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Abstract

We model the provision of owner-occupied versus rental housing services as a competitive search economy where households have private information over their expected duration. Owning solves the private information problem at the cost of double search. With public information, households with low vacancy hazard rates pay lower rents and search in thicker markets. With private information, housing is under-provided to long-duration households to discourage short-duration households from searching there. If a household has a high enough expected duration, rental distortions become large enough that she prefers to own. Customizing a house ameliorates the information problem while rent control exacerbates it.
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1 Introduction

There is a long list of plausible frictions that may create meaningful differences in the value of owning versus renting a home to a household. Many of the frictions that favor renting, such as the higher transactions costs of buying and selling a house and the downpayment constraints in the mortgage market, appear in one form or another in nearly all life cycle models with a homeownership choice[1].

However, there is little consensus on the frictions that favor owning. Tax wedges may offer one motive for owning (as in Diaz and Luengo-Prado (2008); Gervais (2002)). Other, more "fundamental," frictions used in models include a user cost premium of renting over owning, perhaps due to excessive utilization of housing services on the part of renters (as in Henderson and Ioannides (1983)), amplifications to the perceived volatility of rents (Berkovec and Fullerton (1992)), a housing ladder with only owner-occupied housing on the top rungs (Ortalo-Magne and Rady (2006); Rios-Rull and Sanchez-Marcos (2007)), and a warm glow to owning (Iacoviello and Pavan (2009); Kiyotaki et al. (2008)).[2] While "intuitive", it is not yet clear what the size and ultimate source of these various frictions are. Most, like differential housing supply and warm glows, are likely equilibrium outcomes rather than inputs.

In this paper, we build a model of endogenous differential housing supply, where the equilibrium outcome is differential housing supply: owner-occupied housing is only offered in some (sub)markets while rental housing is only offered in others. Later, by including an option to customize a house in an extension to the model, we show that the economy can endogenously give rise to "warm glows:" a higher flow utility from living in an otherwise identical (to the econometrician) owner-occupied house.

Since owning and renting are just labels for different (perhaps many different) contracts to provide housing services, we model the homeownership decision and the properties of rental contracts as an outcome of a contracting problem. In the baseline model, houses are ex-ante identical and households differ only according to their expected duration of search, which may be private information. Homeowners (which may be households or landlords) post contracts for housing services which specify a (potentially duration-dependent) price for housing services as well as whether, after eventual separation, the current owner or the future occupant is responsible for finding the next tenant (a "rental" or "owning" contract, respectively).

Within the housing market in this economy, households can direct their search to a specific

[1]e.g. Campbell and Cocco (2007); Chambers, Garriga and Schlagenhauf (2009*a*,*b*); Cocco (2005); Diaz and Luengo-Prado (2008); Fisher and Gervais (2007); Gervais (2002); Amior and Halket (2011); Iacoviello and Pavan (2009); Kiyotaki, Michaelides and Nikolov (2008); Li and Yao (2007); Rios-Rull and Sanchez-Marcos (2008)

[2]One class of frictions that may work both ways is risk in the housing market, as in Sinai and Souleles (2005).

type of contract (so that each type of contract is its own submarket) and are bilaterally matched to houses within that submarket subject to the frictions from competitive search theory (Moen (1997) and Shimer (1996)). In equilibrium, vacancies in a particular submarket adjusts so that the expected return to adding a new house in any submarket is the same.

The lone ex-ante difference in households in our economy is their expected duration in a house. There is a long literature looking at mobility and homeownership choices. Deng, Gabriel and Nothaft (2003) and Gabriel and Nothaft (2001) find considerable variation across households and Metropolitan Statistical Areas in rental vacancy rates and durations. Boehm, Herzog Jr. and Schlottmann (1991), Cameron and Tracy (1997), Haurin and Gill (2002) and Kan (2000) all find relationships between mobility hazards and homeownership.

We also are following a growing literature by looking at housing in a search or matching framework (e.g. Albrecht, Anderson, Smith and Vroman (2007); Albrecht, Gautier and Vroman (2010); Caplin and Leahy (2008); Ngai and Tenreyro (2009); Piazzesi and Schneider (2009); Wheaton (1990)). To our knowledge, we are the first to look at both renting and owning in such a framework and the first to look jointly at renting and owning with adverse selection[3]. Our work looks at contracts to supply housing services[4] when there are search frictions and asymmetric information and thus extends the work of Guerrieri, Shimer and Wright (2010) to include dynamic contracts in a competitive search equilibrium with adverse selection[5]. In our equilibrium, contracts can be dynamic while the markets themselves are in steady-state. Concurrently and complementarily, Chang (2011) and Guerrieri and Shimer (2012) examine environments where the markets can change dynamically, however all contracts are one-time exchanges (purchases and sales of assets).

Our main results are twofold. First, an incentive problem in rental markets distorts market tightnesses[6] compared to the public information benchmark. In the economy where households' expected durations are public information, households with low vacancy hazard rates (long-duration households) pay lower rental rates and search in less tight markets than households

[3] Hubert (1995); Miceli and Sirmans (1999) have models with renters and adverse selection in which long-term tenants have declining rent schedules while Barker (2003) shows that if households have inelastic demand for housing, those that expect to stay longer do not usually get discounts on their rent. Brueckner (1994) presents a model with adverse selection and evidence that banks use menus of mortgage points and interest rates to obtain information on a household's expected mobility.

[4] and in this sense compliments the work on optimal mortgage design in owner-occupied markets (contracts for loans backed by housing services) by Piskorski and Tchistyi (2011, 2010)

[5] Delacroix and Shi (2007) and Albrecht et al. (2010) have adverse selection problems where the side posting the price has full information. Here, as in Guerrieri et al. (2010), the side directing its search has the superior information.

[6] Markets are less tight if households on average take less time to find a house, or equivalently if landlords take longer on average to fill a vacancy.

with high hazard rates.

When expected durations are private information, on the contrary, long-duration households search in tighter markets than short-duration households, thus they spend more time on average searching for a house (per separation spell), but pay even lower rental rates once matched. (The unique equilibrium is separating.) The intuition behind the result is that in equilibrium housing is under-provided to long-duration households so as to discourage short-duration households from searching there. In this sense, private information causes housing scarcity in some rental markets.

In our economy, owning a house solves the private information problem by internalizing the separation hazard in the optimal search problem of the household, but requires double search - an owner that wants to move must first search for a buyer for her house before searching for her own new home. Our second result is that households that expect to stay in their house long enough choose to own rather than rent. The distortions implied by the incentive problem in the rental market pile-up: the deviations from first-best due to private information (compared to the public information benchmark) are larger in markets where the long-duration households search. Meanwhile the owning contract is always incentive compatible while the extra cost of double search tends to diminish with expected duration. If a household has a high enough expected duration, the distortions in the rental market due to the information problem dominate the double search cost in the owning market so that it prefers to own the house even though owning is assumed to use a less efficient (in a first-best sense) search technology. In equilibrium, households with different expected durations in their houses search in different submarkets: there are a variety of owning and renting submarkets which differ according to price and market tightness.

A policy of rent control predictably leads to a lower supply of rental housing and tighter markets in the regulated market in both public and private information cases. With private information however, the effects on the regulated market spill into the unregulated market, leading to lower supply and tighter markets there as well. This happens even though there is no excess demand in any market (as in e.g. Fallis and Smith (1984)); all markets are in equilibrium. Instead, by worsening the allocation for low-duration households, rent control exacerbates the information problem, making it more costly for higher-duration households to screen the low-duration households.

In the final part of our paper, we give the economy access to a technology which permits the building of non-conforming, i.e. customized, houses; which we model as giving a higher utility flow at some cost to the matching probability. We show that customization appeals most to long-duration households. So, unlike rent control, the customization technology offers an additional way to relax the incentive compatibility constraints in the rental market; thus there may be "over-customization" in the rental market relative to the public information benchmark. Also, since the appeals of owning and customization are each increasing in expected duration, more owners than renters tend to customize. If customization is observable to an econometrician using hedonics, than the owner-occupiers will appear to live in houses with more amenities, otherwise they will appear to get a warm glow from owning (that is, they would appear to get a higher utility flow from the same observable set of house attributes).

Our work on customization is a sort of companion to House and Ozdenoren (2008). In their model of durable goods, goods that are more durable conform more to average tastes due to resale concerns. They cite "McMansions" (which are owner-occupied) as an example of a generic durable good. In our model, durable goods more or less conform based on the expected duration of the match (rather than the duration of the good). The typical owner-occupied house is actually relatively varied compared to rental housing in our economy since, endogenously, owner-occupiers expect to be matched longer with their house.

The remainder of this paper is as follows: Section 2 presents economies of renting with public and then private information; Section 3 presents the owning technology and the equilibrium with owning and renting; Section 4 presents a numerical example and the effects of rent control while section 5 presents the customization technology. Section 6 concludes by commenting briefly on three points: how our economy here could be extended to include optimal rental contracts; how other wedges in the owning market could play a similar role to the one played by double search here; and the challenges of testing housing ladder theories due to the poor quality of data on rental vacancies. Most proofs are in the Appendix.

2 Rental market

2.1 Preferences and technology

Time is continuous and the horizon is infinite. There is a measure one of households indexed by their type $i \in \mathbb{I} = \{1, 2, .., I\}$ and a large set of landlords or builders. Let π_i be the fraction of households of type i in the population, for all i. If a landlord decides to participate in the market, she pays a cost H in units of utility to build a house but then houses are costless to maintain; if she doesn't participate, she gets a payoff equal to 0. Households receive a flow utility of h when they occupy a house and 0 when they do not. Households and landlords each discount at the same rate $\rho < 1$. We assume $h > \rho H$.

Households that are currently occupying a house separate with it at a hazard rate $\gamma : \mathbb{I} \to \Gamma \subset \Re_+$, at which point a separated household no longer receives any utility from living in that particular house. Without loss of generality, we assume that γ is strictly decreasing. We will often refer to a household of type i as having a hazard $\gamma_i = \gamma(i)$. We denote $\bar{\gamma} \equiv \gamma_1$ and $\underline{\gamma} \equiv \lim_{i \to I} \gamma_i$ so that $\Gamma = [\underline{\gamma}, \bar{\gamma}]$. We will also derive some analytical and computational results for the special case where $\{\gamma_i\}_{i=1}^{\infty}$ is dense in Γ. For lack of a better term, we refer to this special case as the differential-γ case.

A rental contract $w \in \mathrm{W}$ specifies a flow rent, possibly contingent on type, paid by the household to the landlord if matched. The contract ends in the case of separation.[7] We consider two cases. In the first, a household's type is publicly observable and so contracts are also free to have type-specific rents. However, we will show that in equilibrium, only one type is lured by each contract. In the second case, a household's type is private information. In this case, by the revelation principle, we assume that landlords post a contract which contains direct revelation mechanisms for each type, without loss of generality. Following Guerrieri et al. (2010), we will show that we can assume without loss of generality that landlords post contracts with type-independent mechanisms. More precisely, in the private information case the equilibrium with contracts is payoff equivalent to the equilibrium with degenerate mechanisms, offering the same rent to each household. This will eventually simplify the notation greatly.

The matching process between households and landlords is frictional. At any given time landlords post a single contract at zero cost and households direct their search to the most attractive contracts.[8]

Associated with any contract w, let u be the measure of households directing their search to w and v be the measure of landlords posting w. Define $\theta = u/v$ as the market tightness associated with contract w, $\theta : \mathrm{W} \to \Re_+$. Households find a house at rate $\alpha_h(\theta)$ where $\alpha_h : \Re_+ \to \Re_+$ and α_h is decreasing in θ. Landlords fill a vacancy at rate $\alpha_l(\theta)$, where $\alpha_l : \Re_+ \to \Re_+$ is increasing in θ. We assume that $\alpha_l(\theta) = \theta\alpha_h(\theta)$, that is equivalent to constant returns to scale in matching, and $\alpha_h(0) = \alpha_l(\infty) = \infty$ and $\alpha_h(\infty) = \alpha_l(0) = 0$. We assume that the elasticity of $\alpha_l(\theta)$, $\varepsilon(\theta) \equiv \frac{\theta}{\alpha_l(\theta)} \frac{d\alpha_l(\theta)}{d\theta}$ is constant: $\varepsilon(\theta) = \varepsilon$.

Let ψ_i be the share of households of type i applying to any given contract w. That is $\psi(w) = \{\psi_1(w), \psi_2(w), ..., \psi_I(w)\} \in \Delta^I$, where Δ^I is the I-dimensional unit simplex, $\psi : \mathrm{W} \to \Delta^I$. The market tightness $\theta(w)$ and the share of households applying to w, $\psi(w)$, associated with every contract w are determined in equilibrium.

Let $V_r(\gamma_i, r, \theta)$ and $Z_r(\gamma_i, r, \theta)$ be the expected values of living in a house and searching for

[7] We will restrict our attention to rental contracts with a fixed flow rent. With some loss of simplicity but with no change in our qualitative results, we could endogenously rule out payments contingent on separation if we also imposed limited commitment constraints on both the household and landlord. See section 6 for some discussion of fully dynamic contracts.

