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Equilibrium Search and the Impact of Equal Opportunities for Women.

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Equilibrium Search and the Impact of Equal Opportunities for Women*

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Abstract

This paper develops a new equilibrium model of two-sided search where agents have multiple attributes and general payoff functions. The model can be applied to several substantive issues. Here we use it to provide a novel understanding of the separate effects of equal opportunities for women in the labor market and improved contraception on female education, employment, and timing of first births after World War II. We find that the diffusion of the pill might have played an important role in explaining the observed rise in female education and employment since the 1960s. But without equal opportunities, these changes would have not occurred.

JEL Classification: C6, J0, J1, N3
Keywords: Two-sided search; Marriage; Female labor supply; Pill; Age at First Birth

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1. Introduction

This paper develops a new equilibrium marriage model of two-sided search where ex-ante heterogenous individuals have multiple characteristics. In the existing search literature, the restriction to a single characteristic is almost universal as the aggregation problem is severe. Heterogeneous singles use different match strategies and equilibrium requires that each individual's strategy must be a best response to the aggregated match strategies of the opposite sex. We use our extended equilibrium approach to provide new insights on the separate impacts of equal opportunities for women in the labor market and improved contraceptive technologies on female education choice, labor market participation rates, and the timing of first births.

It takes time in the marriage market to find the right life-partner. Furthermore singles who choose to be more selective in their choice of life-partner inevitably delay the time at which they might successfully match. Of course more popular singles, those who receive many proposals, might afford to be relatively selective. Following seminal work by Becker (1973, 1974), there is a large literature on equilibrium marriage markets with random search, e.g. Lu and McAfee (1996), Burdett and Coles (1997, 1999), Eeckhout (1999), Bloch and Ryder (2000), Chade (2001), Chade and Ventura (2002), Smith (2006), Gautier, Svarer, and Teulings (2010), Coles and Francesconi (2011), and Díaz-Giménez and Giolito (2013). Much of this literature considers when equilibrium might generate positive assortative matching.[1] An equilibrium search framework, however, is also ideal to analyze how changes in labor market opportunities and contraceptive technologies affect marriage market outcomes and female labor supply.

After World War II, the U.S. labor market witnessed dramatic changes in female education choice and female participation rates. Albanesi and Olivetti (2009) describe how the development of infant formula and improved medication freed the nursing mother from the home. With households increasingly sustained by new labor-saving appliances and a doubling of real wages from the mid 1930s to 1960, married women in the 1950s had a strong incentive to substitute their time to the booming labor market (Greenwood, Seshadri, and Vandenbrouke 2005; Greenwood, Seshadri, and Yorukoglu 2005).[2]

[1]The marriage market approach typically assumes non-transferable utility. A labor market context instead assumes utility is transferable with Nash bargaining. Contributions in this literature include Burdett and Coles (1999), Shimer and Smith (2000), Teuling and Gautier (2004), Eeckhout and Kircher (2011), and Jacquemet and Robin (2013). See Coles and Francesconi (2011) for a discussion of the two approaches.

[2]Among the empirical studies that emphasize a positive impact of wartime work on women's employment, see Acemoglu, Autor, and Lyle (2004), and Goldin and Olivetti (2013).

But many firms at that time implemented discriminatory employment practices toward women. For instance, prior to the 1950s, “marriage bars” in occupations such as doctors, lawyers, teachers, and clerks, were commonly used to restrict the employment prospects of women. Kessler-Harris (1982, 2001) and Goldin (1990, 1991) provide fascinating accounts of women’s struggle for equal opportunities in the U.S. labor market. Changes in public policies and attitudes affected female labor market behavior, occupations, and earnings (Beller 1982; Harrison 1988; Rosen 2000; Costa 2000; Fernandez, Fogli, and Olivetti 2004; Goldin 2006). In this paper, we refer to the combination of technological developments in the home (which freed women from domestic chores), the labor market legislative reforms (culminating in the 1963 Equal Pay Act and the 1964 Civil Rights Act) and the underlying political and cultural environment in which they were conceived (e.g., the Commission on the Status of Women, the Equal Employment Opportunity Commission, and the growth of feminism), as “the arrival of equal opportunities for women”.

Following Aiyagari, Greenwood, and Guner (2000) and Fernandez, Guner, and Knowles (2005), we assume household consumption is a joint public good and match payoffs depend on “love”, considered as match specific random draws.[3] Our model not only identifies the distribution effects arising from the arrival of equal opportunities, it also considers the differential impact of different female attributes on female education choice. For example, men have long placed a high value on female beauty. But the arrival of equal opportunities allows the highly talented (and well paid) single woman to bring something more to the marriage. Our framework therefore allows us to consider how non-economic factors, such as beauty, might affect the female incentive to invest in college education. Similarly, it is well known that educated women, on average, marry later and have their first child later (Hotz, Klerman, and Willis 1997; Goldin and Katz 2002). Young women will then take all such factors into account when making their education choices.

An important, new feature of the paper is that it focuses on women’s age at first birth rather than age at first marriage as the measure of partnership formation. Age at first birth is not only consistently measured over decades, it is also arguably the point at which a long-term partnership is truly cemented. By restricting attention to women who ever had a child, we find that the distribution of age at first birth in the United States has hardly changed since 1830. This is intriguing as there have been both dramatic

[3] The first example of an equilibrium two-sided search model with idiosyncratic match payoffs is Burdett and Wright (1998).

changes in female fertility, education and labor market participation choices over the previous century, and radical improvements in contraceptive technologies.

Focusing on age at first birth also reveals a novel insight into match behavior. Using data from the 2000 U.S. Census we find that, conditional on having a child, the average age at first birth is 28.4 years among college educated women and 23.3 years among non-college women. Yet the standard deviation of age at first birth is the same for the two groups of women, at about 4.6 years. This is surprising: should educated women have considerably longer search spells, one would have expected such spells to exhibit also a greater standard deviation. Of course heterogeneity in match rates implies the estimated standard deviation is not an unbiased estimate of the average completed search spell. Nevertheless, we find that the gap between the average match spell and the measured standard deviation is always small (by birth cohort and within a given education category). This result strongly suggests that college educated and non-college educated women experience roughly the same average search spells. From this we infer that the gap in age at first birth arises as female college students delay entry into the marriage market until after they complete their studies (see also Lundberg and Pollak 2013).

We also consider how contraceptive innovations affect labor market outcomes. As pointed out by Goldin and Katz (2002, p. 765), it is hard empirically to separate out the effect of the birth control pill ("the pill") from the contemporaneous surge of feminism and the emergence of equal opportunities during the 1960s and 1970s. There was, however, another major contraceptive innovation earlier in the 20th century. Following the discovery of a new latex production technology in 1919, the top fifteen U.S. condom manufacturers were, by 1931, producing 1.44 million latex condoms per day (Tone 2001). Noting this innovation occurred prior to the arrival of equal opportunities, it is interesting it did not have a significant impact on female economic activity.[4] We argue the difference in impact of the pill and the latex condom on female education and labor market participation rates is due to the existence, or otherwise, of equal opportunities for women. In other words, the effect of the pill on female behavior is best understood as an interaction effect with the arrival of equal opportunities.

The next section describes some key motivating facts. Section 3 describes an equi-

[4]Going further back in time, the invention of vulcanized rubber in 1844 also resulted in more reliable and cheaper condoms (Guinnane 2011). Vulcanized rubber was also used for the production of diaphragms and other barrier methods in the late nineteenth century. None of these innovations turned out to have an effect on the distribution of the age at first birth.

librium marriage model of two-sided search with multiple attributes and section 4 establishes existence of equilibrium for the general case. Section 5 considers a special case which directly addresses the impact of equal opportunities and of innovations in contraceptive technologies on female economic activity. Section 6 illustrates those issues with a numerical example and Section 7 concludes. The census and survey data used in our analysis are described in the Appendix.

2. Motivating Facts

The removal of marriage bars had a gradual, but major, impact on female labor supply. For example, in 1920, only one in ten of all married women in the labor force were teachers and clerical workers. By 1970, that proportion had risen to 40 percent. Figure 1 shows the employment rates of all women aged 25–34 depending on their marital status and motherhood. Single (never married) women in this age group have always had relatively high labor market participation rates. Indeed, since 1940, the participation rate of these women has remained stable at about 80 percent. In contrast, following the 1950s, there was a dramatic surge in the labor market participation rates of young married women, and especially those of young mothers.

Career options had a large impact on the return to education. Rather than anticipating to be in the labor market for only a few years while single and childless, young women could instead expect a lifelong return to investing in a qualification. Over the period 1940–2009, Figure 2 reports the fraction of women aged 25–34 with a college education and so describes the average education choice of women in the 15–24 age group made in the previous decade. As young women anticipated working more in the labor market, more chose to invest in a college education.[5] Taken together, Figures 1 and 2 demonstrate the coincident increase in female education and employment following the 1950s.

