Multidimensional and Persistent Poverty: Methodological Approaches to Measurement Issues

Yadira Díaz

A Thesis Submitted for the Degree of Doctor of Philosophy in Economics

Institute for Social and Economic Research
University of Essex

August, 2016

Summary

Multidimensional deprivation and persistent poverty are important research areas within the poverty measurement literature. Still, both encompass measurement issues for which methodological solutions are yet to be analysed. The thesis that I present here analyses three specific measurement issues, identified as relevant within these research areas, and proposes methodological approaches to tackle each of them.

First, it evaluates the effect of different demographic population structures on societal multidimensional deprivation incidence comparisons. The results of this evaluation demonstrate that societal multidimensional comparisons reflect not only differences in relative deprivation but also differences in the demographic composition of the societies to be compared. These differences in the demographic structure of the population, thus, confound societal multidimensional deprivation comparisons. To tackle this comparability problem, the application of direct and indirect standardisation methods is proposed and analysed in this context.

Second, it studies the effect of differences in need, exhibited across individuals from different demographic population subgroups or households of different sizes and compositions, on multidimensional deprivation incidence profiles. To address differences in needs and enhance individual or household comparability, I propose a family of multidimensional deprivation indices that describes how much deprivation two demographically heterogeneous units with different needs must exhibit to be catalogued as equivalently deprived. The obtained empirical results demonstrate that neglecting differences in needs yields biased multidimensional deprivation incidence profiles. The results also shed light on the ability of my proposed family of measures to capture these differences in need effectively.

Third, this thesis analyses the reliability of persistent poverty measures in the presence of survey non-response. The obtained empirical results indicate that persistent poverty measures based on balanced panel estimates that do not account for the relationship between survey non-response and the socioeconomic status of the household provide a significantly biased picture of the intertemporal phenomenon.

The methodologies that I present in this thesis are meant foremost to be easy to implement and understand by policymakers. As such, they are proposed as methodological tools to improve the measurement and analysis of poverty in the policy context.

Acknowledgements

I am truly in debt to Paul Clarke, supervisor and graduate director of the Institute for Social and Economic Research, for his patience and invaluable guidance during this journey.

Thank you to my family, who has been a bastion of joy and support. To Vasileios for his companionship, and to life for all the new friends that I have met while developing this research. I am as well thankful to my Colombian friends, whom inspired me to take this path.

Declarations

No part of this thesis has been submitted for another degree.

The 4th chapter that I present in this thesis has been developed as part of a co-authored paper with Stephen Pudney, PhD supervisor of this particular chapter. An earlier version of the co-authored paper appeared in the ISER working paper series (number 2013-22). All remaining work in this thesis is my own.

An earlier version of Chapter 3 appeared in the ECINEQ working paper series number 387 from December 2015

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Chapter 1

Introduction

Multidimensional and persistent poverty are examples of distinct poverty measurement methods. They have a different conceptual basis and identify different populations as the most deprived. While multidimensional poverty assesses multiple dimensions of well-being and identifies as the most deprived those who exhibit the larger number of these dimensions in deprivation, persistent poverty assesses the ability to consume a market commodity basket across time and defines as the most deprived those who lack the resources to access such a basket, repeatedly over time.

They are both of special interest in the current policy arena. On one hand, the usage of multidimensional indices of poverty has been gaining interest in the international context as well as in country-specific settings. Two examples of this are: the global Multidimensional Poverty Index, launched by the United Nations Development Program in 2010, and published annually since then (Alkire et al. 2014); and the Colombian Multidimensional Poverty Index, proposed by Angulo et al. (2016) and in use by the Colombian government to track multidimensional deprivation in the national territory yearly since 2010.

Although the terms 'multidimensional poverty' and 'multidimensional deprivation' are used interchangeably in the literature, henceforth in this thesis, I opt using the term 'multidimensional deprivation' to refer to indices that count the multiple deprivations jointly observed across a selected unit of analysis and, based on this counting procedure, identify the poor as the most deprived population. Examples of this long-standing literature are studies such as Townsend (1979), Atkinson & Bourguignon (1982), Mack et al. (1985), Callan et al. (1993), Feres & Mancero (2001), Atkinson (2002), Alkire & Foster (2011), and Aaberge & Brandolini (2014).

On the other hand, in terms of persistent poverty, households that suffer poverty persistently over time are understood by this type of poverty measurement method to be in worse off conditions than households where this phenomenon occurs on a transient basis. As such, recent research has focused its interest on operationalizing chronic poverty as a persistent pattern of poverty state over time. Examples of this growing literature in persistent poverty measurement are Baulch & Hoddinott (2000), Jalan & Ravallion (2000), Yaqub (2003), Hulme & Shepherd (2003), Foster (2009), Calvo & Dercon (2009), Bossert & Chakravarty (2012), Gradin et al. (2012) and Mendola et al. (2012).[1]

Despite the policy relevance of multidimensional deprivation and persistent poverty as methods to measure poverty, they still embed measurement issues where methodological solutions are to be analysed.

The selection of the methodological approaches to tackle measurement issues is crucial as it may dramatically determine the results of the exercise. In general, there is agreement that the measurement of poverty corresponds to an objective evaluation of the standard of living of a particular society(ies) at a moment of time(s). But, it is also well known that such an evaluation involves various different steps and decisions, which can be approached by alternative methodologies that shape the result (Sen 1979).

The thesis that I present here investigates three specific measurement issues, identified as relevant when measuring multidimensional deprivation and persistent poverty. This is the first effort in the poverty measurement literature that analyses and proposes methodological solutions for each of them. The following paragraphs summarise the three identified measurement issues and the proposed methodological approaches to tackle them.

In terms of multidimensional deprivation measurement, the most commonly used analytical technique is to compare societies' performance across time and

[1]The terms 'chronic poverty' and 'persistent poverty' are used interchangeably in the literature, both referring to repeated poverty states over time.

geographical areas. Nonetheless, societies have a different size and distribution of the population by demographic factors such as age, gender and household size.

Multidimensional measurement methodologies, as the ones proposed by Tsui (2002), Bourguignon & Chakravarty (2003), Seth (2009, 2013), Alkire & Foster (2011), and Rippin (2010) among others, are meant to allow meaningful societal multidimensional comparisons. This is accomplished by making societal measures non-sensitive to the scale of the population and expressing them on a per capita bases. Still, they do not address the challenges that different structures of the population might be placing on these comparisons. Individuals from different population subgroups are assumed to have no other relevant differences than the characteristics included within the measurement process.

As a result, current societal multidimensional deprivation incidence comparisons might not only resemble relative deprivation differences but also differences in the demographic structure of the populations to be compared.

These differences in the demographic structure of the population are understood in the context of Chapter 2 to be a confounding factor. Chapter 2 analyses, therefore, the comparability problem that these demographic confounding factors pose over societal multidimensional deprivation incidence comparisons.

To tackle this comparability problem, I propose the application of direct and indirect standardisation methods. The behaviour of non-standardised versus standardised multidimensional measures is empirically assessed in the chapter, while using demographic household surveys from the Maldives, Ukraine, Jordan, Dominican Republic and Armenia. The advantages and disadvantages of standardisation methods in this context are also discussed.

The empirical results of this chapter indicate that the use of non-standardised comparisons of multidimensional deprivation incidences across societies with different distributions of the population could be producing inaccurate rankings. To perform more meaningful societal comparisons, I propose conducting them under standardised bases.

Subsequently, in Chapter 3, I continue investigating multidimensional deprivation incidence comparisons but in this instance, at either the individual or the household level. Chapter 3 analyses, then, the challenges embedded in comparing either individuals from different demographic population subgroups or households of different size and composition, without accounting for the fact that these demographically heterogeneous units have differences in needs. Multidimensional deprivation comparisons in the presence of these differences in needs have yet to be analysed by the multidimensional measurement literature.

This chapter proposes a family of multidimensional deprivation indices that explicitly takes into account observed differences in needs across demographically heterogeneous units (i.e., either households of different size and composition or individuals of different population subgroups). The proposed counting family of multidimensional indices of this chapter builds upon the Alkire and Foster methodology of poverty measurement (Alkire & Foster 2011) and draws from the one-dimensional parametric equivalence scale literature. It aims to describe how much deprivation two demographically heterogeneous units with different needs must exhibit to be catalogued as equivalently deprived.

Furthermore, my proposed family of measures describes, under equivalent normative considerations, the burden that multidimensional deprivation places on each unit of analysis. For instance, under an absolute normative perspective where each deprivation has an equal absolute value, multidimensional deprivation is described through count-based approaches to measurement. Conversely, under a relative normative perspective that conceives each unit of analysis as equivalently valuable, multidimensional deprivation is described in terms of share-based approaches to measurement. Intermediate normative perspectives, in contrast, lead to the expression of multidimensional deprivation as a mixture of count-based and share-based approaches to measurement.

To evaluate the effect of these different approaches to measurement (count-based, share-based and intermediate) on multidimensional deprivation incidence profiles, I construct counterfactuals using the 2013 Paraguayan household to disentangle how much of the differences in multidimensional deprivation incidence profiles are observed because unaddressed differences in needs.

The obtained empirical results of this evaluation demonstrate that unaddressed differences in needs yield multidimensional deprivation incidence profiles to reflect not only illegitimate differences in deprivation but also differences in needs that should be tackled by the measurement process. Failure to take differences in needs into account was found to cause biased multidimensional incidence profiles. These results also shed light on the ability of the proposed measures of this chapter to effectively capture these differences in need.

On the other hand, persistent poverty measures are based upon panel data sets that naturally suffer from survey non-response. Traditional approaches to tackle survey non-response use weighting systems to correct for such a survey non-response. However, they generally assume the survey non-response pattern is unrelated to the outcome of interest, which in this case is persistent poverty.

Assuming survey non-response not related with the household socio-economic characteristics seems, nonetheless, unrealistic. As a result, Chapter 4 focuses on analysing the reliability of expenditure-based inter-temporal poverty measures in the presence of survey non-response. In particular, the behaviour of two of the members of the Foster (2009) family of persistent poverty measures are assessed: the persistent poverty headcount ratio and the duration adjusted persistent poverty headcount. Both measures are analysed in the presence of survey non-response and for the particular case of the 2007-2010 Peruvian household national panel survey (named in Spanish as the ENAHO).

Here, the survey non-response effect over persistent poverty is understood as a problem of partial observability. As such, we have complete observability of households observed in each of the waves of the panel and partial observability of the households that have at least one wave with survey non-response. Then, the probability distribution of the observed poverty spell counts and non-poverty spell counts revealed by the survey is used to obtain upper and lower bounds of the behaviour of persistent poverty measures.

These derived limits look to identify how persistent poverty measures would have behaved in absence of survey non-response. They make no-assumption of the behaviour of persistent poverty in the absence of survey non-response. Not surprisingly, in the context of the ENAHO, these no-assumption bounds result in being wide.

Following Nicoletti et al. (2011)s methodological approach to analysing poverty rates in the presence of missing data, in this chapter, two visible and credible assumptions are proposed to narrow the no-assumption identification region: an instrumental variable (IV) restriction, and a monotone instrumental variable (MIV) restriction.

While the IV restriction assumes a set of field-work variables statistically independent of the households poverty status but strongly related to survey non-response, the MIV restriction assumes a set of geographical data to be a monotonic descriptor of the population socio-economic status and the population socio-economic status to increase along as persistent poverty decreases. These two assumptions seem credible and plausible when analysing persistent poverty.

The obtained identification regions, once these two restrictions are placed, result in considerably narrower regions than the no-assumption regions. When comparing the identification power of the MIV and the IV restriction, although an MIV restriction places a conceptually weaker assumption than the IV restriction, the region obtained upon placing this MIV restriction results in being narrower than the obtained one upon placing the IV restriction. This result indicates that, in the context of the ENAHO panel, the use of 2 IV plausible covariates had not as good of identification power as the rich set of 36 MIV covariates that was available for this context.

The final results of this chapter demonstrate that the width of the bounds varies across measures and cut-off points, appearing as the most reliable measure the duration adjusted persistent poverty headcount that uses higher cut-off points. The results of the improved bounds also indicate that standard non-response weighted estimations of Peruvian persistent poverty represent a considerable underestimated picture of the inter-temporal phenomenon.

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Chapter 2

Societal multidimensional comparisons: How much does demography matter?

Abstract

Current societal multidimensional deprivation incidence comparisons can reveal not only relative deprivation differences but also differences in the demographic structure of the populations to be compared. These differences in the demographic structure of the population, thus, confound multidimensional deprivation incidence rates comparisons. This chapter analyses the comparability problem that demographic confounding factors, such as age, gender, or household size, pose over societal multidimensional deprivation comparisons. To address this comparability problem, direct and indirect standardisation procedures are proposed. This chapter assesses the behaviour of non-standardised versus standardised multidimensional deprivation headcount rates using demographic household surveys from the Maldives, Ukraine, Jordan, the Dominican Republic, and Armenia. The advantages and disadvantages of standardisation methods are discussed in this context. Our empirical results indicate that the use of non-standardised multidimensional comparisons could produce inaccurate rankings. Meaningful comparisons must be conducted on a standardised basis.

Keywords: Direct standardisation, Indirect standardisation, Multidimensional deprivation

JEL codes: C43, I32, D63, J19

Acknowledgements: I would like to thank Paul Clarke, supervisor and graduate director of the Institute for Social and Economic Research, for his guidance and comments during the development of this chapter. This chapter has also benefited from the comments of Maria Iacovou of University of Cambridge.

2.1 Introduction

Interest in measuring development by assessing multiple well-being dimensions has been growing. Examples of this interest in the international arena include the Human Development Index (HDI) and the global Multidimensional Poverty Index (MPI), launched by the United Nations Development Program (UNDP) in 1990 and 2010, respectively, and published annually since then (Malik 2013, Alkire et al. 2014). In addition to cross-country indices, a plethora of country-specific multidimensional indices of deprivation also exists. Two examples are the Colombian Multidimensional Poverty Index (CMPI), proposed by Angulo et al. (2016), and the Mexican Multidimensional Poverty Initiative (Coneval 2010).[1]

In the context of these policy-oriented multidimensional indices of deprivation, the most commonly used analytical technique is to compare societies' performance across time and geographical areas. For instance, the 2014 global MPI compares 108 countries in terms of the joint distribution of household multiple deprivations using data from 2002 until 2013. Similarly, the CMPI has been used since 2010 by the Colombian government to annually assess multidimensional deprivation for urban and rural areas.

Societies, however, have different sizes and population distributions by demographic factors such as age, gender, and household size. Although theoretically developed families of multidimensional deprivation and multidimensional welfare indices, such as those proposed by Alkire & Foster (2011) and Bourguignon & Chakravarty (2003), are meant to allow meaningful comparisons of differently sized populations, they still do not address the challenges that different structures of the population might pose for these comparisons.

Current multidimensional deprivation methodologies address societal comparisons by, on one hand, making societal measures non-sensitive to the scale of

[1]Throughout this chapter, we use the term 'multidimensional deprivation to refer to multidimensional measures that aggregate several deprivation indicators into a single index that is used to identify the most deprived population. In the literature, this concept is also termed 'multidimensional poverty (Alkire & Foster 2011, Aaberge & Brandolini 2014). We opt for the former expression to differentiate multidimensional deprivation indices from other types of multidimensional indices where the indicators are not necessarily expressed as deprivations and the aggregation and identification of the poor are also done in a different manner. For a complete discussion of different approaches available in the literature for multidimensional poverty measurement, see Aaberge & Brandolini (2014).

the population and expressing them on a per capita basis. Also on the other hand, they consider individuals within each society as having no other relevant differences than the characteristics included within the measurement process, i.e., *anonymous* in the measurement process. This is the case of family of indices as the proposed by Seth (2013), Bourguignon & Chakravarty (2003), Tsui (2002) and Alkire & Foster (2011), among others.

Unfortunately, in practice, using an anonymity axiom, which considers no other individuals' characteristic as relevant for the measurement process can be problematic in the context of policy-oriented applications of multidimensional measurement. Two factors make this anonymity axiom problematic. We describe the two of them below for the case of multidimensional indices of deprivation.[2]

First, some of the indicators included within multidimensional measures vary systematically across demographic population subgroups. Child mortality, school attendance, or labour market indicators, among others, are good examples of one-dimensional indicators that are traditionally included within multidimensional indices and are strongly dependent on demographic characteristics, such as age or gender. For instance, mortality depends systematically on age: according to World Health Organization figures, in 2013, 74% of the global probability of dying by the age of five was concentrated between birth and age 1. As such, the burden that mortality places cannot be considered as equivalent across children of different age and gender. This issue has been also discussed by Duclos & Tiberti (2016).

Second, most of the multidimensional policy-oriented indices are built upon several indicators that are recorded for specific demographic population subgroups, and thus the number of total deprivations that an individual can possibly score, and the subsequent deprivation evaluation, both structurally vary across those population subgroups. For instance, four out of the 10 indicators that constitute the global MPI are recorded for specific age and gender population subgroups. Likewise, 10 out of the 15 indicators that make up the CMPI are recorded for specific age groups.

[2]Henceforth, we focus our analysis on multidimensional indices of deprivation because they are more commonly used across policy applications than multidimensional indices of welfare. They allow the straightforward use of intrinsic ordinal policy-oriented indicators such as the presence or absence of unemployment and school attendance, among others.

These two types of variations in the deprivation rate across demographic subgroups cause any randomly chosen individual to exhibit a probability of being multidimensionally deprived that is not identically distributed across different population subgroups. Therefore, a measure non-sensitivity to re-arrangements of multidimensionally deprived individuals across demographic subgroups is not necessarily desirable because it does not take into account unavoidable and fair differences in the risk to be multidimensionally deprived.

While some of the sources of the variation of the probability of a given individual to be deprived can be considered legitimate/fair, others can be considered illegitimate/unfair. We would like societal multidimensional deprivation incidence comparisons, nonetheless, to reflect exclusively the differences that refer to unfair sources, not differences that can be considered legitimate. In this context, differences in the structure of two populations at a moment in time are catalogued as a legitimate source of differences in multidimensional deprivation incidence because they reflect an unavoidable difference among societies and reflect differences in needs and preferences rather than unfair disadvantages.[3]

As such, if we compare two societies that do not have differences in relative multidimensional deprivation incidence across demographic population subgroups but only differences in their population structure, multidimensional deprivation indices that portray differences in multidimensional societal rates among these two societies, despite no observation of a difference in relative deprivation between the two, are said to be confounded by demographic group differences in composition.

Specifically, consider two societies with no difference in relative deprivation by age range population groups across them. If we compare these two societies in terms of the proportion of multidimensionally deprived population with a multidimensional index that provides greater deprivation recording possibility among young cohorts, although the relative deprivation by age range population groups

[3]Following Fleurbaey & Schokkaert (2009), to catalogue differences in health outcomes as legitimate and illegitimate, differences in achievement levels (such as health or educational attainment) are seen as caused by myriad factors, some of which can be catalogued as producing fair differences and others as producing unfair differences. In particular, for the case of health and healthcare inequalities, Fleurbaey & Schokkaert (2009) defined as legitimate or fair those differences attributed to causes that fall under individuals' responsibility. Legitimate differences in this context therefore correspond to those derived from preferences.

in the two societies is equivalent, the societal evaluation will reveal a greater incidence of multidimensional deprivation in the society that has a greater proportion of young people.

Because the ultimate goal of multidimensional deprivation measurement is to capture unfair disadvantages, we can assert that multidimensional deprivation incidence comparisons that reflect not only relative deprivation differences but also differences in the demographic structure of the populations to be compared are confounded by these demographic group differences in composition.

In light of this, the natural approach to providing meaningful comparisons could be either to compare specific rates for demographic homogeneous subgroups or to make the comparison under either the same population demographic structure or the same subgroup-specific incidence rates. These two latter approaches correspond to direct and indirect standardisation procedures, respectively, and are the methodological approach proposed in this chapter to enhance societal multidimensional comparisons.

Even though standardisation procedures have been developed and continuously applied by health and demographic scholars since 1844 (Neison 1844, Wolfenden 1923, Yerushalmy 1951, Anderson et al. 1998, Feinleib 1992, Naing 2000, Ahmad et al. 2001, Schokkaert & Van de Voorde 2009, O'Connell et al. 2011), current cross-country comparisons of multidimensional indices in the literature do not use standardisation procedures to produce accurate comparisons across contexts with dissimilar population distributions by demographic factors such as age or household size. This is the case for the aforementioned global MPI and CMPI or the proposal for a Latin American MPI that aims to cover 30 countries in total (Santos 2014).

To assess the presence of standardisation techniques in the multidimensional literature, we conducted a systematic search. Indeed, we used the specific keywords 'multidimensional' and 'standardisation' to look for standardisation efforts in English-written, peer-reviewed articles, books, and reports from governmental and international organisations. Our search covered articles published since 2000 that addressed social well-being in low and middle income countries. Our search spanned the full-text sources across several electronic databases (EconBase-Elsevier, Ingenta, Social Science Research Network, ProQuest, Encore, Jstor, Science Direct, Springerlink) as well as the Web page of the United Nations Development Program and the Oxford Poverty and Human Initiative. We found that standardisation efforts have been suggested only by Mazumdar (2003) to enable consistent cross-country comparisons across time for the HDI. In particular, Mazumdar (2003) studied the effect of the specification of the Human Development Index on the relative position of each country within the rank of all studied countries and proposed a variation on its measurement methodology.[4]

In contrast with Mazumdar (2003), we propose the application of standardisation procedures for multidimensional deprivation indices that do not alter their initial configuration. In other words, our proposed methods enable meaningful comparisons of societies with dissimilar demographic population distributions across policy multidimensional indices that use the up-to-date measurement technologies available in the literature.

Within the multidimensional literature, we are the first to point out this comparability problem and to propose the use of standardised indices to address it. Our standardised figures are not yet meant to replace crude rates but rather to provide a useful technique for application in cases where societal comparisons are required. The use of our methods will produce demographically standardised comparable indicators that are easily interpretable and applicable in the policy context.

[4]The first version of the HDI evaluated the mean of each of the four indicators taken into account for each country (life expectancy at birth, mean years of schooling, expected years of schooling, and gross national per capita income) and used the minimum values and maximum values found across countries to describe the position of each country within the distribution of all evaluated countries by year. The second version of the HDI used yearly minimums and maximums for each assessed indicator. Mazumdar (2003) demonstrated that using yearly minimums and maximums for each indicator produced inconsistent results for comparisons across time. The scholar also showed that the changing behaviour of living standards across time yields inconsistent results when using fixed minimums and maximums across time and countries. Mazumdar (2003) proposed a bridging approach between the moving and the fixed minimums and maximums method.

2.2 Multidimensional poverty: Background

Multidimensional indices incorporate several dimensions into a single indicator to assess well-being across societies. This section provides background information on multidimensional indices of poverty. Later, we discuss the application of our proposed methodology to the multidimensional indices described in this section.

Within the multidimensional literature, there are two alternative procedures for aggregating dimensions and identifying the deprived population: the *welfare approach* and the *counting approach*. The welfare approach combines several dimensions into a single index through an additive process. Under this approach, a threshold to be applied over the index is created to differentiate between poor and non-poor populations. A particular welfare multidimensional index can be understood as a weighted mean of achievements. This approach has been developed by Bourguignon & Chakravarty (2002), Bourguignon & Chakravarty (2003), Seth (2009) and Seth (2013), among others.

By contrast, the counting approach, as its name suggests, counts the number of dimensions in which people suffer deprivation. The identification of the multidimensionally deprived population is achieved by defining how many dimensions of deprivation a person should exhibit to be categorised as multidimensionally deprived. The most well-known counting methodology corresponds to the one proposed by Alkire & Foster (2011). Henceforth, we abbreviate the Alkire & Foster (2011) methodology to AF methodology or simply AF.

Efforts have been made, within literature, to analyse both approaches (welfare and counting) under a common framework as Atkinson (2003) attempts. However, as pointed out by Aaberge & Brandolini (2014), this discussion is still inconclusive. We use counting-based multidimensional measures for our analysis, as this approach is the most straightforward applicable when it comes to distributions of ordinal indicators. This type of measures provides informative figures in the policy context. Still, our proposed standardisation procedures could be similarly applied to welfare multidimensional indices that are expressed as population rates.

Table 2.1, below, contains an example of a counting multidimensional index. We use this particular example throughout this chapter to illustrate our argument.

The first column of Table 2.1 lists each considered well-being dimension. The second column lists the indicators used to capture each dimension. Column 3 indicates the population subgroup where the indicator in column 2 is relevant / applicable. Lastly, the fourth column describes the criteria used to identify as deprived or not deprived a person from the applicable population in terms of the column 2 indicator. For instance, school attendance is used to capture the education dimension. This indicator is applicable for the population aged between 5 and 19 years old. Using this indicator, a person is identified as deprived in this dimension if she or he is not attending school.

Applying the AF methodology to our example, we firstly identify the population suffering deprivation in each indicator and configure, therefore, four deprivation indicators. Following this, we calculate a weighted sum of the four considered deprivation indicators to produce a single metric, C. As such, C, hence, denotes the weighted sum of experienced deprivations. The relative importance within C of each deprivation is set by the weights included within brackets in the Table 2.1; the sum of them is equal to 1. Lastly, we identify as multidimensionally deprived anyone whose C exceeds a k-threshold. The k-threshold corresponds, consequently, to the weighted sum of total possible deprivations that are considered as sufficient to identify a person as multidimensionally deprived.

The selection of the k-threshold and the weights, in general, is devised by each of the multidimensional indices. For instance, the global CMPI uses as multidimensional threshold $k = 33\%$ of its total weighted sum of deprivations (Alkire et al. 2014), where the weights correspond to a nested structure of equal dimensional weights and within dimension equal weight per indicator. Like the global MPI, the Colombian MPI uses a nested structure of equal weights and a k-threshold of 33%, while also implementing robustness checks thought k-dominance analysis for a broader set of k-thresholds (Angulo et al. 2016). For a discussion of different alternative procedures to set weights in a multidimensional index see Decancq & Lugo (2013).

If we define, as the AF suggests, a multidimensionally deprived any person that has at least a k weighted sum of deprivations; where k is a threshold that allows to identify multidimensional deprivation; then, the multidimensional deprivation status of any given person i can be characterized by a dichotomous

Table 2.1: Example of multidimensional index: Dimensions, indicators, weights, applicable population and deprivation criteria

Well-being dimension (1)	Indicator (2)	Population subgroup where the indicator is applicable (3)	A person from the applicable population subgroup is deprived if: (4)
Health	Antenatal care [0.33]	Under 5 years old children	Her/his mother did not get access to at least four antenatal care appointments when she was pregnant with her/him.
Education	School attendance [0.17]	5 - 19 years old population	Is not attending school.
	Educational achievement [0.17]	15 - 24 years old population	Has not completed at least 9 years of education.
Dwelling conditions	Material of floor, type of toilet and access to water [0.33]	Any person	Lacks from flooring different from earth or sand or has no adequate toilet** or lacks access to adequate water supply***

Notes: Numbers within square brackets below the label of each indicator denote the relative importance (weight) assigned to this particular indicator within the overall indicator; the sum of them equals 1. **Adequate toilet: Toilet different from: pit latrine without slab/open lit, composting toilet, bucket toilet, hanging toilet/latrine, no facility/bush/field or other. Or a shared improved sanitation (flush toilet or improved/ventilated latrine). ***Adequate water: Water different from unprotected well or spring, tanker truck, cart with small tank, surface water or other. If the source is improved it must be within 30 minutes walking distance. If the source is bottled water, it is considered deprived only if this is also the source of non-drinking water.

indicator, p_i, that takes values of one when the individual is multidimensionally deprived and zero otherwise. As an example, in our multidimensional index, a 16 year-old child can be possibly accounted as dimensionally deprived in school attendance, educational achievement and dwelling conditions. In case this child is not attending school or has not completed at least 9 years of education, and is living in housing that lacks flooring different from earth or sand, or has no adequate toilet, or lacks access to an adequate water supply, then this child is deprived in the three dimensions where he or she can possibly score. Now, using the proposed system of weights, this child scores a C-weighted sum of deprivations of 0.67. If we use the AF identification procedure and set the k threshold of multidimensional deprivation at 40%, this means that this child result in be categorized as multidimensionally deprived and his/her p_i indicator takes the value of one.

As a result, for a society made of N individuals the headcount ratio or multidimensional deprivation incidence rate is expressed as: $H = Q/N$, where Q is the total number of multidimensionally deprived people and can be expressed as $Q = \sum_1^N p_i$.

Here in this chapter, for illustrative purposes, we apply our proposed standardisation method specifically over this H-headcount ratio, using a k-threshold of 40%. However, our methods can also be applied to any multidimensional index expressed as a population ratio, and able to produce the same societal rate regardless of whether they are based on either individual or sub-population grouped data i.e., a decomposable index[5]. Absence of decomposability do not enable societal rates to be derived from group specific sub-population figures. In those cases, our proposed direct or indirect standardisation method cannot be applied.

2.3 The comparability problem

In this section, we analyse the comparability problem that demographic confounding factors, such as age, gender or household size, pose over multidimensional indices of deprivation. We first discuss this comparability problem in relation to measures of multidimensional deprivation that use individuals as their unit of analysis. Then, we extend the analysis to household-based measures.

2.3.1 Individual-based measures

When it comes to literature, multidimensional deprivation is traditionally measured considering the individuals as anonymous in the measurement process. Anonymity, in this case, implies that no single individual or group of individuals has greater emphasis over the measure. This axiom warrant societal metrics being not altered by any rearrangement of the population. Thus, the measures are not meant

[5] A index is said to be decomposable if it can be expressed as a weighted average of subgroups estimates, where weights are subgroup population shares. The poverty measurement literature refers to this property as 'decomposability' (Foster et al. 1984, Tsui 1999, Alkire & Foster 2011) or 'subgroup decomposability' (Bourguignon & Chakravarty 2003)

to be sensitive to re-shuffles of the population. This is the case of multidimensional indices families such as the ones proposed by Tsui (2002), Bourguignon & Chakravarty (2003), Alkire & Foster (2011) and Seth (2013).

However, considering the individuals as anonymous is seen as problematic in the case of multidimensional indices of deprivation that do not account for differences in needs across demographic sub-population groups.

In particular, different needs of demographic sub-population groups imply that while, for instance, a child can be defined as deprived of education because she or he does not attend basic education, a person older than 20 years can be defined as deprived in such a dimension not because she or he does not attend school, but rather because she or he does not know how to read and write for example. In terms of the occupation dimension, while a person older than 18 years can be defined as deprived if she or he is seeking a job but does not have access to one, in contrast, a child younger than 11 years can be defined as deprived if she or he is forced to work. These are two examples of the heterogeneous needs across different demographic sub-population groups.

Multidimensional indices of deprivation portray these heterogeneous needs by defining each indicator of interest as relevant to be measured in a particular applicable population subgroup. For instance, take our example of the multidimensional measurement of deprivation. Indeed, we can see that each of the four considered indicators is relevant to a specific sub-population group (see Table 2.1, column 3). School attendance is relevant for those in the population who are 5 to 19 years old, while antenatal care is relevant for children born during the last five years. Hence, the applicable population for each indicator varies by age, while there are population subgroups that are accounted as deprived within it and those that are not.

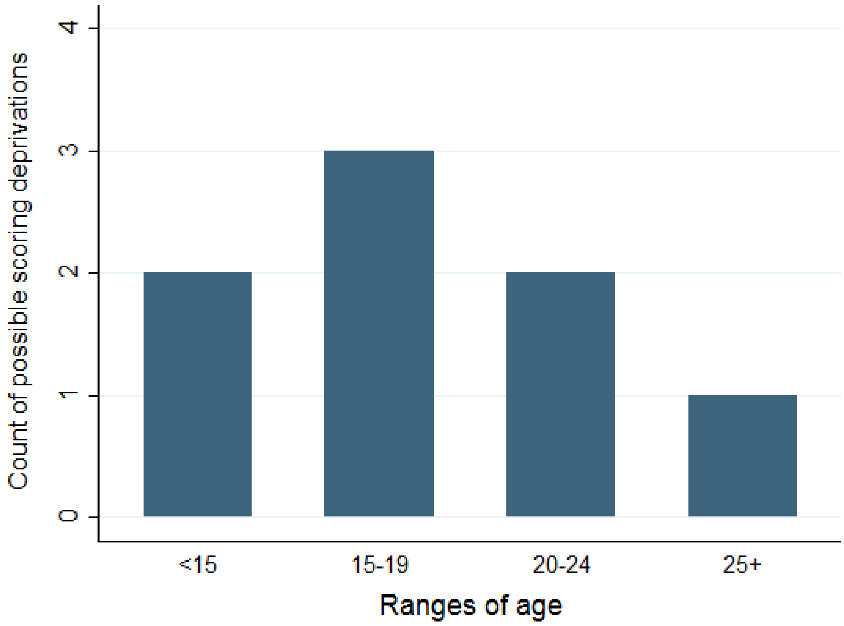
The fact that each of the used indicators is informative when it is evaluated over a specific sub-population groups is admittedly a feature of most of the public policy indicators. For instance, one of the targets of the millennium development goals, launched by the United Nations Development Program, is the reduction of the under-five years old mortality rate by two thirds, between 1990 and 2015. Another target is to achieve universal access to reproductive health for all pregnant women, by 2015. This illustrates that most of the policy indicators are tracked over specific sub-population groups, the first over children under-five years old and the second over pregnant women.

In consequence, since a multidimensional measure combines together indicators, applicable in different sub-population groups, by design the multidimensional index allows different scoring possibilities according to sub-population groups. The combination of indicators that a multidimensional measure takes into account, together with their applicable populations, yields a total number of deprivations that an individual is accounted for. This total number of deprivations (i.e., the scoring possibility of individuals) varies by demographic sub-population group. As such, the populations that exhibit the larger scoring possibility within the multidimensional index can be said that the measure considers them as more important than others that have lower scoring possibility. Figure 2.1 displays the total number of deprivations that each population subgroup, by age range, is accounted for in our example of a multidimensional index. While the 25 years old and older population can score a maximum of one deprivation, out of the four considered indicators, the population from 15 to 19 years old can score up to three deprivations. In general, our measure produces a greater scoring possibility among populations younger than 25 years.

Different scoring possibilities by sub-population groups is a characteristic of most of the multidimensional measures currently in use. Example of this are the Colombian Multidimensional Poverty Index (Angulo et al. 2016) or the International MPI (Alkire et al. 2014). In the case of the CMPI, while an adult person can score up to 11 possible deprivations, a 15- years old youth is able to score up to 14 possible deprivations and a 5- years old child or younger can score up to 7 possible deprivations.

Although current multidimensional methodologies characterize individuals as anonymous in the measurement process, the design features of the currently used applications of them create structural differences of the possibility of a given individual of being multidimensionally deprived across age ranges. As such, if you switch one person from one demographic sub-population group to another, the societal measure will not remain unchanged. As an example, if in our Table 2.1 indicator we switch a 14-year old child to the 15 to 19-year old population, since

Figure 2.1: Count of possible scoring deprivations across age ranges in our example of multidimensional index



this latter group has the possibility of scoring a greater number of deprivations, then the societal measure does not remain unaltered.

This feature resembles that societal multidimensional metrics are sensitive to demographic re-arrangements in the population. It implies that some sub-population groups have greater importance than others in the multidimensional measurement process. We term this empirical regularity as 'demographic sensitivity'. Indeed, below we formalise this concept for the decomposable multidimensional deprivation societal H measure introduced in Section 3.2.

Demographic sensitivity. Consider a society made of $i = 1, 2, \ldots, N$ individuals and $j = 1, 2, \ldots, J$ different demographic sub-population groups. For example, take two gender groups (female and male), and two age groups (population younger than 25 years old and population 25 years old and older). Then, we obviously have that $J = 4$. Each i-individual belongs to one subgroup exclusively. The proportion of individuals belonging to each j-demographic sub-population group is denoted by s_j, where the sum of them across society adds to one, which for our example is: $s_1 + s_2 + s_3 + s_4 = 1$. The population size of each subgroup is denoted by n_j and the sum of n_j across each groups satisfies $N = \sum_{j=1}^{J} n_j$. Because H is a decomposable measure, it can be expressed as: $H = \sum_{j=1}^{J} s_j h_j$, where h_j is the subgroup specific headcount ratio ($h_j = \sum_i p_i/n_j$). In case of a rearrangement of the population distribution such that $s'_j > s_j$, as for example ageing of the population, the share of the elderly population increases and the share of young cohorts decreases. Whenever this rearrangement of the population distribution produces a change in the societal measurement, i.e. $H' \neq H$, we then say that the H societal measure is demographically sensitive. This change in H can occur even though the subgroup specific headcount ratios remained unaltered: $h'_j = h_j, \ \forall j = 1, 2, \ldots, J$.

Intuitively, demographic sensitivity is observed when the person's scoring possibility varies structurally across sub-population groups and therefore the evaluated deprivation headcount differ across these specific population subgroups. Then, demographic sensitivity relies on the fact that for any h_j and h_l-subgroup specific headcount ratio, we have $h_j \neq h_l$ for any $j, l \in J$ and $l \neq j$.

As a result of this empirical regularity, the anonymity axiom becomes undesirable. On the contrary, we would like comparisons of multidimensional deprivation across societies to not reflect these differences in the distribution of the population but only differences in multidimensional deprivation incidence across homogeneous population subgroups.

We follow by discussing the case of household-based measures below.

2.3.2 Household-based measures

If we change the unit of analysis for the index in Table 2.1 from individuals to households, then we identify as deprived in each indicator any household with at least one deprived household member in such a condition. Under this situation, the smaller the household's size, the lower the number of possible deprivations to score within the multidimensional metric. Then, similar to the individual-based case, the headcount ratio varies structurally across sub-population groups, in this case across household size.

Moreover, take two groups of differing household sizes: households with less than five persons and households with five or more persons. Next, compare a particular society at two different moments in time. In between these time periods, a rearrangement in the population by household size occurs. For instance, a demographic transition lowers the average household size. As a consequence of this change, the proportion of households with five or more persons decreases and the proportion of households with less than five persons increases. Even though the mean deprivation headcount of each of these two groups of households could have remained unaltered, this demographic change results in a decrease of the societal headcount evaluation of our example of index. Then, we can assert that H in this case result in being sensitive to rearrangements in the distribution of the population by household size, and therefore demographically sensitive.

Furthermore, the possibility of a household in household-based multidimensional measures being multidimensionally deprived varies not only through household size but also by household composition. For instance, take our multidimensional index across two households, both made up of three persons: household A consists of one child under 15 years old and two adults older than 25 years, while household B is composed of three persons between 20 and 24 years old. The structure of our multidimensional index example produces that the household A can possibly score up to three deprived dimensions and household B only up to two deprived dimensions; this even both households have the same size.

We follow describing possible sources of demographic sensitivity, either for individual-based or household-based measures.

2.3.3 Sources of demographic sensitivity

Whenever societal measures aggregate units (either individuals or households) with heterogeneous needs, then these diverse needs lead to differences, by demographic sub-population group, in the incidence of the considered deprivation and in the multidimensional assessment. Societal measures that may not hold demographic sensitivity are multidimensional measures, whereby any person, irrespective of his or her demographic characteristics can always possibly score the total number of possible deprivations. In other words, multidimensional indices designed to evaluate deprivation on specific needs homogeneous demographic sub-populations, such as children or women, could register the same possible scoring possibilities across demographic groups. This is the case, for instance of, the child multidimensional deprivation index used by Roche (2013) or the women multidimensional index implemented by Alkire et al. (2013). Hence, they might not be demographically sensitive. This is because all of the considered indicators within those indices refer to the same demographic population group, and therefore, all the considered children or women have the same deprivation scoring possibility.

However, worth bear in mind that demographic sensitivity might be also present even in the one-dimensional space and within the sub-population where the indicator is defined as relevant. For instance, school attendance, similar to the mortality rate example from the introduction, varies by age, even within school aged children (children between 5 and 18 years old). As older cohorts are more prone to enter the labour market rather than remain within the formal education system, school attendance decreases with age. This trend is present in both developing and developed countries (Barro & Lee 2001). Another example is labour market indicators, which have demonstrated important differences by sex and age within the working age population. Hence, it is not necessarily true that all multidimensional indices that focus on evaluating deprivation on specific homogeneous sub-population groups are non-demographically sensitive. Typically, most of the indicators included in a multidimensional measure could be said are demographically sensitive, even within the sub-population where the measuring of them is relevant.

As a result of the inclusion of demographically sensitive indicators, most of the multidimensional indices might be considered demographically sensitive. Whenever at least one of the indicators considered within the multidimensional metric can be catalogued as such, demographic sensitivity might also be present in the joint distribution of the multiple dimensions considered in the multidimensional metric.

In presence of demographic sensitivity and given that decomposable multidimensional indices evaluated at the society level add units across demographic sub-population groups, where each sub-population group is *weighted* by its relative size within the society, comparisons between societies on a given multidimensional index indicate not only their different relative deprivation, but also their dissimilar distribution of the population by demographic factors. If the purpose of the analysis is to compare relative deprivation across societies, demography in these cases confounds the deprivation comparison. We observe, therefore, a comparability problem.

One possible way to address this comparability problem is to assess several group specific rates. However, when there is more than two sub-population groups and several societies, such assessment could become cumbersome. A solution to this is found in the statistical, demographical and epidemiological literature. Examples of them can be found in the following studies: Wolfenden (1923), Feinleib (1992); Neison (1844), Yerushalmy (1951), Anderson et al. (1998), Naing (2000), O'Connell et al. (2011), Schokkaert & Van de Voorde (2009). These scholars use direct and indirect standardisation methods to tackle a similar comparability problems for the case of death rates. Likewise, the approach that we propose in this chapter is to apply direct or indirect standardisation techniques to address this comparability problem. We now describe this proposed approach in Section 2.4 below, following which we empirically illustrate, in Section 2.5, the effect that demographic factors, such as household size and age, have when comparing multidimensional deprivation across societies with different population distributions.

2.4 Multidimensional standardised comparisons

Standardisation procedures date back to the nineteenth century. In 1984, Neison proposed the use of direct and indirect standardisation to accurately compare death rates across Great Britain's districts. Neison advocated this method in response to a previously proposed method that urged for these rates to be calculated as population average age at death. Neison (1844) observed the problem of age-driven indicators and their use in comparisons across societies with marked difference in the distribution of their populations. To resolve the issue, he employed opposite and alternative methods. The first method, later called direct standardisation, involves calculating each district's death rate by using the age-specific death rates but assuming the population distribution of another different district. The second alternative proposed by Neison was to apply to each community the age specific death rates of the standard population, while using the specific population distribution of the communities to assist comparison across districts. This procedure was later termed indirect standardisation.

