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Risk Aversion in a Model of Endogenous Growth

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Abstract

Despite the evidence on incomplete financial markets and substantial risk being borne by innovators, current models of growth through creative destruction predominantly model innovators' as risk neutral. Risk aversion is expected to reduce the incentive to innovate and we might fear that without insurance innovation completely disappears in the long run. The present paper introduces risk averse agents into an occupational choice model of endogenous growth in which insurance against failure to innovate is not available. We derive a clear negative relationship between the level of risk aversion and long run growth. Surprisingly, we show that in an equilibrium there exists a cut-off value of risk aversion below which the growth rate of the mass of innovators tends to a strictly positive constant. In this case, innovation persists on the long run and consumption per capita grows at a strictly positive rate. On the other hand, for levels of risk aversion above the cut-off value, the economy eventually stagnates.

Keywords: Endogenous Growth, Risk Aversion, Occupational Choice

JEL Classification: O40, O41, O43

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1. Introduction

The outcome of the individual process of innovation is inevitably uncertain, and modern models of growth through creative destruction allow for such uncertainty. However, despite the fact there is wide evidence that agents are risk averse, these models assume that investment in R&D is not associated to any risk[2]. Common arguments for this simplification are that the risk inherent in performing R&D can either be perfectly hedged against, or that R&D is performed by risk neutral firms. In fact, perfect insurance against R&D risk is theoretically unlikely, due to problems of asymmetric information between innovators and investors, and/or problems of moral hazard (Akerlof (1970), Arrow (1962)). Empirically, a funding gap for R&D has been well-documented even for developed economies, especially for small and new firms (for recent surveys, see Hall (2002) or Hall and Lerner (2010)[3]), and capital markets appear to be imperfect, see, e.g., Card et al. (2007)[4].

The rate of technological progress and consequently the growth rate of consumption per capita crucially depend on the resources devoted to innovation. Without perfect capital markets to finance R&D, the level of risk aversion of agents is likely to impact on the allocation of resources and on the economy's long-run growth rate. The aim of the present paper is to analyze the extent of this impact.

Our model is based on Eaton and Kortum (2001) and Kortum (1997). Agents are born with an endowment of labor that they supply inelastically at birth. The

[2]See, e.g., Aghion and Howitt (1992), Grossman and Helpman (1991b), Kortum (1997), Segerstrom (1998) and Segerstrom et al. (1990) for a non-exhaustive list.

[3]Further examples include Evans and Jovanovic (1989) who find evidence that wealth is positively linked to the likelihood of becoming an entrepreneur, and Caggese (2012), who empirically estimates that increased uncertainty has a large negative effect on risky investments by entrepreneurial firms.

[4]Incidentally, in neoclassical growth models, effects of imperfect insurance of income risk on growth have been analyzed quite extensively. See, e.g., Aiyagari (1994), Angeletos (2007), and the references therein. Generally, the literature has shown that while labor income risk increases precautionary savings, capital income risk can have an ambiguous effect on savings. In Aghion et al. (2010), tighter credit lowers mean growth through its effect on the cyclical composition of investment.

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length of an agent's life is uncertain. Following Yaari (1965) and Blanchard (1985), each agent faces a constant Poisson death rate. A fraction of the labor endowment is specific and can only be supplied to the production of output. Agents face a discrete occupational choice about the supply of their remaining labor: They can either work in the production sector, or become researchers. While the wage in the production sector is certain, the returns of a researcher are uncertain, and in particular, an unsuccessful researcher does not earn any return. Successful researchers are compensated with the expected present value of their innovation[5]. Agents can smooth their consumption through saving, but as their entire income occurs at the beginning of their lives, they are unable to borrow. The assumption of a single income simplifies the analysis considerably, and allows us to abstract from wealth effects in the occupational choice decision.

The lack of insurance with respect to research success allows us to derive a clear and stark relationship between risk aversion and growth. We derive a cut-off value of risk aversion above which the economy stagnates. Consequently, there exists an upper bound on the stock of research and on average consumption per capita. These bounds are decreasing in the level of risk aversion. However, at or below this cut-off value, stagnation is not an equilibrium. On an asymptotically balanced growth path, both average consumption per capita and the level of technology grow without bounds. For levels of risk aversion strictly below the cut-off, on an asymptotically balanced growth path the measure of researchers will grow at a positive rate in the long run. This rate is increasing in the rate of population growth, though strictly below it, and decreasing in the level of risk aversion. To summarize, while risk aversion does indeed depress the growth rate of the economy compared to a risk neutral setting, even without any form of insurance complete stagnation of the economy does not necessarily

[5] An important aspect in the innovation and patent literature is the question of appropriability of innovations. In our model, a successful innovator can perfectly reap the benefits of his innovation.

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occur.

The cut-off value we derive corresponds to a coefficient of relative risk aversion of unity. Empirically, the value of the coefficient of risk aversion (which in our model is the inverse of the intertemporal elasticity of substitution) is still debated. Many authors such as Campbell (1999), Kocherlakota (1996), Patterson and Pesaran (1992), Vissing-Jørgensen and Attanasio (2003), Alan and Browning (2010) or Alan et al. (2009) estimate coefficients of risk aversion well above unity, or equivalently, elasticities of substitutions below unity. See also Attanasio and Weber (2010) for a recent survey. On the other hand, Mulligan (2002) or Gruber (2006) estimate elasticities of substitution above unity, while the results of Gourinchas and Parker (2002) and Yogo (2004) are inconclusive. While our model remains agnostic about the empirical value of risk aversion, it does stress that the qualitative behavior of the economy in the long run critically depends on it.

Our model belongs to the endogenous growth literature[6], and it is not our aim to provide a comprehensive review of this literature here. Within this literature, it is most closely related to recent contributions by García-Peñalosa and Wen (2008) and Zeira (2011), both of which model risk averse agents in occupational choice models[7]. Zeira (2011) models the endogenous formation of patent races for innovations of different levels of difficulty. In an extension, he introduces a model with finitely-lived agents and logarithmic utility in which some form of insurance is granted to innovators by assuming that they always work a fraction of their time in the production sector. He shows that risk aversion can lead to over-researching of "easy" innovations, as these are less

[6]Such as Romer (1986), Romer (1990), Aghion and Howitt (1992),Grossman and Helpman (1991b), Segerstrom (1998), Jones (1995), Jones (2005), Kremer (1993), or Alcalá and Ciccone (2004), for a non-exhaustive list. Some very good overviews of the main theories of endogenous growth can be found in, e.g., Barro and Sala-i-Martin (2003), Grossman and Helpman (1991a), Aghion and Howitt (1998) and more recently Acemoglu (2009).

[7]Also related, though not dealing with risk aversion as such, are Cozzi and Giordani (2011), who study *ambiguity aversion* of innovators and find that higher ambiguity aversion leads to lower R&D efforts.

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risky, where riskiness is defined not over innovative success, but over winning the patent race. As such, both the environment and the question studied differ substantially from our paper. While Zeira focuses on the allocation of resources into different types of innovation, we study the choice between a production and a research sector. Foremost, however, the main contribution of our paper lies in deriving the relationship between the level of risk aversion and its qualitative effects on the economy (stagnation vs. growth). As in Zeira the level of risk aversion is fixed to unity, such a relationship is not derived.

García-Peñalosa and Wen (2008) are the closest to our own model, as their paper focuses on the effects of redistributive taxation on growth and inequality if agents are risk averse. They show that through insurance effects, redistributive taxes may indeed increase growth. This result is driven by the same intuition underlying our own results; the redistributive tax acts as a social insurance for unsuccessful innovators. The innovation process they model is built on Aghion and Howitt (1992), i.e. in contrast to us, they consider fixed inventive steps, a constant population, and the probability to innovate is independent of the stock of ideas. This implies that their model shares the prediction of strong scale effects of Aghion and Howitt (1992), and the growth rate of the economy is an increasing function of the number of researchers. As such, any variable that impacts the level of research, also impacts the growth rate of the economy. Within our model, we can separate effects on levels from effects on growth rates. Most importantly, García-Peñalosa and Wen (2008) focus on the importance of redistribution on growth, while our focus is on the interplay between risk aversion and the occurrence or lack of long-run growth. García-Peñalosa and Wen (2008) exclusively consider values of risk aversion that are below the cut-off value above which we find that the economy stagnates. We instead, are able to show that the existence of long-run growth hinges critically on the value of risk aversion.

