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DOCTORAL THESIS

The Political Economy of Strategic Communication

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Declaration of Authorship

I, Federico VACCARI, declare that this thesis titled, "The Political Economy of Strategic Communication" and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
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Abstract

This thesis contains three chapters exploring the implications of strategically biased information on political outcomes.

The first chapter studies how a politically motivated media outlet misreports information in order to endorse its preferred candidate during an election. The task of identifying the reporting strategy through which an interested outlet can influence the decision of voters is non-trivial as there are many ways in which this can be done. I show that there is only one plausible equilibrium, where the media outlet "pools" information in a way that sways the decision of the median voter – and therefore of a majority of electors.

The second chapter investigates how media bias skews electoral competition and produces distortions in the process of policy formation. I develop a model of communication with endogenous policy-making. Candidates running for office know that information passes through the lens of an interested media outlet before reaching the electorate. This generates tension between pandering to the voter with a populist policy, or pleasing the outlet with a biased policy. I show that the implications of media bias are not confined to distortions of the voters' choice at the ballot box, but they propagate back to the process of policy-making.

In the third chapter, I study to what extent competing forces in the market for news are beneficial for voters. I explore a model where (i) media outlets compete for influence by providing alternative views of the same stories, and (ii) relevant information spreads quickly, and eventually voters listen to all viewpoints. In equilibrium, both media outlets reveal their private information with positive probability, and misreport otherwise. I find that even though competition triggers more news distortions, it always outperforms monopoly: "diversity of opinion" has a value independently of the additional media outlet's bias – even if it is extremely biased.

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Chapter 1

Influential Misreporting in Politics

1.1 Introduction

Mass media are the voters' primary source of policy-relevant information. Therefore, the media play an essential role in the political process. A recent survey shows that about three-quarters of U.S. adults (78%) learned about the 2016 U.S. presidential election from TV-based sources and about two-thirds (65%) from digital sources like news websites and social networks. However, news providers have their own political interests that originate from, e.g., the fingertips of journalists, editorial boards, or from their contributors. This situation makes room for an agency problem: the media may affect electoral outcomes exactly throughout their strategic provision of information.

Indeed, there is a widespread concern among the public that the media are not neutral providers but rather biased reporters of inaccurate, partisan and distorted news. A 2011 study reports that "Fully 66% [of the interviewed] say news stories often are inaccurate, 77% think that news organizations tend to favor one side, and 80% say news organizations are often influenced by powerful people and organizations".[1]

The media engage in news distortion even though misreporting information is a costly activity. For example, media that wittingly deliver biased news can incur a loss of reputation, audience, future profits, and influence. The reporting of false or distorted political information is fined and punished by governments.[2] Although electors are aware of the potential conflicts of interest and the political predisposition

[1] "Just 25% say that news organizations get the facts straight". Surveys by Pew, 2016 and Pew, 2011.

[2] See, e.g., the UK's Representation of the People Act 1983, Section 106, and the Germany's recently approved "NetzDG" law.

of the media, their skepticism is not sufficient to prevent distorted news to affect political outcomes: empirical evidence suggests that while voters can manage to discount biased reports, they are still influenced at the ballot box.[3]

How can a news organization exploit mass communication to exert influence on collective decisions and policy outcomes? How is it possible to achieve influence through the delivery of unbalanced, partisan and false reports whilst rational voters are fully conscious of the conflicts of interest?

I address these questions with a model of strategic communication between an informed and biased media outlet (the sender) and an uninformed electorate (the receivers). The outlet privately knows the relative quality of two exogenous alternatives and, before an election takes place, it delivers a public report about it. Importantly, the outlet can misreport its private information, but at a cost. Then, fully rational and Bayesian voters have to cast a ballot for one of the two alternatives in a majority-rule election. The alternative receiving the most votes is eventually implemented.

The voters and the media outlet might have different preferences regarding the alternatives. However, they share a common interest for "quality": the higher the relative quality is, the better is one alternative with respect to the other. I shall hereafter refer to such relative score as "valence" (Stokes, 1963). Valence is an element of vertical differentiation of the two alternatives, and is orthogonal to the ideological or political dimension of preferences. For instance, it can represent the relative intelligence, ability, or campaign performance of two candidates running for office.

There are situations where the media outlet is in disagreement with a majority of voters, and fully revealing information about valence would result in the implementation of its worst alternative. In these cases, the outlet is willing to pay a cost in order to change the political outcome, e.g., by delivering a news report exaggerating the quality of its favourite candidate. In particular, the outlet benefits from doing so whenever the gains from inducing the implementation of its preferred alternative offset the costs from misreporting. However, voters are fully aware of the media

[3]DellaVigna and Kaplan, 2007 find that, depending on the audience measure, Fox News influenced the voting behaviour of 3-28% of its audience, convincing them to vote Republican. Groseclose and Milyo, 2005 measure media bias in the news reports of major media outlets in the U.S. They find that, on average, outlets tend to be biased towards the left of the political spectrum.

outlet's incentives, and therefore become skeptical and discount any report that is potentially distorted. Hence, an "effective" endorsement would require the delivery of even more expensive messages.

In any equilibrium there are always states where the media outlet misreports its private information. The voters cannot completely neutralize and filter out news distortions, and with some positive probability they are persuaded to implement the best alternative for the outlet when it is also the less preferred by a majority of electors. In particular, there are no fully revealing equilibria where the voters always understand what is the true realized value of valence.

There exists only one unique robust equilibrium.[4] In this equilibrium, the media outlet pools information by delivering the same news report for every intermediate realizations of valence. Upon observing the "pooling report", the median voter is exactly indifferent between the two alternatives. Instead, for relatively extreme realizations of valence, the media outlet reveals truthfully its private information. With (ex-ante) positive probability the outlet induces the implementation of its best alternative, while under complete information the other alternative would be selected by a majority of voters. In these cases, I say that the media outlet has "persuaded" the electorate, and an "election overthrowing" has occurred.

However, there are cases where the media outlet engages in costly misreporting even though it agrees with a majority of electors on which alternative is best. These "white lies" are delivered for two related reasons: (i) skeptical voters discount news reports, and therefore "weak" endorsements are understood as being evidence against the outlet's preferred alternative; (ii) in order to effectively persuade a majority of voters, the pooling report has to be delivered also when there is agreement. Otherwise, voters would understand that such report is transmitted only when there is a political conflict of interest with the media outlet, making the endorsement ineffective.

The main results of this paper concern the extent to which an informed media outlet persuades an electorate and affects political outcomes in equilibrium, and how news are strategically misreported. The approach I use in this paper is different from related works, where misreporting or "lying" is assumed to be either without any

[4]The introduction of misreporting costs transforms cheap talk communication into a costly signalling game, at a cost of having a large multiplicity of equilibria. Uniqueness is obtained by applying Cho and Kreps, 1987's Intuitive Criterion refinement.

consequence (as in cheap talk games) or impossible at all (as in verifiable disclosure and persuasion models).

In contrast with previous related work, this paper is the first to perform some intuitive and testable comparative statics on the cost of misreporting information within a political environment. Furthermore, the theoretical findings of this paper are in line with recent empirical evidence on media bias and its effects on political outcomes.[5]

1.2 Related Literature

The present paper contributes to the literature exploring the political economy of media bias. This body of research employs demand side and supply side models in order to study the implications of biased information on political outcomes.[6]

In demand side models, the media seek to maximize profits, while their concerns for political outcomes is irrelevant or negligible.[7] The media's incentive to please their audience or to cater to more profitable segments of voters eventually triggers biased news reporting, which might affect political outcomes.[8] Within this tradition, Gentzkow and Shapiro, 2006 develop a model where consumers are uncertain about the quality of information provided by media firms. Hence, in order to establish a reputation as high-quality news providers, media firms slant reports toward the prior of their readership. Similarly to the present paper, they find that an increase in the likelihood of ex-post news verification mitigates such distortion.

Mullainathan and Shleifer, 2005 and Bernhardt, Krasa, and Polborn, 2008 study a behavioural model where readers have preferences for news that are consistent with their preconceptions. Profit maximizing "newspapers" slant their reports in order to cater to the confirmatory preferences of their audience. They show that competition might increase media bias. In Strömberg, 2004, media firms operate under increasing

[5]Other theoretical models with misreporting or "lying" costs are Ottaviani and Squintani, 2006, Kartik, Ottaviani, and Squintani, 2007, Kartik, 2009 and Chen, 2011. See Section 1.5 for a discussion on the match between the main results of this paper and the related empirical evidence.

[6]Extensive surveys on the topic are provided by, among others, Strömberg, 2015, Gentzkow, Shapiro, and Stone, 2014, Sobbrio, 2014, Prat and Strömberg, 2013 and Gentzkow and Shapiro, 2008.

[7]Empirical evidence from Gentzkow and Shapiro, 2010 suggest that newspapers' slant is similar to the position that would be chosen by profit-maximizing firms.

[8]Calvert, 1985 and Suen, 2004 show that rational decision makers exhibit a demand for information that conforms to their priors.

return to scale. Therefore, issues that concern large groups are more appealing and widely covered, while minority groups and special interests tend to be neglected. This bias translates into a policy bias: the media affect public policy because they provide the channel through which politicians convey campaign promises to the electorate.

In contrast with the above-mentioned work, the present paper focuses on the media outlet's concern for policy outcomes. With a "supply side" model of media bias, information is distorted because of the intrinsic preferences and motivations of, e.g., agents who work for news organizations like journalists, editors and owners. Following this approach, Shapiro, 2016 develops a model where the establishment of balanced media reporting leads to an inefficient provision of information and to persistent public ignorance. Special interests take advantage of this practice and exploit it to influence public opinion and, indirectly, policies. In Baron, 2006, journalists have the opportunity to advance their careers and exercise influence by delivering biased reports. Media outlets can hire journalists at a lower wage when they grant them more discretion to slant stories. Even though the provision of biased news yields a lower demand, Baron, 2006 finds that the profits of a high-bias media outlet can be larger than those of a low-bias one.

Duggan and Martinelli, 2011 study the optimal "slant" of a media outlet who seeks to maximize the probability of election of either an incumbent or a challenger. They find that the media outlet is less informative when it favours the frontrunner, and that an unbiased outlet is never socially optimal. In Alonso and Camara, 2016 an "information controller" can influence collective decisions through the design of a public signal. They study how different voting rules and the distribution of voters' preferences affect the optimal design of such signal. Under simple majority voting rule, a majority of voters might be strictly worse off due to the information supplied by the controller.

The most closely related paper is Chakraborty and Ghosh, 2016. They develop a supply-side model of media bias to study how an informed outlet can influence the choice of voters at the ballot box and affect the process of policy-making. Before the election takes place, the outlet delivers a cheap talk message about the valence of two candidates running for office. This message can affect the voters' decision only if the ideological difference between the median voter and the outlet is not too large. However, at the outset, the candidates simultaneously make a binding commitment to policy proposals. Therefore, the endorsements and the conflict of interest are endogenously determined. The present paper differs from Chakraborty and Ghosh, 2016 in that here (i) alternatives are exogenous, and (ii) misreporting information is possible, but at a cost. This allows me to focus on how the outlet persuades voters for different levels of misreporting costs rather than for different conflicts of interest.

The key feature that distinguishes the present paper from the related literature is how communication is modelled. Previous work assumes that misreporting information has either no direct consequences or it is simply not possible. For example, Baron, 2006, Bernhardt, Krasa, and Polborn, 2008, Chan and Suen, 2009, and Gul and Pesendorfer, 2012 belong to the former cheap talk tradition; Strömberg, 2004, Besley and Prat, 2006, Duggan and Martinelli, 2011, Alonso and Camara, 2016 make use of the latter approach. In contrast, I allow the media outlet to misreport, but at a cost that depends on the magnitude of misrepresentation.

To this end, my paper touches upon the literature on communication with lying costs. Kartik, 2009 departs from the canonical "cheap talk" setting of Crawford and Sobel, 1982 by introducing an exogenous direct cost for misreporting information. In equilibrium, (almost) every type of sender reports to be of a higher type than it would under complete information. Since the state space is bounded above, lower types separate, while higher types pool into one or more segments. Similarly, Kartik, Ottaviani, and Squintani, 2007 study costly lying within an unbounded state space, obtaining language inflation with full separation.[9]

Differently from this literature, I consider a model with two alternatives and partial agreement; that is, there are some states where the sender and the receiver agree on which alternative is best. In addition, in this literature the sender is assumed to play "monotonic" strategies, that is, to (weakly) "exaggerate" its type whenever it would benefit from being considered to be of an higher type. In the present paper, this "language inflation" endogenously arises in the unique robust equilibrium, without assuming monotone strategies.

[9]Chen, 2011 and Ottaviani and Squintani, 2006 model cheap talk communication without direct lying costs but with possibly non-strategic players. In such settings, receiver's naivety induces costs that are similar to lying costs.

1.3 The Model

1.3.1 Set-Up

There are two types of decision makers: one expert, e, and a unit mass of voters, where each voter is indexed by $i \in [0,1]$. I shall refer to the expert as the "media outlet" or simply as the outlet. There is a state of the nature $\theta \in \Theta \equiv \mathbb{R}$, which I refer to as "valence". Valence is randomly drawn from a common knowledge well-behaved probability density function p with support on the real line. Figure 1.1 illustrates the timeline of the game. There are two sequential stages: (i) in the first "reporting stage" the realization θ of a random variable $\tilde{\theta}$ is privately observed by the expert, where $\Theta \equiv \mathbb{R}$ is the state space.[10] Then, the expert delivers a public report $r \in \mathbb{R}$ to the whole electorate, with the literal or exogenous meaning *"I report that $\theta = r$"*;[11] (ii) the game ends with the "voting stage", where each voter sincerely casts a ballot b_i for one of two alternatives, L and R, in a majority-rule election. I shall refer to the alternatives as the "candidates". Each candidate c is described by an exogenous policy proposal q_c, for $c \in \{L, R\}$, where $q_L < q_R$. The platform of the winning candidate is eventually implemented. The valence θ represents the relative quality of candidate L with respect to candidate R. While θ is a private information of the expert, everything else is common knowledge.

Each decision maker j has an ideal bliss policy $h_j \in \mathbb{R}$, and the electorate is described by the cumulative distribution of bliss policies across voters, F_h. The utility of each voter i is an additively separable combination of standard single peaked policy preferences and candidates' relative quality θ. The utility of the expert e is similar to that of the voters, but with the following key difference: it bears a cost $kC(r,\theta)$ for misreporting its private information from θ to r, where $k \geq 0$ is a scalar parameter.[12] $C(r,\theta)$ is twice continuously differentiable with $C_{rr} > 0 > C_{r\theta}$. Truthful reporting is costless, i.e., $C(\theta,\theta) = 0$, and $C(r,\theta) > 0$ for every $r \neq \theta$. I shall hereafter

[10] I indicate with $\tilde{\theta}$ a random variable $\tilde{\theta} : \Theta \to X$, where Θ is the state space and X a measurable space. Θ is endowed with a probability density measure p. I indicate a generic realization of the random variable $\tilde{\theta}$ with $\theta \in X$. Here $X = \Theta \equiv \mathbb{R}$.

[11] The model allows for different interpretations of the intrinsic meaning of reports. However, it is easier to think of a report as being a common-language literal statement about the expert's private information.

[12] Because the expert cannot commit to a reporting rule, the special case where $k = 0$ is tantamount to a standard cheap talk framework.

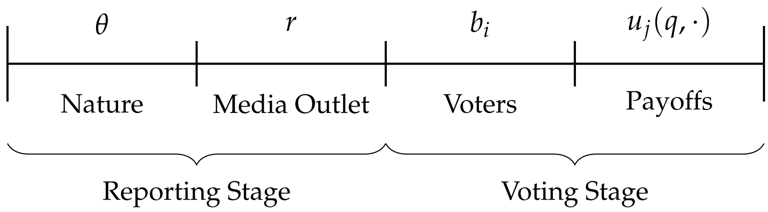


FIGURE 1.1: Stages of the game and timing structure.

assume that the misreporting cost is quadratic, i.e. $C(r, \theta) = (r - \theta)^2$. The following equations (1.1) and (1.2) show the additive separable and single peaked utilities of each voter i and the expert e,

$$u_i(b_i, \theta, \mathbf{q}) = \begin{cases} u_i^L = -\gamma(h_i - q_L)^2 + \theta & \text{if } b_i = L \\ u_i^R = -\gamma(h_i - q_R)^2 & \text{if } b_i = R \end{cases} \tag{1.1}$$

$$u_e(\{b_i\}_i, \theta, r, \mathbf{q}) = \begin{cases} -\lambda(h_e - q_L)^2 + \theta - k(r - \theta)^2 & \text{if } V_L \geq \frac{1}{2} \\ -\lambda(h_e - q_R)^2 - k(r - \theta)^2 & \text{otherwise} \end{cases} \tag{1.2}$$

Where γ and λ are positive intensity parameters weighting the relative importance of policies with respect to valence, $\mathbf{q} = (q_L, q_R)$, and $V_L = \int_{\{i:b_i=L\}} di$ is the size of voters casting a ballot for candidate L.[13]

Given the proposed policies $\mathbf{q} = (q_L, q_R)$, a (pure) reporting strategy for the expert is a function $\psi : \Theta \to \mathbb{R}$, which assigns a report $r \in \mathbb{R}$ to each realization of valence $\theta \in \Theta$. A (pure) voting strategy for each voter i is a function $\nu_i : \mathbb{R} \to \{L, R\}$, assigning a ballot $b_i \in \{L, R\}$ to every possible report r. A belief function for the voters is a function $p : \mathbb{R} \to \Delta(\Theta)$ which, given any media outlet's report r, yields a posterior belief $p(\cdot|r)$. I shall indicate the expected valence given posterior beliefs as $\mu(\theta|r) = \mathbb{E}_p[\theta|r]$. An equilibrium is a weak Perfect Bayesian Equilibrium (wPBE), defined as follows.

Definition 1 (weak Perfect Bayesian Equilibrium)**.** *A weak Perfect Bayesian Equilibrium (wPBE) is a reporting strategy for the media outlet $r = \psi(\theta)$, beliefs for the voters $p(\theta|r)$, and a sequentially rational voting rule $\nu_i(\psi(\theta))$ for each voter i, such that,*

[13]Because the present model focuses on the media outlet's policy-motivation, I do not directly consider its concern for profits and circulation. However, the misreporting cost $kC(r, \theta)$ can be interpreted as a continuation value accounting for the expected future loss of profits and audience.

- *given the voting rule $\nu_i(\psi(\theta))$, $i \in [0,1]$, and the realized valence θ, the media outlet delivers a report that maximizes its utility. For $r = \psi(\theta)$, $u_e(\nu_m(r), \theta, r, \mathbf{q}) \geq u_e(\nu_m(r'), \theta, r', \mathbf{q})$ for every other $r' \neq r$;*

- *the voters' beliefs $p(\theta|r)$ are Bayesian whenever possible;*

- *given beliefs $p(\theta|r)$, the voting rule ν_i is sequentially rational.*

1.3.2 Definitions and Assumptions

In this section, I discuss some definitions, terms, and assumptions that will be used hereafter. I show that in any equilibrium the media outlet can restrict its attention on persuading the median voter, as this is a necessary and sufficient condition to affect the electoral outcome. In order to simplify the analysis, I shall assume that the media outlet is more left-leaning with respect to a majority of voters. Lastly, I define when a report is "effective" in persuading voters and affecting the political outcome.

