Tariffs, Trade and Productivity: A Quantitative Evaluation of Heterogeneous Firm Models*

(Short title: Tariffs, Trade and Productivity)

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September 30, 2014

Abstract

We examine the quantitative predictions of heterogeneous firm in the context of the Canada - US Free Trade Agreement (CUSFTA) of 1989. We compute predicted increases in trade flows and measured productivity and compare them to the post-CUSFTA increases observed in the data. Most models predict increases in measured productivity that are too low by an order of magnitude relative to predicted increases in trade flows. A multi-product firm extension that allows for within-firm productivity increases has the potential to reconcile model predictions with the data.

Since the seminal contribution by Melitz (2003), heterogeneous firm models have become a widely used instrument in the 'toolkit' of international economists. These models

*This work builds upon our earlier (unpublished) paper "Trade Liberalization and Heterogeneous Firm Models: An Evaluation Using the Canada - US Free Trade Agreement". We are grateful to Andy Bernard, Gordon Hanson, Miklós Koren, Sam Kortum, Marc Melitz, Alex Michaelides and Ralph Ossa for helpful discussions. We would also like to thank seminar participants at Banco de España, CEU, Chicago, CORE, the EEA Málaga Meeting, EUI, Fundación Rafael del Pino, Koç University, LSE, Munich, NBER Summer Institute, GIST and Vienna for comments and suggestions. All errors remain ours. Cuñat gratefully acknowledges financial support by the Austrian Science Fund (FWF #AP23424-G11). Corresponding author: Holger Breinlich, University of Essex, Department of Economics, Wivenhoe Park, Colchester CO4 3SQ, United Kingdom; Email address: hbrein@essex.ac.uk.

10.1111/ecoj.12218

were motivated by a number of stylized facts: (*i*) the existence of large productivity differences among firms within the same industry; (*ii*) the higher productivity of exporting firms as compared to non-exporting firms; (*iii*) the large levels of resource reallocations across firms within industries following trade liberalization reforms; and (*iv*) the resulting gains in aggregate industry productivity. In a generalization of the Krugman (1979, 1980) model, the introduction of within-industry productivity heterogeneity and beachhead costs enables this class of models to produce equilibria and comparative statics along the lines of these facts.

While these models are thus *qualitatively* consistent with available empirical evidence, a thorough evaluation of their *quantitative* predictions with regards to trade liberalization is still at an early stage. This is despite the fact that the models' quantitative predictions for the link between trade liberalization and changes in aggregate productivity or trade flows are of first-order importance for economic policy and welfare analysis. In this paper, we attempt to provide such an evaluation. We go beyond the stylized facts listed above and ask to what extent a range of heterogeneous firm models in the tradition of Melitz (2003) are able to quantitatively replicate the changes in trade flows and productivity associated with a specific trade liberalization episode.

We do so in the context of the Canada - US Free Trade Agreement of 1989 (CUSFTA). As we explain in more detail in Section 2 below, CUSFTA is an ideal setting for the quantitative evaluation of trade liberalization episodes.[1] First, it was a 'pure' trade liberalization in the sense that it was not accompanied by any other important economic reform, nor was it a response to a macroeconomic shock. Second, it was also largely unanticipated since its ratification by the Canadian parliament was considered to be uncertain as late as November 1988.[2] Third, the main instrument of liberalization were tariff cuts which are easily quantifiable and have a direct theoretical counterpart in all the models we analyse. Finally, there is a substantial amount of reduced-form evidence that CUSFTA has had a significant causal impact on both trade flows and productivity

[1] Also see Trefler (2004).

[2] See Breinlich (2008) for a discussion of this point. Frizzell *et al.* (1989) provide a detailed account of the political context in which the agreement was signed.

in the Canadian manufacturing sector (*e.g.* Head and Ries, 1999, 2001; Trefler, 2004).

The goal of our analysis is to evaluate to which extent different versions and extensions of Melitz's heterogeneous firm model can replicate the magnitude of trade flow and productivity increases we observe in Canada in the post-CUSFTA period (1988-1996). The baseline model we use for our analysis is a version of Chaney (2008), who extends Melitz (2003) to multiple asymmetric countries and industries as well as asymmetric trade barriers between countries. We write the model's equilibrium conditions in changes following Dekle *et al.* (2008). This allows us to express predicted increases in trade flows and measured productivity as functions of initial trade shares, the actual observed tariff cuts as well as a small number of additional parameters. We compute these predictions for around 200 Canadian manufacturing sectors and compare means, variances and co-variances of these increases across sectors to the trade flow and productivity increases observed in the data. Throughout, we pay close attention to construct model predictions which are directly comparable to the data. We do so by mimicking the procedures used by Statistics Canada in computing measured trade and productivity growth as closely as possible in the construction of our theoretical moments.[3]

Our central result is that our benchmark model is inherently incapable of matching both trade and productivity increases. This is true when we use sectorial parameter estimates obtained from other data sources, or when we choose parameters to minimize deviations between theoretical and empirical moments via a simple GMM procedure. The predicted increase in trade flows for a given change in tariffs is always much too large relative to the predicted increase in measured productivity. Put differently, if we choose parameters to match trade flows, the model substantially underpredicts the growth in measured productivity we observe in the data.

We explore the robustness of our results in a number of ways, such as using different approaches to computing measured productivity growth or modelling tariff cuts in the model. We also investigate whether the baseline model's poor performance is due to the fact that it abstracts from many important real-world determinants of trade and produc-

[3]Section 3 and Appendix A discuss in detail how measured real productivity growth arises in our modeling frameworks despite the presence of fixed markups.

tivity growth (*e.g.*, technological progress unrelated to trade liberalization, or changes in non-tariff barriers and physical transport costs). We first show that allowing for contemporaneous changes in other trade barriers cannot resolve the fundamental mismatch of trade and productivity growth. Secondly, we remove a number of sources of variation from the data which are absent from our model. For example, we first-difference the data to remove time-invariant trends in productivity and trade flow increases. We also project the data on sectorial-level tariff cuts as in Trefler (2004) and use the predicted values for a comparison to our model's predictions. That is, we only use the variation in the data associated with tariff cuts, which is directly comparable to the key mechanism in our model (where tariff reductions are the only exogenous driver of trade and productivity growth).[4] These procedures lead to a better fit of the model to the data, but the overall discrepancies remain large.

Having established the inability of our baseline model to simultaneously match trade and productivity increases, we ask which variations in modelling features bring the model's predictions closer to the data. We experiment with versions of our baseline model allowing for free entry, tradable intermediate inputs, general equilibrium effects operating through wages, and endogenous firm-level productivity through adjustments in product scope as in Bernard *et al.* (2011). We find that free entry and general equilibrium effects do not markedly improve the model's performance. Introducing tradable intermediates helps somewhat, but formal over-identification tests in our GMM framework still reject this model variant. The only model that is capable of providing a good fit to the data and of passing our over-identification tests is the multi-product firm extension. We interpret these results as evidence for the need to explicitly model within-firm productivity increases when constructing quantitative trade models capable of explaining first-order features of trade liberalization episodes. This is in line with a number of recent studies highlighting within-firm productivity effects in response to freer trade, in the context of CUSFTA but also of other trade liberalization episodes (*e.g.*, Bustos, 2011; Lileeva and

[4] We always perform the same transformation on the actual and the model-generated data to preserve comparability. We explain this approach in more detail in Section 4.

Trefler, 2011).[5]

Our research contributes to several related strands in the literature. The first are papers concerned with the design and testing of a new generation of computable general equilibrium (CGE) models (*e.g.*, Balistreri *et al.*, 2011; Corcos *et al.* 2012). This new generation of CGE models tries to improve the predictive performance of earlier CGE models by explicitly modelling firm-level heterogeneity.[6] Our paper highlights a fundamental problem many of these models face when trying to predict the effects of a reduction of trade barriers – the inability to match both trade and productivity increases, the two variables which have been the focus of most existing theoretical and empirical analyses of trade liberalization episodes. We also contribute to this literature by performing a comparative evaluation of a wide range of popular trade models, rather than focusing on the performance of one particular version. Finally, we look at both within- and out-of-sample predictions and employ formal statistical tests to evaluate model performance, rather than only comparing the model predictions and data in a relatively ad hoc fashion.

Secondly, we contribute to the rapidly growing literature on quantitative trade models (*e.g.*, Eaton and Kortum, 2002; Alvarez and Lucas, 2007; Hsieh and Ossa, 2011; Levchenko and Zhang, 2011; Arkolakis *et al.,* 2012; Ossa, 2012; and Costinot and Rodríguez-Clare, 2013, for a recent overview). One of the key purposes of these papers is to compute the gains from trade in different gravity-type models and to relate the magnitude of the predicted gains to specific model features. Obviously, the usefulness of these exercises depends crucially on the empirical validity of the underlying modelling frameworks in terms of their quantitative (rather than just qualitative) predictions. We point out that a class of widely used quantitative trade models has difficulties matching basic adjustment patterns to freer trade, and show which model modifications provide a

[5] Given that the number of free parameters in the above models varies, we also look at the out-of-sample predictions of our models. That is, we estimate parameters on the pre-liberalization period (1980-1988) and compare the models' predictions for the post-liberalization (1988-1996) period, thus controlling for potential problems of overfitting. Still, we find that the multiproduct extension of our baseline model performs best.

[6] See Kehoe (2005) for an evaluation of the (poor) quantitative performance of some of these earlier models.

better fit to the data.

There is a also a much smaller number of papers which have recently evaluated other aspects of the quantitative performance of models in the tradition of Melitz (2003). For example, Lawless (2009) and Eaton *et al.* (2011) note the inability of these models to explain several features of firm-level data such as the fact that firms do not enter markets according to an exact hierarchy or that exporters sell more at home than predicted. Armenter and Koren (2014) show that Melitz-type models cannot match both the size and the share of exporters given the observed distribution of total sales. Chaney (2013) points out that they are unable to simultaneously match a number of stylized facts regarding the distribution of the geographic location and the number of foreign markets accessed by different firms. These papers all make important contributions to improving various quantitative predictions of Melitz-type models in the *cross-section*; but they do not provide evidence for the quantitative performance of these models in predicting the effects of trade liberalization. As we have argued above, we see this aspect as central for economic policy and welfare analysis, and the success of Melitz-type models in explaining post-liberalization changes in productivity and trade *qualitatively* has certainly contributed to their popularity. Put differently, even if Melitz-type models fail to match important cross-sectional facts, they might still provide reasonably precise predictions for the consequences of trade liberalization. Likewise, even if a model matches all relevant cross-sectional facts, it does not automatically follow that its trade liberalization predictions will be adequate. A good quantitative cross-sectional performance is neither necessary nor sufficient for quantitatively accurate time-series predictions with respect to trade liberalization and a separate investigation is thus required.[7] Finally, we again also add to this last group of related papers by introducing formal over-identification tests and an analysis of both within- and out-of-sample predictions.

The rest of the paper is structured as follows. In Section 2, we provide background information on CUSFTA and take a first look at the increases in trade flows and measured

[7]Milton Friedman famously argued that '... theory is to be judged by its predictive power for the class of phenomena which it is intended to explain'. (Friedman, 1953, p.8). In our view, the central aim of Melitz-type models is to make sense of, and predictions for, the reaction of an economy to reductions in trade barriers. Other (cross-sectional) predictions are also relevant but more secondary.

productivity we observe in the data. Section 3 discusses our baseline model and how we compute our theoretical predictions. Section 4 evaluates this model's quantitative predictions and shows why the model is inherently incapable of matching our empirical moments. In Section 5 we discuss different extensions of our baseline model and show that allowing for endogenous firm-level productivity is one way of reconciling models of the class of Melitz (2003) with the evidence. Section 6 concludes.

1 Empirical Setting

Negotiations for CUSFTA started in May 1986, were finalized in October 1987 and the treaty was signed in early 1988. The agreement came into effect on 1 January 1989, which was also the date of the first round of tariff cuts. Tariffs were then phased out over a period of up to ten years with some industries opting for a swifter phase-out.

Figure 1 shows that these tariff reductions were accompanied by strong increases in Canadian trade flows (imports plus exports to/from the U.S.) and measured labour productivity.[8] The average Canadian trade flow increase over the period 1988 to 1996 was 118%, while the increase in labour productivity was 30%. This compares to growth rates of only 44% (trade) and 17% (labour productivity) for the pre-liberalisation period, 1980-1988. Figure 1 also displays a high degree of heterogeneity in trade flow and productivity changes across the 203 sectors in our data in the post-liberalization period. For example, industries at the 5th percentile of the distribution of productivity changes observed a decrease of close to -12% over the 1988-1996 period, or -1.5% per year. In contrast, industries at the 95th percentile saw productivity increase by over 80% in total or 7.7% per year. Likewise, trade flow changes range from -14% (-1.9% p.a.) at the 5th percentile to over +400% (22% p.a.) at the 95th percentile. Using differences-in-differences estimation and instrumental variables techniques, Trefler (2004) demonstrates a causal link between these changes and the extent of tariff cuts across sectors.

[8]We use data for 203 Canadian manufacturing sectors from Trefler (2004), who uses Statistics Canada as his original data source. We compute growth rates from data expressed in 1992 Canadian dollars using 4-digit industry price and value added deflators and the 1992 US-Canadian Dollar exchange rate. Labor productivity is calculated as value added in production activities divided by total hours worked by production workers. See section 4 for additional details on data construction.

In the light of this evidence, we focus on model predictions regarding average changes in trade and productivity and their dispersion across sectors. This is in line with the view that models of trade liberalisation should at least correctly predict average increases in trade and productivity, as well as being able to account for the strong sectorial heterogeneity evident in the data.[9] Table 1 summarizes our empirical moments. Besides the mean and the variance of trade flow and productivity increases, we also look at the covariance between these increases across sectors. That is, we will be comparing the first and second moments of these variables to their theoretical counterparts in our models.

Before moving on to a description of our baseline model, we discuss some possible objections to our approach of comparing model predictions to empirical moments based on trade and productivity growth. Most importantly, the only (exogenous) driver of trade and productivity growth in our models are tariff cuts. In contrast, other determinants are likely to be present in the data, which might make a direct comparison between theoretical and empirical moments uninformative. We have several replies to this objection.

First, several aspects of CUSFTA suggest that it is a reasonable abstraction to rely on models with relatively simple, tariff-reduction driven, data generating processes. In particular, tariff cuts were by far the most important tool of liberalization under CUSFTA. In contrast, non-tariff barriers remained unchanged after 1988 in the sense that the corresponding provisions in CUSFTA only amounted to a reconfirmation of earlier multilateral obligations under the General Agreement on Tariffs and Trade (GATT).[10] CUSFTA also had a 'natural experiment' character in the sense that it was not accompanied by any other important economic reform, nor was it a response to a macroeconomic shock (see Trefler, 2004). This implies that the presence of other, unmodelled, determinants of trade and productivity growth should be less important than during other

[9] Note that this is a less demanding test than asking the model to exactly match trade and productivity growth in all sectors. As we will see, however, the key problem of most of our models is to get the relative response of trade and productivity growth right. This is what prevents these models from matching the data, be it sector-by-sector or on average. A further advantage of relying on moments computed across sectors is that we can implement formal statistical tests within a GMM framework, once we impose the necessary cross-sectoral parameter restrictions (see Section 3 below).

[10] See, in particular, Chapters 5, 6 and 13 of CUSFTA (1988) on National Treatment, Technical Standards and Government Procurement. All of these measures also have in common that they are not sector-specific and as such are unlikely to be correlated with tariff cuts. (We discuss the issue of omitted variables correlated with tariff reductions below and in Section 4.)

liberalization episodes, and the resulting deviations between model predictions and data less substantial.

We think that these points make 'tariff-reductions only' models a useful starting point for our evaluation. But the presence of other, unmodelled determinants of trade and productivity growth in the data is of course still likely. This is why in Section 4 we experiment at length with different procedures of removing variation from the data which is likely to be driven by factors absent from our models. Most importantly, we show that our results go through when we only rely on variation in the data associated with reductions in tariffs. Here again, the 'natural experiment' character of CUSFTA is useful because it makes the variation in tariff cuts largely exogenous. Indeed, Trefler (2004) experiments with different instrumental variable strategies and, using the same tariff data as in this paper, finds no evidence for endogeneity problems in the corresponding Hausman tests.

One remaining concern with relying on tariff cuts as the key driver of trade and productivity growth in our models is that US-Canadian manufacturing tariffs were already relatively low in 1988. Trefler (2004) discusses this issue at length, and shows that two factors make it nevertheless plausible that CUSFTA generated the strong observed trade and productivity responses which he finds and which we have discussed above. While average Canadian manufacturing tariffs against the United States were only around 8% in 1988, this average hid a substantial amount of sectorial heterogeneity. In fact, more than a quarter of Canadian industries were protected by tariffs in excess of 10%. These industries also tended to be characterised by low profit margins, implying that the 1988 tariff wall was high and that its removal could be expected to lead to important selection and trade effects within Canada. Similar arguments apply to the import tariffs faced by Canadian firms exporting to the United States which also showed a strong variation across sectors (although the average initial tariff was somewhat lower here, at approximately 4%).

Finally, we note that even if a large part of the trade and productivity gains after 1988 was driven by factors correlated with, but distinct from, tariff reductions, this is unlikely to rescue our baseline and most of our augmented models. For example, we show in Section 4 that allowing for changes in trade barriers other than tariffs faces the same problems of simultaneously matching trade and productivity increases. If we vary such trade barriers to exactly match trade growth, we still substantially underestimate productivity growth, and vice versa.

2 Description of Baseline Model

In this section, we outline our baseline model, which is a version of Chaney (2008). We describe the model setup and how we derive our equilibrium conditions in changes. We then discuss how to construct theoretical predictions from the model which are comparable to the empirical moments we observe in the data (see Table 1).[11]

Model Setup and Equilibrium Conditions

There are many countries, denoted by h and j.[12] Each country admits a representative agent, with quasi-linear preferences

$$U_j = \sum_{i \in I} m_{ij} \ln Q_{ij} + A_j, \tag{1}$$

where $m_{ij} > 0$ and i denotes industries. A_j denotes consumption of a homogeneous final good. Q_{ij} denotes a Dixit-Stiglitz aggregate (manufacturing) final good i:

$$Q_{ij} = \left[\int_{\gamma \in \Gamma_{ij}} q_{ij}(\gamma)^{\rho_i} d\gamma \right]^{\frac{1}{\rho_i}}, \tag{2}$$

where $\rho_i \in (0, 1)$ and $\sigma_i \equiv 1/(1 - \rho_i)$ denotes the elasticity of substitution between any two varieties. Choosing good A as the numéraire, utility maximisation on the upper level yields demand functions $A_j = Y_j - \sum_i m_{ij}$ and $E_{ij} \equiv P_{ij} Q_{ij} = m_{ij}$, where Y_j is

[11] Given that this model is a straightforward extension of Chaney (2008), we keep the description of the model set up to a minimum and devote more space to the construction of the theoretical moment. Further details about the model are contained in the Online Appendix to this paper (available at http://privatewww.essex.ac.uk/~hbrein/TheAppendix_20130717.pdf).

[12] When considering bilateral variables, we adopt the convention that h and j refer to exporting and importing countries, respectively.

total expenditure per consumer. In the manufacturing goods sector, utility maximisation yields demand function $q_{ihj}(\gamma) = p_{ihj}(\gamma)^{-\sigma_i} P_{ij}^{\sigma_i - 1} m_{ij}$.

The homogeneous good is made with labour l and a linear technology $A = l_A$ identical across countries. Manufacturing varieties are made with the production function $q_i(\gamma) = \gamma l_i(\gamma)$, where γ denotes (firm-specific) productivity. γ is iid across firms within an industry. For tractability purposes, we assume γ to be distributed Pareto with shape parameter a_γ^i and location parameter k_γ^{ij}. We assume the same shape parameter for an industry across countries, but allow it to vary across industries. The location parameter is allowed to vary across industries and countries. Producers of the homogeneous good and the final goods Q_i operate in a perfectly competitive environment. Producers of varieties in the manufacturing industry have instead monopoly power over their own varieties.

The homogeneous good is traded freely; supplying it to any market involves no costs. We consider equilibria in which all countries produce positive amounts of this good, thus leading to the equalization of wages across countries. (We normalize wages to one.) The final goods Q_i are not traded; they are produced and supplied under perfectly competitive conditions. For the varieties produced by the manufacturing industries, we assume iceberg trade costs, which take the form $\tau_{ihj} = (1 + c_{ihj})(1 + t_{ihj})$ for $j \neq h$ and $\tau_{ijj} = 1$. In this expression, tariff barriers are denoted by t_{ihj} and any other trade costs between country h to country j by c_{ihj}. We can safely ignore tariff revenue for now, given the quasi-linear utility assumption above. A manufacturing industry-i firm based in country h faces a fixed cost F_{ihj} of supplying country j. Fixed costs are in terms of the destination country's labour. We assume these labour services are provided by a "services sector" that operates under perfect competition and with a linear technology that turns one unit of labour into one unit of the fixed cost.[13] Fixed and variable trade costs are allowed to vary across industries. We assume there is no free entry in the manufacturing sectors: there is a given mass of firms M_{ih} that pick a draw from the distribution of γ prior to any decision. The labour market is perfectly competitive.

[13]Most of the activities associated with entering foreign markets are best described as service activities, such as conducting market studies or setting up distribution networks.

We now proceed to the formal treatment of the model, which consists of three steps:[14] (i) First we show how to express the model's industry equilibrium outcomes of interest as functions of the model's parameters and of the "productivity thresholds" typical of the Melitz model. (ii) We then express the growth rates of these industry outcomes in terms of the changes in parameter values (the change in trade costs τ_{ihj}), the resulting growth rates of the productivity thresholds, a few of the model's parameters (e.g., a_γ^i and σ_i), and the levels of bilateral trade volumes (which subsume the rest of the model's parameters). (iii) Finally, we show how to manipulate the growth rates of the model's equilibrium conditions so as to obtain changes in the productivity thresholds as a function of changes in τ_{ihj}, which will proxy for the trade liberalization, the shape parameter a_γ^i, and the levels of bilateral trade volumes.

The pricing decision over the variety produced by a country-h firm with productivity γ is the usual mark-up over marginal cost. Well-known manipulation of firm revenue and profit functions yields the following expression for the threshold value of productivity γ_{ihj}^* that leads country-h firms to select into market j:

$$\gamma_{ihj}^* = \frac{\sigma_i}{\sigma_i - 1}\frac{\tau_{ihj}}{P_{ij}}\left(\frac{\sigma_i F_{ihj}}{m_{ij}}\right)^{\frac{1}{\sigma_i - 1}}. \tag{3}$$

The expected revenue and expected profit that a country-h firm obtains in country j, conditional upon selecting into that market, are respectively

$$E\left[r_{ihj}(\gamma)|\, \gamma > \gamma_{ihj}^*\right] = \frac{a_\gamma^i \sigma_i}{a_\gamma^i - \sigma_i + 1} F_{ihj}, \tag{4}$$

$$E\left[\pi_{ihj}(\gamma)|\, \gamma > \gamma_{ihj}^*\right] = \frac{\sigma_i - 1}{a_\gamma^i - \sigma_i + 1} F_{ihj}. \tag{5}$$

The mass of industry-i, country-h firms that select into market j is given by $N_{ihj} = \left(k_h^i/\gamma_{ihj}^*\right)^{a_\gamma^i} M_{ih}$. Country-$h$ exports to country j can be expressed as $X_{ihj} = N_{ihj} E\left[r_{ihj}(\gamma)|\, \gamma > \gamma_{ihj}^*\right]$. The industry's aggregate sales are $R_{ih} = \sum_j X_{ihj}$. Industry employment can be easily

[14] We thank Ralf Ossa for helpful comments and suggestions on this part of the model.

shown to be $L_{ih} = \left[(\sigma_i - 1) / \sigma_i \right] R_{ih}$.[15] The price level P_{ij} is given by

$$P_{ij} = \left[\frac{a_\gamma^i}{a_\gamma^i - \sigma_i + 1} \sum_h N_{ihj} \left(\frac{\sigma_i}{\sigma_i - 1} \frac{\tau_{ihj}}{\gamma_{ihj}^*} \right)^{1-\sigma_i} \right]^{\frac{1}{1-\sigma_i}} . \tag{6}$$

Melitz (2003) defines industry productivity as

$$\tilde{\gamma}_{ih} = \left[\sum_j \frac{N_{ihj}}{\sum_j N_{ihj}} \left(\tilde{\gamma}_{ihj} \right)^{\sigma_i - 1} \right]^{\frac{1}{\sigma_i - 1}} , \tag{7}$$

where

$$\tilde{\gamma}_{ihj} = \frac{1}{1 - G_{ih}\left(\gamma_{ihj}^* \right)} \int_{\gamma_{ihj}^*}^{\infty} \gamma^{\sigma_i - 1} g_{ih}(\gamma) = \left(\frac{a_\gamma^i}{a_\gamma^i - \sigma_i + 1} \right)^{\frac{1}{\sigma_i - 1}} \gamma_{ihj}^* . \tag{8}$$

$G(\gamma)$ denotes the distribution function of γ.

Define $\hat{x} \equiv x'/x$ as a gross growth rate, where x and x' denote, respectively, the values of a variable before and after the trade liberalization:

$$\hat{X}_{ihj} = \hat{N}_{ihj} = \left(\hat{\gamma}_{ihj}^* \right)^{-a_\gamma^i} , \tag{9}$$

$$\hat{R}_{ih} = \hat{L}_{ih} = \sum_j \frac{X_{ihj}}{\sum_j X_{ihj}} \hat{X}_{ihj} , \tag{10}$$

$$\hat{P}_{ij} = \left[\sum_h \frac{X_{ihj}}{\sum_h X_{ihj}} \hat{N}_{ihj} \left(\hat{\tau}_{ihj} \right)^{1-\sigma_i} \left(\hat{\gamma}_{ihj}^* \right)^{\sigma_i - 1} \right]^{\frac{1}{1-\sigma_i}} . \tag{11}$$

Substituting out terms in the price index equation leads to

$$\hat{P}_{ij} = \left[\sum_h \frac{X_{ihj}}{\sum_h X_{ihj}} \hat{\tau}_{ihj}^{-a_\gamma^i} \right]^{-1/a_\gamma^i} . \tag{12}$$

We can use the system (12) to solve for the growth rates of the price levels $\hat{P}_{ij}$ as a function of the changes in transport costs $\hat{\tau}_{ihj}$. From equation (3), we can solve for $\hat{\gamma}_{ihj}^*$ as a function of $\hat{P}_{ij}$ and $\hat{\tau}_{ihj}$,

$$\hat{\gamma}_{ihj}^* = \hat{\tau}_{ihj} / \hat{P}_{ij} , \tag{13}$$

[15] Implicit here is the assumption that the labor necessary to provide the fixed costs F_{ihj} is not part of the manufacturing industry's employment. We think of the fixed cost as services being provided by some other sector that operates under perfectly competitive conditions. As discussed, examples include conducting market studies or setting up distribution networks in foreign markets.

and thereafter generate predictions for the industry aggregates of interest.