Given this radical change in female economic activity, one might expect a similarly large change in the timing of first births. It is well documented that there has been a long-run decline in fertility (Guinnane 2011). Moreover, Bailey (2006) finds that early legal access to the pill reduced the probability of a birth before age 22 by about 15 percent. Yet if we consider the whole population of women who ever had a child, the story is quite

[5]This structure suggests increasing returns to scale in lifetime earnings: a doubling of a woman's participation rate and of her investment in education more than doubles her expected lifetime earnings. This non-convexity can generate very large substitution effects.

different. Conditional on women who had their first child by age 40,[6] Figure 3 shows the median age at first birth has been remarkably stable, being around 23.5 years ever since the 1830s. That the 10-th and the 90-th percentiles have also been stable suggests the entire distribution has been largely invariant to the increase in female education and participation rates. Perhaps even more surprisingly, this distribution is also invariant to the decline in fertility and major improvements in contraceptive technologies, including the pill during the 1960s and 1970s and the latex condom during the 1920s and 1930s. Figure 1 establishes the latex condom innovation had no (or, at best, only a modest) impact on female labor market participation rates.[7]

3. The Model

Basic Setup

We consider a continuous-time, infinite horizon family formation model with frictions. Only steady state equilibria are considered. There are two sexes, male and female indexed by $s = m, f$ respectively, and a continuum of agents of both sexes. All partnerships involve one man and one woman and there is an equal measure of unpartnered men and women in the singles market. The parameter $\lambda > 0$ denotes the rate at which any single meets a potential partner of the opposite sex. For simplicity, all agents are infinitely lived, they discount the future at rate $r > 0$, and partnerships last forever (no divorce).

Each male is described by a vector of characteristics $\underline{x}_m \in \Omega_m$, and each female is characterized by $\underline{x}_f \in \Omega_f$. Should a male $\underline{x}_m$ match with a female $\underline{x}_f$, the male obtains payoff $U_m(\underline{x}_m, \underline{x}_f) + \theta^m/r$, while the female enjoys payoff $U_f(\underline{x}_m, \underline{x}_f) + \theta^f/r$. U_m and U_f are bounded functions for all $(\underline{x}_m, \underline{x}_f) \in (\Omega_m \times \Omega_f)$ and describe their attribute-dependent payoffs. Each θ^s, $s = m, f$, represents a "love" draw, considered as an independent random draw from the exogenous c.d.f. $H(\cdot)$ with support $[\underline{\theta}, \overline{\theta}]$. For ease of exposition, we assume $\overline{\theta} = +\infty$, although none of the results hinges on this assumption. We assume the surplus function $S(\widetilde{\theta}) = \int_{\widetilde{\theta}}^{+\infty} [1 - H(\theta)] d\theta$ exists and note it is a positive, continuous, decreasing and convex function with $\lim_{\widetilde{\theta} \to +\infty} S(\widetilde{\theta}) = 0$. The flow payoff while single is $u_m(\underline{x}_m) \geq 0$ for males and $u_f(\underline{x}_f) \geq 0$ for females, which are bounded functions for all

[6] This choice of age cutoff is driven by data availability. See the Appendix.

[7] Eckstein and Lifshitz (2011) find the contribution of reduced fertility in explaining long-term changes in female employment is small. Comparatively little is known about the relationship between age at first birth and mother's labor force participation and education choices. Early descriptive work includes Happel, Hill, and Low (1984) and Cigno and Ermisch (1989). Since the 1960s, childbearing has shifted to later ages (e.g., Hotz, Klerman, and Willis 1997; Francesconi 2002), but much of this empirical work does not focus on the changes in first birth distributions over a long time horizon.

$(\underline{x}_m, \underline{x}_f) \in (\Omega_m \times \Omega_f)$.

Let $G_m(\underline{x}_m)$ describe the distribution of male attributes $\underline{x}_m$ across single men and $G_f(\underline{x}_f)$ denote the distribution of female attributes $\underline{x}_f$. There are search frictions and contacts are random. Should a single male $\underline{x}_m$ meet a single woman, her attributes are considered a random draw from $G_f(\cdot)$. Similarly from her perspective, his attributes $\underline{x}_m$ are a random draw from $G_m(\cdot)$. Given a contact, each observes the other's attributes and draws his/her independent love values, θ^m or θ^f. A match is formed only if both agree to it, otherwise they separate and continue searching.

If two singles agree to form a match, they permanently exit the singles market and have two children, a son who inherits his father's characteristics $\underline{x}_m$, and a daughter who inherits her mother's characteristics $\underline{x}_f$. As we only consider steady states, there is no further loss in generality by assuming each child instantaneously grows up and immediately enters the singles market. Burdett and Coles (1999) refer to this as the 'clones assumption'.[8] The clones assumption is convenient as it implies the population distribution of singles is invariant to the match strategies of singles. It thus usefully abstracts from inter-cohort competition for partners: by accepting a match and exiting the pool of singles, the clones rule implies this decision does not affect the match opportunities of future singles. The approach is relevant as the birth cohort distribution is then endogenously determined. For example, types who never marry, and so do not have children, are absent from the birth cohort. Conversely, those who match quickly are over-represented in the birth cohort.

Preliminaries

Let $V^m(\underline{x}_m)$ denote the value of a single male with attributes $\underline{x}_m$, and $V^f(\underline{x}_f)$ be the value of a single female with characteristics $\underline{x}_f$. Given contact with female $\underline{x}_f$, the single male $\underline{x}_m$ will propose a long term relationship as long as his match payoff $U_m(\underline{x}_m, \underline{x}_f) + \theta^m/r$ exceeds the value of remaining single $V^m(\underline{x}_m)$. This yields a reservation love value

$$\widetilde{\theta}^m = r\left[V^m(\underline{x}_m) - U_m(\underline{x}_m, \underline{x}_f)\right],$$

where the single male will propose if and only if $\theta^m \geq \widetilde{\theta}^m$. Note this reservation match value is type specific; i.e. $\widetilde{\theta}^m = \widetilde{\theta}^m(\underline{x}_f|\underline{x}_m)$ describes the reservation match strategy of

[8] An alternative is to assume each child's attribute is a mix of his/her parent's characteristics plus a random element. Similarly, one might wish to endogenize the fertility choice. Both extensions are potentially important research projects, but go beyond the scope of the current paper.

each male $\underline{x}_m$. The male match propensity

$$P^m(\underline{x}_f|\underline{x}_m) = 1 - H(\widetilde{\theta}^m) \tag{1}$$

thus describes the probability with which a male $\underline{x}_m$ will propose to a female $\underline{x}_f$ given contact. Note that any attribute $\underline{x}_f$ which raises the male match payoff $U_m(\underline{x}_m, \underline{x}_f)$ implies a one-for-one fall in the reservation love value $\widetilde{\theta}^m(\underline{x}_f|\underline{x}_m)$ and thus a higher match propensity.

The same argument applies for women. Given contact with male $\underline{x}_m$, the single female $\underline{x}_f$ has reservation love value

$$\widetilde{\theta}^f = r\left[V^f(\underline{x}_f) - U_f(\underline{x}_f, \underline{x}_m)\right],$$

and her match propensity

$$P^f(\underline{x}_m|\underline{x}_f) = 1 - H(\widetilde{\theta}^f), \tag{2}$$

describes the probability with which she is willing to match with male $\underline{x}_m$. Any characteristic $\underline{x}_m$ which raises the female match payoff $U_f(\underline{x}_f, \underline{x}_m)$ implies the woman is more likely to propose a match.

Given these match propensities, we now determine the set of values $V^s(\cdot)$ for each $x_s \in \Omega_s$, $s = m, f$. Recall that each single male meets a potential female partner at rate λ whose trait $\underline{x}_f$ is considered a random draw from $G_f(\cdot)$. Standard arguments imply $V^m(\underline{x}_m)$ is identified by the Bellman equation:

$$rV^m(\underline{x}_m) = u_m(\underline{x}_m) + \lambda \int_{\underline{x}_f \in \Omega_f} \Pi^m(\underline{x}_m, \underline{x}_f) P^f(\underline{x}_m|\underline{x}_f) dG_f(\underline{x}_f),$$

where

$$\Pi^m(\underline{x}_m, \underline{x}_f) = \int_{\underline{\theta}}^{\overline{\theta}} \max\left[U_m(\underline{x}_m, \underline{x}_f) + \theta^m/r - V^m(\underline{x}_m), 0\right] dH(\theta^m). \tag{3}$$

Random matching implies the male contacts a single female $\underline{x}_f \in \Omega_f$ at rate $\lambda dG_f(\underline{x}_f)$, who is willing to match with probability $P^f(\underline{x}_m|\underline{x}_f)$, and he then makes expected surplus $\Pi^m(\underline{x}_m, \underline{x}_f)$ by the contact, where that surplus depends on whether his realised match payoff $U_m(\cdot) + \theta^m/r$ exceeds $V^m(\cdot)$. Integrating (3)by parts implies $\Pi^m(\underline{x}_m, \underline{x}_f) = S(\widetilde{\theta}^m)$, where $\widetilde{\theta}^m$ is his reservation love value. Hence, $V^m = V^m(\underline{x}_m)$ solves

$$rV^m = u_m(\underline{x}_m) + \frac{\lambda}{r}\int_{\underline{x}_f\in\Omega_f} S\Big(r\left[V^m - U_m(\underline{x}_m, \underline{x}_f)\right]\Big)P^f(\underline{x}_m|\underline{x}_f)dG_f(\underline{x}_f). \tag{4}$$

Equation (4) is key. For any male $\underline{x}_m \in \Omega_m$ and corresponding female proposal strategies $P^f(\cdot) \in [0,1]$, (4) is an implicit function for V^m. The right hand side of (4) is strictly positive at $V^m = 0$ and is a continuous decreasing function which limits to $u_m(\underline{x}_m)$ as $V^m \to \infty$. It follows that $V^m(\underline{x}_m)$ always exists, is unique, and satisfies $V^m(\underline{x}_m) \geq u_m(\underline{x}_m)/r$.