[8] Matching is bilateral, thus every household can only apply to one contract, but he can use mixed strategies.

a house[9], respectively, to the households of type i applying to any given contract, w with rental payment for that type of r. $\theta = \theta(w)$ is the market tightness associated with the contract w. Then:

$$
\begin{aligned}
\rho V_r(\gamma_i, r, \theta) &= h - r + \gamma_i (Z_r(\gamma_i, r, \theta) - V_r(\gamma_i, r, \theta)) \\
\rho Z_r(\gamma_i, r, \theta) &= \frac{\alpha_l(\theta)}{\theta} (V_r(\gamma_i, r, \theta) - Z_r(\gamma_i, r, \theta))
\end{aligned}
$$

Let $Y_r(\gamma_i, r, \theta)$ and $X_r(w, \theta)$ be the expected values of an occupied house when matched with a type i and a vacant house, respectively, to the landlord:

$$
\begin{aligned}
\rho Y_r(\gamma_i, r, \theta) &= r + \gamma_i (X_r(w, \theta) - Y_r(\gamma_i, r, \theta)) \\
\rho X_r(w, \theta) &= \alpha_l(\theta) \sum_{i \in \mathbb{I}} \psi_i(w) (Y_r(\gamma_i, r_i, \theta) - X_r(w, \theta))
\end{aligned}
$$

where $\psi_i(w)$ is the share of households of type i applying to the contract w, specifying rent r_i for that type, and θ is the market tightness associated with that contract.

Solving for the flow value of searching $\rho Z_r(\gamma_i, r, \theta)$ and posting $\rho X_r(w)$ gives:

$$
\rho Z_r(\gamma_i, r, \theta) = \frac{\alpha_l(\theta)}{\theta(\rho + \gamma_i) + \alpha_l(\theta)} (h - r) \tag{1}
$$

$$
\rho X_r(w, \theta) = \left(1 + \alpha_l(\theta) \sum_{i \in \mathbb{I}} \frac{\psi_i(w)}{\rho + \gamma_i}\right)^{-1} \alpha_l(\theta) \sum_{i \in \mathbb{I}} \frac{\psi_i(w) r_i}{\rho + \gamma_i} \tag{2}
$$

Notice that $\rho Z_r(\gamma_i, r, \theta) < 0$ if $r > h$, $\forall i$ and $\forall \theta > 0$, thus no household would apply to a contract that imposes a flow rent r higher than the flow utility from housing h. Similarly, $X_r(w, \theta) < H$ if $r_i < \rho H$ for all i for which $\psi_i(w)\theta > 0$.

2.2 Equilibrium with public information

A competitive search equilibrium satisfies the following conditions in every submarket: (i) landlords maximize expected profits; (ii) free entry (new entrants earn zero profits in expectation); (iii) households direct their search to the most convenient posted vacancy; (iv) $\theta = \theta(w)$ is consistent with rational expectations in equilibrium but also for any possible deviation w'.

More precisely, a landlord offering $w' \neq w$ expects that households apply until the market tightness θ' implies an expected value for the household equal to the outside option Z_r, that is taken as given by the (atomistic) firm. Formally:

[9] These are the values of searching and living in the same market, repeatedly *ad infinitum*

Definition 1. *A competitive search equilibrium with renting and public information is a vector* $\{Z_r^{*i}\}_{i\in\mathbb{I}}$*, a set of contracts* $\mathrm{W}_\mathrm{r}^* \subseteq \mathrm{W}^\mathbb{I}$ *each of which specifies a rent* r_i *for each* $i \in \mathbb{I}$*, a function* $\theta_r^* : \mathrm{W}^\mathbb{I} \to \Re_+$*, a measure* λ *on* $\mathrm{W}^\mathbb{I}$ *with support* W_r^**, and a function* $\psi : \mathrm{W}^\mathbb{I} \to \Delta^I$ *satisfying, for each* $i \in \mathbb{I}$*:*

(i) Landlords' profit maximization and free entry:

$$\left(1+\alpha_l(\theta_r^*(w))\sum_{i\in\mathbb{I}}\frac{\psi_i(w)}{\rho+\gamma_i}\right)^{-1}\alpha_l(\theta_r^*(w))\sum_{i\in\mathbb{I}}\frac{\psi_i(w)r_i}{\rho+\gamma_i} \le \rho H$$

with equality if $w \in \mathrm{W}_\mathrm{r}^*$*.*

(ii) Households' optimal search:

$$\textit{Let} \quad Z_r^{*i} \equiv \max_{w'\in\mathrm{W}_\mathrm{r}^*}\frac{1}{\rho}\frac{\alpha_e(\theta_r^*(w'))}{\theta_r^*(w')(\rho+\gamma_i)+\alpha_e(\theta_r^*(w'))}(h-r_i')$$

Then $\forall\, w \in \mathrm{W}^\mathbb{I}$

$$Z_r^{*i} \ge \frac{1}{\rho}\frac{\alpha_e(\theta_r^*(w))}{\theta_r^*(w)(\rho+\gamma_i)+\alpha_e(\theta_r^*(w))}(h-r_i)$$

with equality if $\theta_r^*(w) > 0$ *and* $\psi_i(w) > 0$*.*

(iii) market clearing:

$$\int_{\mathrm{W}_\mathrm{r}^*}\psi_i(w)\theta_r^*(w)d\lambda(w) = \pi_i \quad \forall i \in \mathbb{I}$$

The equilibrium definition imposes restrictions on the off-equilibrium beliefs of the landlords. The optimal search value of any type-i household is defined over the set of contracts posted in equilibrium W_r^* only, but under any deviating contract $w' \notin \mathrm{W}_\mathrm{r}^*$, landlords expect market tightness $\theta_r^*(w')$ to adjust to make all types of households weakly worse off.

We can distinguish competitive equilibria according to whether there are contracts which attract more than one type in equilibrium.

Definition 2. *A* separating competitive equilibrium *is any competitive equilibrium where for all* $w \in \mathrm{W}_\mathrm{r}^*$ *and for all* i*,* $\psi_i(w) > 0$ *implies* $\psi_i(w) = 1$*. A* pooling equilibrium *is any competitive equilibrium that is not separating. Two competitive equilibria (indexed by A and B) are* allocatively equivalent *if for all* $i \in \mathbb{I}$ *and* $w^A \in \mathrm{W}_\mathrm{r}^{*\mathrm{A}}$*,* $\psi_i(w^A) > 0$ *implies there exists a* $w^B \in \mathrm{W}_\mathrm{r}^{*\mathrm{B}}$ *with* $\psi_i(w^B) > 0$ *such that* $r_i^A = r_i^B$ *and* $\theta_r^{*A}(w^A) = \theta_r^{*B}(w^B)$ *and vice versa.*

Lemma 1. *If there exists a pooling competitive equilibrium with public information, then there exists an allocatively equivalent separating competitive equilibrium.*

Proof. See Appendix. □

In separating competitive equilibria, the market endogenously segments into submarkets, one for any different type i of households. Thus without loss of generality we can assume that a contract w in a separating competitive equilibrium contains a menu of rents where only one rent $r_i < h$ and thereafter label $w = r_i$. This also pins down the measure of landlords posting the contract w to households of type i, given by $v(w) = \frac{\gamma_i \pi_i}{\alpha_l(\theta_r^*(w)) + \gamma_i \theta_r^*(w)}$.

2.2.1 Characterization

A necessary and sufficient condition for a separating competitive search equilibrium is the following:[10]

Proposition 1. *For any type i of households, a posted contract w_r^{*i} and the associated market tightness $\theta_r^{*i} \equiv \theta_r^*(w_r^{*i})$ are part of an equilibrium allocation if and only if they solve the following constrained maximization problem, R_i:*

$$\begin{aligned} \max_{w_i, \theta_i} \quad & \frac{\alpha_l(\theta_i)}{\theta_i(\rho + \gamma_i) + \alpha_l(\theta_i)}(h - w_i) \\ s.t. \quad & \frac{\alpha_l(\theta_i)}{\rho + \gamma_i + \alpha_l(\theta_i)} w_i \geq \rho H \end{aligned}$$

The equilibrium allocation maximizes the expected value of search of any type-i household conditional on the firms making non-negative profits.

Proposition 2. *A solution to R_i exists for each i. The solution is unique.*

Proof. See Appendix. □

Lemma 2. *In the solution to R, for all $i, j \in \mathbb{I}$ with $i \neq j$, $\theta_r^{*i} \neq \theta_r^{*j}$*

Proof. Using the constraint with equality to substitute for w_r^{*i}, the first order condition implies the following equilibrium condition for the market tightness:

$$\frac{h}{\rho H} = 1 + \frac{1}{\theta_r^{*i}} \frac{\varepsilon}{1 - \varepsilon} + \frac{\rho + \gamma_i}{\alpha_l(\theta_r^{*i})(1 - \varepsilon)} \tag{3}$$

The implicit solution for θ_r^{*i} is strictly increasing in γ_i. □

[10] See e.g. Acemoglu and Shimer (1999) for a proof, with one caveat to the proof of sufficiency: in our setting, even if mechanisms in W_r^* are separating, other mechanisms in $\mathrm{W}^{\mathbb{I}}$ can be pooling. It is straightforward to use the argument in the proof of Lemma 1 to show that if the sufficiency conditions are met for a separating competitive search with separating-only mechanisms then they will be met here too.

Lemma 3. *Any competitive equilibrium with public information is a separating competitive equilibrium.*

Proof. Follows immediately from Lemmas 1 and 2 and Proposition 1. $\square$

The equilibrium values of the flow rent w_r^{*i} and the household's expected value ρZ_r^{*i} are given by:

$$w_r^{*i} = \frac{\rho + \gamma_i + \alpha_l(\theta_r^{*i})}{\alpha_l(\theta_r^{*i})} \rho H$$
$$\rho Z_r^{*i} = \frac{1}{\theta_r^{*i}} \frac{\varepsilon}{1-\varepsilon} \rho H$$

We have the following comparative static results as γ_i varies:

Result 1. *In equilibrium, as the separation hazard γ_i increases:*

(i) the market tightness θ_r^{*i} increases;

(ii) the flow rent w_r^{*i} increases;

(iii) the expected value to households ρZ_r^{*i} decreases.

Proof. See Appendix. $\square$

Thus, households with lower expected durations face tighter markets and higher rents once matched and as a consequence have lower search values.

Analytically, for the differential-γ case, by differentiating[11] the equilibrium condition (3) we obtain:

$$\frac{d\theta_r^*}{d\gamma} = \frac{1}{\varepsilon} \frac{\theta_r^{*2}}{\theta_r^*(\rho+\gamma) + \alpha_l(\theta_r^*)} > 0$$
$$\frac{dw_r^*}{d\gamma} = \frac{\rho H}{\theta_r^*(\rho+\gamma) + \alpha_l(\theta_r^*)} > 0$$
$$\frac{dZ_r^*}{d\gamma} = -Z_r^* \frac{\theta_r^*}{\varepsilon(\theta_r^*(\rho+\gamma) + \alpha_l(\theta_r^*))} < 0$$

[11] We need to explicitly define our notion of differentiation. Let $f : \mathbb{N} \to \Re$ and $g : range(f) \to G \subseteq \Re$. Define

$$\frac{\partial g}{\partial f}|_q = \lim_{q' \to q} \frac{g(q') - g(q)}{q' - q}$$

where $q, q' \in range(f)$. The total derivative is defined analogously.

2.3 Renting with private information

The equilibrium allocation in the public information case implies that every type $j < I$ strictly prefers to search in a higher ($i > j$) type's market if she was offered the higher type's contracted rent. In this section, we assume that the type of the household, i, is known only by the household. So, the public information allocation will not be incentive compatible under private information.

A mechanism in this setting would be a set of rents $\{r\}_{i\in\mathbb{I}}$. However, from the households value of being matched, it is clear that the only mechanism compatible with truth telling offers the same rent to any reported type.

Lemma 4. *A contract is incentive compatible if and only if it offers the same rent to any reported type.*

Proof. Follows from the household's value of being matched to a contract. □

So we can safely associate any incentive compatible contract w with its associated rent (and thus can assume $w \in [\rho H, h]$). We define the equilibrium following and extending the definition in Guerrieri et al. (2010) to a dynamic setting.

Lemma 5. *Sorting: $\forall i$, $w \in [\rho H, h]$, $\theta \geq 0$, and $\epsilon > 0$, there exists a couple $(w', \theta') \in B_\epsilon(w, \theta(w))$, with $w' < w$ and $\theta' > \theta$, such that*

$$Z_r(\gamma_j, w', \theta') > Z_r(\gamma_j, w, \theta) \ , \ \forall \ \gamma_j \leq \gamma_i \quad \textit{and} \quad Z_r(\gamma_j, w', \theta') < Z_r(\gamma_j, w, \theta) \ , \ \forall \ \gamma_j > \gamma_i$$

Proof. Follows from equation 1. □

The sorting condition is sufficient to have a separating equilibrium and differs from the condition in Guerrieri et al. (2010) in that it involves local perturbations in both the contract w and the market tightness θ.

