Contractual Signalling, Relationship-Specific Investment and Exclusive Agreements*

Luís Vasconcelos†

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Abstract

I analyze a model of hold-up with asymmetric information at the contracting stage. The asymmetry of information concerns the value of trade with external parties. I show that contractual signalling and efficiency of investment can conflict if only quantity is contractible. This conflict generates inefficient equilibria in terms of investment. Contracting on exclusivity in addition to quantity resolves the conflict and consequently eliminates the inefficiency of investment.

Keywords: Relationship-specific investment, asymmetric information, exclusivity.

JEL Classification: L14, L40, D82, K21.

1 Introduction

Many relationships are formed under asymmetric information. When two or more parties meet to agree the terms of a relationship, some may have private information on the value of the relationship. For example, in vertical relationships, a final good producer contracting with a

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†University of Essex and Nova School of Business and Economics. Contact information: lvasco@essex.ac.uk. University of Essex, Department of Economics, Wivenhoe Park, Colchester, Essex CO4 3SQ, UK.

specific supplier may have private information regarding potential trade with an alternative supplier. Similarly, in labor market relationships, an employer offering a job to a worker may have private information about the fit of the worker to the job position, or about the possibility of subsequently finding a worker that fits better. As was emphasized by Myerson (1983) and Maskin and Tirole (1992), if the parties with private information participate in the design of the contract (or the terms of the relationship if established in an informal way), the contract's terms may reveal some of their private information to the other parties. Because of this information transmission effect, the design of the contract assumes a strategic role not present when contracting parties have symmetric information. When investment in the relationship is important, this role is in addition to that of providing the parties with the right incentives to invest that is typical to the hold-up problem literature.

This paper considers a model of hold-up with asymmetric information at the (ex-ante) contracting stage, where traders may (ex-post) renegotiate the terms of trade. It identifies how exclusivity agreements may improve efficiency. In particular there is a buyer (the principal) and a supplier (the agent) where the supplier makes an own-investment which is unverifiable and match specific.[1] An important feature of the analysis is the buyer might instead trade with an external supplier (e.g., buy a generic version of the good) where the value of such trade is stochastic. With symmetric information on the potential value of external trade, as in standard models, the hold-up problem is solved by carefully setting the quantity in a contract whose terms specify only a transfer and the quantity traded. With asymmetric information, however, where the principal is better informed on the value of this outside option, such contracts are no longer efficient in that the agent does not invest the socially optimal level. Instead efficiency is achieved by introducing an exclusivity clause, a clause which restricts the principal to trade only with the agent. Of course it may occur that the ex-post realized value of outside trade is sufficiently high that it is more efficient for the principal to trade with the external supplier. What is essential, however, is that the exclusivity clause implies the principal must first negotiate with the agent to trade with the external supplier.

The efficiency-enhancing effect of exclusivity identified here rationalizes the use of contracts that specify both quantity and exclusivity. Furthermore, it is important for the following two

[1] Focusing on own-investment by the agent (also called selfish investment) which affects only the agent's value of trade, as opposed to investment that affects both the principal and the agent's valuations of trade (often called cooperative investment), allows us to better assess the effect of asymmetry of information at the contracting stage on efficiency of investment. In contrast with the case of selfish investment, a contract ensuring efficient cooperative investment may not exist even when information is symmetric at the contracting stage (see for example Che and Hausch, 1999).

reasons. First, in contrast to Segal and Whinston (2000), it rationalizes the use of exclusive contracts in situations of hold-up with pure relationship-specific investments. Motivated by informal discussions (in anti-trust and exclusive contracts) on whether exclusive provisions foster relationship-specific investments, Segal and Whinston (2000) show that exclusivity does not affect investments that are fully relationship-specific, when information is symmetric at the contracting stage.[2] Second, it contributes to the unsettled debate on whether exclusive agreements should be contractually allowed by courts or not. In this specific matter, a long-standing concern of courts is that exclusive contracts serve anticompetitive purposes, and consequently prevent efficiency.

The emergence of equilibria with inefficient investment when only quantity is contractible is due to a conflict between using the contract to provide the agent with the right incentives to invest and using it to signal information to extract surplus. In particular, I show that when the principal expects a low value of trade with the external party, she may initially commit to trade an excessively high quantity with the agent. The principal does so to signal an expected low outside option and, consequently, to convince the agent to accept a contract that allows her to appropriate more of the surplus generated by the relationship with the agent. Such a commitment successfully signals a low outside option because it is less costly to a principal with a low outside option to do it than it is to a principal who a high outside option. The problem of committing to trade such a high quantity with the agent is that it leads the agent to overinvest in the relationship. The agent does so to protect his disagreement payoff (i.e., the payoff if the initial contract is enforced) in the event of a contract renegotiation.

If the parties can contract also on an exclusive-dealing provision, the conflict between signalling information to extract surplus and investment incentives can be resolved. This is for two reasons. First, because contracted exclusivity serves as a signal of the principal's information about the value of her outside option. This is because it is more costly to a principal who expects a high value of trade with an external party to initially commit to trade exclusively with the agent than it is to a principal who expects that value of trade to be low. Second, because in contrast to contracted quantity, exclusivity does not directly affect the agent's investment decision. Thus, when both quantity and exclusivity are contractible, the principal

[2] Segal and Whinston (2000) concerns mainly the case where parties cannot contractually specify a quantity in advance. The case where parties can specify both a quantity and an exclusivity level is studied only in Section 6 of that paper. In De Meza and Selvaggi (2007), the authors show that exclusivity may affect relationship-specific investments. Their result differs from that in Segal and Whinston (2000) because they consider a different bargaining game. Our effect is totally different from that in De Meza and Selvaggi (2007), as it stems from the existence of asymmetric information at the contracting stage.

can set contracted quantity to induce optimal investment by the agent, and adjust contracted exclusivity, without affecting the agent's investment decision, to signal information and extract surplus. Consider again the case of the principal who expects a low value of trade with the external party. Efficiency of investment can be achieved by proposing a contract that prescribes simultaneously a quantity that induces efficient investment by the agent and full exclusivity. Since exclusivity signals a low outside option for the principal, the principal has no need to distort contracted quantity upward to signal this information.

Both the contractual distortions and the effect of contractibility of exclusivity on relationship-specific investment highlighted here are novel in the literature. This is because the existing literature on the hold-up problem (e.g., Grossman and Hart, 1986; Hart and Moore, 1990; Chung, 1991; Rogerson, 1992; MacLeod and Malcomson, 1993; Aghion et al., 1994; Edlin and Reichelstein, 1996; Che and Hausch, 1999; Schmitz, 2002; Hori, 2006; Watson, 2007; Zhao, 2008; and Buzard and Watson, 2012), and in particular that on the interaction between exclusivity and relationship-specific investment (e.g., Segal and Whinston, 2000; and De Meza and Selvaggi, 2007), has focused on situations where parties' information is symmetric at the initial contracting stage. This paper extends the literature on the hold-up problem to the case in which there is asymmetric information at the contracting stage. In the hold-up problem literature (with symmetric information at the contracting stage), the contract is typically designed with one goal: to provide the right incentives to invest. The presence of asymmetric information at the contracting stage introduces a new role for the contract: signalling information to extract surplus.

This paper is also related to the literature on contract design by an informed party. This literature can be divided into two groups. The first group focuses on the characterization (in a general way) of the equilibrium contract proposal by an informed principal in a principal-agent relationship (e.g., Myerson, 1983; Maskin and Tirole, 1990; Maskin and Tirole, 1992; and Beaudry and Poitevin, 1993). The modelling approach in this paper is in the spirit of that in Maskin and Tirole (1992). In the context of the model in this paper, I extend their analysis to the case in which the agent makes a noncontractible investment decision. This extension is not a trivial one. Maskin and Tirole (1992) assume that all payoff-relevant variables are contractible. In their model, the agent's beliefs about the principal's type affect only the agent's decision to accept the contract proposed by the principal. In here, the agent's beliefs at the end of the contracting phase are still important. They affect the agent's investment decision, which in turn affects the principal's payoff (and preferences over contracts). The second group of this literature has studied contract design by an informed party in more concrete settings (e.g., Aghion and Bolton, 1987; Aghion and Hermalin, 1990; Spier, 1992; and Nosal, 2006).[3] The articles in this literature have not studied specifically the relationship between contractual signalling and relationship-specific investment.

The paper is structured as follows. In Section 2, I present the model. In Section 3, I obtain preliminary results and analyze the benchmark case of symmetric information contracting. In Section 4, I show the existence of inefficient equilibria in terms of investment when contracts specify only quantity and how contractibility of exclusivity resolves this inefficiency. In Section 5, I present concluding remarks.

2 The Model

There is a principal (e.g., a buyer) and an agent (e.g., a supplier) who contract on the terms of trade.[4] Both know that ex-post the principal has the possibility of instead trading with a second (external) agent. The value of trade with this external agent is denoted V_E and is stochastic. Specifically there are two states of the world, $j \in \{L, H\}$, and V_E is considered a random draw from c.d.f. $F_j(.)$ where we assume $F_L(.)$ strictly first-order stochastically dominates $F_H(.)$. The benchmark case supposes the principal and agent both observe the state j before contracting. The interesting case instead supposes asymmetric information, where the principal observes the state j (her type) but the agent only knows that state $j = H$ occurs with probability π_H, otherwise $j = L$ with probability $\pi_L = 1 - \pi_H$. For simplicity, it is assumed that V_E is always non-negative.

Following Maskin and Tirole (1992), the (informed) principal offers a menu of contracts to the agent. The agent either agrees to this menu, or rejects the offer and obtains a zero payoff. If the agent accepts the menu, the (informed) principal chooses one of the contracts listed in that menu. This is the contract that governs the relationship between the principal and the agent. A contract specifies an up-front transfer $t \in \mathbb{R}$ from the agent to the principal, a quantity $q \in [0, 1]$, and a level of exclusivity $e \in E \subseteq [0, 1]$. Quantity q denotes the probability that the principal and the agent must trade. The exclusivity variable e denotes the probability that

[3]For example, Aghion and Hermalin (1990) use a contract signalling model to show that imposing legal restrictions on private contracts can enhance efficiency. Spier (1992) identifies a reason for contractual incompleteness by showing that an informed principal can signal information by deliberately proposing an incomplete contract to an agent. Nosal (2006) considers a situation of contract signalling when studying the incentives of a principal to acquire private information before contracting with an agent.

[4]Trade can be interpreted here as a transaction of a good or, for example, as the joint implementation of a project.

the agreement is exclusive; i.e., that the principal cannot trade with an external party.[5] When exclusivity is contractible, $E = [0, 1]$. Noncontractible exclusivity is modeled by imposing $E = \{0\}$.

