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Risk-neutral pricing in a behavioural framework

by

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Abstract

This thesis investigates three issues related to risk-neutral pricing. The first aspect investigated is the effect of discretization and truncation errors on risk-neutral moments, as defined in Bakshi, Kapadia and Madan (2003). It proposes exact solutions for the finite integrals in the volatility, cubic and quartic contracts and compares its accuracy approach with the interpolation-extrapolation approach. It yields more accurate estimates for risk-neutral skewness and kurtosis for those assets which exhibit the *"volatility smirk"*. By contrast, for those assets dominated by the *"forward skew"*, the exact approach outperforms the interpolation-extrapolation approach for skewness only.

The second issue investigated is the skewness preference. It seeks to explain the positive skewness preference through heterogeneous beliefs and overconfidence. An overconfident group longs more skewness in the positively skewed portfolio, over-estimates the value of this portfolio, causes heterogeneous beliefs and yields a positive skewness preference.

The final issue investigated is the relation between risk-neutral kurtosis and returns. The relation can be either positive or concave and cannot be explained bu heterogeneous beliefs and overconfidence. There are important causality effects between skewness and kurtosis and evidence is presented that the relation between kurtosis and returns may not be independent.

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Chapter 1

Introduction

Option pricing is a notoriously challenging topic. The dominant approach adopts a risk-neutral framework. Within this framework, the fair value of an asset today is equal to the discounted expected value of its future payoffs where the relevant discount rate is the risk-free interest rate. Thus all assets have the same expected rate of return in this framework since prices do not depend on risk. There has been much research in the last two decades on risk-neutral moments since researchers have shown that these moments may have the capability to predict returns.

The extant literature employs the seminal Bakshi, Kapadia and Madan (2003) (hereafter BKM) method for calculating risk-neutral moments. The moments are evaluated as functions of three particular integrals but, thus far, nobody has derived closed form solutions for these. Instead, researchers have employed approximation approaches for these integrals which are the volatility, cubic and quaric contracts in a finite domain. These contracts are defined as contigent claims with payoffs equal to future second, third and fourth powers of the log stock returns. They are designed to capture the implied variance, skewness amd kurtosis, respectively, of the underlying asset. The BKM method has the advantage over other approaches that is model free and so is not dependent on any specific option pricing model.

The BKM approach uses approximation methods for evaluating the volatility, cubic and quartic contracts. While these result in pricing errors, the presumption to date is that the interpolation-extrapolation approach makes negligible the effect of these errors. The obvious next question is whether there are exact solutions for these integrals that would result in a more accurate evaluation of the risk-neutral higher moments. Relatedly, the existing literature has produced mixed findings on the ability of skewness to predict future returns and there is no overall explanation for these contradictory results. This suggests the need for further investigation into the relationship between skewness and returns. Finally, there are very few research findings on the relationship between risk-neutral kurtosis and returns. The overall goal of this thesis is to answer these questions and contribute to the extant literature in a number of ways. Before highlighting the main contributions of this thesis, the next section provides some background to the context for the thesis by reviewing the relevant literature.

1.1 Literature

One key assumption in the BKM approach is that continuous strike prices can be defined from zero to infinity which leads to various approximation errors. Existing studies (Jiang and Tian (2005), Chang et al. (2012) among others) argue that if one interpolates between and extrapolates beyond the quoted strike prices, the effect of these errors on implied skewness and volatility will be negligible. However they do not study the effect of these errors on kurtosis to give a complete picture for their effect on the entire risk-neutral distrbution.

Other studies that investigate skewness preferences find that risk-neutral skewness can predict returns. The results are mixed however. Conrad, Dittmar and Ghysels (2013), Bali and Murray (2013), and Bali, Cakici and Whitelaw (2011) find a positive skewness preference. In other words, investors accept lower returns for positively skewed assets. By contrast, Rehman and Vilkov (2012), Xing, Zhang and Zhao (2010), Cremers and Weinbaum (2010), and Stilger, Kostakis and Poon (2017) establish a negative skewness preference which implies that investors accept lower returns for negatively skewed assets.

Despite the fact that the results are mixed and there is no agreed overall explanation for skewness preferences, there is also no separate explanation for the positive or negative skewness preference. Bali and Murray (2013) show that there is not a risk-based explanation for a positive skewness preference. This is intuitively correct since prices do not depend on risk in a risk-neutral world. Stilger et al. (2017) come close to an explanation but not a full one. The negative skewness preference is attributed to the relatively overpriced stocks which exhibit the most negative skewness. However there are unexplained parts. In their Table 2 they find that the top quintile portfolio is overvalued compared to the bottom, but in their Table 8 thay find that the most negatively skewed portfolio is the most overpriced.

In a non-risk-neutral framework linked to over-valuation however, Miller (1977), Morris (1996), Chen et al. (2001), and Viswanathan (2001) argue that, if pessimistic investors are kept out of the market, prices will reflect a more optimistic valuation. They find the bigger the disagreement about a stock's fair value, the more they overvalue the stock which yields lower future returns. These findings indicate that the dispersion in beliefs may cause lower returns.

The most plausible explanation given for over-valuation in a behavioural framework is overconfidence. Overconfidence leads to investors over-estimating their private signals and holding more units of the risky assets which results in over-valuation (De Long, Shleifer, Summers and Waldman (1990), Scheinkman and Xiong (2003), and Nagel (2005)). A similar finding is documented even when investors are risk-neutral (Kyle and Wang (1997) and Wang (1998)).

There is a strand in the risk-neutral literature which connects heterogeneous beliefs to risk-neutral skewness. Buraschi et al. (2006) show that belief differences can affect risk-neutral skewness. Friesen et al. (2012) find that stocks with greater belief differences are more negatively skewed. Finally, to our knowledge there is only one study which examines the capability of risk-neutral kurtosis to predict returns. Only Conrad et al. (2013) find a positive relation between kurotsis and returns. They also establish that, when they control for interactions between volatility, skewness and kurtosis, the evidence for an independent relation between kurtosis and returns is relatively weak.

1.2 Contributions

This thesis makes three original contributions to the literature. The first contribution in Chapter 2 of this thesis is the proposal of exact solutions for the the finite integrals in volatility, cubic and quartic contracts, which avoids the discretization errors (that arise from the absence of exact solutions as shown in the numerical integration literature). It results in a more accurate estimation of risk-neutral skewness and kurtosis for assets dominated by the *"volatility smirk"*. By contrast it yields more accurate estimates for skewness only for other assets that exhibit the *"forward skew"*. The second original contribution of this thesis is that Chapter 3 sheds light on the reasons to what could be the cause of the positive skewness preference. It extends the previous literature by linking skewness preference to heterogeneous beliefs and overconfidence in a behavioural framework. Investors go long skewness more for positively skewed assets which results in overvaluation and yields lower returns. This is consistent with the theoretical models in Barberis and Huang (2008) and Brunnermeier, Gollier and Parker (2007). They show that investors who prefer positively skewed distributions may hold concentrated positions in right skewed assets which will increase the demand for these assets and reduce their expected returns.

The final contribution is in Chapter 4 which extends the work of Conrad et al. (2013) in which they study the relation between risk-neutral kurtosis and returns. Chapter 4 studies this relation and seeks to explain the relation between kurtosis and returns through heterogeneous beliefs and overconfidence. The novel finding is that the relation can be either positive or concave and holds only when one controls for variation in all moments except for skeweness. These relations can not be explained by overconfidence and heterogeneous beliefs. Finally, there are important causality effects and evidence for an independent relation between kurtosis and returns is relatively weak, consistent with Conrad et al. (2013).

1.3 Chapter Preview

The remainder of the thesis is organized as follows. Chapter 2 proposes exact solutions for the finite integrals in volatility, cubic and quaric contracts which avoids the discretization errors. It studies the accuracy of the exact apporach in comparison to the interpolation-extrapolation approach. The conjecture is that the exact approach yields more accurate estimates for skewness and kurtosis (skewness only) when the volatility smirk (forward skew) is present.

Chapter 3 investigates skewness preference when heterogeneous beliefs are present. It seeks to explain the positive skewness preference documented in the literature. The findings support the argument that overconfidence causes heterogeneous beliefs which result in an overvaluation. This may help explain why positively skewed assets yield lower returns.

Chapter 4 investigates the relation between risk-neutral kurtosis and returns following a similar approach as in Chapter 3. The results are different however compared to the ones in Chapter 3 related to heterogeneous beliefs and overconfidence. The relation can be either positive or concave, but cannot be explained by heterogeneous neliefs and overconfidence however. In addition, the findings indicate that there may not be an independent relation between kurtosis and returns.

Finally, Chapter 5 briefly summarizes the key findings of the thesis and offers directions for future research areas.

Chapter 2

An exact approach to valuing risk-neutral moments from option prices

2.1 Introduction

The key participants in derivatives markets are hedgers, speculators and arbitrageurs. Hedgers aim to avoid exposure to risk. Arbitrageurs are looking for risk-less profit by taking advantage of arbitrage opportunities. Speculators take risky positions in the market in order to make profits. While profits could be extremely high, the probability of realising losses is also high as well. Speculators use derivatives in order to realise profits, whereas hedgers use them to minimize risk.

The put-call ratio is a well-known sentiment indicator which is the total daily put volume divided by the daily call volume for an equity. If more calls (puts) are traded than puts (calls), this is an optimistic (pessimistic) indicator. In other words optimists (pessimists) expect the price of the underlying to go up (down). (Bandopadhyaya and Jones (2008), Billingsley and Chance (1998)). As a result when one trades calls (puts) only, she is an optimistic (pessimistic) speculator or hedger, depending on their positions in options, expecting prices to go up (down).

The past two decades have seen a burgeoning interest in utilizing option-implied volatility, skewness, and kurtosis for predicting stock returns. A non-exhaustive list includes Bali and Hovakimian (2009), Conrad, Dittmar and Ghysels (2013), Dennis and Mayhew (2002), Han (2008), Xing, Zhang and Zhao (2010) and the references therein. This research exploits the information content of options for the purpose of stock return valuation. To date, the most widely adopted method is due to Bakshi, Kapadia and Madan (2003) (BKM henceforth) that infers risk-neutral moments from a wide range of out-of-the-money (OTM) traded option prices in a model-free framework. BKM draw upon the theoretical derivation in Bakshi and Madan (2000) which shows that any asset payoff can be spanned and priced using positions in options across a wide range of strike prices. Empirically, the method is model-free and easy to implement. It has become the standard methodology for extracting risk-neutral moments from option prices.[1]

At the center of this literature is the issue of how to extract risk-neutral moments from forward looking options in an accurate and reliable way. This is important since risk-neutral moments incorporate information about expectations and existing studies show that they can predict future market events. Bates (1991) uses S&P 500 futures options up to 1987, in order to investigate whether the market predicted the 1987 crash. He finds that crash was anticipated in the options market two months in advance. Lynch and Panigirtzoglou (2008) use S&P 500 options for the 1985 to 2001 period and conclude that risk-neutral distribution (RND) responds to market events but are not very useful for forecasting them. *"The RND provides exceptional detail about investors expectations."* (Birru and Figlewski (2012, p. 151)).

An important step in the BKM procedure is the evaluation of three contingent claims, namely the volatility contract V, the cubic contract W, and the quartic contract X. These contracts are defined as contingent claims with payoffs equal to the future second, third, and fourth powers of the logarithmic asset returns, respectively. They are expressed in terms of a series of integrals that can

[1] The BKM method is adopted in Bali and Murray (2013), Coakley, Dotsis, Liu and Zhai (2014), Conrad et al. (2013), Han (2008), Rehman and Vilkov (2012) among others.

be evaluated via an approximation approach such as the rectangural, trapezoidal or the Simpson. Existing studies concede that discretization and truncation errors are introduced, since Bakshi et al. (2003) make the assumption of continuous strike prices from 0 to ∞. They argue that, if one interpolates between and extrapolates beyond strike prices, these errors converge to zero (Dennis and Mayhew (2002, 2009), Jiang and Tian (2005), Chang et al. (2012)).

The contribution of this chapter is that it proposes exact solutions for the volatility, cubic and quartic contracts in the Baksi, Kapadia and Madan (2003) framework when they lie in a finite domain. It compares the accuracy of the exact approach with the interpolation-extrapolation which has been shown to be the most accurate hitherto. The conjecture is that the exact approach outperforms the interpolation-extrapolation approach for risk-neutral skewness and kurtosis for those assets which exhibit the *"volatility smirk"* generating a fattter left tail, whereas it yields more accurate estimates for skewness only, for assets dominated by the *"forward skeew"* resulting in a fatter right tail.

To show that, it employs two popular models in our simulation experiments, the Variance-Gamma model by Madan, Carr and Chang (1998) and the stochastic volatility model by Heston (1993). These models have the advantage of providing closed-form expressions for their risk-neutral skewness and kurtosis to use for benchmarks. The results show that the exact approach yields more accurate skewness and kurtosis estimates in the stochastic volatility model. The implication changes slightly for the Variance-Gamma model however, in which the exact approach outperforms the interpolation-extrapolation approach for skewness only.

In an effort to explain and link the results to existing research, I study which model reflects the *"volatility smirk"* or the *"forward skew"*, at least for the chosen set of parameters. In order to do this, I perform a t-test on the null hypothesis of the Black and Scholes (1973) implied volatilities of the right tail being singificantly higher or lower than the left, at their respective moneyness levels. The results provide strong evidence that the Variance-Gamma model exhibits the *"forward skew"* generating a fatter right tail. By contrast, the stochastic volatility model exhibits the *"volatility smirk"* generating a fatter left tail. I conclude that the exact approach yields more accurate estimates for skewness and kurtosis (skewness only) for assets which demonstrate the volatility smirk (forward skew) market.

The remainder of this paper is organized as follows. Section 2.2. outlines the theoretical derivation of the risk-neutral moments in Bakshi et al. (2003), and explains the role of discretization errors in the errors of the risk-neutral moments compared to the truncation errors. In Section 2.3, I develop a method that provides exact solutions for the three contracts when they lie in a finite domain. In Section 2.4 I perform a series of numerical experiments to study the accuracy of the exact approach. Section 2.5 discusses the role of the *"volatility skew"*. Section 2.6 performs robustness tests. Finally, Section 2.7 concludes. Formulae derivations are given in Appendices.

2.2 Theoretical Background

In this section I provide the theoretical background in Bakshi et al. (2003) and explain the role of discretization and truncation errors in the accuracy of risk-neutral moments. A reader who is familiar with the BKM framework could skip section 2.2.1.

2.2.1 The BKM Framework

The fundamental calculus theorem implies that for a twice differentiable function H and any fixed F:

$$
\begin{aligned}
H(S) &= H(F) + 1_{S>F}\int_F^S \frac{dH}{du}du - 1_{S<F}\int_S^F \frac{dH}{du}du \\
&= H(F) + 1_{S>F}\int_F^S \left[\frac{dH}{dF} + \int_F^u \frac{d^2H}{dv^2}dv\right]du - 1_{S<F}\int_S^F \left[\frac{dH}{dF} - \int_u^F \frac{d^2H}{dv^2}dv\right]du
\end{aligned}
$$

Applying Fubini's theorem:

$$
\begin{aligned}
H(S) &= H(F) + \frac{dH}{dF}(S-F) + 1_{S>F}\int_F^S \int_v^S \frac{d^2H}{dv^2} du dv + 1_{S<F}\int_S^F \int_S^v \frac{d^2H}{dv^2} du dv \\
&= H(F) + \frac{dH}{dF}(S-F) + 1_{S>F}\int_F^S \frac{d^2H}{dv^2}(S-v) dv + 1_{S<F}\int_S^F \frac{d^2H}{dv^2}(v-S) dv \\
&= H(F) + \frac{dH}{dF}(S-F) + 1_{S>F}\int_F^\infty \frac{d^2H}{dv^2}(S-v)^+ dv + 1_{S<F}\int_0^F \frac{d^2H}{dv^2}(v-S)^+ dv
\end{aligned}
$$

Setting $F = S_0$ and $v = K$ the payoff function can be spanned in the following way,

$$
H[S] = H_S[S_0] + (S - S_0)H_S[S_0] + \int_{S_0}^{\infty} H_{SS}[K](S-K)^+ dK + \int_0^{S_0} H_{SS}[K](K-S)^+ dK, \quad (2.1)
$$

where $H_S[S_0]$ ($H_{SS}[K]$)) represents the first-order (second-order) derivative of the payoff function H with respect to S evaluated at some point S_0 (strike price), $(S-K)^+ = \max(S-K, 0)$ and $(K-S)^+ = \max(K-S, 0)$. Intuitively, the position in options allows one to purchase the curvature of the payoff function. The equation for the payoff economy is an exact equation and not an approximation.

Taking the τ-period stock return as $R(t,\tau) = \ln[S(t+\tau)] - \ln[S(t)]$, BKM define the volatility, cubic and quartic contracts as $R^2(t,\tau)$, $R^3(t,\tau)$, $R^4(t,\tau)$, respectively. The fair payoffs are defined as $V(t,\tau) \equiv E_t^*\{e^{-r\tau}R^2(t,\tau)\}$, $W(t,\tau) \equiv E_t^*\{e^{-r\tau}R^3(t,\tau)\}$, $X(t,\tau) \equiv E_t^*\{e^{-r\tau}R^4(t,\tau)\}$, respectively, where E_t^* is the expectation operator under the risk-neutral measure.

Applying the martingale pricing operator in equation (2.1),

$$
\begin{aligned}
E_t^*[e^{-r\tau}H[S]] &= [H(S_0) - S_0 H_S(S_0)]e^{-r\tau} + H_S[S_0]S_0 \\
&+ \int_{S_0}^{\infty} H_{SS}[K]C(t,\tau;K)dK + \int_0^{S_0} H_{SS}[K]P(t,\tau;K)dK
\end{aligned}
$$

Using the properties that $C(t,\tau;K) = \int_\Omega e^{-r\tau}(S-K)^+ dK$, $P(t,\tau;K) = \int_\Omega e^{-r\tau}(K-S)^+ dK$ and performing standard differentiation steps for $H(S)$ the prices for the three contracts are given by the following equations under all martingale pricing measures:[2]

$$V(t,\tau) = \int_{S(t)}^{\infty} \frac{2(1-\ln(\frac{K}{S(t)}))}{K^2} C(t,\tau;K)dK + \int_{0}^{S(t)} \frac{2(1+\ln(\frac{S(t)}{K}))}{K^2} P(t,\tau;K)dK,$$

$$\begin{aligned} W(t,\tau) &= \int_{S(t)}^{\infty} \frac{6\ln(\frac{K}{S(t)}) - 3(\ln(\frac{K}{S(t)}))^2}{K^2} C(t,\tau;K)dK \\ &- \int_{0}^{S(t)} \frac{6\ln(\frac{S(t)}{K}) + 3(\ln(\frac{S(t)}{K}))^2}{K^2} P(t,\tau;K)dK, \end{aligned}$$

$$\begin{aligned} X(t,\tau) &= \int_{S(t)}^{\infty} \frac{12(\ln(\frac{K}{S(t)}))^2 - 4(\ln(\frac{K}{S(t)}))^3}{K^2} C(t,\tau;K)dK \\ &+ \int_{0}^{S(t)} \frac{12(\ln(\frac{S(t)}{K}))^2 + 4(\ln(\frac{S(t)}{K}))^3}{K^2} P(t,\tau;K)dK, \end{aligned}$$

where K is the strike price, S_t is the price of the underlying asset at a time t, τ is the time to maturity of the option, and $C(t,\tau;K)$ and $P(t,\tau;K)$ are the prices of OTM European call and put options, respectively.[3] The risk-neutral moments are given by the following equations,

$$\begin{aligned} RNV &= e^{rt}V - \mu^2, \\ RNS &= \frac{e^{rt}(W - 3\mu V) + 2\mu^3}{(e^{rt}V - \mu^2)^{\frac{3}{2}}}, \\ RNK &= \frac{e^{rt}X - 4\mu e^{rt}W + 6e^{rt}\mu^2 V - 3\mu^4}{(e^{rt}V - \mu^2)^2}, \end{aligned}$$

where RNV is the risk-neutral volatility, RNS the risk-neutral skewness, RNK the risk-neutral kurtosis, $\mu = e^{rt} - 1 - \frac{e^{rt}}{2}V - \frac{e^{rt}}{6}W - \frac{e^{rt}}{24}X$, and r is the risk-free rate.