Definition 3. *A competitive search equilibrium with renting and private information is a vector $\{Z_p^{*i}\}_{i\in\mathbb{I}}$, a set of rents (i.e. incentive compatible contracts) $\mathrm{W}_\mathrm{p}^* \subseteq [\rho H, h]^{\mathbb{I}}$, a measure λ on $[\rho H, h]$ with support W_p^*, a function $\theta_p^* : [\rho H, h] \to \Re_+$ and a function $\psi : [\rho H, h] \to \Delta^I$ satisfying:*

(i) landlords' profit maximization and free entry: for any $w \in [\rho H, h]$

$$\left[1 + \left(\alpha_l(\theta_p^*(w)) \sum_{i\in\mathbb{I}} \frac{\psi_i(w)}{\rho + \gamma_i}\right)^{-1}\right]^{-1} w \leq \rho H$$

with equality if $w \in \mathrm{W}_\mathrm{p}^*$.

(ii) households' optimal search: Let

$$Z_p^{*i} \equiv \max_{w' \in \mathrm{W}_\mathrm{p}^*} \frac{1}{\rho} \frac{\alpha_l(\theta_p^*(w'))}{\theta_p^*(w')(\rho+\gamma_i)+\alpha_l(\theta_p^*(w'))}(h-w')$$

Then $\forall w \in [\rho H, h]$ *and* $\forall \gamma_i$

$$Z_p^{*i} \geq \frac{1}{\rho} \frac{\alpha_l(\theta_p^*(w))}{\theta_p^*(w)(\rho+\gamma_i)+\alpha_l(\theta_p^*(w))}(h-w)$$

with equality if $\theta_p^*(w) > 0$ *and* $\psi_i(w) > 0$.

(iii) market clearing:

$$\int_{\mathrm{W}_\mathrm{p}^*} \psi_i(w)\theta_p^*(w)d\lambda(w) = \pi_i \quad \forall i$$

As in the public information case, the equilibrium definition imposes conditions on the off-equilibrium beliefs of the landlords. Heuristically, a landlord considering whether to post a deviating contract w' imagines an initial market tightness $\theta = 0$. If no households is willing to apply, then $\theta = 0$ and the deviation is not profitable. Otherwise, some households apply, increasing market tightness θ, until only one type of household i is indifferent about the deviating w' and all others j (weakly) prefer their equilibrium contracts. This in turn pins down the share ψ_i of households applying to that contract.

2.3.1 Equilibrium and Characterization

The characterization of the equilibrium with private information is equivalent to the public information equilibrium with an extra incentive compatibility constraint that imposes that no other types of households j are attracted to the contract w_i. In the next proposition, we show that at the optimum, for all $i > 1$, only the marginal incentive compatibility constraints $IC(i-1,i)$ bind: every type $(i-1)$ is indifferent between his own contract and the contract offered to the type i with marginally higher expected duration.

Proposition 3. *Let the problem* (PR) *be defined by the following constrained maximization problem* (PR_i)*, for any* $i \in \mathbb{I}$*:*

$$\begin{aligned}
& \max_{\theta \in \Re_+, w \in \Re_+} Z_r(\gamma_i, w, \theta) \\
s.t. \quad & \frac{\alpha_l(\theta)}{\rho+\gamma_i+\alpha_l(\theta)} w \geq rH \\
\text{and} \quad & Z_r(\gamma_j, w, \theta) \leq Z_r(\gamma_j, w_p^{*j}, \theta_p^{*j}) \quad \text{for all } j \neq i \qquad [IC(j,i)]
\end{aligned}$$

where w_p^{*i}, θ_p^{*i} is an optimal solution for i.
The solution of (PR) exists and is unique. Moreover, only the marginal incentive compatibility constraints $IC(i-1,i)$ bind, for all $i > 1$:

$$Z_r(\gamma_{i-1}, w_p^{*i}, \theta_p^{*i}) = Z_r(\gamma_{i-1}, w_p^{*,i-1}, \theta_p^{*,i-1}) \quad \textit{and}$$
$$Z_r(\gamma_j, w_p^{*i}, \theta_p^{*i}) < Z_r(\gamma_j, w_p^{*j}, \theta_p^{*j}) \quad \forall\, j \neq i, i-1$$

Proof. See Appendix. □

Thus, for the type with the highest separation hazard, $\gamma_1 = \bar{\gamma}$, the equilibrium allocation is the same as the one with public information. Then, the problem is solved iteratively for all other types:

(i) For $i = 1$, w_p^{*1} and θ_p^{*1} solve R_1

(ii) For each $i > 1$, w_p^{*i} and θ_p^{*i} are the solutions to

$$\begin{aligned} &\max_{\theta \in \Re_+, w \in \Re_+} Z_r(\gamma_i, w, \theta) \\ \text{s.t.} \quad &\frac{\alpha_l(\theta)}{\rho + \gamma_i + \alpha_l(\theta)} w \geq \rho H \\ \text{and} \quad &Z_r(\gamma_{i-1}, w_p^{*i}, \theta_p^{*i}) \leq Z_r(\gamma_{i-1}, w_p^{*,i-1}, \theta_p^{*,i-1}) \end{aligned}$$

We are now ready to prove the existence and uniqueness of the equilibrium and characterize the equilibrium allocation:

Proposition 4. *There exists a unique separating equilibrium. A set of contracts $\{w_p^{*i}\}_{\mathbb{I}}, \quad w_p^{*i} \in [\rho H, h]$ and market tightnesses $\{\theta_p^{*i}\}_{\mathbb{I}}, \quad \theta_p^{*i} \equiv \theta_p^*(w_p^{*i}) \equiv \theta_i$ associated with their respective types γ_i are part of the equilibrium allocation if and only if they solve the problem PR.*

Proof. See Appendix. □

We have the following comparative static results as γ_i varies.

Result 2. *In equilibrium, as the separation hazard γ_i increases:*

(i) the market tightness θ_p^{*i} decreases;

(ii) the flow rent w_p^{*i} increases;

(iii) the expected value to households ρZ_p^{*i} decreases;

Proof. See Appendix. □

Analytically, for the differential-γ case, then:

$$\begin{aligned}
\frac{d\theta_p^*}{d\gamma} &= -\frac{1}{\rho+\gamma}\left[(1-\varepsilon)\frac{\rho Z_p^*}{\rho H} - \frac{\varepsilon}{\theta_p^*}\right]^{-1} < 0 \\
\frac{dw_p^*}{d\gamma} &= \frac{(1-\varepsilon)\rho Z_p^*}{\alpha_l(\theta_p^*)}\left[(1-\varepsilon)\frac{\rho Z_p^*}{\rho H} - \frac{\varepsilon}{\theta_p^*}\right]^{-1} > 0 \\
\frac{dZ_p^*}{d\gamma} &= -Z_p^*\frac{\theta_p^*}{\theta_p^*(\rho+\gamma)+\alpha_l(\theta_p^*)} < 0
\end{aligned}$$

Contrary to the public information case, low-γ types search in tighter markets in equilibrium, and pay lower rents if matched. In this way landlords are able to optimally (with the least cost) separate types of households by posting contracts w_p^{*i} lower than the first-best optimum w_r^{*i} to those that expect to stay longer, at the cost of higher market tightness θ_p^{*i}.

Households that expect to stay longer are less affected by a higher market tightness (and thus longer expected search times), because they expect to separate from the house and pay the search cost less frequently. On the other hand, those that expect to stay longer are more affected by a lower rent w because they expect to be matched a higher fraction of time for any given market tightness θ. The combination of these two factors implies that the second best allocation dictates tighter markets for those that expect to stay longer, contrary to the first best allocation.

3 Owning market

An owning contract simply specifies an up-front payment P paid by the household to the landlord, which may vary across markets. Preferences and technology (except the search technology) are the same as in the rental market. In particular, households derive the same flow utility h if they own or rent the house, and landlords (i.e. builders) pay the same building cost H to enter the market.

As will become clear below, absent some further friction, owning would efficiently solve the private information problem and all markets would be owner-occupied markets[12]. To provide heterogeneity, we assume that if a homeowner needs to move, she first must sell the house before she can search for a new one to buy. Thus, there is an extra friction in the owning market that takes the form of double search.[13] Moreover, we assume that housing cannot be resold in a different market.[14] This implies that an owner leaving her house posts a contract in the same

[12] The $i = 1$ type would be indifferent between owning and renting.

[13] There are plenty of other potential candidate frictions. For instance, a (possibly heterogeneous) "financing cost" which provides some disutility at the time of purchase or throughout occupancy.

[14] Geographical separation in markets would be one way to prevent homeowners from selling in a different market from which they bought.

owning market and thus (in equilibrium) at the same price paid to buy it.[15]

Builders only have to sell a new house. It is important to notice that the owning market is not affected by the private information friction, because a household that buys the house, an owner, fully internalizes the expected search cost eventually paid in the case of separation, contrary to a renter. In other terms, the builder's expected value of posting in an owning market with tightness θ a contract for sale at price P is simply given by:

$$X_o(P,\theta) = \frac{\alpha_l(\theta)}{\rho + \alpha_l(\theta)} P \tag{4}$$

Notice that 4 is independent of γ_i.

The values of searching as a buyer, living and searching as a seller in a market with market tightness θ and price P for a household of type i, respectively, are given by:

$$\begin{aligned}
\rho Z_o(\gamma_i, P, \theta) &= \alpha_h(\theta)(V_o(\gamma_i, P, \theta) - Z_o(\gamma_i, P, \theta) - P) \\
\rho V_o(\gamma_i, P, \theta) &= h + \gamma_i(S_o(\gamma_i, P, \theta) - V_o(\gamma_i, P, \theta)) \\
\rho S_o(\gamma_i, P, \theta) &= \alpha_l(\theta)(P + Z_o(\gamma_i, P, \theta) - S_o(\gamma_i, P, \theta))
\end{aligned}$$

When a separation shock hits the owner (with hazard rate γ_i), the household incurs two search costs, one on each side of the market. Solving for the flow value of searching as a buyer gives:

$$\rho Z_o(\gamma_i, P, \theta) = \left(1 + \frac{\rho + \gamma_i}{\alpha_l(\theta)/\theta} + \underbrace{\frac{\gamma_i}{\rho + \alpha_l(\theta)}}_{\text{wedge from double search}}\right)^{-1} \left[h - \left(1 + \frac{\gamma_i}{\rho + \alpha_l(\theta)}\right)\rho P\right]$$

The wedge will imply that, in the case of public information, renting is always preferred to owning. Moreover, the effect of the wedge is larger the higher is the separation hazard γ_i (for a given θ): households that move more often pay the double search costs more often.

3.1 Equilibrium with only owning

Neither builders nor owners when selling care about the types of the buyers in the market in which they have posted. So owning markets do not depend on whether households' types are public or private information. The equilibrium definition of the owning market is similar to the equilibrium in the rental market with private information in that contracts (prices) are not type-specific: each market offers just one contract price. The market endogenously segments into submarkets and we can characterize the equilibrium allocation using an equivalent constrained maximization problem.

[15] This assumption greatly simplifies the analysis.

Definition 4. *A competitive search equilibrium with owning is a vector* $\{Z_o^{*i}\}_{i\in\mathbb{I}}$*, a set of prices* $P^* = \{P^{*i}\}_{i\in\mathbb{I}} \in [H, h/\rho]^{\mathbb{I}}$*, a measure* λ *on* $[H, h/\rho]$ *with support* P^**, and functions* $\theta_o^* : [H, h/\rho] \to \Re_+$ *and* $\psi : [H, h/\rho] \to \Delta^I$ *satisfying:*

(i) Builders' profit maximization and free entry:

$$\frac{\alpha_l(\theta_o^*(P))}{\rho + \alpha_l(\theta_o^*(P))} P \leq H$$

with equality if $P \in \mathrm{P}^*$.

(ii) Households' optimal search:

Let $$Z_o^{*i} \equiv \max_{P' \in P^*} \frac{1}{\rho}\left(1 + \frac{\rho + \gamma_i}{\alpha_l(\theta_o^*(P'))/\theta_o^*(P')} + \frac{\gamma_i}{\rho + \alpha_l(\theta_o^*(P'))}\right)^{-1}\left[h - \left(1 + \frac{\gamma_i}{\rho + \alpha_l(\theta_o^*(P'))}\right)\rho P'\right]$$

Then $\forall P \in [H, h/\rho]$ *and* $i \in \mathbb{I}$

$$Z_o^{*i} \geq \frac{1}{\rho}\left(1 + \frac{\rho + \gamma_i}{\alpha_l(\theta_o^*(P))/\theta_o^*(P)} + \frac{\gamma_i}{\rho + \alpha_l(\theta_o^*(P))}\right)^{-1}\left[h - \left(1 + \frac{\gamma_i}{\rho + \alpha_l(\theta_o^*(P))}\right)\rho P\right]$$

with equality if $\theta_o^*(P) > 0$ *and* $\psi_i(P) > 0$.

