Simultaneous Reporting of Credit Ratings*

Luís Vasconcelos† Tianxi Wang‡

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Abstract

We show that requiring a credit rating agency to rate several new issuers simultaneously discourages rating inflation and increases the credibility of its ratings. When the agency rates issuers simultaneously, giving one more good rating lowers the credibility of all the good ratings it gives, diminishing issuers' willingness to pay for them and consequently the rating fee. If the number of issuers is large, this effect ensures an allocation that asymptotically approaches the first best. While the effect is present under sequential rating, it is weaker than under simultaneous rating. We show that it may be worthwhile to synchronize the issuance of corporate bonds so as to benefit from simultaneous rating.

Keywords: Credit Rating Agencies, Simultaneous Rating, Credibility of Credit Ratings, Synchronization of Debt Issuance.

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†Department of Economics, University of Essex, Wivenhoe Park, Colchester, Essex CO4 3SQ, UK. Email: lvasco@essex.ac.uk

‡Department of Economics, University of Essex, Wivenhoe Park, Colchester, Essex CO4 3SQ, UK. Email: wangt@essex.ac.uk

1 Introduction

Credit rating agencies play an important role in the functioning of financial markets. They evaluate issuers' creditworthiness and contribute to a more efficient allocation of capital. The performance of this role by rating agencies critically depends on the credibility of their ratings. In other words, their ratings must be trusted. After all, as emphasized by the rating agencies themselves, their ratings constitute only an opinion on borrowers' ability to repay their debts.

However, after the 2008 financial crisis, the impartiality of rating agencies and the credibility of their ratings have been widely and openly questioned. Doubts on the credibility of credit ratings have been cast in great part because of concerns and evidence of rating inflation during the years leading to the crisis. For example, in the case of Alt-A mortgage-backed securities (MBS), about 10% of the tranches issued in the period 2005-2007 rated safest –triple AAA–were either downgraded to junk status or lost their principal by 2009. The case of CDO bonds was no better. More than 71.3% of such bonds had the same fate despite being initially rated as Aaa.[1,2] Credit rating agencies have also been involved in lawsuits for the issues of rating inflation. In 2008 a group of investors led by the Abu Dhabi Commercial Bank sued Moody's and Standard & Poor's, accusing them of collaborating with Morgan Stanley in arranging for some of its financial products to receive triple-A ratings, even though much of the underlying collateral was low-quality or subprime mortgage debt.[3] More recently, in February 2013, the U.S. Department of Justice sued Standard & Poor's accusing it of inflating ratings associated with mortgage securities for the purpose of gaining market share.[4]

The current practice in the financial markets is that rating agencies rate new issuers on the market (and investors decide whether to invest in each issuer) sequentially, i.e. as they appear on the market. We argue in this paper that rating inflation can be discouraged and the credibility of ratings improved by requiring a credit rating agency to report the ratings of a pool of new issuers *simultaneously* so that investors make decisions on whether to take *any one* of their offers only after observing the ratings given to *all of them*. In particular, we show that it may be worthwhile to synchronize the issuance of corporate bonds so as to

[1] See the "Final Report of the National Commission on the Causes of the Financial and Economics Crisis in the United States", pages 228-229. This report is authenticated U.S. government information.

[2] As a reference point, observe that the historic cumulative default rate (up to 2007) of corporate bonds rated AAA by Standard & Poor's is only 0.6%. In the case of Moody's the analogous figure is 0.52%. (See the "House Report 110-835 - Municipal Bond Fairness Act" of September 2008, U.S. Government Printing Office.)

[3] The case is Abu Dhabi Commercial Bank et al v. Morgan Stanley & Co et al, U.S. District Court, Southern District of New York, No. 08-07508. The parties reached a settlement agreement in April 2013. The settlement amount was almost 9.5 million dollars.

[4] The case is United States of America v. McGraw-Hill Companies, Inc and Standard and Poor's Financial Services LLC, U.S. District Court, General District of California, No. CV13-00779.

benefit from simultaneous rating.

The key effect is that simultaneous rating generates a negative link between the number of good ratings that a rating agency reports and the value of each of its good ratings. In other words, for a given pool of issuers simultaneously rated by the agency, the more issuers receive a good rating, the less credible a good rating becomes. Because of this negative link, the rating agency faces a trade-off. By giving one more good rating it earns one more rating fee, but that lowers the credibility of all the good ratings it gives, which lowers the issuers' willingness to pay for its ratings (and consequently its rating fees). Because of this trade-off, which arises endogenously, the rating agency has an incentive to issue a limited number of good ratings.

We show that even when the rating agency rates only two issuers simultaneously, the case with the lowest possible "level" of simultaneity in the assignment of ratings, this negative link enables the agency to create value in a situation where it would not if it rated the issuers sequentially. The negative link is particularly effective when the pool of issuers simultaneously rated is large. In this case, the value created by the rating agency asymptotically approaches the first-best total surplus.

One key element of simultaneous rating is that investors decide whether to take one offer of bond issuance only after having observed the ratings of *all* the offers in the pool. This contrasts with the current practice of *sequential rating* where the decision on whether to invest in one issuance is made when the investors observe its rating *alone* and possibly the ratings of past issuances. One may expect that the negative link is also present in the case of sequential rating, as investors can adjust their trust of the present rating based on their observation of the past ratings given by the same agency. We show the negative link is present under sequential rating, but it is less effective than under simultaneous rating.

In fact we show that even if investors observe not only the past ratings but also the performance of the projects rated in the past, in which case the reputation mechanism is present, the value created under sequential rating can be at most a fraction of the first-best surplus. Moreover, this fraction is independent of the discount factor. By contrast, as mentioned above, the value created by ratings under simultaneous rating approaches the first-best surplus. This contrast suggests that it might be worthwhile to change the way in which credit rating is done in order to realize the benefit of simultaneous rating. Specifically rating should be done once in a period when a number of new issuers have entered the market, so that they can be simultaneously rated; and in the report the rating agency should disclose the number of issuers that seek a rating from it and the distribution of ratings that it gives to them. Doing so would lead to synchronization of bond issuance. Therefore, this paper suggests another benefit to synchronizing bond issuance, which practitioners have been considering mainly for the benefit of improving liquidity.[5]

[5] See the article "The Debt Penalty" in the *Financial Times*, 11 September 2013.

One obvious cost of rating new issuers periodically is that for those that enter the market early on the access to the financial market and the implementation of their projects are delayed. Nevertheless, given the frequency with which rating agencies issue new ratings, this delay need not to be long. A period of one week might suffice. Over 2001-2010, Moody's, S&P and Fitch issued on average per week, respectively, 285, 275 and 141 corporate bond ratings.[6] Using these numbers, we perform a numerical simulation to compare sequential rating with simultaneous rating where ratings are issued once a week (e.g. the ratings of each week's projects are simultaneously reported at the end of the week). While the surplus generated by sequential rating can be at most 90% of the surplus associated with the first-best allocation, the surplus generated by weekly (simultaneous) rating would be no less than 95% of the first-best surplus for agencies that rate as many issuers as Moody's or S&P and no less than 93% for agencies of the size of Fitch. These results indicate an efficiency gain of weekly simultaneous rating relative to sequential rating of approximately five percentage points in the case of Moody's and S&P, and of three percentage points in the case of Fitch. In all the three cases, the benefit stemming from the economy of scale brought about by simultaneous rating more than compensates the cost of delays in the implementation of projects.

The remainder of the paper is organized as follows. In Section 2, we review the related literature. In Section 3, we present the model used to study simultaneous rating. This model is analyzed in Section 4, where we highlight the importance of the negative link for the credibility of ratings. In Section 5, we analyze sequential rating and its shortcomings. Section 6 discusses the implementation of simultaneous rating in practice. In Section 7 we discuss some robustness issues related to our key findings and Section 8 concludes. All the proofs are given in the Appendix.

2 Related Literature

We show that simultaneous reporting of credit ratings generates a mechanism that discourages rating inflation by a rating agency, thereby contributing to increase the credibility of its ratings. Other mechanisms have been considered in the literature. The most notable of them is the reputation mechanism – see, among others, Kuhner (2001); Mathis, McAndrews and Rochet (2009); Bar-Isaac and Shapiro (2013); Bolton, Freixas and Shapiro (2012); and Frenkel (2015). The reputation mechanism highlights that a rating agency will refrain from giving a good rating to a bad issuer because of the concern that its future credibility and profit will be damaged following the (likely) default by the issuer.[7] This mechanism relies on two factors: repeated interaction and the comparison between the ratings obtained by issuers

[6] These numbers are obtained from the FISD data set.

[7] Interestingly, Frenkel (2015) shows that when a rating agency has two reputations–one with investors and another with issuers–reputation concerns may actually lead to the inflation of ratings by the rating agency.

and their ex-post performance. The mechanism studied in this paper depends on neither of them. It works in a static environment and even when issuers' performance is not observed by investors. While in most of this paper we abstract from the reputation mechanism, we also analyze the case where both mechanisms are present. In particular, we show that a credit rating agency can create more value under simultaneous rating than under sequential rating even when the reputation mechanism is present.[8]

The literature has also highlighted that the credibility of credit ratings could be improved by addressing, in the first place, the conflict of interest that may generate the bias in the ratings. One source of the conflict of interest is the fact that credit rating agencies are paid by issuers – precisely those who they rate – and that such payment usually occurs only if the issuer accepts the rating. Griffin and Tang (2011), for example, provide empirical evidence of rating inflation due to conflict of interest by comparing the CDO assumptions made by the ratings department and by the surveillance department within the same rating agency. Xia and Strobl (2012) provide empirical evidence for rating inflation due to the issuer-pay model by comparing the ratings issued by Standard & Poor's which follows the issuer-pay model to those issued by the Egan-Jones Rating Company which adopts the investor-pay model.

One way of solving such conflict of interests is to require that investors, rather than issuers, pay for credit ratings, as was the case before the 1970s. In a similar spirit, Mathis, McAndrews and Rochet (2009) advocate a new business model in which the platforms where the securities are traded pay for the securities' ratings. Yet another way of reducing the above mentioned conflict of interest is to require issuers to pay an up-front rating fee before receiving their ratings so that the fee is not a concern to the rating agency when it decides on the issuers' ratings. This suggestion has been partially implemented in the Cuomo agreement of 2008.[9] In the present paper, we assume that issuers pay for their ratings, that rating fees can depend on ratings, and that the credit rating agency does not disclose a (bad) rating that the issuer refuses to accept. Hence, we purposely analyze the potential benefits of simultaneous rating in an environment that is highly conducive to rating inflation. We show that simultaneous rating can complement these insitutional arrangements.

Several articles have also studied the impact of competition on the credibility and informativeness of ratings provided by a credit rating agency or more generally by a certifier (e.g., Lizzeri, 1999; Miao, 2009; Skreta and Veldkamp, 2009; Camanho, Deb and Liu, 2012; Bolton, Freixas and Shapiro, 2012). While Lizzeri (1999) shows that competition between

[8]In this literature and in our paper the credit quality of the issuers is assumed to be exogenous. Goldstein and Huang (2017), however, consider how it may be affected by credit ratings through a feedback loop. They show that because of this feedback loop the presence of credit rating agencies may reduce economic efficiency.

[9]In 2008, Andrew Cuomo, then the New York State Attorney General, reached an agreement with the three major rating agencies, Moody's, S&P, and Fitch. The rating agencies agreed that, in rating Residential MBSs, they would charge up-front fees and disclose every rating deal, even those that were not accepted by the issuers. For more details see the Attorney General's office press release at http://www.ag.ny.gov/press-release/attorney-general-cuomo-announces-landmark-reform-agreements-nations-three-principal.

certifiers can lead to full information revelation, Skreta and Veldkamp (2009), Camanho, Deb and Liu (2012) and Bolton, Freixas and Shapiro (2012) show that competition between credit rating agencies can in fact decrease the informativeness of credit ratings and the reputation of the rating agencies.[10] In Skreta and Veldkamp (2009) and Bolton, Freixas and Shapiro (2012), this is because competition allows for credit rating shopping. In Camanho, Deb and Liu (2012) it is because it hinders rating agencies' ability to sustain a high reputation. An important difference amongst these articles is that Lizzeri (1999) assumes that the certifier can commit to a disclosure rule and the other articles assume that it cannot.[11] While we abstract from competition issues in this paper, we also assume that rating agencies cannot commit to a disclosure rule. They are free to give an issuer any rating after having observed its credit quality.

Finally, the paper is related with a strand of literature that explores the idea that information transmission and efficiency may be enhanced by joining the decisions on several matters (e.g., Damiano, Li and Suen, 2008; Chakraborty and Harbaugh, 2007; Jackson and Sonnenschein, 2007). The papers in this literature, however, ignore the specificities of the market for credit ratings and, as such, fail to identify the negative link which plays a central role in the present paper.[12]

3 Model

The model presented here captures the case of simultaneous rating. In the model, a credit rating agency evaluates the creditworthiness of several firms simultaneously, and then investors decide whether or not to invest in each firm. We later adapt the model to analyze the case of sequential rating.

Suppose there are $N \geq 2$ penniless issuers (firms), a pool of investors, and one credit rating

[10] The empirical evidence on the effect of competition on ratings' quality is mixed. Becker and Milbourn (2011) find that increased competition due to Fitch's entry in the credit ratings market in 1997 as a global and full-service ratings agency coincides with lower quality ratings from the incumbents S&P and Moody's. Like S&P and Moody's, Fitch is an issuer-paid rating agency. In contrast, Xia (2014) finds that the entry in the credit ratings market of EJR, an investor-paid rating agency, led to an improvement in S&P's ratings quality.

[11] Another important difference is that Lizzeri (1999) takes a mechanism design approach to the modelling of the certifier. Faure-Grimaud, Peyrache and Quesada (2009) is another example of an article that follows this line of modelling certifiers. They also use a mechanism design approach to model the certifier and assume that the certifier can commit to a disclosure rule.

[12] Damiano, Li and Suen (2008) compare an agency that rates several clients separately with one that rates all clients together. However, they do not model the market for credit ratings and consider a costly signaling model as in Spence (1973) where a rating affects the agency's payoff in itself. By contrast, we consider a model of cheap-talk where ratings affect the rating agency only through equilibrium market interactions. Chakraborty and Harbaugh (2007) analyze a communication model between a sender and a receiver. Jackson and Sonnenschein (2007) use a mechanism design approach to analyze a social choice problem.

agency (CRA hereafter).[13] All agents are risk neutral and protected by limited liability, and the risk free interest rate is normalized to zero. Each issuer has one investment project and seeks to finance it. A project requires an investment of one unit of funds and either succeeds and returns R, or fails and returns nothing. Projects can be of two types: good (g) or bad (b). A project of type $i \in \{g, b\}$ succeeds with probability q_i. We assume that good projects have a positive net present value (NPV), while bad projects destroy value, i.e.,

$$q_b R < 1 < q_g R. \tag{1}$$

In what follows, we denote by V_i the value created by a project of type i, i.e., $V_i \equiv q_i R - 1$ for all $i \in \{g, b\}$. It is common knowledge that ex ante a project is good with probability p and projects' qualities are independent. We assume

$$(pq_g + (1-p)q_b)\, R < 1, \tag{2}$$

which means that financing a project at random destroys value.