In this model, a decrease in country j's own import tariffs triggers an increase in imports and a reduction in country j's price level, thereby reducing the revenues (and profits) obtained by country j's firms in their domestic market. This crowds out some low-productivity firms, thus raising average industry productivity, (7). A reduction in the trade barriers that country j's firms face in their export markets has an ambiguous effect on (7). On the one hand, firms that were not exporting previously (thus with productivity lower than that of old exporters) become exporters. This reduces the average productivity of country j's exporters. On the other hand, the relative mass of exporters over non-exporters rises; since the former are on average more productive than the latter, this effect contributes positively to industry productivity.

Notice that this model minimizes the number of channels for the transmission of changes in trade barriers to changes in industry productivity. In comparison with Melitz (2003), for example, the no-free-entry assumption shuts down the possibility of any effects via changes in M_{ij}. The quasi-linear preferences eliminate general-equilibrium effects via changes in the relative demands of manufacturing goods; and the assumption that the homogeneous good is produced by all countries in equilibrium shuts down any effects via the labour market, as it leads to $w_j = 1$ for all j. (We allow for these additional channels below.)

Finally, we note that expression (7) measures theoretical productivity, which is conceptually different from the measured productivity we observe in the data and on which our descriptive statistics and empirical moments from Section 2 are based. As we will see next, however, theoretical and measured productivity growth are very similar in practice, so that the intuition just provided will continue to hold once we move to measured productivity and trade flows.

Construction of Theoretical Moments

We now construct theoretical counterparts of our empirical moments (mean, variances and covariance of industry-level real growth rates of trade flows and productivity). We try to stay as close as possible to the procedures used by Statistics Canada to assure comparability between theoretical and empirical moments.

We compute real growth rates of measured labour productivity growth by deflating value added per worker with a suitable producer price index (PPI). In our baseline model, value added growth equals revenue growth because there are no intermediate inputs. Thus, measured productivity growth is equal to:

$$MPG_{ih} = \frac{\widehat{R_{ih}/L_{ih}}}{\widehat{PPI}_{ih}} = \left(\widehat{PPI}_{ih}\right)^{-1}. \tag{14}$$

Note that in this basic productivity measure, any measured productivity growth will come from changes in the PPI, as the variations in revenue and employment exactly offset each other. In our robustness checks below, we will also look at additional sources of (measured) productivity gains.

Similarly, real growth in bilateral trade flows between countries h and j is defined as:

$$MTG_{ihj} = \frac{\hat{X}_{ihj}}{\widehat{PPI}_{ih}} \frac{X_{ihj}}{X_{ihj} + X_{ijh}} + \frac{\hat{X}_{ijh}}{\widehat{PPI}_{ij}} \frac{X_{ijh}}{X_{ihj} + X_{ijh}}. \tag{15}$$

Note that we follow Statistics Canada's approach to use PPIs to deflate export sales.[16]

Both growth rates require a suitable PPI deflator. In Appendix A, we provide a more detailed description of how Statistics Canada calculated PPIs at the sectorial level during our sample period, and how their procedure can be replicated in our model. In essence, Statistics Canada's relevant PPIs were based on sample surveys of currently active firms and gave more weight to larger producers. They also used so-called factory gate prices which exclude any costs associated with transport, distribution, subsidies, taxes or tariffs. We compute a theoretical PPI which captures these features while preserving a tight link to theoretical productivity. Specifically, we use the factory-gate price charged by the firm

[16] See Statistics Canada (2001). For a few sectors, export price indices were used but for the vast majority of sectors in our data, Statistics Canada relied on PPIs during our sample period. Also note that exports in our data are valued at free-on-board prices which exclude charges for shipping services incurred abroad, but might include other parts of the overall trade costs such as information or regulatory compliance costs. Here, we use the value of trade flows inclusive of trade costs, although we will also present results excluding them in our robustness checks.

with average productivity, $p(\tilde{\gamma}_{ih}) = [(\sigma_i - 1)/\sigma_i] w_h/\tilde{\gamma}_{ih}$,[17] where

$$\tilde{\gamma}_{ih} = \left[\sum_j \frac{N_{ihj}}{\sum_j N_{ihj}} \left(\tilde{\gamma}_{ihj}\right)^{\sigma_i - 1}\right]^{\frac{1}{\sigma_i - 1}} \tag{16}$$

and

$$\tilde{\gamma}_{ihj} = \frac{1}{1 - G_{ih}\left(\gamma^*_{ihj}\right)} \int_{\gamma^*_{ihj}}^{\infty} \gamma^{\sigma_i - 1} g_{ih}(\gamma)\, d\gamma = \left(\frac{a^i_\gamma}{a^i_\gamma - \sigma_i + 1}\right)^{\frac{1}{\sigma_i - 1}} \gamma^*_{ihj}. \tag{17}$$

As noted by Melitz (2003), $\tilde{\gamma}_{ihj}$ can be interpreted as a weighted average of firm productivities, where the weights reflect the relative output shares of firms. Also note that $\tilde{\gamma}_{ihj}$ is calculated as an average across active firms, reflecting the sampling procedure of Statistics Canada explained in Appendix A. We thus obtain our theoretical PPI as:

$$\widehat{PPI}_{ih} = \hat{p}(\tilde{\gamma}_{ih}) = \frac{p'(\tilde{\gamma}'_{ih})}{p(\tilde{\gamma}_{ih})} = \left(\frac{\tilde{\gamma}'_{ih}}{\tilde{\gamma}_{ih}}\right)^{-1}, \tag{18}$$

where the growth rate $\tilde{\gamma}'_{ih}/\tilde{\gamma}_{ih}$ can be written as

$$\frac{\tilde{\gamma}'_{ih}}{\tilde{\gamma}_{ih}} = \left\{\left[\sum_j \left(\frac{N'_{ihj}}{N_{ihj}} \frac{N_{ihj}}{\sum_{j'} N_{ihj'}}\right)\right]^{-1} \sum_j \left[\frac{N'_{ihj}}{N_{ihj}} \left(\frac{\tilde{\gamma}'_{ihj}}{\tilde{\gamma}_{ihj}}\right)^{\sigma_i - 1} \frac{N_{ihj}\left(\tilde{\gamma}_{ihj}\right)^{\sigma_i - 1}}{\sum_{j'} N_{ihj'}\left(\tilde{\gamma}_{ihj'}\right)^{\sigma_i - 1}}\right]\right\}^{\frac{1}{\sigma_i - 1}}. \tag{19}$$

Expression (19) requires the number of exporters from country h to country j in sector i, which we do not observe in our data. We show in Appendix A that bilateral sector specific exports (X_{ihj}) can be used as a proxy for N_{ihj} under the additional assumption that the fixed market entry costs (F_{ihj}) are proportional to some observable destination-specific factor that is exogenous to our model.[18]

From (14), (15) and (19), we compute MPG_{ih} and MTG_{ihj} separately for each of the 203 sectors in our data as a function of changes in tariffs ($\hat{\tau}_{ihj}$), initial trade flows (X_{ihj}) and the remaining parameters $\theta_i = \left\{a^i_\gamma, \sigma_i\right\}$. We then calculate our theoretical moments as means, variances and covariances across sectors.[19]

[17] We are grateful to Marc Melitz for pointing this out. See Ghironi and Melitz (2005) for a related discussion.

[18] We use sector-destination absorption (m_{ij}) in the calibration of our baseline model, although in practice almost identical results are obtained if we use destination market population size or GDP.

[19] For example, mean trade growth is calculated as $m_{1,model}(\theta) = \frac{1}{I}\sum_{i=1}^{I} MTG_{ihj}$

Regarding the choice of θ_i, we pursue two alternative approaches. We first use sector-specific estimates of θ_i derived from data not used in the calibration of our model. For our baseline model, we derive estimates for σ_i from the ratio of revenues to operating profits using firm-level data from Compustat North America. Estimates for a_γ^i are obtained in two steps. First, we estimate the Pareto shape parameter of the industry *sales* distribution (a_r^i) using industry-specific concentration ratios. We then use the fact that in our model $a_\gamma^i = a_r^i \times (\sigma_i - 1)$ to obtain estimates for a_γ^i. For more details on these estimation procedures, see Appendix B.

Our second approach is to choose θ_i so as to match our empirical moments via GMM estimation. In order for this exercise to be meaningful, we restrict parameters to be equal across sectors ($\theta_i = \theta$). Given that our benchmark model has two remaining parameters and we have five empirical moments, this overidentifies the model and allows us to test the validity of our moment restrictions. Formally, the GMM estimator of θ is given by

$$\hat{\theta}_{gmm} = \arg\min_{\theta} g(\theta) = \arg\min_{\theta} \left\{ \mathbf{m}(\theta)' W_n \mathbf{m}(\theta) \right\}, \tag{20}$$

where $\mathbf{m}(\theta) = [m_1(\theta) ... m_K(\theta)]'$ and $m_k(\theta) = m_{k,data} - m_{k,model}(\theta)$ are the individual moments. W_n is a (positive definite) weighting matrix to be estimated in a first step. We compute a first step estimate $\hat{\theta}_0$ by setting $W_n = W_n^0 = I$. We then use $\hat{\theta}_0$ to compute the optimal weighting matrix

$$W_n^{opt} = \left[\frac{1}{I} \frac{1}{I-1} \sum\nolimits_{i=1}^{I} \mathbf{m}_n\left(\hat{\theta}_0\right) \mathbf{m}_n'\left(\hat{\theta}_0\right) \right]^{-1} \tag{21}$$

and obtain $\hat{\theta}_{gmm}$ by setting $W_n = W_n^{opt}$ in (20). The best way to understand our GMM estimation approach is as a test of the model's basic ability to match the empirical moments of interest. As we will see, all but one of our models will fail even this most basic test.

3 Evaluation of Baseline Model

We now evaluate our baseline model's quantitative predictions and show that the model is inherently incapable of matching our empirical moments.

Data

Our baseline analysis uses sectorial-level data on trade flows, production, labour productivity per worker and tariffs for 203 Canadian manufacturing sectors for the period 1988 to 1996. In our robustness checks, we will also use data for the pre-liberalisation period (1980 to 1988). Note that production data is needed to calculate internal trade flows as the value of production minus exports (see Wei, 1996).

All Canadian data are from Statistics Canada as prepared by Trefler (2004).[20] We also require comparable data for the United States and a third country ('Rest of the World', or RoW). We define RoW here as Japan, the United Kingdom and (West) Germany, Canada's three largest trading partners after the United States in 1988.[21] Data for the United States and RoW are from Trefler (2004), the U.S. Census Bureau (see Schott, 2010) and UNIDO's Industrial Statistics Database.

We convert all data to the 4-digit level of the Canadian Standard Industrial Classification of 1980. Value data are expressed in 1992 Canadian dollars using the US-Canadian Dollar exchange rate and 4-digit industry price and value added deflators. To ensure compatibility with our choice of numéraire, we further normalize all value data by Canadian industry-level wages, proxied by total annual earnings per worker. Data on exchange rates, deflators and wages are also from Trefler (2004).

[20] These data are available from Daniel Trefler's homepage at http://www-2.rotman.utoronto.ca/~dtrefler/files/Data.htm.

[21] Together with the United States, these three countries accounted for approximately 85% of Canadian exports and imports in 1988, the year before the implementation of CUSFTA (and for more later on). Note that having a third country in the empirical estimation is important to capture possible trade diversion effects. Adding more countries, however, would not add new insights and would complicate the computational aspects of our estimation.

Baseline Results

Table 2 reports results for the theoretical moments computed for our baseline model. For comparison, the first row restates the empirical moments from Table 1 which we are trying to match.

In row (2), we present the model's predictions when we use estimates for a_γ^i and σ_i estimated on external data sources. We report the mean and standard deviation of these parameter estimates further down in the table (panel 'Parameters', 'Data (mean, sd)'). Our parameter estimates for σ_i are comparable to other estimates in the literature. For example, Broda and Weinstein (2006) estimate an average of $\sigma = 4.0$ across 256 SITC-3 goods between 1990 and 2001. Likewise, the mean across our estimates for the shape parameter of industry sales distributions is $a_r = 2.1$. Using Compustat data on the sales of US listed firms, Chaney (2008) estimates $a_r = 2.0$.

The model's predictions are substantially out of line with what we observe in the data, however. The model does not generate strong enough increases in either trade or productivity, with the predictions for productivity being particularly far off. For example, the model predicts a mean productivity increase over the period 1988-1996 of just 1.4%, whereas the increase in the data is 30.4%. For comparison, we predict about a quarter (30.9%) of the actual 118% average increase in trade flows.

In row (4), we choose parameters to minimize (weighted) deviations between theoretical and empirical moments, following the GMM approach outlined above.[22] As expected, the model does better in this case but there is still a substantial shortfall in the mean and variance of productivity increases across sectors (we do better for trade flows now). Also note that the optimization procedure pushes the parameter values up to $a_\gamma = 14.2$ and $\sigma = 8.5$. The shape parameter (a_γ) is precisely estimated, but the same is not true for the estimated elasticity of substitution (σ). Finally, the last two rows of Table 2 report

[22]In row (3), we also report predictions based on our first-step estimates (using the identity matrix as our weighting matrix). These give equal weight to all moments and ignore the moment covariance structure. As such, these predictions are more directly comparable to the ones presented in row (2) and show to what extent the optimal choice of parameters improves upon predictions based on externally estimated parameters. (Although we note that the externally estimated parameters vary by sector and could, in principle, lead to more accurate predictions.)

the value of the GMM objective function at its minimum ($g\left(\hat{\theta}_{gmm}\right)$).[23] Given that our baseline model is over-identified (five moments and two parameters), we can also use $g\left(\hat{\theta}_{gmm}\right)$ as the basis for a test of overidentifying restrictions (see Greene, 2000). Under the null that $\hat{\theta}_{gmm} = \theta_{true}$, the GMM objective function follows a $\varkappa^2$-distribution with three degrees of freedom. The corresponding p-value (reported underneath the GMM objective) indicates that we can reject this null hypothesis at the 1%-level.

What explains the inability of the model to simultaneously match trade and productivity moments? A somewhat superficial answer is that the model simply does not generate enough trade and productivity growth for the values of a_γ and σ estimated from external data sources. But this does not explain why we cannot match our empirical moments when we are allowed to freely choose these parameters in our GMM estimation. Here, the underlying reasoning becomes more subtle and hinges on the model's inability to match *relative* trade and productivity growth.[24] This is easiest to see for the case of a symmetric trade liberalisation between two symmetric countries, although the following intuition also carries through to the general asymmetric case used for our results in Table 2. The symmetry assumption implies that $\hat{X}_{ihj} = \hat{X}_{ijh} = \hat{X}_i$, $X_{ihj} = X_{ijh} = X_i$, and $\widehat{PPI}_{ih} = \widehat{PPI}_{ij} = \widehat{PPI}_i$, so that we obtain:

$$\frac{MTG_{ihj}}{MPG_{ih}} = \frac{\hat{X}_i/\widehat{PPI}_i}{\left(\widehat{PPI}_i\right)^{-1}} = \hat{X}_i. \tag{22}$$

Thus, the ratio of trade to productivity growth is simply the nominal growth rate of trade flows. From (9), this is a power function of the change in the export productivity cutoff ($\hat{\gamma}^*_{ihj}$) with the exponent equal to $-a^i_\gamma$. Given that we have $\hat{\gamma}^*_{ihj} < 1$ and estimates of a^i_γ of on average 14.2 (see Table 2), this implies that the ratio of predicted trade to productivity growth will be large. Indeed, from Table 2, the predicted mean increase in trade flows is

[23] Note that the GMM optimisation takes into account the full moment variance-covariance matrix (W^{opt}_n). Thus, it contains more information than the simple comparison of moments in lines (1)-(3). This also explains why the theoretical moments can all be smaller than the empirical moments at the optimized parameter values.

[24] As we will see below, a similar reason explains why we cannot 'save' our baseline model by arguing that our (external) estimates of a_γ and σ are biased, or that we cannot expect our simple model to match all of the observed trade and productivity growth.

22 times larger than the predicted mean increase in measured productivity when relying on external parameter estimates for a_γ and σ. By contrast, the corresponding ratio in the data is only around four.

Furthermore, the predicted ratio is increasing in a_γ. This is important for our GMM estimation because it implies that a higher a_γ will have two effects. First, it leads to a stronger decrease in the domestic price index for a given tariffs reduction (see (12)) and thus to a larger change in domestic and export cutoffs (see (13)). This leads to higher growth in measured productivity and trade flows. At the same time, however, a higher a_γ increases the ratio of trade to productivity growth exponentially. As a consequence, if we increase a_γ far enough to match measured productivity growth, we substantially overestimate trade growth.[25]

Measured trade and productivity growth are of course also influenced by σ, which enters the PPI used to deflate both measures. But in practice changes in σ are quantitatively unimportant in the sense that they do not move the GMM objective function by much.[26] Figure 2 illustrates this point by plotting deviations of the first empirical and theoretical moments (mean productivity and trade growth) against a_γ and σ.

Robustness Checks I: Measurement Issues and Outliers

Tables 3-6 show results for a first set of robustness checks. In Table 3, we move back to predictions based on externally estimated parameter values (which vary by sector). This time, however, we change our sector-level estimates of σ_i and a_r^i by factors which are common across sectors. For example, lines 2-3 changes σ_i to $\sigma_{new,i} = f \times \sigma_{old,i}$ where f is the factor denoted in the first column. The idea behind these changes is to investigate whether systematic bias in our sector-level estimates of σ_i and a_r^i could explain the model's poor performance.[27] Note that because a_γ^i is calculated as $a_\gamma^i = a_r^i \times (\sigma_i - 1)$, changing

[25] As we will see below, increases in a_γ have an even larger impact on the relative *variances* of trade and productivity growth, reinforcing the problem we have just described for means.

[26] This also explains why σ is estimated with little precision, as can be seen from the high standard error reported in Table 2.

[27] We need to impose a common factor across sectors for the variations in σ_i and a_r^i. Otherwise, this exercise would amount to trying to match five moments with 2×203 parameters (one σ_i and one a_r^i per sector). Such a degree of underidentifcation would make a comparison between data and theoretical predictions rather meaningless.

σ_i also leads to a corresponding variation in the shape parameter of the productivity distribution, a_γ^i.

In lines 2-3 of Table 3, we change σ_i by a factor of $f = 1.5$ and $f = 2$, respectively. As expected from the discussion in the last subsection, increasing σ_i and thus a_γ^i leads to slightly higher productivity gains, but increases the mean and variance of trade flows by much more. As a results, at $\sigma_{new,i} = 1.5 \times \sigma_{old,i}$, the model predicts about 60% of the observed mean increase in trade flows, but already overpredicts the trade flow variance by 25%. At $\sigma_{new,i} = 2 \times \sigma_{old,i}$, we overpredict mean trade increases by around 15% and the variance by a factor of 10, but still only obtain a predicted mean increase in productivity of 1.7% and a variance of 0.0002 (or $1/500th$ of the actual variance). In line 4, we go one step further and choose f in $\sigma_{new,i} = f \times \sigma_{old,i}$ to exactly match the mean growth rate of trade flows. Again, this leads to a substantial overprediction in terms of the variance of trade growth rates, but does not generate nearly enough productivity growth.

Lines 5-7 repeat the same exercise with changes in a_r^i, which in turn lead to changes in $a_\gamma^i = a_r^i \times (\sigma_i - 1)$. The results are again similar.[28] Increasing a_r^i helps to match the mean trade flow increases, but cannot generate enough productivity increases. This is of course just a reconfirmation of the intuition we gave in the last section. Our baseline model does not get the *relative* impact of tariff changes on trade and productivity growth right. Thus, changing parameter values to match the average level of one of these growth rates is of no help in matching moments based on the other growth rate.[29,30]

We next examine the sensitivity of our results to outliers, by dropping all sectors which fall within the top or bottom 5% of either the trade or productivity growth distributions. This drops 42 sectors, leaving us with 161 observations. Panel A of Table 4 show how

[28] Note that changing σ and a_r by the same factor f increases a_γ by more in the case of varying σ. Varying σ also has an independent impact on measured trade and productivity growth. As discussed in the last section, however, this impact is quantitatively less important, explaining the relatively similar results in lines 2-4 and 5-7.

[29] Simultaneously varying a_r and σ by different factors is also possible, but would lead to similar results as our baseline GMM estimates in Table 2.

[30] The same point can also be made in a slightly different way. In unreported results, we show that one can also choose a_r or σ to exactly match trade growth rates, sector by sector. One can then look at predicted growth rates of productivity and compare them to the data, again sector by sector. While this approach ignores higher moments (variances and covariances) and does not allow for formal overidentification tests in a GMM framework, the predictive failure of the model is again quite evident: predicted productivity growth rates are too low by an order of magnitude.

this changes the empirical and theoretical moments. (Note that we now only compute theoretical moments based on 161 sectors.) Dropping outliers reduces mean increases in trade flows and productivity and, in particular, the variance of trade flow increases. Still, the model is only able to match a fraction of the variation observed in the data, and does again particularly poorly with regards to productivity. In Panel B, we only drop the 5% of sectors with the highest trade or productivity growth (21 sectors, leaving 182 observations). This does of course work in favour of the model, but its predictive performance remains poor.

In the next robustness check, we modify the computation of our theoretical moments in a way that leads to larger productivity gains. So far, we have valued firm revenue at destination-specific rather than factory gate prices. We now follow Statistics Canada's procedures yet more closely and compute both revenue and trade growth at factory-gate prices, i.e., excluding trade costs. This leads to the following expressions for measured trade and productivity growth:

$$MPG_{ih}^{FG} = \frac{\hat{R}_{ih}^{m}/\hat{L}_{ih}}{\widehat{PPI}_{ih}} = \left(\widehat{PPI}_{ih}\right)^{-1}\left(\sum_{j}\frac{X_{ihj}/\tau_{ihj}}{\sum_{j}X_{ihj}/\tau_{ihj}}\frac{\hat{X}_{ihj}}{\hat{\tau}_{ihj}}\right)\left(\sum_{j}\frac{X_{ihj}}{\sum_{j}X_{ihj}}\hat{X}_{ihj}\right)^{-1}, \tag{23}$$

$$MTG_{ihj}^{FG} = \frac{\hat{X}_{ihj}^{m}}{\widehat{PPI}_{ih}}\frac{X_{ihj}^{m}}{X_{ihj}^{m}+X_{ijh}^{m}} + \frac{\hat{X}_{ijh}^{m}}{\widehat{PPI}_{ij}}\frac{X_{ijh}^{m}}{X_{ihj}^{m}+X_{ijh}^{m}}, \tag{24}$$

where $\hat{R}_{ih}^{m}$ denotes measured revenue growth which is now different from $\hat{R}_{ih}$ as it is valued at factory gate prices. Likewise, we have $\hat{X}_{ihj}^{m} = \hat{X}_{ihj}/\hat{\tau}_{ihj}$ and $X_{ijh}^{m} = X_{ihj}/\tau_{ihj}$. Note that any reduction in tariffs will now automatically lead to an increase in measured revenue and trade growth in the data.

Table 5 presents results for this alternative measurement approach. Compared to Table 2, the differences are only minor. As expected, we achieve higher productivity growth. But we are still an order of magnitude below the actually observed growth rates. In addition, the new approach also leads to higher trade flow increases which makes it more difficult to simultaneously match both trade and productivity moment. This is evident from the results for the internally optimised parameter values, where we obtain a GMM objective function value very close to the baseline results.

Our final robustness check in this section uses a different modelling of tariffs. So far, we have followed the approach in most of the literature of treating tariffs as being isomorphic to physical transportation costs in our formulation of overall trade costs (see Section 3.1). We now explicitly model tariffs as a payment deducted from the firm' s revenue. This brings about a number of changes to the equilibrium conditions of our model. We briefly outline the most important ones here and refer the reader to Appendix C for a full exposition of the modified model.

Most importantly, the firm's market-specific profit function can now be written as:

$$\pi_{ihj} = \frac{p_{ihj}}{1+t_{ihj}} q_{ihj}\left(p_{ihj}\right) - \tau_{ihj} q_{ihj}\left(p_{ihj}\right)\frac{1}{\gamma} - f_{ihj}, \tag{25}$$

where p_{ihj} denotes the price paid by the consumers of the importing country. This modification leads to the following equilibrium conditions for price indices and productivity cut-offs (expressed in changes):

$$\hat{P}_{ij} = \left[\sum_h \frac{T_{ihj}X_{ihj}}{\sum_h T_{ihj}X_{ihj}}\left(\hat{T}_{ihj}\right)^{1-\frac{\sigma_i}{\sigma_i-1}a_\gamma^i}\right]^{-1/a_\gamma^i}, \tag{26}$$

$$\hat{\gamma}^*_{ihj} = \frac{\left(\hat{T}_{ihj}\right)^{\frac{\sigma_i}{\sigma_i-1}}}{\hat{P}_{ij}}, \tag{27}$$

where $T_{ihj} \equiv 1+t_{ihj}$. Similar to before, we can use (26) to solve for price index changes as a function of tariff changes. Using (27) we can then solve for changes in the productivity cut-offs. These are sufficient to calculate changes in trade flows and industry revenues:

$$\hat{X}_{ihj} = \hat{N}_{ihj} = \left(\hat{\gamma}^*_{ihj}\right)^{-a_\gamma^i}, \tag{28}$$

$$\hat{R}_{ih} = \hat{\Pi}_{ih} = \hat{L}_{ih} = \sum_j \frac{X_{ihj}}{\sum_j X_{ihj}}\hat{X}_{ihj}. \tag{29}$$

Note that for the purpose of our estimation, the key change is that the parameter σ_i now enters the price index and productivity cut-off equilibrium conditions. Given that we noted before that the impact of variations in σ_i on the theoretical moments was quantitatively unimportant in our baseline model, this modification should, in principle, allow the model to match the data better. This is because σ_i now directly enters the productivity cut-offs (and thus nominal trade flow increases), rather than only entering measured trade and productivity growth through the theoretical PPI.

