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Estimating Value-at-Risk using a Multivariate Copula-Based Volatility Model.

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Abstract

This paper proposes a multivariate copula-based volatility model for estimating value-at-Risk in banks of some selected European countries by combining Dynamic Conditional Correlation (DCC) multivariate GARCH (M-GARCH) volatility model and copula functions. Non-normality in multivariate models is associated with the joint probability of the univariate models' marginal probabilities —the joint probability of large market movements, referred to as *tail dependence*. In this paper, we use copula functions to model the tail dependence of large market movements and test the validity of our results by performing back-testing techniques. The results show that the copula-based approach provides better estimates than the common methods currently used and captures VaR well based on the differences in the numbers of exceptions produced during different observation periods at the same confidence level.

Keywords: value-at-risk, dynamic conditional correlation, GARCH, volatility, copulas, volatility, risk management.

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1 Introduction

Value-at-Risk (VaR) is a standard risk measure in financial risk management and is widely used in the banking sector and other financial institutions. It summarises the worst possible loss of a portfolio of financial assets at a given confidence level (CL) over a given time period. European banks began adopting VaR in the early 1990s. International bank regulators also influenced the development and use of VaR when the Basel Committee on Banking Supervision chose VaR as the international standard method for evaluating the market risk of a portfolio of financial assets for regulatory purposes (on Banking Supervision, 1996).

There are many methods to estimate VaR (see Holton (2014), Jorion (2007), Malz (2011) and the references therein), and the most common methods used by banks are the variance-covariance method (also known as the parametric or Delta Normal approach, was developed by J.P. Morgan using its RiskMetrics in 1993), historical simulation, and Monte Carlo simulation. Due to their simplicity, variance-covariance methods appear to have been pervasive in the banking sector with over three-quarters of banks using them for calculating VaR (Drehmann, 2007). These methods are based on the assumption that the asset returns are independently and identically normally distributed, which of course, may not be the case in reality. This assumption contradicts empirical evidence, which shows that in many cases (for example see Sheikh and Qiao (2010)), financial asset returns are not independent and normally distributed —financial asset returns are, in fact, leptokurtic and fat-tailed, leading to underestimation or overestimation of VaR, as extremely large positive and negative asset returns are more likely in practice than normally distributed models predict.

Another drawback of these methods is in the estimation of the conditional volatility of financial returns. Most financial asset returns exhibit heavy tails (as explained earlier) with respect to conditional volatility over time. Berkowitz et al. (2011) also showed evidence of changing volatility and non-normality using desk-level data from a large international commercial bank. VaR models are highly dependent on the type of volatility model used. A good volatility model should be able to capture the behaviour of the tail distribution of asset returns, be easily implemented for a wide range of asset returns, and be easily extensible to portfolios with many risk factors of different kinds (Malz, 2011). For multivariate volatility models of VaR, we must focus on the tail dependence, which is the principal factor associated with non-normality.

Therefore, the purpose of this paper is to investigate the reliability of a VaR model constructed using a DCC M-GARCH volatility model and copulas.

The rest of the paper is structured as follows: Section 2 presents an overview of M-GARCH volatility models, the DCC model, and why they are important in VaR estimation. In Section 3, we introduce copulas (elliptical and Archimedean) and Sklar's theorem. Section 4 presents invariant measures used in measuring the dependence structure. Section 5 presents empirical procedures and results of the VaR estimates. In Section 6, we discuss and present the results of back-testing techniques; this is followed by a summary and conclusion in Section 7.

2 Multivariate Volatility Models

Financial asset returns often demonstrate volatility clustering. Therefore, volatility plays an important role in VaR estimation. Many volatility models have been proposed, for example a generalised autoregressive conditional heteroskedasticity (GARCH) model and its extension have been used to capture the effects of volatility clustering and asymmetry in VaR estimation. Numerous studies have applied a variety of univariate GARCH models in VaR estimation (see So and Philip (2006), Berkowitz and Obrien (2002), and McNeil and Frey (2000)). In addition, Kuester et al. (2006) provides an extensive review of VaR estimation methods with a focus on univariate GARCH models. The results of all these studies suggest that GARCH models provide more accurate VaR estimates than traditional methods.

Because financial applications typically deal with a portfolio of assets with several risk factors (as considered in this study), a multivariate GARCH model would be very useful for

VaR estimation. Univariate VaR models focus on an individual portfolio, whereas the multivariate approaches explicitly model the correlation structure of the covariance or volatility matrix of multiple asset returns over time.

Numerous multivariate GARCH models have since been developed (see Bollerslev et al. (1994), Engle and Kroner (1995), Fengler and Herwartz (2008), Tsay (2013) and the references therein). Bauwens et al. (2006) divides multivariate GARCH (M-GARCH) models into three categories: (i) direct generalisation of univariate GARCH models (e.g., exponentially-weighted moving average (EWMA), vector error correction (VEC), BEKK, etc.), (ii) linear combinations of univariate GARCH models (e.g., generalised orthogonal GARCH (GO-GARCH), principal component GARCH (PGARCH), etc.), and (iii) nonlinear combinations of univariate GARCH models (e.g., dynamic conditional correlation (DCC) and constant conditional correlation (CCC)). Most volatility models fail to satisfy the positive definite conditions of the covariance matrix of asset returns. In this paper, we employ the DCC model in our analysis because of some conditions (to be discussed later) that will guarantee the conditional volatility matrix to be positive-definite almost surely. For more details on M-GARCH models, see (Silvennoinen and Teräsvirta, 2009) and (Ghalanos, 2015).

2.1 The DCC model

In the DCC model, the volatility matrix, Σ_t, consists of a marginal standardised vector of the series η_t, where $\eta_t = \frac{a_{it}}{\sigma_{it}}$, and σ_{it} is the conditional volatility series obtained using GARCH(1,1) for $i = 1, \ldots, N$. We can then represent the conditional volatility or covariance matrix as

$$\Sigma_t = D_t \rho_t D_t, \tag{1}$$

and the conditional correlation matrix is given by

$$\rho_t = D_t^{-1} \Sigma_t D_t^{-1}, \tag{2}$$

where D_t is the diagonal matrix of the k conditional volatilities of the stock returns; that is, $D = diag\left\{\sqrt{\sigma_{11,t}}, \ldots, \sqrt{\sigma_{kk,t}}\right\}$, and $\sigma_{ij,t}$ is the (i,j)th element of the volatility matrix (Tsay, 2005).

Tse and Tsui (2002) and Engle (2002) propose two types of DCC models.

The DCC model of Tse and Tsui ($DCC_T(m)$) is given by

$$\rho_t = (1 - \theta_1 - \theta_2)\bar{\rho}_t + \theta_1 \rho_{t-1} + \theta_2 \psi_{t-1}, \tag{3}$$

where $\theta_i \in R^+$, $0 \leq \theta_1 + \theta_2 < 1$ for $i = 1, 2$. $\bar{\rho}_t$ is a k x k unconditional correlation matrix of η_t. ψ_{t-1} is a k x k correlation matrix of the most recent returns that depends on $\{\eta_{t-1} \ldots \eta_{t-m}\}$ and is defined as

$$\psi_{ij,t-1} = \frac{\Sigma_{k=1}^{m} \eta_{i,t-k}\eta_{j,t-k}}{\sqrt{(\Sigma_{k=1}^{m} \eta_{i,t-k}^2)(\Sigma_{k=1}^{m} \eta_{j,t-k}^2)}}. \tag{4}$$

If $m > k$, then ψ_{t-1} and hence ρ_t are guaranteed to be positive-definite.