Finally, our model is related, albeit less closely, to work on inequality in wealth and occupational choice under imperfect capital markets, such as Baner-

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jee and Newman (1991), Banerjee and Newman (1993), and Galor and Zeira (1993)[8]. While we share with this literature the assumption of imperfect capital markets, conceptually we differ substantially. In the above literature, imperfect capital markets affect outcomes because agents are *ex ante* heterogeneous in wealth. In our model, agents are homogeneous in endowments and the lack of capital markets affects the growth rate through a lack of insurance.

The remainder of the paper is organized as follows. Section 2 introduces the optimization problem of consumers in the economy, while section 3 details the production side of the economy, including the innovation process and the value of R&D. Our results on equilibrium growth rates are derived in section 4. Section 5 concludes.

2. Consumers

2.1. Endowments

The economy is populated by a mass L_t of agents, with a (gross) population growth rate $n \geq 0$. Following Yaari (1965) and Blanchard (1985), each agent faces a Poisson death rate of $\upsilon \in (0, \infty)$[9]. Each agent is endowed with $1+o$ units of labor, which he supplies inelastically at the instant he is born, τ. Out of his labor endowment, o units are production-specific, and can only be supplied in the production of consumption goods[10]. The agent chooses between supplying his remaining one unit of labor in the production of consumption goods and becoming a researcher[11]. Income in production is certain and the wage rate is

[8] See also Aghion and Bolton (1997), Ghatak and Jiang (2002), or Mookherjee and Ray (2003) and the references therein.

[9] The introduction of a death rate is in itself innocuous. We do not wish to model infinitely lived agents who make repeated work/research decision at each t, among other reasons as in such a setup it is difficult to justify that agents would be incapable of borrowing against their future (expected) income.

[10] This assumption implies that unsuccessful researchers have a minimum income out of which they can consume.

[11] Units of labor are indivisible in our model.

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unity[12]. The pay-off from being a researcher is uncertain.

Successful researchers are those with an idea whose efficiency surpasses the current state of the art; we denote such ideas as innovations. The probability to innovate has two terms. First, it depends on the probability that an idea arrives to the researcher, which is a Poisson Process with an exogenous arrival rate of α. Second, on the probability that efficiency of the idea is above the state of the art, which is determined endogenously in the model.

Successful researchers are compensated with the expected value of their innovation. This is increasing in the efficiency improvement that the innovation represents, relative to the previous state of the art, and in the expected length that the innovation itself will remain the state of the art[13]. Unsuccessful researchers have zero income apart from the wage return of their o units of labor they supply for production.

2.2. Preferences and budget sets

Agents are risk averse and aggregate the available goods in a Cobb-Douglas fashion. They cannot borrow. This very strict borrowing constraint arises naturally in the current setup in which expected future income of any agent after his birth is zero. Agents in debt would never be able to repay their debts. A less restrictive setup would equip agents with a unit flow of labor at each t. However, allowing for the possibility of multiple innovations during a lifetime would severely complicate the utility maximization problem, and distract from our focus of the effect of risk aversion without insurance.[14]

[12]The increased efficiency in production through research will translate into lower prices and increase the real wages of production workers.

[13]Our equilibrium properties are not affected if we assume instead that the compensation paid to the innovator depends on the *realized* efficiency improvement of his innovation

[14]We would have to keep track of all past decisions of agents, as well as whether their past research had been successful. See, e.g., Levhari and Srinivasan (1969), Merton (1969), or Samuelson (1969). Note that the natural borrowing constraint of a finitely lived agent would be zero also in this case, as the worst possible outcome would be for an agent to always be an unsuccessful researcher.

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However, agents can save any unconsumed income to smooth consumption over their (expected) lifetime. We assume that there exists a financial intermediary that acts as an economy-wide mutual fund, and that holds the ownership claims of all firms that operate in the economy. This fund plays two roles. Its first role is in re-allocating resources between agents and firms, and over time. It enables agents to deposit their income, in return for interest payments, which allows them to smooth their consumption over their expected lifetime. Upon receiving their income in τ, each agent i deposits his income with the intermediary. We denote deposits as $A_{i\tau|\tau}$. Deposited income pays an interest of r_t, which is the rate at which future profits of firms are discounted. Accruing profits are used to pay interests on deposits and to satisfy withdrawals made for consumption. In this way, the intermediary re-allocates the resources across agents at each t, and the resource constraint of the economy implies that the entire production is consumed each t. The second role of the fund is as an intermediary between researchers and firms. Researchers who have found an innovation can sell this to the fund for its present expected value, which gets credited to their deposits with the fund. The intermediary then gives the right to use the innovation in production to a firm. In this way, the only aspect in which workers, successful researchers, and unsuccessful researchers differ, is in their initial deposits, $A_{i\tau|\tau}$, with the fund.

Due to uncertain lifetimes, an agent typically will die while holding positive deposits with the intermediary, and we assume that his deposits become property of the intermediary on this occasion. Death occurs with probability υ at t, independent of an agent's current age. As this is the flow rate with which the agent loses his assets, the intermediary will compensate him for this risk by paying him a return of $r_t + \upsilon$ on his deposits. Under these assumptions, the expected utility of agent i born at time τ can be expressed as

$$U_{i\tau} = E_\tau \left[\int_\tau^\infty e^{-(\rho+\upsilon)(t-\tau)} \frac{\left[exp \int_0^1 ln\ x_{it|\tau}(j)dj \right]^{1-\sigma} - 1}{1-\sigma} dt \right] \tag{1}$$

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where $x_{it|\tau}(j)$ is the quantity that agent i, born at τ, consumes of good j at time t. ρ is the subjective discounting factor, which is assumed to be constant. Future consumption is discounted both by this factor as well as υ as this gives the rate at which future consumption is lost due to death. The level of risk aversion is given by $\sigma \geq 0$. The individual maximizes (1) with respect to $\{x_{it|\tau}(j)\}$ subject to the following constraints:

$$\dot{a}_{it|\tau} = (r_t + \upsilon - \frac{\dot{P}_t}{P_t})a_{it|\tau} - X_{it|\tau}, \tag{2a}$$

$$a_{it|\tau} \geq 0, \quad X_{it|\tau} \geq 0, \tag{2b}$$

$$\lim_{t\to\infty} e^{-(\bar{r}_{t\tau}+\upsilon-\overline{\frac{\dot{P}}{P}}_{t\tau})(t-\tau)} a_{it|\tau} = 0, \tag{2c}$$

where

$$\overline{\frac{\dot{P}}{P}}_{t\tau} \equiv \frac{1}{t-\tau}\int_{\tau}^{t} \frac{\dot{P}_s}{P_s}\, ds \qquad \text{and}$$

$$\bar{r}_{t\tau} \equiv \frac{1}{t-\tau}\int_{\tau}^{t} r_s\, ds$$

are the average growth rate of the price index and the average interest rate between t and τ respectively. $a_{it|\tau}$ is the amount of savings the agent has deposited at t in real terms, i.e., $a_{it|\tau} = \frac{A_{it|\tau}}{P_t}$, P_t is the aggregate price index in the economy and $X_{it|\tau}$ is the consumption index of agent i. The initial deposits that an agent has, $A_{i\tau|\tau}$, depends on his choice of occupation. We take the wage to be the numeraire. $A_{i\tau|\tau}$ then is equal to $1+o$ if he works in production, equal to the expected value of the innovation (V_τ, which will be determined later) plus o if he is a successful researcher, and o if he is an unsuccessful researcher. If an idea arrives that surpasses the technological frontier, innovation is immediate. Note that the uncertainty about the initial amount of deposits is the only uncertainty that the agent faces. Once this is resolved, his lifetime

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utility becomes certain. The values of deposits and consumption at each t depend on when an agent is born, and are therefore indexed by the cohort τ.

2.3. Individual and aggregate demand

The problem of maximizing (1) subject to the constraints in (2) is entirely standard. Agents first decide how to allocate their income across their expected lifetime and then, given the allocation of income, decide how to allocate expenditure across varieties of goods. Let the optimal price and consumption indices be

$$P_t = exp \int_0^1 ln\ p_t(j)\ dj \tag{3}$$

and

$$X_{it|\tau} = exp \int_0^1 ln\ x_{it|\tau}(j)\ dj. \tag{4}$$

respectively.