This game is in every respect a signalling game, where the media outlet e is the sender and each voter i is a receiver. The media outlet has private information about valence and therefore, according to the usual jargon, I will hereinafter refer to θ as being the "type" of outlet, even though it is a relative feature of the candidates.

From equation (1.1) we can see that a generic decision maker j prefers candidate L over candidate R only if the valence is high enough, in particular, when it exceeds a threshold. I define the "partisan endorsement threshold" for each voter i and the expert e, as follows,

$$\gamma\tau_i(h_i, \mathbf{q}) := \gamma(2h_i - q_L - q_R)(q_R - q_L) \tag{1.3}$$

$$\lambda\tau_e(h_e, \mathbf{q}) := \lambda(2h_e - q_L - q_R)(q_R - q_L) \tag{1.4}$$

If valence takes values that are greater than the partisan endorsement threshold, $\theta > \gamma\tau_i$, then voter i prefers candidate L over candidate R. Such threshold summarizes in one simple score the political predisposition of player j given (i) her ideological bliss h_j, (ii) policy proposals $\mathbf{q}$, and (iii) the relative importance of policy to valence γ, and λ. Higher partisan thresholds indicate that player j is less (more) likely to endorse candidate L (R).

The assumption that voters cast a ballot sincerely is without loss of generality: truth-telling is a dominant strategy under majority voting when there are only two alternatives.[14] I also assume that a voter that is indifferent between the two alternatives casts a ballot for candidate L.[15] Therefore, the sequentially rational voting rule ν_i is such that voter i casts a ballot for candidate L only if, given a report r, she expects the valence to be (weakly) higher than her partisan threshold, $\mu(\theta|r) \geq \gamma\tau_i$. Otherwise, she would cast a ballot for R.

The median voter is defined as $m := \left\{i : F_h(h_i) = \frac{1}{2}\right\}$, with bliss policy h_m. Since all voters share the prior $p(\theta)$ and gather the same information r, they also share the same posterior $p(\theta|r)$ and expectation $\mu(\theta|r)$. Given voters' utility in equation (1.1) and the tie rule, if the median voter prefers an alternative over the other, then a majority of voters does so. That is, if $\mu(\theta|r) \geq (<)\gamma\tau_m$, then $V_L \geq (<)\frac{1}{2}$, and the policy q_L (q_R) is implemented. Hence, in order to "effectively" influence the political outcome, the expert should deliver a report that persuades the median voter.

I define the "swing valence" $\hat{\theta}$ as the lowest value of valence for which, given that voters sincerely cast a ballot according to a sequentially rational voting rule, candidate L would win the election and implement the policy q_L. Given equilibrium beliefs, any report r that induces an expectation $\mu(\theta|r) \geq (<)\hat{\theta}$ would result in the implementation of the policy q_L (q_R). That is, the swing valence is defined as being equal to the median voter's partisan endorsement threshold,

$$\hat{\theta} := \gamma\tau_m(h_m, \mathbf{q}) \tag{1.5}$$

Given that swinging the median voter's decision is sufficient to sway the political outcome, the distribution of ideologies in the electorate F_h does not affect the media outlet's equilibrium reporting strategy.[16]

I say that the media outlet is "unbiased" if the the median voter and the outlet always agree on which alternative is best. This is the case when the they have the

[14] In this case, the sincere voting assumption actually helps to prune out some not plausible strategic voting behaviour. Further, in this setting voters do not have an incentive to abstain.

[15] In equation (1.2) it is assumed that ties are solved in favour of candidate L. That is, whenever $V_L = \int_{\{i:b_i=L\}} di \geq \frac{1}{2}$, candidate L wins and the policy q_L is implemented.

[16] Because the decision of the median voter is what ultimately determines the electoral outcome, the present model can accommodate scenarios where the expert wants to persuade a small committee or a single decision maker. Hence, the assumption of "large election" with a continuum of voters is without loss of generality.

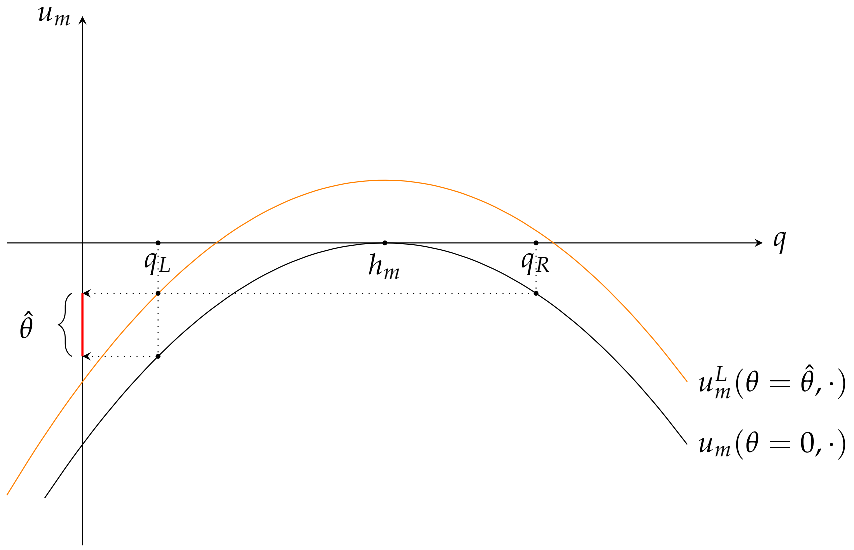


FIGURE 1.2: The swing valence $\hat{\theta}$ is the realization of the valence score that makes the median voter indifferent between the two alternatives. When valence θ is higher than the swing valence $\hat{\theta}$, a majority of voters prefers alternative L to alternative R. The swing valence is equivalent to the partisan endorsement threshold of the median voter $\gamma\tau_m$.

same partisan threshold, $\lambda\tau_e = \gamma\tau_m$.[17] An unbiased media outlet would always fully reveal its private information in equilibrium, and the median voter would always believe the outlet's report. Because misreporting is costly, there are no "babbling" equilibria where the voters ignore the report of the outlet and the outlet reports meaningless messages.

This paper is concerned with a "biased" media outlet. When the median voter and the outlet have different partisan endorsement thresholds, $\lambda\tau_e \neq \gamma\tau_m$, there are realizations of valence for which they disagree on which alternative is best. In these cases, the media outlet is also in disagreement with a majority of voters. Formally, disagreement occurs when $\theta \in (\min\{\lambda\tau_e, \gamma\tau_m\}, \max\{\lambda\tau_e, \gamma\tau_m\})$.

Hereafter, I shall assume that the media outlet is (ex-ante) more likely to support the candidate L (with policy q_L) with respect to the median voter and a majority of electors. In terms of partisan thresholds, I am assuming that $\lambda\tau_e < \gamma\tau_m$.[18] Figure 1.3 depicts the area of disagreement when the media outlet is more left-leaning than a majority of voters.

[17]The media outlet can be unbiased even though it has a different preferred policy than the median voter. Indeed, the outlet is unbiased when its bliss policy is $h_e = \frac{\gamma}{\lambda} h_m - \frac{\gamma-\lambda}{2\lambda}(q_L + q_R)$.

[18]This assumption is without loss of generality. The case where $\lambda\tau_e > \gamma\tau_m$ would lead to similar results.

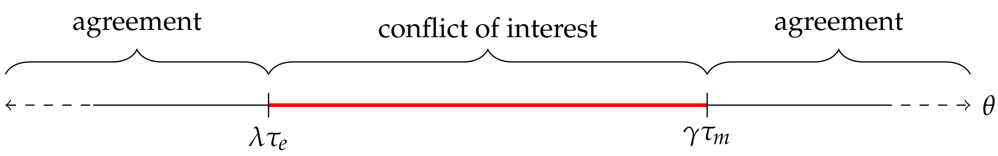


FIGURE 1.3: In red, the area of disagreement when the media outlet is more left-leaning than the median voter. For all realizations of valence that take place in the red area, the media outlet prefers candidate L while a majority of voters prefer candidate R.

Given an equilibrium, a report r is hereafter said to be "effective" for type θ if, in such equilibrium, delivering r yields the implementation of type θ's favourite alternative. Otherwise it is said to be ineffective. The set of equilibrium effective reports for type θ is $\mathcal{E}(\theta)$.[19] Effectiveness (or ineffectiveness) is an equilibrium property of a report for a specific type of expert.[20] More formally, given an equilibrium reporting rule ψ and a type of media outlet θ, the set $\mathcal{E}(\theta)$ of effective reports is defined as,

$$\mathcal{E}(\theta) := \begin{cases} \left\{r \in \mathbb{R} : V_L(r) \geq \frac{1}{2}\right\} & \text{if } \theta > \lambda\tau_e \\ \left\{r \in \mathbb{R} : V_L(r) < \frac{1}{2}\right\} & \text{if } \theta < \lambda\tau_e \\ \varnothing & \text{if } \theta = \lambda\tau_e \end{cases} \tag{1.6}$$

Given an equilibrium and a type θ for which truthful reporting is ineffective, while there exist an effective report $r \neq \theta$, $r \in \mathcal{E}(\theta)$. Depending on the state-dependent misreporting cost $C(r,\theta) = k(r-\theta)^2$, the expert might find it profitable to deliver such counterfeit report. The following definition describes the set of types which would find it convenient to misreport their private information by conveying a specific report r while truthful reporting is ineffective.

Definition 2 (Lowest and highest misreporting types)**.** *Consider an equilibrium and a type θ for which truthful reporting is ineffective, $\theta \notin \mathcal{E}(\theta)$, while there is an effective report $r \neq \theta$, $r \in \mathcal{E}(\theta)$. The lowest type $\underline{\vartheta}(r) \leq r$ and the highest type $\overline{\vartheta}(r) \geq r$ of an expert that*

[19] The tie rules ensure that from the expert's perspective the equilibrium outcome of the election is never random. Therefore, the set $\mathcal{E}(\theta)$ is well defined for every $\theta \in \Theta$.

[20] When $\theta = \lambda\tau_e$, the media outlet is indifferent between the two alternatives. Therefore, $\mathcal{E}(\lambda\tau_e) = \varnothing$ only by convention.

find (weakly) profitable to misreport to r are,[21]

$$\underline{\vartheta}(r) = \begin{cases} r + \frac{1}{2k}\left[1 - \sqrt{1 + 4k(r - \lambda\tau_e)}\right] & \text{if } \lambda\tau_e < \theta \\ r - \frac{1}{2k}\left[1 + \sqrt{1 + 4k(\lambda\tau_e - r)}\right] & \text{if } \lambda\tau_e > \theta \end{cases} \tag{1.7}$$

$$\overline{\vartheta}(r) = \begin{cases} r + \frac{1}{2k}\left[1 + \sqrt{1 + 4k(r - \lambda\tau_e)}\right] & \text{if } \lambda\tau_e < \theta \\ r - \frac{1}{2k}\left[1 - \sqrt{1 + 4k(\lambda\tau_e - r)}\right] & \text{if } \lambda\tau_e > \theta \end{cases} \tag{1.8}$$

The set of types $\theta \in \left(\underline{\vartheta}(r'), \overline{\vartheta}(r')\right)$ finds it more profitable to misreport and deliver an effective report $r' \neq \theta$ rather than reporting truthfully an ineffective $r = \theta$, while types $\underline{\vartheta}(r')$, $\overline{\vartheta}(r')$, and r' are just indifferent. These thresholds are straightforwardly derived by solving a quadratic equation in θ from equation (1.2).

1.4 Solving the Model

1.4.1 Equilibrium

Given that voters are fully aware of the conflict of interest, a natural question is whether the expert is ever going to invest in costly misreporting. If in equilibrium voters can neutralize any attempts of persuasion, then the outlet would economize by always reporting truthfully its private information. At this point we still do not know if an equilibrium exists at all, but the following proposition shows that if it exists, then it must entail some misreporting.[22]

Lemma 1 (Equilibrium misreporting)**.** *In every wPBE the media outlet misreports its private information at some $\theta \in \Theta$.*

Proof. Suppose that the pair reporting rule - system of beliefs $(\psi(\theta) = \theta, p(\theta|r = \theta) = 1)$ for every $\theta \in \Theta$ is a wPBE. All types $\theta \in \left(\underline{\vartheta}\left(\hat{\theta}\right), \hat{\theta}\right)$ have a strict incentive to deviate from the truthful reporting rule by delivering $r = \hat{\theta}$. □

[21]One may wonder whether the solutions for $\underline{\vartheta}(r)$ and $\overline{\vartheta}(r)$ are always real. As we will see later, this is not a concern in any equilibrium due to the weakly increasing property of any incentive compatible reporting rule (see Proposition 4).

[22]Infinite signalling games might suffer from non-existence issues. In these cases, Manelli, 1996 suggests to solve the problem by adding cheap talk communication in addition to the costly signals. I discuss the implication of this approach in the present model at the end of Appendix 1.5.2.

The idea behind the above result is the following. Suppose that there is an equilibrium with truthful reporting at every state, $\psi(\theta) = \theta$ for all $\theta \in \Theta$. As a consequence, voters would correctly infer the realized state of nature, and the political outcome would be the same as under perfect information.[23] However, there are situations where the media outlet can profitably deviate from the prescribed truthful strategy by misreporting its private information. For realizations of valence that are just below the partisan threshold of the median voter $\gamma\tau_m$, the outlet can "inflate" valence by delivering a report $r > \theta$, so as to sway a (weak) majority of voters. Misreporting would entail the payment of a small cost relative to the gain from inducing the implementation of a different policy. Therefore, there cannot exist equilibria where the media outlet never misreports its private information.

A wPBE is said to be "fully revealing" if, given the outlet's report, voters perfectly learn the realized valence at every state $\theta \in \Theta$. The misreporting equilibrium of Lemma 1 has a straightforward implication: voters cannot always understand what is the media outlet's private information.

Corollary 1 (No Full Revelation)**.** *There are no wPBE where the media outlet fully reveals its private information.*

Proof. Suppose that there is a wPBE where beliefs are such that $p\left(\theta|\psi(\theta)\right) = 1$ for every $\theta \in \Theta$. In order for that to be an equilibrium, it must be that $\psi(\theta) = \theta$ for every $\theta \in (\lambda\tau_e, \gamma\tau_m)$. If $\psi(\theta') \neq \theta'$ for some $\theta' \in (\lambda\tau_e, \gamma\tau_m)$, then the outlet would be better off by reporting truthfully. However, as shown in Lemma 1, all types $\theta \in \left(\underline{\vartheta}\left(\hat{\theta}\right), \hat{\theta}\right)$ have a strict incentive to deviate from the truthful reporting rule. □

Because misreporting is costly, every fully revealing equilibria must entail truthful reporting whenever there is conflict of interest. Otherwise, the outlet would be fruitlessly spending resources to misreport, as voters eventually learn the state and select the candidate that is preferred by a majority of electors. No matter what are the voters' equilibrium beliefs, deviating to truthful reporting in such states would yield either the same or a better outcome for the outlet, at no cost. Therefore, there cannot

[23]Actually, in Lemma 1 it is sufficient that each voter casts a ballot for L when $\theta \geq \gamma\tau_i$ and for R otherwise. This weaker condition on posterior beliefs allows for truthful reporting without full revelation of the state in equilibrium. Equilibrium beliefs $p(\theta|r)$ can be such that the expectation is $\mu(\theta|r) \geq \gamma\tau_i$ when $\theta \geq \gamma\tau_i$ and $\mu(\theta|r) < \gamma\tau_i$ otherwise. The same holds for Corollary 1.

be equilibria where the outlet's private information is always fully revealed to the voters.

A similar argument leads to the observation that there are no babbling equilibria: that is, equilibria where voters ignore the information delivered by the media outlet. The idea is straightforward: in equilibrium if voters neglect every report, then the outlet must economize by always reporting truthfully. Since voters are sincere, information never harms them, and thus would be better off by conditioning on the expert's report.

Lemma 2 (No babbling equilibrium)**.** *There are no wPBE where the choice of the voters is independent from the report of the media outlet.*

Proof. Suppose there exists a wPBE where $\nu_i(r) = \nu_i$ for all $r \in \mathbb{R}$ and all $i \in [0,1]$. In order to be an equilibrium, it must be that $\psi(\theta) = \theta$ for every $\theta \in \Theta$. However, given that reporting rule, it is sequentially rational to have $\nu_i = L\ (R)$ if $r = \theta \geq (<) \gamma\tau_i$. $\square$

There exist infinitely many wPBE for this game, mainly sustained by pathological off-path beliefs. The following is an example of unreasonable off-path equilibrium beliefs. Consider a wPBE and a convex set of reports (r', r'') such that $r' > \hat{\theta} = \gamma\tau_m$ and the misreporting costs k are $\frac{\gamma\tau_m - \lambda\tau_e}{(r' - \gamma\tau_m)^2} < k < \frac{4\left(\frac{r''+r'}{2} - \lambda\tau_e\right)}{(r''-r')^2}$. The "lowest misreporting type" that could deliver r' is higher than the swing valence, $\underline{\vartheta}(r') > \hat{\theta}$. This indicates that no type of media that is lower than $\hat{\theta} = \gamma\tau_m$ could profitably deliver a report in (r', r''). However, equilibrium beliefs $p(\theta|r)$ are such that the expected value of valence given any report $r \in (r', r'')$ is lower than the swing valence, $\mu\left(\theta | r \in (r', r'')\right) < \hat{\theta}$, while $\mu(r'|\theta) = \mu(r''|\theta) > \hat{\theta}$. Therefore, given the sequentially rational voting rule ν_i, a majority of voters would cast a preference for candidate R after observing a report $r \in (r', r'')$. However, given that $\lambda\tau_e < \gamma\tau_m < \underline{\vartheta}(r')$, there is no type of media outlet $\theta < \lambda\tau_e$ supporting candidate R that could profitably deliver a report $r \in (r', r'')$.

Only types of media outlet that agree with a majority of electors in supporting candidate L could possibly deliver a report in (r', r''). Yet, given the implausible beliefs of voters, every report $r \in (r', r'')$ would result in the election of candidate R and the implementation of the policy q_R. Every type of outlet $\theta \in (r', r'')$ has to misreport either to r' or to r'' even though truthful reporting would be sufficiently discriminatory. Such behaviour is sustained by an obviously unjustified skepticism of voters, who believe that the media outlet delivers reports $r \in (r', r'')$ when a majority of electors prefer candidate R. Reports in (r', r'') are never delivered in equilibrium.