For the model proposed by Engle ($DCC_E(m)$), the correlation matrix (Eq.(2)) depends on two parameters, θ_1 and θ_2, controlled by

$$\Sigma_t = (1 - \theta_1 - \theta_2)\bar{\Sigma_t} + \theta_1 \Sigma_{t-1} + \theta_2 \eta_{t-1}\eta_{t-1}', \tag{5}$$

and Σ_t is a positive-definite matrix. $\bar{\Sigma_t}$ is the unconditional covariance matrix of η_t, $\theta_i \in R^+$, $0 < \theta_1 + \theta_2 < 1$ for $i = 1, 2$ (Tsay, 2013).

3 Copulas

Non-normality in multivariate models is associated with the joint probability of the univariate models' marginal probabilities, that is, the joint probability of large market movements referred to as *tail dependence*. The VaR estimation for a portfolio of assets can become very difficult due to the complexity of joint multivariate distributions. To overcome these problems, we use the copula theory, which enables us to construct a flexible multivariate distribution with different margins and different dependence structures; this allows the joint distribution of a portfolio to be free from assumptions of normality and linear correlation. Additionally, copulas can easily capture extreme dependencies such as tail dependence, while the normal distribution assumes no extreme dependencies.

The copula theory was first developed by Sklar (1959) and later introduced to the finance literature by Embrechts and McNeil (1999), Frey et al. (2001), and Li (1999). Consequently, Embrechts et al. (2002) introduced the application of copula theory to financial asset returns and Patton (2004) expanded the framework of the copula theory with respect to the time varying nature of financial dependence schemes. The copula theory has also been used in risk management to measure the VaR of portfolios, including both unconditional (Cherubini and Luciano (2001), Embrechts and Lindskog (2003), and Cherubini et al. (2004)) and, recently, conditional distributions (Silva Filho et al. (2014), Huang et al. (2009) and Fantazzini (2008)).

In this paper, we take advantage of copula theory and develop a copula-based volatility model.

For the purpose of estimating the VaR, we use the following version of Sklar's theorem as given by Cherubini et al. (2004).

Theorem 1 *Sklar's theorem: Let $F_1(x_1), \dots, F_n(x_n)$ be known marginal distribution functions. Then, for every $x = (x_1, \dots, x_n) \in \Re^{\pm n}$,*

- *if c is any subcopula whose domain contains Ran $F_1 \times$ Ran $F_2 \times \ldots \times$ Ran F_n, then*

$$c(F_1(x_1), F_2(x_2), \ldots, F_n(x_n))$$

is a joint distribution function with margins $F_1(x_1), F_2(x_2), \ldots, F_n(x_n)$, and

- *if F is a joint distribution function with margins $F_1(x_1), F_2(x_2), \ldots, F_n(x_n)$, there exists a unique subcopula c, with domain Ran $F_1 \times$ Ran $F_2 \times \ldots \times$ Ran F_n, such that*

$$F = c(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)).$$

If $F_1(x_1), F_2(x_2), \ldots, F_n(x_n)$ are continuous, the subcopula is a copula; if not, there exists a copula C such that

$$C(u_1, u_2, \ldots, u_n) = c(u_1, u_2, \ldots, u_n)$$

for every $(u_1, u_2, \ldots, u_n) \in$ Ran $F_1 \times$ Ran $F_2 \times \ldots \times$ Ran F_n.

Where $\Re^{\pm n} = [-\infty, +\infty]^n$ and $Ran\ F$ = range of the function F.

Definition 1 *An $n-$dimensional copula $C(u_1, u_2, u_3, \ldots, u_n)'$ is a distribution function on $\boldsymbol{I}^n$ with standard uniform marginal distributions (Tsay, 2013).*

Consider a random vector $X = (x_1, \ldots, x_n)'$, with margins $F(x_1), \ldots, F(x_n)$; then, from Theorem 1,

$$F(x_1, \ldots, x_n) = C(F(x_1), \ldots, F(x_n)). \tag{6}$$

C is unique if $F(x_1), \ldots, F(x_n)$ are continuous; otherwise, C is uniquely determined on $\mathbf{I}^n$ ($\mathbf{I}$ $= [0, 1]$; a unit interval on the real line.). On the other hand, if C is a copula and $F_1, \ldots, F_n$ are univariate distribution functions, Eq.(6) is a joint distribution function with margins $F_1, \ldots, F_n$ (Ghalanos, 2015), (Tsay, 2013).

Definition 2 *Each copula $C(u_1, \ldots, u_n)$ has a density $c(u_1, \ldots, u_n)$ related to it and defined as*

$$c(u_1, \ldots, u_n) = \frac{\partial_n C(u_1, \ldots, u_n)}{\partial u_1, \ldots, \partial u_n}, \tag{7}$$

and the density function for the copula is

$$f(x_1, \ldots, x_n) = c(F_1(x_1, \ldots, F_n(x_n)) \prod_{i=1}^{n} f_i(x_i) \tag{8}$$

in $\boldsymbol{I}^n$ for a continuous random variable, where f_i are the marginal densities

(Cherubini et al., 2004), (Ghalanos, 2015).

Bob (2013) and Cherubini et al. (2011) discuss two commonly used families of copulas in financial applications: the elliptical and the Archimedean copulas.

3.1 Elliptical Copulas

The most common elliptical copulas are the Gaussian and the Student's t copulas, which are symmetric. The dependence structure is determined by the standardised correlation or dispersion matrix

$$R = \begin{pmatrix} 1 & \ldots & \rho_{1,n} \\ \vdots & \ddots & \vdots \\ \rho_{n,1} & \ldots & 1 \end{pmatrix} \tag{9}$$

because of the invariant property of copulas. $\rho_{i,j}$ is the dispersion parameter, which can be set to either Kendall's tau or Spearman's rho, as discussed later.

Consider a symmetric positive definite matrix (Eq.9) with $diag(R) = (1, 1, \ldots, 1)^T$; we can represent the multivariate Gaussian copula (MGC) as

$$C_R^{G_a} = P(\Phi(X_1) \leq u_1, \ldots, \Phi(X_n) \leq u_n) = \Phi_R(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_n)), \tag{10}$$

where Φ_R is the standardised multivariate normal distribution and Φ_R^{-1} is the inverse standard univariate normal distribution function of u with correlation matrix R. If the margins are normal, then the Gaussian copula will generate the standard Gaussian joint distribution function with density function

$$c_R^{G_a}(u_1, u_2, \ldots, u_n) = \frac{1}{|R|^{\frac{1}{2}}} exp\left(-\frac{1}{2}\varsigma^T(R^{-1} - I)\varsigma\right), \tag{11}$$

where $\varsigma = (\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_n))^T$.

On the other hand, the multivariate Student's t copula (MTC) can be represented as

$$T_{R,v}(u_1, \ldots, u_n) = t_{R,v}(t_v^{-1}(u_1), \ldots, t_v^{-1}(u_n)) \tag{12}$$

with density function

$$c_{R,v}(u_1, \ldots, u_n) = |R|^{-\frac{1}{2}} \frac{\Gamma(\frac{v+n}{2})}{\Gamma(\frac{v}{2})} \left(\frac{\Gamma(\frac{v}{2})}{\Gamma(\frac{v+1}{2})}\right)^n \frac{(1 + \frac{1}{v}\varsigma^T R^{-1}\varsigma)^{-\frac{v+n}{2}}}{\prod_{j=1}^n \left(1 + \frac{\varsigma_j^2}{v}\right)^{-\frac{v+1}{2}}}, \tag{13}$$

where $t_{R,v}$ is the standardised Student's t distribution with correlation matrix R and v degrees of freedom.