After the principal and agent agree on the contract, the agent makes an investment $a \in \mathbb{R}_0^+$, which affects the agent's value of trade with the principal. This value is given by $v_A(a)$, where $v_A' > 0$. The cost for the agent of investing a is $\psi(a)$, where $\psi' > 0$. The principal values trade with the agent as v_P. The agent's payoff is additive in the investment cost. Both parties' payoffs are quasi-linear in money. Thus, in addition to any money transfers (and investment costs), if the principal and agent trade with each other, they obtain values of v_P and $v_A(a)$, respectively. For future convenience, the value of trade between the principal and agent is denoted $V(a) \equiv v_P + v_A(a)$.[6]

After the agent invests, the uncertainty about V_E is resolved. The principal and the agent observe the value of V_E and, if the initial contract prescribes an inefficient level of trade, they have the opportunity to renegotiate trade to the efficient level. As in Edlin and Reichelstein (1996), Che and Hausch (1999), Segal and Whinston (2000) and Segal and Whinston (2002), it is assumed that the bargaining shares of the principal and the agent during renegotiation are exogenously specified.[7] It is also assumed that the external party with whom the principal can alternatively trade receives no surplus. This would be consistent, for instance, with a case of competition among many external parties who are willing to trade with the principal in the event she does not trade with the agent. More specifically, at the renegotiation stage, the principal and agent receive each one half of the renegotiation surplus in addition to their disagreement payoffs. The disagreement payoffs of the principal and agent are the payoffs in the event they do not reach a renegotiation agreement and the initial contract is executed.[8] Given

[5] The quantity and exclusivity variables can be interpreted as proportions of trade capacity. Under this interpretation, quantity q represents the proportion of the trade capacity of the principal that is contractually allocated to the agent, and exclusivity e represents the proportion of the remaining $(1-q)$ of the trade capacity of the principal that cannot be traded with an external party. The assumption that e is a proportion is not crucial. All the results in the paper hold if contracts can only prescribe full exclusivity ($e = 1$) or full non-exclusivity ($e = 0$).

[6] For example, if the principal is a buyer, the agent is a supplier with production cost $c(a)$, and the buyer needs at most one unit of the seller's product, then v_P corresponds to the buyer's valuation of the seller's product and $v_A(a) = -c(a)$. The value created if the buyer and the seller trade is $V(a) = v_P + v_A(a) = v_P - c(a)$.

[7] An implicit assumption in the model is that the agent gains some bargaining power during the relationship. This corresponds to situations where by investing in preparation for trade or by direct contact with the principal, the agent learns more about the principal (e.g., about technology employed, financial position, negotiation strategies) leaving him in a better position in future negotiations.

[8] The assumption that the principal and the agent have equal bargaining shares at the renegotiation stage is not crucial. All the results remain unchanged if instead of $1/2$ we consider that the agent's bargaining share is $\lambda \in (0, 1)$. The important assumption is that the agent has some (strictly positive) bargaining power at the

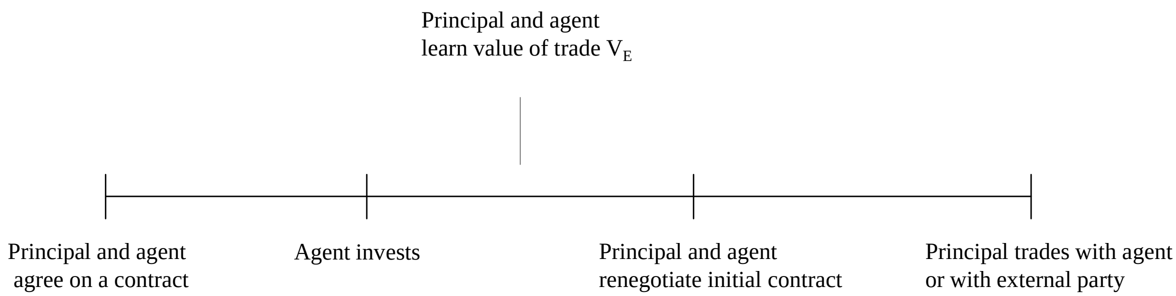

Figure 1: The sequence of events.

contract $c = (t, q, e)$, the disagreement payoffs (ignoring sunk investment costs) are $qv_A(a) - t$ for the agent and $qv_P + (1-q)(1-e)V_E + t$ for the principal. The renegotiation surplus is the difference between the efficient total surplus and the sum of the disagreement payoffs. Since the efficient total surplus (also ignoring sunk investment costs) is $\max\{V(a), V_E\}$, the agent's post-renegotiation payoff given initial contract $c = (t, q, e)$, investment a, and trade valuation V_E is given by

$$\begin{aligned} u^A(c; a, V_E) &= qv_A(a) - t + \frac{1}{2}[\max\{V(a), V_E\} - (qv_A(a) - t) - (qv_P + (1-q)(1-e)V_E + t)] - \psi(a) \\ &= \frac{1}{2}\max\{V(a), V_E\} - \frac{1}{2}[q(v_P - v_A(a)) + (1-q)(1-e)V_E] - t - \psi(a). \end{aligned} \tag{1}$$

Similarly, the principal's post-renegotiation payoff can be written as

$$u^P(c; a, V_E) = \frac{1}{2}\max\{V(a), V_E\} + \frac{1}{2}[q(v_P - v_A(a)) + (1-q)(1-e)V_E] + t. \tag{2}$$

Observe that as a result of renegotiation, the principal and the agent may not trade even if they initially agreed on a contract. However, despite renegotiation, the original contract still matters because it affects the distribution of *ex-post* surplus, which in turn is important for surplus extraction by the principal and investment by the agent. The sequence of events is illustrated in Figure 1.

The *ex-ante* (before uncertainty about V_E is resolved) expected payoffs of the principal and agent, which are relevant at the contracting and investing stages, depend on state j, contract c, and investment a and are given, respectively, by $U_j^P(c; a) = \mathbb{E}[u^P(c; a, V_E) \mid j]$ and

renegotiation stage. Otherwise, his payoff would not depend on the private information of the principal, in which case there is no need for the principal to signal her private information to be able to extract surplus from the agent.

$U_j^A(c;a) = \mathbb{E}[u^A(c;a,V_E) \mid j]$. It follows from direct observation of (1) and (2) that for all $j \in \{L,H\}$, $a \in \mathbb{R}_0^+$ and $c \in C$,

$$U_j^A(c;a) + U_j^P(c;a) = S_j(a), \tag{3}$$

where $S_j(a) \equiv \mathbb{E}[\max\{V(a), V_E\} \mid j] - \psi(a)$ is the efficient total surplus (hereinafter, total surplus) in state j given investment level a. This property of the expected payoffs of the principal and agent is a consequence of efficient renegotiation and will be used in the analysis of the equilibrium outcomes.

The first-best level of investment given state j is given by $a_j^0 \equiv \arg\max_a S_j(a)$. The agent's optimal investment, which depends on the contract c agreed with the principal and his beliefs b_H that $j = H$, is given by $a^*(c, b_H) \equiv \arg\max_a (1-b_H)U_L^A(c;a) + b_H U_H^A(c;a)$. Observe that the agent's beliefs b_H may differ from the prior π_H. The agent may revise his beliefs after observing the menu of contracts proposed by the principal and again after observing the contract chosen by the principal amongst those listed in the menu. It is assumed that $S_j(a)$ and $U_j^A(c;a)$ are concave in a for all $j \in \{L,H\}$, and that both a_j^0 and $a^*(c,b_H)$ are strictly positive. It is also assumed that $S_j(a)$ is differentiable in a and $U_j^A(c;a)$ is twice continuously differentiable in a for all $j \in \{L,H\}$.[9]

In the analysis that follows, the equilibrium concept used is the Perfect Bayesian Equilibrium (PBE).[10]

2.1 A Special Case

I present here a special case of the model which will be helpful to illustrate some of the results obtained. Let the agent's value of trade with the principal and cost of investment be, respectively, $v_A(a) = \phi a$ and $\psi(a) = a^2/2$, where $\phi > 0$. Let the principal's value of trade with the external party V_E take two values only: $v_P - \mu$ and $v_P + \mu$, where $\mu > 0$. Also let the state of the world affect the distribution of V_E in the following way: in state j, $V_E = v_P - \mu$ with probability p_j and $V_E = v_P + \mu$ with probability $1 - p_j$. Consider, in addition, the following parametric assumptions: (i) $\mu < v_P$; (ii) $\mu > \phi^2$; and (iii) $p_H > p_L$. Assumption (i) ensures that V_E is always positive. Assumption (ii) ensures that for all relevant levels of investment,

[9] Concavity of $S_j(a)$ and $U_j^A(c;a)$ in a ensures that $a^*(c,b_H)$ and a_j^0 are unique. Differentiability of $S_j(a)$ and $U_j^A(c;a)$ in a and the fact that $a^*(c,b_H)$ and a_j^0 are strictly positive imply that $a^*(c,b_H)$ and a_j^0 are characterized by the usual first-order conditions. Finally, the fact that $U_j^A(c;a)$ is twice continuously differentiable in a ensures that $a^*(c,b_H)$ changes smoothly with the contractual variables.

[10] See Fudenberg and Tirole (1991) for a precise definition of a PBE.

it is efficient (*ex-post*) for the principal to trade with the external party if $V_E = v_P + \mu$. Note that if $V_E = v_P - \mu$, then necessarily $v_P + v_A(a) > V_E$. Thus, under this realization of V_E, it is efficient *ex-post* for the principal to trade with the agent. If instead $V_E = v_P + \mu$, then whether $v_P + v_A(a) > V_E$ depends on the agent's investment level; in particular, on whether $a > \mu/\phi$. The assumption that $\mu > \phi^2$ ensures that $a_j^0 < \mu/\phi$ for all j and that $a^*(c, b_H) < \mu/\phi$ for all c and b_H. Finally, Assumption (*iii*) implies that the distribution of V_E in state L strictly first-order stochastically dominates that in state H.

3 Preliminary Analysis and the Benchmark Case of Symmetric Information

Before we delve into the analysis of equilibrium contracts and investment, it is useful and instructive to characterize the agent's investment decision and the way the agent's investment affects the principal's expected payoff. It is also instructive to analyze the benchmark case of symmetric information contracting.

The agent's investment decision. Given contract c and belief b_H, the agent chooses investment so as to maximize his expected payoff $(1 - b_H)U_L^A(c; a) + b_H U_H^A(c; a)$. Since

$$U_j^A(c; a) = \frac{1}{2}\mathbb{E}[\max\{V(a), V_E\} \mid j] - \frac{1}{2}[q(v_P - v_A(a)) + (1 - q)(1 - e)\mathbb{E}[V_E \mid j]] - \psi(a) - t, \quad (4)$$

the agent's optimal investment is characterized by the first-order condition

$$v_A'(a)[(1 - b_H)P_L(a) + b_H P_H(a) + q]/2 = \psi'(a), \quad (5)$$

where $P_j(a) = \Pr[v_A(a) + v_P \geq V_E \mid j]$. Observe that $P_j(a)$ is the ex-ante probability that in state j and given investment level a, trade between the principal and the agent is efficient. Because parties agree on an efficient level of trade during ex-post renegotiation, $P_j(a)$ is the ex-ante probability that the principal and the agent trade ex-post. Thus, it can be interpreted as the ex-ante probability of success of the relationship between the principal and the agent.

