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Stochastic Unemployment with Dynamic Monopsony

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Stochastic unemployment with dynamic monopsony.

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Abstract

This paper considers an equilibrium labour market with on-the-job search where heterogeneous firms, both by productivity and size, post wages and choose optimal hiring strategies. There are aggregate and firm specific productivity shocks. Industry dynamics are rich. By comparing the market outcome to the competitive allocation, simple numerical examples establishes how dynamic monopsony generates excessive job-to-job turnover, excessive job destruction rates at low productivity firms and so generates "too high" unemployment. It explains why gross hire flows and gross separation flows may be large and volatile, yet yield an unemployment process which is highly persistent.

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1 Introduction

Menzio and Shi (2011) demonstrate for the U.S. that job-to-job transitions are large, volatile and markedly procyclical. Fallick and Fleishman (2004) further report for the U.S. that around 40% of new hires are filled by job-to-job transitions. In this paper we illustrate using numerical examples why such turnover not only has a major impact on the unemployment dynamics of the economy, it has important consequences for government policy.

A central issue within the dynamic monopsony framework is whether firms can wage discriminate between identical workers. Burdett and Mortensen (1998) presumes not: that equal pay legislation requires firms must pay the same wage to equally productive workers doing the same job (regardless of race, gender etc). In contrast the sequential auction approach of Postel-Vinay and Robin (2002) allows perfect wage discrimination which then allows firms to fully respond to outside offers. Here we use dynamic monopsony to refer to the no wage discrimination case. In this case wage setting (monopsonistic) firms face a trade-off between paying low company wages and low worker retention rates. Should a worker receive a preferred outside offer, the worker simply quits and the firm chooses recruitment effort to hire replacement employees who are always hired at the company wage.

Moscarini and Postel-Vinay (2013) and Coles and Mortensen (2015) [CM from now on] extend the Burdett and Mortensen (1998) framework to costly firm hiring and stochastic shocks. CM in addition allows rich industry dynamics, where new start-up companies enter the market over time while older (less productive firms) exit the market and there are firm specific productivity shocks. In these frameworks, a macro-model of equilibrium unemployment is obtained by simply integrating over the (endogenously evolving) firm size distribution.

Using the CM framework we compare the dynamic monopsony equilibrium to the competitive allocation and show why dynamic monopsony generates too high unemployment. The reason is not because firm hiring rates are too low. Rather dynamic monopsony generates excessive quit rates where a job-to-job transition implies a job is destroyed at the previous employer (and replacing a worker who quits is a costly, time consuming process). In contrast the (steady state) competitive allocation corresponds to a single Walrasian wage and there is no wasteful worker turnover.

We illustrate with numerical examples the four key insights of this approach:

- it is more efficient that a firm hires from the unemployment pool than poach an employed worker who has already been recruited at some cost. The competitive (steady state) equilibrium finds the Auctioneer announces a single wage (the shadow price of an unemployed worker) and there are no job-to-job transitions. Instead equilibrium wage dispersion with dynamic monopsony induces wasteful job turnover as already employed workers on low wages seek better paid employment;

- by generating excessive job-to-job quit rates, dynamic monopsony causes too high job destruction rates at low productivity firms. A numerical example establishes why unemployment is significantly higher than in the competitive outcome;

- as aggregate hiring rates increase in an economic recovery, job-to-job quits also increase. This not only crowds out the re-employment prospects of the unemployed, excessive job destruction rates at low productivity firms implies unemployment is slow to fall. As job-to-job flows automatically track hire flows, unemployment remains persistently high in an economic recovery;

- the equilibrium distribution of wages paid differs from that of the competitive allocation and so distorts the distribution of employment across heterogeneous firms.

Moscarini and Postel-Vinay (2013) was the first to consider stochastic equilibria with dynamic monopsony and undirected search. For the most part, it considers exogenous contact rates and establishes the existence of rank preserving equilibria. When hiring is instead costly and endogenously chosen, the framework is much more complex for the distribution of employment across firms, G, is a relevant state variable (it determines the distribution of worker quit rates) is infinitely dimensional and evolves endogenously depending on firm recruitment choices. Dealing with this state space issue is a major problem. There are currently three successful theoretical approaches.

Menzio and Shi (2011) attack this issue in a directed on-the-job search framework. One key assumption is that it assumes firms commit to pay marginal product and post job fees which potential employees must pay on being hired (see Stevens (2004)). As employees are always paid marginal product, their quit decisions are then jointly efficient. Directed on-the-job search by employed workers also ensures that job-to-job transitions do not crowd out the re-employment prospects of the unemployed. By also making a standard free entry of vacancies assumption, Menzio and Shi (2011) then show the equilibrium firm and worker values are independent of G : so-called block recursivity. This strong result makes numerical computation of market equilibrium remarkably straightforward. As the market is constrained efficient, however, there is no interesting role for labor market policy.

Lise and Robin (2015) instead adopt the sequential auction approach introduced by Postel-Vinay and Robin (2002), augmented with a competitive vacancy market. Like Menzio and Shi (2011), quits are jointly efficient though for a different reason: firms Bertrand compete on values paid should an employee receive an outside offer. Also like Menzio and Shi (2011), competitive vacancy creation yields the strong result that the equilibrium surplus to a match can be computed independently of G. With two sided heterogeneity and a one-firm/one job technology assumption, it identifies a most elegant framework for understanding how mismatch, in a frictional assignment framework, evolves over the cycle.