In practice, however, this additional impact channel only leads to minor improvements in the model's predictive performance, as is evident from Table 6. The reason for this is that a_γ^i and $1/\sigma_i$ tend to move our moments in the same directions. Thus, the increased impact σ_i now has is not useful in matching the data. Figure 3 illustrates this by plotting deviations of theoretical from empirical moments against a_γ^i and σ_i, as Figure 2 did for our baseline model. We note that the tendency to move theoretical moments in similar ways also explains the reduction in the precision with which the parameter a_γ^i is now estimated (although σ_i is now of course estimated with a lower standard error).[31]

Robustness Checks II: Is Our Baseline Model Too Stylized?

We now return to the issue of whether our baseline model abstracts from too many real-world features to make a comparison with the data informative. We argued in Section 2 that several aspects of CUSFTA made it a reasonable abstraction to rely on models with relatively simple, tariff-reduction-driven data generating processes. Nevertheless, an important concern is that the observed post-1988 changes in trade and productivity are simply too large to be explained by tariff reductions alone, and that other factors must have been present in the process generating the observed data. Since such factors are absent from our model, one might not be surprised that the model falls short of generating sufficient trade and productivity responses. We try to address this concern in several ways in this subsection.

We start by progressively removing sources of variation from the data which are arguably absent from the model. First, we take first differences in growth rates between the post- and pre-liberalisation period (1980-1988 and 1988-1996, respectively). The

[31] In an additional robustness check (not reported), we also constructed a PPI deflator by giving equal weight to all active firms, rather than overweighting larger firms (see Appendix A for details). This yielded very similar results to the one in Table 2, with a GMM objective function value of 91.9835 compared to 91.7885 for the baseline model.

purpose of this exercise is to eliminate time-invariant factors from the data which are absent from our model, such as technological progress leading to ongoing productivity growth. Indeed, first-differenced growth rates are less than half as large on average as growth rates in levels (see Table 7, first line). To assure comparability with these 'cleaned' data and our theoretical predictions, we perform a similar procedure when generating data from our model. That is, we separately calculate predictions for the pre- and the post-liberalization period in the same way described above for our baseline model. For the 1980-1988 period, we use initial trade flows for 1980 and observed tariff cuts between 1980 and 1988. (The remaining parameters, a_γ and σ, are assumed to remain constant over the entire period 1980-1996.) We then first-difference the generated data across the two periods in the same way we differenced the actual data.[32]

Secondly, we implement a difference-in-differences strategy similar to Trefler (2004). We regress first differences of trade and productivity growth (as calculated above) on first differences in tariff cuts, and compute predicted values from these two regressions. We then use the model to generate data for both the pre- and post-liberalization period (as described above) and run the same regressions on the generated data. We again compute predicted values and compare them to the predicted values from the regressions using the actual data. The purpose of this approach is to only use variation which is correlated with tariff cuts. Since this is the driving force in the model's data generating process, we would expect the model to perform much better when focusing on this source of variation only. The first line of Table 8 shows that this approach does indeed lead to further substantial reductions in empirical mean growth rates, especially for productivity.

Table 7 presents the full results for first differences, Table 8 for the difference-in-differences approach. As discussed, first-differencing the data reduces the magnitude of all moments with the exception of the variance of (first-differenced) productivity growth rates. However, the first-differenced theoretical moments are also smaller, so that we only

[32] An alternative approach would be to directly compare the 1988-1996 model predictions to the first-differenced data. This is not strictly correct, however, because there were small (GATT-driven) tariff reductions in the 1980-1988 period, too. These generate positive, if small, growth in trade and productivity in our model, which in turn lead to differences between first-differenced and 1988-1996 model growth rates. In practice, however, results are very similar for this alternative approach (available from the authors).

obtain a small reduction in the percentage difference with the first two empirical moments (mean growth rates). The reduction in the covariance difference is more substantial, but differences in variances actually go up slightly. Thus, the model's overall performance is similar to our baseline results. This is true when we use externally estimated data (row 2) and when we choose parameters to match the empirical moments (rows 3-4). This lack of improvement is also reflected in the GMM objective function value which is basically unchanged compared to the baseline results in Table 2.

The difference-in-differences approach fares better. We now get much closer to observed trade flow changes even when using externally estimated parameters (we match 75% of the mean increase and 50% of the variance). We also do better for mean productivity increases (we match 30% of the observed increase). However, we are still an order of magnitude below the actual variance of productivity increases and the covariance between trade and productivity increases. The better ability of the model to match the 'cleaned' data is also reflected in a lower GMM objective function value, although we still reject the null that the moment restrictions implied by our model are valid at the 1% level.

A remaining concern is that tariff reductions might be correlated with other factors present in the data, but absent from our model. As discussed in Section 2, there are a priori few reasons to believe that such omitted variables were important during the implementation of CUSFTA. We also have econometric evidence from Trefler (2004) that endogeneity issues related to tariff reductions are unlikely to be a major problem in our data (see Section 2). Nevertheless, we explore possible implications for our results in the following. A natural candidate for an omitted variable are changes in other trade costs which we assumed to be constant in our baseline simulation. These changes could be due to reductions in non-tariff barriers (including more efficient border procedures), reductions in physical transport costs over the sample period, or even a reduction in the uncertainty regarding possible future tariff hikes.[33] If such variables were indeed important, using only the variation in the data associated with tariff cuts is still not a

[33] See footnote 10 for details.

'fair' comparison because part of this variation will still be driven by factors not present in the model.

It is easy to show, however, that allowing for such correlations is not sufficient to rescue our model. In Section 3, we defined trade costs as consisting of a tariff (t_{hj}) and a component comprising all other trade costs (c_{hj}), such that $\tau_{hj} = (1 + c_{hj})(1 + t_{hj})$. We do not observe changes in $(1 + c_{hj})$ but can make a number of assumptions which should, in principle, help the model to generate larger trade and productivity gains. Our first approach is to work with sectorial-level estimates of a and σ as before and assume that the change in $(1 + c_{CAN,US})$ and $(1 + c_{US,CAN})$ is proportional to observed reductions of US and Canadian import tariffs, respectively. That is, for $h, j \in \{CAN, US\}$, we assume that $\left(1 + c'_{hj}\right) / (1 + c_{hj}) = c\left(1 + t'_{hj}\right) / (1 + t_{hj})$. The change in τ_{hj} will thus be $\hat{\tau}_{hj} = c\left[\left(1 + t'_{hj}\right) / (1 + t_{hj})\right]^2$.[34] The second approach uses our GMM framework but allows for a third parameter in addition to a_γ and σ – mean changes in $(1 + c_{hj})$. That is, $\bar{c} = \left(1 + c'_{hj}\right) / (1 + c_{hj})$ and $\hat{\tau}_{hj} = \bar{c}\left(1 + t'_{hj}\right) / (1 + t_{hj})$. We now try to minimize our GMM objective function through varying a_γ, σ and $\bar{c}$. Note that the first approach, in particular, is similar in spirit to our previous robustness check of varying σ and a_r by factors which are common across sectors.[35]

Table 9 presents the results for both robustness checks. In lines 2-4, we show theoretical moments for different values of c. Lines 2 and 3 use values of c which lead to theoretical predictions of mean trade growth rates which are too low and too high, respectively. As expected, decreases in c (i.e., stronger reductions in other trade costs) lead to higher trade and productivity growth. Line 4 uses $c = 0.98$ which allows us to exactly match mean trade growth. At this value, however, we overestimate the variance of trade growth by a factor of five, but still only achieve less than one eight of the observed mean productivity growth, and only one hundredth of the variance of productivity growth. Line

[34] We assume that trade costs other than tariffs for exports and imports to and from the rest of the world remain unchanged. Allowing for less than perfect correlation between c_{hj} and t_{hj} is also possible but does not change the following results qualitatively. Note that if c_{hj} and t_{hj} are completely uncorrelated, our previous approach of using tariff-cut related variation will again be valid.

[35] Again, we restrict the variation in c_{hj} to be governed by one additional parameter only. As discussed in footnote 27, allowing for more flexibility would make our model underidentified and our approach much less meaningful.

5 shows moment deviations for our first-step GMM estimates. While we now vary a_γ, σ and $\bar{c}$, we do not see major improvements as compared to our baseline GMM results in Table 2.

The intuition for these negative results is similar in all cases. Allowing for changes in non-tariff trade costs only allows the model to generate larger increases in both trade and productivity; it does nothing to help address the model's problem of getting the *relative* growth rates right. This holds true even when we vary a_γ, σ and $\bar{c}$ simultaneously because a and $\bar{c}$ have similar impacts on our theoretical moments and do not allow the model to generate sufficient variation in trade and productivity growth.

The effects are more subtle for the full GMM estimation. As before, the presence of our weighting matrix W_n^{opt} implies that the estimation now places much less weight on the trade moments, and in particular on the deviation from the variance of trade growth across sectors. This somewhat alleviates the problem that increases in $\bar{c}$ cannot generate enough productivity growth because of the implied deviation from the trade growth variance. As a result, we obtain much lower values for the GMM objective function than in our baseline estimation. However, from a statistical point of view the model is still rejected at the 1%-level. We also note that the parameter estimates are quite extreme. We find very low estimates for a_γ and σ, and an implied reduction in non-tariff trade costs $(1 + c_{hj})$ of more than 50%. This does not seem plausible given the absence of major changes in transportation technology over the sample period and the rather minor reductions in non-tariff barrier agreed to in CUSFTA.

One final possibility is that there are factors present in the data, but absent from the model, which lead to increases in productivity, but not trade growth, and happen to be correlated with tariff reductions over the sample period. We cannot definitely exclude this possibility because we do not see a way of 'cleaning' our data of such factors that is compatible with our theoretical framework.[36] We note, however, that the solution we will eventually propose relies on a related mechanism. As we show below, one way of

[36] Trefler (2004) controls for productivity trends in the US and includes business cycle controls based on aggregate movements in GDP and exchange rates. We did not adopt this approach because our model predicts that such variables would themselves depend on CUSFTA-induced tariff reductions, making them unsuitable as controls.

matching model predictions and data is to allow for sources of within-firm productivity growth which are triggered by tariff reductions.

4 Model Extensions

We now move on to a number of more major modifications of our basic modelling framework. The goal of this section is to explore which extensions are most promising in terms of improving the baseline model's predictive performance. As all the extensions we consider are well known in the literature, we focus on an exposition of the most important modifications. We also outline how our main equilibrium conditions and our trade and productivity measures change, and explain the economic intuition behind these changes. A detailed exposition of the different models is available in the paper's on-line appendix.[37]

Free Entry

The "free-entry model" is identical to the "baseline model" but for the assumption of a given mass of potential entrants, M_{ij}. We now allow for firms to decide whether to enter the market at the fixed cost F_{ij} (before they pick a draw of γ from its distribution). This adds a free-entry condition to the model which sets expected firm profits equal to the fixed entry cost F_{ij}. As a consequence, we also obtain an additional set of equations when we express the equilibrium conditions in changes:

$$1 = \sum_j \frac{X_{ihj}}{\sum_n X_{ihn}} \hat{\tau}_{ihj}^{-a_\gamma^i} \hat{P}_{ij}^{a_\gamma^i}, \tag{30}$$

$$\hat{P}_{ij}^{-a_\gamma^i} = \sum_h \frac{X_{ihj}}{\sum_h X_{ihj}} \hat{M}_{ih} \hat{\tau}_{ihj}^{-a_\gamma^i}. \tag{31}$$

The first equation above is the free-entry condition in growth rates; the second equation is the price index equation in growth rates. In comparison with (12), the growth in the mass of firms $\hat{M}_{ih}$ is now an argument in the determination of price indices. These equations can be solved for $\hat{P}_{ij}$ and $\hat{M}_{ij}$, which in turn can then be used to generate the

[37] Available at: http://privatewww.essex.ac.uk/~hbrein/TheAppendix_20130717.pdf. Also see Costinot and Rodriguez-Clare (2013) and Redding and Melitz (2013) for recent surveys of a number of heterogeneous firm models.

model's predictions for all other variables of interest.

Adding free entry implies an additional effect of trade liberalization on industry productivity and trade flows, as the mass M_{ij} reacts to changes in tariff barriers, with a decrease if import barriers fall and an increase if export barriers fall. Other things equal, an increase in M_{ij} leads to higher average productivity as low-productivity entrants decide not to produce. Similarly, an increase (decrease) in M_{ij} increases (decreases) the number of exporters and leads, ceteris paribus, to more (less) exports.

Thus, allowing for free entry has an a priori ambiguous effect on trade flows, as well as on theoretical and (through changes in the PPI) measured productivity. Whether we observe an overall increase depends on whether the effect of lower US import tariffs (which raises M_{CAN}) outweighs the effect of lower Canadian import tariffs (which lowers M_{CAN}). This ambiguity is reflected in the results in Table 10, where we actually observe a slightly lower increase in average Canadian productivity when using externally estimated parameter values (row 2). Thus, allowing for free entry does not help with improving the model's predictive performance with regards to productivity. We do predict slightly higher trade flow increases, but remain far off our target of 118%.[38]

The fact that allowing for free entry only marginally affects model predictions also explains that the free-entry model is not noticeably better than the baseline model at matching our empirical moments when we can choose parameter values optimally (rows 3-4). Indeed, the GMM objective function value is only slightly lower than the one reported in Table 2 (83.30 compared to 91.79).

General Equilibrium

In our second model extension, we replace the quasi-linear utility function of the baseline model with a Cobb-Douglas utility function

$$U_j = \prod_{i \in I} (Q_{ij})^{\mu_{ij}} , \tag{32}$$

[38] Note that changes in trade flows are influenced by changes in both M_{US} and M_{Canada} Thus, trade and productivity growth need not move in the same direction as compared to the baseline model.

where $\mu_{ij} > 0$, $\sum_i \mu_{ij} = 1$. We also assume free entry and remove the numéraire sector.[39]

In analytical terms, the most important changes implied by this model are (i) the presence of wages both as unknowns and as a relevant variable in many of the equations that pin down industry outcomes; (ii) the presence of labour market clearing within the equilibrium conditions. For the sake of brevity, we omit a detailed description of the equilibrium conditions in growths rates here. In the On-line Appendix we show that we can obtain predictions for all growth rates of interest by manipulating the growth rates of the price index, the free-entry condition and the labour market clearing condition.[40]

In this general equilibrium version of our model, the effects of trade liberalization now also operate via changes in the demand for labour and its subsequent effect on wages. A lowering of US import tariffs leads to a higher demand for Canadian exports, which in turn raises Canadian labour demand and (with a fixed labour supply) wages. Ceteris paribus, this increases production costs, dampening the overall increase in Canadian exports but also driving some of the less productive Canadian firms out of the market. A reduction in Canadian import tariffs has the opposite effect through a reduction in domestic demand for Canadian producers. Compared to the baseline model, this lowers wages and production costs, dampening the productivity increasing effect of tougher import competition from the US.

Note that these wage effects operate in addition to the free-entry effects described in the last subsection, but also modify them. For example, a reduction in Canadian wages in response to lower Canadian import tariffs will also dampen the decline in the number of potential entrants (M_{ij}). Thus, Canadian exports will decline by less compared to a

[39] Allowing for free entry and Cobb-Douglas preferences while keeping the numéraire sector (that is, fixing all wages to 1) yields results identical to those of our "free-entry" model. This is due to the fact that, besides labor income being the same across the two models, the free-entry conditions in both models lead to the same price levels.

[40] Ossa (2014) points out that the choice of numéraire matters in the presence of aggregate trade imbalances. This is not an issue for our models with quasi-linear preferences because aggregate trade deficits are absorbed by our homogeneous final good A; but it is relevant for the present general equilbrium extension. Ossa suggests adjusting the raw data to eliminate trade imbalances. Unfortunately, it is unclear how to extend his procedure to labor productivity, our second key data input. It also seems likely that cleaning our trade data as suggested by Ossa would not change our results qualitatively. This is because our empirical moment, total trade growth (export plus imports), would basically remain unchanged. Trade flows also influence model predictions through initial trade shares, but they do not have a quantitatively important influence on the relative growth rate of trade and productivity (see the discussion on pp. 14-15).

situation without a wage response, and productivity will drop by less.

A priori, the expected change in our model predictions is thus again ambiguous compared to both the baseline and the free-entry version of our model. Table 11 shows that trade and productivity growth are indeed very similar to the free-entry version when we use externally estimated parameter values (row 2). The same is true when we choose parameter values to match our empirical moments. Our key statistic, the GMM objective function value is practically identical to the one for the free-entry version. We conclude that allowing for general equilibrium wage effects is quantitatively unimportant in matching the data.

Intermediate Inputs

In the third extension of our model, we assume that the production of manufacturing varieties requires both labour and intermediate inputs:

$$q_{ij}(\gamma) = \gamma \left[\frac{Q_{ij}^{input}(\gamma)}{\alpha_i}\right]^{\alpha_i} \left[\frac{l_{ij}(\gamma)}{1-\alpha_i}\right]^{1-\alpha_i}, \tag{33}$$

where Q_{ij}^{input} denotes the amount of the aggregate manufacturing good used as an intermediate input, and $\alpha_i \in [0,1)$.[41]

In this case, the price and expenditure equations in changes can be rewritten as

$$\hat{P}_{ij}^{\frac{a_\gamma^i}{\sigma_i-1}} = \left(\hat{E}_{ij}\right)^{\frac{\sigma_i - a_\gamma^i - 1}{(1-\sigma_i)^2}} \left[\sum_h \frac{X_{ihj}}{\sum_m X_{imj}} \hat{\tau}_{ihj}^{-a_\gamma^i} \frac{1}{\hat{P}_{ih}^{\alpha_i a_\gamma^i}}\right]^{\frac{1}{1-\sigma_i}}, \tag{34}$$

$$\hat{E}_{ij} = \frac{1}{E_{ij}}\left[m_{ij} + P_{ij} Q_{ij}^{input} \hat{P}_{ij}^{-a_\gamma^i \alpha_i} \left(\sum_h \frac{X_{ijh}}{\sum_h X_{ijh}} \hat{\tau}_{ijh}^{-a_\gamma^i} \hat{P}_{ih}^{a_\gamma^i} \hat{E}_{ih}^{\frac{a_\gamma^i}{\sigma_i-1}}\right)\right]. \tag{35}$$

This yields a system of non-linear equations in $\hat{P}_{ij}$ and $\hat{E}_{ij}$. Once we solve for $\hat{P}_{ij}$ and $\hat{E}_{ij}$, we can solve for the growth rates of the variables of interest. The expression for expenditure is more elaborate now because it now also encompasses purchases of intermediates (i.e., $E_{ij} = m_{ij} + P_{ij} Q_{ij}^{input}$).

[41] For simplicity, we abstract from interindustry input-output linkages (see Caliendo and Parro, 2012, for such an extension). Note that in order to isolate the effect of allowing for intermediates, we have also switched back to the no free-entry, no general equilibrium case.

Labour productivity is now value added per worker,

$$\frac{VA_{ij}}{L_{ij}} = \frac{R_{ij} - P_{ij}Q_{ij}^{v}}{L_{ij}}, \tag{36}$$

and its measured growth rate is

$$MPG_{ih}^{int} = \frac{\widehat{VA_{ih}/L_{ih}}}{\widehat{PPI}_{ih}} = \left(\hat{P}_{ij}\right)^{-\alpha_i} \left(\tilde{\gamma}'_{ih}/\tilde{\gamma}_{ih}\right). \tag{37}$$

A comparison with expressions (14) and (18) reveals that allowing for intermediate inputs adds $\left(\hat{P}_{ij}\right)^{-\alpha_i}$ as an additional source of measured productivity growth. Intuitively, the availability of cheaper (imported) intermediate inputs leads to a stronger decrease in the domestic PPI for a given change in the productivity of the 'average' firm ($\tilde{\gamma}'_{ih}/\tilde{\gamma}_{ih}$), and thus to stronger increases in measured productivity. Note, however, that increases in $\tilde{\gamma}'_{ih}/\tilde{\gamma}_{ih}$ will tend to be lower than in the baseline model. This is because lower input costs mean that some of the less productive firms can stay in the market, ceteris paribus.

Table 12 shows that the overall impact on productivity is positive. When we use externally estimated parameter values (row 2), we more than double predicted productivity growth as compared to the baseline model. However, a large gap between predicted and actual productivity gains remains (2.95% vs. 30.41%). The results also reveal that allowing for intermediates increases predicted trade flows as intermediates make up a growing proportion of international trade. Thus, while the presence of intermediate inputs allows us to obtain larger productivity increases, it also leads to stronger trade growth. This again makes it difficult for the model to simultaneously match trade and productivity growth and explains why our GMM approach is still unsuccessful in matching the empirical moments (rows 3 and 4). While the GMM objective function is 50% lower than in the baseline model, the overidentification test still rejects at the 1%-level.[42]

[42] Note that the model with intermediates has one additional parameter (α), so that we lose one degree of freedom as compared to the baseline model. This is taken into account in the reported p-value which in any case is substantially below the 1% level.

Multi-product Firms

The final extension we consider is to introduce multi-product firm features as modelled in Bernard *et al.* (2011) into the baseline model.[43] As in Bernard *et al.* (2011), we introduce an additional layer into our utility function by modelling final goods (Q_i) as a continuum of products which are imperfect substitutes in demand. Within each product, firms supply horizontally differentiated varieties. While firms produce one variety of each product, they can supply a range of products. In addition to productivity (γ), firms now also draw 'product attributes' (λ) for the continuum of products which act as demand shifters. We assume that λ is Pareto distributed with location parameter k_λ and shape parameter a_λ. Firms observe their γ and λ and decide whether to pay the additional fixed costs associated with entering different markets and products. As we explain in the On-line Appendix, the derivation of our equilibrium conditions and productivity measures is similar to the baseline model. The main difference is that they now contain a third parameter (a_λ) which governs productivity and trade growth rates in addition to a_γ and χ.[44]

Intuitively, the "multi-product model" reinforces the between-firm reallocation effect on productivity with a within-firm reallocation effect as a response to trade liberalization. Firms reallocate resources from product-varieties with (now loss-making) low attributes to product-varieties with (more profitable) high attributes, thus leading to higher firm-level productivity. As we show in the On-line Appendix, this leads to a stronger decrease in the industry PPI and thus a more pronounced increase in industry productivity. At the same time, the additional within-firm productivity effect also reduces the increase in imports as domestic firms become more productive relative to foreign exporters.

As seen in Table 13, this combination of effects makes the multi-product firm model quite successful in matching the observed data. We are now able to simultaneously match productivity and trade flow increases by choosing the appropriate model parameters.[45]

[43] Apart from the multi-product firm features described below, we thus switch back to the assumptions of the benchmark model. That is, we assume a given mass M_{ij}, impose $\alpha_i = 0$ for all sectors, and assume quasilinear preferences and the presence of a numéraire good.

[44] χ governs the substitutability of product varieties and is the equivalent of σ in our baseline model in terms of its role in the estimation procedure.

[45] Note that we do not have estimates for our parameters obtained from external sources. This would

Indeed, our overidentification test is now unable to reject the model at conventional levels of statistical significance.[46] We see this as an indication that sources of within-firm productivity increases need to be added to our baseline model in order to solve the problem of simultaneously matching trade and productivity growth rates in the wake of CUSFTA. While we have used the multi-product firm model of Bernard *et al.* (2011) to achieve these within-firm productivity gains, our conjecture is that other modelling frameworks will yield similar results. For example, within-firm productivity gains could also be achieved through technological upgrading in response to trade liberalization (see Bustos (2011)). As we noted in the introduction to this paper, the presence of within-firm productivity effects has indeed been documented for CUSFTA by Trefler (2004) and Lileeva and Trefler (2011). But there is evidence that such effects were important in other trade liberalization episodes as well, including Argentina (see Bustos, 2011) and India (see Topalova and Khandelwal, 2011).

One concern with the multi-product extension is that we have now one more parameter at our disposition. This will make it easier to match our empirical moments within a given sample, but might not necessarily lead to the best out-of-sample predictions (this is the classic 'overfitting' problem).

In order to evaluate whether this is an issue in the present context, we also perform the following out-of-sample test of our baseline model and the four extensions discussed above. We first estimate the model parameters on the pre-liberalisation period (1980-1988). We then use these estimates to obtain trade and productivity growth predictions for the post-liberalization period (1988-1996) and recompute the GMM objective function with these new predictions.[47] If overfitting were a problem, we would expect a higher value for the multi-product firm model than for the other extensions. Table 14 shows that this is not the case – the multi-product firm model continues to outperform all other extensions.

require firm-level data similar to the data available to Bernard *et al.* (2011). Unfortunately, we do not have access to such data for Canada or the US.

[46] Note that the p-value in Table 13 has been adjusted to account for the loss of one degree of freedom due to the fact that now have three parameters.

[47] To ensure comparability, we use the same weighting matrix as in the original (post-liberalization) GMM estimation.

5 Conclusions

In this paper, we examined the quantitative predictions of heterogeneous firm models à la Melitz (2003) in the context of the Canada - US Free Trade Agreement (CUSFTA) of 1989. We computed predicted increases in trade flows and measured productivity across a range of standard models and compared them to the post-CUSFTA increases observed in the data.