The quality of an issuer's project is known to the issuer only. However, issuers can hire the CRA to rate their projects before they seek funds from the investors. Each issuer decides whether to hire the CRA without observing the decision of the other issuers. If hired by an issuer, the CRA observes the quality of the issuer's project at no cost.[14] After observing the quality of the projects of all the issuers that solicited a rating, the CRA decides for each of them a rating $r \in$ {good,bad}. This decision determines the total number of good ratings k the CRA plans to issue. Then the CRA communicates to each issuer individually the decision on its rating and on the total number of good ratings k. Issuers that reject the rating owe nothing to the CRA and remain unrated. Issuers that accept the rating agree to pay the CRA a rating fee f and have their rating publicly disclosed. Since at this time issuers are penniless, they pay the fee ex post with the revenue of their projects. Rating fees are not observed by investors, and investors learn that an issuer has hired the CRA only if a rating for that issuer is publicly disclosed.

Upon observing *all* the ratings issued by the CRA (if any), investors decide which issuers they will fund (if any). If investors decide to fund an issuer they demand a repayment C, which by limited liability the issuer will pay if and only if its project succeeds. In other words, the issuer defaults if its project fails. We assume the market for capital is long on the

[13] We consider the case of a monopolist rating agency because it allows us to highlight the benefits of the simultaneity in the reporting of credit ratings in the simplest possible way. In Section 7, we discuss the implications of competition between rating agencies for our mechanism.

[14] The assumption that the CRA observes the quality of a project at no cost is made for simplicity of exposition. The assumption that the CRA evaluates a firm's project only if hired by the firm rules out the possibility of unsolicited credit ratings. For an analysis of why rating agencies may wish to issue unsolicited ratings and their impact on welfare see Fulghieri, Strobl and Xia (2013).

supply side. This means that investors have more funds than those that can be absorbed by firms and, as a result, they are satisfied with an expected net return of zero. Specifically, if investors believe that a given project (issuer) is good with probability ϕ, they will require a repayment C to invest in the project such that

$$(\phi q_g + (1-\phi)q_b)\, C - 1 = 0. \tag{3}$$

The lower the belief ϕ that the project is good, the higher the repayment they demand. For sufficiently low values of ϕ, that repayment exceeds R, in which case they refrain from investing in the project. The ratings issued by the CRA are important because they may affect the investors' beliefs about the quality of a project.

Given the investors' funding decisions, the funded projects (if any) are implemented. We assume that if a project succeeds, the issuer first pays investors the agreed repayment, then pays the CRA the agreed fee, and keeps the remainder as profit. If a project fails, because of limited liability, all parties obtain nothing. Hence, the expected payoff of an issuer whose project is of quality q, who pays f to the CRA for a rating, and whose project is financed at a repayment C is $q(R - C - f)$. The payoff of an issuer whose project is not financed is zero. Finally, the expected profit for the CRA from selling a rating at a fee f to an issuer of quality q is qf if his project is financed and zero if it is not.

We complete the description of the model by specifying how rating fees are set. We assume $f = \alpha(R - C)$, where $\alpha \in (0, 1]$. The proportion α (which can be interpreted as the commission rate usually charged by rating agencies) reflects the bargaining power of the CRA relative to the issuers.[15] The case of a CRA with full bargaining power that makes take it or leave it offers to issuers corresponds to the case where $\alpha = 1$. Observe that the rating fees considered here exhibit the following three features. First, they are paid ex post, out of the revenue of the project. Second, the fee paid by an issuer is related to his cost of finance, C. Third, an issuer pays the fee only if he accepts the rating offered by the CRA. However, the main message of the paper, namely that simultaneous rating enables the rating agency to create more economic value than sequential rating, is robust to alternative specifications of the rating fee. It continues to hold if: (a) the fee is of the same format as above but paid ex ante when the issuer accepts the rating (assuming the issuer has some funds to pay it); or (b) the fee is of a fixed amount (i.e., independent of C) and again paid when the issuer accepts the rating; or (c) the fee is paid up front when the issuer requests a rating and still does not know what the rating will be, as in the Cuomo agreement. We analyze case (a) in the discussion of Proposition 1, and cases (b) and (c) in Subsection 7.1 where we discuss the

[15] We will focus on equilibria where unrated issuers are not financed obtaining zero profit. Therefore, if a rating allows an issuer of quality q to finance his project at a repayment $C < R$, the (expected) value of the rating for the issuer is $q(R - C)$. The current rating fee implies than the CRA appropriates a proportion α of this value.

robustness of the results obtained in the paper.

The strategies of the issuers, CRA, and investors can be summarized as follows. Based on the quality of their project, an issuer first decides whether to request the CRA a rating and then whether to accept or reject it. The CRA, on the other hand, decides on the rating of the issuers who request one based on the qualities of their projects. Finally, investors decide on which projects to invest and on the repayment required based on the projects' ratings. We use Perfect Bayesian Equilibrium (PBE) as the equilibrium concept and focus on equilibria where a good rating signals good quality and a bad rating bad quality.[16] That is, we focus on equilibria where investors believe that an unrated issuer or an issuer with a bad rating is of good quality with a probability no greater than the prior p. Given this, issuers pay only for a good rating, and reject any offer of a bad rating.

4 Simultaneous Rating

In this section, we characterize the equilibrium outcomes in the model described above. We are interested in the CRA's rule for rating issuers, the informativeness of the CRA's ratings, and the efficiency in the allocation of funds. We analyze separately two cases regarding the total number of issuers. To illustrate how and why credit ratings can be informative and create value under simultaneous rating, we first analyze the case of two issuers (i.e., $N = 2$). We then analyze the case of a large number of issuers and provide asymptotic results on the value created by the CRA and on the efficiency.

But before proceeding to that analysis, it is useful to briefly mention two benchmark cases, one in which no CRA operates on the market, the other in which there is only one issuer (i.e. $N = 1$). In both cases, no value is created in equilibrium: either no project is financed, or the probability of a good project being financed and the probability of a bad project being financed are such that, in expectation, the value created by the implementation of good projects is completely dissipated by the implementation of bad projects. In the first benchmark case, this is essentially because funding an issuer at random destroys value. Regarding the second case, suppose the rating created value and therefore the CRA obtained a positive fee for it. Then the CRA would offer a good rating to the issuer regardless of his quality. But this means the rating would be uninformative and could not create value, contradicting the initial supposition.

[16] In the current framework the ratings of the CRA are no more than cheap talk. As a result, there exists an equilibrium in which they are totally disregarded by investors and the CRA just "babbles" (i.e., assigns them in a totally random way). Also, it is possible that equilibria in which investors believe that a good rating means something bad about the issuer's quality and a bad rating something good exist. In this paper, we discard all these equilibria.

4.1 Two Issuers

Suppose $N = 2$. Our first observation is that credit ratings cannot perfectly reveal the quality of all issuers. In other words, there is no equilibrium in which the CRA gives a good rating to an issuer if and only if the issuer is good.

To see this, suppose to the contrary that such an equilibrium exists. In it, investors necessarily believe that a project is good when the CRA rates it as good. That is, $\phi = 1$ for any project that receives a good rating. This means by (3) that investors require a repayment $C = 1/q_g < R$ to finance a project with a good rating. But given this repayment, in the event that no project is good, instead of offering no good rating obtaining no fee as the equilibrium prescribes, the CRA is better off deviating by offering a good rating to both issuers. By doing so, it obtains $f = \alpha(R - 1/q_g)$ for each good rating and an expected profit of $2q_b\alpha(R - 1/q_g) > 0$.

Credit ratings cannot fully eliminate the asymmetry of information between issuers and investors, but they can reduce it and create value. Let ϕ_k denote the investors' belief that an issuer with a good rating has a good project when k good ratings are issued by the CRA. Similarly, let C_k denote the repayment demanded by investors to fund an issuer with a good rating when k good ratings are issued by the CRA. Finally, let $\Delta \equiv q_g - q_b$, and observe that condition (2) can be written as $p < -V_b/(\Delta R)$ and that the ex-ante probability that at least one project is good is $1 - (1-p)^2 = p\,(2-p)$. We can claim the following.

Proposition 1 *Suppose $N = 2$. If $p\,(2-p) \leq -V_b/(\Delta R)$, then there is no equilibrium in which the CRA creates value. If $p\,(2-p) > -V_b/(\Delta R)$, however, there exist a continuum of value-creating equilibria. In any of these equilibria, the credibility of a good rating decreases and the repayment demanded by investors increases with the number of good ratings, i.e. $\phi_1 > \phi_2$ and $C_1 < C_2$. Moreover, in all but one of these equilibria, with a positive probability the CRA gives both issuers a good rating and both are financed at $C_2 < R$.*

When the ex-ante probability that *at least* one issuer is good is sufficiently low (i.e. lower than $-V_b/(\Delta R)$), it is very likely that all issuers on the market are bad and that any issuer the CRA rates as good is actually bad. In this case, investors will not trust the CRA's ratings and no value will be created. If instead the ex-ante probability that at least one issuer on the market is good exceeds $-V_b/(\Delta R)$, the ratings issued by the CRA can be credible enough to create value. Indeed, in this case, there exist a continuum of value-creating equilibria.[17]

The same two reasons explain why the CRA creates value in all the value-creating equilibria. The first is the existence of a negative link between the credibility (and value) of a good rating and the number of good ratings issued by the CRA. As stated in Proposition 1,

[17] A complete characterization of the equilibria in which the CRA creates value is given in the proof of Proposition 1 and the Online Appendix.

$\phi_1 > \phi_2$ and $C_1 < C_2$ in all value-creating equilibria, which implies that the rating fee if only one good rating is issued, $\alpha(R - C_1)$, is greater than that if two are issued, $\alpha(R - C_2)$. This link discourages rating inflation: by issuing one more good rating the CRA may collect one more fee, but the rating fee obtained per good rating is lower. The second reason is that, conditional on the decision to issue any given number of good ratings, the CRA gives them to the best issuers on the market. In this case with two issuers, this means that if the CRA issues only one good rating, it always gives it to a good issuer if there is one. The CRA does so because the expected profit from giving the good rating to a good issuer, $q_g\alpha(R - C_1)$, is greater than that from giving it to a bad issuer, $q_b\alpha(R - C_1)$.

Because of the first reason, the CRA does not give a good rating to all issuers regardless of their qualities. And because of the second reason, an issuer with a good rating is more likely to be good than an issuer without a good rating. Together, they imply that a good rating signals a quality better than the prior, giving good issuers a chance of being financed. Indeed, if the CRA gives only one issuer a good rating, this issuer will be the better of the two and it will be worth financing it. Interestingly, even though the ex-ante probability of both issuers being good is (considerably) lower than the probability of at least one issuer being good, in all but one equilibrium, with a positive probability the CRA rates both issuers as good and the ratings are trusted such that investors finance both issuers.

We conclude the analysis of this two-issuer case with two remarks. First, the timing of the rating fee payment matters for none of the two reasons why the CRA creates value. Therefore, it has no effect on the mechanism highlighted in the paper. Because in the model issuers are penniless at the outset, they can only pay the rating fees ex post with the revenue of their projects. If instead they had some funds and rating fees were paid at the moment of the issuance of the ratings (i.e., before projects being financed), the rating fee paid by an issuer of quality q would be $qf = \alpha q(R - C)$, since at that moment the value of a good rating for the issuer would be $q(R - C)$. Clearly, the negative link would still be present, as the rating fee would still be negatively related with C and hence with k. Moreover, also in this case, the CRA would be better off giving a good rating to a good issuer than to a bad issuer, as $\alpha q_g(R - C) > \alpha q_b(R - C)$.

Second, in our model the CRA is better off giving a good rating to a good issuer than to a bad issuer because a good issuer pays more (in expectation) for the rating. However, in reality, rating agencies may prefer to give a good rating to a good issuer than to a bad issuer also for other reasons. One of them is the preservation of a good reputation. Since a bad issuer is more likely to default than a good issuer, rating agencies will be more reluctant to give a good rating to a bad issuer than to a good issuer. Interestingly, what is important for the mechanism highlighted in the paper is that rating agencies have a preference for good issuers, not the specific reason for it.

4.2 A Large Number of Issuers

We now consider the case of a large number of issuers and obtain asymptotic results on the value created by the CRA under simultaneous rating.

We focus on equilibria where investors finance all the issuers with a good rating at a fixed repayment of $C < R$, so long as the total number of good ratings issued by the CRA does not exceed a threshold k. In other words, the CRA is asked to recommend at most k out of the N projects. The negative link, therefore, is that a good rating is worth $R - C$ ex post (conditional on the success of the issuer's project) if the total number of good ratings is no greater than k, otherwise it is worthless. In such equilibria the CRA does not miss the opportunity to earn one more rating fee and always recommends k issuers. As the CRA issues the same number of good ratings regardless of the number of good projects, we call equilibria of this type pooling equilibria. There is, however, a slight abuse of language in doing so. Pooling occurs only regarding the total number of good ratings issued by the CRA. The identity of the issuers that are rated as good depends on the realization of issuers' types. Finally, as argued above, the CRA will optimally give good ratings first to good issuers, and only if it cannot find k of them, it will fill the gap by giving the remaining good ratings to bad issuers. That is, the CRA will indeed recommend the *best* k amongst the N projects.

If the number of issuers N is sufficiently large, an equilibrium of this type always exists. Roughly speaking, by the Law of Large Numbers, the CRA will not have difficulty finding Np good projects. Thus, if k is close to Np, investors will correctly believe that a project recommended by the CRA is good with a high probability. In fact, for the same number of issuers N, pooling equilibria with a different threshold k coexist. An important aspect about pooling equilibria with different thresholds of good ratings is that they differ in the associated expected total surplus (i.e. in the level of efficiency). Given a threshold k, relative to the first-best allocation in which a project is financed if and only if it is good, there are two types of loss. One occurs whenever the number of good projects on the market is greater than k, in which case some of them do not receive a good rating, are not financed, and their NPV is lost. The other occurs when the number of good projects on the market is smaller than k, in which case the CRA will fill the gap with bad projects, which are then financed and destroy value. An increase in the value of k reduces the first type of loss but increases the second. For any given number of issuers N, the optimal value of k balances these two types of loss.

The next proposition characterizes how asymptotically the optimal k depends on N and how the efficiency in the optimal pooling equilibrium (i.e., the pooling equilibrium with the optimal k) approaches that of the first-best allocation.

Proposition 2 *Consider the case where the number of issuers N is large. The optimal threshold is asymptotically $k = Np + \lambda\sqrt{Np(1-p)}$, where λ is implicitly defined by $\Phi(\lambda) =$*

$V_g/(V_g - V_b)$ with $\Phi(.)$ denoting the c.d.f. of the standard Normal distribution. Moreover, in the pooling equilibrium with this optimal threshold: (i) the expected value created asymptotically approaches the expected value of the first-best allocation; and (ii) the probability that a project with a good rating is indeed good approaches one. In both cases, the approximation is in the order of $N^{-1/2}$.

