Lucas and Stokey (1983) model details

Alex Clymo & Andrea Lanteri

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1 Environment

The government's budget constraint is

$$q_t b_t + \tau_t l_t \geq g_t + b_{t-1}$$

Where $b > 0$ is government debt. To avoid transversality issues the government faces exogenous borrowing limits

$$\bar{b}_l \leq b_t \leq \bar{b}_h$$

The private sector can be summarised by

$$q_t = \beta \frac{u_{c_{t+1}}}{u_{c_t}}$$

$$\frac{v_{l_t}}{u_{c_t}} = 1 - \tau_t$$

$$c_t + g_t = l_t$$

Combining these:

$$u_{c_t} b_{t-1} - u_{c_t} c_t + v_{l_t} l_t \leq \beta u_{c_{t+1}} b_t$$

2 Full commitment solution

Constraint:

$$u_{c_0} b_{-1} \leq \sum_{t=0}^{\infty} \beta^t \left(u_{c_t} c_t - v_{l_t}(c_t + g_t) \right) \tag{1}$$

Maximisation:

$$L = \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - v(c_t + g_t) \right) + \Delta \left(\sum_{t=0}^{\infty} \beta^t \left(u_{c_t} c_t - v_{l_t}(c_t + g_t) \right) - u_{c_0} b_{-1} \right) \tag{2}$$

Period $t > 0$:

$$(1 + \Delta)(u_{c_t} - v_{l_t}) + \Delta(u_{c_t, c_t} c_t - v_{l_t, l_t} l_t) = 0 \tag{3}$$

Period 0:

$$(1+\Delta)(u_{c_0} - v_{l_0}) + \Delta(u_{c_0,c_0}c_0 - v_{l_0,l_0}l_0 - u_{c_0,c_0}b_{-1}) = 0 \tag{4}$$

For constant g we have $c_0 = c_0(b_{-1}, \Delta)$ and $c_t = c(b_{-1}, \Delta)$ for $t > 0$. Solve for Δ by noting that the (binding) implementability becomes:

$$u_{c_0}b_{-1} = u_{c_0}c_0 - v_{l_0}(c_0 + g) + \frac{\beta}{1-\beta}\left(u_c c - v_l(c+g)\right) \tag{5}$$