Starting from a version of Chaney (2008), we found that this model was not able to simultaneously match both trade and productivity increases. This was true when we used sectorial parameter estimates obtained from other data sources, or when we chose parameters to minimize deviations between theoretical and empirical moments via a simple GMM procedure. Our basic result were also robust to different ways of computing predicted productivity and trade growth, and to comparing model predictions and data in ways which eliminate a number of unmodelled determinants of trade and productivity increases. In each case, the fundamental problem remained that predicted increases in trade flows for a given change in tariffs were much too large relative to the predicted increase in measured productivity.

We also considered different extensions of our basic framework by allowing for free entry, tradable intermediate inputs, general equilibrium effects operating through wages, and endogenous firm-level productivity through adjustments in product scope as in Bernard *et al.* (2011). Free entry and general equilibrium effects did not markedly improve the model's performance. Introducing tradable intermediates helped somewhat, but formal over-identification tests in our GMM framework still rejected this model variant. The only model that is capable of providing a good fit to the data and of passing our over-identification tests was the multi-product firm extension. We interpret these results as evidence for the need to explicitly model within-firm productivity increases when constructing quantitative trade models capable of explaining first-order features of trade liberalization episodes.

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Appendix

A PPI Deflators

In the following, we describe how Statistics Canada calculated producer price indices (PPIs) during our sample period, which are used to convert current to constant prices entries in our data. We then outline how we apply that procedure to the computation of theoretical moments in our setting.[48]

During our sample period (1980-1996), Statistics Canada computed current price entries for 243 industries of which 211 industries are in the manufacturing sector. For manufacturing, there was also a more disaggregated commodity level, the so-called Principal Commodity Group Aggregation (PCGA), for which prices and shipment values were available. There were 1057 PCGAs in total which served as the starting point for constructing deflators.

In a first step, Statistics Canada computed PCGA price indices via the following sampling procedure. Each month, Statistics Canada obtained price quotes from important producers and from a random sample of smaller producers of a given PCGA manufacturing product. From these quotes, an average price was calculated.[49] Price quotes were always based on so-called factory gate prices which excluded any costs associated with transport, distribution, subsidies, taxes or tariffs. The particular weights used in computing the average across price quotes varied from PCGA to PCGA, but generally more weight was given to producers accounting for a larger fraction of industry output. Yearly average prices were then computed as arithmetic averages over the 12 monthly average prices. The sample of firms used for obtaining price quotes was updated every December. That is, Statistics Canada drew a new sample of smaller producers from the currently active firms. If any producer went out of business or dropped a product, Statistics Canada chose a still active producer/product as a replacement. By construction

[48] The following is based on Statistics Canada (1991; 1993; 2001; 2012a; 2012b).

[49] The exact number of price quotes obtained varied slightly over time. Currently, 3 to 15 price quotes are obtained for each PCGA. This number was slightly smaller in the earlier parts of our sample period but the basic methodology described in the following did not change substantially (see Statistics Canada, 2001, 2012a).

(and by necessity), the sample from which price quotes were obtained was thus based on the set of currently active firms. In a second step, Statistics Canada combined PCGA price indices into industry-level PPIs using current shipment values as weights.[50] The number of PCGAs indices used as inputs varied across industries but was generally low, at around 4-5 PCGAs per industry.

The choice of an appropriate deflator in the computation of our theoretical moments depends on what we consider the most appropriate counterpart in the data to the 'industries' in the model. In the data, an industry at the level of aggregation we are working at comprises on average only 4-5 PCGAs. In contrast, there were around 40,000 establishments in Canadian manufacturing in 1988, or around 200 per industry. This means that each PCGA product will be produced by dozens or even hundreds of producers. Thus, it seems appropriate to associate product varieties in our model with varieties of PCGA products in the data (with each firm producing one variety of a PCGA product). For the multi-product firm version of our model, it would seem natural to associate products with PCGAs and product varieties with PCGA varieties.

Hence, if we associate model varieties with PCGA product varieties, our theoretical PPI should be calculated following the random sampling procedure outlined above. That is, we calculate an average price in each period based on a set of active domestic producers in the period. As discussed, the way in which individual price quotes are weighted varies by PCGA, but generally gives more weight to producers with larger market shares.

Thus, we compute a theoretical PPI which captures these features while preserving a tight link to theoretical productivity. Specifically, we use the factory gate price charged by the firm with average productivity, $p\left(\tilde{\gamma}_{ih}\right) = \left[\sigma_i / \left(\sigma_i - 1\right)\right] w_h / \tilde{\gamma}_{ih}$, where

$$\tilde{\gamma}_{ih} = \left[\sum_j \frac{N_{ihj}}{\sum_j N_{ihj}} \left(\tilde{\gamma}_{ihj}\right)^{\sigma_i - 1} \right]^{\frac{1}{\sigma_i - 1}} \tag{A.1}$$

[50]Note that Statistics Canada did not use chain price indices during our sample period. This seems to be different from current methods used in other countries such as the United States (see Statistics Canada, 2001; Burstein and Cravino, 2014).

and

$$\tilde{\gamma}_{ihj} = \frac{1}{1 - G_{ih}\left(\gamma^*_{ihj}\right)} \int_{\gamma^*_{ihj}}^{\infty} \gamma^{\sigma_i - 1} g_{ih}(\gamma)\, d\gamma = \left(\frac{a^i_\gamma}{a^i_\gamma - \sigma_i + 1}\right)^{\frac{1}{\sigma_i - 1}} \gamma^*_{ihj}. \tag{A.2}$$

As noted by Melitz (2003), $\tilde{\gamma}_{ihj}$ can be interpreted as a weighted average of firm productivities, where the weights reflect the relative output shares of firms. Also note that $\tilde{\gamma}_{ihj}$ is calculated as an average across active firms, reflecting the sampling procedure of Statistics Canada. Thus,

$$\frac{p'_{ih}}{p_{ih}} = \frac{p\left(\tilde{\gamma}_{ih}\right)}{p\left(\tilde{\gamma}'_{ih}\right)} = \frac{\tilde{\gamma}'_{ih}}{\tilde{\gamma}_{ih}}, \tag{A.3}$$

where

$$\frac{\tilde{\gamma}'_{ih}}{\tilde{\gamma}_{ih}} = \left\{\left[\sum_j \left(\frac{N'_{ihj}}{N_{ihj}} \frac{N_{ihj}}{\sum_{j'} N_{ihj'}}\right)\right]^{-1} \sum_j \left[\frac{N'_{ihj}}{N_{ihj}} \left(\frac{\tilde{\gamma}'_{ihj}}{\tilde{\gamma}_{ihj}}\right)^{\sigma_i - 1} \frac{N_{ihj}\left(\tilde{\gamma}_{ihj}\right)^{\sigma_i - 1}}{\sum_{j'} N_{ihj'}\left(\tilde{\gamma}_{ihj'}\right)^{\sigma_i - 1}}\right]\right\}^{\frac{1}{\sigma_i - 1}}. \tag{A.4}$$

We do not have data for N_{ihj}, but under the assumption that entry costs are proportional to some observable destination-specific factor that is exogenous to the model (such as m_{ij} in our baseline model):

$$\frac{N_{ihj}}{\sum_j N_{ihj}} = \frac{X_{ihj}/\left(\frac{a^i_\gamma \sigma_i}{a^i_\gamma - \sigma_i + 1}\right) F_{ihj}}{\sum_j X_{ihj}/\left(\frac{a^i_\gamma \sigma_i}{a^i_\gamma - \sigma_i + 1}\right) F_{ihj}} = \frac{X_{ihj}/m_{ij}}{\sum_j X_{ihj}/m_{ij}}. \tag{A.5}$$

From (8) and $N_{ihj} = \left(k^i_h/\gamma^*_{ihj}\right)^{a^i_\gamma} M_{ih}$,

$$\tilde{\gamma}_{ihj} = \left(\frac{a^i_\gamma}{a^i_\gamma - \sigma_i + 1}\right)^{\frac{1}{\sigma_i - 1}} \gamma^*_{ihj} = \left(\frac{a^i_\gamma}{a^i_\gamma - \sigma_i + 1}\right)^{\frac{1}{\sigma_i - 1}} k^i_h \left(M_{ih}\right)^{\frac{1}{a^i_\gamma}} N_{ihj}^{-\frac{1}{a^i_\gamma}}. \tag{A.6}$$

We can approximate

$$\frac{N_{ihj}\left(\tilde{\gamma}_{ihj}\right)^{\sigma_i - 1}}{\sum_{j'} N_{ihj'}\left(\tilde{\gamma}_{ihj'}\right)^{\sigma_i - 1}} = \frac{\left(X_{ihj}/m_{ij}\right)^{\frac{1 - \sigma_i + a^i_\gamma}{a^i_\gamma}}}{\sum_{j'}\left(X_{ihj'}/m_{ij'}\right)^{\frac{1 - \sigma_i + a^i_\gamma}{a^i_\gamma}}}. \tag{A.7}$$

In an (unreported) robustness check, we also experimented with giving equal weight to the prices charged by active firms. This means that changes in the PPI are not affected by market share reallocations among active producers but will still reflect changes in the set of active firms.[51] The resulting PPI is:[52]

$$p_{ih}^{m} = \frac{1}{1 - G_{ih}\left(\gamma_{hh}^{*}\right)} \int_{\gamma_{ihh}^{*}}^{\infty} p_{ih}\left(\gamma\right) dG_{ih}\left(\gamma\right) = \frac{\sigma_i}{\sigma_i - 1} \left(\frac{k_{\gamma}^{ih}}{\gamma_{ihh}^{*}}\right)^{-a_{\gamma}^{i}} \int_{\gamma_{ihh}^{*}}^{\infty} \gamma^{-1} dG_{ih}\left(\gamma\right) = \frac{\sigma_i}{\sigma_i - 1} \frac{a_{\gamma}^{i}}{a_{\gamma}^{i} + 1} \left(\gamma_{ihh}^{*}\right)^{-1} \tag{A.8}$$

with growth rate

$$\hat{p}_{ih}^{m} = \left(\hat{\gamma}_{ihh}^{*}\right)^{-1}. \tag{A.9}$$

In practice, this alternative approach to constructing the PPI yielded very similar results to the ones in Table 2, with a GMM objective function value of 91.9835 compared to 91.7885 for the baseline model.

B Estimation Procedure for a_{γ}^{i} and σ_i

This appendix describes how we obtain estimates for the elasticity of substitution (σ_i) and the shape parameter of the Pareto distribution of productivities (a_{γ}^{i}) from data sources not used in the model calibration.

We start by noting that total sales by exporting firms can be expressed as $r_{ih}\left(\gamma\right) = \sum_j r_{hj}(\gamma) = \Lambda_1 \gamma^{\sigma-1}$, which is proportional to $\gamma^{\sigma-1}$ (the term Λ_1 is constant across firms). Since γ is distributed Pareto with shape parameter a_{γ}, sales are distributed Pareto with shape parameter $a_{r}^{i} = a_{\gamma}^{i} / \left(\sigma_i - 1\right)$ and cut-off $k_{r}^{i} = \Lambda_1 \left(k^{i}\right)^{\sigma_i - 1}$. Thus, we can estimate a_{r}^{i} and σ_i, and then recover a_{γ}^{i}.

[51] It is unclear how frequently Statistics Canada updated price weights in the earlier years of our sample (Statistics Canada, 1991; 1993) so this procedure might have relevance here. But price quotes could of course only be obtained from active firms, so that changes in the set of active firms will influence price changes.

[52] We are assuming here that (i) the prices used to compute this average price index are also measured at factory gates, and (ii) all firms are sampled with the same probability (hence the lower limit γ_{ihh}^{*} in the integral sign and the lack of firm-specific weights on individual firm-specific prices).

Obtaining of σ from Firm-level Data

In our baseline model, operating profits (that is, profits net of fixed costs) are

$$\pi^{o}(\gamma) = \frac{r(\gamma)}{\sigma}. \tag{B.1}$$

We use data on operating profits (π^o) and revenue (r) for US and Canadian firms from Compustat North America and Compustat Global. We proxy π^o as operating income before depreciation and r as net sales.[53] From (B.1) we can obtain estimates of σ for each firm in our data. Industry-specific estimates of σ are calculated as the median across all firms within each of our 203 manufacturing industries.

Obtaining a_r from Sales Data

Aggregate sales for firms with sales equal or larger than r_x are (assuming $a_r > 1$):

$$R_{r_x} = \int_{r_x}^{\infty} r v(r) dr = \frac{a_r k_r^{a_r}}{a_r - 1} (r_x)^{1-a_r}. \tag{B.2}$$

Take the sales value r_x that corresponds to the x-th largest firm. The fraction n_{r_x} of firms that are bigger than or equal to this firm is $n_{r_x} = 1 - V(r_x)$. Hence, $r_x = k_r n_{r_x}^{-(1/a_r)}$. Taking the ratio to the $y-$th largest firm's sales eliminates k_r: $\frac{r_x}{r_y} = \left(\frac{n_{r_y}}{n_{r_x}}\right)^{1/a_r}$. We do not have data on r_x, but we know the sales volume R_{r_x} defined above (total shipments times the appropriate concentration ratio):

$$\left(\frac{R_{r_x}}{R_{r_y}}\right)^{1/(1-a_r)} = \left(\frac{n_{r_y}}{n_{r_x}}\right)^{1/a_r}. \tag{B.3}$$

Solving for a_r,

$$a_r = \frac{(\ln n_{r_y} - \ln n_{r_x})}{(\ln R_{r_x} - \ln R_{r_y}) + (\ln n_{r_y} - \ln n_{r_x})}. \tag{B.4}$$

[53] Information on these variables is contained in Compustat North America data items 12 (net sales) and 13 and 189 (operating income before depreciation and administrative expenses; note that we do not include the latter in the computation of costs). For Compustat Global, net sales are contained in data item 1 and operating profits are calculated as operating income plus depreciation plus administrative expenses (data items 14 plus 11 plus 189).

If firm x is larger than firm y, we have $n_{r_y} > n_{r_x}$ and $R_{r_y} > R_{r_x}$. Thus, $a_r > 1$ from above as long as $\left(\ln R_{r_x} - \ln R_{r_y}\right) + \left(\ln n_{r_y} - \ln n_{r_x}\right) > 0$, which holds by construction.

We use information from Statistics Canada on the output share accounted for by the top 4 and 8 enterprises in each Canadian manufacturing industry in our data. Multiplying these shares with total industry output (R_d) we obtain the total output of the top 4 and top 8 enterprises which we use as proxies for R_{r_x}. Note that using comparable data for the US yields qualitatively similar results to the ones reported in Table 2. (Recall that we are imposing a common shape parameter across countries, so that either of these two data sources can be used.)

C Alternative Modeling of Tariffs

Our assumptions about preferences, technology, market power, labour markets and entry are the same as in our baseline model (see Section 3). As before, we also assume that homogeneous good is traded freely; supplying it to any market and entering the market involves no costs. The final goods Q are still not traded and supplying them or entering the (domestic) market involves no costs either. A manufacturing industry-i firm based in country h faces the same fixed cost F_{ihj} of supplying country j as in the baseline model.

The key difference to the baseline model is that we now assume that for the varieties produced by the manufacturing industries, iceberg trade costs take the form $\tau_{ihj} = (1 + c_{ihj})$ for $j \neq h$ and $\tau_{ijj} = 1$. As before, h and j denote the exporting and importing country, respectively and $c_{ihj} > 0$ denotes "natural" transport costs. Note that iceberg trade costs now exclude policy-induced trade barriers. We model be separately in the form of ad-valorem tariffs $t_{ihj} > 0$ (with $T_{ihj} \equiv 1 + t_{ihj}$).

This changes the firm's profit maximisation problem to problem to

$$\max \pi_{ihj} = \frac{p_{ihj}}{1+t_{ihj}} q_{ihj}\left(p_{ihj}\right) - \tau_{ihj} q_{ihj}\left(p_{ihj}\right) \frac{1}{\gamma} - f_{ihj} = \frac{1}{1+t_{ihj}} p_{ihj}^{1-\sigma_i} P_{ij}^{\sigma_i - 1} E_{ij} - \tau_{ihj} p_{ihj}^{-\sigma_i} P_{ij}^{\sigma_i - 1} E_{ij} \frac{1}{\gamma} - F_{ihj}, \tag{C.1}$$

where p_{ihj} denotes the price paid by the consumers of the importing country. The first order condition yields

$$p_{ihj} = \frac{\sigma_i}{\sigma_i - 1} \tau_{ihj} T_{ihj} \frac{1}{\gamma}. \tag{C.2}$$

The resulting expression for the threshold value of productivity γ^*_{ihj} that leads country-h firms to select into market j is:

$$\gamma^*_{ihj} = \frac{\sigma_i}{\sigma_i - 1} \frac{\tau_{ihj}}{P_{ij}} \left(\frac{\sigma_i F_{ihj}}{m_{ij}} \right)^{\frac{1}{\sigma_i - 1}} T_{ihj}^{\frac{\sigma_i}{\sigma_i - 1}}. \tag{C.3}$$

The average productivity of country-h firms exporting to market j, defined as in Melitz (2003), can be expressed as

$$\tilde{\gamma}_{ihj} = \left(\frac{a^i_\gamma}{a^i_\gamma - \sigma_i + 1} \right)^{\frac{1}{\sigma_i - 1}} \gamma^*_{ihj}. \tag{C.4}$$

The expected revenue and expected profit that a country-h firm obtains in country j, conditional upon selecting into that market, are respectively

$$E\left[r_{ihj}(\gamma) | \gamma > \gamma^*_{ihj} \right] = r_{ihj}(\tilde{\gamma}_{ihj}) = \frac{a^i_\gamma T_{ihj}^{-\sigma_i}}{a^i_\gamma - \sigma_i + 1} \left(\frac{\sigma_i}{\sigma_i - 1} \tau_{ihj} \frac{1}{P_{ij} \gamma^*_{ihj}} \right)^{1-\sigma_i} E_{ij} = \frac{a^i_\gamma \sigma_i}{a^i_\gamma - \sigma_i + 1} F_{ihj}, \tag{C.5}$$

$$E\left[\pi_{ihj}(\gamma) | \gamma > \gamma^*_{ihj} \right] = \frac{r_{ihj}(\tilde{\gamma}_{ihj})}{\sigma_i} - F_{ihj} = \frac{\sigma_i - 1}{a^i_\gamma - \sigma_i + 1} F_{ihj}. \tag{C.6}$$

Country-h exports to country j can be expressed as

$$X_{ihj} = N_{ihj} r_{ihj}(\tilde{\gamma}_{ihj}) = N_{ihj} \frac{a^i_\gamma \sigma_i}{a^i_\gamma - \sigma_i + 1} F_{ihj}. \tag{C.7}$$

The industry's aggregate sales are then

$$R_{ih} = \sum_j X_{ihj} = \sum_j N_{ihj} \frac{a^i_\gamma \sigma_i}{a^i_\gamma - \sigma_i + 1} F_{ihj}. \tag{C.8}$$

The mass of country-h firms that select into market j is given by

$$N_{ihj} = \left(\frac{k^{ih}_\gamma}{\gamma^*_{ihj}} \right)^{a^i_\gamma} M_h. \tag{C.9}$$

Expected profits, aggregated across all destination markets, are

$$\Pi_{ih} = \sum_j prob\left(\gamma > \gamma^*_{ihj}\right) E\left[\pi_{ihj}(\gamma) | \gamma > \gamma^*_{ihj}\right] = \sum_j \left(\frac{k^{ih}_\gamma}{\gamma^*_{ihj}}\right)^{a^i_\gamma} \frac{\sigma_i - 1}{a^i_\gamma - \sigma_i + 1} F_{ihj}. \quad \text{(C.10)}$$

Industry profits are therefore

$$M_{ih}\Pi_{ih} = M_{ih}\sum_j \left(\frac{k^{ih}_\gamma}{\gamma^*_{ihj}}\right)^{a^i_\gamma} \frac{\sigma_i - 1}{a^i_\gamma - \sigma_i + 1} F_{ihj} = \frac{\sigma_i - 1}{a^i_\gamma \sigma_i} \sum_j X_{ihj}. \quad \text{(C.11)}$$

Industry employment can be easily shown to be

$$L_{ih} = M_{ih}E\left[l_{ihj}(\gamma)\right] = M_{ih}\sum_j \left(\frac{k^{ih}_\gamma}{\gamma^*_{ihj}}\right)^{a^i_\gamma} E\left[l_{ihj}(\gamma) | \gamma > \gamma^*_{ihj}\right] = a^i_\gamma M_{ih}\Pi_{ih}. \quad \text{(C.12)}$$

and the price level P_{ij} is given by

$$P_{ij} = \left[\frac{a^i_\gamma}{a^i_\gamma - \sigma_i + 1}\sum_h N_{ihj}\left(\frac{\sigma_i}{\sigma_i - 1}\frac{\tau_{ihj}T_{ihj}}{\gamma^*_{ihj}}\right)^{1-\sigma_i}\right]^{\frac{1}{1-\sigma_i}}. \quad \text{(C.13)}$$

The expressions for industry level of growth rates are unchanged except for the price index equation:

$$\hat{P}_{ij} = \left[\sum_h \frac{T_{ihj}X_{ihj}}{\sum_h T_{ihj}X_{ihj}}\hat{N}_{ihj}\left(\hat{T}_{ihj}\right)^{1-\sigma_i}\left(\hat{\gamma}^*_{ihj}\right)^{\sigma_i - 1}\right]^{\frac{1}{1-\sigma_i}}. \quad \text{(C.14)}$$

It is easy to show that

$$\hat{P}_{ij} = \left[\sum_h \frac{T_{ihj}X_{ihj}}{\sum_h T_{ihj}X_{ihj}}\left(\hat{T}_{ihj}\right)^{1-\frac{\sigma_i}{\sigma_i - 1}a^i_\gamma}\right]^{-1/a^i_\gamma}. \quad \text{(C.15)}$$

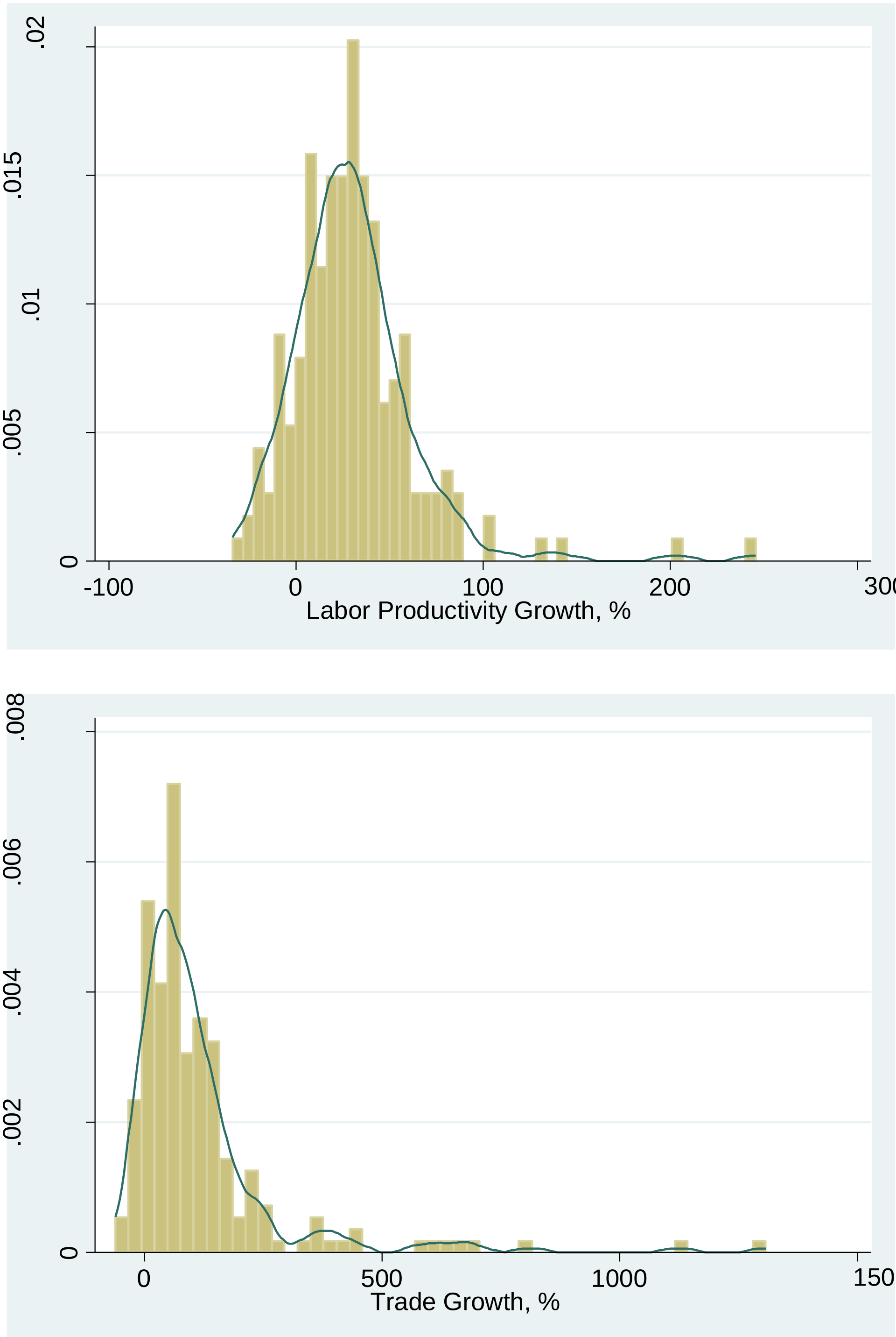
We can use the system (C.15) to solve for the growth rates of the price levels $\hat{P}_{ij}$ as a function of the changes in tariffs $\hat{T}_{ihj}$. From equations (C.3), we can solve for $\hat{\gamma}^*_{ihj}$ as a function of $\hat{P}_{ij}$ and $\hat{T}_{ihj}$,

$$\hat{\gamma}^*_{ihj} = \frac{\left(\hat{T}_{ihj}\right)^{\frac{\sigma_i}{\sigma_i - 1}}}{\hat{P}_{ij}}. \quad \text{(C.16)}$$

and thereafter generate predictions for the industry aggregates of interest.