Then the first order conditions yield the familiar Euler Equation

$$\frac{\dot{X}_{it|\tau}}{X_{it|\tau}} = \frac{1}{\sigma}\left[r_t - \rho - \frac{\dot{P}_t}{P_t}\right]. \tag{5}$$

and consumption at every t can be expressed as

$$X_{it|\tau} = \frac{a_{it|\tau}}{\mu_t}, \tag{6}$$

where

$$\mu_t = \int_t^\infty exp\{[(1-1/\sigma)\left(\overline{\frac{\dot{P}}{P}}_{st} - \bar{r}_{st}\right) - \rho/\sigma - \upsilon](s-t)\}\ ds. \tag{7}$$

Note that in the case of $\sigma = 1$, μ_t simplifies considerably, to $\mu_t = \frac{1}{\rho+\upsilon}$. All agents of cohort τ consume according to (6), regardless of their occupation.

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The only factor in which their consumption plans differ is the amount of initial deposits, $a_{i\tau|\tau}$.

Given equations (5) and (6), consumption at any point in time can be expressed as

$$X_{it|\tau} = \frac{a_{i\tau|\tau}}{\mu_\tau} \cdot e^{\frac{1}{\sigma}(\bar{r}_{t\tau} - \rho - \overline{\frac{\dot{P}}{P}}_{t\tau})(t-\tau)} \tag{8}$$

3. Supply

3.1. Production and innovation

The production and innovation side of the economy is a closed economy version of the model introduced by Eaton and Kortum (2001). Minor changes with respect to this paper are our introduction of initial labor endowments o, the fact that at each t only newly-born agents are active, and the introduction of an initial technology level, T_0. Here, we summarize the main results of Eaton and Kortum (2001) for convenience under these adjustments. Detailed derivations and proofs can be found in the original paper.

Time is continuous. The economy produces at each t a continuum of goods, indexed by $j \in [0, 1]$. Labor is the only input in production. How many units of good j can be produced with a unit input of labor depends on the highest efficiency level in production, $z(j)$, and differs across goods j. $z(j)$ represents the state of the art of production in sector j, and $\{z(j)|j \in [0, 1]\}$ is referred to as the technological frontier of the economy. The frontier is common knowledge. Innovative activity in the economy is focused on expanding this frontier.

Firms turn labor input into consumption goods, and compete *à la* Bertrand. In each sector j, there will be a single active firm that can use an idea with efficiency $q(j) > z(j)$ and make positive profits by charging the marginal costs

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of the competitor producing with efficiency $z(j)$, $\frac{1}{z(j)}$[15].

Researchers obtain ideas about how to produce goods more efficiently. Let R_t denote the measure of researchers at t, and let

$$R_t = \beta_t n L_t, \tag{9}$$

i.e., β_t is the fraction of the active population that are researchers. Denote the growth rate of researchers as g_t. To each researcher, ideas arrive as a Poisson process with parameter α[16]. Therefore, the stock of ideas (i.e., the level of technology) evolves according to

$$\dot{T}_t = \alpha R_t, \tag{10}$$

and

$$T_t = \alpha \int_0^t R_s \, ds + T_0, \tag{11}$$

where T_0 is the initial level of technology.

Research is undirected, and efficiencies of new ideas, $q(j)$, are drawn from the Pareto distribution, $H(q) = 1 - q^{-\theta}$, which is the same for all sectors[17].

Under these assumptions, Eaton and Kortum (2001) derive the technological frontier and link it to production costs. As prices are set to marginal costs, they are able to show that the price index is itself a function of production costs, and can be expressed as a function of the stock of research,

$$P_t = \gamma \, T_t^{-1/\theta}, \tag{12}$$

where γ is Euler's constant. It follows that:

[15] We assume that if consumers are faced with identical prices for goods produced with different efficiency levels, they will always buy the good with the higher efficiency level.
[16] This parameter can be seen as the efficiency of research.
[17] The parameter $\theta > 1$ governs the variation in efficiencies of production.

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$$\frac{\dot{P}_t}{P_t} = -\frac{1}{\theta}\frac{\dot{T}_t}{T_t}, \tag{13}$$

i.e., the overall price index decreases at a rate that is proportional to the rate of technological progress in the economy.

3.2. Profits and value of R&D

Profits from improvements in the technological frontier determine the value of an innovation. Eaton and Kortum (2001) show that total profits of firms, Π_t, can be expressed as a function of total expenditure in the economy and the distribution of the inventive step m of new ideas, as these determine the markups over costs. They also show that this distribution, $H(m)$, is Pareto and independent of time.

Let $y_{it|\tau}$ be an individual's optimal expenditure at t. Let $Y_t = \int_0^t \int_i y_{it|\tau}\, di\, d\tau$ be total expenditure at time t. Given that the markups, m's, are drawn from $H(m)$, total profits at t are

$$\Pi_t = Y_t \int_0^1 [1 - m(j)^{-1}]\, dj = Y_t \int_1^\infty (1 - m^{-1})\, dH(m) = \frac{Y_t}{1+\theta}. \tag{14}$$

Total expenditure at t is equal to consumption, which is equal to total output, Y_t, which in turn is the sum of total profit and total wage income in the economy at t. Total wage income depends on the measure of individuals who choose to work in the production sector. Individuals only earn income the moment they are born, τ. The measure of agents that are born at time t is nL_t. Out of these, a fraction $(1 - \beta_t)$ will work in the production sector. Given the wage and additional labor endowments, o, total income in the economy is

$$\begin{aligned} Y_t &= (1 - \beta_t + o)nL_t + \frac{Y_t}{1+\theta} \qquad \Rightarrow \\ Y_t &= \frac{1+\theta}{\theta}(1 - \beta_t + o)nL_t. \end{aligned} \tag{15}$$

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The value of an innovation, V_t, is equal to the present discounted value of the future streams of profits that it grants. Apart from profit flows, $\Pi_t = \frac{Y_t}{1+\theta}$, this value has to take into account the discount rate (r_t), the change in price level, and the probability of the profit flows ending. Patent rights to an innovation become void at any $s > t$ if a better idea arrives, which occurs with probability T_t/T_s. This implies that the value of an innovation is given by

$$V_t = \frac{P_t}{1+\theta} \int_t^\infty e^{-\bar{r}_{st}(s-t)} \frac{Y_s}{P_s} \frac{T_t}{T_s} ds = \frac{P_t}{\theta} \int_t^\infty e^{-\bar{r}_{st}(s-t)} \frac{(1-\beta_s+o)nL_s}{P_s} \frac{T_t}{T_s} ds. \quad (16)$$

Finally, combining (15) and (12) yields average consumption per capita at t, which is increasing in the stock of ideas:

$$x_t = \frac{Y_t/L_t}{P_t} = \frac{\frac{1+\theta}{\theta}(1-\beta_t+o)n}{\gamma T_t^{-1/\theta}}. \quad (17)$$

Both the value of an innovation, (16), and consumption per capita, (17), differ from their respective values in Eaton and Kortum (2001) through additional labor endowments, o, and the fact that only a fraction n of the total population is earning any income at time t.

4. Equilibrium

4.1. Labor market optimality

Denote by W_t the utility of working in the production sector and by EU_t the expected utility of research. Initial assets can take three distinct values, i) $1+o$ if agent i works in production, ii) $V_\tau + o$ if agent i is a successful researcher, and iii) o if agent i is an unsuccessful researcher. Combining equations (8) and (7) for consumption at time τ and using the appropriate level of initial assets, for all $\tau \in t$,

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$$W_\tau = \begin{cases} \dfrac{1}{1-\sigma}\left[\left(\dfrac{1+o}{P_\tau}\right)^{1-\sigma}\mu_\tau^\sigma - \dfrac{1}{\rho+\upsilon}\right] & \text{if } \sigma \neq 1 \text{ and } \sigma \geq 0, \\ \\ \dfrac{1}{\rho+\upsilon}\left[\ln(1+o) - \ln(P_\tau) + \ln(\rho+\upsilon)\right] + I_\tau & \text{if } \sigma = 1, \end{cases} \tag{18}$$

and

$$EU_\tau = \begin{cases} \dfrac{1}{1-\sigma}\left\{\dfrac{\alpha}{T_\tau}\left[\left(\frac{V_\tau+o}{P_\tau}\right)^{1-\sigma}\mu_\tau^\sigma - \frac{1}{\rho+\upsilon}\right] + \left(1-\dfrac{\alpha}{T_\tau}\right)\left[\left(\frac{o}{P_\tau}\right)^{1-\sigma}\mu_\tau^\sigma - \frac{1}{\rho+\upsilon}\right]\right\} & \text{if } \sigma \neq 1 \text{ and } \sigma \geq 0, \\ \\ \dfrac{\alpha}{T_\tau}\left[\dfrac{\ln(V_\tau+o) - \ln(P_\tau) + \ln(\rho+\upsilon)}{\rho+\upsilon} + I_\tau\right] + \\ \qquad + \left(1-\dfrac{\alpha}{T_\tau}\right)\left[\dfrac{\ln(o) - \ln(P_\tau) + \ln(\rho+\upsilon)}{\rho+\upsilon} + I_\tau\right] & \text{if } \sigma = 1, \end{cases} \tag{19}$$

where $I_\tau \equiv \int_\tau^\infty (\bar{r}_{t\tau} - \rho - \overline{\frac{\dot{P}}{P}}_{t\tau})(t-\tau)e^{-(\rho+\upsilon)(t-\tau)}\, dt$.