In the asymptotically optimal pooling equilibrium, k is equal to the unconditional mean of the number of good projects, Np, adjusted by λ times the standard deviation of the number of good projects. The magnitude of this adjustment depends on the value created by a good project and the value destroyed by a bad project. Specifically, as the value created by a good project V_g increases relative to the value destroyed by a bad project $-V_b$, λ increases and the optimal k is higher. The intuition for this result is simple. As V_g increases relative $-V_b$, the loss from leaving good projects unfinanced becomes more important relative to the loss from financing bad projects. Therefore, it is optimal to increase k. In the special case in which a good project creates as much value as a bad project destroys, i.e. when $V_g = -V_b$, $\lambda = 0$ and the optimal k equals the expected number of good projects, Np.

The analysis has so far highlighted the potential gains generated by a CRA when rating issuers simultaneously. Proposition 2, in particular, shows that those gains can be very large. While the first-best allocation is never attainable, when the number of issuers is large the difference between the value created by the CRA and the first-best total surplus becomes negligible.

What drives this result is the negative link between the value of a good rating and the number of such ratings. One may wonder whether the negative link also works in the case of sequential rating, and, in case it does, whether it works equally well. A consideration based on statistics would suggest an affirmative answer to both questions. For example, Proposition 2 is largely driven by the Law of Large Numbers, which applies to dynamic settings equally well. That is, it applies equally well to the case of throwing the same coin 1000 times and to the case of throwing 1000 coins of the same attributes once. However, this consideration ignores the fact that the CRA's incentives under sequential rating may differ from those under simultaneous rating. As we will see, because of this difference in incentives, the value created by the CRA under sequential rating can approach only a *fraction* of the first-best total surplus.

5 Sequential Rating

Consider now a dynamic version of the model where the CRA rates issuers sequentially. More specifically, suppose there are N periods and in each period a different issuer seeks to finance its project, which matures in the same period. All the other aspects of the model remain unchanged. Hence, in any given period, the issuer decides whether to obtain a credit rating from the CRA; if the issuer asks for a rating, the CRA observes the quality of its project and rates it; the issuer decides whether to accept the rating and then seeks funding from investors. As before, the investors have abundant funds and are satisfied with a zero net return (because the project matures within the period). The period discount factor is $\beta \in (0, 1)$ and is the same for all agents. Finally, there is a saving asset with a gross return rate of $1/\beta$ over a period so that investors are indifferent between consumption and saving in each period.

Suppose the investors in each period observe the rating given in that period and the entire history of ratings given by the CRA. As in the baseline model, they do not observe the performance of previously financed projects. (The case where they do will be analyzed later.) Thus, the credibility investors give in any given period to a good rating depends only on the history of ratings. We begin by analyzing the case where $N = 2$ and then the case where N is infinite. As before, no value is created if no CRA operates on the market.

5.1 Two Issuers

Suppose there are two periods and one issuer per period. We can use backward induction to characterize the equilibrium interaction between the CRA, issuers and investors.

In the second period, the CRA's rating cannot create value. The situation is identical to the second benchmark case discussed at the beginning of Section 4 where the CRA rates one issuer only. As discussed, no value is created in this case. But the CRA cannot create value in period one either. The decisions made by the CRA and the investors in the first period have no impact on their second-period payoffs: in the second period they obtain zero, as no value is created. This means that the reasoning applied to analyze the second period also applies to the first period. Since credit ratings can create value in neither period, they cannot create value at all. For convenience of exposition, we state this without further proof in the next proposition.

Proposition 3 *The CRA creates no value if rating is sequential and $N = 2$.*

This proposition constitutes the first step in highlighting the difference between simultaneous rating and sequential rating. With two issuers, ratings can create value under simultaneous rating but not under sequential rating. The reason for the difference is that under sequential rating (with two issuers), the negative link between the number of good ratings issued and the fee obtained for a good rating is not present. The CRA's decision to offer a good rating in the second period has *no* effect on the fee received for a good rating in the first period. This means that the negative link is not present in the last period. And since rating generates no fee in the second period regardless of the first-period rating decisions, the negative link is also not present in the first period. Without it the CRA has no incentive to limit the number of good ratings issued. Its ratings lack credibility and create no value.

The unraveling argument that underpins the results in the case of two issuers holds for any finite number of issuers. As such, the CRA cannot create value under sequential rating when the number of issuers N is finite.

One may wonder whether the CRA could solve the problem of the lack of credibility of its ratings under sequential rating if it could commit ex-ante to issue only a limited number of good ratings. The answer to the question is negative. Suppose $N = 2$ and the CRA can commit ex-ante to issue only one good rating over the two periods. If the CRA does not rate the first period issuer as good, it will rate the second period issuer as good regardless of its quality (if it obtains a positive fee for the good rating). This implies that investors will never trust a good rating issued in the second period, and such rating is necessarily worthless. As a result, the CRA will always issue a good rating in period one. But applying the same reasoning, we conclude that a good rating issued in period one must also be worthless. Hence, commitment by the CRA to issue a limited number of good ratings is not sufficient to ensure the credibility of its ratings under sequential rating.

5.2 An Infinite Number of Issuers

Suppose now that there are an infinite number of periods and in each period a new issuer enters the market. In other words, the CRA expects to be in the rating business forever. As the CRA expects to rate issuers indefinitely, its rating decision in any given period can potentially affect the value of its ratings in all future periods, creating room for the negative link to emerge and for the CRA to create value. We derive below an upper bound for the value created by the CRA and compare it with the value created by the CRA under simultaneous rating.

Before presenting such upper bound, we discuss how equilibria where the CRA creates value might emerge. The CRA creates value only if its ratings are at least partially credible; and its ratings are credible only if the CRA does not give every type of issuer a good rating. Therefore, there must exist a cost for the CRA of issuing a good rating. Such cost stems from a reduction in the credibility of the CRA's future ratings, which creates a negative link between the issuance of a good rating and the value of future ratings. In value-creating equilibria, investors adjust their trust in a rating based on the history of the CRA's ratings. For example, as in the pooling equilibria considered in subsection 4.2, they may check the ratio of the number of good ratings issued in the past to the number of issuers that came to the market, k/N, and adjust their trust in a rating accordingly. Investors may adjust their beliefs given the history of ratings in other ways, leading to different equilibria with a possibly different value created by the CRA. The next proposition establishes an upper bound for the value created by the CRA across all equilibria. Note that the discounted first-best total surplus, namely, the value created when in each period the project is financed if and only if it is good, is $pV_g/(1-\beta)$. In what follows we let $V_s^{FB} \equiv pV_g/(1-\beta)$.

Proposition 4 *For any $\beta < 1$, the value created by the CRA under sequential rating is no larger than $(1 - q_b/q_g) \times V_s^{FB}$.*

Thus, at most a fraction $1 - q_b/q_g$ of the first-best total surplus can be realized under sequential rating. This result, which is independent of the value of the discount factor β, contrasts with that obtained in the case of simultaneous rating. As stated in Proposition 2, the value created by the CRA under simultaneous rating asymptotically approaches the first-best value. Two factors explain the difference.

First, the effect of the negative link is stronger under simultaneous rating than under sequential rating, providing the CRA with stronger disincentives for rating inflation. Under simultaneous rating, the CRA's decision to issue one additional good rating lowers the value of *all* the ratings issued by the CRA. Under sequential rating, that decision can lower only the value of the ratings issued by the CRA in the *future*; it does not affect the rating fees collected by the CRA in the past. This is the same reason why the CRA fails to create value under sequential rating when the number of issuers is finite. In that case, in the last period, as there is no future for the CRA, the negative link completely breaks down.

Second, good projects are more likely to be abandoned under sequential rating than under simultaneous rating. This is because the order with which good and bad projects appear on the market matters under sequential rating. To briefly illustrate this point, consider the event where k out of N projects are good. Suppose also that under sequential rating, after issuing a good rating, the CRA has to wait l periods until it can issue another good rating that will be trusted by investors. If the k good projects appear on the market consecutively, there is a congestion of good projects and many will necessarily be abandoned. Under simultaneous rating, the order with which good projects appear is irrelevant, and there is no loss of surplus due to congestion of good projects.

5.3 The Reputation Mechanism

It is well known that reputational concerns play an important role in disciplining rating agencies against rating inflation, thereby contributing to increase the credibility and value of credit ratings. Yet, we show here that even when the reputation mechanism is present, in general, the maximum value created by ratings under sequential rating is still only a fraction of the first-best surplus.

Suppose $N = \infty$ and investors observe the outcome of the previously financed projects in addition to the history of ratings given by the CRA. Since investors can compare the ratings of past projects with their performance and adjust the credibility they give to the CRA's ratings accordingly, the reputation mechanism is present. We recompute an upper bound for the value created by the CRA in this setting. Recall that the value of the first-best allocation is $V_s^{FB} \equiv pV_g/(1-\beta)$.

Proposition 5 *For any $\beta < 1$, the value created by the CRA under sequential rating when investors observe the performance of past projects is no larger than $(q_g - q_b)/(q_g - q_g q_b) \times V_s^{FB}$.*

The proof of this proposition is similar to that of Proposition 4 and is relegated to an online appendix. The upper-bound for the value created reaches the value of the first-best allocation only if $q_g = 1$, i.e. only in the special case where a good project never fails. In this case, a failure of a project recommended (i.e. rated good) by the CRA indicates the CRA recommended a bad project. If the discount factor β is sufficiently high, the first-best allocation can be implemented by the investors playing a trigger strategy where they fully trust the CRA as long as no project recommended in the past has failed, and never trust the CRA again if otherwise. If $q_g < 1$, however, a good project fails with a positive probability, as does a bad project. This trigger strategy wrongly punishes the CRA with some probability and is unable to generate the first-best allocation. Actually, as $(q_g - q_b)/(q_g - q_g q_b) < 1$ when $q_g < 1$, Proposition 4 implies that no strategy can attain the first-best allocation in any equilibrium. And at most a fraction $(q_g - q_b)/(q_g - q_g q_b)$ of the first-best surplus can be generated. Once again, this contrasts with the case of simultaneous rating where the value created by the CRA asymptotically approaches the value of the first-best surplus.

6 Implementation of Simultaneous Rating in Practice

The analysis of simultaneous rating done so far assumes that several issuers are already on the market seeking to finance their projects. But in the real world issuers enter the market sequentially. Firms approach the financial market as opportunities to develop new projects emerge. Since issuers enter the market sequentially, the implementation of simultaneous rating in practice requires waiting until several of them have entered the market. Waiting, however, is costly. Issuers entering the market early on have to wait to be rated and to obtain funding from investors, which delays their projects. Thus, there is a trade-off with the implementation of simultaneous rating: it generates an economy of scale in rating as illustrated in the two previous sections, but it may also cause delays in the implementation of projects.

6.1 The basic trade-off

To formalize this trade-off, consider the setting of the previous section where there are an infinite number of periods and one issuer entering the market in each period. Let the length of waiting be $\widehat{N}$ periods. That is, the issuers arriving on the market in periods 1, 2, ... $\widehat{N}-1$ wait until period $\widehat{N}$ and then, in that period, all the $\widehat{N}$ issuers (including the issuer that arrives in period $\widehat{N}$) are simultaneously rated by the CRA and issue bonds to investors. Similarly, the issuers arriving in periods $\widehat{N}+1$, $\widehat{N}+2, ...,$ $2\widehat{N}-1$ wait until period $2\widehat{N}$ to be simultaneously rated, and so on. By Proposition 2, for $\widehat{N}$ large, the expected value created (in a cycle of $\widehat{N}$ periods) is a difference of order $1/\sqrt{\widehat{N}}$ away from the first-best value (when the issuers are already in the market) $\widehat{N}pV_g$; more precisely, the value created is approximately $\widehat{N}pV_g \times (1 - c \times 1/\sqrt{\widehat{N}})$, where $c > 0$ is independent of $\widehat{N}$.[18] Hence, evaluated at the first period of the cycle, the value created with these $\widehat{N}$ projects under simultaneous rating is $V_{\widehat{N}} \approx \beta^{\widehat{N}-1} \times \widehat{N}pV_g(1 - c/\sqrt{\widehat{N}})$, while the first-best value where they are financed immediately as they enter the market if they are good is $V^{FB} = pV_g \cdot (1 + \beta + ...\beta^{\widehat{N}-1})$. The efficiency relative to the first-best for these $\widehat{N}$ projects is thus given by

$$S_{\widehat{N}} \equiv \frac{V_{\widehat{N}}}{V^{FB}} \approx \frac{\beta^{\widehat{N}-1}}{(1+\beta+...\beta^{N-1})/\widehat{N}}(1-\sqrt{\frac{1}{\widehat{N}}}c) \equiv \widetilde{S}_{\widehat{N}}. \tag{4}$$

Observe that $S_{\widehat{N}}$ is also the ratio of the value created by simultaneous rating every $\widehat{N}$ periods to the first-best value, when all the projects are taken into account, i.e. when all cycles of $\widehat{N}$ periods are taken into account. The first term of $\widetilde{S}_{\widehat{N}}$ represents the relative cost of waiting incurred by issuers. The denominator, $(1+\beta+...\beta^{\widehat{N}-1})/\widehat{N}$, is the average discount factor in the first-best case where a good project arriving in a given period t is immediately implemented and thus its present value in period one is discounted with factor β^{t-1}. The numerator, $\beta^{\widehat{N}-1}$, is the discount factor in the case of simultaneous rating, where all projects are implemented in period $\widehat{N}$ and thus their value is discounted by $\beta^{\widehat{N}-1}$. The second term, measures the efficiency in rating relative to the first-best, whose loss, as noted earlier, is in order of $\sqrt{1/\widehat{N}}$.

Using this formalization, we can clearly see how the length of waiting, $\widehat{N}$, affects the benefit of simultaneous rating and the cost of waiting. The first term of $\widetilde{S}_{\widehat{N}}$ decreases with $\widehat{N}$, which captures the fact that the cost of waiting increases with $\widehat{N}$. The second term increases with $\widehat{N}$, which captures the economy of scale associated with simultaneous rating: the greater the number of issuers rated simultaneously, the smaller the efficiency loss relative to the first-best value. It is not difficult to show that at a unique $\widehat{N}$ these two effects

[18] While we use here the asymptotic approximation, observe that in the case of the binomial distribution this involves only a small error even for low values of N. For example, for a binomial with probability of success of 1/2 and $N = 16$, the difference between the true cdf and its the asymptotic approximation never exceeds 0.002 (see Mosteller, Rourke and Thomas, 1961, page 277).

are perfectly balanced and $\widetilde{S}_{\widehat{N}}$ is maximized. Moreover, this optimal $\widehat{N}$ increases with the discount factor β and goes to infinity as β goes to one because a higher value of β implies a lower waiting cost and, as β approaches one, the waiting cost approaches zero.