As job fees are ruled out by assumption and firms cannot wage discriminate, quits in CM are not jointly efficient and equilibrium does not have the "block recursive" property. Equilibrium is instead a complex fixed point problem where firm wage and hiring strategies must be a best response to the market collective and in a stochastic environment. CM identifies a tractable framework by assuming hiring costs exhibit constant returns to scale - that should a larger firm with twice as many employees wish to hire twice as many new hires, then its hiring costs are simply double. CM then shows in a Markov perfect (Bayesian) equilibrium that the (finite) vector $\mathbf{N}$, where N_i describes total employment in firms in state i, is a sufficient statistic for the infinitely dimensional firm size distribution function G. Equilibrium can then be directly computed using value function iteration. Here we illustrate the underlying principles using simple numerical examples.

The paper is structured as follows. Sections 2-4 quickly describe the CM framework and equilibrium. Section 5 restricts attention to steady state and compares the equilibrium properties of dynamic monopsony to the competitive allocation. Section 6 considers non-steady state dynamics and, by computing the impulse response function, identifies how the economy adjusts to a favourable (permanent) aggregate productivity shock.

2 The CM Model.

Time is continuous, denoted $t \in [0, \infty)$. There is a unit measure of agents who are risk neutral, equally productive, infinitely lived and discount the future at rate $r > 0$. Each agent is either (i) employed earning some wage w, (ii) unemployed with home productivity $b \geq 0$ or (iii) an entrepreneur trying to start-up a new company.

Each firm is risk neutral with productivity $p = p(x, \theta)$ where $x \in [0, 1]$ is a firm specific productivity parameter and $\theta \in [\underline{\theta}, \overline{\theta}]$ an aggregate parameter. There are constant returns to scale: a firm with (integer) n employees generates flow revenue $np(x, \theta)$. $p(.)$ is increasing in both arguments, where the lowest productivity state $\underline{p} = p(0, \underline{\theta}) > b$ and $\overline{p} = p(1, \overline{\theta})$ denotes maximal productivity. For the moment assume $p(.)$ is strictly increasing in x but we relax this assumption for the case of finite firm productivity states.

There is firm turnover: new firms are created at an exogenous rate $\mu > 0$ while existing firms die at rate $\delta(x, \theta) \geq 0$ where $\delta(.)$ is decreasing in x (i.e. more productive firms have greater survival rates). At start-up, a firm's initial productivity x is a random draw from c.d.f. $\Gamma_0(.)$. Conditional on survival, each firm x receives a firm specific productivity shock at rate $\gamma \geq 0$, whereupon its new productivity $x' \in [0, 1]$ is considered a random draw from c.d.f. $\Gamma_1(.|x)$. Throughout we assume first order stochastic dominance in Γ_1 and $\Gamma_1(0|0) = 1$ which implies the lowest productivity state is absorbing. Firm productivity x is private information to the firm.

θ evolves stochastically according to a Poisson process with arrival rate $\alpha \geq 0$ and new productivity draw $\theta' \in [\underline{\theta}, \overline{\theta}]$ considered a random draw from c.d.f. $H(\theta'|\theta)$. We assume the aggregate productivity parameter θ is common knowledge.

Firms post wages, cannot precommit to future wages and equal treatment (no discrimination) requires a firm must pay equally productive employees the same wage. As productivity x is private information, a firm's posted wage w signals x. Should an employee receive an outside job offer at wage w', the worker uses these wage signals to predict future wages at the two firms and takes employment at the firm which is perceived to offer greater expected value. Once a worker rejects a job, there is no recall.

If a firm with n employees recruits new employees at rate H, its foregone revenues due to the costly recruitment process is $p(x, \theta)C(H, n)$.[1] Assuming

[1] Recent empirical work suggests that "adjustment costs" of this form explain the data

constant returns in $C(.)$, hiring costs are equivalent to $np(x,\theta)c(h)$, where $h = H/n$ is the firm's hire rate per employee, $c(h) \equiv C(h,1)$ and $c(.)$ is assumed continuously differentiable, strictly convex with $c(0) = c'(0) = 0$.

We abstract from job search effort: all agents receive job offers at the same rate $\lambda(.)$ and, with random contacts, we let $F(.)$ denote the distribution of the value of job offers made by firms. Both of these objects are endogenously determined and evolve stochastically.

Let $1 - G_t(x)$ denote the measure of workers employed at firms with productivity parameter no less than $x \in [0,1]$ at date t. Hence $U_t = G_t(0)$ is the number who are not employed and $G_t(1) = 1$. Agents who are not employed choose either to be home producers (with flow output b) or entrepreneurs. Let $E_t \leq U_t$ denote the measure of non-employed agents who choose to be entrepreneurs. There is perfect crowding out: each entrepreneur successfully starts up a new firm at rate μ/E_t. Should an entrepreneur successfully create a new start-up, he/she sells the start-up company for its value and becomes the firm's first employee. In this way, each start-up begins life with $n = 1$ and $x \sim \Gamma_0$. Throughout we assume μ/b sufficently small that some unemployed workers always choose to be home producers and so $E_t < U_t$. This simplifies as it ensures no employed worker wishes to quit into unemployment to become an entrepreneur.

3 Markov Perfect (Bayesian) Equilibria.

As CM fully characterise equilibrium, here we only sketch the relevant argument. We restrict attention to separating Markov perfect (Bayesian) equilibria [MPBE from now on] in which the optimal wage strategies of firms are continuous and strictly increasing in x. This is a natural case for, as in a first price auction, more productive firms (buyers) bid strictly higher wages and workers sell their services to the buyer who posts the highest price. Indeed the information structure and random contacts assumption yields a first price auction structure with private independent values. It is thus highly tractable.