Define $E_t = \frac{EU_t}{W_t}$ as the expected utility of research relative to working in the production sector at time t. This measure determines the agents' choice between R&D and work. In particular, the labor market allocation is optimal for all agents if

$$R_t = \begin{cases} 0 & \text{if } E_t < 1 \text{ and } 0 \leq \sigma \leq 1, \\ 0 & \text{if } E_t > 1 \text{ and } \sigma > 1, \\ \in [0, nL_t] & \text{if } E_t = 1, \\ nL_t & \text{if } E_t > 1 \text{ and } 0 \leq \sigma \leq 1, \\ nL_t & \text{if } E_t < 1 \text{ and } \sigma > 1. \end{cases} \tag{20}$$

The optimal labor market allocation is determined by the path of research stock, $\{T_t\}$, which in turn determines the evolution of the price level, the expected value of an innovation, and the probability to innovate. The path of research stock is an equilibrium if it yields the path of research, $\{R_t\}$, necessary

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to generate it.

4.2. Interest rate

In endogenous growth models of the type on which our model is based on, the economy typically exhibits a balanced growth path along which both the fraction of researchers in the economy, β, and the interest rate, r, are constant. Our first result establishes that the constancy of the interest rate indeed hinges on the fact that the fraction of researchers is constant over time.

Lemma 1. *On any equilibrium path such that the average growth rate of researchers converges to a constant,* $\lim_{t\to\infty} \bar{g}_{st} = g$*, the average interest rate between* t *and* $s \geq t$ *converges to a constant,*

$$\lim_{t\to\infty} \bar{r}_{st} = \frac{g}{\theta}(2\sigma - 1) + \sigma n + \rho. \tag{21}$$

If on the equilibrium path $\beta_t = \beta$ *holds for all finite* $t \geq \hat{t} \geq 0$*,* $\bar{r}_{st} = \frac{g}{\theta}(2\sigma - 1) + \sigma n + \rho$ *holds for all* $s \geq t \geq \hat{t}$*.*

Proof. See Appendix Appendix A. □

The convergence result in Lemma 1 relies on the fact that for any constant growth rate of researchers, g, the fraction of researchers will converge to a constant. In particular, β_t will converge to zero if $g < n - \upsilon$ and may take any value β in the limit if the growth rate of researchers converges to $g = n - \upsilon$. For finite t, a constant interest rate is again conditional on $\beta_t = \beta$, which in finite time can only occur if either $g_t = n - \upsilon$ or $R_t = 0$ for all t.

Under risk neutral preferences, an equilibrium path exists on which $g_t = n - \upsilon$ and $\beta_t = \beta > 0$ for all t. It is not obvious that such an equilibrium path also exists with risk averse agents. However, Lemma 1 establishes that the interest rate will converge to a constant for a large class of research growth rates. For any level of risk aversion σ and limit growth rate of researchers g, there exists

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a private discount factor ρ such that the interest rate converges to a constant in the long run that is large enough to ensure that the expected value of an innovation is finite.

4.3. Growth rate of researchers

We are now in a position to derive the equilibrium path of the economy.

Definition 1. Given initial values of the stock of research and the labor force, T_0 and L_0, as well as population growth rate $n \geq 0$ and death rate $\upsilon \geq 0$, with $n \geq \upsilon$, an equilibrium is a path of the stock of research, $\{T_t\}$, such that for all t: (i) the stock of research evolves according to equation (10), (ii) the path of research satisfies (20), (iii) the price level satisfies equation (12), (iv), the expected value of an innovation satisfies equation (16), (v) the probability to innovate is α/T_t, and (vi) consumption demand is equal to output.

As the interest of this paper is to identify the effect of risk aversion on occupational choice decisions and hence research growth, we first derive the equilibrium of our model for risk neutral agents, $\sigma = 0$.

Lemma 2. *Let $\sigma = 0$. The economy exhibits a balanced growth path along which the measure of researchers grows at rate $g = n - \upsilon$ and average consumption per capita grows at rate $\frac{g}{\theta}$.*

Proof. See Appendix Appendix B □

The derivations of equilibrium are standard and straightforward if $g = n - \upsilon$. In particular, we know that the interest rate is constant over time and our model trivially yields the same results with respect to research as Eaton and Kortum (2001). Lemma 2 reiterates known results that neither the introduction of work-specific labor endowments, nor the perpetual-youth aspect of the model affect the steady-state properties of the model. Any differences in the equilibrium research path in our model are therefore entirely due to risk aversion of agents. We find that, compared to the case of risk neutral agents, the introduction

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of risk aversion leads to non-trivial alterations in the equilibrium properties of the economy. Indeed, the economy exhibits qualitatively different equilibria, depending on the exact level of risk aversion.

Theorem 1. *Assume that $\sigma > 1$ and denote $\bar{T} = \alpha \frac{1}{1-\left(\frac{1+o}{o}\right)^{1-\sigma}}$. The behavior of research on an equilibrium path depends on the initial stock of research, T_0:*

1. *If $T_0 \geq \bar{T}$, there exists a unique equilibrium path of research such that $R_t = 0 \ \forall \ t$.*
2. *If $T_0 < \bar{T}$, $R_t = 0$ for every t is not an equilibrium; some research will be undertaken at some t. T_t is then increasing, and $\bar{T}$ is the maximum stock of research that can be reached in the economy.*

Proof. See Appendix Appendix C □

Theorem 1 establishes that there exists a clear cut-off value of σ, i.e., $\sigma = 1$, above which growth in the economy eventually stagnates. There exists an upper level of technology above which the economy can never grow. The qualitative result is particularly strong as it does not depend on the exact level of risk aversion σ, nor on how much exogenous "security" agents have through their labor endowments o. These parameters impact the level of $\bar{T}$, but not the qualitative result of eventual stagnation.[18] However, it is straightforward to show that the level of $\bar{T}$ is increasing in both α and o, and decreasing in σ. This result is in stark contrast to the qualitative equilibrium behavior of the economy if $0 < \sigma \leq 1$, as we now show.

Lemma 3. *Assume $0 < \sigma \leq 1$. Along an equilibrium path of research, it cannot be that $\lim_{t \to \infty} T_t = T$. There does not exist a t_0 such that for all $t > t_0$, $R_t = 0$.*

Proof. See Appendix Appendix C □

Lemma 3 shows that for low enough values of risk aversion, some research will be undertaken, and in particular, on an equilibrium path there does not

[18] As research success determines the entire consumption profile over an agents' lifetime once and for all, our results are also robust to the introduction of preferences of the Epstein-Zin type. We are grateful to an anonymous referee for pointing this out.

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exist an upper bound on the stock of research. The proof, given in full in Appendix Appendix C, proceeds by contradiction. We show that there exists no finite value of T to which T_t may converge, such that a path of research along which the measure of researchers converges to zero will be an equilibrium. Unfortunately, while we are able to establish that low levels of risk aversion imply that research does never cease altogether, it is not obvious how the equilibrium research path looks for finite t. By Lemma 1, we know that a constant growth rate of researchers will lead to a constant interest rate in finite time only if either no research is undertaken at all, or if the growth rate of researchers is equal to the population growth rate. The latter, however, can be shown to violate the optimal labor market allocation[19]. Consequently, even if an equilibrium path of research exists along which the measure of researchers grows at a constant rate (possibly zero or negative), the interest rate on this path will not be constant, and it is not obvious how the expected value of an innovation evolves on this path. We are able instead to make additional statements on the asymptotic behavior of the economy. We begin by defining the concept of an asymptotically balanced growth path.

