[13]

$$\sum_{T \in \mathcal{T}_i, e_T} \sum_{z_T} x_i(T, e_T, z_T) = 1, \forall i \in N \tag{1}$$

where $\mathcal{T}_i := \{T \subset N | i \in T\}$.

Consistency (Team-formation Constraints): Suppose fraction (or the probability) of i being matched with $(\{i, j\}, (e_i, e_j), (z_i, z_j))$ is 1/3. The fraction (or probability) of j being matched with the team has to be 1/3 too. In other words, the probabilities have to be consistent across population. Let $x_T(e_T, z_T)$ to be the mass of team T with (e_T, z_T). Then the following formalizes the consistency of team-formation probabilities.

$$x_i(T, e_T, z_T) = x_T(e_T, z_T), \forall i, T, e_T, z_T \tag{2}$$

Resource Constraint and Incentive Compatibility Constraints: Since the aggregate uncer-

[12]Note that only z_i in z_T does influence i's utility. Another formulation based on probability $x_i(T, e_T, z_i)$ is found in Song (2006). The convention here is adopted for simplicity of model description.

[13]Technically speaking, $x_i(\cdot)$ is a probability density function since the domain is a continuum set. However, they are considered to be probability mass functions. Moreover, they actually have finite support by Carathéodory's Theorem. Technical details are in appendices.

tainty is washed out by the Law of large numbers, the resource constraint can be written as

$$\sum_T \sum_{e_T} \sum_{z_T} \sum_q \left[\sum_{i \in T} z_i(q) \right] \varphi_T(q; e_T) x_T(e_T, z_T) \leq \sum_T \sum_{e_T} \sum_{z_T} \sum_q q \varphi_T(q; e_T) x_T(e_T, z_T) \tag{3}$$

The left hand side of the inequality is the allocation of commodities to all individuals, and the right hand side is the entire produce of the economy.

Define the following where $e'_i | e_T$ denotes e_T with e_i being replaced by e'_i.

$$DG_i(e'_i; T, e_T, z_i) := \left[\sum_q v_i(z_i(q)) \varphi_T(q; e'_i | e_T) - C_i(e'_i) \right] - \left[\sum_q v_i(z_i(q)) \varphi_T(q; e_T) - C_i(e_i) \right].$$

Deviation gain $DG_i(e'_i; T, e_T, z_i)$ represents the gain of utility by choosing effort e'_i instead of e_i. For the assignment/allocation to be incentive compatible, $DG_i(e'_i; T, e_T, z_i) \leq 0$ is required for all e'_i. The incentive compatibility condition is written as

$$DG_i(e'_i; T, e_T, z_i) x_T(e_T, z_T) \leq 0, \forall i, T : |T| > 1, z_T, e_T, e'_i. \tag{4}$$

Notice that, if $x_T(e_T, z_T) = 0$, $DG_i(e'_i; T, e_T, z_i) \leq 0$ does not have to hold. Single person teams do not have the moral hazard problem: this is to use the single person team as a benchmark case without the moral hazard problem.

Definition 1 *Probabilistic assignment/allocation* $(x_i(T, e_T, z_T), x_T(e_T, z_T))_{i \in T, T \in \Omega}$ *is* <u>*feasible*</u> *if it satisfies (1), (2), (3), (4),* $x_i(T, e_T, z_T) \geq 0, x_T(e_T, z_T) \geq 0$

Objective Function: The expected utility for individual i is

$$U_i(x_i(\cdot)) := \sum_{T \in \mathcal{T}_i, e_T} \sum_{z_T} \left[\sum_{q \in Q} v_i(z_i(q)) \varphi_T(s; e_T) - C_i(e_i) \right] x_i(T, e_T, z_T),$$

since $x_i(T, e_T, z_T)$ is the probability that individual i is in team (T, e_T) with commodities z_T. The objective function of the planner is the weighted sum of $U_i(x_i(\cdot))$ with $\lambda := (\lambda_i)_{i \in T}$,

$$\sum_{i \in N} \lambda_i U_i(x_i(\cdot)).$$

Definition 2 *A probabilistic assignment/allocation* $(x_i(\cdot), x_T(\cdot))_{i\in I, T\subset N}$ *is* *incentive constrained efficient* *if a probabilistic assignment/allocation solves the planner's problem, i.e.*

$$(x_i(\cdot), x_T(\cdot)) \in \operatorname{argmax}\left\{\sum_{i\in N} \lambda_i U_i(\tilde{x}_i(\cdot)) : (\tilde{x}_i(\cdot), \tilde{x}_T(\cdot)) \textit{ is feasible }\right\}$$

Assumption 1 *The domain of the planner's problem is not empty.*

A sufficient condition for Assumption 1 is the possibility of autarky. Then, the domain contains at least one assignment/allocation, the autarky. Alternatively, it can be assumed that everybody can become a single person team to make the classical exchange economy.

Proposition 1 *A solution of the planner's problem exists.*

Proof. See Appendix A.2 ∎

3.2 Decentralization of Efficient Assignment/Allocation

The dual linear programing of the planner's problem specifies a market environment replicating the solution of the planner. The environment includes goods (contracts and commodities), prices of the goods (Lindahl prices on the contracts and anonymous prices on the commodities), markets (the contract market, the futures market of commodities, the spot market of commodities), timing of the markets, contractual agreement (exclusiveness of contracts), randomization devices, and contract arbitrageurs. Detailed derivation of the market environment by the duality of linear programming is provided in Appendix A.1, and I state the market environment here without the derivation.

Contracts, Commodities, and Markets: Contract (T, e_T, z_T) specifies who are matched, what effort is, and what the payoffs are. The structure of team (T, e_T) has externality over the individuals inside the team, since the implemented effort enters the individuals' utilities through probability $\varphi_T(q; e_T)$. In that sense, individuals in a team "consume" the team as a public good. For the efficient allocation of the public good, it is known that Lindahl price is used; hence, prices of contracts are Lindahl ones denoted by $p_i(T, e_T, z_T)$.

The implementation of probabilistic assignment in a decentralized environment requires a market for probabilities, or a lottery market, as is often called. Suppose that individual i purchased 1/3 probability of contract (T, e_T, z_T). I write $x_i(T, e_T, z_T) = 1/3$, and $p_i(T, e_T, z_T)x_i(T, e_T, z_T)$ is the expenditure on the purchase. By combining the purchased probabilities , a lottery of individual i is formed. "Lottery is traded" and "probabilities are traded" are interchangeably used. Randomization devices are required to implement the lotteries. It is well-known that Lindahl prices could be negative. Nonetheless, I say that *i buys probability for (T, e_T, z_T) at price $p_i(T, e_T, z_T)$* even when $p_i(T, e_T, z_T) < 0$.