3.2 Archimedean Copulas:

Archimedean copulas are built via a generator as

$$C(u_1, \ldots, u_n) = \varphi^{-1}(\varphi(u_1) + \ldots + \varphi(u_n)) \tag{14}$$

with density function

$$c(u_1, \ldots, u_n) = \varphi^{-1}(\varphi(u_1) + \ldots + \varphi(u_n)) \prod_{i=1}^{n} \varphi^{'}(u_i), \tag{15}$$

where φ is the copula generator and φ^{-1} is completely monotonic on $[0, \infty]$. That is, φ must be infinitely differentiable with derivatives of ascending order and alternative sign such that $\varphi^{-1}(0) = 1$ and $\lim_{x \to +\infty} \varphi(x) = 0$ (Cherubini et al., 2011). Thus, $\varphi^{'}(u) < 0$ (i.e., φ is strictly decreasing) and $\varphi^{''}(u) > 0$ (i.e., φ is strictly convex).

Archimedean copulas are very useful in risk management analysis because they capture an asymmetric tail dependence between financial asset returns. The most common are Gumbel (1960), Clayton (1978) and Frank (1979) copulas (Yan et al., 2007).

The Gumbel copula captures upper tail dependence, is limited to positive dependence, and has generator function $\varphi(u) = (-\ln(u))^{\alpha}$ and generator inverse $\varphi^{-1}(x) = \exp(-x^{\frac{1}{\alpha}})$. This will generate a Gumbel n-copula represented by

$$C(u_1, \ldots, u_n) = \exp\left\{ -\left[\sum_{i=1}^{n} (-\ln u_i)^{\delta} \right]^{\frac{1}{\delta}} \right\} \quad \delta > 0. \tag{16}$$

The generator function for the Clayton copula is given by $\varphi(u) = u^{-\alpha} - 1$ and generator inverse $\varphi^{-1}(x) = (x+1)^{-\frac{1}{\alpha}}$, which yields a Clayton n-copula represented by

$$C(u_1, \ldots, u_n) = \left[\sum_{i=1}^{n} (u_i^{-\alpha} - n + 1\right]^{-\frac{1}{\alpha}} \quad \alpha > 0. \tag{17}$$

The Frank copula has generator function $\varphi(u) = \ln\left(\frac{\exp(-\alpha u)-1}{\exp(-\alpha)-1}\right)$ and generator inverse $\varphi^{-1}(x) = -\frac{1}{\alpha}\ln\left(1 + e^{x}(e^{-\alpha} - 1)\right)$, which will result in a Frank n-copula represented by

$$C(u_1, \ldots, u_n) = -\frac{1}{\alpha} \ln\left\{1 + \frac{\prod_{i=1}^{n}(e^{-\alpha u_i} - 1)}{(e^{-\alpha} - 1)^{n-1}}\right\} \quad \alpha => 0, \tag{18}$$

(Cherubini et al., 2004).

4 Measuring Dependence

The traditional way to measure the relationship between markets and risk factors is by looking at their linear correlations, which depend both on the marginal and joint distributions of the risk factors. If there is no linear relationship —in the case of non-normality —the results might be misleading (see Cherubini et al. (2011)). In this situation, nonparametric invariant measures that are not dependent on marginal probability distributions are more appropriate. Copulas measure a form of dependence between pairs of risk factors (i.e., asset returns) known as concordance using invariant measures.

Two observations (x_i, y_i) and (x_j, y_j) from a vector (X, Y) of continuous random variables are concordant if $(x_i - x_j)(y_i - y_j) > 0$ and discordant if $(x_i - x_j)(y_i - y_j) < 0$. Large values of X are paired with large values of Y and small values of X are paired with small values of Y as the proportion of concordant pairs in the sample increases. On the other hand, the proportion of concordant pairs decreases as large values of X are paired with small values of Y and small values of X are paired with large values of Y (Alexander, 2008).

The most commonly used invariant measures are Kendall's tau and Spearman's rho.

Consider n paired continuous observations (x_i, y_i) ranked from smallest to largest, with the smallest ranked 1, the second smallest ranked 2 and so on. Then, Kendall's tau is defined as the sum of the number of concordant pairs minus the sum of the number of discordant pairs divided by the total number of pairs, i.e., the probability of concordance minus the probability of discordance:

$$\tau_{X,Y} = P[(x_i - x_j)(y_i - y_j) > 0] - P[(x_i - x_j)(y_i - y_j) < 0] = \frac{C - D}{C + D}, \tag{19}$$

where C is the number of concordant pairs below a particular rank that are larger in value than that particular rank, and D is the number of discordant pairs below a particular rank that are smaller in value than that particular rank.

Spearman's rho, on the other hand, is defined as the probability of concordance minus the probability of discordance of the pair of vectors (x_1, y_1) and (x_2, y_3) with the same margins. That is,

$$\rho_{X,Y} = 3(P[(x_1 - x_2)(y_1 - y_3) > 0] - P[(x_1 - x_2)(y_1 - y_3)] < 0).$$

The joint distribution function of (x_1, y_1) is $H(x, y)$, while the joint distribution function of (x_2, y_3) is $F(x), G(y)$ because x_2 and y_3 are independent (Nelsen, 2007). Alternatively,

$$\rho_{X,Y} = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2 - 1)},$$

where d is the difference between the ranked samples.

Nelsen (2007) has shown that Kendall's tau and Spearman's rho depend on the vectors (x_1, y_1), (x_2, y_2) and (x_1, y_1), (x_2, y_3), respectively, through theirs copulas C, and that the following relationship holds:

$$\tau_{X,Y} = 4\int_0^1\int_0^1 C(u, v)dC(u, v) - 1$$

$$\rho_{X,Y} = 12 \int_0^1 \int_0^1 C(u,v)dudv - 3.$$

5 Empirical Analysis

5.1 Data

We investigate the performance of an M-GARCH DCC copula-based volatility model in VaR estimation using daily closing stock prices of 47 banking sector stocks from seven different EU countries —Germany, the UK, Sweden, France, Italy, Spain, and Greece —over the period from 31 December 2004 to 31 December 2015. All data are from DataStream and consist of 2870 observations.

The daily log return series of the stocks are calculated via

$$r_t = \left[log\left(\frac{S_{1,t+\tau}}{S_{1,t}}\right), \ldots, log\left(\frac{S_{N,t+\tau}}{S_{N,t}}\right)\right] = (r_{1t}, \ldots, r_{Nt}), \qquad (20)$$

where N represents the number of stocks in the sample. Figures 1 to 7 shows time series plots of daily log returns series for the different countries. From the plots, we can observe the presence of volatility clustering. That is, small changes in volatility tend to be followed by small changes for a prolonged period of time and large changes in volatility tend to be followed by large changes for a prolonged period of time. Basic statistics of the stock returns are reported in percentages in Table 1. From the table, we see that the stock returns are far from being normally distributed, as indicated by their high excess kurtosis and skewness. We confirm this by running a multivariate ARCH test, as described by Tsay (2013), on the log returns at 5% significance. The results confirm the presence of conditional heteroskedasticity in the daily log return series with a $P - value$ equal to or close to zero.

