An Overview of Optimal and sub-Optimal Detection Techniques for a Non Orthogonal Spectrally Efficient FDM

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Abstract: Spectrally Efficient non orthogonal Frequency Division Multiplexing (SEFDM) Systems occupy less bandwidth than equivalent orthogonal FDM (OFDM). However, enhanced spectral efficiency comes at the expense of an increased complexity in the signal detection. In this work, we present an overview of different detection techniques that trade the error performance optimality for the signal recovery computational effort. Linear detection methods like Zero Forcing (ZF) and Minimum Mean Squared Error (MMSE) offer fixed complexity but suffer from a significant degradation of the Bit Error Rate (BER). On the other hand optimal receivers like Sphere Decoders (SD) achieve the optimal solution in terms of error performance. Notwithstanding, their applicability is severely constrained by the SEFDM signal dimension, the frequency separation between the carriers as well as the noise level in the system.

1. Introduction.

Increased interest in spectrally efficient multicarrier systems has been evident over the past few years. Indeed, in the beginning of the current decade, Rodrigues and Darwazeh in [1] and Xiong in [2] introduced practical implementations of multicarrier systems that occupied half the bandwidth of an equivalent OFDM. However, these multicarrier systems had considerable limitations since the detection was possible for real alphabets (e.g. BPSK) only.

Later, Rodrigues and Darwazeh in [3] and Hamamura and Tachikawa afterwards [4] proposed FDM systems that deliberately violate the orthogonality principle by having the subcarrier frequency separation smaller than the inverse of the FDM signalling period. Consequently, these systems offered the potential of considerable bandwidth efficiency by conveying the same information of conventional OFDM systems in a fraction of their bandwidth.

Interestingly, Rusek and Anderson have very recently shown [5] that the concept of reducing the frequency separation between orthogonal subcarriers is the dual case of the Mazo's time-domain transmission technique whose aim is to transmit information faster than the Nyquist rate [6]. In addition, they have proved that in presence of AWGN the frequency separation of the subcarriers can be reduced up to a limit (the dual of the so called Mazo's limit) with no impact on the minimum Euclidean distance between the transmit vectors. As a consequence, one ought not to expect any degradation in the performance of spectrally efficient FDM systems, in relation to OFDM systems, up to this frequency separation limit. Notwithstanding, the reliable detection of the information of such FDM systems is still a very challenging issue since the optimal maximum likelihood detection is overly complex.

This paper provides an overview of detection techniques for the non orthogonal SEFDM introduced in [3]. In particular, standard sub-optimal Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) detectors are applied and compared to various versions of optimal Sphere Decoders (SD) in terms of Bit Error Rate (BER). In addition, a complexity comparison between a typical and a regularised version of SD is given.

The paper is organised as follows: In section 2 we describe the SEFDM system as introduced in [3]. In section 3 we discuss about the various detection methods that are investigated. Section 4 presents a range of simulation results unveiling the merits and demerits of these techniques. Finally, section 5 summarises our conclusions.

2. SEFDM Model Description

The original SEFDM transceiver is described in [3]. A high data rate input stream is split into N parallel low data rate streams. The latter modulate, according to a specific modulation scheme of level M, N SEFDM subcarriers $f_n(t)$, $n = 0, \ldots, N-1$, whose frequency separation Δf is only a fraction of the inverse of the SEFDM symbol period T, i.e.,

$$\Delta f = \frac{\alpha}{T}, \; with \;\; \alpha < 1. \tag{1}$$

Thus, the required bandwidth is reduced by a factor $1\text{-}\alpha$, at the expense of the loss of orthogonality between the carriers. The transmitted signal, in an SEFDM symbol period, is given by

$$s(t) = \frac{1}{\sqrt{T}} \sum_{n=0}^{N-1} S_n f_{a,n}(t) = \frac{1}{\sqrt{T}} \sum_{n=0}^{N-1} S_n e^{j2\pi n \Delta f t}, \tag{2}$$

where S_n represents the n^{th} modulation symbol. In order to demonstrate the SEFDM system spectral gain, the spectrum of the transmitted FDM signal is depicted in Fig. 1 for $\alpha = 1$ (OFDM), $\alpha = 0.75$ (25% spectral gain) and $\alpha = 0.5$ (50% spectral gain). For these cases, the evaluation of the noise equivalent bandwidth provides similar spectral gains to the ones observed above. Assuming the only impairment introduced by the communication channel is Additive White Gaussian Noise (AWGN) $n(t)$, the received signal $r(t)$ can be expressed as:

$$r(t) = s(t) + n(t). \tag{3}$$

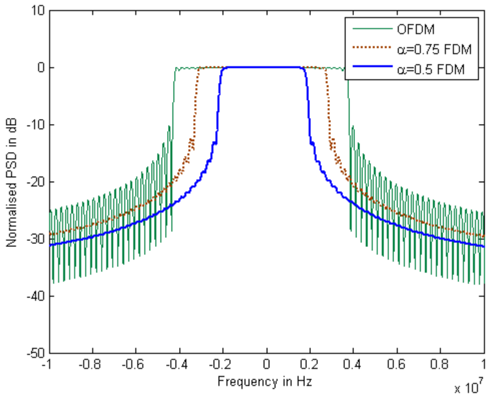

Fig. 1: FDM Spectrum of N=32 and T=4μsec FDM schemes for α={1,3/4,1/2}.

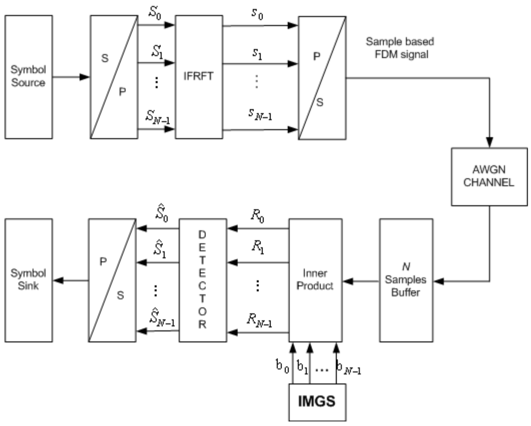

Fig. 2: SEFDM modem.

The proposed receiver consists conceptually of two stages. The first stage uses a bank of N correlators to extract N sufficient statistics from the received signal. The second stage uses a detector. The particular choice of the correlation functions, $b_i(t)$ with $i=0,1\ldots N-1$, at the receiver outer stage is driven by two important requirements: (i) The correlation functions should be orthonormal in order to prevent colouring of $n(t)$, and (ii) That detection of the SEFDM signal could be computationally facilitated. Both requirements are met by generating an orthonormal base that spans the SEFDM signal space using the Iterative Modified Gram Schmidt (IMGS) orthonormalization method [7]. We underline the use of IMGS instead of the classic version of the GS algorithm because of its superior performance in terms of numerical stability.

We also note it is possible to relate the vector of sufficient statistics to the vector of information symbols using the following linear statistical model

$$\mathbf{R} = \mathbf{MS} + \mathbf{N}, \tag{4}$$

where $\mathbf{R} = [R_i]$ is the vector of the N observation statistics, $\mathbf{S} = [S_i]$ is the vector of the N transmitted symbols, $\mathbf{M} = [M_{ij}]$ is the $N\times N$ covariance matrix of the SEFDM carriers and the orthonormal base, and $\mathbf{N} = [N_i]$ is a vector containing N independent Gaussian noise samples of zero mean and covariance matrix $\sigma^2\mathbf{I_N}$ ($\mathbf{I_N}$ being an identity matrix of $N\times N$ dimension). The elements of $\mathbf{R}$ and $\mathbf{M}$ are given by

$$\begin{aligned} R_i &= \int_0^T r(t) b_i^*(t)dt, \qquad & i = 0,1\ldots N-1, \\ M_{ij} &= \int_0^T f_{\alpha,i}(t) b_j^*(t)dt, \qquad & i, j = 0,1\ldots N-1. \end{aligned} \tag{5}$$

Finally, it is interesting to note that the typical implementation of an SEFDM system mimics the IFFT-FFT based implementation of an OFDM system [4]. For example, an Inverse Fractional Fourier Transform (IFrFT), with quadruple the complexity of a conventional IFFT, could be used for the SEFDM signal generation [8]. A block diagram of a possible SEFDM modem is illustrated in Fig. 2.