By symmetry, $V^f = V^f(\underline{x}_f)$ is given by

$$rV^f = u_f(\underline{x}_f) + \frac{\lambda}{r}\int_{\underline{x}_m\in\Omega_m} S\Big(r\left[V^f - U_f(\underline{x}_f, \underline{x}_m)\right]\Big)P^m(\underline{x}_f|\underline{x}_m)dG_m(\underline{x}_m). \tag{5}$$

Expressions (4) and (5) thus uniquely determine the value of being single for each sex, given the match propensities of the opposite sex.

In the application of Sections 5 and 6, we focus on female education choice after the emergence of equal opportunities for women. As the model demonstrates, each woman's investment in education not only takes into account the direct financial benefits and costs of investing in education (i.e., how it affects the flow value of being single $u_f(\underline{x}_f)$ and the expected value of marriage $U_f(\cdot)$), but also on how it affects marriage prospects $P^m(\underline{x}_f|\underline{x}_m)$. Of course, the collective change in female match strategies $P^f(\underline{x}_m|\underline{x}_f)$ through the introduction of equal opportunities also changes the value of being a single man $V^m = V^m(\underline{x}_m)$ and thus changes the entire match structure.

Before defining and establishing the existence of equilibrium, it is useful to detail how changes in match strategies affect agent values. Specifically, consider a woman $\underline{x}_f$ and two different match strategies by men, $P_1^m(\cdot)$ and $P_0^m(\cdot)$. Let $V_1^f(\underline{x}_f)$ and $V_0^f(\underline{x}_f)$ denote the solutions to (5) with P^m being equal to P_1^m and P_0^m, respectively.

Lemma 1. If $P_1^m(\underline{x}_f|\underline{x}_m) \geq P_0^m(\underline{x}_f|\underline{x}_m)$ for all $\underline{x}_m \in \Omega_m$, then $V_1^f(\underline{x}_f) \geq V_0^f(\underline{x}_f)$.

Lemma 1 simply says that a woman $\underline{x}_f$ is better off when all men are more likely to propose to her. The result follows immediately from (5). For any given V^f and noting the surplus function is positive, the right hand side of this expression is increasing in P^m. Thus, V^f solving (5) must increase with an increase in P^m.

Lemma 1 is important as it allows us to identify an upper bound for the value for each sex. Clearly, by Lemma 1, the ideal situation for each male $\underline{x}_m$ is that all women are willing to match with him; i.e., $P^f = 1$ for all $\underline{x}_f \in \Omega_f$. Now consider his ideal match $\underline{x}_f^*$ defined as

$$\underline{x}_f^*(\underline{x}_m) = \arg\max_{\underline{x}_f \in \Omega_f} U_m(\underline{x}_m, \underline{x}_f).$$

It follows that $V^m(\underline{x}_m) \leq \overline{V}^m(\underline{x}_m)$, where

$$r\overline{V}^m = u_m(\underline{x}_m) + \frac{\lambda}{r} S\Big(r\left[\overline{V}^m - U_m(\underline{x}_m, \underline{x}_f^*)\right]\Big), \tag{6}$$

as $\overline{V}^m$ describes the value of being single in a market where all women want to propose to him and every woman is also his ideal match, i.e., $\underline{x}_f = \underline{x}_f^*$, for all $\underline{x}_f \in \Omega_f$. The assumptions on $S(\cdot)$ guarantee that $\overline{V}^m$ defined by (6) exists and $\overline{V}^m \geq u_m(\underline{x}_m)/r$. The above analysis has thus established the following lemma.

Lemma 2. For any male $\underline{x}_m \in \Omega_m$ and female proposal strategies $P^f(\cdot) \in [0, 1]$, the solution for $V^m(\underline{x}_m)$ exists, is unique, and is bounded with

$$V^m(\underline{x}_m) \in \Big[\frac{u_m(\underline{x}_m)}{r}, \overline{V}^m(\underline{x}_m)\Big].$$

Armed with Lemmas 1 and 2, we are now in a position to define and establish the existence of an equilibrium.

4. Equilibrium

A Matching Equilibrium requires the set of values V^m and V^f are consistent with the match propensities P^m and P^f of the opposite sex, while those match propensities P^m and P^f are optimal given V^m and V^f. That is, equilibrium is the set of functions $\{P^m, P^f, V^m, V^f\}$ over $(\underline{x}_m, \underline{x}_f) \in (\Omega_m \times \Omega_f)$ which satisfy the functional equations (1)–(2) and (4)–(5).

Existence is established by considering the following fixed point problem. Suppose $V_k^m(\cdot) = V^m(\underline{x}_m)$ for all $\underline{x}_m \in \Omega_m$ describes the equilibrium set of male values. Equation (1) then implies the equilibrium male match propensities

$$P_k^m(\underline{x}_f|\underline{x}_m) = 1 - H\left(r[V_k^m(\underline{x}_m) - U_m(\underline{x}_m, \underline{x}_f)]\right), \tag{7}$$

for each $\underline{x}_m \in \Omega_m$. Given these male match propensities $P_k^m(\cdot)$, the Bellman equation (5) then uniquely yields $V^f(\underline{x}_f)$ for each $\underline{x}_f \in \Omega_f$. Let $V_k^f(\cdot)$ denote this solution. Given

$V^f = V_k^f(\cdot)$, equation (2) then yields female match propensities

$$P_k^f(\cdot) = 1 - H\big(r[V_k^f(\underline{x}_f) - U_f(\underline{x}_f, \underline{x}_m)]\big). \tag{8}$$

Of course given these female match propensities $P_k^f(\cdot)$, (4) then uniquely determines $V^m(\underline{x}_m)$ for all $\underline{x}_m \in \Omega_m$. Let $V_{k+1}^m(\cdot)$ denote this updated solution. The above thus identifies a functional mapping $V_{k+1}^m(\underline{x}_m) = T[V_k^m(\underline{x}_m)]$. Equilibrium requires finding its fixed point where $V^m(\underline{x}_m) = T[V^m(\underline{x}_m)]$ for all $\underline{x}_m \in \Omega_m$. The existence proof uses the following monotonicity property.

Lemma 3. [Monotonicity] If $V_k^m(\underline{x}_m) \geq V_{k+1}^m(\underline{x}_m)$ for all $\underline{x}_m \in \Omega_m$, then $T[V_k^m(\underline{x}_m)] \geq T[V_{k+1}^m(\underline{x}_m)]$ for all $\underline{x}_m \in \Omega_m$.

Proof. As $H(\cdot)$ is an increasing function, $V_k^m(\underline{x}_m) \geq V_{k+1}^m(\underline{x}_m)$ for all $\underline{x}_m \in \Omega_m$ and (7) implies $P_k^m(\cdot) \leq P_{k+1}^m(\cdot)$ for all $\underline{x}_m \in \Omega_m, \underline{x}_f \in \Omega_f$. Lemma 1 establishes $V_k^f(.) \leq V_{k+1}^f(.)$ for all $\underline{x}_f \in \Omega_f$. As $H(\cdot)$ is an increasing function, $V_k^f(\cdot) \leq V_{k+1}^f(\cdot)$ for all $\underline{x}_f \in \Omega_f$ and (8) implies $P_k^f(\cdot) \geq P_{k+1}^f(\cdot)$ for all $\underline{x}_f \in \Omega_f, \underline{x}_m \in \Omega_m$. Lemma 1 establishes $T[V_k^m(\underline{x}_m)] \geq T[V_{k+1}^m(\underline{x}_m)]$ for all $\underline{x}_m \in \Omega_m$ and so the map $T[\cdot]$ is monotonic. ■

The proof of Lemma 3 reflects the following structure of the model. When all single men are better off, each man becomes more selective in the marriage market: given a contact with a single woman, each male proposes with a lower probability. Lemma 1 then implies every single woman is worse off as it is harder for her to find a male who is willing to form a permanent partnership. Now, when all single women are worse off, each becomes less selective in the marriage market and their match propensities increase. Lemma 1 then implies every single man is better off. The proof of Lemma 3 uses this feedback mechanism to establish monotonicity of the map $T[V]$. But these feedback effects also suggest multiple equilibria may be possible where men, say, prefer an equilibrium in which women are less selective (and men more selective) while women prefer another equilibrium in which men are less selective (and women more selective). Establishing existence of an equilibrium is now straightforward.

Theorem 1. A Matching Equilibrium exists.

Proof. Existence follows by repeated iteration of the map $V_{k+1}^m(\underline{x}_m) = T[V_k^m(\underline{x}_m)]$, starting at the upper bound $V_0^m(\cdot) = \overline{V}^m(\cdot)$. As $V_1^m(\underline{x}_m) = T[V_0^m(\underline{x}_m)] \leq \overline{V}^m(\underline{x}_m)$ for all $\underline{x}_m \in \Omega_m$, Lemma 3 and an induction argument imply a sequence of decreasing values $V_{k+1}^m(\underline{x}_m) \leq V_k^m(\underline{x}_m)$ for each $\underline{x}_m \in \Omega_m$, $k = 0, 1, 2, ...$ As this sequence is bounded below by $u_m(\underline{x}_m)/r$, a limit point exists for every $\underline{x}_m \in \Omega_m$. As this limiting set of values is the required fixed point, an equilibrium must exist. This completes the proof of the Theorem. ■

The proof of Theorem 1 suggests a straightforward numerical procedure which is guaranteed to converge to an equilibrium, although there is no guarantee that the equilibrium is unique. It is worth stressing, however, that if multiple equilibria exist where all men prefer one equilibrium (labeled as equilibrium 1) over another (equilibrium 2), then monotonicity implies the above iteration will converge to equilibrium 1. Conversely, if we were to start the iteration at the lower bound, where $V_0^m(\cdot) = u_m(\underline{x}_m)/r$, monotonicity implies the iteration would instead converge to equilibrium 2.