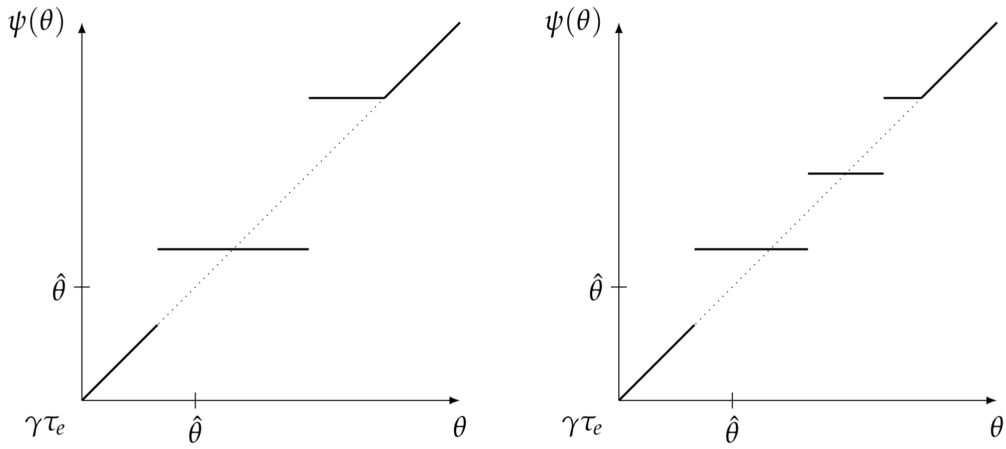
There are two main drawbacks of employing the solution concept of weak Perfect Bayesian Equilibrium as described in Definition 1.3.1. The first is that for this class of games the set of strategy profiles that can be supported as an equilibrium is usually very large. This is a crucial problem as multiplicity severely limits the predictive power of the model. The second drawback is that some of these equilibria might prescribe insensible behavior from the players as illustrated in the example above. In order to prune out pathological equilibria and introduce some robustness, I focus on wPBE that survives the Cho and Kreps, 1987's Intuitive Criterion (IC) refinement.[24]

The refinement consists of two main steps. In the first step, for every out of equilibrium report I construct the set of types of media outlet for which the equilibrium pay-off does not dominate the highest achievable utility they could potentially achieve by delivering such off-path report. For instance, following the above example, for every $r \in (r', r'')$ I define the set of types for which reporting r cannot be equilibrium dominated. Intuitively, I want to restrict my attention to types of media outlet for which deviating to an off equilibrium report r could potentially grant them a higher utility than sticking to the strategy that is prescribed by the equilibrium.

In the second step, for every out of equilibrium report r, beliefs are restricted to the set of types for which such report is not equilibrium dominated. If there exists a type of media outlet θ, and an out of equilibrium report r, such that deviating yields an higher pay-off for every sequentially rational voting rule given the restricted beliefs, then the original equilibrium fails the Intuitive Criterion and it is pruned. Figure 1.4.1 shows two equilibrium reporting rules that fail the test.

There exists only one unique "hybrid" equilibrium reporting rule that passes the Intuitive Criterion test, and it exists always. In such equilibrium, extreme types separate by reporting truthfully, while a convex set of intermediate types pool by inflating their private information. Intermediate types pool to the same "counterfeit" report in a way that it just sways the decision of the median voter.

[24]In Appendix 1.5.1 I show the incentive compatibility properties of generic equilibrium reporting rules.



(A) 2-steps equilibrium reporting rule (B) Multi-steps equilibrium reporting rule

FIGURE 1.4: Equilibrium reporting strategies not surviving the Intuitive Criterion refinement.

It is widely known that the Intuitive Criterion might fail to prune unreasonable pooling when there are more than two types. Given that in the present model there is a continuum of types, and misreporting behaviour occurs precisely via central pooling, a natural question is whether such wPBE is robust to stronger refinements. I show that such equilibrium survives Cho and Kreps, 1987's D1 and D2, Banks and Sobel, 1987's Divinity and Universal Divinity, and Kohlberg and Mertens, 1986's Never a Weak Best Response (NWBR).

The following Proposition 1 shows the unique equilibrium surviving the Intuitive Criterion (IC) refinement. I define $r = \theta^*$ as the report such that if the realization of valence is between θ^* and the lowest misreporting type that can deliver θ^*, then the expected value of valence is exactly the swing report $\hat{\theta}$.

Proposition 1 (Equilibrium). *The IC-refined wPBE is a pair of reporting rule – system of beliefs (ψ^*, p^*) and a voting rule v_i^* for all $i \in [0,1]$, such that,*[25]

$$\psi^*(\theta) = \begin{cases} \theta^* & \text{if } \theta \in (\underline{\vartheta}(\theta^*), \theta^*) \\ \theta & \text{otherwise} \end{cases} \tag{1.9}$$

[25]In this specification, out of equilibrium beliefs are drawn from the idea that voters avail themselves of an IC refinement-like procedure upon observing an unexpected report. However, this characterization would work for any posterior $p^*(\theta|r)$ such that $\mu^*(\theta|r') < \hat{\theta}$ for all $r' \in (\underline{\vartheta}(\theta^*), \theta^*)$.

$$\mu^*(\theta|r) = \begin{cases} \mathbb{E}_p\left[\tilde{\theta}|\underline{\vartheta}(\theta^*) \le \theta \le \theta^*\right] = \hat{\theta} & \textit{if } r = \theta^* \\ \mathbb{E}_p\left[\tilde{\theta}|\underline{\vartheta}(\theta') \le \theta \le \frac{\theta'+\theta^*}{2}\right] < \hat{\theta} & \textit{if } r = \theta' \in (\underline{\vartheta}(\theta^*), \theta^*) \\ \theta & \textit{otherwise} \end{cases} \tag{1.10}$$

where θ^* is the unique report $r \in \mathbb{R}$ such that $\mathbb{E}_p\left[\tilde{\theta}|\underline{\vartheta}(\theta^*) \le \theta \le \theta^*\right] = \hat{\theta}$, and the voting rules $\{v_i^*(r)\}_i$ are sequentially rational. Such equilibrium always exists, it is unique and robust to the Kohlberg and Mertens, 1986's NWBR refinement.

Proof. See Appendix 1.5.2.

Figure 1.5 illustrates the shape of the IC-refined equilibrium reporting rule. The media outlet disagrees with a majority of voters when valence is $\theta \in (\lambda\tau_e, \hat{\theta})$. In the lower range of the conflict area, when $\theta \in (\lambda\tau_e, \underline{\vartheta}(\theta^*)]$, the outlet finds that misreporting effectively to θ^* would be too expensive. Therefore, the outlet opts for reporting truthfully, inducing the implementation of its less favourite candidate, R. Instead, when $\theta \in (\underline{\vartheta}(\theta^*), \hat{\theta})$, the outlet delivers $r = \theta^*$, which is effective in that it induces the implementation of its favourite candidate, L. The report θ^* has the following equilibrium property: it is delivered both when $\theta < \gamma\tau_m$ and when $\theta \ge \gamma\tau_m$, in a way that induces voters to assess that valence is such that the median voter is indifferent between the two alternatives. When $\theta < \gamma\tau_m$, an "election overthrowing" takes place: the media outlet's preferred candidate, L, wins the election and implements q_L even though, under complete information, a majority of voters would prefer candidate R with policy q_R.

When $\theta \in \left[\hat{\theta}, \theta^*\right)$, the media outlet engages in costly misreporting although it agrees with a majority of electors on which canidate is best. This phenomenon, which I refer to as "white lies", occurs for two reasons: (i) voters are aware that the outlet can misreport information, and therefore become skeptical when a left-leaning outlet endorses the left-leaning candidate. In these cases, if the expert were to reveal its private information, its report would be discounted to a point where its less desirable alternative is implemented; and (ii) in order for a report to successfully persuade the median voter, it must be that sometimes it is delivered when there is actual agreement. Otherwise, in equilibrium, voters would understand that such report is always delivered when $\theta < \hat{\theta}$, neutralizing any attempt of "election overthrowing".

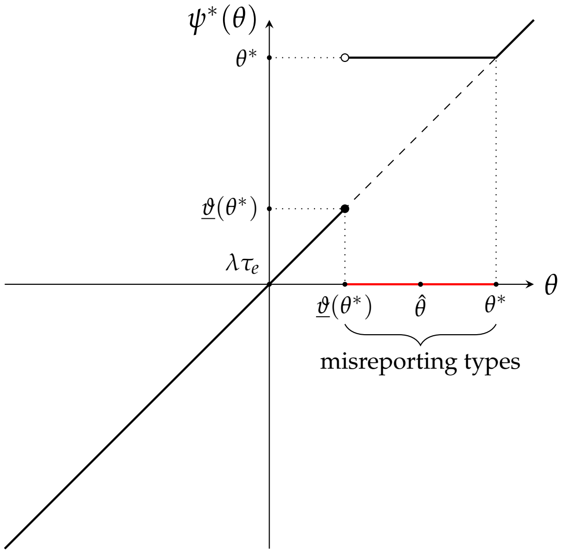


FIGURE 1.5: Reporting strategy in the unique robust equilibrium. Information is pooled around the swing valence, and truthfully reported otherwise.

In equilibrium, information is "jammed" for every realization of valence $\theta \in (\underline{\vartheta}(\theta^*), \theta^*)$. In contrast, when $\theta \in (-\infty, \lambda\tau_e] \cup [\theta^*, +\infty)$, truthful reporting is discriminating enough: the media outlet does not need to invest in costly misreporting in order to induce the implementation of its favourite alternative. When $\theta \in (-\infty, \lambda\tau_e]$, the outlet reports truthfully the state and voters fully believe the outlet's report. In these cases, a left-leaning media outlet delivers a report claiming that the "rightish" candidate is the best alternative for a majority of voters. Because such endorsements are profitable only when $\theta \leq \lambda\tau_e$, voters are not skeptical and take the outlet's report at face value. Similarly, when $\theta \in [\theta^*, +\infty)$, the left-leaning outlet delivers a report endorsing the "leftish" candidate. Despite the conflict of interest, such report does not generate skepticism as it is discriminating enough: there is no type $\theta < \hat{\theta}$ that would find it profitable to deliver a report $r > \theta^*$.[26] Therefore, the media outlet can just report its private information truthfully.[27] Figure 1.6 shows for which states misreporting results in an election overthrowing.

Proposition 1 shows that the equilibrium reporting strategy of the media outlet depends on the intensity of the misreporting costs k. Therefore, the extent to which the

[26]Types of outlet θ' that are just above θ^* report truthfully even though the same message could have been delivered by some other type $\theta < \hat{\theta}$, that is, $\underline{\vartheta}(\theta') < \hat{\theta}$. However, in equilibrium, all types $\theta \in (\underline{\vartheta}(\theta'), \theta^*)$ prefer to misreport to $r = \theta^*$ as it is effective, but cheaper.

[27]However, because of pathological off-path beliefs, in non-robust wPBE it is possible that also types $\theta \in (-\infty, \lambda\tau_e] \cup [\theta^*, +\infty)$ engage in misreporting information.

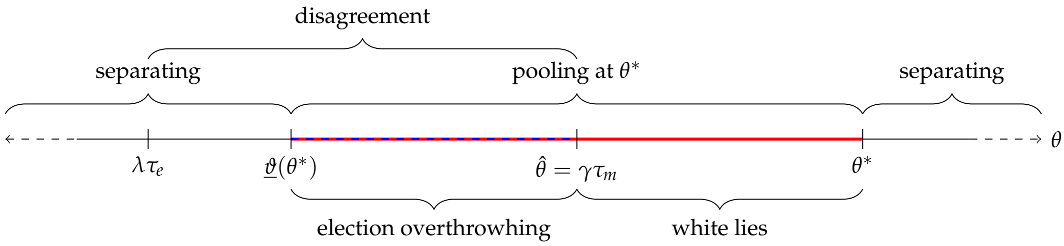


FIGURE 1.6: Election overthrowing and white lies in equilibrium. The media outlet misreports for all realizations $\theta \in (\underline{\vartheta}(\theta^*), \theta^*)$, where it delivers the same pooling report θ^*. However, persuasion occurs only in the subset $\theta \in (\underline{\vartheta}(\theta^*), \hat{\theta})$ where there is disagreement.

outlet persuades the voters and induces an election overthrowing is also a function of such costs. The "persuasion ratio" $\rho(k)$ is the ex-ante probability that an election overthrowing occurs given that there is a conflict of interest between the outlet and a majority of voters. Formally,

$$\rho(k) = \frac{\int_{\underline{\vartheta}(\theta^*)}^{\hat{\theta}} p(\theta) d\theta}{\int_{\lambda\tau_e}^{\hat{\theta}} p(\theta) d\theta} \tag{1.11}$$

Voters are fully aware of the media outlet's cost of misreporting information. Therefore, when k decreases, they might anticipate that misreporting is cheaper and become less trusting of any suspicious report. However, this heightened skepticism is not enough to prevent the media outlet to increase its influence over the electoral outcome. With lower costs, the outlet can pool a larger set of realizations of valence. In contrast, higher costs naturally limit the outlet's persuasive power. These intuitions are formally described by Lemma 3.

Lemma 3 (Persuasion ratio)**.** *As the misreporting costs k decrease (increase), the persuasion ratio $\rho(k)$ monotonically decreases (increases). As $k \to 0^+$, $\rho(k) \to 1$ (almost full persuasion). As $k \to +\infty$, $\rho(k) \to 0^+$ (almost full revelation).*

Proof. See the end of Appendix 1.5.2.

As misreporting costs k shrink to zero, that is, when communication is close to cheap talk, election overthrowing occurs at (almost) every state of nature at which there is disagreement.[28] On the other hand, prohibitive costs yield almost an absence of persuasion. As misreporting becomes increasingly expensive, it is more difficult for the media outlet to persuade voters even if they become more trusting. At the limit, when fabricating information is infinitely costly, the media outlet cannot do better than fully revealing its private information.

1.4.2 Cheap Talk and Almost Cheap Talk

In this section, I derive the "cheap talk" equilibrium of this game where, in contrast to the present model, communication is completely costless. In canonical models of cheap talk communication (Crawford and Sobel, 1982; Green and Stokey, 2007), a privately informed sender delivers a message to an uninformed receiver. The latter has all the decision making power, and the sender is affected by the decision of the receiver. However, there is a conflict of interest in that the sender prefers a different action than the receiver for almost every state of the world. In standard settings, communication occurs just once, and between one sender and one receiver.[29] This class of models have the following salient features: (i) the sender cannot commit to a reporting rule, and (ii) messages are pay-off irrelevant.

In equilibrium, the sender transmits only noisy and partial information to the receiver. In the extreme case, there can be "babbling" communication: the receiver disregards any message delivered by the sender, and therefore the sender delivers only uninformative messages. On the other hand, under relatively mild degrees of conflicts of interests, there are "partitional" equilibria where some information is transmitted. In these equilibria, the sender reports that the realized state of nature is within a subset of the state space.

The present paper presents a model of communication between a media outlet (sender) and a voter (receiver). The main differences with traditional cheap talk models are that (i) messages are not costless, but might have a direct impact on the

[28]This is not necessarily true when the support of $p(\theta)$ has a finite upper bound $\phi > \gamma\tau_m$ (case where $\phi < \gamma\tau_m$ are trivial). In that case, the outlet misreports to $r = z^*$ when $\theta > z$, where $z := \left\{t \in (\lambda\tau_e, \gamma\tau_m) \cup \varnothing | \mathbb{E}_p \left[\tilde{\theta} | t < \theta < \phi\right] = \gamma\tau_m\right\}$, and $z^* := \{r \in \mathbb{R} | \underline{\vartheta}(r) = z\}$. When $k = 0$, communication is influential only if $\mathbb{E}_p \left[\tilde{\theta} | \theta > \lambda\tau_e\right] \geq \gamma\tau_m$.

[29]There are many exceptions, e.g., see Morris, 2001 for repeated communication and Krishna and Morgan, 2001a; Gilligan and Krehbiel, 1989; Battaglini, 2002 for communication with multiple senders.

sender's pay-off, (ii) the conflict of interest occurs only in a strict subset of the state space, and (iii) the receiver has a binary action space.

As follows, I consider the cheap talk version of the present model where communication is costless and messages do not directly impact on the utility of the media outlet. Formally, this is the special case where $k = 0$. Apart from that, the structure and timeline remain the same as before: the expert privately observes $\theta \in \Theta$, and then delivers a public report $r \in \mathcal{R}$ to the whole electorate, where $\mathcal{R}$ is an abstract set of messages.[30] I assume that $\mathcal{R}$ contains at least two possible reports, that is, $|\mathcal{R}| \geq 2$. Voters condition their assessment of valence to the cheap talk report, and then cast a ballot for one of the two candidates in a majority-rule election. The candidate receiving more votes wins, and eventually implements her policy proposal.

In every "cheap talk" equilibrium, the outlet can convey its endorsement, but cannot credibly deliver the intensity of its preferences. Indeed, any type of media outlet $\theta \geq (<) \lambda\tau_e$ has the same incentives to deliver whatever it takes to induce a majority of electors to cast a ballot for L (R). Eventually, in equilibrium, the outlet can communicate only which candidate it is endorsing. This is in contrast with the case with positive misreporting costs where in equilibrium higher reports signal that valence is higher.

Equilibrium reports can only bear the two following meanings: "*I endorse L*" and "*I endorse R*". I will refer to such messages with, respectively, `l` and `r`.[31] In any cheap talk equilibrium, the expert reports $\psi(\theta) = \mathtt{l}$ when $\theta \geq \lambda\tau_e$ and $\psi(\theta) = \mathtt{r}$ otherwise. Communication is *influential* if $\nu_m(r)$ is not constant along the equilibrium path.[32] This is possible only if, given the distribution of valence p, the conflict of interest between the outlet and the median voter is not too large. More formally, the median voter should expect the valence to be high enough given that the outlet is endorsing candidate L; that is, $\mathbb{E}_p \left[\tilde{\theta} | \theta \geq \lambda\tau_e\right] \geq \hat{\theta}$. The following Proposition 2 formally states the above observations.

[30] I use the abstract set $\mathcal{R}$ instead of $\mathbb{R}$ for the message space because with cheap talk communication the meaning of the messages is defined only in equilibrium.

[31] To be more precise, the expert could also be indifferent between the two candidates and therefore communicate "*I am indifferent*". However, such message is not necessary in equilibrium.

[32] Along the equilibrium path refers to reports that in equilibrium are delivered with positive probability. See Definition 3 in Appendix.

Proposition 2 (Cheap talk). *An influential cheap talk equilibrium is a pair of reporting rule – system of beliefs* (ψ_c^*, p_c^*)*, and a sequentially rational voting rule* $v_i^*(r)$ *for all* $i \in [0,1]$*, such that,*

$$\psi_c^*(\theta) = \begin{cases} \mathtt{l} & \text{if } \theta \geq \lambda\tau_e \\ \mathtt{r} & \text{otherwise} \end{cases} \tag{1.12}$$

$$\mu_c^*(\theta|r) = \begin{cases} \mathbb{E}_{p_c^*}\left[\tilde{\theta}|\theta \geq \lambda\tau_e\right] \geq \hat{\theta} & \text{if } r = \mathtt{l} \\ \mathbb{E}_{p_c^*}\left[\tilde{\theta}|\theta < \lambda\tau_e\right] < \hat{\theta} & \text{if } r = \mathtt{r} \end{cases} \tag{1.13}$$

Proof. The reporting rule and beliefs follow from the above discussion. Since I am assuming $\lambda\tau_e < \gamma\tau_m$, it is always the case that $\mathbb{E}_p\left[\tilde{\theta}|\theta < \lambda\tau_e\right] < \hat{\theta} = \gamma\tau_m$. Upon observing the report $r = \mathtt{r}$, a majority of voters casts a ballot for R. If $\mathbb{E}_p\left[\tilde{\theta}|\theta \geq \lambda\tau_e\right] < \hat{\theta}$, then R would always be elected independently from the expert's report, thus making the equilibrium communication to be non-influential. In such cases voters take a decision as if they were merely relying on their prior. Therefore, in order for the cheap talk equilibrium to be influential, it must be that $\mathbb{E}_p\left[\tilde{\theta}|\theta \geq \lambda\tau_e\right] \geq \hat{\theta}$. □

When communication is costless, there are two extreme and polarized results: either the media outlet has no influence at all, as if it does not exists, or it has full power in determining the political outcome, as if it is the only voter deciding upon the result of the election. In contrast, with positive misreporting costs, the media outlet retains always some influence, but its persuasive power is limited by the cost of misreporting and the voters' understanding of its incentives.