From (5), one obtains that the agent's investment decision depends on contract c only through quantity q. Thus, henceforth I use $a^*(c, b_H)$ and $a^*(q, b_H)$ interchangeably. Moreover, since $P_H(a) \geq P_L(a)$ for all a, the agent's investment level increases with b_H. That is, the more the agent believes that the trade relationship with the principal will be successful, the more he is willing to invest in it. Finally, note that the agent's investment decision increases with the contracted quantity q. When contracted quantity is high, the agent's disagreement payoff at the renegotiation stage, $qv_A(a) - t$, is very sensitive to his value of trade with the principal $v_A(a)$. Therefore, his incentives to invest are also high in order to protect his disagreement payoff. I state without further proof these results in the following Lemma.

Lemma 1 *The agent's investment decision $a^*(q, b_H)$ is increasing with contracted quantity q and with the agent's belief b_H.*

Agent's investment and principal's payoff. Although the agent's investment does not affect the principal's value of trade with the agent v_P, it affects her expected payoff. Notice that

$$U_j^P(c; a) = \frac{1}{2}\mathbb{E}[\max\{V(a), V_E\} \mid j] + \frac{1}{2}[q(v_P - v_A(a)) + (1-q)(1-e)\mathbb{E}[V_E \mid j]] + t. \tag{6}$$

A higher investment by the agent then affects the principal's expected payoff through two channels: by increasing the total surplus (first term in (6)), which is a positive effect, and by increasing the agent's disagreement payoff ($-qv_A(a)$ in the second term of (6)), which is a negative effect on the principal's expected payoff. Differentiating $U_j^P(c; a)$ with respect to a, we obtain

$$v_A'(a)[P_j(a) - q]/2. \tag{7}$$

Therefore, which of the effects is the dominant one depends on the relative values of the contracted quantity and probability of success of the relationship. In particular, when contracted quantity is zero, the "total surplus effect" dominates, and therefore the principal's payoff is increasing with investment. When contracted quantity equals one (the maximum value it can take), the reverse occurs. Finally, note from (7) and the fact that $P_H(a) \geq P_L(a)$ for all a, that the principal's payoff responds more positively to agent's investment when $j = H$, i.e. when the probability of success of the relationship is high. For future convenience, I state without further proof this result in the following Lemma.

Lemma 2 *For all a_1, $a_2 \in \mathbb{R}_0^+$ such that $a_2 \geq a_1$ and for all $c \in C$, $U_H^P(c; a_2) - U_H^P(c; a_1) \geq U_L^P(c; a_2) - U_L^P(c; a_1)$.*

The benchmark case of symmetric information contracting. Suppose that both the principal and the agent know the state j at the contracting and investment stages. In this case, the principal's problem consists of choosing the contract that maximizes her expected payoff taking into account the individual rationality constraint and the investment decision of the agent, i.e., given state j the principal solves

$$\begin{aligned} &\max_{c,a} U_j^P(c;a) \\ &\text{s.t.} \quad (i) \quad U_j^A(c;a) \geq 0 \\ &\qquad\quad (ii) \quad a \in \arg\max_{a'} U_j^A(c;a') \end{aligned}$$

Because U_j^P and U_j^A are quasilinear in t, constraint (i) must bind in any solution to this problem. Moreover, since the sum of the expected payoffs of the principal and agent is necessarily identical to the total surplus (i.e., (3) must hold), the principal's problem can be rewritten as $\max_{q,e,a} S_j(a)$ s.t. $a \in \arg\max_{a'} U_j^A(0,q,e;a')$. Hence, the principal always proposes the contract that induces the agent to invest as efficiently as possible and uses the transfer t to extract all the surplus from the agent.

In the case of symmetric information, the first-best level of investment is always implementable. Because $S_j(a) = \mathbb{E}[\max\{V(a), V_E\} \mid j] - \psi(a)$ (and by assumption $S_j(a)$ is differentiable in a and $a_j^0 > 0$), a_j^0 satisfies the first-order condition

$$v_A'(a_j^0)P_j(a_j^0) = \psi'(a_j^0). \tag{8}$$

From the first-order condition that characterizes the agent's investment decision (see (5)), we obtain that the agent's investment when he knows that the state is j necessarily satisfies

$$v_A'(a)[P_j(a) + q]/2 = \psi'(a). \tag{9}$$

Comparing (8) and (9), we obtain that the principal can induce the agent to invest the first-best level of investment in state j by choosing $q = P_j(a_j^0) \equiv q_j^0$, i.e., by setting in state j the contracted quantity equal to the probability of success of the relationship evaluated at the first-best investment a_j^0. Thus, when information is symmetric, investment is efficient in equilibrium (first-best) and the principal receives the first-best total expected surplus $S_j(a_j^0)$, for all $j \in \{L, H\}$. Contractibility of exclusivity is not needed for this result. For future convenience, let c_j^0 denote the contract chosen by the principal in state j when information is symmetric and exclusivity is not contractible. Hence, $c_j^0 = (t_j^0, q_j^0)$ where t_j^0 is such that $U_j^A(c_j^0; a_j^0) = 0$.

I next come back to the case of the principal with private information at the contracting and investment stages.

4 Contractual Signalling and Relationship-Specific Investment

In this section, I characterize equilibrium contracting between the principal and the agent and equilibrium agent's investment. As a consequence of *ex-post* renegotiation, trade is always efficient. This is because the levels of trade and exclusivity prescribed in the initial contract can always be changed (without cost) to their efficient levels after uncertainty about V_E has vanished. In contrast, the agent's investment decision is irreversible at the renegotiation stage. Hence, efficiency of investment is not ensured by renegotiation. The literature on the hold-up problem with symmetric information at the contracting stage and the benchmark case studied above show that the inefficiency of investment can be resolved (or mitigated) if parties choose a contract that provides them with the right incentives to invest. In the present setting, because of asymmetry of information, the principal uses the contract not only to provide incentives to invest, but also to signal information to the agent in order to extract surplus. As we shall see below, these two roles of contracting can conflict with one another, and contractibility of exclusivity may play an important role in solving this conflict.

4.1 Quantity Contracts

Let us focus first on the case of quantity contracts. Suppose that $E = \{0\}$, meaning that exclusivity is not contractible. Thus, a contract is a transfer-quantity pair, i.e., $c = (t, q)$. These contracts are often referred to as specific performance contracts. The main purpose here is to show that the investment level may be inefficient in equilibrium when the principal proposes this type of contracts. The main result appears in Proposition 2 at the end of the subsection. I follow a procedure similar to that of Maskin and Tirole (1992). I first define and characterize a specific type of allocation—the *best separating allocation*.[11] I then use it to characterize equilibria.

Definition 1 *A menu of contracts $\{\widehat{c}_L, \widehat{c}_H\}$ constitutes the best separating allocation if and only if, for all $j \in \{L, H\}$,*

[11] This is the counterpart of the RSW allocation in Maskin and Tirole (1992) in the present setting with noncontractible relationship-specific investments.

$$U_j^P(\widehat{c}_j; a^*(\widehat{c}_j, \beta_j)) = \max_{\{c_L, c_H\}} U_j^P(c_j; a^*(c_j, \beta_j)) \tag{10}$$

$$\begin{array}{llll} s.t. & (i) & U_L^P(c_L; a^*(c_L,0)) \geq U_L^P(c_H; a^*(c_H,1)) & (IC_L) \\ & (ii) & U_H^P(c_H; a^*(c_H,1)) \geq U_H^P(c_L; a^*(c_L,0)) & (IC_H) \\ & (iii) & U_r^A(c_r; a^*(c_r,\beta_r)) \geq 0,\ r = L, H & (IR_r) \end{array}$$

where $\beta_L = 0$ *and* $\beta_H = 1$.

The best separating allocation is obtained by performing two independent maximizations, one for the principal of type L (to obtain $\widehat{c}_L$) and one for the principal of type H (to obtain $\widehat{c}_H$). In each, the principal maximizes her payoff within the set of menus that are incentive compatible for the principal, and regardless of the principal's type, yield the agent a non-negative payoff. Note two things. First, incentive compatibility depends on the agent's investment decisions following the principal's choice of contract c in $\{c_L, c_H\}$, which in turn depends on the agent's beliefs. In the definition of the best separating allocation we implicitly assume that the agent's beliefs are: $b_H = 0$ after observing contract choice c_L and $b_H = 1$ after observing contract choice c_H (hereinafter, *separating beliefs*). Second, a best separating allocation is itself incentive compatible given these separating beliefs.[12] These two facts have two implications. First, although obtaining the best separating allocation involves performing two independent maximizations, the best separating allocation $\{\widehat{c}_L, \widehat{c}_H\}$ solves the problem presented in (10) for both $j = L$ and $j = H$. Second, following the proposal of the best separating allocation by the principal, there is always a *continuation* equilibrium in which the agent accepts the proposal and the principal of type j chooses contract $\widehat{c}_j$ from $\{\widehat{c}_L, \widehat{c}_H\}$, for all $j \in \{L, H\}$. These two properties will be used below to obtain the best separating allocation and to characterize equilibrium contracting and investment.

In the rest of the paper, I impose the following condition. That is, I focus on the cases in which it holds.

Condition 1 *The agent's expected payoff* $(1-b_H)U_L^A(c; a^*(c, b_H)) + b_H U_H^A(c; a^*(c, b_H))$ *increases with* b_H *when* c *specifies quantity* q_L^0.

Condition 1 is a condition on the agent's preferences regarding the principal's type. It says that when the contract specifies quantity q_L^0, the agent prefers that the principal is of type H

[12] For a formal statement and proof of this result see Proposition 4 in the Appendix.

rather than of type L.[13] In our model, whether the agent prefers a principal of type L or of type H depends on the contract under consideration. Indeed, when only quantity is contractible, the agent prefers a principal of type H (with a low expected value of trade with the external party) to a principal of type L (with a high expected value of trade with the external party) if the contract specifies a sufficiently low quantity. The opposite occurs if the contract specifies a sufficiently high quantity. The intuition is as follows. An increase in the principal's value of trade with the external party has two effects on the agent's payoff. First, it contributes to increase the agent's payoff because the agent is able to appropriate some of that extra value when renegotiating the initial contract. Second, it contributes to decrease the agent's payoff: as the principal's value of trade with the external party increases, the disagreement payoff of the principal increases; this improves the principal's position during renegotiation of the initial contract allowing the principal to appropriate more of the surplus. Contracted quantity affects the magnitude of this second effect. When the contract prescribes a sufficiently low quantity, the principal commits to trade only a small quantity with the agent. Thus, even if the contract is enforced, the principal is free to trade a high quantity with an external party, implying that the principal's disagreement payoff increases significantly with the principal's value of trade with the external party. In this case, the second effect is dominant and the agent prefers a principal with a lower value of trade with the external party (principal of type H). When contracted quantity is high, the opposite occurs. The second effect becomes muted or negligible. In that case, the agent prefers a principal with a higher value of trade with the external party (principal of type L).