Each firm in an MPBE is described by (x, n, θ, G). Let $\Pi(x, n, \theta, G)$ denote its expected discounted profit. As there are constant returns to scale, the optimal wage strategies $w = w(x, \theta, G)$ are independent of firm size. Below

better than the more traditional "recruiting cost" specification. For example, see Sala et al. (2012) and Christiano et al (2012).

we establish the empirically well-known large firm wage effect, that wages are positively correlated with (though here not caused by) firm size.

As equilibrium $w(.)$ is strictly increasing in x, the firm's wage offer fully reveals its productivity x. Thus given any announced wage $w' \in [w(0,.), w(1,.)]$, Bayes rule implies each employee infers its employer has productivity $x = \widehat{x} \in [0,1]$ solving $w(\widehat{x}, \theta, G) = w'$. If a firm posts a "too low" wage $w' < w(0,.)$, there is no firm state which corresponds to this posted wage. Existence of an equilibrium requires belief $\widehat{x}(w',.) = 0$. If instead the firm posts wage $w' > w(1,.)$, we set belief $\widehat{x}(w',.) = 1$. Out-of-equilibrium beliefs in this latter case play no important role.

As equilibrium wages paid do not depend on firm size, we can let $W(x, \theta, G)$ denote a worker's equilibrium lifetime discounted utility when employed at firm (x, n, θ, G). Employees, of course, do not observe x. Given any posted wage w', each employee first updates productivity belief $\widehat{x}(w',.)$ as defined above and then infers corresponding value of employment $W' = W(\widehat{x}, \theta, G)$. We let $V_u(\theta, G)$ denote the value of being non-employed which, with free entry into entrepreneurship, must also describe the value of being an entrepreneur. CM formally establish the following Claim.

Claim 1. Optimal worker search:

(i) Unemployed workers use a reservation wage strategy $R = b$; i.e. they accept any job which offers wage $w \geq b$ and prefer being unemployed otherwise;

(ii) Employed workers on wage $w \geq b$ quit to an outside offer if and only if it offers a higher wage $w' > w$.

Workers always quit to a higher outside wage offer as they infer the outside firm has higher productivity and, as wages are increasing in productivity, anticipate higher wages at that firm in the entire future. Workers have reservation wage b as the unemployed and employed have equal access to the same job search technology (e.g. Lise (2013)). Of course with endogenous job search effort, unemployed workers would choose greater search effort (as their return to search is larger) and receive job offers at a faster rate. The extension to endogenous search effort, however, is beyond the scope of this paper.[2] Given such search strategies, we now determine the equilibrium wage and hiring strategies of firms.

[2] CM also show that Claim 1 requires state $x = 0$ must also be absorbing. Otherwise there is a positive probability of a favourable productivity shock and the option value of remaining employed would imply $R < b$.

4 Equilibrium Firm Behaviour.

Firm (x, n, θ, G) chooses wage w' and recruitment rate h' to solve the recursive Bellman equation:

$$[r+\delta(x,\theta)]\Pi(x,n,.) = \max_{w',h'\geq 0} \left\langle \begin{array}{c} n[p(x,\theta)-w'] - np(x,\theta)c(h') \\ +nh'\left[\Pi(x,n+1,.)-\Pi(x,n,.)\right] \\ +nq(\widehat{x},.)\left[\Pi(x,n-1,.)-\Pi(x,n,.)\right] \\ +\gamma\int_0^1 \left[\Pi(x',n,.)-\Pi(x,n,.)\right] d\Gamma_1(x'|x) \\ +\alpha\int_{\underline{\theta}}^{\overline{\theta}} \left[\Pi(x,n,\theta',.)-\Pi(x,n,\theta,.)\right] dH(\theta'|\theta) + \frac{\partial\Pi}{\partial t} \end{array} \right\rangle .$$

The first line describes flow profit, the second is the capital gain by successfully recruiting a new employee, the third is the profit loss though a quit (where the quit rate $q(\widehat{x}, \theta, G)$ depends on the induced belief $\widehat{x} = \widehat{x}(w', .)$), the fourth the gain through a state-x productivity shock, the last the gain through a state-θ productivity shock. The last term captures the effect on $\Pi(.)$ through the stochastic evolution of G.

The constant returns structure implies $\Pi(x, n, \theta, G) = nv(x, \theta, G)$ where $v(x, \theta, G)$ is the profit per employee in firm (x, n, θ, G). The Bellman equation above reduces to the following Bellman equation for $v(.)$:

$$(r+\delta(.)+\gamma+\alpha)v(x,.) = \max_{w',h'\geq 0} \left\langle \begin{array}{c} p(x,\theta)+[h'v(x,.)-p(x,\theta)c(h')] \\ -[w'+q(\widehat{x}(w',.),.)v(x,.)] \\ +\gamma\int_0^1 v(x',.)d\Gamma_1(x'|x) \\ +\alpha\int_{\underline{\theta}}^{\overline{\theta}}[v(x,\theta',.)]dH(\theta'|\theta)+\frac{\partial v}{\partial t} \end{array} \right\rangle . \quad (1)$$

(1) implies the firm's optimal hire strategy $h = h(x, \theta, G)$ satisfies

$$c'(h) = \frac{v(x,\theta,G)}{p(x,\theta)}, \quad (2)$$

and so is independent of firm size. Note this yields a form of Gibrat's law: conditional on survival, the net growth rate of firm (x, n, θ, G) is $[h(x, \theta, G) - q(x, \theta, G)]$ which depends on the firm's state (x, θ, G) but is otherwise independent of firm size n.