Commodities are priced by the spot market price vector $\phi \in \mathbb{R}^L_{++}$. In other words, the value of $z_i(q) \in \mathbb{R}^L$ is $\phi z_i(q)$ at the spot market. Accordingly, output q of team (T, e_T) has the value of $\phi\varphi_T(q; e_T)q$ in the futures market. The futures market and the spot market are cleared together after uncertainty on output q is resolved. Mixture of the futures and the spot markets is called *ex-post market*.

Contractual Agreement (Exclusiveness of Contracts):

Individuals are not allowed to trade commodities privately in the futures market. If the private access to the futures market were possible, they will try to smooth consumption over states; hence, the contracted effort cannot be enforced. This exclusiveness is further discussed in Section 4.2.

Players of the Economy: Individuals are pure price-takers: they maximize their utilities given prices of contracts, $p_i(T, e_T, z_T)$, without considering the incentive compatibility constraints.

For no arbitrage condition in the lottery market, perfectly competitive contract arbitrageurs are derived. Contract arbitrageurs are expected money maximizers. They design contract, (T, e_T, z_T), and trade it with individuals for profit. Their revenue by selling a contract is $\sum_{i \in T} p_i(T, e_T, z_T)$. They trade commodities in the futures market to ensure the delivery of commodities $\sum_{i \in T} z_i(q)$ for each output state q. Therefore, the total expenditure on the purchase of the futures is $\sum_q [\sum_{i \in T} z_i(q) - q]\phi\varphi_T(q; e_T)$. Note that the delivery of the futures trade is dependent upon output q: if team (T, e_T, z_T) is not formed, no commodities are delivered since no q is realized. Hence, their objective is to design contract (T, e_T, z_T) to maximize

$$\sum_{i \in T} p_i(T, e_T, z_T) - \sum_{q \in Q} [\sum_{i \in T} z_i(q) - q]\phi\varphi_T(q; e_T).$$

They are restricted to trade only incentive-compatible contracts. The reason of the restriction is described in Appendix A.1.4. Since contract arbitrageurs are under perfect competition, contracts sold on equilibrium would balance insurance and incentive optimally.

These contract arbitrageurs could be understood as firms: their profits are in principle redistributed to the agents in the economy, that is, they should be owned by the individuals in the economy. However, note that they are constant returns to scale firms: doubling the sales of contracts doubles the profit. Thus, contract arbitrageurs will end up with zero profit in equilibrium; hence, the specification of the redistribution of profits to individuals would be irrelevant.

The economy can be described without contract arbitrageurs as in the following: when individuals trade contracts, the members of a potential team (that is formed with certain probability by a common randomization device) trade commodities in the futures market to ensure the delivery of the contract. Therefore, trade of contracts becomes a complicated process of contracting with many potential members and of trading in the futures market at the same time. It is likely that brokers emerge to simplify the complicated trade; contract arbitrageurs are interpreted as such.

Timing of Markets: The timing of the markets and the players' behavior are summarized below.

The 1st stage: Individuals and contract arbitrageurs trade probabilities of contracts. Contract arbitrageurs trade also commodities in the futures market to be able to deliver what is promised in the contract.

The 2nd stage: Contracts are picked by randomization devices.

The 3rd stage: Individuals choose effort.

The 4th stage: Outcomes are realized, and commodities are awarded to individuals according to the contracts. The delivery of contract arbitrageurs' futures market trade and the spot market trade of single-person teams clear the ex-post market.

Formal definition of equilibrium is stated.

Definition 3 (Definition of Equilibrium) **1. Individual Optimization:** *Individuals buy probabilities of contracts at per unit-probability price of $p_i(T, e_T, z_T)$. Once contract (T, e_T, z_T) is picked by the randomization device, individuals join the team, and choose effort. After the realization of outcome q, commodities are awarded to the members depending on the outcome.*

Formally, the individuals' problems are

$$\max_{e_i'} \max_{\xi_i(T,e_T,z_T)} \sum_{(T,e_T,z_T)} \left[\sum_{\mathbf{s}} v_i(z_i(q))\varphi_T(q;e_i'|e_T) - C_i(e_i') \right] \xi_i(T,e_T,z_T)$$

$$s.t. \sum_{(T,e_T,z_T)} p_i(T,e_T,z_T)\xi_i(T,e_T,z_T) = 0 \quad and \quad \sum_{(T,e_T,z_T)} \xi_i(T,e_T,z_T) = 1$$

2. **Contract-arbitrageur's Optimization:** *Competitive contract arbitrageurs trade probabilities of contracts at price of $p_i(T,e_T,z_T)$. They also trade commodities in the futures market to prepare for the exercise of the contract. Formally, the contract arbitrageurs' problems are*

$$\max_{\xi_T(e_T,z_T)} \sum_{e_T,z_T} \left[\sum_{i\in T} p_i(T,e_T,z_T) - \phi \sum_q [\sum_{i\in T} z_i(q) - q]\varphi_T(q;e_T) \right] \xi_T(e_T,z_T)$$

$$s.t.\ \xi_T(e_T,z_T) > 0 \text{ only if } (T,e_T,z_T) \text{ is incentive compatible.}$$

3. **Clearance of Commodity Market:** *The ex-post market clears.*

$$\sum_T \sum_{e_T} \sum_{z_T} \sum_q [\sum_{i\in T} z_i(q) - q]\varphi_T(q;e_T)\xi_T(e_T,z_T) \leq 0$$

4. **Contractual Team Market Clearance:** *The contractual team-formation market clears in the sense that the purchased probabilities are consistent across the population.*

$$\xi_i(T,e_T,z_T) = \xi_T(e_T,z_T), \forall i \in T, T, e_T, z_T$$

Note that individuals face probability constraints $\sum \xi_i(T,e_T,z_T) = 1$ while contract arbitrageurs do not face such condition (i.e. the only restriction for contract arbitrageurs is $\xi_T(e_T,z_T) \in \mathbb{R}_+$); hence, the contract arbitrageurs are constant returns to scale firms. Also note that individuals are pure price-takers: they maximize their utilities given the price of contracts without considering the incentive compatibility constraint.

Theorem 1 (Welfare theorems) [**The first Welfare theorem**] *A competitive equilibrium is incentive-constrained efficient.* [**The second Welfare theorem**] *Planner's probabilistic assignment/allocation can be decentralized without any money transfer for some weight profile $\lambda \gg 0$ under a mild technical condition.*

Proof. See Appendix A.1. ∎

The mild technical condition in the second Welfare theorem is discussed in Song (2007).