Whether Condition 1 is satisfied depends both on the specific values (or distribution) of the trade valuations and on the specifications of the agent's investment. Consider, for example, the special case of the model presented in Section 2.1 where V_E can take two values ($v_P - \mu$ with probability p_j and $v_P + \mu$ with probability $1 - p_j$, in state j), $v_A(a) = \phi a$ and $\psi(a) = a^2/2$. In this case, $q_L^0 = p_L$. Condition 1 is satisfied if and only if $p_L \leq \mu/(2\mu - \phi^2)$. This condition is always satisfied when for example $p_L \leq 1/2$. Condition 1 allows us to concentrate our attention on the payoffs in state H. As we will see below, it has two implications. First, the payoff of the principal in state L associated with the best separating allocation is the first-best total surplus $S_L(a_L^0)$. Second, in state L the principal can ensure herself at least $S_L(a_L^0)$, regardless of the agent's beliefs. I return to this issue at the end of Section 4.2, where I briefly discuss how the

[13] A similar assumption is imposed in Maskin and Tirole (1992) who, in their analysis of the principal-agent relationship with an informed principal, assume that the agent prefers that the principal is of a higher type for almost all contracts (see the second paragraph of Section 2 of Maskin and Tirole, 1992).

results may change when Condition 1 does not hold.

I now characterize the best separating allocation and the payoff of the principal associated with it. In what follows, let $\widehat{U}_L^P$ and $\widehat{U}_H^P$ denote the principal's payoffs associated with the best separating allocation

Lemma 3 *Contract* $\widehat{c}_L = c_L^0$ *and* $\widehat{U}_L^P = S_L(a_L^0)$*. If* $U_L^P(c_L^0, a_L^0) \geq U_L^P(c_H^0, a_H^0)$*, then* $\widehat{c}_H = c_H^0$ *and* $\widehat{U}_H^P = S_H(a_H^0)$*. Otherwise,* $\widehat{c}_H = (\widehat{t}_H, \overline{q})$ *where* $\overline{q} > q_H^0$ *and* $\widehat{t}_H = U_H^A(0, \overline{q}; a^*(\overline{q}, 1))$*, and* $\widehat{U}_H^P = S_H(a^*(\overline{q}, 1)) < S_H(a_H^0)$*.*

Lemma 3 is proved in the Appendix. Under Condition 1, given separating beliefs, it is always possible to construct an incentive compatible menu of contracts satisfying the agent's individual rationality constraint in both states L and H (i.e., a menu satisfying the constraints of problem (10)), with a contract c_L that leaves the principal of type L with the first-best total surplus $S_L(a_L^0)$. The only contract c_L compatible with that payoff and constraint IR_L is precisely contract c_L^0, which specifies quantity q_L^0 that induces efficient investment by the agent and transfer t_L^0 that allows the principal to extract all the surplus from the agent in state L (i.e., IR_L binds).[14] This is the contract $\widehat{c}_L$ associated with the principal of type L in the best separating allocation.

Consider now the case of the principal of type H. Given the observation made above that the best separating allocation $\{\widehat{c}_L, \widehat{c}_H\}$ must solve (10) for both types of principal, after knowing that $\widehat{c}_L = c_L^0$ we can restrict without loss of generality to menus of the type $\{c_L^0, c_H\}$ when solving (10) for $j = H$. The solution to this (new) problem critically depends on whether contracts c_L^0 and c_H^0 satisfy constraint IC_L, i.e. on whether $U_L^P(c_L^0, a_L^0) \geq U_L^P(c_H^0, a_H^0)$. In the special case of the model presented in Section 2.1, for example, this condition is satisfied if and only if $p_L \leq \mu(2p_H - 1)/\phi^2$.[15]

If $U_L^P(c_L^0, a_L^0) \geq U_L^P(c_H^0, a_H^0)$, contracts c_L^0 and c_H^0, which induce first-best investment and allow full surplus extraction by the principal in both states L and H, are also incentive compatible. So, they constitute the best separating allocation. In this case, the investment level is efficient and the payoff of the principal of type H associated with the best separating allocation is the first-best total surplus $S_H(a_H^0)$.

[14] Observe that, given state L, if the principal's payoff equals the first-best total surplus, then the agent's payoff is non-negative only if investment is efficient, i.e., a_L^0. Moreover, when the agent's beliefs are $b_H = 0$, only a contract specifying quantity $q_L = q_L^0$ induces the agent to choose the first-best investment a_L^0.

[15] Thus, for example, if $p_H < 1/2$ (i.e., if p_H is not too large), then $\mu(2p_H - 1)/\phi^2 < 0$ and this condition will not be satisfied. In contrast, if $p_H \simeq 1$ (i.e., p_H is sufficiently large), then $\mu(2p_H - 1)/\phi^2 > 1$ and this condition necessarily holds. Note that by assumption $\mu > \phi^2$.

However, if $U_L^P(c_L^0, a_L^0) < U_L^P(c_H^0, a_H^0)$, the contracts that induce first-best investment and allow full surplus extraction in both states L and H are *not* incentive compatible. In this case, IC_L (and not IR_H) is the binding constraint of problem (10) for $j = H$ when contract c_H specifies quantity $q_H = q_H^0$. If c_H specifies quantity q_H^0 and satisfies IC_L, the agent will be left with positive surplus. A tension between efficiency in investment and surplus extraction emerges. The optimal contract for the principal of type H is obtained by distorting upward the quantity specified in c_H and adjusting the transfer in a way that leaves the principal of type L indifferent, i.e. IC_L continues to bind. Doing so increases the payoff of the principal of type H because an increase in contracted quantity is less costly (more beneficial) to the principal of type H than to the principal of type L. This is so because of two reasons. First, because it is less costly to the principal of type H (who expects a low value of trade with the external party) to initially commit to trade a higher quantity with the agent than it is to the principal of type L (who expects a high value of trade with the external party). This is a direct effect. Second, because a higher contracted quantity induces more investment by the agent, and the principal of type H values investment by the agent more than the principal of type L. This is an indirect investment effect.

Hence, in this case, the solution to the problem involves a contract c_H with quantity $\overline{q} > q_H^0$. In contrast to the previous case, the outcome associated with the best separating allocation is inefficient in terms of investment: to appropriate more of the surplus generated, the principal of type H sets an excessively high quantity, $\overline{q} > q_H^0$, leading the agent to overinvest in the relationship. This completes the derivation of the best separating allocation. I now proceed to the characterization of equilibrium outcomes.

As argued above, following the proposal of the best separating allocation $\{\widehat{c}_L, \widehat{c}_H\}$ by the principal, there is a continuation equilibrium in which the agent accepts the principal's proposal and then the principal chooses contract $\widehat{c}_L$ if she is of type L and contract $\widehat{c}_H$ if she is of type H. Hence, the remaining question is whether both types of principal proposing $\{\widehat{c}_L, \widehat{c}_H\}$ followed by this separating continuation equilibrium constitutes an equilibrium of the overall game, i.e., whether there exist beliefs and continuation equilibria off-the-equilibrium path such that no type of principal gains by deviating and proposing a menu of contracts different from $\{\widehat{c}_L, \widehat{c}_H\}$. The next proposition, which is proved in the Appendix, clarifies this question. In what follows, let M denote the set of finite menus of contracts.

Proposition 1 *If both types of principal propose a menu $m \in M$, followed by a continuation equilibrium (after menu proposal) in which the principal's payoffs $\widetilde{U}_L^P$ and $\widetilde{U}_H^P$ are such that*

$\widetilde{U}_j^P \geq \widehat{U}_j^P$ *for all* $j \in \{L, H\}$*, then there are off-the-equilibrium path beliefs such that this proposal and continuation equilibrium constitutes an equilibrium outcome of the overall game.*

An implication of Proposition 1 is that both types of principal proposing the best separating allocation, followed by the respective separating continuation equilibrium, always constitutes an equilibrium of the game. This fact, together with Lemma 3, leads directly to the first part of the following proposition.

Proposition 2 *Suppose that quantity is contractible, but not exclusivity (*$E = \{0\}$*). Then, if* $U_L^P(c_L^0, a_L^0) < U_L^P(c_H^0, a_H^0)$ *there exist inefficient equilibria where the agent overinvests relative to the first-best level of investment in state* H*. Moreover, for some specifications of the model, only inefficient equilibria exist.*

Proposition 2 is proved in the Appendix. To illustrate this proposition, consider again the special case of the model presented in Section 2.1. As mentioned above, if $p_L > \mu(2p_H - 1)/\phi^2$ (and, of course, if $p_L \leq \mu/(2\mu - \phi^2)$ so that Condition 1 is satisfied), $U_L^P(c_L^0, a_L^0) < U_L^P(c_H^0, a_H^0)$ and the best separating allocation involves overinvestment by the agent in state H. So, in this case, there exist inefficient equilibria where the agent overinvest in state H. In these equilibria, the principal of type H offers to the agent a contract where she commits to trade with the agent an excessively high quantity. The principal does so to signal a low outside option. By signalling this information, the principal convinces the agent to accept a contract that allows her to appropriate more of the surplus generated by their relationship. But given such a contract, the agent overinvests in the relationship to improve his default position (payoff if initial contract is enforced) in the event of a renegotiation. Interestingly, when the (prior) probability π_H that the principal is of type H is sufficiently high, these equilibria coexist with efficient equilibria where the agent invests the first-best level of investment. However, for lower values of π_H, there are parameter values for which no efficient equilibria in terms of investment exist. This is the case, for example, when $p_L = p_H/2$, $0.4 < p_H \leq 0.5$, and π_H is sufficiently low.

Proposition 2 is important not only because of the specific efficiency implications that it has, but also because it emphasizes that surplus extraction and efficiency of investment can in fact conflict with one another when parties contract under asymmetric information. I next allow exclusivity to be contractible and show it has an important role in mitigating this conflict.

4.2 Quantity and Exclusivity Contracts

Suppose now that both quantity and exclusivity are contractible, i.e., $E = [0, 1]$. In this case, a contract is a triple $c = (t, q, e)$. To characterize the equilibrium allocations and payoffs, I start by presenting in Lemma 4 lower bounds for the principal's equilibrium payoffs. Then, in Proposition 3, I present the equilibrium payoffs themselves and characterize equilibrium investments. The following Lemma is proved in the Appendix.