Time to Partnership by Birth Cohort

Each single woman $\underline{x}_f \in \Omega_f$ matches at equilibrium rate

$$\lambda_f(\underline{x}_f) = \lambda \int_{\underline{x}_m \in \Omega_m} P^f(\underline{x}_m|\underline{x}_f) P^m(\underline{x}_f|\underline{x}_m) dG^m(\underline{x}_m),$$

where matching requires a double coincidence of wants. The clones rule implies the gross inflow of new single females is

$$g = \int_{\underline{x}_f \in \Omega_f} \lambda_f(\underline{x}_f) dG^f(\underline{x}_f).$$

If $\Phi_f(\underline{x}_f)$ denotes the distribution of characteristics $\underline{x}_f$ across each female birth cohort, then

$$d\Phi_f(\underline{x}_f) = \frac{\lambda_f(\underline{x}_f) dG^f(\underline{x}_f)}{g}.$$

Individuals who match rapidly (i.e., have higher $\lambda_f(\underline{x}_f)$) are over-represented in the birth cohort. Conversely those who never match are absent from the birth cohort. As each individual $\underline{x}_f$ has expected unmatched spell equal to $1/\lambda_f(\underline{x}_f)$ then the average unmatched spell across all women in any given birth cohort is

$$\mu_f = \int_{\underline{x}_f \in \Omega_f} \frac{1}{\lambda_f(\underline{x}_f)} d\Phi_f(\underline{x}_f) = 1/g.$$

The same argument applies equally to men. As g describes both the inflow of new single men and new single women it follows that, by birth cohort, the average unmatched spell must be the same for both sexes. This is interesting because, as in Burdett and Coles

(1997), equilibrium can have the property that all men marry (i.e., $\lambda_m = \lambda_m(\underline{x}_m) > 0$ for all $\underline{x}_m \in \Omega_m$), while some women never marry (i.e., $\lambda_f = \lambda_f(\underline{x}_f) = 0$, for some $\underline{x}_f \in \Omega_f$). In that case and across the entire population, the average time to partnership for women must be greater than that for men (in fact, it is unboundedly large). But this cannot occur when restricting attention to birth cohorts where population weights are scaled by match frequencies.

Although the mean unmatched spell must be the same across sexes in any given birth cohort, the variance is different as

$$
\begin{aligned}
\sigma_f^2 &= \int_{\underline{x}_f \in \Omega_f} \left[\frac{2}{\lambda_f(\underline{x}_f)^2}\right] d\Phi_f(\underline{x}_f) - \left[\frac{1}{g}\right]^2 \\
&= \frac{1}{g} \int_{\underline{x}_f \in \Omega_f} \left[\frac{2}{\lambda_f(\underline{x}_f)} - \frac{1}{g}\right] dG^f(\underline{x}_f),
\end{aligned}
$$

and thus it depends on $\lambda_f(\cdot)$. Obviously, $\sigma_f = 1/g$ when all individuals match at the same rate, that is, when $\lambda_f(\underline{x}_f) = g$ for all $\underline{x}_f \in \Omega_f$. As $1/\lambda_f$ is a convex function of λ_f, however, disperse match rates imply $\sigma_f > 1/g$, and this raises the standard deviation of time to first partnership. The estimated standard deviation of age at first partnership, both for men and for women, thus identifies an upper bound for the average search spell, $1/g$.

5. An Application: Equal Opportunities and the Pill

We use the model of Section 3 to understand changes in women's behavior due to the emergence of equal opportunities and the diffusion of the pill. To focus on female behavior, we keep the male side of the market deliberately simple. Each male is characterized by a scalar, $\underline{x}_m = \{y\}$, which describes his labor market earnings. Earnings across single men have distribution $G^m(\cdot)$ and support $[\underline{y}, \overline{y}]$.

Each single woman is described by a pair of attributes $\underline{x}_f = (n, \alpha)$. The term α denotes an ability variable that identifies her potential earnings in the labor market, while n captures her innate attractiveness to the opposite sex. Goldin and Katz (2002) describe n as nurturing skills, Burdett and Coles (1997) describe it as pizzazz, and Cole, Mailath, and Postlewaite et al. (1993) develop an "aristocratic" equilibrium where n describes aristocratic rank. Here we refer to n as attractiveness or desirability, where a higher n raises the male payoff to the match. Let $\widetilde{G}^f(\cdot)$ denote the ex-ante distribution of attributes (n, α) across single women.

Each woman (n, α) makes an education choice $e \in \{0, 1\}$ prior to entering the marriage market, with $e = 1$ indicating university education and $e = 0$ otherwise. For convenience, we refer to the first group as college educated and to the second as college uneducated. The cost of educational investment, which is the same for all, is denoted by c, $c > 0$. Given this choice, her wage w in the labor market is: (i) $w = \underline{y}$ if there are unequal labor market opportunities, (ii) $w = \alpha$ if there are equal opportunities and $e = 0$ (uneducated), (iii) $w = \gamma\alpha$ if there are equal opportunities and $e = 1$ (educated), where $\gamma > 1$ describes the wage return to education. As the education choice is sunk ex-post, each woman in the marriage market is described by her ex-post characteristics (n, w). Hence, given the ex-ante distribution of female characteristics $\widetilde{G}^f(n, \alpha)$, the optimal education choices of women yield $G^f(n, w)$ defined as the distribution of ex-post female attributes (n, w) in the marriage market.

Family consumption is assumed to be a joint public good and joint labor supply is chosen efficiently. A match between a male y and a female (n, w) yields the following lifetime payoffs:

$$U^m = \frac{\beta\,[y + \max[w - \phi, 0]] + n + F}{r}; \qquad U^f = \frac{\beta\,[y + \max[w - \phi, 0]] + F}{r},$$

where β, ϕ, and F are positive parameters. The male's flow payoff in the match depends on joint labor market earnings $y + \max[w - \phi, 0]$ where ϕ describes the opportunity cost of child care, i.e., the female partner continues in the labor market if her earnings $w \geq \phi$. We assume $\phi > \underline{y}$, so that a married woman who only earns $\underline{y}$ will not participate in the labor market and so becomes a "homemaker". Conversely, a high earning woman with $w > \phi$ will participate and purchase child services at cost ϕ.[9] Joint earnings are deflated by β, $0 < \beta \leq 1$. The case $\beta = 1$ assumes "two can live as cheaply as one", while β below 1 embeds an equivalence scale greater than one: that is, partners raising a family on joint income y can afford the same lifestyle as a single on income βy. F describes the flow utility yielded by raising a family and is assumed to be the same for both sexes.[10]

The female match payoff has an identical structure, the only difference being that her match payoff does not depend on her own desirability n. Her attractiveness thus increases her partner's match payoff but does not directly increase her own. Single women,

[9] An alternative specification, as considered in Coles and Francesconi (2011), is that the higher earner participates in the labor market, while the lower earner only works if the added earnings compensate for child care costs.

[10] Allowing F to be gender specific is potentially important, but this is an extension left to future research.

however, benefit from having greater attractiveness as this improves their chances of attracting a more desirable mate.

The flow payoff of a single male is $u_m(y) = y + u$, whereas the flow payoff to a single woman is $u_f(n, w) = w + u$. Following Goldin and Katz (2002), we assume improved contraception raises u, the flow value of being single relative to the flow value of marriage; that is, improved contraception reduces the cost to delaying partnership formation.

Matching Equilibria with Unequal Opportunities

In this context, a labor market with unequal opportunities means a woman can only earn wage $\underline{y}$. This can be considered as an extreme version of the "marriage bar".

As child care costs $\phi > \underline{y}$, each married woman optimally becomes a homemaker. The match payoffs for men and women in this case reduce respectively to

$$U^m = \frac{\beta y + n + F}{r}; \quad U^f = \frac{\beta y + F}{r}.$$

This payoff structure implies married women withdraw from the labor market to raise the family. Because the model implies her return to college education is zero, the costly investment in education is a strictly dominated strategy.

Let $V^m(y)$ denote the value of a single male with income y and $V^f(n)$ denote the value of a single female with attractiveness n, noting that her ability α is not payoff relevant. The analysis for the general case now applies. Given contact with female n, a single male y has reservation love value $\widetilde{\theta}^m = rV^m(y) - \beta y - n - F$ and corresponding match propensity

$$P^m(n|y) = 1 - H(rV^m(y) - \beta y - n - F). \tag{9}$$

Similarly given contact with male y, a single female n has reservation love value $\widetilde{\theta}^f = rV^f(n) - \beta y - F$ and corresponding match propensity

$$P^f(y|n) = 1 - H(rV^f(n) - \beta y - F). \tag{10}$$

Given (10), the single male y enjoys value $V^m = V^m(y)$ satisfying

$$rV^m = u + y + \frac{\lambda}{r}\int_{\underline{n}}^{\overline{n}} S(rV^m - \beta y - n - F)P^f(y|n)dG^f(n), \tag{11}$$

where $G^f(n)$ describes the distribution of attractiveness across single women. Likewise the single female n enjoys value $V^f = V^f(n)$ satisfying

$$rV^f = u + \underline{y} + \frac{\lambda}{r}\int_{\underline{y}}^{\overline{y}} S(rV^f - \beta y - F)P^m(n|y)dG^m(y). \tag{12}$$

A Matching Equilibrium is the set of functions $\{P^m, P^f, V^m, V^f\}$ satisfying the functional equations (9)–(12). Theorem 1 establishes a Matching Equilibrium exists.