(iii) market clearing:

$$\int_{P^*} \psi_i(P)\theta_o^*(P) d\lambda(P) = \pi_i \quad \forall i$$

As in the economies with renting, the equilibrium in the owning economy can be found by solving a constrained optimization problem iteratively by type.[16] The optimal market tightness conditional on owning for each type, θ_o^{*i}, is the solution to the following equation[17]:

$$(\rho + \gamma_i + \alpha_l)\left[\frac{\rho + \alpha_l}{\alpha_l} - \frac{\rho\varepsilon}{\theta_o^{*i}} \frac{\rho + \gamma_i + \alpha_l}{(\rho + \gamma_i)(\rho + \alpha_l)}\right] + \varepsilon = \frac{h}{\rho H}\left[\frac{\varepsilon\alpha_l\alpha_l\gamma_i}{(\rho + \alpha_l)\theta_o^{*i}(\rho + \gamma_i)} + (\rho + \alpha_l + \varepsilon)\right]$$

3.2 Equilibrium with both renting and owning

We are now ready to study the equilibrium problem in the housing market with private information.

Landlords/builders are free to enter in both the rental and the owning market. If they enter, they pay a building cost H and post a contract in one market. Households have private

[16] That is, a similar version of either Proposition 1 or 3 holds. Furthermore, it is also easy to show that a similar type of incentive compatibility constraint as the one in Proposition 3 never binds.

[17] where α_l implies $\alpha_l(\theta_o^{*i})$

information over their expected duration of stay, or mobility hazard rate γ, and direct their search to their preferred postings.

In the appendix, we formally define a competitive equilibrium with private information and both renting and owning. The equilibrium with both renting and owning can be characterized by the iterative solutions to a problem analogous to those with only owning or renting[18]:

$$Z_{po}^{*i} \equiv \max_{\{rent,own\}} \left\{ \tilde{Z}_p^i \equiv \max_{\tilde{\theta}_p^i \in \Re_+, w_i \in [\rho H, h]} Z_r(\gamma_i, w_i, \tilde{\theta}_p^i), \max_{\theta_o^i \in \Re_+, P_i \in [H, h/\rho]} Z_o(\gamma_i, P_i, \theta_o^i) \right\}$$

$$\text{s.t.} \quad \frac{\alpha_l(\tilde{\theta}_p^i)}{\rho + \gamma_i + \alpha_l(\tilde{\theta}_p^i)} w_i \geq \rho H$$

$$P_i = \frac{\rho + \alpha_l(\theta_o^i)}{\alpha_l(\theta_o^i)} H$$

$$Z_{po}^{*i-1} \geq Z_r(\gamma_{i-1}, w_i, \tilde{\theta}_p^i) \quad \text{for all } i > 1$$

Result 3. *(i) The equilibrium expected value of search in the owning market Z_o^{*i} is lower than the equilibrium expected value of search in the rental market with public information Z_r^{*i}, and the equilibrium market tightness is lower:*

$$Z_o^{*i} < Z_r^{*i} \quad \textit{and} \quad \theta_o^{*i} < \theta_r^{*i} \quad \forall i$$

(ii) If $\underline{\gamma} = 0$, then there exists a threshold $\tilde{i} \leq I$ such that $\forall i > \tilde{i}$ $Z_o^{*i} > \tilde{Z}_p^{*i}$.

Proof. See Appendix. □

The equilibrium in the private information rental market for the highest-γ type is the same as in the public information case, thus Result 3 implies that households with the lowest expected durations always prefer to search in the rental market, even if information is private. The maximands to the owning part of characterization are identical to the solutions for the owning-only economy above.

For households with lower γ's, the equilibrium in the (private information) rental market is increasingly distorted with respect to the first best (public information) equilibrium. Moreover, the extra search friction in the owning market is less severe as γ decreases, because long duration households move less frequently. At the limit, $\gamma \to 0$, the household stays in the same location forever and is not affected by the double search friction: the expected values of search in the owning market and in the public information rental market are the same. Moreover, for low enough γ, owning-occupied markets dominate rental markets (with private information).

[18]We omit the proof, however it is similar to the case with only renting

4 Example and application to rent control

As a parametrization, we set $\rho = .05$, $h = .1$, $H = 1$, $\alpha_l = \theta^\varepsilon$ with $\varepsilon = .5$, and we allow for $\gamma \in [.2, .7]$, that is expected durations between 1.4 and 5 years, approximately. Figure 1 plots the market tightness, or queue length, and the flow rent (or housing price) as a function of γ in the three economies: renting with public information, renting with private information and owning.

The queue length increases as γ increases in the case of renting with public information and in the owning economy (and it is shorter in the latter case), while it decreases as γ increases in the renting economy with private information, because low γ-types signal themselves by waiting longer. In both renting economies, the flow rent increases with γ, as shown in figure 2: it increases faster in the private information case to offset the positive effect of the longer queue length faced by low γ-types on landlords' profits. The housing price in the owning economy markets, expressed in flow terms, decreases slightly as γ increases.

Finally, figure 3 shows the expected value of searching for a house as a function of γ in the three markets: renting with public information, renting with private information and owning. The value of renting with public information is always higher than the other cases (and it coincides with the private information renting for the highest value of γ). The expected value increases as γ decreases in all markets, but it increases less in the private information renting market.

4.1 Rent control

We continue the example by analyzing the same economy but with the addition of a very stylized rent control policy. Here rent control is just a simple rent ceiling (which we set to 90 percent of the highest rent in the uncontrolled economy). Figures 4, 5 and 6 show, respectively, the queue length, rents and expected value of searching in the controlled economy.

In the case of public information, the rent control policy distorts the markets for the shortest-duration households the most; their queues lengthen considerably and the supply of regulated housing falls. In fact, if the rent ceiling were lower, it is possible that the distortions to these households' markets are high enough to push them into ownership. All rental markets that had rents above the ceiling in the uncontrolled economy now have rents at the ceiling rate. However, because markets segment perfectly with public information, rent control does not affect the uncontrolled rental markets that already had low rents.

With private information, all rental markets are affected by the ceiling even though only the low-duration households have rents at the ceiling rate. That the controlled market affects the uncontrolled markets is not due to some households leaving the controlled market for an uncontrolled one, as in e.g. Fallis and Smith (1984) (where there is excess demand in the controlled market) and Weibull (1983) (where there is no excess demand) nor to misallocation of high-quality housing (as hinted at in Glaeser and Luttmer (2003) and examined more broadly for the case of China by Wang (2011)). Here, there is no excess demand; the controlled market is in "equilibrium", albeit an inefficient one[19]. Instead, rent control exacerbates the private information problem by making the low-duration households worse-off in their own market, tightening the incentive compatibility constraint. Queues in all rental markets are higher and the supply of rental housing is everywhere lower. The lower expected value of searching in the rental market also leads to more ownership, which, unlike in the case of public information, occurs with any binding rent ceiling.

Obviously rent control is not a welfare-improving policy in our economy. In fact, the Pareto optimal policy would be a system of market dependent lump-sum taxes and transfers to households that effectively shares the surplus that the longer-duration households have over the shorter-duration ones in the public information economy[20]. Rather than focusing on these policies though, we instead next endow the economy with a customization technology which in equilibrium helps screen low-duration types.

5 Customization

As we have seen, the private information problem can be decentralized in a rather easy way: some houses are for sale while other houses are for rent. It is "easy" for a household to direct its search in this case. In this section, we relax the assumption that all houses offer the same utility flow to all households and that this utility flow is observable prior to a match. There are generally many attributes, like specific location, the quality of the light in the house and so forth, that are often only observable in person. Tastes for these particular attributes can vary - some households value a quiet residential street more than others. To capture some of this, we add a customization technology similar to ones used in random-search models of housing (e.g. Arnott (1989); Igarashi (1991)). We assume that customization raises the flow utility that a household gets from the house at a cost of reduced matching.

[19] There is no excess demand or supply at the controlled rent, and in that sense the controlled market is in equilibrium (as in Weibull (1983)). However landlords would enter into the market offering a higher rent and lower implied market tightness, if they could. Therefore neither the public nor private information controlled market allocations are competitive equilibriums as defined above. Rather they are competitive equilibriums to economies with the added restriction that $w \in [\rho H, \bar{w}]^{\mathbb{I}}$, where $\bar{w}$ is the rent ceiling.

[20] This optimum can potentially replicate the first best queues and rents if the masses of long-duration households are large enough.

Formally, a house can be customized or not. An uncustomized house gives a utility flow of h. A customized house has a variety τ located on a circle of circumference 1. Households have idiosyncratic tastes over varieties, denoted by ι and known only by the household. A household of taste ι living in a customized house of variety τ receives a utility flow of $h + c$ (with $c > 0$) if $d(\tau, \iota) < \frac{1}{2\delta}$, where $\delta > 1$, and receives a flow of 0 otherwise.[21]

Tastes are distributed uniformly over the population and independently of type i. We assume that a contract can specify whether or not a house has been customized but not the variety of customization. The variety of a particular house is not known to a prospective renter or owner until after the household is matched with the house. At this point the household observes the variety of the house and can then reject the match (and thus the contract) and continue to search.

Lastly, we assume that when houses are built, the builders know the measure of customized houses in the economy but do not observe the distribution of existing varieties. Thus builders pursue symmetric mixed strategies with regards to variety choice and the resulting distribution of varieties is uniform.

Our assumptions mean that: i) if a household chooses to search in a customized market, it will optimally choose to search there until it is matched with a house for which it is well-matched (i.e. gets $h + c$ utility flow from); ii) acceptable matches in a customized market with a mass of u searchers and a mass v postings will occur at a rate $m_c(u, v) = m(u, v)/\delta$.

5.1 Customization in rental markets

The flow value of searching in a customized rental market for a given γ_i, w and market tightness θ is given by $\rho Z_c(\gamma_i, w, \theta)$ (and likewise the flow value of vacancy is $\rho X_c(\gamma_i, w, \theta)$).[22]

$$\rho V_c(\gamma_i, w, \theta) = h + c - w + \gamma(Z_c(\gamma_i, w, \theta) - V_c(\gamma_i, w, \theta))$$
$$\rho Z_c(\gamma_i, w, \theta) = \frac{\alpha_l(\theta)}{\delta\theta}(V_c(\gamma_i, w, \theta) - Z_c(\gamma_i, w, \theta))$$

$$\rho Y_c(\gamma_i, w, \theta) = w + \gamma(X_c(\gamma_i, w, \theta) - Y_c(\gamma_i, w, \theta))$$
$$\rho X_c(\gamma_i, w, \theta) = \frac{\alpha_l(\theta)}{\delta}(Y_c(\gamma_i, w, \theta) - X_c(\gamma_i, w, \theta))$$

With public information, for any market that customizes, the equilibrium conditions for

[21] $d : [0, 1) \times [0, 1) \to \Re_+$ with $d(\tau, \iota) = \min\{|\tau - \iota|, 1 + \min\{\tau - \iota, \iota - \tau\}\}$

[22] To keep notation as light as possible, we note that only separating equilibria are possible here and drop X_c's dependency on Ψ

market tightness, rents and search value for each type ($\theta_{cr}^{*i}, w_{cr}^{*i}, Z_{cr}^{*i}$, respectively) are

$$\frac{h+c}{\rho H} = 1 + \frac{1}{\theta_{cr}^{*i}}\frac{\varepsilon}{1-\varepsilon} + \frac{\delta(\rho+\gamma_i)}{\alpha_l(\theta_{cr}^{*i})(1-\varepsilon)}$$
$$w_{cr}^{*i} = \frac{\delta(\rho+\gamma_i)+\alpha_l(\theta_{cr}^{*i})}{\alpha_l(\theta_{cr}^{*i})}\rho H$$
$$\rho Z_{cr}^{*i} = \frac{1}{\theta_{cr}^{*i}}\frac{\varepsilon}{1-\varepsilon}\rho H$$

Note that δ influences the flow value only through the equilibrium queue length.

The overall equilibrium value of search for a household with public information renting only but with the choice of customization is then the upper envelope of Z_{cr}^{*i} and Z_r^{*i}. Finally, customization with public information is a normal good in the sense that if any type prefers their customized market to their (shadow) uncustomized one, then all types with longer expected durations will also prefer their respective customized markets:

Result 4. *If there exists an $\tilde{i}$ such that $Z_{cr}^{*\tilde{i}} \geq Z_r^{*\tilde{i}}$, then $Z_{cr}^{*i} > Z_r^{*i}$ for all $i > \tilde{i}$.*

Proof. See Appendix. □

5.2 Customization in the owning market

The analysis of the owning market with customization is similar to the case without customization. For any type i, price P and market tightness θ:

$$\rho Z_{co}(\gamma_i, P, \theta) = \frac{\alpha_h(\theta)}{\delta}(V_{co}(\gamma_i, P, \theta) - Z_{co}(\gamma_i, P, \theta) - P)$$
$$\rho V_{co}(\gamma_i, P, \theta) = h + c + \gamma_i(S_{co}(\gamma_i, P, \theta) - V_{co}(\gamma_i, P, \theta))$$
$$\rho S_{co}(\gamma_i, P, \theta) = \frac{\alpha_l(\theta)}{\delta}(P + Z_{co}(\gamma_i, P, \theta) - S_{co}(\gamma_i, P, \theta))$$

The market tightness in a given market is determined from the builders' zero profit condition:

$$P = \frac{\rho\delta + \alpha_l(\theta)}{\alpha_l(\theta)}H$$

5.2.1 Customization with private information

The problem of customization when information is private follows similarly. We skip the definition of a competitive equilibrium and turn immediately to how to solve for its unique allocation.