6.2 A Numerical Simulation

In this numerical simulation, we compare sequential rating with simultaneous rating where ratings are issued once a week (i.e., the ratings of each week's projects are simultaneously reported at the end of the week). Giving reasonable values to the parameters, we obtain that it is worthwhile to wait one week to rate issuers simultaneously.

Proposition 5 offers an upper bound for the value created under sequential rating. That upper bound, which holds for all values of $\beta < 1$ and takes into account repuational effects, is $(q_g - q_b)/(q_g - q_g q_b) \times V_s^{FB}$. Dividing it by V_s^{FB}, we obtain that the maximum possible efficiency of sequential rating relative to the first-best allocation is

$$E_s = \frac{q_g - q_b}{q_g - q_g q_b}.$$

We will compare this measure of efficiency with its counterpart in the case of simultaneous rating done weekly.

Instead of using an asymptotic approximation for the value created by simultaneous rating in a cycle of a week, we use the actual value. Suppose N issuers (i.e., projects) appear on the market every week and consider an equilibrium where k of them receive a good rating and are financed. The expected value created, evaluated at the end of the week is

$$V(k, N) = kV_g \sum\nolimits_{n=k}^{N} P_n + \sum\nolimits_{n=0}^{k-1} P_n(nV_g + (k-n)V_b),$$

where P_n is the probability that n out of the N projects are good.[19] If we let β_d denote the daily discount factor, the efficiency of simultaneously rating in a given week (i.e., in a period of 5 business days) relative to the first-best allocation is given by

$$E_w = \frac{\beta_d^4 V(k, N)}{\frac{N}{5} p V_g (1 + \beta_d + ... + \beta_d^4)}.$$

The numerator is the value created under simultaneous rating evaluated at the beginning of the week. The denominator is the value created in the first-best allocation, where the good projects (out of the $N/5$ that enter the market daily) are financed without delay.[20] Clearly,

[19] In our model a project is good with probability p and therefore $P_n = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}$. The equilibria used here are the pooling equilibria described in Section 4.2 where k issuers receive a good rating regardless of the number of good issuers on the market.

[20] We assume no intraday discounting in this simulation.

E_w is also the efficiency of weekly simultaneous rating relative to the first-best allocation when all cycles of one week are taken into account.

To compute the efficiency measures E_s and E_w, we use the following parameter values. For the number of projects N, we use the number of corporate bond ratings issued by the three major ratings agencies. Using the FISD data set, we obtain that during the period 2001-2010, Moody's, S&P and Fitch issued on average per week, respectively, 285, 275 and 141 corporate bond ratings. We compute E_w for each of these three values. We set the yearly discount factor $\beta = 0.95$, which means that $\beta_d = \beta^{\frac{1}{360}} = 0.999\,86$.

To obtain an estimate for the probability of success (no default) of good projects q_g, we use the global corporate average cumulative default rate (1981- 2014) of all issues rated that appears in the S&P's study "2014 Annual Global Corporate Default Study And Rating Transitions" (see table 24 in pages 56 and 57).[21] Since this average default rate is 0.1078, we set $q_g = 1 - 0.1078 = 0.892\,2$. The reason for using the average default rate of all issues rated instead of the issues with a good rating only, is that in our model good projects represent all those with a positive NPV. Thus, assuming real financial markets are efficient (at least to some degree), it is reasonable to assume these projects are typically financed (even if they do not receive a top rating). In contrast, since bad projects in our model represent those with a negative NPV, it is reasonable to assume they are usually not financed. This also means that it is not possible to obtain directly from the data an estimate for the probability of default of bad projects. Therefore, we set somewhat arbitrarily q_b equal to roughly half of q_g, i.e. we set $q_b = 0.45$. Similarly, it is not easy to obtain an estimate for p and we set $p = 0.5$. We set $R = 1.\,489\,3$ based on an estimate of the return on capital in the US in the period 1995-2007.[22] Finally, we use the asymptotically optimal k, as given in Proposition 2.

Given the above parameter values, we obtain that $E_s = 0.901$. Thus, the value created under sequential rating is approximately 90% of the value created in the first-best allocation. This measure is independent of the frequency with which new projects enter the market (i.e., it is independent of N). Regarding the efficiency of simultaneous rating, we obtain that $E_w = 0.952$ if $N = 285$ (Moody's), $E_w = 0.951$ if $N = 275$ (S&P), and $E_w = 0.932$ if $N = 141$ (Fitch). These results indicate an efficiency gain of weekly simultaneous rating relative to sequential rating of approximately 5 percentage points in the case of Moody's and S&P, and of 3 percentage points in the case of Fitch. In all cases, the benefit stemming from the economy of scale in rating more than compensates the cost of any delays in the

[21] This study is publicly available on the S&P's webpage (https://www.standardandpoors.com/).

[22] More specifically, R is calculated in the following way: $R = (ROC/P_K) \times 1/(1 - \beta \times (1 - \delta) \times (1 + g))$, where ROC is the return on capital, P_K is the price of capital in terms of the consumption good, δ is the depreciation rate of capital, and g is the productivity growth rate. The values for the ROC (11.4%) and $1/P_K$ (0.979) are taken from Chou, et al. (2016)–see Table 1–; the value for g (2.5%) is taken from Jorgenson, et al. (2008)–see Figure 1. They are all for the US for the period 1995-2007. As for δ, it is set at 5%.

implementation of projects.[23]

Finally, observe that the difference $E_w - E_s$ constitutes a *lower bound* for the efficiency gains of adopting simultaneous rating once a week. Recall that E_s is an *upper bound* for the level of efficiency under sequential rating, while E_w does not take into account the reputation mechanism. If this mechanism is taken into account, the efficiency level of simultaneously rating issuers once a week will exceed E_w.

7 Discussion and Extensions

In this section, we discuss the robustness of our key findings and explore some possible extensions of our setting.

7.1 Rating Fees

The rating fee in the baseline model is paid ex post if the issuer's project is financed and successful. In the discussion of Proposition 1 in Section 4.1, we discuss the case where the rating fee is paid ex ante when the issuer accepts the rating. In both cases, it is assumed that the rating fee depends on the cost of finance and on the quality of the issuer. We discuss here two alternative types of rating fee: (i) a fixed fee (i.e. independent of the cost of finance and of the issuer's quality) paid when the issuer accepts the rating; and (ii) a fixed fee paid up front when the issuer requests a rating. In both cases, we suppose issuers have retained earnings with which they can pay the fee.

Fixed fees paid when issuers accept a rating. The benefits of simultaneous rating identified in the paper continue to exist in this case. The pooling equilibria discussed in Section 4.2 continue to exist, and the asymptotic results of Proposition 2 continue to hold. Even though the rating fee is fixed, there is still a negative link between the number of good ratings issued and the value of a good rating: if the number of good ratings exceeds a certain threshold (defined by the equilibrium), investors will not trust the CRA's ratings, and no issuer will pay the corresponding fixed fee for a rating. Because of this negative link, the CRA issues only a limited number of good ratings.

With fixed fees, however, good and bad issuers pay the same for a rating, and it is only weakly optimal for the CRA to give the good ratings to the best issuers on the market. Since the CRA's incentive to give good ratings to good issuers is weak, the pooling equilibria mentioned above are less robust. Yet, the benefits of simultaneous rating relative to sequential

[23] Note that a reduction in the discount factor β has a very small effect on the cost of delaying projects, as the maximum period an issuer has to wait for a rating is only four days. For example, taking into account only this effect (i.e., ignoring the effect that β might have on R), a reduction in β to 0.93 leaves E_w essentially unchanged. And recall that E_s does not depend on β.

rating remain. Under sequential rating, because of such weak incentives, the CRA never creates value. If in a period the CRA's rating is trusted and the CRA can collect the fee by giving a good rating, it will give the rating regardless of the quality of that period's issuer. As such, the CRA's ratings are never trusted and cannot create value.

This analysis abstracts from reputational effects for the CRA. If they are present, the CRA will strictly prefer to give a good rating to a good issuer (who defaults with low probability) than to a bad issuer (who defaults with high probability), even if both pay the same fee. Hence, the presence of reputational effects restores the robustness of the equilibria identified in Section 4.2 and the associated asymptotic results under simultaneous rating. Reputational effects can also (partially) restore the credibility of ratings under sequential rating. Nevertheless, as shown in the next section, the value created by the CRA when reputational effects are taken into account can be at most a fraction of the first-best total surplus. While the result is derived with rating fees that are paid ex post, it also applies in the case of fixed ratings fees that are paid ex ante. Thus, the benefit of simultaneous rating the paper highlights holds even when the rating fee is fixed and paid ex-ante.

Fixed fees paid up front when issuers request a rating. Thus far, we have assumed that issuers who reject the rating offered by the CRA do not pay the rating fee. The CRA's rating decision in such cases is directly linked to its revenue, creating a strong incentive for rating inflation. One way of breaking the link is to require that all issuers who request a rating to the CRA pay the (fixed) rating fee up front. This is what Moody's, S&P and Fitch agreed to do in rating Residential MBSs as part of the Cuomo agreement of 2008.

Suppose, as in the Cuomo agreement, that issuers must pay the rating fee to the CRA up front when they request a credit rating. Both under simultaneous rating and under sequential rating there exists one equilibrium in which the CRA reports truthfully the quality of the issuers who request a rating. Moreover, in this equilibrium, only the issuers with a good project approach the CRA for a rating and their projects are financed at a repayment of $1/q_g$. In other words the first-best allocation is attained. Since the fee is paid up front, the CRA is indifferent about the rating given to the issuer, and truthfully reporting the issuer's type to investors is an optimal decision for the CRA. If we focus on this equilibrium, there is no gain of simultaneous rating relative to sequential rating. However, there is a problem with this equilibrium: it is not robust to collusion between the CRA and the issuers. Since the CRA's ratings are fully trusted by investors, the CRA has an incentive to approach issuers who do not request a rating and offer them a good rating in exchange for the rating fee; and those issuers have an incentive to accept the offer. But if investors anticipate this behavior, they will no longer fully trust the CRA's ratings and the equilibrium collapses. Hence, when collusion between the CRA and issuers cannot be prevented, requiring that rating fees be paid up front may not ensure credible ratings.

By contrast, the mechanism proposed in this paper–simultaneous rating–is collusion proof.

In all the value-creating equilibria under simultaneous rating analyzed in Section 4 (where rating fees are paid ex post), the CRA's rating decisions maximize the joint profit of the coalition formed by the agency and the issuers.

An important question is whether simultaneous rating works when rating fees are paid up front (even if collusion between the CRA and the issuers cannot be prevented). Even in this case it works. Pooling equilibria similar to those described in Section 4.2 where the CRA issues a fixed number of good ratings k exist. If k is set optimally, which is around Np as stated in Proposition 2, then good issuers request a rating to the CRA and bad issuers do so with a certain small probability so that the total number of issuers that request a rating hits closely k by the Law of Large Numbers. In these equilibria, the CRA fills the quota of k good ratings first with good issuers and then with bad issuers, as it gains nothing by doing otherwise. Bad issuers are willing to pay the up-front fee because there is a chance that they receive a good rating due to lack of good issuers. If the number of issuers who request a rating is smaller than k, the CRA can invite some issuers to request a rating (as collusion cannot be prevented). However, with the optimal number of good ratings k, the proportion of the bad issuers who receive a good rating is very small and goes to zero as the number of issuers goes to infinity. Once again, the value created by the CRA asymptotically approaches the value created in the first-best allocation.

7.2 Commitment by Investors in a Finite Horizon

We have seen that ratings bear no credibility and create no value under sequential rating with a finite number of periods. As discussed in Section 5.1, the inability to create value persists even if the rating agency can commit to issue only a certain number of good ratings.

By contrast, the CRA's ratings could create value under sequential rating if investors could commit ex ante to finance a given number of issuers. Indeed, part of the value-creating equilibria identified in Proposition 1 would be recovered. To see this, suppose investors commit to finance at a certain repayment $C < R$ one issuer with a good rating. The optimal strategy for the CRA is as follows: in the first period, give the issuer a good rating if and only if the issuer is good; if a good rating is not issued in period one, give a good rating to the issuer in the second period regardless of its quality. The decision in the second period is optimal because the CRA obtains a profit of $q \times \alpha(R - C) > 0$ by issuing a good rating, while it obtains zero profit otherwise. Regarding the decision in the first period, if the issuer is good the CRA is better off giving the issuer a good rating than waiting since $q_g \times \alpha(R - C) > E(q) \times \alpha(R - C)$, and if the issuer is bad the CRA is better off waiting because $q_b \times \alpha(R - C_1) < E(q) \times \alpha(R - C)$. Thus, the CRA gives only one good rating and gives it to a good issuer if there is one, which is the same outcome as in one particular equilibrium identified in Proposition 1. Observe that if the period one issuer does not receive a good rating, the period two issuer receives a good rating regardless of its quality. Hence, financing it generates a loss to the investors (compensated by the gain of financing an issuer rated as good in period one which is good for sure). If investors lacked commitment power, they would not finance the project in period two and the entire equilibrium would collapse.

When N is large, a similar argument can be applied to show that if investors can commit to finance k projects with a good rating, then the results given in Proposition 2 – especially the asymptotic approximation to the first best allocation – can be recovered under sequential rating. It is unlikely, however, that such commitment can be made in the corporate bonds market where investors are dispersed and change over time.

7.3 Competition in the Market for Credit Ratings

While a complete analysis of the case where several CRAs compete for issuers is out of the scope of this paper, we briefly discuss one potential implication of competition for the benefits of simultaneous rating.

One point of the paper is to show that there is an economy of scale associated with simultaneous rating. Recall that the CRA creates no value if it rates only one issuer. However, as shown in Section 4.1, credit ratings can create value if the CRA rates two issuers. Also, as shown in Section 4.2, for a large number of issuers, the value created by a CRA asymptotically approaches the first-best total surplus. Competition is likely to reduce the magnitude of this economy of scale. For example suppose there are two CRAs and competition between them leads to a split of the rating market. The scale of each CRA is reduced and part of the benefit of the economy of scale is lost. To illustrate this, consider the case of a large number N of issuers studied in Section 4.2. In the case of a monopolist CRA, the loss in expected surplus (relative to the first-best) is the order of $c\sqrt{N}$, for some $c > 0$.[24] If the market share of the two CRAs is α and $1-\alpha$, respectively, the surplus loss is in the order of $c\sqrt{\alpha N}+c\sqrt{(1-\alpha)N}$. While in both cases the ratio of the loss to the first-best is in the order of $1/\sqrt{N}$, such loss is clearly greater in the case of two CRAs than in the case of a monopolist CRA, since $\sqrt{N} < \sqrt{\alpha N} + \sqrt{(1-\alpha)N}$.