As $P^m(n|y)$ increases with n, Lemma 1 implies women are better off with greater attractiveness because single men are more likely to propose a match. But by raising their option value of being single, they become more selective: $P^f(y|n)$ is decreasing in n. Marriage in this case is then sorted by attractiveness, where more attractive women are unlikely to marry low earning men. The overall effect of increased attractiveness on match rate is ambiguous, however, as men are more willing to match with an attractive woman while she is more likely to decline their proposal.

So how does improved contraception, considered as an increase in u, affect behavior? Given unequal labor market opportunities, women have no incentive to invest in a costly education as each anticipates spending her time in the home sector as a homemaker. Thus improved contraception, in the absence of equal opportunities, has *no* impact on female education rates.

Given the proposal strategies $P^s(\cdot)$, $s = f, m$ then, ceteris paribus, an increase in u implies the value of being single increases. This, however, implies both sexes become more selective and the corresponding fall in $P^s(\cdot)$ makes both sexes worse off. A simple contradiction argument establishes some singles must become better off as u increases. But it is not necessarily the case that all must be better off in a matching equilibrium.[11] Indeed, we cannot rule out, say, the possibility that an increase in u makes men so much less willing to commit to a long term partnership, that the advent of improved contraception makes all women worse off (it becomes much harder to find a committed long-term male partner).

[11] A straightforward counterexample exists when H is degenerate. In that case, equilibrium matching has the class structure as described in Burdett and Coles (1997). An increase in u implies all the class boundaries shift up and an individual on a boundary is necessarily made worse-off by dropping down a class.

Matching Equilibria with Equal Opportunities

With equal opportunities, female investment in education generates two benefits. First, it improves her own welfare directly (a better standard of living). Second, it improves her marital prospects. To separate out these two returns to education, define female "fitness" z as $z = n + \beta \max[w - \phi, 0]$. The male and female payoffs to a match can then be rewritten as

$$U^m = \frac{\beta y + z + F}{r}; \qquad U^f = \frac{\beta y + \beta \max[w - \phi, 0] + F}{r}.$$

As men now rank women according to their fitness z, rather than their attractiveness n, it is useful to relabel each female (n, w) by her equivalent characteristics (z, w).

Given the distribution of male earnings $G^m(y)$ and ex-post female characteristics $G^f(z, w)$, it is easy to formulate the value function for each sex and describe the corresponding match propensities. Theorem 1 then establishes a Matching Equilibrium exists. What is interesting in this case is characterizing the equilibrium education choices of women where education choice $e \in \{0, 1\}$ maps female characteristics (n, α) to ex-post attributes (z, w), noting that male match propensities $P^m(z|y)$ depend only on female fitness. For convenience, we refer to women with ex-post wage $w > \phi$ as "career women": career women always participate in the labor market. Women with $w < \phi$, as in the world with unequal opportunities, are instead referred to as "homemakers": when married, homemakers withdraw from the labor market and raise the family.

For a woman with sufficiently high ability, $\alpha > \phi/\gamma$, investment in education yields two direct benefits: (i) the increase in earnings raises her standard of living in every state, that is, she becomes a career woman with $w = \gamma\alpha > \phi$, and (ii) it increases her fitness in the singles market from $z = n + \beta \max[\alpha - \phi, 0]$ to $z = n + \beta[\gamma\alpha - \phi]$ which, by increasing male match propensities, improves her chances of finding a good match. For a woman with lower ability $\alpha < \phi/\gamma$, investment in education has a very limited return. This investment does not improve her fitness in the marriage market and so does not improve her marital prospects. Her return to education is therefore short-lived: it increases her wage only for the time when she remains single.

A little algebra establishes that the reservation match value of a woman (z, w) given contact with male y satisfies

$$\widetilde{\theta}^f = u + w - \beta y - \beta \max[w - \phi, 0] - F + \frac{\lambda}{r} \int_{\underline{y}}^{\overline{y}} S(\widetilde{\theta}^f) P^m(z|y) dG^m(y).$$

Consider now a woman with intermediate ability $\alpha \in [\phi/\gamma, \phi]$. If she does not invest in education, her wage $\alpha < \phi$ implies she becomes a homemaker upon marriage. Her reservation match value when $e = 0$, denoted $\widetilde{\theta}_0^f$, is given by

$$\widetilde{\theta}_0^f = u + \alpha - \beta y - F + \frac{\lambda}{r} \int_{\underline{y}}^{\overline{y}} S(\widetilde{\theta}_0^f) P^m(n|y) dG^m(y). \tag{13}$$

If instead she invests in education, her wage increases to $w = \gamma\alpha$ and her fitness to $z = n + \gamma\alpha - \phi$. Her reservation match value in this case becomes

$$\widetilde{\theta}_1^f = u + [\gamma\alpha - \beta(\gamma\alpha - \phi)] - \beta y - F + \frac{\lambda}{r} \int_{\underline{y}}^{\overline{y}} S(\widetilde{\theta}_1^f) P^m(z|y) dG^m(y). \tag{14}$$

There are two differences between these two reservation value equations. First, as shown in (14), by investing in education, a woman gains greater fitness $z > n$ and so enjoys improved marital prospects. Second, the opportunity wage cost of forming a match is different. The uneducated woman in (13) gives up flow income α to become a homemaker. The college educated woman instead gives up flow income $\gamma\alpha$ but, by remaining in the labor market, continues to enjoy a share $\beta(\gamma\alpha - \phi)$ of her continuation earnings. As $\alpha < \phi$, $\gamma > 1$, and $\beta \leq 1$, it is easy to show $[\gamma\alpha - \beta(\gamma\alpha - \phi)] > \alpha$; i.e. the opportunity wage cost through forming a match is greater for the college educated woman. Standard arguments imply $\widetilde{\theta}_1^f > \widetilde{\theta}_0^f$, and educated women are more selective in the marriage market. This occurs not only because they have a higher opportunity wage cost to forming a match, but also because they enjoy more proposals.

6. Numerical Example

As the diffusion of the pill coincided with the emergence of (more) equal opportunities for women in the labor market, it is hard to separate out empirically their effects on female education and employment rates.[12] This section aims to quantify the extent to which improved contraception additionally enhances female education rates when women face equal opportunities in the labor market.

[12] As mentioned in the first two sections, Goldin and Katz (2002) and Bailey (2006) provide empirical evidence based on strategies which try to identify the impact of the pill. See also Bailey (2010).

Parametric Specifications

Table 1 describes the baseline parameters and functional forms used in our numerical example. The choice $\gamma = 1.04$ implies a return to education of 4 percent. The income equivalence scale for a married couple with two children is typically set equal to 2.1 (e.g., OECD 2008), which implies $\beta = 1/(2.1) = 0.48$.

In the standard job search literature, the exponential distribution of wage offers is an important special case: for r arbitrarily small, an exogenous fall in the job offer rate implies the job seeker's reservation wage falls so that the expected job search spell remains unchanged (e.g., Mortensen and Pissarides 1999). Figure 3 shows the family formation process is consistent with this property: the female distribution of age at first birth has changed little over the last 150 years, even though society, and particularly the quality of effective contraception, has changed enormously. To replicate this feature of the data, we assume $H(\cdot)$ is exponential with parameter δ.

Based on the 2000 U.S. Census data, we assume the distribution of male wages is lognormal with mean \$10.44 and standard deviation \$0.687. We truncate the distribution at two standard deviations from the mean, so that $\underline{y} = \$9,608$ and $\overline{y} = \$135,131$ per annum, and renormalize the probability weights appropriately. The distribution of female abilities is the same, but deflated by $\gamma = 1.04$. Thus if all women invest in education, equal opportunities implies women enjoy the same distribution of wages.

The distribution of female abilities is assumed to be orthogonal to the distribution of female attractiveness or desirability.[13] As there is no objective information on the distribution of attractiveness n in the population (Buss 2003), we suppose there are just two levels of attractiveness, $\{-\frac{\overline{n}}{2}, \frac{\overline{n}}{2}\}$, where each type occurs with equal probability. We use L to label women with low attractiveness, $-\frac{\overline{n}}{2}$, and H to label those with high attractiveness, $\frac{\overline{n}}{2}$.

The relative magnitude of δ to $\overline{n}$ determines the extent to which "beauty is in the eye of the beholder". If $\overline{n}$ is large, all men are keen to match with the most attractive women, while if $\overline{n}$ is small all men search for their (idiosyncratic) true loves. Due to the exponential assumption on $H(\cdot)$, $1/\delta$ is the standard deviation of love draws. We set $\overline{n} = 1/\delta$, so that the relative magnitudes of these desires are of the same order. We normalize $u = 1/\delta$, noting that $(F - u)$ describes the net increase in flow utility through starting a family. Finally, the annual child care cost ϕ is set to \$10,732 per child

[13] Although Hamermesh and Biddle (1994) find evidence of a positive impact of workers' physical looks on their earnings, we simplify by abstracting from this effect.