Solving iteratively, for any type i, with θ_{cp}^{*i} and w_{cp}^{*i} the argmaxs for customized renting, θ_{up}^{*i} and w_{up}^{*i} the argmaxs for uncustomized renting, θ_{co}^{*i} and P_{co}^{*i} the argmaxs for customized owning, θ_{uo}^{*i} and P_{uo}^{*i} the argmaxs for uncustomized owning, and Z_{cu}^{*i} the maximum over all options:

$$
\begin{aligned}
Z_{cu}^{*i} \equiv \max\Bigg\{ & \max_{\theta_{cp}\in\Re_+, w_{cp}\in\Re_+} Z_c(\gamma_i, w_{cp}, \theta_{cp}), \max_{\theta_{up}\in\Re_+, w_{up}\in\Re_+} Z_r(\gamma_i, w_{up}, \theta_{up}), \\
& \max_{\theta_{co}\in\Re_+, P_{co}\in\Re_+} Z_{co}(\gamma_i, P_{co}, \theta_{co}), \max_{\theta_{uo}\in\Re_+, P_{uo}\in\Re_+} Z_o(\gamma_i, P_{uo}, \theta_{uo}) \Bigg\} \\
\text{s.t.} \quad & \frac{\alpha_l(\theta_{up})}{\rho + \gamma_i + \alpha_l(\theta_{up})} w_{up} \geq \rho H \\
& \frac{\alpha_l(\theta_{cp})}{\delta(\rho + \gamma_i) + \alpha_l(\theta_{cp})} w_{cp} \geq \rho H \\
& P_{co} \frac{\alpha_l(\theta_{co})}{\rho\delta + \alpha_l(\theta_{co})} = H \\
& P_{uo} \frac{\alpha_l(\theta_{uo})}{\rho + \alpha_l(\theta_{uo})} = H \\
& \quad Z_c(\gamma_j, w_c, \theta_c) \leq Z_{cu}^{*j} \quad \text{for all } j < i \\
& \quad Z_r(\gamma_j, w_u, \theta_u) \leq Z_{cu}^{*j} \quad \text{for all } j < i
\end{aligned}
$$

We analyze numerically some properties of the equilibrium in the following example.

5.3 Example continued

We continue the above example (without rent control) by adding $\delta = 1.35$ and $c = .01$. Figure 7 shows the value of searching in each rental market with private info, ρZ_{cp}^*, ρZ_{up}^*. There is a kink in ρZ_{cp}^* and the customized queue length path; the incentive compatibility constraint does not bind in customized market for the lowest types and thus queue lengths can fall as γ decreases for as long as the constraint doesn't bind (as in figure 8). However, these markets are non-existent in equilibrium as the values of searching in the customized markets are dominated by the uncustomized markets' values for these types.

For higher types the values of search in the uncustomized and customized markets are nearly the same (although the customized market is slightly better): for any type, slightly worse types are searching in their own customized markets where their search value is higher than it otherwise would be if there were only uncustomized markets. This relaxes the incentive compatibility constraint in the uncustomized market (relative to the case with only that market) nearly to the value of the customized market's one. However, the value of search in the uncustomized market is still slightly below because it is still harder to properly incentivize lower types in an uncustomized market and so distortions using the queue length are larger.

Figures 9, 10, 11 plot the upper envelopes over the values of customized versus uncustomized, and the queues and rents in all markets. There are several points worth noting.

First, private information leads to "over-customization" in the rental market: some markets are customized with private information where the types' corresponding market with public information would not customize. As in the simpler economy without customization but with private information, the market uses longer search times to screen away shorter duration households from the long duration households' markets. In the economy with the customization technology, there are two ways to lengthen search times: lengthen queues and customizing. So customization has two benefits with private information (higher flow utility and better screening) which leads it to be adopted for types that would not have adopted it under public information.

Second, in general there may be four regions (from low types to high types in γ-space) where first the private information equilibrium is uncustomized renting, then customized renting, then uncustomized owning, then finally customized owning for the highest types. However, the example shows that one or more of the regions may not exist for particular parameterizations. In our example (which turns out to be qualitatively typical), there are no uncustomized owner-occupied houses in equilibrium and relatively fewer customized houses "available" for renters. The average homeowner is this example gets a higher flow utility from living in his house $(h+c)$ than does the average renter. An econometrician who did observe this customization would think that homeowners get a warm glow from owning.

6 Conclusion

We build a competitive search equilibrium model of housing tenure choices where households have private information over their expected duration, and we study the properties of rental and owning markets in a search equilibrium. Owning a house solves the private information problem but at the cost of double search: owners that move to another location must sell their house before searching for another one. We show that both markets endogenously segment into submarkets, one for every type of households.

In the rental markets, households that expect to stay longer search in thinner markets in order to discourage more footloose households from searching in the same market. Relative to the first-best, the distortions in the rental market with private information increase with expected duration. On the other hand, the wedge due to double search in the owning market decreases with expected duration. As a result, households that expect to stay longest in their houses will be the ones that choose to own (if any choose to own).

Rent control leads to distortions in both controlled and uncontrolled markets by exacerbating incentive compatibility constraints when information is private. A customization technology that raises the utility from housing at the cost of a lower probability of a match can help screen low-duration types. The extra screening leads to over-customization in the private information rental markets relative to the public information benchmark. However, since the appeal of customization is higher for households that expect to stay in their house longer, owner-occupiers tend to customize more.

Though the rental contracts considered here are limited to constant, duration-independent rents, it would be relatively straightforward to consider duration-dependent contracts (and thus fully optimal) subject to additional limited participation constraints that, absent a separation shock, neither the landlord nor the household's continuation values in the contract fall below their outside options of search. Optimal duration-dependent contracts could achieve the first best as long as households remain risk-neutral. If households were risk-neutral, the optimal rent contract with private information would feature an upfront payment to the landlord followed by a constant rent $w = \rho H$. However, Barker (2003) finds little evidence for declining rent schedules. If households are risk-averse, we suggest (without proving) that the equilibrium contracts offered in such an economy may otherwise have many of the same qualitative features as those presented above.

Other scopes for extension include considering other wedges in the owning market other than double search. For instance, one could assume a (potentially heterogenous) flow cost of owning due to borrowing constraints. As long as any mooted wedge does not increase too quickly with expected duration, those with the highest expected durations will choose to own.

Theories with housing ladders are essentially theories where an otherwise identical house is available only in either the rental or owner-occupied market. Sadly, data on rental vacancy durations across narrowly defined markets are, to our knowledge, poor. So direct measures of housing ladders remain elusive.

7 Appendix

Definition of Competitive Equilibrium With Renting and Owning

Definition 5. *A competitive search equilibrium with renting, owning and private information is a set of vectors* $\{Z_{po}^{*i}, Z_o^i, \tilde{Z}_p^i\}_{i\in\mathbb{I}}$*, a set of incentive compatible rents* $\tilde{\mathrm{W}}_\mathrm{p}^* \subseteq [\rho H, h]^{\mathbb{I}}$*, a set of prices* $P^* = \{P^{*i}\}_{i\in\mathbb{I}} \in [H, h/\rho]^{\mathbb{I}}$*, a measure* λ_r *on* $[\rho H, h]$ *with support* $\tilde{\mathrm{W}}_\mathrm{p}^*$*, a measure* λ_o *on* $[H, h/\rho]$ *with support* P^**, functions* $\tilde{\theta}_p^* : [\rho H, h] \to \Re_+$ *and* $\theta_o^* : [H, h/\rho] \to \Re_+$ *and functions* $\psi_r : [\rho H, h] \to \Delta^I$ *and* $\psi_o : [H, h/\rho] \to \Delta^I$ *satisfying:*

(i) Landlords' profit maximization and free entry: for any $w \in [\rho H, h]$

$$\left[1 + \left(\alpha_l(\tilde{\theta}_p^*(w)) \sum_{i\in\mathbb{I}} \frac{\psi_{r,i}(w)}{\rho+\gamma_i}\right)^{-1}\right]^{-1} w \le \rho H$$

with equality if $w \in \tilde{\mathrm{W}}_\mathrm{p}^*$*.*

(ii) Builders' profit maximization and free entry: for any $P \in [H, h/\rho]$

$$\frac{\alpha_l(\theta_o^*(P))}{\rho + \alpha_l(\theta_o^*(P))} P \le H$$

with equality if $P \in P^*$*.*

(iii) Households' optimal search: Let

$$\tilde{Z}_p^i \equiv \max_{w' \in \tilde{\mathrm{W}}_\mathrm{p}^*} \frac{1}{\rho} \frac{\alpha_l(\tilde{\theta}_p^*(w'))}{\tilde{\theta}_p^*(w')(\rho+\gamma_i) + \alpha_l(\tilde{\theta}_p^*(w'))}(h - w')$$

$$Z_o^i \equiv \max_{P' \in P^*} \frac{1}{\rho}\left(1 + \frac{\rho+\gamma_i}{\alpha_l(\theta_o^*(P'))/\theta_o^*(P')} + \frac{\gamma_i}{\rho+\alpha_l(\theta_o^*(P'))}\right)^{-1}\left[h - \left(1 + \frac{\gamma_i}{\rho + \alpha_l(\theta_o^*(P'))}\right)\rho P'\right]$$

$$\text{and} \quad Z_{po}^{*i} = \max\{Z_o^i, \tilde{Z}_p^i\} \ \forall\ i \in \mathbb{I}$$

Then $\forall w \in [\rho H, h]$ *and* $\forall \gamma_i$

$$Z_{po}^{*i} \ge \frac{1}{\rho} \frac{\alpha_l(\tilde{\theta}_p^*(w))}{\tilde{\theta}_p^*(w)(\rho+\gamma_i) + \alpha_l(\tilde{\theta}_p^*(w))}(h - w)$$

with equality if $\tilde{\theta}_p^*(w) > 0$ *and* $\psi_{r,i}(w) > 0$*. And* $\forall P \in [H, h/\rho]$ *and* $\forall \gamma_i$

$$Z_{po}^{*i} \ge \frac{1}{\rho}\left(1 + \frac{\rho+\gamma_i}{\alpha_l(\theta_o^*(P))/\theta_o^*(P)} + \frac{\gamma_i}{\rho+\alpha_l(\theta_o^*(P))}\right)^{-1}\left[h - \left(1 + \frac{\gamma_i}{\rho + \alpha_l(\theta_o^*(P))}\right)\rho P\right]$$

with equality if $\theta_o^*(P) > 0$ *and* $\psi_{o,i}(P) > 0$*.*

(iv) market clearing:

$$\int_{\tilde{\mathrm{W}}_{\mathrm{p}}^*} \psi_{r,i}(w)\tilde{\theta}_p^*(w)d\lambda_r(w) + \int_{P^*} \psi_{o,i}(P)\theta_o^*(P)d\lambda_o(P) = \pi_i \quad \forall i$$

Proofs not in the main text

Proof of Lemma 1

Let w be any contract in any pooling equilibrium for which there exists $i \neq j$ and $\psi_i(w) > 0$, $\psi_j(w) > 0$. The landlord takes the expected values $\rho Z_r(\gamma_i, r_i, \theta(\mathrm{w}))$ and $\rho Z_r(\gamma_j, r_j, \theta(\mathrm{w}))$ of the two types as given.

A landlord cannot make strictly lower expected profits from either type. If she could, then a deviating contract would be the menu that does not offer an attractive rent to that type. By rational expectations, the expected queue length must be the same and so the landlord will make strictly higher expected profits, a contradiction. Therefore:

$$\frac{\alpha_l(\theta(\mathrm{w}))}{\rho + \gamma_i + \alpha_l(\theta(\mathrm{w}))} r_i = \frac{\alpha_l(\theta(\mathrm{w}))}{\rho + \gamma_j + \alpha_l(\theta(\mathrm{w}))} r_j = \rho H \tag{5}$$

The lemma follows trivially from there. □

Proof of Proposition 2

We want to prove the existence and uniqueness of the solution of the "unconstrained" maximization problem. We follow the following steps (and drop dependence on i)

The landlord's zero profit constraint (ZPC) constraint holds with equality for each type: Suppose not. We can increase Z by decreasing w and/or θ in a ball $B_\varepsilon(w_r^*, \theta_r^*)$ and still meet the constraint for ε small enough. Thus (w_r^*, θ_r^*) is not a maximum.

Existence. We can impose the ZPC with equality: $\theta_r^{zpc}(\gamma, w) = \alpha^{-1}\left(\frac{(\rho+\gamma)\rho H}{w-\rho H}\right)$. The maximization problem simplifies to: $\max_{w\in[\rho H,h]} Z_r^{zpc}(\gamma, w) = Z_r(\gamma, w, \theta_r^{zpc}(\gamma, w))$. Note that as $w \to \rho H$, $\theta_r^{zpc}(\gamma, w) \to \infty$ and $\frac{\alpha(\theta_r^{zpc})}{\theta_r^{zpc}}(\gamma, w) \to 0$, thus $Z_r^{zpc}(\gamma, w = \rho H) = 0$. The objective function is continuous and the constraint set is compact.