3. Optimal and Suboptimal Detection

The optimum ML detection of SEFDM reduces to the following combinatorial optimisation problem:

$$\begin{aligned} &\text{minimise } \left\|\mathbf{R} - \mathbf{MS}\right\|^2, \\ &\text{subject to } \mathbf{S} \in Q^N, \end{aligned} \tag{6}$$

where $\|\cdot\|$ denotes the Euclidean norm and Q^N is the feasible set of the problem comprising of all the possible N-tuples of the data symbols. It is apparent that the above problem can be solved through an exhaustive comparison amid the values of the cost function for all the feasible points. However, the complexity of such a solution would be of exponential order over the constellation cardinality and the number of the SEFDM carriers, i.e. $O(M^N)$.

Thus, a first simple solution to the complexity problem could come from the relaxation of the feasible set Q^N. In particular, after discarding the constraint of eq. (6) so that the elements of **S** can be any real numbers, the SEFDM detection reduces to an unconstrained Least Squares (LS) problem that has the solution $\mathbf{\acute{S}}=\mathbf{M}^{-1}\mathbf{R}$. Since the elements of $\mathbf{\acute{S}}$ are any real numbers, a slice operation is then applied to recover the originally transmitted symbols **S**. Such a kind of detection is well known as Zero Forcing.

A slightly different approach to ZF is MMSE detection. In his case, the cost function of the previous unconstrained LS problem is regularised so that big solutions are penalised as follows:

$$\text{minimise } \|\mathbf{R}-\mathbf{MS}\|^2+\varepsilon\|\mathbf{S}\|^2, \tag{7}$$

where ε is called the regulator and in MMSE is set to be equal to *1/SNR* (*SNR* is the Signal to Noise Ratio). The solution to (7) is given by $\hat{\mathbf{S}}=(\mathbf{M}^H\mathbf{M}+\varepsilon\mathbf{I})^{-1}\mathbf{M}^H\mathbf{R}$, where $(\cdot)^H$ denotes the Hermitian of a matrix and **I** is the identity matrix of $N\times N$ size. As in ZF case, the final MMSE estimate is derived after the application of a slicer on the $\hat{\mathbf{S}}$.

Although ZF and MMSE have a fixed polynomial complexity (i.e. $O(N^3)$), both of them suffer from an overly amplification of noise that results into a severe degradation of their error performance.

Should we constrain the feasible set so that **S** takes values in the $\{\pm1\pm j\}$ N-tuples, optimality is achieved using Sphere Decoding, which is a dynamic programming algorithm proposed by Finchke and Pohst [9] for the fast solution of integer LS (ILS) problems. Consequently, eq. (6) could be reduced to the following:

$$\begin{aligned}&\text{minimise } \|\mathbf{R}-\mathbf{MS}\|^2,\\&\text{subject to } \|\mathbf{R}-\mathbf{MS}\|^2\le C,\ \mathbf{S}\in\{\pm1\pm j\}^N,\end{aligned} \tag{8}$$

where C is the so called SD radius parameter. Despite the fact that SD guarantees an optimal solution, its main drawback is that its computational efficiency depends on the noise and the bad conditioning of the projections matrix **M** [10]. An efficient solution to the last problem could be given by regularising, similarly to MMSE, the cost function of eq. (8) before implementing the typical SD algorithm.

In the next paragraph, we demonstrate by simulation comparisons between ZF, MMSE detectors and standard and regularised versions of Sphere Decoders.

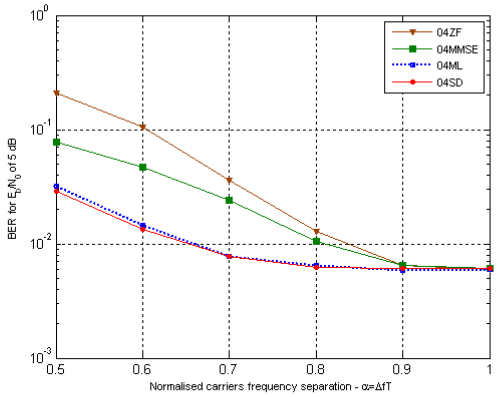

Fig. 3: Error performance of various detection techniques versus α. E_b/N_0 was set to 5 dB.