[2]The first fundamental asset pricing theorem states that if a market is arbitrage-free then a risk-neutral measure exists. Moreover, if this market is complete the risk-neutral measure is unique (Cochrane (2001)). Markets are not complete in the real world however.There are studies which support this (Duffie (1987) and Geanakoplos and Mas-Colell (1989)).

[3]One may look at the appendix in Bakshi et al. (2003) for a more detailed derivation of these equations.

The risk-neutral moments thus inferred exhibit obvious advantages. They are model-free and hence there is no need to rely on any particular option pricing model. They are derived from a wide range of OTM call and put options which enjoy better liquidity and information content than options at other moneyness levels (Bollen and Whaley (2004)). As a result, the method is widely adopted in the literature and has become the industry standard for estimating risk-neutral moments from option prices.[4]

2.2.2 The role of discretization errors in the valuation of risk-neutral moments

Bakshi et al. (2003) make the assumption of continuous strike prices from zero to infinity. Actual strike prices are not continuous and not from zero to infinity. As a result, discretization and truncation errors are introduced. Discretization errors arise from the absence of closed-form solutions whereas truncation errrors arise from the fact that strike prices lie in a finite domain.

Existing studies argue that the effect of these errors is negligible in the valuation of risk-neutral moments if one interpolates between and extrapolates beyond strike prices for a relatively large domain of integration.[5] They interpolate (extrapolate) to mitigate the effect of discretization (truncation) errors, following the numerical integration literature. They implicitely assume however that the effect of truncation errors in each tail has the same impact on the valuation of risk-neutral moments in the BKM framework. This is a reasonable assumption to make, since volatility, cubic and quartic contracts are *log* functions which go to $-\infty$ (∞) when strike prices go to zero (∞). One might argue that truncation errors of the right and left tail are offsetting. However these contracts are divided by K^2 which goes to zero (∞) as the strike price goes to ∞ (zero). In other words, the truncation errors in one tail may have a higher impact on the valuation of risk-neutral moments.

There has been a debate about which is the most accurate way of dealing with these errors. The curve fitting method has been shown to be the most effective one (Shimko (1993), Ait Sahalia

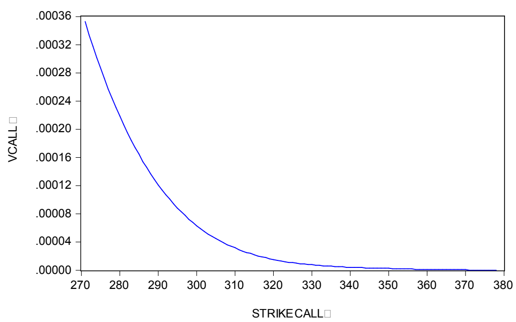
[4]See for example, Bali and Murray (2013), Conrad et al. (2013), and Han (2008)

[5]See Chang et al. (2012), Jiang and Tian (2005) among others

and Lo (1998)). Existing studies using this method, convert the listed option prices to implied volatilities using the Black and Scholes (1973) formula, fit a cubic spline at implied volatilities, extract interpolated implied volatilities using the fitted cubic spline and convert the interpolated implied volatilities to Black and Scholes prices. Then they extrapolate by assuming that implied volatility is flat beyond the minimum and maximum strike price and equal to the implied volatility at the minimum (maximum) for those options with strike prices lower (higher) than the minimum (maximum) strike price. Finally, they evaluate risk-neutral moments using these interpolated-extrapolated Black and Scholes option prices.

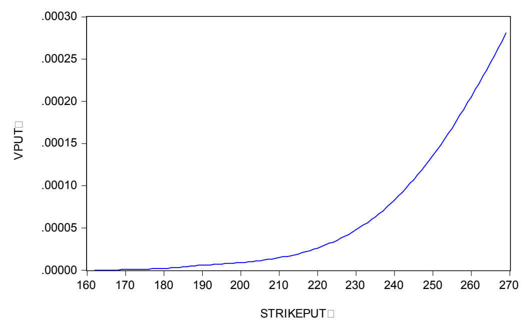
When one employs the extrapolation approach in the BKM framework, they multiply the extrapolated Black and Scholes option prices with the respective fraction in each integral for each contract. There is a negative relation between the window of extrapolation and the Black and Scholes option price. In other words, deep OTM Black and Scholes option prices, which have very low values, are multiplied with the respective fraction in each integral for each BKM contract. This may result in very low values, close to zero, for the extrapolated BKM contracts. Since existing studies show that the interpolation-extrapolation approach yields zero errors in valuing risk-neutral volatility and skewness, the interpolation approach which tackles discretization errors, is enough to yield accurate estimates. To study whether the extrapolation aporoach adds almost zero contracts in the BKM framework, at least for short maturity options, I plot each interpolated-extrapolated volatility, cubic and quartic contract along the strike price dimension. Each contract is evaluated as the interpolated-extrapolated Black and Scholes price at the respective strike price, multiplied with the fraction in the respective BKM contract. I follow Jiang and Tian (2005) and choose the spot price to be 270 and a risk free rate 5%. I assume initially that options follow the Heston (1993) model with a 3-month time to expiry. Call (put) strike prices are 275 to 310 with a \$5 increment and 325 and 350 (200, 220 and 225 to 265 with a \$5 increment). I use $\kappa = 2$, $\theta = 0.04$ $\rho = -0.5$ and instantaneous variance equal to 0.04 as in Chang et al. (2012). Then I apply the interpolation-extrpolation approach at these options and a domain of integration $[162, 378]$ for strike prices is chosen with \$1 increment. The graph is in Figure 2.1.

Figure 2.1: The volatility contract V, the cubic contract W, and the quartic contract X from OTM options

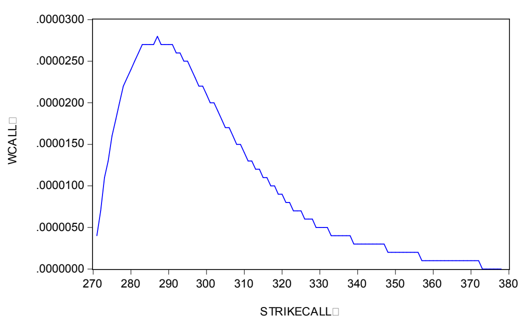


(a)

V from OTM calls

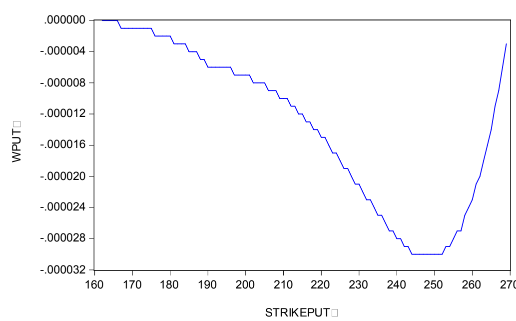


(b) V from OTM puts

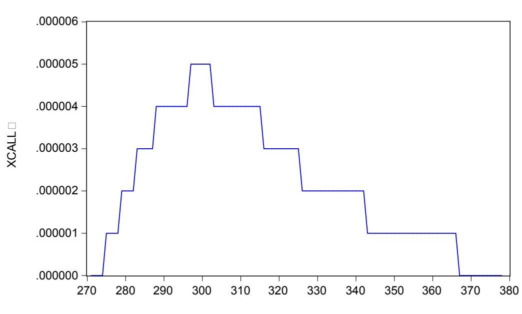


(c)

W from OTM calls

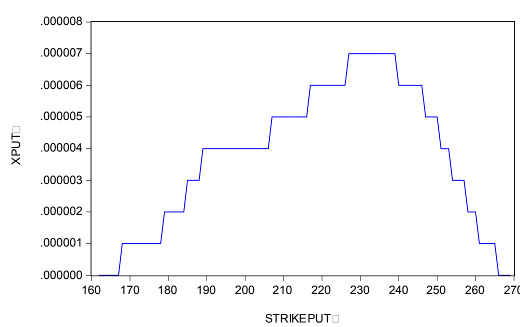


(d) W from OTM puts



(e)

X from OTM calls



(f)

X from OTM puts

First, Figure 2.1 shows that the graph of each call contract is symmetric to the graph of the put contract, with respect to the spot price of the underlying stock. In other words call and put contracts decrease at the same speed as the OTM moneyness increases. As a result truncation errors of the right tail have the same impact on the valuation on the risk neutral moments with the truncation errors of the left tail.

Second, there is a negative relation between the value of contracts and the strike prices. The aim of the extrapolation approach is to generate tails consistent with the empirical framework. This is not achieved however and contradicts empirical findings such as the volatility smile and fatter tails than the normal distrbution. The volatility and quartic contract which aim to capture volatility and kurtosis respectively go to zero for deeper OTM options, instead of taking higher values to capture fat tails and the volatility smile. After a certain strike price the contracts take zero values. As a result, discretization errors may be the main reason behind the mispricing in the valuation of risk-neutral moments. This highlights the need for a method which has no discretization errors that could result in a more accurate estimation for risk-neutral moments despite the fact that the approximation errors for the volatility, cubic and quartic contracts can be small.[6]

2.3 An exact approach

This section outlines a new exact approach for calculating the risk-neutral moments from OTM European option prices by providing exact solutions for the integrals in the volatility, cubic and quartic contracts. Following Bakshi et al. (2003), let $(\Omega, F_{t,t\in[o,\tau]}, \mu)$ denote a filtered probability space, where Ω is the set of all possible realizations of the stochastic economy between time 0 and τ, F_t is a filtered σ-algebra and μ is a risk-neutral measure. The price of an option at time t can

[6] The numerical integration literature shows that when one employs an approximation approach to evaluate an integral of a continuous and differentiable function, errors will be small. The volatility, cubic and quartic contracts are log functions which are continuous and differentiable. As a result the errors will be small. See Atkinson (1989) and Kharab and Guenther (2006) among others.

be expressed as,

$$P(t, S(t)) = \frac{1}{D(t)} E[D(\tau)P(\tau, S(\tau))|F_t] \tag{2.2}$$

where τ is the maturity date of the option, E is the expectation operator and D is the discount factor which follows the process $dD(t) = -r(t)D(t)$, P is the future payoff, and r is a stochastic interest rate process. Equation (2.2) states that the option price at time t is the discounted expectation of the option price at the maturity date when the expectation is conditional on a σ-algebra at a specific time t. In other words, when one wants to compute the European option price at a time prior to the maturity date, one does so at a specific time t so that the interest rate and the discount factor can be determined. The price of the option is subsequently independent of the discount factor. Furthermore, since the option is European style and cannot be exercised except on the maturity date, the time to maturity can also be determined. Hence, the only variable in the integral is the term dK, which means one is integrating over a range of different strike prices K. All other variables can be considered as constants and taken outside the integral.

First, I calculate the risk-neutral moments at a specific time t for a stock which has N OTM European call options written on it with N different strike prices $(K_1, K_2, ..., K_N)$. The stock price is $S(t)$ at time t. As all options are OTM call options, we have $S(t) < K_1 < K_2 < ... < K_N$. We have,

$$\begin{aligned} V_c &= \int_{S(t)}^{K_1} 2C_1 \frac{(1 - \ln(\frac{K}{S(t)}))}{K^2} dK + \int_{K_1}^{K_2} 2C_2 \frac{(1 - \ln(\frac{K}{S(t)}))}{K^2} dK + \cdots \\ &+ \int_{K_{N-1}}^{K_N} 2C_N \frac{(1 - \ln(\frac{K}{S(t)}))}{K^2} dK, \end{aligned} \tag{2.3}$$

where C_i denotes the price of the ith OTM call option. Given that at a specific time t, one can observe market option prices and their time to maturity, I set $S(t) = S_0$ for all t until the option maturity date. I can then take these two variables outside the integrals as known variables and directly integrate over a wide range of strike prices as follows,

$$\begin{aligned} V_c &= 2C_1 \int_{S_0}^{K_1} \frac{(1-\ln(\frac{K}{S_0}))}{K^2} dK + 2C_2 \int_{K_1}^{K_2} \frac{(1-\ln(\frac{K}{S_0}))}{K^2} dK + \cdots \\ &+ 2C_N \int_{K_{N-1}}^{K_N} \frac{(1-\ln(\frac{K}{S_0}))}{K^2} dK. \end{aligned} \tag{2.4}$$

In equation (2.4), all integrals have exact solutions. For example, the first integral on the RHS can be written as,

$$\int_{S_0}^{K_1} \frac{(1-\ln(\frac{K}{S_0}))}{K^2} dK = \frac{\ln(\frac{K}{S_0})}{K} \Big|_{S_0}^{K_1} . \tag{2.5}$$

Hence, equation (2.4) has the following exact solution,

$$V_c = 2C_1 \frac{\ln(\frac{K}{S_0})}{K} \Big|_{S_0}^{K_1} + 2C_2 \frac{\ln(\frac{K}{S_0})}{K} \Big|_{K_1}^{K_2} + \cdots + 2C_N \frac{\ln(\frac{K}{S_0})}{K} \Big|_{K_{N-1}}^{K_N} . \tag{2.6}$$

Similarly, the first integral for the cubic contract W and the quartic contract X can be written as,

$$\int_{S_0}^{K_1} \frac{6\ln(\frac{K}{S_0}) - 3(\ln(\frac{K}{S_0}))^2}{K^2} dK = \frac{3(\ln(\frac{K}{S_0}))^2}{K} \Big|_{S_0}^{K_1}, \tag{2.7}$$

$$\int_{S_0}^{K_1} \frac{12(\ln(\frac{K}{S_0}))^2 - 4(\ln(\frac{K}{S_0}))^3}{K^2} dK = \frac{4(\ln(\frac{K}{S_0}))^3}{K} \Big|_{S_0}^{K_1} . \tag{2.8}$$

Hence, the cubic and quartic contracts also have exact solutions,

$$W_c = 3C_1 \cdot \frac{(\ln(\frac{K}{S_0}))^2}{K} \Big|_{S_0}^{K_1} + 3C_2 \cdot \frac{(\ln(\frac{K}{S_0}))^2}{K} \Big|_{K_1}^{K_2} + \cdots + 3C_N \frac{(\ln(\frac{K}{S_0}))^2}{K} \Big|_{K_{N-1}}^{K_N}, \tag{2.9}$$

$$X_c = 4C_1 \cdot \frac{(\ln(\frac{K}{S_0}))^3}{K} \Big|_{S_0}^{K_1} + 4C_2 \frac{(\ln(\frac{K}{S_0}))^3}{K} \Big|_{K_1}^{K_2} + \cdots + 4C_N \cdot \frac{(\ln(\frac{K}{S_0}))^3}{K} \Big|_{K_{N-1}}^{K_N} . \tag{2.10}$$

Analytical solutions to equations (2.5), (2.7), and (2.8) are in Appendix B.

Likewise, for M OTM put options written on a stock, one can sort the price of the underlying stock and the strike prices of the OTM put options in decreasing order $S_0 > K_M > ... > K_2 > K_1$. Based on the same assumptions, the volatility contract V, the cubic contract W, and the quartic contract X can be expressed as follows,

$$V_p = -(2P_M \frac{\ln(\frac{S_0}{K})}{K} |_{K_M}^{S_0} + 2P_{M-1} \frac{\ln(\frac{S_0}{K})}{K} |_{K_{M-1}}^{K_M} + \cdots + 2P_1 \frac{\ln(\frac{S_0}{K})}{K} |_{K_1}^{K_2}), \tag{2.11}$$

$$W_p = 3P_M \frac{(\ln(\frac{S_0}{K}))^2}{K} |_{K_M}^{S_0} + 3P_{M-1} \frac{(\ln(\frac{S_0}{K}))^2}{K} |_{K_{M-1}}^{K_M} + ... + 3P_1 \frac{(\ln(\frac{S_0}{K}))^2}{K} |_{K_1}^{K_2}, \tag{2.12}$$

$$X_p = -(4P_M \frac{(\ln(\frac{S_0}{K}))^3}{K} |_{K_M}^{S_0} + 4P_{M-1} \frac{(\ln(\frac{S_0}{K}))^3}{K} |_{K_{M-1}}^{K_M} + ... + 4P_1 \cdot \frac{(\ln(\frac{S_0}{K}))^3}{K} |_{K_1}^{K_2}). \tag{2.13}$$

The volatility, cubic, and quartic contracts are the sum of the respective contracts from OTM calls and OTM puts,

$$V = V_c + V_p, \tag{2.14}$$

$$W = W_c + W_p, \tag{2.15}$$

$$X = X_c + X_p. \tag{2.16}$$

To summarize, this novel approach provides exact solutions for the integrals involved in the computation of the volatility, cubic, and quartic contracts, which in turn form the basis for evaluating risk-neutral moments and has two advantages over existing approaches. First it has no discretization errors since it provides exact solutions for the integrals in the volatiltiy, cubic and quartic contracts (See Atkinson (1989)). Second there is no need for interpolation to mitigate the effect of discretization errors which results in less noise incorporated in the risk-neutral distribution. When one fits a curve, it must pass through every knot point which makes the curve to encapsulate all noise in the risk-neutral distribution (Figlewski (2010)).

2.4 Numerical experiments

I perform numerical experiments to compare the estimation accuracy between the exact approach and the interpolated-extrapolated rectangular method used in Bakshi et al. (2003). In particular, I specify parameter values and generate option prices assuming that they follow the Variance-Gamma

(VG) model of Madan, Carr and Chang (1998) and the stochastic volatility model by Heston (1993).

2.4.1 The Variance-Gamma process

The VG process is obtained by evaluating Brownian motion with drift at a random time given by a gamma process.

$$b(t;\theta,\sigma) = \theta t + \sigma W(t) \tag{2.17}$$

where W(t) is a standard Brownian motion. The process $b(t;\theta,\sigma)$ is Brownian motion with drift θ and volatility σ. Returns follow a normal distribution conditional on the time which has a gamma density.

The VG option pricing model of Madan et al. (1998), which nests the famed Black and Scholes (1973) model as a special case, is suitable for the purposes of this study because it offers closed-form solutions for the risk-neutral moments I want to use as a benchmark.[7] Note that when I assume that option prices follow this model, I use this as a mapping tool between option prices and risk-neutral moments. It does not necessarily indicate that option prices follow this model.

Madan et al. (1998) derives closed-form solution for the risk-neutral skewness and kurtosis as follows,

$$\text{Risk-neutral skewness} = (2\theta^3\nu^2 + 3\sigma^2\theta\nu)t \tag{2.18}$$

$$\text{Risk-neutral kurtosis} = (3\sigma^4\nu + 12\sigma^2\theta^2\nu^2 + 6\theta^4\nu^3)t + (3\sigma^4 + 6\sigma^2\theta^2\nu + 3\theta^4\nu^2)t^2 \tag{2.19}$$

I compute a range of call option prices following the option pricing model given by equations (25)-(28) in Madan et al. (1998). I use the global adaptive quadrature rule to evaluate the integral in

[7] In section 4.2 the importance for the choice of a benchmark is discussed since one may draw different conclusions on whether interpolated-extrapolated errors converge to zero or not.

their equation (25) because it has been shown to perform well for a general spectrum of functions that behave *"well"* and *"bad"* (Press et al. (2007)). I derive put option prices via the put-call parity. Equation (1) states that the economy consists of a bond, a stock, OTM and at-the-money (ATM) call and put options. This is also stated explicitely in Appendix 1 in Carr and Madan (2000). An ATM option is the one in which the strike and the price of the underlying are the same. As a result, ATM options do not play any role in the payoff function of the economy because the volatiltiy, cubic and quartic contracts take zero values. When one uses the put-call parity to evaluate the price of a call or a put option, one of the assumptions is that these options have the same strike price. In other words, in order to evaluate the price of an OTM call (put) option, an in-the-money (ITM) put (call) option must be utilized. To be consistent with the assumptions in the BKM framework, I assume that ITM options exist in the economy, they do not play any role in the payoff function however.