The first method, direct standardisation, provides the rate that would have been observed if the societies to be compared had the same standardising factor structure or weighting system as discussed on page 27. Age-direct-standardised death rates depict the death rate that would have been observed if the societies under comparison had had the standard population age distribution.

While direct methods require the use of the specific sub-populations rates, which are not known in every case, indirect standardisation provides the rate that would be observed if each society had its own age distribution but the same incidence as the standard population.

These two methods allow for comparisons of societal rates without confounding the provided ranking with the different weighting systems imposed by the structures of the population. Direct and indirect standardization methods, as a matter of fact, are still widely used by the epidemiological literature. Examples include studies such as: Doak et al. (2012), Fellman & Eriksson (2013), Chen et al. (2012), Jones-Smith et al. (2011), and Yang et al. (2011). All of these studies use standardisation techniques to perform more accurate comparisons across demographically differently structured population groups. We propose the use of these two standardisation procedures to produce meaningful societal multidimensional deprivation incidence comparisons.

In fact, when the purpose of the analysis is to compare two different societies in terms of deprivation, two necessary conditions enforce the use of standardisation procedures. First, the indicator to be compared across societies is demographically sensitive to confounding factors, such as age, sex or households size. Second, the societies to be compared have different distributions of population by those confounding factors. Only when both conditions are true at the same time do we advocate the use of standardisation procedures. Still, since most of the indicators used for policy purposes are demographically sensitive, absence of the first condition is highly unrealistic. On the other hand, for the second condition, if the demographic distributions across societies are not significantly different, demographic standardisation loses its purpose.

Worth noting, however, that multidimensional metrics, as any other socioeconomic indicator, can be associated with socio demographic characteristics not necessarily related with the multidimensional measurement process, but indeed, the societal measure results in being sensitive to them. For instance, consider the h_j headcount ratio of multidimensional deprivation for a j sub-population group using the index example, as significantly higher than the h_l for a sub-population group l. In this case j and l are income poor and non-poor households, respectively. We also know that $h_j > h_l$. Take now society A and B, such that society A has greater proportion of income poor population than society B, and the distribution of the population across both societies differ.

If we compare the distribution of the population of Society A and Society B by income groups, these two distributions of the population differ. Also, the proportion of multidimensionally deprived households by income poor and non-poor group differ. These are the two necessary conditions that make meaningful the use of the type of standardization methods that we propose in this chapter. However, income groups shall not be understood as a demographic confounding factor to which for standardize multidimensional deprivation incidence rates. This is due to the fact that the multidimensional index example is not meant to be, by design, non-sensitive to changes across the distribution of the population by income groups. The application of other types of analytical methods, such as selection of observable techniques, could produce more plausible results from which to infer conclusions in this example.

In consequence, we highlight that only when multidimensional indices result to be sensitive to demographic characteristics involved in the measurement process, or characteristics that depict heterogeneous needs across sub-population groups with regards to the indicators included within the index, our proposed standardisation techniques are meant to enhance societal comparability. Whenever the measurement process fails to produce societal figures non-sensitive to changes in the structure of the population across these demographic confounding factors, the proposed methodology of this chapter is suitable to be implemented.

On the other hand, and as discussed by O'Donnell et al. (2007), standardisation methods are either not meant to provide causal inference, they still remain within the descriptive analysis. Regardless, they definitely furnish more accurate societal rankings.

We continue, in this section, first describing the *direct* standardisation procedure that we propose, following which we present the proposed analogous *indirect* standardization method.

2.4.1 Direct multidimensional standardisation

In this section we describe how a direct standardisation procedure applies to the H-headcount ratio of the AF family of counting multidimensional measures. As previously mentioned, we opt to exemplify our proposed procedures for the H-headcount ratio, although this can be applied to all members of the family of measures.

For the purpose of the direct standardisation procedure, we use the fact that H is decomposable and h_j is the headcount ratio of the j specific demographic population subgroup, such that $h_j = q_j/n_j$, where q_j is the number of multidimensional deprived population in the subgroup j and n_j the population size of each subgroup such that $N = \sum_{j=1}^{J} n_j$.

There is necessary as many J as subgroups made according to the standardising factor(s) deemed to be important. For instance, considering age as our standardising factor, we can define the J groups from the standard population as five years cohorts where the first group comprises 5- year olds, and the second group comprises 5-10-year olds, and so forth. Moreover, in the case of our multidimensional index example, one could use the applicable population of each of the indicators and build the relevant group ages: 0-5 years old, 5-14 years old, 15-19, 20-24 and 25 years old and older.

Using the selected J sub-population groups, the direct standardised headcount ratio (DSH) is expressed as:

$$DSH = \sum_{j=1}^{J} s_j^* \frac{q_j}{n_j}, \tag{2.1}$$

where s_j^* corresponds to the share of the j sub-population group in the standard population. *DSH* provides information about the multidimensional incidence that a society would have experienced had it had its own h_j subgroup specific headcount ratios but the population distribution of the standard population. We say that *DSH* is the demographically standardised comparable version of the H headcount ratio.

There are two important characteristics of this technique that are worth mentioning. First, the subgroup specific rates before and after standardisation reflect the same relative incidence to which the sub-population group is exposed. Secondly, using a procedure like this to compare two societies means that the pre-existing relations across societies for the sub-population group rates are held by the standardised rates. According to Wolfenden (1923) and Yerushalmy (1951), it is found, by way of a direct standardisation procedure, that these two characteristics must be held because they ensure that the relative incidence of arguably homogeneous and comparable groups remains unaltered and comparable.

Our proposed direct multidimensional standardisation procedure does ensure that the relative incidence across sub-population groups and across societies is identical before and after standardisation. Indeed, to obtain direct standardised figures, we only propose to evaluate the societal incidence by altering the set of subgroup population shares $s_j \forall J$. We keep unadjusted both the pre-existent relation within sub-population groups incidence and the pre-existent relation across societies in terms of the incidence of each sub-population group. In consequence, a direct multidimensional standardisation method produces consistent results across sub-population groups and societies.

Nonetheless, standardised indices values do not have inherent meaning alone, they do not replace the actual rate value, nor report the state of each society. Standardised indices provide the relative deprivation state across societies in demographically comparative bases. Wolfenden (1923) is among the first scholars to have pointed out the absence of inherent meaning among standardised rates. More recently, Anderson et al. (1998) also discusses this issue while evaluating the implementation of new global age direct standardised death rates.

The second possible course of action is to use, rather than a direct multidimensional standardisation method, an indirect multidimensional standardisation method. This second proposed standardisation technique is described below.

2.4.2 Indirect multidimensional standardisation

Indirect standardisation, in the absence of these subgroup specific rates, uses the factor specific incidence of the standard population, which must be known. This form of standardisation is more rarely used than direct standardisation.

Following the notation of Anderson et al. (1998), for indirect standardised mortality rates, we express our Indirect Standardised Multidimensional Headcount Ratio (*ISH*) as follows:

$$\mathit{ISH} = \frac{Q}{\sum_{j=1}^{J} h_j^* n_j}. \tag{2.2}$$

The numerator in Equation (2.2) indicates simply how spread multidimensional deprivation is in the observed society. The denominator, on the other hand, corresponds to the expected multidimensional count of persons if the subgroup specific incidence rates of the standard population prevail over the observed society. This standardisation method requires from the observed society, in addition to its population distribution, either the Q count of multidimensional deprived people or alternatively its H headcount ratio and N population size ($Q = HN$).

In the next section, we assess the empirical effects of the proposed direct standardisation technique on societal multidimensional deprivation incidence comparisons. In the interest of brevity, we restrict the empirical analysis of the next section to the assessment of the proposed direct standardisation method. Nonetheless, the indirect standardisation tool can be similarly analysed. Section 2.6 in this chapter discusses the implications of selecting either the direct or the indirect standardisation tool. In this same section 2.6, straightforward criteria to apply when choosing the standard population are discussed as well.

2.5 Assessing the empirical effects of direct standardisation

To assess the effect of the proposed direct standardisation technique of this chapter, this section compares the results of unstandardised multidimensional comparisons with regards to their standardised results. In particular, we analyse the ranking and relative size differences obtained when comparing multidimensional deprivation incidence across societies with different structure of the population. This analysis is performed through unstandardised and standardized comparisons. This section provides four standardisation analytical examples. First, we use a household-based measure to compare the unstandardised rates with the household size standardised version of them. Second, using an individual-based multidimensional measure we compare results standardising by ranges of age. Third, we use a household-based measure to illustrate a standardisation procedure that uses two standardising characteristics: household-size and age. Fourth, this section finalizes by providing an age-standardization example for time comparisons.

2.5.1 Data

The empirical analysis of this chapter uses a cross-country micro data setting based on the Demographic Household Surveys (DHS), developed by Measure DHS (DHS 2014). The DHS are repeated cross sectional household-based surveys, run since 1986 and usually carried out every five years. These international comparable surveys include more than 260 surveys of 90 countries from the developing world. Since its creation, the data sets have been available for public use.

These surveys include household and individual data for several socioeconomic topics such as child health, education, family planning, fertility, gender/ domestic violence, HIV prevalence and maternal health, among others. The majority of the topics included in the DHS questionnaires are comparable across countries, still, country-specific data are also included in the DHS.

We use DHS data from the Maldives 2009, Ukraine 2007, Jordan 2007, Dominican Republic 2007, Armenia 2005 and Armenia 2010. We selected these countries because they have comparable levels of development, but possess dissimilar distribution of the population by ranges of age and household size. According to the 2014 UNDP global Multidimensional Poverty Index, all of the countries in our analysis have multidimensional deprivation incidence lower than 10%. Also, across the five countries, the proportion of population aged 25 years or older ranges from 40.0% to 73.2%, with Jordan representing the smallest proportion and Ukraine the largest proportion of this population.

These six cross-sectional DHS surveys use a probabilistic sample design. The sampling frame used by each survey was the population and housing census of the correspondent country. The Armenia 2010 and Armenia 2005 DHS surveys use a census master sample as a sample frame for enumeration areas. In terms of sample size, all six DHS surveys have large sample sizes (between 7,500 and 35,700 households) and survey designs that enable nationally representative estimates for a variety of indicators. In addition, the surveys also allow estimations across urban and rural areas and other different geographical aggregations depending on the country. Specifically, among the six surveys, that with the lowest sample size is the Maldives 2009 DHS, which was collected over 7,515 households. The Maldives 2009 DHS' sample produces representative results for urban and rural areas, six geographical areas and each of the 21 subsequent country geographical regions, known as *atoll*. On the other hand, the largest among the six surveys is the Dominican Republic 2007 DHS with 35,700 total selected households; this sample allows for the disaggregation of country figures into 32 geographical areas (known as *provincias*).

2.5.2 Example: direct standardisation on household-size

As a first example, in this section, we demonstrate the utility of our proposed direct standardisation procedure to compare household-based multidimensional indices of deprivation across societies with different population structures by household size.

For such an endeavour, we compare the Maldives in 2009 and Ukraine in 2007. These two countries have similar multidimensional deprivation incidence, as measured by the 2014 UNDP global MPI.[6] The two countries exhibit dissimilar population distributions by household size.[7]

Using the DHS micro-data from the two countries, we first compare their population distribution by groups of household size. Figure 2.2.a plots these results. As expected, a dissimilar population distribution by household size is observed between the two countries. While 60.7% of Ukraine's population lives in a household comprising three or less persons, less than 7% of the Maldives population lives in a household of one to three persons. In the Maldives, a higher percentage of the population lives in larger households, 61.1% of the Maldives population belongs to households with seven or more persons, whereas in Ukraine, this subgroup represents less than 3.0% of the total population.

Subsequently, we compare the proportion of the multidimensionally deprived population for both countries across household size group using the household-based example of the multidimensional index introduced in Section 3.2 and Table 2.1. Figure 2.2.b displays these results. For the sake of simplicity, we use a k-threshold of 40%. However, similar results can be observed using other k-thresholds. The light bars in the figure correspond to the Maldives, while the dark bars correspond to Ukraine.

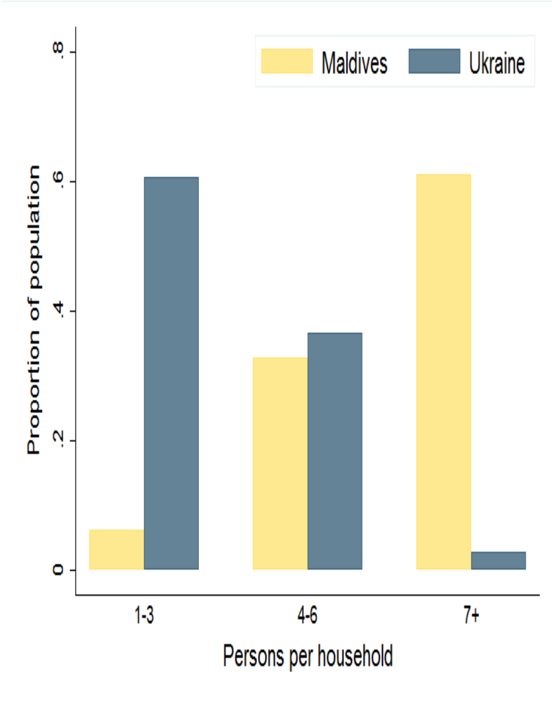
The figure shows that the mean proportion of the multidimensionally deprived population is larger among larger households and smaller among smaller households in both countries. The multidimensional deprivation incidence increases as household size increases. For instance, 25.4% of the population that belongs to households with seven or more persons in Ukraine is living under multidimensional deprivation conditions. In the Ukraine, however, when the household size decreases to one to three persons, this proportion is only 1.4%.

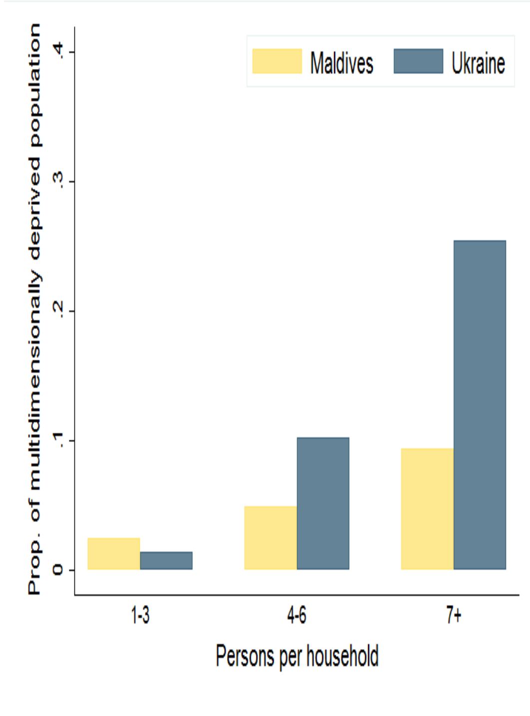
In addition, we observe that the only subgroup-specific h-headcount ratio that is greater in the Maldives than in Ukraine is exhibited among the population that lives in households with three or fewer persons (Figure 2.2.b). While this

[6]According to the 2014 UNDP global Multidimensional Poverty Index, in 2009, 5.2% of the Maldives population lived under multidimensional poor conditions, while in 2007, 2.2% of Ukraine's population lived under the same conditions.

[7]According to DHS micro-data, while 45% of the Maldives population is 25 years old or older, in Ukraine, this proportion is 73%.

Figure 2.2: H-headcount ratio across groups of household size: Maldives and Ukraine Example





Source: Author's calculations based on DHS micro-data. Note: Point estimates developed using the weighting system provided by Measure DHS. H-headcount ratio estimates developed using a k-threshold of 40%.

population subgroup represents in the Maldives only 5.9% of the population, in Ukraine, it represents about 60% of the population.

When it comes to societal incidence of multidimensional deprivation, 5.6% of the population in Ukraine is catalogued as multidimensionally deprived and 7.5% in the Maldives. Although Ukraine exhibits a greater incidence of multidimensional deprivation in two out of the three analysed sub-population groups, Maldives is the country with a larger societal H headcount. This result is driven by the fact that the greatest subgroup-specific deprivation incidence rate is found in both countries across larger households. But, the country with the largest proportion of the population living in households consisting of seven or more persons dominates the results. Household size arises, therefore, as a demographic confounding factor.

Worth remarking here that household size arises as a confounding factor because the subgroup specific h-headcount ratio varies structurally across household size and the structure of both populations also vary systematically by household size. Practitioners might find context-specific situations where multidimensional deprivation incidence does not necessarily increases as household size increases, but in contrast other not necessarily linear shapes are observed. For instance, an U shape which reflects higher multidimensional deprivation incidence among small and large households and lower multidimensional deprivation incidence among households with an average size. Another example can be tick shapes. In any of these alternative contexts, still the method proposed in this chapter results of relevance because there exists a systematic variation of multidimensional deprivation incidence across household size and also the population of the societies to be compared have different structures by household size.

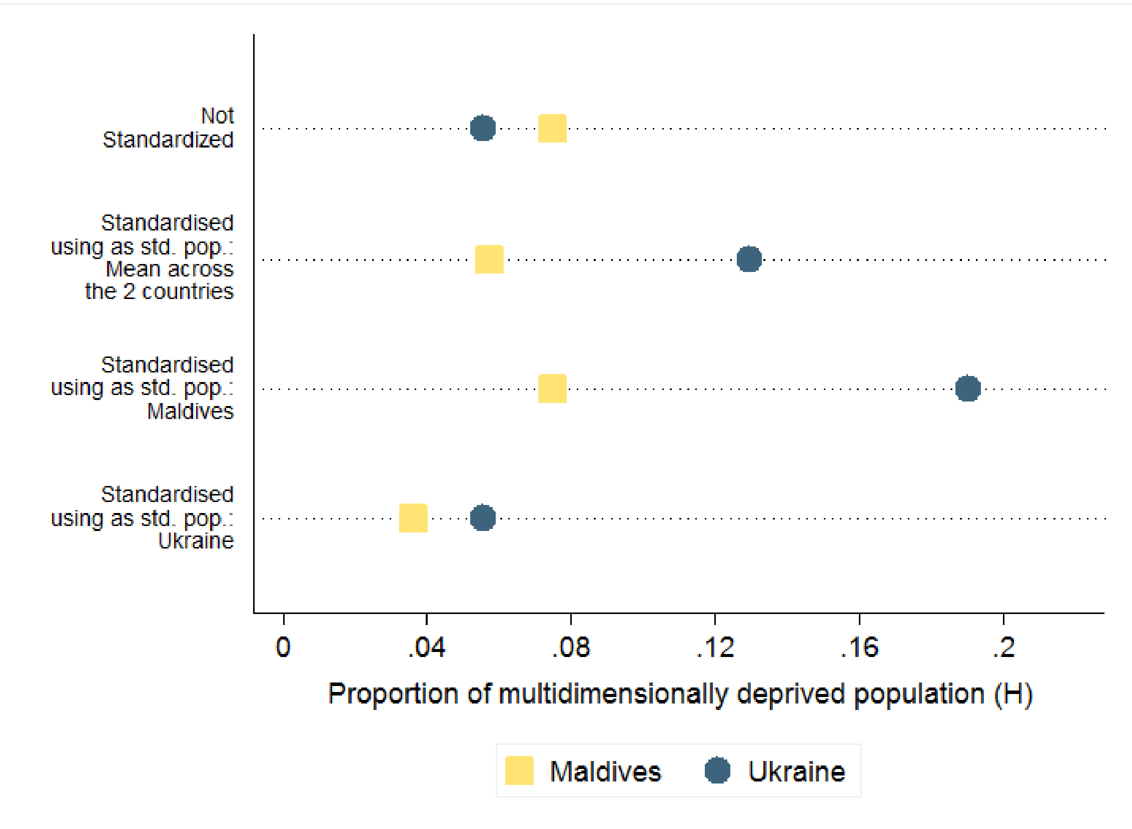
To tackle this comparability problem, we continue illustrating our proposed direct standardisation technique. Figure 2.3 illustrates the societal proportion of multidimensionally deprived population in the Maldives and Ukraine. It includes both unstandardised and standardised H-headcount ratios. For the standardised figures we use three different and alternative standard populations. From top to bottom, the first horizontal line plots the non-standardised headcount ratio for both countries. The subsequent horizontal lines in the figure plot the direct household size standardised H across the three different distributions of the population used as standard. The distributions of population used as standard in this example are: i) The mean distribution of the population across both countries (second horizontal line of the plot); ii) The population distribution of the Maldives (third horizontal line of the plot); and iii) the population distribution of Ukraine (fourth horizontal line of the plot). Squared markers in the plot indicate the results obtained for the Maldives, while circle markers correspond to results obtained from Ukraine.

Unstandardised figures indicate that the Maldives has two percentage points greater H incidence than Ukraine. Once we account for comparing both countries' group specific relative deprivation under the same structure of the population, the standardised figures show that Ukraine is relatively worse off than the Maldives. This observation is consistent across the three different standard populations used in the analysis.

Indeed, the three different distributions of the population used as standard rank in the same order both countries. With this said however, the size of the resulting difference in H among both countries varies depending on the population distribution utilised as standard. It is observed in Figure 2.3 that the use of the Maldives as a standard population to standardise H, indicates that the difference in multidimensional incidence across both countries is 11.5 percentage points. In contrast, the use of Ukraine as standard population signifies that this difference corresponds to only 2.0 percentage points.

The use of a standard population with greater numbers in households containing seven or more persons (as with the Maldives) assigns greater weight to deprivation concentrated in this sub-population group. In contrast, using Ukraine's distribution as the standard population, and one which is more concentrated on households with three or less persons, assigns greater weight to this population subgroup. Indeed, the index that we use as example results in observing greater evaluated deprivation across households with greater household size. In light of

Figure 2.3: Effects of direct household size standardisation



Source: Author's calculations based on DHS micro-data. Note: Point estimates developed using the weighting system provided by Measure DHS. H-headcount ratio estimates developed using a k-threshold of 40%.

this, the use of the Maldives as the standard population produces greater H standardised figures than the use of Ukraine.

Nonetheless, while standardised methods produce demographically standardised comparable figures, the evaluated ranking and point estimations are still sensitive to the population used as standard. This is the main drawback of this type of method. It poses an undoubtedly relevant question in terms of which population distribution should be used as standard to perform the comparisons. The selection of the standard population requires careful analysis. In the Section 2.6.2 ahead on this chapter, we discuss the implications of such selection and the proposed criteria to use as a guide in making this selection.

2.5.3 Example: direct standardisation on age

As a second example, we now present the application of our proposed direct standardisation procedure for an individual-based measure, standardising by ranges of age. For this analysis we construct the index example introduced in Section 3.2, here at the individual level and using the DHS micro-data surveys of Ukraine 2007, Jordan 2007, Dominican Republic 2007 and Armenia 2005.

The purpose of the analysis is to compare individual-based multidimensional deprivation across these four countries that, although sharing comparable levels of development, have dissimilar population distributions. In this case we use three different standard population distributions. We selected the first two standard population distributions to analyse results as those that could register the possible most extreme opposite cases, among the four countries. This enables to analyse the possible range of variation for the standardised figures. We first use as standard the society among the four countries that has the highest concentration of younger persons, namely Jordan. The second standard used is the population with the highest number of older persons, which is Ukraine. Both distributions are extreme but still credible, as they are not unrealistic. They correspond to the real distribution of the population of these two countries. Finally, we use as third possible standard population, an intermediate approach, which is the mean distribution of the population across the five countries.

Figure 2.4 below reports the results obtained without any standardisation, and results from the three different standardisation applications. The horizontal axis in the figure refers to the H resulting metric of multidimensional deprivation, while the vertical axis includes the results obtained for each country across different standardisation procedures. Circle filled markers denote the unstandardised H figures, hollow circular markers correspond to standardised figures using the average population of the five countries, triangle markers to standardised figures using Ukraine as the standard population and square markers refer to standardisation using Jordan as the standard.

By pairs of countries, unstandardised figures indicate that multidimensional deprivation is greater in Jordan than in Ukraine, while the three proven standardised figures signify that Ukraine exhibits greater multidimensional deprivation than Jordan. This is similar to the case of Armenia and Dominican Republic, where unstandardised results indicate the opposite order to that obtained by the proven standardised results: Armenia exhibits greater multidimensional deprivation incidence than Dominican Republic.

Results indicate that the ranking using unstandardised metrics does not correspond to the ranking produced by standardised figures. Unstandardised figures signify that amongst the four countries, Dominican Republic is the most deprived and Ukraine the least. However, standardised figures using either Jordan as standard population, or the mean across the four countries, produce a ranking where Armenia is the most deprived and Jordan the least. On the other hand, when using Ukraine as the standard population, the least deprived country is Jordan; a result that is consistent with the previous two observations.

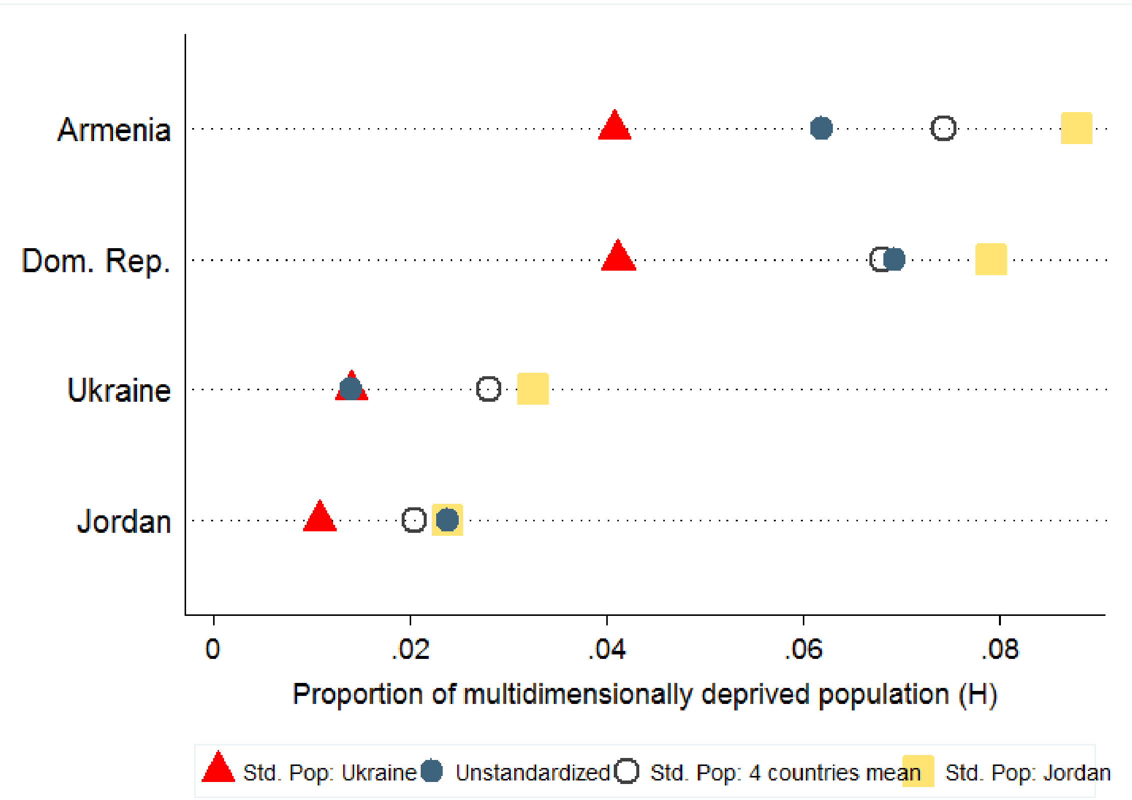
The use of a standard population with a higher concentration of younger population produces more consistent results than using as standard a population with a higher concentration of older persons. The standardised ranking is robust to the use of a standard population that concentrates a greater proportion of the population where the indicator places greater emphasis.

On the other hand, in terms of size, we observe in Figure 2.4 that the sensitivity of the age which *DSH* point estimates to the standard population is considerable. A direct standardisation that uses Ukraine as standard produces a demographically standardised comparable H rate for Armenia of 4.1%, while the use of Jordan as standard produces a demographically standardised comparable H rate of 8.8% for the same country. Thus, the size of the rate using Jordan as standard is more than twice the value of that seen when using Ukraine as standard.

In light of these results and given that the multidimensional index that we use as example enables the population of persons younger than 25 years have a higher number of deprivations (as discussed on page 22 and displayed in Figure 2.1), while the population of persons 25 years and older to have a lower number of deprivations. A standard population concentrated on young persons, such as Jordan, produces greater multidimensional deprivation rates.

In contrast, given that Ukraine is among the four countries, the one that exhibits lower proportion of young population, when we use Ukraine as the standard population, lower deprivation rates are produced than any of the other standard-

Figure 2.4: Direct age standardisation effects based on three different standard populations



Source: Author's calculations based on DHS micro-data. Note: Point estimates developed using the weighting system provided by Measure DHS. H-headcount ratio estimates developed using a k-threshold of 40%.

ised rates. Notice that, while the use of Ukraine as standard (diamond markers in Figure 2.4) produces the lowest *DSH* within each country, the use of Jordan as standard (square markers in Figure 2.4) produces, instead, the highest *DSH*.

As a result, measures that use different standard populations are not comparable with each other. A standard population concentrated in younger cohorts produces a metric driven by the deprivation experienced by these ages. Conversely, the use of a standard population concentrated in older cohorts allows the metric to be driven by the deprivation observed in older ages.

When we use the mean distribution of the five countries as standard, we obtain results which fall between the two previously mentioned situations. This latter approach is therefore an intermediate one. This type of standard avoids demographic overweighting of deprivation among either tail of the age distribution and still portrays a credible distribution of the population.

2.5.4 Example: direct standardisation on household size and age

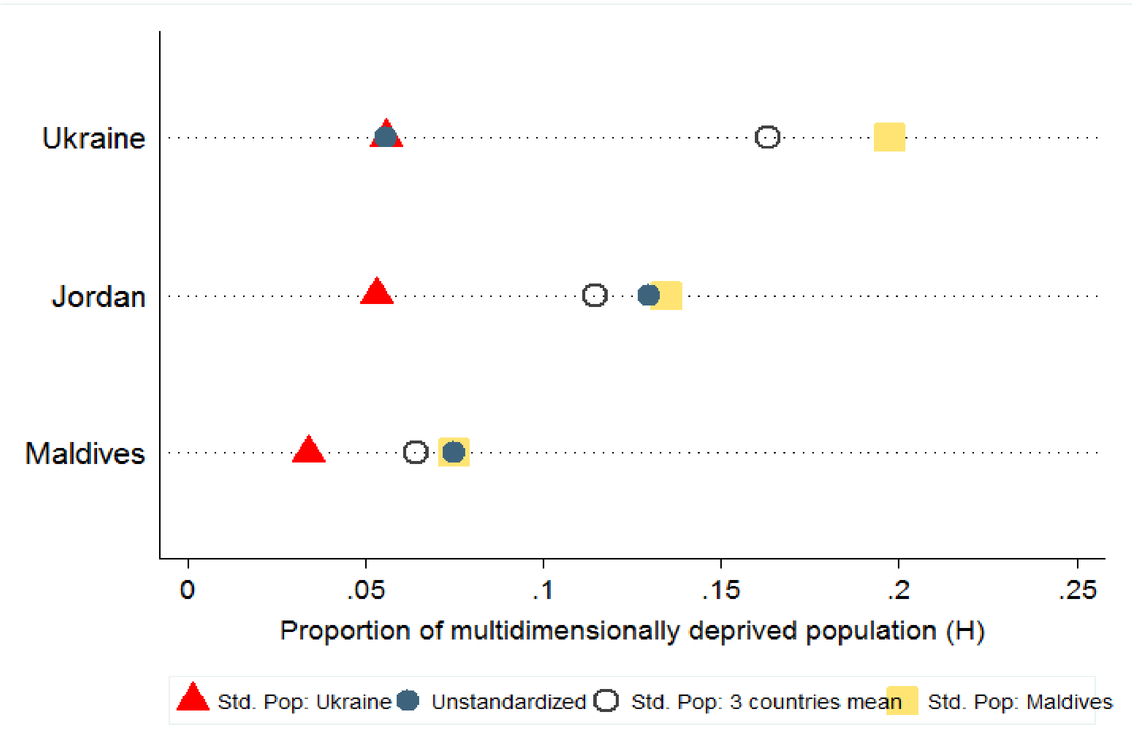
Since most of the policy multidimensional index currently in use have selected households as their unit of analysis and the indicators included within them vary across different population subgroups by age ranges, we present here an example of direct standardisation on two combined characteristics: age and household size.

Figure 2.5 includes the results of this example. It uses DHS 2007 information to evaluate our household-based multidimensional measure in Jordan, Ukraine and the Maldives. We present the resulting H headcount at a k-threshold of 40%, before and after standardising by household size and age. The standardisation was performed through three different standard populations: i) The mean distribution of the population across the three countries (circle hollow markers in the plot); ii) The population distribution of Ukraine (triangle markers); and iii) The population distribution of the Maldives (square markers in the plot).

The ranking shown by the three countries is robust across the three proven standard populations in use. By pair of countries, while unstandardised figures indicate that Jordan faces seven percentage points greater incidence than Ukraine, the three standardised comparisons suggest that Ukraine is facing greater relative multidimensional deprivation incidence than Jordan. Likewise, when comparing Ukraine and the Maldives, the three standardised figures produce a consistent ranking among each other, thus signifying that Ukraine is facing greater relative deprivation than the Maldives. Conversely, unstandardised evaluations indicate the opposite: the Maldives has greater incidence multidimensional deprivation than Ukraine.

The use of a standardisation procedure does provide more accurate comparisons. The standardised results are, in fact, demographically comparable. Nonetheless, standardisation procedures that use more than one standardising characteristics or many categories within the same standardising factor require careful analysis of the precision of the results. More specifically, the proposed standardisation techniques to be applied with multidimensional indices rely on the development of contingency tables, where each cell contains the mean incidence by population subgroup. Then, the smaller the number of sample cases that

Figure 2.5: Example of multidimensional deprivation incidence (H), unstandardised and directly standardised by household size and age



Source: Author's calculations based on DHS micro-data. Note: Point estimates developed using the weighting system provided by Measure DHS. H-headcount ratio estimates developed using a k-threshold of 40%.

each cell contains, the larger the standard errors associated with those means will be.

Still, another advantage of direct standardisation, in comparison with analysing homogeneous sub-population groups separately, could be that standardised societal results might have lower standard errors than those observed in each cell. However, the greater the number of standardising categories is, the greater the standard errors of the overall societal measure will be. The standard errors for each particular indicator included in the multidimensional index, as with the whole societal index vary, nonetheless, from context to context. Sample variability should therefore be studied under case-based circumstances.

2.5.5 Example: time comparisons using age-standardised results

In this section, we analyse the implications of implementing a direct standardisation method for time comparisons. We illustrate this case by using the Armenia 2005 DHS and comparing it with Armenia 2010 DHS. The results of this exercise are presented in Table 2.2. The first column of Table 2.2 reports the proportion of the population under 25 years old, while the second column presents the unstandardised results using a k threshold of 20%.

As reported in column 1, Armenia experienced a reduction in the proportion of the population under 25 years old between 2005 and 2010. Between 2005 and 2010, the proportion of the population younger than 25 years old fell 5.23 percentage points. In 2005, 39.27% of the Armenian population was younger than 25 years old. This proportion corresponded to 34.04% in 2010. This is an example of a society with two different population distributions across time. We select a time difference of only five years, first because the DHS data are available regularly every five years and second to assess how much standardisation techniques are worth utilising under not necessarily considerable demographic changes.

We observe in 2010 a structure of the population by age ranges in Armenia different from that observed in 2005. In addition, we have seen that the multidimensional index example in use produces a multidimensional deprivation incidence that varies across age ranges. Hence, the standardisation technique proposed in the chapter appears worth utilising to furnish time comparisons which are not confounded by changes in the structure of the population.

In this case, demographically standardised comparable results are accomplished by standardising H on age. Here, we standardise Armenia's H multidimensional incidence using three alternative standard populations: i) the mean population distribution across evaluated periods of time; ii) the population distribution observed in 2005; and iii) the population distribution population observed in 2010. Columns 4 through 6 in Table 2.2 include the standardised version of these results.

The Armenian unstandardised multidimensional incidence rate, as indicated by column 2 in Table 2.2, displays a reduction of 4.15 percentage points between 2005 and 2010. The trend observed in the same period across the three different standardised comparisons is unchanged, regardless of the standard population used. Each of these four results indicates that Armenia registered a reduction in the multidimensional deprivation incidence across time.

The unstandardised results indicate that the reduction in the multidimensional deprivation incidence observed between 2010 and 2005 in the country correspond to 4.15 percentage points (p.p.). However, the three age-standardised results indicate that this reduction is not as large as the unstandardised result suggests. Such an empirical result demonstrates that the unstandardised reported reduction accounts not only for the reduction in relative multidimensional deprivation but also for the demographic change, i.e., the reduction in the proportion of the population younger than 25 years old.

Armenia's greater concentration of the population under 25 years of age in 2005 in comparison to 2010 produces a larger 2005 multidimensional deprivation incidence than that which would have been observed if it had a population distribution similar to the one experienced in 2010. This is the counter-factual question answered by the direct standardised results reported by columns 3 and 5 of Table 2.2. Conversely, the lower proportion of Armenia's under-25 population in 2010 produces a lower multidimensional deprivation incidence than the one the country would have observed if it had had a population distribution more similar to that of 2005 (see columns 3 and 4 in Table 2.2).

Table 2.2: Age direct standardised results for H across time, Armenia

	Proportion of population under 25 years old	Unstandardised results	Standardised results		
			Standard: Mean 2005, 2010	Standard: 2005	Standard: 2010
	(1)	(2)	(3)	(4)	(5)
2005	39.27	6.20	5.73	6.20	5.24
2010	34.04	2.05	2.29	2.52	2.05
Difference (p.p)	-5.23	-4.15	-3.44	-3.67	-3.19
Difference (%)	-13.33	-66.90	-60.00	-59.28	-60.89

Source: Author's calculations based on DHS micro-data. Note: Point estimates developed using the weighting system provided by Measure DHS. H-headcount ratio estimates developed using a k-threshold of 40%.

As a result, when comparing time, the more sensible approach is to select an intermediate distribution of the standard population, e.g. the mean of the populations to be compared across a wide range of years. National projections of population divided by age group are an important source of information for this purpose. This procedure enables accurate comparisons without the need to frequently change the standard population. This approach is employed by Ahmad et al. (2001) and Anderson et al. (1998) in their analysis of worldwide death rates.

2.5.6 Sample variability

Multidimensional deprivation is traditionally measured using samples of households or individuals which allow to assess the multiple deprivations that a particular household / individual suffers at the same time. In use of these surveys, point estimates of unstandardised rates of multidimensional deprivation incidence are subject to sample variability that must be taken into account when assessing societal rankings of multidimensional deprivation incidence.

The rigorous assessment of the sample variability that point estimates exhibit requires to take into account the sample design of each survey because different units might have had different selection probabilities.

On one hand, for the case of unstandardised rates most of the statistical software provide computational tools that allow estimating the standard errors that H societal rates and the h_j subgroup specific headcount ratios exhibit, while taking into account the design of the survey. The estimation of such standard errors allow us statistical inference.

On the other hand, given that the *DSH* -direct standardized headcount provides information about the multidimensional incidence that a society would have experienced had it had its own h_j subgroup specific headcount ratios but the population distribution of the standard population, the variance of the *DSH* can be defined as the weighted average of the variances that each h_j subgroup specific headcount ratios exhibit. In this weighted average, the weights are the population shares of the standard population, which are assumed as invariant; also, the h_j subgroup specific headcount ratios are assumed independent across population subgroups. This is as well the approach used by Anderson et al. (1998) and Curtin & Klein (1995) to estimate the variance of age-adjusted death rates.

Table 2.3 presents the obtained confidence intervals for the household size direct standardised results developed in Section 2.5.2. We observe that the confidence interval of the unstandardised H-rate of Maldives does not overlap the confidence interval of the unstandardised H-rate of Ukraine. Then, the 2.21 p.p difference between these two countries is statistically significant, being Maldives the most deprived.

On the contrary, when H is standardized using three different standard populations we observe that the *DSH* rate of Ukraine is always above the *DSH* rate of Maldives, being this difference statistically significant in the case of using Maldives as the standard population or the mean distribution of both countries.

We observe that the variance of the *DSH* is higher than the variance of unstandardised rates because the usage of the shares of the standard population as weights in the estimation of the variance. For instance, while the unstandardised and the standardized H point estimate of Maldives is equivalent when using Maldives as the standard population, the confidence intervals of the standardized rate are wider than the obtained for the unstandardised variant.

In general, to avoid striking increments in the estimated variance of standardized rates, it is recommended to base estimations on large enough population subgroups.

Table 2.3: Household size direct standardised results for the H-Proportion of multidimensionally deprived population, Maldives and Ukraine

Standard population	Maldives			Ukraine		
	Mean	Ll	Ul	Mean	Ll	Ul
	(1)	(2)	(3)	(4)	(5)	(6)
Maldives	7.50	6.23	8.76	18.98	13.77	24.18
Ukraine	3.53	2.59	4.46	5.29	3.96	6.61
Mean population distribution of Maldives and Ukraine	5.73	4.60	6.86	12.89	8.91	16.86
Unstandardised	7.50	6.45	8.54	5.29	4.67	5.90

Source: Author's calculations based on DHS micro-data. Note: Point estimates developed using the weighting system provided by Measure DHS. H-headcount ratio estimates developed using a k-threshold of 40%. LI and Ul refer to the lower and upper 95% confidence limits developed using the standard errors derived from the Taylor-linearised variance estimation that takes into account the clustered probabilistic sample design of each survey.

Still, the use of an standard population that emphasizes the population sub-groups that register lower h_j-incidence rates and lower variance reduces the overall estimate of the variance. This is observed when applying the distribution of the population of Ukraine (which emphasizes households consisting of 1-3 persons) as standard over the h_j-incidence rates of Maldives.

2.6 Context-specific definitions

This section first discusses the natural concern that arises while implementing a direct or indirect standardisation procedure to compare societal figures of multidimensional indices, which especially relates to the selection of the standard population to be used by these methods. Secondly, it discusses the normative definitions embedded when selecting either a direct or an indirect standardisation technique in the context of multidimensional deprivation indices. These two discussions aim to guide the use of the proposed methods of this chapter in context-specific applications.