Lemma 4 *Suppose that quantity and exclusivity are contractible ($E = [0, 1]$). Then, in any equilibrium, the payoff of the principal of type j is at least the first-best expected total surplus $S_j(a_j^0)$, for all $j \in \{L, H\}$.*

Contractibility of exclusivity plays no role in ensuring to the principal of type L the first-best total surplus $S_L(a_L^0)$. Under Condition 1, the principal of type L always achieves this payoff even if exclusivity is not contractible. In contrast, in the case of the principal of type H, it is the fact that exclusivity is contractible that allows the principal to construct a contract that guarantees her the first-best total surplus $S_H(a_H^0)$.

To illustrate the role of exclusivity when the principal is of type H, consider the expected payoff of the agent given contract $(t = 0, q, e)$, state j and investment a. This payoff can be written as

$$U_j^A(0, q, e; a) = \frac{1}{2}\mathbb{E}[\max\{V(a), V_E\} \mid j] - \frac{1}{2}[q(v_P - v_A(a)) + (1 - q)(1 - e)\mathbb{E}[V_E \mid j]] - \psi(a).$$

When the contract prescribes full exclusivity, i.e., $e = 1$, the agent's expected payoff is affected by state j only through the term $\mathbb{E}[\max\{V(a), V_E\} \mid j]/2$. Therefore, from the fact that F_L first-order stochastically dominates F_H, it follows that for all a and q

$$U_L^A(0, q, e = 1; a) \geq U_H^A(0, q, e = 1; a). \tag{11}$$

Intuitively, when the principal promises full exclusivity, the agent is better off when the principal has a high outside option (state L), since he can appropriate part of it at the renegotiation stage by threatening to enforce the contract and prevent the principal from trading with the external party.

From (11), it follows that the agent's expected payoff $(1 - b_H)U_L^A(c; a) + b_H U_H^A(c; a)$ when $a = a^*(c, b_H)$ and $c = (0, q, e = 1)$ is decreasing in his belief b_H, for any given quantity q in c. In particular, this holds for $q = q_H^0$. This implies that regardless of the agent's beliefs, he always accepts contract $(t, q_H^0, e = 1)$ in which $t = U_H^A(0, q_H^0, 1; a^*(q_H^0, 1))$, i.e., his expected payoff when his beliefs are $b_H = 1$ (his worst possible payoff across beliefs). Hence, exclusivity allows the principal to construct a contract in which the agent is better off in state L than in state H. This is also possible when exclusivity is not contractible if the principal sets a sufficiently high quantity in the contract. The problem in doing so, however, is that she distorts the agent's investment decision.

I now turn to the question of equilibrium payoffs and investments.

Proposition 3 *Suppose that quantity and exclusivity are contractible ($E = [0, 1]$). Then, in any equilibrium, investment levels are efficient (first-best in both states) and the principal always appropriates the first-best total surplus, i.e., the equilibrium payoff of the principal in state j is $S_j(a_j^0)$ for all $j \in \{L, H\}$.*

Proposition 3, which is proved in the Appendix, establishes that in any equilibrium when the principal can contractually use both quantity and exclusivity, the investment levels are efficient in both states L and H. To illustrate why efficiency is always obtained when both quantity and exclusivity are contractible (as opposed to the case when only quantity is contractible), consider the characterization of the best separating allocation presented in the previous section when $U_L^P(c_L^0, a_L^0) < U_L^P(c_H^0, a_H^0)$ and, therefore, IC_L is the binding constraint of problem (10) for the principal of type H. Recall that, in that case, investment is inefficient because the principal sets an excessively high quantity ($\overline{q} > q_H^0$) in order to signal information and extract more surplus from the agent. When exclusivity is contractible, instead of increasing quantity above q_H^0, which induces the agent to overinvest in the relationship, the principal can set quantity q_H^0 and use (increase) exclusivity to move along the IC_L constraint, signal her type, and extract surplus from the agent. Surplus extraction can be achieved in this way because exclusivity signals a low expected value of trade with the external party and does not directly affect the agent's investment decision, implying that signalling information through exclusivity does not interfere with provision of investment incentives.

The preceding analysis shows that contractibility of exclusivity has important efficiency implications. I conclude this section with a brief discussion of the robustness of the results obtained. The analysis has focused on the cases in which Condition 1 holds. However, the basic mechanism through which contractibility of exclusivity affects equilibrium and efficiency – allowing the principal to signal information about the value of trade with the external party – is present even when Condition 1 is not satisfied. Note, for example, that contractibility of exclusivity allows the principal to appropriate the entire efficient total surplus in state H

irrespective of Condition 1 being satisfied. More generally, it allows the principal to design contracts that more easily satisfy the incentive compatibility constraints in the definition of the best separating allocation – see (10).[16] Another important assumption is that on the structure of the principal's private information. It is assumed that in state L the distribution of the value of trade of the principal with an external party dominates in a first-order stochastic sense the distribution of that valuation in state H. Again, the basic mechanism through which contractibility of exclusivity affects equilibrium and efficiency is present if, for example, we consider instead the weaker assumption that the expected value of the principal's trade with the external party is higher in state L than in state H. Exclusivity signals low outside option in this case because, as under the assumption of first-order-stochastic dominance, it is more costly to a principal who expects a high value of trade with an external party to commit initially to trade exclusively with an agent than it is to a principal who expects a low value of trade with an external party. The above discussion suggests that the role of exclusivity highlighted in the paper holds even when not making those assumptions. However, obtaining formal results without them becomes difficult.

5 Conclusion

The literature on contractual solutions to the hold-up problem has focused on situations where the parties to the contract have symmetric information when contracting about future transactions. In this paper, I depart from this literature by examining a situation in which the party that designs the contract has relevant private information at the contracting phase. I show that because of information concerns, the contract designer may distort the contract's terms relative to those that induce efficient investment in order to signal information and appropriate more of the surplus generated. I also show that the ability to include exclusive clauses in the contract may play an important role in eliminating these distortions and, consequently, the inefficiency of investment.

Regarding the literature on the effect of exclusive contracts on relationship-specific investment, the analysis in this paper complements that in Segal and Whinston (2000) and De Meza and Selvaggi (2007) where information is symmetric at the contracting stage. Following a cooperative approach to model renegotiation, Segal and Whinston (2000) show that renegotiable exclusivity contracts have no effect on relationship-specific investment. De Meza and Selvaggi

[16] In particular, the principal can achieve this by proposing contracts with a higher exclusivity level when the principal expects a low value of trade with external parties.

(2007) point out that if the bargaining solution to renegotiation is non-cooperative, exclusivity may affect relationship-specific investments. The present paper contributes to this literature by providing a novel channel through which exclusive agreements affect relationship-specific investments. Concretely, because exclusivity signals information, it helps to mitigate the conflict between signalling information and providing incentives to invest that is present when parties contract under asymmetric information.

On a more practical level, this paper offers an explanation as to why contracts often specify simultaneously both a quantity to be traded in the future and an exclusivity clause. It also offers an explanation as to why firms voluntarily bind themselves by committing to trade exclusively with another firm. More specifically, the results in the paper suggest that asymmetry of information about external trade values may be an important driver of the use of exclusivity contracts. This is an implication that, in principle, can be empirically identified. Perhaps one way to do so is to analyze whether exclusivity is more likely to emerge in situations where parties are contracting about transactions for the first time than in situations where parties have traded regularly in the past, as in the former cases asymmetry of information is more likely than in the latter cases. The effect of exclusivity that I highlight here may also be important for policy design. The major (and unsettled) debate on that front is on whether exclusive agreements serve anticompetitive purposes and, as a consequence, on whether they should be contractually allowed. By showing that contractibility of exclusivity may enhance efficiency of investment, this paper suggests that a policy that systematically prohibits exclusive contacts may be misguided.

Finally, the analysis is this paper has focused on the case in which the principal has private information about her value of trade with an external party. However, it can be easily extended to the case where the principal's private information is about her value of trade with the agent (v_P in the model). Interestingly, all the results in Section 4.1 continue to hold in this case, meaning that in some cases inefficient equilibria in terms of investment exist if the principal and agent use quantity contracts only. However, in this case the use of exclusivity clauses has *no* effect on the efficiency of investment. In fact, it is totally irrelevant as it does not affect the set of equilibrium outcomes relative to the case where only quantity contracts are used.[17]

[17] A formal analysis of this case can be found in Vasconcelos (2008). A distinction has been made in the contract-theory literature between public and private actions (see Watson, 2007, and Buzard and Watson, 2012). As shown by Watson (2007), this distinction is important because, in some cases, considering trade actions as public instead of as individual actions, restricts implementation. While in this paper trade actions are modeled as public actions this irrelevance of exclusivity continues to hold even if the principal's action to trade with the external party is modeled as an individual action.

This observation allows us to establish an interesting parallelism between the results in Segal and Whinston (2000) and those in the present paper. In Segal and Whinston (2000), exclusivity has no effect on relationship-specific investments. Exclusivity affects only investments by the contracting parties that have an impact on the value of trade with external parties. Similarly, the results in the present paper suggest that contractibility of exclusivity affects equilibrium outcomes and investment only if the private information of the contracting parties concerns the value of trade with an external party.

Appendix

This appendix is organized as follows. I start by formally stating and proving the claim made in the text that a best separating allocation is incentive compatible for the principal given separating beliefs. This is done in Proposition 4 and its proof. I then state and prove two new lemmas, Lemma 5 and Lemma 6, which are used in the proofs of the lemmas and propositions in the text. Finally, I prove the lemmas and propositions in the text, with the exception of Lemma 1 and Lemma 2 which, as mentioned in the text, are stated without further proof.

Proposition 4 *A best separating allocation $\{\widehat{c}_L, \widehat{c}_H\}$ is incentive compatible for the principal given the separating beliefs.*

Proof. I only show here that $U_H^P(\widehat{c}_H; a^*(\widehat{c}_H, 1)) \geq U_H^P(\widehat{c}_L; a^*(\widehat{c}_L, 0))$. The proof that $U_L^P(\widehat{c}_L; a^*(\widehat{c}_L, 0)) \geq U_L^P(\widehat{c}_H; a^*(\widehat{c}_H, 1))$ is perfectly analogous and therefore omitted. Let $\{\widehat{c}_L, \widehat{c}_H\}$ be a solution to problem (10) for $j = L$. First, because $\{\widehat{c}_L, \overline{c}_H\}$ must satisfy the IC_H constraint in the problem, we obtain that $U_H^P(\overline{c}_H; a^*(\overline{c}_H, 1)) \geq U_H^P(\widehat{c}_L; a^*(\widehat{c}_L, 0))$. Second, because the constraints of problems (10) for $j = L$ and for $j = H$ coincide, $\{\widehat{c}_L, \overline{c}_H\}$ also satisfies all the constraints of problem (10) for $j = H$. Optimality of $\widehat{c}_H$ implies that $U_H^P(\widehat{c}_H; a^*(\widehat{c}_H, 1)) \geq U_H^P(\overline{c}_H; a^*(\overline{c}_H, 1))$. From this inequality and the fact shown above that $U_H^P(\overline{c}_H; a^*(\overline{c}_H, 1)) \geq U_H^P(\widehat{c}_L; a^*(\widehat{c}_L, 0))$, it follows that $U_H^P(\widehat{c}_H; a^*(\widehat{c}_H, 1)) \geq U_H^P(\widehat{c}_L; a^*(\widehat{c}_L, 0))$. ■

Lemma 5 $U_L^A(t, q = 1, e = 0; a) \geq U_H^A(t, q = 1, e = 0; a)$ *for all* $t \in \mathbb{R}$ *and all* $a \in \mathbb{R}_0^+$.