Country		Stocks									
France		F.BNP	F.SGE	F.CRDA	F.KNF	F.CC	F.CAI	F.CAMO	F.LAV	F.CAIV	F.CPD
	Mean	0.0006	-0.0151	-0.0215	-0.0019	0.0006	-0.0108	-0.0102	-0.0011	-0.0103	-0.0166
	Variance	0.0647	0.0807	0.0764	0.0959	0.0197	0.0145	0.0262	0.0267	0.0225	0.0202
	Stdev	2.5427	2.8399	2.7645	3.0969	1.4027	1.2029	1.6182	1.6340	1.5003	1.4212
	Skewness	34.4305	6.9738	27.7890	60.0710	62.1017	7.6754	-43.2826	-46.9617	5.3467	-1.5654
	Excess Kurtosis	867.8928	680.0429	624.0289	1209.1365	821.6399	678.3979	476.7556	1107.4525	601.0420	702.8488
Italy		I.ISP	I.UCG	I.MB	I.PMI	I.BMPS	I.BP	I.UBI	I.BPE	I.CE	I.BPSO
	Mean	-0.0023	-0.0519	-0.0078	-0.0250	-0.1260	-0.0635	-0.0278	-0.0129	-0.0022	-0.0132
	Variance	0.0671	0.0829	0.0429	0.0773	0.0870	0.0814	0.0566	0.0574	0.0517	0.0335
	Stdev	2.5905	2.8797	2.0722	2.7800	2.9488	2.8539	2.3799	2.3966	2.2734	1.8305
	Skewness	-25.1383	-16.0663	3.8658	29.6811	-24.7777	2.9842	-4.4575	24.7941	1.4148	58.9718
	Excess Kurtosis	656.5973	589.0746	320.6962	383.9322	891.5712	461.0507	273.2804	392.4517	408.2981	553.0105
UK		HSBA	BARC	LLOY	RBS	STAN	CIHL				
	Mean	-0.0124	-0.0306	-0.0407	-0.0971	-0.0112	-0.1171				
	Variance	0.0293	0.1030	0.1077	0.1504	0.0594	0.0762				
	Stdev	1.7122	3.2098	3.2823	3.8785	2.4376	2.7600				
	Skewness	-33.6697	143.8658	-105.4936	-840.1325	31.6077	9.0031				
	Excess Kurtosis	1690.7965	4021.7880	3727.5413	23552.6326	1308.5010	4940.1441				
Germany		D.DBK	D.CBK	D.IKB	D.MBK	D.OLB	D.UBK				
	Mean	-0.0320	-0.0782	-0.1287	0.0008	-0.0457	0.0879				
	Variance	0.0615	0.0904	0.1529	0.0550	0.0478	0.0466				
	Stdev	2.4803	3.0066	3.9100	2.3445	2.1863	2.1586				
	Skewness	32.5034	-13.9034	151.9830	31.5160	83.5634	-31.7385				
	Excess Kurtosis	1022.5506	873.5286	1914.4094	1145.6160	2008.7958	1984.3534				
Greece		G.PIST	G.PEIR	G.EFG	G.ETE	G.ATT	G.ELL				
	Mean	-0.1603	-0.3371	-0.3377	-0.2872	-0.2045	-0.0642				
	Variance	0.2287	0.2876	0.3162	0.2487	0.2818	0.0448				
	Stdev	4.7826	5.3624	5.6234	4.9866	5.3087	2.1157				
	Skewness	-12.5182	-105.8321	-61.0233	-103.6301	-59.4886	-7.5496				
	Excess Kurtosis	827.1241	1063.2666	865.8341	1044.0274	1317.2709	1519.9341				
Spain		E.SCH	E.BBVA	E.BSAB	E.BKT	E.POP					
	Mean	-0.0070	-0.0154	-0.0199	0.0117	-0.0702					
	Variance	0.0463	0.0444	0.0359	0.0512	0.0529					
	Stdev	2.1519	2.1078	1.8940	2.2617	2.2991					
	Skewness	20.4736	32.7311	71.6545	49.2691	43.5654					
	Excess Kurtosis	828.4460	668.4718	623.2511	304.9592	494.7029					
Sweden		W.NDA	W.SVK	W.SWED	W.SEA						
	Mean	0.0202	0.0234	0.0109	0.0104						
	Variance	0.0419	0.0347	0.0631	0.0643						
	Stdev	2.0471	1.8633	2.5116	2.5361						
	Skewness	52.6835	12.2476	-20.9301	5.3782						
	Excess Kurtosis	657.9097	690.7580	906.9637	1278.0779						

Table 1: Basic statistics of stock returns (in percentages). High excess kurtosis and skewness implies stock returns are not normally distributed.

5.2 Modelling the volatility matrix and copula parameters

We obtained the volatility matrix $\sum_t$, which consists of the marginal standardised residuals $\{\eta_{i,t}\}_{t=1}^{T}$, by applying the M-GARCH DCC model to the log return series and setting the conditional distribution of the standardised residuals to the Student's t distribution to account for the heavy tails.

Copula parameters are estimated by the canonical maximum likelihood (CML) method (Cherubini et al., 2004). That is, we use pseudo-observations of the standardised residuals to estimate the marginals and then estimated the copula parameters by means of maximum likelihood estimation (MLE):

$$\hat{\theta}_2 = ArgMax_{\theta_2} \sum_{t=1}^{T} \ln c(\hat{F}_1(x_{1t}), \ldots, \hat{F}_n(x_{nt}); \theta_2). \tag{21}$$

The best copula to use for our VaR modelling is selected by comparing MLE values. We select two models, one from each copula family. Table 2 shows the MLEs and copula parameter values for both Archimedean and elliptical copulas.

	Archimedean Copula			**Elliptical Copula**	
Stocks from	Gumbel	Clayton	Frank	Gaussian	Student's t
UK	1691 (1.30)	**1813** (0.47)	1810 (2.33)	3834 (ρ_n)	**4047** (ρ_t)
Germany	81.47 (1.05)	**152.5** (0.10)	105.2 (0.43)	986.6 (ρ_n)	**1018** (ρ_t)
Greece	**3861** (1.55)	3609 (0.81)	3825 (3.86)	4717 (ρ_n)	**5092** (ρ_t)
Spain	4817 (1.88)	4616 (1.34)	**4952** (5.73)	5936 (ρ_n)	**6362** (ρ_t)
France	1352 (1.16)	**1615** (0.24)	1298 (1.32)	4789 (ρ_n)	**5059** (ρ_t)
Italy	6959 (1.45)	6816 (0.70)	**7024** (3.55)	8846 (ρ_n)	**9493** (ρ_t)
Sweden	**3747** (1.95)	3142 (1.32)	3715 (6.00)	**4057** (ρ_n)	4047 (ρ_t)

Table 2: MLE and copula parameter (in parentheses) values. The best copula that fits the data (in bold) is selected based on the highest MLE value.

5.3 Modelling the marginal distributions

Next, we specify the desired marginal distributions, which we set to a Student's t distribution, and generate 10000 simulations from the fitted copula. Our choice of Student's t distributions for the margins is because a multivariate ARCH test on the standardised residuals $\{\eta_{i,t}\}_{t=1}^{T}$ still indicates the presence of volatility clustering. We then reintroduce the GARCH(1,1) model and convert the daily simulated data with t-margins to daily risk factor returns.That is,

$$x_{i,t} = \mu_i + \hat{\sigma}_{i,t}\zeta_{i,t} \text{ , and}$$
$$\hat{\sigma}_t^2 = \alpha_0 + \alpha_1\xi_{t-1}^2 + \beta_1\sigma_{t-1}^2, \tag{22}$$

where $i = 1, \ldots n$ represents the stocks in each country, $t = 1 \ldots T$ represents the length of the original data $= 2869$, and $\zeta_{i,t}$ are the daily simulated observations from the copulas with t-margins. α_0, α_1, and β_1 are the GARCH parameters, μ_i are the unconditional means of the risk factors, and $\hat{\sigma_i}$ are estimates of the conditional volatilities of the risk factors from the M-GARCH DCC model.

5.4 Estimating VaRs

For each country, we apply the risk factor mappings to construct a simulated portfolio of returns consisting of all stocks represented by

$$\bar{R}_{p,t} = E\left(R_{p,t}\right) = \sum_{t=1}^{N} w_i E\left(R_{i,t}\right), \tag{23}$$

where

$$w_i = \frac{\sum_{j=1}^{n} x_{ij}}{\sum_{j=1}^{N} C_j} \quad , \quad \text{and} \quad \sum_{i=1}^{N} w_i = 1 \qquad 1 \leq i \leq n, \;\; 1 \leq j \leq N.$$

$\bar{R}_{p,t}$ is the expected return of the portfolio at time t; x_{ij} is the amount invested in asset i; C_j is the total dollar amount invested; and w_i the weight of asset i. We assume equal weights.