2.6.1 Choosing the standard population

Our proposed standardisation techniques enable more accurate comparisons of current policy developed indices across societies or across time. In the case of direct standardisation, this is possible by fixing as constant the distribution of the population to use for the comparisons. The selection of this standard population is therefore crucial, although there is no conceptual justification for this choice. This lack of theory can lead to arbitrariness.

In fact, for mortality rates, following Kleinman (1992) and Anderson et al. (1998), the main drawback of an age-direct standardisation procedure is found when the subgroup specific rates show divergent trends or different relative magnitudes, thus leading to a different overall trend of societal figures that is strictly dependent on the chosen standard population. In particular, standard populations with greater emphasis on specific cohorts stress the rates experienced on such ages or population specific subgroups.

Following Rosenberg et al. (1992), statistical and non-statistical considerations arise when it comes to the factors that should be taken into account when selecting a standard population for standardised indicators. In particular, based on the normative criteria and developments done by the mortality rates standardisation literature (Wolfenden 1923, Kleinman 1992, Ahmad et al. 2001, Anderson et al. 1998), we discuss six guidelines that we found as criteria to be taken into account when choosing a standard population to compare multidimensional indices of deprivation across societies.

1. The standard population to be chosen varies depending on the purpose of the analysis. Each analysis seeks to compare different societies and thus different standard populations may have to be used. For instance, the standard population to enable comparisons across time within the same country will intuitively not be the same chosen standard as that used for a cross country setting. While in the first case the most advisable procedure is the use of a standard population that resembles the mean structure of the population of the country across time, in a cross country setting the best practice is the use of a cross country mean structure of the population as standard. In general, the standard population to be used must be chosen based on the requirements of each of the analyses to be performed. There is no single standard population to be used in any case.

2. The standard distribution should reflect a credible structure of the population of the societies to be compared. If the purpose of the analysis is to compare multidimensional deprivation across low-income countries using a standard population that resembles high-income countries, then population distribution will underweight events that are relevant to stress across the former cases. As an example, according to World Health Organization data (WHO 2014) and the World Bank income groups classification used by WHO, while in 2012 13.12% of Japan's population is younger than 15 years old, 49.99% of the population in Niger is concentrated in this age cohort. A direct standardisation exercise to compare low-income countries, using as standard the distribution of the population exhibited by 2012 Japan will provide relatively higher weight to events observed among the older population, while also weighting lower events observed across the young population. As such, a credible standard population to compare developing countries corresponds to a structure that resembles the mean distribution of the countries to be compared, for instance.

In fact, the examples provided by Sections 2.5.3 and 2.5.4, have shown that a standard population that uses the mean of the countries to compare shows more consistent results than standard populations that can be considered as extreme structures of the population. Our finding is consistent with previous studies that have analysed the sensitivity of standardised metrics to different standard populations. An example are the results obtained by both Rosenberg et al. (1992) and Curtin et al. (1980), whom indeed, after analysing the sensitivity of age-standardised death rates to compare states within the United States, found that the ranking provided by standardised figures that use as standard the more common structure of the population shows the most robust rankings across different standards. They also found that the use of unusual structures changes the ranking considerably.

3. When very different population structured societies need to be compared, an intermediate or mean approach is desirable. Following from the previous example of Japan and Niger, for worldwide comparisons, the use of Japan as the standard population will focus the attention of the indicator on events that occur amongst the older population, although the use of Niger, for instance, produces the greatest attention on events observed on persons younger than 15 years old. Worldwide exercises that imply the comparison of several dissimilar societies in terms of their population structure can be more consistently performed by the use of an average world population. In fact, this is the approach proposed by the World Health Organization to compare worldwide age-standardised mortality rates (Ahmad et al. 2001).

4. The use of the average structure across time might be a good practice when time comparisons are expected to be performed. The World Health Organization, for instance, proposed the use of world average structure of the population between 2000 and 2025 as the standard population for the future comparative worldwide death rates (Ahmad et al. 2001). According to Ahmad et al. (2001), the use of an average world population, as well as a time series of observations, removes the effect of historical events on population's age composition. As the example provided in Section 2.5.5 reveals, when comparing Armenia 2005 and 2010 for multidimensional incidence, either comparing the society using the structure of the first year, i.e. 2005, or the last year of the comparison (2010) maintains the overall trend. Yet, a sensible approach in this case might be to select the intermediate structure, i.e. the mean distribution of the population across both years as it does not provide the greater emphasis given by one or the other distribution.

5. A fix year's population structure to standardize multidimensional deprivation incidence rates can be useful to shed light on specific counter-factual questions of the behaviour of multidimensional deprivation incidence. For instance, in the example of age standardized time comparisons analysed in Section 2.5.5, the use of Armenia's 2010 structure as standard shows that the 2005 unstandardised multidimensional deprivation incidence results are larger than that which the country would have observed if it had a population distribution similar to the one experienced in 2010.

6. A chosen standard with a population concentrated among the groups in which the multidimensional measure has lower emphasis provides unsatisfactory results. For instance, as we have shown in Sections 2.5.3 and 2.5.4, using a standard population concentrated in older citizens to compare a multidimensional index that emphasizes deprivation on young ages, reduces the variation spectrum of societal figures. This, in turn, provides a ranking based on the deprivations that the indicator had defined as having relatively lower importance.

7. Easy to use and interpret. As pointed out by Rosenberg et al. (1992), standardised indicators are used as policy analytical tools to compare relative deprivation across societies. They must, therefore, be easy to explain and easily usable by a broad audience. Standard populations based on methods of common knowledge of policy audience and public are found to be an advantage.

2.6.2 Selecting a standardisation technique

While the direct standardised multidimensional estimate reveals the incidence rate that a society would have observed if it had the population distribution of the standard, an indirect standardised multidimensional estimate reveals the incidence rate that a society would have observed if it had the sub-population group deprivation incidences of the standard population. Both correspond to different normative approaches.

To analyse which standardisation technique, either direct or indirect, could be more suitable for each context, the framework set up by Fleurbaey & Schokkaert (2009) to analyse unfair inequalities in health and health care is of special interest. In the framework, the scholars catalogued differences in health outcomes caused by myriad factors, some of which can be catalogued as producing fair/legitimate differences and others as producing unfair/illegitimate differences. They defined as legitimate or fair those differences attributed to causes that fall under individuals' responsibility. Legitimate differences in this context therefore correspond to those derived from preferences.

Following the analysis of Fleurbaey & Schokkaert (2009), the two standardisation techniques reflect different ethical conditions. While a direct standardisation enables us to observe standardised figures that do not reflect variations among them because of legitimate causes, an indirect procedure enables us to observe standardised figures that reflect the same outcome if two societies have the same level of illegitimate differences. As the scholars indicated, both ethical conditions are desirable. Indirect standardisation rules out illegitimate differences in deprivation, so standardised figures indicate any remaining 'fairness gap (a term used by Fleurbaey & Schokkaert (2009) to describe this procedure). Direct standardisation, in contrast, rules out legitimate differences.

However, as the scholars discussed, it is not possible to achieve both situations at the same time using either technique. In particular, in a direct standardisation context, it is possible that no difference in multidimensional deprivation incidence between two societies will be observed because they have identical legitimate sources of deprivation (i.e., an identical population distribution), so no legitimate difference in deprivation remains. However, the two societies may still have illegitimate differences in deprivation, and the standardised results would depict them as equivalently deprived. Scholars might find this situation ethically undesirable, so they might opt to use an indirect standardisation technique. Nonetheless, this decision is context specific, and it is advised to be taken in light of the particular analysis to be done.

2.7 Concluding remarks

The current paper studies the effect of dissimilar population structures on societal multidimensional comparisons. Through this paper, we show that comparisons of multidimensional deprivation incidence rates across societies with dissimilar population structures produce misleading rankings of relative deprivation. We discuss reasons for standardisation, providing examples of circumstances that merit the use of standardisation techniques. Last, we provide guidelines for the selection of the standardisation technique and the standard population to choose when standardised multidimensional indices will be in use.

We argue that current multidimensional deprivation incidence rates fail to reveal relative deprivation exclusively when comparing societies with different population distributions. These comparisons can be misleading. Direct and indirect standardisation techniques are presented as plausible and desirable methods to produce demographically comparable rates. This is achieved through the use of a standard population distribution, which enables meaningful comparisons across societies with different population distributions.

Unstandardised H rates, in comparison to either household size-standardised rates or age-standardised rates, show a different multidimensional deprivation societal ranking. For instance, while the household-based crude H ratio indicates greater relative deprivation in Maldives in comparison to Ukraine, all three proven household size-standardised H rates signify the greater relative deprivation of Ukraine in comparison to Maldives. The results are also consistent with the multidimensional deprivation rate observed across groups of household size. These results demonstrate the advantages of our proposed standardisation procedures over the use of unstandardised rates when comparisons of societies with dissimilar population distributions are to be analysed.

Although direct or indirect standardisation is useful for comparisons across societies, their main limitation is based on the sensitivity of the results to the standard population used. We compare a multidimensional index across four countries: Jordan, Ukraine, Dominican Republic, and Armenia. The countries have similar levels of development but dissimilar population distributions. We find that the size of the estimates, along with the ranking produced by standardised measures, is sensitive to the standard population in use. Standardised multidimensional metrics based on different standard populations are found not to be comparable at all.

Multidimensional comparisons across time were also assessed. The results show that multidimensional deprivation incidence comparisons across time must be addressed on a standardised basis whenever important demographic changes have taken place. Demographic changes may alter the magnitude of the evaluated reductions in multidimensional deprivation. Moreover, comparisons both across time and across societies might be even more sensitive to differentials across the distribution of the populations by confounding factors such as age or household size. Our results demonstrate that accurate conclusions can be derived through standardised analysis.

Any chosen standard with a higher proportion of its population concentrated among age groups in which a greater number of dimensions are relevant weights events in these groups in a similarly higher proportion. As a result, a chosen standard with a population concentrated among these groups would be appropriate for events that more concentrated there. In addition, while the use of an unusual population structure as the standard changes the societal ranking considerably, standardised figures based upon a common structure of the population show robust rankings across different standards. Along with these two guidelines, we provide in Section 2.6.2 five other criteria to follow in choosing the standard population.

Standardised multidimensional figures do not reflect the magnitude of multidimensional deprivation for each specific society. Standardised figures are not meant to replace crude rates. Indeed, it is only when making comparisons that they are necessary and meaningful.

As such, the methodology proposed in this paper can be seen as useful to enhance societal multidimensional comparisons of already designed indices that do not take into account neither differences in needs across population subgroups nor differences in the demographic structure of the populations to be compared.

Still, other possible methodological approach could be via the use of weighting strategies that take into account differences in the structures of the populations to be compared. An example of this later approach can be seen in Hildebrand et al. (2015), whom control for differences in demographic structure and employment of two populations using a non-parametric sample reweighting technique. Or, a third possible course of action could be to take into account observed differences in needs across individuals or household whilst constructing the index to be used. This is the approach that the next chapter of this thesis follows.

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Chapter 3

Differences in needs and multidimensional deprivation measurement

Abstract

Individuals from different demographic population subgroups and households of different size and composition exhibit different needs. Multidimensional deprivation comparisons in the presence of these differences in needs have yet to be analysed. This chapter proposes a family of multidimensional deprivation indices that explicitly takes into account observed differences in needs across demographically heterogeneous units (i.e., either households of different size and composition or individuals of different population subgroups). The proposed counting family of multidimensional indices builds upon the Alkire and Foster methodology of poverty measurement (J. Public Econ. 95:476−487, 2011) and draws from the one-dimensional parametric equivalence scale literature. It aims to describe how much deprivation two demographically heterogeneous units with different needs must exhibit to be catalogued as equivalently deprived. Through microsimulation techniques, applied over the 2013 Paraguayan household survey, the measurement approaches contained in the proposed family of measures of this chapter are evaluated. The obtained results demonstrate that neglecting differences in needs yields biased multidimensional deprivation incidence profiles. Results also shed light on the ability of the proposed measures of this chapter to effectively capture these differences in needs.

Keywords: Multidimensional deprivation, poverty measurement, equivalence scales, heterogeneous households, individual heterogeneity.

JEL codes: D63, I32

Acknowledgements: I would like to thank Paul Clarke, supervisor and graduate director of the Institute for Social and Economic Research, for his guidance and thoughtful comments during the development of this chapter. An earlier version of this manuscript appeared in the ECINEQ working paper series number 387 from December 2015. This chapter has benefited from the comments received from Suman Seth and Sabina Alkire from the Oxford Poverty and Human Initiative-OPHI. I am thankful for the helpful comments received from Erik Schokkaert, Frank Cowell and participants of the 2015 ECINEQ meeting.

3.1 Introduction

There is increasing interest in measuring poverty by assessing deprivation in multiple dimensions of well-being rather than by exclusively evaluating the ability to consume market commodities. Within this growing literature, most of the applications of multidimensional deprivation measurement use the Alkire & Foster (2011) method and either individuals or households as the unit of analysis.[1]

However, differences in needs are present when measuring multidimensional deprivation across either individuals from different demographic sub-population groups or households of different sizes and compositions. While pregnant women, for instance, need access to antenatal health services, school-age children need access to basic education services. Deprivation of antenatal health services is thus relevant only to pregnant women, and access to basic education services is only relevant to school-age children. Similarly, households without children are not necessarily deprived in the absence of educational services and vaccinations, just as households without pregnant women are not necessarily deprived because of a lack of antenatal health services. Differences in needs therefore pose comparability challenges when measuring multidimensional deprivation across demographically heterogeneous units, such as households of different sizes and compositions or individuals of different age ranges and genders.

In the previous chapter, I investigated the effect of these differences in needs when comparing multidimensional deprivation across societies of different demographic composition and proposed standardisation methods to enhance societal multidimensional comparisons. In this chapter, I focus on studying the effect of differences in needs when comparing multidimensional deprivation across households of different sizes and compositions or individuals of different age ranges and gender.

[1]Although the terms ‘multidimensional deprivation’ and ‘multidimensional poverty’ are used interchangeably in literature, throughout this chapter, the term ‘multidimensional deprivation’ is used to refer to indices that count the multiple deprivations jointly observed across a selected unit of analysis and, based on this counting procedure, identify the poor as the most deprived population. Examples of this long-standing literature are studies such as Townsend (1979), Atkinson & Bourguignon (1982), Mack et al. (1985), Callan et al. (1993), Feres & Mancero (2001), Atkinson (2002), Alkire & Foster (2011), and Aaberge & Brandolini (2014*b*).

A plethora of methods and techniques that account for differences in needs can be found in the one-dimensional welfare literature. Examples of this literature include Kapteyn & Van Praag (1978), Pollak & Wales (1979), Blundell & Lewbel (1991), Coulter et al. (1992*b*), Cowell & Mercader-Prats (1999), Duclos & Mercader-Prats (1999), and Ebert & Moyes (2003). They aim to provide societal profiles based on comparable household-based aggregates of income or expenditure obtained through the use of *equivalence scales*.

In contrast to the one-dimensional welfare literature, comparisons of multidimensional deprivation between demographically dissimilar units have yet to be described. In fact, theoretically developed families of multidimensional indices such as those proposed by Tsui (2002), Bourguignon & Chakravarty (2003), Alkire & Foster (2011), and Seth (2013) have been developed exclusively using the individual as the unit of analysis, and do not discuss the arising comparability problems that heterogeneity in needs across units might pose.

This chapter proposes a family of indices that measures multidimensional deprivation across demographically heterogeneous units while explicitly taking into account differences in needs across them. The proposed approach extends the Alkire & Foster (2011) counting family of multidimensional poverty indices, providing a wider set of indices that aims to adjust for observable differences in needs across demographically heterogeneous units. This is the methodological contribution of this chapter to the multidimensional measurement literature.

The choice of the individual or the household as the unit of analysis is not arbitrary. It involves a normative decision to be made during the multidimensional measurement process. Household-based measures conceive households as cooperative units that jointly face the deprivation suffered by the household members, as, for instance, Angulo et al. (2016) discussed regarding the selection of the household as the unit of analysis for the Colombian Multidimensional Poverty Index. Individual-based measures, in contrast, allow the unmasking of differences in multidimensional deprivation across demographic sub-population groups, such as the case of gender differences analysed by Vijaya et al. (2014) for Karnataka, India or by Agbodji et al. (2013) for Burkina Faso and Togo.[2]

[2]For a broad discussion of the different normative decisions embedded in multidimensional measurement, see Alkire, Foster, Seth, Santos, Roche & Ballon (2015).

The family of indices that I propose in this chapter allows multidimensional deprivation to be measured using either individuals or households as the unit of analysis. The choice of individual or household is therefore open to be made according to the context of each application.

In the case of household-based multidimensional measures, the purpose of accounting for differences in needs is to enable pairs of households and thus different populations of households to be compared on a more equivalent basis. Similarly, in the individual-based case, the indices proposed in this paper aim to enable multidimensional deprivation comparisons of any two individuals and hence of different populations of individuals.

Furthermore, my proposed family of measures allows describing, under equivalent normative considerations, the burden that multidimensional deprivation places on each unit of analysis (either households or individuals). For instance, under an absolute normative perspective where each deprivation has an equivalent absolute value, the burden that multidimensional deprivation places is described through a deprivations count-based approach to measurement. Conversely, under a relative normative perspective that conceives each household / individual as equivalently valuable, the burden that multidimensional deprivation places is described in terms of a share-based approach to measurement. Intermediate normative perspectives, in contrast, lead to the expression of multidimensional deprivation as a mixture of count-based and share-based approaches to measurement.

To evaluate the effect of these different approaches to measurement (count-based, share-based and intermediate) on multidimensional deprivation incidence profiles, I construct counterfactuals using the 2013 Paraguayan household to disentangle how much of the differences in multidimensional deprivation incidence profiles are observed because unaddressed differences in needs. The results of this evaluation demonstrate that unaddressed differences in needs yield multidimensional deprivation incidence profiles to reflect not only differences in deprivation but also differences in needs that for the purposes of this paper are considered as a legitimate source of variation and should be therefore tackled by the measurement process. Failure to take differences in needs into account was found to cause biased multidimensional incidence profiles. Results also shed light on the ability of my proposed measures to effectively capture these differences in need.

3.2 Background

The starting point of this chapter is the background literature that analyses welfare comparisons in the presence of heterogeneous needs. This section presents an overview of this literature and the equivalence scale notion that seeds the family of indices proposed in this chapter, along with a description of the relevant multidimensional measurement background literature.

3.2.1 Welfare comparisons in the presence of heterogeneous needs

A plethora of methods and techniques from the one-dimensional literature attempts to assess welfare and inequality rankings while taking into account differences in needs between households. Examples of this in the literature are Kapteyn & Van Praag (1978), Pollak & Wales (1979), Blundell & Lewbel (1991), Coulter et al. (1992*b*), Cowell & Mercader-Prats (1999), Duclos & Mercader-Prats (1999), and Ebert & Moyes (2003). Within this literature, these technologies are known as *equivalence scales*. Their relevance, as pointed out by Cowell & Mercader-Prats (1999), is crucial for inequality and social welfare comparisons: "equivalence scales, by providing an interpersonally comparable measure of living standards, play a central role in the assessment of social welfare and income inequality. Failure to take account of the relationship between nominal and equivalized income can give a biased picture of both inequality and social welfare" (Cowell & Mercader-Prats 1999, p. 409).

In particular, equivalence scales have been used to allow the construction of societal measures of welfare and inequality based on comparable household measurements of income or expenditure (Fisher 1987, Muellbauer 1974). These scales intend to reflect the amount of income required for households of different sizes and compositions to have the same welfare level (Pollak & Wales 1979, Nelson 1993). An important emerging fact from reading this literature is that there is no universally correct equivalence scale. Different procedures are justified according to different circumstances.

From the empirical perspective, two main approaches to construct equivalence scales can be recognised: equivalence scales drawn from *econometric* techniques and equivalence scales that use *parametric* approaches. For a review of both branches of the empirical literature, see Cowell & Mercader-Prats (1999) and Flückiger (1999). Both econometric and parametric approaches are based on different normative values that determine the results. While econometric approaches vary across different functional forms used to model household preferences, parametric approaches are based on the selection of a set of parameters to typify the size and composition of the household. The following will briefly describe both approaches.

The most common econometric techniques implemented to derive equivalence scales consist of modelling demand functions using household budget data and then estimating the effect that non-income characteristics have over such demand (Coulter et al. 1992*a*). However, as Pollak & Wales (1979) pointed out, these type of scales are based on a household's demand preferences already constrained on the household demographic composition. Moreover, according to Blundell & Lewbel (1991, pp.50), scales revealed from demand data are based on conditional preferences, regardless of whether households choose demands and demographic attributes simultaneously, sequentially or independently. These types of equivalence scales are referred by Pollak & Wales (1979) as 'conditional' equivalence scales.

Conversely, 'unconditional' equivalence scales refer to the variation in income that households of different sizes and compositions require to achieve the same welfare level. However, this variation should be derived independently from the observed demographic profile of the household. According to Pollak & Wales (1979, pp.217), to derive unconditional scales, "welfare analysis must compare the well-being of a family in alternative situations which differ with respect to its demographic profile as well as its consumption pattern". In this vein, unconditional equivalence scales are not directly observable. For this particular type of scale, studies such as Blundell & Lewbel (1991), or, more recently, Hausman & Newey (2013), focus on estimating those unobserved parameters by using counterfactual techniques and applying sensible identifying assumptions.

The parametric approaches, on the other hand, have focused on providing a measurement approach that first takes into account the elasticity of the needs with respect to household size and then the different household compositions. Examples of these parametric technologies can be found in Atkinson & Bourguignon (1987), Buhmann et al. (1988), Coulter et al. (1992*b*), and Cowell & Mercader-Prats (1999). A general approach of this type of equivalence scale is analysed by Buhmann et al. (1988) and Coulter et al. (1992*b*), in which they express household adjusted income (y_h) as a function of the observed household income (x_h), the size of the household (q_h) and a scale relativity parameter (θ):

$$y_h = \frac{x_h}{(q_h)^\theta}. \tag{3.1}$$

In this approach, needs are expressed in terms of the size of the household, and the scale relativity parameter varies from no adjustment of the household income by needs ($\theta = 0$) to a complete adjustment portrayed by the per capita household income ($\theta = 1$).

The family of measures proposed in this chapter draws from this parametric equivalence scale literature. Similar to the one-dimensional equivalence scale of Eq.(3.1), we use a parametric approach to measurement and emphasise needs under a scale relativity parameter θ. The proposed family of measures enhances multidimensional deprivation comparisons across either households of different sizes and compositions or individuals from different demographic sub-population groups. The approach aims to describe how much deprivation demographically heterogeneous units must exhibit to be catalogued as equivalently deprived. It allows societal multidimensional indices based on more comparable profiles than those available in multidimensional measurement literature.

3.2.2 Multidimensional deprivation measurement

Several conceptual approaches exist to measure well-being, and each chooses its specific conceptual focus: resources (income or others), basic needs, Sen's functionings or capabilities (Sen 1993), rights, happiness and so on. In particular, the family of multidimensional measures proposed in this chapter can be applied by different conceptual approaches.

However, the conceptual focus of any index and the selection of dimensions and indicators correspond to a normative selection to be taken for each specific context. For instance, the index currently in use by the Colombian government to track multidimensional poverty (the Colombian Multidimensional Poverty Index - CMPI) chose as focus a standard of living concept within which dimensions and indicators were selected (Angulo et al. 2016). The CMPI considered household deprivations as constitutive elements that describe the lack of a minimum standard of living. In particular, dimensions and indicators were selected by Angulo et al. (2016) using various criteria that range from literature-review-revealed relevant living standards for the Colombian context, to identified governmental priorities, and availability and reliability assessment of the data to be used. Another example is the Grenadian Living Conditions Index (GLCI) currently in use by the Grenadian government to target the most deprived population as eligible for social programs (Díaz et al. 2015). The GLCI also uses a living standard concept from which selected dimensions and indicators. But in contrast to the CMPI, the GLCI defined dimensions and indicators under a set of criteria correspondent to the targeting purpose of the measure. For example, the GLCI explicitly excluded from the set of indicators, variables that could be object of misreporting or that refer to a narrow time frame window to avoid capturing transient household living conditions. For a detailed discussion of the conceptual space, dimensions and indicators, to be chosen in the context of multidimensional deprivation measurement, see Alkire, Foster, Seth, Santos, Roche & Ballon (2015).

Within the multidimensional literature, two alternative procedures identify the poor population and aggregating dimensions: the 'welfare approach' and the 'counting approach'. The first combines several dimensions into a single variable and sets a threshold to differentiate between poor and non-poor populations. The welfare approach has been studied by Bourguignon & Chakravarty (2003), Seth (2009), and Seth (2013), among others.

By contrast, the counting approach, as its name indicates, counts the number of dimensions in which persons suffers deprivation, and the identification of the poor person relies on defining how many dimensions must be deprived for someone to be categorized as multidimensionally deprived. Examples of these types of measures and analysis are proposed by Townsend (1979), Atkinson & Bourguignon

(1982), Mack et al. (1985), Callan et al. (1993), Feres & Mancero (2001), Atkinson (2002), Aaberge & Brandolini (2014*b*), and Alkire & Foster (2011). Efforts have been made within the literature, such as Atkinson (2003), to analyse both approaches (welfare and counting) under a common framework. However, as pointed out by Aaberge & Brandolini (2014*a*), this discussion is still inconclusive.

The family of measures proposed in this chapter stands, specifically, within the counting multidimensional deprivation literature and builds upon Alkire & Foster (2011)'s methodology. For brevity, I henceforth refer to the multidimensional poverty measurement method proposed by Alkire & Foster (2011) using the abbreviation 'AF' or 'AF methodology'. I now continue describing this multidimensional deprivation measurement methodology, using a slightly modified notation.

3.2.3 The AF methodology

Consider a population consisting of $I \geq 1$ individuals evaluated across $J \geq 2$ indicators or dimensions. The AF method begins by defining an $I \times J$ matrix $\mathbf{A} = [a_{ij}]$, where each row corresponds to an individual and each column to the indicators quantifying the individuals' achievements such as education level, nutrition, health status, etcetera.[3] More precisely, the cell a_{ij} of the matrix $\mathbf{A}$ quantifies for the i-individual the j achievement. Each column is either a cardinal or an ordinal achievement indicator.[4]

The AF methodology defines the i individual as deprived in the j dimension by placing a threshold z_j over a_{ij}. Then, whenever $a_{ij} < z_j$ the i individual is said

[3]In general, greater values of an achievement indicator refer to better-off conditions, and lower values of it refer to worse-off conditions.

[4]A cardinal indicator is such that any of its values measures the size of the achievement. This means that the comparison between any two given observed points of a cardinal indicator can be commensurate with the difference between their respective sizes. For instance, years of education is a cardinal achievement indicator because having two years of education can be considered double the number of one year of education. In contrast, an ordinal indicator does not allow measuring the size of the achievement, but rather only indicates a particular ordering between situations. An example of an ordinal achievement indicator is the self-assessment of health status, which takes the categories of "very poor", "poor", "good", and "very good". Note that in this case, we are unable to evaluate the 'size' of the situation. If we compare two observations, for instance, one person having very good health and another person having very poor health, we do not observe the size of the difference between the two situations. In this latter case, we only know that the first person has better off self-assessed health status than the second one, but we do not know the size of the difference in self-assessed health status between the two persons.

to be j-deprived and the breadth of the suffered deprivation is described by:

$$g_{ij}^{\alpha} = \begin{cases} \left(\dfrac{z_j - a_{ij}}{z_j}\right)^{\alpha} & \text{if } a_{ij} < z_j \\ 0 & \text{otherwise,} \end{cases} \tag{3.2}$$

where $\alpha \geq 0$ is the poverty aversion parameter. The α parameter, first introduced in the poverty measurement literature by Foster et al. (1984) and used by Alkire & Foster (2011), assigns greater emphasis to the most deprived or lowest achieving individuals. The greater the value of α, the larger the accentuation of g_{ij}^{α} on the most deprived.

However, if the achievement variable is ordinal, the g_{ij}^{α} indicator is valid only for $\alpha = 0$, and g_{ij}^{0} takes the value of either 1 or 0, indicating the presence or absence of deprivation. Hence, as Alkire & Foster (2011) also discussed, any g_{ij}^{α} with $\alpha > 0$ can be defined only for cardinal indicators.

Given that most of the public policy indicators in current use are ordinal, our proposed methodology restricts g_{ij}^{α} strictly to the case of $\alpha = 0$. Henceforth, we denote it as simply g_{ij}.

The application of the z_j thresholds over the $\mathbf{A}$ matrix results in an $I \times J$ deprivation matrix $\mathbf{G} = [g_{ij}]$. Each row of the $\mathbf{G}$ matrix corresponds to an i individual and each column to a binary indicator of presence or absence of deprivation in each dimension.

The AF methodology continues by aggregating deprivations across dimensions for each i person with a c_i metric:

$$c_i = \sum_{j \in J} g_{ij}. \tag{3.3}$$

Then, a threshold k to identify the multidimensionally deprived is placed over the c_i metric. As a result, any i individual satisfying $c_i \geq k$ is identified as multidimensionally deprived.

Subsequently, the g_{ij} element from the $\mathbf{G}$ matrix is censored to zero in case the i individual is identified as not multidimensionally deprived, namely $g_{ij}(k) = 0$ for any i individual that satisfies $c_i < k$. Thus, $g_{ij}(k)$ denotes the i row and j column element of the $\mathbf{G}$ matrix after the identification of the multidimensionally deprived.

To obtain societal metrics, the simplest measure that AF proposes is the H-multidimensional deprivation incidence. This first metric corresponds to the proportion of people identified as multidimensionally deprived using the k threshold.

The second most important societal metric that AF proposes and that is currently in use by most of the applications of the method is the M_0-adjusted headcount ratio. AF defines the adjusted headcount ratio as $M_0 = \mu(g_{ij}(k))$, where $\mu(g_{ij}(k))$ corresponds to the average $g_{ij}(k)$ for $i = 1, 2, \ldots, I$ and $j = 1, 2, \ldots, J$.

3.3 The proposed family of multidimensional deprivation indices

This section motivates and presents the proposed family of multidimensional deprivation indices, as an extension of the AF methodology, which explicitly takes into account differences in needs among demographically heterogeneous units. The AF methodology and multidimensional methodologies available in the literature, such as Tsui (2002), Bourguignon & Chakravarty (2003), Seth (2009, 2013), and Rippin (2010) have all been developed using individuals as the unit of analysis and do not analyse the comparability problems that differences in needs might bring to multidimensional deprivation measurement.

In particular, when measuring deprivation, demographic heterogeneity plays a central role in the definition of what can be considered a lack of a minimum achievement. Children, for instance, can be considered deprived when they are not accessing basic education services, unlike adults, who can be considered deprived in the same education dimension when they do not know how to read and write. As another example, while adult populations that do not have access to job opportunities despite seeking them can be defined as deprived in employment, children cannot be defined as deprived in the absence of employment.

A long-standing tradition of policy indicators evaluates deprivation for each particular achievement over a specific sub-population of interest. For instance, one of the Millennium Development Goals launched by the United Nations Development Programme and adopted by several countries to be achieved by 2015 is universal primary education. Another MDG is universal access to reproductive health. Both access to primary education and access to reproductive health services are relevant for measurement only among their particular applicable populations, which are children 6 to 15 years of age and pregnant women, respectively.

When it comes to measuring multidimensional deprivation, these differences in needs, reflected by the different populations where each indicator is applicable to be measured, bring comparability challenges to measuring how many dimensions in deprivation a particular individual or household might exhibit to be catalogued as multidimensionally deprived.

The applied multidimensional deprivation literature addresses these differences in needs by restricting individual-based measures of multidimensional deprivation to the analysis of demographically homogeneous individuals or, in the case of household-based measures, by either assuming the same set of needs across households or ignoring the fact that demographically dissimilar households have significantly different needs.

For instance, in terms of the individual-based applied multidimensional literature, a majority of these studies focus on measuring multidimensional deprivation among either children or adult populations. Examples of child multidimensional deprivation include studies such as Roelen et al. (2010), Roche (2013), Trani & Cannings (2013), Trani et al. (2013), and Qi & Wu (2014). Examples of studies that focus on multidimensional deprivation among an adult population include Oshio & Kan (2014) and Solaymani & Kari (2014).

In contrast, household-based applications of multidimensional deprivation measurement identify as most deprived those households that exhibit the largest number of dimensions in deprivation. Examples of such an empirical approach are the global MPI launched by the United Nations Development Program (Alkire et al. 2014), the Mexican official methodology of poverty measurement (Coneval 2010), the Colombian Multidimensional Poverty Index (Angulo et al. 2016), and the Chilean National Multidimensional Poverty Index (MDS 2014). Along with these policy-oriented indices, there is an applied academic literature in which multidimensional deprivation is analysed using the household as the unit of analysis and assuming the same set of needs across households. Examples include Alkire

& Seth (2015), Alkire & Santos (2014), Ayuya et al. (2015), Bader et al. (2016), Cavapozzi et al. (2015), Mitra (2016), Alkire, Roche, Seth & Summer (2015), and Yu (2013).

However, the larger and more demographically heterogeneous a household is, the greater its needs might be. Thus, small and demographically homogeneous households might register a systematically lower number of dimensions in deprivation, and conversely, larger and demographically heterogeneous households can exhibit a systematically larger number of dimensions in deprivation.

The proposed method of this chapter enables the measurement of multidimensional deprivation across heterogeneous units (i.e., households of different sizes and compositions or individuals from different demographic sub-population groups) while taking into account observable differences in need. The following sections describe these methods. Specifically, Section 3.3.1 below presents some basic definitions, subsequently Section 3.3.2 continues describing the proposed method when selecting household as the unit of analysis and then, as an extension of household-based measures, the individual-based method is presented.

3.3.1 Basic definitions

The proposed methodology of this chapter begins by defining for each j achievement the sub-population group for which it is relevant to be measured. We call this the *applicable population* for achievement j, and we will measure the presence or absence of the j deprivation only within this set of sample units. This feature of our methodology captures individual differences in needs, corresponding to the traditional approach in the policy context to tracking indicators. With this feature, we bridge the gap between theoretically developed multidimensional indices and policy-oriented single indicators design.

This feature is formalized with an $I \times J$ matrix of applicable populations that we call $\mathbf{S}$. There are as many as J applicable sub-population groups, and any two applicable populations are not necessarily mutually exclusive. The cell s_{ij} of the matrix $\mathbf{S}$ is an indicator variable that takes a value of 1 if and only if the i-individual belongs to the applicable population of the j-achievement, and 0 if and only if the i-individual does not belong to the applicable population of the j-achievement. For instance, according to the Millennium Development Goals access to primary education is relevant to be measured among school-age children, then, cell s_{ij} takes a value of 1 whenever the i-individual is aged 6 to 15 years old and zero in case the individual is outside this age range.

Any observed j achievement for the i person that does not belong to the applicable population of such achievement is, therefore, defined as unimportant for the measurement process. Thus, the g_{ij} individual dimensional deprivation indicator evaluated on its applicable population is denoted by $g_{ij}(s_j)$ and takes the form of:

$$g_{ij}(s_j) = \begin{cases} 1 & \text{if } a_{ij} < z_j \text{ and } s_{ij} > 0 \\ 0 & \text{otherwise,} \end{cases} \tag{3.4}$$

where s_j denotes the applicable population of the j achievement.

3.3.2 Household-based metrics

Considering household as the unit of multidimensional deprivation analysis implies understanding the burden that deprivation places as shared among household members. For instance, while child mortality refers to a particular episode that is suffered by children, using the household as the unit of deprivation analysis implies that this episode is understood as a phenomenon that not only affects children but also the household as a whole. The living conditions and behaviours of household members contribute into reducing or increasing the frequency of such situation, and the burden of the episode is faced collectively by the household.

In fact, there is a growing literature on the measurement of deprivation, which takes into account the intrahousehold externalities that arise from the presence of a deprived household member. This is the case of the proposed approach of Basu & Foster (1998) to measure literacy by taking into account not only individuals ability to read and write but also the additional advantage that illiterate households members have from the presence of literate members in the household. Extensions have been developed by Subramanian (2004), Subramanian (2008) and Chakravarty & Majumder (2005), and a similar approach but for the case of the unemployment rate has been proposed by Basu & Nolen (2008).

In this vein, also household-based decision making, and in general collective-based decision making has being broadly studied by the economic literature. Examples are the studies of Chiappori (1992) and Corfman & Lehmann (1987), who model and analyse this type of decision making process.

Yet, there is no consensual empirical evidence of whether or not households behave as a collective unit. A classic example in the literature of risk pooling evidence among household members to protect the collective unit from adverse shocks is the Townsend (1994) study of three poor high-risk Indian villages. In that particular setting, Townsend (1994) found that contemporaneous household consumption is not dramatically influenced by transitory shocks, such as unemployment or sickness. Conversely, there is also evidence that individual risk is only partially pooled among household members because competitive objectives among them might arise. Examples of this evidence are studies such as Hayashi et al. (1996), Doss (2001), and Dercon & Krishnan (2000). This latter literature suggests absence of full risk pooling among members but partial and heterogeneous risk pooling depending on characteristics such as age, gender, and cultural traditions, in each of those analysed contexts.

As such, the proposed methodology of this chapter enables using either the household or the individual as the unit of analysis. It recognizes that selecting individuals or households have embedded different normative criteria that need to be analysed and defined according to the purposes of each particular application. In the particular case of household-based measures, my methodology follows the intuition of the economic literature on the measurement of deprivation as a household-based phenomenon, and are developed under the premise that household members jointly face deprivation, whenever it occurs to a particular member. As a matter of fact, my household-based measures represent the burden that individual deprivation places over the household as a collective unit, and they are approached through combining the deprivation profiles of household members. The following paragraphs formalize this proposed methodology.

Consider then that each individual belongs to a particular h household, and each household contains q_h household members. The d_{hj}^{β}-dimensional deprivation indicator for the h household and the j dimension is, thus, defined as:

$$d_{hj}^{\beta} = \begin{cases} \left(\sum_{i \in q_h} g_{ij}(s_j)\right)^{\beta} & \text{if } \sum_{i \in q_h} g_{ij}(s_j) > 0 \\ 0 & \text{otherwise,} \end{cases} \tag{3.5}$$

where $\beta \in \{0, 1\}$ is the parameter of aversion to deprivation. Larger values of β assign increasing value to the most deprived dimensions (i.e, those with greatest number of j deprived household-members). Whenever $\beta = 0$, then household dimensional deprivation is expressed by a $\{0, 1\}$ indicator of absence or presence of at least one j deprived household-member. On the other hand, if $\beta = 1$, then dimensional deprivation is expressed by the count of deprived household members in the j dimension.

The β parameter of aversion to deprivation is analogous to the α parameter of poverty aversion introduced by Foster et al. (1984) and used by Alkire & Foster (2011) to assign increasing value to those dimensions with biggest shortfall gap ratio $((z_j - a_{ij})/z_j)$. Similar to the α parameter of the AF method, whenever $\beta = 0$, dimensional deprivation is expressed as an indicator of presence or absence of deprivation in the j dimension. However, while in the AF method $\alpha > 0$ can be used only in case the j dimension is captured by a cardinal achievement indicator, here $\beta = 1$ commensurates the household deprivation breadth in the j dimension, without necessarily enforcing the use of cardinal achievement indicators and in terms of the number of j deprived household-members.

As a result of the ordinal nature of most of policy indicators, current household-based applications of the AF method have been restricted to measure the burden that dimensional deprivation places on the household by indicating the presence or absence of at least one household member under deprivation in this dimension. This particular approach corresponds to using d_{hj}^{0} to express dimensional deprivation, which is setting $\beta = 0$.

The use of $\beta = 0$, however, does not allow household metrics to be sensitive to increments in the number of deprived persons in an already deprived dimension. For instance, when evaluating access to primary education, a household with two school-aged children, one child attending school and the other not attending, registers $d^0_{hj} = 1$. Now, if this same household, as a result of a deprivation increment, increases its number of children who are not attending school to two, its d^0_{hj} indicator remains invariant.

In contrast, our proposed methodology enables expressing household dimensional deprivation with any $\beta = 1$, which produces a measure of dimensional deprivation that is sensitive to increments in the number of deprived persons in already deprived dimensions. For instance, in the example of the previous paragraph, if we evaluate school attendance in the household with one deprived school-age child, then $d^1_{hj} = 1$. But if the household has two children deprived of school attendance, then $d^1_{hj} = 2$, a value that is twice as large that of the initial case. I further discuss and illustrate this advantage of the proposed method when discussing the properties of our societal metrics in Section 3.6 ahead on.

Still, not every household has the same set of dimensional needs. In fact, the number of j applicable household members generally varies across households. To account for this, we define n^β_{hj} to be the size of the h household needs on the j dimension:

$$n^\beta_{hj} = \begin{cases} \left(\sum_{i \in q_h} s_{ij}\right)^\beta & \text{if } \sum_{i \in q_h} s_{ij} > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{3.6}$$

Two important cases are obtained by setting $\beta = 0$ or $\beta = 1$: n^0_{hj} indicates whether the household has need in the j dimension or not (i.e., has at least one household member that could suffer deprivation in such dimension); and n^1_{hj} informs the number of household members that exhibit need in the j dimension. For instance, in our same example of school attendance, since the h household has two school-age children, then we know that $n^0_{hj} = 1$ and $n^1_{hj} = 2$.

Using this n^β_{hj}-dimensional size of household needs from Eq. (3.6) we can express the size of household multidimensional needs as:

$$N^\beta_h = \sum_{j \in J} n^\beta_{hj}, \tag{3.7}$$

where N^0_h counts the number of dimensions that the h-household exhibit as need and N^1_h counts the number of achievements that the h-household exhibit as need.