Numerical experiment results

I follow Jiang and Tian (2005) on the choice of the price of the underlying stock and the strike price structure. They use as a prototype the strike price structure of of SPX on September 23, 1988. I choose the price of stock to be 270 and call (put) strike prices to be 275 to 310 with a $5 increment and 325 and 350 (200, 220 and 225 to 265 with a $5 increment). The sampling date is not important since I do not use observed option prices in the simulation study as argued in Jiang and Tian (2005).

I study the accuracy of the exact approach in comparison to the interpolation-extrapolation approach. I evaluate the volatility, cubic and quartic contracts using a rectangural approximation as in BKM and Bali and Murray (2013).[8] I follow the Jiang and Tian (2005) approach in employing the interpolation-extrapolation method but I slightly depart from them because it has one drawback related to volatility smile, a phenomenon documented in an empirical setting. When they

[8] I also evaluate these contracts using the Simpson method as in Stilger et al. (2017). The results are qualitatively the same and reported in Appendix B.

extrapolate, they assume that implied volatility is flat beyond the minimum and maximum strike prices. However when the volatility smile is present, deeper OTM call and put options exhibit higher volatility than less deep OTM options. As a result, the extrapolation method will yield an under-estimation for the implied volatility beyond the range in which strike prices lie. In order to alleviate the effect of a flat implied volatility and make the simulation more realistic, I extrapolate beyond the minimum and maximum strike price using the fitted cubic spline between the minimum and maximum strike price. The use of cubic spline provides an exact fit to known implied volatilities (Jackwerth (1999)). This method of extrapolation will conform closer to the volatility smile beyond the minimum and maximum strike price than a flat implied volatility and generate fatter tails.[9]

Existing studies which employ the interpolation-extrapolation approach in the BKM framework, implicitely assume that there is no mispricing in options since their simulation experiments are performed in the absence of noise. In other words there are sources of errors that were not incorporated in previous simulation experiments. Therefore I extend these studies by adding white gaussian noise in option prices in both approaches. By doing so, I seek to reflect the existence in real data of a bid-ask spread and other possible sources of error in the recorded prices as in Ait Sahalia and Lo (1998). Noise is generated once.[10] Once again, in the name of a realistic simulation, I add noise with a different structure among calls and puts to reflect different mispricing. Doran, Fondor and Jiang (2013) find that mispricing among put options is limited compared to call options. A similar finding is also presented in Bali and Murray (2013). When they construct delta-vega neutral portfolios in order to isolate the effect of skewness on valuation, the delta from call options deviates from their target and they attribute this to a mispricing in the right tail of the risk-neutral distribution. Therefore lower noise standard deviation is chosen for puts, since there is evidence that they are less mispriced. Noise follows $N(0, 0.125)$ ($N(0, 0.0625)$) for call option

[9] I also perform the same simulation experiment assuming that implied volatility is flat beyond the minimum and maximum strike price and the results remain qualitatively the same. They also support the argument that my way of extrapolation generates fatter tails. They are reported in the robustness section.

[10] In Appendix C I perform the same simulation experiment in the absence of noise and the results remain qualitatively the same.

prices higher (less) than \$3 as in Ait-Sahalia and Lo (1998). With respect to put options, noise follows $N(0, 0.1)$ ($N(0, 0.05)$) for prices higher (less) than \$3.

According to the Central Limit Theorem risk-neutral distribution should converge to the normal distribution as the time horizon increases. There are studies however that support the opposite. Ait-Sahalia and Lo (1998), Carr and Wu (2003) and Neumman and Skiadopoulos (2013) find that skewness and kurtosis do not converge to zero and three respectively, as the horizon increases. For example, Neumman and Skiadopoulos (2013) report in their Table 1 values of skewness and kurtosis higher in magnitude as the horizon increases. Similar findings are also present in Borochin, Chang and Wu (2017). In an effort to reconcile the mixed results on skewness preference, they find that skewness becomes more negative as time to maturity increases and the relation between skewness and returns is different across the horizon dimension. Jiang and Tian (2005) find that there is a positive relation between volatility errors and time horizon. Dennis and Mayhew (2002, 2009) and Chang et al. (2012) who evaluate risk-neutral moments in the BKM framework, work with one maturity date. Dennis and Mayhew use 1-month options, whereas Chang et al use 3-month options. Motivated by these studies which highlight the impact of the time horizon on risk-neutral moments and their potential effect on their relation with returns, this chapter extends existing research in the BKM framework and incorporates three sets of maturity dates, 3, 6 and 9 months.

In order to study the accuracy of each apporach, errors have to be evaluated. I evaluate the error for each moment as the difference between the risk-neutral moment value returned by each method and the respective closed form solution, $moment - closed\ form\ solution$ as in Jiang and Tian (2005). Chang et al. (2012) show in their Figure 8 that when one chooses a large domain of integration to apply the interpolation-extrapolation approach, a \$5 increment among strike prices is enough to yield almost zero skewness and volatility error. Dennis and Mayhew (2002) who simply interpolate-extrapolate at strike prices and do not fit a cubic spline at implied volatilities, use a smaller domain of integration than Chang et al. (2012). They find that an \$1 increment yields almost zero skewness error. I choose \$1 increment among strike prices for the interpolation-extrapolation approach in all simulation experiments, to cover both cases. I also choose the interpolated-extrapolated domain of integration to be $[162, 378]$ as in Table 1 in Jiang and Tian (2005).[11]

In the Variance-Gamma model, the parameter σ is the volatility of the underlying Brownian motion, ν is the variance rate of the gamma time change, and θ controls the drift in the Brownian motion. I choose $\sigma = 0.25$ as in Jiang and Tian (2005), $\theta = -0.14$ and $\nu = 0.16$ the average values of the weekly empirical estimates in Table 2 in Madan et al. (1998). I use 3, 6 and 9-month options in the simulation study, therefore I use $t = 3/12$, $t = 6/12$ and $t = 9/12$ respectively. For these values, the closed form solutions for skewness and kurtosis are 0 except for the 9-month kurtosis which is 0.01. One might reasonably argue that the choice of the Variance-Gamma model is not so realistic since the risk-neutral distribution in an empirical setting is higher than 3 (Conrad et al. (2013)). I also use the Heston (1993) option pricing model in section 2.4.2 which exhibits kurtosis higher than 3. Table 2.1 reports the results of this simulation experiment.

The results show that the exact approach outperforms the interpolation-extrapolation approach for skewness. The exact approach yields smaller errors than the interpolation-extrapolation approach for all maturity dates. The opposite holds for kurtosis however. The interpolation-extrapolation approach performs better in this case.

Consistent with the theoretical framework of the Variance-Gamma model which makes the assumption that the Central Limit Theorem holds, both approaches imply that the risk-neutral distribution converges to the normal distribution as time to maturity increases. Skewness and kurtosis go to zero and three respectively as one is moving from a 3-month to a 9-month distribution. This contradicts the findings in Carr and Wu (2003) however. It is not suprising though since they develop a model which deliberately violates the Central Limit Theorem assumptions. As expected, the findings also contradict the empirical findings in Neumman and Skiadopoulos (2013) who find that skewness and kurtosis do not converge to zero and three respectively, as the horizon increases. Forresi and Wu (2005) find that volatility smirk becomes steeper as time to maturity increases in an empirical setting. As Carr and Wu (2003) point out, the implied volatility smirk obtained from

[11]Note that this domain of integration is larger than the one Dennis and Mayhew (2002, 2009) and Chang et al. (2012) use.

Table 2.1. Risk-neutral moment values and errors for the Variance-Gamma process

Table 2.1 reports the risk-neutral skewness (RNS) and kurtosis (RNK) values and errors of the exact (EX) and the interpolation-extrapolation (IE) approach when 3, 6 and 9-month options follow the Variance-Gamma process. The error for each approach is evaluated as the difference between the moment value and the respective closed form solution. The price of stock is chosen to be 270 and structural parameters $\sigma = 0.25$, $\theta = -0.14$, $\nu = 0.16$. Call (put) strike prices are 275 to 310 with a \$5 increment and 325 and 350 (200, 220 and 225 to 265 with a \$5 increment) for the exact approach. The domain of integration is $[162, 378]$ and \$1 increment among strike prices is chosen for the interpolation-extrapolation approach. White gaussian noise is generated once and added in option prices by imposing the restriction that it will yield positive prices for the extremely deep interpolated-extrapolated OTM 3-month put options. Noise follows $N(0, 0.125)$ ($N(0, 0.0625)$) for call option prices higher (less) than \$3. With respect to put options, noise follows $N(0, 0.1)$ ($N(0, 0.05)$) for prices higher (less) than \$3. Panel A reports the errors and values of skewness and kurtosis for a 3-month risk-neutral distribution. Panel B reports the errors and values of skewness and kurtosis for a 6-month risk-neutral distribution. Panel C reports the errors and values of skewness and kurtosis for a 9-month risk-neutral distribution.

	RNKIE	RNSIE	RNKEX	RNSEX
Panel A. 3-month risk-neutral distribution				
Moment Error	4.56	0.48	5.21	0.2
Moment Value	4.56	0.48	5.21	0.2
Panel B. 6-month risk-neutral distribution				
Moment Error	3.19	0.21	3.84	0.06
Moment Value	3.19	0.21	3.84	0.06
Panel C. 9-month risk-neutral distribution				
Moment Error	2.67	0.05	3.14	-0.04
Moment Value	2.68	0.05	3.15	-0.04

the Variance-Gamma model flattens out very quickly as maturity increases, once again due to the validity of the Central Limit Theorem in this model.

The results are also consistent with Konikov and Madan (2002). They show that the absolute value of skewness decreases like the reciprocal of the square-root of maturity, while kurtosis decreases with the reciprocal of maturity for the Variance-Gamma model. For example, the 6 and 9-month exact skewness are closer to each other than their respective values of kurtosis. A similar pattern is observed for the interpolation-extrapolation approach.

Suprisingly both approaches yield positive skewness except for the exact in the case of a 9-month distribution. Theta controls for skewness. We choose negative theta which should yield negative skewness. The risk-neutral distribution exhibits positive skewness instead. Evidence is provided in section 5 that this may be due to the so called *"forward skew"* in which the right tail is fatter than the left tail of the risk-neutral distribution.

Table 1 shows that the exact approach yields more accurate estimates for skewness only. The Variance-Gamma model makes the unrealistic assumption however, that the Central Limit Theorem holds. An other unrealistic assumption is that implied volatility is constant. The aim of this study is to study the acuracy of the exact approach compared to the interpolation-extrapolation approach in a realistic environment. Put differently, which method will perform better in an empirical setting. As a result I study further the accuracy of both approaches using the stochastic volatiltiy model by Heston (1993). At least this model can be considered more realistic than the Variance-Gamma since it makes the assumption that volatility is not constant.

2.4.2 The Heston (1993) model

This section studies the accuracy of the exact compared to the interpolation-extrapolation approach when options follow the Heston (1993) stochastic volatility model. First it highlights the importance of a benchmark and then discusses the simulation results.

The choice of a benchmark

Das and Sundaram (1999) study the term structure of the volatilty smile and smirk using two models. The first model they use is the one in which jump is introduced in returns. For the second, they allow volatility to be stochastic. Their conjecture is that both models exhibit some pattern but it is not in line with what is observed in the real world. Stochastic volatility models do a better job than jumps however. Therefore I turn my attention to the Heston (1993) model which allows the volatility of returns to evolve stochastically. The returns $x_t = log(\frac{S_t}{S_0})$ are given in the following stochastic form,

$$dx_t = \alpha dt + \sqrt{V_t} dW_t \tag{2.20}$$

$$dV_t = \kappa(\theta - V_t)dt + \eta\sqrt{V_t} dB_t \tag{2.21}$$

$$dB_t dW_t = \rho dt \tag{2.22}$$

where,

α is the current volatility to its long term mean

κ is the rate at which V_t reverts to θ

θ is the long variance

η is the volatility of volatility

dW_t and dB_t are two Wiener processes

Das and Sundaram (1999) in order to study the term structures of the stochastic volatilty models and the ones with jump in returns, propose closed form solutions for skewness and kurtosis. Skewness and kurtosis of log returns for the stochastic volatility model conditional upon a time horizon $[t, t + h]$ are given as follows,

$$
\begin{aligned}
Skewness &= \left(\frac{3\eta\rho e^{0.5\kappa h}}{\sqrt{\kappa}}\right)\cdot\left(\frac{\theta(2-2e^{\kappa h}+\kappa h+\kappa h e^{\kappa h})-V_t(1+\kappa h-e^{\kappa h})}{(\theta(1-e^{\kappa h}+\kappa h e^{\kappa h})+V_t[e^{\kappa h}-1])^{3/2}}\right) &(2.23)\\
Kurtosis &= 3\left[1+\eta^2\left(\frac{\theta A_1-V_t A_2}{B}\right)\right] &(2.24)
\end{aligned}
$$

where, letting $y = \kappa h$,

$A_1 = [1 + 4e^y - 5e^{2y} + 4ye^y + 2ye^{2y}] + 4\rho^2[6e^y - 6e^{2y} + 4ye^y + 2ye^{2y} + y^2e^y]$,

$A_2 = 2[1 - e^{2y} + 2ye^y] + 4\rho^2[2e^y - 2e^{2y} - 2ye^y + y^2e^y]$,

$B = 2\kappa[\theta(1 - e^y + ye^y) + V_t(e^y - 1)]^2$.

Chang et al. (2012) in order to study equity risk measured by beta, extend existing studies by extracting betas from options. Their approach has an advantage over other studies since it is extracted from options which are forward looking and have the ability to predict future market events (Bates (1991), Birru and Figlewski (2012)). They find that option-implied betas that were extracted for equities with liquid options often outperform the historical market beta in predicting the future beta in the following period. The method they use to evaluate option implied volatilty and skewness is the interpolation-extrapolation approach in the BKM framework. They study the accuracy of the interpolation-extrapolation approach using a simulation experiment when options follow the Heston (1993) model. In order to define a benchmark for their 3-month skewness, they perform a Monte Carlo experiment and simulate 250,000 price paths to calculate log-returns in their Appendix B. Their benchmark value for skewness is -0.46 and show that the interpolated-extrapolated error is zero. Using the same parameters as in Chang et al. (2012), evaluating skewness using the closed form solution Das and Sundaram (1999) propose however, skewness is -0.36. In other words the interpolated-extrapolated errors do not converge to zero for a different choice of benchmark.

A natural question rises then is which benchmark is more accurate. I argue that the Das and

Sundaram (1999) closed form expressions are more accurate, relying on numerical integration literature. When the accuracy of an approximation approach is tested, their benchmark is the exact value of the integral (See Kharab and Guenther (2006), ch. 10).

Simulation experiment results

This section studies the accuracy of the exact approach compared to the interpolation-extrapolation approach when options follow the Heston (1993) process. I use the same price of the underlying, strike price and noise structure and volatility as in Table 2.1. I use $\kappa = 2$, $\theta = 0.04$ $\rho = -0.5$ and instantaneous variance equal to 0.04 as in Chang et al. (2012). To define a benchmark for skewness and kurtosis, I use equations (23) and (24) respectively. Table 2.2 reports the results of this simulation experiment.

In a world which allows volatility to evolve stochastically, the exact approach yields more accurate estimates for skewness and kurtosis, especially for shorter maturity distributions.

Kurtosis converges to three at a slower rate compared to the rate of convergence in the Variance-Gamma model. This is consistent with Backus, Foresi, and Wu (1997) and Das and Sundaram (1999). Das and Sundaram (1999) prove this through their proposition 4. Kurtosis is a hump-shaped function of time to maturity, increasing from zero to a maximum and then decreasing back to three again. Our results indicate that the exact approach complies with the proposition 4 because it exhibits a hump-shaped behaviour. By contrast the interpolated-extrapolated kurtosis fails to account for this hump-shaped behaviour since it follows a monotone pattern. As time to maturity increases, kurtosis decreases, highlighting possibly the effect of discretization errors.

By contrast skewness does not follow a hump-shaped pattern with respect to time to maturity. Both approaches yield more negative values of skewness as the time horizon increases. In a working version of their 1999 published study, Das and Sundaram (1998) find the values of time to maturity which maximize the absolute values of conditional skewness and kurtosis. They show in their Tables 2 and 3 that the time to maturity which maximises skewness and kurtosis is not the same. It is

Table 2.2. Risk-neutral moment values and errors for the Heston (1993) model

Table 2.2 reports the risk-neutral skewness (RNS) and kurtosis (RNK) values and errors of the exact (EX) and the interpolation-extrapolation (IE) approach when 3, 6 and 9-month options follow the Heston (1993) stochastic volatility model. The error for each approach is evaluated as the difference between the moment value and the respective closed form solution. The price of stock is chosen to be 270 and structural parameters $\sigma_v = 0.25$, $\theta = 0.04$, $k = 2$, $\rho = -0.5$ and $V = 0.04$. Call (put) strike prices are 275 to 310 with a \$5 increment and 325 and 350 (200, 220 and 225 to 265 with a \$5 increment) for the exact approach. The domain of integration is $[162, 378]$ and \$1 increment among strike prices is chosen for the interpolation-extrapolation approach. White gaussian noise is generated once and added in option prices by imposing the restriction that it will yield positive prices for the extremely deep interpolated-extrapolated OTM 3-month put options. Noise follows $N(0, 0.125)$ ($N(0, 0.0625)$) for call option prices higher (less) than \$3. With respect to put options, noise follows $N(0, 0.1)$ ($N(0, 0.05)$) for prices higher (less) than \$3. Panel A reports the errors and values of skewness and kurtosis for a 3-month risk-neutral distribution. Panel B reports the errors and values of skewness and kurtosis for a 6-month risk-neutral distribution. Panel C reports the errors and values of skewness and kurtosis for a 9-month risk-neutral distribution.

	RNKIE	RNSIE	RNKEX	RNSEX
Panel A. 3-month risk-neutral distribution				
Moment Error	1.05	-0.03	-0.67	0.01
Moment Value	4.47	-0.42	2.75	-0.38
Panel B. 6-month risk-neutral distribution				
Moment Error	0.32	-0.04	0.28	-0.02
Moment Value	3.95	-0.52	3.91	-0.5
Panel C. 9-month risk-neutral distribution				
Moment Error	-0.03	-0.04	-0.02	-0.02
Moment Value	3.71	-0.56	3.71	-0.54

not suprising then, that a different pattern is followed between skewness and kurtosis accross the maturities we choose. The time to maturity that maximizes skewness may be in longer maturities. It is worth noting that the 3-month interpolated-extrapolated value of skewness is close to the value in Chang et al. (2012). They find a value of -0.46 whereas I find a value of -0.42. There are slight differences between mine and their approach however. They use the trapezoidal approximation to evaluate the volatility, cubic and quartic contracts, I use the rectangural approximation. I add noise in option prices, they do not. They also use different spot price and strike price structure.
The results provide evidence that the exact approach may perform better in a market in which volatility evolves stochastically. By contrast, one might reasonably argue that there is not enough evidence for the validity of this argument due to the results in Das and Sundaram (1999) in which a stochastic volatility model does not capture completely the volatility smile and smirk. Another one might reasonably argue again that the results are inconclusive since I left an important question unexplained which could raise computational issues. For example, why both approaches yield positive skewness for the Variance-Gamma model whereas they yield negative skewness for the stochastic volatility model? In both cases, negative values of the respective parameters which control for skewness were chosen. The next section aims to clarify these issues by studying the *"volatility skew"* for these models.