4 Characterization of Contracts

4.1 Replacement of Principals by Futures Market

Holmstrom argues that a team requires a principal to minimize moral hazard: (i) waste of resource at certain state of output is required to give better incentive to team members, (ii) commitment of the waste can be enforced only by bringing a third party, a principal, and (iii) the waste becomes positive profit to the principal. However, the futures market trade changes the results in general equilibrium perspective.

Firstly, the waste of output is understood as transfer of resource from one state to another state in the current framework: the output in certain state is traded to buy output in another state by contract arbitrageurs.[14] Each team has output q, money $\sum_{i \in T} p_i(T, e_T, z_T)$, and allocation of $\sum_{i \in T} z_i(q)$ to the members. The value of the output and the money, and the allocation are different in general, i.e. $\sum_{i \in T} p_i(T, e_T, z_T) + \phi q \neq \phi \sum_{i \in T} z_i(q)$. In that sense, *ex-post* budget balance does not hold for each state. Nevertheless, through the futures market, ex-post budget balance can be said to hold: the difference between the output and the consumption $\left[\sum_{i \in T} z_i(q) - q\right]$ is purchased at price $\phi \left[\sum_{i \in T} z_i(q) - q\right] \varphi_T(q; e_T)$ in the futures market before the realization of q. The budget for such futures market trade is financed by $\sum_{i \in T} p_i(T, e_T, z_T)$, i.e.

$$\sum_{i \in T} p_i(T, e_T, z_T) = \phi \sum_q [\sum_{i \in T} z_i(q) - q] \varphi_T(q; e_T),$$

which is also the zero profit condition for contract arbitrageurs as constant returns to scale firms.

And, consumption $\sum_{i \in T} z_i(q)$ is possible by output q and the delivery of the futures market

[14] Principals of Holmstrom are not involved in physical production, but only arrange contracts; hence, contract arbitrageurs are parallels to the principals in his model. These terms are used interchangeably.

trade $\sum_{i\in T} z_i(q) - q$, i.e.

$$\sum_{i\in T} z_i(q) = q + [\sum_{i\in T} z_i(q) - q].$$

The waste or the excess in his model should not be understood literally: the waste and the excess are transfer of the resource across states through the futures market trade.[15]

Secondly, the positive profit of the principal is not true. Competition of principals will drive down their profit to zero by not leaving residual through the trade in the futures market.

Lastly, the enforcement problem can be resolved through the futures market. Holmstrom argues the following:

> Suppose something less than [pre-specified outcome] $x^*(a)$ is produced. *Ex-post* it is not in the interest of any of the team members to waste some of the outcome. But if it is expected that penalties will not be enforced, we are back in the situation with budget-balancing, and the free-rider problem [or, equivalently, multi-side moral hazard problem] reappears.

In the context of general equilibrium model, the futures market is used as a commitment device instead of the principal. Since $[q - \sum_{i\in T} z_i(q)]$ is delivered to the counter-parties at output state q, there remains nothing to renegotiate over.

The assumption of continuum of identical teams makes it possible to create consumption gap across states. Even though continuum does not exist in the real world, such gap should be achievable using the rich set of financial instruments.

The mechanism enforcing incentive compatibility constraints is similar to that of Holmstrom's in the sense that it changes the consumption gap across states. However, organizational implication is quite different: principals are not required in teams when the futures market is utilized.

Positive implication of a certain kind of financial market (here, the futures market) has been exhibited. The futures market can replace the role of the principal as a commitment device. Holmstrom's team is in an extreme environment where interaction with outside of the team (the

[15]Note that output has to be known to the market for the delivery of the futures market trade, as the output of Holmstrom's team has to be known to the members.

futures market trade) is not possible; hence, the principal receives positive profit even if they are perfectly competitive. There is no room and no role for the principal as someone who is inherently different from the individuals. If a principal is required for some other reason, production technology $\varphi_T(q; e_T)$ would include her. And, she would be treated exactly the same way as other individuals are.

4.2 Arbitrage Opportunity and Exclusive Conracts

It is well-known that arbitrage opportunity exists when prices are non-linear. The prices of contracts are linear in probability space[16], but not in commodity space, i.e. there exist no $p_i(T, e_T)$ satisfying the following in general[18].

$$p_i(T, e_T, z_T) = p_i(T, e_T) + \phi \sum_{q \in Q} z_i(q) \varphi(q; e_T), \forall z_T. \tag{5}$$

In other words, if an individual could access the futures market of commodities, the individual could have received more utility than that of the planner's assignment/allocation. This will, in turn, change the prices, and the planner's assignment/allocation cannot be implemented; hence, inefficiency by the definition of efficiency. Exclusiveness of contracts is another requirement for efficiency derived from the dual linear program. Song (2007) has detailed illustration why equation (5) is not possible in general. The necessity of exclusiveness to decentralize constrained efficient allocations is not a unique result of this paper.[19] However, the novelty here is to find it by the price system that is derived from the dual linear programming, instead of the direct observation

[16]The space of contract prices is linearized by an infinite dimensional vector space of non-negative measures on a compact set in a similar way as in Cole and Prescott (1997). Formally, individual i's feasible set is the set of non-negative measures defined on $\Omega \times \mathcal{E}^{|N|} \times \mathbb{R}^{L \times |Q| \times |N|}$, $\mathbf{M}[\Omega \times \mathcal{E}^{|N|} \times \mathbb{R}^{L \times |Q| \times |N|}]$.[17] The proper price of a contract is a continuous function(al) defined on $\mathbf{M}[\Omega \times \mathcal{E}^{|N|} \times \mathbb{R}^{L \times |Q| \times |N|}]$ with weak-$*$ topology.

[18]For the linearity, $p_i(T, e_T)$ has to satisfy each equality for each z_T, which is not possible in general. Also note that only z_i in z_T influences i's utility, which is why there is no other z_j on the right hand side.

[19]For example, Fisher (1992) shows that prohibition of insider trading is Pareto preferred if, and only if, a moral hazard problem exists. Tommasi and Weinschelbaum (2004) show that, if the private access to an insurance market were possible, an economic agent will try to smooth consumption over states; hence, second-best effort cannot be enforced.

of the incentive compatibility constraints. This exclusive contractual agreement might explain the prohibition of insider trading. Thus, only the occupations subject to a moral hazard problem are predicted to prohibit insider trading.