Outcome-wise, the equilibrium with negligible misreporting costs ($k \to 0^+$) convergences to the influential cheap talk equilibrium.[33] However, the presence of even infinitesimal misreporting costs makes each report to be a discriminating signal for every positive intensity of misreporting costs $k > 0$.[34] Therefore, at the limit, the equilibrium reporting rule has a different structure than in the cheap

[33] In the Kartik, 2009's model of costly lying, as the intensity of misreporting costs vanishes, the equilibrium converges to the finest Crawford and Sobel, 1982's cheap talk equilibrium partition. This also happens in Chen, 2011 when the share of non-strategic players approaches zero. Chen, Kartik, and Sobel, 2008 use this limiting result to justify the selection of the most informative equilibrium partition in cheap talk games.

[34] In the present model, there always exists a babbling equilibrium when $k = 0$. This is standard in cheap talk games. In contrast, for any positive $k > 0$, communication is always influential. Indeed, for any upper bound ϕ of the support of $p(\theta)$ such that $\phi > \gamma\tau_m$, there always exists, for every $k > 0$, a message m such that, $m := \left\{r \in \mathbb{R}|\mathbb{E}_p\left[\tilde{\theta}|\underline{\vartheta}(r) < \theta < \min\{\phi, r\}\right] = \hat{\theta}\right\}$.

talk version. For instance, the media outlet can make literal, precise statements about the relative quality of candidates. Therefore, it can convey the intensity of its preferences. For relatively extreme realizations of valence, the outlet reports truthfully its private information. When misreporting occurs, information is pooled for relatively intermediate realizations of valence, inducing the median voter to be indifferent.

However, outcome-wise, the way communication takes place is irrelevant. Every report $r \geq (<)\theta_0^*$ is equivalent to a cheap talk endorsement for candidate L (R). In terms of expected pay-off and political outcomes, the two models are equivalent. Figure 1.7 illustrates the difference between the pure cheap talk and the "almost" cheap talk equilibrium reporting rule. The following Proposition 3 formally shows the equilibrium reporting rule as misreporting costs approach zero.

Proposition 3 (Almost cheap talk)**.** *The almost cheap talk IC-wPBE, where $k \to 0^+$, is a pair of reporting rule – system of beliefs (ψ_0^*, p_0^*) and a sequentially rational voting rule $v_i^*(r)$ for all $i \in [0,1]$, such that,*

$$\psi_0^*(\theta) = \begin{cases} \theta_0^* & \text{if } \theta \in (\lambda\tau_e, \theta_0^*) \\ \theta & \text{otherwise} \end{cases} \tag{1.14}$$

$$\mu_0^*(\theta|r) = \begin{cases} \mathbb{E}_{p_0^*}\left[\tilde{\theta}|\lambda\tau_e \leq \theta \leq \theta_0^*\right] = \hat{\theta} & \text{if } r = \theta_0^* \\ \mathbb{E}_{p_0^*}\left[\tilde{\theta}|\lambda\tau_e \leq \theta \leq \frac{\theta'+\theta_0^*}{2}\right] < \hat{\theta} & \text{if } r = \theta' \in (\lambda\tau_e, \theta_0^*) \\ \theta & \text{otherwise} \end{cases} \tag{1.15}$$

where θ_0^* is the unique $r \in \mathbb{R}$ such that $\mathbb{E}_p\left[\tilde{\theta}|\lambda\tau_e \leq \theta \leq \theta_0^*\right] = \hat{\theta}$. Such equilibrium always exists, it is unique, and robust to NWBR.

Proof. For every finite report $r > \lambda\tau_e$, $\lim_{k\to 0^+} \underline{\vartheta}(r) = \lambda\tau_e$. By Lemma 12, for every positive $k > 0$ there always exists one unique r^* such that,

$$r^* = \left\{r : \mathbb{E}_p[\tilde{\theta}|\lambda\tau_e \leq \theta \leq r] = \hat{\theta}\right\}$$

The rest of the proof, including uniqueness and robustness to refinements, follows the one for the equilibrium described in Proposition 1. □

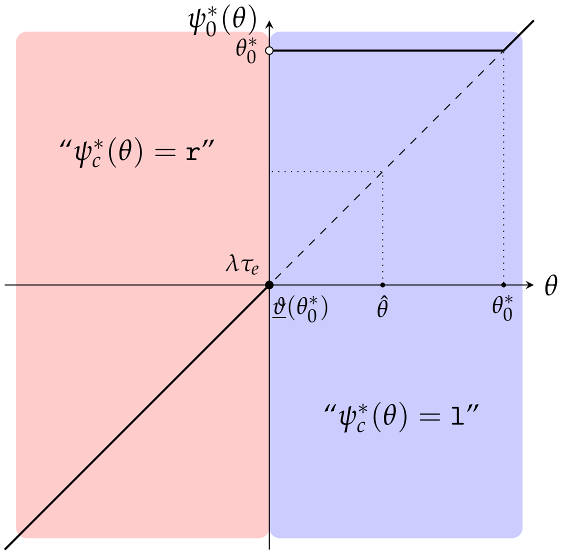


FIGURE 1.7: Cheap Talk and Almost Cheap Talk equilibrium reporting rule. When $k = 0$, the outlet delivers an endorsement for candidate L (R) for every realization of valence in the blue (red) area.

1.5 Conclusion

In this paper, I present a model of strategic communication between a privately informed media outlet (the sender) and an uninformed electorate (the receivers). Before the election takes place, the outlet delivers a public report about the relative quality (valence) of two candidates running for office. In contrast to traditional models of communication, here (i) misreporting information is possible, but costly, and (ii) for some states of nature there is complete agreement between the sender and the receiver. I show that, in every equilibrium, there are situations where the outlet misreports its private information. As a consequence, there are always cases where the outlet's favourite candidate wins the election while, under complete information, the other candidate would be preferred by a majority of voters.

The introduction of misreporting costs transforms cheap talk communication into a costly signalling game. Signalling games typically yield a plethora of equilibria, mostly sustained by unreasonable off-path beliefs. However, there is only one unique equilibrium that survives a relatively mild refinement, the Intuitive Criterion (Cho and Kreps, 1987). In the unique refined equilibrium, the media outlet either reports truthfully or "exaggerates" the quality of its favourite candidate. For intermediate states, the outlet pools information about valence. Upon observing the "pooling" report, the median voter is indifferent between the two candidates. Otherwise, for more extreme states, the outlet truthfully reveals its private information.

When a left-leaning outlet delivers reports favouring the rightish alternative, it is always credible and it never needs to misreport. In contrast, when endorsing the leftist alternative, the outlet has to provide reports that are discriminating enough. This implies that there are situations where the outlet has to misreport even though it agrees with a majority of voters that candidate L is the best alternative.

In the cheap talk version of the present model there are two extreme results: either every report is lost in a babbling speech and ignored by the voters, or the outlet is always influential and can always implement its favourite alternative. In contrast, the introduction of misreporting costs yields a result that stands in between these two extremes: the media outlet always retains some influence because of its private ownership of relevant information, but as long as misreporting costs are positive, it cannot always implement its favourite alternative. As misreporting costs shrink to zero and become negligible, the equilibrium outcome converges to its corresponding cheap talk version.

The existence of an equilibrium with limited but influential misreporting has direct empirical implications. Empirical evidence suggests that even though voters can filter and discount unbalanced, partisan and biased reports, media bias affects their voting behaviour. DellaVigna and Kaplan, 2007 find a significant effect of the introduction of Fox News on the vote share in Presidential elections. Depending on the audience measure, Fox News convinced 3-28% of its audience to vote Republican. Chiang and Knight, 2011 find that newspapers' endorsements are influential to a certain extent which depends on the direction of the bias. They show that voters are more likely to support the endorsed candidate, but reduce their reliance on biased reporting when the newspaper has the same political stance as the candidate. This is in line with the theoretical findings in this paper, where a left biased media outlet is always trusted when endorsing the rightish alternative.

This paper is a first step in exploring how a politically motivated media outlet can strategically distort information in order to influence electoral outcomes.

While this is a reasonable first step, there is room for future research. If other media outlet could release information about different and orthogonal dimensions of valence *after* the expert, then the position of the median voter would be uncertain. On the other hand, the assumption that the expert has monopolistic ownership of private information could be relaxed by introducing a competitor with the same information. This latter direction allows to inquire also on the role of experts' profit motivation and voters' abstention. I assumed the expert has complete information about the state of the world. It would be interesting to study its incentives of information acquisition when acquiring information is costly, and its effect on the equilibrium reporting rule. Further, the available alternatives could be endogenised by allowing candidates to strategically champion them at the outset of the electoral competition. I leave this endeavour for future research.

Appendix

1.5.1 Incentive Compatibility

Here I study the incentive compatibility of the equilibrium reporting rule $\psi(\theta)$. I do not impose any individual rationality constraint in line with the interpretation that the expert cannot make its utility independent of voters' collective choice. I define $\psi^-(\delta) = \lim_{\theta \to \delta^-} (\psi(\theta))$ and $\psi^+(\delta) = \lim_{\theta \to \delta^+} (\psi(\theta))$.

Proposition 4 (Incentive compatibility). *Any incentive compatible reporting rule $\psi(\theta)$ must satisfy the followings: (i) $\psi(\theta)$ is weakly increasing in θ; (ii) if $\psi(\theta)$ is strictly increasing and continuous on an open interval (θ', θ''), then $\psi(\theta) = \theta$ on (θ', θ''); (iii) there exists an open set containing $\lambda\tau_e$ where $\psi(\theta) = \theta$; (iv) if $\psi(\theta)$ is discontinuous at δ, the jump must satisfy:*

(a) $u_e(\delta, \psi^-(\delta)) = u_e(\delta, \psi^+(\delta))$

(b) $\delta \in \left\{ \frac{\psi^+(\delta)+\psi^-(\delta)}{2}, \underline{\vartheta}(\psi^+(\delta)), \overline{\vartheta}(\psi^-(\delta)) \right\}$

(c) $\underline{\vartheta}(\psi^+(\delta)) \leq \delta \leq \overline{\vartheta}(\psi^-(\delta))$

Before proving Proposition 4, I state the following observations and lemmas.

Observation 1. *In any incentive compatible reporting rule, type $\theta = \lambda\tau_e$ always reports truthfully.*

Type $\theta = \lambda\tau_e$ is completely indifferent between candidate L and R. For such type, truthful reporting is a strictly dominant strategy.

Observation 2. *In any incentive compatible reporting rule, if type θ reports $\psi(\theta) \neq \theta$ then it must be that $\psi(\theta) \in \mathcal{E}(\theta)$. Further, all reports r' such that,*

$$r' \in (\min\{2\theta - \psi(\theta), \psi(\theta)\}, \max\{2\theta - \psi(\theta), \psi(\theta)\})$$

must be ineffective, $r' \notin \mathcal{E}(\theta)$.

To see this, note that if $\psi(\theta) \neq \theta$ is not effective, then it is strictly dominated by truthful reporting. Instead, if $r \in \mathcal{E}(\theta)$, then it would be strictly dominated by any other $r' \in \mathcal{E}(\theta)$ such that $(r' - \theta)^2 < (r - \theta)^2$.

Lemma 4. *In any incentive compatible reporting rule, for any two different types which are strictly endorsing the same candidate, the reporting rule $\psi(\theta)$ is weakly increasing in θ. That is, for either $\lambda\tau_e < \theta' < \theta''$ or $\theta' < \theta'' < \lambda\tau_e$, it must be that $r(\theta') \leq (\theta'')$.*

Proof. Consider any two types such that $\lambda\tau_e < \theta' < \theta''$. Suppose $r(\theta') > r(\theta'')$. For $\theta \in \{\theta', \theta''\}$ there are three possible cases:

(a) $r(\theta'), r(\theta'') \notin \mathcal{E}(\theta)$

(b) $r(\theta'), r(\theta'') \in \mathcal{E}(\theta)$

(c) $r(\theta') \in \mathcal{E}(\theta)$ and $r(\theta'') \notin \mathcal{E}(\theta)$, or $r(\theta') \notin \mathcal{E}(\theta)$ and $r(\theta'') \in \mathcal{E}(\theta)$

Firstly consider case (a): since both reports are ineffective it must be that, for any incentive compatible reporting rule, they truthfully reveal expert's type. Hence, $r(\theta') = \theta' < \theta'' = r(\theta'')$. This is in contradiction with the assumption that $r(\theta') > r(\theta'')$.

In case (b) both reports are effective. If $r(\theta'') \leq \theta'$ then θ'' would profitably deviate by reporting $r(\theta')$. If $r(\theta') \geq \theta''$ then θ' would profitably deviate by reporting $r(\theta'')$. If $\theta' \leq r(\theta'') < r(\theta') \leq \theta''$, then both types would profitably deviate by delivering the other's report.

In case (c) one report is effective while the other is not. Take the instance where $r(\theta') \in \mathcal{E}(\theta)$ and $r(\theta'') \notin \mathcal{E}(\theta)$. We have that $r(\theta'')$ must be truthful, that is $r(\theta'') = \theta''$. Hence, by assumption $r(\theta') > \theta'' > \theta'$. But if θ' is willing to misreport up to $r(\theta')$, then also θ'' should, because it would bear smaller misreporting costs while sharing the same political preference. This is in contradiction with θ'' reporting truthfully or θ' misreporting to $r(\theta')$. The instance where $r(\theta') \notin \mathcal{E}(\theta)$ and $r(\theta'') \in \mathcal{E}(\theta)$ is similar.

Further, the case $\theta' < \theta'' < \lambda\tau_e$ can be proved leading to contradictions in a similar fashion. Therefore, any equilibrium reporting rule is weakly increasing for any types endorsing the same candidate. □

Lemma 5. *In any incentive compatible reporting rule, if $\psi(\theta)$ is strictly monotonic and continuous on an open interval (θ', θ''), then $\psi(\theta) = \theta$ on (θ', θ'').*

Proof. Suppose it is not. If $\psi(\theta) > \theta$ for some $\theta \in (\theta', \theta'')$, then $\exists\, \epsilon > 0$ such that types θ and $\theta - \epsilon$ support the same candidate and $\theta < r(\theta - \epsilon) < \psi(\theta)$. The latter inequality is due to continuity, strict monotonicity and Lemma 4. Since $\psi(\theta) > \theta$, it has to be that $\psi(\theta) \in \mathcal{E}(\theta)$ (see Observation 2). Similarly, since $r(\theta - \epsilon) > \theta > \theta - \epsilon$ it has to be that $r(\theta-\epsilon) \in \mathcal{E}(\theta-\epsilon)$. This implies that, in such equilibrium, both reports make the two types' favourite candidate to win. But $(\psi(\theta)-\theta)^2 > (r(\theta-\epsilon)-\theta)^2$, hence type θ would be better off deviating to $r(\theta-\epsilon)$. Hence it would not be incentive compatible. The proof for $\psi(\theta) < \theta$ is similar. $\square$

Lemma 6. *In any incentive compatible reporting rule, there cannot be a discontinuity at $\theta = \lambda\tau_e$.*

Proof. Suppose there is a discontinuity at $\lambda\tau_e$, which means that,

$$r^-(\lambda\tau_e) = \lim_{\theta\to\lambda\tau_e^-}(\psi(\theta)) \neq \lim_{\theta\to\lambda\tau_e^+}(\psi(\theta)) = r^+(\lambda\tau_e)$$

By Lemma 4 and Observation 1, $r^-(\lambda\tau_e) < r^+(\lambda\tau_e)$. A direct requirement of incentive compatibility is that $u_e(\lambda\tau_e, \psi^-(\lambda\tau_e)) = u_e(\lambda\tau_e, \psi^+(\lambda\tau_e))$, which implies that $r^-(\lambda\tau_e) < \lambda\tau_e < r^+(\lambda\tau_e)$. Indeed, if one of the limits is equal to $\lambda\tau_e$ while the other is different, type $\theta = \lambda\tau_e$ could never be indifferent due to misreporting costs. We therefore must have that $\lambda\tau_e = \frac{r^+(\lambda\tau_e)+r^-(\lambda\tau_e)}{2}$. Consider a type $\theta' = \lambda\tau_e + \epsilon$, with $0 < \epsilon < \frac{r^+(\lambda\tau_e)-\lambda\tau_e}{2}$. Since by Lemma 4 the reporting rule is weakly increasing we have that $r(\theta') > \theta'$, with the difference being $r(\theta') - \theta' \geq r^+(\lambda\tau_e) - \lambda\tau_e - \epsilon > 0$. Any such type has an incentive to deviate by reporting anything in $r \in (\lambda\tau_e - \varepsilon, \lambda\tau_e)$ with $0 < \varepsilon < r^+(\lambda\tau_e) - \lambda\tau_e - 2\epsilon$. To see this, note that also any type $\theta'' = \lambda\tau_e - \epsilon$ is misreporting. If they are doing so, it must be that any $r' \in (\lambda\tau_e - \epsilon, \lambda\tau_e)$ is effective for types θ' but not for types θ'', i.e. $r' \in \mathcal{E}(\theta')$. Any such discontinuity is therefore not incentive compatible. $\square$

Lemma 7. *In any incentive compatible reporting rule there exists an open set containing $\lambda\tau_e$ where $\psi(\theta) = \theta$.*

Proof. By Observation 1 we know that $r(\lambda\tau_e) = \lambda\tau_e$. Moreover, by Lemma 6 there cannot be a discontinuity at $\lambda\tau_e$. This implies that there always exists an open set (θ', θ'') containing $\lambda\tau_e$ where $\psi(\theta)$ is continuous. There are two possible cases: either $\psi(\theta)$ is strictly increasing in (θ', θ'') or it is a flat step where $\psi(\theta) = \lambda\tau_e$ on (θ', θ''). To see this, note that by Lemma 5 if $\psi(\theta)$ is strictly increasing then $\psi(\theta) = \theta$ on (θ', θ''), otherwise if it is not it must be constant at $\lambda\tau_e$ by Observation 1. Suppose the latter case is true. If $\theta \in (\theta', \theta'') \setminus \{\lambda\tau_e\}$ do not report truthfully, it must be that $\theta \notin \mathcal{E}(\theta)$

and $\lambda\tau_e \in \mathcal{E}(\theta)$. This yields an immediate contradiction, since it is not possible that $\lambda\tau_e \in \mathcal{E}(\theta)$ for both types in $(\theta', \lambda\tau_e)$ and in $(\lambda\tau_e, \theta'')$, since they disagree on what candidate is better. Therefore it must be that $\psi(\theta) = \theta$ for all $\theta \in (\theta', \theta'')$. $\square$

Lemma 8 (Discontinuity)**.** *In any incentive compatible reporting rule, if the reporting rule $\psi(\theta)$ is discontinuous at δ, the jump must satisfy: (a) $u_e(\delta, \psi^-(\delta)) = u_e(\delta, \psi^+(\delta))$; (b) $\delta \in \left\{\frac{\psi^+(\delta)+\psi^-(\delta)}{2}, \underline{\vartheta}(\psi^+(\delta)), \overline{\vartheta}(\psi^-(\delta))\right\}$; (c) $\underline{\vartheta}(\psi^+(\delta)) \leq \delta \leq \overline{\vartheta}(\psi^-(\delta))$.*

Proof. Part (a) is a direct requirement of incentive compatibility. Part (b) states that, given the left and right limits of the reporting rule, the discontinuity δ must occur at one of the element of the set $\left\{\frac{\psi^+(\delta)+\psi^-(\delta)}{2}, \underline{\vartheta}(\psi^+(\delta)), \overline{\vartheta}(\psi^-(\delta))\right\}$. Suppose that $\psi^+(\delta), \psi^-(\delta) \notin \mathcal{E}(\delta)$. Because of Observation 2, it must be that both reports are truthful. Therefore, at the limit, they are both equal to δ, contradicting that there is a discontinuity. Hence, at least one of the limits must belong to $\mathcal{E}(\delta)$. Consider as an instance a discontinuity at $\delta > \lambda\tau_e$ where I assume $\psi^-(\delta) \notin \mathcal{E}(\delta)$ and $\psi^+(\delta) \in \mathcal{E}(\delta)$. Since $\psi^-(\delta)$ is ineffective, it must be $\psi^-(\delta) = \delta$. From part (a) I obtain that,

$$-\gamma(h_e - q_R)^2 = -\gamma(h_e - q_L)^2 + \delta - k(\psi^+(\delta) - \delta)^2$$

which leads to $\delta = \underline{\vartheta}(\psi^+(\delta))$. Suppose instead that $\psi^-(\delta) \in \mathcal{E}(\delta)$ and $\psi^+(\delta) \notin \mathcal{E}(\delta)$. As before it must be that $\psi^+(\delta) = \delta$, eventually leading to $\delta = \underline{\vartheta}(\psi^-(\delta)) < \psi^-(\delta) < \psi^+(\delta) = \delta$. This contradiction is due to to Lemma 4 and the assumption of discontinuity at δ. Considering the case $\delta > \lambda\tau_e$ eventually leads to $\delta = \overline{\vartheta}(\psi^-(\delta))$. In case $\psi^+(\delta), \psi^-(\delta) \in \mathcal{E}(\delta)$, condition (a) imposes that $(\psi^+(\delta) - \delta)^2 = (\psi^-(\delta) - \delta)^2$, which yields $\delta = \frac{\psi^+(\delta)+\psi^-(\delta)}{2}$.