The one day VaR at time t with α% confidence level is simply the $1-\alpha$ percentile of the distribution of the simulated portfolio returns below which lies $(1-\alpha)$% of the observations and above which lies α% of the observations. Thus, we are α% confident that in the worst case scenario, the losses on the portfolio will not exceed the $1-\alpha$ quantile. Table 3 shows VaR estimates with confidence levels $\alpha = 99\%, 95\%$, and 90%, based on the selected Archimedean and elliptical copulas for the constructed portfolios.

		Copula Family	
Portfolio	CL	Archimedean	Elliptical
UK		Clayton	Student's t
	99%	-7.52	-7.85
	95%	-3.41	-3.76
	90%	-2.19	-2.39
Germany		Clayton	Student's t
	99%	-6.24	-5.31
	95%	-3.52	-2.90
	90%	-2.53	-2.13
Greece		Gumbel	Student's t
	99%	-14.82	-18.88
	95%	-7.23	-8.68
	90%	-5.04	-5.71
Spain		Frank	Student's t
	99%	-6.12	-7.02
	95%	-3.75	-3.78
	90%	-2.79	-2.65
France		Clayton	Student's t
	99%	-5.11	-4.54
	95%	-2.49	-2.52
	90%	-1.81	-1.77
Italy		Frank	Student's t
	99%	-5.76	-7.35
	95%	-3.77	-4.07
	90%	-2.87	-2.78
Sweden		Gumbel	Gaussian t
	99%	-6.44	-8.11
	95%	-3.53	-3.37
	90%	-2.34	-2.47

Table 3: VaR estimates (in percentages) for the portfolio constructed from returns generated using Archimedean and elliptical copulas.

6 Back-testing

To check that the model does not overestimate or underestimate risk, we do back-testing on the model. This involves comparing the estimated VaRs for a given number of days T to the subsequent portfolio returns. The number of days N in which the loss on the portfolio exceeds VaR is recorded as the number of exceptions or failures. Too many exceptions implies the VaR model underestimates the level of risk on the portfolio, and too few exceptions implies the model overestimates risk. The number of exceptions should be reasonably close to $T(1-c)\%$ (c = confidence level), depends on the choice of c, and follows a binomial distribution

$$f(x) = \binom{T}{x} p^x q^{T-x} \tag{24}$$

with mean $= pT$ and variance $= pqT$; $q = 1-p$ and $p = 1-c$ (Best, 2000).

6.1 Back-testing methods

The most popular back-testing methods include the standard normal hypothesis (or failure rate) test , Kupiec's (1995) "proportion of failures" (POF) test, Basel's (1996*a*) "traffic light" test, and Christoffersen's (1998) test. We employ the standard normal hypothesis test, Basel's "traffic light" test, and Kupiec's POF test to check the reliability of the VaR models.

6.2 Standard normal hypothesis test

From the central limit theorem and with sufficiently large T, Eq.(24) can be approximated by the normal distribution

$$z = \frac{x - pT}{\sqrt{pqT}} \approx N(0,1), \tag{25}$$

which is also the test statistic for a standard normal hypothesis test to assess the reliability of the VaR model (Jorion, 2007). The VaR model is rejected if $z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$ for a two-tailed test and if $z > z_\alpha$ for a one-tailed test. $\alpha = 1-c$, and $z_{\alpha/2}$ and z_α are the cutoff values for the inverse standard normal cumulative distribution of α and $\alpha/2$, respectively. Tables 5 and 6 show back-testing results based on the standard normal hypothesis test.

6.3 Basel "traffic light" test

The Basel "traffic light" approach to back-testing the VaR was originally proposed by the Basel Committee of Banking and Supervision (BCBS) and is described in (on Banking Supervision, 1996). In a new accord, the BCBS further came up with a set of requirements that the VaR model must satisfy for it to be considered a reliable risk measure (Resti, 2008). That is, (i) VaR must be calculated with 99% confidence, (ii) back-testing must be done using a minimum of a one year observation period and must be tested over at least 250 days, (iii) regulators should be 95% confident that they are not erroneously rejecting a valid VaR model, and (iv) Basel specifies a one-tailed test —it is only interested in the underestimation of risk. Table 4 shows the acceptance region for the Basel "traffic light" approach.

Zone	Number of Exceptions	CumulProb
Green	≤ 4	89.22%
Yellow	5	95.88%
	6	98.63%
	7	99.60%
	8	99.89%
	9	99.97%
Red	≥ 10	99.99%

Table 4: Acceptance region for Basel "traffic light" approach for back-testing VaR models. $CL = 99\%$, $T = 250$ (Jorion, 2007).

Depending on the number of exceptions, the bank is placed in a red, green or yellow zone. Test results based on this test are shown in Table 7

6.4 Kupiec's POF test

Kupiec defined an approximate 95% confidence region whereby the number of exceptions produced by the model must be within this interval for it to be considered a reliable risk measure. The confidence region is approximated using a chi-square distribution with one degree of freedom, defined as

$$LR_{PF} = 2ln\left[\frac{\left(1-\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^{N}}{q^{T-N}p^{N}}\right]. \tag{26}$$

Assuming one degree of freedom and at a 5% confidence level, the chi-square value is 3.841. Hence, by equating Eq.(26) to 3.841 and solving for N, we obtain two values of N; the rejection region is $[x_1, x_2]$. The VaR model is rejected if $N \notin [x_1, x_2]$ and accepted if $N \in [x_1, x_2]$ (Holton, 2002). Results based on this test are shown in Tables 8 and 9.

		Exceptions		**VaR Coverage (%)**		**Z-score**		**Accept one tail?**		**Accept two tail?**	
Portfolio	CL	250	500	250	500	250	500	250	500	250	500
UK	99%	28	27	0.98	0.94	-0.13	-0.32	Yes	Yes	Yes	Yes
	95%	141	138	4.94	4.81	-0.21	-0.47	Yes	Yes	Yes	Yes
	90%	277	264	9.65	9.20	-0.62	-1.43	Yes	Yes	Yes	Yes
Germany	99%	27	26	0.94	0.91	-.32	-.50	Yes	Yes	Yes	Yes
	95%	137	132	4.78	4.60	-0.55	-0.98	Yes	Yes	Yes	Yes
	90%	266	259	9.27	9.03	-1.24	-1.66	Yes	Yes	Yes	Yes
Greece	99%	27	27	0.94	0.94	-0.32	-0.32	Yes	Yes	Yes	Yes
	95%	143	140	4.98	4.88	-0.04	-0.30	Yes	Yes	Yes	Yes
	90%	282	279	9.83	9.27	-0.30	-0.49	Yes	Yes	Yes	Yes
Spain	99%	27	27	0.94	0.94	-0.32	-0.32	Yes	Yes	Yes	Yes
	95%	142	142	4.95	4.95	-0.12	-0.12	Yes	Yes	Yes	Yes
	90%	283	283	9.86	9.86	-0.24	-0.24	Yes	Yes	Yes	Yes
France	99%	26	26	0.91	0.91	-0.50	-0.50	Yes	Yes	Yes	Yes
	95%	140	130	4.88	4.53	-0.30	-1.15	Yes	Yes	Yes	Yes
	90%	272	253	9.48	8.82	-0.93	-2.11	Yes	Yes	Yes	Yes
Italy	99%	27	27	0.94	0.94	-0.32	-0.32	Yes	Yes	Yes	Yes
	95%	143	143	4.98	4.98	-0.04	-0.04	Yes	Yes	Yes	Yes
	90%	283	281	9.86	9.79	-0.24	-0.37	Yes	Yes	Yes	Yes
Sweden	99%	27	25	0.94	0.87	-0.32	-0.69	Yes	Yes	Yes	Yes
	95%	141	137	4.91	4.78	-0.21	-0.55	Yes	Yes	Yes	Yes
	90%	280	269	9.76	9.38	-0.43	-1.11	Yes	Yes	Yes	Yes

Table 5: Testing the reliability of the VaR model based on a standard normal hypothesis test. Returns generated using selected Archimedean copulas; time horizon = 1 day, 250-day and 500-day observation periods.