The individuals in teams have to commit to the exclusiveness of contracts. This commitment is different from that of the waste in Holmstrom's model. In order to break the commitment of Holmstrom's, all the members of a team need to agree to renegotiate, i.e. renegotiation possibility is the source of the commitment problem. The way to eliminate the negotiation possibility in it is to bring a third party (principal) who would have conflicting interest (acquisition of the waste). On the other hand, an unilateral action of *anonymous* access to the futures or the spot market is sufficient to break the commitment to the exclusiveness. Therefore, bringing a third party does not solve the problem. The commitment to the exclusiveness can be enforced only through monitoring or legal enforcement. Those can be done only in non-anonymous close relationship. In a sense, the exclusiveness distinguishes the activities inside teams (contract) and the activities outside teams (commodity trade); hence, it defines the boundary of teams.

5 Conclusion

Contracts among members of a team can be compared to commodity trade: both are formed by competition. However, members of a team, unlike *anonymous* commodity traders, maintain an intimate relationship. These two similar but distinct competitive environments are interrelated. A team's commodity trade with the outside will be affected by the team members' preferences and/or discipline of the team. At the same time, the process of forming teams through contracts will be influenced by the value of output, which is determined in the commodities markets. The focus of this paper is to analyze how such contracts, in equilibrium, are shaped by the interrelation between the two competitive environments.

The futures market trade makes it possible to transfer resources across states. Therefore, the penalty waste in Holmstrom (1982) is replaced by a transfer of resources. Since there is no need for the waste, there is no need for a principal. This exhibits efficiency implication of certain kinds of financial contracts in an economy with moral hazard. The prices of contracts, derived from the dual linear programming, are non-linear. Thus contracts must be exclusive.

In summary, the contractual arrangements restrain moral hazard problem in teams through futures market trade and exclusive contracts. This echoes Alchian and Demsetz (1972) who state that the essence of firms is the nexus of contracts restraining the behavior of transactors. Exclusiveness defines the boundaries of the teams by distinguishing activities inside the teams (contract) and activities outside the teams (the futures markets trade).

A contract is a mechanism design for the members of a team. While Holmstrom's contract is a mechanism with ex-post budget constraint, the contract in the current paper has binding ex-ante budget constraint: the futures market trade is self-financed, and ex-post budget constraint is obtained only through the delivery of the futures market trade. In other words, ex-ante budget constraint is a more reasonable assumption in mechanism design when a certain kind of financial market exists. Therefore, the exercise here hints the following: the consideration of the interaction between mechanisms – rather than the consideration of only a self-contained mechanism – could generate different results in mechanism design literature.

A Proofs

A.1 Characterization of Equilibria and Proof of Theorem 1

Section A.1.1 and A.1.2 outline the dual analysis. Section A.1.3, A.1.4, and A.1.5 interpret the first and the second dual constraints as the individuals' and the contract arbitrageurs' maximization problems.

A.1.1 Dual Linear Programming

Let y_i be the dual variable for the primal constraint (1), $p_i(T, e_T, z_T)$ for (2), ϕ for (3), and $\alpha_i(e_i'|T, e_T, z_T)$ for (4). Then the following is the dual linear program.

$$
\begin{aligned}
(D) \quad \min \quad & \sum_{i \in N} y_i \\
s.t. \quad & y_i \geq \lambda_i \left[\sum_q v_i(z_i(q)) \varphi_T(q; e_T) - C_i(e_i) \right] - p_i(T, e_T, z_T) \\
& 0 \geq \sum_{i \in T} p_i(T, e_T, z_T) + \phi \sum_q [q - \sum_{i \in T} z_i(q)] \varphi_T(q; e_T) - \sum_{i \in T} \sum_{e_i'} \alpha_i(e_i'|T, e_T, z_T) DG_i(e_i'; T, e_T, z_i) \\
& \phi, \alpha_i(e_i'|T, e_T, z_T) \geq 0
\end{aligned}
$$

Proposition 2 (Fundamental Theorem of Linear Programming) *(i) There exists a solution for the primal linear program. (ii) There exists a solution for the dual linear program. (iii) The primal and the dual programs attain the same value.*

Proof. See Appendix A.2 ∎

Proposition 3 (Complementary Slackness)

$$
x_i(T, e_T, z_T) \left\{ y_i - \left[\lambda_i \left(\sum_q v_i(z_i(q)) \varphi_T(q; e_T) - C_i(e_i) \right) - p_i(T, e_T, z_T) \right] \right\} = 0
$$

$$
x_T(e_T, z_T) \left\{ \sum_{i \in T} p_i(T, e_T, z_T) + \phi \sum_q [q - \sum_{i \in T} z_i(q)] \varphi_T(q; e_T) - \sum_{i \in T} \sum_{e_i'} \alpha_i(e_i'|T, e_T, z_T) DG_i(e_i'; T, e_T, z_i) \right\} = 0
$$

Proof. The results are direct application of Complementary Slackness of linear programming. ∎

Lemma 1 *For an optimal solution $(x_i(\cdot), x_T(\cdot))$ of the planner's problem,*

$$
\begin{aligned}
y_i &= \lambda_i \sum_{T, e_T, z_T} \left[\sum_q v_i(z_i(q)) \varphi_T(q; e_T) - C_i(e_i) \right] x_i(T, e_T, z_T) - \sum_{T, e_T, z_T} p_i(T, e_T, z_T) x_i(T, e_T, z_T) \\
0 &= \sum_{i \in T} p_i(T, e_T, z_T) + \phi \sum_q [q - \sum_{i \in T} z_i(q)] \varphi_T(q; e_T) \text{ for } (T, e_T, z_T) \text{ such that } x_T(e_T, z_T) > 0
\end{aligned}
$$

Proof. Let $x_i(\cdot)$ and $x_T(\cdot)$ be an optimal solution of the planner's problem. By Complementary Slackness of linear programming, I derive

$$
x_i(T, e_T, z_T) \left\{ y_i - \left[\lambda_i \left(\sum_q v_i(z_i(q)) \varphi_T(q; e_T) - C_i(e_i) \right) - p_i(T, e_T, z_T) \right] \right\} = 0
$$

Summing it over (T, e_T, z_T) proves the first equality.

From complementary slackness of linear programming,

$$0 = x_T(e_T, z_T) \left\{ \sum_{i \in T} p_i(T, e_T, z_T) + \phi \sum_q [q - \sum_{i \in T} z_i(q)] \varphi_T(q; e_T) - \sum_{i \in T} \sum_{e'_i} \alpha_i(e'_i | T, e_T, z_T) DG_i(e'_i; T, e_T, z_i) \right\}$$

From the duality of linear programming, $\alpha_i(e'_i|T, e_T, z_T)DG_i(e'_i; T, e_T, z_i)x_T(e_T, z_T) = 0$;hence, the last term is zero. Therefore, the desired result is shown. ∎

Lemma 2 *There exists a weight profile* $\lambda := (\lambda_k) \geq 0$ *such that*

$$\sum_{T, e_T, z_T} p_i(T, e_T, z_T) x_i(T, e_T, z_T) = 0, \forall i \in N.$$

Proof. See Appendix A.3 ∎

Assumption 2 *There exists* $\lambda \gg 0$ *satisfying Lemma 2.*

Song (2007) discusses Assumption 2, and provide a mild technical condition for the existence of such $\lambda \gg 0$.