We next compute the worker quit function $q(.)$. As offers are random and workers only quit from lower productivity firms, each equilibrium job offer by firm x, which offers wage $w = w(x, .)$, is only accepted with probability

$G(x)$. Hence to hire at equilibrium rate $H = nh(x, \theta, G)$, the firm must make job offers at rate $nh(x, \theta, G)/G(x)$. Aggregating across firms, noting there is a unit measure of workers, implies each worker receives a job offer at rate

$$\lambda(\theta, G) = \int_0^1 \frac{h(x, \theta, G)}{G(x)} dG(x), \tag{3}$$

where $dG(x)$ describes the measure of workers employed at type x firms. The same aggregation argument implies equilibrium quit function

$$q(x, \theta, G) = \int_x^1 \frac{h(z, \theta, G)}{G(z)} dG(z), \tag{4}$$

as employed workers at firm (believed to be in state) x only quit to more productive firms $x' > x$ (which offer higher wages). If G is differentiable, the firm's marginal quit rate is thus:

$$\frac{\partial q}{\partial x} = -\frac{h(x, \theta, G)G'(x)}{G(x)}. \tag{5}$$

The Bellman equation (1) implies the firm's wage strategy minimizes the sum of the wage bill and turnover costs; i.e.

$$w(x, .) = \arg\min_{w'} \left[w' + q(\widehat{x}, \theta, G)] v(x, .) \right]. \tag{6}$$

CM establish the following wage equation.

Claim 2: Equilibrium wage equation.

If G is differentiable, equilibrium $w(.)$ is the solution to the initial value problem:

$$\frac{\partial w}{\partial x} = \frac{h(x, .)G'(x)}{G(x)} v(x, .) \text{ for all } x \in [0, 1] \tag{7}$$

with $w(0, .) = b$.

The wage equation (7) describes the dynamic monopsony wage trade-off: a marginally higher wage paid by firm x marginally reduces its employee quit rate by $h(x, .)G'(x)/G(x)$ with corresponding value $v(x, .)$ should the employee choose not to quit. This wage equation has the same structure as that of a first price auction with independent private values. Less productive firms, say those in state $x_L < x$, bid lower wage $w_L = w(x_L, .) < w(x, .)$ as they have lower employee value $v_L = v(x_L, .)$. At this wage point w_L, firm

x finds the return to reducing its quit rate exceeds the profit loss by paying higher wages; i.e.

$$\frac{h(x_L,.)G'(x_L)}{G(x_L)}v(x,.) > \frac{h(x_L,.)G'(x_L)}{G(x_L)}v_L = \frac{\partial w(x_L)}{\partial x}.$$

It thus bids higher wage $w > w_L$. Of course it does not raise its wage beyond equilibrium wage $w(x,.)$ as the reduced quit rate no longer fully compensates for the cost of paying higher wages. Given this wage structure, an MPBE can now be formally defined.

Definition of Equilibrium: An MPBE is the set $< v, h, q, w, \widehat{x} >$ such that for all $x \in [0,1]$ and (θ, G):

(Di) employee value $v(x,\theta,G)$ satisfies (1);

(Dii) hire strategy $h(x,\theta,G)$ satisfies (2);

(Diii) quit function

$$q(x,\theta,G) = \int_x^1 \frac{h(z,\theta,G)dG(z)}{G(z)}, \tag{8}$$

(Div) wage strategies $w(x,\theta,G)$ satisfy Claim 2, where

(Dv) beliefs $\widehat{x}(w',\theta,G)$ are rational, satisfying

$$\begin{aligned} w(\widehat{x},\theta,G) &= w' \text{ when } w' \in [w(0,\theta,G), w(1,\theta,G)]\,; \\ \widehat{x} &= 0 \text{ when } w' < w(0,\theta,G); \\ \widehat{x} &= 1 \text{ when } w' > w(1,\theta,G); \end{aligned}$$

(Dvi) (θ, G) are Markov processes which evolve consistently with the equilibrium hire and quit strategies.

5 Steady State MPBE and the Competitive Allocation.

As there is no "matching function" in the above, the competitive allocation is well-defined. To identify the social inefficiency due to dynamic monopsony, we compare in this section the steady state MPBE outcome with no shocks ($\alpha = \gamma = 0$) to the (steady state) competititve allocation.

For ease of exposition, suppose death rate $\delta(x) = \delta_0 > \mu$ is the same for all firms and that Γ_0 is uniform. Standard turnover arguments then imply steady state employment distribution

$$G(x) = \frac{\delta_0 - \mu[1-x]}{\delta_0 + q(x)}. \tag{9}$$

Proposition 1. In steady state and if $p(.)$ is differentiable, an MPBE implies $\{v(.), q(.)\}$ satisfy:

$$\frac{dv}{dx} = \frac{[1-c(h)]\, p'(x)}{r + \delta_0 + q - h}, \tag{10}$$

$$\frac{dq}{dx} = -\frac{h\mu\,[\delta_0 + q]}{[\delta_0 - \mu(1-x)]\,[\delta_0 + q - h(x)]}, \tag{11}$$

where $h = h(x)$ solves $c'(h) = v(x)/p(x)$. At $x = 0$, $(v, q) = (v_0, q_0)$ where v_0 satisfies:

$$(r + \delta_0 + q_0)v_0 = p(0) - b + \max_{h'} \left[h'v_0 - p(0)c(h')\right], \tag{12}$$

and $q_0 > 0$ is tied down by the endpoint condition $q(1) = 0$.