		Exceptions		VaR Coverage (%)		Z-score		Accept one tail?		Accept two tail?	
Portfolio	CL	250	500	250	500	250	500	250	500	250	500
UK	99%	27	27	0.94	0.94	-0.32	-0.32	Yes	Yes	Yes	Yes
	95%	142	141	4.95	4.91	-0.12	-0.21	Yes	Yes	Yes	Yes
	90%	281	273	9.79	9.52	-0.37	-0.87	Yes	Yes	Yes	Yes
Germany	99%	27	27	0.94	0.94	-.32	-.32	Yes	Yes	Yes	Yes
	95%	140	132	4.88	4.60	-0.30	-0.98	Yes	Yes	Yes	Yes
	90%	279	264	9.72	9.20	-0.47	-1.36	Yes	Yes	Yes	Yes
Greece	99%	23	23	0.80	0.80	-1.07	-1.07	Yes	Yes	Yes	Yes
	95%	122	121	4.25	4.22	-1.84	-1.92	Yes	Yes	Yes	Yes
	90%	253	245	8.82	8.54	-2.11	-2.61	Yes	Yes	Yes	Yes
Spain	99%	27	27	0.94	0.94	-0.32	-0.32	Yes	Yes	Yes	Yes
	95%	142	137	4.95	7.78	-0.12	-0.55	Yes	Yes	Yes	Yes
	90%	279	271	9.72	9.45	-0.49	-0.99	Yes	Yes	Yes	Yes
France	99%	27	27	0.94	0.94	-0.32	-0.32	Yes	Yes	Yes	Yes
	95%	139	135	4.84	4.71	-0.38	-0.72	Yes	Yes	Yes	Yes
	90%	280	286	9.76	9.34	-0.43	-1.18	Yes	Yes	Yes	Yes
Italy	99%	27	27	0.94	0.94	-0.32	-0.32	Yes	Yes	Yes	Yes
	95%	140	140	4.88	4.88	-0.30	-0.30	Yes	Yes	Yes	Yes
	90%	282	275	9.83	9.59	-0.74	-0.30	Yes	Yes	Yes	Yes
Sweden	99%	27	25	0.94	0.87	-0.32	-0.69	Yes	Yes	Yes	Yes
	95%	141	134	4.91	4.67	-0.21	-0.81	Yes	Yes	Yes	Yes
	90%	278	263	9.69	9.17	-0.55	-1.49	Yes	Yes	Yes	Yes

Table 6: Testing the reliability of the VaR model based on standard normal hypothesis test. Returns generated using selected elliptical copulas; time horizon = 1 day; 250-day and 500-day observation periods.

Portfolio	CopulaFamily	Exceptions 250	Exceptions 500	CumulProb OneTail % 250	CumulProb OneTail % 500	CumulProb TwoTail % 250	CumulProb TwoTail % 500	Zone OneTail 250	Zone OneTail 500	Zone TwoTail 250	Zone TwoTail 500	Accept OneTail? 250	Accept OneTail? 500	Accept TwoTail? 250	Accept TwoTail? 500
UK	Archimedean	28	27	49.81	42.32	99.96	99.91	Green	Green	Yellow	Yellow	Yes	Yes	Yes	Yes
	Elliptical	27	27	42.32	42.32	99.91	99.91	Green	Green	Yellow	Yellow	Yes	Yes	Yes	Yes
Germany	Archimedean	27	26	42.32	35.01	99.91	98.82	Green	Green	Yellow	Yellow	Yes	Yes	Yes	Yes
	Elliptical	27	27	42.32	42.32	99.91	99.91	Green	Green	Yellow	Yellow	Yes	Yes	Yes	Yes
Greece	Archimedean	27	27	42.32	42.32	99.91	99.91	Green	Green	Yellow	Yellow	Yes	Yes	Yes	Yes
	Elliptical	23	23	16.52	16.52	98.80	98.80	Green	Green	Yellow	Yellow	Yes	Yes	Yes	Yes
Spain	Archimedean	27	27	42.32	42.32	99.91	99.91	Green	Green	Yellow	Yellow	Yes	Yes	Yes	Yes
	Elliptical	27	27	42.32	42.32	99.91	99.91	Green	Green	Yellow	Yellow	Yes	Yes	Yes	Yes
France	Archimedean	26	26	35.00	35.00	99.82	99.82	Green	Green	Yellow	Yellow	Yes	Yes	Yes	Yes
	Elliptical	27	27	42.32	42.32	99.91	99.91	Green	Green	Yellow	Yellow	Yes	Yes	Yes	Yes
Italy	Archimedean	27	27	42.32	42.32	99.91	99.91	Green	Green	Yellow	Yellow	Yes	Yes	Yes	Yes
	Elliptical	27	27	42.32	42.32	99.91	99.91	Green	Green	Yellow	Yellow	Yes	Yes	Yes	Yes
Sweden	Archimedean	27	25	42.32	28.14	99.91	99.65	Green	Green	Yellow	Yellow	Yes	Yes	Yes	Yes
	Elliptical	27	25	42.32	28.14	99.91	99.65	Green	Green	Yellow	Yellow	Yes	Yes	Yes	Yes

Table 7: Testing the reliability of the VaR model based on BCBS requirements. Time horizon = 1 day; 250- and 500-day observation periods. CumulProb = cumulative probability.

		Exceptions		Test Statistic		Accept?	
Portfolio	CL	250	500	250	500	250	500
UK	99%	28	27	0.02	0.10	Yes	Yes
	95%	141	138	0.04	0.22	Yes	Yes
	90%	277	264	NaN	NaN	No	No
Germany	99%	27	26	0.10	0.26	Yes	Yes
	95%	137	132	0.31	0.99	Yes	Yes
	90%	266	259	NaN	NaN	No	No
Greece	99%	27	27	0.1	0.1	Yes	Yes
	95%	143	140	0.00	0.09	Yes	No
	90%	282	279	NaN	NaN	No	No
Spain	99%	27	27	0.10	0.10	Yes	Yes
	95%	142	142	0.02	0.02	Yes	Yes
	90%	283	283	NaN	NaN	No	No
France	99%	26	26	0.26	0.26	Yes	Yes
	95%	140	130	0.09	1.37	Yes	Yes
	90%	272	253	NaN	NaN	No	No
Italy	99%	27	27	0.10	0.10	Yes	Yes
	95%	143	143	0.00	0.00	Yes	Yes
	90%	283	281	NaN	NaN	No	No
Sweden	99%	27	25	0.10	0.50	Yes	Yes
	95%	141	137	0.04	0.31	Yes	Yes
	90%	280	269	NaN	NaN	No	No

Table 8: Testing the reliability of the VaR model based on Kupiec's POF coverage test. Returns generated using Archimedean copulas. Time horizon = 1 day; 250-day and 500-day observation periods.