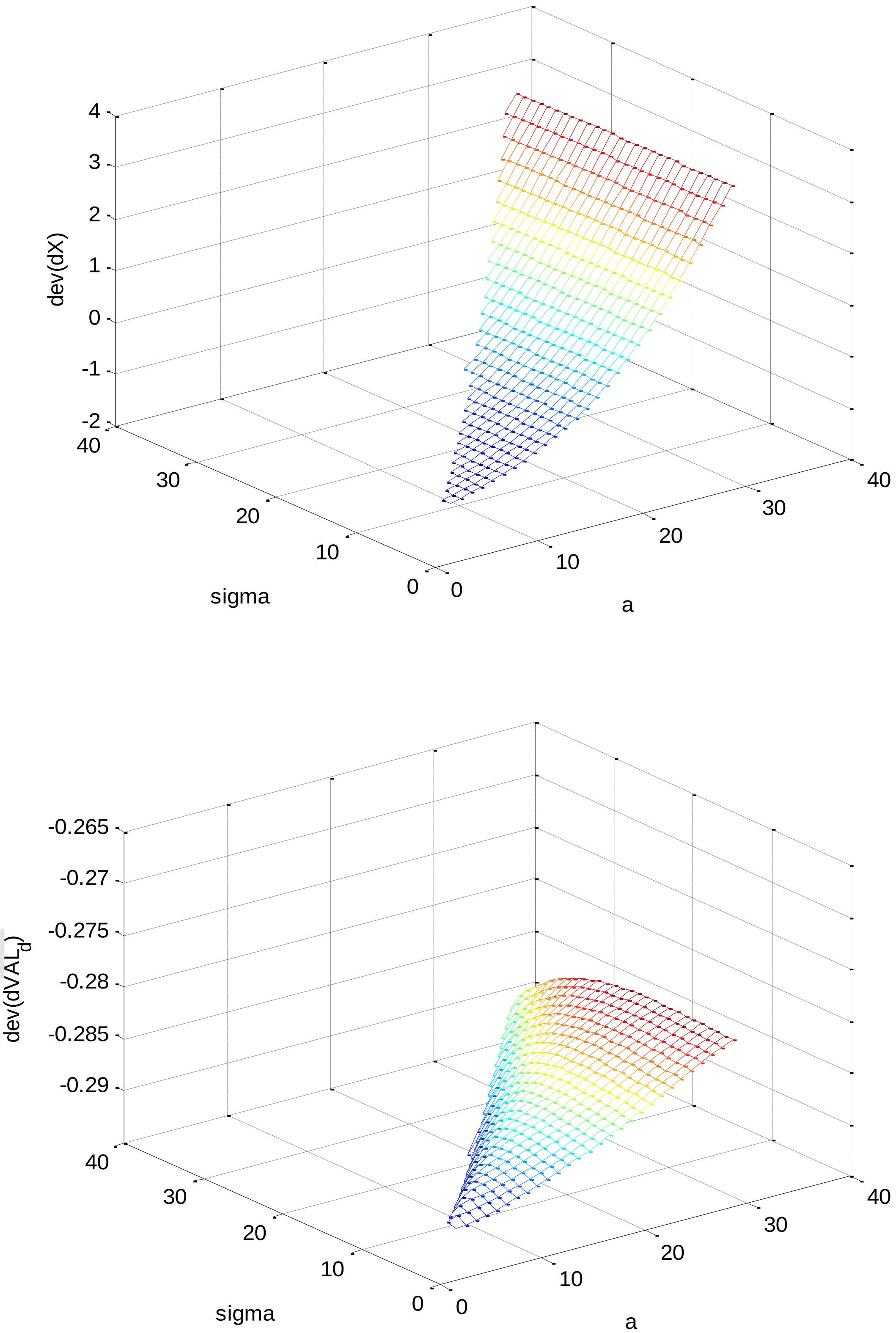
Figures and Table

Figure 1: *Increases in Trade Flows and Labor Productivity in Canada, 1988-1996*



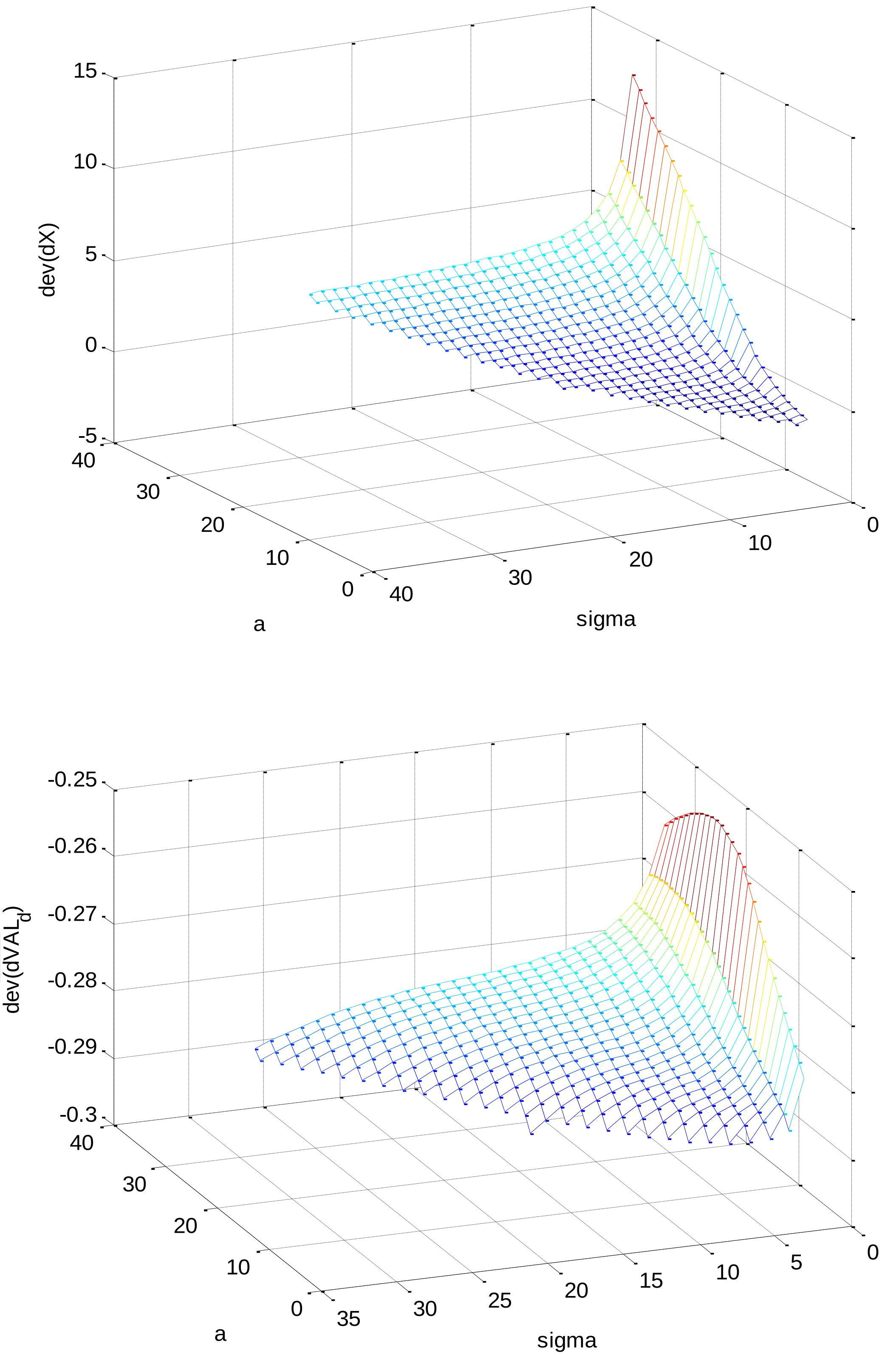
Notes: Figures show trade and labor productivity growth at the sectoral level (203 sectors) in Canadian manufacturing, 1988 to 1996. Trade is measured as Canadian exports plus imports,labor productivity is calculated as value added in production activities divided by total hours worked by production workers(see Section 4.1 for details). All data are expressed in 1992 Canadian dollars using 4-digit industry price and value added deflators, and the 1992 US-Canadian exchange rate.

Figure 2: *Moment Deviations as a Function of a_Y and σ (Trade and Productivity Growth, First Moment; Baseline Model)*



Notes: See Section 4 for details.

Figure 3: *Moment Deviations as a Function of a_γ and $\boldsymbol{\sigma}$ (Trade and Productivity Growth, First Moment; Alternative Modeling of Tariffs)*



Notes: See Section 4 for details.

Table 1: *Empirical Moments to Be Matched*

Moment	Mean(dX)	Mean(dVAL)	Cov(dX,dVAL)	Var(dX)	Var(dVAL)
Data	1.1820	0.3041	0.1007	3.0130	0.1153

Notes: Table shows empirical moments to be matched by our theoretical models. dX denotes trade growth and dVAL labor productivity growth (see Figure 1 and Section 4 for details).

Table 2: *Results for Baseline Model (Matching Growth Rates 1988-1996)*

Moments	Mean(dX)	Mean(dVAL)	Cov(dX,dVAL)	Var(dX)	Var(dVAL)
(1) Data	1.1820	0.3041	0.1007	3.0130	0.1153
(2) Model – Observed Parameter Values	0.3088	0.0139	0.0007	0.3373	0.0002
(3) Model – Optimised Parameter Values (First Step)	1.1408	0.0177	0.0039	3.0185	0.0002
(4) Model – Optimised Parameter Values (GMM)	1.0538	0.0174	0.0034	2.4004	0.0002

Parameters	Optimised (value, SE)	Data (mean, sd)
Σ	8.4942	3.4611
	(121.9231)	(0.8765)
a_γ	14.2482	5.0951
	(1.1253)***	(2.6032)
GMM objective (p-value)	91.7885	
	(0.00000)	

Notes: dX denotes trade growth and dVAL labor productivity growth (see Section 4 for details).

Table 3: *Results for Baseline Model (Higher a_γ and σ)*

Moments	Mean(dX)	Mean(dVAL)	Cov(dX,dVAL)	Var(dX)	Var(dVAL)
(1) Data	1.1820	0.3041	0.1007	3.0130	0.1153
(2) Model – Observed Parameter Values (σ x1.5)	0.7005	0.0153	0.0041	3.7584	0.0002
(3) Model – Observed Parameter Values (σ x2)	1.3797	0.0169	0.0164	29.4970	0.0002
(3) Model – Observed Parameter Values (σ x1.88, match mean trade growth)	1.1820	0.0165	0.0125	18.6332	0.0002
(4) Model – Observed Parameter Values (a_r x1.5)	0.5674	0.0149	0.0022	1.9818	0.0002
(5) Model – Observed Parameter Values (a_r x 2)	0.9529	0.0158	0.0077	9.6694	0.0002
(6) Model – Observed Parameter Values (a_r x2.23, match mean trade growth)	1.1820	0.0163	0.0122	18.5781	0.0002

Notes: dX denotes trade growth and dVAL labor productivity growth (see Section 4 for details).

Table 4: *Results for Baseline Model (Drop Outliers)*

Panel A: drop top and bottom 5%

Moments	**Mean(dX)**	**Mean(dVAL)**	**Cov(dX,dVAL)**	**Var(dX)**	**Var(dVAL)**
(1) Data	0.9737	0.2858	0.0372	0.6675	0.0453
(2) Model – Observed Parameter Values	0.2893	0.0153	0.0005	0.1440	0.0002
(3) Model – Optimised Parameter Values (First Step)	0.7657	0.0171	0.0007	0.7436	0.0002
(4) Model – Optimised Parameter Values	0.8085	0.0183	0.0015	0.8770	0.0002

Parameters	**Optimised (value, SE)**	**Data (mean, sd)**
Σ	7.0894	3.5161
	(72.9391)	(0.9040)
a_γ	12.5859	5.1936
	(0.8968)***	(2.5479)
GMM objective (p-value)	107.378	
	(0.00000)	

Panel B: drop top 5% only

(5) Data	0.8739	0.2495	0.0542	0.7049	0.0553
(6) Model – Observed Parameter Values	0.2679	0.0143	0.0006	0.1325	0.0002
(7) Model – Optimised Parameter Values (First Step)	0.7365	0.0162	0.0013	0.7526	0.0002
(8) Model – Optimised Parameter Values	0.7606	0.0172	0.0021	0.8324	0.0002

Parameters	**Optimised (value, SE)**	**Data (mean, sd)**
Σ	7.1215	3.5011
	(80.0526)	(0.8933)
a_γ	12.6347	5.1776
	(0.9331)***	(2.5549)
GMM objective (p-value)	112.996	
	(0.00000)	

Notes: dX denotes trade growth and dVAL labor productivity growth (see Section 4 for details).

Table 5: *Results for Baseline Model (Prices at Factory Gate)*

Moments	**Mean(dX)**	**Mean(dVAL)**	**Cov(dX,dVAL)**	**Var(dX)**	**Var(dVAL)**
(1) Data	1.1820	0.3041	0.1007	3.0130	0.1153
(2) Model – Observed Parameter Values	0.3857	0.0171	0.0005	0.4862	0.0002
(3) Model – Optimised Parameter Values (First Step)	1.1631	0.0198	0.0016	3.0156	0.0003
(4) Model – Optimised Parameter Values (GMM)	1.0653	0.0196	0.0014	2.3443	0.0003

Parameters	**Optimised (value, SE)**	**Data (mean, sd)**
Σ	8.0972	3.4611
	(127.1901)	(0.8765)
a_γ	13.2980	5.0951
	(1.3123)***	(2.6032)
GMM objective (p-value)	90.6741	
	(0.00000)	

Notes: dX denotes trade growth and dVAL labor productivity growth (see Section 4 for details).

Table 6: *Results for Baseline Model (Alternative Modeling Approach to Tariffs)*

Moments	**Mean(dX)**	**Mean(dVAL)**	**Cov(dX,dVAL)**	**Var(dX)**	**Var(dVAL)**
(1) Data	1.1820	0.3041	0.1007	3.0130	0.1153
(2) Model – Observed Parameter Values	0.5154	0.0191	0.0029	1.3670	0.0003
(3) Model – Optimised Parameter Values (First Step)	1.1584	0.0330	0.0058	3.0195	0.0009
(4) Model – Optimised Parameter Values (GMM)	1.0705	0.0323	0.0050	2.3986	0.0009

Parameters	**Optimised (value, SE)**	**Data (mean, sd)**
Σ	2.0101	3.4611
	(1.5114)	(0.8765)
a_γ	7.1208	5.0951
	(5.3689)	(2.6032)
GMM objective (p-value)	85.6412	
	(0.00000)	

Notes: dX denotes trade growth and dVAL labor productivity growth (see Section 4 for details).

Table 7: *Results for Baseline Model (First Differences 1980-1988 to 1988-1996)*

Moments	**Mean(dX)**	**Mean(dVAL)**	**Cov(dX,dVAL)**	**Var(dX)**	**Var(dVAL)**
(1) Data	0.4621	0.1095	0.0493	0.6642	0.1193
(2) Model – Observed Parameter Values	0.1367	0.0062	0.0011	0.0649	0.0002
(3) Model – Optimised Parameter Values (First Step)	0.5216	0.0112	0.0078	0.6409	0.0004
(4) Model – Optimised Parameter Values (GMM)	0.4772	0.0104	0.0064	0.5376	0.0003

Parameters	**Optimised (value, SE)**	**Data (mean, sd)**
Σ	11.1387	3.4611
	(488.2403)	(0.8765)
a_γ	19.9583	5.0951
	(3.8789)***	(2.6032)
GMM objective (p-value)	95.5912	
	(0.00000)	

Notes: dX denotes trade growth and dVAL labor productivity growth (see Section 4 for details).

Table 8: *Results for Baseline Model (Diff-in-Diff Predicted Values)*

Moments	**Mean(dX)**	**Mean(dVAL)**	**Cov(dX,dVAL)**	**Var(dX)**	**Var(dVAL)**
(1) Data	0.1457	0.0104	0.0087	0.0576	0.0044
(2) Model – Observed Parameter Values	0.1072	0.0032	0.0008	0.0292	0.0000
(3) Model – Optimised Parameter Values (First Step)	0.1487	0.0035	0.0013	0.0537	0.0000
(4) Model – Optimised Parameter Values (GMM)	0.1411	0.0017	0.0005	0.0483	0.0000

Parameters	**Optimised (value, SE)**	**Data (mean, sd)**
Σ	8.2290	3.4611
	(1.9631)***	(0.8765)
a_γ	7.2313	5.0951
	(0.0983)***	(2.6032)
GMM objective (p-value)	42.3101	
	(0.00000)	

Notes: dX denotes trade growth and dVAL labor productivity growth (see Section 4 for details).

Table 9: *Results for Baseline Model (Changes in Non-Tariff Trade Costs)*

Moments	Mean(dX)	Mean(dVAL)	Cov(dX,dVAL)	Var(dX)	Var(dVAL)
(1) Data	1.1820	0.3041	0.1007	3.0130	0.1153
(2) Model – Observed Parameter Values (c=1.1)	0.2055	0.0027	0.0073	1.5017	0.0004
(3) Model – Observed Parameter Values (c=0.9)	2.4642	0.0679	0.0481	65.7687	0.0029
(4) Model – Observed Parameter Values (c=0.98))	1.1820	0.0367	0.0225	15.0791	0.0011
(3) Model – Optimised Parameter Values (First Step)	1.1878	0.0187	0.0037	3.0124	0.0003
(4) Model – Optimised Parameter Values (GMM)	0.8924	0.2948	-0.0116	0.0582	0.0849

Parameters	Optimised (value, SE)
Σ	1.0100 (0.5206)*
a_γ	0.7209 (0.1059)***
c	0.4322 (0.0364)***
GMM objective (p-value)	9.7516 (0.0076)

Notes: dX denotes trade growth and dVAL labor productivity growth (see Section 4 for details).

Table 10: *Results for Baseline Model with Free Entry*

Moments	Mean(dX)	Mean(dVAL)	Cov(dX,dVAL)	Var(dX)	Var(dVAL)
(1) Data	1.1820	0.3041	0.1007	3.0130	0.1153
(2) Model – Observed Parameter Values	0.3455	0.0076	0.0007	0.3877	0.0001
(3) Model – Optimised Parameter Values (First Step)	1.2533	0.0091	0.0116	3.2869	0.0002
(4) Model – Optimised Parameter Values (GMM)	1.1451	0.0090	0.0089	2.4802	0.0001

Parameters	Optimised (value, SE)	Data (mean, sd)
σ	8.3205 (90.6883)	3.4611 (0.8765)
a_γ	13.1861 (0.8983)	5.0951 (2.6032)
GMM objective (p-value)	83.2979 (0.0000)	

Notes: dX denotes trade growth and dVAL labor productivity growth (see Section 5 for details).

Table 11: *Results for the 'General Equilibrium' Extension*

Moments	**Mean(dX)**	**Mean(dVAL)**	**Cov(dX,dVAL)**	**Var(dX)**	**Var(dVAL)**
(1) Data	1.1820	0.3041	0.1007	3.0130	0.1153
(2) Model – Observed Parameter Values	0.3533	0.0088	0.0005	0.3863	0.0001
(3) Model – Optimised Parameter Values (First Step)	1.1310	0.0098	0.0147	3.2430	0.0001
(4) Model – Optimised Parameter Values (GMM)	0.9590	0.0119	0.0122	1.9985	0.0002

Parameters	**Optimised (value, SE)**	**Data (mean, sd)**
Σ	7.1959	3.4611
	(68.4995)	(0.8765)
a_γ	11.5299	5.0951
	(0.0796)***	(2.6032)
GMM objective (p-value)	83.5599 (0.0000)	

Notes: dX denotes trade growth and dVAL labor productivity growth (see Section 5 for details).

Table 12: *Results for the 'Intermediate Inputs' Extension*

Moments	**Mean(dX)**	**Mean(dVAL)**	**Cov(dX,dVAL)**	**Var(dX)**	**Var(dVAL)**
(1) Data	1.1820	0.3041	0.1007	3.0130	0.1153
(2) Model – Observed Parameter Values	0.3541	0.0295	0.0100	0.5171	0.0020
(3) Model – Optimised Parameter Values (First Step)	1.2208	0.0466	0.0245	2.9930	0.0013
(4) Model – Optimised Parameter Values (GMM)	1.4581	0.1470	0.2139	3.7357	0.0393

Parameters	**Optimised (value, SE)**	**Data (mean, sd)**
Σ	12.9013	3.4611
	(0.0211)***	(0.8765)
a_γ	11.9700	5.0951
	(0.0018)***	(2.6032)
α	0.9871	0.7065
	(0.0025)***	(0.0918)
GMM objective (p-value)	44.8265 (0.0000)	

Notes: dX denotes trade growth and dVAL labor productivity growth (see Section 5 for details).

Table 13: *Results for the Multiproduct-Firm Model*

Moments	**Mean(dX)**	**Mean(dVAL)**	**Cov(dX,dVAL)**	**Var(dX)**	**Var(dVAL)**
(1) Data	1.1820	0.3041	0.1007	3.0130	0.1153
(2) Model – Observed Parameter Values	--	--	--	--	--
(3) Model –Optimised Parameter Values (First Step)	1.2392	0.2738	0.1552	3.0034	0.0976
(4) Model – Optimised Parameter Values (GMM)	1.2175	0.2855	0.1549	2.8239	0.1073

Parameters	**Optimised (value, SE)**	**Data (mean, sd)**
X	1.1276 (0.0786)***	--
a_γ	8.1552 (1.1506)***	--
a_λ	4.9922 (0.5831)***	--
GMM objective (p-value)	3.3434 (0.1879)	

Notes: dX denotes trade growth and dVAL labor productivity growth (see Section 5 for details).

Table 14: *Out-of-Sample Predictions*

Model	**Parameters**	**Moments**	**d.o.f.**	**GMM objective in-sample (p-value)**	**GMM objective out-of-sample**
Baseline	2	5	3	91.7885 (0.0000)	99.2693
Free entry	2	5	3	83.2979 (0.0000)	102.1245
General equilibrium	2	5	3	83.5599 (0.0000)	90.4165
Intermediates	3	5	2	44.2633 (0.0000)	106.2829
Multiproduct	3	5	2	3.3434 (0.1879)	42.7024

Notes: See Section 5 for details.

Online Appendix to: Tariffs, Trade and Productivity: A Quantitative Evaluation of Heterogeneous Firm Models

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September 29, 2014

1 Introduction

This appendix provides a detailed mathematical treatment of the models discussed in the paper. For simplicity, each model is presented in a self-contained manner.

2 Baseline model

2.1 Assumptions

2.1.1 Preferences

There are many countries. Each country admits a representative agent, with preferences

$$U = \sum_{i \in I} m_i \ln Q_i + A, \tag{1}$$

where $m_i > 0$. A denotes consumption of a homogeneous final good. Q_i denotes consumption of a Dixit-Stiglitz aggregate (manufacturing) final good i:[1]

$$Q_i = \left[\int_{\gamma \in \Gamma_i} q_i(\gamma)^{\rho_i} d\gamma \right]^{\frac{1}{\rho_i}}, \tag{2}$$

where $\rho \in (0,1)$ and $\sigma \equiv 1/(1-\rho)$ denotes the elasticity of substitution between any two varieties. Choosing good A as the numéraire, utility maximization on the upper level yields demand functions $A = Y - \sum_i m_i$ and $Q_i = m_i / P_i$, where Y is total expenditure per consumer. In the manufacturing goods sector, utility maximization yields demand function $q_i(\gamma) = p_i(\gamma)^{-\sigma} P_i^{\sigma-1} m_i$.

[1] Wherever possible, we dispense with industry index i and with country indexes.

2.1.2 Technology

The homogeneous good is made with labor l and a linear technology $A = l_A$ that is identical across countries. Manufacturing varieties are made with the production function $q_i(\gamma) = \gamma l_i(\gamma)$, where γ denotes (firm-specific) total factor productivity. γ is iid across firms within an industry. For tractability purposes, we assume γ to be distributed Pareto with shape parameter a_γ and location parameter k_γ. We assume the same shape parameter for an industry across countries, but allow it to vary across industries. The location parameter is allowed to vary across industries and countries.

2.1.3 Market power

Producers of the homogeneous good and the final goods Q operate in a perfectly competitive environment. Producers of varieties in the manufacturing industry have instead monopoly power over their own varieties.

2.1.4 Fixed and transport costs

The homogeneous good is traded freely; supplying it to any market and entering the market involves no costs. We consider equilibria in which all countries produce positive amounts of this good, thus leading to the equalization of wages across countries. (We normalize wages to one.) The final goods Q are not traded; supplying them or entering the (domestic) market involves no costs either.

For the varieties produced by the manufacturing industries, we assume iceberg transport costs, which take the form $\tau_{hj} = (1 + c_{hj})(1 + t_{hj})$ for $j \neq h$ and $\tau_{jj} = 1$. (h and j denote the exporting and importing country, respectively.) c_{hj} denotes "natural" transport costs, and t_{hj} denotes policy-induced trade barriers. We can safely ignore tariff revenue, given the quasi-linear utility assumption above.

A manufacturing industry-i firm based in country h faces a fixed cost F_{hj} of supplying country j. Fixed costs are in terms of the destination country's labor.

Fixed costs and transport costs are allowed to vary across industries and country-pairs.

2.1.5 Entry

We assume there is no free entry in the manufacturing sectors: there is a given mass of firms M that pick a draw from the distribution of γ prior to any decision.

2.1.6 Labor market

The labor market is perfectly competitive.