2.5 The role of the volatility skew

The *"volatility skew"* refers to the differences in implied volatilities across option types and strike prices within a maturity expiration cycle. It can be either the *"volatility smirk"* or the *"forward skew"*.
"At a given maturity level, the Black and Scholes (1973) implied volatilities for out-of-the-money puts are much higher than those of out-of-the-money calls. This phenomenon is commonly referred to as the volatility smirk." (Carr and Wu (2003, p. 753)).
Since the US market crash in 1987, the volatility smirk, also known as the *"reverse skew"*, has

been studied by researchers. Early studies try unsuccessfully to develop models accounting for the volatility smirk. Some examples of these studies are Merton (1976) and the Variance-Gamma model of Madan et al. (1998). These models however make the assumption that the Central Limit Theorem holds. As a result the volatilty smirk flattens out quickly as time to maturity increases which contradicts empirical studies which find that volatility smirk becomes steeper across the maturity dimension as in Forresi and Wu (2005).

Significant progress has been made since then. Carr and Wu (2003) extend previous studies which work with symmetric α-stable models by developing an asymmetric α-stable model. Symmetric models have the problem of generating infinite price moments and hence potentially infinite call option values. The cleverly designed study in Carr and Wu (2003) imposes a maximum negative skewness in the symmetric model, in order to obtain finite conditional moments and hence finite call values. Eventually they show that their asymmetric framewok captures the volatility smirk in US index options.

In an other study Forresi and Wu (2005) study whether the volatility smirk is a US or a global phenomenon. Their study includes 12 global indices and conclude that volatility smirk is a global phenomenon suggesting that the left tail is fatter than the right tail of the distribution implying a *"crash-o-phobia"*. Investors are willing to pay more to buy put options than calls.

Highlighting the importance of the volatility smirk, it is worth to note that it is not observed only in the equity market. There is supportive evidence of its existence in the currency market as well. Bakshi, Carr, and Wu (2008) identify stochastic discount factors in international economies from currency returns and currency options. They find that even though the aggregate uncertainty of an economy can receive both positive and negative shocks, only downside jumps are priced.

By contrast, there is not as much information about an other phenomenon documented in the commodity market known as the *"forward skew"* (Fackler and King (1990), Sherrick et al (1996)). When the forward skew is present, the implied volatility for options at the lower strikes are lower than the implied volatility at higher strikes. Relating this to the definition Carr and Wu (2003) give for the volatility smirk, it simply states that the Black and Scholes implied volatilities of OTM calls are higher than the Black and Scholes implied volatilities of OTM puts at their respective moneyness levels. The forward skew is usually observed for agricultural commodities like wheat and soybean possibly reflecting an upside risk.

In summary the volatility smirk simply implies that the left tail of the distribution is fatter than the right tail resulting in negative skewness. Its positive curvature implies fatter tails than the normal distribution. Since the Variance-Gamma model fails to capture the volatility smirk, across the maturity dimension, as indicated by other studies and yields positive skewness in mine, maybe the Black and Scholes (1973) implied volatilities for OTM puts are lower than those of OTM calls. Put differently, the Variance-Gamma model may exhibit the forward skew at least for the set of parameters we choose. By contrast, if the volatility smirk is present in the Heston (1993) model, this may help explain why the exact and the interpolation-extrapolation approach yield negative skewness in this case.

The exact approach yields more accurate estimates for skewness and kurtosis for the stochastic volatility model. If the stochastic volatility options exhibit the volatility smirk, I can conclude that the exact method may outperform the interpolation-extrapolation approach for those assets in which the volatility smirk dominates. By contrast, if Variance-Gamma options exhibit the forward skew, the exact approach may yield more accurate estimates for skewness only, for those assets which exhibit the forward skew. Therefore the next hypothesis is tested,

Hypothesis 1: The exact (interpolation-extrapolation) approach yields more accurate estimates for skewness and kurtosis (kurtosis) for those assets that exhibit the volatility smirk (forward skew) than the interpolation-extrapolation (exact) approach.

In order to test Hypothesis 1, one must first define a proxy for the moneyness level. Following existing studies,[12] I use the following proxy for moneyness,

$$d = \frac{ln(K/F)}{\sigma\sqrt{\tau}} \tag{2.25}$$

[12]See Carr and Wu (2003), Backus, Foresi and Wu (1997) among others.

where, K is the strike price, F is the forward price, σ is the implied volatilty and τ is the time to maturity.

Backus, Foresi and Wu (1997) show that this proxy allows the slope and curvature of the volatility smirk to be transparently related to the skewness and kurtosis of the underlying risk-neutral distribution. On the choice of σ, I choose 0.25 as in Table 2.1. This contradicts the framework in the stochastic volatility model and can be viewed unrealistic in a sense that volatility is not constant. However, as Carr and Wu (2003) point out, the use of a constant volatility provides a simple interpretion of this proxy as roughly how many standard deviations the log strike is away from the log forward price in the Black and Scholes model and has become an industry convention. I evaluate the absolute value of d in order to be able to compare the moneyness among call and put options since it projects the location of put options in the right tail. I cannot obtain identical values of moneyness, therefore I define a call and put option to have the same moneyness if their proxy absolute difference is less than or equal to 0.01. The moneyness proxy d is extracted from the interpolated-extrapolated Black and Scholes options in order to obtain a relatively large sample in which one may draw more reliable statistical inferences.

I evaluate the differences, $Diff$ = *Black and Scholes OTM Call implied volatilty - Black and Scholes OTM Put implied volatilty*, for each call and put option that has the same moneyness level and these differences are regressed on a constant. A significantly positive (negative) $Diff$ reflects the forward skew (volatility smirk). The sample consists of 60 (76) [89] $Diff$ for the 3 (6) [9] -month risk-neutral distribution. Table 2.3 reports the results of this t-test.

Table 2.3 reports the major contribution of this chapter. In the case of assets in which the volatilty smirk dominates, the exact approach outperforms the interpolation-extrapolation approach for risk-neutral skewness and kurtosis.[13] By contrast, for other assets which exhibit the forward skew, the exact approach yields more accurate estimates for skewness but not for kurtosis in which the interpolation-extrapolation approach performs better.[14]

[13]The volatiltiy smirk dominates due to the significantly negative values of $Diff$ in the stochastic volatilty model.

[14]The forward skew dominates because of the results that the Variance-Gamma $Diff$ are significantly

Table 2.3. The volatilty skew t-test

Table 2.3 reports the results of a t-test whether the difference ($Diff$) *Black and Scholes OTM Call implied volatilty - Black and Scholes OTM Put implied volatilty* at their respective moneyness levels, is significantly different than zero for the interpolation-extrapolation approach, when 3, 6 and 9-month options follow the Variance-Gamma (VG) and the Heston (1993) (HS) process. In order to perform the t-test, the differences are regressed on a constant. The sample consists of 60 (76) [89] observations for the 3 (6) [9] -month distribution which results in 59 (75) [88] degrees of freedom. The price of stock is chosen to be 270. Call (put) strike prices are 275 to 310 with a \$5 increment and 325 and 350 (200, 220 and 225 to 265 with a \$5 increment) for the exact approach. The domain of integration is $[162, 378]$ and \$1 increment among strike prices is chosen for the interpolation-extrapolation approach. Structural parameters are $\sigma = 0.25$, $\theta = -0.14$, $\nu = 0.16$ for the Variance-Gamma model. and $\sigma_v = 0.25$, $\theta = 0.04$, $k = 2$, $\rho = -0.5$ and $V = 0.04$ for the Heston (1993) model. White gaussian noise is generated once and added in option prices by imposing the restriction that it will yield positive prices for the extremely deep interpolated-extrapolated OTM 3-month put options. Panel A reports the intercept values (in %) along with their t-statistic for a 3-month risk-neutral distribution. Panel B reports the intercept values (in %) along with their t-statistic for a 6-month risk-neutral distribution. Panel C reports the intercept values (in %) along with their t-statistic for a 9-month risk-neutral distribution.

	Intercept	t-statistic
Panel A. 3-month risk-neutral distribution		
$VG\ Diff$	5.17	29.47
$HS\ Diff$	-2.18	-18.02
Panel B. 6-month risk-neutral distribution		
$VG\ Diff$	3.42	23.01
$HS\ Diff$	-2.87	-15.59
Panel C. 9-month risk-neutral distribution		
$VG\ Diff$	2.17	23.15
$HS\ Diff$	-2.85	-20.53

The pattern in the intercept values of the forward skew may help explain the distribution behaviour for the Variance-Gamma model across the maturity dimension. The forward skew becomes weaker as time to maturity increases which makes skewness and kurtosis decrease and converge to the normal distribution. In a similar fashion, the hump-shaped pattern in the Heston intercepts across the maturity dimension may help explain why kurtosis follows a hump-shaped behaviour as proven in the proposition 4 in Das and Sundaram (1999). The volatilty smirk becomes steeper as one moves from a 3 to a 6-month ditribution and starts to flatten out after that. The same pattern may help explain why Backus, Foresi, and Wu (1997) and Das and Sundaram (1999) find that a stochastic volatility process slows down the speed of convergence to normality, but does not stop it.

The forward skew in the Variance-Gamma model possibly explains why both approaches yield positive skewness. It generates a fatter right tail which results in positive skewness. The volatility smirk possibly explains in a similar way why both approaches yield negative skewness in the stochastic volatility model. It generates a fatter left tail which results in negative skewness.

The results may also help explain the bias introduced when one evaluates skewness by an individual option. Dennis and Mayhew (2002) find that if one uses one call (put) Black and Scholes skewness is positive (negative). When skewness is extracted from calls (puts) only, a fatter right (left) tail is generated since this yields automatically a forward skew (volatilty smirk).

The forward skew and the volatility smirk may help explain the findings in Liu and Heijden (2016). To my knowledge, it is the only study that works in an extensive simulation framework incorporating different parameters across different models in order to study the accuracy of different approaches, including the interpolation-extrapolation, in evaluating risk-neutral moments. Their conjecture is that errors can be largely unquantifiable and can be in any direction, either positive or negative. Maybe different parameters across different models result in either a forward skew or a volatility smirk which could potentially yield errors in any direction. The magnitude of errors may depend on how steep or flat the volatility skew is across the parameter and model dimension.

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positive.

In summary the exact approach yields more accurate estimates for skewness and kurtosis for assets dominated by the volatility smirk, whereas the interpolation-extrapolation approach performs better for kurtosis only, for assets dominated by the forward skew. The different patterns in the forward skew and the volatilty smirk may help explain the puzzling findings with respect to the behaviour of the return distribution in existing models.

2.6 Robustness tests

This section reports the results of two robustness tests. First, it studies the accuracy of the exact approach compared to the interpolation-extrapolation method when implied volatility is flat beyond the minimum and maximum strike price. Second, it employs the exact approach in an empirical setting to study whether the distribution it returns, is consistent with existing studies. The results are robust across these tests.

Flat implied volatility

Existing studies assume that volatility is flat beyond the minimum and maximum strike price and equal to the volatility at the minimum and maximum strike price. The next robustness test studies the accuracy of the exact compared to the interpolation-extrapolation approach in this framework. Table 2.4 reports the results of this test.

The results remain qualitatively the same except for one case. This is the 6-month Heston error in which the interpolation-extrapolation error yields a smaller error. However this is probably due to the peculiar negative relation between the interpolated-extrapolated kurtosis and time to maturity, rather than capturing the features of the underlying distribution. This can be noted from the 9-month distribution in which the exact approach performs better. In further support of this argument I perform the same simulation experiment for 12-month options, the results are not reported however, but are available upon request. The exact kurtosis error is -0.43. By contrast

Table 2.4. Flat implied volatility beyond the minimum and maximum strike price

Table 2.4 reports the risk-neutral skewness (RNS) and kurtosis (RNK) values and errors of the exact (EX) and the interpolation-extrapolation (IE) approach when 3, 6 and 9-month options follow the Variance-Gamma (VG) and the Heston (1993) (HS) stochastic volatility model by making the assumption that implied volatilty is flat beyond the minimum and maximum strike price and equal to the volatilty at the minimum and maximum strike price. The error for each approach is evaluated as the difference between the moment value and the respective closed form solution. The price of stock is chosen to be 270 and structural parameters $\sigma_v = 0.25$, $\theta = 0.04$, $k = 2$, $\rho = -0.5$ and $V = 0.04$. Call (put) strike prices are 275 to 310 with a \$5 increment and 325 and 350 (200, 220 and 225 to 265 with a \$5 increment) for the exact approach. The domain of integration is $[162, 378]$ and \$1 increment among strike prices is chosen for the interpolation-extrapolation approach. White gaussian noise is generated once and added in option prices by imposing the restriction that it will yield positive prices for the extremely deep interpolated-extrapolated OTM 3-month put options. Noise follows $N(0, 0.125)$ ($N(0, 0.0625)$) for call option prices higher (less) than \$3. With respect to put options, noise follows $N(0, 0.1)$ ($N(0, 0.05)$) for prices higher (less) than \$3. Panel A reports the errors and values of skewness and kurtosis for a 3-month risk-neutral distribution. Panel B reports the errors and values of skewness and kurtosis for a 6-month risk-neutral distribution. Panel C reports the errors and values of skewness and kurtosis for a 9-month risk-neutral distribution.

	RNKIE	RNSIE	RNKEX	RNSEX
Panel A. 3-month risk-neutral distribution				
VG Moment Error	4.5	0.45	5.21	0.2
VG Moment Value	4.5	0.45	5.21	0.2
HS Moment Error	0.71	-0.04	-0.67	0.01
HS Moment Value	4.13	-0.43	2.75	-0.38
Panel B. 6-month risk-neutral distribution				
VG Moment Error	3.17	0.18	3.84	0.06
VG Moment Value	3.17	0.18	3.84	0.06
HS Moment Error	0.19	0.03	0.28	-0.02
HS Moment Value	3.82	-0.45	3.91	-0.5
Panel C. 9-month risk-neutral distribution				
VG Moment Error	2.62	0.05	3.15	-0.04
VG Moment Value	2.63	0.05	3.15	-0.04
HS Moment Error	-0.2	0.02	-0.02	-0.02
HS Moment Value	3.53	-0.5	3.71	-0.54

the interpolated-extrapolated error is -0.59.

It is worth noting that all interpolated-extrapolated kurtosis values are lower than the ones in Tables 2.1 and 2.2. This supports the initial argument, that my way of extrapolation generates fatter tails than the flat implied volatility assumption.

The exact approach in an empirical setting

This section studies whether the exact approach, when employed in an empirical setting, yields a negatively skewed distribution with tails fatter than the normal. It also studies whether the left tail is fatter than the right implying the *"crash-o-phobia"* as documnted by other studies. To do that, it evaluates skewness and kurtosis using calls or puts only.

I employ the database of OptionMetrics with respect to US equity options with 1-month time to expiry, for the period between January 1996 and October 2012.[15] The delta, vega, strike price, best bid, and best offer of all options are retrieved from OptionMetrics. I extract the spot prices of underlying stocks from the Center for Research in Security Prices (CRSP), and daily risk-free rates from the data library of Kenneth French.

The risk-neutral moments are calculated on the first trading day after the monthly expiration day using OTM options which expire in the following month. I allow a lag of one day to compute the risk-neutral moments to avoid microstructure bias as in Bali and Murray (2013). Ni, Pearson and Poteshman (2005) find that a bias may be present since option trading changes the prices of underlying stocks and on expiration dates stock prices cluster at their respective strike prices. I identify an OTM call (put) option to be one that has a delta closest to 0.1 (-0.1) as in Bali and Murray (2013). I require at least two OTM call and two OTM options to be available to calculate the risk-neutral skewness and kurtosis for each stock/month expiration combination following Dennis and Mayhew (2002) who show that skewness error is zero in this case. I compute the price of

[15]Since the end of 2013 Essex Business School does not have access to Optionmetrics due to budget constraints. Optionmetrics is updated once a year. I started my phd in October 2013. As a result, the update restriction in Optionmetrics and the no longer subscription by Essex Business School did not allow me to update the database to a closer date when this thesis was submitted (September 2016).

Table 2.5. The exact approach in the US equity market

Table 2.5 reports the mean, the 5th, 25th, 50th, 75th, and 95th percentiles, for risk-neutral skewness and kurtosis when they are extracted from 1-month US equity OTM calls only, OTM puts only and OTM puts and calls simultaneously. The volatility, cubic and quartic contracts are evaluated using the exact appproach and the risk-neutral moments are calculated on the first trading day after the expiration date using options with one-month to expiry. The sample period is from January 1996 to October 2012.

	Mean	5%	25%	50%	75%	95%
OTM calls						
RNS	3.90	2.88	3.44	3.89	4.34	4.97
RNK	3.19	-15.13	-1.93	5.26	8.65	15.37
OTM puts						
RNS	-3.22	-4.36	-3.70	-3.19	-2.72	-2.17
RNK	10.97	4.82	7.59	10.33	13.78	19.07
OTM puts and calls						
RNS	-1.20	-3.22	-2.05	-1.20	-0.33	0.83
RNK	4.92	-7.78	2.14	5.05	8.20	16.35

an option as the average of bid and offer prices. I apply the same filtering rule as in Bali and Murray(2013).[16]

The final dataset consists of 62,631 stock/month expiration combination compared with the total of 57,537 employed by Bali and Murray (2013). Table 2.5 reports the summary statistics for the risk-neutral skewness and kurtosis.

The exact approach extracts information about the risk-neutral distribution consistent with existing studies. When skewness and kurtosis are extracted from call and put options, the distribution is negatively skewed and has fatter tails than the normal (Bali and Murray (2013), Conrad et al. (2013)). The left tail of the risk-neutral distribution is fatter than the right implying *"crash-o-*

[16] I remove options with a missing bid or offer price, a non-positive bid price, an offer price less than or equal to the bid price, a spread bid-ask spread less than the minimum spread (0.05 for options with prices below 3.00, and 0.10 for options with prices greater than or equal to 3.00). One of the assumptions for computing the risk-neutral moments is that one works under the martingale pricing measure. Thus, I remove all options which violate arbitrage conditions. Following Bali and Murray (2013), I require that the bid price for call option must be less than the spot price minus the strike price and the offer price be at least as large as the spot price minus the present value of the strike. For put options, the bid price must be less than the strike and the offer price be at least as large as the strike price minus the spot price.

phobia" consistent with Forresi and Wu (2005).

When skewness is extracted from calls (puts) only, skewness is positive (negative) consistent with Dennis and Mayhew (2002). This is also in line with Bandopadhyaya and Jones (2011) and Billingsley and Chance (1998). Intutively positive (negative) skewness implies expectations for prices to go up (down). Bandopadhyaya and Jones (2008) and Billingsley and Chance (1998) find that when more calls (puts) are traded than puts (calls), this reflects optimistic (pessimistic) expectations.

Suprisingly, when kurtosis is extracted from calls only, roughly 25% of the sample exhibits negative kurtosis. A similar finding is also documented in Bali, Hu and Murray (2017). They study the relation between risk-neutral moments and monthly expected returns. They download analyst price target data from the Institutional Brokers Estimate System (I/B/E/S) unadjusted Detail History database and evaluate these returns by taking the average of all price target implied expected returns from price targets announced during the given month. They find that there is a positive relation between risk-neutral moments and ex-ante expected returns. One of the proxies they use to evaluate risk-neutral kurtosis is the sum of OTM call and put implied volatiltiies minus the sum of the ATM call and put implied volatilities. They report in their Table 3 that at least 5% of their sample exhibits negative kurtosis implying that there are ATM call options with implied volatilities higher than their OTM call counterparts. In other words, for some stocks, volatility does not *"smile"*, it possibly *"frowns"* resulting in negative kurtosis. However for most of the stocks, the volatility smile dominates since the average value of kurtosis is higher than 3. The same finding is documented in this chapter as well.