Proof. By taking expectations of (1), we obtain that $U_j^A(t, 1, 0; a) = \mathbb{E}[\max\{V(a), V_E\} \mid j]/2 - [v_P - v_A(a)]/2 - \psi(a) - t$. Clearly, $U_L^A(t, q = 1, e = 0; a) \geq U_H^A(t, q = 1, e = 0; a)$ if and only if $\mathbb{E}[\max\{V(a), V_E\} \mid L] \geq \mathbb{E}[\max\{V(a), V_E\} \mid H]$, which follows directly from the fact that $\max\{V(a), V_E\}$ is an increasing function of V_E and F_L first-order stochastically dominates F_H. ■

Lemma 6 *Suppose that IC_L is the binding constraint of problem (12) (presented in the Proof of Lemma 3 below) when $q = q_H^0$. Then there exists $\overline{q} \in [q_H^0, 1]$ such that IC_L is the binding constraint for $q_H \in [q_H^0, \overline{q}]$ and IR_H is the binding constraint for $q_H \in [\overline{q}, 1]$. Moreover, If a contract $c = (t, q)$ satisfies simultaneously $U_L^P(c; a^*(c, 0)) \leq S_L(a_L^0)$ and $U_H^P(c; a^*(c, 0)) > S_H(a^*(\overline{q}, 1))$ then $q > \overline{q}$.*

Proof. I start by showing the first part of the lemma. Constraints IC_L and IR_H of problem (12) can be written as $t_H \leq S_L(a_L^0) - U_L^P(0, q_H; a^*(q_H, 1))$ and $t_H \leq U_H^A(0, q_H; a^*(q_H, 1))$, respectively. The right-hand side of both inequalities is continuous in q_H. Moreover, $U_H^A(0, q_H; a^*(q_H, 1)) > S_L(a_L^0) - U_L^P(0, q_H; a^*(q_H, 1))$ when $q_H = q_H^0$, since by assumption IC_L is the binding constraint when $q_H = q_H^0$. Hence, it suffices to show that $U_H^A(0, q_H; a^*(q_H, 1)) \leq S_L(a_L^0) - U_L^P(0, q_H; a^*(q_H, 1))$ when $q_H = 1$ and that $U_H^A(0, q_H; a^*(q_H, 1)) - [S_L(a_L^0) - U_L^P(0, q_H; a^*(q_H, 1))]$ decreases with q_H in $[q_H^0, 1]$. To obtain the former condition, simply note that $U_H^A(0, 1; a^*(1, 1)) \leq U_L^A(0, 1; a^*(1, 1)) \leq S_L(a_L^0) - U_L^P(0, 1; a^*(1, 1))$, where: (i) the first inequality follows directly from Lemma 5 (in this appendix); and (ii) the second inequality follows from (3) and the fact that $a_L^0 = \arg\max_a S_L(a)$. I next show the latter condition. Using (1) and (2) and the fact that $S_j(a^*(q_H, 1)) = \mathbb{E}[\max\{V(a^*(q_H, 0), V_E)\} \mid j] - \psi(a^*(q_H, 0))$, we obtain that $U_H^A(0, q_H; a^*(q_H, 1)) - [S_L(a_L^0) - U_L^P(0, q_H; a^*(q_H, 1))] = -S_L(a_L^0) + \frac{1}{2} S_L(a^*(q_H, 1)) + \frac{1}{2} S_H(a^*(q_H, 1))] + \frac{1}{2}(1 - q_H)(\mathbb{E}[V_E \mid L] - \mathbb{E}[V_E \mid H])$. First, note that both $S_L(a^*(q_H, 1))$ and $S_H(a^*(q_H, 1))$ decrease with q_H when $q_H \geq q_H^0$, since: (i) $a^*(q, 1)$ is increasing in q (Lemma 1); (ii) $a^*(q, 1) \geq a_j^0$ $\forall j \in \{L, H\}$ when $q \geq q_H^0$; and (iii) S_j is concave. Finally, note that $(1 - q_H)(\mathbb{E}[V_E \mid L] - \mathbb{E}[V_E \mid H])$ decreases with q_H, since the fact that F_L strictly first-order stochastically dominates F_H implies that $\mathbb{E}[V_E \mid L] > \mathbb{E}[V_E \mid H]$.

I next show the second part of the lemma. Given a contract $c = (t, q)$, conditions $U_L^P(c; a^*(c, 0)) \leq S_L(a_L^0)$ and $U_H^P(c; a^*(c, 0)) > S_H(a^*(\overline{q}, 1))$ are equivalent to $S_H(a^*(\overline{q}, 1)) - U_H^P(0, q; a^*(q, 0)) < t \leq S_L(a_L^0) - U_L^P(0, q; a^*(q, 0))$. Hence, they hold simultaneously only if $S_H(a^*(\overline{q}, 1)) - S_L(a_L^0) < U_H^P(0, q; a^*(q, 0)) - U_L^P(0, q; a^*(q, 0))$. Since by Lemma 2 the right-hand side of this inequality increases with q, it suffices to show that the inequality is not satisfied when $q = \overline{q}$. To obtain this, note that $S_H(a^*(\overline{q}, 1)) - S_L(a_L^0) = U_H^P(0, \overline{q}; a^*(\overline{q}, 1)) - U_L^P(0, \overline{q}; a^*(\overline{q}, 1)) \geq U_H^P(0, \overline{q}; a^*(\overline{q}, 0)) - U_L^P(0, \overline{q}; a^*(\overline{q}, 0))$, where: (i) the first equality follows from the fact that $S_H(a^*(\overline{q}, 1)) = U_H^P(0, \overline{q}; a^*(\overline{q}, 1)) + U_H^A(0, \overline{q}; a^*(\overline{q}, 1))$ and the fact that IC_L is the binding constraint of problem (12) when $q_H = q_H^0$, i.e., $S_L(a_L^0) = U_L^P(0, \overline{q}; a^*(\overline{q}, 1)) + U_H^A(0, \overline{q}; a^*(\overline{q}, 1))$; and (ii) the inequality follows from the fact that $U_H^P(0, \overline{q}; a^*(\overline{q}, b_H)) - U_L^P(0, \overline{q}; a^*(\overline{q}, b_H))$ increases with b_H, which is an implication of Lemma 2. ■

Proof of Lemma 3. The proof is given by the following steps.

Step 1: $\widehat{c}_L = c_L^0$.

This is proved by showing that the menu of contracts $\{c_L^0, \overline{c}_H = (\overline{t}_H, q_L^0)\}$, where $\overline{t}_H$ is such that $U_H^P(\overline{c}_H; a^*(q_L^0, 1)) = U_H^P(c_L^0; a^*(q_L^0, 0))$, constitutes a solution to problem (10) for $j = L$. By construction, $\{c_L^0, \overline{c}_H\}$ satisfies IR_L and IC_H. From: (i) Lemma 2, (ii) the fact that $a^*(q_L^0, 1) > a^*(q_L^0, 0)$, and (iii) quasilinearity of U_j^P in t for all $j \in \{L, H\}$; it follows that $U_L^P(\overline{c}_H; a^*(q_L^0, 1)) - U_L^P(c_L^0; a^*(q_L^0, 0)) \leq U_H^P(\overline{c}_H; a^*(q_L^0, 1)) - U_H^P(c_L^0; a^*(q_L^0, 0)) = 0$. Thus, $\{c_L^0, \overline{c}_H\}$ satisfies IC$_L$. Now, note that condition $U_H^P(\overline{c}_H; a^*(q_L^0, 1)) = U_H^P(c_L^0; a^*(q_L^0, 0))$ is equivalent to $t_L^0 - \overline{t}_H = U_H^P(0, q_L^0; a^*(q_L^0, 1)) - U_H^P(0, q_L^0; a^*(q_L^0, 0))$. Since $P_H(a) \geq q_L^0 \; \forall a \in [a^*(q_L^0, 0), a^*(q_L^0, 1)]$, we obtain that $U_H^P(0, q_L^0; a)$ increases with $a, \forall a \in [a^*(q_L^0, 0), a^*(q_L^0, 1)]$ (see (7)), which implies that $t_L^0 - \overline{t}_H > 0$. This, in turn, implies that $U_H^A(\overline{c}_H; a^*(q_L^0, 1)) \geq U_H^A(c_L^0; a^*(q_L^0, 1))$. Moreover, by Condition 1, $U_H^A(c_L^0; a^*(q_L^0, 1)) \geq U_L^A(c_L^0; a^*(q_L^0, 0)) = 0$. Hence, $U_H^A(\overline{c}_H; a^*(q_L^0, 1)) \geq 0$, meaning that $\{c_L^0, \overline{c}_H\}$ also satisfies IR$_H$. Clearly, $\{c_L^0, \overline{c}_H\}$ solves problem (10) for $j = L$, as is satisfies all its constraints and $U_L^P(c_L^0; a^*(q_L^0, 0)) = S_L(a_L^0)$, which is the maximum value that the objective function can take without violating constraint IR$_L$.

Step 2: *If* $U_L^P(c_L^0, a_L^0) \geq U_L^P(c_H^0, a_H^0)$*, then* $\widehat{c}_H = c_H^0$.

If $U_L^P(c_L^0, a_L^0) \geq U_L^P(c_H^0, a_H^0)$, the menu of contracts $\{c_L^0, c_H^0\}$ satisfies all the constraints of problem (10) for $j = H$. (Usually in this type of problem constraint IC$_H$ is not binding. I ignore it in the remainder of this proof. A detailed proof that the solutions derived here satisfy it can be found in Vasconcelos, 2008). Clearly, $\{c_L^0, c_H^0\}$ solves the problem, as $U_H^P(c_H^0; a^*(q_H^0, 1)) = S_H(a_H^0)$, which is the maximum value that the objective function can take without violating constraint IR$_H$.

Step 3: *If* $U_L^P(c_L^0, a_L^0) < U_L^P(c_H^0, a_H^0)$*, then* $\widehat{c}_H = (\widehat{t}_H, \overline{q})$ *where* $\overline{q} > q_H^0$ *and* $\widehat{t}_H = U_A^A(0, \overline{q}; a^*(\overline{q}, 1))$.