(Macartney and Laughlin 2011).

The remaining parameters of the model (c, δ, λ, F) are tied down by requiring the data generated by the model fit the following moments. First, the education cost c must ensure 25 percent of women complete college education (in line with the 2000 Census data). Second, the standard deviation of love draws, $1/\delta$, must be such that the correlation of pre-marital incomes across matched partners is 0.5.[14] Note in particular that a very small value of $1/\delta$ implies love plays a negligible role in matching. In that case married couples sort by incomes. At the opposite extreme, a large value of $1/\delta$ implies love is all and there is then no sorting by incomes. Requiring the correlation of pre-marital incomes across matched partners is 0.5 thus ties down δ.

Third, the last two parameters (λ, F) are chosen so that the model generated match rates are as close as possible to the empirical conditional age at first birth distributions by birth cohort. For women with high school qualifications or lower, and conditional on having at least one child, the average age at first birth is 23.3 years with standard deviation $\sigma_u = 4.56$. For college educated women, the conditional average age is higher, at 28.4 years, but the corresponding standard deviation is virtually the same, $\sigma_e = 4.57$. The identification problem, of course, is that age at first birth cannot pin down the time to match. That these standard deviations are the same, however, suggests that average match rates by education status may not be very different.

One interpretation of these data is that college educated singles delay search for a permanent partner until after they complete their studies (Lundberg and Pollak 2013). For example, we could impose that high school graduates enter the marriage market at age 18, while college graduates enter at age 22. But a difficulty is that the implied average time to first birth is sensitive to this arbitrary specification. To identify (λ, F), therefore, we calibrate the model only to the standard deviations of the conditional age at first birth and let the calibrated model back out the average times to match. We thus search over (λ, F) to minimize the loss function $\ell = (\widehat{\sigma}_u - 4.56)^2 + (\widehat{\sigma}_e - 4.57)^2$, where the "hat" denotes the moments generated by the model, requiring all the other parameters fit the data described above.

[14] Fernandez, Guner, and Knowles (2005) use the correlation between spouses' years of schooling to identify marital sorting. Across 34 countries, they find this correlation ranging between 0.32 (Australia) and 0.76 (Colombia), with an average of 0.60, and for the U.S. they find a correlation of 0.63. Using SIPP data, Jacquemet and Robin (2012) report a correlation between spouses' wages of 0.30. With data from the National Longitudinal Survey of Youth, Hess (2004) reports a correlation between partners' observed income of 0.22. The correlation between partners' pre-marital wages (y, w) is harder to compute. Our estimates using data from the Panel Study of Income Dynamics and the British Household Panel Survey range between 0.45 and 0.53.

Table 2 describes our calibrated parameter values for (c, δ, λ, F). Table 3 describes the implied mean and standard deviation of unmatched spells by birth cohort and education.

Model Outcomes

Despite generating wide variation in female match rates (see Figure 4 below), the model finds $\sigma_u \simeq \mu_u$ and $\sigma_e \simeq \mu_e$ within a birth cohort. As the estimated standard deviation of age at first birth appears to be a good proxy for the average search spell, the model suggests the following decomposition of average female behavior. High school graduates enter the marriage market at age 18.1 years. It takes them, on average, $\widehat{\mu}_u = 4.4$ years to form a permanent long-term partnership which, with an additional nine month pregnancy, yields 23.3 years as the average age at first birth. College educated women, instead, enter the marriage market at age 22.9 years, experience an average unmatched spell $\widehat{\mu}_e = 4.7$ years which, following a 9 month pregnancy, translates into 28.4 years as the average age at first birth.

The implied value for $F - u$ means that each child yields a flow value of \$23,850 per year. This is large and reflects the large child care costs. Love also matters, as otherwise marital sorting would be more highly correlated by income. The standard deviation of θ is \$8,900 per year, which implies that searching for love is worthwhile.

Obviously it is better for a woman to have higher ability *and* to be more attractive. An interesting equilibrium feature with equal opportunities, however, is that the attractiveness premium $V_H(\alpha) - V_L(\alpha)$ falls steeply with ability (see Figure 5). When annuitized at the market rate of 4 percent, the added yearly flow value for an H-type at $\alpha = \underline{\alpha}$ is \$8,860. As we set $\overline{n} = \$8,900$, H-types with low ability seem able to fully monetize the males' valuation for their added desirability. They achieve this by being more selective in the marriage market; i.e., they use their attractiveness to seek a more favorable match. At the top end of the ability spectrum, however, the added flow value for an H-type is only \$1,562 per year. Though positive, this return is dwarfed by their earnings. Compared to low ability women, high ability women find increased attractiveness is not so valuable.

Figure 4 describes the corresponding equilibrium match rates $\lambda_L(\alpha)$ and $\lambda_H(\alpha)$ by ex-ante ability. High ability women and highly attractive women both enjoy higher proposal rates. They match, however, at very different rates. Regardless of having low or high attractiveness, female match rates decline as ability α becomes sufficiently large. Conversely, for any ability α, match rates always increase with attractiveness. The difference is that a high ability woman enjoys high earnings while single and, as marriage imposes a tax $(1-\beta)$ on her earnings, she takes time in her search for a favorable match. In contrast, being highly attractive improves one's marital opportunities, but that benefit is only realized upon marriage. A highly attractive woman, therefore, "cashes in" relatively early in order to enjoy sooner a more comfortable married lifestyle.

As the attractiveness premium $V_H(\alpha) - V_L(\alpha)$ is strongly decreasing in ability, this has a surprising and interesting implication for female educational choice. In our example, we find that the top 28 percent ability women with L attractiveness invest in education. But only the top 22 percent of women in the ability distribution with H attractiveness invest in education. This gap arises because although the marginal L-investor has lower ability, and so for her the increase in wage through education is smaller, her corresponding increase in fitness has a greater impact on her matching prospects.

We now use this numerical simulation to assess the impact of equal opportunities and advances in contraception technology on market outcomes.

Equal Opportunities — With unequal (zero) opportunities in the labor market, women do not invest in education. As male preferences then depend only on female desirability and love, it is no surprise that being more attractive is then very important to single women. Figure 5 compares equilibrium payoffs V_H and V_L with unequal opportunities (the two flat lines) to the payoffs $V_H(\alpha)$ and $V_L(\alpha)$ obtained with equal opportunities. To ease interpretation, we report annuitized values at the market rate $r = 0.04$.

The presence of equal opportunities generates a steep increase in the return to female ability. It also attenuates the attractiveness premium, i.e., $V_H - V_L$ falls at every ability level. The vast majority of women are better off with equal opportunities, especially those endowed with higher levels of ability. But not all are better off. In particular, the bottom 13 (and 11) percent in the ability distribution among H-type (and L-type) women are worse off. This is because, for these lower ability women, the introduction of equal opportunities brings little value to their labor market prospects and worsens their prospects in the marriage market.

Improved Birth Technology — Recall that an improved contraceptive technology is considered as an increase in u, which increases the flow value of being single relative to the flow value of being married. Given attractiveness n, let $\alpha(n|\Theta)$ denote the critical ability level at which a woman is indifferent to investing in education, where Θ denotes the parameter set of the model. If this marginal investor chooses not to invest in education, her ex-post characteristics are $z^- = n + \beta \max[\alpha - \phi, 0]$ and $w^- = \alpha$. If instead she invests in education, her ex-post characteristics are $z^+ = n + \beta \max[\gamma\alpha - \phi, 0]$ and $w^+ = \gamma\alpha$. Thus the marginal investor α is identified by

$$V^f(z^+, \gamma\alpha) - V^f(z^-, \alpha) = c.$$

Now given an optimal match strategy, the Envelope Theorem implies an increase in u, ceteris paribus, raises the value of being single, $V^f(\cdot)$, proportional to the time she expects to remain single. Thus, the impact of increased u on the net return to education $V^f(z^+, \gamma\alpha) - V^f(z^-, \alpha)$ is proportional to the difference in the expected time to marriage for each educational choice. The data and the model, however, indicate that the average unmatched spells for educated and uneducated women are not very different. The numerical example finds that increasing u, by itself, has a negligible impact on female schooling decisions.

This exercise, however, ignores the data interpretation suggested above: that women who invest in education delay entering the marriage market, from the age of 18.1 years to the age of 22.9. Suppose for example that, without the pill, flow utility while single u is \$500 per year lower, a decrease of roughly 1.7 percent of average male earnings. Consistency requires us to adjust the education cost by $\Delta c = 4.8\Delta u = \$2,400$ to reflect the dollar equivalent reduction in utility faced while single and studying at college.

Given these changed values for both u and c, computations in our numerical example yield a reasonably large effect on education. In fact, for both attractiveness types, female education rates decline by two percentage points, so that the average university education rate falls from 25 to 23 percent (an eight percent reduction). This change arises as the estimated (psychic) cost of education increases by 5 percent, from \$47,000 to \$49,400. This suggested change in college education rates is consistent with the evidence presented by Goldin and Katz (2002) and Bailey (2006). But the interpretation is different: the pill encourages higher female education rates by reducing the psychic cost of attending college. Nevertheless without equal opportunities, women would have far less incentive to invest in a costly college education.

7. Conclusion

This paper makes two significant contributions. First it introduces a new equilibrium model of search and matching in which men and women have multiple attributes and general payoff functions. This new framework is extremely rich and offers considerable scope for future research. Second, our specific application yields new insights into how equal labor market opportunities for women and improved contraceptive technologies affected women's education and employment decisions as well as the timing of their first births.