Portfolio	CL	Exceptions 250	Exceptions 500	Test Statistic 250	Test Statistic 500	Accept? 250	Accept? 500
UK	99%	27	27	0.10	0.10	Yes	Yes
	95%	142	141	0.02	0.04	Yes	Yes
	90%	281	273	NaN	NaN	No	No
Germany	99%	27	27	0.10	0.10	Yes	Yes
	95%	140	132	0.09	0.99	Yes	Yes
	90%	279	264	NaN	NaN	No	No
Greece	99%	23	23	1.22	1.22	Yes	Yes
	95%	122	121	3.55	3.90	Yes	No
	90%	253	245	NaN	NaN	No	No
Spain	99%	27	27	0.10	0.10	Yes	Yes
	95%	142	137	0.02	0.31	Yes	Yes
	90%	279	271	NaN	NaN	No	No
France	99%	27	27	0.10	0.10	Yes	Yes
	95%	139	135	0.15	0.53	Yes	Yes
	90%	280	268	NaN	NaN	No	No
Italy	99%	27	27	0.10	0.10	Yes	Yes
	95%	140	140	0.00	0.00	Yes	Yes
	90%	282	275	NaN	NaN	No	No
Sweden	99%	27	25	0.10	0.50	Yes	Yes
	95%	141	134	0.04	0.67	Yes	Yes
	90%	278	263	NaN	NaN	No	No

Table 9: Testing the reliability of the VaR model based on Kupiec's POF coverage test. Returns generated using elliptical copulas. Time horizon = 1 day; 250- and 500-day observation periods.

7 Summary and Conclusion

Because VaR models attempt to capture the behaviour of asset returns in the left tail, it is important that the model is constructed such that it does not underestimate the proportion of outliers and hence the true VaR. The normality assumption of asset returns might severely underestimate the true VaR because extreme values are assumed to be very unlikely to occur. Therefore for a reliable VaR model, it is important to take into account the choice of time horizon, the observation period, and the type of volatility models being used. We construct our VaR model using copulas with a DCC M-GARCH volatility model for a time horizon of one day. We then check the reliability of the model by back-testing on a window of 250 and 500 observation periods and record the number of exceptions produced.

As the observation period increases from 250 to 500 days at a 99% confidence level, the number of exceptions produced is unchanged using elliptical copulas with the exception of

Sweden, which exhibits a difference of 2. With Archimedean copulas, there is a difference of 1, 1, and 2 for the UK, Germany and Sweden, respectively, at the 99% confidence level. The number of exceptions produced is also very close to the expectation (i.e., 29) with the exception of Greece for the elliptical Student's-t copula. This suggests that the model at a 99% confidence level captures VaR extremely well because the difference in exceptions is very minimal or zero.

At the 95% confidence level, there is a significant difference between the number of exceptions produced in some of the countries when using 250- and 500-observation periods for both Archimedean and elliptical copulas. Although the number of exceptions is quite close to the expectation, 143, the significant difference indicates that there is greater room for error in estimating VaR at the 95% compared with the 99% confidence level. However, back-testing results indicate that the VaR model performed quite well at the 95% confidence level except for the models of Greece in the 500-observation period.

At a 90% confidence level, the difference in the number of exceptions is quite high for both copula families. The tail dependence structure of the portfolio returns is not quite accounted for and hence, the model fails to capture extreme events.

Back-testing results from Kupiec's POF test confirm the above analysis. The model is accepted at the 99% and 95% confidence levels for both copula families except for in Greece, where the elliptical Student's-t copula rejects the model at a 95% confidence level with a 500-day observation period. At a 90% confidence level, Kupiec's POF test rejects the model for both copula families.

For the standard normal hypothesis test and Basel "traffic light" test, we perform one- and two-tailed tests. Although Basel is only concerned with the underestimation of risk, we performed a two-tailed test to make sure that the model does not overestimate risk and thereby result in excess capital being provided (Best, 2000). The model is accepted in all cases for both one-tailed and two-tailed tests. However, for both Archimedean and elliptical copulas, the Basel "traffic light" test places the VaR model in the yellow zone for a two-tailed test when using 250- and 500-observation periods, suggesting that there might be some instances of overestimation of risk since for a one-tailed test the model falls in the green zone in all instances.

It is also important to note that the standard normal hypothesis test and the Basel "traffic light" test do not specify whether the number of exceptions produced is too small. That is, the model will not be rejected if the number of exceptions is too low, which would lead to overestimation of VaR. Kupiec's POF test rejects the model if the number of exceptions produced is too high or too low. This is why the standard normal hypothesis test and Basel "traffic light" test fail to reject the model at the 90% confidence level. Thus, from the banks perspective, Kupiec's POF test is preferable and superior because it accounts for both underestimation and overestimation of risk. Back-testing results also suggest that the type of copula family used (i.e., elliptical or Archimedean) to model the dependence structure does not have a strong effect when dealing with quantile VaRs. The results are quite similar based on the number of exceptions produced.

This study suggests a challenging yet possible development in the world of risk management, that is, to design a model based on the Basel requirements that detects when the number of exceptions produced by a VaR model is too low.

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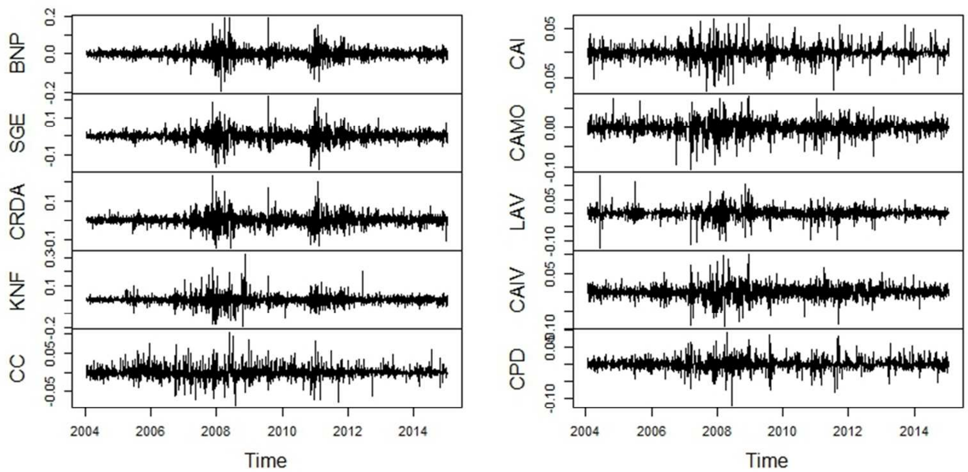
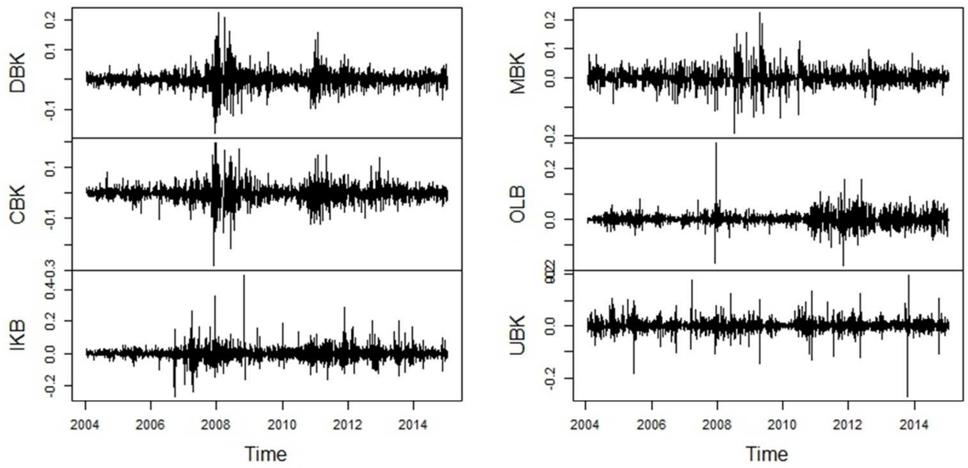


Time plots of daily log return series for French stocks indicating presence of volatility clustering.

291x164mm (72 x 72 DPI)



Time plots of daily log return series for German stocks indicating presence of volatility clustering.