The second stage of our proposed methodology consists of aggregating household deprivations across dimensions, discounted by needs, to obtain multidimensional profiles. In particular, we propose measuring the burden that multidimensional deprivation places on the household with a functional form that enables capturing either count-based, shared-based or a mixture of these two types of measures. In this vein and following Cowell & Mercader-Prats (1999) and Buhmann et al. (1988) one-dimensional equivalence scale presented in Eq. (3.1) from page 68, we express the burden of multidimensional deprivation as:

$$m_h^{\beta,\theta} = \begin{cases} \dfrac{\sum_{j \in J} d_{hj}^{\beta}}{\left(\sum_{j \in J} n_{hj}^{\beta}\right)^{\theta}} & \text{if } \displaystyle\sum_{j \in J} n_{hj}^{\beta} > 0 \\ 0 & \text{otherwise,} \end{cases} \tag{3.8}$$

where $\theta \in [0, 1]$ is a deprivation response scale parameter that reflects the relativity of the response of the burden of deprivation to the scale of household needs. In the case that $\theta = 0$, we are in the presence of a count-based approach, and no discounting in needs is applied at all. Thus, the household is assumed as not receiving any advantage from the cooperative unit, and therefore, the burden that deprivation places on the household is not lightened to any degree from the scale of the needs. On the other hand, when $\theta = 1$, we are using a share-based approach. While the count-based structure places greater emphasis on larger households without accounting for any possible scale economy that might arise at this level, the share-based approach places greater emphasis on small households because they are more prone to registering the maximum possible burden of deprivation. Values of θ different than 0 or 1, aim to describe $m^{\beta,\theta}$ through an intermediate approach that lie in between of count-based and share-based perspectives. Henceforth, the expression $m^{\beta,\theta}$ is used to refer to the different values of the measure defined in Eq. (3.8) for the whole population of households as β or θ varies.

Current household-based policy-oriented indices that use the AF method measure the burden of multidimensional deprivation through the $m^{0,0}$ metric, which corresponds to counting the number of dimensions in deprivation. However, the $m^{0,0}$ metric does not discount by household needs at all. It does not differentiate the deprivation burden of non-deprived and non-applicable dimensions.

This induces observing a systematically lower burden of multidimensional deprivation across small and demographically homogeneous households, as I empirically demonstrate in Section 3.4.

In contrast, my proposed $m_h^{\beta,\theta}$ family of measures allows not only a count-based approach to measurement, which corresponds to setting $\theta = 0$; but also takes into account heterogeneous household needs within and across dimensions through using any $0 < \theta \leq 1$. In fact, whenever θ is set in such interval, the burden of household multidimensional deprivation is discounted by the household needs and takes into account the scale advantages that the household receives to lighten the burden that deprivation places on it.

3.3.3 More on the $m_h^{\beta,\theta}$ proposed family of measures

At this point, the $m^{\beta,\theta}$ family of measures has been obtained upon first aggregating individuals' deprivation at the household level for each dimension and then aggregating deprivations across dimensions. This particular strategy is termed as a *first-individuals-then-dimensions* aggregating order. Nonetheless, a second possible course of action can consist of first aggregating dimensions in deprivation at the individual level to obtain individual multidimensional profiles and then aggregating across individuals to obtain household metrics. This second approach is referred as a *first-dimensions-then-individuals* aggregating order.

Each particular order leads to a different set of measures. My proposed household-based methodology is restricted to the use of a *first-individuals-then-dimensions* aggregating order. Table 3.1 describes the idea behind the four key most intuitive metrics that $m^{\beta,\theta}$ captures on the basis of this selected aggregating order. Only the members of the proposed family of household measures that use $\beta = 1$ are non-sensitive to the order in which they are constructed.

Though both aggregating orders enable household dimensional deprivation metrics to be cardinal rather than providing merely ordinal profiles, the first-individuals-then-dimensions selected order prevents invisibility of the multiple dimensions of deprivation. In other words, the opposite order would conduce the expression of $m_h^{0,\theta}$ in terms of the number of household members with at least one j deprived dimension, disregarding the number of dimensions of deprivation

Table 3.1: Resulting measure of the $m_h^{\beta,\theta}$-burden of multidimensional deprivation across a selected combination of parameters and using a first-individuals-then-dimensions aggregating order

Combination of parameters	Resulting $m_h^{\beta,\theta}$ measure
$\beta = 0$, $\theta = 0$	Count of dimensions with at least one household member under deprivation. Measure comparable with the c metric of the AF method and termed the *dimensions-count-based* approach to measurement.
$\beta = 0$, $\theta = 1$	Share of possibly deprived dimensions. Measure termed the *dimensions-share-based* approach to measurement.
$\beta = 1$, $\theta = 0$	Count of household deprivations. Measure termed the *deprivations-count-based* approach to measurement.
$\beta = 1$, $\theta = 1$	Share of household possible deprivations. Measure termed the *deprivations-share-based* approach to measurement.

that the household may be exhibiting. Thus, it would not evidence the many different j dimensions of deprivation suffered by households at the same time. In addition, using a first-individuals-then-dimensions as the selected order enables my proposed family of measures to encompass the AF approach to measurement. Then, it allows comparability with regard to current household-based applications.

In the selected first-individuals-then-dimensions aggregating order (Table 3.1), the use of $\beta = \{0, 1\}$ switches $m_h^{\beta,\theta}$ between being a count of household dimensions of deprivation (i.e., whether someone in the household is deprived, $\beta = 0$) and being a count of those members who are deprived ($\beta = 1$). Deprivation aversion captured by $\beta > 0$ assigns greater value to the most deprived dimensions.

In contrast, $\theta = \{0, 1\}$ switches $m_h^{\beta,\theta}$ between being a count-based measure of household deprivation (i.e., one in which the denominator is switched off, $\theta = 0$) and being a share-based measure ($\theta = 1$). Values of θ different from 0 and 1 aim to describe $m^{\beta,\theta}$ as an intermediate approach between share-based and count-based measures.

In general, count-based and share-based measures can be considered to capture two different conceptions of inequality. While count-based measures depict an ‘absolute’ conception of inequality, share-based measures a ‘relative’ conception of inequality. According to Kolm (1976*a*,*b*) and Shorrocks (1983), in the context of income inequality, a relative measure of inequality is one that remains invariant under a variation of income in the same proportion for all incomes in society. In addition, according to scholars, an absolute measure of inequality does not change under an equal absolute variation of income for all incomes in society. Absolute and relative measures of inequality have been analysed by the inequality literature under a common framework as alternative approaches to measurement; examples include the studies of Kolm (1976*a*,*b*), and Shorrocks (1983). Intermediate indices of inequality have also been analysed by literature. Examples include Bossert & Pfingsten (1990) and Chakravarty & Tyagarupananda (2009), which express intermediate inequality indices as a mixture of relative and absolute measures of inequality.

In the specific case of the $m^{\beta,\theta}$ family of indices proposed in this chapter, I follow the embodied intuition of the inequality literature and express the $m^{\beta,\theta}$-burden of multidimensional deprivation in terms of a θ parameter that allows us to capture different conceptions of inequality. The use of a count-based approach to measurement assigns an equal absolute value to each dimension or each deprivation. Whereas, under a share-based approach to measurement, the household burden of multidimensional deprivation is expressed in relation to the potential number of dimensions or deprivations that the household could possibly suffer.

Although the discussion about the pertinence of absolute, relative, or intermediate indices to analysis of the distribution of the population within a particular achievement might date from the 1970s, there remains little agreement about which approach is more pertinent for any society, mostly because they are based on value judgements about what can be considered just or unjust, so any decision must be context specific.

We now proceed to describe the method proposed in this chapter to identify the most deprived households.

3.3.4 Identification of the multidimensionally deprived

For a given combination of β and θ, households exhibiting at least a k burden of multidimensional deprivation are identified as the multidimensionally deprived. Parameter k represents the multidimensional deprivation threshold above of which the most deprived household are observed. The k threshold takes values between zero and the maximum possible observable $m_h^{\beta,\theta}$. For instance, applications as the Colombian index of multidimensional poverty have set k, under a combination of statistical methods and empirical findings, as the 33% of the maximum weighted sum of dimensions on deprivation (Angulo et al. 2016). A similar 33% cut-off point over the weighted sum of deprivations have been used by Alkire et al. (2014) for the global MPI and by Battiston et al. (2013) for a proposed index in the context of six Latin American countries. The plausible k is to be defined according to the context of each application.

Having set the k threshold, it naturally arises a binary indicator of presence or absence of multidimensional deprivation, p_h, as follows:

$$p_h = \begin{cases} 1 & \text{if } m_h^{\beta,\theta} \geq k \\ 0 & \text{otherwise.} \end{cases} \tag{3.9}$$

While applications of the AF method sort households under the basis of $m^{0,0}$ and households satisfying $m_h^{0,0} > k$ get identified as the multidimensionally deprived, the proposed methodology of this chapter enables the identification of the most deprived to be done under the basis of any $m^{\beta,\theta}$. The implications that different $m^{\beta,\theta}$ measures have on identifying the multidimensionally deprived are investigated and discussed in Section 3.4. We continue presenting the proposed methodology for aggregating household multidimensional deprivation at the society level.

3.3.5 The family of societal measures

Suppose that R is the total number of households. Then, as proposed by Alkire & Foster (2011), the simplest metric to represent the overall society multidimensional deprivation incidence is:

$$H = \mu(p_h), \tag{3.10}$$

where $\mu(p_h)$ corresponds to the average value of p_h for $h = 1, 2, \ldots, R$. In line with the AF methodology, H corresponds to the rate of societal multidimensional deprivation incidence.

Since the proposed methodology of this chapter allows identifying the multidimensionally deprived population under the basis of any $m^{\beta,\theta}$, we denote the proportion of multidimensionally deprived population identified on the basis of a particular $m_h^{\beta,\theta}$ metric as $H(m^{\beta,\theta})$. One case, worth highlighting, is the H proportion of multidimensionally deprived population identified on the basis of the AF' $m^{0,0}$ sorting metric, which is henceforth denoted as $H(m^{0,0})$.

On the other hand, to construct societal metrics of the average burden that multidimensional deprivation places across households, and as the AF method proposes, we censor to zero any $m_h^{\beta,\theta}$ for non-multidimensionally deprived households, namely, $m_h^{\beta,\theta} = 0 \ \forall \ h \ s.t. \ p_h = 0$. We denote, therefore, the household burden of multidimensional deprivation after the identification of the multidimensionally deprived with the k threshold as $m^{\beta,\theta}(k)$. As a result, the societal mean burden of multidimensional deprivation is defined as:

$$MD^{\beta,\theta} = \mu(m^{\beta,\theta}(k)), \tag{3.11}$$

where $\mu(m^{\beta,\theta}(k))$ corresponds to the average value of $m^{\beta,\theta}(k)$ for $h = 1, 2, \ldots, R$. In this case, our $MD^{0,0}$ metric corresponds to the M_0 metric of the Alkire & Foster (2011) method. In comparison to the M_α family of measures of the AF method, our proposed $MD^{\beta,\theta}$ constitutes a broader set of measures that takes into account count-based, share-based and intermediate approaches to measure the burden that multidimensional deprivation places on the household.

In general, given the ordinal nature of policy indicators, most current applications on the Alkire & Foster (2011) method are able to describe societal multidimensional deprivation through $H(m^{0,0})$ and $MD^{0,0}$. Our proposed approach, in contrast, allows describing the multidimensional deprivation in terms of any $H(m^{\beta,\theta})$ and $MD^{\beta,\theta}$ with $\beta \in \{0, 1\}$ and $\theta \in [0, 1]$.

The range of variability and the characterization that make the proposed family of societal measures satisfactory for the purposes of multidimensional deprivation measurement are investigated and discussed in Section 3.6 ahead on.

3.3.6 Weights

For completeness purposes and to guide applications where dimensions have different relative importance across each-other, in this section we introduce and describe a weighting system to differentiate these relative importances. We therefore, introduce the $\mathbf{w} = (w_1, w_2, \ldots, w_J)$ vector of non-negative importance weights, where $w_j \geq 0$ denotes the relative importance weight for the j achievement in the overall deprivation evaluation, and satisfies $\sum_{j=1}^{J} w_j = 1$. This weighting system can be used to aggregate deprivations across the J dimensions and obtain the burden of multidimensional deprivation as:

$$\mathbb{m}_{\mathbb{h}}^{\beta,\theta} = \begin{cases} \dfrac{\sum_{j \in J} w_j d_{hj}^{\beta}}{\left(\sum_{j \in J} w_j n_{hj}^{\beta}\right)^{\theta}} & \text{if } \displaystyle\sum_{j \in J} w_j n_{hj}^{\beta} > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{3.12}$$

This $\mathbb{m}_{\mathbb{h}}^{\beta,\theta}$-burden of multidimensional deprivation represents the $\mathbf{w}$ scaled variant of Eq.(3.8). The application of the $\mathbf{w}$ dimensional weights produces, subsequently, societal measures H and $MD^{\beta,\theta}$ to be updated using this $\mathbf{w}$ scaled variant of $m_h^{\beta,\theta}$.

The selection of these dimensional weights can be devised according to the purpose of the measure and by different alternative procedures such as normative selection or data-driven techniques. For a discussion of alternatives to setting weights in a multidimensional index, see Decancq & Lugo (2013).

3.3.7 The individual-based scenario

Whenever individuals, rather than households, are selected as the unit of multidimensional deprivation analysis, differences in needs are observed across different demographic sub-population groups, as for instance across population from different ranges of age or gender. While pregnant women, for instance, need to access to antenatal health services, school aged children need to access to basic educative services. Deprivation in antenatal health services is, therefore, relevant to be measured exclusively across pregnant women, as it is access to basic educative services across school aged children.

Given that individuals from different demographic sub-population groups exhibit differences in needs, current applications of the Alkire & Foster (2011) multidimensional deprivation method, that use the individual as the unit of analysis, tackle these differences in needs by restricting the analysis to arguably homogeneous demographic sub-population groups and a set of comparable indicators. For instance, Oshio & Kan (2014) studied the association between multidimensional poverty and health among individuals aged 20 to 59 years old, using for the multidimensional poverty index indicators such as: low education attainment, non-coverage to public pension and household income. Batana (2013), on the other hand, studied multidimensional poverty in fourteen Sub-Saharan African countries by restricting the analysis to women between 15 and 49 years old and using indicators of assets, access to health services, schooling and empowerment. Also, examples of multidimensional poverty focused on children are Trani & Cannings (2013), Qi & Wu (2014), Roelen et al. (2010), and Trani et al. (2013) for Western Darfur (Sudan), China, Vietnam and Afghanistan, respectively. To date in my knowledge, no application of individual-based multidimensional deprivation taking into account the whole age range of the population has been carried out.

The methodology introduced here can be used to enable individual-based multidimensional deprivation measurement in presence of different needs across demographically heterogeneous sub-population groups. In the context of this chapter, this approach is named as the individual-based scenario, as it is derived as a special case of the previously described household-based measures.

Particularly, in this proposed individual-based scenario each household in the society is assumed as consisting of one member, which simply implies each person is in its own household. The afore-presented household-based measures are consequently derived. Hence, the dimensional deprivation indicator and the burden of multidimensional deprivation, both are obtained without aggregating at the household level.

Specifically, given that the d^{β}_{hj}-dimensional deprivation indicator for the h household in the j dimension, was developed as an aggregation of the household members' $g_{ij}(s_j)$ individual deprivation indicators to the power of β (Eq.(3.5));

then, this aggregation and the β parameter have no relevance in an individual-based scenario because in this case the resulting measure is always a binary variable of presence or absence of deprivation, which is simply $g_{ij}(s_j)$. Consequently, the $m_h^{\beta,\theta}$-burden of multidimensional deprivation for the h household (Eq.(3.8)), becomes also non-sensitive to different values of β and expressed independently for each i individual. We denote this variant of Eq.(3.8) as m_i^θ.

Still, the use of the θ parameter in the individual-based scenario expresses the responsiveness of deprivation to the size of the individual's needs. Similar to the household-based case, in the individual-based scenario, the use of the θ parameter allows expression of the multidimensional deprivation burden that the i individual suffers, either as a count of dimensions on deprivation, a proportion of dimensions of deprivations or any mixture of these two types of measures.

The use of this individual-based scenario naturally produces an identification of the most deprived to be done sorting individuals with any m_i^θ measure and defining as multidimensionally deprived those satisfying $m_i^\theta > k$. Societal measures H and MD, are therefore, developed using the individual-based variants of the measures.

The individual-based proposed approach with $\theta = 0$, worth noting, corresponds to the individual-based AF methodology. In this case the proportion of multidimensionally deprived individuals is expressed by $H(m_i^0)$ and the MD^0 metric results equivalent to the AF metric M_0.

Another approach to measure individual-based multidimensional deprivation might be, for instance, setting a weighting system to account for the observed heterogeneous needs. This means using a dimensional weighting system ($\mathbf{w} = (w_1, w_2, \ldots, w_J)$) differentiated by sub-population groups. With such an approach, it is possible to ensure that each sub-population group exclusively weights their relevant indicators, such that the sum across w_j adds to 1, for each sub-population group. It is worth noting, however, that in this case each dimension results in not having the same normative value across individuals in society and m^θ is always restricted to the share-based approach.

We now proceed to evaluate throughout different methods the implications of using different possible measures to identify the multidimensionally deprived population.

3.4 Evaluating measures

Examples of applications of the Alkire & Foster (2011) method that select household as the unit of analysis are Alkire et al. (2014), Alkire, Roche, Seth & Summer (2015), Angulo et al. (2016), Alkire & Seth (2015), Alkire & Santos (2014), Ayuya et al. (2015), Bader et al. (2016), Cavapozzi et al. (2015), Mitra (2016), Alkire, Roche, Seth & Summer (2015), and Yu (2013). The above literature measures the burden of multidimensional deprivation through the household count of deprived dimensions, strategy termed in Table 3.1 the *dimensions-count-based* approach to measurement. In this section, I evaluate the effects on multidimensional deprivation profiles of using such an approach and compare it to those obtained using other members of the family of measures proposed in this chapter. The analysis is carried out making use of the data that is presented in the next section.

3.4.1 Data

For the empirical analysis in this chapter, a household-based multidimensional deprivation index is built using the 2013 Paraguayan Household Survey (PHS). The PHS is a cross-sectional living conditions survey that has been collected yearly since 1984 by the Paraguayan National Statistical Department. Referred to as the *Encuesta Permanente de Hogares*, it captures a broad range of living condition indicators. The survey provides national estimates for income poverty, inequality, and some key quality of life descriptors. The questionnaire of the PHS 2013 includes information regarding education, health, the labour market, individual income, dwelling conditions, and international migration and a special module for agriculture and forestry activities.

The PHS 2013 used a two-stage, clustered probabilistic sample design that was stratified in the first stage by 31 geographical domains. The strata corresponded to

Table 3.2: Example of multidimensional indicator: Dimensions, indicators, weights, applicable population and deprivation criteria

Well-being dimension	Deprivation indicator	Applicable population where the indicator is relevant to be measured	A person from the applicable population is deprived if:
Health	Health insurance non-coverage	Any person	Does not have access to health insurance coverage.
	Non-access to health services	Any person that was sick or had an accident during the 90 days previous to the interview	Did not receive institutional care*.
Education	Non-school attendance	5 - 17 years old population	Is not attending school.
	Low educational achievement	Population 18 years old and over	Has less than 9 years of completed education.
Dwelling conditions	Sub-standard housing	Any person	Lacks at least 2 of the following 3 dwelling conditions: flooring different from earth or sand; adequate material of ceilings**; and adequate material of walls***.

Notes: *Institutional care corresponds to attention received by a professional health worker (physicist, nurse, dentist or professional midwife) in private or public health institution (It is not a health care institution: pharmacy, empirical medicine man store, own house, other's house). **Inadequate ceiling material refers to the following: Straw, eternit, clapboard, palm trunk, cardboard, rubber, packaging timber, other. ***Inadequate wall materials refer to the following: wattle, mud, wood, palm trunk, cardboard, rubber, wood, another material, or no wall at all.

rural and urban areas of 15 out of the total 17 Paraguayan counties (*departamentos*) and the national capital of Asunción. The sample allows for total national, urban, and rural area estimates, as well as for disaggregation throughout seven geographic domains. The first geographic domain corresponds to Asunción, the Paraguayan capital city. The next five domains correspond to the national counties of San Pedro, Caaguazú, Itapúa, Alto de Paraná, and Central. The seventh and last domain corresponds to the aggregation of the 12 remaining Paraguayan countries. In 2013, the PHS was collected from a sample of 21,207 persons across 5,424 households.

Table 3.2 describes the items included within the multidimensional deprivation index constructed for the analysis purposes of this chapter. In particular, this index example captures information on health, education, and dwelling conditions across five deprivation indicators: health insurance non-coverage, non-access to health services, non-school attendance, low educational achievement, and substandard housing.

Note that here, for illustrative purposes, a deprivation indicator of substandard housing was included. In such a case, the applicable population of this indicator corresponds to any household member and is defined as deprived whenever the housing lacks from at least 2 of the 3 considered dwelling conditions (flooring different from earth or sand, adequate material of ceilings, and adequate material of walls).

Of the 21,207 interviewed individuals for PHS 2013, we excluded from the analysis 264 observations that do not belong to the household unit (i.e., domestic personnel), and 34 observations were also excluded because of non-response to at least one of the five considered indicators. Thus, our effective sample comprises 20,909 interviewed persons across 5,423 households.

3.4.2 Observed multidimensional deprivation incidence profiles

As described on Section 3.3.2, a dimensions-count-based approach implies measuring household dimensional deprivation in terms of whether or not there is at least one household member facing deprivation, and subsequently, households are compared in terms of the number of deprived dimensions. Multidimensionally deprived households are those exhibiting a majority of these deprived dimensions.

Table 3.3 presents the proportion of households with at least one deprived household member in each of the five dimensions considered in this application. This corresponds to the mean d^0_{hj}-dimensional deprivation indicator across the 5,423 observed Paraguayan households by household size. Reading the table by lines, it can be seen that larger households exhibit a larger proportion of dimensional deprivation than smaller households. The dimensions more prone to this effect are health insurance non-coverage, non-access to health services, non-school attendance, and low educational achievement. The positive relation between household size and dimensional deprivation is observed because the number of persons in the applicable population increases as the household size increases. Take, for instance, the non-school attendance indicator in Table 3.3, which is applicable for children 5 to 17 years of age. One-person households are rarely composed by this population subgroup because school-age children cannot form a household. Therefore, the proportion of households consisting of one person that are dimensionally deprived in school attendance is 0%. Conversely, 21.4% of households consisting of seven or more persons are deprived of school attendance because they contain in average 4 children.

If, subsequently, household dimensions of deprivation are counted and the

Table 3.3: Proportion of households with at least one deprived person from the applicable population (%)

	Persons per household							Total
	1	2	3	4	5	6	7 or more	
(1) Health insurance non-coverage	70.3	75.2	79.9	81.0	84.3	91.4	93.8	81.3
(2) No access to health services	12.5	17.7	15.1	19.7	20.1	23.6	29.6	19.0
(3) Non-school attendance	0.0	1.9	2.9	4.4	5.8	9.7	21.4	5.5
(4) Low educational achievement	61.4	64.8	57.8	65.4	68.6	78.3	88.3	67.0
(5) Sub-standard housing	25.5	25.0	18.5	19.9	23.5	24.9	34.2	23.3
Sample number								
Number of households	593	836	1,135	1,108	771	466	514	5,423
% of individuals	2.8	8.0	16.3	21.2	18.4	13.4	19.9	100

Source: Author's calculations based on 2013 PHS.

multidimensionally deprived households are those with the largest count of these dimensions on deprivation, larger and more heterogeneous households tend to be identified as the most deprived. The following paragraphs elaborate further on this.

With the purpose of comparing the multidimensionally deprived population of households identified using different $m^{\beta,\theta}$ measures, households are sorted on the basis of each $m^{\beta,\theta}$ score and the first 40% most deprived (2,168 households) are identified as multidimensionally deprived. The population of households identified as the most deprived using the dimensions-count-based approach ($m^{0,0}$) is compared with regard to those obtained using the other three $m^{\beta,\theta}$ measures described in Table 3.1: the dimension-share-based approach ($m^{0,1}$), the deprivations-count-based approach ($m^{1,0}$), and the deprivations-share-based approach ($m^{1,1}$).

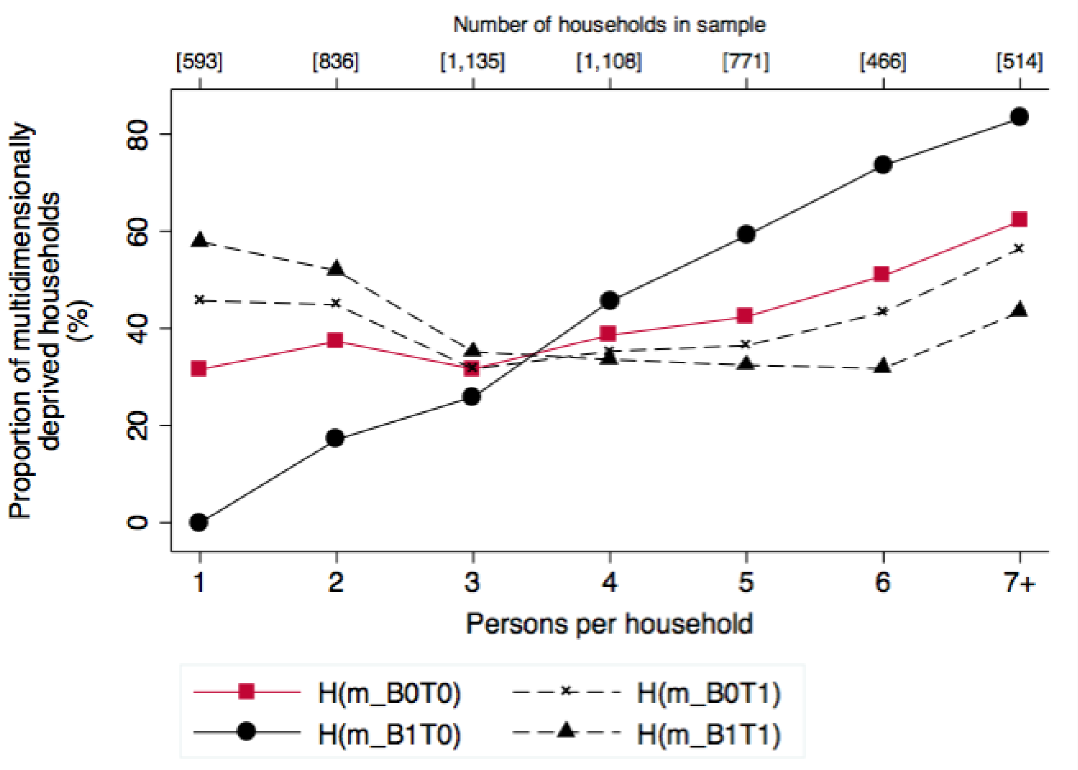
Note that identifying a fixed share of the population (40% in this case) as the most deprived is different from placing a particular k multidimensional threshold over the $m^{\beta,\theta}$ score. Given that the range of variability of $m^{\beta,\theta}$ varies along the β and θ parameters, the use of a fixed share of households enables us to compare the different deprived populations on an equal basis. The particular 40% share of households arose as a plausible natural breaking point in the distribution of deprivations observed by the multidimensional index in the analysis. Nonetheless, in Section **??** I test the robustness of the obtained results under other different possible shares of the population.

Figure 3.1 plots the obtained H-multidimensional deprivation incidence by household size for the four $m^{\beta,\theta}$. No adjustment by differences in needs corresponds to measures that use $\theta = 0$: the dimensions-count-based and the deprivations-count-based approaches. In the figure, the results obtained upon sorting households under a dimensions-count-based approach ($m^{0,0}$) are plotted by square markers. The profile obtained on the basis of a deprivations-count-based approach ($m^{1,0}$) is plotted by circle markers in the figure. The vertical axis corresponds to the proportion of households of each size identified as multidimensionally deprived. For instance, out of the total observed 514 households consisting of seven or more persons, in about 80% of them are identified as multidimensionally deprived when a deprivation-count-based approach is used.

As expected, the results indicate that the H-multidimensional deprivation incidence varies across household size and measures. The profiles obtained upon $m^{\beta,\theta}$ measures that do not account for needs ($m^{0,0}$ and $m^{1,0}$) show the greatest proportion of multidimensionally deprived among large households, as well as, the lowest proportion among small households. In particular, when using the AF-proposed $m^{0,0}$, households consisting of seven or more persons register 29.2 percentage points more multidimensional deprivation incidence than households consisting of one person.

Any $\theta > 0$ enables the burden of household multidimensional deprivation to be adjusted by household needs, increasing the amount of the adjustment as θ increases. Then, contrary to count-based approaches, a deprivations-share-based approach (triangle markers in the figure) produces 57.8% of households consisting of one person being catalogued as multidimensionally deprived and 43.6% of households consisting of seven or more persons being catalogued as multidimensionally

Figure 3.1: Proportion of multidimensionally deprived households, $H(m^{\beta,\theta})$, across household size



Source: Author's calculations based on 2013 PHS.

deprived. Thus, in this case, a 14.2 p.p. higher incidence of multidimensional deprivation is observed among smaller households than across larger households.

These descriptive statistics suggest that identifying the most deprived on the basis of a household burden of multidimensional deprivation not adjusted by household needs results in greater H-deprivation incidence among larger households. Multidimensional deprivation incidence among larger households reduces as the adjustment by the size of the needs increases. The use of different $m^{\beta,\theta}$ measures to sort households produces different profiles of multidimensional deprivation incidence, and these results are driven by the size of the household needs.

What should we make of these differences? On one hand—as particular studies from the one-dimensional equivalence scale literature suggest—one could argue that there is no correct or incorrect equivalence scale and that different measures are justified according to different circumstances (Cowell & Mercader-Prats 1999, pg.409). In this vein, the selection of the measure to describe household multidimensional deprivation constitutes a context-specific normative definition. While count-based approaches ($\theta = 0$) give either to each dimension (using $\beta = 0$) or to each deprivation ($\beta = 1$) an equal absolute value in the measurement of the burden of multidimensional deprivation, deprivation share-based approaches ($\theta = 1$) give an equal absolute value to each household, disregarding its demographic composition and taking into account the scale economies that arise at this level.

An intermediate normative perspective corresponds to setting the θ parameter between these two solutions. The value of θ reflects the responsiveness of the burden of deprivation to the scale of needs; values of θ close to zero convey a lower response of the burden of multidimensional deprivation to the size of the needs. Conversely, values of θ close to one convey a greater response of the burden of deprivation to the size of the needs.

On the other hand, researchers can consider—as I do in this chapter—differences in need as a 'legitimate' source of variation in the observed multidimensional deprivation profiles that should be tackled by the measurement process. Following the framework set up by Fleurbaey (2008) in social choice on equity, responsibility, and fairness, and in particular the proposed approach of Fleurbaey & Schokkaert (2009) to analyse fair and unfair health and healthcare inequalities, differences in achievement levels (such as health or educational attainment) are considered as caused by myriad factors, some of which can be catalogued as producing fair/legitimate differences and others as producing unfair/illegitimate differences. In particular, for the case of health and healthcare inequalities, Fleurbaey & Schokkaert (2009) defined as legitimate or fair those differences attributed to causes that fall under individuals' responsibility. Legitimate differences in this context correspond, therefore, to those derived from preferences.

In light of this framework, one can argue differences in multidimensional deprivation measurements should not arise from legitimate causes and we should therefore in our methodologies account for the differences that needs bring, as well as for any of other legitimate causes, such as differences in preferences. In this chapter, for the sake of simplicity and as a first effort in the literature to account for differences in needs, we focus on accounting strictly for them. The effect that other sources of fair/legitimate differences, such as preferences, could have over multidimensional deprivation incidence profiles is left for further research. Analysis of the relation between multidimensional poverty and preferences can be found in Decancq et al. (2014).

Therefore, we evaluate how effectively each of the $m^{\beta,\theta}$ measures accounts for differences in needs. The methodology for approaching such evaluation and the results are presented in the following sections.

3.4.3 Method

To determine the ability of any multidimensional deprivation measure to account for differences in needs, we contemplate direct and indirect standardization techniques. As proposed by Fleurbaey & Schokkaert (2009), we consider both standardization techniques in light of their embedded ethical conditions and implications. As such, a desirability condition that resembles an indirect standardization technique is set out here to be attained by a multidimensional deprivation incidence profile. Based on this condition, we determine how much of the observed profile results from differences in needs and this regard its performance is evaluated. The next paragraph describes this condition.

Desirability condition. An *unbiased* multidimensional deprivation incidence profile is such that it is unable to distinguish between two population groups that have no systematic differences in deprivation between each other but only different sets of needs. As such, any two households in a household-based scenario or any two individuals in an individual-based scenario with no systematic difference in deprivation between the two of them must be classified equivalently as either multidimensionally deprived or non-multidimensionally deprived, regardless of the size of their needs.

Multidimensional deprivation incidence profiles that are unable to equivalently classify (as multidimensionally deprived or non-multidimensionally deprived) two households with differences in $m^{\beta,\theta}$, strictly caused by differences in needs, are said to provide a *biased* picture of societal multidimensional deprivation incidence.

If we can confirm that a particular $m^{\beta,\theta}$ measure is able to provide an equivalent measurement for any two households with no systematic differences in deprivation but only differences in needs, we also know that a multidimensional profile based on such an $m^{\beta,\theta}$ measure portrays differences in incidence that are not driven by differences in needs.[5]

Now, given that differences in deprivation originating strictly from differences in needs, cannot be straightforwardly differentiated from factual observed multidimensional deprivation incidence profiles because we do not know from the observed profile how much of the observed differences are due to differences in needs and how much of them are due to the measurement approach. We use a static microsimulation technique to generate a counterfactual deprivation profile in which

[5] Another possible course of action could be using an alternative condition. This alternative condition can be set out in light of a direct standardization procedure. It would define as unbiased multidimensional deprivation incidence profile such that is unable to distinguish two population groups with no systematic differences in needs. However, as Fleurbaey & Schokkaert (2009) discussed, if this alternative condition is satisfied, it is possible that no difference in multidimensional deprivation incidence between two population groups will be observed because they have identical size of needs. This, despite these two populations still might have significant differences in deprivation. If such condition is attained the measure would depict these differently deprived populations under an equivalent multidimensional deprivation incidence. For the purposes of this chapter, this situation is considered ethically undesirable, so we deliberately focus on evaluating our measures only in terms of the selected desirability condition.

the observed differences are strictly due to differences in needs. In such counterfactual scenario, no systematic difference in deprivation exists but only differences in needs.

Then, the evaluation of of our $m^{\beta,\theta}$ measures is approached as a 'controlled experiment' (a term used by Figari et al. (2014) to describe microsimulation techniques) with the data to determine the ability of each measure to observe such counterfactual state of things and therefore to portrait an unbiased incidence profile.

The counterfactual scenario of no systematic difference in deprivation is created, in use of the 2013 Paraguayan Household Survey, by setting as invariant the characteristics of the household that describe differences in need and distributing deprivation completely at random across individuals and households. In other words, we fix the characteristics of the sample members (including whether or not they are members of applicable population subgroups) and then, for each j-dimension, we randomly allocate whether or not they are in deprivation. The random allocation is performed by sampling without replacement from the observed deprivation so that the total number of deprived people is the same in the counterfactual and factual samples.

The random distribution of deprivation emulates no systematic difference because is not related to any individual or household characteristics and thus is not a result of an underlying behaviour or characteristic. By building a (counterfactual) population in which there is no difference in deprivation resulting from these causes, we can determine whether a multidimensional incidence profile based on a particular $m^{\beta,\theta}$ measure is able to make an unbiased comparison.

Any multidimensional deprivation incidence profile satisfying the desirability condition, must exhibit no relation between multidimensional deprivation incidence and the size of household needs in this counterfactual scenario. Thus, we approach the evaluation of each profile in the counterfactual state of no systematic difference via a comparison of multidimensional deprivation incidence and the size of households needs. For this purpose, we use the linear regression $p_h = \rho + \delta N_h^0$, where p_h is the binary indicator of the presence or absence of multidimensional deprivation in the h-household, ρ is the intercept term, N_h^0 the count number of dimensions that the h-household needs, and δ is the regression coefficient of interest. This δ regression coefficient captures the difference in p_h-multidimensional deprivation incidence that can be attributed to the size of household needs. A profile that satisfies the desirability condition must reflect no difference in multidimensional deprivation incidence given by households' different needs.[6]

One could argue, nonetheless, that because of the randomness of the allocation of deprivation, a particular population subgroup might have a larger incidence of deprivation than another, simply as a result of this randomness. To overcome these possible random differences among population subgroups, the counterfactual scenario with no illegitimate difference in deprivation among households was simulated 1,000 times; each simulation or trial being independent from the other. The resulting collection of estimates approximates the distribution of the index over the counterfactual scenario's outcomes. The results that we describe below correspond to the distribution of these 1,000 independent simulations. For completeness and replicability purposes, Appendix A includes the implemented pseudo-code for these simulations.

3.4.4 Results

In this section, I present the microsimulation results of multidimensional profiles developed under $m^{\beta,\theta}$ measures that do not adjust the burden of multidimensional deprivation by differences in needs (using $\theta = 0$). I compare these results with those obtained from measures that adjust by differences in need (measures with $\theta > 0$). At this stage, it should be recalled that θ reflects the response of the burden of deprivation to the scale of household needs. Values of θ close to zero reflect a low

[6]Different approaches can nonetheless be used to measure the size of household needs, as for example household size. Still, the N_h^0-count number of dimensions that the h-household needs is our preferred measure of the size of household needs to be used for this evaluation because with such an approach the number of persons in the household and its composition is taken into account with respect to the dimensions captured by the multidimensional index. For a simple example consider households A and B, both consisting of two persons each. Household A, consisting of one adult person and one toddler. In the index example, this household may be scored as deprived in four out of the five considered dimensions. In contrast, household B, consisting of one adult and a 10-year-old child, may be scored as deprived in all five considered dimensions. In this case, household size does not capture the difference in possible deprivations that these two households of the same size have.

response of the burden of deprivation to the scale of household needs and values of θ close to one reflect a greater adjustment of the burden of deprivation by the size of household needs. This θ parameter was included to account for differences in needs when comparing household's multiple deprivations, as the applied literature on income and expenditure household-based measures does to compare household's welfare.[7]

Figure 3.2 on page 102 plots the results of this evaluation. The horizontal axis in the figure corresponds to the range of θ parameters used to calculate the $m^{\beta,\theta}$ measure. The first value of this range corresponds to $\theta = 0$ (no adjustment for the size of household needs), the adjustment by the size of household needs increases as θ increases. The last value on the right-hand side of the horizontal axis corresponds to $\theta = 1$. The vertical axis in the figure represents the magnitude in percentage points of the estimated δ regression coefficient of the effect that the N_h^0-size of household needs has on p_h.

One estimated δ regression coefficient is obtained in each of the 1,000 simulations, thus, each δ coefficient measures the strength of the relationship between p_h-multidimensional deprivation incidence and the size of household needs in the counterfactual scenario of no ilegitimate difference in deprivation. The 1,000 obtained δ coefficients describe the distribution of this relation in the (counterfactual) population in which there is no difference in deprivation resulting from unfair causes. The mean of this obtained regression coefficient across the 1,000 simulations is used as measure of central tendency of the behaviour of δ.

In Figure 3.2, each marker represent this central tendency measure of the δ regression coefficient obtained from using a particular $m^{\beta,\theta}$ measure. The shaded zone around the markers represents the range of variability of 95% of these 1,000 obtained estimates of δ. Any measure that properly accounts for legitimate differences in needs is, ideally, expected to have a distribution with a mean of zero and a narrow spread (such as 95% of the values within that narrow interval).

As observed, the mean of the obtained δ regression coefficient across the 1000 simulations, when using $m^{0,0}$ to sort and identify households is 17.8 percentage points (p.p.), with a range of variability of 95% of its values between 16.3 and 19.4

[7]Two examples of this literature are Buhmann et al. (1988) and Coulter et al. (1992*b*)

p.p. This result indicates, that comparing households on the basis of the widely used AF dimensions-count-based approach ($m^{0,0}$) does not permit an unbiased incidence profile. The simulation results of using this metric show a distribution of estimates far above the desirable zero mean, and their values are concentrated around this positive mean.

Similarly, the mean across the 1,000 simulations of the δ regression coefficient between p_h and the size of the needs, obtained when measuring the burden of multidimensional deprivation on the basis of the deprivations-count-based approach to measurement ($m^{1,0}$), results to be 21.9 p.p, with 95% of its values between 20.7 and 23.2 p.p.

A positive δ regression coefficient observed across all the 1,000 counterfactual scenarios when measuring the burden of multidimensional deprivation by any of these two metrics (the dimensions-count-based approach and the deprivations-count-based approach) indicates that these both metrics produce multidimensional deprivation incidence p_h to be correlated with the size of the household needs. This occurs even when households do not have any illegitimate difference in deprivation among them.

When the $m^{0,0}$ metric is used to sort and identify multidimensionally deprived households, an additional dimension that households exhibit as need increases by an average of 17.8 p.p. the ability of the household to be classified as multidimensionally deprived. Similarly, when $m^{1,0}$ is used to sort households, an additional possible household scoring dimension increases multidimensional deprivation incidence by an average of 21.9 p.p.

These results demonstrate that count-based measures cause any two households with different sizes of household needs to show different multidimensional deprivation incidence even if there is no illegitimate difference in deprivation between the two of them. Thus, these two metrics proved unable to properly capture a state in which there are no unfair differences in deprivation between households.

On the other hand, sorting households using a share-based approach to measurement, either an $m^{0,1}$ or an $m^{1,1}$ metric, does not permit unbiased multidimensional deprivation incidence profiles. The distribution of the obtained δ regression coefficient in these two cases is concentrated far below zero, and the interval of

95% of their values is narrow around the negative mean of the 1,000 obtained δ regression coefficients. An negative mean across the simulations of the δ regression coefficient, indicates that the metric used to sort and identify households does not effectively addressed differences in need. It produces multidimensional deprivation incidence to decrease systematically as the size of household needs increases.