Consider problem (10) for $j = H$. Because the best separating allocation $\{\widehat{c}_L, \widehat{c}_H\}$ necessarily solves (10) for both when $j = L$ and when $j = H$ and we already know that $\widehat{c}_L = c_L^0$, we can restrict without loss of generality to menus of the type $\{c_L^0, c_H\}$. Thus, ignoring constraint IC$_H$, solving problem (10) for $j = H$ amounts to solving

$$\begin{aligned} &\max_{t_H, q_H} U_H^P(t_H, q_H; a^*(q_H, 1)) && && (12)\\ &\text{s.t.} \quad (i) \quad S_L(a_L^0) \geq U_L^P(t_H, q_H; a^*(q_H, 1)) && (\mathrm{IC}_L) \\ &\qquad\;\; (ii) \quad U_H^A(t_H, q_H; a^*(q_H, 1)) \geq 0 && (\mathrm{IR}_H) \end{aligned}$$

In any solution to problem (12), at least one of the constraints IC$_L$ or IR$_H$ is binding. If this were not the case, then it would be possible to increase t_H by an arbitrarily small amount $\varepsilon > 0$ and still have all the constraints in the problem satisfied (including IC$_H$) while increasing the objective function, which would be a contradiction. Because $U_L^P(c_L^0, a_L^0) < U_L^P(c_H^0, a_H^0)$, IC$_L$ is the binding constraint of problem (12) when $q_H = q_H^0$. By Lemma 6 (in this appendix), there exists $\overline{q} \in [q_H^0, 1]$ such that IC$_L$ is the binding constraint for $q_H \in [q_H^0, \overline{q}]$ and IR$_H$ is the binding constraint for $q_H \in [\overline{q}, 1]$. Let $\mathcal{U}_H^P(q_H)$ denote the function obtained by replacing t_H

in the objective function of problem (12) with its value obtained from the binding constraint given q_H. That is, $\mathcal{U}_H^P(q_H) = U_H^P(0, q_H; a^*(q_H, 1)) - U_L^P(0, q_H; a^*(q_H, 1)) + S_L(a_L^0)$ if $q_H < \overline{q}$ and $\mathcal{U}_H^P(q_H) = S_H(a^*(q_H, 1))$ if $q_H \in [\overline{q}, 1]$. Solving problem (12) consists of maximizing $\mathcal{U}_H^P$. First, note that $\mathcal{U}_H^P$ is decreasing for $q_H \in [\overline{q}, 1]$. This is because S_H is concave, S_H reaches its maximum value at a_H^0 (which is equal to $a^*(q_H^0, 1)$), and the facts that $a^*(q, 1)$ increases with q and $\overline{q} \geq q_H^0$. Next, note that $\mathcal{U}_H^P$ increases with q_H when $\overline{q} < q_H^0$. To see this, observe that

$$\begin{aligned}\partial\mathcal{U}_H^P(q_H)/\partial q_H &= \partial[U_H^P(0, q_H; a^*(q_H, 1)) - U_L^P(0, q_H; a^*(q_H, 1))]/\partial q_H \\ &\quad +\partial[U_H^P(0, q_H; a^*(q_H, 1)) - U_L^P(0, q_H; a^*(q_H, 1))]/\partial a \times \partial a^*(q_H, 1)/\partial q_H.\end{aligned}$$

The first term is equal to $\{\mathbb{E}[V_E \mid L] - \mathbb{E}[V_E \mid H]\}/2$ and is positive. The second term is equal to $v_A'(a^*(q_H, 1))[P_H(a^*(q_H, 1)) - P_L(a^*(q_H, 1))] \times \partial a^*(q_H, 1)/\partial q_H$ and is non-negative since $\partial a^*(q_H, 1)/\partial q_H \geq 0$ (see Lemma 1) and $P_H(a) \geq P_L(a)$ for all a. Hence, the solution to problem (12) is given by $q_H = \overline{q}$ and the contract of the principal of type H associated with the best separating allocation is $\widehat{c}_H = (\widehat{t}_H, \overline{q})$, where $\widehat{t}_H$ is such that IR_H binds.[18] The payoff of the principal associated with this contract is $S_H(a^*(\overline{q}, 1)) < S_H(a_H^0)$. ■

Proof of Proposition 1. This proof consists of showing that there exist off-the-equilibrium path beliefs and continuation equilibria such that no deviation to another menu m' is profitable. Let $b_H(m')$ and $b_H(c, m')$ denote the agent's beliefs that the principal is of type H after observing, respectively, that the principal proposed a menu $m' \in M$ and that principal chose contract c among those she proposed in m'. Note that the concept of perfect Bayesian equilibrium imposes no restriction on beliefs $b_H(m')$ off-the-equilibrium path. However, even off-the-equilibrium path, it requires: (i) that $b_H(c, m')$ be consistent with beliefs $b_H(m')$ and prescribed equilibrium play by the principal when choosing a contract from m' (*consistency of beliefs*); and (ii) that after the principal's proposal of m', the prescribed equilibrium play by the principal (choice of c in m') and the agent (decision to accept or reject m' and of investment level a) be optimal given beliefs $b_H(m')$ and $b_H(c, m')$ and the other player's strategy (*sequential rationality*). Thus, any beliefs and continuation equilibrium used in this proof must satisfy these requirements.

Suppose that the principal deviates and proposes to the agent a menu $m' \in M$ such that for all beliefs $b_H(m')$ and all continuation equilibria the agent accepts it. (A deviation to a menu that is rejected by the agent is trivially not optimal to the principal.) I next show that there

[18] The solution to problem (12) cannot involve a contract specifying a quantity smaller than q_H^0, since $\text{IC}(\theta_L)$ is also the binding constraint for $q_H < q_H^0$.

are also beliefs $b_H(m')$ and a continuation equilibrium following the proposal of m' in which the payoff to the principal of type j is no larger than $\widehat{U}_j^P$, $\forall j \in \{L, H\}$. This is done in three steps. In the first two, I derive properties that menu m' must satisfy.

Step 1: $U_L^P(c; a^*(c, 0)) \leq \widehat{U}_L^P$ $\forall c \in m'$.

To see this, consider the following beliefs and continuation equilibrium: $b_H(m') = 0$; $b_H(c, m') = 0$ $\forall c \in m'$; each type of principal chooses the contract in m' that maximizes her expected payoff given that $b_H(c, m') = 0$. Since $\widehat{U}_L^P = S_L(a_L^0)$, m' must satisfy $U_L^P(c; a^*(c, 0)) \leq \widehat{U}_L^P$ $\forall c \in m'$, otherwise the payoff to the principal of type L would be strictly larger than $S_L(a_L^0)$, which by (3) (efficient *ex-post* renegotiation) would imply a strictly negative expected payoff to the agent from accepting m'. This would violate the assumption that m' is always accepted by the agent.

Step 2: *Let* $C_H = \{c \in m' : U_H^P(c; a^*(c, 0)) > \widehat{U}_H^P\}$. *If* $C_H \neq \varnothing$, *then* $U_H^P(c; a^*(c, 1)) \leq \widehat{U}_H^P$ $\forall c \in C_H$.

This is shown by contradiction. Suppose to the contrary that $U_H^P(c; a^*(c, 1)) > \widehat{U}_H^P$ for some $c \in C_H$. Consider in this case the following beliefs and continuation equilibrium: $b_H(m') = 1$; $b_H(c, m') = 1$ if $c \in C_H$ and $b_H(c, m') = 0$ if $c \in m' \backslash C_H$; each type of principal chooses the contract from m' that maximizes her payoff given beliefs $b_H(c, m')$. In this continuation equilibrium, the principal of type H chooses a contract in C_H and has payoff $\max\limits_{c \in C_H} U_H^P(c; a^*(c, 1)) > \widehat{U}_H^P$. If $\widehat{U}_H^P = S_H(a_H^0)$ (i.e., $U_L^P(c_L^0, a_L^0) \geq U_L^P(c_H^0, a_H^0)$ and IR_H is the binding constraint in problem (12) when $q_H = q_H^0$), then by (3) (efficient *ex-post* renegotiation) the agent's expected payoff is negative. If $\widehat{U}_H^P = S_H(a^*(\overline{q}, 1))$ (i.e., $U_L^P(c_L^0, a_L^0) < U_L^P(c_H^0, a_H^0)$ and IC_L is the binding constraint in problem (12) when $q_H = q_H^0$), then by Lemma 6 all the contracts in C_H specify a quantity $q > \overline{q}$ and so does the contract chosen by the principal of type H. From the fact that $S_H(a^*(q, 1)) < S_H(a^*(\overline{q}, 1))$ $\forall q > \overline{q}$ and (3) (efficient *ex-post* renegotiation), it follows that also in this case the agent's expected payoff must be negative. Thus, both when $\widehat{U}_H^P = S_H(a_H^0)$ and when $\widehat{U}_H^P = S_H(a^*(\overline{q}, 1))$, the agent's expected payoff is negative. This is a contradiction as m' is by assumption a menu that is accepted by the agent for all beliefs $b_H(m')$ and continuation equilibrium.

Step 3: *There exist beliefs* $b_H(m')$ *and a continuation equilibrium following the proposal of* m' *in which the payoff to the principal of type* j *is no larger than* $\widehat{U}_j^P$, $\forall j \in \{L, H\}$.

There are two possible cases regarding menu m' which I consider separately. Suppose first that $C_H = \varnothing$. This means that $U_H^P(c; a^*(c, 0)) \leq \widehat{U}_H^P$ $\forall c \in m'$. From Step 1, we know that $U_L^P(c; a^*(c, 0)) \leq \widehat{U}_L^P$ $\forall c \in m'$. Hence, $U_j^P(c; a^*(c, 0)) \leq \widehat{U}_j^P$ $\forall c \in m'$ and $\forall j \in \{L, H\}$. Consider the following beliefs and continuation equilibrium: $b_H(m') = 0$; $b_H(c, m') = 0$ $\forall c \in m'$; each type of principal chooses the contract in m' that maximizes her expected payoff given that $b_H(c, m') = 0$. Clearly, in this continuation equilibrium the payoff of the principal of type j is no larger than $\widehat{U}_j^P$ $\forall j \in \{L, H\}$ and the result trivially holds.