Definition 2. Let a path of research $\{R_t\}$ and average consumption per capita $\{x_t\}$ be an equilibrium path. This path is an *asymptotically balanced growth path* if $\lim_{t\to\infty} g_t = g$ and $\lim_{t\to\infty} \frac{\dot{x}_t}{x_t} = \frac{\dot{x}}{x}$.

As we know from Lemma 1 that the interest rate converges to a constant whenever $g_t \to g$, we can state the properties of an asymptotically balanced growth path.

Theorem 2. *Assume that the economy is on an asymptotically balanced growth path. On this path, the level of risk aversion affects the growth rate of researchers as follows:*

1. *If* $0 < \sigma < 1$, *in long run equilibrium it must be that the growth rate of*

[19]The proof of this statement is trivial and therefore omitted. Setting $g = n - \upsilon$, β_β, and $r_t = r$ in the labor market optimality condition (20), it is straightforward to show that the utility of research relative to work, E_t, is decreasing over time.

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researchers converges to $g = (1-\sigma)(n-\upsilon) > 0$. Both the measure of researchers, R_t, and the stock of ideas, T_t, grow without bounds.

2. *If $\sigma = 1$, in long run equilibrium it must be that the measure of researchers is a strictly positive constant. The stock of ideas, T_t, grows without bounds, but its growth rate converges to zero.*

Proof. See Appendix Appendix D. □

The long run equilibrium results of Theorem 2 show that along an asymptotically balanced growth path, the measure of researchers will always be strictly positive, and their growth rate converges to a constant. In line with our result that growth ceases if $\sigma > 1$, Theorem 2 shows that also for $\sigma \leq 1$, research activity in equilibrium is, in the long run, decreasing in σ. For $\sigma < 1$, the growth rate of researchers on the asymptotically balanced growth path is strictly decreasing in σ, while for $\sigma = 1$, the measure of researchers on this path converges to a constant. Note that the growth rate of researchers is never affected by the labor endowments o that are supplied in production. Growth rates are only ever affected by the value of σ, and the growth rate of the population. However, we show in Appendix Appendix D that for $\sigma = 1$, the measure of researchers to which R_t converges in the long run is strictly increasing in o.

4.4. Consumption per capita

Average consumption per capita is given by equation (17), i.e.,

$$x_t = \frac{1+\theta}{\theta}(1-\beta_t+o)n\frac{1}{\gamma}T_t^{1/\theta} \tag{22}$$

and its growth rate is

$$\frac{\dot{x}_t}{x_t} = -\frac{\dot{\beta}_t}{1-\beta_t} + \frac{1}{\theta}\frac{\dot{T}_t}{T_t}. \tag{23}$$

The following results on the growth rate and the level of x_t follow immediately from our results on the equilibrium research path derived in the preceding section.

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Corollary 1. 1. *Assume* $0 < \sigma \leq 1$ *and that the economy is on an asymptotically balanced growth path. On this path,*

$$\lim_{t\to\infty} x_t = \infty$$
$$\lim_{t\to\infty} \frac{\dot{x}_t}{x_t} = \frac{(1-\sigma)(n-\upsilon)}{\theta}.$$

2. *If* $\sigma > 1$*, there exists a maximum level of average consumption per capita, which depends on* $\bar{T} = \frac{\alpha}{1-\left(\frac{1+o}{o}\right)^{1-\sigma}}$*:*

$$x_t = \frac{1+\theta}{\theta}(1+o)n\frac{1}{\gamma}\bar{T}^{1/\theta}$$

Corollary 1 implies that the qualitative results on the effect of risk aversion on the path of research translate one for one into impacts on average consumption per capita. In particular, average consumption per capita grows without bounds if $\sigma \leq 1$, and is bounded if $\sigma > 1$. At the cut-off value of $\sigma = 1$, the stock of research, and hence average consumption per capita grows without bounds. However, as the measure of researchers converges to a constant in this case, growth is linear, and the growth rate of x_t converges to zero. Finally, as x_t depends on T_t, we observe level effects of increasing risk aversion even if $\sigma > 1$. An increase in σ lowers $\bar{T}$ and therefore lowers the maximum level of average consumption per capita that can be achieved.

5. Conclusions

The vast majority of the work on endogenous growth shares the assumption that agents are risk neutral. While this is a valid assumption under perfect capital markets, problems of either asymmetric information or moral hazard are likely to prohibit perfect capital markets, a result that is theoretically plausible and supported by empirical evidence. Lacking perfect capital markets, the natural aversion to risk characterizing individual innovators is expected to affect

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the conclusions obtained with traditional endogenous growth models with risk neutral agents.

In our paper risk averse agents are introduced in a model of occupational choice based on Eaton and Kortum (2001) and Kortum (1997). Under risk neutrality in these models, consumption growth per capita ultimately depends on the population growth rate and technological parameters. The equilibrium balanced growth path encompasses a constant fraction of the population to become researchers. Risk averse agents will clearly be more hesitant to engage into innovation. We show that the equilibrium balanced growth path results of, e.g., Eaton and Kortum (2001) and Kortum (1997), do not hold for risk averse agents that cannot insure themselves. No matter the level of risk aversion, an equilibrium path on which the measure of researchers grows at the same rate as population does not exist. This result implies that there exists no steady state in which consumption per capita would grow at a constant rate in finite time. Given these results, we could expect that innovation would in the long run completely disappear. Instead we show that even with risk averse agents and complete lack of insurance against failure in innovation, there exists a cut-off value of risk aversion below which a positive growth rate of researchers in the long run is compatible with an asymptotically balanced growth path. Indeed, if an equilibrium exists, it must be that on the equilibrium path the growth rates of researchers and of the stock of ideas converge to a positive constant for low levels of risk aversion. We find that the value of this growth rate depends positively on population growth and negatively on the level of risk aversion. For agents with risk aversion above the cut-off, the stock of ideas approaches a constant, and no innovation takes place anymore once this level is reached. However, the level of this stock of ideas itself depends negatively on the level of risk aversion. As average consumption per capita in equilibrium depends on the stock of ideas, this cut-off along an equilibrium path is also observed in the growth rate (and level) of consumption per capita.

The results of the paper highlight the negative effect of risk aversion on

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growth but also show that some innovation is still possible even in the complete absence of insurance markets. While on an equilibrium path the fraction of researchers asymptotically converges to zero if insurance does not exist, both the measure of researchers and the level of technology can grow without bounds if risk aversion is low enough.

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Appendix A. Interest rate derivation

As there exists no savings technology in the economy, the resource constraint requires that at each point in time, all produced output is also consumed. By (15), output at t is given by

$$\frac{1}{\gamma} T_t^{1/\theta} \frac{1+\theta}{\theta} (1 - \beta_t + o) n L_t. \tag{A.1}$$

From the consumers' optimization problem, total consumption at time t is

$$X_t = \frac{a_t}{\int_t^\infty exp\{[(1-1/\sigma)\left(-\frac{1}{\theta}\bar{g}_{st} - \bar{r}_{st}\right) - \rho/\sigma - \upsilon](s-t)\}\, ds} \tag{A.2}$$

where a_t it total wealth in the economy at time t, and we use the fact that the price level is a function of the stock of research, and grows at rate $-\frac{1}{\theta} g_t$. This, in turn, is equal to the present discounted value of the future stream of income, adjusted for the price level,

$$a_t = \int_t^\infty \frac{1}{\gamma} T_s^{1/\theta} e^{-\bar{r}_{st}(s-t)} \frac{1+\theta}{\theta} (1 - \beta_s + o) n L_s \, ds. \tag{A.3}$$

Equating (A.2) and (A.3), we find that the following condition needs to hold for the resource constraint to be satisfied:

$$(1 - \beta_t + o) = \frac{\int_t^\infty e^{[\frac{1}{\theta}\bar{g}_{st} + n - \upsilon - \bar{r}_{st}](s-t)} (1 - \beta_s + o)\, ds}{\int_t^\infty exp\{[(1-1/\sigma)\left(-\frac{1}{\theta}\bar{g}_{st} - \bar{r}_{st}\right) - \rho/\sigma - \upsilon](s-t)\}\, ds} \tag{A.4}$$

To prove Lemma 1, we first show that for any $t \geq \hat{t}$ with $\hat{t} \geq 0$, a constant average interest rate is the equilibrium interest rate if either $g_t = g = n - \upsilon$ or $R_t = 0$, i.e., $g_t = 0$ and $R_{\hat{t}} = 0$. To this end, note that for $g = n - \upsilon$, the fraction of researchers is constant, $\beta_t = \beta$, while if $R_{\hat{t}} = 0$ and $g = 0$, for any $t \geq \hat{t}$, $\beta_t = \beta_{\hat{t}} = \beta = 0$. In either case, equation (A.4) becomes