8 Conclusion

We argue in this paper that requiring a rating agency to report the ratings of several issuers simultaneously may discourage rating inflation, increase the credibility and value of the agency's ratings, and lead to a more efficient allocation of funds. This is because simultaneous reporting of credit ratings allows for a link between the CRA's decisions on the ratings of different issuers: by giving more good ratings, the rating agency lowers the credibility of

[24] See the proof of Proposition 2 for the complete mathematical expression of this loss and its derivation.

its ratings and the fee it can charge for a good rating. We show that there is an economy of scale associated with simultaneous rating. When the number of issuers simultaneously rated is sufficiently large, the surplus generated in equilibrium asymptotically approaches the first-best surplus. We also show that while this link between the number of good ratings issued and the value of a good rating may be present when issuers are rated (and financed) sequentially, it is less effective than under simultaneous rating.

From a more practical point of view, the paper's findings suggest an extra benefit to the idea of synchronizing bond issuance, which practitioners have been considering mainly for its potential benefits to corporate bonds liquidity. Synchronization of bond issuance may allow the benefits of simultaneous rating highlighted in the paper to work, increasing the value created by ratings. There are potential costs associated the implementation of simultaneous rating. One of them is the potential delay in the implementation of projects, since firms may have to wait more for a credit rating and for the funds needed to implement their projects. In this respect, the periodicity with which credit ratings can be reported is important, as it affects the magnitude of both the benefits and the cost of the scheme.

9 Appendix

Proof of Proposition 1. The proof is given in three steps. We begin in Step 1 by showing that in any value-creating equilibrium, if such an equilibrium exists, (i) $C_1 < C_2$ (and $\phi_1 > \phi_2$), and (ii) if the CRA issues only one good rating, the CRA gives it to a good issuer if there is one. In Step 2, we show that no value-creating equilibrium exists if $p\,(2-p) \leq -V_b/(\Delta R)$. Finally, in Step 3, we construct for the case where $p\,(2-p) > -V_b/(\Delta R)$ a continuum of value-creating equilibria, in all but one of which the CRA issues two good ratings on the equilibrium path. When p is sufficiently high, in addition to the equilibria identified in this proof, there exists another equilibrium. Also in that equilibrium, the CRA issues two good ratings on the equilibrium path. These are all the equilibria of the game. See the Online Appendix for a complete analysis and characterization of the equilibria.

Step 1: For claim (i): Suppose to the contrary that there exists a value-creating equilibrium in which $C_1 \geq C_2$. Because the CRA creates value in this equilibrium, $\min\{C_1, C_2\} < R$. Thus, $C_2 < R$. Since $C_1 \geq C_2$ and $C_2 < R$, the CRA issues two good good ratings regardless of the profile of the issuers' qualities (q_1, q_2): by issuing two good ratings it obtains $(q_1 + q_2) \times \alpha(R - C_2)$, while by issuing one good rating it obtains $\max\{q_1, q_2\} \times \alpha \max\{R - C_1, 0\}$, and $(q_1 + q_2) \times \alpha(R - C_2) > \max\{q_1, q_2\} \times \alpha(R - C_2) \geq \max\{q_1, q_2\} \times \alpha \max\{R - C_1, 0\}$. As the CRA rates both issuers as good irrespective of their qualities, in this equilibrium $\phi_2 = p$. It follows from (3) and (2) that $C_2 = 1/(pq_g + (1-p)q_b) > R$, which contradicts the previous assertion that $C_2 < R$. Thus, $C_1 < C_2$ in any value-creating equilibria. Since $C_1 < C_2$ and by (3) C_k is connected with ϕ_k by $C_k = 1/[\phi_k q_g + (1-\phi_k) q_b]$, $\phi_1 > \phi_2$.

For claim (ii): As in any value-creating equilibrium $C_1 < C_2$, then $C_1 < R$. Hence, if in such an equilibrium the CRA issues only one good rating, it obtains $q_g \times \alpha(R - C_1)$ if it gives the rating to a good issuer, while it obtains $q_b \times \alpha(R - C_1)$ if it gives the rating to a bad issuer. The former payoff is larger because $q_g > q_b$ and $R - C_1 > 0$. Thus, the CRA gives the good rating to a good issuer if there is one.

Step 2: Suppose to the contrary that there exists a value-creating equilibrium when $p(2-p) \leq -V_b/(\Delta R)$. In this equilibrium, as the CRA creates value, $C_1 < R$ (as shown in Step 1). Hence, for any given number of good issuers $n \in \{0, 1, 2\}$, it obtains a positive payoff by issuing one good rating. Since the CRA obtains nothing by issuing no good ratings, choosing $k = 0$ is never optimal. Hence the CRA either chooses $k = 1$ or $k = 2$. Let μ_n denote the probability that the CRA chooses $k = 1$ in state n. Hence, the CRA chooses $k = 2$ with probability $1 - \mu_n$ in state n. Moreover, let P_n denote the ex ante probability of state n occurring, that is, $P_0 = (1-p)^2$, $P_1 = 2p(1-p)$ and $P_2 = p^2$. Since investors update their beliefs according to Bayes' rule (whenever possible) and when $n = 1$ the CRA gives the good rating to the good issuer if it chooses $k = 1$, then investors' posteriors must satisfy

$$\phi_1 = \frac{P_1\mu_1 + P_2\mu_2}{P_0\mu_0 + P_1\mu_1 + P_2\mu_2} \tag{5}$$

$$\phi_2 = \frac{P_1(1-\mu_1)/2 + P_2(1-\mu_2)}{P_0(1-\mu_0) + P_1(1-\mu_1) + P_2(1-\mu_2)}, \tag{6}$$

whenever the denominator is positive. We next consider separately three cases that jointly capture all the possible forms that the equilibrium can take and show that none can be an equilibrium.

First, suppose $\mu_n = 1$ for all $n \in \{0, 1, 2\}$, that is, the CRA issues only one good rating in all the states. Using (5), we obtain $\phi_1 = p(2-p)$. Since by assumption $p(2-p) \leq -V_b/(\Delta R)$, we obtain that $\phi_1 \leq -V_b/(\Delta R)$, which implies that $C_1 = 1/[\phi_1 q_g + (1-\phi_1) q_b] \geq R$, contradicting the fact that in any value-creating equilibrium $C_1 < R$.

Second, suppose (i) $\mu_n < 1$ for at least one n and (ii) $\mu_n > 0$ for at least one n (not necessarily the same n), that is, both $k = 1$ (one good rating is issued) and $k = 2$ (two good ratings are issued) are on the equilibrium path. In this case, both $C_1 < R$ and $C_2 < R$, necessarily. It follows that both $\phi_1 > -V_b/(\Delta R)$ and $\phi_2 > -V_b/(\Delta R)$. Using (5) and (6), this implies that

$$\begin{aligned} P_1\mu_1 + P_2\mu_2 &> [-V_b/(\Delta R)] \times [P_0\mu_0 + P_1\mu_1 + P_2\mu_2] \\ P_1(1-\mu_1)/2 + P_2(1-\mu_2) &> [-V_b/(\Delta R)] \times [P_0(1-\mu_0) + P_1(1-\mu_1) + P_2(1-\mu_2)]. \end{aligned}$$

Summing both sides and using the fact that $P_0 + P_1 + P_2 = 1$, we obtain that $P_1(1+\mu_1)/2 +$

$P_2 > -V_b/(\Delta R)$, which, as $P_1+P_2 \geq P_1(1+\mu_1)/2+P_2$, leads to $P_1+P_2 > -V_b/(\Delta R)$, which is contradictory with $P_1 + P_2 = p\,(2-p) \leq -V_b/(\Delta R)$.

Third, suppose $\mu_n = 0$ for all the n, that is, the CRA always issues two good ratings. Since the CRA obtains a positive fee by doing so, $C_2 < R$. However, as $\mu_n = 0$ for all n, by (6), $\phi_2 = P_1/2 + P_2 < P_1 + P_2 \leq -V_b/(\Delta R)$, which implies $C_2 > R$, a contradiction.

Step 3: In this step we prove the following: If $p\,(2-p) > -V_b/(\Delta R)$, for each $x \in [u, 1]$, with $u = \max\{0, 1 - \frac{(1-p)^2}{p^2}\frac{1-q_b/q_g-V_b}{V_g}\}$, there exists a value-creating equilibrium in which $\mu_2 = x$, $\mu_1 = 1$, and $\mu_0 = f(x)$, where $f' > 0$ and $f(1) = 1$. Observe that except for the equilibrium in which $\mu_2 = \mu_0 = 1$, in all the other equilibria, ex ante the CRA chooses $k = 2$ (i.e. issues two good ratings) with probability $P_0(1-\mu_0) + P_2(1-\mu_2) > 0$. The proof is given in the following steps.

Step 3.1. In this step we present necessary and sufficient conditions for an equilibrium that will be used in the following steps. Let π_k^n denote the CRA's expected profit from issuing $k \in \{1,2\}$ good ratings in state $n \in \{0,1,2\}$. Recall that in a value-creating equilibrium $C_1 < R$ and if the CRA chooses $k = 1$ when $n = 1$ the CRA gives the good rating to the good issuer (see Step 1). Hence, $\pi_1^0 = q_b \times \alpha(R - C_1)$, and $\pi_2^0 = 2q_b \times \alpha \max\{R - C_2, 0\}$; $\pi_1^1 = \pi_1^2 = q_g \times \alpha(R - C_1)$; $\pi_2^1 = (q_b + q_g) \times \alpha \max\{R - C_2, 0\}$, and $\pi_2^2 = 2q_g \times \alpha \max\{R - C_2, 0\}$.

A profile $\{\mu_n; \phi_k, C_k\}_{n=0,1,2;k=1,2}$ constitutes a value-creating equilibrium if and only if $C_1 < R$ and the following two equilibrium conditions hold:

(EC1) Given the CRA's strategy $\{\mu_n\}_{n=0,1,2}$, the investors' beliefs satisfy Bayes rule whenever possible, that is, (5) and (6) are satisfied whenever the denominator is different from zero; and the repayment investors request satisfies

$$C_k = \frac{1}{\phi_k q_g + (1-\phi_k)q_b}; \tag{7}$$

and

(EC2) Given the investors' belief and strategy $\{\phi_k, C_k\}_{k=1,2}$, for any $n \in \{0,1,2\}$: if $\pi_1^n > \pi_2^n$ then $\mu_n = 1$, if $\pi_1^n < \pi_2^n$ then $\mu_n = 0$, and if $\mu_n \in (0,1)$, then $\pi_1^n = \pi_2^n$.

Step 3.2. We show that if a profile $\{\mu_0, \mu_1 = 1, \mu_2; \phi_1, \phi_2, C_1 < R, C_2\}$ satisfies (EC1) and

$$R - C_1 = 2(R - C_2), \tag{8}$$

then it constitutes a value-creating equilibrium. To prove this, we only need to show that the profile also satisfies (EC2). To see this, observe that $C_1 < R$ and (8) imply that $C_2 < R$. Next observe that if $C_1, C_2 < R$, then $\pi_1^0 = \pi_2^0 \Leftrightarrow q_b \times \alpha(R - C_1) = 2q_b \times \alpha(R - C_2) \Leftrightarrow$ (8); and $\pi_1^2 = \pi_2^2 \Leftrightarrow q_g \times \alpha(R - C_1) = 2q_g \times \alpha(R - C_2) \Leftrightarrow$ (8). Since $\pi_1^0 = \pi_2^0$ and $\pi_1^2 = \pi_2^2$, any $\mu_0, \mu_2 \in [0,1]$ satisfy (EC2), as any $\mu_0, \mu_2 \in [0,1]$ is consistent with the CRA's optimal decision. Moreover, as $2(R-C_2) > \frac{q_g+q_b}{q_g}(R-C_2)$, (8) implies $R-C_1 > \frac{q_g+q_b}{q_g}(R-C_2) \Leftrightarrow q_g(R-C_1) > (q_g+q_b)(R-C_2) \Leftrightarrow \pi_1^1 > \pi_2^1$. Hence $\mu_1 = 1$ satisfies (EC2).

Step 3.3. We show that if $p\,(2-p) > -V_b/(\Delta R)$, then for any $\mu_2 \in [u,1]$, there exists a $\mu_0 = f(\mu_2) \in [0,1]$, with $f' > 0$ and $f(1) = 1$, ϕ_1, ϕ_2, $C_1 < R$ and C_2 such that the profile $\{\mu_0, \mu_1 = 1, \mu_2; \phi_1, \phi_2, C_1 < R, C_2\}$ satisfies (EC1) and (8). Therefore, by Step 3.2 this profile constitutes a value-creating equilibrium. To show this, we consider separately two cases regarding the value of μ_2.

Case 1: Suppose first that $\mu_2 = 1$. In this case, let $\mu_0 = 1$. Hence, $\mu_n = 1$ for all n and $k = 2$ is off-equilibrium path, meaning that (6) is not applicable. By (5), $\phi_1 = p\,(2-p)$. If $p\,(2-p) > -V_b/(\Delta R)$, then $\phi_1 > -V_b/(\Delta R)$ and thus by (7) $C_1 = 1/[\phi_1 q_g + (1-\phi_1)q_b] < R$. Since (6) is not applicable, any value of C_2 is consistent with condition (EC1). Hence we can pick one that satisfies equation (8).

Case 2: Suppose now that $\mu_2 < 1$. In this case, both $k = 1$ and $k = 2$ are on equilibrium path. As $\mu_1 = 1$, (5) and (6) imply that

$$\phi_1 = \frac{P_1 + \mu_2 P_2}{\mu_0 P_0 + P_1 + \mu_2 P_2} \quad \text{and} \quad \phi_2 = \frac{(1-\mu_2)P_2}{(1-\mu_0)P_0 + (1-\mu_2)P_2}.$$

It follows from these two equations that

$$\mu_0 = \frac{1-\phi_1}{\phi_1} \cdot \frac{P_1 + P_2\mu_2}{P_0} \quad \text{and} \quad 1-\mu_0 = \frac{1-\phi_2}{\phi_2} \cdot \frac{P_2(1-\mu_2)}{P_0}. \tag{9}$$

Hence, μ_2 satisfies:

$$\frac{1-\phi_1}{\phi_1} \cdot \frac{P_1 + P_2\mu_2}{P_0} + \frac{1-\phi_2}{\phi_2} \cdot \frac{P_2(1-\mu_2)}{P_0} = 1. \tag{10}$$

From (7) it follows that for all $k = 1, 2$,

$$\frac{1-\phi_k}{\phi_k} = \frac{q_g C_k - 1}{1 - q_b C_k}. \tag{11}$$

Since $0 < \phi_k < 1$ and $C_k < R < 1/q_b$, this equation implies $C_k > 1/q_g$ in a value-creating equilibrium. Substituting (11) for $k = 1, 2$ into (10) and rearranging, we obtain

$$\frac{q_g C_1 - 1}{1 - q_b C_1} \cdot (P_1 + P_2\mu_2) + \frac{q_g C_2 - 1}{1 - q_b C_2} \cdot P_2(1-\mu_2) = P_0. \tag{12}$$

From (8), $C_2 = (C_1 + R)/2$. Substituting it into (12), we obtain

$$g(C_1; \mu_2) \equiv \frac{q_g C_1 - 1}{1 - q_b C_1} \cdot (P_1 + P_2\mu_2) + \frac{q_g (C_1 + R)/2 - 1}{1 - q_b (C_1 + R)/2} \cdot P_2(1-\mu_2) = P_0. \tag{13}$$

A value-creating equilibrium exists if and only if there is a $\mu_2 \in [0,1]$ such that equation (13) has a solution in $C_1 \in [1/q_g, R)$. We next show that for each $\mu_2 \geq \max\{0, 1 - \frac{(1-p)^2}{p^2}\frac{1-q_b/q_g-V_b}{V_g}\} = u$, such a solution exists and is unique.