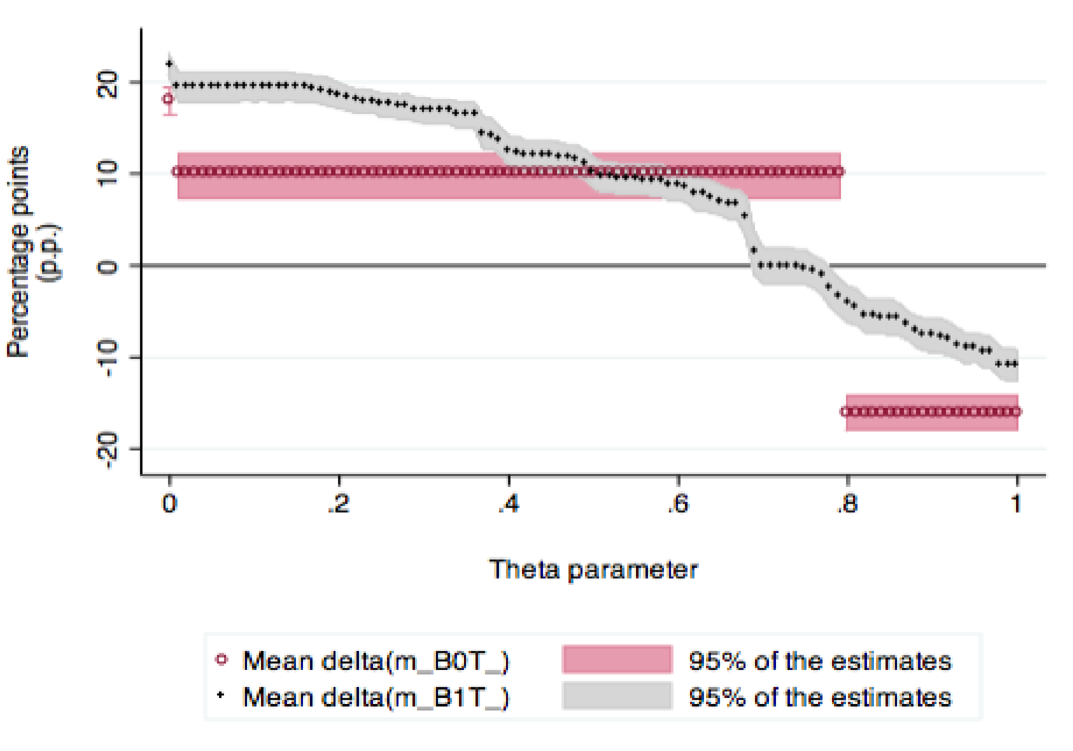
For instance, the use of a deprivations-share-based approach to measurement ($m^{1,1}$) to sort households produces a distribution of the 1,000 obtained δ regression coefficient concentrated around -10.9 p.p., and the distribution of 95% of the estimates varies between -12.6 and -9.1 p.p. This means that, even when there is no difference in illegitimate deprivation between households, the use of an $m^{1,1}$ measure to sort and identify multidimensionally deprived households produces an additional dimension that the household exhibits as a need to reduce the ability of this household to be classified as multidimensionally deprived at 10.9 p.p.

Whereas count-based approaches cause a biased picture of household-based multidimensional deprivation profiles, larger and more heterogeneous households are more likely to be identified as the most deprived. Share-based approaches invert these results, producing also a biased picture of household-based multidimensional deprivation profiles. In the latter case, in contrast to count-based approaches, small and homogeneous households tend to be more likely to be identified as the most deprived, but only about half as often as in count-based approaches.

Nonetheless, sorting households in these counterfactual states based on any $m^{\beta,\theta}$ measures that use $\beta = 1$ and a value θ between 0.69 and 0.77 satisfies the desirability condition for the particular case of 2013 Paraguayan index example. Any of these metrics produces a distribution of the obtained 1,000 δ regression coefficients between $p_h(m_h^{\beta,\theta})$ and N_h^0 with values very close to zero and a narrow spread of the distribution around this value. These results suggest that, in the case of the 2013 Paraguayan example, those metrics enable us to depict as equivalently deprived households with no illegitimate difference in deprivation but only differences in needs among them.

Measuring the household multidimensional deprivation based on a burden with a larger aversion to deprivation parameter, such as $\beta = 1$, in comparison to measuring it with a smaller aversion to deprivation parameter, such as $\beta = 0$,

Figure 3.2: Simulation results: distribution of the obtained δ regression coefficient in percentage points (p.p.) when using $m^{\beta,\theta}$ to sort and identify the most deprived households



Source: Author's calculations based on 2013 PHS. Notes: Estimated population means based on a sample of 5,423 households. Results obtained by simulating 1,000 independent times a random allocation of deprivation across the observed households, keeping constant the demographic configuration of the households and the societal amount of deprivation in each indicator. Shaded areas denote 95% of the obtained δ estimates. The lower limit corresponds to the δ value at the 0.025 percentile and the upper limit to the δ value at the 0.975 percentile.

shows the distribution of the estimated δ coefficient increasing the adjustment by the size of the needs as long as we increase the θ deprivation response scale parameter.

In summary, the results shown in this section indicate that neglecting differences in needs and in particular the use of a dimensions count-based approach to measurement yields biased household-based multidimensional deprivation incidence profiles. Other different combinations of β and θ to describe the burden of household multidimensional deprivation in the context of the 2013 Paraguayan application have proved to reveal unbiased multidimensional deprivation incidence profiles. The degree to which we must account for these differences in need therefore stands out as relevant.

Still, the afore-discussed results correspond to the Paraguayan index example without applying any **w** dimensional weighting system and identifying the 40%

most deprived households as the multidimensionally deprived population. The next section analyses the robustness of these 'baseline' obtained results under alternative considerations.

3.4.5 Alternative specifications

The first set of alternative specifications analysed in this section aims to build the index through a combination of indicators that 'balance' the applicable population subgroups within each dimension. This type of balancing procedure was proposed by Alkire (2015) as an alternative methodological approach to account for differences in need. It implies each well-being dimension to account all population subgroups with one applicable deprivation indicator. Note that, when introducing the five considered indicators for the Paraguayan illustration (Table 3.2 above), only the dwelling conditions dimension includes a set of indicators that balance the applicable populations subgroups. In contrast, the access to health services and education dimensions both include a set of indicators that together do not cover all population subgroups.

As such, to illustrate the effect and implications of implementing balancing procedures of the type proposed by Alkire (2015), the first alternative specification analysed here consists of balancing the access to health services dimension. Specifically, the access to health services dimension can be considered as 'unbalanced' in the baseline configuration of the index because the indicator of non-access to health services when needed applies exclusively to persons that were sick or had an accident during the 90 days before the interview, and such a dimension does not include any additional indicator for persons that were not sick or did not have an accident during the 90 days before the interview. Implementing a balancing procedure in this dimension, thus implies either excluding the non-access to health services indicator from the index, or including an indicator applicable exclusively to the population that were not sick and had not had an accident during the 90 days before the interview. Here, the first approach is implemented because the latter would be conducive to including an indicator that is neither straightforwardly intuitive nor relevant for the purposes of policy.

The results of the δ relation coefficient between p_h and the size of household needs obtained using the Paraguayan index under this first alternative specification (i.e. excluding the non-access to health services when needed indicator) and across 1,000 counterfactual scenarios of no systematic difference in deprivation but only differences in need are shown in the second row of Table 3.4 (Specification A). The magnitude of δ results decrease while the number of indicators applicable to specific population subgroups decrease in the index. Whereas the baseline specification consists of aggregating five indicators, of which three describe differences in needs across demographic population subgroups, this alternative Specification A includes only two out of these three indicators. This reduction in indicators, in the case of the dimensions count-based approach to measurement, results in a mean δ 7.6 p.p. smaller than that observed in the baseline specification.

It is worth noting that this first alternative specification of the index balances the access to health services and the dwelling conditions dimensions, but the education dimension remains unbalanced.

To balance this remaining dimension, an additional indicator applicable exclusively to children under five years of age is included. The additional deprivation indicator included corresponds to whether or not a 04 years old child has been registered into the national identification system. The results of this specification are shown in the third row of Table 3.4 (Specification B).

The results of this specification of the index show a larger mean δ relation than that obtained by Specification A. We observe, therefore, the strength of the relation between multidimensional deprivation incidence and the size of household needs increasing/decreasing as the number of indicators that depict differences in need across population subgroups increases/decreases.

The results of Specification B also indicate, interestingly, that despite population subgroups per each dimension being completely balanced across indicators, the obtained distribution of the δ-relation between multidimensional deprivation incidence and the size of household needs is located far above zero with its values concentrated around 16.9 p.p. This result proves that a dimensions count-based approach to measurement ($m^{0,0}$) is unable to classify equivalently any two households with no systematic difference in deprivation but only different needs. This is even though balancing procedures across dimensions and indicators have taken place.

However, one might argue that a weighting structure to avoid giving more importance to some dimensions over others could enhance this type of balancing procedure outcome. Hence, the third alternative specification tested here imposes on top of Specification B a nested weighting structure, which means each dimension and each indicator within each dimension has an equal relative importance in the index. The results of this fourth specification are shown in the fourth row of Table 3.4 (Specification C). It is seen in the table that this weighting structure rather worsens the results, it increases in about 70% the size of the δ-relation with regard to the unweighted specification (Specification B).[8] This result is observed because the implemented weighting system of Specification C reduces the importance in the index of the indicators that apply to any person (health insurance non-coverage and substandard dwelling conditions) and increases the importance of indicators that capture differences in needs across population subgroups.

Another possible course of action to implement a balancing procedure could consist of applying a set of weights that vary across population subgroups that exhibit different sets of needs. In the Paraguayan example, this procedure implies imposing over the balanced specification of the index (Specification B) a set of weights such that the sum of relative importances across indicators is one in each population group that is accounted through a different set of indicators. Note that here weights are applied over individuals' deprivations rather than over household's dimensions. The results of this D-alternative balancing specification are shown in the fifth row of Table 3.4. The δ relation between multidimensional deprivation incidence and the size of household needs in the implemented counterfactual scenarios results in this D-case being equivalent to using the unweighted balanced specification of the index (Specification B).

Now, in terms of different shares of the population to identify the multidimensionally deprived population, while the baseline results are based on identifying the first 40% most deprived households, here as alternative specifications 20% and 30% of the total population of households are identified as the multidimensionally

[8] An unweighted specification across five indicators implies in practice that each indicator has one fifth of relative importance in the whole index.

Table 3.4: Simulation results: distribution of the obtained δ regression coefficient in p.p. when using $m^{\beta,\theta}$ to sort and identify the most deprived households

	Specification	Dimensions count-based ($m^{0,0}$)			Deprivations share-based ($m^{1,1}$)		
		Mean δ	Ll	Ul	Mean δ	Ll	Ul
		(1)	(2)	(3)	(4)	(5)	(6)
	Baseline						
	Two unbalanced dimensions & 40% of share	17.9	16.5	19.5	-10.9	-12.7	-9.2
	Alternative specifications						
A	One unbalanced dimension	10.3	7.8	12.8	-16.9	-19.5	-14.2
B	Balanced specification: All dimensions are balanced	16.9	15.2	18.7	-9.6	-11.3	-7.6
C	Weighted balanced specification	29.0	25.7	32.2	-14.4	-17.9	-10.7
D	Using an individual weighting system	17.0	15.2	18.6	-9.5	-11.3	-7.5
E	20% of share	12.0	10.9	13.3	-8.0	-9.6	-6.5
F	30% of share	16.3	14.9	17.6	-8.9	-10.5	-7.3

Source: Author's calculations based on 2013 PHS. Notes: Estimated population means based on a sample of 5,423 households. Results obtained by simulating 1,000 independent times a random allocation of deprivation across the observed households, keeping constant the demographic configuration of the households and the societal amount of deprivation in each indicator. Columns (1) and (4) correspond to the mean δ regression coefficient across these 1,000 simulations. Columns (2) and (5) correspond to the δ value at the 2.5 percentile of the distribution of δ coefficients across the 1,000 performed simulations. Columns (3) and (6) correspond to the δ value at the 97.5 percentile.

deprived. Results are shown in the last two rows of Table 3.4. As expected, the size of the δ relation coefficient is sensitive to the share of population identified as multidimensionally deprived, it decreases as the share of identified multidimensionally deprived population decreases.

In summary, these alternative specifications, consistently with Section 3.4.4's findings, demonstrate that measuring the burden of multidimensional deprivation without accounting for differences in need, as the dimensions count-based approach to measurement does, produces a biased multidimensional deprivation incidence profile. It captures not only relevant differences in deprivation but also unaddressed differences in needs. Though a deprivations share-based approach addresses differences in needs, this approach to measurement overshoots the results. The obtained δ estimates that use this relative approach to measurement result concentrated not around the desired zero mean, but around negative values. It conduces multidimensional deprivation incidence to decrease systematically as the size of the household needs increases. Still, these negative mean δ values are about half the size of those obtained by the dimensions count-based metric.

We continue in the next section briefly discussing the case when multidimensional deprivation is evaluated rather than at the household at the individual level; the methodology presented in Section 3.3.7 is named 'the individual-based scenario'.

3.4.6 The individual-based scenario

This section illustrates the empirical behaviour of the multidimensional deprivation measurement methodology proposed in this chapter in an individual-based scenario and evaluates its proposed measures. For this purpose, the same 2013 PHS indicators used for analysis in previous sections are used here. However, as described when outlining the methodology for individual-based multidimensional deprivation measurement in the presence of differences in needs (Section 3.3.7), household-based aggregates are not pursued here. In contrast, in the individual-based scenario, each individual is considered its own household and the burden of multidimensional deprivation is measured by an m^{θ} metric. Measuring the burden of multidimensional deprivation without accounting for differences in need imply setting $\theta = 0$, which is simply the number of dimensions of deprivation that each individual exhibits. This corresponds to the AF method.

Table 3.5 presents for each 2013 PHS considered deprivation indicator the number of observed persons in its applicable population and the proportion of deprived persons within it. These five deprivation indicators are subsequently combined to depict the burden that multidimensional deprivation places on each individual. As a result, we obtain the m^{θ} index that takes values according to the used θ parameter of responsiveness of deprivation with regard to the level of needs.

A burden of multidimensional deprivation that does not account for the size of individual needs counts the number of deprivations that each individual exhibits. However, given that the accounted needs vary across three population subgroups

Table 3.5: Observed individual dimensional deprivation

Deprivation indicator	Applicable population where the indicator is relevant	Number of observed persons in the applicable population	Proportion (%) of deprived persons in the applicable population
Health insurance non-coverage	Any person	20,909	71.9
No access to health services	Any person that was sick or had an accident during the 90 days previous to the interview	7,199	23.8
Non-school attendance	5 - 17 years old population	5,706	6.7
Low educational achievement	18 years old population and over	13,406	47.9
Sub-standard housing	Any person	20,909	24.6

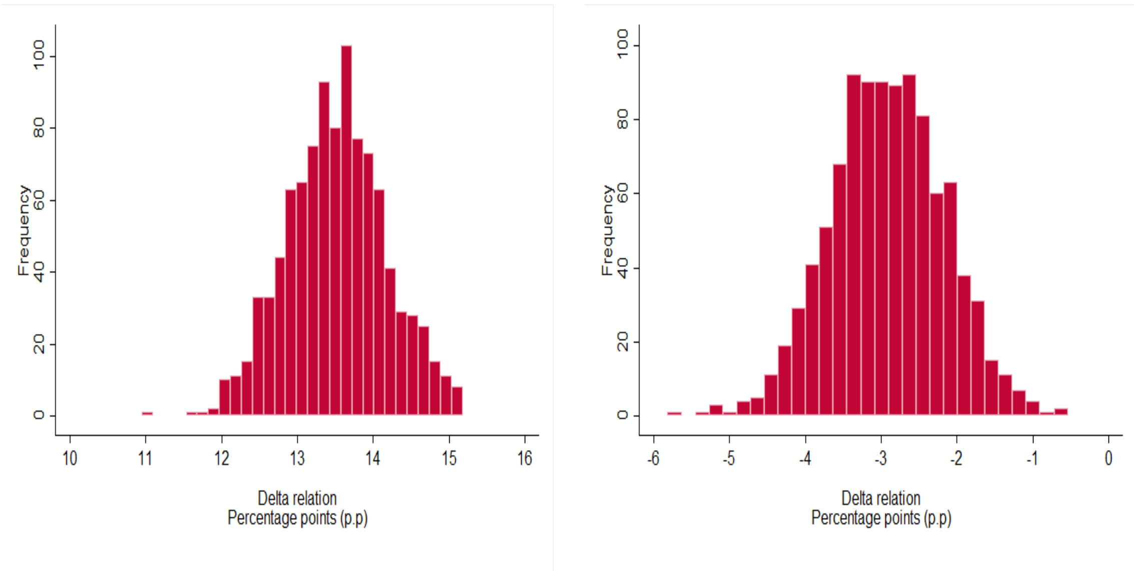
Source: Author's calculations based on 2013 PHS.

(children under five years of age, 5- to 17-year-old children, and those 18 years of age and over), the use of m^0 leads to a smaller mean burden of multidimensional deprivation among the population groups with a smaller number of accounted dimensions. This is the case for children under five years of age. This population subgroup is recorded as having need with respect to three out of the five considered indicators, whereas children from 5 to 17 years of age and the population 18 years of age and older may be recorded as exhibiting a in four out of the five considered indicators.

The proposed methodology of this chapter seeks to enable multidimensional deprivation measurement in the presence of differences in need. The same methodological approach used for the household-based scenario is followed here. Observed differences in the unadjusted burden of multidimensional deprivation across demographic heterogeneous groups capture, in addition to illegitimate differences in deprivation, differences in deprivation caused by legitimate and unavoidable differences in needs. Thus, to evaluate measures, we use the desirability condition outlined on page 96, which states that any two individuals with no illegitimate difference in deprivation between them must have the same multidimensional deprivation incidence.

Figure 3.3: Simulation results: distribution of the obtained δ regression coefficient in percentage points (p.p.) using two different m^θ measures

(a) m^0 (b) m^1



Source: Author's calculations based on 2013 PHS. Notes: Estimated population means based on a sample of 20,909 individuals. Results obtained by simulating 1,000 independent times a random allocation of deprivation across the observed individuals, keeping constant the demographic configuration of the population and the societal amount of deprivation in each indicator.

In a counterfactual scenario with no illegitimate difference in deprivation among individuals but only differences in need across them, we analyse the relationship between the multidimensional deprivation incidence (p_i) and the size of individual needs. Similar to the household-based analysis, here I approach this analysis via the linear regression $p_i = \rho + \delta N_i$, where N_i represents the size of individuals multidimensional needs and is measured by the number of achievements, i.e., indicators, the individual exhibits as needs.

Figure 3.3 plots the evaluation results of m^0 and m^1. The distribution of obtained δ estimate coefficient, when using m^0 to rank individuals, register positive values across all the 1,000 simulated scenarios and ranges between 11.0 and 15.2 p.p. In these counterfactual scenarios of randomly allocated deprivation, one additional dimension increases the multidimensional deprivation incidence by an average of 13.5 p.p. These results indicate that the m^0 metric leads to a biased multidimensional deprivation incidence profile.

A multidimensional profile based on a share-based measure does not satisfy the desirability condition. As observed from Figure 3.3.b, the estimates of the δ

regression coefficient across the 1,000 simulations range between -5.8 and -0.5 p.p., and the mean of these estimates is concentrated at 2.9 p.p. below zero.

The evaluation results of these two metrics (m^0 and m^1) were obtained upon identifying as multidimensionally deprived the 40% most deprived population. Other alternative population shares were also used to identify the multidimensionally deprived population (30% and 20%), and the results proved robust under these other two population shares.

3.5 Context-specific definitions

Multidimensional deprivation measurement embeds, as does any poverty measurement process, several assumptions and normative definitions that vary from context to context. In fact, Sen (1979) indicates the following as a good practice for the general poverty measurement exercise: 'There is very little alternative to accepting the element of arbitrariness in the description of poverty, and making that element as explicit as possible.' (Sen 1979, p.288). In light of this, in this section I discuss the most relevant context-specific definitions embedded in the proposed multidimensional deprivation family of indices.

3.5.1 Unit of multidimensional deprivation analysis

The first normative selection required when measuring multidimensional deprivation is the unit of analysis where multidimensional deprivation is evaluated. The two most common approaches are selecting either households or individuals as the unit of analysis. The proposed methodology of this article allows selecting either of these two different units. While individual-based measures allow the unmasking of differences in multidimensional deprivation across demographic subpopulation groups, household-based measures conceive households as co-operative units that jointly face the deprivation suffered by the household members.

Considering household as the unit of multidimensional deprivation analysis implies understanding the burden that deprivation places as shared among household members. For example while child mortality refers to a particular episode that is suffered by children, using the household as the unit of deprivation analysis implies that this episode is understood as a phenomenon that not only affects children but also the household as a whole. The living conditions and behaviours of household members contribute to reducing or increasing the frequency of such situations, and the burden of the episode is faced collectively by the household.

An example of a household-based approach to measurement where possible intrahousehold externalities arising from the presence of non-deprived household members is the proposed approach of Basu & Foster (1998) for the case of literacy. In this case, the scholars proposed taking into account in the literacy measurement not only individuals' ability to read and write but also the additional advantage that literate households members bring to illiterate members in the household. Extensions have been developed by Subramanian (2004), Subramanian (2008), and Chakravarty & Majumder (2005), and a similar approach but for the case of the unemployment rate has been proposed by Basu & Nolen (2008).

As such, the proposed methodology of this chapter enables using either the household or the individual as the unit of analysis. It recognizes that selecting individuals or households have embedded different normative criteria that need to be analysed and defined according to the purposes of each particular application.

3.5.2 Defining needs

Implicit in my approach is that an individual can be regarded as deprived by a particular indicator only if it measures an achievement that can be viewed as something that this individual legitimately needs. Needs differ across dimensions of multidimensional deprivation by population subgroup. For example while adults who do not have work opportunities despite looking for them can be catalogued as employment deprived, children cannot be catalogued as deprived in the absence of employment. Conversely, children under 11 years old who are forced to work would be catalogued as deprived. Children are accountable on other deprivations that are relevant to them, such as access to education services. As such, adults and children have different sets of needs. While adults need access to job opportunities and are considered employment deprived whenever they do not have access to them, children need access to basic school services and are considered educationally deprived if they lack such access.

Needs are, thus, incorporated into my multidimensional deprivation family of indices by excluding from the calculations all dimensions that do not correspond to needs for a particular individual. As such, by setting the applicable population subgroups where relevant to measure presence or absence of deprivation in each well-being indicator, the practitioner formalizes whom is defined as legitimately needing each of these dimensions. These differences in need make visible the normative definition of legitimate and illegitimate differences in achievement levels. Whereas differences in achievement level within the applicable population subgroup of each indicator are set as illegitimate, differences within the non-applicable population subgroup are catalogued as fair and are therefore tackled by the measurement process.

In consequence, setting the applicable populations results in being a key normative decision in my proposed approach. It is suggested that they be made using context-specific norms of what is considered desirable and undesirable in each of the dimensions included within the multidimensional index, or available international indicator definitions.

For example in the education dimension, deprivation indicators may be defined using the ranges of ages suitable to measure enrolment and school lag according to each country. Another example is child labour. The International Labour Organization describes in its regulations the age ranges defined as suitable to measure this kind of deprivation and the activities and time duration that are considered as acceptable for this matter. Still, these definitions are context specific and should be tailored with special care.

3.5.3 Choosing the combination of parameters to describe the burden of multidimensional deprivation

The selection of parameters to be used to measure the burden that multidimensional deprivation places over the household also constitutes an important context-specific definition. There is no correct or incorrect selection of parameters. However, each combination produces different household rankings and therefore it can be utilized according to the circumstances.

To select the most appropriate combination of parameters, two different approaches can be employed. First, the combination of parameters can be selected from a normative perspective. While count-based approaches ($\theta = 0$) give either to each dimension (using $\beta = 0$) or to each deprivation ($\beta = 1$) an equal absolute value in the measurement of the burden of multidimensional deprivation, share-based approaches ($\theta = 1$) give an equal absolute value to each household, regardless of their demographic composition and size, and taking into account the possible scale economies that arise at this level. An intermediate normative perspective approach corresponds to a θ parameter in between these two solutions. The value of θ reflects the responsiveness of the burden of deprivation to the scale of need, and values of θ close to zero convey a lower response of the burden of multidimensional deprivation to the size of need. Conversely, values of θ close to one convey a greater response of the burden of deprivation to the size of need.

On the other hand, the second possible course of action corresponds to determining the combination of parameters that enables non-biased societal multidimensional deprivation incidence profiles. This combination of parameters can be obtained, as discussed in Section 3.4, by simulating a counterfactual scenario of no illegitimate difference in deprivation. The evaluation of the measures in such a scenario enables determination of whether or not a particular combination of β and θ to describe the burden of multidimensional deprivation enables any two households with different sets of needs but no systematic difference in deprivation to be classified as equivalently deprived.

The selection of parametric values of β and θ to describe the burden of multidimensional deprivation under the proposed methodology of this chapter is advised in light of robustness checks using different multidimensional deprivation thresholds and specifications.

3.6 Concluding remarks

This chapter proposed a family of multidimensional deprivation indices that takes into account differences in need that demographically heterogeneous units (i.e. either households of different size and composition or individuals of different population subgroups) exhibit. The proposed family of indices is meant to be applicable for the purposes of policy and suitable for contexts where multidimensional deprivation is aimed to be measured through a wide range of indicators that describe differences in needs.

To measure the burden of multidimensional deprivation, different approaches to measurement that range from count-based (absolute) to share-based (relative) and intermediate approaches are used in this chapter. These different approaches to measurement to sort and identify the multidimensional deprived population produce significantly different multidimensional deprivation profiles. As such, they were evaluated using the 2013 PHS and counterfactual scenarios of no systematic differences in deprivation across households/individuals but only differences in needs. Measures able to catalogue households/individuals as equivalently deprived in these scenarios are said to be portraying an unbiased multidimensional deprivation profile.

Multidimensional deprivation measures, which do not address differences in needs, as for example the dimensions AF's count-based approach, were found to yield biased multidimensional deprivation incidence profiles. In general, count-based approaches produced larger multidimensional deprivation incidence among households with larger sizes of needs, despite having no systematic difference in deprivation. Share-based approaches, in contrast, produced larger multidimensional deprivation incidence among households with smaller sizes of needs. The degree to which we must account for these differences in need, therefore, stands out as relevant.

To evaluate the robustness of these results, the behaviour of the measures was analysed using different alternative specifications of the index to address differences in needs. Balancing procedures as proposed by Alkire (2015), which imply in each well-being dimension accounting for all population subgroups with one applicable deprivation indicator, were also discussed and evaluated as alternative methodological approaches. The results of this chapter proved to be robust under all of these alternative considerations.

In the context of the Paraguayan implemented index example, particular members of our proposed family of measures of the burden of multidimensional deprivation demonstrated the ability to depict as equivalently deprived households with no systematic difference in deprivation but only different sets of needs. They, therefore, confirmed providing an unbiased multidimensional deprivation incidence profile in this specific context.

This chapter also evaluates the proposed family of measures in terms of their properties. The results of this evaluation demonstrate the proper orientation of the family of measures and the desirable non-sensitivities, so we can conclude our proposed technology is adequate for the purposes of poverty measurement.

However, within this used framework the limitations that a parametric equivalence scale of this type can be recognized. First, considering that the ultimate purpose of a multidimensional measure of deprivation is to capture unfair disadvantage, differences in deprivation due to other fair sources are not accounted. Further research is required to disentangle the effect that other sources of legitimate differences might have over multidimensional deprivation incidence profiles, such as preferences or needs not necessarily based on the still limited number of observable attributes (i.e. household size, composition, or age and gender) that are addressed in this chapter. In addition, differences in need are due to be analysed in the context of multidimensional deprivation technologies that take into account the complementarity and substitutability that might arise among dimensions.

Second, as analysed by Pollak & Wales (1979), Fisher (1987), and Blundell & Lewbel (1991), for the one-dimensional equivalence scale case, a household's current demographic composition that leads to differences in need might be driven by previous deprivation status as well. For example a particular household consisting of two adults and five children might be this size not only because both adults have a preference for many children, but also because they did not have access to pregnancy prevention education or could not afford to use some form of birth control. Then, household composition not only reflects needs or preferences, catalogued in this chapter as producing fair differences in deprivation among households, but also current household compositions might be a reflection of avoidable and unfair previous states of deprivation. This is a complex issue that is left for further research.

Furthermore, the proposed methodology of this chapter measures multidimensional deprivation either at the individual or at the household level without addressing the intrahousehold bargaining power and allocation of resources that might be conducive to ameliorating or intensifying each individual's burden of deprivation. In particular, the one-dimensional welfare comparisons literature has shown that households do not behave as a single unit but rather under collective instances where household's behaviour is the outcome of the joint decisions of its members (Chiappori 1992, Browning et al. 2013, Mazzocco 2005). As such, further research is required to provide a multidimensional measurement technology able to account for these intrahousehold differences and their effect on the burden that multidimensional deprivation places on each individual.

Nonetheless, following Elster & Roemer (1991, pp.1), who exhorts any notion of well-being to be based on appropriately operationalized interpersonal comparisons, and also to be adequate for the purposes of distributive justice. The family of measures presented in this chapter contributes to the multidimensional deprivation measurement literature by enhancing the comparability across households or individuals that exhibit different needs, as well as being adequate for the purposes of multidimensional deprivation measurement.[9]

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[9] Appendix B. describes the characteristics of the proposed family of measures that make it suitable for the purpose of multidimensional deprivation measurement.

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Appendix A. Pseudocode simulations

```
                                             Pseudocode_sim
**********************************************************************
* Terminology
**********************************************************************
Dimensions : noseguro saluate noasiste noedu dwelling
Variables
I_*        : Individual deprivation indicators (g_ij)
pr_*       : Applicable population subgroups per indicator (s_ij)
hhid       : Household identification
Scalars
A_*        : Societal observed amount of deprivation
C_*        : Size of the applicable population subgroup
p03        : Household head identificator (one per household)

***********************************************************************
* Counterfactual scenario of no ilegitimate difference in deprivation
***********************************************************************
* The replications of each counterfactual scenario
local repe = 1000
local sim=0
while `sim'<=`repe'-1 {
        * Data at the individual level
        use database, clear

        foreach X in noseguro saluate noasiste noedu {
                * Assigning a random number to individuals within the j-applicable population
                gen random = runiform() if I_`X'!=. & pr_`X'>0 & pr_`X'!=.
                sort random
                gen temp=_n
                * The simulated deprivation distribution
                gen R_`X'=0 if temp<=C_`X' & I_`X'!=. & pr_`X'==1
                replace R_`X'=1 if temp<=A_`X' & I_`X'!=. & pr_`X'==1
                drop temp random
        }
        gen random = runiform() if I_dwelling!=. & pr_dwelling==1 & p03==1
        sort random
        gen temp=_n
        gen temp_dwelling=0 if temp<=C_dwelling & I_dwelling!=. & pr_dwelling==1 & p03==1
        replace temp_dwelling=1 if temp<=A_dwelling & I_dwelling!=. & pr_dwelling==1 & p03==1
        bysort hhid: egen R_dwelling=mean(temp_dwelling)
        drop temp random

        * Calculating measures in the counterfactual scenario
        foreach X in noseguro saluate noasiste noedu dwelling {
                * The beta parameter of deprivation aversion
                foreach b of numlist 0 1 {
                        * Household dimensional deprivation indicator
                        capture drop temp
                        bysort hhid: egen temp=total(R_`X')
                        gen dh_B`b'_j`X'=(temp)^`b'
                        replace dh_B`b'_j`X'=0 if temp<=0
                        * Size of household dimensional needs
                        capture drop temp
                        bysort hhid: egen temp=total(pr_`X')
                        gen nh_B`b'_j`X'=(temp)^`b'
                        replace nh_B`b'_j`X'=0 if temp<=0
                }
        }

        * Data at the household level
        keep if p03==1

        * Size of household multidimensional needs
        egen needs_B1=rowtotal(nh_B1_j*)
        egen needs_B0=rowtotal(nh_B0_j*)

        foreach b of numlist 0 1 {
                * The theta parameter of scale response of deprivation to needs
                foreach Theta of numlist 0(0.01)1 {
                        local t=round(`Theta'*100 ,1)
                        * Household multidimensional deprivation
                        egen m_B`b'T`t'=rowtotal(dh_B`b'_j*)
                        replace m_B`b'T`t'=m_B`b'T`t'/(needs_B`b'^`Theta')
                        replace m_B`b'T`t'=0 if needs_B`b'<=0
                        * Identification of the multidimensionally deprived
                        egen orderB`b'T`t'=rank(m_B`b'T`t'), unique
                        gen p_s40_B`b'T`t'=(orderB`b'T`t'>=3254)
                        * The delta regression coefficient
                        regress p_s40_B`b'T`t' needs_B0
                        sca delta_B`b'T`t'=(_b[needs_B0])
                }
        }
        local sim=`sim'+1
}

                                                 Page 1
```

Appendix B. On the properties

This Appendix describes the proposed family of measures of the chapter in terms of the characteristics that make it subject to evaluation as suitable for the purpose of multidimensional deprivation measurement.

Following the classification of properties for income or expenditure-based poverty measures proposed by Foster (2006) and generalising this classification for multidimensional poverty measures, in this section I first investigate the properties that make the societal measures proposed in this chapter non-sensitive to some aspects of the distribution, namely *Scale invariance*, *Anonymity*, *Replication invariance*, and *Focus*. Subsequently, the features that reflect a proper orientation of these societal measures, namely the *Dominance properties*, are analysed. This section completes the discussion, examining how *Transfers*, *Decomposability*, and *Continuity* behave in this context.

Scale invariance

The income or expenditure poverty measurement literature has traditionally used a *scale invariance* or *normalization* property to ensure societal measures are expressed in relation to the poverty line. In particular, this is the approach used by Foster et al. (1984) and described in detail by Foster (2006). Along with the *replication invariance* and *anonymity* property, which I discuss ahead on this section, this scale invariance property has been used in literature to enable comparisons of the incidence, depth and severity of poverty across societies of different sizes.

For the particular case of the H and $MD^{\beta,\theta}$ measures of multidimensional deprivation proposed in this chapter, the range of variability vary along β and θ vary. As such, excursively H and $MD^{\beta,1}$ measures, which in fact correspond to a relative approach to measurement, are invariant to the size of the population to be compared. They, therefore, take values from the interval $[0, 1]$. While H expresses the incidence of multidimensional deprivation in relation to the size of the population of households, $MD^{\beta,1}$ expresses multidimensional deprivation in relation to the size of household needs.[10]

[10]Exception are then the $MD^{\beta,\theta}$ metrics that use $\theta \in [0, 1)$. They do not lead societies with different sizes to reach the same $MD^{\beta,\theta}$ value whenever all i household members are j deprived.

In particular, in an individual-based scenario (i.e., any i individual is its own household), if all individuals in society are identified as multidimensionally non-deprived, then $H = 0$. Similarly, in the household-based scenario, if all households in the society are non-multidimensionally deprived, then $H = 0$. However, in such household-based scenario, multidimensional deprivation is not evaluated at the individual level. Therefore, in such a case, $H = 0$ does not mean that all individuals in the society are non-multidimensionally deprived; it reflects that all individuals in society belong to a non-multidimensionally deprived household.

On the other hand, in an individual-based scenario if all individuals in the society are identified as multidimensionally deprived, then $H = 1$; whereas in the household-based scenario $H = 1$ implies that all individuals in the society belong to a multidimensionally deprived household.

To illustrate the range of scale of the most important societal measures of the proposed methodology of this chapter, the Paraguayan index example is used. In particular, the first row within Table 3.6 below in page 140 shows the observed H and $MD^{\beta,\theta}$, where $\beta = \{0, 1\}$ and $\theta = \{0, 1\}$ in the 2013 PHS. To develop this observed case, any h household satisfying $m_h^{1,0.87} > 0.65$ is identified as multidimensionally deprived. Then, 40% of the 2013 Paraguayan households in the sample is identified as multidimensionally deprived.[11] The results on only these five metrics are analysed as they are the most important societal measures of the methodology proposed in this chapter. Henceforth in this section I focus on the analysis of these five societal measures.

Subsequently, two scenarios worth analysing are simulated: first, the scenario where all household members are assumed as non-deprived in all their relevant indicators. Second, the scenario where all household-members are assumed as deprived in all their relevant deprivations. In each of the simulations, any h household satisfying $m_h^{1,0.87} > 0.65$ is identified as multidimensionally deprived. The second

This result is consistent with the absolute or intermediate measurement approach used in these latter metrics.

[11] Although this particular measure of the household burden of multidimensional deprivation ($m_h^{1,0.87}$) was selected in light of the findings of Section 3.4 above, here in this section is used only for illustrative purposes. As such, the examples presented in this section can be equivalently derived from different $m^{\beta,\theta}$ measures and different k-thresholds.

and third rows in Table 3.6 include the results of the fully non-deprived scenario and fully deprived scenario, respectively.

As expected, the fully non-deprived scenario results in all measures having a value of zero (Row (2) in the table). In contrast, in the fully deprived scenario, as is seen from Row (3) in the table, the $MD^{\beta,\theta}$ measures that use a shared based approach to measurement, namely the $MD^{0,1}$ and $MD^{1,1}$, exhibit a value of 1.0, as well as the H metric. The value $H = 1$ indicates that 100% of households result in being identified as multidimensionally deprived, and $MD^{0,1} = 1$ indicates that households in society have an average of 100% of their applicable dimensions in deprivation. Similarly, $MD^{1,1} = 1$ indicates that households in society have on average 100% of their applicable achievements in deprivation.

Both $MD^{0,0}$ and $MD^{1,0}$ measures, in the fully deprived scenario take the value of the societal mean of household needs. For instance, in the table this fully deprived scenario shows $MD^{0,0} = 4.2$ and $MD^{1,0} = 12.6$, meaning that, households in society have in average 4.2 dimensions in deprivation and 12.6 deprived achievements; values that in turn represent the average societal size of household needs.

Population replication invariance

This property makes societal measures comparable across differently sized populations. In terms of the family of measures proposed in this chapter, this characteristic implies that, for a particular society made of R households, if we replicate $t \geq 2$ times these R households, the society level multidimensional measures, H and $MD^{\beta,\theta}$, will remain unaltered for any combination of β and θ parameters.

To illustrate this proposed characteristic for the H and $MD^{\beta,\theta}$ societal measures, in the context of the Paraguayan index example, I replicate t times the 5,423 PHS 2013 observed households. In this case t was defined as a random integer number that takes values from the set of integer numbers $\{2, 1000\}$. After such replication of the Paraguayan households in sample, I evaluate H and $MD^{\beta,\theta}$. This replication was repeated 1,000 independent times. The mean H and $MD^{\beta,\theta}$ obtained across these 1,000 independent replications is shown in Row (4) of Table 3.6. Any measure sensitive to replications of the population would show a non-zero difference between Row (4) and Row (1) of the table. As expected, any of the five analysed measures result in being sensitive to replications of the population of households. This result illustrates Population replication invariance in the five analysed measures.

Anonymity

The poverty measurement literature and, in particular, the multidimensional literature characterise some families of societal measures under a *symmetry* or *anonymity* property. For instance, according to Alkire & Foster (2011), the symmetry property that their family of measures uses ensures that societal metrics are not being constructed under the basis of greater emphasis on some population subgroups over others. This property is also used by multidimensional measures, such as the ones proposed by Tsui (2002), Bourguignon & Chakravarty (2003), and Seth (2013), among others. Bourguignon & Chakravarty (2003) defined their measures as symmetric since any person's characteristics, other than the multiple well-being dimensions considered for the measure, are set as not relevant in the measurement process of their measures. Similarly, Seth (2013) suggested that the identities of the individuals are not ethically significant in the measurement process. As such, individuals within society are considered anonymous. I henceforth refer to this measurement property as *anonymity*.

In practice, however, assuming anonymity of individuals that exhibit different needs, without accounting for these differences in need, results in biased multidimensional incidence profiles. The text that follows elaborates further on this.

As discussed in previous sections, needs differ across dimensions of multidimensional deprivation by population subgroup. While a particular population subgroup can be catalogued as deprived in a certain j dimension because it lacks an achievement level that is considered as needed, this does not necessarily mean that all demographic sub-population groups that lack such an achievement level can be catalogued as deprived. Therefore, an individual can only be regarded as deprived by a particular indicator if it measures an achievement which can be viewed as something this individual legitimately needs.

In the methodological approach of this chapter, differences in needs are tackled throughout the measurement process. Neglecting heterogeneity in needs, as the empirical findings of Section 3.4.4 above have demonstrated, does not enable unbiased multidimensional deprivation incidence profiles. Then, assuming anonymity across individuals without taking into account their heterogeneity in needs led to a biased picture of the incidence of societal multidimensional deprivation.

In the one-dimensional welfare measurement literature, as pointed out by Coulter et al. (1992*a*), heterogeneity in needs has been tackled by either measuring each persons well-being through a common metric that incorporates the information on heterogeneity and then aggregating across persons using the anonymity property or, alternatively, by dropping the anonymity property and accounting for the heterogeneity with, for instance, a weighting system that reflects those heterogeneous needs. The approach that I present in this chapter follows the first methodological strategy. Heterogeneity in needs across units is tackled by the measurement process and then the anonymity property is used.

Therefore, societal measures H and $MD^{\beta,\theta}$, for any combination of β and θ parameters, are meant to be non-sensitive to permutations of the units where the identification of the multidimensionally deprived population occurs. In the individual-based scenario, this means that societal measures are non-sensitive to rearrangements of individuals across the population. Similarly, in the household-based case, societal measures are meant to be non-sensitive to permutations of households within society and implicitly are also non-sensitive to permutations of individuals within households. These two characteristics of societal measures are termed, for the purposes of this chapter, as *Household anonymity* and *Within household anonymity*, respectively.

To illustrate these two characteristics of H and $MD^{\beta,\theta}$, I simulate in the context of the 2013 Paraguayan household-based index example these two types of permutations of the population. First, 1,000 independent and random permutations of the population of households are simulated; second, 1,000 independent and random permutations of individuals within each household are simulated. The observed deprivation in each household and its demographic configuration is kept as constant. Then, H and $MD^{\beta,\theta}$ are evaluated in the observed case and after each simulation.

The observed mean of these five societal metrics before any permutation correspond to the observed case, Row (6) from Table 3.6 shows the mean of each of the five analysed metrics across the 1,000 simulated permutations of households, and Row (6) shows the obtained mean across 1,000 simulations of random rearrangements of individuals within each household.

Any measure sensitive to permutations of households would show a non-zero difference between Row (6) and Row (1). Similarly, any measure sensitive to permutations of individuals within the household would show a non-zero difference between Row (8) and Row (1). The results shown in the table illustrate that, as expected, any of the five analysed measures is sensitive to permutations of households across society or to permutations of individuals within households.

Worth noting that permutations of individuals across households, in the household-based scenario, resemble either demographic changes or transfers across units, to which the proposed household-based measures of this chapter are sensitive. I further elaborate on these sensitivities on page 134 and page 139, when discussing the proposed dominance properties and how transfers behave in this context.

Focus

An individual-based family of measures, such as the one proposed by Alkire & Foster (2011), considers as non-relevant the sensitivity of societal measures to two types of increments in achievement levels: first, increments of achievement levels in the non-multidimensionally deprived population, and second, increments of achievement levels in non-deprived dimensions. The authors termed the ability of their measures to be non-sensitive to these two types of increments as *poverty focus* and *deprivation focus*, respectively.