In line with earlier empirical research, our numerical example finds the introduction of the pill contributed to the rise in female education and labor force participation rates observed since the 1950s. But we argue those increases would not have occurred without equal opportunities in the labor market. This is demonstrated by contraceptive innovations preceding World War II which, with unequal opportunities, had only a very limited impact on female education rates and female labor supply.

An important, previously undocumented feature of the data is that the distribution of women's age at first birth, *conditional* on having a child, has hardly changed over the last 150 years. We know, however, that there has also been a steep increase in female education rates and educated women, on average, delay childbirth by about five years. Taken together these empirical facts appear inconsistent. The reconciliation is that we have analyzed the distribution of age at first birth conditional on having at least one child, i.e., Figure 3 describes the distribution of completed spells, while women who heavily invest in education are more likely to remain childless (Rowland 2007; Gobbi 2013). Although beyond the scope of the current paper, this raises an interesting and potentially important issue for future research.

Appendix

Our data sources, which we use for the discussion in the Introduction and the numerical exercise in Section 6, come from the Integrated Public Use Microdata Series (IPUMS) of the decennial censuses (Ruggles et al. 2010) and the National Survey of Family Growth (NSFG).

IPUMS — We use decennial censuses from 1850 to 2000 (1890 is missing), and the 2009 American Community Survey sample, which is a 1 percent national random sample of the population. Direct information on age at first birth is not available. We therefore construct it using the variable ELDCH (age of the eldest own child in household) in conjunction with BIRTHYR (year of birth) and AGE. Detailed information on education, which is grouped in 12 different categories in the variable EDUC, is not available before the 1940 Census. Data on labor force participation come from the variable LABFORCE and cover all census years with the exception of 1850, in which the information for women is not recorded. For our analysis we select only women who are aged between 15 and 44 for the age at first birth statistics (or aged between 25 and 34 for the education and labor market statistics). The sample size therefore varies across censuses and outcomes. The smallest sample is for labor force participation in 1860 with just under 15,000 women (Figure 1), while the largest is for age at first birth in 2000 with over 3 million women (Figure 3).

Figure 1 shows the trends in the proportion of women aged 25–34 who are in paid employment, expressed in percentage terms. Unemployed women and women who are out of the labor force in that age range are in the base category. Besides all women, the figure also shows the trends for mothers and for women who are single and never married.[15] The red vertical lines are drawn in correspondence of the diffusion of birth technology improvements. That is, the 1860 line corresponds to vulcanized rubber condoms, which followed the invention of vulcanized rubber in 1844 and its application to the production of condoms in 1855 (Guinnane 2011); the 1940 line represents the 1936 legalization of contraceptive use for family planning, which followed the introduction of the latex technology in 1919; the 1965 and 1980 lines capture the diffusion of the pill, which after the introduction of *Enovid* in 1960 took time before a full liberalization of access to oral contraception (Goldin and Katz 2002; Bailey 2006, 2010).

Figure 2 shows the proportion of women aged 25–34 who have a university degree or higher qualification. Red vertical lines are drawn in correspondence of 1940, 1965 and 1980. Figure 3 reports the distribution of the age at first birth (median, 10th and 90th percentiles) for all women by mother's year of birth. These statistics were obtained from life table analysis (Kalbfleisch and Prentice 2002). In this case, each of the vertical lines is predated by 20–25 years to reflect the birth cohorts of women who would have had a greater exposure to the birth technology improvements.

NSFG — Our analysis uses Cycles 1–7 collected by the National Center for Health Statistics in 1973, 1976, 1982, 1988, 1995, 2002, and 2006–2010. Each sample contains information on women aged 15–44.

The surveys for cycles 1–5 (up to 1995) were based on personal interviews conducted in the homes of a national sample of women in the civilian, noninstitutionalized popu-

[15] Because data on marital status are available only since the 1880 census and not before, the employment series for single, never married women can only be constructed from that year onwards.

lation of the United States. The main purpose of the 1973–1995 surveys was to provide reliable national data on marriage, divorce, contraception, infertility, and the health of women and infants. Cycles 1 and 2 (1973 and 1976) collected information only on currently or formerly married females, so they do not allow us to construct series for single, never married women.

Cycle 6 (2002), in which both men and women were interviewed, was based on an area probability sample. The sample represents the household population of the United States, 15–44 years of age. The survey sample is designed to produce national data, not estimates for individual states. In-person interviews were completed with 7,643 female respondents (and 4,928 males). The response rate was 79 percent overall — 80 percent for females and 78 percent for males. Cycle 7 (2006–2010) is similar to cycle 6, except that interviews in this case were done 48 weeks of every year for 4 years, from June 2006 through June 2010. This cycle interviewed a nationally representative sample of 22,682 women and men 15–44 years of age living in households in the United States.

Each cycle contains direct information on each of the variables of interest (age at first birth, education, and labor force participation) in all data collection years. As in the case of IPUMS, sample sizes vary across outcomes and cycles. The smallest sample is for the labor force participation series of never married women in 1982 with 507 observations (Figure 1), while the largest is for age at first birth for the cohort of women born between 1951 and 1955 with over 8,000 observations (Figure 3). As already mentioned, because cycles 1 and 2 did not sample never married women, the employment rate series in Figure 1 for this group of women starts from cycle 3 (1982). For each of the Figures 1–3, the same considerations we discussed earlier apply.

As mentioned in the text, the choice of the age cutoff used for the analysis underlying Figure 3 is driven by data availability. Raising the cutoff to 44 years (the oldest birth cohort available in the NSFG) does not change the results in Figure 3. Choosing a higher age cutoff would not be possible in the NSFG, and problematic for the Census data where direct information on age at first birth must be constructed using mother's age and the age of the eldest child living in the household. This child in fact may not be the first born. In this case, then, the statistics in Figure 3 could be lower bounds. Lowering the cutoff at ages below 40 reduces the problem related to mother's co-residence with her first child, but raises an issue of selection for more recent cohorts of women, especially those who were more likely to be exposed to the availability of oral contraceptives.

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Table 1: Baseline parameters and functional forms

r	4 percent (per year)
γ	1.04
β	0.48
$H(\cdot)$	Exponential, with parameter δ
$G^m(y)$	Lognormal, with $\mu_m = \$10.44$ and $\sigma_m = \$0.69$
$[\underline{y}, \overline{y}]$	[\$9,608, \$135,131]
$G^f(\alpha \vert n)$	$G^m(\cdot)$ rescaled with $\alpha = y/1.04$
$G^f(n \vert \alpha)$	Binomial, with draws from $\{-\frac{\overline{n}}{2}, \frac{\overline{n}}{2}\}$
ϕ	\$10,730 per year, per child
u	$1/\delta$
$\overline{n}$	$1/\delta$
Proportion of college graduates	0.25

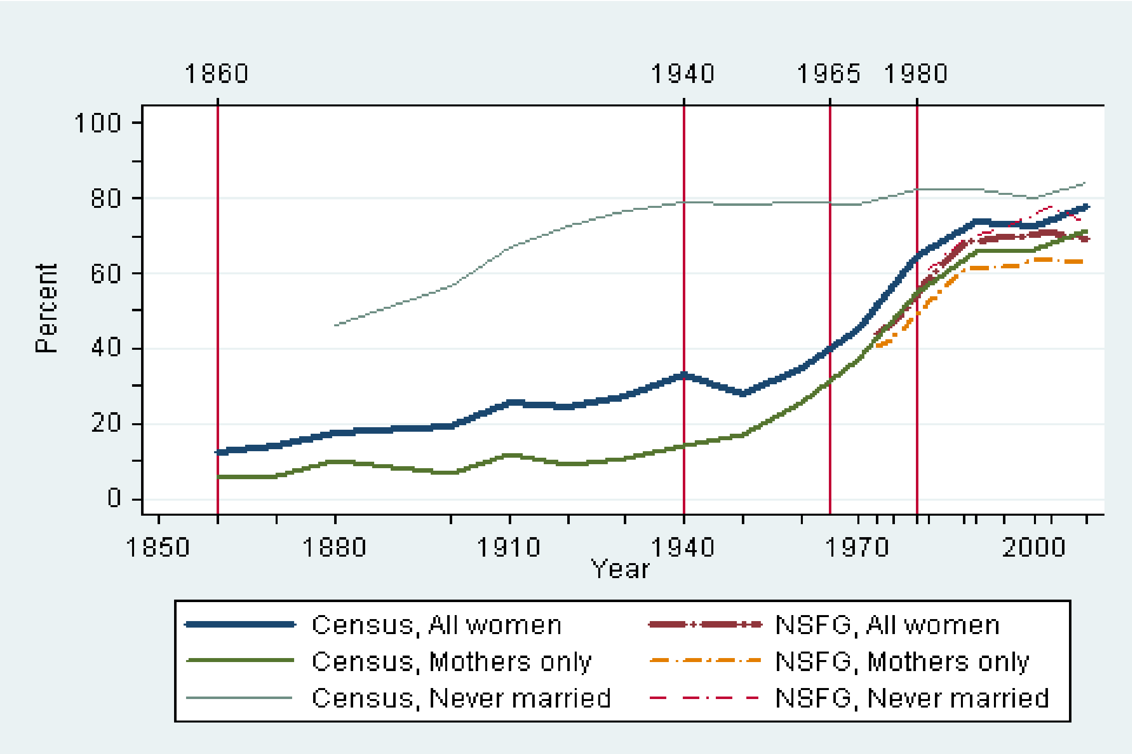
Table 2: Fitted Parameters

Parameter	Estimate
λ	6.0 per year
$\frac{1}{\delta}$	\$8,900
$F - u$	\$47,700
c	£47,000.