Part (c) simply states that type δ must find reporting either $\psi^+(\delta)$ or $\psi^-(\delta)$ to be at least as convenient as truthful reporting, but not less. Suppose instead that type δ strictly prefers truthful reporting, which happens when $\underline{\vartheta}(\psi^+(\delta)) > \delta > \overline{\vartheta}(\psi^-(\delta))$. Then there exists an open neighbourhood of types around δ which strictly prefer to report truthfully as well. Indeed, any report $r \in (\psi^-(\delta), \psi^+(\delta))$ is ineffective by construction. But it contradicts the existence of a discontinuity of the reporting rule at δ. This part puts a constraint in the height of any discontinuity where $\delta = \frac{\psi^+(\delta)+\psi^-(\delta)}{2}$, which cannot be too long in order to be incentive compatible. In particular, it has to be that the height of the jump $\psi^+(\delta) - \psi^-(\delta) \leq \underline{\vartheta}(\psi^+(\delta)) + \overline{\vartheta}(\psi^-(\delta))$. $\square$

The following is the proof of Proposition 4.

Proof. Part (i) states that $\psi(\theta)$ is weakly increasing. Lemma 4 shows that this is true between any two types endorsing the same candidate. However it does not rule out the possibility that the reporting rule might take higher values to the left of $\theta = \lambda\tau_e$ than to the right. However, by Lemma 6 we know that there cannot be a discontinuity at $\theta = \lambda\tau_e$. Furthermore, by Observation 1, we also know that $\psi(\lambda\tau_e) = \lambda\tau_e$. Hence Lemma 4 extends over any two types and the reporting rule passes trough $\lambda\tau_e$. Part (ii) is proved by Lemmas 4 and 5, where the former proves the (weakly) increasing behaviour of the rule and the latter proves its truthful behaviour under the stated conditions. Part (iii) and (iv) are directly proved by, respectively, Lemmas 7 and 8. □

1.5.2 IC-refined wPBE

Before proving Proposition 1, I introduce the following lemmas.

Lemma 9. *In any IC-refined wPBE, $\forall\, \theta \leq \lambda\tau_e$, $r^*(\theta) = \theta$ and $V_L < \frac{1}{2}$.*

Proof. For $\theta = \lambda\tau_e$ the claim is trivial due to incentive compatibility. Suppose there exists an IC-refined wPBE where $r = \theta' < \lambda\tau_e$ is never reported. This implies that, in any wPBE, $\mu^*(\theta|\theta') \geq \hat{\theta}$, $\theta' \notin \mathcal{E}(\theta')$ and $r^*(\theta') \in \mathcal{E}(\theta')$. I now introduce some further notation which is useful for applying the Intuitive Criterion. I define the set of all pure strategy best responses for the median voter m to report r for beliefs $p(\cdot|r)$ such that $p(T|r) = 1$ as follows,[35]

$$B_m(T,r) = \bigcup_{p:p(T|r)=1} \arg\max_{b_m} \int_{\theta\in\Theta} p(\theta|r) u_m(b_m,\theta,\mathbf{q}) d\theta \tag{1.16}$$

Fix an equilibrium payoff for the expert, u_e^*. For each report r, define $J(r)$ as follows,

$$J(r) = \left\{ \theta \in \Theta \middle| u_e^*(\theta) \leq \max_{b_m \in B(\Theta,r)} u_e(b_m,\theta,r,\mathbf{q}) \right\} \tag{1.17}$$

Consider what happens if some type unexpectedly reports $r = \theta' < \lambda\tau_e$. The highest possible incentive compatible maximal element of $J(\theta')$ is $\overline{\vartheta}(\theta')$.[36] In words, given

[35] For $T = \varnothing$, set $B_m(\varnothing,r) = B_m(\theta,r)$.

[36] $\overline{\vartheta}(\theta') = \max J(\theta')$ when in a wPBE type $\overline{\vartheta}(\theta')$ delivers a truthful and ineffective reports and voters respond to such deviation in a way that makes R the winner.

the off-equilibrium report $r = \theta'$, the highest type for which such report cannot be equilibrium dominated is $\overline{\vartheta}(\theta')$. Since $\overline{\vartheta}(r) < \lambda\tau_e < \hat{\theta} = \gamma\tau_e$ for any $r < \lambda\tau_e$, the best choice of the median voter (and thus for a majority of the electorate) restricting her beliefs in any possible $J(\theta')$ is always to cast a ballot for R, implying $V_L < \frac{1}{2}$. More formally, according to the Intuitive Criterion refinement, type θ' could deviate from $r(\theta') \neq \theta'$ to truthful reporting because of the following,

$$u_e^*(\theta') < \min_{b_m \in B_m(J(\theta'),\theta')} u_e(b_m, \theta', \theta', \mathbf{q}) \tag{1.18}$$

This is true for any $\theta' < \lambda\tau_e$. Therefore, in any IC-refined wPBE all $r \leq \lambda\tau_e$ are reported and Bayes' rule implies that for any such report R wins. This nails down the equilibrium reporting rule for any $\theta \leq \lambda\tau_e$ to be truthful, $r^*(\theta) = \theta$. □

Lemma 10 (Equilibrium discontinuity)**.** *In any IC-refined wPBE the misreporting rule exhibits at least one discontinuity.*

Proof. Lemma 9 says that R wins whenever $\theta \leq \lambda\tau_e$. By Proposition 4 we also know that around $\lambda\tau_e$ there is an open set containing $\lambda\tau_e$ where $\psi(\theta) = \theta$. Therefore, in the IC-refined wPBE, R wins for types in $(-\infty, \theta')$ for some $\theta' \in (\lambda\tau_e, \hat{\theta})$. To see this notice that in any equilibrium, by Bayes' rule, if $\psi(\theta) = \theta$ then voters correctly infer $\mu(\theta|r) = \theta < \hat{\theta}$. However, by Lemma 2, R cannot always win independently from the expert report. There must exists a report $r > \lambda\tau_e$ such that $r \in \mathcal{E}(\theta)$ for any $\theta > \lambda\tau_e$. Take $\theta' := \min\{\theta : r^*(\theta) \in \mathcal{E}(\theta) \text{ for } \theta > \lambda\tau_e\}$. Because of weak monotonicity, $r^*(\theta') \geq \theta'$. If $r^*(\theta') = \theta'$, then all $\theta \in (\underline{\vartheta}(\theta'), \theta')$ have a strict incentive to misreport to $r^*(\theta')$, contradicting the definition of θ' itself. Therefore it must be that $r^*(\theta') > \theta' = \underline{\vartheta}(r^*(\theta'))$, creating a discontinuity in the equilibrium reporting rule at $\theta = \theta'$. □

Lemma 11 (Equilibrium discontinuity)**.** *In any IC-refined wPBE, the reporting rule exhibits a discontinuity at $\delta = \underline{\vartheta}(\theta^*)$ where $\psi^-(\underline{\vartheta}(\theta^*)) = \underline{\vartheta}(\theta^*)$, $\psi^+(\underline{\vartheta}(\theta^*)) = \theta^*$, $\psi(\theta) = \theta^* \ \forall \ \theta \in (\underline{\vartheta}(\theta^*), \theta^*)$ and,*

$$\theta^* = \min\left\{r \in \mathbb{R} \middle| \mathbb{E}_p\left[\tilde{\theta} | \underline{\vartheta}(\theta^*) \leq \theta \leq \theta^*\right] = \hat{\theta}\right\} \tag{1.19}$$

Proof. Consider the first discontinuity in Lemma 10. Say that, in an IC-refined wPBE, types $\theta \in (\underline{\vartheta}(\theta^*), \theta'')$ for $\theta'' \in [\theta^*, \underline{\vartheta}(\theta^*)]$ report $r^* = \theta^*$. Note that if $\underline{\vartheta}(\theta^*)$ is willing to report θ^* then it must be that all types in $(\underline{\vartheta}(\theta^*), \theta^*)$ are willing to do so as well, and the same is true for types in (θ^*, θ'') if $\theta'' > \theta^*$. In order for θ^* to be effective in equilibrium, it must be that, because of Bayes' rule, $\mathbb{E}_p\left[\tilde{\theta}|\underline{\vartheta}(\theta^*) \leq \theta \leq \theta''\right] \geq \hat{\theta}$. Suppose $\theta'' > \theta^*$. In this case, a discontinuity in the equilibrium reporting rule must occur at θ''. Since $\theta'' > \hat{\theta}$, such discontinuity is between two effective reports, θ^* and r'. Notice that it must be $r' > \theta''$. Lemma 8 implies that $\theta'' = \frac{\theta^* + r'}{2}$. If type θ'' is willing to misreport to r', then types in $(\theta'', \overline{\vartheta}(r'))$ are strictly willing to do so, meaning that in any wPBE reports $r \in (\theta'', \overline{\vartheta}(r'))$ are never observed. Applying the Intuitive Criterion, consider instead what happens when voters observe a report $\theta''' \in (\theta'', \overline{\vartheta}(r'))$. The lowest type belonging to $J(\theta''')$ is precisely $\theta'' > \hat{\theta}$, yielding the median voter (and thus a majority of voters) always to vote for candidate L. Since for type θ''', $u_e^*(\theta''') < \min_{b_m \in B_m(J(\theta'''), \theta''')} u_e(b_m, \theta''', \theta''', \mathbf{q})$, any such wPBE fails the Intuitive Criterion test. Therefore, in any IC-refined wPBE, $\theta'' = \theta^*$. Suppose now that θ^* is such that $\mathbb{E}_p\left[\tilde{\theta}|\underline{\vartheta}(\theta^*) \leq \theta \leq \theta^*\right] > \hat{\theta}$. In such cases it is always possible to pick a $\theta' \in (\hat{\theta}, \theta^*)$ such that $\mathbb{E}_p\left[\tilde{\theta}|\underline{\vartheta}(\theta') \leq \theta \leq \frac{\theta' + \theta^*}{2}\right] \geq \hat{\theta}$. Applying the Intuitive Criterion we have that $u_e^*(\theta') < \min_{b_m \in B_m(J(\theta'), \theta')} u_e(b_m, \theta', \theta', \mathbf{q})$. Hence, in any IC-refined wPBE, it must be that $\mathbb{E}_p\left[\tilde{\theta}|\underline{\vartheta}(\theta^*) \leq \theta \leq \theta^*\right] = \hat{\theta}$.

Suppose the distribution of valence F_θ allows for the existence of two reports $r'' > r'$ both satisfying $\mathbb{E}_p\left[\tilde{\theta}|\underline{\vartheta}(r') \leq \theta \leq r'\right] = \mathbb{E}_p\left[\tilde{\theta}|\underline{\vartheta}(r'') \leq \theta \leq r''\right] = \hat{\theta}$. Notice that it must be that $\underline{\vartheta}(r') < \underline{\vartheta}(r'') < \hat{\theta} < r' < r''$. Any wPBE which prescribes types $\theta \in (\underline{\vartheta}(r''), r'')$ to report $r = r''$ would then fail the Intuitive Criterion test, since $u_e^*(\theta') < \min_{b_m \in B_m(J(\theta'), \theta')} u_e(b_m, \theta', \theta', \mathbf{q})$. This completes the proof. □

Lemma 12 (Just effective misreport)**.** *There always exists a unique report $r = \theta^*$ such that $\mathbb{E}_p\left[\tilde{\theta}|\underline{\vartheta}(\theta^*) \leq \theta \leq \theta^*\right] = \hat{\theta}$.*

Proof. Set $g(x) = \mathbb{E}_p\left[\tilde{\theta}|\underline{\vartheta}(x) \leq \theta \leq x\right]$ and $y = \left\{\theta \in \Theta : \underline{\vartheta}(\theta) = \hat{\theta}\right\} = \left(\hat{\theta} + \sqrt{\frac{\hat{\theta} - \lambda\tau_e}{k}}\right)$. Given the continuity of f_θ and F_θ, $\frac{\partial g(x)}{\partial x}$ exists and is continuous. Hence, $g(x)$ is continuously differentiable and thus continuous in $(\hat{\theta}, y)$. Since $g\left(\hat{\theta}\right) < \hat{\theta}$ and $g(y) > \hat{\theta}$, by the intermediate value theorem there exists at least one $x \in (\hat{\theta}, y)$ such that $g(x) = \hat{\theta}$.

Since for every $x \in (\lambda\tau_e, +\infty)$ we have $\frac{\partial\vartheta(x)}{\partial x} \in (0,1)$ it follows that $g(x)$ is strictly increasing in x and therefore there is only one x such that $g(x) = \hat{\theta}$. $\square$

Lemma 13 (No further misreports). *In any IC-refined wPBE there is only one convex set of types that misreport. After that, the reporting rule exhibits truthful reporting.*

Proof. Consider the equilibrium discontinuity described in Lemma 11. Suppose there is an IC-refined wPBE where a report $r = \theta' > \theta^*$ is never delivered in such equilibrium. It must be that $r = \theta' \notin \mathcal{E}(\theta)$ and $r^*(\theta') \in \mathcal{E}(\theta)\ \forall\ \theta > \lambda\tau_e$. But if $r = \theta$ were to be observed by voters, the lowest type belonging to $J(\theta')$ is greater than $\theta^* > \hat{\theta}$. Applying the Intuitive Criterion, such report would make a majority of voters to cast a ballot in favour of L. Hence, $u_e^*(\theta') < \min_{b_m \in B_m(J(\theta'),\theta')} u_e(b_m, \theta', \theta', \mathbf{q})$, which implies that in any IC-refined wPBE all $\theta \geq \theta^*$ are reported in equilibrium. This in turn implies that $r^*(\theta) = \theta\ \forall\ \theta > \theta^*$. $\square$

Lemma 14 (Refinements). *The IC-refined wPBE in Proposition 1 is robust to NWBR in signalling games.*

Proof. I need to introduce some notation first. Define the set of mixed best responses of the median voter to report r and any beliefs with support in T as,

$$MB_m(p,r) = \Delta \left\{ \arg\max_{b_m} \int_{\theta\in\Theta} p(\theta|r) u_m(b_m, \theta, \mathbf{q}) d\theta \right\} \tag{1.20}$$

and let $MB_m(T,r) = \bigcup_{p:p(T|r)=1} MB_m(p,r)$.[37] I use β_m to indicate a probability distribution over A_m, that is $\beta_m \in \Delta\{L, R\}$. Define the set of mixed best responses of the median voter to report r and any beliefs concentrated on T that make θ strictly prefer r to her equilibrium strategy as,

$$D(\theta, T, r) = \bigcup_{p:p(T|r)=1} \{\beta_m \in MB_m(p,r) | u_e^*(\theta) < u_e(\beta_m, \theta, r, \mathbf{q})\} \tag{1.21}$$

and let $D^o(\theta, T, r)$ be similarly defined by the set of mixed best responses of the median voter to report r and any beliefs concentrated on T that make θ just indifferent to her equilibrium strategy. Following Cho and Kreps, 1987, p. 206 a type-report pair

[37] Note that $MB_m(T,r)$ is not the set of all probability distributions over $B_m(T,r)$.

can be deleted under NWBR in signalling games if,

$$D^o(\theta,\Theta,r) \subseteq \bigcup_{\theta'\neq\theta} D(\theta',\Theta,r) \tag{1.22}$$

Suppose a report $r \in (\underline{\vartheta}(\theta^*),\theta^*)$ is unexpectedly delivered. Note that, given the assumed tie rule, the median voter never mixes, but instead,

$$MB_m(p,r) = \begin{cases} L & \text{if } \mu(\theta|r) \geq \hat{\theta} \\ R & \text{otherwise} \end{cases} \tag{1.23}$$

For every type θ, $\bigcup_{\theta'\neq\theta} D(\theta',\Theta,r) = \{L\}$. Types $\theta \in (\underline{\vartheta}(r),r)$ have $D(\theta,\Theta,r) = \{L\}$ and $D^o(\theta,\Theta,r) = \{\varnothing\}$ while types $\theta \in \left\{\underline{\vartheta}(r), \frac{r+\theta^*}{2}\right\}$ have $D(\cdot) = \{\varnothing\}$ and $D^o(\cdot) = \{L\}$. Applying NWBR in signalling games, I prune all types $\theta \in (-\infty, \underline{\vartheta}(r)] \cup \left[\frac{r+\theta^*}{2}, +\infty\right)$ and remain with the set $J(r) = \left(\underline{\vartheta}(r), \frac{r+\theta^*}{2}\right)$. Since $\mathbb{E}_p\left[\tilde{\theta}|\underline{\vartheta}(r) \leq \theta \leq r\right] < \hat{\theta}$, there is no type θ such that $u_e^*(\theta) < \min_{b_m\in B_m(J(r),\theta)} u_e(b_m,\theta,r,\mathbf{q})$. Therefore, the IC-refined wPBE is robust to the NWBR in signalling games refinement. This also implies that it is robust to Banks and Sobel, 1987's Divinity and Universal Divinity refinements and Cho and Kreps, 1987's D1 and D2.[38] □

The proof for Proposition 1 is the following.