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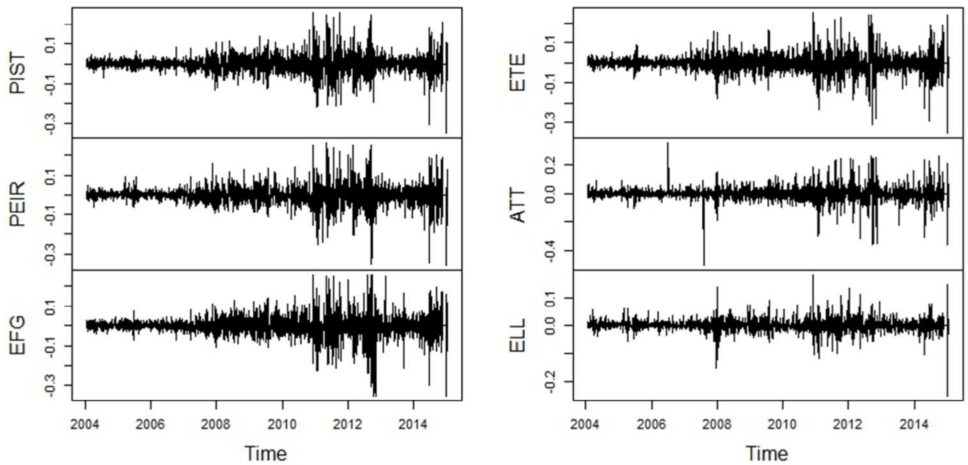

Time plots of daily log return series for Greek stocks indicating presence of volatility clustering.

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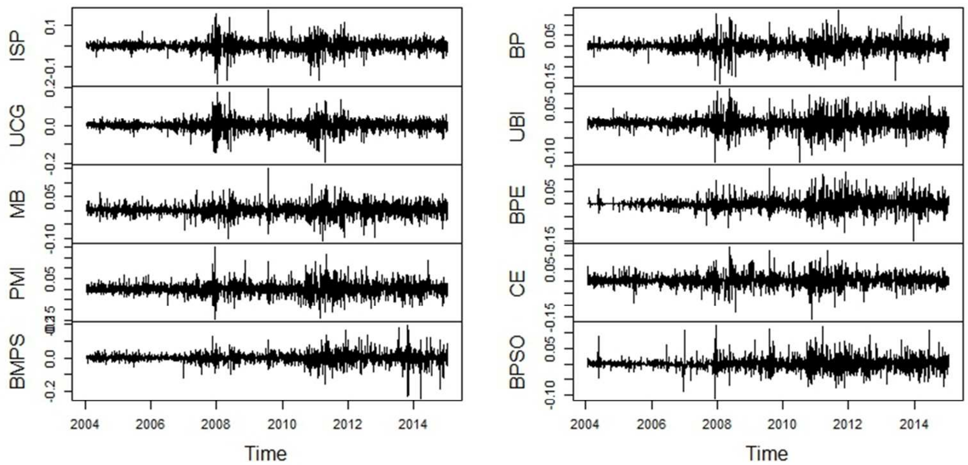

Time plots of daily log return series for Italian stocks indicating presence of volatility clustering.

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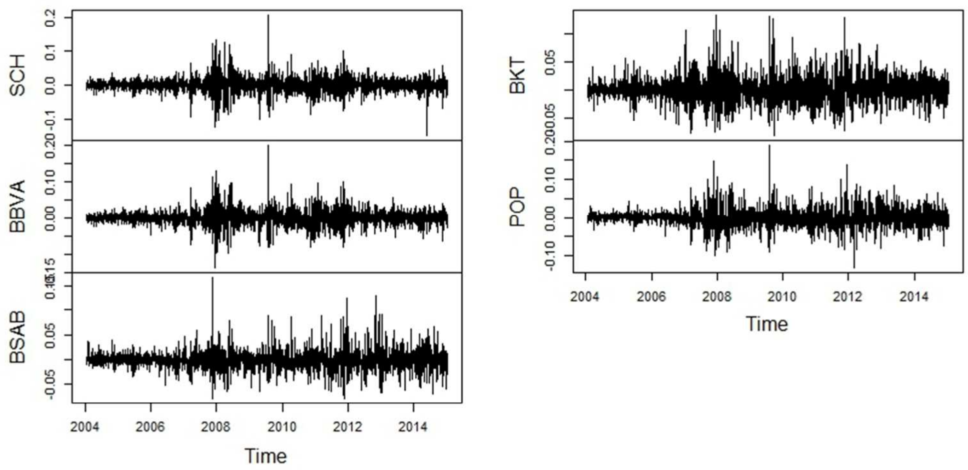

Time plots of daily log return series for Spanish stocks indicating presence of volatility clustering.

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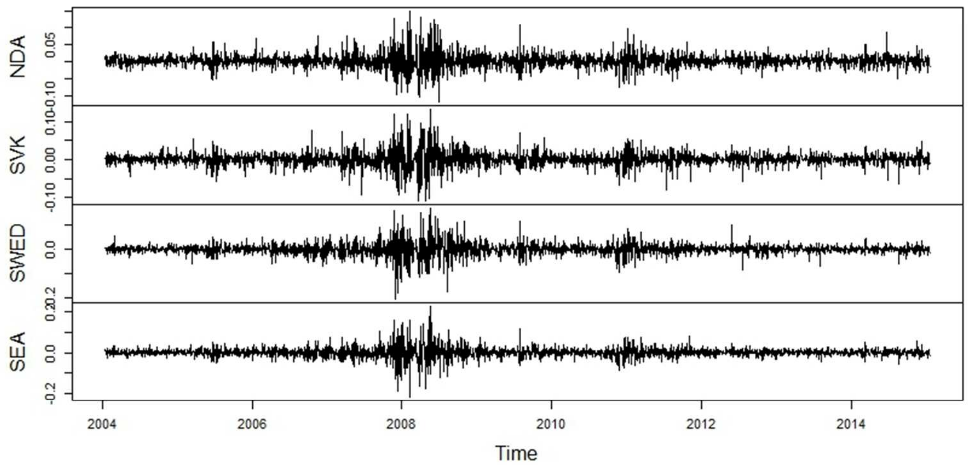

Time plots of daily log return series for Swedish stocks indicating presence of volatility clustering.

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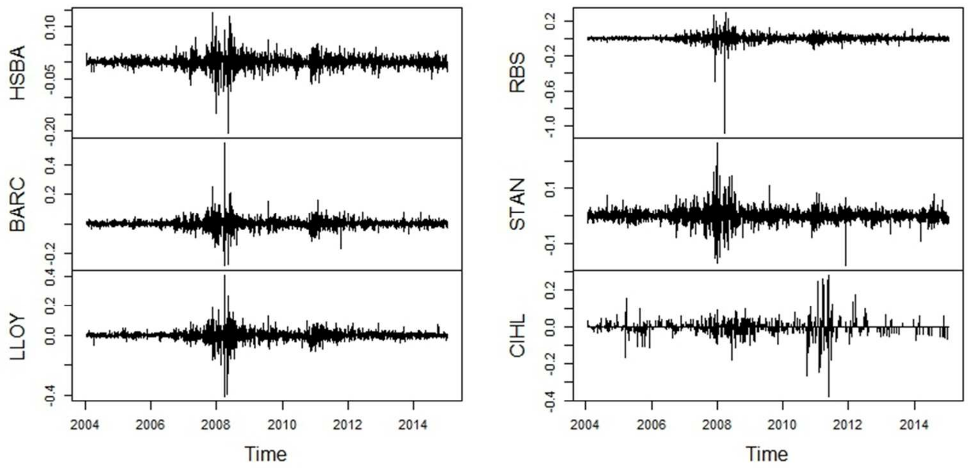

Time plots of daily log return series for UK stocks indicating presence of volatility clustering.

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