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$$\int_t^\infty e^{[-(1-1/\sigma)\left(\frac{g}{\theta}+\bar{r}_{st}\right)-\rho/\sigma-\upsilon](s-t)}\,ds = \int_t^\infty e^{[\frac{g}{\theta}+n-\upsilon-\bar{r}_{st}](s-t)}\,ds \tag{A.5}$$

which holds iff

$$\begin{aligned} -(1-1/\sigma)\left(\frac{g}{\theta}+\bar{r}_{st}\right)-\rho/\sigma-\upsilon &= \frac{g}{\theta}+n-\upsilon-\bar{r}_{st} \\ \bar{r}_{st} &= \frac{g}{\theta}(2\sigma-1)+\sigma n+\rho. \end{aligned} \tag{A.6}$$

We proceed by showing that for any other value of g, the resource constraint is not satisfied if $\bar{r}_{st} = r$. The argument is by contradiction. Suppose that $g_t = g \neq (n-\upsilon)$ and $\bar{r}_{st} = r$. Then, equation (A.4) becomes

$$(1-\beta_t+o) = \frac{\int_t^\infty e^{[\frac{g}{\theta}+n-\upsilon-r](s-t)}(1-\beta_s+o)\,ds}{\int_t^\infty exp\{[-(1-1/\sigma)\left(\frac{g}{\theta}+r\right)-\rho/\sigma-\upsilon](s-t)\}\,ds} \tag{A.7}$$

Only if either $g = n-\upsilon$ or $R_t = 0$ for all t do we observe that $\beta_t = \beta$. For all other values of g, the growth rate of β_t is $g_\beta = g-(n-\upsilon)$. In this case, simplifying (A.7) leads to

$$\begin{aligned} (1-\beta_t+o)\frac{1}{(1-1/\sigma)\left(\frac{g}{\theta}+r\right)+\rho/\sigma+\upsilon} &= (1+o)\frac{1}{-\frac{g}{\theta}-(n-\upsilon)+r}-\beta_t\frac{1}{-g\frac{1+\theta}{\theta}+r} \\ (1+o)\frac{\frac{g}{\theta}(1/\sigma-2)-n+\frac{1}{\sigma}(r-\rho)}{[(1-1/\sigma)\left(\frac{g}{\theta}+r\right)+\rho/\sigma+\upsilon][r-\frac{g}{\theta}-(n-\upsilon)]} &= \beta_t\frac{\frac{g}{\theta}(1/\sigma-2-\theta)+\frac{1}{\sigma}r-\frac{\rho}{\sigma}-\upsilon}{[(1-1/\sigma)\left(\frac{g}{\theta}+r\right)+\rho/\sigma+\upsilon][r-g\frac{1+\theta}{\theta}]} \end{aligned} \tag{A.8}$$

As the right-hand side of (A.8) depends on time through β_t, and the left-hand side does not, this condition can only hold if the numerators of both the left- and the right-hand side are zero. The numerator of the left-hand side of (A.8) is zero iff

$$r = \frac{g}{\theta}(2\sigma-1)+\rho+\sigma n, \tag{A.9}$$

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while the numerator of the right-hand side is zero iff

$$r = \rho + \sigma\upsilon + \frac{g}{\theta}(2\sigma - 1 + \theta\sigma). \tag{A.10}$$

Combining (A.9) and (A.10), we find that they are equal only if $g = n - \upsilon$, which contradicts our original assumption. I.e., there exists no other constant value g other than $n - \upsilon$ (or zero research) for which the interest rate is constant for finite t.

To prove the convergence result of the Lemma, note that if $\lim_{t\to\infty} g_t = g$, then $\lim_{t\to\infty} \beta_t = \beta \in (0,1)$, where $\beta = 1$ if $g > n - \upsilon$ and $\beta = 0$ if $g < n - \upsilon$. In this case, we can work with the limit of equation (A.4), which is

$$\lim_{t\to\infty}(1 - \beta_t + o) = \frac{\lim_{t\to\infty} \int_t^\infty e^{[\frac{1}{\theta}\bar{g}_{st} + n - \upsilon - \bar{r}_{st}](s-t)}(1 - \beta_s + o)\, ds}{\lim_{t\to\infty} \int_t^\infty exp\{[(1 - 1/\sigma)\left(-\frac{1}{\theta}\bar{g}_{st} - \bar{r}_{st}\right) - \rho/\sigma - \upsilon](s-t)\}\, ds}$$

$$(1 - \beta + o) = \frac{(1 - \beta + o)\int_t^\infty e^{[\frac{1}{\theta}\lim_{t\to\infty}\bar{g}_{st} + n - \upsilon - \lim_{t\to\infty}\bar{r}_{st}](s-t)}\, ds}{\int_t^\infty exp\{[(1 - 1/\sigma)\left(-\frac{1}{\theta}\lim_{t\to\infty}\bar{g}_{st} - \lim_{t\to\infty}\bar{r}_{st}\right) - \rho/\sigma - \upsilon](s-t)\}\, ds} \tag{A.11}$$

and

$$\int_t^\infty exp\{[(1 - 1/\sigma)\left(-\frac{g}{\theta} - \lim_{t\to\infty}\bar{r}_{st}\right) - \rho/\sigma - \upsilon](s-t)\}\, ds = \int_t^\infty e^{[\frac{1}{\theta}g + n - \upsilon - \lim_{t\to\infty}\bar{r}_{st}](s-t)}\, ds \tag{A.12}$$

which is identical to the condition in (A.5) except that it depends on the limit of $\bar{r}_{st}$ rather than $\bar{r}_{st}$ itself. Consequently, equation (A.12) is satisfied iff

$$\lim_{t\to\infty}\bar{r}_{st} = \frac{g}{\theta}(2\sigma - 1) + \sigma n + \rho. \tag{A.13}$$

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Appendix B. Equilibrium with risk neutrality

If agents are risk neutral, consumers will be indifferent to the allocation of consumption over time if the interest rate is $r = \rho + \frac{\dot{P}_t}{P_t}$. In this case, it is trivial that all output will be consumed at each t. The (expected) utilities of work and research are, respectively,

$$W_t = \frac{1+o}{P_t} - \frac{1}{\rho + \upsilon} \tag{B.1}$$

$$EU_t = \frac{\alpha}{T_t}\frac{V_t}{P_t} + \frac{o}{P_t} - \frac{1}{\rho + \nu} \tag{B.2}$$

and a labor allocation in which both workers and researchers are active is optimal if, $E_t = 1$, i.e.,

$$1 = \frac{\alpha}{T_t} V_t \tag{B.3}$$

It is straightforward to show that $g = n - \upsilon$ satisfies this equation. The value of an innovation, V_t, is,

$$\begin{aligned} V_t &= \frac{(1-\beta+o)nL_t}{\theta} \int_t^{\infty} e^{[\frac{1}{\theta}(n-\upsilon)-r](s-t)}\, ds \\ &= \frac{(1-\beta+o)n}{\theta r - (n-\upsilon)} L_t \\ &= \frac{(1-\beta+o)n}{\theta\rho - 2(n-\upsilon)} L_t \end{aligned} \tag{B.4}$$

and $T_t = T_0 e^{(n-\upsilon)t}$, i.e., labor market optimality is satisfied if

$$1 = \frac{\alpha}{T_0}\frac{(1-\beta+o)n}{\theta\rho - 2(n-\upsilon)} L_0. \tag{B.5}$$

Finally, the growth rate of average consumption per capita, from equation (17), is $\frac{g}{\theta}$. The economy follows a balanced growth path with $g = n - \upsilon$.