Proof. The following lemmas prove results that must hold in any IC-refined wPBE. Lemma 9 shows that $r^*(\theta) = \theta \ \forall \ \theta \in (-\infty, \gamma\tau_e]$. The properties of the first discontinuity and pooling area are shown in Lemma 11. More precisely, pooling occurs because types $\theta \in (\underline{\vartheta}(\theta^*),\theta^*)$ misreport to $r^* = \theta^*$, while $\underline{\vartheta}(\theta^*)$ is just indifferent between misreporting and truthful reporting and θ^* is determined by $\theta^* = \min\left\{r \in \mathbb{R} \middle| \mathbb{E}_p\left[\tilde{\theta}|\underline{\vartheta}(\theta^*) \leq \theta \leq \theta^*\right] = \hat{\theta}\right\}$. Lemma 13 states that $r^*(\theta) = \theta \ \forall \ \theta \in [\theta^*, +\infty)$, while Lemma 12 guarantees that a report $r = \theta^*$ such that the expectation $\mathbb{E}_p\left[\tilde{\theta}|\underline{\vartheta}(\theta^*) \leq \theta \leq \theta^*\right] = \hat{\theta}$ always exists and is unique. All together these lemmas nail down the IC-refined wPBE to one unique possible equilibrium reporting rule. Lemma 14 shows the robustness of the equilibrium to several refinements. □

[38] See discussion in Van Damme, 1987, Chapter 10.

Infinite signaling games may not have equilibria. Manelli, 1996 solves this existence problem by adding cheap talk to the communication framework. While existence is not an issue in the current paper, the reader might wonder whether different types could separate through an added cheap talk dimension. An alternative equivalent approach is to assume a rich language space as in Kartik, 2009. This latter approach assumes that there are many ways to report the same information. Formally, for each type θ there is a set of reports R_θ such that any $r \in R_\theta$ has the literal meaning "my type is θ". When type θ reports $r \in R_{\theta'}$ for $\theta' \neq \theta$, it bears a misreporting cost of $kC(\theta', \theta)$. The rich language assumption means that $|R_\theta| = \infty \; \forall \; \theta \in \Theta$. [39] Indeed, in Kartik, 2009's model of costly lying there might be multiple pools where high types deliver different reports belonging to the same set R_1. The following Lemma 15 shows that, for the equilibrium in Proposition 1, neglecting segmentation is without loss of generality.

Lemma 15 (Segmentation)**.** *A single pool in $(\underline{\vartheta}(\theta^*), \theta^*)$ constitutes an equilibrium even when allowing for a rich report space or adding a cheap talk dimension. Multiple pools can be sustained in equilibrium, but lead to the same equilibrium outcome as the single pool.*

Proof. In the IC-refined wPBE of Proposition 1, types in $(\underline{\vartheta}(\theta^*), \theta^*)$ pool by reporting $r = \theta^*$. Since such report induces the election of candidate L, there is no type in the pool that has a strict incentive to differentiate itself. However, a segmentation of the set of pooling types can constitute an equilibrium. As an instance, consider a partition of $(\underline{\vartheta}(\theta^*), \theta^*)$ in n sets, $\{T_i\}_{i=1}^n$, such that $r \in R_{\theta^*} \; \forall \; \theta \in \bigcup_{i=1}^n T_i$ but types belonging to different sets deliver different messages. If $\mathbb{E}_p[\tilde{\theta}|\theta \in T_i] = \hat{\theta}$ for every $i \in \{1, \ldots, n\}$ then such segmentation would be an equilibrium. If $\mathbb{E}_p[\tilde{\theta}|\theta \in T_j] < \hat{\theta}$ for some j, then all types $\theta \in T_j$ would deviate by reporting a different message in R_{θ^*}. Since this separation occurs either through different reports with the same misreporting costs or the same report coupled with a cheap talk message, the outcome of the game is the same in all cases, for all types. □

The following is the proof of Lemma 3.

[39] See Kartik, 2009 for a comprehensive formalization.

Proof. From equation (2), considering a report $r > \lambda\tau_e$ independent of k, we have the following derivatives,

$$\frac{\partial \underline{\vartheta}(r)}{\partial r} = \left(1 - \frac{1}{\sqrt{1+4k(r-\lambda\tau_e)}}\right) \in (0,1) \tag{1.24}$$

$$\frac{\partial \underline{\vartheta}(r)}{\partial k} = -\frac{1}{2k^2}\left[1 - \sqrt{1+4k(r-\lambda\tau_e)}\right] - \frac{r-\lambda\tau_e}{k\sqrt{1+4k(r-\lambda\tau_e)}} > 0 \tag{1.25}$$

the latter equation is always positive when $k > 0$ and $r > \lambda\tau_e$, which is the case. Since θ^* is implicitly defined as,

$$\theta^* := \left\{ t \in \mathbb{R} | \int_{\underline{\vartheta}(t)}^{t} \theta p(\theta) d\theta - \gamma\tau_m \int_{\underline{\vartheta}(t)}^{t} p(\theta) d\theta = 0 \right\} \tag{1.26}$$

I define the function $g(\theta^*)$ as follows,

$$g(\theta^*) := \int_{\underline{\vartheta}(\theta^*)}^{\theta^*} \theta p(\theta) d\theta - \gamma\tau_m \int_{\underline{\vartheta}(\theta^*)}^{\theta^*} p(\theta) d\theta = 0 \tag{1.27}$$

By the implicit function theorem,

$$\frac{d\theta^*}{dk} = -\frac{\frac{\partial g(\theta^*)}{\partial k}}{\frac{\partial g(\theta^*)}{\partial \theta^*}} = -\frac{\frac{\partial \underline{\vartheta}(\theta^*)}{\partial k} p(\underline{\vartheta}(\theta^*))[\gamma\tau_m - \underline{\vartheta}(\theta^*)]}{p(\theta^*)[\theta^* - \gamma\tau_m] - \frac{\partial \underline{\vartheta}(\theta^*)}{\partial \theta^*} p(\underline{\vartheta}(\theta^*))[\underline{\vartheta}(\theta^*) - \gamma\tau_m]} < 0 \tag{1.28}$$

A simple application of the chain rule shows that, $\frac{d\underline{\vartheta}(\theta^*)}{dk} = \frac{\partial \underline{\vartheta}(\theta^*)}{\partial \theta^*}\frac{d\theta^*}{dk} + \frac{\partial \underline{\vartheta}(\theta^*)}{\partial k} > 0$. Therefore, as k increases, θ^* increases and $\underline{\vartheta}(\theta^*)$ decreases monotonically. As $k \to 0^+$, $\underline{\vartheta}(\theta^*) \to \lambda\tau_e$ and the persuasion ratio tends to 1,

$$\lim_{k \to 0^+} \rho(k) = \frac{\int_{\lambda\tau_e}^{\hat{\theta}} p(\theta) d\theta}{\int_{\lambda\tau_e}^{\hat{\theta}} p(\theta) d\theta} = 1 \tag{1.29}$$

Similarly, $\lim_{k \to +\infty} \underline{\vartheta}(\theta^*) = \theta^*$, and since θ^* has to satisfy (1.26) for any $k > 0$, both $\underline{\vartheta}(\theta^*)$ and θ^* converge to $\hat{\theta} = \gamma\tau_m$. This gives us,

$$\lim_{k \to +\infty} \rho(k) = \frac{\int_{\hat{\theta}}^{\hat{\theta}} p(\theta) d\theta}{\int_{\lambda\tau_e}^{\hat{\theta}} p(\theta) d\theta} = 0 \tag{1.30}$$

□

1.5.3 Pay-off Relevance

Definition 3 (Informative, influential and relevant communication)**.** *I say communication is* informative *if $p(\theta|r)$ is not constant along the equilibrium path. Communication is* influential *if $\nu_m(r)$ is not constant along the equilibrium path.*[40] *Communication is* pay-off relevant *(for the receiver) if $\mathbb{E}_p[u_m(\nu^*(\psi^*(\theta)),\theta)] > \max_{b_m \in A_m} \mathbb{E}_p[u_m(b_m,\theta)]$. Informative and influential communication are a necessary condition for pay-off relevance.*

Lemma 16 (Pay-off relevant communication)**.** *Communication between the expert and the median voter is always pay-off relevant.*

Proof. Consider the median voter's expected pay-off $U_m(\cdot)$ in case where there is no expert or without communication.

$$U_m^p = \max_{b_m \in \{L,R\}} \mathbb{E}_p[u_m(b_m,\theta,\mathbf{q})] = \begin{cases} -\gamma(h_m - q_L)^2 + \mathbb{E}_p[\tilde{\theta}] & \text{if } \mathbb{E}_p[\tilde{\theta}] \geq \gamma\tau_m \\ -\gamma(h_m - q_R)^2 & \text{if } \mathbb{E}_p[\tilde{\theta}] < \gamma\tau_m \end{cases} \tag{1.31}$$

In contrast, her ex-ante pay-off before consulting the expert is,

$$\begin{aligned} U_m^r = \mathbb{E}_p\left[u_m\left(\nu^*\left(\psi^*(\theta)\right),\theta,\mathbf{q}\right)\right] = F_\theta\left(\underline{\vartheta}(\theta^*)\right) \cdot \left\{-\gamma(h_m - q_R)^2\right\} + \\ + \left[1 - F_\theta\left(\underline{\vartheta}(\theta^*)\right)\right] \cdot \left\{-\gamma(h_m - q_L)^2 + \mathbb{E}_p\left[\tilde{\theta}|\theta > \underline{\vartheta}(\theta^*)\right]\right\} \end{aligned} \tag{1.32}$$

Consider first the case where the prior p is such that $\mathbb{E}_p[\tilde{\theta}] < \gamma\tau_m$. In order for $U_m^r > U_m^p$ it has to be that $\mathbb{E}_p[\tilde{\theta}|\theta > \underline{\vartheta}(\theta^*)] > \gamma\tau_m$, which is always true given that in equilibrium θ^* is such that $\mathbb{E}_p[\tilde{\theta}|\underline{\vartheta}(\theta^*) \leq \theta \leq \theta^*] = \gamma\tau_m$. Consider now the case where the prior is such that $\mathbb{E}_p[\tilde{\theta}] \geq \gamma\tau_m$. $U_m^r > U_m^p$ implies that $F_\theta\left(\underline{\vartheta}(\theta^*)\right) \cdot \left\{\mathbb{E}_p[\tilde{\theta}|\theta > \underline{\vartheta}(\theta^*)] - \gamma\tau_m\right\} < \left(\mathbb{E}_p[\tilde{\theta}|\theta > \underline{\vartheta}(\theta^*)] - \mathbb{E}_p[\tilde{\theta}]\right)$. For the same reason as before, this implies $F_\theta\left(\underline{\vartheta}(\theta^*)\right) < 1$ which is always true. Therefore, for the median voter communication is always pay-off relevant, meaning that the presence of a privately informed but strategic and biased expert improves her expected utility with respect to the absence of communication. □

[40] Along the equilibrium path means for reports that in equilibrium are delivered with positive probability.

Chapter 2

Influential News and Policy-Making

2.1 Introduction

One of the most common criticism against the media is that they are ideologically biased and can strategically distort news in order to achieve political influence. Indeed, there is substantial evidence that media bias has an impact on voters' decision at the ballot box.[1] Such policy-motivated media are willing to sacrifice profits in order to achieve political influence. Indeed, misreporting information comes at a cost, e.g., from a loss of reputation, audience and profits. Legislators might intervene by raising further the cost of misreporting information. Consider as an instance the Representation of the People Act 1983 (Chapter 2, Part II, Section 106),

> A person who, or any director of any body [...] which – (a) before or during an election, (b) for the purpose of affecting the return of any candidate at the election, makes or publishes any false statement of fact in relation to the candidate's personal character or conduct shall be guilty of an illegal practice.

More recently, the German parliament has approved a law that imposes a fine of up to €50 million to social media firms that fail to remove illegal content such as fake news and hate speech. The law was proposed mainly due to a concern about the negative effects of news distortion on politics.[2]

[1] See DellaVigna and Kaplan, 2007, Gerber, Karlan, and Bergan, 2009 and Chiang and Knight, 2011.

[2] The Netzwerkdurchsetzungsgesetz (or NetzDG) was approved on June 30, 2017, and took full effect on January 1, 2018.

The implications of media bias are not confined to distortions of the voters' choice at the ballot box, but from there they spread and propagate back to the process of policy-making. Indeed, during electoral competition, the presence of an influential media outlet generates a tension between pandering to the voter with a populist policy, or pleasing the media with a biased policy. Therefore, media bias skews electoral competition and produces distortions in policy selection and outcomes.

In this paper, I study the implications of strategic news reporting over the process of policy-making in a Downsian framework with a rational but uninformed representative voter. Before the election takes place, an informed policy-motivated media outlet delivers a public and potentially distorted news report about the relative quality of the two candidates running for office. In contrast to canonical models of communication, the media outlet bears a cost of misreporting its private information that is increasing in the magnitude of misrepresentation. At the outset, policies are endogenously championed by the two competing office-motivated candidates: an incumbent and a challenger.[3]

In equilibrium, the incumbent proposes a policy that is relatively more "populist" with respect to that of the challenger, who sets forth a proposal that is more "biased" toward the media outlet's preferred policy. Both candidates advance more populist proposals when the misreporting costs are higher, and more biased policies when costs are lower. This is due to the fact that when costs are lower, it is more beneficial to look for the support of an influential outlet. The "Median Voter Theorem" breaks down, and when misreporting costs are low enough, the proposals of both candidates converge to the media outlet's best policy.

A consequence of the endogenous process of policy-making is that the media outlet's favourite candidate always wins the election: in addition to seeking the endorsement of the media outlet, the challenger reacts to the proposal of the incumbent with the policy that grants the highest possible influence to the outlet. Such proposal can be neither too populist nor too biased. With the former, she would lose the support of the media outlet; the latter would increase excessively the conflict of interest between the voter and the outlet, resulting in less effective endorsements.

[3]While I analyze a sequential policy-making stage where the incumbent moves first, I will show that some results obtain when policy proposals are simultaneous.

Almost paradoxically, when misreporting costs are low enough, the media outlet is highly influential but it fully reveals its private information. This is because low costs yield convergence of proposals to the outlet's favourite policy, which in turns eradicates any conflict of interest with the voter. As politicians are offering exactly the same policy, both the media outlet and the voter agree on electing the candidate with the greatest quality. In contrast, for higher costs of misreporting, the media outlet engages in information fabrication and persuades the voter to elect its preferred candidate. However, the voter is always better off with higher misreporting costs, even though she might receive less information about quality. This is because the loss from electing the "wrong" candidate is more than compensated by the gain from implementing more populist policies.

Therefore, a regulator concerned about the welfare of the voter might enact interventions directed toward increasing the misreporting costs. In order to produce any effect, such interventions have to be substantial. When costs are already low, a lenient intervention would not alter the incentive of the candidates to cater to the media outlet's preferred policy.[4] However, I show that there might be some resistance from the government to enact substantial and effective regulations. Indeed, the incumbent is systematically disadvantaged from the presence of an influential media outlet, as in equilibrium the challenger can always get the outlet's endorsement.[5] Therefore, governments currently holding office have the incentive to eliminate such disadvantage by setting low misreporting costs at the expense of the voter. This result suggests that this type of interventions should be enacted by a third-party that is independent from the government in charge.

2.2 Related Literature

This paper contributes to the strand of literature exploring the political consequences of media bias. In particular, it collocates within the work employing a supply-side

[4] This could explain the high fines imposed by the NetzDG in Germany, a trend that is recently followed by other countries such as Russia, Philippines, Singapore, Kenya, and Venezuela among others (see the Human Rights Watch).

[5] The incumbent could seek the support of the media outlet by proposing an extremely biased policy. However, the challenger would react by advancing an extremely populist policy, which would profitably please the voter and weaken the outlet's endorsement. Green-Pedersen, Mortensen, and Thesen, 2017 provide empirical evidence of the incumbent disadvantage in media coverage.

approach, where media bias originates from the intrinsic preferences and motivations of, e.g., agents who work for news organizations like journalists, editors and owners. Papers belonging to this strand are, among others, Baron, 2006, Duggan and Martinelli, 2011, Alonso and Camara, 2016, and Shapiro, 2016.[6] However, most of this literature neglects the effect of media bias on the process of policy-making as they consider exogenous policies.[7] In contrast, I include a stage of strategic policy selection, which allows me to explore how news distortion spills over the process of policy-making.

This paper also relates to a body of literature studying the welfare effects and regulation of the media market. In this strand, Baron, 2006 provides a welfare analysis and finds a role for regulation within a model where media bias originates from privately informed journalists with both ideological motivations and career concerns. As already pointed before, this work does not account for the policy distortion generated by media bias. In contrast, I provide a welfare and regulatory analysis that crucially hinges on the policy-making stage.

Mullainathan and Shleifer, 2005 explore a demand-driven model of media bias where readers have preferences for news that are consistent with their prior beliefs. Because in their work biased reporting directly originates from the self-confirmatory preferences of the readers, it is not clear whether and how such bias should be taken into account by a regulator. In contrast, this paper features rational and Bayesian voters that are fully aware of the media outlet's preferences over policies and its ability to fabricate information.

Most of the interventions in the media market are directed toward regulating ownership concentration. An early work studying the welfare effect of competition is provided by Steiner, 1952. Perego and Yuksel, 2015 show how competition in the market for news can have negative welfare consequences due to the type of information produced in equilibrium by news organizations. Anderson and McLaren, 2012 develop a supply-side model where, in equilibrium, the media outlet either

[6]In demand-driven models of media bias, profit-maximising news organisations either conform to their audience's preconceptions or cater to more profitable segments of voters, eventually affecting policy outcomes as a by-product. Papers belonging to this tradition are, among others, Gentzkow and Shapiro, 2006, Mullainathan and Shleifer, 2005, Bernhardt, Krasa, and Polborn, 2008, Strömberg, 2004, Gul and Pesendorfer, 2012 and Perego and Yuksel, 2015.

[7]Chakraborty and Ghosh, 2016 are an exception. I shall discuss their work below.

completely withholds or fully transmits information to the decision maker. They provide a merger policy and find that competition increases the amount of information transmitted. Gul and Pesendorfer, 2012 make use of a demand-side model of media bias to explore the relationship between competition and divergence of policy platforms.

All these papers assume that news providers either can say whatever they want without any direct consequence (cheap talk), or simply cannot lie at all (verifiable disclosure). Therefore, they cannot assess the consequences of interventions directed toward limiting the news providers' profitability in distorting information. In contrast to this body of work, in the present paper I allow the media outlet to misreport information, but at a cost. Hence, I can investigate the welfare effects of interventions on the media outlet's misreporting cost.

Brocas, Carrillo, and Wilkie, 2011 provide the only related work where information can be strategically garbled at a cost. They adopt Blackwell, 1951's "comparison of experiments" in order to model the strategic information transmission of policy-motivated news providers, and deliver a welfare analysis of the media market under different levels of competition. News providers have preferences over the policy outcome, and pay a "reputational cost" whenever they suppress or withhold information that is detrimental to their interests. In contrast to their work, in the present paper news reports cannot be ordered in the Blackwell sense, and policies are endogenous as well as the media outlet's preferences over outcomes. They find that when the cost of garbling information increases, the amount of bias decreases and the welfare of consumers increases. In contrast, I show that this might not be the case when accounting for the endogenous process of policy formation.