A.1.2 Outline of Dual Linear Programming Analysis

The planner's problem is interpreted as a revenue maximization problem. The inputs for the planner's problem are individuals, contracts, and commodities. Incentive compatibility constraints can be interpreted as technological constraint.[20] The revenue of the planner is the welfare of the economy. Dual variable y_i of individual i's probability constraint (equation (1)) measures the value of individual i. In other words, the planner has willingness to pay y_i amount of welfare to bring (or, equivalently, "purchase") an infinitesimally additional individual i; hence, y_i should be the welfare that individual i enjoys in the economy. Dual variable $p_i(T, e_T, z_T)$ of the team-formation constraint (equation (2)) measures the value of contract (T, e_T, z_T). For the planner to "purchase" the contract, $p_i(T, e_T, z_T)$ should be paid; hence, price of the contract in the market would be $p_i(T, e_T, z_T)$. Dual variable ϕ of the resource constraint (inequality (3)) measures the value of commodity. Again, ϕ should be the price of commodities in the economy. Lastly, the prices of incentives are derived from the last primal constraints (inequality (4)), which is discussed in Section A.1.4.

[20] Incentive compatibility constraints are treated differently from other constraints; however, the incentive compatibility constraints could be interpreted in a same way. The input that the planner has for the incentive compatibility is "util" to relax incentive compatibility. Even though the endowment of "util" for the planner is zero, the dual variable of it measures the planner's willingness pay for additional util.

The following sections show that, if any environment of Definition 3 (Definition of Equilibrium) is violated, the dual inequalities fail to be interpreted as the maximization problems. Therefore, Definition 3 is a minimal set of conditions for efficiency.

A.1.3 Individuals' Choice: the first dual constraint

By summing up the first dual constraints with arbitrary probability $\xi_i(T, e_T, z_T)$, I get

$$y_i \geq \lambda_i \sum_{T,e_T,z_T} \left[\sum_{q \in Q} v_i(z_i(q))\varphi_T(q; e_T) - C_i(e_i) \right] \xi_i(T, e_T, z_T) - \sum_{T,e_T,z_T} p_i(T, e_T, z_T)\xi_i(T, e_T, z_T). \qquad \text{[indv]}$$

If $\xi_i(T, e_T, z_T)$ is equivalent to the optimal solution of the planner, $x_i(T, e_T, z_T)$, then the inequalities become equalities by Lemma 1.

The dual variable of the first constraint in the primal linear program, y_i, is the value of individual i to the planner. Therefore, y_i/λ_i is the equilibrium utility since individual i's utility enters the planner's objective function with weight λ_i.

Inequality [indv] is interpreted as the individual i's maximization problem. From Lemma 2 and Assumption 2, pick $\lambda \gg 0$ such that $\sum_{T,e_T,z_T} p_i(T, e_T, z_T)x_i(T, e_T, z_T) = 0$: money expenditure on the probabilities is zero if it is the same to the planner's solution. Also, for $\sum p_i(\cdot)\xi_i(\cdot) \leq 0$ (a feasible probability), $y_i \geq \lambda_i \sum [\sum v_i(z_i(q))\varphi_T(q; e_T) - C_i(e_i)]\, \xi_i(\cdot) - \sum p_i(\cdot)\xi_i(\cdot) \geq \lambda_i \sum [\sum v_i(z_i(q))\varphi_T(q; e_T) - C_i(e_i)]\, \xi_i(\cdot)$ (i.e., a suboptimality). I.e., if individual i has chosen a different probability than that of the planner's, the purchase would be suboptimal or infeasible. Also, the purchased contracts are incentive compatible by the primal constraints of the incentive compatibility constraints. Therefore, the above inequality [indv] summarizes the individual i's optimization in Definition 3.

In equilibrium, the individuals of a same type purchase the same probabilities on contracts; hence, individuals of the same type have same kind of randomization devices. I call it a *common randomization device.*

The realized utility after the realization of contract (T, e_T, z_T) is

$$\frac{y_i}{\lambda_i} + \frac{1}{\lambda_i} p_i(T, e_T, z_T) = \left[\sum_{q \in Q} v_i(z_i(q))\varphi_T(q; e_T) - C_i(e_i) \right]$$

In general the second term in the left-hand side of the equality is not zero. Therefore, if non-degenrate lottery is used, individual i's utility is different across the realization of (T, e_T, z_T). Efficiency typically requires individuals of the same type to obtain different utility levels when assigned to different teams.

A.1.4 Contract Arbitrageurs' Choice: the second dual constraints

The second dual constraint is

$$0 \geq \sum_{i \in T} p_i(T, e_T, z_T) - \phi \sum_q \left[\sum_{i \in T} z_i(q) - q \right] \varphi_T(q; e_T) - \sum_{i \in T} \sum_{e'_i} \alpha_i(e'_i | T, e_T, z_T) DG_i(e'_i; T, e_T, z_i) \qquad \text{[arb]}$$

If $x_T(e_T, z_T) > 0$, the inequality becomes an equality, and the last term is zero by Lemma 1.

The second dual constraint is interpreted as the contract arbitrageurs' maximization problem. There is no probability constraint for contract arbitrageurs, i.e. they are a freely available input to the planner. The value of a freely available input must be zero, so contract arbitrageurs receive zero profit unlike individuals. The description of contract arbitrageurs' trade is already detailed in Section 4.1.

The last term of [arb] is the shadow value of the incentive compatibility constraints. A literal interpretation of [arb] is the following: the seller's profit internalizes the shadow value of the incentive compatibility constraints. If $x_T(e_T, z_T) > 0$, the last term in [arb] is zero by Complementary Slackness. And, the contract arbitrageur would receive zero profit by Lemma 1. If the arbitrageur chooses non-incentive compatible contract (T, e_T, z_T), the profit (including the incentive cost represented by the shadow value) would be smaller than zero by the inequality.

Apparently, the shadow values of the incentive compatibility constraints are imaginary. A realistic story instead of the literal interpretation is the following: if an arbitrageur sold a non-incentive compatible lottery, the contract would not deliver the promised allocation to the members of the team because of the moral hazard problem. Therefore, some individuals would not buy the contract to be a member of the team. If the team were picked up by a randomization device, she would not have enough team members. Therefore, the contract arbitrageur has no way but to default. Knowing the possibility of the default, even the individuals who preferred the non-incentive compatible contract would not buy it either.