The solution is interior. From above, $Z_r^{zpc}(\gamma, w = \rho H) = 0$ and it is easy to show that $Z_r^{zpc}(\gamma, w = h) = 0$. Moreover, $Z_r^{zpc}(\gamma, w) > 0$ for all $w \in (\rho H, h)$.

Uniqueness. Analytically, it is easier to solve the equivalent problem $\max_{\theta \in \Re_+} Z_r(\gamma, w_r^{zpc}(\gamma,\theta),\theta)$, where w_r^{zpc} satisfies the ZPC. The objective function is non-negative iff $\alpha \geq \frac{(\rho+\gamma)\rho H}{h-\rho H}$, or equivalently $\theta \geq \alpha^{-1}\left(\frac{(\rho+\gamma)\rho H}{h-\rho H}\right)$, and $\lim_{\theta\to\infty} Z_r(\gamma, w_r^{zpc}(\gamma,\theta),\theta) = 0$. Since the objective function is continuously differentiable on $\Re_+$, the first-order condition is necessary for an optimum:

$$\frac{h}{\rho H} = 1 + \frac{1}{\theta_r^*}\frac{\varepsilon}{1-\varepsilon} + \frac{\rho+\gamma}{\alpha_l(\theta_r^*)(1-\varepsilon)} \tag{6}$$

The right-hand side of (6) is a decreasing, continuous, function in θ. Thus, there is only one solution θ^* of the maximization problem. □

Proof of Result 1

From (6), θ_r^* is increasing in γ, so from the zero-profit condition for landlords, w_r^* is increasing in γ. So Z_r^* is decreasing in γ. □

Proof of Proposition 3

We go through the following steps:

The $IC(j,i)$ with $j > i$, never binds; a type with $\gamma_j < \gamma_i$ never wants to deviate to the i-contract. Any contract and associated market-tightness for a type i is also feasible for any type $j > i$.

For all $\{PR_i\}$, the ZPC binds and, for $i > 1$, at least one IC must bind.

By contradiction. Suppose not. If no constraint ever binds, then Z_p^{*i} is maximized by setting $w = \theta = 0$, but that violates the ZPC. If only the ZPC binds, then the problem is equivalent to the unconstrained one, but in that case the optimal contract associated with higher i (lower γ_i) is always preferred by all $j < i$, thus the IC is violated. If one $IC(j,i)$ binds but not the ZPC, then by the sorting condition we can pick a couple $(w,\theta) \in B_\varepsilon((w_p^{*i}, \theta_p^{*i}))$ such that the ZPC still holds and both types i and j are strictly better off, thus that is not a solution.

$\{PR_1\}$ is equivalent to the first best problem

Follows from the previous results.

There exists an unique solution to $\{PR_i\}$ for all $i > 1$. At the optimum, only the marginal IC is binding, $IC(i-1, i)$.

We prove this iteratively.

First step. The solution for $i = 1$ is the first best allocation: $Z_p^{*1} = Z_r^{*1}$, $\theta_p^{*1} = \theta_r^{*1}$ and $w_p^{*1} = w_r^{*1}$.

Iterative step. Consider the problem PR_i for type $i > 1$. We go through two sub-steps.

i Assume first that only the marginal IC is binding, $IC(i-1, i)$. By the previous analysis, this must be the case, in particular, for $i = 2$. The constrained optimum Z_p^{*i}, market tightness θ_p^{*i} and rent w_p^{*i} must satisfy the ZPC and $IC(i-1, i)$. Thus, θ_p^{*i} and w_p^{*i} satisfy the following non-linear system in θ and w:

$$X(\gamma_i, w, \theta) = H$$

$$Z_r(\gamma_{i-1}, w, \theta) = Z_p^{*(i-1)}$$

We can express w as a function of θ in both equations:

$$w = w_{zpc}(\gamma_i, \theta) = \left(1 + \frac{\rho + \gamma_i}{\alpha}\right)\rho H \tag{7}$$

$$w = w_{icc}(\gamma_{i-1}, \theta) = h - \left(1 + \frac{\rho + \gamma_{i-1}}{\alpha/\theta}\right)\rho Z_p^{*(i-1)} \tag{8}$$

Equation (8) is the indifference curve of type $(i-1)$ that by construction goes through the optimal point $(\theta_p^{*(i-1)}, w_p^{*(i-1)})$. Moreover, at $(\theta_p^{*(i-1)}, w_p^{*(i-1)})$ landlords make zero profits in the market for type $(i-1)$, thus they make strictly positive profits with households of type i. It implies that, at $\theta_p^{*(i-1)}$, the zero profit curve in the market for type i (7) is met for a lower value of the rent, $w < w_p^{*(i-1)}$. Thus:

$$w_{zpc}(\gamma_i, \theta_p^{*(i-1)}) < w_{icc}(\gamma_{i-1}, \theta_p^{*(i-1)})$$

At the limit, $w_{zpc} > w_{icc}$:

$$\lim_{\theta^{zp} \to 0} w^{zp} = \infty > h - \rho Z_r^{*1} = \lim_{\theta^{ic} \to 0} w^{ic}$$

$$\lim_{\theta^{zp} \to \infty} w^{zp} = \rho H > -\infty = \lim_{\theta^{ic} \to \infty} w^{ic}$$

Thus, they cross at least twice, one time on the left and one time on the right of the point $(\theta_p^{*(i-1)}, w_p^{*(i-1)})$.

It is easy to show that:

Result 5. *The expected value of a type i increases as θ increases on the indifference curve of a type j, with $i > j$ ($\gamma_i < \gamma_j$), and viceversa; moreover, the two types have the same expected values at $\theta = 0$.*

Intuitively, a higher market tightness affects more the type with higher moving probability. This implies that the expected value of type i is maximized at the crossing point with higher θ and lower w, and it is higher than the optimal expected value of type $(i-1)$:

$$\theta_p^{*i} > \theta_p^{*(i-1)}$$
$$w_p^{*i} < w_p^{*(i-1)}$$
$$Z_p^{*i} > Z_p^{*(i-1)}$$

This solves the problem for $i = 2$.
(ii) In general, we need to show that no other $IC(i-k, i)$ binds, with $i > 2$ and $k > 1$. Suppose by way of contradiction that it does bind. We can assume, from substep (i), that (only) the marginal incentive compatibility constraints bind for all $j < i$, in particular $IC(i-k, i-k+1)$. Thus, type $(i-k)$ is indifferent between the pairs $(\theta_p^{*(i-k)}, w_p^{*(i-k)})$, $(\theta_p^{*(i-k+1)}, w_p^{*(i-k+1)})$ and $(\theta_p^{*i}, w_p^{*i})$. Since the pair $(\theta_p^{*(i-k+1)}, w_p^{*(i-k+1)})$ is feasible for type i (the zero profit condition for type i is not binding), by result 5 type i chooses optimally a higher θ and lower w:

$$\theta_p^{*i} > \theta_p^{*(i-k+1)} > \theta_p^{*(i-k)}$$
$$w_p^{*i} < w_p^{*(i-k+1)} < w_p^{*(i-k)}$$

But then, by the same argument, type $(i-k+1)$ would prefer $(\theta_p^{*i}, w_p^{*i})$ to $(\theta_p^{*(i-k+1)}, w_p^{*(i-k+1)})$, violating the incentive compatibility constraint $IC(i-k+1, i)$. Thus $(\theta_p^{*i}, w_p^{*i})$ is not incentive compatible. A contradiction. □

Proof of Proposition 4

The proof is divided into two main parts. Part (1) proves that, if an allocation solves (PR), then there exists a competitive search equilibrium with that allocation. Part (2) proves that any equilibrium allocation solves (PR). From Proposition 3, it follows that the equilibrium exists and is unique.

Part (1)
The proof is by construction. Let $\{w_p^{*i}, \theta_p^{*i}\}_{\mathbb{I}}$ be a solution to the (PR) problem. Construct the candidate equilibrium allocation as follows:

$$Z_p^{*i} = Z_r(\gamma_i, w_p^{*i}, \theta_p^{*i}) \quad \forall i$$
$$W_p^{*} = \{w_p^{*i}\}_{\mathbb{I}}$$

Let the functions θ_p^* and Ψ be defined over the entire set $[\rho H, h]$ as follows:

$$\theta_p^*(w): \quad \frac{\alpha(\theta_p^*(w))}{\theta_p^*(w)} = \min_{j \in \mathbb{I}} \left[\frac{h-w}{\rho Z_p^{*j}} - 1 \right]^{-1} (\rho + \gamma_j)$$

$$\psi_k(w) = 1 \quad \text{implies} \quad k = \arg\min_{j \in \mathbb{I}} \left[\frac{h-w}{\rho Z_p^{*j}} - 1 \right]^{-1} (\rho + \gamma_j)$$

If there is more than one solution k that minimizes that equation, choose the largest one. The definition of the function $\Psi(w)$ then implies $\psi_j(w_i^*) = 0$ for all $j \neq k$. The expression for ρZ_p^{*i} implies:

$$\theta_p^*(w_p^{*i}) = \theta_p^{*i} \quad \forall w_p^{*i} \in W_p^*$$

$$\psi_i(w_p^{*i}) = 1 \quad \forall w_p^{*i} \in W_p^*$$

The first equation is derived by noting that if the expression is minimized for $j \neq i$, then j strictly prefers the i-optimal contract to the j-optimal contract, a contradiction. The second equation follows, noting that, by the properties of the constrained optimum, the equation is minimized by i and $(i-1)$ only. Finally, the measure of landlords posting w_p^{*i} is consistent with market tightness $\Theta(w_p^{*i})$:

$$\lambda(w_p^{*i}) = \frac{\psi_i}{\theta_p^*(w_p^{*i}) + \frac{\alpha(\theta_p^*(w_p^{*i}))}{\gamma_i}} \quad \forall w_p^{*i} \in W_p^*$$

and $\lambda(w) = 0$ if $w \notin W_p^*$.

We prove that this allocation satisfies all the equilibrium conditions:

(i) Landlords' profit maximization and free entry.

By construction, the ZPC holds with equality $\forall w \in W_p^*$. Consider $w \notin W_p^*$, $w \in [\rho H, h]$ and assume, by contradiction:

$$\left[1 + \left(\alpha_l(\theta_p^*(w)) \sum_{i \in \mathbb{I}} \frac{\psi_i(w)}{\rho + \gamma_i} \right)^{-1} \right]^{-1} w > \rho H$$

This implies $\theta_p^*(w) > 0$ and there exists j with $\psi_j(w) > 0$ and

$$\left[1 + \frac{\rho + \gamma_j}{\alpha_l(\theta_p^*(w))} \right]^{-1} w > \rho H$$

By construction of $\Psi(w)$, $\psi_j(w) = 1$ and $\psi_k(w) = 0$ $\forall k \neq j$. Then, by construction of $\Theta(w)$:

$$\frac{\alpha(\theta_p^*(w))}{\theta_p^*(w)} = \left[\frac{h-w}{\rho Z_p^{*j}} - 1 \right]^{-1} (\rho + \gamma_j) \leq \left[\frac{h-w}{\rho Z_p^{*k}} - 1 \right]^{-1} (\rho + \gamma_k) \quad \forall k$$

And the inequality holds strictly for all $k > j$.

So, the couple $(w, \theta_p^*(w))$ satisfies all the constraints of the problem (P_j) and guarantees the optimal value Z_p^{*j} to j and strictly positive profits to landlords. By continuity and the sorting condition, there exists a couple $(w', \theta') \in B_\varepsilon(w, \theta_p^*(w))$, with $w' < w$ and $\theta' > \theta_p^*(w)$ such that $Z_r(\gamma_j, w', \theta') > Z_p^{*j}$ and the ZPC and IC's are satisfied. A contradiction.

(ii) Households' optimal search.

By construction, $Z_p^{*i} = \max_{w \in W_p^*} Z_r(\gamma_i, w, \theta_p^*(w))$, $\theta_p^*(w_p^{*i}) > 0$ and $\psi_i(w_p^{*i}) > 0$. Moreover, by the construction of $\theta_p^*(w)$, for all $w \in [\rho H, h]$, $Z_p^{*i} \geq \frac{1}{\rho} \frac{\alpha_l(\theta_p^*(w))}{\theta_p^*(w)(\rho+\gamma_i)+\alpha_l(\theta_p^*(w))}(h - w)$.

(iii) Market clearing.

Follows directly by construction.

Part (2)

Part (i) of the equilibrium definition implies that $\theta_p^*(w) > 0$ for all $w \in W_p^*$, and part (iii) implies that for each i $\exists$ $w \in W_p^*$ such that $\psi_i(w) > 0$. It follows that, $\forall i$, $\exists$ $w \in W_p^*$ such that $\theta_p^*(w) > 0$ and $\psi_i(w) > 0$, thus from condition (ii) $Z_r(\gamma_i, w, \theta_p^*(w)) = Z_p^{*i}$.