Proof of Proposition 1. Substitute out G in (8) using (9) to get (11). (10) follows by differentiating the Bellman equation (1) with respect to x, using the Envelope Theorem and noting $w'(x) = -q'(x)v(x)$ by Claim 2. Putting $x = 0$ in (1) and $w(0) = b$ yields (12). This completes the proof of Proposition 2.

The (steady state) MPBE is the solution to a pair of first order differential equations for (v, q) along with a pair of boundary conditions for $v(0)$ and $q(1)$. (10) determines $v(x)$ which depends on employee quit rates $q(.)$. Associated with those values $v(x)$ are firm hiring rates $h(x)$ where $c'(h) = v(x)/p(x)$. (11) determines the equilibrium quit function $q(.)$ given those hiring rates $h(.)$. Equilibrium is a fixed point where each firm's value $v(.)$ depends on its employee quit rate $q(.)$, while quit rates depend on the values of all other firms and their associated recruitment rates $h(.)$.

It is easy to show the type-x firm size distribution is Pareto with average firm size

$$\overline{n}(x) = \frac{\delta_0}{\delta_0 + q(x) - h(x)}. \tag{13}$$

Note firm size is unboundedly large if $h(x) = \delta_0 + q(x)$. In the numerical example below, it turns out that $h(x) - \delta_0 - q(x) \to 0$ as $x \to 1$ and so a handful of the most productive firms become very large corporations. This singularity makes problematic a formal existence proof. Numerical simulations, however, are perfectly straightforward.

It is easy to show the conditions of Proposition 1 imply hire rates $h(.)$ increase in x and quits $q(.)$ decrease. Thus (13) implies firm size is increasing, on average, with x. We thus obtain the large firm wage effect - that larger firms, on average, pay higher wages - even though here wages do not depend on firm size. This structure also implies workers, on average, quit from small to large firms. In contrast to Moscarini and Postel-vinay (2013) however, it does not imply workers only ever quit from small to large firms. Quits also occur from large to small whenever an outside offer is received from a smaller but higher growth, start-up company.

Now compare this outcome to the competitive allocation. As all workers are identical, the Walrasian auctioneer posts a single wage ω equal to the shadow price of an unemployed worker. As all workers are paid the same, there is no job-to-job turnover: all hires are from the unemployment pool. The resulting allocation is efficient as all investment costs are firm specific and are paid by firms.

Define the participation margin x^C where $p(x^C) = \omega$. Low productivity firms with $x < x^C$ are inactive. Firms with $x \geq x^C$ instead enjoy value v^C given by:

$$(r + \delta_0)v^C(x) = p(x) - \omega + \max_{h'} \left[h'v^C(x) - p(x)c(h') \right], \tag{14}$$

where each employee is used in the recruitment process to hire additional new employees. The solution for $v^C(x)$ determines the firm's hiring rate $h^C(x)$ which in turn yields average firmsize

$$\overline{n}^C(x) = \frac{\delta_0}{\delta_0 - h^C(x)}, \tag{15}$$

as quit rates are zero. As $[\mu/\delta_0]dx$ is the steady measure of firms with rank $x' \in [x, x + dx] \subseteq [0, 1]$, steady state employment is

$$N^C = \frac{\mu}{\delta_0} \int_{x^C}^{1} \overline{n}^C(x)dx.$$

There are two types of competitive allocation, either

- (i) $N^C < 1$ (positive unemployment) and $\omega = b$, or

- (ii) $N^C = 1$ (full employment) and $\omega \geq b$.

We demonstrate the cost of dynamic monopsony using a simple numerical example where Γ_0 is uniform and $p(.)$ is linear with $p(x) = b + x[\overline{p} - b]$. Following Mortensen and Nagypal (2007), suppose $b = 0.7$ and set $\overline{p} = 1.3$ so that the expected productivity of a new start-up is one. Using a month as the reference unit of time, we set $r = 0.04/12$ and $\delta_0 = 0.025$ (which then matches the empirical exit rate of employed workers into unemployment). Setting $\mu = \delta_0/5$ implies average firm size δ_0/μ =5 employees (with full employment). Merz and Yashiv (2007) estimate cubic hiring costs $c(h) = c_0 h^3$. We choose c_0 so the competitive allocation corresponds to full employment at market wage $\omega = 1$. Table 1 compares the resulting competitive allocation to that of the MPBE as described in Proposition 1.[3]

Rank x	0	0.5	0.75	0.99	1
Wage $w(x)$	0.70	0.71	0.74	0.82	1.00^-
Quit $q(x)$	0.015	0.013	0.011	0.007	0
Hire $h(x)$	0	0.018	0.021	0.025^-	0.025
Firm size $\overline{n}(x)$	0.62	1.22	1.73	3.43	∞
Competitive wage ω	1	1	1	1	1
Competitive $h^C(x)$	0	0	0.015	0.024	0.025^-
Competitive $\overline{n}^C(x)$	0	1	2.51	40.1	104,700

Table 1: MPBE with a continuum of firm types.

In this example all firms in the MPBE are active, enjoy greater profit than in the competitive allocation (they post low wages $w < \omega$) and so recruit at a greater rate, $h(x) > h^C(x)$. The MPBE finds the bottom 75% of firms by productivity pay wages very close to $b = 0.7 \ll \omega$. Although the equilibrium quit rates from these firms are high, their marginal quit rates are low and so wages only rise slowly in this range. As $x \to 1$, competition between large firms implies wages increase more quickly and a handful of the most productive firms pay wages close to the competitive wage.