Also contract arbitrageurs do not want to sell other incentive compatible contract than the solution of the planner's, since the profit would not go up by the following Lemma.

Lemma 3 *If (T, e_T, z_T) is incentive compatible, then*

$$0 \geq \sum_{i \in T} p_i(T, e_T, z_T) - \phi \sum_q \left[\sum_{i \in T} z_i(q) - q \right] \varphi_T(q; e_T).$$

Also, the equality holds if $x_i(T, e_T, z_T) > 0$.

Proof. From inequality [arb], it is enough to show the following, which is true by the incentive compatibility.

$$0 \geq \sum_{i \in T} \left[\sum_{e'_i} \alpha_i(e'_i | T, e_T, z_T) DG_i(e'_i; T, e_T, z_i) \right] \xi_T(e_T, z_T) \qquad \blacksquare$$

Therefore, inequality [arb] summarizes the contract arbitrageurs' optimization.

A.1.5 Single Person Team: Price of Contract

For the case of single person teams, the second dual constraint becomes

$$-p_i(\{i\}, e_i, z_i) \geq -\phi \sum_{q \in Q} [z_i(q) - q]\varphi(q; e_i)$$

Then, the first dual constraint becomes

$$y_i \geq \lambda_i \sum_{q \in Q} v_i(z_i(q))\varphi(q; e_i) - p_i(\{i\}, e_i, z_i) \Rightarrow y_i \geq \lambda_i \sum_{q \in Q} v_i(z_i(q))\varphi(q; e_i) - \phi \sum_{q \in Q} [z_i(q) - q]\varphi(q; e_i) \quad \text{[single]}$$

Again, the inequality is equality if $x_i(\{i\}, e_i, z_i) > 0$.

Definition 4 *Subdifferential of $\lambda v_i(\cdot)$ at z is*

$$\partial(\lambda_i v_i(z)) := \{p | \lambda_i v_i(z) - pz \geq \lambda_i v_i(z') - pz', \forall z'\}.$$

Subdifferentiability is a generalization of differentiability when $v_i(\cdot)$ is concave[21]. Therefore, if $v_i(\cdot)$ is differentiable and concave, then $\phi \in \partial(\lambda_i v_i(z))$ implies $\phi = \nabla(\lambda_i v_i(z))$: the optimization condition for individual i facing price ϕ.

Inequality [single] is equivalent to $(\phi\varphi(q; e_i))_{q \in Q} \in \partial \left(\lambda_i \sum_{q \in Q} v_i(z_i(q))\varphi(q; e_i) \right)$, which is in turn equivalent to $(\phi\varphi(q; e_i))_{q \in Q} = \nabla \left(\lambda_i \sum_{q \in Q} v_i(z_i(q))\varphi(q; e_i) \right)$ when $v_i(z_i)$ is differentiable. Suppose one i was picked to form a single person team. Commodity $(z_i(q))_{q \in Q}$ is already at an optimal point. Therefore, allowing the i to trade commodities in the futures market does not disrupt efficiency: the individual would choose the same allocation to that of the planner's. Inequality [single] also imlies $\phi \in \partial(\lambda_i v_i(z_i))$, because the utility function is separable over the states. Therefore, allowing individual i in single-person team to trade in the spot market does not disrupt efficiency either.

Contract arbitrageurs for single person teams can be understood as a kind of personal asset managers. In order to understand single person teams without the contract arbitrageurs, the futures market is required to open for the individuals in single person teams in the 3rd stage.

[21]Notice the following equivalence. $\lambda_i v_i(z_i) - \tilde{\phi} z_i \geq \lambda_i v_i(z_i') - \tilde{\phi} z_i' \Leftrightarrow \tilde{\phi}(z_i' - z_i) \geq \lambda_i v_i(z_i') - \lambda_i v_i(z_i) \Leftrightarrow \tilde{\phi}(z_i' - z_i)/||z_i' - z_i|| \geq (\lambda_i v_i(z_i') - \lambda_i v_i(z_i))/||z_i' - z_i||$ where $||\cdot||$ is $\mathbf{L}^{L \times |Q|}$ norm in $\mathbb{R}^{L \times |Q|}$.

A.2 Infinite Dimensional Linear Programming

A.2.1 Basic Concepts and Notations

Let Z_T be a compact set in $\mathbb{R}^{L\times|Q|\times|T|}$

- $\mathbf{C}(Z_T)$: Banach space of continuous functions on Z_T equipped with the supremum norm, $\|f\| := \sup_{z\in Z_T} |f(z)|$.
- $\mathbf{M}(Z_T)$: Banach space of countably additive Borel measures on Z_T equipped with the total variation norm; i.e. $\|\nu\| = \sup_\pi \sum_{C_i} |\nu(C_i)|$ over all finite measureable partitions π of Z_T.

Let $m \in \mathbf{M}(Z_T)$ be the *Lesbegue* measure. Let R_T be a compact disc in $\mathbb{R}^{L\times|Q|\times|T|}$, $R_T = \{z \in \mathbb{R}^{L\times|Q|\times|T|} | \langle z, z\rangle^{\frac{1}{2}} \leq R_T^{L\times|Q|\times|T|}\}$ with a slight abuse of notation. Define measures

$$x_i(T, e_T), x_T(e_T) \in \mathbf{M}(R_T).$$

For the notational simplicity, I also define $U_i(z_i; e_T) := \sum_{q\in Q} v_i(z_i(q))\varphi_T(q; e_T) - C_i(e_i)$, and $U_i(z_i; e_i'|e_T) := \sum_{q\in Q} v_i(z_i(q))\varphi_T(q; e_i'|e_T) - C_i(e_i')$ when i is engaged in team (T, e_T) and deviating to e_i'.

$\langle f, x\rangle = \int f dx$ is well-defined with $f \in \mathbf{C}(R_T)$ and $x \in \mathbf{M}(R_T)$, and $\langle \cdot, \cdot\rangle$ is a bilinear operation. I write $\langle f, x\rangle_E^{R_T} = \int_E f dx$ for Borel set $E \in \mathcal{B}(R_T)$. When $E = R_T$, I write $\langle f, x\rangle^{R_T} := \langle f, x\rangle_{R_T}^{R_T}$. Also, define $\langle \mathscr{I}, x\rangle := \int 1 \cdot dx$.