2.7 Conclusion

Bakshi, Kapadia and Madan (2003) propose a model-free method to evaluate risk-neutral moments. They make the assumption however, that strike prices are continuous and from zero to infinity. This results in descretization and truncation errors. Existing studies argue that the effect of these errors will be negligible in evaluating the risk-neutral moments if one employs the interpolation-extrapolation approach.

I employ the interpolation-extrapolation approach in the Bakshi et al. (2003) framework and show in Figure 1 that the extrapolated contracts go to zero as the window of the extrapolation becomes larger due to the low values of Black and Scholes option prices, indicating that discretization errors are probably the underlying cause of errors in evaluating risk-neutral moments.

This study proposes a new approach to valuing risk-neutral moments from option prices which avoids discretization errors. It provides exact solutions for the volatility, cubic and quartic contracts defined in Bakshi et al. (2003) when they lie in a finite domain. It studies the accuracy of the exact solutions compared to the interpolation-extrapolation approach which has been shown to be the most accurate hitherto. In the simulation horserace, the Variance-Gamma and the stochastic volatility model are chosen because they have closed-form expressions for their skewness and kurtosis one can use as *"reliable"* benchmarks. The results show that for the chosen set of parameters, the volatilty smirk (forward skew) is present in the stochastic volatility (Variance-Gamma) model. This leads to the conclusion that the exact aproach may outperform the interpolation-extrapolation approach for risk-neutral skewness and kurtosis (skewness) for assets dominated by the volatility smirk (forward skew).

Appendix A

Analytical Solution to Integrals (2.5), (2.7), and (2.8)

We let $()'$ denote the first derivative of the function inside the parentheses with respect to strike price K. Equation (2.5) can be solved as follows,

$$
\begin{aligned}
\int_{S_0}^{K_1} \frac{(1-\ln(\frac{K}{S_0}))}{K^2} dK &= \int_{S_0}^{K_1} \frac{1-\ln(\frac{K}{S_0})\cdot 1}{K^2} dK = \int_{S_0}^{K_1} \frac{K\cdot\frac{1}{K}-\ln(\frac{K}{S_0})\cdot 1}{K^2} dK \\
&= \int_{S_0}^{K_1} \frac{K\cdot\ln(\frac{K}{S_0})' - \ln(\frac{K}{S_0})\cdot(K)'}{K^2} dK \\
&= \frac{\ln(\frac{K}{S_0})}{K} \Big|_{S_0}^{K_1} = \frac{\ln(\frac{K_1}{S_0})}{K_1} - 0 = \frac{\ln(\frac{K_1}{S_0})}{K_1}
\end{aligned}
$$

With respect to equation (2.7),

$$
\begin{aligned}
\int_{S_0}^{K_1} \frac{6\ln(\frac{K}{S_0}) - 3(\ln(\frac{K}{S_0}))^2}{K^2} dK &= \int_{S_0}^{K_1} \frac{6\ln(\frac{K}{S_0})\cdot 1 - 3(\ln(\frac{K}{S_0}))^2\cdot 1}{K^2} dK \\
&= \int_{S_0}^{K_1} \frac{6\ln(\frac{K}{S_0})\cdot(K\cdot\frac{1}{K}) - 3(\ln(\frac{K}{S_0}))^2\cdot 1}{K^2} dK \\
&= \int_{S_0}^{K_1} \frac{(3(\ln(\frac{K}{S_0}))^2)'\cdot K - 3(\ln(\frac{K}{S_0}))^2\cdot(K)'}{K^2} dK \\
&= \frac{3(\ln(\frac{K}{S_0}))^2}{K} \Big|_{S_0}^{K_1} = \frac{3(\ln(\frac{K_1}{S_0}))^2}{K_1} - 0 \\
&= \frac{3(\ln(\frac{K_1}{S_0}))^2}{K_1}
\end{aligned}
$$

Similarly, for equation (2.8),

$$\begin{aligned}
\int_{S_0}^{K_1} \frac{12(\ln(\frac{K}{S_0}))^2 - 4(\ln(\frac{K}{S_0}))^3}{K^2} dK &= \int_{S_0}^{K_1} \frac{12(\ln(\frac{K}{S_0}))^2 \cdot 1 - 4(\ln(\frac{K}{S_0}))^3 \cdot 1}{K^2} dK \\
&= \int_{S_0}^{K_1} \frac{12(\ln(\frac{K}{S_0}))^2 \cdot (K \cdot \frac{1}{K}) - 4(\ln(\frac{K}{S_0}))^3 \cdot 1}{K^2} dK \\
&= \int_{S_0}^{K_1} \frac{(4(\ln(\frac{K}{S_0}))^3)' \cdot K - 4(\ln(\frac{K}{S_0}))^3 \cdot (K)'}{K^2} dK \\
&= \frac{4(\ln(\frac{K}{S_0}))^3}{K} \,|_S^{K_1} = \frac{4(\ln(\frac{K_1}{S_0}))^3}{K_1} - 0 \\
&= \frac{4(\ln(\frac{K_1}{S_0}))^3}{K_1}
\end{aligned}$$

Appendix B

The Simpson Approximation

This Appendix studies the accuracy of the exact approach compared to the interpolation-extrapolation approach when one uses the Simpson method to evaluate the volatility, cubic and quartic contracts. The Simpson approximation is given by the following formula,

$$\int_{\alpha}^{\beta} f(x)dx = \frac{\beta-\alpha}{6}(f(\alpha)+f(\beta)+4f(\frac{\alpha+\beta}{2})). \tag{2.26}$$

Assume that I work with a stock, N_c OTM call options, and N_p OTM put options. All options are written on the stock. I define the strike price differences for call options as $\Delta K_1^c = K_1^c - S$ and $\Delta K_i^c = K_i^c - K_{i-1}^c$ when $i \in \{2, ..., N_c\}$. In a similar way, for the put options I define the strike price differences as $\Delta K_1^p = S - K_1^p$ and $\Delta K_i^p = K_{i-1}^p - K_i^p$ when $i \in \{2, ..., N_p\}$.

Using equation (2.26), the Simpson rule for the three integrals are as follows,

$$\begin{aligned} V &= g_c(K_1^c)C_1\Delta K_1^c + \sum_{i=2}^{N_c}\frac{\Delta K_i^c}{6}[g_c(K_i^c)C_i + g_c(K_{i-1}^c)C_{i-1} + 4\frac{(C_i+C_{i-1})}{2}g_c(\frac{K_i^c+K_{i-1}^c}{2})] \\ &+ g_p(K_1^p)P_1\Delta K_1^p + \sum_{i=2}^{N_p}\frac{\Delta K_i^p}{6}[g_p(K_i^p)P_i + g_p(K_{i-1}^p)P_{i-1} + 4\frac{(P_i+P_{i-1})}{2}g_p(\frac{K_i^p+K_{i-1}^p}{2})] \end{aligned}$$

$$\begin{aligned} W &= h_c(K_1^c)C_1\Delta K_1^c + \sum_{i=2}^{N_c}\frac{\Delta K_i^c}{6}[h_c(K_i^c)C_i + h_c(K_{i-1}^c)C_{i-1} + 4\frac{(C_i+C_{i-1})}{2}h_c(\frac{K_i^c+K_{i-1}^c}{2})] \\ &- (h_p(K_1^p)P_1\Delta K_1^p + \sum_{i=2}^{N_p}\frac{\Delta K_i^p}{6}[h_p(K_i^p)P_i + h_p(K_{i-1}^p)P_{i-1} + 4\frac{(P_i+P_{i-1})}{2}h_p(\frac{K_i^p+K_{i-1}^p}{2})]) \\ X &= l_c(K_1^c)C_1\Delta K_1^c + \sum_{i=2}^{N_c}\frac{\Delta K_i^c}{6}[l_c(K_i^c)C_i + l_c(K_{i-1}^c)C_{i-1} + 4\frac{(C_i+C_{i-1})}{2}l_c(\frac{K_i^c+K_{i-1}^c}{2})] \\ &+ l_p(K_1^p)P_1\Delta K_1^p + \sum_{i=2}^{N_p}\frac{\Delta K_i^p}{6}[l_p(K_i^p)P_i + l_p(K_{i-1}^p)P_{i-1} + 4\frac{(P_i+P_{i-1})}{2}l_p(\frac{K_i^p+K_{i-1}^p}{2})] \end{aligned}$$

where

$$
\begin{aligned}
g_c(K) &= \frac{2(1 - \ln[\frac{K}{S}])}{K^2} \\
g_p(K) &= \frac{2(1 + \ln[\frac{S}{K}])}{K^2} \\
h_c(K) &= \frac{6\ln[\frac{K}{S}] - 3(\ln[\frac{K}{S}])^2}{K^2} \\
h_p(K) &= \frac{6\ln[\frac{S}{K}] - 3(\ln[\frac{S}{K}])^2}{K^2} \\
l_c(K) &= \frac{12(\ln[\frac{K}{S}])^2 - 4(\ln[\frac{K}{S}])^3}{K^2} \\
l_p(K) &= \frac{12(\ln[\frac{S}{K}])^2 - 4(\ln[\frac{S}{K}])^3}{K^2}
\end{aligned}
$$

I use the same parameters, spot price and strike price structure as in Tables 2.1 and 2.2. Table B1 reports the risk-neutral skewness and kurtosis values and errors for the exact approach and the interpolation-extrapolation approach when one uses the Simpson method to evaluate the volatility, cubic and quartic contracts.

The results are robust even when one uses the Simpron method to evaluate the three contracts. The exact approach yields more accurate estimates for skewness and kurtosis for the Variance-Gamma model and it outperforms the interpolation-extrapolation approach with respect to skewness only, for the stochastic volatility model.

Table B1. The Simpson approximation

Table B1 reports the risk-neutral skewness (RNS) and kurtosis (RNK) values and errors of the exact (EX) and the interpolation-extrapolation (IE) approach when 3, 6 and 9-month options follow the Variance-Gamma (VG) and the Heston (1993) (HS) stochastic volatility model by evaluating the volatilty, cubic and quartic contracts using the Simpson method. The error for each approach is evaluated as the difference between the moment value and the respective closed form solution. The price of stock is chosen to be 270 and structural parameters $\sigma_v = 0.25$, $\theta = 0.04$, $k = 2$, $\rho = -0.5$ and $V = 0.04$. Call (put) strike prices are 275 to 310 with a \$5 increment and 325 and 350 (200, 220 and 225 to 265 with a \$5 increment) for the exact approach. The domain of integration is $[162, 378]$ and \$1 increment among strike prices is chosen for the interpolation-extrapolation approach. White gaussian noise is generated once and added in option prices by imposing the restriction that it will yield positive prices for the extremely deep interpolated-extrapolated OTM 3-month put options. Noise follows $N(0, 0.125)$ ($N(0, 0.0625)$) for call option prices higher (less) than \$3. With respect to put options, noise follows $N(0, 0.1)$ ($N(0, 0.05)$) for prices higher (less) than \$3. Panel A reports the errors and values of skewness and kurtosis for a 3-month risk-neutral distribution. Panel B reports the errors and values of skewness and kurtosis for a 6-month risk-neutral distribution. Panel C reports the errors and values of skewness and kurtosis for a 9-month risk-neutral distribution.

	RNKIE	RNSIE	RNKEX	RNSEX
Panel A. 3-month risk-neutral distribution				
VG Moment Error	4.35	0.47	5.21	0.2
VG Moment Value	4.35	0.47	5.21	0.2
HS Moment Error	4.25	-0.42	-0.67	0.01
HS Moment Value	0.83	-0.03	2.75	-0.38
Panel B. 6-month risk-neutral distribution				
VG Moment Error	3.08	0.2	3.84	0.06
VG Moment Value	3.08	0.2	3.84	0.06
HS Moment Error	0.29	0.03	0.28	-0.02
HS Moment Value	3.92	-0.45	3.91	-0.5
Panel C. 9-month risk-neutral distribution				
VG Moment Error	2.58	0.05	3.14	-0.04
VG Moment Value	2.59	0.05	3.15	-0.04
HS Moment Error	-0.17	-0.02	-0.02	-0.02
HS Moment Value	3.56	-0.54	3.71	-0.54

Appendix C

Absence of noise

This Appendix studies the accuracy of the exact approach compared to the interpolation-extrapolation approach in the absence of noise. The same procedure is followed as in Tables 1 and 2. Table C1 reports the results of this test.

The results remain qualitatively the same. The exact approach outperforms the interpolation-extrapolation approach for skewness and kurtosis (skewness) for the stochastic volatility (Variance-Gamma) model.

Table C1. No noise

Table C1 reports the risk-neutral skewness (RNS) and kurtosis (RNK) values and errors of the exact (EX) and the interpolation-extrapolation (IE) approach when 3, 6 and 9-month options follow the Variance-Gamma (VG) and the Heston (1993) (HS) stochastic volatility model in the absence of noise in the market. The error for each approach is evaluated as the difference between the moment value and the respective closed form solution. The price of stock is chosen to be 270 and structural parameters $\sigma_v = 0.25$, $\theta = 0.04$, $k = 2$, $\rho = -0.5$ and $V = 0.04$. Call (put) strike prices are 275 to 310 with a \$5 increment and 325 and 350 (200, 220 and 225 to 265 with a \$5 increment) for the exact approach. The domain of integration is $[162, 378]$ and \$1 increment among strike prices is chosen for the interpolation-extrapolation approach. White gaussian noise is generated once and added in option prices by imposing the restriction that it will yield positive prices for the extremely deep interpolated-extrapolated OTM 3-month put options. Noise follows $N(0, 0.125)$ ($N(0, 0.0625)$) for call option prices higher (less) than \$3. With respect to put options, noise follows $N(0, 0.1)$ ($N(0, 0.05)$) for prices higher (less) than \$3. Panel A reports the errors and values of skewness and kurtosis for a 3-month risk-neutral distribution. Panel B reports the errors and values of skewness and kurtosis for a 6-month risk-neutral distribution. Panel C reports the errors and values of skewness and kurtosis for a 9-month risk-neutral distribution.

	RNKIE	RNSIE	RNKEX	RNSEX
Panel A. 3-month risk-neutral distribution				
VG Moment Error	4.49	0.5	5.25	0.27
VG Moment Value	4.49	0.5	5.25	0.27
HS Moment Error	1.21	-0.07	-0.69	-0.01
HS Moment Value	4.63	-0.46	2.73	-0.4
Panel B. 6-month risk-neutral distribution				
VG Moment Error	3.18	0.21	3.82	0.06
VG Moment Value	3.18	0.21	3.82	0.06
HS Moment Error	0.26	-0.01	0.22	-0.01
HS Moment Value	3.89	-0.49	3.85	-0.49
Panel C. 9-month risk-neutral distribution				
VG Moment Error	2.66	0.05	3.12	-0.04
VG Moment Value	2.67	0.05	3.13	-0.04
HS Moment Error	-0.04	-0.03	-0.03	0
HS Moment Value	3.69	-0.55	3.70	-0.52

Chapter 3

Investor heterogeneity, overconfidence, and skewness preference in options market

3.1 Introduction

The key participants in derivatives markets are hedgers, speculators and arbitrageurs. Hedgers aim to avoid exposure to risk. Arbitrageurs are looking for risk-less profit by taking advantage of arbitrage opportunities. Speculators take risky positions in the market in order to make profits. While profits could be extremely high, the probability of realising losses is also high as well. Speculators use derivatives in order to realise profits, whereas hedgers use them to minimize risk.

Bakshi, Kapadia, and Madan (2003) [hereafter BKM] develop a methodology for calculating risk-neutral moments (RNMs) which are extracted from option prices. It is the most widespread method employed by researchers for calculating risk-neutral moments.[1] Several researchers have tested the ability of risk-neutral skewness to predict returns. The results are mixed though. On the one

[1] See, for example, Bali and Murray (2013), Dennis and Mayhew (2002), and Han (2008).

hand, Conrad Dittmar and Ghysels (2013), Bali and Murray (2013), and Bali Cakici and Whitelaw (2011) find a negative relation while, on the other, Rehman and Vilkov (2012), Xing, Zhang and Zhao (2010), Cremers and Weinbaum (2010), and Stilger, Kostakis and Poon (2017) find a positive relation.

Coakley, Dotsis, Liu and Zhai (2014) find a positive (negative) relation between sentiment measures and the risk-neutral skewness for growth (value) index options. They implicitely use heterogeneity to show that the relation between skewness and sentiment can be either positive or negative. Diether et al. (2002), Baik et al. (2003), and Doukas et al. (2004) find that value stocks have greater belief heterogeneity than growth stocks. Buraschi et al. (2006) show that belief differences can affect risk-neutral skewness. Friesen et al. (2012) find that stocks with greater belief differences are more negatively skewed.

The objective of this chapter is to shed light on a possible explanation for the capability of RNS in predicting stock returns by linking it to overconfidence and dispersion in beliefs. Intuitively, in the risk-neutral framework, today's fair value of an asset is equal to the discounted expected value of the future payouts of the asset, where the discount rate is the risk-free rate. As a result all financial assets in a risk-neutral economy have the same expected return since prices do not depend on risk. Therefore one might argue that it is not possible for overconfidence and dispersion in beliefs to prevail in this economy since all investors expect the same return for all assets.

Friedman's (1953) Market Selection Hypothesis (MSH hereafter) states that traders with inaccurate forecasts will be eliminated from the market and eventually the effect of their beliefs on prices will be neutralized. A supporter of MSH might argue that even if one makes the assumption that there is a possibility for overconfidence and dispersion in beliefs to develop when investors are risk-neutral, this would not persist and would have no effect on prices. By contrast, Miller (1977) and many other price-optimist models[2] argue that, if pessimistic investors are kept out of the market, prices will reflect a more optimistic valuation. They find that the bigger the disagreement about a stock's fair value the lower its future returns. By contrast, Shapiro (2009) argues that it

[2]See, for example, Morris (1996), Chen et al. (2001), and Viswanathan (2001).

is not the imposition of short-sale constraints which cause the overvaluation but the evolution of heterogeneous beliefs.

Many explanations have been given for overvaluation. Merton (1987) and Heaton and Lucas (2000) explain it in a rational framework by increasing opportunities for diversification. De Long, Shleifer, Summers and Waldman (1990), Scheinkman and Xiong (2003), and Nagel (2005) attribute the overvaluation to overconfidence in a behavioural framework. Investors are overconfident because they overestimate the precision of their signals which results in the development of differing opinions and holding more units of the risky asset leading to its overvaluation. However in a risk-neutral market there is no risk-premium incorporated in prices. Kyle and Wang (1997) assumes that the investors are risk-neutral and come to the same conclusion. Overconfident investors hold more units of the asset which makes it overvalued. Wang (1998) shows as well that overconfident investors can still make higher profits in the absence of a risk premium. On the one hand, there is a strand in the literature which provides strong evidence that risk-neutral skewness and heterogeneous beliefs are connected. On the other, there is another strand in the literature in which heterogeneous beliefs can cause an overvaluation of a stock which may lead to lower returns respectively.

This chapter's contribution is that it can rationalise the positive skewness preference in terms of overconfidence and differences in beliefs. The overconfident investors hold more units of the positively skewed portfolio where the dispersion in beliefs is greater and cause overvaluation. As a result this portfolio yelds lower returns. The results are consistent with those of Bali and Murray (2013) and Conrad et al. (2013). The findings indicate that overvaluation is not because of risk-premium, consistent with Wang (1998) and Kyle and Wang (1997). As a result, this produces an upward bias which makes the market value of a stock higher than its true value and results in lower returns. The overconfident investors hold more units of the stock which makes the asset overvalued.

The remainder of this chapter is organized as follows. Section 3.2 explains how I construct and develop my motivation. It also presents the way I form the skewness asset to develop delta-vega neutral portfolios and how returns are calculated. Section 3.3 provides the details of the dataset and an empirical analysis of the results. In section 3.4 I perform robustness tests and section 3.5

concludes.

