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Supplementary Material to: An Algorithm to Compute the Polar Decomposition of a 3×3 Matrix

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1 Operation counts for Algorithms 3.2 and 3.5

We give detailed operation counts for Algorithm 3.2 and Algorithm 3.5 in the different cases that can arise. In the tables, M denotes a multiplication or division, A an addition or subtraction, C a comparison, and O a different operation, such as a square root. We do not include in our operation counts the cost of copying an element or swapping two elements in an array. The lines in italics and marked with an asterisk are mutually exclusive: either one or the other is executed.

In Tables 1.2 and 1.3 the operation counts for the block LDL^T factorization assume that the first two pivots are taken as 1×1 pivots based on the largest diagonal element. A detailed analysis shows that this is always possible and is numerically stable.

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Table 1.1 Detailed operation count for Algorithm 3.2.

Task	Operations			
Compute $\|A\|_F$	$9M$	$8A$		$1O$
Scale A to norm 1	$9M$			
Form B		$12A$		
Compute b via PLU factorization of B	$23M$	$14A$	$6C$	
Compute $\det A$ (and its sign) via PLU factorization	$11M$	$5A$	$6C$	
Decide whether to use Newton or analytic method			$1C$	
*Compute λ_1 via analytic formula**	$13M$	$7A$	$1C$	$6O$
*Compute λ_1 via Newton's method (per iteration)**	$13M$	$10A$	$1C$	
Compute B_s		$4A$		
LDL^T factorization of B_s	$15M$	$9A$	$6C$	
Form v	$9M$	$7A$		$1O$
Form Q from v	$18M$	$12A$		
Check if sign of Q needs changing			$1C$	
Form H from Q	$18M$	$12A$		
Scale back H	$6M$			
TOTAL (assuming analytic method to get λ_1)	$131M$	$90A$	$21C$	$8O$

Table 1.2 Detailed operation count for Algorithm 3.5 for $b \geq 1-\tau_2$ and $\log_{10} u_{22} > -7.18$.

Task	Operations			
Compute $\|A\|_F$	$9M$	$8A$		$1O$
Scale A to norm 1	$9M$			
Form B		$12A$		
Compute b via PLU factorization of B	$23M$	$14A$	$6C$	
Decide if b is close to 1			$1C$	
Compute $\det A$ (and its sign) via P_1LUP_2 factorization	$11M$	$5A$	$15C$	
Check value of $\log_{10} \|u_{22}\|$			$1C$	$1O$
Estimate number of iterations	$2M$	$1A$		$2O$
Decide whether to use Newton or analytic method			$1C$	
*Compute λ_1 via analytic formula**	$13M$	$7A$	$1C$	$6O$
*Compute λ_1 via Newton's method** (per iteration)	$13M$	$10A$	$1C$	
Compute B_s		$4A$		
Block LDL^T factorization of B_s	$14M$	$13A$	$5C$	
Compute $\det D$	$2M$	$1A$		
Form starting guess for v	$9M$	$7A$		$1O$
Inverse iteration, per iteration	$24M$	$17A$		$1O$
Form Q from v	$18M$	$12A$		
Check if sign of Q needs changing			$1C$	
Form H from Q	$18M$	$12A$		
Scale back H	$6M$			
Total (assuming analytic method to get λ_1 and n iterations)	$(134+24n)M$	$(96+17n)A$	$31C$	$(11+n)O$

Table 1.3 Detailed operation count for Algorithm 3.5 for $b \geq 1-\tau_2$ and $\log_{10} u_{22} \leq -7.18$.

Task	Operations			
Task	Operations			
Compute $\|A\|_F$	$9M$	$8A$		$1O$
Scale A to norm 1	$9M$			
Form B		$12A$		
Compute b via PLU factorization of B	$23M$	$14A$	$6C$	
Decide if b is close to 1			$1C$	
Compute $\det A$ (and its sign) via P_1LUP_2 factorization	$11M$	$5A$	$15C$	
Check value of $\log_{10} \|u_{22}\|$			$1C$	$1O$
Decide whether to use Newton or analytic method			$1C$	
Compute λ_1 via analytic formula*	$13M$	$7A$	$1C$	$6O$
Compute λ_1 via Newton's method (per iteration)*	$13M$	$10A$	$1C$	
Compute B_s		$4A$		
Block LDL^T factorization of B_s	$14M$	$13A$	$5C$	
Compute $\det D$	$2M$	$1A$		
Form V	$2M$	$2A$		
Subspace iterations	$110M$	$72A$		$8O$
Project	$38M$	$26A$		
Find eigenvector and project back	$19M$	$9A$	$1C$	$3O$
Form Q from v	$18M$	$12A$		
Check if sign of Q needs changing			$1C$	
Form H from Q	$18M$	$12A$		
Scale back H	$6M$			
TOTAL (assuming analytic method to get λ_1)	$292M$	$197A$	$32C$	$19O$