Definition 5 *The assignment $(x_i(T, e_T), x_T(e_T))$ is feasible if*

$$\sum_{T\in\mathcal{T}_i, e_T\in\mathcal{E}^{|T|}} \langle \mathscr{I}, x_i(T, e_T)\rangle^{R_T} = 1, \forall i \in N$$
$$x_i(T, e_T)(E) = x_T(e_T)(E), \forall i \in N, \forall (T, e_T), \forall E \in \mathcal{B}(R_T)$$
$$\sum_{T,e_T} \sum_q \left\langle [\sum_{i\in N} z_i^s - q], x_T(e_T) \right\rangle^{R_T} \varphi_T(q; e_T) \leq 0,$$
$$\left([U_i(\cdot; e_i'|e_T) - U_i(\cdot; e_T)]\, x_T(e_T)\right)(E) \leq 0, \forall i \in N, \forall (T, e_T), \forall e_i', \forall E \in \mathcal{B}(R_T)$$

Definition 6 *The planner's problem is to find $(x_i(T, e_T), x_T(e_T))$ to attain*

$$g = \sup \sum_i \lambda_i \sum_{T,e_T} \langle U_i(\cdot; e_T), x_i(T, e_T)\rangle^{R_T} \quad s.t. \quad (x_i(T, e_T), x_T(e_T)) \text{ is feasiible}$$

A.2.2 Derivation of the Dual

From the following equality, the result follows.

$$\begin{aligned}
\mathcal{L} &= \sum_i \lambda_i \sum_{T,e_T} \langle U_i(\cdot;e_T), x_i(T,e_T)\rangle^{R_T} \\
&\quad + \sum_i y_i \left(1 - \sum_{T,e_T} \langle \mathscr{I}, x_i(T,e_T)\rangle^{R_T}\right) - \sum_i \sum_{T,e_T} \langle p_i(T,e_T), x_i(T,e_T) - x_T(e_T)\rangle^{R_T} \\
&\quad -\phi \sum_{T,e_T} \sum_q \left\langle [\sum_{i\in T} z_i^s - q], x_T(e_T) \right\rangle^{R_T} \varphi_T(q;e_T) - \sum_i \sum_{T,e_T} \sum_{e_i'} \langle \alpha_i(e_i'|T,e_T), DG_i(e_i';T,e_T,\cdot)\cdot x_T(e_T)\rangle^{R_T} \\
&= \sum_i y_i + \sum_i \sum_{T,e_T} \langle [\lambda_i U_i(\cdot;e_T) - y_i - p_i(e_T)], x_i(T,e_T)\rangle^{R_T} \\
&\quad + \sum_{T,e_T} \sum_q \left\langle \sum_{i\in T} p_i(e_T) - \phi[\sum_{i\in T} z_i(q) - q]\varphi_T(q;e_T), x_T(e_T) \right\rangle^{R_T} \\
&\quad - \sum_{T,e_T} \sum_{i\in T} \sum_{e_i'} \langle \alpha_i(e_i'|T,e_T)\cdot DG_i(e_i';T,e_T,\cdot), x_T(e_T)\rangle^{R_T} \qquad \blacksquare
\end{aligned}$$

A.2.3 Finite Support of Allocation and Proof for Proposition 1

The space of commodity is finite-dimensional, $\mathcal{E}$ is assumed to be finite, and Q is also finite. Therefore, support of $x_i(\cdot)$ and $x_T(\cdot)$ is finite from Carathéodory Theorem on convexification.

Proof. Since support of $x_i(\cdot)$ and $x_T(\cdot)$ is finite, maximum exists. ∎

A.2.4 Proof of Proposition 2 (Fundamental Theorem of Linear Programming)

It is direct by applying Gretsky, Ostroy, and Zame (1992) to the described infinite dimensional linear program.

A.3 Proof of Lemma 2

Let $\Gamma(\lambda)$ denote the planner's linear program with weight λ, and $\Gamma^{-1}(\lambda)$ denote the dual linear program. Let $F(\lambda)$ denote the feasible set for $\Gamma(\lambda)$, and let

$$\psi(\lambda) := -\left(\sum_{T,e_T} \sum_{z_T} p_i(T,e_T,z_T) x_i(T,e_T,z_T)\right)_{i\in N} \quad \text{where} \quad x_i(\cdot) \in \operatorname{argmax} \Gamma(\lambda), p_i(\cdot) \in \operatorname{argmin} \Gamma^{-1}(\lambda)$$

The set of all such vectors, denoted by $P(\lambda)$, is non-empty by Proposition 2.

Before getting into the proof, I prove a few lemmas.

Lemma 4 $y_i \geq 0$

Proof. Replace the resource constraint of the planner by

$$\sum_{T:|T|>1}\sum_{e_T}\sum_{z_T}\sum_{q}[\sum_{i\in T} z_i(q)-q]\varphi_T(q;e_T)x_T(e_T,z_T)\le 0.$$

Also let $U_i(x_i(\cdot))$ be

$$U_i(x_i(\cdot)) := \sum_{z_T}\sum_{T,e_T:|T|>1}\left[\sum_{q\in Q} v_i(z_i(q))\varphi_T(q;e_T)-C_i(e_i)\right]x_i(T,e_T,z_T).$$

In other words, if i is a single person team, the consumption is given by $z_i(q)\equiv 0$. Then the probability constraint is written as an inequality constraint

$$\sum_{T,e_T:|T|>1}\sum_{z_T} x_i(T,e_T,z_T)\le 1, \forall i\in N.$$

Therefore, $y_i \ge 0$ since the dual value of a primal inequality constraint is always non-negative.

The solution of the new planner's program is trivially in the domain of the original planner's program. The planner's solution of the original program cannot be smaller than that of the new one. Therefore, the value of individual i cannot go down even after the origianl resource constraint is introduced. So $y_i \ge 0$. ∎

Lemma 5 *Let $(p_i(\cdot), x_i(\cdot), x_T(\cdot))$ and $(\rho_i(\cdot), \xi_i(\cdot), \xi_T(\cdot))$ to be two solutions of the primal and the dual linear programs. The prices can be modified without changing the optimal value such that the following holds.*

$$\begin{aligned}&\sum_{T,e_T}\sum_{z_T} p_i(T,e_T,z_T)x_i(T,e_T,z_T)+\sum_{T,e_T}\sum_{z_T}\rho_i(T,e_T,z_T)\xi_i(T,e_T,z_T)\\ =&\sum_{T,e_T}\sum_{z_T} p_i(T,e_T,z_T)\xi_i(T,e_T,z_T)+\sum_{T,e_T}\sum_{z_T}\rho_i(T,e_T,z_T)x_i(T,e_T,z_T).\end{aligned}$$

Proof. For (T,e_T,z_T) such that $x_i(T,e_T,z_T)=0$ and $\xi_i(T,e_T,z_T)>0$, decrease $p_i(T,e_T,z_T)$ so that

$$y_i=\lambda_i\left[\sum_q v_i(z_i(q))\varphi_T(q;e_T)-C_i(e_t)\right]-p_i(T,e_T,z_T)$$

Note that this shift of the price does not change the individual i's choice of probabilities in the environment of transferable utility (TU) since each individual still weakly prefers the original probability before the shift. Also, it does not influence contract arbitrageurs' choice either since the revenue only went down for (T,e_T,z_T) that used to be chosen with zero probability.