2.2 Firm-level and industry outcomes

The pricing decision over the variety produced by a country-h firm with productivity γ is the usual mark-up over marginal cost. Well-known manipulation of firm revenue and profit functions yields to the following expression for the threshold value of productivity γ^*_{hj} that leads country-h firms to select into market j:

$$\gamma^*_{hj} = \frac{\sigma}{\sigma-1}\frac{\tau_{hj}}{P_j}\left(\frac{\sigma F_{hj}}{m_j}\right)^{\frac{1}{\sigma-1}}. \tag{3}$$

The average productivity of country-h firms exporting to market j, defined as in Melitz (2003), can be expressed as

$$\tilde{\gamma}_{hj} = \left(\frac{a_\gamma}{a_\gamma-\sigma+1}\right)^{\frac{1}{\sigma-1}}\gamma^*_{hj}. \tag{4}$$

The expected revenue and expected profit that a country-h firm obtains in country j, conditional upon selecting into that market, are respectively

$$E\left[r_{hj}(\gamma)|\,\gamma>\gamma^*_{hj}\right] = r_{hj}\left(\tilde{\gamma}_{hj}\right) = \frac{a_\gamma\sigma}{a_\gamma-\sigma+1}F_{hj}, \tag{5}$$

$$E\left[\pi_{hj}(\gamma)|\,\gamma>\gamma^*_{hj}\right] = \frac{r_{hj}\left(\tilde{\gamma}_{hj}\right)}{\sigma} - F_{hj} = \frac{\sigma-1}{a_\gamma-\sigma+1}F_{hj}. \tag{6}$$

Country-h exports to country j can be expressed as

$$X_{hj} = N_{hj}r_{hj}\left(\tilde{\gamma}_{hj}\right) = N_{hj}\frac{a_\gamma\sigma}{a_\gamma-\sigma+1}F_{hj}. \tag{7}$$

The industry's aggregate sales are then

$$R_h = \sum_j X_{hj} = \sum_j N_{hj}\frac{a_\gamma\sigma}{a_\gamma-\sigma+1}F_{hj}. \tag{8}$$

The mass of country-h firms that select into market j is given by

$$N_{hj} = \left(\frac{k_h}{\gamma^*_{hj}}\right)^{a_\gamma}M_h. \tag{9}$$

Expected profits, aggregated across all destination markets, are

$$\Pi_h = \sum_j prob\left(\gamma>\gamma^*_{hj}\right)E\left[\pi_{hj}(\gamma)|\,\gamma>\gamma^*_{hj}\right] = \sum_j\left(\frac{k_h}{\gamma^*_{hj}}\right)^{a_\gamma}\frac{\sigma-1}{a_\gamma-\sigma+1}F_{hj}. \tag{10}$$

Industry profits are therefore

$$M_h\Pi_h = M_h\sum_j\left(\frac{k_h}{\gamma^*_{hj}}\right)^{a_\gamma}\frac{\sigma-1}{a_\gamma-\sigma+1}F_{hj} = \frac{\sigma-1}{a_\gamma\sigma}\sum_j X_{hj}. \tag{11}$$

Industry employment can be easily shown to be

$$L_h = M_h E\left[l_{hj}(\gamma)\right] = M_h \sum_j \left(\frac{k_h}{\gamma^*_{hj}}\right)^{a_\gamma} E\left[l_{hj}(\gamma)\middle|\, \gamma > \gamma^*_{hj}\right] = a_\gamma M_h \Pi_h. \quad (12)$$

The price level P_j is given by

$$P_j = \left[\frac{a_\gamma}{a_\gamma - \sigma + 1} \sum_h N_{hj} \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{hj}}{\gamma^*_{hj}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \quad (13)$$

Melitz (2003) defines industry productivity as

$$\tilde{\gamma}_h = \left[\sum_j \frac{N_{hj}}{\sum_j N_{hj}} \left(\tilde{\gamma}_{hj}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}. \quad (14)$$

2.3 Growth rates

The growth rates of the industry's aggregates can be expressed as functions of parameter values, changes in τ_{hj}, changes in thresholds γ^*_{hj}, and the levels of bilateral trade X_{hj}. Define $\hat{x} \equiv x'/x$ as a gross growth rate, where x and x' denote, respectively, the values of a variable before and after the trade liberalization:

$$\hat{X}_{hj} = \hat{N}_{hj} = \left(\hat{\gamma}^*_{hj}\right)^{-a_\gamma}, \quad (15)$$

$$\hat{R}_h = \hat{\Pi}_h = \hat{L}_h = \sum_j \frac{X_{hj}}{\sum_j X_{hj}} \hat{X}_{hj}, \quad (16)$$

$$\hat{P}_j = \left[\sum_h \frac{X_{hj}}{\sum_h X_{hj}} \hat{N}_{hj} \left(\hat{\tau}_{hj}\right)^{1-\sigma} \left(\hat{\gamma}^*_{hj}\right)^{\sigma-1}\right]^{\frac{1}{1-\sigma}}. \quad (17)$$

As a measurable proxy for productivity growth, we will consider the growth rate of deflated value added per worker,

$$\frac{\hat{R}_h}{\hat{L}_h} \frac{p\left(\tilde{\gamma}_h\right)}{p\left(\tilde{\gamma}'_h\right)} = \frac{\hat{R}_h}{\hat{L}_h} \frac{\tilde{\gamma}'_h}{\tilde{\gamma}_h} = \frac{\tilde{\gamma}'_h}{\tilde{\gamma}_h}, \quad (18)$$

where we use the price charged by the firm with average productivity $p\left(\tilde{\gamma}_h\right)$ as a deflator. (See Section 3 and Appendix A in the paper for a detailed discussion of the appropriate choice of deflator.) The growth rate $\tilde{\gamma}'_h/\tilde{\gamma}_h$ can be written as

$$\frac{\tilde{\gamma}'_h}{\tilde{\gamma}_h} = \left[\left(\sum_j \left(\frac{N'_{hj}}{N_{hj}} \frac{N_{hj}}{\sum_{j'} N_{hj'}}\right)\right)^{-1} \sum_j \left[\frac{N'_{hj}}{N_{hj}} \left(\frac{\tilde{\gamma}'_{hj}}{\tilde{\gamma}_{hj}}\right)^{\sigma-1} \frac{N_{hj}\left(\tilde{\gamma}_{hj}\right)^{\sigma-1}}{\sum_{j'} N_{hj'}\left(\tilde{\gamma}_{hj'}\right)^{\sigma-1}}\right]\right]^{\frac{1}{\sigma-1}}. \quad (19)$$

We do not have data for N_{hj}, but under the assumption that entry costs are proportional to some observable destination-specific factor that is exogenous to the model (such as m_j, GDP, population, area,...):

$$\frac{N_{hj}}{\sum_{j'} N_{hj'}} = \frac{X_{hj}/\left(\frac{a_\gamma \sigma}{a_\gamma - \sigma + 1}\right) F_{hj}}{\sum_{j'} X_{hj'}/\left(\frac{a_\gamma \sigma}{a_\gamma - \sigma + 1}\right) F_{hj'}} = \frac{X_{hj}/m_j}{\sum_{j'} X_{hj'}/m_{j'}}. \tag{20}$$

From (4) and (9),

$$\tilde{\gamma}_{hj} = \left(\frac{a_\gamma}{a_\gamma - \sigma + 1}\right)^{\frac{1}{\sigma-1}} \gamma^*_{hj} = \left(\frac{a_\gamma}{a_\gamma - \sigma + 1}\right)^{\frac{1}{\sigma-1}} k_h \left(M_h\right)^{\frac{1}{a_\gamma}} N_{hj}^{-\frac{1}{a_\gamma}}. \tag{21}$$

We can approximate

$$\frac{N_{hj}\left(\tilde{\gamma}_{hj}\right)^{\sigma-1}}{\sum_{j'} N_{hj'}\left(\tilde{\gamma}_{hj'}\right)^{\sigma-1}} = \frac{\left(X_{hj}/m_j\right)^{\frac{1-\sigma+a_\gamma}{a_\gamma}}}{\sum_{j'}\left(X_{hj'}/m_{j'}\right)^{\frac{1-\sigma+a_\gamma}{a_\gamma}}}. \tag{22}$$

2.4 Predictions

It is easy to show that

$$\hat{P}_j = \left[\sum_h \frac{X_{hj}}{\sum_h X_{hj}} \hat{\tau}_{hj}^{-a_\gamma}\right]^{-1/a_\gamma}. \tag{23}$$

We can use the system (23) to solve for the growth rates of the price levels $\hat{P}_j$ as a function of the changes in transport costs $\hat{\tau}_{hj}$. From equations (3), we can solve for $\hat{\gamma}^*_{hj}$ as a function of $\hat{P}_j$ and $\hat{\tau}_{hj}$,

$$\hat{\gamma}^*_{hj} = \hat{\tau}_{hj}/\hat{P}_j, \tag{24}$$

and thereafter generate predictions for the industry aggregates of interest.

3 Free entry

3.1 Assumptions

3.1.1 Preferences

There are many countries. Each country admits a representative agent, with preferences

$$U = \sum_{i \in I} m_i \ln Q_i + A, \tag{25}$$

where $m_i > 0$. A denotes consumption of a homogeneous final good. Q_i denotes consumption of a Dixit-Stiglitz aggregate (manufacturing) final good i:[2]

$$Q_i = \left[\int_{\gamma \in \Gamma_i} q_i(\gamma)^{\rho_i} d\gamma \right]^{\frac{1}{\rho_i}}, \tag{26}$$

where $\rho \in (0, 1)$ and $\sigma \equiv 1/(1-\rho)$ denotes the elasticity of substitution between any two varieties. Choosing good A as the numéraire, utility maximization on the upper level yields demand functions $A = Y - \sum_i m_i$ and $Q_i = m_i/P_i$, where Y is total expenditure per consumer. In the manufacturing goods sector, utility maximization yields demand function $q_i(\gamma) = p_i(\gamma)^{-\sigma} P_i^{\sigma-1} m_i$.

3.1.2 Technology

The homogeneous good is made with labor l and a linear technology $A = l_A$ that is identical across countries. Manufacturing varieties are made with the production function $q_i(\gamma) = \gamma l_i(\gamma)$, where γ denotes (firm-specific) total factor productivity. γ is iid across firms within an industry. For tractability purposes, we assume γ to be distributed Pareto with shape parameter a_γ and location parameter k_γ. We assume the same shape parameter for an industry across countries, but allow it to vary across industries. The location parameter is allowed to vary across industries and countries.

3.1.3 Market power

Producers of the homogeneous good and the final goods Q operate in a perfectly competitive environment. Producers of varieties in the manufacturing industry have instead monopoly power over their own varieties.

3.1.4 Fixed and transport costs

The homogeneous good is traded freely; supplying it to any market and entering the market involves no costs. We consider equilibria in which all countries produce positive amounts of this good, thus leading to the equalization of wages across countries. (We normalize wages to one.) The final goods Q are not traded; supplying them or entering the (domestic) market involves no costs either.

For the varieties produced by the manufacturing industries, we assume iceberg transport costs, which take the form $\tau_{hj} = (1 + c_{hj})(1 + t_{hj})$ for $j \neq h$ and $\tau_{jj} = 1$. (h and j denote the exporting and importing country, respectively.) c_{hj} denotes "natural" transport costs, and t_{hj} denotes policy-induced trade barriers. We can safely ignore tariff revenue, given the quasi-linear utility assumption above.

A manufacturing industry-i firm based in country h faces a fixed cost F_{hj} of supplying country j. Fixed costs are in terms of the destination country's labor.

[2] Wherever possible, we dispense with industry index i and with country indexes.

Fixed costs and transport costs are allowed to vary across industries and country-pairs.

3.1.5 Entry

We assume free entry: the mass of firms M_{ji} active in an industry is the result of firms comparing expected profits with the fixed cost F_{ji} (in terms of country j's labor) that they have to pay in order to pick a draw from the distribution of γ. F_{ji} is allowed to vary across industries and countries.

3.1.6 Labor market

The labor market is perfectly competitive.

3.2 Firm-level and industry outcomes

The pricing decision over the variety produced by a country-h firm with productivity γ is the usual mark-up over marginal cost. Well-known manipulation of firm revenue and profit functions yields to the following expression for the threshold value of productivity γ^*_{hj} that leads country-h firms to select into market j:

$$\gamma^*_{hj} = \frac{\sigma}{\sigma-1}\frac{\tau_{hj}}{P_j}\left(\frac{\sigma F_{hj}}{m_j}\right)^{\frac{1}{\sigma-1}}. \tag{27}$$

The average productivity of country-h firms exporting to market j, defined as in Melitz (2003), can be expressed as

$$\tilde{\gamma}_{hj} = \left(\frac{a_\gamma}{a_\gamma-\sigma+1}\right)^{\frac{1}{\sigma-1}}\gamma^*_{hj}. \tag{28}$$

The expected revenue and expected profit that a country-h firm obtains in country j, conditional upon selecting into that market, are respectively

$$E\left[r_{hj}(\gamma)|\,\gamma>\gamma^*_{hj}\right] = r_{hj}\left(\tilde{\gamma}_{hj}\right) = \frac{a_\gamma\sigma}{a_\gamma-\sigma+1}F_{hj}, \tag{29}$$

$$E\left[\pi_{hj}(\gamma)|\,\gamma>\gamma^*_{hj}\right] = \frac{r_{hj}\left(\tilde{\gamma}_{hj}\right)}{\sigma} - F_{hj} = \frac{\sigma-1}{a_\gamma-\sigma+1}F_{hj}. \tag{30}$$

Country-h exports to country j can be expressed as

$$X_{hj} = N_{hj}r_{hj}\left(\tilde{\gamma}_{hj}\right) = N_{hj}\frac{a_\gamma\sigma}{a_\gamma-\sigma+1}F_{hj}. \tag{31}$$

The industry's aggregate sales are then

$$R_h = \sum_j X_{hj} = \sum_j N_{hj}\frac{a_\gamma\sigma}{a_\gamma-\sigma+1}F_{hj}. \tag{32}$$

The mass of country-h firms that select into market j is given by

$$N_{hj} = \left(\frac{k_h}{\gamma^*_{hj}}\right)^{a_\gamma} M_h. \tag{33}$$

The free-entry condition sets expected profits equal to the fixed cost F_h:

$$\sum_j prob\left(\gamma > \gamma^*_{hj}\right) E\left[\pi_{hj}(\gamma) \middle| \gamma > \gamma^*_{hj}\right] = F_h. \tag{34}$$

Industry employment can be easily shown to be

$$L_h = M_h E\left[l_{hj}(\gamma)\right] = M_h \sum_j \left(\frac{k_h}{\gamma^*_{hj}}\right)^{a_\gamma} E\left[l_{hj}(\gamma) \middle| \gamma > \gamma^*_{hj}\right] = \frac{\sigma-1}{\sigma} R_h. \tag{35}$$

The price level P_j is given by

$$P_j = \left[\frac{a_\gamma}{a_\gamma - \sigma + 1} \sum_h N_{hj} \left(\frac{\sigma}{\sigma-1} \frac{\tau_{hj}}{\gamma^*_{hj}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{36}$$

Melitz (2003) defines industry productivity as

$$\tilde{\gamma}_h = \left[\sum_j \frac{N_{hj}}{\sum_j N_{hj}} \left(\tilde{\gamma}_{hj}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}. \tag{37}$$

3.3 Growth rates

The growth rates of the industry's aggregates can be expressed as functions of parameter values, changes in τ_{hj}, changes in thresholds γ^*_{hj}, and the levels of bilateral trade X_{hj}. Define $\hat{x} \equiv x'/x$ as a gross growth rate, where x and x' denote, respectively, the values of a variable before and after the trade liberalization:

$$\hat{X}_{hj} = \hat{N}_{hj} = \left(\hat{\gamma}^*_{hj}\right)^{-a_\gamma} \hat{M}_h, \tag{38}$$

$$\hat{R}_h = \hat{L}_h = \sum_j \frac{X_{hj}}{\sum_j X_{hj}} \hat{X}_{hj}, \tag{39}$$

$$\hat{P}_j = \left[\sum_h \frac{X_{hj}}{\sum_h X_{hj}} \hat{N}_{hj} \left(\hat{\tau}_{hj}\right)^{1-\sigma} \left(\hat{\gamma}^*_{hj}\right)^{\sigma-1}\right]^{\frac{1}{1-\sigma}}. \tag{40}$$

As a measurable proxy for productivity growth, we will consider the growth rate of deflated value added per worker,

$$\frac{\hat{R}_h}{\hat{L}_h} \frac{p\left(\tilde{\gamma}_h\right)}{p\left(\tilde{\gamma}'_h\right)} = \frac{\hat{R}_h}{\hat{L}_h} \frac{\tilde{\gamma}'_h}{\tilde{\gamma}_h} = \frac{\tilde{\gamma}'_h}{\tilde{\gamma}_h}, \tag{41}$$

where we use the price charged by the firm with average productivity $p(\tilde{\gamma}_h)$ as a deflator. (See Section 3 and Appendix A in the paper for a detailed discussion of the appropriate choice of deflator.) The growth rate $\tilde{\gamma}'_h/\tilde{\gamma}_h$ can be written as

$$\frac{\tilde{\gamma}'_h}{\tilde{\gamma}_h} = \left[\left(\sum_j \left(\frac{N'_{hj}}{N_{hj}} \frac{N_{hj}}{\sum_{j'} N_{hj'}}\right)\right)^{-1} \sum_j \left[\frac{N'_{hj}}{N_{hj}} \left(\frac{\tilde{\gamma}'_{hj}}{\tilde{\gamma}_{hj}}\right)^{\sigma-1} \frac{N_{hj}\left(\tilde{\gamma}_{hj}\right)^{\sigma-1}}{\sum_{j'} N_{hj'}\left(\tilde{\gamma}_{hj'}\right)^{\sigma-1}}\right]\right]^{\frac{1}{\sigma-1}}. \tag{42}$$

We do not have data for N_{hj}, but under the assumption that entry costs are proportional to some observable destination-specific factor that is exogenous to the model (such as m_j, GDP, population, area,...):

$$\frac{N_{hj}}{\sum_j N_{hj}} = \frac{X_{hj}/\left(\frac{a_\gamma \sigma}{a_\gamma-\sigma+1}\right) F_{hj}}{\sum_j X_{hj}/\left(\frac{a_\gamma \sigma}{a_\gamma-\sigma+1}\right) F_{hj}} = \frac{X_{hj}/m_j}{\sum_j X_{hj}/m_j}. \tag{43}$$

From (28) and (33),

$$\tilde{\gamma}_{hj} = \left(\frac{a_\gamma}{a_\gamma - \sigma + 1}\right)^{\frac{1}{\sigma-1}} \gamma^*_{hj} = \left(\frac{a_\gamma}{a_\gamma - \sigma + 1}\right)^{\frac{1}{\sigma-1}} k_h \left(M_h\right)^{\frac{1}{a_\gamma}} N_{hj}^{-\frac{1}{a_\gamma}}. \tag{44}$$

We can approximate

$$\frac{N_{hj}\left(\tilde{\gamma}_{hj}\right)^{\sigma-1}}{\sum_{j'} N_{hj'}\left(\tilde{\gamma}_{hj'}\right)^{\sigma-1}} = \frac{\left(X_{hj}/m_j\right)^{\frac{1-\sigma+a_\gamma}{a_\gamma}}}{\sum_{j'}\left(X_{hj'}/m_{j'}\right)^{\frac{1-\sigma+a_\gamma}{a_\gamma}}}. \tag{45}$$

3.4 Predictions

Manipulating the growth rates of the free-entry condition and the price level,

$$1 = \sum_j \frac{X_{hj}}{\sum_n X_{hn}} \hat{\tau}_{hj}^{-a_\gamma} \hat{P}_j^{a_\gamma}, \tag{46}$$

$$\hat{P}_j^{-a_\gamma} = \sum_h \frac{X_{hj}}{\sum_h X_{hj}} \hat{M}_h \hat{\tau}_{hj}^{-a_\gamma}. \tag{47}$$

Once we have the values of $\hat{P}_j$ and $\hat{M}_j$, we can generate the model's predictions for all variables of interest.

4 Intermediate inputs

4.1 Assumptions

4.1.1 Preferences

There are many countries. Each country admits a representative agent, with preferences

$$U = \sum_{i \in I} m_i \ln Q_i^c + A, \tag{48}$$

where $m_i > 0$. A denotes consumption of a homogeneous final good. Q_i^c denotes consumption of a Dixit-Stiglitz aggregate (manufacturing) final good Q_i:[3]

$$Q_i = \left[\int_{\gamma \in \Gamma_i} q_i(\gamma)^{\rho_i} d\gamma \right]^{\frac{1}{\rho_i}}, \tag{49}$$

where $\rho \in (0,1)$ and $\sigma \equiv 1/(1-\rho)$ denotes the elasticity of substitution between any two varieties. Choosing good A as the numéraire, utility maximization on the upper level yields demand functions $A = Y - \sum_i m_i$ and $Q_i = m_i/P_i$, where Y is total expenditure per consumer. In the manufacturing goods sector, utility maximization yields demand function $q_i(\gamma) = p_i(\gamma)^{-\sigma} P_i^{\sigma-1} m_i$.

4.1.2 Technology

The homogeneous good is made with labor l and a linear technology $A = l_A$ that is identical across countries. Manufacturing varieties are made with the following production function:

$$q_i(\gamma) = \gamma \left[\frac{Q_i^{input}(\gamma)}{\alpha} \right]^{\alpha} \left[\frac{l(\gamma)}{1-\alpha} \right]^{1-\alpha}. \tag{50}$$

Q_i^{input} denotes the amount of the aggregate manufacturing good used as an intermediate input; $\alpha \in [0,1)$; γ denotes (firm-specific) total factor productivity and is iid across firms within an industry. For tractability purposes, we assume γ to be distributed Pareto with shape parameter a_γ and location parameter k_γ. We assume the same shape parameter for an industry across countries, but allow it to vary across industries. The location parameter is allowed to vary across industries and countries.

4.1.3 Market power

Producers of the homogeneous good and the final goods Q operate in a perfectly competitive environment. Producers of varieties in the manufacturing industry have instead monopoly power over their own varieties.

[3] Wherever possible, we dispense with industry index i and with country indexes.

4.1.4 Fixed and transport costs

The homogeneous good is traded freely; supplying it to any market and entering the market involves no costs. We consider equilibria in which all countries produce positive amounts of this good, thus leading to the equalization of wages across countries. (We normalize wages to one.) The final goods Q are not traded; supplying them or entering the (domestic) market involves no costs either.

For the varieties produced by the manufacturing industries, we assume iceberg transport costs, which take the form $\tau_{hj} = (1 + c_{hj})(1 + t_{hj})$ for $j \neq h$ and $\tau_{jj} = 1$. (h and j denote the exporting and importing country, respectively.) c_{hj} denotes "natural" transport costs, and t_{hj} denotes policy-induced trade barriers. We can safely ignore tariff revenue, given the quasi-linear utility assumption above.

A manufacturing industry-i firm based in country h faces a fixed cost F_{hj} of supplying country j. Fixed costs are in terms of the destination country's labor.

Fixed costs and transport costs are allowed to vary across industries and country-pairs.

4.1.5 Entry

We assume there is no free entry in the manufacturing sectors: there is a given mass of firms M that pick a draw from the distribution of γ prior to any decision.

4.1.6 Labor market

The labor market is perfectly competitive.