Table 3: Mean and Standard Deviation of Unmatched Spells

	μ_u	σ_u	μ_e	σ_e
Data	–	4.56	–	4.57
Model	4.43	4.48	4.68	4.72

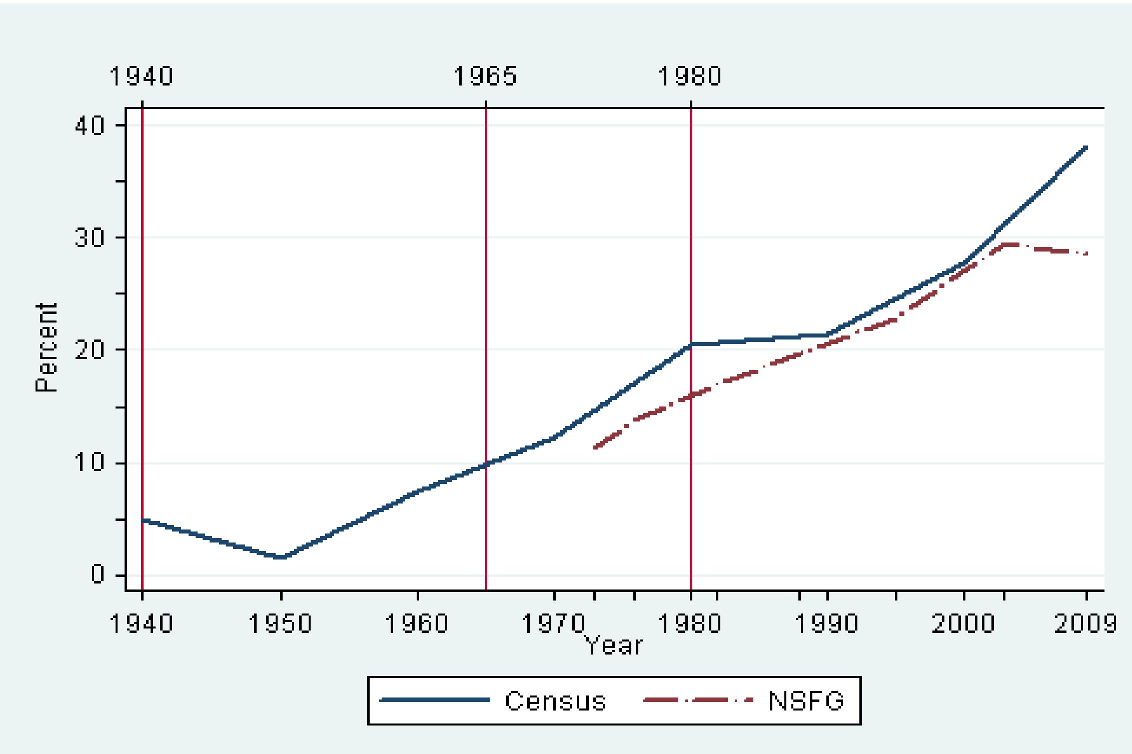
Figure 1: Female Employment Rate, Women aged 25-34



Sources: IPUMS Census 1850–2000, 2009 ACS sample; NSFG, Cycles 1-7.

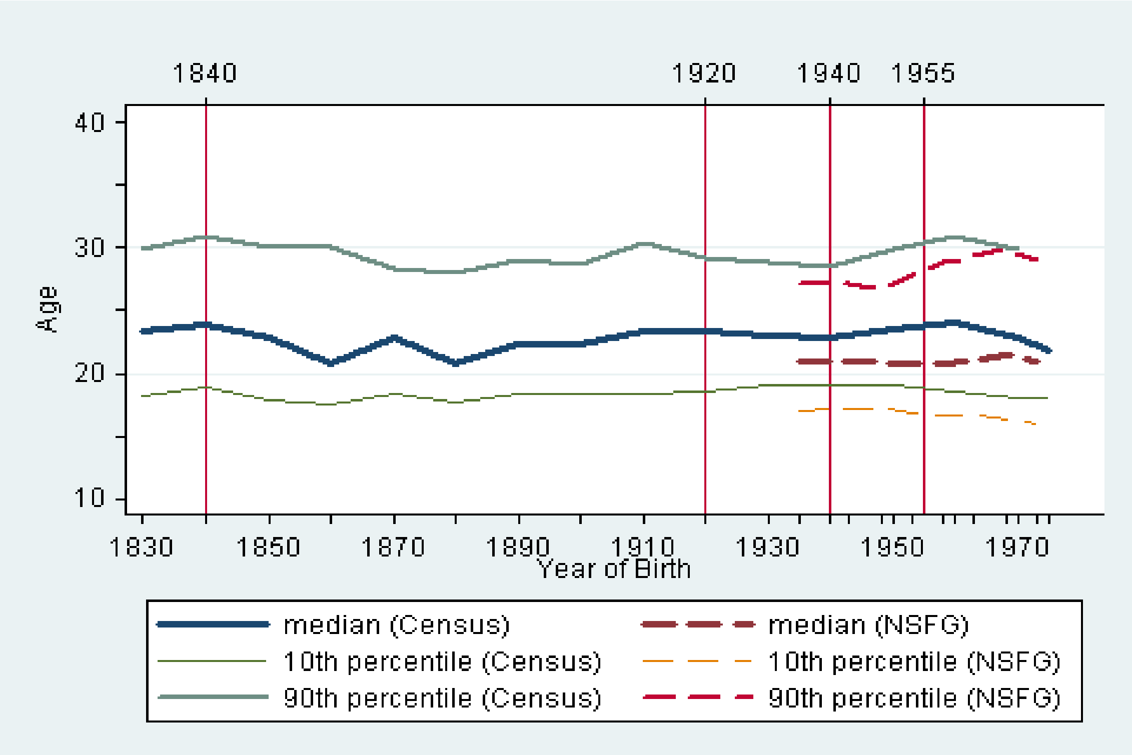
Note: See the Data Appendix for a description of the data and the variables used.

Figure 2: Fraction of College Graduate Women, Women aged 25-34



Note: See the note to Figure 1.

Figure 3: Age at First Birth by Mother's Birth Cohort



Note: See the note to Figure 1.

Figure 4: Equilibrium Match Rates by Attractiveness Type

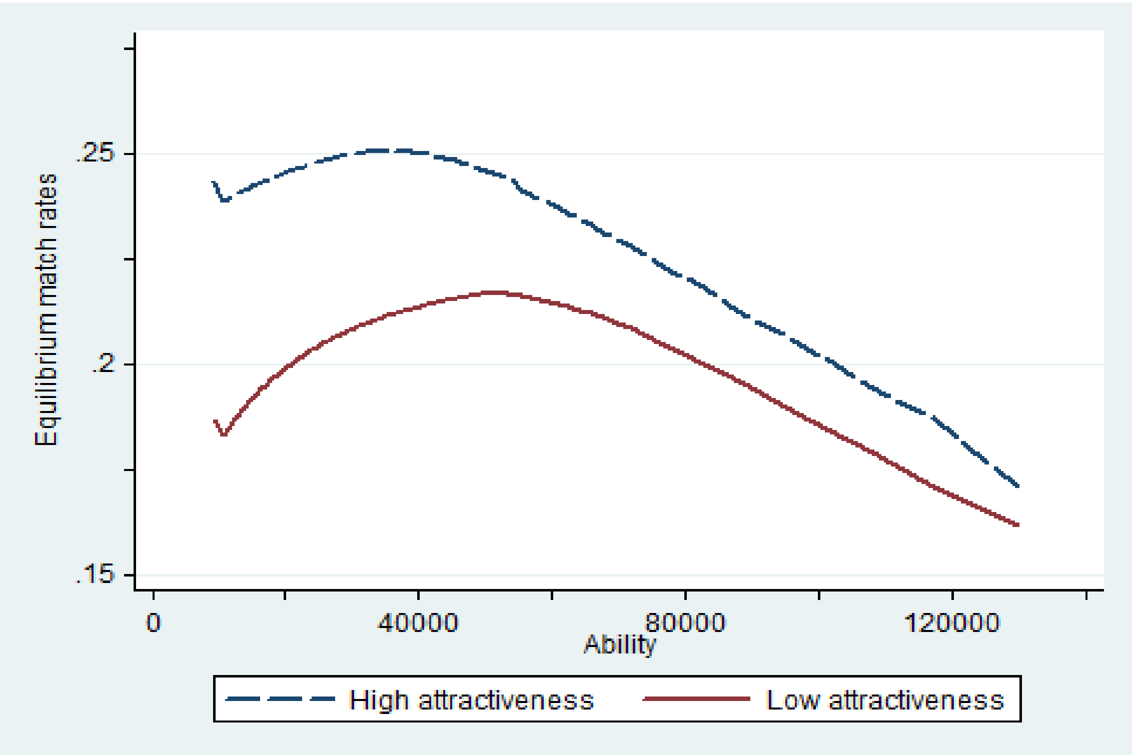


Figure 5: Equilibrium Payoffs by Attractiveness Type, With Unequal and Equal Opportunities