We go through four steps to show that the equilibrium allocation solves the constrained maximization problem P_i, for all i:

(i) The ZPC is satisfied.

Let $w_p^{*i} \in W_p^*$ and $\theta_p^{*i} \equiv \theta_p^*(w_p^{*i})$, with $\psi_i(w_p^{*i}) > 0$. Suppose by contradiction that the ZPC is not satisfied:

$$\left[1 + \frac{\rho + \gamma_i}{\alpha_l(\theta_p^{*i})}\right]^{-1} w_p^{*i} < \rho H$$

Then, by equilibrium condition (i) and by noting that expected profits are decreasing in γ, there exists a $k > i$ such that:

$$\left[1 + \frac{\rho + \gamma_k}{\alpha_l(\theta_p^{*i})}\right]^{-1} w_p^{*i} < \rho H$$

By the sorting condition, $\exists$ $(\theta', w') \in B_\varepsilon$, with $\theta' > \theta$ and $w' < w$ s.th.:

$$Z_r(\gamma_j, w', \theta') > Z_r(\gamma_j, w_p^{*i}, \theta_p^{*i}) \quad \forall j \geq k$$
$$Z_r(\gamma_j, w', \theta') < Z_r(\gamma_j, w_p^{*i}, \theta_p^{*i}) \quad \forall j < k$$

Thus, for all $j < k$, $Z_r(\gamma_j, w', \theta') < Z_r(\gamma_j, w_p^{*i}, \theta_p^{*i}) \leq Z_p^{*j}$ by equilibrium condition (ii). But then condition (ii) and $\theta' > 0$ imply $\psi_j(w') = 0, \forall\, j < k$. It follows:

$$\left[1 + \left(\alpha_l(\theta') \sum_{i \in \mathbb{I}} \frac{\psi_i(w')}{\rho + \gamma_i}\right)^{-1}\right]^{-1} w' \geq \left[1 + \frac{\rho + \gamma_h}{\alpha_l(\theta')}\right]^{-1} w' > \rho H$$

where the last inequality holds for ε small enough. Thus, (w', θ') is a profitable deviation for the landlord. A contradiction.

(ii) IC's are satisfied.
Consider again $w_p^{*i} \in W_p^*$, $\theta_p^{*i} \equiv \theta_p^*(w_p^{*i}) > 0$ and $\psi_i(w_p^{*i}) > 0$. By equilibrium condition (ii), applied to all types j, it must be that:

$$Z_r(\gamma_j, w_p^{*i}, \theta_p^{*i}) \leq Z_p^{*j} \quad \forall j$$

Thus, the incentive compatibility constraints $IC(j, i)$ are satisfied $\forall j$.

(iii) The equilibrium value is equal to Z_p^{*i}, as defined in equilibrium condition (ii).
Again, it follows directly from condition (ii), since $\theta_p^*(w_p^{*i}) > 0$ and $\psi_i(w_p^{*i}) > 0$.

(iv) The equilibrium allocation solves P_i.
Let $\bar{Z}_r^i$ be the value from the competitive equilibrium allocation for each i. Suppose there exists a (w, θ) which respects the constraints for PR_i and is better: $X_r(w, \theta) \geq H$, $Z_r(\gamma_i, w, \theta) > \bar{Z}_r^i$ and $Z_r(\gamma_j, w, \theta) \leq \bar{Z}_r^j$ for $j < i$.
Take $w' \in B_\epsilon(w)$ such that $X_r(w', \theta) > X_r(w, \theta)$, $Z_r(\gamma_i, w', \theta) > \bar{Z}_r^i$ and $Z_r(\gamma_j, w', \theta) \leq \bar{Z}_r^j$ for $j < i$. There exists a $B_{\epsilon'}(w', \theta)$ such that for all $(\hat{w}, \hat{\theta}) \in B_{\epsilon'}(w', \theta)$, $X_r(\hat{w}, \hat{\theta}) > X_r(w, \theta)$ and $Z_r(\gamma_i, \hat{w}, \hat{\theta}) > \bar{Z}_r^i$.
By sorting (relative to (w', θ)), there exists $(w'', \tilde{\theta}) \in B_{\epsilon'}(w', \theta)$ such that $Z_r(\gamma_i, w'', \tilde{\theta}) > \bar{Z}_r^i$ and $Z_r(\gamma_j, w'', \tilde{\theta}) < \bar{Z}_r^j$ for $j < i$. Note that $w'' < w'$ and $\tilde{\theta} > \theta$.
The equilibrium θ for the rent w'' according to the competitive equilibrium: $\theta_p^*(w'') \geq \tilde{\theta}$. So $Z_r(\gamma_j, w'', \theta_p^*(w'')) < \bar{Z}_r^j$ for $j < i$ and $X_r(w'', \theta_p^*(w'')) \geq X_r(w'', \tilde{\theta}) \geq X_r(w', \theta) > H$. So the allocation which gave $\bar{Z}_r^i$ was not an equilibrium allocation. □

Proof of Result 2

Start from the two equations for the constrained optimum and write them in Δ-form:

$$w(\gamma_{i+1} - \Delta) = \left(1 + \frac{\rho + \gamma_{i+1} - \Delta}{\alpha(\gamma_{i+1} - \Delta)}\right) \rho H$$
$$w(\gamma_{i+1} - \Delta) = h - \left(1 + \frac{\rho + \gamma_{i+1}}{\alpha(\gamma_{i+1} - \Delta)/\theta(\gamma_{i+1} - \Delta)}\right) \rho Z_p^{*(i+1)}$$

where $\alpha(\gamma_{i+1}-\Delta)=\alpha(\theta(\gamma_{i+1}-\Delta))$. We can then derive (dropping the subscripts $i+1$ and using the notation $\alpha_h=\alpha/\theta$):

$$\frac{w(\gamma)-w(\gamma-\Delta)}{\Delta}=\frac{\rho H}{\alpha(\gamma-\Delta)}-\frac{\alpha(\gamma)-\alpha(\gamma-\Delta)}{\Delta}\frac{\rho+\gamma}{\alpha(\gamma)\alpha(\gamma-\Delta)}\rho H$$
$$\frac{w(\gamma)-w(\gamma-\Delta)}{\Delta}=\frac{(\rho+\gamma)\rho Z_p^*}{\alpha_h(\gamma)\alpha_h(\gamma-\Delta)}\frac{\alpha_h(\gamma)-\alpha_h(\gamma-\Delta)}{\Delta}$$

Taking $\lim_{\Delta\to 0}$ and rearranging:

$$\frac{\partial w}{\partial \gamma}=\frac{\rho H}{\alpha}\left[1-\varepsilon\frac{\frac{\partial\theta}{\partial\gamma}}{\theta}(\rho+\gamma)\right]$$
$$\frac{\partial w}{\partial \gamma}=-(\rho+\gamma)(1-\varepsilon)\frac{\frac{\partial\theta}{\partial\gamma}}{\alpha}\rho Z_p^*$$

Solving for θ' and w':

$$\frac{\partial\theta}{\partial\gamma}=-\frac{1}{\rho+\gamma}\left[(1-\varepsilon)\frac{\rho Z_p^{*(i+1)}}{\rho H}-\frac{\varepsilon}{\theta}\right]^{-1}$$
$$\frac{\partial w}{\partial\gamma}=\frac{(1-\varepsilon)\rho Z_p^*}{\alpha}\left[(1-\varepsilon)\frac{\rho Z_p^*}{\rho H}-\frac{\varepsilon}{\theta}\right]^{-1}$$

Thus:

$$\frac{\partial w}{\partial\theta}=-(\rho+\gamma)\frac{1-\varepsilon}{\alpha}\rho Z_p^*<0$$

θ_p^* is increasing in γ implies:

$$\rho Z_p^*>\frac{1}{\theta_p^*}\frac{\varepsilon}{1-\varepsilon}\rho H$$

$$\frac{\partial\theta_p^*}{\partial\gamma}<0\quad\forall\gamma<\gamma_I$$
$$\frac{\partial w_p^*}{\partial\gamma}>0\quad\forall\gamma<\gamma_I$$

They go to ∞ for $\gamma=\gamma_I$. $\partial w/\partial\theta$ at the border is well defined:

$$\frac{\partial w_p^*}{\partial\theta_p^*}=-(\rho+\gamma)\frac{\varepsilon}{\theta_p^{*I}\alpha(\theta_p^{*I})}\rho H\quad\text{for }\gamma=\gamma_I\qquad\square$$

Proof of Result 3

Define:

$$A=\theta(\rho+\gamma)+\alpha$$
$$B=\frac{\alpha\gamma}{\rho+\alpha}$$

The value of searching in owning market ρZ_o and in the renting market with public information ρZ_r (after plugging in the firms' zero profit condition) can be expressed as follows:

$$\rho Z_r = A^{-1}\big[\alpha(h-\rho H)-(\rho+\gamma)\rho H\big]$$
$$\rho Z_o = (A+B)^{-1}\big[\alpha(h-\rho H)-(\rho+\gamma)\rho H\big]$$

Thus, $Z_r > Z_o$ for any value of θ. It follows immediately that the optimal value in the renting market with public information is higher than the one in the owning market: $Z_r^* > Z_o^*$.
The FOCs of the two problems are respectively:

$$\frac{h}{\rho H} = 1 + \frac{\rho+\gamma}{\alpha_r^*}\big[1-\frac{\varepsilon}{\varepsilon_A}\big]^{-1}$$
$$\frac{h}{\rho H} = 1 + \frac{\rho+\gamma}{\alpha_o^*}\big[1-\frac{\varepsilon}{\varepsilon_{A+B}}\big]^{-1}$$

where ε_A and ε_{A+B} are the elasticities with respect to θ of A and $A+B$, respectively. Moreover:

$$\varepsilon_{A+B} = \frac{A}{A+B}\varepsilon_A + \frac{B}{A+B}\varepsilon_B$$

And:

$$\varepsilon_A \equiv \frac{\theta A'}{A} = \frac{\theta(\rho+\gamma)+\varepsilon\alpha}{\theta(\rho+\gamma)+\alpha} \quad \in\ (0,1)$$
$$\varepsilon_B \equiv \frac{\theta B'}{B} = \rho\varepsilon \quad \in\ (0,1)$$

The FOC are necessary (since $\theta = 0$ or $\theta = \infty$ is never optimal for $\gamma > 0$), so at the optimum θ_o^*, $\varepsilon_{A+B} > \varepsilon$. Also, for all θ, $\varepsilon_A > \varepsilon$ and $\varepsilon > \varepsilon_B$. So $\varepsilon_A > \varepsilon_{A+B}$.

To show that the RHS of both FOCs are decreasing in θ, notice that:

$$\varepsilon_A = 1-(1-\varepsilon)\left(1+\frac{\rho+\gamma}{\alpha_h}\right)^{-1}$$
$$\frac{A}{A+B} = \left[1+\frac{\gamma}{\rho+\alpha}\left(1+\frac{\rho+\gamma}{\alpha_h}\right)^{-1}\right]^{-1}$$
$$\frac{\partial\varepsilon_{A+B}}{\partial\theta} = \frac{\partial\varepsilon_A}{\partial\theta}\frac{A}{A+B} + \frac{\partial(\frac{A}{A+B})}{\partial\theta}(\varepsilon_A-\varepsilon_B)$$

Staring at the expressions for ε_A and $A/(A+B)$, it is easy to show that they are both increasing in θ.[23] This in turn implies that ε_{A+B} is increasing in θ as well:

$$\frac{\partial\varepsilon_A}{\partial\theta} > 0$$
$$\frac{\partial\varepsilon_{A+B}}{\partial\theta} > 0$$

[23] ε_A is a negative function of α_h, that in turn is a negative function of θ. $A/(A+B)$ is given by the inverse of the product of two functions: one is a negative function of α and thus of θ, the other is a positive function of α_h and thus a negative function of θ. As a result, the inverse is a negative function of θ.

Thus the RHS of the two FOCs, say RHS_r and RHS_o, are decreasing in θ (as the product of two decreasing functions in θ). But since $\varepsilon_{A+B} < \varepsilon_A$:

$$\mathrm{RHS}_o > \mathrm{RHS}_r \quad \forall\theta$$
$$\frac{\partial \mathrm{RHS}_i}{\partial \theta} < 0$$

The condition $\mathrm{RHS}_o(\theta_o^*) = \mathrm{RHS}_r(\theta_r^*)$ implies that $\theta_o^* < \theta_r^*$.

Furthermore, rearranging the FOC for owning and differentiating,

$$\frac{\rho H}{h-\rho H} d\gamma = \left[\alpha_o'(1-\frac{\varepsilon}{\varepsilon_{A+B}}) + \varepsilon\alpha_o \frac{d\varepsilon_{A+B}}{d\theta}\frac{1}{\varepsilon_{A+B}^2}\right] d\theta$$

From the FOC for owning, we know that $\frac{\varepsilon}{\varepsilon_{A+B}} < 1$ (otherwise $\theta_o^* < 0$). So $\frac{d\theta_o^*}{d\gamma} > 0$. Using the envelope condition,

$$\frac{d\rho\tilde{Z}_o}{d\gamma} = \frac{\partial\rho\tilde{Z}_o}{\partial\gamma} = -\rho Z_o^*\left[\frac{\rho H}{C_o} + \frac{\theta_o^* + \frac{\alpha_o}{\rho+\alpha_o}}{A_o+B_o}\right] \tag{9}$$

where $C_o = \alpha(h-\rho H) - (\rho+\gamma)\rho H$.