The key insight, however, is that dynamic monoposony generates high unemployment. In this illustrative example steady state unemployment is

[3] files Econometrica1 solves for co in the Planner's problem. Econometrica5 solves for the MPBE, Econometrica7 computes the statistics.

surprisingly high, being close to 50%. The reason is neither low hiring rates nor too few active firms.[4] Instead (13) and (15) reveal that high quit turnover implies low steady state employment. This arises simply because a job-to-job transition destroys a job at the previous employer. As replacing an employee who quits takes time and resources, the (steady state) competitive allocation sets wasteful job-to-job transtions to zero by ensuring all firms pay the same competitive wage. The cost of dynamic monopsony is thus clear: equilibrium wage dispersion generates excessive quit rates as low paid workers seek better pay, which then generates excessive job destruction. The next section now identifies the implications of such turnover on equilibrium unemployment dynamics.

6 Finite Firm Types and Stochastic Equilibrium.

In general a stochastic equilibrium is not tractable as $(\theta, G(.))$ is infinitely dimensional. CM identifies the following simplification. Suppose $S \geq 1$ aggregate productivity states, $\theta \in \{\theta_1, \theta_2, .., \theta_S\}$ and, conditional on a shock, let $\pi_{ss'}$ denote the transition probabilities. Suppose $I \geq 1$ firm productivities. The interval $[0, 1]$ is partitioned into a grid $(x_{i-1}, x_i] \subset [0, 1]$ with $x_0 = 0$, $x_{i-1} < x_i$ and $x_I = 1$. Firms with rank $x \in (x_{i-1}, x_i] \subseteq [0, 1]$ are referred to as state i firms, where each has the same productivity $p_i(\theta)$, death rate $\delta_i(\theta)$ and transition rates $\gamma_{ii'}$ between firm productivity states where $\gamma_i = \Sigma_{i' \neq i} \gamma_{ii'}$. Let γ_{0i} describe the fraction of start-ups who begin as state-i firms. The wage $w_i(\theta, .)$ is defined as the equilibrium wage paid by firm $x = x_{i-1}$ (i.e. it is the lowest wage paid by type i firms) and $q_i(\theta, .)$ its corresponding equilibrium quit rate.

As each state i firm has the same production opportunities, they each enjoy equal value v_i in an MPBE. Let $\mathbf{v} = (v_1, v_2, ..., v_I)$ denote the $i \times 1$ vector of firm values. Define the generic type i hire function

$$h_i^*(v, \theta) = \arg\max_{h'} \left[h'v - p_i(\theta)c(h') \right].$$

Hence $h_i^*(v, \theta)$ with $v = v_i$ describes the optimal hiring rate of type i firms.

[4] dynamic monopsony implies too many firms are active (low productivity firms survive by paying wages below the competitive wage $\omega = 1$) and enjoy higher values v (through paying low wages) and so choose higher hiring rates.

Let $N_i = G(x_i) - G(x_{i-1})$ denote total employment in firms of type i, and note unemployment $U = 1 - \Sigma_i N_i$. CM establishes the critical result: that the finite vector $(\theta, \mathbf{N})$ is a sufficient statistic for $(\theta, G(.))$ in an MPBE. Furthermore value functions $v_i(.)$ are identified by a simple set of functional equations which can be solved using standard value function iteration. Claim 3 describes the result.

Claim 3. Over (arbitrarily small) time period $\Delta > 0$, an MPBE implies vector values $\mathbf{v} = \mathbf{v}(\theta, \mathbf{N})$ are the fixed point to the following map:

$$\begin{aligned} v_i(\theta, \mathbf{N}) &= \max_{h \geq 0} \begin{bmatrix} p_i(\theta) - w_i + hv_i(\mathbf{N}, \theta) - p_i(\theta)c(h) \\ +\alpha E[v_i(\theta', \mathbf{N})|\theta] + \sum_{j \neq i} \gamma_{ij} v_j(\theta, \mathbf{N}) \end{bmatrix} \Delta \\ &\quad + e^{-(r+\delta_i+q_i+\alpha+\gamma_i)\Delta} v_i(\theta, \mathbf{N}^{\Delta}), \end{aligned} \tag{16}$$

for $i = 1, .., I$, where

$$w_i = b + \sum_{j=1}^{i-1} v_j h_j^*(v_j, \theta) \left[\ln \frac{U + N_1 + ... + N_j}{U + N_1 + .. + N_{j-1}} \right], \tag{17}$$

$$q_i = \sum_{j=i}^{I} h_j^*(v_j, \theta) \ln[\frac{U + N_1 + ... + N_j}{U + N_1 + .. + N_{j-1}}], \tag{18}$$

and $\mathbf{N}^{\Delta} = \mathbf{N} + \dot{\mathbf{N}}(\theta, \mathbf{N})\Delta$ with

$$\begin{aligned} \dot{N}_i &= \mu\gamma_{0i} + [U + N_1 + ... + N_{i-1}] h_i^* \ln[\frac{U + N_1 + ... + N_i}{U + N_1 + .. + N_{i-1}}] \\ &\quad - \delta_i N_i - q_{i+1} N_i + \Sigma_{j \neq i} \gamma_{ji} N_j - \Sigma_{j \neq i} \gamma_{ij} N_i \end{aligned} \tag{19}$$

Claim 3 describes a closed set of recursive equations for $v_i(\boldsymbol{\theta}, \mathbf{N})$ where (17) is found by integrating forward the wage equation (7) from $x = 0$, while (18) is found by integrating backward the quit function (8) from $x = 1$. Note the fixed point identifies the equilibrium hiring strategies of each state i firm given the quit rates $q(.)$ induced by the hiring strategies of all firms. This set of Bellman equations can be solved numerically by value function iteration. Here we report an illustrative example with three firm types, $I = 3$. Although the example is not stochastic, we illustrate the impulse response of the economy to a permanent (favourable) productivity shock.