3.2 Methodology

This section describes how incomplete financial markets allow investor heterogeneity to play a role in the pricing of option portfolios. It then outlines the formation of the PUTCALL asset building upon Bali and Murray (2013).

3.2.1 Incomplete market

The first fundamental asset pricing theorem states that if a market is arbitrage-free then a risk-neutral measure exists. Moreover, if this market is complete the risk-neutral measure is unique (Cochrane (2001)). Under these two assumptions, today's fair value of an asset can be calculated as follows:

$$V_0 = \beta(0, \tau) \cdot E^Q(V_\tau), \tag{3.1}$$

where V_0 and V_τ are value of the asset today and the value of the asset a future date τ respectively, E is the expectation operator, Q denotes the risk-neutral measure, and $\beta(0, \tau)$ is the discount factor from now until τ.

A market is said to be complete if information is perfect and there is a price for every asset in every possible state of the world. However actual information is not perfect. There are studies which support that financial markets are not complete (Duffie (1987) and Geanakoplos and Mas-Colell (1989)). Blume and Easley (2006) show that the MSH may fail when a market is incomplete whereas Shapiro (2009) suggests that heterogeneous beliefs are persistent. Taking into account the first fundamental asset pricing theorem and the findings in Blume and Easley (2006) and in Shapiro (2009), when one uses market data they work with multiple risk-neutral measures due to the market incompleteness. Hence the incompleteness allows for overconfidence among investors which leads to dispersion in their beliefs as they disagree on today's fair value of an asset.[3] Since heterogeneous beliefs are persistent (Shapiro (2009)), it may have a significant impact on prices. One situation is that we have two risk-neutral measures E_1^Q and E_2^Q such that:

$$V_1 = \beta(0,\tau) \cdot E_1^Q(V_\tau) \tag{3.2}$$

$$V_2 = \beta(0,\tau) \cdot E_2^Q(V_\tau) \tag{3.3}$$

Equations (2) and (3) show that although investors do not disagree on the future price of an asset, they disagree with regard to today's fair value. The incompleteness of the market allows for misvaluation to develop among investors. Overconfident investors assign higher probabilities to relatively high future prices to an asset and make the interval in which the fair value may lie in narrower. They hold more units of this asset which results in overvaluation and lower future return.

In summary, the incompleteness of the market allows for the development of overconfidence and heterogeneous beliefs with respect to the fair value of an asset. As a result, a group of investors may hold more units of an asset and this may lead to the overvaluation of an asset.

3.2.2 Asset formation

I follow Bali and Murray (2013) in constructing delta-vega neutral portfolios with positions in stocks and options. A delta-vega neutral portfolio is a portfolio which is immune to changes of the underlying asset price (i.e delta-neutral) and to changes in the implied volatility of the stock (i.e vega-neutral). Hence skewness matters for the pricing of these portfolios.[4] I begin by finding the out-of-the-money (OTM) call and put options as in Bali and Murray (2013). I identify an OTM call (put) option to be one that has a delta closest to 0.1 (-0.1). If there are insufficient data for the

[3] One might argue that seacrh cost issue could be the cause of heterogeneous beliefs, however it is beyond the scope of this paper and, to my knowledge, there is no research which connects heterogeneous beliefs to search cost issue.

[4] In robustness section I run Fama and MacBeth (1973) regressions which controls for other moments and results remain qualitatively the same.

above observations on a business day, they are removed from our sample. I use υ and Δ to denote option vega and delta, respectively. So, for example, $\Delta_{C,OTM}$ refers to the delta of an OTM call option. I retrieve these option Greeks from the OptionMetrics database.[5]

Removing insufficient data to construct delta-vega meutral portfolios, may give rise to survivorship bias. Numerous studies have studied the impact of survivorship bias on the momentum effect.[6] They find that the presence of the momentum effect is subject to whether this bias is present or not. In an other study more related to ours, Jacobs et al. (2017) find that skewness is among the most important cross-sectional determinants of momentum. As a result, survivorship bias may play a role in the robustness of a positive or a negative skewness preference. However sample sizes in skewness preference studies are large enough to mitigate the effect of a survivorship bias. For example Conrad et al. (2013) use 3,722,700 option-day combinations over the time period January 1996 through December 2005 and Bali and Murray (2013) use 57,537 stock/month expiration combinations for the period January 1996 - October 2010. Both of these studies find the same skewness preference despite the different periods they cover.

PUTCALL asset

The PUTCALL asset consists of positions in both put and call options nd the underlying stock. Specifically, I take a long position of $Pos^{PC}_{C,OTM} = 1$ contract of the OTM call, a position of $Pos^{PC}_{P,OTM} = -\upsilon_{C,OTM}/\upsilon_{P,OTM}$ contracts in the OTM put, and a stock position of $Pos = -(Pos^{PC}_{C,OTM}\Delta_{C,OTM} + Pos^{PC}_{P,OTM}\Delta_{P,OTM})$ shares of the stock. The position in the OTM put is constructed in a way to remove any exposure of the asset to changes in the implied volatility of the underlying security as the sum of the vega exposures of the options times the position sizes is 0. The position in the stock is set to the negative of the sum of the option delta exposures times the position sizes in order to remove any exposure to changes in the price of the underlying stock. As a result skewness matters in valuation. The value of the PUTCALL asset changes whenever there is a change in the left or

[5] One might argue that Greeks are not reliable and maybe i should use spot-strike ratio. Following Bali and Murray (2013), I choose Greeks instead of spot-strike ratios in order for the OTM options to have strike prices at approximately the same location in the cumulative distribution function of the future stock returns.

[6] See Grundy and Martin (2001), Demir et al. (2004) and Henker et al. (2010)

right tail of the risk-neutral density.

Computation of skewness asset

When computing the skewness of the PUTCALL asset, I depart from the Bali and Murray (2013) method in the following respect which is related to the evaluaion of the volatility, cubic and quartic contracts as defined in Bakshi et al. (2003). They employ a rectangular approximation whereas I adopt the method proposed in the second chapter since it has been shown to be more accurate numerically for risk-neutral skewness.[7]

Finally, I follow Goyal and Saretto (2009) and Bali and Murray (2013) in calculating skewness asset excess returns as follows:

$$\text{excess return} = \frac{\text{Payoff} - \text{Price}}{|\text{Price}|} - (e^{rt} - 1), \tag{3.4}$$

where Payoff is the sum of all positions multiplied by the payoffs of all securities comprising the skewness assets at option expiry date, Price is the sum of all positions multiplied by the market prices of the securities comprising the skewness asset when they are created, and r is the average risk-free rate over the holding period.[8]

Proxy for investor heterogeneity

I quantify investor heterogeneity using four different proxies. A popular proxy often adopted in the literature for investor belief is the size of a firm (see, for example, Diether et al. (2002) and Baik and Park (2003)). The intuition is that financial analysts and media coverage tend to follow large firms and, as a result, there is more information in the marketplace and investors demonstrate less dispersion in their beliefs about large and well-covered firms compared to small firms.

The second proxy is related to the first but from the adverse selection perspective. Chung et al.

[7]A robustness test is performed using the rectangural approach and the results remain qualitatively the same.

[8] The options usually expire on the Saturday after the third Friday of each month. Hence we have a holding period of two months and we use the average risk-free rate over the holding period.

(1995) argue that market makers observe the number of financial analysts for a particular firm and this helps them determine the level of information asymmetry and deduce the extent of the adverse selection problem. Hence market makers are able to quote a narrow bid-ask spread on stocks with a large number of financial analysts because these are the stocks with least information asymmetry and are least likely to suffer from the adverse selection. As a result, investor heterogeneity is negatively related to the number of financial analysts, which is negatively related to the bid-ask spread. Hence there is a positive relation between investor heterogeneity and the bid-ask spread.

The two final proxies adopted are the OTM put-call volume difference and the OTM put-call open interest difference. Option volume is the number of contracts being exchanged by traders whereas open interest is the number of options that are still open. The original proxies in the literature are the ratios between the OTM put and call volume and that between the OTM put and call option interest (see Friesen et al. (2014) and Billingsley and Chance (1998)). However, I modify the two proxies since some options have zero volume and/or open interest. Hence, if the OTM put-call volume difference is negative (positive), it indicates that more calls (puts) are being traded than puts (calls) and this in turn implies optimism (pessimism). In the same way, if the difference between the OTM put and call open interest is negative (positive), it indicates optimism (pessimism) among investors. Numerous studies link trading volume to investor preferences and show that high trading volume reflects overconfidence.[9] I choose to include zero volume options in my analysis since it may refer to those investors who demonstrate a low level of overconfidence.

Another suitable proxy for heterogeneous beliefs could be the variance of analysts' forecast as in Diether et al. (2003). Unfortunately Essex Business School does not subscribe to I/B/E/S where one can download data to perform the analysis. However Diether et al. (2003) conclude that small stocks are the most heterogeneous. Therefore, using the size of a firm as a proxy for heterogeneous beliefs may cover this case too.

[9] See Statman et al. (2006), Gervais and Odean (2001).

3.3 Data and empirical analysis

3.3.1 Data and Descriptive Statistics

I employ the equity options database of OptionMetrics for the US market with 1-month time to maturity for the period between January 1996 to October 2012.[10] The delta, vega, strike price, best bid, and best offer of 1-month options are retrieved from OptionMetrics. I extract the spot prices of underlying stocks from the Center for Research in Security Prices (CRSP), and daily risk-free rates from the data library of Kenneth R. French.

The approach proposed in the second chapter is used to calculate the volatility, cubic and quartic contracts from option prices which provides more accurate estimates for risk-neutral skewness. The risk-neutral skewness is calculated on the first trading day after the monthly expiration day using OTM options which expire in the following month.[11] I allow a lag of one day to compute the risk-neutral moments to avoid microstructure bias as in Bali and Murray (2013). Ni et al. (2005) find that a bias may be present since option trading changes the prices of underlying stocks and on expiration dates stock prices cluster at their respective strike prices. I use one-month options and require at least two OTM call and two OTM put options to be available to calculate the RNS for each stock/month expiration combination. Dennis and Mayhew (2002) show that it is more accurate to evaluate skewness when one uses at least 2 OTM call and 2 OTM put options. I compute the price of an option as the average of bid and offer prices. I remove options with a missing bid or offer price, a non-positive bid price, an offer price less than or equal to the bid price, a spread bid-ask spread less than the minimum spread (0.05 for options with prices below 3.00, and 0.10 for options with prices greater than or equal to 3.00).

One of the assumptions for computing the risk-neutral moments is that one works under the mar-

[10]Unfortunately, due to budget constraints, Essex Business School does not subscribe to Optionmetrcs since the end of 2013. Optionmetrics database is updated once a year, therefore I was not able to update the sampling period up to 2016 when this thesis was submitted.

[11] The first period is used as the formation period. This follows Bali and Murray (2013).

tingale pricing measure. Thus, I remove all options which violate arbitrage conditions. Following Bali and Murray (2013), I require that the bid price for call option must be less than the spot price and the offer price be at least as large as the spot price minus the strike. For put options, the bid price must be less than the strike and the offer price be at least as large as the strike price minus the spot price. Any other variable with invalid data is omitted from the sample.

Following Bali and Murray (2009) and Goyal and Saretto (2009), I calculate risk-neutral skewness on the first trading day after option expiry to avoid potential microstructural noise. I form skewness assets and sort them into quintile portfolios based on the risk-neutral skewness on the second trading day after option expiry. On the first trading after the expiration day I calculate skewness. The next trading day I form skewness assets that expire next month and sort them in portfolios based on skewness I calculate the previous day. The portfolios are held unchanged until the expiration date.[12] I sort skewness assets into quintile portfolios based on their risk-neutral skewness. The final dataset consists of 62,631 stock/month expiration combination for all assets compared with the total of 57,537 employed by Bali and Murray (2013). Table 3.1 reports the summary statistics for the Deltas of the options, the portfolio positions in the stock and the options, and the risk-neutral moments.

I notice that even though I target a Delta of 0.1 for an OTM call option, the average is 0.12 as I work with market data. The call option average Delta deviating from the target is also documented in Table 1 in Bali and Murray (2013). However, I am closer in achieving the target Delta for put options with an average of -0.10 for OTM put options when the target is -0.1 as in Bali and Murray (2013).

Bali and Murray attribute this deviation to a mispricing for call options. Doran, Fondor and Jiang (2013) find that mispricing among put options is limited compared to call options. The results in Table 3.1 support these findings as well.

Positions in the stock and the options in Table 3.1 show significant variation. For example, the

[12] One of the assumptions that Bakshi, Kapadia and Madan (2003) make is that options are European so we do not consider early exercise.

Table 3.1. Summary statistics for option Deltas, option positions, and the risk-neutral moments

This table reports the mean, the minimum, the 5th, 25th, 50th, 75th, and 95th percentiles, and the maximum for option Deltas, option positions in the stock and the OTM options for the PUTCALL asset. The risk-neutral skewness (RNS) is also reported which is calculated on the first trading day after the expiration date using options with one-month to expiry. The sample period is from January 1996 to October 2012.

	Mean	Min	5%	25%	50%	75%	95%	Max
PUTCALL Asset								
OTM Call Delta	0.12	0.02	0.03	0.07	0.12	0.18	0.24	0.26
OTM PUT Delta	-0.10	-0.26	-0.23	-0.16	-0.09	-0.05	-0.03	-0.02
Position in OTM Put	-1.54	-6.66	-4.14	-2.02	-1.15	-0.69	-0.35	-0.15
Position in Stock	-0.24	-1.05	-0.42	-0.33	-0.23	-0.14	-0.08	-0.04
RNS	-1.20	-5.41	-3.22	-2.05	-1.20	-0.33	0.83	1.98

position in OTM stock (put) ranges from -1.05 to -0.04 (-6.66 to -0.15). Skewness of the risk-neutral distribution is negative consistent with existing studies.

3.3.2 Skewness Preference and Heterogeneous Beliefs

Table 3.2 contains the set of results that underline one of the major contributions of our paper. This table reports the summary statistics for skewness asset excess returns for each quintile portfolio, with P1 (P5) corresponding to the most negatively skewed (least negatively skewed or positively skewed) assets. As I have options in the skewness assets, the leverage in options means that the excess returns can be less than -100% or more than 100%, as in Table 2 of Bali and Murray (2013). This table indicates a monotone pattern in the excess returns from P1 to P5. The return differential is -6.77% per month with a Newey-West t-statistic of -4.12. The risk-adjusted return, the Alpha from the Capital Asset Pricing Model (CAPM), the Fama-French three-factor model, and the Carhart four-factor model, is always highly significant at the 1% level. CAPM alpha is the intercept from the regressions of $P5 - P1$ return on the market risk premium. FF3 alpha intercept

Table 3.2. Summary statistics for skewness asset returns

This table reports the mean, the minimum, the 5th, 25th, 50th, 75th, and 95th percentiles, and the maximum of monthly excess returns in percent for quintile portfolios formed on risk-neutral skewness (RNS). P1 contains assets with the lowest (most negative) RNS while P5 contains assets with the highest (least negative or most positive) RNS. The RNS and RNK are calculated for each stock on the first trading day after the expiration date using options with one-month to expiry, and the portfolios are formed on the same day. The portfolios are held unchanged until the expiration date to compute excess returns. The table also reports risk-adjusted alphas from the Capital Asset Pricing Model of Sharpe (1964), the Fama and French (1993) 3-factor model and the Carhart (1997) 4-factor model. The Newey and West (1987) t-statistics are reported in the parentheses. The sample period is from January 1996 to October 2012.

	Mean	Min	5%	25%	50%	75%	95%	Max
PUTCALL Asset								
P1	-0.48	-618.25	-16.34	-1.69	2.49	7.71	16.81	56.75
P2	-1.97	-412.26	-19.94	-3.45	0.51	5.87	14.68	99.08
P3	-3.31	-520.47	-19.27	-5.13	-0.59	4.66	13.40	83.10
P4	-3.12	-239.36	-18.53	-6.00	-1.16	3.42	10.92	101.13
P5	-7.26	-509.58	-31.53	-7.63	-2.78	1.74	7.78	40.15
P5-P1	-6.77	(-4.12)						
CAPM α	-6.91	(-4.18)						
FF3F α	-7.17	(-4.27)						
FFC4F α	-7.33	(-4.89)						
Option component	-1.89	(-0.01)						
Stock component	-0.03	(-5.47)						

from the regressions of $P5 - P1$ return on the market risk premium, size and value factor. FF3 alpha intercept from the regressions of $P5 - P1$ return on the market risk premium, size, value and momentum factor. Second, the findings also indicate that investors have positive skewness preference as portfolios with higher skewness offer lower returns on average than those with lower skewness consistent with Bali and Murray and Conrad et al. (2013).

Although skewness assets consist of options and stocks, it is the position in stocks that drive our results. This can be seen in the more significant t-statistic for the stock component. The position for the option component is insignificant whereas the stock component has a Newey-West t-statistic of -5.47.

How can I rationalize the empirical findings that investors exhibit positive skewness preference? My conjecture is that this is due to overconfidence and heterogeneity about the fair value of the stock. The evidence is reported in Table 3.3.

Each month I calculate the average value of each proxy for each quintile portfolio. For example I calculate the average size of firms comprising the P5 portfolio each month. I do the same for the P1 skeweness portfolio. Then, each month I calculate the difference $P5 - P1$ and regress it on a constant. The coefficients together with the Newey-West t-statistics are reported in Panel A.

The results indicate that the bid-ask spread difference is insignificant but the market capitalization of stocks in P5 is significantly lower than that in P1 with a t-statistic of -3.65. Also, the difference between call and put options for P5 is larger then P1, indicating that P5 is the portfolio with heterogeneous investors. As a result P5 is the portfolio with the lowest average return.

Overconfident traders believe that their signals about the fair value of an asset are very accurate. As a result they hold more units of the asset which makes it overvalued. Therefore one way of testing how overconfident traders invest, is to identify the skewness portfolio in which they long more skewness.

Each month I calculate the average value of each position for each quintile portfolio and regress the difference of the position for P5 and P1 on a constant. The coefficients together with the Newey-West t-statistics are reported in Panel B.

Table 3.3. Portfolio positions and proxy for investor heterogeneous belief

In Panel A, we report proxies for heterogeneous beliefs such as the market capitalization (Mkt Cap in millions), the spread, and the option volume and open interest are average time series differences between P5 and P1. In Panel B we report the positions in stock and options as the average time series differences between P5 and P1. The Newey and West (1987) t-statistics are reported in the parentheses. The sample period is from January 1996 to October 2012.

	PUTCALL Asset
Panel A	
Market Cap	-51,776
	(-3.65)
Stock spread	-0.01
	(-1.22)
OTM spread	-0.00
	(-1.80)
Option volume difference	46.49
	(3.39)
Option open interest difference	329.49
	(3.79)
Panel B	
Position in stock	-0.02
	(-8.92)
Position in OTM put	0.24
	(12.09)

Investors short more stocks and less put options for the P5 portfolio.[13] In other words they long more skewness for this portfolio and this reflects overconfidence.[14] This is consistent with the models in Barberis and Huang (2008) and Brunnermeier, Gollier and Parker (2007). They show that investors who prefer positively skewed distributions may hold concentrated positions in right skewed assets which will incresase the demand for theses assets and reduce their expected returns.