The proof that $\frac{d\tilde{Z}_p^*}{d\gamma} > \frac{dZ_r^*}{d\gamma}$ is as follows: For any given $\tilde{\gamma} \in \Gamma$, define the constant $k \equiv Z_r^*(\tilde{\gamma}) - \tilde{Z}_p^*(\tilde{\gamma})$. Note that the function $Z_r^* - k = \tilde{Z}_p^*$ at $\tilde{\gamma}$ and $\frac{d(Z_r^*-k)}{d\gamma} = \frac{dZ_r^*}{d\gamma}$. Also, $\forall\Delta > 0$, $Z_r^*(\tilde{\gamma}-\Delta) - k > \tilde{Z}_p^*(\tilde{\gamma}-\Delta)$.

Finally, $\frac{dZ_o^*}{d\gamma}$ is continuous in γ. Since $Z_o^* < Z_r^*$ $\forall\gamma > 0$ and $Z_o(0) = Z_r(0)$, continuity of the first derivative guarantees that there exists a $\tilde{\gamma}$ such that, for all $\gamma < \tilde{\gamma}$, $\frac{dZ_o^*}{d\gamma} < \frac{dZ_r^*}{d\gamma}$. This means that there is at most one crossing point between Z_o^* and $\tilde{Z}_p^*$ on the interval $(0,\tilde{\gamma}]$ and that they definitely cross here if they haven't crossed before. □

Proof of Result 4

From the first-order conditions for renting with and without customization, respectively:

$$\frac{d\theta_r^*}{d\gamma} = \left(\frac{\varepsilon(\rho+\gamma)}{\theta_r^*} + \frac{\varepsilon\alpha_l(\theta_r^*)}{(\theta_r^*)^2}\right)^{-1}$$
$$\frac{d\theta_{cr}^*}{d\gamma} = \left(\frac{\varepsilon(\rho+\gamma)}{\theta_{cr}^*} + \frac{\varepsilon\alpha_l(\theta_{cr}^*)}{\delta(\theta_{cr}^*)^2}\right)^{-1}$$

Suppose there exists a $\tilde{i}$ such that $Z_{cr}^{*\tilde{i}} = Z_r^{*\tilde{i}}$. Then $\theta_{cr}^{*\tilde{i}} = \theta_r^{*\tilde{i}}$ and (with slight abuse of notation) $\frac{d\theta_r^{*\tilde{i}}}{d\gamma} < \frac{d\theta_{cr}^{*\tilde{i}}}{d\gamma}$. Thus $\theta_r^*(\gamma)$ and $\theta_{cr}^*(\gamma)$ can cross at most once. □

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Figure 1: Market tightness (queue length)

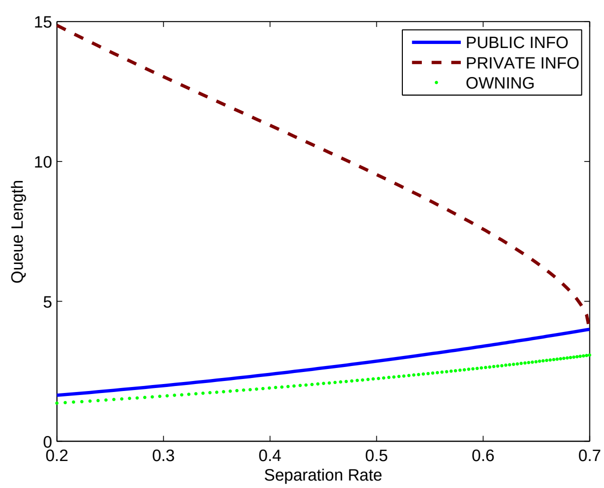

Figure 2: Flow rent

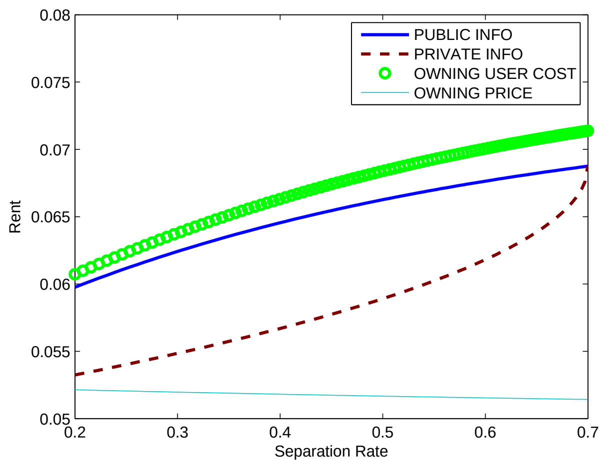

Figure 3: Expected value of searching in all markets

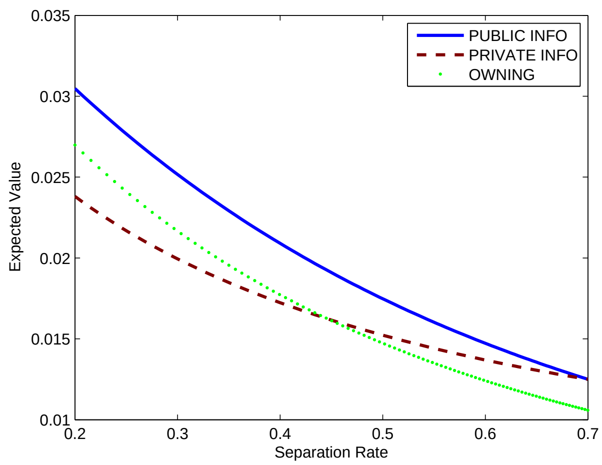

Figure 4: Market tightness with and without rent control (queue length)

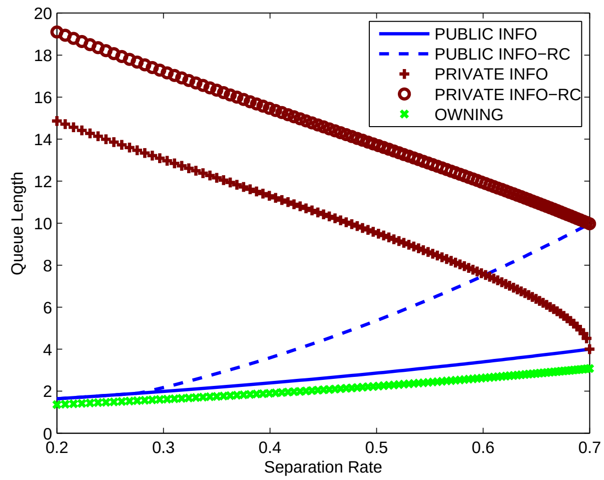

Figure 5: Flow rent with and without rent control

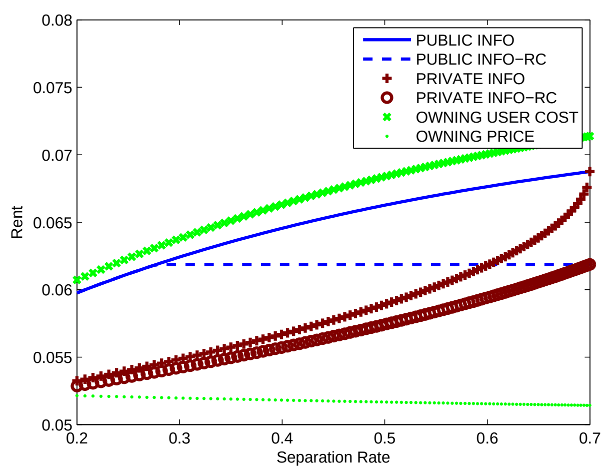

Figure 6: Expected value of searching in all markets with and without rent control

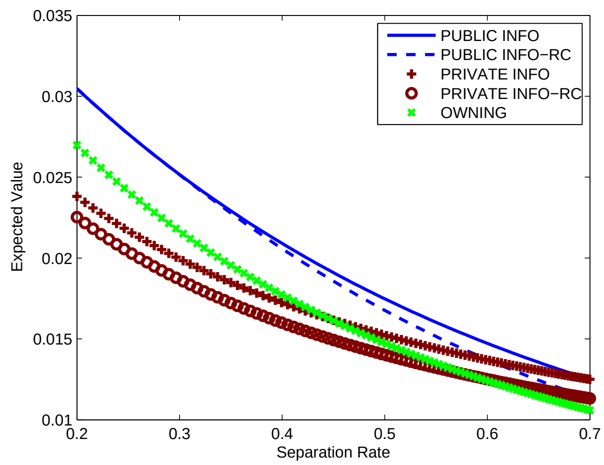

Figure 7: Flow value for renting with private information and option to customize

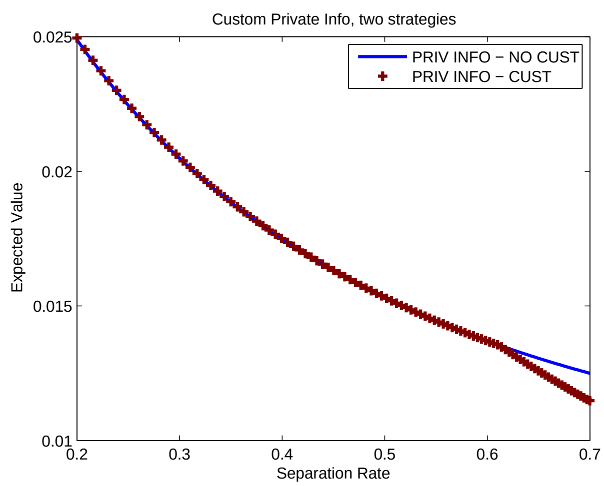

Figure 8: Market tightness (queue length) for renting with private information and option to customize

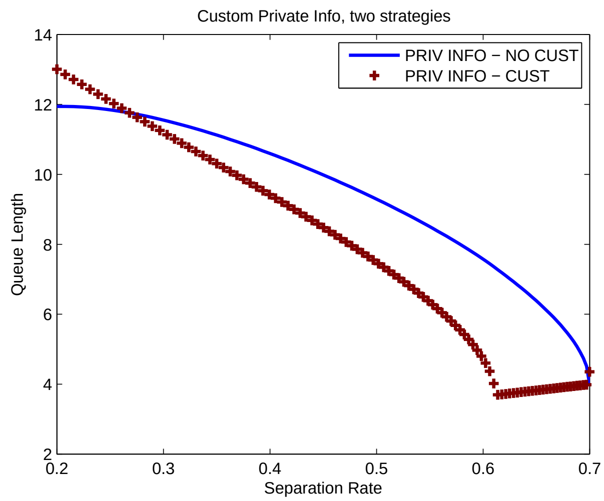

Figure 9: Flow value of search with all choices

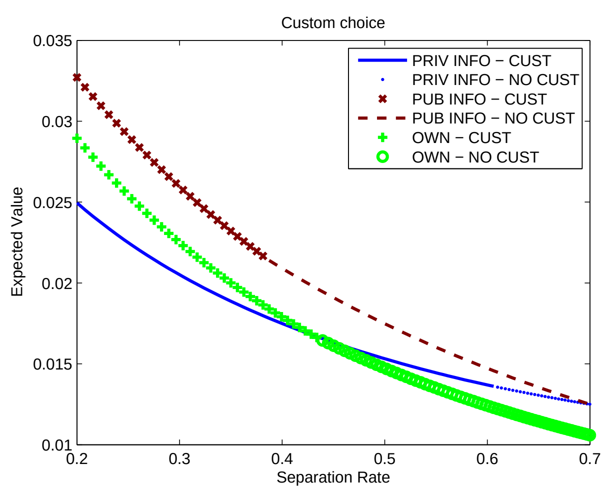

Figure 10: Market tightness (queue length) with all choices

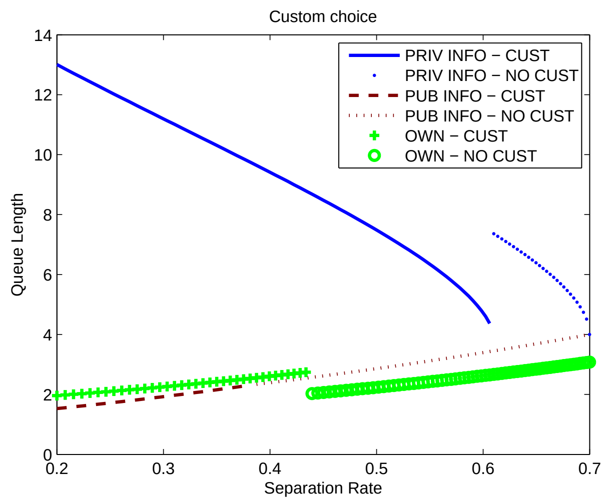

Figure 11: Rent with all choices