4.2 Firm-level and industry outcomes

The pricing decision over the variety produced by a country-h firm with productivity γ is the usual mark-up over marginal cost. Well-known manipulation of firm revenue and profit functions yields to the following expression for the threshold value of productivity γ^*_{hj} that leads country-h firms to select into market j:

$$\gamma^*_{hj} = \frac{\sigma}{\sigma - 1} \frac{\tau_{hj} P_h^{\alpha}}{P_j} \left(\frac{\sigma F_{hj}}{E_j} \right)^{\frac{1}{\sigma-1}}, \tag{51}$$

where $E_j = m_j + P_j Q_j^{input}$. The average productivity of country-h firms exporting to market j, defined as in Melitz (2003), can be expressed as

$$\tilde{\gamma}_{hj} = \left(\frac{a_\gamma}{a_\gamma - \sigma + 1} \right)^{\frac{1}{\sigma-1}} \gamma^*_{hj}. \tag{52}$$

The expected revenue and expected profit that a country-h firm obtains in country j, conditional upon selecting into that market, are respectively

$$E\left[r_{hj}(\gamma)|\,\gamma > \gamma_{hj}^{*}\right] = r_{hj}\left(\tilde{\gamma}_{hj}\right) = \frac{a_{\gamma}\sigma}{a_{\gamma}-\sigma+1}F_{hj}, \tag{53}$$

$$E\left[\pi_{hj}(\gamma)|\,\gamma > \gamma_{hj}^{*}\right] = \frac{r_{hj}\left(\tilde{\gamma}_{hj}\right)}{\sigma} - F_{hj} = \frac{\sigma-1}{a_{\gamma}-\sigma+1}F_{hj}. \tag{54}$$

Country-h exports to country j can be expressed as

$$X_{hj} = N_{hj} r_{hj}\left(\tilde{\gamma}_{hj}\right) = N_{hj}\frac{a_{\gamma}\sigma}{a_{\gamma}-\sigma+1}F_{hj}. \tag{55}$$

The industry's aggregate sales are then

$$R_h = \sum_j X_{hj} = \sum_j N_{hj}\frac{a_{\gamma}\sigma}{a_{\gamma}-\sigma+1}F_{hj}. \tag{56}$$

The mass of country-h firms that select into market j is given by

$$N_{hj} = \left(\frac{k_h}{\gamma_{hj}^{*}}\right)^{a_{\gamma}} M_h. \tag{57}$$

Expected profits, aggregated across all destination markets, are

$$\Pi_h = \sum_j prob\left(\gamma > \gamma_{hj}^{*}\right) E\left[\pi_{hj}(\gamma)|\,\gamma > \gamma_{hj}^{*}\right] = \sum_j \left(\frac{k_h}{\gamma_{hj}^{*}}\right)^{a_{\gamma}} \frac{\sigma-1}{a_{\gamma}-\sigma+1}F_{hj}. \tag{58}$$

Industry profits are therefore

$$M_h\Pi_h = M_h \sum_j \left(\frac{k_h}{\gamma_{hj}^{*}}\right)^{a_{\gamma}} \frac{\sigma-1}{a_{\gamma}-\sigma+1}F_{hj} = \frac{\sigma-1}{a_{\gamma}\sigma}\sum_j X_{hj}. \tag{59}$$

Industry employment can be easily shown to be

$$L_h = M_h E\left[l_{hj}(\gamma)\right] = M_h \sum_j \left(\frac{k_h}{\gamma_{hj}^{*}}\right)^{a_{\gamma}} E\left[l_{hj}(\gamma)|\,\gamma > \gamma_{hj}^{*}\right] = (1-\alpha)\,a_{\gamma} M_h \Pi_h. \tag{60}$$

Similarly, industry demand for intermediate input Q^{input} can be shown to be

$$Q_h^{input} = M_h E\left[Q_h^{input}(\gamma)\right] = \frac{\alpha a_{\gamma}}{P_h} M_h \Pi_h.$$

The price level P_j is given by

$$P_j = \left[\frac{a_{\gamma}}{a_{\gamma}-\sigma+1}\sum_h N_{hj}\left(\frac{\sigma}{\sigma-1}\frac{\tau_{hj}P_h^{\alpha}}{\gamma_{hj}^{*}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{61}$$

Melitz (2003) defines industry productivity as

$$\tilde{\gamma}_h = \left[\sum_j \frac{N_{hj}}{\sum_j N_{hj}} \left(\tilde{\gamma}_{hj}\right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}}. \tag{62}$$

4.3 Growth rates

The growth rates of the industry's aggregates can be expressed as functions of parameter values, changes in τ_{hj}, changes in thresholds γ^*_{hj}, and the levels of bilateral trade X_{hj}. Define $\hat{x} \equiv x'/x$ as a gross growth rate, where x and x' denote, respectively, the values of a variable before and after the trade liberalization:

$$\hat{X}_{hj} = \hat{N}_{hj} = \left(\hat{\gamma}^*_{hj}\right)^{-a_\gamma}, \tag{63}$$

$$\hat{R}_h = \hat{\Pi}_h = \hat{L}_h = \sum_j \frac{X_{hj}}{\sum_j X_{hj}} \hat{X}_{hj}, \tag{64}$$

$$\hat{P}_j = \left[\sum_h \frac{X_{hj}}{\sum_h X_{hj}} \hat{N}_{hj} \left(\hat{\tau}_{hj}\right)^{1-\sigma} \left(\hat{P}^\alpha_h\right)^{1-\sigma} \left(\hat{\gamma}^*_{hj}\right)^{\sigma-1} \right]^{\frac{1}{1-\sigma}}. \tag{65}$$

Our measurable proxy for industry productivity, deflated value added per worker, is now defined as[4]

$$\frac{VA_h}{p\left(\tilde{\gamma}_h\right) L_h} = \frac{R_h - P_h Q_h^{input}}{p\left(\tilde{\gamma}_h\right) L_h}. \tag{66}$$

Taking growth rates,

$$\frac{\hat{R}_j R_j - P_j Q_j^{input} \hat{\Pi}_j}{VA} \frac{1}{\left(\hat{P}_j\right)^\alpha \hat{L}_j} \frac{\tilde{\gamma}'}{\tilde{\gamma}} = \left(\hat{P}_j\right)^{-\alpha} \frac{\tilde{\gamma}'}{\tilde{\gamma}} \tag{67}$$

The growth rate $\tilde{\gamma}'_h/\tilde{\gamma}_h$ can be written as

$$\frac{\tilde{\gamma}'_h}{\tilde{\gamma}_h} = \left[\left(\sum_j \left(\frac{N'_{hj}}{N_{hj}} \frac{N_{hj}}{\sum_{j'} N_{hj'}} \right) \right)^{-1} \sum_j \left[\frac{N'_{hj}}{N_{hj}} \left(\frac{\tilde{\gamma}'_{hj}}{\tilde{\gamma}_{hj}} \right)^{\sigma-1} \frac{N_{hj} \left(\tilde{\gamma}_{hj}\right)^{\sigma-1}}{\sum_{j'} N_{hj'} \left(\tilde{\gamma}_{hj'}\right)^{\sigma-1}} \right] \right]^{\frac{1}{\sigma-1}}. \tag{68}$$

We do not have data for N_{hj}, but under the assumption that entry costs are proportional to some observable destination-specific factor that is exogenous to the model (such as m_j, GDP, population, area,...):

$$\frac{N_{hj}}{\sum_j N_{hj}} = \frac{X_{hj}/\left(\frac{a_\gamma \sigma}{a_\gamma - \sigma + 1}\right) F_{hj}}{\sum_j X_{hj}/\left(\frac{a_\gamma \sigma}{a_\gamma - \sigma + 1}\right) F_{hj}} = \frac{X_{hj}/m_j}{\sum_j X_{hj}/m_j}. \tag{69}$$

[4] See Section 3 and Appendix A in the paper for a detailed discussion of the appropriate choice of deflator.

From (52) and (57),

$$\tilde{\gamma}_{hj} = \left(\frac{a_\gamma}{a_\gamma - \sigma + 1}\right)^{\frac{1}{\sigma-1}} \gamma^*_{hj} = \left(\frac{a_\gamma}{a_\gamma - \sigma + 1}\right)^{\frac{1}{\sigma-1}} k_h \left(M_h\right)^{\frac{1}{a_\gamma}} N_{hj}^{-\frac{1}{a_\gamma}}. \tag{70}$$

We can approximate

$$\frac{N_{hj}\left(\tilde{\gamma}_{hj}\right)^{\sigma-1}}{\sum_{j'} N_{hj'}\left(\tilde{\gamma}_{hj'}\right)^{\sigma-1}} = \frac{\left(X_{hj}/m_j\right)^{\frac{1-\sigma+a_\gamma}{a_\gamma}}}{\sum_{j'}\left(X_{hj'}/m_{j'}\right)^{\frac{1-\sigma+a_\gamma}{a_\gamma}}}. \tag{71}$$

4.4 Predictions

The price equation in changes can be rewritten as

$$\hat{P}_j^{\frac{a_\gamma}{\sigma-1}} = \left(\hat{E}_j\right)^{\frac{\sigma-a_\gamma-1}{(1-\sigma)^2}} \left[\sum_h \frac{X_{hj}}{\sum_m X_{mj}} \hat{\tau}_{hj}^{-a_\gamma} \frac{1}{\hat{P}_h^{\alpha a_\gamma}}\right]^{\frac{1}{1-\sigma}}, \tag{72}$$

where

$$\hat{E}_j = \frac{1}{E_j}\left[m_j + P_j Q_j^{input} \hat{P}_j^{-a_\gamma \alpha}\left(\sum_h \frac{X_{jh}}{\sum_h X_{jh}} \hat{\tau}_{jh}^{-a_\gamma} \hat{P}_h^{a_\gamma} \hat{E}_h^{\frac{a_\gamma}{\sigma-1}}\right)\right]. \tag{73}$$

This yields a system of non-linear equations in $\hat{P}_j$ and $\hat{E}_j$. Once we have $\hat{P}_.$ and $\hat{E}$, we can solve for the remaining growth rates in the following order: $\hat{\gamma}^*_{jj}$, $\hat{\gamma}^*_{hj}$, $\hat{N}_{hj}$, $\hat{X}_{hj}$, $\hat{\Pi}_j$, and thus $\hat{R}_j$, $\hat{L}_j$ and $\hat{Q}_j^{input}$.

5 General equilibrium

5.1 Assumptions

5.1.1 Preferences

There are many countries. Each country admits a representative agent, with preferences

$$U = \prod_i Q_i^{\mu_i}, \tag{74}$$

where $\mu_i > 0$, $\sum_i \mu_i = 1$. Q_i denotes consumption of a Dixit-Stiglitz aggregate (manufacturing) final good i:[5]

$$Q_i = \left[\int_{\gamma \in \Gamma_i} q_i(\gamma)^{\rho_i} d\gamma\right]^{\frac{1}{\rho_i}}, \tag{75}$$

where $\rho \in (0,1)$ and $\sigma \equiv 1/(1-\rho)$ denotes the elasticity of substitution between any two varieties.

[5] Wherever possible, we dispense with industry index i and with country indexes.

5.1.2 Technology

Manufacturing varieties are made with the production function $q_i(\gamma) = \gamma l_i(\gamma)$, where γ denotes (firm-specific) total factor productivity. γ is iid across firms within an industry. For tractability purposes, we assume γ to be distributed Pareto with shape parameter a_γ and location parameter k_γ. We assume the same shape parameter for an industry across countries, but allow it to vary across industries. The location parameter is allowed to vary across industries and countries.

5.1.3 Market power

Producers of the final goods Q operate in a perfectly competitive environment. Producers of varieties in the manufacturing industry have instead monopoly power over their own varieties.

5.1.4 Fixed and transport costs

The final goods Q are not traded; supplying them or entering the (domestic) market involves no costs either.

For the varieties produced by the manufacturing industries, we assume iceberg transport costs, which take the form $\tau_{hj} = (1 + c_{hj})(1 + t_{hj})$ for $j \neq h$ and $\tau_{jj} = 1$. (h and j denote the exporting and importing country, respectively.) c_{hj} denotes "natural" transport costs, and t_{hj} denotes policy-induced trade barriers. We ignore tariff revenue for comparability purposes with the rest of the models.[6]

A manufacturing industry-i firm based in country h faces a fixed cost F_{hj} of supplying country j. Fixed costs are in terms of the destination country's labor.

Fixed costs and transport costs are allowed to vary across industries and country-pairs.

5.1.5 Entry

We assume free entry: the mass of firms M_{ji} active in an industry is the result of firms comparing expected profits with the fixed cost F_{ji} (in terms of country j's labor) that they have to pay in order to pick a draw from the distribution of γ. F_{ji} is allowed to vary across industries and countries.

5.1.6 Labor market

The labor market is perfectly competitive. In each country j, L_j units of labor are supplied inelastically.

[6] Modeling tariff revenue in terms of lump-sum transfers to the levying country's consumers does not change the model's quantitative implications significantly. Results available from the authors upon request.

5.2 Firm-level and industry outcomes

The pricing decision over the variety produced by a country-h firm with productivity γ is the usual mark-up over marginal cost. Well-known manipulation of firm revenue and profit functions yields to the following expression for the threshold value of productivity γ^*_{hj} that leads country-h firms to select into market j:

$$\gamma^*_{hj} = \frac{\sigma}{\sigma-1}\frac{\tau_{hj}w_h}{P_j}\left(\frac{\sigma w_j F_{hj}}{\mu_j w_j L_j}\right)^{\frac{1}{\sigma-1}}. \tag{76}$$

The average productivity of country-h firms exporting to market j, defined as in Melitz (2003), can be expressed as

$$\tilde{\gamma}_{hj} = \left(\frac{a_\gamma}{a_\gamma - \sigma + 1}\right)^{\frac{1}{\sigma-1}} \gamma^*_{hj}. \tag{77}$$

The expected revenue and expected profit that a country-h firm obtains in country j, conditional upon selecting into that market, are respectively

$$E\left[r_{hj}(\gamma)|\,\gamma > \gamma^*_{hj}\right] = r_{hj}\left(\tilde{\gamma}_{hj}\right) = \frac{a_\gamma \sigma}{a_\gamma - \sigma + 1} w_j F_{hj}, \tag{78}$$

$$E\left[\pi_{hj}(\gamma)|\,\gamma > \gamma^*_{hj}\right] = \frac{r_{hj}\left(\tilde{\gamma}_{hj}\right)}{\sigma} - F_{hj} = \frac{\sigma - 1}{a_\gamma - \sigma + 1} w_j F_{hj}. \tag{79}$$

Country-h exports to country j can be expressed as

$$X_{hj} = N_{hj} r_{hj}\left(\tilde{\gamma}_{hj}\right) = N_{hj}\frac{a_\gamma \sigma}{a_\gamma - \sigma + 1} w_j F_{hj}. \tag{80}$$

The industry's aggregate sales are then

$$R_h = \sum_j X_{hj} = \sum_j N_{hj}\frac{a_\gamma \sigma}{a_\gamma - \sigma + 1} w_j F_{hj}. \tag{81}$$

The mass of country-h firms that select into market j is given by

$$N_{hj} = \left(\frac{k_h}{\gamma^*_{hj}}\right)^{a_\gamma} M_h. \tag{82}$$

The free-entry condition sets expected profits equal to the fixed cost F_h:

$$\sum_j prob\left(\gamma > \gamma^*_{hj}\right) E\left[\pi_{hj}(\gamma)|\,\gamma > \gamma^*_{hj}\right] = w_h F_h. \tag{83}$$

Industry employment can be easily shown to be

$$L_h = M_h E\left[l_{hj}(\gamma)\right] = M_h \sum_j \left(\frac{k_h}{\gamma^*_{hj}}\right)^{a_\gamma} E\left[l_{hj}(\gamma)|\,\gamma > \gamma^*_{hj}\right] = a_\gamma M_h F_h = \frac{\sigma-1}{\sigma}\frac{R_h}{w_h}. \tag{84}$$

The labor market clearing condition sets L_h equal to the demand for labor (inclusive of fixed costs):

$$L_j = \sum_i M_{ij} \left(1 + a_\gamma^i\right) F_j^i + \sum_i \left[\frac{a_\gamma^i - \sigma^i + 1}{w_j a_\gamma^i \sigma^i} \left(\sum_h X_{hj}^i \right) \right]. \tag{85}$$

Thus,

$$w_j L_j = \sum_i \left[\frac{\left(1 + a_\gamma^i\right)\left(\sigma^i - 1\right)}{a_\gamma^i \sigma^i} \left(\sum_h X_{jh}^i \right) \right] + \sum_i \left[\frac{a_\gamma^i - \sigma^i + 1}{a_\gamma^i \sigma^i} \left(\sum_h X_{hj}^i \right) \right]. \tag{86}$$

The price level P_j is given by

$$P_j = \left[\frac{a_\gamma}{a_\gamma - \sigma + 1} \sum_h N_{hj} \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{hj} w_h}{\gamma_{hj}^*} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{87}$$

Melitz (2003) defines industry productivity as

$$\tilde{\gamma}_h = \left[\sum_j \frac{N_{hj}}{\sum_j N_{hj}} \left(\tilde{\gamma}_{hj}\right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}}. \tag{88}$$

5.3 Growth rates

The growth rates of the industry's aggregates can be expressed as functions of parameter values, changes in τ_{hj}, changes in thresholds γ_{hj}^*, and the levels of bilateral trade X_{hj}. Define $\hat{x} \equiv x'/x$ as a gross growth rate, where x and x' denote, respectively, the values of a variable before and after the trade liberalization:

$$\hat{X}_{hj} = \hat{w}_j \hat{N}_{hj} = \hat{w}_j \left(\hat{\gamma}_{hj}^*\right)^{-a_\gamma} \hat{M}_h, \tag{89}$$

$$\hat{R}_h = \sum_j \frac{X_{hj}}{\sum_j X_{hj}} \hat{X}_{hj}, \tag{90}$$

$$\hat{L}_h = \hat{R}_h / \hat{w}_h, \tag{91}$$

$$\hat{P}_j = \left[\sum_h \frac{X_{hj}}{\sum_h X_{hj}} \hat{N}_{hj} \left(\hat{\tau}_{hj}\right)^{1-\sigma} \left(\hat{w}_h\right)^{1-\sigma} \left(\hat{\gamma}_{hj}^*\right)^{\sigma-1} \right]^{\frac{1}{1-\sigma}}. \tag{92}$$

As a measurable proxy for productivity growth, we will consider the growth rate of deflated value added per worker,

$$\frac{\hat{R}_h}{\hat{L}_h} \frac{p\left(\tilde{\gamma}_h\right)}{p\left(\tilde{\gamma}_h'\right)} = \frac{\hat{R}_h}{\hat{w}_h \hat{L}_h} \frac{\tilde{\gamma}_h'}{\tilde{\gamma}_h} = \frac{\tilde{\gamma}_h'}{\tilde{\gamma}_h}, \tag{93}$$

where we use the price charged by the firm with average productivity $p(\tilde{\gamma}_h)$ as a deflator. (See Section 3 and Appendix A in the paper for a detailed discussion of the appropriate choice of deflator.) The growth rate $\tilde{\gamma}'_h/\tilde{\gamma}_h$ can be written as

$$\frac{\tilde{\gamma}'_h}{\tilde{\gamma}_h} = \left[\left(\sum_j \left(\frac{N'_{hj}}{N_{hj}} \frac{N_{hj}}{\sum_{j'} N_{hj'}}\right)\right)^{-1} \sum_j \left[\frac{N'_{hj}}{N_{hj}} \left(\frac{\tilde{\gamma}'_{hj}}{\tilde{\gamma}_{hj}}\right)^{\sigma-1} \frac{N_{hj} \left(\tilde{\gamma}_{hj}\right)^{\sigma-1}}{\sum_{j'} N_{hj'} \left(\tilde{\gamma}_{hj'}\right)^{\sigma-1}}\right]\right]^{\frac{1}{\sigma-1}}. \tag{94}$$

We do not have data for N_{hj}, but under the assumption that entry costs are proportional to some observable destination-specific factor that is exogenous to the model:

$$\frac{N_{hj}}{\sum_{j'} N_{hj'}} = \frac{X_{hj} / \left(\frac{a_\gamma \sigma}{a_\gamma - \sigma + 1}\right) w_j F_{hj}}{\sum_{j'} X_{hj'} / \left(\frac{a_\gamma \sigma}{a_\gamma - \sigma + 1}\right) w_{j'} F_{hj'}} = \frac{X_{hj}/w_j L_j}{\sum_{j'} X_{hj'}/w_{j'} L_{j'}} \tag{95}$$

where L_j is our choice of factor exogeneous to the model. Multiplying by w_j yields GDP in the model which we use as the empirical proxy for $w_j L_j$. From (77) and (82),

$$\tilde{\gamma}_{hj} = \left(\frac{a_\gamma}{a_\gamma - \sigma + 1}\right)^{\frac{1}{\sigma-1}} \gamma^*_{hj} = \left(\frac{a_\gamma}{a_\gamma - \sigma + 1}\right)^{\frac{1}{\sigma-1}} k_h \left(M_h\right)^{\frac{1}{a_\gamma}} N_{hj}^{-\frac{1}{a_\gamma}}. \tag{96}$$

We can approximate

$$\frac{N_{hj} \left(\tilde{\gamma}_{hj}\right)^{\sigma-1}}{\sum_{j'} N_{hj'} \left(\tilde{\gamma}_{hj'}\right)^{\sigma-1}} = \frac{\left(X_{hj}/w_j L_j\right)^{\frac{1-\sigma+a_\gamma}{a_\gamma}}}{\sum_{j'} \left(X_{hj'}/w_{j'} L_{j'}\right)^{\frac{1-\sigma+a_\gamma}{a_\gamma}}} \tag{97}$$

5.4 Predictions

The growth rates of the free entry condition, the price level and the labor market clearing condition solve for the variables we need to generate predictions $(\hat{M}_j, \hat{w}_j, \hat{P}_j)$.

$$\sum_j \frac{X_{hj}}{\sum_n X_{hn}} \left(\hat{\gamma}^*_{hj}\right)^{-a_\gamma} \hat{w}_j = \hat{w}_h, \tag{98}$$

$$\hat{P}_j = \left[\sum_h \frac{X_{hj}}{\sum_m X_{mj}} \hat{N}_{hj} \left(\hat{\tau}_{hj}\right)^{1-\sigma} \left(\hat{w}_h\right)^{1-\sigma} \left(\hat{\gamma}^*_{hj}\right)^{\sigma-1}\right]^{\frac{1}{1-\sigma}}, \tag{99}$$

$$\hat{w}_j \hat{L}_j = \frac{A}{A+B} \hat{A} + \frac{B}{A+B} \hat{B}, \tag{100}$$

where

$$\hat{\gamma}^*_{hj} = \frac{\hat{\tau}_{hj}\hat{w}_h}{\hat{w}_j}\hat{\gamma}^*_{jj}, \tag{101}$$

$$\hat{\gamma}^*_{jj} = \frac{\hat{w}_j}{\hat{P}_j}, \tag{102}$$

$$\hat{N}_{hj} = \left(\hat{\gamma}^*_{hj}\right)^{-a}\hat{M}_h, \tag{103}$$

and

$$A = \sum_i \left[\frac{\left(1+a^i_\gamma\right)\left(\sigma^i-1\right)}{a^i_\gamma\sigma^i}\left(\sum_h X^i_{jh}\right)\right], \tag{104}$$

$$B = \sum_i \left[\frac{a^i_\gamma-\sigma^i+1}{a^i_\gamma\sigma^i}\left(\sum_h X^i_{hj}\right)\right], \tag{105}$$

$$\hat{A} = \frac{1}{A}\sum_i \left[\frac{\left(1+a^i_\gamma\right)\left(\sigma^i-1\right)}{a^i_\gamma\sigma^i}\left(\sum_h X^i_{jh}\right)\left(\widehat{\sum_h X^i_{jh}}\right)\right], \tag{106}$$

$$\hat{B} = \frac{1}{B}\sum_h \left[\frac{a^i_\gamma-\sigma^i+1}{a^i_\gamma\sigma^i}\left(\sum_h X^i_{hj}\right)\left(\widehat{\sum_h X^i_{hj}}\right)\right]. \tag{107}$$

5.5 Other general equilibrium effects

If we consider trade deficits, then expenditure on industry i becomes

$$E_{ji} = \mu_{ji}\left(w_jL_j + TD_j\right). \tag{108}$$

Productivity thresholds are now

$$\gamma^*_{hji} = \frac{\sigma}{\sigma-1}\frac{\tau_{hji}w_h}{P_{ji}}\left(\frac{\sigma w_j F_{hji}}{E_{ji}}\right)^{\frac{1}{\sigma-1}}, \tag{109}$$

with growth rates

$$\hat{\gamma}^*_{hji} = \frac{\hat{\tau}_{hji}\hat{w}_h}{\hat{w}_j}\hat{\gamma}^*_{jji}, \tag{110}$$

$$\hat{\gamma}^*_{jj} = \frac{\hat{w}_j}{\hat{P}_j}\left(\frac{\hat{w}_j}{\hat{E}_{ji}}\right)^{\frac{1}{\sigma-1}}. \tag{111}$$

The growth rate of E_j is

$$\hat{E}_{ji} = \frac{w_jL_j}{\left(w_jL_j+TD_j\right)}\hat{w}_j + \frac{TD_j}{\left(w_jL_j+TD_j\right)}\frac{TD'_j}{TD_j}. \tag{112}$$

6 Multi-product firms[7]

6.1 Assumptions

$$U_j = \sum_{i \in I} \mu_{ji} \ln Q_{ji} + A_j, \tag{113}$$

where i denotes industries and j denotes countries. Let us call Q_{ji} "final goods" and A_j the "numéraire" good. Utility maximization yields demand functions $A = Y - \sum_i m_i$ and $Q_i = m_i / P_i$, where Y is total expenditure per consumer.

The homogeneous good is made with labor l and a linear technology $A = l_A$ that is identical across countries. It is traded freely: supplying it to any market and entering the market involves no costs. We consider equilibria in which all countries produce positive amounts of this good, thus leading to the equalization of wages across countries. (We normalize wages to one.)

Final goods are made with a continuum of "products", which are imperfect substitutes in demand:

$$Q_{ji} = \left(\int_0^1 C_{jik}^{\nu} dk \right)^{\frac{1}{\nu}}, \tag{114}$$

where $\nu \in (0, 1)$ and k denotes product.

$$P_{ji} = \left(\int_0^1 P_{jik}^{1-\kappa} dk \right)^{\frac{1}{1-\kappa}}. \tag{115}$$

Within each product-industry, firms supply horizontally differentiated "varieties" of the product:[8]

$$C_{jik} = \left[\sum_h \int_{\omega \in \Omega_{hjik}} \left[\lambda_{hjik} (\omega) \, c_{hjik} (\omega) \right]^{\rho} d\omega \right]^{\frac{1}{\rho}}, \tag{116}$$

where $\rho \in (0, 1)$; h denotes the product's country of origin; and $\chi \equiv 1/(1-\rho) > \kappa \equiv 1/(1-\nu)$; $\lambda_{hjik}(\omega)$ captures "product attributes", to be discussed below. The ideal price index associated to C_{jik} is

$$P_{jik} = \left[\sum_h \int_{\omega \in \Omega_{hjik}} \left[\frac{p_{hjik}(\omega)}{\lambda_{hjik}(\omega)} \right]^{1-\chi} d\omega \right]^{\frac{1}{1-\chi}}. \tag{117}$$

There is a given mass of firms in each country/industry. (That is, we are in the no-free-entry case.) After observing their γ and λ, firms decide whether they want to pay fixed costs to select into different markets/products: (i) Firms based in country h face a fixed cost F_{hji} of supplying country j. (ii) Besides F_{hji}, in order to supply one variety of a product in country j, country-h firms must pay

[7] This section draws upon Bernard, A., S.J. redding and P.K. Schott (2011): "Multi-product Firms and Trade Liberalization," Quarterly Journal of Economics, 126(3), pp.1271-1318.

[8] Firms are assumed to operate only within one industry.

fixed cost f_{hji}. (Firms cannot supply more than one variety of each product by assumption.) (*iii*) Fixed costs are in terms of the destination country's labor.

There is a firm-specific constant marginal cost of production for each product: $q_{hjik}(\gamma, \lambda_{hjik})/\gamma$ units of labor are employed in country h to supply $q_{hjik}(\gamma, \lambda_{hjik})$ units of output of product k to market j. γ is the "ability" (that is, productivity) of a firm. For tractability purposes, we assume γ to be distributed Pareto with shape parameter a_γ and location parameter k_γ. We assume the same shape parameter for an industry across countries, but allow it to vary across industries. The location parameter is allowed to vary across industries and countries.

Product attributes are independently distributed across the unit continuum of symmetric products. We make the "common product attributes" assumption that product attributes vary across products but are the same across countries ($\lambda_{hjik} = \lambda_{hik}$). For tractability purposes, we assume λ to be distributed Pareto with shape parameter a_λ and location parameter k_λ. We assume the same shape parameter for an industry across countries, but allow it to vary across industries. The location parameter is allowed to vary across industries and countries.

For the varieties produced by the manufacturing industries, we assume iceberg transport costs, which take the form $\tau_{hj} = (1 + c_{hj})(1 + t_{hj})$ for $j \neq h$ and $\tau_{jj} = 1$. (h and j denote the exporting and importing country, respectively.) c_{hj} denotes "natural" transport costs, and t_{hj} denotes policy-induced trade barriers. We can safely ignore tariff revenue, given the quasi-linear utility assumption above.

Fixed costs and transport costs are allowed to vary across industries and country-pairs.