Observe that g is continuous and $g_1' > 0$ for all $C_1 \in (1/q_g, R]$. Moreover,

$$g(R;\mu_2) = \frac{V_g}{-V_b}\cdot(P_1 + P_2\mu_2) + \frac{V_g}{-V_b}\cdot P_2(1-\mu_2) = \frac{V_g}{-V_b}\cdot(P_1+P_2).$$

Thus, since $p\,(2-p) > -V_b/(\Delta R)$, then $P_1 + P_2 > (-V_b)/(V_g - V_b) \Leftrightarrow P_1 + P_2 > (-V_b)/V_g \cdot P_0 \Leftrightarrow g(R) > P_0$. It follows that equation (13) has a solution in $C_1 \in [1/q_g, R)$ (and that solution is unique) if and only if $g(1/q_g) \leq P_0$. Since

$$g(1/q_g;\mu_2) = \frac{V_g}{1 - q_b/q_g - V_b}\cdot P_2(1-\mu_2),$$

then $g(1/q_g) \leq P_0$ if and only if $\mu_2 \geq \max\{0, 1 - \frac{P_0}{P_2}\frac{1-q_b/q_g-V_b}{V_g}\} = u$. Hence, we obtain the existence of the equilibrium. For a given $\mu_2 \in [u, 1)$, the equilibrium is as follows: $C_1 = C_1(\mu_2)$, where $C_1(\mu_2)$ is the solution to (13) given μ_2; $C_2 = (C_1(\mu_2) + R)/2$, and from (9) and (11), we obtain that μ_0 is given by

$$\mu_0 = \frac{q_g C_1(\mu_2) - 1}{1 - q_b C_1(\mu_2)}\cdot\frac{P_1 + P_2\mu_2}{P_0} := f(\mu_2). \tag{14}$$

Observe that $\mu_0 \geq 0$ as $1/q_g \leq C_1(\mu_2) < R < 1/q_b$. The fact that $\mu_0 \leq 1$ follows from $f' > 0$ and $f(1) = 1$, which we prove next. The fact that $f'(\mu_2) > 0$ follows from the fact that $C_1'(\mu_2) > 0$, which holds true because $C_1'(\mu_2) = -\frac{g_2'}{g_1'}$ and we have seen $g_1' > 0$ while $g_2' = [\frac{q_g C_1 - 1}{1 - q_b C_1} - \frac{q_g(C_1+R)/2-1}{1-q_b(C_1+R)/2}] \times P_2 < 0$. Finally, to see that $f(1) = 1$, observe that from the second equation of (9), $\mu_0 = 1$ if $\mu_2 = 1$. Therefore, when $\mu_2 \to 1$, the equilibrium in case 2 converges to the equilibrium in case 1. ■

Proof of Proposition 2. Considering that the CRA obtains a share α of the surplus of the project, by maximizing its payoff, it also maximizes the social surplus, which is calculated below. Given the number of issuers N, if the realized number of good issuers is $n \in \{0, 1, .., N\}$, the ex-post value created (or total surplus generated) in a pooling equilibrium with threshold k is kV_g if $n \geq k$ and $nV_g + (k-n)V_b$ if $n < k$. Thus, the expected total surplus (before n is realized) associated with k is given by

$$\begin{aligned} V(k,N) &= \Pr(n \geq k) \times kV_g + \sum_{n=0}^{k-1} P_n(nV_g + (k-n)V_b) \\ &= \Pr(n \geq k) \times kV_g + \Pr(n < k) \times kV_b + (V_g - V_b) \times \sum_{n=0}^{k-1} P_n n \\ &= kV_g - \Pr(n < k)k \times (V_g - V_b) + (V_g - V_b) \times \sum_{n=0}^{k-1} P_n n \\ &= kV_g - (V_g - V_b)[\Pr(n < k)k - \sum_{n=0}^{k-1} P_n n], \end{aligned}$$

where P_n denotes the ex-ante probability that n issuers are good. The optimal k maximizes $V(k,N)$. Finding the optimal k is not a trivial problem. Therefore, we next use an asymptotic approximation of $V(k,N)$.

We show in the Online Appendix (see Lemma 7) that

$$\frac{V(k,N)}{N} = pV_g + \sqrt{\frac{p(1-p)}{N}}\{\lambda_N V_g - (V_g - V_b)[\Phi(\lambda_N)\lambda_N + \phi(\lambda_N)]\} + o(\frac{1}{\sqrt{N}}),$$

where $\lambda := \frac{k-Np}{\sqrt{Np(1-p)}} \in R$, and $\Phi(.)$ and $\phi(.)$ denote the c.d.f and density functions, respectively, of the Standard Normal. Hence, asymptotically, given N, the optimal k, or equivalently the optimal λ, is obtained by solving:

$$\max_{\lambda} \lambda V_g - (V_g - V_b)[\Phi(\lambda)\lambda + \phi(\lambda)].$$

Observe that $\Phi'(\lambda) = \phi(\lambda)$ and $\phi'(\lambda) = -\lambda\phi(\lambda)$. The first-order condition associated with this maximization problem is $V_g - (V_g - V_b)\Phi(\lambda) = 0$, from which it follows that the optimal λ does not depend on N and is implicitly defined by the condition $\Phi(\lambda^*) = V_g/(V_g - V_b)$. Hence, asymptotically, the optimal k is $k^* = Np + \lambda^*\sqrt{Np(1-p)}$ and the expected total surplus evaluated at the optimal pooling equilibrium is

$$V(k^*) = N\left(pV_g - \sqrt{\frac{p(1-p)}{N}}(V_g - V_b)\phi(\lambda^*)\right).$$

Since the first-best expected total surplus is $V^{FB}(N) = NpV_g$, we obtain that

$$\frac{V(k^*,N)}{V^{FB}(N)} = 1 - \sqrt{\frac{1-p}{Np}}(1 - \frac{V_b}{V_g})\phi(\lambda^*), \tag{15}$$

which means that $V(K^*,N)$ asymptotically approaches $V^{FB}(N)$ in the order of $N^{-1/2}$. Finally, observe that under the optimal k, the probability that a project with a good rating is indeed good when N is sufficiently large is given by

$$\begin{aligned}
\phi_{k^*}^N &= \Pr(n \geq k^*) \times 1 + \sum\nolimits_{n=0}^{k^*-1} P_n \frac{n}{k^*} \\
&= 1 - \Phi(\lambda^*) + \frac{N}{k^*} \times \sum\nolimits_{n=0}^{k^*-1} P_n \frac{n}{N} + O(\frac{1}{\sqrt{N}}) \\
&= 1 - \Phi(\lambda^*) + \frac{1}{p + \lambda^* \sqrt{\frac{p(1-p)}{N}}} \times [p\Phi(\lambda^*) - \sqrt{\frac{p(1-p)}{N}} \phi(\lambda^*)] + O(\frac{1}{\sqrt{N}}) \\
&= 1 - \Phi(\lambda^*) + \frac{1}{p} \times (1 - \lambda^* \sqrt{\frac{1-p}{Np}}) \times [p\Phi(\lambda^*) - \sqrt{\frac{p(1-p)}{N}} \phi(\lambda^*)] + O(\frac{1}{\sqrt{N}}) \\
&= 1 - \Phi(\lambda^*) + (1 - \lambda^* \sqrt{\frac{1-p}{Np}}) \times [\Phi(\lambda^*) - \sqrt{\frac{(1-p)}{Np}} \phi(\lambda^*)] + O(\frac{1}{\sqrt{N}}) \\
&= 1 - \Phi(\lambda^*) + \Phi(\lambda^*) + O(\frac{1}{\sqrt{N}}) \\
&= 1 + O(\frac{1}{\sqrt{N}}).
\end{aligned}$$

It converges to one in the order of $N^{-1/2}$. Finally, the fact that this probability converges to one ensures that all projects with a good rating are actually good with a probability close to one and are thus financed in equilibrium, as was initially supposed. ■

Proof of Proposition 4. In the model, the CRA obtains a fixed share of the social surplus. Hence the equilibria in which the CRA obtains the maximum equilibrium payoff are also those with the maximum social surplus among all the equilibria.

Let h_t denote the history of ratings issued by the CRA until the beginning of period t, H_t denote the set of all such histories, and E denote the set of Perfect Bayesian Equilibria of the game. Let also v_{e,h_t} denote the continuation value to the CRA in equilibrium e at the beginning of period t given history h_t. Observe that the continuation value of the CRA at the beginning of a given period depends only on the public history up to that period. This is because this value is fully determined by the sequence of repayments required by investors in the future and such repayments depend only on the (public) history of ratings. We can define the set $\mathcal{V} = \{v = v_{e,h_t} | e \in E, h_t \in H_t, t = 1, 2, ...\}$. This is the set of all the continuation values to the CRA in any Perfect Bayesian equilibrium. $\mathcal{V}$ is non-empty, since there always exists an equilibrium in which investors never trust the CRA's ratings and the CRA never issues a rating, which implies that $0 \in \mathcal{V}$. Moreover, $\mathcal{V}$ is bounded from above, since the value created by the CRA cannot exceed αV_s^{FB}. Let $\overline{v} \equiv \sup \mathcal{V}$, which exists and is finite, since $\mathcal{V}$ is non-empty and bounded from above. If $\overline{v} = 0$, then the proposition trivially holds. We next prove the proposition for the case where $\overline{v} > 0$, that is, we prove that $\overline{v} \leq \alpha \times (1 - q_b/q_g) \times V_s^{FB}$. Before we proceed, observe that to find an upper bound for

$\overline{v}$ there is no loss in considering only equilibria where the CRA always gives a good rating to an issuer if he is good, as its continuation value is lower otherwise.

By definition of $\overline{v}$, it follows that for any given $\varepsilon \in (0, (1-\beta)\overline{v})$, there exist $v \in \mathcal{V}$ such that $v > \overline{v} - \varepsilon$. Consider the equilibrium, e, and the history h_t such that v_{e,h_t} equals this v. Let ϕ and C denote, respectively, the investors' beliefs and the repayment requested by investors to finance the issuer in (the current) period t (namely the first period of the subgame after history h_t) if he obtains a good rating from the CRA. Let v_0 denote the continuation value to the CRA at the beginning of period $t+1$ if it does not give a good rating in period t, and v_1 denote that value if it does. Finally, let μ denote the equilibrium probability that the CRA recommends the issuer in period t in the event he is bad.

Observe first that $C < R$, otherwise the CRA obtains zero in period t, implying that $v = \beta v_0 \leq \beta \overline{v}$, which contradicts $v > \overline{v} - \varepsilon > \overline{v} - (1-\beta)\overline{v}$. Observe also that we can write,

$$\begin{aligned} v &= p \times [q_g \times \alpha(R-C) + \beta v_1] + (1-p) \times \{\mu \times [q_b \times \alpha(R-C) + \beta v_1] + (1-\mu) \times \beta v_0\} \\ &= [pq_g + (1-p)\mu q_b] \times \alpha(R-C) + \gamma \times \beta v_1 + (1-\gamma) \times \beta v_0, \end{aligned} \tag{16}$$

where $\gamma := p + (1-p)\mu$ is the ex ante probability that the CRA issues a good rating in period t. Given the CRA's rating strategy in period t, consistency of beliefs implies $\phi = p/(p+(1-p)\mu)$, which by (3) implies that

$$C = \frac{p + (1-p)\mu}{pq_g + (1-p)\mu q_b}. \tag{17}$$

Substituting it into (16), we obtain

$$v = \alpha[pV_g + (1-p)\mu V_b] + \gamma\beta v_1 + (1-\gamma)\beta v_0. \tag{18}$$

Next, observe that because $C < R$, then $\mu < 1$. Thus, contingent on the period-t issuer being bad, the CRA must not be strictly better off rating him as good than not doing so, that is, $q_b \cdot \alpha(R-C) + \beta v_1 \leq \beta v_0$, or $\beta v_1 \leq \beta v_0 - q_b \cdot \alpha(R-C)$. Combining this inequality with (18) and using the fact that by definition $v_0 \leq \overline{v}$, it follows that $v \leq \alpha[pV_g + (1-p)\mu V_b - \gamma q_b(R-C)] + \beta v_0 \leq \alpha f(\mu) + \beta \overline{v}$, where $f(\mu) := pV_g + (1-p)\mu V_b - \gamma q_b(R-C)$. From this and the fact that $\overline{v} - \varepsilon < v$, it follows that $\overline{v} - \varepsilon/(1-\beta) < \alpha \times f(\mu)/(1-\beta)$ for any $\varepsilon \in (0, (1-\beta)\overline{v})$, which implies that $\overline{v} < \max_{0 \leq \mu \leq 1} f(\mu)/(1-\beta) + \varepsilon/(1-\beta)$, and since this inequality holds for any $\varepsilon > 0$, we have:

$$\overline{v} \leq \alpha \times \max_{0 \leq \mu \leq 1} f(\mu)/(1-\beta). \tag{19}$$

We next calculate $\max_{0 \leq \mu \leq 1} f(\mu)$. Using (17) and $\gamma := p + (1-p)\mu$, we can write

$f(\mu) = pV_g + (1-p)\mu V_b - [p + (1-p)\mu] \cdot q_b[R - \frac{p+(1-p)\mu}{pq_g+(1-p)q_b\mu}]$. Then,

$$f' = -(1-p)\left(\frac{p(q_g - q_b)}{pq_g + (1-p)q_b\mu}\right)^2 < 0.$$

Therefore, $\max_{0\leq\mu\leq1} f(\mu) = f(0) = pV_g(1 - q_b/q_g)$. It follows by (19) that $\overline{v} \leq \alpha \times (1 - q_b/q_g) \times pV_g/(1-\beta) = \alpha \times (1 - q_b/q_g) \times V_s^{FB}$. This completes the proof. ■

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10 Online Appendix

This online appendix consists of a characterization and derivation of all the equilibria of the game under simultaneous rating when $N = 2$, the derivation of the asymptotic approximation of $V(k, N)$ used in the proof of Proposition 2, and the proof of Proposition 5 in the paper.

10.1 Full Characterization of the Equilibria under Simultaneous Rating when $N = 2$

We present here a complete analysis and characterization of the equilibria. Parts of the material presented in the proof of Proposition 1 may be appear repeated here. We begin with a preliminary analysis on the necessary and sufficient conditions for a value-creating equilibrium. We then present and prove a proposition that offers a complete characterization of all those equilibria.