Appendix C. Existence of (active) research equilibrium

Proof of Theorem 1:

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For the proof of part 1 of Theorem 1, note that if $\sigma > 1$, a research path of $R_t = 0$ for all t is an optimal labor allocation if and only if for all t

$$\begin{aligned} \frac{\alpha}{T_t}\left[(V_t+o)^{1-\sigma} - o^{1-\sigma}\right] + o^{1-\sigma} &> (1+o)^{1-\sigma} \\ T_t &> \alpha\frac{(V_t+o)^{1-\sigma} - o^{1-\sigma}}{(1+o)^{1-\sigma} - o^{1-\sigma}} \end{aligned} \tag{C.1}$$

where the reversal of the sign is due to the fact that $(1+o)^{1-\sigma} - o^{1-\sigma} < 0$ if $\sigma > 1$. For this condition to be met for all t, it must be that it holds for the maximal value that the RHS can take. As with $R_t = 0$, the value of an innovation is

$$V_t = \frac{(1+o)nL_t}{\theta[r-(n-\upsilon)]}, \tag{C.2}$$

the RHS of (C.1) is strictly increasing in t, and therefore reaches its maximum value as $t \to \infty$, in which case $R_t = 0$ for all t is an optimal labour market allocation if

$$T_t = T_0 \geq \alpha\frac{1}{1-\left(\frac{1+o}{o}\right)^{1-\sigma}} = \bar{T}. \tag{C.3}$$

$\bar{T}$ is the level of the stock of research such that agents are indifferent between research and work only in the limit as $t \to \infty$. For all finite t, if $T_t \geq \bar{T}$, agents strictly prefer to work over engaging in R&D. If the initial stock of research is at least $\bar{T}$ and agents have "high" levels of risk aversion, the probability to innovate is so small that no agent is ever willing to take that risk, and all agents always choose to work in the production sector.

These derivations also show that for any $T_t < \bar{T}$, the equilibrium labor market condition for zero research, (C.1), will be violated at some t. In particular, if we assume $R_t = 0$ for all t and

$$T_t = \alpha\frac{1}{1-\left(\frac{1+o}{o}\right)^{1-\sigma}} - \epsilon \tag{C.4}$$

with $\epsilon > 0$, there exists a finite value of t, $\hat{t}$, at which the expected utility of research is identical to the utility of work, which shows that for any $t > \hat{t}$, $R_t = 0$ is no labor market equilibrium:

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$$\alpha \frac{1}{1-\left(\frac{1+o}{o}\right)^{1-\sigma}} - \epsilon = \alpha \frac{1-\left[\frac{(1+o)nL_{\hat{t}}}{o\theta(r-(n-\upsilon))}+1\right]^{1-\sigma}}{1-\left(\frac{1+o}{o}\right)^{1-\sigma}}$$

$$1-\left[1-\left(\frac{1+o}{o}\right)^{1-\sigma}\right]\frac{\epsilon}{\alpha} = 1-\left[\frac{(1+o)nL_{\hat{t}}}{o\theta(r-(n-\upsilon))}+1\right]^{1-\sigma}$$

$$(1+o)nL_{\hat{t}} = \left\{\left[\frac{\epsilon}{\alpha}\left(1-\left(\frac{1+o}{o}\right)^{1-\sigma}\right)\right]^{\frac{1}{1-\sigma}} - 1\right\} o\theta(r-(n-\upsilon))$$

$$\hat{t} = \frac{1}{n-\upsilon}\ln\left\{\left\{\left[\frac{\epsilon}{\alpha}\left(1-\left(\frac{1+o}{o}\right)^{1-\sigma}\right)\right]^{\frac{1}{1-\sigma}} - 1\right\}\frac{o\theta(r-(n-\upsilon))}{(1+o)n}\right\} \tag{C.5}$$

This result proves that as long as $T_t < \bar{T}$, some research will occur at some point in time, and combined with our earlier result on the strict preference of work over research whenever $T_t > \bar{T}$, we know that $\bar{T}$ is the highest level of the stock of research that may be reached if $\sigma > 1$. This completes the proof of Theorem 1.

Proof of Lemma 3:

The proof of the Lemma is straightforward by contradiction. For $\lim_{t\to\infty} T_t = T$ to hold, it must be that $\lim_{t\to\infty} R_t = 0$. From Lemma 1, we know that if $R_t \to 0$, the fraction of researchers and the average interest rate converge to constants. From equations (18) and (19), zero research constitutes an optimal labor market allocation in the long run if

$$\lim_{t\to\infty}\frac{\alpha}{T_t}\left[(V_t+o)^{1-\sigma}+o^{1-\sigma}\right] \quad < (1+o)^{1-\sigma} \quad \text{if } \sigma < 1, \tag{C.6}$$

$$\lim_{t\to\infty}\frac{\alpha}{T_t}\left[\ln(V_t+o)+\ln(o)\right] \quad < \ln(1+o)-\ln(o) \quad \text{if } \sigma = 1. \tag{C.7}$$

We know that with zero research in the limit, $\lim_{t\to\infty} T_t = T$, and that the value of an innovation in the limit is

$$V_t = \frac{(1+o)nL_t}{\theta[r-(n-\upsilon)]} \quad \forall t \geq \hat{t}, \tag{C.8}$$

which grows without bounds. This implies that (C.6) and (C.7) will be violated in the long run, and we have reached a contradiction.

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Appendix D. Growth rate of researchers

We prove Theorem 2 by showing that no other path of research is compatible with equilibrium in the long run. It is easiest to consider the cases $0 < \sigma < 1$ and $\sigma = 1$ separately:

Case 1: $0 < \sigma < 1$

Labor market allocation in the limit is optimal if

$$R_t = \begin{cases} nL_t & \text{if} \quad \lim\limits_{t\to\infty} \frac{\alpha(V_t+o)^{1-\sigma}}{T_t} - \lim\limits_{t\to\infty} \frac{\alpha o^{1-\sigma}}{T_t} > (1+o)^{1-\sigma} - o^{1-\sigma} \\ \in [0, nL_t] & \text{if} \quad \lim\limits_{t\to\infty} \frac{\alpha(V_t+o)^{1-\sigma}}{T_t} - \lim\limits_{t\to\infty} \frac{\alpha o^{1-\sigma}}{T_t} = (1+o)^{1-\sigma} - o^{1-\sigma} \\ 0 & \text{if} \quad \lim\limits_{t\to\infty} \frac{\alpha(V_t+o)^{1-\sigma}}{T_t} - \lim\limits_{t\to\infty} \frac{\alpha o^{1-\sigma}}{T_t} < (1+o)^{1-\sigma} - o^{1-\sigma} \end{cases} \tag{D.1}$$

For any g_t such that $\lim\limits_{t\to\infty} g_t = g$, we know that:

$$\lim_{t\to\infty} \frac{\dot{T}_t}{T_t} = g \tag{D.2}$$

$$\lim_{t\to\infty} \bar{r}_{st} = \frac{g}{\theta}(2\sigma - 1) + \sigma n + \rho \tag{D.3}$$

Making use of these convergence results in equation (16), the expected value of an innovation in the limit is

$$\lim_{t\to\infty} V_t = \frac{(1+o)nL_t}{\theta[r - (n-\upsilon) + g(1-\frac{1}{\theta})]} - \frac{R_t}{\theta r - g}, \tag{D.4}$$

i.e., $\lim\limits_{t\to\infty} V_t = \infty$ if $g < n - \upsilon$.

We already know that zero research at any t or as a limit will violate labor market optimality. Now, we ascertain that a research path along which $R_t = R_0$ or $R_t \to C$, where C is any positive constant, is not compatible with an equilibrium path in the long run either. In this case, $\lim\limits_{t\to\infty} V_t = \infty$ and $\lim\limits_{t\to\infty} T_t = \infty$. This implies that

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$$\lim_{t\to\infty} \quad \alpha\frac{o^{1-\sigma}}{T_t} \quad = 0 \tag{D.5}$$

$$\lim_{t\to\infty} \quad \alpha\frac{(V_t+o)^{1-\sigma}}{T_t} \quad = \frac{\infty}{\infty}. \tag{D.6}$$

By L'Hopital's rule,

$$\lim_{t\to\infty} \alpha\frac{(V_t+o)^{1-\sigma}}{T_t} = \lim_{t\to\infty} \alpha\frac{\frac{(1-\sigma)\frac{\dot{V}_t}{V_t}}{\left(V_t^{\frac{\sigma-1}{\sigma}}+oV_t^{-\frac{1}{\sigma}}\right)^{\sigma}}}{\dot{T}_t} \tag{D.7}$$

where either $\dot{T}_t = \alpha R_0$, if $R_t = R_0$, or $\lim_{t\to\infty} \dot{T}_t \to \alpha C$, if $R_t \to C$. In either case, the limit of the denominator of (D.7) is a constant, and if we define $B = \{R_0, C\}$,

$$\lim_{t\to\infty} \alpha\frac{(V_t+o)^{1-\sigma}}{T_t} = \frac{1}{B}\lim_{t\to\infty} \frac{(1-\sigma)\frac{\dot{V}_t}{V_t}}{\left(V_t^{\frac{\sigma-1}{\sigma}}+oV_t^{-\frac{1}{\sigma}}\right)^{\sigma}} \tag{D.8}$$