As in the previous example, we set $r = 0.04/12$ and $c(.) = c_0 h^3$ with c_0 chosen so that the competitive wage $\omega = 1$ in the Planner's steady state. To generate a reasonable min-mean wage ratio (e.g. Hornstein et al (2011)), we set $b = 0.6$ as then $\omega/b \simeq 1.7$.

The data find large firms rarely die, firm growth rates tend to decline with age while start-ups have relatively low survival rates (e.g. Haltiwanger et al (2013)). To capture such firm turnover, we assume each firm i receives a productivity shock at rate γ whereupon its type falls from i to $i-1$. If the lowest firm type $i = 1$ receives a $\gamma-$shock we presume its productivity $p_0 = b^-$ and it closes down. We assume this is the only form of firm death; i.e. only the lowest productivity firms die and do so at rate γ.

For purposes of illustration, p_i is assumed linearly increasing in i and entry weights γ_{0i} are linearly decreasing in i so that the distribution of start-ups is skewed towards low productivity (i.e. most start-ups have low survival rates). With the additional restriction that $E_0[p_i] = 1$, the 3 type case and linearity requires $p_1 = 0.84, p_2 = 1.08, p_3 = 1.32$ with corresponding entry weights $\gamma_{01} = 1/2, \gamma_{02} = 1/3, \gamma_{03} = 1/6$.

The turnover parameters (μ, γ) are chosen so that steady state unemployment equals 5% with an expected duration of unemployment equal to 3 months. This identifies parameter values $c_0 = 1520.6, \gamma = 0.05603$ and $\mu = 0.0123$.[5]

Table 2 reports the steady state MPBE for these parameter values.

type i	w_i	h_i	q_i	N_i	Average Firm size $\overline{n}_i$	Mean wage$_i$
1	0.600	0.0217	0.0872	0.292	1.35	0.649
2	0.675	0.0354	0.0454	0.381	3.53	0.767
3	0.839	0.0586	0.0190	0.277	7.70	1.045

Table 2: Steady State MPBE with Finite Firm Types.

Note the lowest productivity firms, those in states $i = 1, 2$ announce wages $w < w_3$ which are far below the competitive wage. The final column reports the average wage earned by type i employees and reveals that type 3 employ-

[5]Found by numerical6. Table 2 statistics computed in Dalecalibration1.

ees, on average, earn more than the competitive wage.[6] Dynamic monopsony does not imply all wages paid are necessarily below the competitive wage.

To illustrate the impulse response dynamics of this economy, suppose productivity $p_i(\theta) = \theta p_i$ and the economy starts at the steady state described above with $\theta = 1$. Table 3 describes the adjustment dynamics of the employment distribution $\{N_i(t)\}$ assuming θ increases permanently by 5% at date $t = 0$ while home productivity b is held fixed.[7]

time t	0	6 months	20 months	new steady state
Unemployment	5.0%	4.9%	4.75%	4.5%
N_1	29.2%	29.1%	29.0%	28.9%
N_2	38.1%	38.1%	38.2%	38.4%
N_3	27.7%	27.8%	27.9%	28.2%

Table 3. Impulse Response Function: Employment dynamics

The unemployment process is highly persistent with a half-life of 20 months. To understand why, Table 4 describes how gross hire flows and quit flows change in response to the positive productivity shock. Column t=0^- describes those flows in the initial steady state. In this Table, $[hires_1] \equiv N_1 h_1$ describes total hires (per month) by all type 1 firms while $[quits_1]$ describes total quits by workers employed in type 1 firms.

[6]Using integration by parts, the average wage of workers employed in type i firms is

$$\begin{aligned} \text{Mean wage}_i &= \frac{1}{N_i}\int_{x_{i-1}}^{x_i} w(z,\Omega)dG(z) \\ &= w_i + v_i h_i\left[\frac{G_i}{N_i}\ln\left[\frac{G_i}{G_{i-1}}\right] - 1\right], \end{aligned}$$

where $G_i = U + \Sigma_{j \le i} N_i$. Similarly the average quit rate of workers in type i firms is

$$\text{Mean quit}_i = q_i - \frac{[meanwage_i - w_i]}{v_i}.$$

[7]If b also increases by 5%, then an MPBE implies all wages increase by 5% and there is no change in turnover.

time t	0^-	0^+	6months	20 months	new steady state
Hire Flows					
gross hires	0.0360	0.0369	0.0369	0.0369	0.0370
hires_1	0.0063	0.0066	0.0066	0.0066	0.0065
hires_2	0.0135	0.0138	0.0138	0.0138	0.0138
hires_3	0.0162	0.0164	0.0165	0.0165	0.0166
Quit Flows					
gross quits	0.0317	0.0324	0.0324	0.0326	0.0328
quits_1	0.0175	0.0180	0.0180	0.0180	0.0181
quits_2	0.0117	0.0118	0.0119	0.0120	0.0121
quits_3	0.0025	0.0025	0.0025	0.0026	0.0026

Table 4. Impulse Response Function: Gross Flows.