Recall that in a value-creating equilibrium, (i) both issuers ask for a rating from the CRA because if the CRA serves only one issuer then it cannot create value as we show at the beginning of Section 4; and (ii) if the CRA issues only one good rating, the CRA gives it to a good issuer if there is one (see Proposition 1). Hence, to complete the characterization of the CRA's strategy in a value-creating equilibrium, it is sufficient to characterize the CRA's choice of the number of good ratings it issues given the number of good issuers n. Hence, in the remainder of this section, with some abuse of language, we call the CRA's strategy the profile $\{\mu_n\}_{n=0,1,2}$, where μ_n is the probability that the CRA chooses to issue one good rating–i.e. $k = 1$–in state $n \in \{0, 1, 2\}$. This means that the CRA chooses $k = 2$ with probability $1 - \mu_n$ since $k = 0$ is never optimal for the CRA. As in the main text, the investor's beliefs and strategy is a profile of $\{\phi_k, C_k\}_{k=1,2}$. For convenience of exposition, let π_k^n denote the CRA's expected profit from issuing $k \in \{1, 2\}$ good ratings in state $n \in \{0, 1, 2\}$. Recall than in a value-creating equilibrium $C_1 < R$. Hence, $\pi_1^0 = q_b \times \alpha(R - C_1)$, $\pi_1^1 = \pi_1^2 = q_g \times \alpha(R - C_1)$, $\pi_2^0 = 2q_b \times \alpha \max\{R - C_2, 0\}$; $\pi_2^1 = (q_b + q_g) \times \alpha \max\{R - C_2, 0\}$, and $\pi_2^2 = 2q_g \times \alpha \max\{R - C_2, 0\}$.

Given this, a profile $\{\mu_n; \phi_k, C_k\}_{n=0,1,2;k=1,2}$ constitutes a value-creating equilibrium if and only if:

(i) Given the CRA's strategy $\{\mu_n\}_{n=0,1,2}$, the investors' beliefs satisfy Bayes rule whenever possible, that is,

$$\phi_1 = \frac{P_1\mu_1 + P_2\mu_2}{P_0\mu_0 + P_1\mu_1 + P_2\mu_2} \tag{20}$$

$$\phi_2 = \frac{P_1(1-\mu_1)/2 + P_2(1-\mu_2)}{P_0(1-\mu_0) + P_1(1-\mu_1) + P_2(1-\mu_2)} \tag{21}$$

whenever their denominator is different from zero, and the repayment investors request satisfies

$$C_k = \frac{1}{\phi_k q_g + (1-\phi_k)q_b}, \tag{22}$$

and $C_1 < R$ (if $C_k > R$, they refuse to finance the issuers); and

(ii) Given the investors' belief and strategy $\{\phi_k, C_k\}_{k=1,2}$, for any $n \in (0,1,2\}$, if $\pi_1^n > \pi_2^n$ then $\mu_n = 1$, if $\pi_1^n < \pi_2^n$ then $\mu_n = 0$, and if $\mu_n \in (0,1)$, then $\pi_1^n = \pi_2^n$.

Since ϕ_k is determined by $\{\mu_n\}_{n=0,1,2}$ and C_k is determined by by ϕ_k, in the characterization of the equilibria that follows, we state only $\{\mu_n\}_{n=0,1,2}$, i.e. we state only the CRA's strategy.

Before proceeding, we define two values that will be useful for the characterization of the equilibria. Let $\underline{p}$ be the unique root for p on the interval $[0,1]$ of the following equation

$$2\left(R - \frac{1}{\psi(p)q_g + (1-\psi(p))q_b}\right) = R - \frac{1}{q_g},$$

where $\psi(p) = p^2/[(1-p)^2 + p^2]$; and let

$$u \equiv \max\{0, 1 - \frac{(1-p)^2}{p^2}\frac{1 - q_b/q_g - V_b}{V_g}\}.$$

We can now offer a complete characterization of the value-creating equilibria of the game.

Proposition 6 *Suppose $p\,(2-p) > -V_b/(\Delta R)$. Then, the value-creating equilibria of the game, which depend on the ex-ante probability p that an issuer is good, are as follows:*

(i) If $p \le \underline{p}$, then for each $x \in [u,1]$ there exists an equilibrium in which $\mu_2 = x$, $\mu_1 = 1$ and

$$\mu_0 = \frac{q_g C_1(\mu_2) - 1}{1 - q_b C_1(\mu_2)} \cdot \frac{P_1 + P_2\mu_2}{P_0},$$

where $C_1(\mu_2)$ is the solution for C_1 of

$$\frac{q_g C_1 - 1}{1 - q_b C_1} \cdot (P_1 + P_2\mu_2) + \frac{q_g(C_1+R)/2 - 1}{1 - q_b(C_1+R)/2} \cdot P_2(1-\mu_2) = P_0.$$

This μ_0 increases with μ_2 and $\mu_0 = 1$ at $\mu_2 = 1$.

(ii) If $p > \underline{p}$, then, in addition to the equilibrium above, there is an equilibrium in which $\mu_0 = \mu_2 = 0$ and $\mu_1 = \min\{1, \widehat{\mu}_1\}$, where $\widehat{\mu}_1$ is the unique root for μ_1 on the interval $(0, \infty)$ of the following equation

$$\frac{q_g + q_b}{q_g}(R - \frac{1}{\phi_2(\mu_1)q_g + (1 - \phi_2(\mu_1))q_b}) = R - \frac{1}{q_g},$$

where $\phi_2(\mu_1)$ is given by (21) when $\mu_0 = \mu_2 = 0$.

As we can see, there exists a continuum of equilibria in which the CRA creates value (observe that $u < 1$). Moreover, there exist equilibria in which the CRA gives a good rating to both issuers and investors trust those ratings enough to finance both issuers. We now prove the proposition. The proof is considerably long, as several cases need to be considered separately.

Proof. In any value creating equilibrium, $C_1 < C_2$ and $C_1 < R$. Thus, either $C_1 < R \leq C_2$ or $C_1 < C_2 < R$. We consider each of these cases separately.

Case A: $C_1 < R \leq C_2$. In this case, $\pi_2^n = 0$ for any n. Thus, for all $n \in \{0, 1, 2\}$ the CRA chooses $k = 1$, that is, $\mu_n = 1$. By (20), $\phi_1 = P_1 + P_2 = p\,(2 - p)$. By (22), $C_1 = 1/[(P_1 + P_2)\,q_g + P_0 q_b]$. As $k = 2$ is off-the-equilibrium path, we can choose any value for ϕ_2 that is consistent with $C_2 \geq R$, that is, we can choose any $\phi_2 \leq -V_b/(\Delta R)$. Given $\phi_1 = p\,(2 - p) > -V_b/(\Delta R)$, we have $C_1 < R$. Hence, $\{\mu_n = 1\}_{n=0,1,2}$ underpins some value creating equilibria, which are special case of part (i) of the proposition with $x = 1$.

Case B: $C_1 < C_2 < R$. In this case,

$$\begin{aligned}
\pi_1^0 &\geq \pi_2^0 \Leftrightarrow R - C_1 \geq 2(R - C_2) \\
\pi_1^1 &\geq \pi_2^1 \Leftrightarrow R - C_1 \geq \frac{q_g + q_b}{q_g}(R - C_2) \\
\pi_1^2 &\geq \pi_2^2 \Leftrightarrow R - C_1 \geq 2(R - C_2).
\end{aligned}$$

From direct inspection of these conditions and the fact that $\frac{q_g+q_b}{q_g}(R - C_2) < 2(R - C_2)$ as $R - C_2 > 0$, one obtains that there are three relevant subcases regarding the values of $R - C_1$ and $R - C_2$, which are analyzed separately in next three subcases.

Subcase B.1: Suppose $R - C_1 < ((q_g + q_b)/q_g)(R - C_2) < 2(R - C_2)$. In this case $\pi_2^n > \pi_1^n$ for all $n \in \{0, 1, 2\}$. Hence, if there is an equilibrium in this subcase, in this equilibrium the CRA issues two good ratings regardless of the number of good issuers, that is, $\mu_n = 0$. Bayesian updating by investors implies that $\phi_2 = p$. It follows from (22) that $C_2 = 1/(pq_g + (1 - p)q_b) > R$, which contradicts the supposition that $C_2 < R$ in case B. Hence, there is no value-creating equilibrium in this subcase.

Subcase B.2: Suppose that $((q_g + q_b)/q_g)(R - C_2) \leq R - C_1 < 2(R - C_2)$. In this case, $\pi_2^n > \pi_1^n$ for $n \in \{0, 2\}$ and $\pi_1^1 \geq \pi_2^1$. Hence, if a value-creating equilibrium exists for this case, $\mu_n = 0$ for $n \in \{0, 2\}$. From (20) and (21), $\phi_1 = 1$ – hence $C_1 = 1/q_g$ – and that

$$\phi_2 = \frac{P_1(1 - \mu_1)/2 + P_2}{P_0 + P_1(1 - \mu_1) + P_2}, \tag{23}$$

If $p \leq 1/2$ and hence $P_0 \geq P_2$, then ϕ_2 decreases with μ_1, which implies that $\phi_2 \leq P_1/2 + P_2 = p$. It follows from (22) that $C_2 > 1/(pq_g + (1-p)q_b) > R$, which contradicts the supposition that $C_2 < R$. To find value creating equilibria, we focus on the case with $p > 1/2$. In this case, ϕ_2 increases with μ_1 and hence $C_2 = 1/[\phi_2 q_g + (1 - \phi_2)q_b] := C_2(\mu_1)$ decreases with μ_1. At $\mu_1 = 1$, ϕ_2 reaches

$$\psi(p) := \frac{P_2}{P_0 + P_2} = \frac{p^2}{(1-p)^2 + p^2},$$

and hence C_2 reaches

$$C(p) := \frac{1}{\psi(p)q_g + (1 - \psi(p))q_b}.$$

Note that $\psi'(p) > 0$, $\psi(1/2) = 1/2 < p$ and $\psi(1) = 1$ and hence $C'(p) < 0$, $C(1/2) > \frac{1}{pq_g + (1-p)q_b} > R$ and $C(1) = \frac{1}{q_g}$. Define $\overline{p}$ and $\underline{p}$ respectively by

$$\begin{aligned} \frac{q_g + q_b}{q_g}(R - C(\overline{p})) &= R - \frac{1}{q_g} \\ 2(R - C(\underline{p})) &= R - \frac{1}{q_g}. \end{aligned}$$

Then $\overline{p}$ and $\underline{p}$ uniquely exist and satisfy $1 > \overline{p} > \underline{p} > 1/2$. Depending on the value of p, there are three scenarios to consider.

If $p \geq \overline{p}$: In this scenario,

$$\frac{q_g + q_b}{q_g}(R - C_2(\mu_1)) = R - C_1, \tag{24}$$

that is, $\pi_1^1 = \pi_2^1$, has a unique solution $\mu_1 = \widehat{\mu} \in (0, 1]$ because, if we let $f(\mu_1)$ denote the left hand side of (24), then (i) $f' = -\frac{q_g + q_b}{q_g}C_2' > 0$; (ii) at $\mu_1 = 0$, as $\phi_2 = p$ and hence $C_2 = \frac{1}{pq_g + (1-p)q_b} > R$, we have $f < 0 < R - \frac{1}{q_g} = R - C_1$; and (iii) at $\mu_1 = 1$, $f = \frac{q_g + q_b}{q_g}(R - C(p))|_{p \geq \overline{p}} \geq \frac{q_g + q_b}{q_g}(R - C(\overline{p})) = R - C_1$. Therefore, $\{\mu_0 = 0, \mu_1 = \widehat{\mu}, \mu_2 = 0; \phi_1 = 1,$ ϕ_2 given by (23), $C_1 = 1/q_g, C_2 = C_2(\mu_1)\}$ forms a value creating equilibrium.

If $\underline{p} < p < \overline{p}$: In this scenario, $((q_g + q_b)/q_g)(R - C(p)) < R - C_1 < 2(R - C(p))$, that is, at $\mu_1 = 1$ (and $\mu_0 = \mu_2 = 0$), $\pi_2^n > \pi_1^n$ for $n \in \{0, 2\}$ and $\pi_1^1 > \pi_2^1$. Thus, $\{\mu_0 = 0, \mu_1 = 1, \mu_2 = 0; \phi_1 = 1,$ ϕ_2 given by (23), $C_1 = 1/q_g, C_2 = C(p)\}$ forms a value creating equilibrium.

Note that at $p = \overline{p}$, the root of (24) for μ_1, namely $\widehat{\mu}$, equals 1 and that $\widehat{\mu} > 1$ if $p < \overline{p}$. Thus the two scenarios can be summarized as $\mu_1 = \min(1, \widehat{\mu})$.

If $p \leq \overline{p}$: In this scenario, $R - C_1 \geq 2(R - C(p)) > ((q_g + q_b)/q_g)(R - C(p))$. This cannot be consistent with inequalities $((q_g + q_b)/q_g)(R - C_2) \leq R - C_1 < 2(R - C_2)$, which define this subcase. The subcase with which it is consistent is considered as follows.

Subcase B.3: Suppose that $((q_g + q_b)/q_g)(R - C_2) < 2(R - C_2) \leq R - C_1$. In this case, $\pi_1^n \geq \pi_2^n$ for $n \in \{0, 2\}$ and $\pi_1^1 > \pi_2^1$. Hence, if a value-creating equilibrium exists, $\mu_1 = 1$. If $2(R - C_2) < R - C_1$, then $\pi_1^n > \pi_2^n$ for all $n \in \{0, 2\}$ and in equilibrium $\mu_0 = \mu_2 = 1$. In this scenario, the CRA's strategy is the same as was in Case A. And again, $k = 2$ is off-equilibrium path. The only difference is that now ϕ_2 has to be consistent with $2(R - C_2) < R - C_1$, rather $R - C_2 \leq 0$.