3.3.3 Heterogeneous beliefs and the existing risk-neutral literature

The results may help explain why the empirical results on skewness are mixed. I now discuss the findings of the existing literature in the risk-neutral framework. Dennis and Mayhew (2002) study the source of bias when they calculate skewness using the BKM method. They find that the discreteness of strike prices and the unequal number of OTM calls and puts used to calculate skewness introduces a bias. They propose to use at least two and equal number of OTM calls and puts and options with similar distance between the strike prices and the price of the underlying stock, to minimize this bias as much as possible. There has been a divergence among researchers on this matter. Some use unequal number of OTM calls and puts to calculate skewness despite Dennis and Mayhew show that, using the largest range of strike prices such that the domain of integration is symmetric for the volatility, cubic and quartic contracts, minimizes the bias introduced by asymmetry. Using market data one cannot achieve a perfectly symmetric domain of integration but can make it as symmetric as possible by using an equal number of OTM calls and puts. To my knowledge, only Conrad et al. (2013) state explicitly that they use an equal number of OTM calls and puts.[15] If one uses more puts (calls) than calls (puts), this may reflect pessimistic (optimistic) expectations.[16] This along with the dispersion in beliefs and the overconfidence for the top

[13] A similar pattern is observed in Bali and Murray (2013) in their Table 7.

[14] Recall that the stock component drives the returns.

[15] They also use these options which have the most similar distance from stock to strike price to mitigate the bias introduced by the dicreteness of strike prices.

[16] See Bandopadhyaya and Jones (2011) and Billingsley and Chance (1998).

skewness portfolio may cause an upward or downward bias which leads to a positive or negative skewness preference, respectively.

However previous researchers agree on the high dispersion in beliefs for the top skewness portfolio but used in a different way (i.e a proxy used for dispersion in beliefs such as the size of a firm was used as a liquidity control). BM find in their Table 9 that small firms and options with bigger spreads lie in the top skewness portfolio. Conrad et al. (2013) report in their Table X that small firms and firms with high book-to-market ratio lie in the top skewness portfolio. Diether et al. (2002), Baik and Park (2003) and Doukas et al. (2004) find that stocks with high book-to-market ratios demonstrate greater belief heterogeneity than stocks with low book-to-market ratio. Stilger et al. (2017) report in their Table 2 that the top skewness portfolio consists of small firms and firms with high idiosyncratic volatility. Harris and Raviv (1993) and Diether et al. (2002) find stocks with a higher dispersion of opinion demonstrate higher volatility.

3.4 Robustness tests

This section reports robustness tests on our findings. In particular it studies the robustness of the results when less OTM options are chosen, when they have positive open interest, across different periods, in the presence of transaction costs, when one uses the rectangural approximation to to evaluate the volatility, cubic and quartic contracts and when one controls for the other moments of the risk-neutral distribution.

3.4.1 Different choice of Delta and Positive Open Interest

When the PUTCALL asset was formed, I chose OTM options to be those with delta closest to $|0.1|$. Next I choose OTM options to have delta closest to $|0.2|$. In other words, I choose options which are less OTM as in BM.

Market frictions are important for understanding the cross-section of expected returns.[17] I study

[17]See Miller (1977), Merton (1987), Basak and Cuoko (1998), Shapiro (2002)

Table 3.4. Skewness asset returns when targeting $|\Delta| = 0.2$ for OTM options and when options have positive open interest

In this table, P1 contains assets with the lowest (most negative) RNS while P5 contains assets with the highest (least negative or most positive) RNS calculated for each stock on the first trading day after the expiration date using options with one-month to expiry, and the portfolios are formed on the same day. In Panel A, the OTM options have a Δ close to 0.2 for calls and -0.2 for puts. In Panel B, the options have positive open interest. The portfolios are held unchanged until the expiration date to compute excess returns. The table also reports risk-adjusted alphas from the Capital Asset Pricing Model of Sharpe (1964), the Fama and French (1993) 3-factor model and the Carhart (1997) 4-factor model. The Newey and West (1987) t-statistics are reported in the parentheses. The sample period is from January 1996 to October 2012.

	PUTCALL Asset
Panel A. $\|\Delta\| = 0.2$	
P5-P1	-7.97
	(-5.05)
CAPM α	-8.15
	(-5.13)
FF3F α	-8.37
	(-5.16)
FFC4F α	-8.51
	(-5.86)
Panel B. Positive open interest	
P5-P1	-7.51
	(-4.75)
CAPM α	-7.45
	(-4.84)
FF3F α	-7.39
	(-4.91)
FFC4F α	-7.07
	(-5.28)

the robustness of my findings where market frictions should be less of an issue. I include in the sample those options which have positive open interest as in BM and this restriction yields a total 40,432 stock/month expiration combinations for all assets. Table 3.4 reports the skewness preference for skewness assets when I select options with deltas close to $|0.2|$ and positive open interest.

Table 3.4 shows that even when we target an other delta for OTM options, the initial findings remain qualitatively the same.

3.4.2 Subsample analysis

The dataset contains two crisis periods. The first one is the internet bubble in the late 1990s and the second is the one triggered by the collapse of Lehman Brothers. As the risk-neutral moments of equity stocks are closely associated with and shaped by market events (Birru and Figlewski (2012)), this may change the order of sorting and as a result the skewness preference. Therefore I divide our dataset in two periods to study the effect of the two crises on the skewness preference. The first one is February 1996 - March 2000 as in Conrad et al. (2013) and the second one is July 2007-October 2010 as in Bali and Murray (2013). Table 3.5 reports the skewness preference for each asset in each crisis period.

In each crisis period, the skewness preference remains the same. Furthermore those investors who are overconfident follow the same strategy and long more skewness.

3.4.3 Transaction costs

The findings indicate that market frictions do not affect my results. However I have not evaluated how much an investor would have to pay and keep her investment strategy profitable. To do that I study how much of the quoted half spread (25%, 50%, 75%, 100%) an investor should pay and make profit out of her investment strategy as in Bali and Murray (2013). $Payoff$ in equation (3.4) is calculated now as the sum of all positions multiplied by the payoffs of all securities comprising the skewness asset at option expiry date minus the sum of all half option spread comprising the asset. $Price$ in the same equation is calculated in a similar way. Then I calculate the return difference P1-P5 and use the the FFC4 model to evaluate alphas along with the Newey and West (1987) t-statistic for the P1-P5 portfolio.

Table 3.6 indicates that investors should employ a sophisticated investment strategy to reduce transaction costs for their strategy to remain profitable. Even when they pay 25% of the half spread the investment strategy is not profitable. This is also consistent with Korajczyk and Sadka (2004) who find that trading costs may lead to zero abnormal returns.

Table 3.5. Skewness asset excess returns and proxy for investor heterogeneous belief: Sub-sample analysis

This Table reports excess returns (P5-P1), the risk-adjusted alphas from the Capital Asset Pricing Model of Sharpe (1964), the Fama and French (1993) 3-factor model and the Carhart (1997) 4-factor model and positions in the PUTCALL asset. Panel A is for the period from January 1996 to March 2000 while Panel B is between July 2007 to October 2010. The Newey and West (1987) t-statistics are reported in the parentheses.

	PUTCALL Asset
Panel A. January 1996 - March 2000	
P5-P1	-14.88
	(-3.49)
CAPM α	-16.78
	(-3.56)
FF3F α	-16.41
	(-3.92)
FFC4F α	-16.07
	(-4.92)
Position in stock	-0.02
	(-3.42)
Position in OTM put	0.17
	(4.94)
Panel B. July 2007 - October 2010	
P5-P1	-8.12
	(-3.14)
CAPM α	-8.03
	(-3.19)
FF3F α	-8.83
	(-3.40)
FFC4F α	-8.40
	(-3.79)
Position in stock	-0.03
	(-8.67)
Position in OTM put	0.18
	(5.60)

Table 3.6. Excess returns for skewness excess with transaction cost

This table reports the effect of transaction cost on the profit when we compute P1-P5. We study the effect when an investor has to pay 25%, 50%, 75%, and 100% of half the quoted spread of the options to enter the option positions. The values in the parentheses are the Newey and West (1987) t-statistic.

	0	25%	50%	75%	100%
PUTCALL Asset	6.77	-0.97	-2.02	-3.07	-4.11
	(4.12)	(-10.58)	(-11.03)	(-11.17)	(-11.22)

3.4.4 Skewness preference when one uses the BKM approach

Next we study whether the results are the same when one uses the Bakshi et al. (2003) approach to evaluate the volatility, cubic and quartic contracts. Table 3.7 reports the results of this robustness test.

Results remain qualitatively the same. Overconfident investors hold more units of the P5 portfolio and it is the most heterogeneous. This results in overvaluation and yields a positive skewness preference.

3.4.5 Control of other moments of the distribution

The final robustness test I perform is to study whether the other moments are driving the results. In order to do that, I perform Fama and MacBeth (1973) regressions of the PUTCALL asset returns on skewness and controls for the mean, volatility, and kurtosis of the distribution of future stock returns. I control for the mean of the distribution of stock returns using the log return of the underlying stock during the 1-month (Ret1M). I control for the second and fourth moment by evaluating the implied volatility and kurtosis as in Bakshi et al. (2003). Table 3.8 reports the results of this robustness test.

Table 3.8 supports the intitial findings. Even when one controls for other moments of the risk-neutral distribution, the relation between skewness and returns is negative and significant.

Table 3.7. Skewness preference using the Bakshi et al. (2003) approach

Panel A reports excess returns (P5-P1), the risk-adjusted alphas from the Capital Asset Pricing Model of Sharpe (1964), the Fama and French (1993) 3-factor model and the Carhart (1997) 4-factor model. Panel B reports proxies for heterogeneous beliefs such as the market capitalization (Mkt Cap in millions), the spread, and the option volume and open interest are average time series differences between P5 and P1. Panel C reports the positions in stock and options as the average time series differences between P5 and P1. The Newey and West (1987) t-statistics are reported in the parentheses. The sample period is from January 1996 to October 2012.

	PUTCALL Asset
Panel A	
P5-P1	-5.34
	(-3.12)
CAPM α	-5.58
	(-3.32)
FF3F α	-5.93
	(-3.45)
FFC4F α	-6.02
	(-3.91)
Panel B	
Market Cap	-48,324
	(-2.88)
Stock spread	-0.05
	(-1.36)
OTM spread	-0.02
	(-1.65)
Option volume difference	53.42
	(3.84)
Option open interest difference	381.23
	(2.97)
Panel C	
Position in stock	-0.15
	(-5.47)
Position in OTM put	0.21
	(9.84)

Table 3.8. Controls for other moments of the distribution

Table 3.8 reports the effects of controlling for other moments of the distribution of stock returns in analyzing the ability of skewness (RNS) to predict returns. Controls for the mean by calculating the 1-month return (RET1M) of the underlying stock. It controls for volatility by calculating the Bakshi et al. (2003) volatility (RNV). It controls for kurtosis by calculating the Bakshi et al. (2003) kurtosis (RNK). It reports the results of Fama and MacBeth (1973) regressions, controlling for each of the variables. The values in brackets are the Newey and West (1987) t-statistic.

	PUTCALL Asset
RNS	-0.46
	(-3.35)
RET1M	-0.002
	(-0.21)
RNV	0.03
	(2.14)
RNK	-0.16
	(-3.16)
Intercept	-0.18
	(-0.65)

3.5 Conclusion

Working with US equity options for the period January 1996-October 2012, this paper seeks to explain the positive skewness preference documented in the literature. Results indicate that there is a group which is overconfident. (See De Long, Shleifer, Summers and Waldman(1990), Scheinkman and Xiong (2003), and Nagel (200)). This group overestimates its own private signals which leads to a high dispersion in beliefs. As a result it holds more units of the asset which leads to an overvaluation based on the expectations they demonstrate.

The overconfident investors hold more units of the positively skewed securities because they disagree about their fair value which results in an overvaluation. This yields a positive skewness preference. The findings strongly indicate that the overvaluation takes place in the absence of a risk premium. In other words the investors do not hold more units of the risky asset because it is attractive, but rather because they overestimate their own private signals. The findings are robust across different parameters.

Chapter 4

Heterogeneous beliefs and relation between risk-neutral kurtosis and returns

4.1 Introduction

The characteristics and behaviour of equity options have attracted the attention of researchers, especially in the last decade. Even though huge progress has been made to incorporate the effect of risk-neutral moments in predicting returns, there is not much research about the relation between the higher risk-neutral moments and returns except for skewness and volatility.

Baskhi, Kapadia and Madan (BKM) (2003) develop a methodology for calculating risk-neutral moments (RNMs) which are extracted from option prices. This methodology mitigates higher moment errors associated with historical estimates. It is the most widespread method employed by researchers for calculating risk-neutral moments.[1] In the second chapter I propose a method which reduces to zero the discretization errors and yields more accurate estimates for skewness and kurtosis. Therefore I employ this to calculate the RNMs.

Miller (1977), Morris (1996), Chen et al. (2001), and Viswanathan (2001) argue that, if pessimistic investors are kept out of the market by imposing short-sale constraints, prices will reflect a more

[1] See Bali and Hovakimian (2009), Dennis and Mayhew (2002), Han (2008), Yan (2011)

optimistic valuation. The stock will be overvalued due to the disagreement about its fair value, which will result in lower future returns. Shapiro (2009) shows that it is not the imposition of short-sale constraints which cause the over-valuation but the evolution of heterogeneous beliefs.

In a risk-neutral economy investors expect the same risk-free rate of return. Third chapter explains how the incompleteness of a market allows for misvaluation and heterogeneous beliefs to develop, based on the first fundamental asset pricing theorem. It studies the positive skewness preference documented in existing studies and establishes that there is a group which is overconfident, causes heterogeneous beliefs and overestimates the value of a stock which results in a positive skewness preference.

The objective of this chapter is to study the relation between risk-neutral kurtosis and returns. However there is little research about the relation between risk-neutral kurtosis and returns. To my knowledge, only Conrad et al. (2013) find a positive relation between kurtosis and returns. They find that the relation becomes weaker after controlling for variations on skewness and volatility. This chapter extends the previous work of Conrad et al. (2013) and seeks to explain the relation between kurtosis and returns by linking the results to heterogeneous beliefs and overconfidence.

The findings indicate that there is no clear pattern for the relation between kurtosis and returns. The relation can be either positive or inverse U-shaped. The results indicate that there are important interactions between risk-neutral skewness and kurtosis and these interactions may affect the relation between kurtosis and returns. Consistent with Conrad et al. (2013), there is weak evidence for an independent relation between kurtosis and returns. I find that the relation between kurtosis and returns holds when one controls for variation in all moments except for skeweness and cannot be explained by heterogeneous beliefs and overconfidence.

The remainder of this chapter is organized as follows. Section 4.2 explains how I construct the asset, calculate returns and what proxies are used for heterogeneous beliefs. Section 4.3 provides the details of the dataset and an empirical analysis of the results. Section 4.4 conducts robustness tests and section 4.5 concludes.

4.2 Methodology

This section expains how I construct the assets, calculate returns and what proxies are used for heterogeneous beliefs.

4.2.1 Asset formation and returns

I follow a broadly similar approach to Bali and Murray (2013) to construct the PUTCALL asset. However I construct delta-neutral portfolios instead of delta-vega neutral portfolios as in Bali and Murray (2013). Since kurtosis is the volatility of volatility, i construct delta-neutral to add the effect of volatility on valuation. If one constructs delta-vega neutral, volatility will not play any role in valuation as explained by Bali and Murray (2013).

The asset is very similarly defined to those in Bali and Murray (2013). There are two differences however. The first one is that I take a position in the OTM call which is the inverse of the one in Bali and Murray. The second diference is that I follow Conrad et al. (2013) and depart from BM in using equal OTM call and put options in calculating kurtosis. I use these options which have the most similar distance between the strike and the spot price as in Conrad et al. (2013). The bias introduced in Dennis and Mayhew (2002) by an assymetric domain of integration is mitigated this way. I follow Bali and Murray (2013) and Goyal and Saretto (2009) in calculating returns.

$$\text{excess return} = \frac{\text{Payoff} - \text{Price}}{|\text{Price}|} - (e^{rt} - 1), \tag{4.1}$$

4.2.2 Proxy for heterogeneous beliefs

I use four different proxies to quantify investor heterogeneity. These are the size of a firm[2], the bid-ask spread[3], the OTM put-call difference and the OTM put-call option open interest difference.[4] Stocks with small size are associated with high dispersion in beliefs. Financial analyst and media

[2]See Diether et al. (2002) and Baik and Park (2003).
[3]See Chung et al. (1995).
[4]See Friesen et al. (2014) and Billingsley et al. (1998).

press cover large firms which results in less information for the small firms. This causes divergence in opinions for small firms.

Large spreads also indicate heterogeneous beliefs. Market makers quote a narrow bid-ask spread for those stocks with the least information assymetry. Since these stocks have the least information assymetry, stocks with wide bid-ask spreads demonstrate high dispersion in beliefs.

Finally large OTM put-call differences are associated with high dispersion in beliefs. If the OTM put-call volume difference is negative (positive), it indicates more calls (puts) are being traded than puts (calls) and this implies optimism (pessimism). A similar explanation applies for the other proxy.[5]

4.3 Data and empirical analysis

4.3.1 Data and descriptive statistics

I employ the Optionmetrics database and download equity options from US stocks for the period January 1996 to October 2012. The delta, vega, strike price, best bid, and best offer of all options are retrieved from OptionMetrics. I extract the spot prices of underlying stocks from the Center for Research in Security Prices (CRSP), and daily risk-free rates from the data library of Kenneth French.

I follow the approach in the second chapter in evaluating the volatility, cubic and quartic contracts. This approach has been shown to provide more accurate estimates for risk-neutral skewness and kurtosis. The risk-neutral moments are calculated on the first trading day after the monthly expiration day using OTM options which expire in the following month as in Bali and Murray (2013). I use one-month options and remove options with a missing bid or offer price, a non-positive bid price, an offer price less than or equal to the bid price, a bid-ask spread less than the minimum spread (0.05 for options with prices below 3.00, and 0.10 for options with prices greater

[5] The literature has used ratios instead of differences. I modify the two proxies since some options have zero volume and/or open interest as in the third chapter.

than or equal to 3.00). I remove all these options which violate arbitrage conditions. [6]

Following Bali and Murray (2013) and Goyal and Saretto (2009), I calculate risk-neutral moments on the first trading day after the option expiration date. I form kurtosis assets and sort them into kurtosis portfolios following a different approach than in Conrad et al. (2013). When they sort their portfolios on kurtosis only, they document a positive relation between kurtosis and returns and the high kurtosis portfolio produces higher returns than the low kurtosis portfolio. When they double-sort on kurtosis and volatility, keeping the top tercile volatility constant, the relation between kurtosis and returns is concave. They also find that, after they control for interactions between risk-neutral moments, there is weak evidence for an independent relation between risk-neutral kurtosis and returns. Risk-neutral moments are forward looking and incorporate information about expectations.[7] As a result when one adds other moments in order to sort, they get a clearer picture about each portfolio's expectations. The empirical findings in Conrad et al. (2013) and the extra information one gets about expectations by adding the other two moments in the sorting process, motivates me to sort the portfolios first on kurtosis, then on skewness and finally on volatility. For example, I sort assets based on kurtosis first. Then I form portfolios based on kurtosis terciles. P1(P3) portfolio is the one with the lowest (highest) kurtosis. Then within each tercile I sort assets based on skewness and form tercile portoflios. Finally within each tercile I do the same about volatility. That way I construct $3x3x3$ portfolios for each month. The portfolios are held unchanged until the expiration date. The final dataset consists of 62,631 stock/month expiration combinations for all assets. Table 4.1 reports the summary statistics for the deltas of the options, the portfolio positions in the stock and the options, and the risk-neutral moments.

I notice that even though I target an OTM Delta call of 0.1, the average is 0.12. Bali and Murray (2013) also have the call option average Delta deviating from the target in their Table 1. Positions in the stock and the options in Table 4.1 show significant variation. For example, the position in

[6] Following Bali and Murray (2013) I require that the bid price for call option must be less than the spot price and the offer price be at least as large as the spot price minus the strike. For put options, the bid price must be less than the strike and the offer price be at least as large as the strike price minus the spot price. Any other variable with invalid data is removed from our dataset.