In light of these two properties, the $MD^{\beta,\theta}$ proposed family of measures of this chapter consider non-relevant the sensitivity of societal measures to the following types of increments in achievement levels: i) increments in the j achievement level among individuals that belong to a non-multidimensionally deprived household; ii) increments in the j achievement level among j non-deprived individuals; and iii) increments in the j achievement level among individuals that do not belong to the j applicable population subgroup. Societal measures H and $MD^{\beta,\theta}$, for any combination of β and θ parameters, are meant to be non-sensitive to these type of increments in achievement levels.

The non-sensitivity of the measures to increments in achievement levels among individuals that belong to non-multidimensionally deprived households, in the context of this chapter, is termed *Multidimensional deprivation focus*. In comparison to the AF method, this characteristic is analogous to the poverty focus property proposed by the AF method.

It is worth noting, however, that in a household-based scenario, this multidimensional deprivation focus property enforces societal measures to be non-sensitive to increments in achievements of both deprived and non-deprived individuals that belong to multidimensionally non-deprived households. Identifying the multidimensionally deprived population at the household level is based on considering the household as a single unit. As such, it prevents observing multidimensionally deprived and multidimensionally non-deprived individuals within the same household. It produces measures to be non-sensitive to improvements or declines in achievement levels of deprived individuals that might have a large number of dimensions in deprivation but that do not belong to multidimensionally deprived households. This is in fact the case of any societal measures based on household-based metrics, either $H(m^{0,0})$ and M_0 from the AF method or the proposed $H(m^{\beta,\theta})$ and $MD^{\beta,\theta}$ measures of this chapter.

On the other hand, as discussed when introducing the proposed methodology of this chapter, every achievement is not necessarily relevant to be measured across any i person. Then, the $MD^{\beta,\theta}$ proposed measures uncover this consideration. This means that the $MD^{\beta,\theta}$ family of measures considers non-relevant achievement increments, not only among j non-deprived individuals but also among individuals that do not belong to the j applicable population. In the context of this chapter, this property is termed *applicable deprivation focus*.

To illustrate multidimensional deprivation focus and applicable deprivation focus, I simulate particular increments in achievement levels in the context of the 2013 PHS index example. Each simulation is repeated 1,000 independent times. Societal measures, H and $MD^{\beta,\theta}$ are evaluated before and after each simulated increment. The following describes these simulations and the obtained results.

Multidimensional deprivation focus. An increment in the educational achievement indicator is simulated among individuals that belong to a non-multidimensionally deprived household. As such, 50% of the 18 years old and over individuals that belong to non-multidimensionally deprived households are sampled without replacement. Among them, one additional year of education is simulated. This population corresponds to 4,050 individuals out of the 20,909 individuals in the sample of the PHS 2013. Societal measures are evaluated after each of the 1,000 independent simulations. Row (10) in Table 3.6 shows the mean of the obtained measures after the simulations.[12]

Any measure not satisfying the Multidimensional deprivation focus property would show a non-zero difference between Row (10) and Row (1) of the table. It is observed in Row (11) of the table that the five analysed measures result in being non-sensitive to increments in achievement levels of individuals belonging to multidimensionally non-deprived households. This result illustrates Multidimensional deprivation focus for the five analysed measures.

Applicable deprivation focus. In this case, we illustrate the sensitivity of societal H and $MD^{\beta,\theta}$ to increments in the j achievement level among individuals that do not belong to the j applicable population subgroup. In particular, an increment in one year of education among 50% of the under 18 years old individuals belonging to a multidimensionally deprived household, is simulated 1,000 independent times. This population corresponds to 1,695 individuals in the PHS 2013 sample. The mean H and $MD^{\beta,\theta}$ obtained across the simulations is shown in Row (12) from Table 3.6.

Any measure not satisfying the Applicable deprivation focus property would show a non-zero difference between Row (12) and Row (1) in the table. It is observed in Row (13) of the table that the five analysed societal measures result being non-sensitive to increments in achievement levels among individuals that do not belong to the applicable population subgroup. This result illustrates Applicable deprivation focus for the five analysed measures.

[12]For the purposes of this illustration, the 50% share of this population subgroup was selected to ensure observing a big enough change in the measure displayed with two decimals of precision in Table 3.6. However, the example presented in this section can be analogously derived using different shares of the population or indicators.

We now discuss the set of properties that depict the orientation and desirable sensitivities of our measures, which are termed by Foster (2006) as dominance properties.

Dominance properties

According to Foster (2006), dominance properties are the characteristics of a poverty measurement that describe the ability of the metric to reflect improvements or declines among the poor population. They aim to resemble the proper orientation of societal metrics. In this regard, the characterization of the family of measures of this chapter, which is described below, draws and extends the dominance properties proposed by Alkire & Foster (2011) for multidimensional measures of poverty.

Here, I define three relevant types of improvements in achievement levels: i) *applicable achievement increment*, ii) *deprivation reduction among the multidimensionally deprived*, and iii) *dimensional deprivation reduction among the multidimensionally deprived.*

First, an *applicable achievement increment* occurs whenever the i individual that belongs to the s_j applicable population subgroup increases its a_{ij} achievement level by a constant $\gamma > 0$. This means that the a'_{ij} achievement for the i household-member and the j dimension is obtained by an increment of a constant $\gamma > 0$, such that $a'_{ij} = a_{ij} + \gamma$ for any person i satisfying $i \in s_j$.

Now, let assume that this i individual is deprived in the j dimension and belongs to a multidimensionally deprived household. Then, a *deprivation reduction among the multidimensionally deprived* occurs whenever this i individual increases his/her welfare in the j achievement, and this improvement changes his/her status from deprived to non-deprived.

Hence, in addition to be an applicable achievement increment, a deprivation reduction among the multidimensionally deprived makes this individual, no longer j-deprived due to this welfare improvement. This means that the a'_{ij} achievement for the i individual in the j dimension, obtained by an increment of a constant $\gamma > 0$ such that $a'_{ij} = a_{ij} + \gamma$, for any person i satisfying $i \in s_j$, $a_{ij} < z_j$,

$i \in h\ s.t.\ p_h = 1$, is a deprivation reduction among the multidimensionally deprived whenever $a'_{ij} \geq z_j > a_{ij}$.

Nonetheless, the household to which this i individual belongs might be still having any other household member j-deprived, therefore continuing to be deprived in such dimension. Then, a *dimensional deprivation reduction among the multidimensionally deprived* is an improvement such that it involves an applicable achievement increment that produces a deprivation reduction among the multidimensionally deprived and also a change in the household status from deprived to non-deprived in such a dimension.

This means that the a'_{ij} achievement for the i individual and the j dimension, obtained by an increment of a constant $\gamma > 0$ such that $a'_{ij} = a_{ij} + \gamma$ for any person i satisfying $i \in s_j$, $a_{ij} < z_j$, $i \in h\ s.t.\ p_h = 1$, produces $a'_{ij} \geq z_j > a_{ij}$ and $d^{\beta'}_{hj} = 0$, where the d^{β}_{hj}-dimensional deprivation indicator for the h household and the j dimension before this achievement increment is $d^{\beta}_{hj} > 0$ and after the achievement increment corresponds to $d^{\beta'}_{hj} = 0$.

Having defined these three different types of increases in welfare as relevant, the following three properties to characterise the $MD^{\beta,\theta}$ family of measures arise:

Weak Achievement Monotonicity (WAM). Multidimensional deprivation $MD^{\beta,\theta}$ satisfies weak achievement monotonicity if $MD^{\beta,\theta}$ does not increase due to an applicable achievement increment.

Deprivation Monotonicity (DM). Multidimensional deprivation $MD^{\beta,\theta}$ satisfies deprivation monotonicity if $MD^{\beta,\theta}$ decreases due to a deprivation reduction among the multidimensionally deprived.

Dimensional Deprivation Monotonicity (DDM). Multidimensional deprivation $MD^{\beta,\theta}$ satisfies dimensional deprivation monotonicity if $MD^{\beta,\theta}$ decreases due to a dimensional deprivation reduction among the multidimensionally deprived.

Although not every measure satisfies the three proposed dominance properties, each combination of β and θ parameters enforces different properties. In particular, any $MD^{\beta,\theta}$ is proposed to satisfy WAM and DDM, and $MD^{\beta,\theta}$, where $\beta > 0$ to satisfy DM.

These three properties are illustrated simulating each of the three different types of improvements in welfare in the 2013 Paraguayan example. Each of the three different types of improvements in welfare are simulated 1,000 independent times. After each simulation, the resulting societal H and $MD^{\beta,\theta}$ metrics are evaluated and Table 3.6 presents the obtained average across 1,000 independent simulations performed. The next paragraphs describe these simulations and the obtained results.

According to the definition used in the 2013 PHS multidimensional index example, individuals 18 years of age or older that have less than 9 years of completed education are considered as deprived in educational achievement i.e., having low educational achievement. The 2013 PHS has a sample of 13,389 interviewees that are 18 years of age or older. Before any achievement increment was simulated, this population exhibited an average of 8.9 years of education.

Then, an achievement increment in the educational achievement indicator is simulated by sampling without replacement, out of the total 13,389 observed individuals, 50% of those having less than 8 completed years of education and belonging to a multidimensionally deprived household. This sample corresponds to 1,972 individuals. The number of years of education for each of the sampled individuals is incremented by one. Although this sample experienced this one-year increment, the additional year does not change their deprivation status. After the achievement increment, the 18 years of age and older population had an average of 9.1 years of completed education.

The mean results of societal measures obtained after these 1,000 independent simulations are presented in Row (14) from Table 3.6. Any measure satisfying the WAM property would show a zero or negative difference between the simulated scenario and the observed case. Row (15) from the table, shows this obtained difference. It is observed that any of the five different societal analysed measures increased as a result of an achievement increment. These results indicate that these measures satisfy the proposed WAM property.

After simulating an achievement increment that, although constitutes an improvement in welfare for some individuals but does not alter their deprivation status because is not large enough to remove individual deprivation, I simulate an achievement increment such that deprivation no longer is observed, namely a deprivation reduction among the multidimensionally deprived. In particular, out of the 13,389 PHS 2013 individuals 18 years of age or older, 4,150 are deprived in educational achievement and belong to a multidimensionally deprived household. Out of those 4,150 individuals, 3,533 belong to households that along to be multidimensionally deprived also exhibit more than one deprived person in educational achievement.

To simulate a deprivation reduction among the multidimensionally deprived, a random sample without replacement of 50% of these 3,533 individuals is drawn. Only one person per household belongs to this randomly selected sample. In total, the sample is made up of 719 individuals. Each of the sampled individual change his/her deprivation status from deprived to non-deprived. It is worth noting that although these sampled individuals experience an improvement in welfare that removes their deprivation status in educational achievement, this improvement does not change the household deprivation status because they were not the only household members facing low educational achievement.

Before the simulated deprivation reduction, 47.9% of the 18 years old and older population had low educational achievement; after the simulated deprivation reduction, this rate became 42.5%. The mean result of the 1,000 simulations is displayed in Row (16) of Table 3.6. The results suggest that, keeping constant the households identified as multidimensionally deprived, societal values of $MD^{\beta,\theta}$ with $\beta > 0$ decrease after a deprivation reduction among the multidimensionally deprived. This result illustrates the DM behaviour in my proposed family of indices.

The third type of simulated welfare increment is a dimensional deprivation reduction among the multidimensionally deprived. In the first two types of simulated welfare increments, although the i individual increases her/his welfare due to no longer being deprived in educational achievement, other household members might be still deprived in the same dimension. Therefore, an achievement increment or a deprivation reduction among the multidimensionally deprived does not necessarily change the household status from deprived to non-deprived.

In fact, when analysing the PHS 2013, from the 40.2% of households identified as multidimensionally deprived, 4,150 adults are deprived in educational achievement but only 617 of them belong to a household where they are the only person having low educational achievement. Accordingly, household dimensional deprivation is only removed in virtue of an improvement in welfare when any of those 617 individuals suffer a deprivation reduction among the multidimensionally deprived. To simulate the welfare increment being a dimensional deprivation reduction among the multidimensionally deprived, a random sample of 50% of those 617 individuals is drawn. The sample is made up of 380 individuals.

Before the simulation of dimensional deprivation reduction among the multidimensionally deprived, 67.0% of interviewed households were deprived in educational achievement. After the simulated deprivation reduction, the rate of low educational achievement among household decreased to 60.0%. Row (18) from Table 3.6 shows the mean result of the 1,000 simulations. The results suggest that societal $MD^{\beta,\theta}$ falls due to a dimensional deprivation reduction among the multidimensionally deprived, result consistent with the proposed DDM property.

The results of these simulations, in summary, confirm the proper orientation of the proposed $MD^{\beta,\theta}$ family of indices. While WAM enforces the $MD^{\beta,\theta}$ multidimensional deprivation measures to not increase as a result of any increment in welfare, DDM makes $MD^{\beta,\theta}$ decrease if any multidimensionally deprived household reduces its number of deprived dimensions. DM ensures that $MD^{\beta,\theta}$ decreases if any j-deprived i individual belonging to a multidimensionally deprived household reduces his/her number of suffered deprivations.

One the one hand, in terms of an individual-based scenario and in comparison to the AF set of properties, the proposed WAM and DDM properties of this chapter result equivalently to the AF's proposed *weak monotonicity* and *dimensional monotonicity* properties, respectively.

Worth noting that a desirable dominance characteristic of a multidimensional deprivation measure is the ability of the measure to fall unambiguously under any applicable achievement increment, even if such achievement increment does not remove deprivation. The AF method termed this property as *monotonicity*. Any measure satisfying monotonicity would produce a decrease in the societal value of $MD^{\beta,\theta}$ because an applicable achievement increment. In practical terms, if for instance $MD^{\beta,\theta}$ would satisfy monotonicity, we would observed in the simulation results presented in Row (14) from Table 3.6 a non-zero difference with regards to the observed scenario (Row (1)). However, since my measures are built on the basis of counting deprivations, they are not able to document this type of welfare improvement. In the case of the AF method, any M_α with $\alpha > 0$ is meant to satisfy monotonicity. However, M_α with $\alpha > 0$ are metrics not commonly used in the applied literature because they require all considered achievement indicators to be cardinal. Given the ordinal nature of the majority of policy indicators, the AF's monotonicity property is therefore hardly exhibited.

Another dominance property widely analysed by the income-based poverty measurement literature is the sensitivity of the measures to progressive transfers, which are transfers of income from a poor person to any other person that is poorer. In such a case, poverty measures are desired to decrease as a result of this type of change in the income distribution. This measurement sensitivity has been analysed by Sen (1976) and Kakwani (1980), among others.

For multidimensional measures, on the other hand, Bourguignon & Chakravarty (2002), Foster et al. (2005), Alkire & Foster (2011), and Chakravarty & Silber (2008) have proposed their societal measures to be sensitive to progressive transfers. Nonetheless, in the case of multidimensional deprivation indices, this type of transfer principle is not necessarily compelling for all the dimensions included within a particular index. For instance, it is not a compelling argument to desire sensitivity of the measures to transfers of good health from one individual to another or to desire sensitivity of the measures to transfers of educational achievement from one person to another.

Sensitivity to transfers is relevant when describing multidimensional deprivation through indicators circumscribed to resources, such as monetary or educational resources. In these cases, sensitivity of the societal measures to transfers from one individual with a larger amount of those resources to an individual with smaller amount of them are seen to be desirable for the purposes of distributive analysis, as pointed out by the poverty measurement literature. However, when the deprivation indicators describe a lack of outcomes, such as the absence of good

Table 3.6: Simulation results: Mean H incidence of multidimensional deprivation and mean $MD^{\beta,\theta}$ burden of multidimensional deprivation

	H	$MD^{\beta,\theta}$			
		$\beta = 0$		$\beta = 1$	
		$\theta = 0$	$\theta = 1$	$\theta = 0$	$\theta = 1$
(1) Observed	0.40	1.18	0.28	3.53	0.28
Scale invariance					
(2) Fully non-deprived scenario	0.0	0.0	0.0	0.0	0.0
(3) Fully deprived scenario	1.0	4.2	1.0	12.6	1.0
Population replication invariance					
(4) Simulated	0.40	1.18	0.28	3.53	0.28
(5) Difference (4)-(1)	0.00	0.00	0.00	0.00	0.00
Household anonymity					
(6) Simulated	0.40	1.18	0.28	3.53	0.28
(7) Difference (6)-(1)	0.00	0.00	0.00	0.00	0.00
Within household anonymity					
(8) Simulated	0.40	1.18	0.28	3.53	0.28
(9) Difference (8)-(1)	0.00	0.00	0.00	0.00	0.00
Multidimensional deprivation focus					
(10) Simulated	0.40	1.18	0.28	3.53	0.28
(11) Difference (10)-(1)	0.00	0.00	0.00	0.00	0.00
Applicable deprivation focus					
(12) Simulated	0.40	1.18	0.28	3.53	0.28
(13) Difference (12)-(1)	0.00	0.00	0.00	0.00	0.00
Weak achievement monotonicity					
(14) Simulated	0.40	1.18	0.28	3.53	0.28
(15) Difference (14)-(1)	0.00	0.00	0.00	0.00	0.00
Deprivation monotonicity					
(16) Simulated	0.40	1.18	0.28	3.39	0.27
(17) Difference (16)-(1)	0.00	0.00	0.00	-0.13	-0.01
Dimensional deprivation monotonicity					
(18) Simulated	0.40	1.11	0.27	3.46	0.26
(19) Difference (18)-(1)	0.00	-0.07	-0.01	-0.07	-0.02

Source: Author's calculations based on 2013 PHS. Note: population means developed under the basis of a sample of 5,423 Households. Households satisfying $m^{1,0.87} > 0.65$ are identified as the multidimensionally deprived.

health or nutritional status for example, this desired sensitivity might be losing its purpose.

As Aaberge & Brandolini (2014*a*) pointed out, the analysis of the sensitivity of multidimensional measures to changes in the distribution of deprivations is an area that requires further research. In particular, for the proposed $MD^{\beta,\theta}$ family of indices of this chapter, two types of sensitivities arise as being relevant to analyse with regards to transfers: the sensitivity of societal measures to demographic re-arrangements resembled by permutations of individuals across households and transfers of resources from better off to worse off individuals (within the multidimensionally deprived population). We would like to ensure that societal measures are based on a progressive transfer of resources or individuals across households. However, the complexity involved in the possible compensation dynamics between attributes and the analysis of the mechanisms throughout demographic reconfigurations require more detailed research that is out of the scope of this chapter. For the time being, the dominance outlined properties in this section (WAM, DM and DDM) assure that the proposed $MD^{\beta,\theta}$ measures have the proper orientation if any of these transfers result in either an achievement increment, a deprivation reduction, or a dimensional deprivation reduction.

Decomposability

The poverty measurement literature defines as decomposable any metric that can be expressed as a weighted average of subgroup estimates, where weights are population subgroup shares. The references Foster et al. (1984), Tsui (1999), and Alkire & Foster (2011) refer to this property as *decomposability*, and Bourguignon & Chakravarty (2003) refers to it as *subgroup decomposability*.

Subgroup decomposability allows consistent decompositions of the societal measure into population subgroups. In particular, my H and $MD^{\beta,\theta}$ societal measures are able to be expressed as a weighted average of the multidimensional deprivation level observed across subgroups of households, where the weight of each is the share of households that each subgroup represents.

As an example, if I sort Paraguayan households under the basis of $m^{1,0.87}$ and identify as multidimensionally deprived those satisfying $m^{1,0.87} > 0.65$, this leads to 40.3% of the total 5,423 households being identified as multidimensionally deprived. This result can be decomposed further by sub-population groups such as household sizes or counties. For illustrative purposes, the decompositions by county is shown in Table 3.7 below. The last row in the table corresponds to the overall societal estimate, and all seven previous rows correspond to each subgroup of households by county. If I obtain the share of households by county based on the figures included in Column 1 and use those shares as weights to calculate the weighted sum of each of the measures across household sizes, the results correspond to the overall societal figures. As Alkire & Foster (2011) pointed out for their family of measures, this measurement feature becomes an important technology for policy purposes. It allows the design and evaluation policy interventions for specific population subgroups.

However, two caveats are worth noting. First, an identification of the multidimensionally deprived at the household level produces societal measures to not be decomposable by individual population subgroups (ranges of age, gender, or ethnicity) disregarding the household where they belong. Including current household-based applications of multidimensional deprivation measurement, H and $MD^{\beta,\theta}$ are not the exception in this case.

Table 3.7: County specific Paraguayan results

	Number of households	H	$MD^{\beta,\theta}$			
			$\beta = 0$		$\beta = 1$	
			$\theta = 0$	$\theta = 1$	$\theta = 0$	$\theta = 1$
	(1)	(2)	(3)	(4)	(5)	(6)
Asunción	691	0.1	0.3	0.1	0.8	0.1
San Pedro	469	0.6	1.8	0.4	5.8	0.4
Caaguazú	758	0.6	1.8	0.4	5.5	0.4
Itapúa	456	0.5	1.5	0.3	4.3	0.3
Alto de Paraná	826	0.5	1.5	0.4	4.7	0.4
Central	1,141	0.2	0.5	0.1	1.4	0.1
12 remaining counties	1,082	0.5	1.4	0.3	3.9	0.3
Total Paraguay	5,423	0.4	1.2	0.3	3.5	0.3

Source: Author's calculations based on 2013 PHS. Note: Multidimensionally deprived households are identified as those satisfying $m^{1,0.87} > 0.65$.

Second, following the Alkire & Foster (2011) proposed notion of dimensional decomposability for any of their M_α metrics, the $MD^{\beta,\theta}$ measures with $\theta = 0$ can be decomposed to estimate the contribution of each j dimension in the overall societal measure. Then, $MD^{\beta,0}$ can be, alternatively to Eq. (3.11), expressed as the following:

$$MD^{\beta,0} = \sum_{j \in J} \mu(d_{hj}^{\beta}(k)) \ / \ J, \tag{3.13}$$

where $d_{hj}^{\beta}(k)$ is the household dimensional deprivation indicator censored to zero for any non-multidimensionally deprived household, and $\mu(d_{hj}^{\beta}(k))$ corresponds to the average value of $d_{hj}^{\beta}(k)$ for $h = 1, 2, \ldots, R$. Hence, the contribution of the j dimension in the $MD^{\beta,0}$ societal measure corresponds to $\big(\mu(d_{hj}^{\beta}(k)) \ / \ J \ \big)/MD^{\beta,0}$.

Continuity

Continuity in the multidimensional measurement literature has been used, for instance, by Bourguignon & Chakravarty (2003) to characterise welfare multidimensional measures. According to the scholars, it ensures a well-behaved functional form that would produce no abrupt jumps given changes in achievements. My proposed approach may have more than one possible source of discontinuity. First, it counts dimensions on deprivation and household deprivations, thus, household metrics are counting indicators that belong to the set of natural numbers. Second, given that two identification procedures are used (the first one identifies individuals in deprivation and the second identifies multidimensionally deprived households). Therefore, continuity is not expected to be achieved.

Despite the lack of continuity of the proposed metrics of this chapter, they are still cardinal. The proposed methodology of this chapter exploits the ordinal nature of most of the policy indicators currently in use. Using either count-based, share-based, or a mixture of both approaches, leads to cardinal metrics of the household burden of multidimensional deprivation being developed. The cardinal nature of the proposed $m^{\beta,\theta}$ metrics allow the comparison of the size of this observed burden of multidimensional deprivation across any two given households, which provides policy makers a technology that allows ranking households from most deprived to least deprived. This characteristic reveals an important technique for targeting the most deprived households in developing countries.

Chapter 4

Persistent poverty in the presence of survey non-response: The case of Peru

Abstract

Despite their policy relevance, intertemporal poverty measures could be questionable, as they are based upon panel data sets that naturally suffer from not-at-random survey non-response. This chapter examines the reliability of expenditure-based intertemporal poverty measures in the presence of survey non-response. Two persistent poverty measures are analysed: the persistent-poverty-headcount ratio and the duration-adjusted persistent-poverty-headcount. Using the 2013 Paraguayan panel household survey, we implement a partial identification approach to derive upper and lower bounds for the persistent poverty estimates. The first set of bounds uses the evidence revealed by the survey, so the bounds are wide. We obtained narrower bounds by imposing an instrument and a monotone instrument restriction. The results demonstrate that the width of the bounds varies across measures and cut-off points, with the duration-adjusted persistent-poverty headcount that uses higher cut-off points being more reliable. The results of the improved bounds indicate that standard non-response-weighted estimations of Peruvian persistent poverty represent a considerable underestimated picture of the intertemporal phenomenon.

Keywords: Poverty persistence, Chronic poverty, Survey non-response, Partial identification.

JEL codes: C23, D31, I32.

Acknowledgements: I am thankful to Stephen Pudney (PhD supervisor for this particular chapter) for his guidance, comments, and contributions during the development of this manuscript. The chapter that I present here has been developed as part of a supervisor/student co-authored paper. This chapter does not take into account all the features included in the co-authored paper. I also would like to thank Nancy Hidalgo, Lucia Gaslac, and their team at the Peruvian National Statistics Office, who provided us with the datasets to make this work possible. An earlier version of the co-authored paper appeared as the ISER working paper number 2013-22.

4.1 Introduction

There is general agreement that households that suffer poverty persistently over time are in worse condition than households where poverty occurs on a transient basis. In recent years, given its policy relevance, special efforts have been carried out to operationalise the concept of chronic poverty as a persistent pattern of poverty states over time. For the purposes of this chapter, we use interchangeably the term 'persistent poverty' and 'chronic poverty', both referring to repeated expenditure-based poverty states over time.

Persistent poverty measures have been proposed by Baulch & Hoddinott (2000), Jalan & Ravallion (2000), Yaqub (2003), Hulme & Shepherd (2003), Foster (2009), Calvo & Dercon (2009), Bossert & Chakravarty (2012), Gradin et al. (2012), and Mendola et al. (2012), among others. All these metrics are meant to be constructed as household balanced panel estimates. Household panel surveys are longitudinal studies that observe a set of variables across households over time. They constitute a rich source of information for intertemporal analysis. While households exclusively observed in each wave of the longitudinal survey constitute a balanced panel, households observed in at least one of the waves constitute an unbalanced version of the panel. As such, persistent poverty measures available in literature require households to be observed in each wave of the longitudinal survey; in other words, they are based on the balanced version of the panel survey.

However, panel surveys suffer from survey non-response that tends to increase and accumulate as further waves of interviewing are conducted (Watson & Wooden 2009). This is the case for most household panel surveys. The longest-running longitudinal studies, such as the Panel Study of Income Dynamics from the United States (PSID), the British Household Panel Survey (BHPS), and the German Socio-Economic Panel (GSOEP), lost at least 25% of the original sample after their eighth wave (Watson & Wooden 2009).

Panel surveys in developing countries are no exception. While rare and often based on small sample sizes or a few waves of available data (Jenkins & Siedler 2007), panel surveys in developing countries can suffer from much more dramatic cumulative survey non-response than long-ago developed surveys as the PSID, BHPS, and GSOEP.

In particular, in some developing countries, panel household surveys have been designed using a household follow-up procedure exclusively based on residence. This means that only households that do not leave their residence are interviewed by the consecutive waves of the panel survey. The Peruvian National Panel Household Survey (ENAHO), for instance, follows households on the basis of their housing, but if a household moves out, the ENAHO does not use any follow-up criteria for them or for demographic reconfigurations of them, i.e., household splits. As a result, in 2011, after five waves of implementation of the ENAHO panel, only 14.5% of the initially sampled ENAHO panel households were still observed by the panel.

Other examples of household panel surveys in developing countries that track households based on residence are the Bolivian Pre-School Program Evaluation Household Survey, the Kenyan Ideational Change Survey, the Ethiopian Rural Household Survey, the Indian Rural Economic Development Survey, the Continuous Sub-sample of the Argentinian Permanent Household Survey, and the Brazilian Longitudinal Survey Data of Households in Ouro Preto do Oeste, Rondonia (Alderman et al. 2000, Dercon & Hoddinott 2009, NCAER 1997, Baulch 2011, Hall & Caviglia-Harris 2013).

While studies such as Rosenzweig (2003) have analysed the effect that a follow-up procedure exclusively based on residence has on inferences of economic mobility, little is known with regard to the effect of cumulative survey non-response on persistent poverty estimates. This chapter, as the first in literature, examines the reliability of expenditure-based intertemporal poverty measures in the presence of survey non-response. In the context of the Peruvian household panel data set, ENAHO, we assess the behaviour of two members of the Foster (2009) family of intertemporal poverty measures: the persistent-poverty headcount and the duration-adjusted persistent-poverty headcount, both in the presence of survey non-response.

The traditional approach to tackle survey non-response in longitudinal surveys is through weighting systems that aim to correct for such non-response. This approach assumes the survey non-response process related to observables and unrelated to unobservable characteristics. In the case of persistent poverty measures, balanced panel estimates are constructed using sample non-response weights that correct for observable characteristics that drive the survey non-response. This means that, after controlling for survey non-response caused by observable characteristics, the estimates are assumed to be unrelated with unobservable characteristics of the household that could have determined the survey non-response.

This assumption nonetheless seems unrealistic. Survey non-response in household panels might be driven not only by observable characteristics but also by unobservable characteristics that influence the survey non-response process and are related to household poverty conditions. If this is the case, persistent poverty estimates might occur in the presence of 'missing-not-at-random' processes. According to Little & Rubin (1987)'s taxonomy of missing data patterns, the probability for a household to be missing in a particular wave is most likely related to unobservables that in turn are related to the outcome variables of interest are referred by the authors as missing-not-at-random (MNAR) patterns. As such, persistent poverty estimates developed on the basis of weighting systems that account strictly by observables but not unobservable characteristics can provide biased persistent poverty estimates.

A specific branch of the biostatistics literature tackles the potential bias that this type of missing data patterns might bring to biological outcomes. Examples of this literature are studies such as Kenward (1998), Jansen et al. (2006), Albert & Follmann (2009) and Little (2009). The methods implemented by this type of literature, however, rely on specific assumptions about the pattern and shape of the missingness distribution that cannot be tested. If these assumptions do not hold, the use of such methods might introduce additional bias to the estimates (Enders 2011). For a review of the literature on approaches to address MNAR processes and their implementation on longitudinal surveys, see Enders (2011).

Along with those methods, as pointed out by Horowitz & Manski (1998), this type of data problem can be also catalogued as an *identification* problem. According to Koopmans (1949), identification and statistical inference correspond to two different concepts. While statistical inference refers to the determination of a parameter based on a large-enough sample of observations so that the conclusions of such a parameter rely on the variability of the sample, identification explores the limits of inference in drawing conclusions about the parameter under the hypothesis of full knowledge of the probability distribution of the observations (Koopmans

1949). Using this approach the missing data problem can be understood as a problem of partial observability. As such, the identification regions where the true estimates belong can be determined. The formal characterisation of identification regions was first introduced by Horowitz & Manski (1995) and Manski (2003). Examples of this type of studies on poverty measures include Chavez-Martin del Campo (2004) and Nicoletti et al. (2011).

Using a partial identification approach, there is no need for assumptions over the distribution of the unobservables; instead, conclusions are drawn using the information that the sample reveals. This is the selected framework of the current chapter. The understanding of the survey non-response problem and its effect on persistent poverty estimates as a partial identification setup allows to develop lower and upper bounds based on the probability distribution that the sample reveals and on visible and plausible assumptions rather than non-response corrected point estimates that could be based on unstable assumptions. This is the contribution of the current chapter to the methodological poverty measurement literature.

The chapter is organised as follows. After this introduction, it describes the Peruvian household panel data set and its survey non-response pattern. Then, the selected chronic poverty measures to be analysed are described. We describe the anatomy of the problem that survey non-response poses over persistent poverty estimates in the ENAHO context. Subsequently, throughout Section 4.4, we determine the effect that survey non-response has on persistent poverty estimates in this context. In particular, we first derive the lower and upper bounds that could be learnt from the sample-revealed information and then derive additional sets of bounds after imposing an instrumental variable assumption and a monotone instrumental variable assumption. We finish by discussing the empirical results and presenting conclusions and further research paths.

4.2 Data

The empirical analysis of this chapter uses the Peruvian National Household Survey, which is a survey run yearly by the Peruvian National Institute of Statistics since 1995. It aims to monitor Peruvian households living conditions, and it is known as the ENAHO, *Encuesta Nacional de Hogares*. This section briefly describes the ENAHO and its survey non-response problem.

4.2.1 The ENAHO

The ENAHO household survey covers the Peruvian national territory, including urban and rural areas, 24 counties (*departmentos*), and the constitutional province of Callao. The national official figures of expenditure-based poverty are produced by the Peruvian National Institute of Statistics (whose acronym in Spanish is INEI) on the basis of the information collected by the ENAHO and the technical advice and supervision of an independent poverty committee.

The ENAHO's sample is selected from the national population and housing census frame that the statistical department maintains and is based on the 2007 census-collected data. Independently within counties, the ENAHO's sample is drawn using a multi-stage probabilistic area selection procedure. Specifically, the sample design uses three stages that differ by urban and rural areas.[1] Each ENAHO sampling unit, in the first two stages, is selected with a probability proportional to the sample frame. For the last stage, the number of selected housing units is driven by a selection interval provided to the field team.

Each year, the survey uses 12 sub-samples, each one randomly assigned to each month of the year. Then, the sample is evenly distributed across time and space. On average, from 2007 to 2011, this cross-sectional survey has interviewed 22,353 households per year. To measure changes in the behaviour of some socioeconomic characteristics of the population, the ENAHO maintains a sub-sample of housing units as a panel. At the time the current chapter was written, the ENAHO has available for use two different panels, the first run from 2004 until 2006 and the second run from 2007 until 2011. To perform the empirical analysis described in

[1] In urban areas, the primary sampling units (PSUs) are municipalities inhabited by two thousand or more inhabitants. The secondary sampling unit is the conglomerate, which has, on average, 120 housing units. The tertiary sampling unit is the housing. In rural areas, in contrast, the primary sampling unit is either a municipality inhabited by a population consisting on 500 to less than two thousand persons or a less populated area termed an *Área de empadronamiento rural* (AER). An AER, on average, consists of 100 housing units. The secondary sampling units, in the case of rural areas, are conglomerates comprised 120 housing units or single housing units in particular rural PSUs. Similarly to the urban areas, in the rural areas, the tertiary sampling units correspond to single housing units.

the current chapter, we use the 2007-2011 panel sub-sample of the ENAHO. We continue in the next section by describing this panel sub-sample and the survey non-response that it suffers.

4.2.2 The ENAHO panel survey

The 2007-2011 ENAHO panel began in 2008 with a sub-sample of 7,560 housing units. This sub-sample of housing units was drawn from the 2007 cross-sectional ENAHO by the INEI using 12 probabilistic sub-samples that had an approximate number of conglomerates per department. The 2007 cross-sectional ENAHO sample consists of 25,947 attempts to conduct housing unit interviews.

The 7,560 selected housing units for the ENAHO panel were inhabited in 2007 by 7,769 households. The ENAHO panel yearly follows up with households based strictly on the criterion of residence. This means that there are no follow-up procedures for households that relocate from the housing where they were interviewed for first time by the panel. There is also no follow-up procedure for demographic reconfigurations of households that move out of the initial housing. Only very rare cases of households are followed, and those are households that move to a location within the same geographical area. As a result, the households that remain within the panel are those where at least one household member remains in the housing were she/he was interviewed in the first wave. Each panel household is followed for up to five years.

To refresh and compensate for the households that leave the panel, since 2008, the INEI has included a yearly boost in the number of housing units within the panel. For the purposes of the current chapter, we confine our analysis to the initial drawn panel sample which comprises the attempted to interview 7,769 households in 2007. This set of households is defined in the current chapter as the panel member households.

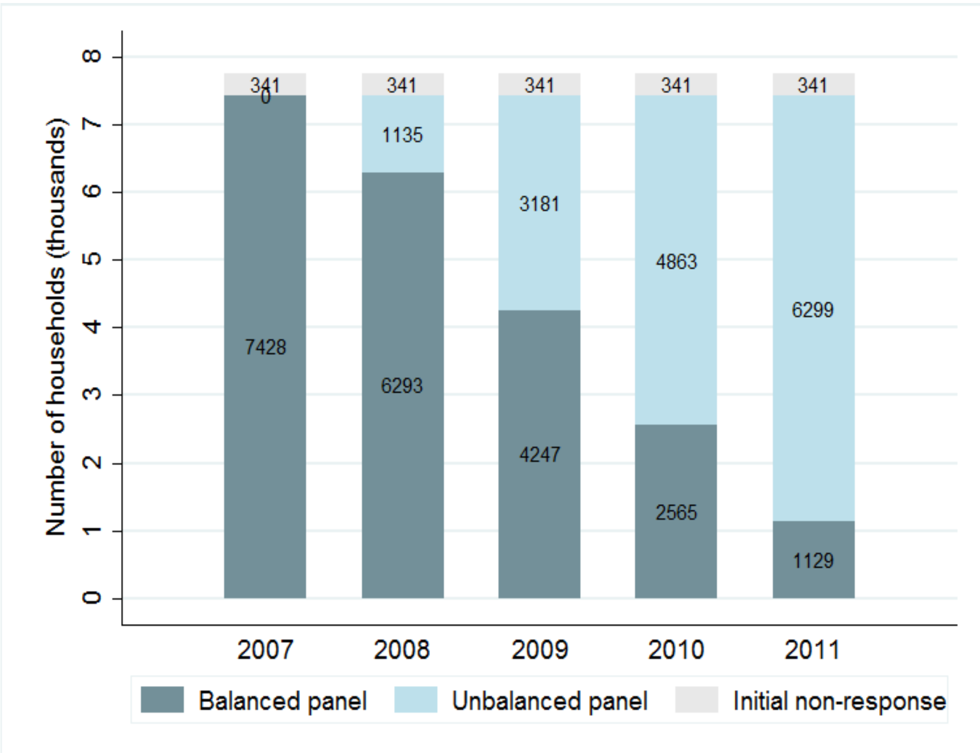
In 2007, out of these 7,769 attempted to interview households, 193 refused an interview and 148 were absent at the time of the interview. Thus, 7,428 households in the sample provided either complete or incomplete interviews. In 2008, 84.7% of the panel households interviewed in 2007 remained within the same residence. After that, in 2009, the share of households that remained in the panel from

2008 was 67.5%. Across the five years of the panel, the share of households that remained in the sample declined year over year. In 2011, only 44.0% of the 2010 observed households were interviewed. As a result, after five waves of implementing the panel, out of the 2007 7,769 attempted to interview households, only 14.5% of them remained within the panel.

As such, an increasing and cumulative pattern of survey non-response is observed through the five years of the 2007-2011 ENAHO panel implementation. Figure 4.1 shows the yearly total number of household interviews attempts from 2007 until 2011 in the ENAHO panel. The darker zone within columns represents the share of balanced panel households, and the lighter zones represent first the unbalanced panel households and then the non-response households. While, in 2008, the balanced panel (households observed in both 2007 and 2008) consisted of 6,293 households, in 2011, the balanced panel (households observed in each of the five years) included 1,129 households.

A cumulative behaviour of survey non-response, such as the one observed in the ENAHO panel, can be also observed in other longitudinal surveys. However,

Figure 4.1: Survey non-response pattern



Source: author's calculations developed on the basis of the 2007-2011 ENAHO panel.

in the ENAHO, this pattern is more accelerated than in other panels because of the conception of the survey as a panel with a household follow-up procedure exclusively based on residence. For instance, while the ENAHO panel lost 85.5% of its original sample of households after five years of implementation, the United Kingdom's BHPS registered a cumulative non-response of 33% (Taylor et al. 2010) after its fifth collected wave. Similar is the case of the GSOEP survey, which, after the fifth year of implementation, registered 22% of the sample lost. After 18 waves of application of the BHPS, the cumulative sample lost was 40%. For the GSOEP, it took 24 waves to reach a cumulative non-response rate of 44% (Watson & Wooden 2009).

The cumulative and increasing survey non-response observed in the ENAHO panel has measurement implications for any indicator or analysis carried out on the basis of balanced panel estimates. In particular, in the current chapter, we are interested in analysing the reliability of persistent poverty measures developed under these circumstances. In the next section, we continue by first defining persistent poverty and then describing the particular measures for which we analyse the effect of survey non-response in the context of the ENAHO panel.

4.3 Measuring persistent poverty in the Peruvian context

Defining chronic poverty is an open debate and implies several value judgements. In the literature, there have been several efforts to operationalise the concept of chronic poverty as a persistent pattern of poverty states over time. Examples include the measures proposed by Baulch & Hoddinott (2000), Jalan & Ravallion (2000), Yaqub (2003), Hulme & Shepherd (2003), Foster (2009), Calvo & Dercon (2009), Bossert & Chakravarty (2012), Gradin et al. (2012), and Mendola et al. (2012). All these measures rely on balanced panel data sets and are income or expenditure based.

As a result and for the purposes of this chapter, we adopt the definition of chronic poverty as a persistent pattern of household intertemporal expenditure-based poverty. This definition implies that the household is selected as the unit of analysis and expenditure as the welfare indicator to track across households and time.

We continue describing this selected welfare indicator and then move on to present the persistent poverty measures that will be used throughout this chapter to characterise chronic poverty.

4.3.1 The welfare indicator

The ENAHO survey provides a comprehensive set of variables that allow the construction of annual household aggregates of either income or expenditure. We opt to use expenditure as welfare indicator rather than income for three reasons.

First, expenditure is seen as an indicator that captures more accurately long-term trends of living standards than income. As such, an expenditure indicator might smooth long-term patterns of welfare more accurately than income. Indeed, when analysing both the income and expenditure Peruvian household-based indicators, we found in the ENAHO cross-sectional survey from 2007-2011 two times larger variability in the annual mean household income than in the annual household expenditure.

Second, from the theoretical point of view, according to Deaton & Zaidi (2002), the ultimate goal of both income and expenditure aggregates is to describe the consumption of the households. However, income is an indicator of the amount of transient resources that households can account for, whereas expenditure is considered an indicator that approximates better the consumption level of the household.[2]

The third and last reason to select expenditure as a welfare indicator rather than income refers to the particular rich source of expenditure items that the ENAHO survey constitutes. The main drawback of using household expenditure drawn from household surveys is that these surveys usually do not capture own-account and non-monetary transactions; which for developing countries might represent a significant share of the economy. For the case of Peru and, in fact, for the ENAHO survey, the expenditure questionnaire captures, along with monetary

[2]For a discussion of the theoretical basis of consumption-based measures of welfare, see Deaton & Zaidi (2002).

expenditure, own-account transactions (self-supplied), non-monetary expenditure, and monetary and non-monetary transfers received from institutions and other households. The capture of all these items in the survey enhances the observed share of household non-monetary expenditure.