χ is assumed to be the same for each product.

6.2 Firm-product profitability

Optimal pricing:

$$p_{hjik}(\gamma, \lambda) = \tau_{hji} \frac{1}{\rho} \frac{w_h}{\gamma}. \tag{118}$$

Since the production technology and elasticity of substitution χ across varieties are the same for each product, all products with productivity γ have the same price. (We can therefore dispense with the product k sub-index.)

Product revenue:

$$r_{hji}(\gamma, \lambda) = (w_h \tau_{hji})^{1-\chi} (\rho P_{ji} \gamma \lambda)^{\chi - 1} E_{ji}, \tag{119}$$

where $E_{ji} = \mu_{ji}$. From (119),

$$\frac{r_{hji}(\gamma, \lambda)}{r_{hj'i}(\gamma, \lambda)} = \left(\frac{\tau_{hji}}{\tau_{hj'i}}\right)^{1-\chi} \frac{\mu_j}{\mu_{j'}} \left(\frac{P_{ji}}{P_{j'i}}\right)^{\chi - 1}. \tag{120}$$

Product profits:

$$\pi_{hji}(\gamma, \lambda) = \frac{r_{hji}(\gamma, \lambda)}{\chi} - f_{hji}. \tag{121}$$

For each firm ability γ, there is a zero-profit cutoff for product attributes, $\lambda^*_{hj}(\gamma)$, for each source country and destination market, such that the firm only supplies the product if $\lambda \geq \lambda^*_{hji}(\gamma)$:

$$r_{hji}\left[\gamma, \lambda^*_{hji}(\gamma)\right] = \chi f_{hji}. \tag{122}$$

From (119) and (122),

$$\lambda^*_{hji}(\gamma) = \left(\frac{\gamma^*_{hji}}{\gamma}\right)\lambda^*_{hji}\left(\gamma^*_{hji}\right). \tag{123}$$

The higher γ, the lower $\lambda^*_{hji}(\gamma)$. The fraction of products supplied by industy i's firm with given γ from country h to market j: $1 - Z\left[\lambda^*_{hji}(\gamma)\right]$.

From (119) and (122),

$$\lambda^*_{hji}(\gamma) = \left[\frac{\chi f_{hji}}{(w_h\tau_{hji})^{1-\chi}(\rho P_{ji})^{\chi-1}E_{ji}}\right]^{\frac{1}{\chi-1}}\gamma^{-1}. \tag{124}$$

6.3 Firm profitability

Since product attributes are independently distributed across the unit continuum of symmetric products, the law of large numbers implies that a firm's expected revenue across the unit continuum of products equals its expected revenue for each product:

$$r_{hji}(\gamma) = \int_{\lambda^*_{hji}(\gamma)}^{\infty} r_{hji}(\gamma,\lambda)\, z(\lambda)\, d\lambda. \tag{125}$$

$$\pi_{hji}(\gamma) = \int_{\lambda^*_{hji}(\gamma)}^{\infty}\left[\frac{r_{hji}(\gamma,\lambda)}{\chi} - f_{hji}\right] z(\lambda)\, d\lambda - F_{hji}. \tag{126}$$

$$\pi_{hji}\left(\gamma^*_{hji}\right) = 0 \Longrightarrow \gamma^*_{hji}. \tag{127}$$

Putting together (127), (122), (126), and (120),

$$\int_{\lambda^*_{hji}\left(\gamma^*_{hji}\right)}^{\infty}\left[\left(\frac{\lambda}{\lambda^*_{hji}\left(\gamma^*_{hji}\right)}\right)^{\chi-1} - 1\right] f_{hji} z(\lambda)\, d\lambda = F_{hji}. \tag{128}$$

This equation implies a recursive structure: $\lambda^*_{hji}\left(\gamma^*_{hji}\right)$ is determined for each source country and destination market independently of price indices and labor endowments (and wages) as a function of fixed costs and other parameters.

Finally, from (119) and (122) one can also show that

$$\gamma^*_{hji} = \Gamma_{hjj'i}\gamma^*_{hj'i}, \tag{129}$$

$$\Gamma_{hjj'i} = \frac{\tau_{hji}}{\tau_{hj'i}}\frac{P_{j'}}{P_j}\left(\frac{f_{hji}}{f_{hj'i}}\frac{\mu_{j'}}{\mu_j}\right)^{\frac{1}{\chi-1}}\frac{\lambda^*_{hj'i}\left(\gamma^*_{hj'i}\right)}{\lambda^*_{hji}\left(\gamma^*_{hji}\right)}. \tag{130}$$

6.4 Productivity

From the appendix to Bernard et al. (2011),[9]

$$\tilde{\gamma}_{hji} \equiv \left[\frac{1}{1-G\left(\gamma^*_{hji}\right)}\int_{\gamma^*_{hji}}^{\infty}\left[\gamma\tilde{\lambda}_{hji}(\gamma)\right]^{\chi-1} g_h(\gamma)\,d\gamma\right]^{\frac{1}{\chi-1}}, \quad (131)$$

$$\tilde{\lambda}_{hji}(\gamma) \equiv \left[\frac{1}{1-Z\left[\lambda^*_{hji}(\gamma)\right]}\int_{\lambda^*_{hji}(\gamma)}^{\infty}\lambda^{\chi-1}z(\lambda)\,d\lambda\right]^{\frac{1}{\chi-1}}. \quad (132)$$

$$\begin{aligned}\tilde{\lambda}_{hji}(\gamma) &= \left[\left(\frac{k_\lambda}{\lambda^*_{hji}(\gamma)}\right)^{-a_\lambda}\frac{a_\lambda k_\lambda^{a_\lambda}}{a_\lambda-\chi+1}\left[\lambda^*_{hji}(\gamma)\right]^{\chi-a_\lambda-1}\right]^{\frac{1}{\chi-1}} = \\ &= \left[\frac{a_\lambda}{a_\lambda-\chi+1}\left[\frac{\chi f_{hji}}{(w_h\tau_{hji})^{1-\chi}(\rho P_{ji})^{\chi-1}E_{ji}}\right]\gamma^{1-\chi}\right]^{\frac{1}{\chi-1}}. \quad (133)\end{aligned}$$

$$\begin{aligned}\tilde{\gamma}_{hji} &= \left[\left(\frac{k_\gamma}{\gamma^*_{hji}}\right)^{-a_\gamma}\frac{a_\lambda}{a_\lambda-\chi+1}\left[\frac{\chi f_{hji}}{(w_h\tau_{hji})^{1-\chi}(\rho P_{ji})^{\chi-1}E_{ji}}\right]\int_{\gamma^*_{hji}}^{\infty}dG(\gamma)\right]^{\frac{1}{\chi-1}} \quad (134)\\ &= \left[\left(\frac{k_\gamma}{\gamma^*_{hji}}\right)^{-a_\gamma}\frac{a_\lambda}{a_\lambda-\chi+1}\left[\frac{\chi f_{hji}}{(w_h\tau_{hji})^{1-\chi}(\rho P_{ji})^{\chi-1}E_{ji}}\right]\left(\frac{k_\gamma}{\gamma^*_{hji}}\right)^{a_\gamma}\right]^{\frac{1}{\chi-1}} = \\ &= \left[\frac{a_\lambda}{a_\lambda-\chi+1}\frac{\chi f_{hji}}{E_{ji}}\right]^{\frac{1}{\chi-1}}\frac{w_h\tau_{hji}}{\rho P_{ji}}.\end{aligned}$$

$$\frac{\tilde{\gamma}'_{hji}}{\tilde{\gamma}_{hji}} = \frac{\hat{\tau}_{hji}}{\hat{P}_{ji}} = \hat{\gamma}^*_{hji}. \quad (135)$$

"Theory-based" industry productivity:

$$\tilde{\gamma}_{hi} = \left[\sum_j \frac{N_{hji}}{\sum_n N_{hni}}\left(\tilde{\gamma}_{hji}\right)^{\chi-1}\right]^{\frac{1}{\chi-1}}. \quad (136)$$

Implicit here is the assumption that parameter values across all the C_{jik} aggregated in equation (114) are identical. As before, the change in $\tilde{\gamma}_{hi}$ is given

[9] See http://www.princeton.edu/~reddings/papers/mpt_webappendix_030.pdf.

by:

$$\frac{\tilde{\gamma}'_{hi}}{\tilde{\gamma}_{hi}} = \left[\frac{\sum_j \frac{N'_{hji}}{\sum_n N'_{hni}} \left(\tilde{\gamma}'_{hji}\right)^{\chi-1}}{\sum_{j'} \frac{N_{hj'i}}{\sum_n N_{hni}} \left(\tilde{\gamma}_{hj'i}\right)^{\chi-1}}\right]^{\frac{1}{\chi-1}} = \left[\sum_j \left[\frac{\frac{N'_{hji}}{\sum_n N'_{hni}} \left(\tilde{\gamma}'_{hji}\right)^{\chi-1}}{\frac{N_{hji}}{\sum_n N_{hni}} \left(\tilde{\gamma}_{hji}\right)^{\chi-1}} \frac{\frac{N_{hji}}{\sum_n N_{hni}} \left(\tilde{\gamma}_{hji}\right)^{\chi-1}}{\sum_{j'} \frac{N_{hj'i}}{\sum_n N_{hni}} \left(\tilde{\gamma}_{hj'i}\right)^{\chi-1}}\right]\right]^{\frac{1}{\chi-1}} =$$

$$= \left[\left(\frac{\sum_n N'_{hni}}{\sum_n N_{hni}}\right)^{-1} \sum_j \left[\frac{N'_{hji}}{N_{hji}} \left(\frac{\tilde{\gamma}'_{hji}}{\tilde{\gamma}_{hji}}\right)^{\chi-1} \frac{N_{hji} \left(\tilde{\gamma}_{hji}\right)^{\chi-1}}{\sum_{j'} N_{hj'i} \left(\tilde{\gamma}_{hj'i}\right)^{\chi-1}}\right]\right]^{\frac{1}{\chi-1}} =$$

$$= \left[\left(\sum_n \left(\frac{N'_{hni}}{N_{hni}} \frac{N_{hni}}{\sum_m N_{hmi}}\right)\right)^{-1} \sum_j \left[\frac{N'_{hji}}{N_{hji}} \left(\frac{\tilde{\gamma}'_{hji}}{\tilde{\gamma}_{hji}}\right)^{\chi-1} \frac{N_{hji} \left(\tilde{\gamma}_{hji}\right)^{\chi-1}}{\sum_{j'} N_{hj'i} \left(\tilde{\gamma}_{hj'i}\right)^{\chi-1}}\right]\right]^{\frac{1}{\chi-1}}. \tag{137}$$

Note that (see (154) below)

$$X_{hji} = N_{hji} \frac{a_\gamma a_\lambda \chi}{\left(a_\gamma - a_\lambda\right)\left(\chi - 1\right)} F_{hji} \tag{138}$$

Under the assumption that market entry costs are proportional to some observable destination-specific factor *that is exogenous to the model* (such as m_{ji}, GDP, population, area,...):

$$\frac{N_{hji}}{\sum_n N_{hni}} = \frac{\left(X_{hji}/F_{hji}\right)^{\frac{(a_\gamma - a_\lambda)(\chi-1)}{a_\gamma a_\lambda \chi}}}{\sum_n \left(X_{hni}/F_{hni}\right)^{\frac{(a_\gamma - a_\lambda)(\chi-1)}{a_\gamma a_\lambda \chi}}} = \frac{X_{hji}/m_{ji}}{\sum_n X_{hni}/m_{ni}}. \tag{139}$$

From (134), (150) and (159):

$$\begin{aligned} \tilde{\gamma}_{hji} &= \left[\frac{a_\lambda}{a_\lambda - \chi + 1} \frac{\chi f_{hji}}{E_{ji}}\right]^{\frac{1}{\chi-1}} \frac{w_h \tau_{hji}}{\rho P_{ji}} \\ &= \gamma^*_{hji} F_{hji}^{-1/a_\lambda} \left(\frac{a_\lambda - \chi + 1}{\chi - 1}\right)^{-1/a_\lambda} f_{hji}^{\frac{1}{a_\lambda}} k_\lambda \left[\frac{a_\lambda}{a_\lambda - \chi + 1}\right]^{\frac{1}{\chi-1}} \\ &= k_\gamma M_{hi}^{1/a_\gamma} N_{hji}^{-1/a_\gamma} \left(\frac{f_{hji}}{F_{hji}}\right)^{1/a_\lambda} k_\lambda \left(\chi - 1\right)^{1/a_\lambda} \left(a_\lambda\right)^{\frac{1}{\chi-1}} \left(a_\lambda - \chi + 1\right)^{\frac{\chi-1+a_\lambda}{a_\lambda(1-\chi)}} \end{aligned} \tag{140}$$

$$N_{hji} \left(\tilde{\gamma}_{hji}\right)^{\chi-1} = M_{hi}^{\frac{\chi-1}{a_\gamma}} N_{hji}^{\frac{a_\lambda-\chi+1}{a_\gamma}} \left(\frac{f_{hji}}{F_{hji}}\right)^{\frac{\chi-1}{a_\lambda}} k_\lambda^{\chi-1} k_\gamma^{\chi-1} \left(\chi - 1\right)^{\frac{\chi-1}{a_\lambda}} a_\lambda \left(a_\lambda - \chi + 1\right)^{-\frac{\chi-1+a_\lambda}{a_\lambda}} \tag{141}$$

Thus,

$$\frac{N_{hji} \left(\tilde{\gamma}_{hji}\right)^{\chi-1}}{\sum_{j'} N_{hj'i} \left(\tilde{\gamma}_{hj'i}\right)^{\chi-1}} = \frac{N_{hji}^{\frac{a_\lambda-\chi+1}{a_\gamma}} \left(\frac{f_{hji}}{F_{hji}}\right)^{\frac{\chi-1}{a_\lambda}}}{\sum_{j'} N_{hj'i}^{\frac{a_\lambda-\chi+1}{a_\gamma}} \left(\frac{f_{hj'i}}{F_{hj'i}}\right)^{\frac{\chi-1}{a_\lambda}}} \tag{142}$$

Assuming that $f_{hji} = F_{hji}$ for all j, j' and as before that market entry costs F_{hji} are proportional to some observable destination-specific factor,

$$\frac{N_{hji}\left(\tilde{\gamma}_{hji}\right)^{\chi-1}}{\sum_{j'} N_{hj'i}\left(\tilde{\gamma}_{hj'i}\right)^{\chi-1}} = \frac{N_{hji}^{\frac{a_\lambda-\chi+1}{a_\gamma}}}{\sum_{j'} N_{hj'i}^{\frac{a_\lambda-\chi+1}{a_\gamma}}} = \frac{\left(X_{hji}/F_{hji}\right)^{\frac{a_\lambda-\chi+1}{a_\gamma}}}{\sum_{j'}\left(X_{hj'i}/F_{hj'i}\right)^{\frac{a_\lambda-\chi+1}{a_\gamma}}} = \frac{\left(X_{hji}/m_{ji}\right)^{\frac{a_\lambda-\chi+1}{a_\gamma}}}{\sum_{j'}\left(X_{hj'i}/m_{j'i}\right)^{\frac{a_\lambda-\chi+1}{a_\gamma}}} \tag{143}$$

Thus,

$$\frac{\tilde{\gamma}'_{hi}}{\tilde{\gamma}_{hi}} = \left[\left(\sum_n\left(\hat{N}_{hni}\frac{X_{hji}/m_{ji}}{\sum_n X_{hni}/m_{ni}}\right)\right)^{-1}\sum_j\left[\hat{N}_{hni}\left(\frac{\tilde{\gamma}'_{hji}}{\tilde{\gamma}_{hji}}\right)^{\chi-1}\frac{\left(X_{hji}/m_{ji}\right)^{\frac{1-\chi+a_\lambda}{a_\gamma}}}{\sum_{j'}\left(X_{hj'i}/m_{j'i}\right)^{\frac{1-\chi+a_\lambda}{a_\gamma}}}\right]\right]^{\frac{1}{\chi-1}} \tag{144}$$

Revenue-based industry (labor) productivity:

$$\frac{VA_{hi}}{L_{hi}} = \frac{R_{hi}}{L_{hi}} \tag{145}$$

The real growth rate of industry labor productivity, deflated by the price of the average firm, is[10]

$$\widehat{\left(\frac{VA_{hi}}{L_{hi}}\right)} = \left(\frac{R_{hi}}{p\left(\tilde{\gamma}_{hi}\right)L_{hi}}\right)/\left(\frac{R'_{hi}}{p\left(\tilde{\gamma}_{hi}\right)' L'_{hi}}\right) = \frac{\hat{R}_{hi}}{\hat{L}_{hi}}\frac{1}{\widehat{p\left(\tilde{\gamma}_{hi}\right)}} \tag{146}$$

where $p\left(\tilde{\gamma}_{hi}\right)$ is the average price of the average (domestic) firm defined as:

$$\begin{aligned} p\left(\tilde{\gamma}_{hi}\right) &= \frac{1}{1-Z\left[\lambda^*_{hji}\left(\tilde{\gamma}_{hi}\right)\right]}\int_{\lambda^*_{hji}\left(\tilde{\gamma}_{hi}\right)}^{\infty} p_{hhik}\left(\gamma,\lambda\right)z\left(\lambda\right)d\lambda \qquad (147)\\ &= \frac{1}{1-Z\left[\lambda^*_{hji}\left(\tilde{\gamma}_{hi}\right)\right]}\int_{\lambda^*_{hji}\left(\tilde{\gamma}_{hi}\right)}^{\infty}\frac{1}{\rho}\frac{w_h}{\tilde{\gamma}_{hi}}z\left(\lambda\right)d\lambda \\ &= \frac{\chi-1}{\chi}\frac{w_h}{\tilde{\gamma}_{hi}} \end{aligned}$$

Note that using geometric weights leads to the same result:

$$\begin{aligned} p\left(\tilde{\gamma}_{hi}\right) &= \left(\frac{1}{1-Z\left[\lambda^*_{hji}\left(\tilde{\gamma}_{hi}\right)\right]}\int_{\lambda^*_{hji}\left(\tilde{\gamma}_{hi}\right)}^{\infty} p_{hhik}\left(\gamma,\lambda\right)^{1-\chi}z\left(\lambda\right)d\lambda\right)^{\frac{1}{1-\chi}} \qquad (148)\\ &= \left(\frac{1}{1-Z\left[\lambda^*_{hji}\left(\tilde{\gamma}_{hi}\right)\right]}\int_{\lambda^*_{hji}\left(\tilde{\gamma}_{hi}\right)}^{\infty}\left(\frac{1}{\rho}\frac{w_h}{\tilde{\gamma}_{hi}}\right)^{1-\chi}z\left(\lambda\right)d\lambda\right)^{\frac{1}{1-\chi}} \\ &= \frac{\chi-1}{\chi}\frac{w_h}{\tilde{\gamma}_{hi}} \end{aligned}$$

Thus,

$$\widehat{\left(\frac{VA_{hi}}{L_{hi}}\right)} = \frac{\hat{R}_{hi}}{\hat{L}_{hi}}\frac{\tilde{\gamma}'_{hi}}{\tilde{\gamma}_{hi}} \tag{149}$$

[10] See Section 3 and Appendix A in the paper for a detailed discussion of the appropriate choice of deflator.

6.5 Number of firms

$$N_{hji} = \left(\frac{k_\gamma}{\gamma^*_{hji}}\right)^{a_\gamma} M_{hi}. \tag{150}$$

$$\hat{N}_{hji} = \left(\hat{\gamma}^*_{hji}\right)^{-a_\gamma}.$$

6.6 Aggregate exports

Using (119) and (124),

$$\begin{aligned}
X_{hji} &= N_{hji} E\left[r_{hji} \middle| \gamma > \gamma^*_{hji}\right] = \frac{N_{hji}}{1 - G\left(\gamma^*_{hji}\right)} \int_{\gamma^*_{hji}}^{\infty} r_{hji}(\gamma)\, dG(\gamma) = \\
&= \frac{N_{hji}}{1 - G\left(\gamma^*_{hji}\right)} \int_{\gamma^*_{hji}}^{\infty} \int_{\lambda^*_{hji}(\gamma)}^{\infty} r_{hji}(\gamma, \lambda)\, dZ(\lambda)\, dG(\gamma) = \\
&= N_{hji} \frac{a_\lambda k_\lambda^{a_\lambda}}{a_\lambda - \chi + 1} E_{ji} \left(\frac{\rho P_{ji}}{w_h \tau_{hji}}\right)^{\chi - 1} \left[\frac{\chi f_{hji}}{\left(w_h \tau_{hji}\right)^{1-\chi} \left(\rho P_{ji}\right)^{\chi - 1} E_{ji}}\right]^{\frac{\chi - a_\lambda - 1}{\chi - 1}} \frac{a_\gamma}{a_\gamma - a_\lambda} \left(\gamma^*_{hji}\right)^{a_\lambda} \\
&= N_{hji} \frac{a_\gamma}{a_\gamma - a_\lambda} \frac{a_\lambda k_\lambda^{a_\lambda}}{a_\lambda - \chi + 1} \left(\chi f_{hji}\right)^{\frac{\chi - a_\lambda - 1}{\chi - 1}} E_{ji}^{\frac{a_\lambda}{\chi - 1}} \left(\frac{\rho P_{ji}}{w_h \tau_{hji}}\right)^{a_\lambda} \left(\gamma^*_{hji}\right)^{a_\lambda}. \qquad (151)
\end{aligned}$$

Implicit in this derivation are the following results:

$$\int_{\lambda^*_{hji}(\gamma)}^{\infty} \lambda^{\chi - 1} dZ(\lambda) = \frac{a_\lambda k_\lambda^{a_\lambda}}{a_\lambda - \chi + 1} \left[\lambda^*_{hji}(\gamma)\right]^{\chi - a_\lambda - 1}, \tag{152}$$

$$\int_{\gamma^*_{hji}}^{\infty} \gamma^{a_\lambda} dG(\gamma) = \frac{a_\gamma k_\gamma^{a_\gamma}}{a_\gamma - a_\lambda} \left(\gamma^*_{hji}\right)^{a_\lambda - a_\gamma}. \tag{153}$$

These derivatives are only well defined if $a_\gamma > a_\lambda > \chi - 1$. Using the expression for the cutoff γ^*_{hji} from below (see (159)):

$$X_{hji} = N_{hji} \frac{a_\gamma a_\lambda \chi}{\left(a_\gamma - a_\lambda\right)(\chi - 1)} F_{hji} \tag{154}$$

6.7 Employment by industry

$$l_{hji}(\gamma, \lambda) = \tau_{hji} q_{hji}(\gamma, \lambda) / \gamma = \rho r_{hji}(\gamma, \lambda). \tag{155}$$

$$l_{hji}(\gamma) = \rho \int_{\lambda^*_{hji}(\gamma)}^{\infty} r_{hji}(\gamma, \lambda)\, dZ(\lambda) = \rho r_{hji}(\gamma). \tag{156}$$

$$L_{hji} = M_{hi} \int_{\gamma^*_{hji}}^{\infty} l_{hji}(\gamma)\, dG(\gamma) = \rho X_{hji}. \tag{157}$$

$$L_{hi} = \rho R_{hi} = \rho \sum_j X_{hji}. \tag{158}$$

6.8 Cutoff

From (127), (124), and (152),

$$\left(\gamma_{hji}^{*}\right)^{a_\lambda} = F_{hji}\left(\frac{a_\lambda-\chi+1}{\chi-1}\right)\chi^{\frac{a_\lambda}{\chi-1}} f_{hji}^{\frac{\chi-a_\lambda-1}{1-\chi}} k_\lambda^{-a_\lambda} E_{ji}^{\frac{a_\lambda}{1-\chi}} w_h^{a_\lambda}\tau_{hji}^{a_\lambda}\rho^{-a_\lambda}P_{ji}^{-a_\lambda}. \tag{159}$$

6.9 Price index

$$\begin{aligned} P_{ji} &= \left[\sum_h M_h \int_{\gamma_{hji}^*}^{\infty}\int_{\lambda_{hji}^*(\gamma)}^{\infty}\left[\frac{p_{hji}(\gamma)}{\lambda_{hji}}\right]^{1-\chi} dZ(\lambda)\, dG(\gamma)\right]^{\frac{1}{1-\chi}} = \\ &= \left[\sum_h \frac{N_{hji}a_\lambda k_\lambda^{a_\lambda}}{a_\lambda-\chi+1}\left(\frac{\rho}{w_h\tau_{hji}}\right)^{\chi-1}\left[\frac{\chi f_{hji}}{\left(w_h\tau_{hji}\right)^{1-\chi}\left(\rho P_{ji}\right)^{\chi-1}E_{ji}}\right]^{\frac{\chi-a_\lambda-1}{\chi-1}}\frac{a_\gamma}{a_\gamma-a_\lambda}\left(\gamma_{hji}^*\right)^{a_\lambda}\right]^{\frac{1}{1-\chi}} = \\ &= \left[\sum_h X_{hji}P_{ji}^{1-\chi}E_{ji}^{-1}\right]^{\frac{1}{1-\chi}}. \end{aligned} \tag{160}$$

6.10 Calibration

Taking growth rates of the price index,

$$\hat{P}_{ji}^{-a_\gamma} = \sum_h \frac{X_{hji}}{\sum_h X_{hji}}\hat{\tau}_{hji}^{-a_\gamma}. \tag{161}$$

$$\hat{\gamma}_{hji}^* = \frac{\hat{\tau}_{hji}}{\hat{P}_{ji}}. \tag{162}$$

$$\hat{N}_{hji} = \left(\hat{\gamma}_{hji}^*\right)^{-a_\gamma}. \tag{163}$$

$$\hat{X}_{hji} = \hat{N}_{hji}. \tag{164}$$

$$\hat{R}_{hi} = \sum_j \frac{X_{hji}}{\sum_j X_{hji}}\hat{X}_{hji}. \tag{165}$$

$$\hat{L}_{hi} = \hat{R}_{hi}. \tag{166}$$

$$\frac{\tilde{\gamma}_{hji}'}{\tilde{\gamma}_{hji}} = \hat{\gamma}_{hji}^*. \tag{167}$$