We are left to consider the scenario that $k = 2$ is on equilibrium path, that is,

$$R - C_1 = 2(R - C_2). \tag{25}$$

and $\mu_n < 1$ for at least one $n \in \{0, 2\}$. Then,

$$\phi_1 = \frac{P_1 + \mu_2 P_2}{\mu_0 P_0 + P_1 + \mu_2 P_2} \quad \text{and} \quad \phi_2 = \frac{(1 - \mu_2) P_2}{(1 - \mu_0) P_0 + (1 - \mu_2) P_2}. \tag{26}$$

It follows from these two equations respectively that

$$\mu_0 = \frac{1 - \phi_1}{\phi_1} \cdot \frac{P_1 + P_2 \mu_2}{P_0} \quad \text{and} \quad 1 - \mu_0 = \frac{1 - \phi_2}{\phi_2} \cdot \frac{P_2(1 - \mu_2)}{P_0}. \tag{27}$$

Hence, μ_2 satisfies:

$$\frac{1 - \phi_1}{\phi_1} \cdot \frac{P_1 + P_2 \mu_2}{P_0} + \frac{1 - \phi_2}{\phi_2} \cdot \frac{P_2(1 - \mu_2)}{P_0} = 1. \tag{28}$$

From (22) it follows that for all $i = 1, 2$,

$$\frac{1 - \phi_i}{\phi_i} = \frac{q_g C_i - 1}{1 - q_b C_i}. \tag{29}$$

Note that as $C_i < R < 1/q_b$ in the subcase, $\phi_i > 0$ implies $C_i < 1/q_g$. Substituting (29) for $k = 1, 2$ into (28) and rearranging, we obtain

$$\frac{q_g C_1 - 1}{1 - q_b C_1} \cdot (P_1 + P_2 \mu_2) + \frac{q_g C_2 - 1}{1 - q_b C_2} \cdot P_2(1 - \mu_2) = P_0. \tag{30}$$

All together, in this scenario a value creating equilibrium exists if and only if there is a $\mu_2 \in [0, 1]$ such that the simultaneous equations of (25) and (30) has a solution for $(C_1, C_2) \in$

$[1/q_g, R] \times [1/q_g, R]$. We next show that for each $\mu_2 \geq \max\{0, 1 - \frac{(1-p)^2}{p^2}\frac{1-q_b/q_g-V_b}{V_g}\}$ such a solution – thus a value creating equilibrium – uniquely exists.

Using (25) to eliminate C_2 in (30), we obtain

$$g(C_1; \mu_2) \equiv \frac{q_g C_1 - 1}{1 - q_b C_1} \cdot (P_1 + P_2\mu_2) + \frac{q_g(C_1 + R)/2 - 1}{1 - q_b(C_1 + R)/2} \cdot P_2(1 - \mu_2) = P_0. \tag{31}$$

Observe that g is continuous and $g_1' > 0$ for all $C_1 \in (1/q_h, R]$. Moreover,

$$g(R; \mu_2) = \frac{V_g}{-V_b} \cdot (P_1 + P_2\mu_2) + \frac{V_g}{-V_b} \cdot P_2(1 - \mu_2) = \frac{V_g}{-V_b} \cdot (P_1 + P_2).$$

Thus, $g(R) > P_0 \Leftrightarrow P_1 + P_2 > (-V_b)/V_g \cdot P_0 \Leftrightarrow P_1 + P_2 > (-V_b)/(V_g - V_b)$, which is equivalent to $p\,(2-p) > -V_b/(\Delta R)$, the condition under which the value creating equilibria exist. It follows that equation (31) has a solution in $C_1 \in [1/q_g, R]$ (and that solution is unique) – denote it by $C_1(\mu_2)$ – if and only if $g(1/q_g) \leq P_0$. Since

$$g(1/q_g; \mu_2) = \frac{V_g}{1 - q_b/q_g - V_b} \cdot P_2(1 - \mu_2),$$

this condition is satisfied if and only if $\mu_2 \geq \max\{0, 1 - \frac{P_0}{P_2}\frac{1-q_b/q_g-V_b}{V_g}\} = \max\{0, 1 - \frac{(1-p)^2}{p^2}\frac{1-q_b/q_g-V_b}{V_g}\}$. With (25), $C_2 \in [1/q_g, R]$ whenever $C_1 \in [1/q_g, R]$. Hence, we obtain the existence of the equilibrium.

Given μ_2, by (27) and (29), $\mu_0 = \frac{q_g C_1(\mu_2) - 1}{1 - q_b C_1(\mu_2)} \cdot \frac{P_1 + P_2\mu_2}{P_0}$. This μ_0 increases with μ_2 because $g_2' = [\frac{q_g C_1 - 1}{1 - q_b C_1} - \frac{q_g(C_1+R)/2-1}{1-q_b(C_1+R)/2}] \times P_2 < 0$ and hence $C_1'(\mu_2) = -\frac{g_2'}{g_1'} > 0$. Moreover, from the second equation of (27), $\mu_0 = 1$ at $\mu_2 = 1$. That is, there is a continuity between these two scenarios of this subcase. ■

10.2 Asymptotic Approximation of $V(k, N)$

We present in this section the derivation of the asymptotic approximation of $V(k, N)$ used in the proof of Proposition 2.

Lemma 7 *Let $\Phi(.)$ and $\phi(.)$ denote the c.d.f and density functions, respectively, of the Standard Normal distribution. Then,*

$$\frac{V(k, N)}{N} = pV_g + \sqrt{\frac{p(1-p)}{N}}\{\lambda V_g - (V_g - V_b)[\Phi(\lambda)\lambda + \phi(\lambda)]\} + o(\frac{1}{\sqrt{N}}),$$

where $\lambda = \frac{k - Np}{\sqrt{Np(1-p)}}$.

Proof. We know that $V(k,N) = kV_g - (V_g - V_b)[\Pr(n<k)k - \sum_{n=0}^{k-1} P_n n]$. This implies that

$$\frac{V(k,N)}{N} = \frac{k}{N}V_g - (V_g - V_b)\frac{\Pr(n<k)k - \sum_{n=0}^{k-1} P_n n}{N}. \tag{32}$$

As $k = Np + \lambda\sqrt{Np(1-p)}$, we can write

$$\frac{k}{N} = p + \lambda\sqrt{\frac{p(1-p)}{N}}. \tag{33}$$

Let $t := \frac{n-Np}{\sqrt{Np(1-p)}}$. By the Central Limit Theorem, asymptotically, $t \sim N(0,1)$ with density $\phi(t) = (\sqrt{2\pi})^{-1}e^{\frac{-t^2}{2}}$ and c.d.f. $\Phi(t) = \int_{-\infty}^{t}\phi(s)ds$. By the Barry-Esseen Theorem, for N large,

$$\Pr(n<k) = \Phi(\lambda) + O(\frac{1}{\sqrt{N}}). \tag{34}$$

Observe also that

$$\begin{aligned}
\sum_{n=0}^{k-1} P_n \frac{n}{N} &= \Pr(n<k)E[\frac{n}{N} \mid n<k] \\
&= \Pr(n<k)E[\frac{Np + t\sqrt{Np(1-p)}}{N} \mid n<k] \\
&= \Pr(n<k)E[p + t\sqrt{\frac{p(1-p)}{N}} \mid n<k] \\
&= \Pr(n<k)p + \sqrt{\frac{p(1-p)}{N}} \times \Pr(n<k)E[t \mid t<\lambda] \\
&= \Pr(n<k)p + \sqrt{\frac{p(1-p)}{N}}\int_{-\infty}^{\lambda} t\phi(t)dt + o(\frac{1}{\sqrt{N}}).
\end{aligned}$$

Using the fact that $\int_{-\infty}^{\lambda}\phi(t)tdt = \int_{-\infty}^{\lambda}\frac{1}{\sqrt{2\pi}}e^{\frac{-t^2}{2}}tdt = -\phi(\lambda)$, we can write

$$\sum_{n=0}^{k-1} P_n \frac{n}{N} = \Pr(n<k)p - \sqrt{\frac{p(1-p)}{N}}\phi(\lambda) + o(\frac{1}{\sqrt{N}}). \tag{35}$$

Using (33), (34) and (35) into (32), we obtain

$$
\begin{aligned}
\frac{V(k,N)}{N} &= V_g \times (p + \lambda\sqrt{\frac{p(1-p)}{N}}) \\
&\quad -(V_g - V_b) \times \left\{ \begin{array}{c} \Pr(n<k) \times (p + \lambda\sqrt{\frac{p(1-p)}{N}}) \\ -[\Pr(n<k) \times p - \sqrt{\frac{p(1-p)}{N}}\phi(\lambda) + o(\frac{1}{\sqrt{N}})] \end{array} \right\} \\
&= pV_g + \sqrt{\frac{p(1-p)}{N}}\lambda V_g - \sqrt{\frac{p(1-p)}{N}}(V_g - V_b)[\Phi(\lambda)\lambda + \phi(\lambda)] + o(\frac{1}{\sqrt{N}}) \\
&= pV_g + \sqrt{\frac{p(1-p)}{N}}\{\lambda V_g - (V_g - V_b)[\Phi(\lambda)\lambda + \phi(\lambda)]\} + o(\frac{1}{\sqrt{N}})
\end{aligned}
$$

which completes the proof. ■

10.3 Proof of Proposition 5

Proof of Proposition 5. Let $r_t \in \{\varnothing, b, g\}$ denote the rating issued by the CRA in period t (where $\varnothing$ indicates that no rating was issued), $\varphi_t \in \{\varnothing, f, s\}$ denote the performance of the project of the issuer of period t (where $\varnothing$ indicates that the project was not funded), $h_t = \{r_\tau, \varphi_\tau\}_{\tau=1}^{t-1}$ denote the rating and performance history until the beginning of period t, H_t denote the set of all such histories, and E denote the set of Perfect Bayesian Equilibria of the game. Let also v_{e,h_t} denote the continuation value to the CRA in equilibrium e at the beginning of period t given history h_t. Observe that the continuation value of the CRA at the beginning of a given period depends only on the public history up to that period. This is because this value is fully determined by the sequence of repayments required by investors in the future and such repayments depend only on the public history of ratings and projects' performance. We can define the set $\mathcal{V} = \{v_{e,h_t} | e \in E, h_t \in H_t, t = 1, 2, ...\}$. This is the set of all the continuation values to the CRA in any Perfect Bayesian equilibrium. The set $\mathcal{V}$ is non-empty, since there always exists an equilibrium in which investors never trust the CRA's ratings and the CRA never issues a rating, implying that $0 \in \mathcal{V}$. Moreover, $\mathcal{V}$ is bounded from above, since the CRA's payoff cannot exceed V_s^{FB}. Hence $\overline{v} \equiv \sup_{v \in \mathcal{V}} v$ exists and is finite. Considering that the CRA obtains a fixed share of α of the social surplus, the social value created by the CRA is thus bounded from above by $\frac{1}{a} \times \overline{v}$. To prove the proposition, hence, it suffices to show that $\overline{v} \leq \alpha \times [(q_g - q_b)/(q_g - q_g q_b)]V_s^{FB}$.

If $\overline{v} = 0$, then the proposition trivially holds. Now consider the case where $\overline{v} > 0$. Before we proceed, observe that to find an upper bound for $\overline{v}$ there is no loss in considering only equilibria where the CRA always gives a good rating to an issuer if he is good, as its continuation value is lower otherwise. By definition of $\overline{v}$, it follows that for any given

$\varepsilon \in (0, (1-\beta)\overline{v})$, there exist $v \in \mathcal{V}$ such that $v > \overline{v} - \varepsilon$. As $v \in \mathcal{V}$, there exists an equilibrium e and history h_t such that $v = v_{e,h_t}$. Focus on period t, which is the first period of the subgame after history h_t. Let ϕ denote the investors' beliefs in this period t about the issuer's quality if he obtains a good rating and C denote the according repayment requested by investors if they finance him. Let v_0, v_1^f and v_1^s denote the equilibrium continuation values to the CRA at the beginning of the next period (i.e. period $t+1$) if in period t, respectively, CRA abstains from giving a good rating (or the rating fails to draw financing), the CRA gives a good rating to the issuer and the issuer's project is financed and fails, and the CRA gives a good rating to the issuer and the issuer's project is financed and succeeds. Finally, let μ denote the probability that the CRA recommends the issuer in period t if he is bad.

Observe that $C < R$, otherwise the CRA obtains zero in period t, implying that $v = \beta v_0 \leq \beta \overline{v}$, which contradicts $v > \overline{v} - \varepsilon > \overline{v} - (1-\beta)\overline{v}$. Since $C < R$, then $\mu < 1$, that is, the CRA abstains from recommending a bad project in period t with a positive probability. (Recall that if $\mu = 1$, $\phi = p$, and by $C = 1/(pq_g + (1-p)q_b) > R$). This means that the value to the CRA from recommending a bad issuer in period t cannot exceed the value from abstaining to recommend the issuer, or

$$f_b + \beta[q_b v_1^s + (1-q_b)v_1^f] \leq \beta v_0, \tag{36}$$

where $f_i = q_i \times \alpha(R-C)$ is the value of the rating fee charged to an issuer of quality $i = g, b$. This implies that

$$v_1^f \leq \frac{\beta v_0 - f_b - \beta q_b v_1^s}{\beta(1-q_b)}. \tag{37}$$

Moreover, the value of the CRA contingent on the period-t issuer being bad is

$$v_b \equiv \max\left\{f_b + \beta[q_b v_1^s + (1-q_b)v_1^f], \beta v_0\right\} = \beta v_0, \tag{38}$$

where the equality follows from (36). Similarly, the value of the CRA contingent on the project being good, as the CRA recommends it with probability one, is

$$v_g = f_g + \beta q_g v_1^s + \beta(1-q_g)v_1^f.$$

The value of the CRA in the subgame after h_t before the type of the period-t issuer is known, namely v, satisfies $v = p \times v_g + (1-p) \times v_b$. From this, (37) and (38), it follows that

$$\begin{aligned} v &\leq p\{f_g + \beta q_g v_1^s + \beta(1-q_g) \cdot \frac{\beta v_0 - f_b - \beta q_b v_1^s}{\beta(1-q_b)}\} + (1-p) \cdot \beta v_0 \\ &= \frac{1}{1-q_b}\{p\Delta\alpha(R-C) + \beta p \Delta v_1^s + \beta[(1-p)(1-q_b) + p(1-q_g)]v_0\}. \\ &\leq \frac{1}{1-q_b}\{p\Delta\alpha(R-C) + \beta(1-q_b)\overline{v}\} \\ &\leq \frac{1}{1-q_b}\{p\Delta\alpha(R-\frac{1}{q_g})\} + \beta\overline{v} \\ &= \frac{\alpha p \Delta V_g}{q_g(1-q_b)} + \beta\overline{v}, \end{aligned} \tag{39}$$

where the second inequality follows from $v_1^s \leq \overline{v}$ and $v_0 \leq \overline{v}$, the third inequality from $C \leq 1/q_g$, and the last equality from $V_g = q_g R - 1$. On the other hand, we have $v > \overline{v} - \varepsilon$. Therefore, from inequality (39) it follows that

$$\begin{aligned} \overline{v} - \varepsilon &< \frac{\alpha p \Delta V_g}{q_g(1-q_b)} + \beta\overline{v} \Leftrightarrow \\ \overline{v} &< \frac{\alpha\Delta}{q_g(1-q_b)} \times \frac{pV_g}{1-\beta} + \frac{\varepsilon}{1-\beta}. \end{aligned} \tag{40}$$

Since the above inequality holds for any $\varepsilon \in (0, (1-\beta)\overline{v})$ and $pv_g/(1-\beta) = V_s^{FB}$, it follows that

$$\overline{v} \leq \frac{\alpha\Delta}{q_g(1-q_b)} \times V_s^{FB} = \alpha \times \frac{(q_g - q_b)}{q_g(1-q_b)} \times V_s^{FB}.$$

This completes the proof. ■