As a result of these three considerations and for the purposes of this chapter, we use the total annual household expenditure derived by the INEI for the ENAHO survey. This household welfare indicator, as collected by the ENAHO survey, takes into account food consumed outside the household; food to be consumed within the household; clothing and shoes; housing rent, fuel, electricity, and housing repairs; furniture and housing maintenance; health services and self-health care; transport and communications; leisure, amenities, and education and cultural services; and other goods and services.[3]

Although the Peruvian official poverty figures provided by the INEI are calculated using as a household aggregate the monthly household per-capita expenditure, we opt instead to express the ENAHO household expenditure in terms of equivalent adults. To obtain expenditure estimates that are comparable across households of different sizes and compositions, we use the equivalence scale proposed by Deaton & Zaidi (2002). To this aim, we use the monthly total expenditure of the i-household divided by an e_i-equivalence factor. This equivalence factor is defined for each i-household as follows: $e_i = (A_i + \varphi C_i)^\theta$, where A_i is the number of adults who belong to the i-household, C_i is the number of children in the i-household, φ is a parametric value that expresses the cost of a child relative to an adult and takes values from zero to one, and θ is the parameter that accounts for the scale economies within households.

To set the values of the φ and θ parameters we follow Deaton & Zaidi (2002)'s recommendations with regard to this selection in the case of developing countries. According to the authors, φ values close to one are devoted to industrialised countries, where the cost of having a child could be arguably high, while values close

[3]The ENAHO expenditure aggregate, however, does not include expenditure on public health or public education, the imputed value of the consumption of durable goods, or the imputed value of the consumption of water taken from the river.

to zero but as low as 0.3 are devoted to the poorest economies. Because Peru is considered an upper middle income country[4], we set $\varphi = 0.65$.

In terms of θ, according to Deaton & Zaidi (2002), values of θ close to one reflect a high share of private goods at home (such as food) and are devoted to the poorest economies, which according to the authors spend as much as three-quarters of their budget on food. In Peru, according to the cross-sectional ENAHO, the share of food expenditure was on average 13.7% of the total 2011 expenditure. Then, we follow the Deaton & Zaidi (2002) recommended approach for richer economies and set $\theta = 0.75$.

Finally, in terms of the poverty threshold to identify poor households, the INEI has available for public use the official expenditure-based poor thresholds that vary across years and geographical areas of the Peruvian national territory. These expenditure-based poor thresholds aim to value the expenditure level that enables household members to access the basic food energy required to carry out moderate activities to survive, in addition to the value of goods and services required by household members to satisfy their needs, such as clothing, footwear, housing rent, fuel use, furniture, appliances, health care, transportation, communications, entertainment, education, culture, and others. These official thresholds indicate the minimum per-capita expenditure level that a household, according to its location and the year of the evaluation, must exhibit to be catalogued as non-poor.

To identify poor households, in this chapter, these official expenditure thresholds are used. However, we re-expressed them as adult-equivalent poverty lines as follows. Since the average official poverty line of each year and each of the 25 Peruvian counties for which the cross-sectional ENAHO allows inferences corresponds to afore-discussed minimum average expenditure expected by a household to be considered out of poverty divided by the average household size of each of these geographical domains. We obtained the average expected expenditure for each of the 25 Peruvian counties and years by multiplying the official poverty line per the mean household size of each of these domains. Then, the adult-equivalent poverty

[4]According to the World Bank's classification of countries, Peru is an upper middle income country that in 2014 registered a per capita gross national income of USD 6,360 (WB 2015).

line corresponds to such expenditure value divided by the mean e_i equivalent factor obtained across the i-households that belong to each domain.

We now continue in the next section presenting the selected measures used in this Chapter to describe persistent poverty.

4.3.2 The selected persistent poverty measures

A variety of persistent poverty measures are available in the literature. Examples include Baulch & Hoddinott (2000), Jalan & Ravallion (2000), Yaqub (2003), Hulme & Shepherd (2003), Foster (2009), Calvo & Dercon (2009), Bossert & Chakravarty (2012), Gradin et al. (2012), and Mendola et al. (2012). They aim to describe intertemporal poverty in a public policy context. An application example that uses these types of measures is the Dercon & Porter (2012)'s study. In the context of rural Ethiopia, Dercon & Porter build and analyse several persistent poverty measures, including the Foster (2009) family of measures. Another application example is the study presented by Dickerson & Popli (2012), which analyses the impact of persistent poverty among children in the UK and considers three different sets of measures: the Bossert & Chakravarty (2012), the Dutta et al. (2011), and the Foster (2009) family of measures.

For the purposes of this chapter, we opt to use the widely known and cited persistent poverty metrics proposed by Foster (2009). In particular, this family of measures incorporates the time dimension into the poverty measurement approach proposed by Foster et al. (1984), which is known as the FGT family of poverty measures.

Consider an i-household and a t-period of time, where $i = 1, 2, \ldots, R$ and $t = 1, 2, \ldots, T$. Further, define y_{it} as the expenditure of the i-household at t-time. Whenever y_{it} lies below a certain z poverty line, the household is considered poor during the period. Thus, the i-household poverty status at t-time is defined as a p_t-binary indicator of the presence or absence of poverty as follows:

$$p_t = \begin{cases} 1 & \text{if } y_{it} < z \\ 0 & \text{otherwise.} \end{cases} \tag{4.1}$$

For a setting of T periods of time, the P-poverty-spell-count observed for the i-household is defined as follows:

$$P = \sum_{t=1}^{T} p_t. \tag{4.2}$$

According to Foster (2009), any i-household satisfying $P \geq C$, where C represents the persistent threshold, is considered persistently poor. Then, the h-persistent poverty indicator for the i household is defined as follows:

$$h = \begin{cases} 1 & \text{if } P \geq C \\ 0 & \text{otherwise,} \end{cases} \tag{4.3}$$

where a value of $h = 1$ indicates that the i-household is identified as persistently poor and a value of $h = 0$ indicates that the i-household is identified as persistently non-poor.

However, persistent poverty can be characterised not merely by this binary measure, as the number of periods under poverty indicates the breadth of intertemporal deprivation. A greater number of periods in poverty depicts greater overall deprivation. Consequently, for households defined as persistently poor, Foster (2009) proposes adjusting the h-persistent poverty indicator with the share of periods under poverty to obtain a k_0-duration-adjusted persistent poverty indicator, which is defined as follows:

$$k_0 = \begin{cases} \dfrac{P}{T} & \text{if } P \geq C \\ 0 & \text{otherwise.} \end{cases} \tag{4.4}$$

Therefore, for a society consisting of R households, the H-persistent-poverty headcount ratio and the K_0-duration-adjusted persistent-poverty headcount are expressed, respectively, as follows:

$$H = \mu(h) \tag{4.5}$$

$$K_0 = \mu(k_0), \tag{4.6}$$

where $\mu(h)$ denotes the average value of the h-persistent poverty indicator and $\mu(k_0)$ denotes the average of the k_0-duration-adjusted persistent poverty indicator, both for $i = 1, 2, \ldots, R$ households.

Although Foster (2009) proposes as members of his chronic poverty family of measures H and any K_α, where $\alpha = \{0, 1, 2\}$ is the poverty aversion parameter, we expressly focus on the analysis of H and K_0 because both measures, H and K_0, describe chronic poverty in terms of the p_t-binary indicator of the presence or absence of poverty in the i household at time t. Conversely, the K_1 chronic poverty gap and the K_2 average severity, both express chronic poverty in terms of the y_{it}-expenditure indicator and the z-poverty line. As such, we leave for future research the analysis of K_α members where $\alpha = \{1, 2\}$.

The metrics of interest of this chapter, the H-persistent-poverty headcount and the K_0-duration-adjusted persistent-poverty headcount, like a majority of other persistent poverty measures available in the literature, require households to be observed over T periods of time.

4.3.3 The weighting system approach used to correct for survey non-response

Out of the initial sample of households selected to integrate the ENAHO panel, 1,129 of them have either 'complete' or 'incomplete' results in each of the five years of the panel survey. Households with complete survey results are those who replied to all the questions applicable to them in the questionnaire. Households with incomplete results are those who replied only to some of their applicable questions in the questionnaire. For any household with either complete or incomplete results in the survey, the INEI computed their annual household expenditure. Then, we observe for each of these 1,129 households the p_t-binary indicator of the presence or absence of poverty for any year $t = 2007, 2008, 2009, 2010$ or 2011.

However, this sample of 1,129 households corresponds to only 14.5% of the total panel sample of households attempted to interview in 2007. The natural question that arises and which is the focus of this chapter is, therefore, how chronic poverty estimates are affected by such survey non-response.

The traditional approach to tackle this problem in longitudinal surveys is through a weighting system that aims to correct the possible bias that survey non-response induces in the estimates and to ensure that the results reflect the structure of the population that the survey frame portrays. In the case of the ENAHO panel, the INEI follows this traditional approach and develops a weighting system that corrects for both the structure of the population and survey non-response. This INEI's official weighting system is available for public use.

The INEI developed this weighting system throughout two stages. The first stage consists of deriving the non-response rate that each secondary sampling unit independently registers, which in the context of the ENAHO corresponds to geographical conglomerates. Each of these conglomerates consists of a specific quarter of the year, county, region, urban or rural area, and a socioeconomic strata.[5] In the second stage, the INEI adjusts the obtained weighting system of the first stage by the population structure. This adjustment is carried out to reflect the population structure and in accordance with the population distribution derived from the census data for the mid-point of each year by county, region, urban and rural area, and socioeconomic strata.

For the purposes of the current chapter, we have available the final and official INEI's weighting system that corrects for survey non-response and the structure of the population. Additional to this set of weights we built a set of weights, that aims to adjust results exclusively for the differences reflected by the structure of the population shown in the official predicted population for the 2007-2011 period (according to census data).

As such, in this paper our analysis are carried out for societal measures of persistent poverty, first in use of the unadjusted estimates. Then, using our own estimated design weights that correct exclusively for the structure of the population, and lastly using the INEI's official weighting system. Figure 4.2 on page 162 presents these results. Sub-figures a. and b. present the results for the H-persistent-poverty headcount and the K_0-duration-adjusted persistent-poverty headcount, both across the five possible persistent thresholds C.

Dotted lines in Figure 4.2 plot the obtained persistent poverty measures on the basis of the 1,129 balanced panel households and without any adjustment

[5]The socioeconomic strata is a categorical variable that classifies geographical conglomerates into five ordered socioeconomic groups. It was developed by the INEI on the basis of a housing index that uses information on the housing itself, households and household members present in each 2007 national population and housing census-observed housing unit. The final strata assigned to characterise each conglomerate was defined by the INEI as the predominant strata observed across the housing units of the conglomerate.

(Unadjusted). These unadjusted results indicate that 61.4% of households in the sample have at least one period out of the five analysed in expenditure-based poverty and 19.0% of households in the sample have five out of five periods in poverty. This corresponds to the H-persistent-poverty-headcount when setting $C = 1$ and $C = 5$, respectively.

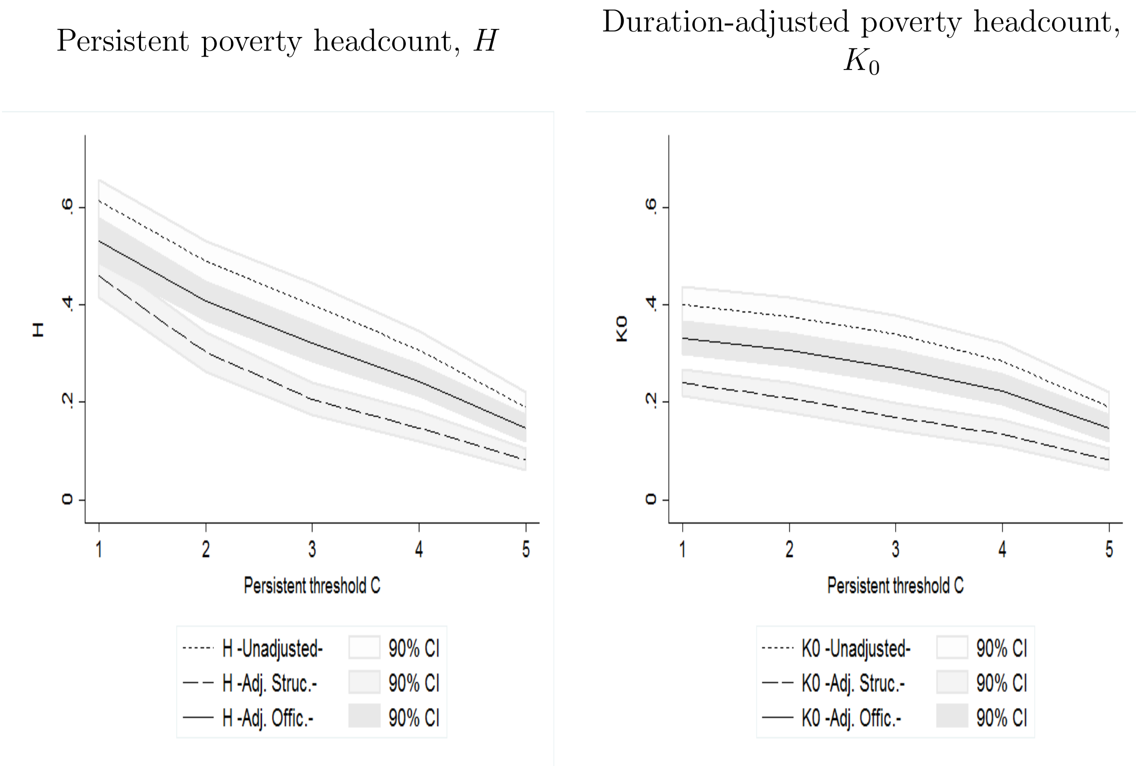
If the estimates are adjusted exclusively by the structure of the population, which are the dashed lines in Figure 4.2 and correspond to using the weighting system that corrects exclusively for the structure of the population (Adj. Struc.), we observe that the adjusted version of the persistent poverty estimates lie below the unadjusted version of the estimates for each different C threshold.

Now, if the chronic poverty estimates are adjusted using INEI's official weighting system available for public use, which correct for survey non-response and the structure of the population (Adj. Offic.), results show that the INEI's correction produces mean estimates of H and K_0 smaller than the unadjusted results by about 7.0 percentage points (p.p.). We see that the solid line in the Figure 4.2 lie below the unadjustead version of the estimates for each C persistent poverty threshold.

In consequence, the two adjusted versions of the chronic poverty estimates, either the version that adjusts exclusively by structure of the population or the one that uses the official INEI's weighting system, both lie below their unadjusted estimate. This result suggests that households that exhibit survey non-response might be systematically better off than those remaining in the sample. As such, household unobservable socioeconomic characteristics that influence the household poverty status might be as well influencing the survey non-response process.

The traditional approach to correct for survey non-response through weighting systems assumes, nonetheless, survey non-response related exclusively to the observable characteristics taken into account in the correction. Thus, if survey non-response is related not only to observable characteristics taken into account for the weighting system that corrects for survey non-response but also to unobservables related to the household poverty status, persistent poverty estimates could provide a biased picture of the intertemporal poverty phenomenon.

Figure 4.2: Persistent poverty balanced panel estimates



Source: Author's calculations developed on the basis of the 2007-2011 ENAHO panel. Notes: Population means developed on the basis of 1,129 households in sample. Lower and upper limit of the population means use a 90% confidence level and were developed under the basis of the weighted bootstrap procedure described in detail in Appendix A.

In the next section, we further investigate whether or not is plausible to assume survey non-response unrelated to unobservables that might influence the household poverty status.

4.3.4 Is plausible to assume absence of confounding unobservables?

The ENAHO panel has in the 2007 sample 7,428 households with either complete or incomplete survey results. Then, we observe for each of these households the p_t-binary indicator of the presence or absence of poverty in $t = 2007$. The unadjusted mean of p_{2007} among these 7,428 interviewed households is 0.444. We henceforth denote the mean poverty rate by $\mu(p_t)$, where $t = 2007, 2008, 2009, 2010, or 2011$. Thus, $\mu(p_{2007}) = 0.444$ and indicates that 44.4% of the panel households interviewed in 2007 were identified as poor in that year.

Out of the households interviewed in 2007, 1,135 left the panel in 2008. Therefore, we only observe one period of time for these 1,135 households, which is 2007. Thus, we observe only p_{2007} for them and not their p_t-poverty status across further years. In contrast, for the group of 1,129 balanced panel households, we observe p_t for each year $t = 2007, 2008, 2009, 2010$, and 2011.

If we calculate and compare the $\mu(p_{2007})$-mean poverty rate for these two different groups of households, we can determine whether they are similar in terms of the incidence of poverty or not. In particular, the $\mu(p_{2007})$-mean poverty rate among the 1,135 households observed exclusively in 2007 is 0.346, while the $\mu(p_{2007})$-mean poverty rate among the 1,129 households that remained in the panel for five years is 0.469. Thus, households that remain in the panel have a larger poverty incidence than those leaving the panel. In particular, households observed in the panel during all five years had a 12.3 p.p. larger poverty rate than those that were observed only in the first wave. This result suggests that better-off households might indeed be systematically leaving the panel, while worse-off households might tend to remain in the sample.

Nonetheless, one could argue that observable characteristics could capture the missing pattern and that controlling for them might restore randomness to the process. According to Little & Rubin (1987)'s taxonomy of missing data patterns, when the probability that a household will be missing, after controlling for observables, is random, it is termed a missing-at-random pattern. As such, estimates developed on the basis of weighting systems that account for observable characteristics assume this particular type of missing pattern.

To investigate whether or not, when accounting for observable covariates, we could still have significant differences in poverty rates as the number of observed periods increases, we calculate the annual mean poverty rate among panel households by the number of observed periods within the panel. We account for possible differences in the missing pattern given by the geographical location of the household using the weighting system that corrects for the population structure described in Section 4.3.3. Table 4.1 below reports these results. Each row in the table shows the $\mu(p_t)$-mean poverty rate calculated for the households available in each year of analysis and across the number of observed periods of time. Integers

Table 4.1: Mean poverty rate $\mu(p_t)$ in any year t between 2007 and 2011 across the number of observed periods

	Number of observed periods					Total
	1	2	3	4	5	
$\mu(p_{2007})$	0.255	0.318	0.363	0.368	0.358	0.332
	[1,135]	[2,016]	[1,685]	[1,463]	[1,129]	[7,428]
$\mu(p_{2008})$	n.d	0.272	0.310	0.336	0.323	0.305
	[0]	[2,016]	[1,685]	[1,463]	[1,129]	[6,293]
$\mu(p_{2009})$	n.d	n.d	0.285	0.313	0.299	0.298
	[0]	[0]	[1,668]	[1,450]	[1,129]	[4,247]
$\mu(p_{2010})$	n.d	n.d	0.236	0.318	0.271	0.297
	[0]	[0]	[14]	[1,449]	[1,129]	[2,592]
$\mu(p_{2011})$	n.d	n.d	0.420	0.078	0.257	0.253
	[0]	[0]	[3]	[27]	[1,129]	[1,159]

Source: Author's calculations developed on the basis of the 2007-2011 ENAHO panel. Notes: Figures were calculated using a weighting system that corrects for the population structure. Integers in square brackets refer to the number of observations used for the mean reported above the bracket. Households with no observed periods were omitted from the table because no poverty figure is observed for them ([341] households).

in square brackets in the table refer to the number of observed households used to calculate the mean reported above the brackets.

Reading Table 4.1 line-to-line from left to right, it is observed that, on average, poverty incidence in larger among households observed three or more periods than households observed only one and two periods. For instance, the 2007 mean poverty rate is smaller among households observed exclusively in 2007 than among those observed during all five years of the panel. This is, households observed in only one period of time have $\mu(p_{2007}) = 0.255$, and households observed in five periods of time have $\mu(p_{2007}) = 0.358$. In 2007, the mean poverty rate among panel households is 10.3 percentage points larger than the rate observed among households that were in the sample only in 2007. Similarly, if we analyse the mean poverty rate in 2008, while households observed in the sample for only two years have $\mu(p_{2008}) = 0.272$, households observed for five years in the sample have $\mu(p_{2008}) = 0.323$.

This result suggests that, after controlling for the structure of the population given by the population's forecast drawn from the census, households leaving the panel sample are still better off than households remaining in the sample. If observability were distributed randomly across poverty status, one would expect a similar mean of poverty incidence across households with a different number of observed periods. However, the mean poverty rate across households is lower in households observed fewer periods of time.

As a result, so far, assuming survey non-response not related to unobservable characteristics that could influence household poverty status seems unrealistic for the ENAHO panel sample. Neglecting these unobservables might produce biased persistent poverty estimates. In the next section, we assess the reliability of the ENAHO persistent poverty estimates under these circumstances.

4.4 Partial identification of persistent poverty measures

Persistent poverty measures are constructed upon balanced panel samples. As such, this type of balanced panel estimates neither consider neither the observable information remaining in the unbalanced panel sub-sample nor the non-observed information that would have helped to describe this phenomenon in households with survey non-response. This section formalises this data problem as a problem of partial observability, and then it derives identification regions that aim to determine how persistent poverty estimates would have behaved in the absence of survey non-response.

4.4.1 Anatomy of the problem of partial observability

Consider the observed P-poverty spell count that the i-household experiences during T periods of time introduced in Eq. (4.2). Then, an analogous useful definition for the subsequent analysis is the number of observed periods of non-poverty, the N-non-poverty spell count:

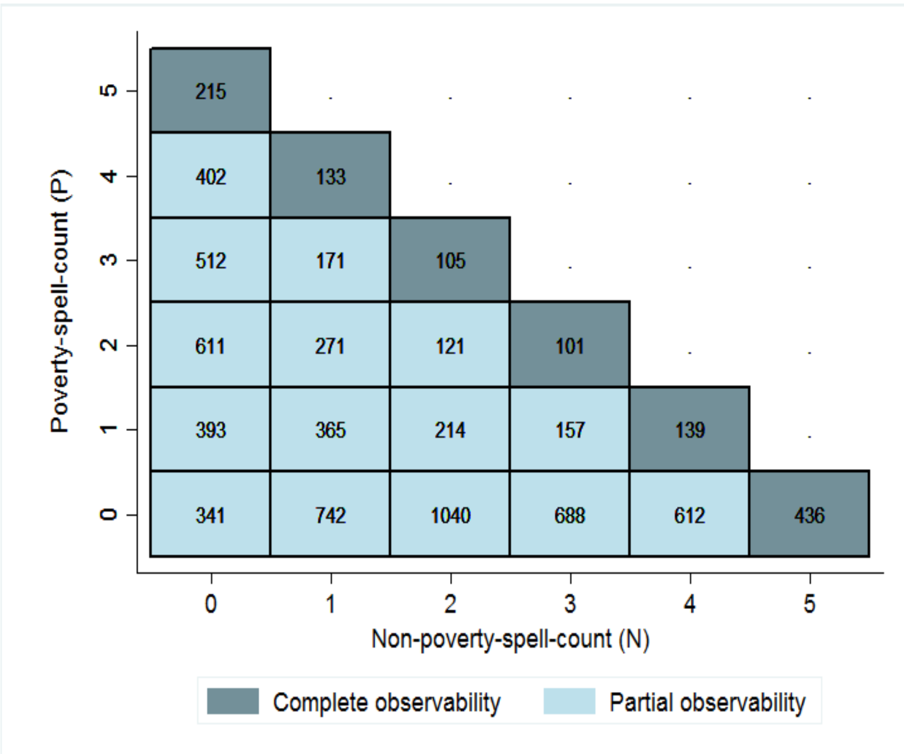
$$N = \sum_{t=1}^{T} (1 - p_t) . \tag{4.7}$$

Any i-household such that the sum of its observed poverty and non-poverty spell counts add up to T is a household with complete observability. Thus, any i-household satisfying $P+N=T$ is observed during all T periods of time and, as such, integrates the balanced version of the panel.

However, there are also households that belong to the portion of the panel that is unbalanced. For any unbalanced panel household, we know that $P+N<T$. This means that we have partial observability of the poverty status of these households across time.

As a result, we encounter a problem of partial observability, where one set of households has complete observability and the other has partial observability. In the context of the ENAHO panel, this problem of partial observability can be visualised as shown in Figure 4.3, where T corresponds to five periods of time: 2007, 2008, 2009, 2010, and 2011. Any household selected for the panel exhibits $P \in \{0,1,2\ldots5\}$ and $N \in \{0,1,2\ldots5\}$. The vertical axis represents the observed

Figure 4.3: Anatomy of the problem: number of households by observed poverty and non-poverty spell count



Source: Author's calculations developed on the basis of the 2007-2011 ENAHO panel. Note: total number of panel households: 7,769.

P-poverty spell count for any i-household over T. The horizontal axis in the figure indicates the observed N-non-poverty spell count. Each cell of the matrix in the figure indicates the number of households that register a combination of P-poverty spells and N-non-poverty spells across the T periods of time. While the cells located on the diagonal refer to any possible combination of P and N that households with complete observability have, cells located in the left lower quadrant of the matrix refer to any possible combination of P and N that households with partial observability exhibit.

Analysing inter-temporal poverty across T periods of time for any i household in terms of the observed values of P and N enables us to express T as:

$$T = P + N + P^* + N^*, \tag{4.8}$$

where P^* and N^* denote the count number of non-observed periods of poverty and non-poverty, respectively. For any household with complete observability, we know that $P^* + N^* = 0$, whereas any household with partial observability satisfies $P^* + N^* > 0$.

The goal is to make inferences with regard to societal measures H and K_0, introduced in Eq. (4.5) and Eq. (4.6), and constructed upon the household indicators h and k_0, respectively. Nonetheless, while the h-persistent poverty indicator and the k_0-duration-adjusted persistent poverty indicator can be straightforwardly calculated for any household with complete observability, for households with partial observability, it is uncertain how they could have been classified (either as persistently poor or non-poor) if they had remained in the panel.

To address this problem, we use the *direct misclassification approach* proposed by Molinari (2008) to study error-ridden discrete variables. The following section describes Molinari (2008)'s proposed approach in the context of the persistent poverty analysis described in this chapter.

4.4.2 The direct misclassification approach

According to Molinari (2008, p.81), the problem of error-ridden discrete variables can be analysed through specifying the relationship between the distribution of the 'true' but unobserved variable and the misclassified representation of it as a linear system of equations, where the coefficient matrix is the matrix of misclassification probabilities.

Following Molinari (2008), we exploit the relation between P and N (which are revealed by the sample) and P^* and N^* (which are unobserved) given by the law of total probabilities and express the problem of partial observability for persistent poverty estimates in terms of its misclassification probabilities. Then, the classification of any given i panel household as persistently poor can be expressed as the sum of conditional probabilities across P, N, P^*, and N^* as follows:

$$\Pr(h = 1) = \sum_{P=0}^{T} \sum_{N=0}^{T} \sum_{P^*=0}^{T} \sum_{N^*=0}^{T} H(P, N, P^*, N^*) \Pr(P, N, P^*, N^*), \tag{4.9}$$

where $H(P, N, P^*, N^*)$ represents the H-persistent poverty headcount introduced in Eq. (4.5) and evaluated on the population subgroup that take the P, N, P^*, N^* values. In other words, $H(P, N, P^*, N^*)$ is the deterministic function H evaluated on each P, N, P^*, N^* population subgroup. Here, in use of Eq. (4.8), we can simplify Eq. (4.9) as follows:

$$\Pr(h = 1) = \sum_{P=0}^{T} \sum_{N=0}^{T-P} \sum_{P^*=0}^{T-P-N} H(P, N, P^*) \Pr(P, N, P^*). \tag{4.10}$$

Nonetheless, as expected, not every term in Eq. (4.10) is observed. Then, following the proposed approach of Molinari (2008) to incorporate prior information into the analysis and based on the definition of the h-persistent poverty indicator, we look into classifying households as persistently poor, as persistently non-poor, or with an unknown outcome (i.e., it is uncertain whether they are persistently poor or non-poor). In addition, persistently poor households are further characterised as households with complete or partial observability.

As a result, any i panel household is classified in exactly one out of four possible subsets of households, which we use for subsequent analysis. These four relevant subsets of households are as follows: i) households unambiguously classified as persistently poor and have complete observability, ii) households that can be classified as persistently poor but exhibit partial observability, iii) households that can be classified as persistently non-poor, and iv) households with an uncertain outcome. These subsets are characterised for any panel survey run over T periods of time as follows.

Consider any i-panel household that can be described in terms of the observed combination of P-poverty spells and N-non-poverty spells across T periods of time. We define S as the set of integers $P \in 0, 1, \ldots, T$ and $N \in 0, 1, \ldots, T$, such that $0 \leq P + N \leq T$. The set S can be partitioned into four subsets of interest, S_1, S_2, S_3, and S_4, such that $S = S_1 \cup S_2 \cup S_3 \cup S_4$. Each subset is defined in terms of the P, N combination that contains the following:

$S_1 = \{P, N : P \geq C, P + N = T\}$, i.e. *persistently poor & complete observability*;

$S_2 = \{P, N : P \geq C, P + N < T\}$, i.e. *persistently poor & partial observability*;

$S_3 = \{P, N : P < C, N > T - C\}$, i.e. *persistently non-poor*;

$S_4 = \{P, N : P < C, N \leq T - C\}$, i.e. *uncertain outcome.*

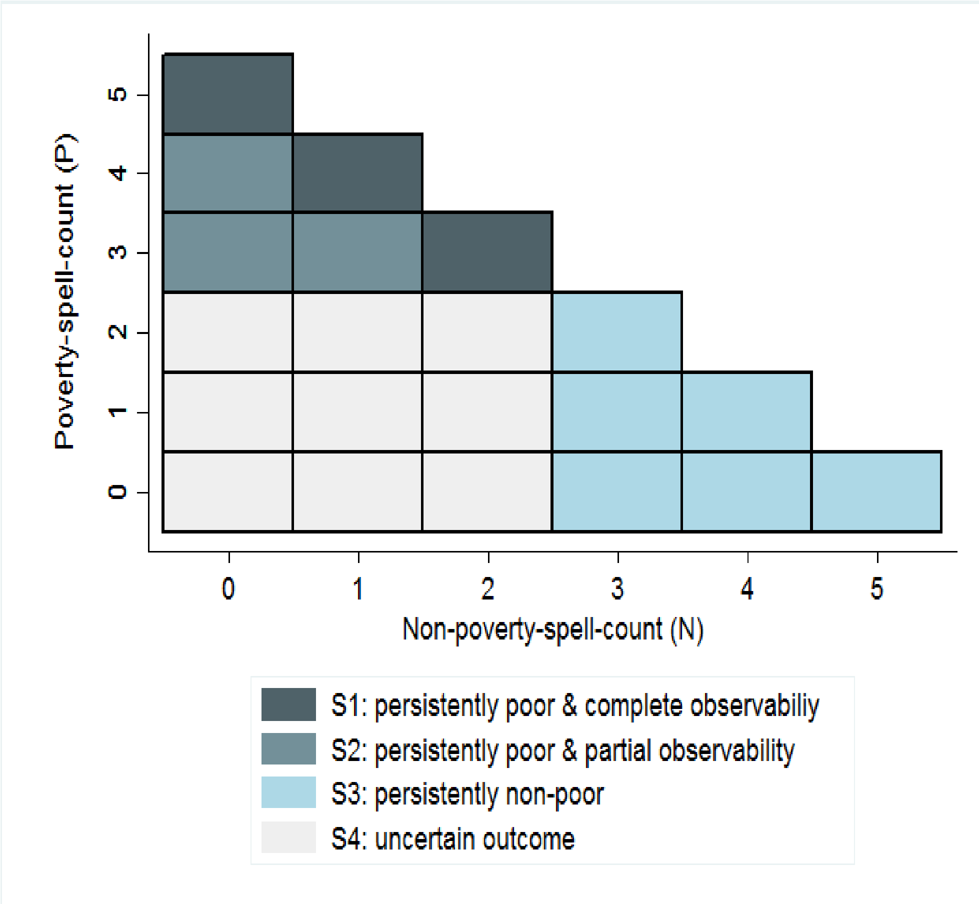
For instance, if in the ENAHO panel we set the persistent poverty threshold at $C = 3$, the four subsets of panel households can be visualised as shown in Figure 4.4. Reading the figure from the darkest zones to the lightest, the darkest zone corresponds to households classified as persistently poor that have complete observability. These are households such that $P \geq 3$ and $P = 5 + N$. Then, households that can be classified as persistently poor and have partial observability are those satisfying $P \geq 3$ and $P = 5 + N$. Following is the set of households that, because of the information that we have in terms of N, we can classify as persistently non-poor. These are households satisfying $N \geq 3$. Finally, households with an uncertain outcome are represented in the figure by the lightest area and are those such that $P < 3$ and $N \leq 2$.

The classification of households into these four sets is subsequently used to express the probability that a household will be identified as persistently poor introduced in Eq. (4.10). Further, simplifying to zero any persistently non-poor household (as we know with certainty that $h = 0$ for them), we obtain the following:

$$\begin{aligned} \Pr(h = 1) = &\sum_{P,N \in S_1} \Pr(P, N|S_1)\Pr(S_1) \\ &+ \sum_{P,N \in S_2} \Pr(P, N|S_2)\Pr(S_2) \\ &+ \sum_{P,N \in S_4} \Pr(P, N|S_4)\Pr(S_4), \end{aligned} \tag{4.11}$$

where households exhibiting a combination $P, N \in S_1$, always have $P^* = 0$. In addition, households exhibiting a combination $P, N \in S_2$ or $P, N \in S_4$, may have integer values of $P^* \in 0, 1, \ldots T - P - N$.

Figure 4.4: Subsets of households of interest in the ENAHO 2007-2011 panel if $C = 3$ by poverty and non-poverty spell count



We use Eq. (4.11) to subsequently derive upper and lower bounds for the persistent poverty measures of interest, H and K_0. In the next section, we present these bounds inferred from the probability distribution of P and N revealed by the survey.

4.4.3 Survey-revealed bounds

This section focuses on presenting the derived upper and lower bounds that characterise the limits of the identification region for the H-persistent-poverty headcount and the K_0-duration-adjusted persistent-poverty headcount. We also present in this section the results obtained upon applying these derived bounds to the specific case of the ENAHO 2007-2011 panel.

The persistent-poverty headcount

Recall that H was defined in Section 4.3.2 as the average value across households in society of the h-persistent poverty indicator for each i-household. In turn, h was defined as the binary indicator of the presence or absence of persistent poverty, where any i household such that $P \geq C$ is classified as persistently poor.

Then, in the case of the h-persistent poverty indicator, any household that registers at least C poverty spells, regardless of whether it has complete or partial observability, can be classified as persistently poor. This means that households that exhibit a combination of P and N that belongs to either the S_1 subset or the S_2 subset exhibit an equivalent value of $h = 1$.

As such, the probability that any i household will be classified as persistently poor introduced in Eq. (4.11) can now be expressed as follows:

$$\begin{aligned}\Pr(h=1) = & \sum_{P,N \in S_1 \cup S_2} \Pr(P, N | S_1 \cup S_2) \Pr(S_1 \cup S_2) \\ & + \sum_{P,N \in S_4} \Pr(P, N | S_4) \Pr(S_4), \end{aligned} \tag{4.12}$$

where the first term on the right-hand side of the equation can be always identified and corresponds to $\Pr(P \geq C)$ and the second term on the right-hand side of the equation is unidentified and corresponds to households with an uncertain outcome.

Nonetheless, with regard to households with an uncertain outcome, at this stage of the analysis we only know that their probability of being classified as persistently poor ranges from zero to one. Thus, the lower and upper bounds for the H-persistent-poverty headcount are as follows:

$$L_H = \Pr(P \geq C) \tag{4.13}$$

$$\begin{aligned} U_H &= \Pr(P \geq C) + \Pr(P < C, N \leq T - C) \\ &= \Pr(N \leq T - C). \end{aligned} \tag{4.14}$$

Next, we present the analogous survey-revealed bounds for the K_0-duration-adjusted persistent-poverty headcount.

The duration-adjusted persistent-poverty headcount:

In this case, recall that K_0 was defined as the average value across households in the society of the k_0-duration-adjusted persistent poverty indicator. In turn, for any i household classified as persistently non-poor, $k_0 = 0$, and for any i household classified as persistently poor, $k_0 = (P + P^*)/T$.

Then, we use the probability of being classified as persistently poor, $\Pr(h = 1)$, given by Eq.(4.11), and the definition of k_0 to express the societal K_0 as follows:

$$\begin{aligned} K_0 \times T = &\sum_{P,N \in S_1} P \times \Pr(P, N|S_1)\Pr(S_1) \\ &+ \sum_{P,N \in S_2} \sum_{P^*=0}^{T-P-N} (P + P^*) \times \Pr(P, N|S_2)\Pr(S_2) \\ &+ \sum_{P,N \in S_4} \sum_{P^*=0}^{T-P-N} (P + P^*) \times \Pr(P, N|S_4)\Pr(S_4), \end{aligned} \tag{4.15}$$

where the first term on the right-hand side of the equation refers to households with complete observability, which can be always identified. The second and third terms, in contrast, refer to households with partial observability. Households with partial observability are unidentified.

To obtain the lower and upper bound for the identification area of K_0, we minimise the terms of Eq.(4.15) that refer to partially observed households for the lower bound, and for the upper bound, we maximise the same terms. As such, we obtain the following:

$$L_{K_0} = \sum_{P,N \in S_1 \cup S_2 \cup S_4} \frac{P}{T} \times \Pr(P, N|S_1 \cup S_2 \cup S_4)\Pr(S_1 \cup S_2 \cup S_4) \tag{4.16}$$

$$\begin{aligned} U_{K_0} = &\sum_{P,N \in S_1} \frac{P}{T} \times \Pr(P, N|S_1)\Pr(S_1) \\ &+ \sum_{P,N \in S_2 \cup S_4} \frac{T - N}{T} \times \Pr(P, N|S_2 \cup S_4)\Pr(S_2 \cup S_4) \end{aligned} \tag{4.17}$$

Having derived and presented lower and upper bounds to characterise the identification region for the H-persistent-poverty headcount and the K_0-duration-adjusted persistent-poverty headcount, we now apply these obtained bounds to the ENAHO panel and analyse the obtained results. These results are presented in the next section.

Bounding ENAHO panel estimates

Figure 4.5 plots the balanced panel point estimates of the chronic poverty figures, their obtained identification region and the 90% confidence interval area of the identification region. These results are displayed separately for for the H-persistent-poverty headcount ratio (Subfigure a.) and the K_0-duration-adjusted persistent-poverty headcount (Subfgure b). The horizontal axis in both figures corresponds to the threshold C used to identify persistent poverty households.

To provide estimates of the identification area consistent with the survey design and therefore with the Peruvian structure of the population, the lower and upper bound of the chronic poverty measures are estimated using the weighting system that adjusts exclusively for the structure of the population, weighting system described in Section 4.3.3.

In terms of sample variability, the lower limit of the confidence interval area corresponds to the lower limit of the 90% confidence interval obtained from the lower bound estimate. Conversely, the upper limit of the confidence interval area corresponds to the upper limit of the 90% confidence interval registered by the upper bound estimate.

The results of the identification areas across different C persistent poverty thresholds for both H and K_0 show a width that decreases along the persistent poverty threshold increases. The larger the persistent poverty threshold, the smaller the width of the identification regions. These results indicate that persistent poverty measures that identify as persistently poor households those that experience a greater number of periods in poverty produce more reliable results than those measures that identify as persistently poor those households with fewer periods in poverty.

In particular, among the five C thresholds used to define persistent poverty, the smallest width of the identification region is exhibited when $C = 5$ for both the H and K_0 metrics. The identification region of the persistent-poverty headcount ratio, when $C = 5$, comprises values of H that range between 0.017 and 0.239

points. Similarly, the identification region of the duration-adjusted persistent-poverty headcount, when using $C = 5$, comprises values of K_0 between 0.096 and 0.239 points. However, these two regions are still wide for the purposes of policy.

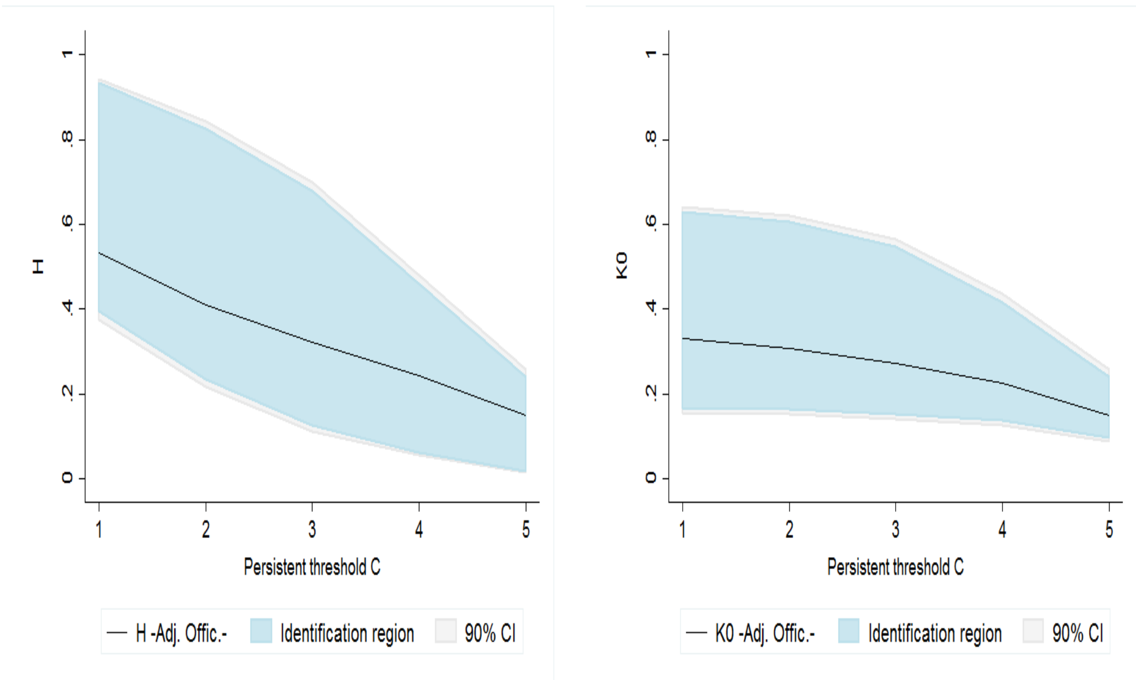
Nonetheless, the bounds obtained for the K_0 metric, in comparison to the resulting bounds of H, are tighter. Thus, persistent poverty estimates based on a K_0 metric provide a more reliable picture of intertemporal poverty than the H estimates.