Because of the change of prices, I derive the following by summing up the dual constraints with probability $\xi_i(T,e_T,z_T)$.

$$y_i=\lambda_i U_i(\xi_i(\cdot))-\sum_{T,e_T,z_T}\xi_i(T,e_T,z_T)p_i(T,e_T,z_T)$$

Since $y_i = \lambda_i U_i(x_i(\cdot)) - \sum_{T,e_T,z_T} x_i(T,e_T,z_T) p_i(T,e_T,z_T)$, I derive

$$\lambda_i U_i(x_i(\cdot)) - \sum_{T,e_T,z_i} x_i(T,e_T,z_T) p_i(T,e_T,z_T) = \lambda_i U_i(\xi_i(\cdot)) - \sum_{T,e_T,z_T} \xi_i(T,e_T,z_T) p_i(T,e_T,z_T).$$

By applying the same procedure to $(\rho_i(T,e_T,z_T), \xi_i(T,e_T,z_T), \xi_T(e_T,z_T))$,

$$\lambda_i U_i(\xi_i(\cdot)) - \sum_{T,e_T,z_T} \xi_i(T,e_T,z_T) \rho_i(T,e_T,z_T) = \lambda_i U_i(x_i(\cdot)) - \sum_{T,e_T,z_T} x_i(T,e_T,z_T) \rho_i(T,e_T,z_T).$$

By summing up the two equalities, the result follows. ∎

Lemma 6 *$P(\lambda)$ is convex-valued.*

Proof. Take two solutions of the linear program,

$$(x_i(T,e_T,z_T), x_T(e_T,z_T), p_i(T,e_T,z_T)) \text{ and } (\xi_i(T,e_T,z_T), \xi_T(e_T,z_T), \rho_i(T,e_T,z_T)).$$

Define

$$\begin{aligned} p_i^\omega(T,e_T,z_T) &:= (1-\omega)p_i(T,e_T,z_T) + \omega \rho_i(T,e_T,z_T) \\ x_i^\omega(T,e_T,z_T) &:= (1-\omega)x_i(T,e_T,z_T) + \omega \xi_i(T,e_T,z_T). \end{aligned}$$

$x_i^\omega(T,e_T,z_T)$ is another solution of the primal linear program, and $p_i^\omega(T,e_T,z_T)$ is another solution of the dual linear program, since they were generated by convex combination.

Finally, the proof is through if the following is shown

$$\sum_{T,e_T,z_T} p_i^\omega(T,e_T,z_T) x_i^\omega(T,e_T,z_T) = (1-\omega) \sum_{T,e_T,z_T} p_i(T,e_T,z_T) x_i(T,e_T,z_T) + \omega \sum_{T,e_T,z_T} \rho_i(T,e_T,z_T) \xi_i(T,e_T,z_T).$$

To save space, define $p_i \cdot x_i := \sum_{T,e_T,z_T} p_i(T,e_T,z_T) x_i(T,e_T,z_T)$. Then the above condition is

$$\begin{aligned} p_i^\omega \cdot x_i^\omega &= (1-\omega) p_i \cdot x_i + \omega \rho_i \cdot \xi_i \\ \Leftrightarrow ((1-\omega)p_i + \omega \rho_i) \cdot ((1-\omega)x_i + \omega \xi_i) &= (1-\omega) p_i \cdot x_i + \omega \rho_i \cdot \xi_i \\ \Leftrightarrow (\omega^2 - \omega) [p_i \cdot x_i + \rho_i \cdot \xi_i - p_i \cdot \xi_i - \rho_i \cdot x_i] &= 0 \end{aligned}$$

The last line follows from Lemma 5. Therefore, $P(\lambda)$ is convex-valued. ∎

$\lambda_i = 0$ implies $\psi_i(\lambda) \geq 0$ by [indv] and $y_i \geq 0$ (Lemma 4). Therefore, $P(\lambda)$ is compact. By lemma 6, $P(\lambda)$ is convex-valued. Also, it can be shown that $x_i(\cdot;\lambda)$ and $p_i(\cdot;\lambda)$ (solutions of $\Gamma(\lambda)$ and $\Gamma^{-1}(\lambda)$) are upper hemi-continuous in λ. (For example, see Champsaur, Drèze, and Henry (1977).) Therefore, $P(\lambda)$ is upper hemi-continuous in λ.

If $P(\lambda)$ contains a zero vector, then $\psi(\lambda)$ is feasible (i.e. $\psi_i(\lambda) \equiv 0$), and it is through. Define a set-valued function T by

$$T(\lambda) = \lambda + P(\lambda) = \{\lambda + \pi | \pi \in P(\lambda)\}.$$

Let A be a simplex in the hyperlane $\{\alpha | \sum_{k \in N} \alpha_k = 1\}$, large enough to contain all sets $T(\lambda)$, $\lambda \in \Lambda = \{\lambda \geq 0 | \sum \lambda_k = 1\}$, as well as Λ itself; the upper-continuity of T makes this possible – i.e., makes $T(\Lambda)$ compact. Extend the definition of T to A by

$$T(\alpha) = T(f(\alpha)), \quad \text{where} \quad f_k(\alpha) = \frac{\max(0, \alpha_k)}{\sum_h \max(0, \alpha_h)}$$

According to Kakutani's theorem, there is a "fixed point" α^* satisfying $\alpha^* \in T(\alpha^*)$. Denote $f(\alpha^*)$ by λ^*. Suppose first that $\alpha^* \neq \lambda^*$. Then $\alpha^* \in A - \Lambda$, and for some i, $\lambda_i^* = 0 > \alpha_i^*$. But $\alpha^* \in T(\lambda^*) = \lambda^* + P(\lambda^*)$, hence $\pi_i^* < 0$ for some $\pi^* \in P(\lambda^*)$. Since $\psi_i(\lambda^*) \geq 0$, the feasible payoff vector $\psi(\lambda^*) - \pi^* \in F(\lambda^*)$ gives player i a positive amount. But this is impossible without side payments, since all his payoffs in $\Gamma(\lambda^*)$ are zero. I conclude that $\alpha^* = \lambda^*$; hence that $0 \in P(\lambda^*)$. Therefore,

$$\psi(\lambda^*) \in F(\lambda^*)$$

Lemma 2 is shown. ∎

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