The most closely related paper is Chakraborty and Ghosh, 2016. They develop a supply-driven model of media bias where, before the election takes place, a policy-motivated media outlet delivers a public report about the candidates' relative quality. At the outset, policies are simultaneously selected by two office-motivated candidates. Their welfare analysis focuses on the conflict of interest between the voter and the media outlet, which is endogenously determined in the policy-making stage. The present paper differs in that (i) news reports are not cheap talk, but the media pays a cost for misreporting, and (ii) the policy-making stage is sequential rather than

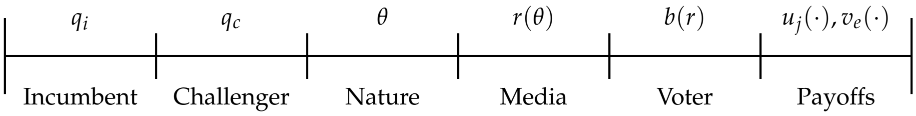


FIGURE 2.1: Timeline.

simultaneous. Introducing misreporting costs allows me to study the consequences of the media outlet's profitability from distorting information. Indeed, such costs are a control variable of regulators, e.g., via the institution of media watchdogs and fines. Furthermore, sequentiality allows me to study to what extent incumbency is an advantage when a media outlet can strategically influence the decision of voters.

2.3 The Model

There are four players: a representative voter, v, two candidates, $\{i, c\}$, and a media outlet, e. The voter has to cast a ballot $b \in \{i, c\}$ for one of the two candidates: the incumbent i or the challenger c. At the outset, each candidate $j \in \{i, c\}$ makes a binding commitment to a policy proposal q_j. Proposals are sequential: the incumbent i firstly proposes $q_i \in \mathbb{R}$, then the challenger c proposes $q_c \in \mathbb{R}$. If candidate j is elected by the voter, her policy q_j is implemented. After the candidates' commitments, but before the election takes place, a media outlet e delivers a news report $r \in \mathbb{R}$ about the relative quality of the two candidates, $\theta \in \Theta \equiv [-\phi, \phi]$. In particular, θ represents the relative quality of the incumbent with respect to the challenger, and is uniformly distributed in Θ.[8] All players share the common uniform cumulative prior $P(\theta)$ with density $p(\theta)$. However, the media outlet privately observes the exact realization of θ. While the voter and the media outlet might have different preferences over the policy spectrum, they agree that the higher is the realized θ, the relatively better is the incumbent with respect to the challenger. Hereafter, I shall refer to θ simply as "quality", meaning relative quality or quality-difference.

Figure 2.1 illustrates the timing structure of the game: (i) the incumbent makes a binding commitment to a policy q_i; (ii) afterwards, the challenger observes q_i and then commits to a policy q_c; (iii) nature selects the state of nature θ, which is privately

[8]Alternatively, $\theta \equiv \theta_i - \theta_c$, where θ_j is the quality of candidate $j \in \{i, c\}$. The distributional assumption captures that candidates are ex-ante symmetrical and the voter holds uninformative prior about their relative quality. However, all results would hold with a general, well-behaved, distribution.

observed only by the media outlet; (iv) the media outlet delivers a news report r about the realized state θ, which is observed by the voter; (v) the voter casts a ballot $b \in \{i, c\}$ for one of the two candidates, and (vi) the policy proposed by the winning candidate is implemented and payoffs are realized.

Candidates are purely office seeking and care only about being in office. Therefore, they select policies that maximize the probability of electoral victory. I assume that winning the elections gives the candidates a utility of 1, while losing gives a utility 0. The utility $u_j(b)$ of candidate $j \in \{i, c\}$ is,

$$u_j(b) = \begin{cases} 1 & \text{if } b = j \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$

The voter v has an ideal "bliss policy" $h_v \in \mathbb{R}$, while the media outlet has a bliss of $h_e \in \mathbb{R}$. I assume, without loss of generality, that $h_e < h_v$. The voter's utility $u_v(\cdot)$ is an additively separable combination of standard single peaked policy preferences and candidates' relative quality θ,

$$u_v\left(b, \theta, \mathbf{q}\right) = \begin{cases} u_v^i = -\gamma(h_v - q_i)^2 + \theta & \text{if } b = i \\ u_v^c = -\gamma(h_v - q_c)^2 & \text{if } b = c \end{cases} \tag{2.2}$$

Where $\gamma > 0$ is a positive intensity parameter weighting the relative importance of policies to quality. Given a pair of policy proposals $\mathbf{q} = (q_i, q_c)$, I define a threshold $\gamma\tau_j$ for player $j \in \{v, e\}$ such that, if quality θ exceeds such threshold, then player j prefers to elect the incumbent rather than the challenger.

Definition 4 (Partisan endorsement threshold). *Given a pair of policies $\mathbf{q} = (q_i, q_c)$, the partisan endorsement of the voter v and the media outlet e are, respectively,*

$$\gamma\tau_v(h_v, \mathbf{q}) := \gamma(2h_v - q_i - q_c)(q_i - q_c) \tag{2.3}$$

$$\gamma\tau_e(h_e, \mathbf{q}) := \gamma(2h_e - q_i - q_c)(q_i - q_c) \tag{2.4}$$

Therefore, given the policies $\mathbf{q}$ and relative quality θ, the media outlet's preferred candidate $m \in \{i, c\}$ is,

$$m(\theta, \mathbf{q}) = \begin{cases} i & \text{if } \theta \geq \gamma \tau_e(\mathbf{q}) \\ c & \text{otherwise} \end{cases} \tag{2.5}$$

A conflict of interest between the voter and the media outlet arises when $\tau_v \neq \tau_m$. In such cases, there are realization of quality such that the preferred candidate of the media outlet differs from that of the voter. I shall assume that the "political utility" $u_e(b, \theta, \mathbf{q})$ of the media outlet is such that,

$$u_e(b = j | m = j) - u_e(b = -j | m = j) = \Delta > 0 \tag{2.6}$$

That is, the media outlet enjoys a higher utility if its favourite candidate m is elected than otherwise.[9] In addition to the political utility $u_e(\cdot)$, the media outlet pays a cost $kC(r, \theta)$ when its private information is θ and it delivers the news report r. The parameter $k \geq 0$ is a scalar measuring the intensity of misreporting costs, and $C(r, \theta) \geq 0$ for every $r \in \mathbb{R}$ and every $\theta \in \Theta$. I shall refer to k simply as the "misreporting costs". Throughout the paper I will assume $C(\cdot)$ to be the square loss function $C(r, \theta) = (r - \theta)^2$.[10] Therefore, the media outlet's "total" utility $v_e(\cdot)$ is a combination of its political utility stemming from which candidate is elected and its cost of delivering a report r,

$$v_e(r, b, \theta, \mathbf{q}) = u_e(b, \theta, \mathbf{q}) - k(r - \theta)^2 \tag{2.7}$$

A strategy for candidate $j \in \{i, c\}$ is a binding commitment to a policy proposal $q_j \in \mathbb{R}$.[11] A reporting strategy for the media outlet is a function $r : \Theta \to \mathbb{R}$ which assigns a report $r \in \mathbb{R}$ to each realization of quality $\theta \in \Theta$. A voting strategy for the

[9] In contrast to the voter, the media outlet does not directly benefit from higher levels of quality in the elected candidate. However, quality enters in the media outlet's preferences in that it determines who is its favourite candidate. This assumption is without loss of generality, and does not alter the qualitative findings of this paper, but helps to maintain the exposition as smooth and clean as possible.

[10] The same qualitative results hold if $C(r, \theta)$ is twice continuously differentiable over $\mathbb{R} \times \Theta$, with $C_{rr} > 0 > C_{r\theta}$ and $C(\theta, \theta) = 0$.

[11] Candidates cannot propose state-contingent policies $q_j(\theta)$. This is consistent with the idea that the uncertainty regarding quality θ is publicly resolved only after the implementation of the winner's proposals, and policies cannot be changed in the short-run.

voter v is a function $b : \mathbb{R} \to \{i, c\}$, assigning a ballot for a candidate $b \in \{i, c\}$ for every possible report $r \in \mathbb{R}$. A belief function for the voter is a mapping $q : \mathbb{R} \to \Delta(\Theta)$ which, given any report $r \in \mathbb{R}$, yields a posterior belief $P(\theta|r)$. I shall indicate the expected valence given a report r as $\mu(\theta|r) = \mathbb{E}_p[\theta|r]$. This paper is concerned with the case where the media outlet is influential, meaning that its reports can affect the voter's decision.[12] An equilibrium is a weak Perfect Bayesian Equilibrium (wPBE).

2.4 Equilibrium

Solving for the equilibrium requires a number of steps. First, I show what is the media outlet's equilibrium reporting strategy given any two fixed exogenous policy proposals. As a second step, I endogenize the candidates' policy proposals as a function of the media outlet's equilibrium strategy. This requires (i) finding the challenger's best response to the incumbent proposal, and (ii) the optimal policy for the incumbent given the challenger's strategy. In this step, both candidates have to account for the media outlet's reporting rule. Lastly, I incorporate the equilibrium policies in the media outlet's equilibrium strategy. I will provide the intuition behind these results in the following paragraphs, and present all the proofs in the Appendix.

2.4.1 News Reporting in Equilibrium

The sub-game concerning the strategic communication between the informed media outlet e to the uninformed voter v constitutes in fact a costly signalling game. Indeed, when $k > 0$, reports are discriminating signals of the outlet's private information. Communication is not "cheap", as messages have a direct impact on the outlet's utility through the cost function $C(r, \theta)$. Importantly, such costs are contingent on the realisation of the state of nature. However, this framework shares many features with the canonical "cheap talk" models: communication is unmediated and the "sender" cannot commit to a reporting rule. Because the media outlet's private information is

[12]The media outlet is influential if the voter's sequentially optimal decision b as a function of the media's report r is not constant along the equilibrium path. Whether communication is influential depends on the conflict of interest between the voter and the media outlet, which in the present model is endogenously determined by the candidates' policy proposals. Hence, there cannot be unilateral and profitable deviation from the candidates' equilibrium strategies that make the media outlet not influential. This requires enough uncertainty on quality, i.e., ϕ has to be large enough.

about the realized quality θ, I will hereafter refer to θ as being the "type" of media outlet, even though it is an actual relative feature of the candidates.

How quality θ is strategically reported in equilibrium crucially depends on which candidate is ex-ante more likely to be supported by the media outlet with respect to the voter. In turn, this depends on which policies $\mathbf{q} = (q_i, q_c)$ are proposed. As an instance, if $\tau_e(\mathbf{q}) < \tau_v(\mathbf{q})$, then the outlet is ex-ante more likely to support the incumbent than the voter. Indeed, for all quality realizations $\theta \in (\gamma\tau_e, \gamma\tau_v)$, the voter prefers to elect the challenger and implement policy q_c, while the outlet prefers the incumbent with policy q_i. In these kind of situations there is a conflict of interest that makes room for the strategic delivery of news: there are some states of nature θ where the media outlet can engage in information "inflation", meaning that it can profitably exaggerate the quality-difference between the incumbent and the challenger. Similarly, when $\tau_v(\mathbf{q}) < \tau_e(\mathbf{q})$, sometimes the outlet will "deflate" quality, reporting a lower value than the realized one, $r < \theta$.

However, the fact that the media outlet can potentially deliver distorted news generates skepticism in the voter. Given a report r, I find the the types of outlet that could profitably deliver a report r by equating the potential gains from misreporting Δ with the cost $k(r-\theta)^2$,

$$l(r) = r - \sqrt{\frac{\Delta}{k}} \tag{2.8}$$

$$h(r) = r + \sqrt{\frac{\Delta}{k}} \tag{2.9}$$

Where $l(r)$ is the "lowest" misreporting type, that is, the lowest type $\theta < r$ that could have profitably delivered r. All the types $\theta < l(r)$ would prefer to report truthfully their private information, as misreporting to r is prohibitively expensive with respect to the potential gain. The "highest" misreporting type is similarly defined.

However, there is no type of outlet $\theta < \gamma\tau_e$ supporting the challenger that would spend resources to inflate quality by delivering a report $r > \gamma\tau_e$, endorsing the incumbent. When accounting for this, I define the "potential misreporting types" as follows,

$$\hat{l}(r) = \max\{l(r), \gamma\tau_e\} \tag{2.10}$$

$$\hat{h}(r) = \min\{h(r), \gamma\tau_e\} \tag{2.11}$$

In Chapter 1 I show that, within this setting, "language monotonicity" naturally arises in the unique equilibrium surviving the Intuitive Criterion refinement: an outlet supporting candidate j misreports by inflating its quality (or by deflating the quality of the opponent $-j$). This is in contrast with the indeterminacy of language in cheap talk equilibria. Therefore, I define the set of potential misreporting types M, given the report r and pair of policy proposals $\mathbf{q} = (q_i, q_c)$, as follows,

$$M(r) = \begin{cases} (\max\{l(r), \gamma\tau_e\}, r) & \text{if } \tau_e < \tau_m \\ (r, \min\{h(r), \gamma\tau_e\}) & \text{otherwise} \end{cases} \tag{2.12}$$

In equilibrium, the media outlet pools different realizations of quality that are close to the voter's partisan threshold $\gamma\tau_v$ by reporting the same message r^*. Information is jammed in a specific way: every misreporting type $\theta \in M(r^*)$ delivers the same pooled report r^*; in turn, the pooled report r^* is such that the expected quality in the set $M(r^*)$ is exactly equal to the voter's partisan threshold $\gamma\tau_v$. Therefore, upon observing the report r^*, the voter is indifferent between casting a ballot for the incumbent and the challenger. The tie is solved in favour of the outlet's preferred candidate.

Proposition 5 (Communication Equilibrium)**.** *There is a unique equilibrium robust to the Intuitive Criterion, where the posterior beliefs $p(\theta|r)$ are according Bayes' rule when possible, and such that the reporting rule $r(\theta)$ and voter's expectation $\bar{\mu}(r)$ are,*

$$r(\theta) = \begin{cases} r^* := \{r \in \mathbb{R} | \mathbb{E}_p[\theta \in M(r)] = \gamma\tau_v\} & \textit{if } \theta \in M(r^*) \\ \theta & \textit{otherwise} \end{cases} \tag{2.13}$$

$$\bar{\mu}(r) = \begin{cases} \gamma\tau_v & \textit{if } r = r^* \in M(r^*) \\ \mu(r) < \gamma\tau_v & \textit{if } r \in M(r^*) \\ \theta & \textit{otherwise} \end{cases} \tag{2.14}$$

Proof. The proof follows the one in Appendix 1.5.2 for the equilibrium described in Proposition 1, Chapter 1. The case $\tau_e > \tau_v$ is proved similarly to the case $\tau_e < \tau_v$. □

The media outlet misreports even when, given the realized quality, it agrees with the voter on which candidate is the best. This is a result of the voter's skepticism:

since every report is discounted, the media outlet has to provide sufficient evidence in favour of its preferred candidate. In addition, in order to persuade the voter, it is necessary to misreport despite the absence of a conflict of interest: only in this way, when observing the report r^*, the voter has to account for the fact that sometimes she should follow the outlet's endorsement.

When the media outlet persuades the voter, it causes an "election overthrowing": the winning candidate would, under perfect information, lose the electoral competition. Persuasion occurs when the relative quality takes values between the voter and the outlet's partisan thresholds. However, there are cases where the media outlet reports truthfully despite the presence of a conflict of interest with the voter. As an instance, when $\tau_e < \tau_v$ this happens if $l(r^*) > \gamma\tau_e$. Persuasion and election overthrowing occurs when $\theta \in [l(r^*), \gamma\tau_v)$, while when $\theta \in (\gamma\tau_e, l(r^*))$ the outlet reports truthfully, letting its favourite candidate to lose.

In contrast, if the misreporting costs are low enough, the media outlet can afford to persuade the voter every time there is a conflict of interest. If $\tau_e < \tau_v$, this occurs when the costs are so low that every type of outlet in the conflict is a "misreporting type", $l(r^*) = r^* - \sqrt{\frac{\Delta}{k}} \leq \gamma\tau_e$. Because quality is uniformly distributed, in equilibrium it has to be that $r^* - \gamma\tau_v = \gamma\tau_v - l(r^*)$. Therefore, if $l(r^*) > \gamma\tau_e$, then $r^* = \gamma\tau_v + \frac{1}{2}\sqrt{\frac{\Delta}{k}}$. The condition $l(r^*) \leq \gamma\tau_e$ can be rewritten as follows,

$$k \leq \frac{\Delta}{4\gamma^2 \left(\tau_v(\mathbf{q}) - \tau_e(\mathbf{q})\right)^2} \tag{2.15}$$

Given policies $\mathbf{q}$, if the misreporting costs k are low enough to satisfy the above condition, then the media outlet can persuade the voter to elect its favourite candidate every time there is disagreement. In these cases, the equilibrium pooling report is $r^* = 2\gamma(\tau_v - \tau_e)$. Such report would induce the voter to have an expectation of $\mathbb{E}_p\left[\theta | \theta \in [\gamma\tau_e, 2\gamma(\tau_v - \tau_e)]\right] = \gamma\tau_v$, and make the voter indifferent between the incumbent and the challenger. Therefore, when the misreporting cost are low enough, the outlet's favourite candidate always wins the election. Alternatively, given a fixed cost k, the same occurs when proposed policies are close enough.[13]

[13]Rearranging the condition, the media outlet obtains full persuasion in the whole disagreement area when $(q_i - q_c)^2 \leq \frac{\Delta}{16\gamma^2(h_v - h_e)^2 k}$.

As follows, I summarize the equilibrium explicit form of the pooling report r^*, the lowest and highest misreporting types $\hat{l}(r^*(\mathbf{q})) := \max\{l(r^*(\mathbf{q})), \gamma\tau_e\}$ and $\hat{h}(r^*(\mathbf{q})) := \min\{h(r^*(\mathbf{q})), \gamma\tau_e\}$ as a function of the misreporting costs k and policy proposals $\mathbf{q}$.

$$r^*(\mathbf{q}) = \begin{cases} \gamma\tau_v + \frac{1}{2}\sqrt{\frac{\Delta}{k}} & \text{if } k > \tilde{k} \text{ and } \tau_e < \tau_m \\ \gamma\tau_v - \frac{1}{2}\sqrt{\frac{\Delta}{k}} & \text{if } k > \tilde{k} \text{ and } \tau_e > \tau_m \\ 2\gamma(\tau_m - \tau_e) & \text{if } k \leq \tilde{k} \\ \gamma\tau_v & \text{if } \tau_e = \tau_m \end{cases} \tag{2.16}$$

$$\hat{l}(r^*(\mathbf{q})) = \begin{cases} \gamma\tau_v - \frac{1}{2}\sqrt{\frac{\Delta}{k}} & \text{if } k > \tilde{k} \\ \gamma\tau_e & \text{if } k \leq \tilde{k} \text{ or } \tau_e = \tau_m \end{cases} \tag{2.17}$$

$$\hat{h}(r^*(\mathbf{q})) = \begin{cases} \gamma\tau_v + \frac{1}{2}\sqrt{\frac{\Delta}{k}} & \text{if } k > \tilde{k} \\ \gamma\tau_e & \text{if } k \leq \tilde{k} \text{ or } \tau_e = \tau_m \end{cases} \tag{2.18}$$

2.4.2 Equilibrium Policy-Making

In this section, I study which policies are proposed in equilibrium by the incumbent i and the challenger c. There is a tension between pandering to the voter, thus looking for the support granted by a popular policy, and pleasing the media so as to gain its persuasive endorsement. The proofs for the best response functions are in Appendix 2.6.1, while the proofs for the policy-making stage are in Appendix 2.6.2.