Across date $t = 0$, the 5% productivity shock generates an immediate 2.5% increase in gross hires. But increased hires by type 2 and 3 firms triggers a correspondingly large increase in quit rates by workers employed in type 1 and 2 firms. In this example, the 2.5% increase in gross hires is almost matched by a 2.2% increase in gross quits. Such "churning" severely crowds out the re-employment prospects of the unemployed and explains why unemployment has such a long half-life. Across the entire adjustment process, gross flows are almost perfectly correlated over time (both increase slowly as unemployment falls).

Table 5 describes how wages change. It computes the average wage of employees in each type i sector, where $[meanwage1]_0$ is the mean wage of type 1 employees in the initial steady state and $[meanwage1]_t$ is their mean wage at date $t > 0$.

time t	0^+	6 months	20 months	new steady state
$\frac{meanwage_t - [meanwage]_0}{[meanwage]_0}$	3.7%	3.8%	4.0%	4.3%
$\frac{[meanwage1]_t - [meanwage1]_0}{[meanwage1]_0}$	1.6%	1.6%	1.7%	1.8%
$\frac{meanwage2 - [meanwage2]_0}{[meanwage2]_0}$	3.6%	3.6%	3.7%	3.9%
$\frac{meanwage3 - [meanwage3]_0}{[meanwage3]_0}$	5.3%	5.4%	5.5%	5.7%

Table 5. Impulse Response Function: Wage Dynamics.

The first row describes the total change in average wages across all employed workers. As b is fixed, a permanent 5% increase in productivity leads average wages to rise by less than 5%. More interestingly, there is a large increase in wage inequality: average type 1 wages increase by only 1.8% while average type 3 wages increase by more than 5%.

7 Conclusion.

The CM framework identifies a rich, equilibrium model of job and labor flows that seems ideal for both macro-policy applications and micro-empirical analysis. In contrast to Menzio and Shi (2011) and Lise and Robin (2015), dynamic monopsony generates inefficiently high unemployment levels. It thus identifies genuine policy implications. In contrast to Moscarini and Postel-Vinay (2013), it also incorporates rich industry dynamics as small start-up companies enter the market over time, some older, existing firms exit and employees keep searching on-the-job for preferred employment prospects. Firm specific productivity shocks not only allow that firm growth rates may decline with age but a skewed entry distribution allows that start-up firms have relatively low survival rates relative to the market, yet have higher average growth rates [e.g. Haltiwanger et al (2013)].

The CM framework does not exhibit an amplification mechanism: small changes in productivity do not yield large changes in (net) firm hiring rates. This is not necessarily an important failure of the model for it can easily accomodate large (exogenous) job destruction shocks (by firm type). Of course following Shimer (2005) it has typically been presumed that job destruction shocks cannot play an important role in the theory of unemployment. Elsby et al (2009), however, document that job destruction shocks are neither small nor acyclical. Furthermore Menzio and Shi (2011) show, with on-the-job search, that large (endogenous) variations in job destruction rates not only generate large variations in unemployment over the cycle, the resulting vacancy dynamics are also data consistent.

References

[1] Burdett, K. and D. T. Mortensen (1998). "Wage Differentials, Employer Size and Unemployment," *International Economic Review* 39: 257-273.

[2] Christiano, L.J., M. S. Eichenbaum and M. Trabandt (2012) "Unemployment and Business Cycles," Northwestern University working paper.

[3] Coles, M.G. and D.T. Mortensen (2015), Equilibrium Labor Turnover, Firm Growth and Unemployment, ISER wp

[4] Elsby, M.W.L., R. Michaels and G. Solon (2009) "The Ins and Outs of Cyclical Unemployment" *American Economic Journal: Macroeconomics,* 1, 84-110.

[5] Fallick, B. and C.A. Fleischman (2004), "Employer-to-Employer flows in the U.S. Labor Market: The Complete Picture of Gross Worker Flows", FEDS wp 2004-34.

[6] Haltiwanger, J.C., R.S. Jarmin and J. Miranda (2013), "Who Creates Jobs? Small vs Large vs Young." *Review of Economics and Statistics*, .

[7] Hornstein, A, P. Krusell and G.L. Violante (2011), "Frictional Wage Dispersion in Search Models: A Quantitative Assessment." *American Economic Review, 101, 7,2873-98.*

[8] Lise, J. (2013) "On-the-Job Search and Precautionary Savings", *Review of Economic Studies*, 2013, 80: 1086-1113.

[9] Lise, J. and J-M. Robin (2015), "The macrodynamics of sorting between workers and firms." Mimeo.

[10] Merz, M, and E Yashiv (2007). "Labor and the Market Value of the Firm," *American Economic Review* 90: 1297-1322.

[11] Menzio, G, and S Shi (2011), "Efficient Search on the Job and the Business Cycle," *Journal of Political Economy* 119: 468—510.

[12] Mortensen, D.T. and E. Nagypal (2007), "More on unemployment and vacancy dynamics", *Review of Economic Dynamics, 10, 327-347.*

[13] Moscarini, G and F Postel-Vinay (2013), "Stochastic Search Equilibrium," *Review of Economic Studies, 1, 1-44.*

[14] Postel-Vinay F., and J. Robin (2002) “Equilibrium Wage Dispersion with Worker and Employer Heterogeneity.” *Econometrica* 70: 2295—350.

[15] Sala, L., U. Soderstrom, and A. Trigari (2012) "Structural and Cyclical Forces in the Labor Market During the Great Recession: Cross-Country Evidence," working paper prepared for the NBER International Seminar on Macroeconomics 2012.

[16] Shimer, R. (2005). “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, 95(1): 25–49.