Results show that the balanced panel point estimates, which use the official weighting system that adjusts for survey non-response and the structure of the population, lie within the obtained identification region. In case the point estimates would have lain outside of the identification region we would have had evidence to assert that these point estimates are providing a biased picture of

Figure 4.5: Survey-revealed bounds

Persistent poverty headcount, H

Duration-adjusted poverty headcount, K_0



Source: Author's calculations developed on the basis of the 2007-2011 ENAHO panel. Notes: point estimates of H and K_0 developed on the basis of 1,129 balanced panel households and using the official weighting systems that the INEI provides. The lower and upper bounds of the identification region were developed on the basis of 7,769 panel households and using the weighting system that adjusts exclusively for the structure of the population. The 90% confidence interval of the identification region is delimited by the 90% confidence lower limit of the lower bound and the 90% confidence upper limit of the upper bound. The confidence limits of the lower and upper bounds were obtained using the weighted bootstrap procedure described in detail in Appendix A.

chronic poverty. But, this is not the case of any of the H and K_0 estimates.

However, given that the identification region obtained upon this survey revealed bounds is still wide, at this stage of the analysis, we cannot determine whether or not the obtained point estimates of H and K_0 provide a reliable picture of the intertemporal phenomenon. We now address tightening these bounds using further visible and plausible restrictions.

4.5 Tighter bounds

The identification region of H and K_0 aims to determine how persistent poverty estimates would have behaved in the absence of survey non-response. The bounds presented above in Section 4.4 correspond to survey-revealed bounds. They were derived upon the probability distribution of the poverty and non-poverty spell counts that we observe in the survey and the possible limit values that non-observed spell counts could have taken if they had been observed.

As such, these survey-revealed bounds make no assumption about the behaviour of either persistent poverty or its relation with survey non-response. Henceforth, we refer to them as the 'no-assumption bounds'.[6] The no-assumption bounds are, however, wide and could become uninformative for policy purposes.

Nonetheless, there is still prior information that can be used to narrow these no-assumption bounds. In particular, two restrictions were found plausible to implement when analysing persistent poverty in the presence of survey non-response: i) an instrumental variable restriction and ii) a monotone instrumental variable restriction. We place these restrictions on the obtained survey-revealed bounds. The next subsections below discuss each of them, along with the results obtained upon applying them in the context of the ENAHO panel.

[6]'No-assumption bounds is the term used by Manski & Pepper (2009) to refer to the known possible limits given by the law of total probabilities of the outcome of interest in the analysis.

4.5.1 Instrumental variable restriction

Consider a random variable Z. If Z is believed unrelated to household poverty status but strongly related to survey non-response, then Z is said to be a valid instrumental variable (IV). Following the Nicoletti et al. (2011) approach to partially identify poverty rates in the presence of missing data, fieldwork information can be used as Z valid instrument for survey non-response. In this context, fieldwork information is considered a valid instrument because there is no credible evidence to indicate that the interviewer's characteristics, such as age, gender, or marital status, could have been influenced by the interviewed households. In addition, these same characteristics serve as good predictors of survey non-response.

Considering that Z is a valid IV implies: assuming Z is *statistically independent* from persistent poverty and related to survey non-response. This assumption is referred to Nicoletti et al. (2011) as an *IV assumption*.[7]

If this IV assumption holds, Manski (2003) shows that, for each value of $z \in Z$, we can characterise the identification region of the H-persistent-poverty headcount as follows:

$$\max_{z \in Z} L(z) \leq H \leq \min_{z \in Z} U(z), \tag{4.18}$$

where $L(z)$ refers to the survey-revealed lower bound of the persistent-poverty measure introduced in Eq. (4.13) and Eq. (4.16) for the H-persistent poverty headcount and the duration adjusted persistent poverty headcount. The z value within parenthesis indicates that the bound is evaluated in the observations where the IV takes the value $z \in Z$. The $\max_{z \in Z}$ operator indicates that the IV lower bound corresponds to the maximum value of $L(z)$ across all observed $z \in Z$.

Similarly, $U(z)$ refers to the survey-revealed upper bound introduced in Eq. (4.14) and Eq. (4.17) for our two chronic poverty measures of interest. The

[7]Different types of distributional assumptions exist in the literature; Manski (2003) analyses these different assumptions and their use under a partial identification methodological approach. While the relation of Z with regard to survey non-response can be easily confirmed empirically, the statistical independence of Z from persistent poverty remains an assumption. The statistical independence assumption defined by Manski (2003, p.28) and applied in the context of our application can be formalised as follows: Z is an IV if, for any value $z \in Z$, it satisfies $\Pr(h = 1|Z = z) = \Pr(h = 1)$.

$\min_{z \in Z}$ operator in Eq. (4.18) indicates that the IV upper bound corresponds to the minimum value of $U(z)$ across all observed $z \in Z$.

Regarding the instruments, we have available information about the collection process of the ENAHO 2007 cross-sectional survey. In particular, we use a categorical variable of the position that the interviewer holds in the survey (supervisor, coordinator, or interviewer) and an indicator of the load of work conducted by each of the interviewers measured as the proportion of interview attempts i.e., visits that the interviewer made in relation to the total number of visits carried out by the 2007 cross-sectional ENAHO.

To judge the plausibility of the statistical independence assumption of the instruments and persistent poverty, we analysed whether or not the number of P poverty spells was balanced over each IV. The mean of each IV was found not statistically significantly different among households with different numbers of poverty spells.

In addition, to confirm that the instruments are good predictors of survey non-response, in an order probit model of the household response rate[8] that uses as predictors the instrumental variables, the two dummies of the interviewer's position in the survey and the workload variable had a coefficient that was statistically significantly different from zero.

This regression analysis and the balanced behaviour of P across the selected IV gave grounds to confirm the plausibility of the IV assumption. Thus, the IV assumption appears credible in this context, so we opt to apply it to restrict the no-assumption bounds.

The use of more than one instrumental variable to apply the IV restriction over the identification region of the persistent poverty estimates requires us to reduce the two selected instruments to a single Z covariate that can be partitioned further to obtain subsets of Z such that $z \in Z$. Given that the interviewer's position in the survey is a categorical variable and the workload is a ratio, we combined the two variables into a single expression by using the workload ratio as a base, which was squared for the cases where the person who carried out the

[8]The response rate of the household is defined as the proportion of years that the household has been observed by the ENAHO panel survey $(P + n/T)$.

survey was a supervisor and cubed for the cases where the person who carried out the survey was an interviewer.

Having defined this Z variable, a trade-off is faced between the number of $z \in Z$ partitions to use and the bias that a large number of partitions might induce because of finite sample bias (Manski & Pepper 2009). In particular, Manski & Pepper (2009) analyse the limits of identification regions based on the application of these types of restrictions through a Monte Carlo experiment. The scholars compared the results given by a sample size of 100, 500, and $1,000$ observations in each partition and found that greater variability along with a greater number of z partitions would produce a considerable increase in the bias of the results.

In the context of the ENAHO, we performed a sensitivity analysis between the results obtained upon using $4, 5, \ldots 14$ partitions of Z. We opt to use 14 partitions because this particular number of partitions allows each partition to comprise at least 500 households in the sample. A sample size of 500 observations per cell is considered large enough to avoid severe finite sample bias. Still, a thorough analysis of the sample variability is required to determine the bias given by the size of the sample.

On the other hand, different approaches can be considered to reduce the two selected instrumental variables to a single Z score or categorical variable. In particular, we consider a k-means procedure to avoid any transformation of the selected IV that allowed us to determine the z partitions of Z. However, the randomness involved in the selection of the partitions in the k-means algorithm and the dissimilar produced size among the z partitions was found inconvenient for the purposes of applying the IV restriction. In addition, a principal component analysis (PCA) was considered to obtain a Z score. However, this dimensionality reduction strategy was found inconvenient because it reduces the variability of the selected instrumental variables, which is in fact what the IV restriction aims to exploit in the context of our persistent poverty analysis. In other words, the PCA analysis might reduce the identification power of our Z variables.

Figure 4.6 plots the results of imposing the IV restriction on the no-assumption identification region. The shaded areas in the figure represent the obtained identification regions. The darkest zone corresponds to the area that implements the

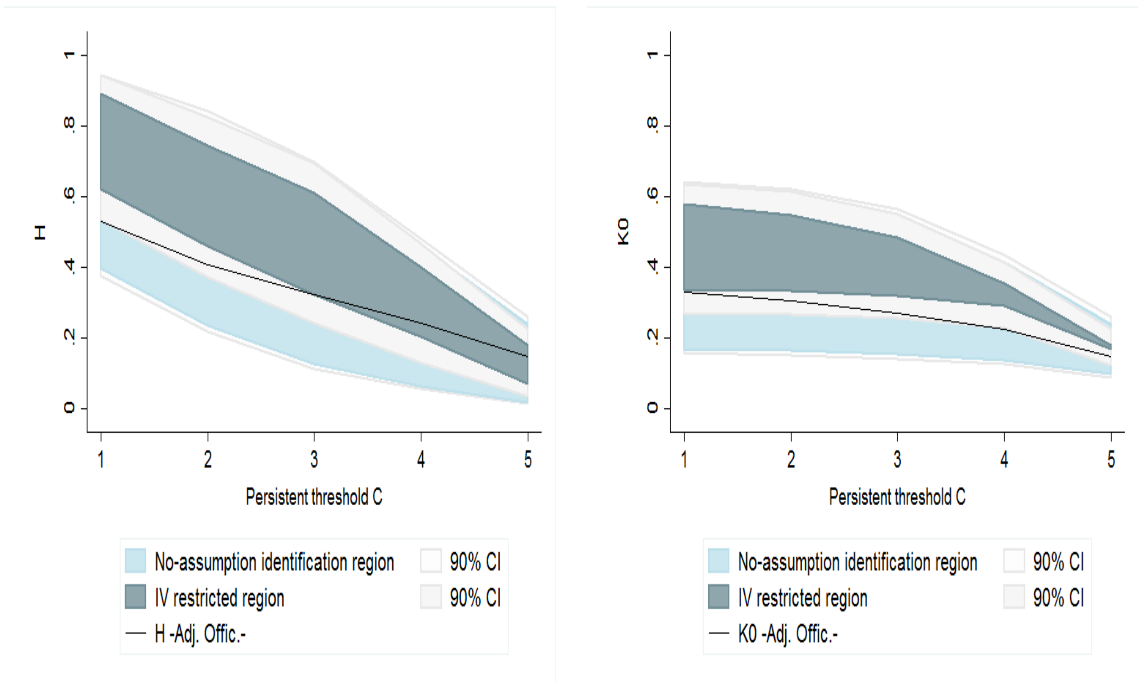
IV restriction, and the lightest zone corresponds to the identification region that remains outside the implemented IV restriction. In the figure we also included the 90% confidence area obtained in use of the lower and upper bound. The lower limit corresponds to the 90% lower limit confidence interval of the the obtained lower bound. Similarly, the upper limit corresponds to the 90% upper limit confidence interval of the upper bound.

The results indicate that the selected Z covariate has identification power, as the identification region is considerably narrower after imposing the restriction. Specifically, the lower bound of both H and K_0 increased by at least 50% of its original value. For the upper bound, the IV restriction produces a reduction of at least 10% of its original value.

The figure also shows that the application of this restriction causes balanced

Figure 4.6: No-assumption and IV bounds

Persistent poverty headcount, H | Duration-adjusted poverty headcount, K_0



Source: Author's calculations developed on the basis of the 2007-2011 ENAHO panel. Notes: point estimates of H and K_0 developed on the basis of 1,129 balanced panel households and using the official weighting systems that the INEI provides. The lower and upper bounds of the identification region were developed on the basis of 7,769 panel households and using the weighting system that adjusts exclusively for the structure of the population. The 90% confidence interval of the identification region is delimited by the 90% confidence lower limit of the lower bound and the 90% confidence upper limit of the upper bound. The confidence limits of the lower and upper bounds were obtained using the weighted bootstrap procedure described in detail in Appendix A.

panel estimates to lie not necessarily within the identification area with an observed downward bias in both H and K_0. This result suggests that the ENAHO balanced panel estimates of persistent poverty result in an underestimation of the size of the phenomenon.

We now describe the second restriction placed on the identification region of H and K_0, a monotone instrumental variable restriction.

4.5.2 Monotone instrumental variable restriction

Consider a random variable X. If X is believed represents an ordered measure of household socioeconomic status that decreases/increases monotonically along it, then the X variable is said to be a monotone instrumental variable (MIV). Following Manski & Pepper (2009, p.S207), using an *MIV assumption*, as this assumption is referred to by the scholars, results in the following context-specific assumptions for the analysis of persistent poverty in the presence of survey non-response. First, it implies assuming that persistent poverty decreases with the socioeconomic status of the household; second, it assumes that X is a monotonic predictor of household socioeconomic status; and third, it also implies assuming that the descriptors of the survey non-response process, which we do not observe, are statistically independent of household socioeconomic status and X.

As such, we require the MIV variables to be socioeconomic descriptors available for any panel household, regardless of whether the household has complete or partial observability. Given that households with partial observability have no available information in the ENAHO survey, our MIV candidates are thus constrained to socioeconomic descriptors of geographical areas.

Assuming that socioeconomic descriptors of geographical areas are monotonic descriptors of household socioeconomic status appears plausible, as does assuming that persistent poverty decreases along these observed descriptors. Therefore, we opt to use such an MIV assumption to restrict the obtained persistent poverty identification regions.[9]

[9] The MIV assumption formalised by Manski & Pepper (2000) and applied in the context of our analysis can be expressed as follows: X represents an ordered measure and is an MIV if, for any x_1, x_2 and x values of X such that $x_2 \geq x \geq x_1$, it satisfies $\Pr(h = 1|X = x_2) \geq \Pr(h = 1|X = x_1), \forall(x_1, x_2) \in X$.

If the MIV assumption holds, Manski & Pepper (2000) shows that, for any x_1, x_2 and x values of X such that $x_2 \geq x \geq x_1$, we can characterise the identification region of the H-persistent-poverty headcount as follows:

$$E\left[\max_{x_1 \leq x} L(x_1)\right] \leq H \leq E\left[\min_{x_2 \geq x} U(x_2)\right], \tag{4.19}$$

where the $\max_{x_1 \leq x}$ operator indicates that the x_1 value of X corresponds to the maximum value that x takes in X and the $\min_{x_2 \geq x}$ operator indicates that the x_2 value of X corresponds to the minimum value that x takes in X. The expectation applies with regards to the distribution of the X random variable.

The available set of MIV candidates comprised 36 variables, 28 of which correspond to socioeconomic indicators of the municipalities where panel households reside. This municipality information was drawn from a rich national administrative register of socioeconomic indicators at the municipality level that the INEI have made available for public use.[10] The remaining eight MIV covariates are dummy indicators that differentiate the national territory among urban and rural areas and regions.

To reduce the 36 MIV variables to a single one, a linear transformation was made over 12 out of the 36 indicators to ensure that all 36 had the same $[0, 1]$ range of scale and the same orientation. This means that all 36 indicators, after the linear transformation, take values from the interval $[0, 1]$ and jointly increase with the household equivalent adult expenditure, as measured in the 2007 ENAHO cross-sectional survey. Subsequently, we add the 36 variables without applying any weighting system to obtain a single X covariate.

In practice, we use as the X variable a simple score, where the minimum value of X corresponds to the lowest observed 'proxy' value of socioeconomic status. Similarly, the maximum observed value of X corresponds to the maximum observed 'proxy' value of socioeconomic status.

[10]Examples of these variables include the number of social organisations per 100 inhabitants, number of librarian personnel (by position in the library) per 1,000 inhabitants, number of library computers connected to the Internet per 1,000 inhabitants, number of library users per 100 inhabitants, number of social program beneficiaries, number of divorces per 1,000 inhabitants, number of Internet kiosks per 100 inhabitants, number of micro and small businesses per 1,000 inhabitants, number of post offices per 1,000 inhabitants, electricity coverage rate, and tons of solid trash collected per 1,000 inhabitants.

To ensure that the MIV restriction is applied to the identification area using a large enough sample size for each $x \in X$, we implement 14 partitions of the X ordered score, where each partition contains at least 500 households. It is worth remarking that each partition contains x values such that $x_1 > x_2 > \ldots > x_{14}$, where x_1 refers to the values that x takes in the first partition of the score, x_2 to the values of x in the second partition, and x_{14} to the values of x in the 14th partition.

Different approaches can be used, nonetheless, to construct the X covariate. For instance, a principal component analysis or a k-means procedure could have been used. We opt for using the score approach to exploit the variability we find across the 36 MIV variables and to ensure the monotonic behaviour of the resulting X, which both are key in the context of applying an MIV restriction.

Figure 4.7 plots the results of imposing the MIV restriction, along with the previously identified no-assumptions region and the corresponding confidence intervals obatined for these identification regions. Reading the graph from the darkest to the lightest shaded areas, the darkest area corresponds to the identification region obtained upon imposing the MIV restriction. Subsequently, we plotted the no-assumption identification region. The lightest shaded area corresponds to the 90% confidence area obtained from the confidence intervals of the lower and upper bound of each region.

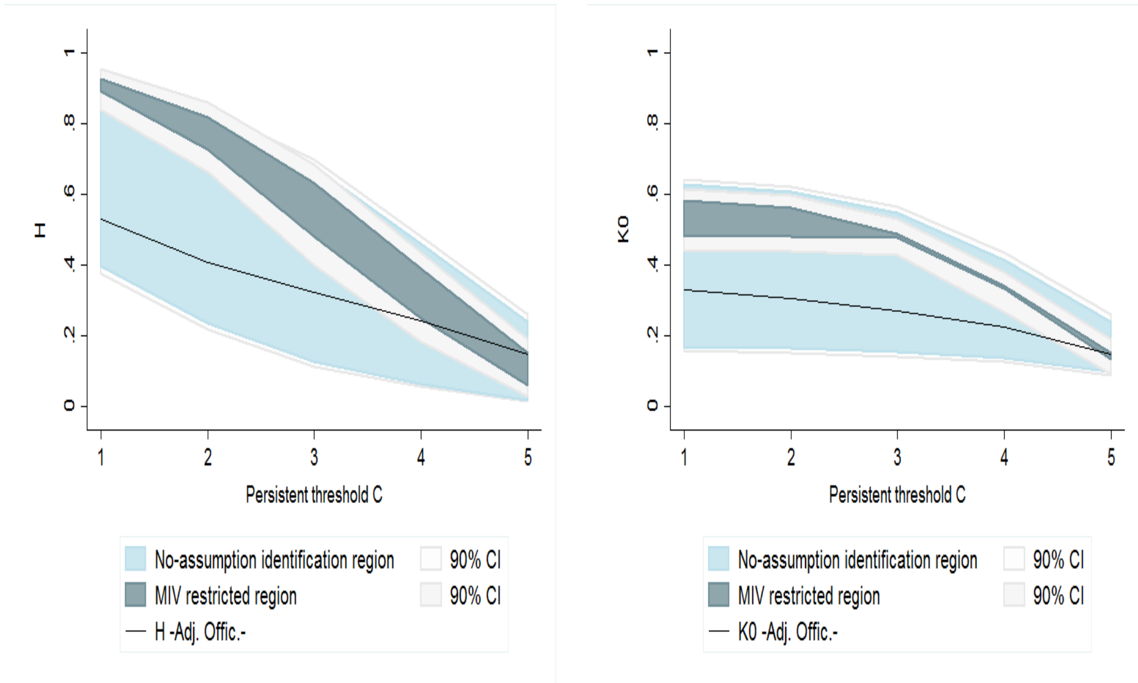
Although an MIV assumption is conceptually weaker than an IV assumption, the figure shows that the MIV identification region is considerably more narrow than the obtained upon applying an IV restriction. This result indicates that, in the context of the ENAHO panel, the identification power of the two used IV variable indicators is lower than the identification power of the 36 MIV indicators.

As a sensitivity analysis, we estimate the results using a lower number of MIVs with subsets of the 36 indicators; the results of this analysis indicate that the identification power of the MIV was the largest when using the complete set of these 36 indicators. However, the results of this analysis also pointed out that the identification power of the MIV restriction does not necessarily increase as the number of MIV indicators in use increases.

Figure 4.7: No-assumption and MIV bounds

Persistent poverty headcount, H

Duration-adjusted poverty headcount, K_0



Source: Author's calculations developed on the basis of the 2007-2011 ENAHO panel. Notes: point estimates of H and K_0 developed on the basis of 1,129 balanced panel households and using the official weighting systems that the INEI provides. The lower and upper bounds of the identification region were developed on the basis of 7,769 panel households and using the weighting system that adjusts exclusively for the structure of the population. The 90% confidence interval of the identification region is delimited by the 90% confidence lower limit of the lower bound and the 90% confidence upper limit of the upper bound. The confidence limits of the lower and upper bounds were obtained using the weighted bootstrap procedure described in detail in Appendix A.

The MIV results also show that the MIV identification region for both H and K_0 metrics, across most of the C persistent poverty thresholds, results in a subset of the IV-restricted identification region. This is, however, not exactly the case for all the C used persistent poverty thresholds. We therefore provide final estimates of the identification region obtained as the intersection of the MIV and IV restrictions. This analysis and results are presented in the next section.

4.5.3 Improved identification region

Improved lower and upper bounds are constructed in this section to describe the narrowest identification area that is revealed upon imposing both: an IV restriction and a MIV restriction. With such an approach, we aim to determine the tightest possible identification region from where is possible to draw credible and reliable persistent poverty estimates.

The improved identification region was obtained selecting the maximal lower bound and minimal upper bound found upon the no assumption region and the IV and MIV restrictions. Other possible course of action could consist of conditioning the no-assumption bounds to both Z and X. This approach was considered nonetheless inconvenient because it requires a decrease in the sample size for each partition used to select lower and upper bounds, which can increase the sample variability and the finite sample bias of the estimates. Because of this consideration, we prefer the selected approach.

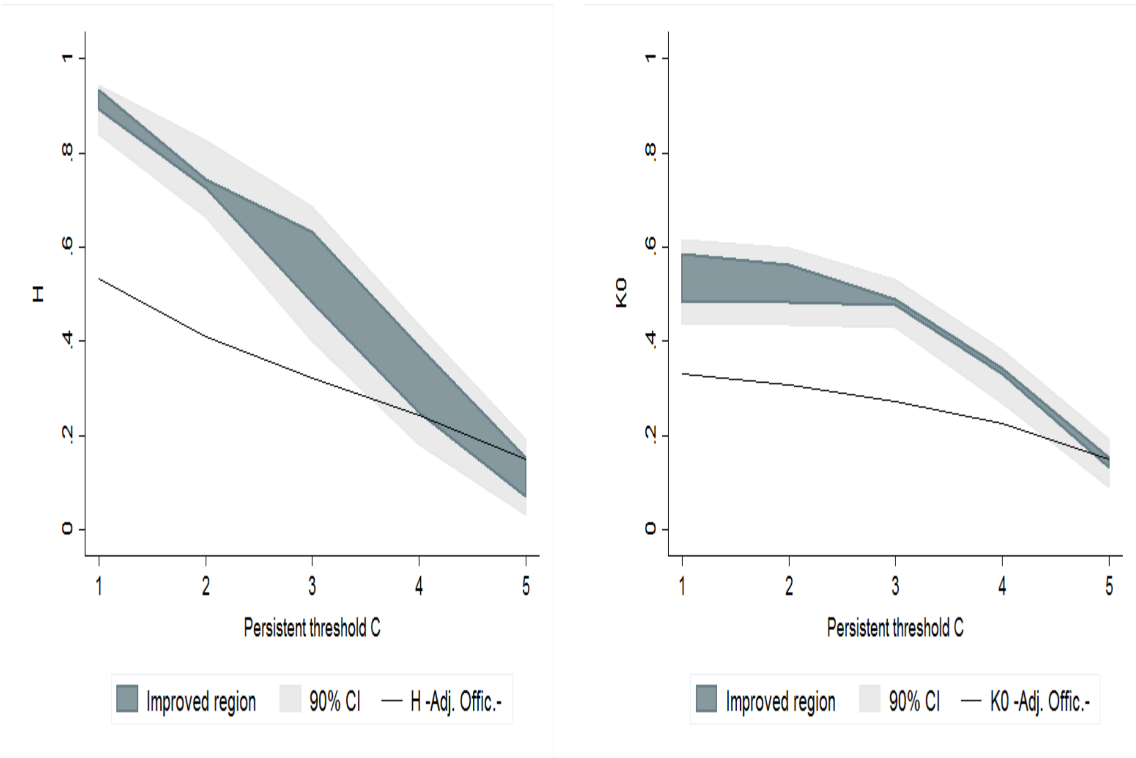
Figure 4.8 presents the obtained final improved identification region for H and K_0. These results indicate that the obtained improved identification region is considerably narrower than the initial no-assumption region. For instance, once restrictions imposed, persistently poor households –identified as these exhibiting at least one poverty spell count– are found to represent in between 84% to 94% of the population of households. The 90% confidence interval lower limit of the lower bound of the identification region lie above the balanced panel estimate 31.0 percentage points. In general, the improved identification region consists of persistent poverty values that do not contain the balanced panel estimates for both H and K_0 metrics. This is the case for any C threshold used to define persistent poverty households, with the exception of the use of $C = 5$ and $C = 4$ for the H-persistent-poverty headcount and $C = 5$ for the $K0$-duration adjusted poverty headcount. This result indicates that balanced panel estimates based on weighting systems that correct for survey non-response, in the case of the ENAHO panel, provide a considerable downwards biased picture of the intertemporal phenomenon.

The obtained improved identification region lies largely above the H and K_0 balanced panel estimates, indicating that, if both persistent poverty measures had been measured in the absence of survey non-response, they would have portrayed a considerably larger size for the phenomenon.

If we compare the difference in percentage points between the lower bound of the improved identification region and the balanced panel estimates for both H and K_0, we observe that this difference decreases as the C persistent poverty

Figure 4.8: Improved bounds

Persistent poverty headcount, H | Duration adjusted poverty headcount, K_0



Source: Author's calculations developed on the basis of the 2007-2011 ENAHO panel. Notes: point estimates of H and K_0 developed on the basis of 1,129 balanced panel households and using the official weighting systems that the INEI provides. The lower and upper bounds of the identification region were developed on the basis of 7,769 panel households and using the weighting system that adjusts exclusively for the structure of the population. The 90% confidence interval of the identification region is delimited by the 90% confidence lower limit of the lower bound and the 90% confidence upper limit of the upper bound. The confidence limits of the lower and upper bounds were obtained using the weighted bootstrap procedure described in detail in Appendix A.

threshold increases. This result indicates that persistent poverty measures that identify persistently poor households in terms of a larger number of poverty spells are more reliable than measures that use fewer spells to identify the most deprived on this basis.

In the next section, we analyse the bias that the bounds might suffer because of the finite feature of the sample.

4.5.4 Finite sample bias

Not only point estimates but also bounds estimates could suffer from finite sample bias. In particular, bounds estimates of the type that we develop in this chapter and apply to the ENAHO panel survey calculate maximums and minimums over samples of relative small sizes. As pointed out by Kreider & Pepper (2007), these procedures can lead to systematically biased estimations, with lower bounds tending to be upward biased and upper bounds tending to be downward biased because of the finite nature of samples.

To correct for this possible bias, Kreider & Pepper (2007) proposes a non-parametric bootstrapped correction to be applied over the obtained lower and upper bound. This section focuses on applying this finite sample bias correction proposed by Kreider & Pepper (2007) to our Section 4.5.3 obtained improved lower and upper bounds.

Specifically, the non-parametric bootstrapped correction proposed by Kreider & Pepper (2007) and discussed as well in Manski & Pepper (2009) consists on bootstrapping the point estimate of the lower and upper bounds, which we do 1,000 independent times as per described in Appendix A. The bias of the bound corresponds to $E^*(B) - B$, where B refers to the estimated bound, $E^*(\cdot)$ is the expectation operator with respect to the bootstrap distribution, and $E^*(B)$ is the mean of the estimated bound using the bootstrap distribution. Then, the biased corrected estimator is expressed as the B-original estimated after subtracting from it the bias, which is: $2B - E^*(B)$.

Table 4.2 presents the point estimates of the improved lower and upper bounds of H and K_0, as well as their corresponding finite-sample bias-corrected estimates and their 90% confidence intervals, which have been obtained using the weighted bootstrap procedure described in detail in Appendix A.

The results indicate that the finite-sample bias-corrected estimates produce a small improvement with regards to the analogous point estimates. This result is consistent with the results obtained by Kreider & Pepper (2007) when implementing this type of finite sample bias correction on employment estimates. Nonetheless, the sample variability shown by the bootstrapped standard errors result to widen the identification area mostly when using the bound obtained upon the IV restriction.

Despite, both restrictions (IV and MIV) were imposed on the basis on cells consisting of minimum 500 households in the sample, we observe larger sample variability from the point estimates obtained from applying an IV restriction than those obtained upon imposing an MIV restriction. This result is driven by the fact that the MIV variables that we use to narrow the identification region consist of indicators that characterise geographical areas, which produce lower finite-sample bias and lower sample variability.

Still, once accounting for the finite sample bias and the sample variability, if we compare the balanced panel estimates of the H-persistent-poverty headcount with the bootstrapped confidence intervals, it is observed that the only estimate that lie within the confidence intervals corresponds to the case when a household is defined as persistently poor because it has at least five or four poverty spells. The remaining H balanced panel estimates result in a considerably downwards-biased

Table 4.2: Improved bounds and 90% confidence intervals

	H		K_0	
	Balanced panel estimate	Bounds	Balanced panel estimate	Bounds
$C = 1$	0.532 [a]	[0.893 , 0.935] [b]	0.331	[0.482 , 0.584]
		[0.892 , 0.934] [c]		[0.480 , 0.583]
		[0.839 , 0.944] [d]		[0.437 , 0.614]
$C = 2$	0.408 [a]	[0.726 , 0.744] [b]	0.306	[0.482 , 0.562]
		[0.724 , 0.741] [c]		[0.480 , 0.561]
		[0.665 , 0.825] [d]		[0.437 , 0.596]
$C = 3$	0.322 [a]	[0.481 , 0.632] [b]	0.271	[0.476 , 0.487]
		[0.476 , 0.631] [c]		[0.473 , 0.487]
		[0.401 , 0.686] [d]		[0.430 , 0.530]
$C = 4$	0.243 [a]	[0.249 , 0.389] [b]	0.224	[0.332 , 0.341]
		[0.247 , 0.389] [c]		[0.331 , 0.341]
		[0.181 , 0.434] [d]		[0.268 , 0.380]
$C = 5$	0.148 [a]	[0.071 , 0.151] [b]	0.148	[0.132 , 0.151]
		[0.070 , 0.150] [c]		[0.127 , 0.150]
		[0.033 , 0.189] [d]		[0.091 , 0.189]

Source: Author's calculations developed on the basis of the 2007-2011 ENAHO panel. Notes: a. Point estimates of the population mean developed on the basis of 1,129 balanced panel households and using the official weighting systems that the INEI provides. b. Point estimates of the lower and upper bound developed on the basis of 7,769 panel households and using the weighting system that adjusts exclusively for the structure of the population. c. Finite-sample bias-corrected estimates d. Bootstrapped 90% confidence intervals of the identification region, where the lower limit corresponds to the 90% confidence lower limit of the lower bound and the upper limit to the 90% confidence upper limit of the upper bound. The confidence limits of the lower and upper bounds were obtained using the weighted bootstrap procedure described in detail in Appendix A.

picture of persistent poverty. Similar is the case of balanced panel estimates of the K_0-duration-adjusted persistent-poverty headcount. Only the estimate that considers persistently poor households that exhibit five poverty spells lie within the bootstrapped confidence intervals.

These results confirm the orientation and considerable size of the bias that balanced panel estimates exhibit when providing persistent poverty measures without accounting for the relationship that survey non-response and the socioeconomic status of the household.

4.6 Conclusions

This chapter derived identification regions for two persistent poverty measures: the persistent-poverty headcount and the duration-adjusted persistent-poverty headcount. We analyse the results of these two measures in the context of the 2007-2011 ENAHO panel. First, a set of survey-revealed bounds was derived. These bounds do not impose any assumptions regarding the behaviour of persistent poverty in the presence of survey non-response. The result is wide.

To narrow the no-assumption identification area, we propose using two visible and credible assumptions. The first restriction imposed, an IV restriction, assumes a set of fieldwork variables statistically independent of household poverty status but strongly related to survey non-response. The second, an MIV restriction, assumes that a set of geographical data is a monotonic descriptor of the population's socioeconomic status and that the population's socioeconomic status will increase as persistent poverty decreases. These two assumptions seem credible and plausible when analysing persistent poverty.

The obtained identification regions, once the restrictions are imposed, are considerably narrower than the no-assumption regions. Although an MIV assumption is conceptually weaker than an IV assumption, the region obtained upon imposing an MIV restriction is narrower than that obtained upon imposing the IV restriction. This result indicates that, in the context of the ENAHO panel, the IV variable indicators used have poorer identification power than the rich set of MIV covariates.

The obtained improved identification regions lie largely above the persistent poverty balanced panel estimates. This result indicates that, if persistent poverty had been measured in the absence of survey non-response, it would have portrayed a considerably larger size for the phenomenon. As such, current balanced panel estimates of persistent poverty are said to portray a considerably downwards-biased picture of the intertemporal phenomenon.

It was also found that persistent poverty measures that identify persistently poor households in terms of a larger number of poverty spells are more reliable than measures that use fewer spells to identify the most deprived on this basis. The width of the identification region of the duration-adjusted headcount is tighter than the identification region of the persistent-poverty headcount. This result leads to the conclusion that the use of such a measure provides a more reliable picture of persistent poverty.

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Appendix A. Sample variability

Sample variability has been analysed for all the point estimates that we provide in this Chapter. This analysis comprised the development of the standard errors for each chronic poverty point estimate and for the lower and upper bound point estimates. We developed a non-parametric estimate of the sample variability using a bootstrap procedure. In this Appendix we describe how this procedure has been carried out.

Let recall that the ENAHO 2007-2011 panel used in this paper is based on a complex sample design and that panel households have been defined in this paper as those attempted to interview as panel households in the first wave of the panel (2007). The 2007 survey uses a stratified and clustered sample design, where households that belong to different clusters and strata had different selection probability.

While chronic poverty point estimates are based on the balanced panel, which is made of 1,129 households observed across the five years of the panel, the lower and upper bounds point estimates are based on the sample of all panel households that comprises 7,769 households.

To obtain estimates from each of these two samples, sampling weights are necessary to ensure the different selection probability across households is taken into account and to allow population figures match the population totals that the Peruvian census registers. As such each sample has available a set of weights that adjusts figures to resemble the structure of the population. These weights were described in Section 4.3.3.

Rao & Wu (1988) demonstrated that bootstrap procedures which do not take into account the sample design underestimated the population variance and produce inconsistent results. Then, to obtain estimates of the sample variability that take into account these different selection probabilities across households, we use the weighted bootstrap methodology proposed by Rao & Wu (1988) and Rao et al. (1992).

Specifically, we follow the procedure described by Rao et al. (1992), which consists on re-scaling weights rather than estimates. To apply this procedure, first, we construct 1,000 independent sets of replicate weights. This ensures that we are re-sampling primary sampling units (PSU) within each correspondent strata and households within each correspondent primary sampling units. Each replicate weighting system resembles a possible re-sampling of the data that maintains the original sampling scheme. In each replicate, all the households belonging to a specific PSU may be removed and other PSU's are retained.

Once the set of 1,000 alternative bootstrapped weights is available, then the population mean of each point estimate is obtained 1,000 independent times and each one uses each of the alternative bootstrapped weights. This methodology was carried out using the STATA software and the commands *bsweights* and *bs4rw*.

We set as confidence level 90%. The lower and upper limit of each population mean presented in this chapter corresponds to the 5th and 95th percentile of the bootstrapped distribution of estimates. In terms of the identification region, the lower limit corresponds to the 90% confidence lower limit of the lower bound and the upper limit to the 90% confidence upper limit of the upper bound.

Chapter 5

Discussion

This thesis has analysed and proposed two different methodologies to address differences in needs in multidimensional deprivation indices. In Chapter 2, I investigated the effect of these differences when comparing multidimensional deprivation across societies of different demographic composition and proposed standardisation methods to enhance societal multidimensional comparisons. In Chapter 3, I studied the effect of differences when comparing multidimensional deprivation across households of different sizes and compositions or individuals of different age ranges and gender. This latter chapter proposed a counting family of multidimensional deprivation indices that describes how much deprivation two demographically heterogeneous units with different needs must exhibit to be catalogued as equivalently deprived.

The proposed methodologies of Chapter 2 and Chapter 3 are the first to analyse the comparability problems that differences in need might bring to current indices of multidimensional deprivation measurement. The empirical results of these two chapters demonstrate that neglecting differences in needs produces biased multidimensional deprivation incidence estimates. The failure of current multidimensional measures to take into account differences in needs might produce inaccurate rankings of either societies with different distribution of the population, households of different sizes and compositions or individuals of different age ranges and gender.

These two proposed methodologies are meant to be easy for policy-makers to implement and understand. While the societal standardization method analysed in Chapter 2 is proposed to be used in contexts in which multidimensional deprivation measures have been designed and are currently in use, the family of equivalized indices of multidimensional deprivation proposed in Chapter 3 is suitable for contexts in which either multidimensional deprivation is measured at the individual level across a wide range of indicators with different applicable population groups, public policies target the household, or risk or resources are arguably pooled across household members.

On the other hand, in Chapter 4, this thesis has analysed the reliability of persistent poverty measures in the presence of survey non-response. Traditional approaches to tackle survey non-response in longitudinal data use non-response weighting systems that either do not account for the relationship that survey non-response has with the socio-economic status of the household or impose assumptions that might be shaping the results. This chapter is the first to analyse persistent poverty estimates, first without imposing any assumption of the behaviour of survey non-response and then imposing two visible and plausible restrictions.

Specifically, two assumptions were found to be credible and plausible when analysing persistent poverty in the presence of survey non-response: an instrumental variable (IV) restriction and a monotone instrumental variable (MIV) restriction. While the IV restriction assumes a set of field-work variables statistically independent of the household's poverty status but strongly related to survey non-response, the MIV restriction assumes a set of geographical data to be a monotonic descriptor of the population's socio-economic status to increase along persistent poverty decreases. The obtained identification regions, once placed within these two restrictions, are considerably narrower than the no-assumption regions.

The empirical results of this chapter indicate that persistent poverty measures based on balanced panel estimates that do not account for the relationship that survey non-response has with the socio-economic status of the household can be considerably biased. In the context of the 2007-2011 Peruvian national household survey panel, I found that estimates that use traditional non-response weighting systems are systematically downwards biased with regard to the identification region that contains the most plausible estimates.

The methodological approach developed in this chapter is proposed for policy contexts to analyse the reliability of persistent poverty measures and to base design policy interventions on more accurate estimates.

In general, these methodological approaches tackle the measurement issues identified by the chapters of this thesis and thus open a broad spectrum of further extensions and research paths. The next paragraphs briefly discuss the research agenda that arises in this regard.

First, in terms of multidimensional deprivation measurement, the family of measures proposed in Chapter 2 has taken into account differences in need while considering them a fair source of differences in multidimensional deprivation incidence. Further research is required to incorporate other sources of fair differences in multidimensional deprivation incidence such as preferences.

Current policy oriented multidimensional measures of deprivation define a basic set of needs to measure deprivation across the population. However, they still do not take into account that each household or individual might assign greater value to some dimensions than to others and might prefer to be deprived of those to which they assign lower value. Current policy-oriented indices capture possible differences in preferences by identifying as the most deprived the population that exhibits the largest number of the dimensions in deprivation at the same time. However, thorough research is required to analyse the effect that preferences might have on multidimensional deprivation incidence profiles and to take these preferences into account.

In addition, the proposed methodologies of Chapter 2 and Chapter 3 do not address the complexities that arise when complementarity and substitutability among dimensions are observed. Examples of multidimensional deprivation measures that take into account these complexities are Aaberge & Brandolini (2014), Bourguignon & Chakravarty (2003) and Seth (2013). Still, these proposed methodologies do not account for differences in needs. Research efforts are required to analyse multidimensional deprivation in the presence of differences and needs, taking into account the complementaries and substitutability among dimensions.

Moreover, as pointed out by Pollak & Wales (1979), Fisher (1987), and Blundell & Lewbel (1991) and discussed in Chapter 3, the current household demographic composition that leads to differences in need might be driven by a previous deprivation status. For instance, a particular household consisting of two adults and five children might be this size not only because both adults have a preference for many children, but also because they did not have access to pregnancy prevention education or could not afford to use birth control. Thus, household composition not only reflects needs or preferences, catalogued in this thesis as producing fair differences in deprivation among households, but also might be a reflection of avoidable and unfair previous states of deprivation. Further research is required to provide an equivalence scale tool to enhance household or individual comparability for multidimensional deprivation measurement that takes into account the relationship between the observed distribution of the population and previous states of deprivation.

Second, in terms of persistent poverty, the analysis of Chapter 4 results in a narrow identification area, which is policy relevant for the case of the duration adjusted persistent poverty headcount measure. However, in the case of the persistent poverty headcount, the resulting identification area is still wide for the purposes of policy. Nonetheless, within the partial identification literature, other two plausible restrictions commonly used referred to by Manski (2003) as a Monotonic Treatment Selection (MTS) and a Monotonic Treatment Response (MTR) could be valuable to study further persistent poverty measures in the presence of survey non-response. Using any of these two restrictions implies assuming persistent poverty to monotonically decrease as survey non-response increases, which the empirical results of Chapter VI suggest might be a realistic assumption.

Also, in terms of policy applications, the analysis performed in Chapter 4 encourages further study of the behaviour of the probability of a household to be poor given that was poor in a previous moment in time. This probability is known in literature as the transition probability. The analysis of the effect that survey non-response or item non-response have on the transitions probabilities might be valuable for the policy arena. In particular, longitudinal data and repeated cross-section data could result useful for such an analysis.

Finally, in the intersection between the topics analysed by the chapters, there is no study in the literature that has compared thoroughly multidimensional deprivation and income poverty rates in the presence of missing data. This analysis could be valuable in guiding policy-makers when using both types of measures in the design and evaluation of policy interventions.

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