Suppose now that $C_H \neq \varnothing$. Consider the following beliefs: $\forall c \in m' \backslash C_H$ let $b_H(c, m') = 0$ while $\forall c \in C_H$ let $b_H(c, m')$ be such that $U_H^P(c; a^*(c, b_H(c, m'))) = \widehat{U}_H^P$. Let us denote these beliefs by $\widetilde{b}_H(c, m')$. Note that they always exist, since (i) $U_H^P(c; a^*(c, 1)) \leq \widehat{U}_H^P$ (by Step 2) and $U_H^P(c; a^*(c, 0)) > \widehat{U}_H^P$ $\forall c \in C_H$, and (ii) $U_H^P(c; a^*(c, b_H))$ is continuous in b_H. Let $\widetilde{c}_L$ denote the best contract in m' for the principal of type L given beliefs $\widetilde{b}_H(c, m)$. Suppose first that $\widetilde{c}_L \notin C_H$, then beliefs $b_H(m') = 0$, $b_H(c, m') = \widetilde{b}_H(c, m')$ $\forall c \in m'$, together with the principal of type L choosing contract $\widetilde{c}_L$ and the principal of type H choosing any contract in C_H constitutes a continuation equilibrium following the proposal of m'. In this continuation equilibrium, the agent accepts menu m' and the payoff of the principal of type j is no larger than $\widehat{U}_j^P$, $\forall j \in \{L, H\}$. Suppose now that $\widetilde{c}_L \in C_H$. In this case, the beliefs $b_H(m') = \widetilde{b}_H(\widetilde{c}_L, m')$, $b_H(c, m') = \widetilde{b}_H(c, m')$ $\forall c \in m'$, together with both types of principal choosing contract $\widetilde{c}_L$ constitutes a continuation equilibrium following the proposal of m'. In this continuation equilibrium, both types of principal have lower payoffs than those associated with the best separating allocation: for the principal of type H this is obvious; while for the principal of type L the result follows from the fact that by Step 1 $U_L^P(\widetilde{c}_L; a^*(\widetilde{c}_L, 0)) \leq \widehat{U}_L^P$, the fact that the expected payoff of the principal of type H decreases with an increase in investment from $a^*(c, 0)$ to $a^*(c, \widetilde{b}_H(c, m'))$ $\forall c \in C_H$, and from Lemma 2. This completes the proof. ■

Proof of Proposition 2. The first part of the proposition follows directly from Lemma 3 and Proposition 1. Hence, this proof consists of showing that for some specifications of the model only equilibria with inefficient investment level exist. In any equilibrium with efficient investment, the principal of type j chooses a contract (t_j, q_j^0), $\forall j \in \{L, H\}$, and the agent learns the principal's type from the choice of contract. The maximum payoff of the principal of type H in an efficient equilibrium is $\max\limits_{t_L, t_H} U_H^P(t_H, q_H^0; a_H^0)$ subject to (i) $U_L^P(t_L, q_L^0; a_L^0) \geq U_L^P(t_H, q_H^0; a_H^0)$ and (ii) $(1 - \pi_H)U_L^A(t_L, q_L^0; a_L^0) + \pi_H U_H^A(t_H, q_H^0; a_H^0)$. Constraint (i) ensures that the principal of type L has no incentive to deviate by choosing contract (t_H, q_H^0) instead of contract (t_L, q_L^0). Under constraint (ii), it is individually rational for the agent to accept the menu with contracts (t_H, q_H^0) and (t_L, q_L^0). The solution to this problem involves $t_H = \pi_H U_H^A(t_H, q_H^0; a_H^0) + (1 - \pi_H)[U_L^A(t_L, q_L^0; a_L^0) + U_L^P(t_L, q_L^0; a_L^0) - U_L^P(t_H, q_H^0; a_H^0)]$. Using (3), this implies a payoff $U_H^P = \pi_H S_H(a_H^0) + (1 - \pi_H)[S_L(a_L^0) + U_H^P(t_H, q_H^0; a_H^0) - U_L^P(t_H, q_H^0; a_H^0)$. This payoff can be compared with the payoff the principal of type H can ensure herself in any equilibrium. There exists

$\widetilde{q} > \overline{q} > q_0^H$ such that $U_H^A(t, \widetilde{q}; a^*(\widetilde{q}, b_H)) \leq U_L^A(t, \widetilde{q}; a^*(\widetilde{q}, b_H))$ for all $t \in \mathbb{R}$ and all $b_H \in [0,1]$. Thus, the principal's proposal of contract $q = \widetilde{q}$ and $t = U_H^A(t, \widetilde{q}; a^*(\widetilde{q}, 1))$ is accepted by the agent regardless of his beliefs. Since $U_H^P(c; a^*(c, b_H))$ decreases with b_H when $q \geq q_H^0$ (see proof of Lemma 4), the minimum payoff for the principal if he proposes contract $q = \widetilde{q}$ and $t = U_H^A(t, \widetilde{q}; a^*(\widetilde{q}, 1))$ is $U_H^P(0, \widetilde{q}; a^*(\widetilde{q}, 1)) + U_H^A(t, \widetilde{q}; a^*(\widetilde{q}, 1)) = S_H(a^*(\widetilde{q}, 1))$. Whether this payoff exceeds the maximum payoff in an efficient equilibrium depends on the parametrization of the model. For the special case of the model presented in Section 2.1, the lowest $\widetilde{q}$ is $(2\mu + \phi^2 p_H)/(4\mu - \phi^2)$. In that case, $S_H(a^*(\widetilde{q}, 1))$ exceeds the maximum payoff U_H^P derived above when $\pi_H = 0$ if

$$\frac{\left(2\mu + \phi^2 p_H\right)\left(8\mu p_H - 2\mu - 3\phi^2 p_H\right)\phi^2}{\left(4\mu - \phi^2\right)^2} > 4\mu\left(2p_H - 1\right)\left(p_H - p_L\right) + \phi^2\left(2p_L - p_H\right)^2.$$

This condition is satisfied, for example, when $2\mu/(8\mu - 3\phi^2) < p_H \leq 1/2$ and $p_L = p_H/2$. In these cases, its left-hand side is positive whereas its right-hand side is non-positive. Since by assumption $\phi^2 < \mu$, then $2\mu/(8\mu - 3\phi^2) \leq 0.4$. Hence, for example, for π_H close to zero, $p_L = p_H/2$ and $0.4 < p_H \leq 0.5$, there exists no equilibria with efficient investment level. ■

Proof of Lemma 4. This proof consists of showing that there exist contracts (one for each type of principal) that when proposed by the principal ensure her the efficient total surplus. Throughout the proof, let $U^A(c; a, b_H) \equiv (1 - b_H)U_L^A(c; a) + b_H U_H^A(c; a)$. Suppose that the principal of type L proposes contract $c_L = (t_L^0, q_L^0, e = 0)$ and the principal of type H proposes contract $c_H = (t_H, q_H^0, e = 1)$ where $t_H = U_H^A(0, q_H^0, 1; a^*(q_H^0, 1))$.

I first show that the agent accepts both contracts regardless of his beliefs, i.e., $U^A(c_j; a^*(c_j, b_H), b_H) \geq 0$ $\forall b_H \in [0,1]$ and $\forall j \in \{L, H\}$. By construction, $U^A(c_L; a^*(c_L, b_H), b_H) = 0$ when $b_H = 0$. Since contract c_L specifies quantity q_L^0, it follows from Condition 1 that $U^A(c_L; a^*(c_L, b_H), b_H)$ increases with b_H. Thus, $U^A(c_L; a^*(c_L, b_H), b_H) \geq 0$ $\forall b_H \in [0,1]$. Similarly, by construction, $U^A(c_H; a^*(c_H, b_H), b_H) = 0$ when $b_H = 1$. Differentiating $U^A(c_H; a^*(c_H, b_H), b_H)$ with respect to b_H (and using the Envelope Theorem), we obtain $U_H^A(c_H; a^*(c_H, b_H)) - U_L^A(c_H; a^*(c_H, b_H))$. From direct inspection of (4) and the fact that contract c_H specifies $e = 1$, this difference is equal to $\mathbb{E}[\max\{V(a^*(c_H, b_H)), V_E\} \mid H]/2 - \mathbb{E}[\max\{V(a^*(c_H, b_H)), V_E\} \mid L]/2$, which is negative since by assumption F_L first-order stochastically dominates F_H. Thus, $U^A(c_H; a^*(c_H, b_H), b_H)$ decreases with b_H, which implies that $U^A(c_H; a^*(c_H, b_H), b_H) \geq 0$ $\forall b_H \in [0,1]$.

I next show that the payoff of the principal of type j following the agent's acceptance of contract c_j is no less than $S_j(a_j^0)$ $\forall j \in \{L, H\}$. By construction of c_L and c_H and the fact that $a^*(q_L^0, b_H = 0) = a_L^0$ and $a^*(q_H^0, b_H = 1) = a_H^0$, it follows that $U_L^P(c_L; a^*(c_L, 0)) = S_L(a_L^0)$ and $U_H^P(c_H; a^*(c_H, 1)) = S_H(a_H^0)$. Hence, to obtain that $U_j^P(c_j; a^*(c_j, b_H)) \geq S_j(a_j^0)$ $\forall b_H \in [0, 1]$ and $\forall j \in \{L, H\}$, it remains only to show that $U_L^P(c_L; a^*(c_L, b_H))$ increases with b_H and $U_H^P(c_H; a^*(c_H, b_H))$ decreases with b_H. Using (7) and the chain rule to differentiate $U_j^P(c_j; a^*(c_j, b_H))$ with respect to b_H, we obtain

$$1/2 \times v_A'(a^*(q_j^0, b_H)) \times [P_j(a^*(q_j^0, b_H)) - q_j^0] \times \partial a^*(q_j^0, b_H)/\partial b_H. \tag{13}$$

By assumption, $v_A' > 0$. By Lemma 1, $\partial a^*(q, b_H)/\partial b_H \geq 0$ $\forall q, b_H \in [0, 1]$. Since $P_j(a)$ increases with a and $a^*(q_j^0, b_H)$ increases with b_H, then $P_j(a^*(q_j^0, b_H))$ increases with b_H $\forall j \in \{L, H\}$. This implies that $P_L(a^*(q_L^0, b_H)) \geq P_L(a^*(q_L^0, 0)) \equiv q_L^0$ and $P_H(a^*(q_H^0, b_H)) \leq P_H(a^*(q_H^0, 1)) = q_H^0$ $\forall$ $b_H \in [0, 1]$. Hence, for $j = L$, (13) is positive and, for $j = H$, it is negative. ■

Proof of Proposition 3. Consider an equilibrium and let $\widetilde{U}_j^P$ denote the principal's payoff in state j in that equilibrium, for all $j \in \{L, H\}$. Lemma 4 implies that $\widetilde{U}_j^P \geq S_j(a_j^0)$ for all $j \in \{L, H\}$. Individual rationality of the agent implies that it is not possible that $\widetilde{U}_j^P \geq S_j(a_j^0)$ for all $j \in \{L, H\}$ and, simultaneously, $\widetilde{U}_j^P > S_j(a_j^0)$ for some $j \in \{L, H\}$. The two preceding results imply that $\widetilde{U}_j^P = S_j(a_j^0)$ for all $j \in \{L, H\}$. From individual rationality of the agent and the fact that $\widetilde{U}_j^P = S_j(a_j^0)$ for all $j \in \{L, H\}$, it follows that investment must be efficient (first-best) in both states L and H. ■

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