The denominator in (D.8) converges to zero. To determine the long run behavior of $\frac{\dot{V}_t}{V_t}$, first note that

$$V_t = \frac{n}{\theta}T_t^{\frac{\theta-1}{\theta}}\int_t^{\infty} e^{-\int_t^s r_x\,dx}(1-\beta_s+o)L_sT_s^{\frac{1-\theta}{\theta}}\,ds \tag{D.9}$$

Define the integral in (D.9) as $J_t \equiv \int_t^{\infty} e^{-\int_t^s r_x\,dx}(1-\beta_s+o)L_sT_s^{\frac{1-\theta}{\theta}}\,ds$. Then the growth rate of V_t can be expressed as

$$\frac{\dot{V}_t}{V_t} = \frac{\theta-1}{\theta}\frac{\dot{T}_t}{T_t} + \frac{\dot{J}_t}{J_t} \tag{D.10}$$

with

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$$\begin{aligned}\frac{\dot{J}_t}{J_t} &= r_t - \frac{(1-\beta_t+o)L_t T_t^{\frac{1-\theta}{\theta}}}{\int_t^\infty e^{-\int_t^s r_x\,dx}(1-\beta_s+o)L_s T_s^{\frac{1-\theta}{\theta}}\,ds} \\ &= r_t - \frac{(1-\beta_t+o)}{\int_t^\infty (1-\beta_s+o)e^{[(n-\upsilon)+\frac{1-\theta}{\theta}\bar{g}_{st}-\bar{r}_{s-t}](s-t)}\,ds}.\end{aligned} \tag{D.11}$$

This implies that

$$\begin{aligned}\lim_{t\to\infty}\frac{\dot{J}_t}{J_t} &= r - \frac{\lim_{t\to\infty}(1-\beta_t+o)}{\int_t^\infty\left[(1+o)e^{[(n-\upsilon)+\frac{1-\theta}{\theta}g-r](s-t)} - \lim_{t\to\infty}\beta_t e^{[\frac{g}{\theta}-r](s-t)}\right]\,ds} \\ &= r - \frac{\lim_{t\to\infty}(1-\beta_t+o)}{\frac{(1+o)}{r-(n-\upsilon)-\frac{1-\theta}{\theta}g} - \frac{\lim_{t\to\infty}\beta_t}{r-\frac{g}{\theta}}} \\ &= r - \left(r-(n-\upsilon)-\frac{1-\theta}{\theta}g\right) \\ &= n-\upsilon+\frac{1-\theta}{\theta}g,\end{aligned} \tag{D.12}$$

since $\lim_{t\to\infty}\beta_t = 0$, and that the growth rate of the value of an innovation, equation (D.10), converges to

$$\lim_{t\to\infty}\frac{\dot{V}_t}{V_t} = \frac{\theta-1}{\theta}g + (n-\upsilon) + \frac{1-\theta}{\theta}g = n-\upsilon. \tag{D.13}$$

Plugging this result into equation (D.8), we find that if the measure of researchers either is a constant, or converges to a constant, the left-hand side of equation (D.1) goes to infinity, under which condition the entire population should engage in R&D, contradicting the assumption that the measure of researchers is constant.

As the measure of researchers on an equilibrium path cannot go to zero and cannot be nor converge to a constant, the measure must increase over time. Consider any $g_t \to g$. Note, this includes exponentially growing measures of researchers as well as measures of researchers that grow at rates lower than exponentially. In this case,

$$\dot{T}_t = \alpha R_t, \tag{D.14}$$

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and $R_t \to \infty$. The long run behavior of the left-hand side of (D.1) is again determined by the limit behavior of $\frac{(V_t+o)^{1-\sigma}}{T_t}$, but now also the derivatives of the numerator and the denominator go to infinity in the limit. We make use of the following relationship:

$$\begin{aligned}
\frac{\alpha}{T_t}\left[(V_t+o)^{1-\sigma}-o^{1-\sigma}\right] &= (1+o)^{1-\sigma}-o^{1-\sigma} \quad \Rightarrow \\
\ln(\alpha)-\ln(T_t)+\ln[(V_t+o)^{1-\sigma}-o^{1-\sigma}] &= \ln[(1+o)^{1-\sigma}-o^{1-\sigma}] \quad \Rightarrow \\
-\frac{\dot{T}_t}{T_t}+\frac{(1-\sigma)(V_t+o)^{-\sigma}\dot{V}_t}{(V_t+o)^{1-\sigma}-o^{1-\sigma}} &= 0 \quad \Rightarrow \\
-\frac{\dot{T}_t}{T_t}+(1-\sigma)\frac{\frac{\dot{V}_t}{V_t}}{1+oV_t^{-1}-o^{1-\sigma}(V_t^{\frac{\sigma-1}{\sigma}}+oV_t^{-\frac{1}{\sigma}})^{\sigma}} &= 0 \quad \Rightarrow \\
\lim_{t\to\infty}-\frac{\dot{T}_t}{T_t}+(1-\sigma)\lim_{t\to\infty}\frac{\frac{\dot{V}_t}{V_t}}{1+oV_t^{-1}-o^{1-\sigma}(V_t^{\frac{\sigma-1}{\sigma}}+oV_t^{-\frac{1}{\sigma}})^{\sigma}} &= 0
\end{aligned}$$

We know that $\frac{\dot{T}_t}{T_t} \to g$, that $V_t \to \infty$ and that $\frac{\dot{V}_t}{V_t} \to n-\upsilon$. Consequently, the only value of g that does not contradict an equilibrium path on which both researchers and workers are active in the long run is $g=(1-\sigma)(n-\upsilon)$. Note that this also shows that R_t cannot grow at a rate below exponential on an equilibrium path, as this would imply $g_t \to g = 0$.

Case 2: $\sigma = 1$

If $\sigma = 1$, the condition for an optimal labor market allocation becomes

$$R_t = \begin{cases} nL_t & \text{if} \quad \lim_{t\to\infty}\frac{\alpha\ln(V_t+o)}{T_t}-\lim_{t\to\infty}\frac{\alpha\ln(o)}{T_t} > \ln(1+o)-\ln(o) \\ \in[0,nL_t] & \text{if} \quad \lim_{t\to\infty}\frac{\alpha\ln(V_t+o)}{T_t}-\lim_{t\to\infty}\frac{\alpha\ln(o)}{T_t} = \ln(1+o)-\ln(o) \\ 0 & \text{if} \quad \lim_{t\to\infty}\frac{\alpha\ln(V_t+o)}{T_t}-\lim_{t\to\infty}\frac{\alpha\ln(o)}{T_t} < \ln(1+o)-\ln(o) \end{cases} \tag{D.15}$$

For any path of researchers such that $T_t \to \infty$, an optimal labor market allocation in the long run requires that

$$\lim_{t\to\infty}\frac{\alpha\ln(V_t+o)}{T_t} = \ln(1+o)-\ln(o) \tag{D.16}$$

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And the left-hand side of (D.16) can be evaluated, similar to the case where $\sigma < 1$, by applying L'Hopital's rule:

$$\lim_{t\to\infty} \frac{\alpha \ln(V_t + o)}{T_t} = \alpha \lim_{t\to\infty} \frac{\frac{\frac{\dot{V}_t}{V_t}}{1+oV_t^{-1}}}{\dot{T}_t} \tag{D.17}$$

We know that $\lim_{t\to\infty} \frac{\frac{\dot{V}_t}{V_t}}{1+oV_t^{-1}} = n - \upsilon$, and that $\lim_{t\to\infty} \dot{T}_t = \alpha R_t$. This implies that (D.16) cannot hold if $g_t \to g > 0$. It can hold, however, if either $R_t = R_0$ or $R_t \to C > 0$. In this case, if we again let $B \equiv \{R_0, C\}$, we find that

$$\lim_{t\to\infty} \frac{\alpha \ln(V_t + o)}{T_t} = \frac{n - \upsilon}{B} \tag{D.18}$$

which is equal to $\ln(1+o) - \ln(o)$ if $B = \frac{n-\upsilon}{\ln(1+o)-\ln(o)}$. Which implies that only constant and positive measures of researchers are compatible with optimal labor market allocation in the long run if $\sigma = 1$.

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