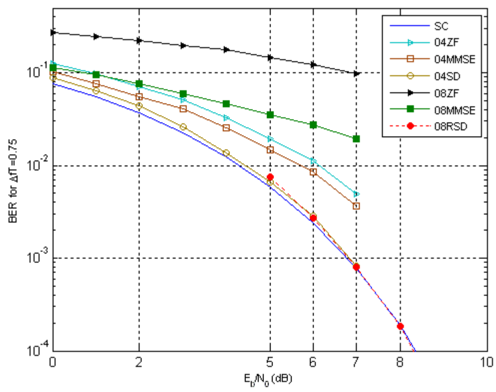

Fig. 4: Error performance of various detection techniques versus E_b/N_0; α was set to 0.75.

4. Simulation Results

We performed simulations for different number of *4*-QAM SEFDM carriers N with frequency separation that ranged from the OFDM ($\alpha=1$) to the half OFDM one ($\alpha=0.5$). In addition, we evaluated the performance of the examined detection techniques for various values of the Energy of the bit over the Noise power density, E_b/N_0.

Fig. 3 shows BER for the different detections for a small dimension, $N=4$, SEFDM signal versus α. E_b/N_0 was set to be equal to *5* dB. It appears that ZF and MMSE significantly diverge from the optimal case. On the contrary, SD result coincides with ML as expected. We also need to underline the fact the optimal case achieves the BER of an equivalent OFDM for α approximately equal to *0.8*. Henceforth, BER gradually degrades even if brute force ML detection is applied.

Fig. 4 presents a similar comparison of BER versus the E_b/N_0 for a fixed $a=0.75$. It is obvious that SD prevails approximating the OFDM performance represented by the single carrier (SC) *4*-QAM curve.

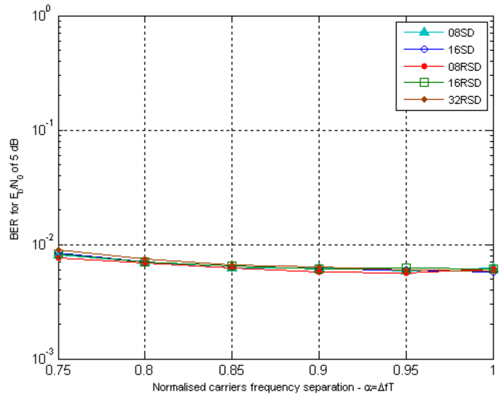

Fig. 5: Error performance of various Sphere Decoders versus α; E_b/N_0 was set to 5 dB.

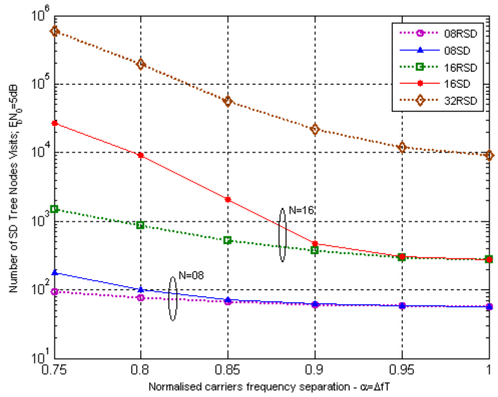

Fig. 6: Complexity comparison versus *a* between SD and RSD detection techniques; E_b/N_0 was set to 5 dB.

Figs 5 and 6 illustrate results for the standard and Regularised SD (RSD) detections. In the former, it is shown that both schemes achieve the same error performance since they both solve equivalent optimisation problems [10]. In the latter, the complexity of both techniques versus *a*, for a fixed E_b/N_0, is evaluated. The number of the visits to the SD tree nodes is used as a measure of comparison. It becomes clear that RSD improves standard SD complexity. In addition, the regularisation benefit enhances as the conditioning of **M** is worsening with the decrease in α and\or the increase of SEFDM signal dimension *N*.

5. Conclusions.

We investigated optimal and sub-optimal detection techniques for a non orthogonal spectrally efficient FDM system. We showed by simulation that relaxations of the ML detection problem like ZF and MMSE suffer from a severe degradation in their error performance due to the overly amplification of the noise. On the other hand Sphere Decoders achieve the optimum solution but their complexity is random depending on the noise and the projections matrix properties. An efficient solution to the latter is given by a regularised version of the SD algorithm. Hence, a tangible detection could be accomplished for a small dimensional SEFDM signal in high *SNR* regimes.

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