[7] See Bates (1995) and Birru and Figlewski (2012)

Table 4.1. Summary statistics for option Deltas, option positions, and the risk-neutral moments

This table reports the mean, the minimum, the 5th, 25th, 50th, 75th, and 95th percentiles, and the maximum for option Deltas, option positions in the stock and the ATM options for the PUTCALL asset. The risk-neutral volatility (RNV), skewness (RNS) and kurtosis (RNK) are reported. They are calculated on the first trading day after the expiration date using options with one-month to expiry. The sample period is from January 1996 to October 2012.

	Mean	Min	5%	25%	50%	75%	95%	Max
PUTCALL Asset								
OTM Delta Call	0.12	0.02	0.03	0.07	0.12	0.18	0.24	0.26
OTM Delta Put	-0.10	-0.26	-0.23	-0.16	-0.09	-0.05	-0.03	-0.02
Position OTM Put	-1.11	-6.65	-2.86	-1.44	-0.87	-0.49	-0.24	-0.15
Position Stock	-0.28	-1.74	-0.61	-0.35	-0.24	-0.17	-0.10	-0.04
RNV	0.019	0.000	0.002	0.006	0.011	0.023	0.061	0.363
RNS	-1.20	-5.41	-3.22	-2.05	-1.20	-0.33	0.83	1.98
RNK	4.92	-19.99	-7.78	2.14	5.05	8.20	16.35	54.96

the OTM put ranges from -6.65 to -0.15. The range for the stock position ranges from -1.74 to -0.04. This range departs from Bali and Murray (2013) since we construct delta-neutral portfolios and not delta-vega. As a result the positions are different.

There is significant variation for risk-neutral moments as well. The risk-neutral distribution is negatively skewed with tails fatter than the normal, consistent with existing studies (Bali and Murray (2013), Conrad et al. (2013)). The average value of skewness is -1.20 and the average value of kurtosis is 4.92.

4.3.2 Kurtosis asset returns

Table 4.2 reports the results which contain one of the major contrubutions in this paper. This table reports the summary statistics for kurtosis asset excess returns for each tercile portfolio, with P1 (P3) corresponding to the assets with the lowest (highest) kurtosis. Long and short positions in options are taken and as a result the excess returns can be less than -100% or more than 100%, as in Table 2 of Bali and Murray (2013).

There is a hump U-shaped relation between kurtosis and returns. A strategy of longing (shorting)

Table 4.2. Summary statistics for kurtosis asset returns

This table reports the mean, the minimum, the 5th, 25th, 50th, 75th, and 95th percentiles, and the maximum of monthly excess returns in percent for tercile portfolios formed on risk-neutral kurtosis (RNK). P1 contains assets with the lowest (most negative) RNK while P3 contains assets with the highest (most positive) RNK. The RNV, RNS and RNK are calculated for each stock on the first trading day after the expiration date using options with one-month to expiry, and the portfolios are formed on the same day. The portfolios are held unchanged until the expiration date to compute excess returns. The table also reports risk-adjusted alphas from the Capital Asset Pricing Model of Sharpe (1964), the Fama and French (1993) 3-factor model, the Carhart (1997) 4-factor model and the differences P2-P1 for the call and putcall asset. The Newey and West (1987) t-statistics are reported in the parentheses. The sample period is from January 1996 to October 2012.

	Mean	Min	5%	25%	50%	75%	95%	Max
PUTCALL Asset								
$P1$	-3.14	-313.14	-21.77	-5.46	-0.80	3.51	12.51	71.79
$P2$	-1.64	-246.82	-20.63	-4.88	-0.04	4.21	12.73	65.34
$P3$	-2.32	-283.79	-20.75	-5.23	-0.60	4.40	13.95	96.75
$P3 - P1$	0.82	(0.98)						
$P2 - P1$	1.50	(2.43)						
CAPM α	1.53	(2.43)						
FF3F α	1.55	(2.47)						
FFC4F α	1.55	(2.49)						
Option Component	20.19	(1.15)						
Stock Component	0.01	(2.28)						

P2 (P1) earns a statistically significant profit of 1.5%. The u-shaped relation I find is in contrast with Conrad et al. (2013) where they find a positive relation between kurtosis and returns. A reason for this, might be that I use the approach in the second approach to evaluate the contracts in the BKM framework. Conrad et al. (2013) use the trapezium rule to evaluate these contracts.

A startegy of longing (shorting) P3 (P1) yields an insignificant relation between kurtosis and returns. This indicates that when one uses approximation methods to evaluate the volatility, cubic and quartic contracts, which may result in a mispriced kurtosis, may not be the only reason we know very little about the relation between kurtosis and returns. Maybe previous researchers looked for a monotonicity pattern. Based on my third chapter's findings, the monotonicity is caused when heterogeneous investors invest in the top or bottom skewness portfolio. What if they decide to invest in the mid-skewness portfolio? The relation will not be monotonic. This may help explain the kurtosis preference I find in Table 4.2.

Next I study whether the option or the stock component drives the returns since portfolios consist of stocks and options. The stock component drives the returns, consistent with the findings in chapter 3, since the t-statistic of the stock component is more significant than the one of the option component.

How can the relation between kurtosis and returns be explained? One might argue that since kurtosis reflects the risk of a stock, I should maybe look for a risk-based explanation. However in the risk-neutral framework, prices do not depend on risk and Bali and Murray (2013) find that there is no risk-based explanation for the relation between skewness and returns.

I find in the third chapter that there is a group which is overconfident, causes heterogeneous beliefs and overvalues the price of a stock. This results in a positive skewness preference. I follow this approach in seeking to explain the relation between kurtosis and returns. The results are reported in Table 4.3.

P2 portfolio is the most heterogeneous since it consists of options with large spreads. This portfolio is the one that exhibits higher returns however. According to heterogeneous belief literature, it should yield lower returns. The results in Panel B indicate that heterogeneous beliefs are not

Table 4.3. Portfolio positions and proxies for investor heterogeneous belief

In Panel A, I report proxies for heterogeneous beliefs such as the market capitalization (Mkt Cap in millions), the spread, and the option volume and open interest are average time series differences between P2 and P1. In Panel B I report the positions in stock and options as the average time series differences between P2 and P1. The Newey and West (1987) t-statistics are reported in the parentheses. The sample period is from January 1996 to October 2012.

	PUTCALL Asset
Panel A	
Market Cap	11,963
	(1.23)
Stock spread	0.02
	(2.60)
OTM spread	0.01
	(6.33)
Option volume difference	-12.49
	(-0.98)
Option open interest difference	-65.31
	(-0.69)
Panel B	
Position in stock	-0.01
	(-0.75)
Position in OTM put	0.06
	(4.66)

caused by overconfidence since investors short more OTM puts for portfolio P1. In other words the U-shape relation between kurtosis and returns can not be explained by heterogeneous beliefs and overconfidence.

4.4 Robustness tests

This section reports robustness tests on our findings. In particular it studies the robustness of the results when less OTM options are chosen, when they have positive open interest, across different periods, for a different way of sorting. It also studies any possible interaction among risk-neutral moments and the robustness of the results when one controls for the other moments of the risk-neutral distribution.

4.4.1 Different choice of Delta and positive open interest

So far, I chose OTM options to be those with delta closest to $|0.1|$. I now choose OTM options to have delta closest to $|0.2|$. In other words, I choose options which are less OTM as in BM. I also study whether the results are robust when market frictions are less of an issue. There is a strand in the literature which focuses on the importance of frictions on portfolio selection and prices.[8] I study the relation between kurtosis and returns when market frictions are less of an issue by choosing those options with positive open interest as in Bali and Murray (2013).

Table 4.4 shows that even when I target an other delta for OTM options, the initial findings remain qualitatively the same. When market frictions are less of an issue the relation remains the same.

4.4.2 Subsample analysis

The dataset contains two crisis periods. The first one is the internet bubble in the late 1990s and the second is that triggered by the collapse of Lehman Brothers. As the risk-neutral moments of equity stocks are closely associated with and shaped by market events (Birru and Figlewski

[8] See Miller (1977), Merton (1987), Basak and Cuoco (1998) among others.

Table 4.4. Kurtosis asset returns when targeting $|\Delta| = 0.2$ for OTM options and when options have positive open interest

This table reports the P2 - P1 excess return. RNK is calculated for each stock on the first trading day after the expiration date using options with one-month to expiry, and the portfolios are formed on the same day. In Panel A, the OTM options have a Δ close to 0.2 for calls and -0.2 for puts. In Panel B, the options have positive open interest. The portfolios are held unchanged until the expiration date to compute excess returns. The table also reports risk-adjusted alphas from the Capital Asset Pricing Model of Sharpe (1964), the Fama and French (1993) 3-factor model and the Carhart (1997) 4-factor model. The Newey and West (1987) t-statistics are reported in the parentheses. The sample period is from January 1996 to October 2012.

	PUTCALL Asset
Panel A. $\|\Delta\| = 0.2$	
CAPM α	1.02
	(1.81)
FF3 α	1.02
	(1.80)
FFC4 α	0.98
	(1.76)
Panel B. Positive open interest	
CAPM α	1.50
	(2.27)
FF3 α	1.52
	(2.33)
FFC4 α	1.55
	(2.54)

Table 4.5. Kurtosis asset excess returns: Sub-sample analysis

This table reports excess returns (P3-P1) for the PUT asset, (P2-P1) for the PUTCALL and CALL asset, the risk-adjusted alphas from the Capital Asset Pricing Model of Sharpe (1964), the Fama and French (1993) 3-factor model and the Carhart (1997) 4-factor model. Panel A is for the period from January 1996 to March 2000 while Panel B is between July 2007 to October 2010. The Newey and West (1987) t-statistics are reported in the parentheses.

	PUTCALL Asset
Panel A. January 1996 - March 2000	
CAPM α	2.77
	(1.51)
FF3 α	2.15
	(1.37)
FFC4 α	3.28
	(1.86)
Panel B. July 2007 - October 2010	
CAPM α	0.79
	(0.58)
FF3 α	0.46
	(0.23)
FFC4 α	-0.48
	(-0.21)

(2012)), this may change the order of sorting and as a result the relation between kurtosis and returns. Therefore I divide our dataset in two periods to study the effect of the two crises on this relation. The first one is February 1996 - March 2000 as in Conrad et al. (2013) and the second one is July 2007-October 2010 as in Bali and Murray (2013). Table 4.5 reports the relation between kurtosis and returns for each asset in each crisis period.

In the dot com crisis period, which covers the period from January 1996 to March 2000, the relation between kurtosis and returns remains the same. The relation is insignificant in the second period however.

4.4.3 Different method of sorting

When I sorted the securities on risk-neutral moments, I sorted first on kurtosis, then on skewness and finally on volatility. Next I change the order and sort first on kurtosis, then on volatility and finally on skewness. Table 4.6 reports the relation between kurtosis and returns for this method of

Table 4.6. Kurtosis asset excess returns from a different sort

In this table I report excess returns (P3 - P1) the risk-adjusted alphas from the Capital Asset Pricing Model of Sharpe (1964), the Fama and French (1993) 3-factor model and the Carhart (1997) 4-factor model, positions of the kurtosis assets, when one sorts first on kurtosis then on volatility and finally on skewness. The sample period is from January 1996 to October 2012. The Newey and West (1987) t-statistics are reported in the parentheses.

	PUTCALL Asset
Panel A	
$P1$	-2.78
$P2$	-1.69
$P3$	1.10
$P3 - P1$	3.88 (4.97)
Panel B	
Market Cap	-4,312
	(-0.36)
Stock spread	0.01
	(1.36)
OTM spread	0.03
	(8.69)
Option volume difference	-18.85
	(-1.37)
Option open interest difference	-115.13
	(-1.09)
Panel C	
Position in stock	0.05
	(14.95)
Position in OTM put	0.18
	(10.54)

sorting.

The relation between kurtosis and returns is different now. The findings now indicate a positive relation between kurtosis and returns, consistent with the Conrad et al. (2013). This change may be due to a relation to a possible interaction among the risk-neutral moments which I study in the next sub-section.

4.4.4 Interaction among risk-neutral moments

Byun and Kim (2013) find that risk-neutral skewness and future volatility are related. In particular they show that these two moments demonstrate a positive relation. Neumann and Skiadopoulos

(2013) find that the the higher order risk-neutral moments can be statistically forecasted and economically exploited for short horizon investments. BKM developed their formulae for the RNMs under all martingale pricing measures. In the martingale framework knowledge of past events never helps predict future events. One might argue that since the RNMs are calculated in this framework, it is not possible to use one moment to forecast the other. However BKM applied the martingale expectation operator for each moment separately when they developed their method. This simply means that their method does not guarantee that a moment is a martingale sequence with respect to another moment. In other words, their method does not ensure that past movement of one moment does not help predict the future movements of another moment. Therefore from an empirical perspective there is indication that the moments may be related to each other and from a methodological perspective there is a gap which allows for the RNMs to mutually interact and this interaction may play a role in asset pricing.

To test the hypothesis of a possible interaction between the RNMs, I perform Granger (1969) causality tests. This test is used to see whether one can use a series to forecast another. First I have to test whether the RNMs are stationary processes. Every month I calculate time series monthly RNMs for all the stocks available comprising this month. By doing that, I test if one can use a moment today to forecast an other moment a month later. Table 4.7 reports the Phillips and Perron (1988), Granger causality tests and the Pearson correlation coefficients of the RNMs for the period March 1996-October 2012.

Table 4.7 shows that the RNMs are stationary consistent with Lynch and Panigirtzoglou (2003). The results show that one can use the one moment to forecast the other in many cases. The RNS can forecast the RNV and RNK and the RNK can forecast the RNV and RNS. RNV is uncorrelated with RNS and RNK since the correlation coefficient is -0.08 and 0.01 respectively. Skewness exhibits a stronger negative correlation with kurtosis since the correlation coefficient is -0.64.

The results shed light to some reasons which may help explain why Bali and Murray (2013) find significant kurtosis for the PUTCALL asset when they perform robustness tests to show that no other moment drives the returns. They attribute this to a mispricing in the right tail of the risk-

Table 4.7. Interaction among risk-neutral moments.

This table reports the results of Philips-Perron unit root test and the Granger-Causality tests for the time series average monthly RNMs. The RNV, RNS and RNK are the risk-neutral volatility, risk-neutral skewness and risk-neutral kurtosis respectively. Panel A reports the p-values. Panel B reports the p-values of the null hypothesis of the Granger causality test. The RNMs on the vertical axis for Panel B are those used to forecast the RNMs on the horizontal axis. Panel C reports the Pearson correlation coefficient of the RNMs. The dataset spans the period January 1996-October 2012.

	PUTCALL Asset		
Panel A. Unit root test			
RNV	-4.11		
	(0.00)		
RNS	-12.65		
	(0.00)		
RNK	-14.15		
	(0.00)		
Panel B. Granger causality test			
	RNV	RNS	RNK
RNV	-	0.81	0.53
RNS	0.00	-	0.01
RNK	0.05	0.00	-
Panel C. Pearson correlation coefficient			
	RNV	RNS	RNK
RNV	1.00	-0.08	0.01
RNS	-0.08	1.00	-0.64
RNK	0.01	-0.64	1.00

Table 4.8. Controls for other moments of the distribution

Table 4.8 reports the effects of controlling for other moments of the distribution of stock returns in analyzing the ability of kurtosis (RNK) to predict returns. Controls for the mean by calculating the 1-month return (RET1M) of the underlying stock. It controls for volatility by calculating the Bakshi et al. (2003) volatility (RNV). It controls for skewness by calculating the skewness (RNS). P2-P1 (P3-P1) reports the excess returns of portfolios when they are sorted on kurtosis first, on skeweness(volatility) then and finally on volatility(skeweness). It reports the results of Fama and MacBeth (1973) regressions, controlling for each of the variables. The values in brackets are the Newey and West (1987) t-statistic.

	P2-P1	**P2-P1**	**P3-P1**	**P3-P1**
RNK	0.22	0.65	0.33	0.98
	(1.45)	(1.98)	(1.28)	(2.15)
RET1M	0.06	0.03	0.04	0.02
	(0.31)	(0.21)	(0.82)	(0.18)
RNV	0.04	0.02	0.06	0.05
	(1.98)	(2.41)	(2.19)	(1.82)
RNS	-0.12		-0.18	
	(-3.36)		(-2.56)	()
Intercept	0.11	-0.06	0.03	(0.02)
	(1.21)	(-1.18)	(0.96)	(1.04)

neutral distribution. There is an interaction among the moments which may change the relation between kurtosis and returns. As Conrad et al. (2013) argue, this interaction makes it difficult to isolate the pricing effect of kurtosis only.

4.4.5 Controlling for other moments

The results in Table 4.7 show that the interaction between skewness and kurtosis is stronger than the one they have with volatility. This indicates that skweness may play a role in the relation between kurtosis and returns. The final robustness test performed is the one that controls for other moments of risk-neutral distribution. In particular it studies the concave and positive relation in two cases, when one controls for the variation in all moments and when one controls for the variation in all moments except for skewness. Table 4.8 reports the results of this robustness test. When one controls for the variation of all moments, the relation between kurtosis and returns is insignificant. When one controls for the variation of all moments except for skeweness, the concave and positive relation is statistically significant, indicating that probably there is not an independent relation between kurtosis and returns, consistent with Conrad et al. (2013).

4.5 Conclusions

Using a sample of US equity options for the period January 1996 - October 2012, this paper studies the relation between kurtosis and returns.

The hitherto study from Conrad et al. (2013) on this relation show a positive relation between kurtosis and returns. It also provides evidence that there is probably not an independent relation implying that other moments, among other factors, play a role too.

The results of this chapter indicate that the relation can be either positive or concave. Motivated by Conrad et al a test on possible interactions among risk-neutral moments shows that there are important interactions between the risk-neutral skewness and kurtosis. Testing whether other moments may drive the relations I find, the results show that , as in Conrad et al, the relation between keurtosis and returns is not independent.

Chapter 5

Concluding remarks

5.1 Conclusions

Over the last two decades a substantial part of the financial lieterature has employed the risk-neutral density framework to study the higher moments of options. The merits of this framework include the incorporation of information about expectations when prices do not depend on risk and the fact that risk-neutral moments are shown to have some capability in predicting returns and future market events. Despite the significant progress made in this area, many findings are mixed and inconclusive.

This thesis makes two overarching contributions relative to the extant literature. One of the major problems in inferring risk-neutral densities is the limited number of strike prices available. These can give rise to discretization and truncation errors, respectively. Existing studies tackle these errors by interpolating between quoted exercise prices and extrapolating beyond the final OTM option strike. The first contrbution in Chapter 2 is to propose exact solutions for the finite integrals in the volatility, cubic and quartic contracts that are germane to evaluate their respective risk-neutral higher moments. The proposed solutions avoids discretization errors. The upshot of these exact solutions is a more accurate estimation of skewness and kurtosis (skewness) for assets which exhibit the volatilty smirk (forward skew).

The second overarching contribution involves combining a new element that is applied to ivestigating risk-neutral skewness and kurtosis in Chapters 3 and 4, respectively. This new element involves adopting a behavioural framework with overconfidence and heterogeneous beliefs to risk-neutral skewness and kurtosis. For instance, the overconfident investors cause high dispersion in beliefs for positively skewed securities which yield an overvaluation and leads to a positive skewness preference.

The final contribution is that the relation between risk-neutral kurtosis and returns can be either positive or concave. This thesis presents evidence that there is not probably an independent relation between kurtosis and returns and there are important interaction especially between skewness and kurtosis. The relations between kurtosis and returns are not explained by overconfidence and heterogeneous beliefs.

5.2 Future research

An immediate avenue for future research emerges from the results in Chapter 3 in which investors disagree about the fair value of a stock. In particular, they disagree about the future payment of dividends. It would be interesting to relax the assumption of no dividends in Bakshi et al. (2003) and give a clearer picture of which investors cause the overvaluation. Are the ones who are expecting payment of dividends or not?

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