Both candidates seek to maximize their chance to win the election. The expected utility of candidate $j \in \{i, c\}$ is therefore $U_j(\mathbf{q}) = P(j \text{ wins } |\mathbf{q})$. In equilibrium, when $\tau_e(\mathbf{q}) < \tau_m(\mathbf{q})$, the incumbent wins if $\theta \geq \hat{l}(r^*)$, hence with probability $P(\theta \geq \hat{l}(r^*))$. To keep the calculation simpler, I apply an affine transformation to each candidate's utility, and denote $V_j(\mathbf{q}) = 2\phi U_j(\mathbf{q}) - \phi$. The incumbent proposes a policy q_i to maximise the (transformed) expected utility $V_i(\mathbf{q})$, where,

$$V_i(\mathbf{q}) = \begin{cases} -\hat{l}(r^*(\mathbf{q})) & \text{if } \tau_e(\mathbf{q}) < \tau_m(\mathbf{q}) \\ -\hat{h}(r^*(\mathbf{q})) & \text{otherwise} \end{cases} \tag{2.19}$$

As the challenger wins when the incumbent loses, her utility is $V_c(\mathbf{q}) = -V_i(\mathbf{q})$, that is,

$$V_c(\mathbf{q}) = \begin{cases} \hat{l}(r^*(\mathbf{q})) & \text{if } \tau_e(\mathbf{q}) < \tau_m(\mathbf{q}) \\ \hat{h}(r^*(\mathbf{q})) & \text{otherwise} \end{cases} \tag{2.20}$$

For example, when $\tau_e < \tau_v$, the incumbent wants to make $\hat{l}(r^*)$ as low as possible, while the challenger wants to maximizes her chances of being in office by making $\hat{l}(r^*)$ as high as possible. Furthermore, the partisan thresholds and the conflict of interest between the outlet and the voter are jointly determined by the pair of policies (q_i, q_c). Therefore, candidates can seek for the support of the voter by proposing a "populist" policy that is close enough to the voter's bliss h_v. Alternatively, they can exploit the media outlet's persuasive power by proposing a "biased" policy that is closer to the outlet's bliss h_e. Hence, policies can be proposed in order to strategically induce $\tau_e < \tau_v$ or $\tau_v < \tau_e$, gathering the support of either the media outlet or the voter.[14] Importantly, the incumbent decides first her proposal q_i.

In the following discussion I shall assume that both candidates make proposals between the outlet and the voter's preferred policies, $q_j \in [h_e, h_v]$ for $j \in \{i, c\}$. This will hold true in equilibrium, as any policy $q_j < h_e$ or $q_j > h_v$ loses the support of both the voter and the media outlet.

How the challenger best responds to the proposal of the incumbent crucially depends on the misreporting costs k. Consider first the case where k is relatively high, $k > \bar{k}$, where,

$$\bar{k} = \frac{\Delta}{\gamma^2(h_v - h_e)^4} \tag{2.21}$$

When the incumbent proposes a relatively populist policy q_i, the challenger reacts by offering a more biased policy $q_c < q_i$, thus seeking the support of the media outlet. In particular, the challenger's policy is more biased but close enough to q_i so as to grant full persuasive power to the outlet. An even more biased policy $q'_c < q_c$ would certainly please the outlet, but it would also increase its conflict of interest with the voter, resulting in a loss of persuasive power. On the other hand, a more populist policy $q''_c > q_c$ would either result in less frequent endorsements or turn the media outlet in favour of the incumbent.

[14] Note that $\tau_e < (>)\tau_v$ when $q_i < (>)q_c$.

However, when the incumbent proposes a policy that is sufficiently close to the media outlet's bliss h_e, the challenger reacts by offering the voter's preferred policy $q_c = h_v$. Reacting with a populist policy is now profitable because it allows to gather the support of the voter while proposing a policy that is substantially different from the one of the incumbent. Such difference creates a larger conflict of interest, which dampens the persuasive power of the media outlet. This strategy would leave the incumbent with both an unpopular policy and weak support from the media outlet.

There are few differences when misreporting costs are lower than $\bar{k}$. For intermediate costs $k \in [\bar{k}/4, \bar{k}]$, the challenger proposes the outlet's preferred policy $q_c = h_e$ as a response to a whole (convex) set of policies proposed by the incumbent. Indeed, the media outlet has a greater persuasive power when misreporting costs are smaller. In turn, the challenger has more incentives to please the media outlet and less incentives to jump for extremely populists policies. In these cases, the best way to seek the support of the media outlet is to offer exactly $q_c = h_e$. Going for a populist platform becomes profitable only when the incumbent proposes something that is very close to the outlet's bliss policy h_e.

Similarly, when costs are relatively low, $k \in [0, \bar{k}/4)$, there are no proposals by the incumbent that are best replied with a populist policy. Any departure from the outlet's preferred policy h_e is severely punished, and the challenger proposes $q_c = h_e$ even when the incumbent does the same, $q_i = h_e$. Figure 2.2 depicts both candidates' best responses for the three levels of misreporting costs k discussed above. The formal argument and proofs are in Appendix 2.6.1.

Consider now the optimal policy proposal of the incumbent. Formally, the incumbent wants to select the policy q_i that maximizes her utility given the best response of the challenger $BR_c(q_i)$, that is, $q_i \in \arg\max_{q_i \in \mathbb{R}} V_i(q_i, BR_c(q_i))$.

For any finite misreporting cost $k > \bar{k}$, the incumbent selects a relatively moderate policy $q_i^* \in (h_e, h_v)$. A more populist policy $q_i' \in (q_i^*, h_v]$ would allow the challenger to get the support of the media outlet with a proposal that is appealing to the voter as well. On the other hand, a more biased policy $q_i'' \in [h_e, q_i^*)$ would allow the challenger to head for a very populist policy $q_c = h_v$. In this way, the challenger would gather the consensus of the voter at the expense of having the media outlet endorsing the incumbent. However, the two proposals would be so different that the

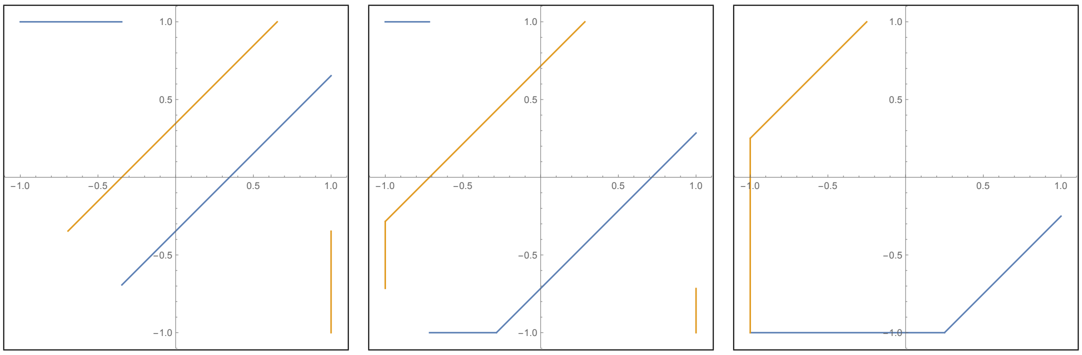


FIGURE 2.2: From the left to the right panel are depicted in blue and yellow the best response functions when the costs k are relatively high, intermediate, and low. The voter and media outlet's preferred policies are, respectively, $h_v = 1$ and $h_e = -1$.

outlet would have a weak grasp on the voter's choice due to an increased conflict of interest. Hence, the incumbent optimally proposes a policy q_i^* that is biased enough to push the challenger best response far from the voter's preferred policy without making room for populist responses.

Proposition 6. *For relatively high level of misreporting costs $k > \bar{k}$, equilibrium policies are,*

$$q_i^* = h_v + \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)} - \sqrt{\frac{1}{\gamma}\sqrt{\frac{\Delta}{k}}} \tag{2.22}$$

$$q_c^* = h_v - \sqrt{\frac{1}{\gamma}\sqrt{\frac{\Delta}{k}}} \tag{2.23}$$

For any finite misreporting cost $k > \bar{k}$, the incumbent proposes more populist policies than the challenger, $q_i^* > q_c^*$. With higher costs, both candidates get closer to the voter's preferred policy, as $\frac{\partial q_j^*}{\partial k} > 0$ for $j \in \{i, c\}$. Furthermore, the policy difference $q_i^* - q_c^* = \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)}$ shrinks when the misreporting costs increase. However, the presence of a persuasive media outlet prevents them to go fully populist and propose exactly h_v. This is in stark difference with canonical models of electoral competition where the "median voter theorem" yields convergence of policy proposals to the voter's bliss h_v. In the present model, this occurs only when misreporting costs are infinitely high, $\lim_{k\to\infty} q_i^* = \lim_{k\to\infty} q_c^* = h_v$.

As the misreporting costs decrease, both equilibrium proposals become more biased toward the media outlet's preferred policy. For intermediate levels of costs

$k \in [\bar{k}/4, \bar{k}]$, the incumbent proposes a relatively biased policy $q_i^* < \frac{h_v+h_e}{2}$. As before, in order to avoid a populist reaction by the challenger, the optimal policy q_i^* is not too close to h_e. However, the incumbent can take advantage of the fact that the challenger cannot do better than pandering to the media outlet, $q_c^* = h_e$. Hence, it "squeezes" the challenger by proposing a policy as close as possible to h_e. In this way, the incumbent can reduce the policy difference $q_i - q_c$, therefore limiting the support that the media outlet gives to the challenger.

Proposition 7. *For relatively intermediate level of misreporting costs, $k \in [\bar{k}/4, \bar{k}]$, equilibrium policies are,*

$$q_i^* = \frac{h_v + h_e}{2} - \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)} \tag{2.24}$$

$$q_c^* = h_e \tag{2.25}$$

When misreporting costs are relatively low, $k < \bar{k}/4$, the challenger proposes $q_c = h_e$ independently of what the incumbent proposes. Because the endorsements of the media outlet are highly influential, populist policies are never appealing. The incumbent cannot do better than proposing $q_i^* = h_e$, as any slightly more populist choice $q_i' > h_e$ would result in a strong, effective endorsement in favour of the challenger. Therefore, in equilibrium both candidates propose the very same policy $q_i^* = q_c^* = h_e$.

Proposition 8. *For relatively low level of misreporting costs, $k \in [0, \bar{k}/4)$, equilibrium policy-making features full convergence to the media outlet's preferred proposal, $q_i^* = q_c^* = h_e$.*

This is the best possible outcome that the media outlet can get as its preferred policy h_e will be implemented with certainty. As I will explain in the next section, when policies converge, there is no conflict of interest and the outlet never engages in costly misreporting. The voter gathers perfect information about the quality at the expense of having worse policies. Figure 2.4.2 shows the two candidates' equilibrium policies for different cost of misreporting.[15]

[15] In this model, the median voter theorem always breaks down, while full policy convergence occurs at the media outlet's preferred policy for $k < \bar{k}/4$. Both results do not depend on the sequentiality of proposals, and obtain as well when proposals are simultaneous. I formally show the argument in Proposition 10 and Proposition 11 in Appendix 2.6.5.

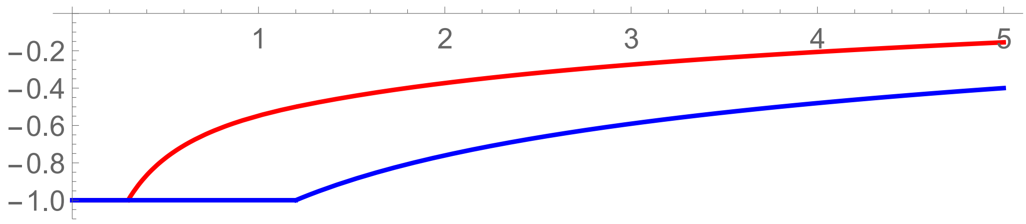


FIGURE 2.3: Policy-making in equilibrium. The policy proposals of the incumbent and the challenger are, respectively, in red and blue. On the horizontal axis, the misreporting costs k. The media outlet's bliss policy is at $h_e = -1$.

2.4.3 Communication with Endogenous Policy-Making

In this section I study the outlet's equilibrium reporting rule with endogenous policy-making. Because the incumbent is relatively more populist than the challenger, $q_i^* \geq q_c^*$, the media outlet always endorses the latter, as $\tau_v(\mathbf{q}^*) \leq \tau_e(\mathbf{q}^*)$. In particular, persuasion and election overthrowing occur when $\theta \in \Big(\gamma\tau_v, \hat{h}(r^*(\mathbf{q}^*))\Big)$. However, policies are always similar enough to grant the outlet full persuasive power in the disagreement area $(\gamma\tau_v, \gamma\tau_e)$. Indeed, the equilibrium policy difference satisfies the condition $(q_i^* - q_c^*)^2 \leq \frac{\Delta}{16\gamma^2(h_v-h_e)^2k}$ for every $k \geq 0$, implying that $\hat{h}(r^*(\mathbf{q}^*)) = \gamma\tau_e$. Hence, the ex-ante probability of persuasion and election overthrowing is positive, $P\left(\theta \in [\gamma\tau_v, \gamma\tau_e] \,|k\right) = \frac{\gamma}{\phi}(h_v - h_e)(q_i^*(k) - q_c^*(k)) > 0$ for every finite $k > \bar{k}/4$. Figure 2.4 illustrates the equilibrium reporting rule with endogenous policies.

However, when the misreporting costs are low, $k \in \left[0, \bar{k}/4\right]$, the equilibrium policies converge to $q_i^* = q_c^* = h_e$. Because the two candidates are proposing exactly the same policy, there is no conflict of interest between the media outlet and the voter as $\tau_e = \tau_v = 0$. In this case, they both agree that the best candidate is the one with the greater relative quality. In this case, the media outlet can truthfully report its private information about θ, and the voter has no reason to be skeptical. When misreporting costs are low, the outlet's favourite candidate will always win, its favourite policy h_e will always be implemented, and it will never have to engage in costly misreporting.

When policies are endogenous as in the present paper, the media outlet's favourite candidate always wins the election. Indeed, since persuasion occurs for all quality levels $\theta \in (\gamma\tau_v, \gamma\tau_e)$, the incumbent (challenger) wins when $\theta \geq (<)\gamma\tau_e$. Almost paradoxically, when the misreporting costs are low, the media outlet reveals truthfully its private information to the voter. Misreporting and persuasion occur only for higher

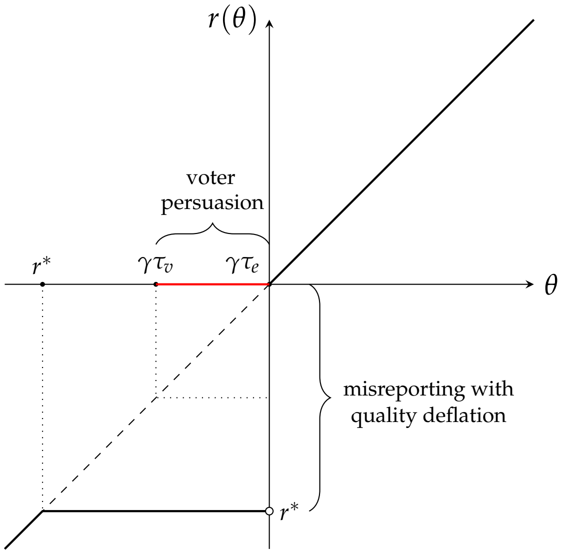


FIGURE 2.4: Equilibrium reporting rule with endogenous policies.

costs, $k > \bar{k}/4$.

2.5 Discussion

In this section I briefly explain some implications of the media outlet's strategic news reporting. I shall consider its impact on the voter's welfare, discuss the effects of a regulator's intervention, and examine the probability of winning the election of the candidates.

2.5.1 Welfare and Regulation

In equilibrium, as the misreporting costs k increase, (i) both candidates propose increasingly populist policies, (ii) the policy-difference shrinks, and therefore (iii) the conflict of interest between the media outlet and the voter decreases. At the extreme case, when the misreporting costs are infinitely high, both candidates propose the voter's preferred policy h_v and the media outlet fully reveals its private information. This supports the idea that higher misreporting costs increase the welfare of the voter as they yield better policies and less persuasion.

However, the policy-difference and conflict of interest annihilate when the misreporting costs are low as well. As outlined before, in this case the voter can obtain full information about the relative quality θ at the expense of having worse policies.

On the other hand, intermediate levels of misreporting costs feature both relatively biased policies (but closer to the voter's bliss h_v) and election overthrowing through persuasion.

A natural question is whether the welfare of the voter is monotonic in the media outlet's misreporting costs. Indeed, if quality is relatively more important with respect to the policy (i.e. $\gamma > 0$ is low enough), the voter might prefer to implement the outlet's favourite policy h_e with certainty, but obtain full information regarding the candidates' quality θ rather than having a slightly better policies but less information.

In Proposition 9 I show that the voter always prefers higher misreporting costs, even when this would result in less information about quality. The foregone utility stemming from electing the wrong candidate with positive probability is more than compensated by having a slightly better policy, even if in a probabilistic sense. Indeed, for intermediate costs $k \in \left[\bar{k}/4, \bar{k}\right]$, $\tau_v < 0 < \tau_e$ and $q_i^* > q_c^* = h_e$. Persuasion occurs for $\theta \in (\gamma\tau_v, \gamma\tau_e)$, and the policy h_e is implemented for all quality $\theta < \gamma\tau_e$. With probability $P(\theta > \gamma\tau_e)$ the voter implements a better policy $q_i^* > h_e$. Otherwise, she would obtain exactly the same policy h_e that she would get with lower misreporting costs. However, she would have less information regarding the relative quality, resulting in a loss of $\left(\frac{\phi-\gamma\tau_e}{2\phi}\right)\frac{\phi+\gamma\tau_e}{2} - \frac{\phi}{4} < 0$. On the other hand, with probability $P(\theta > \gamma\tau_e)$ she would implement $q_i^* > h_e$ rather than h_e, which yields an expected gain of $-\gamma\tau_v(q_i^*, h_e)\frac{\phi-\gamma\tau_e(q_i^*,h_e)}{2\phi} > 0$. For $q_i = q_i^*$, the expected gains are greater than the expected loss from less information about quality, i.e., $-\gamma\tau_v(q_i^*, h_e)\frac{\phi-\gamma\tau_e(q_i^*,h_e)}{2\phi} > \frac{\phi}{4} - \left(\frac{\phi-\gamma\tau_e(q_i^*,h_e)}{2\phi}\right)\frac{\phi+\gamma\tau_e(q_i^*,h_e)}{2}$.

Yet, the welfare of the voter $W_v(k)$ is constant for low misreporting costs, $W_v(k') = W_v(k'')$ for $k', k'' \in \left[0, \bar{k}/4\right]$. This is because the policy convergence toward h_e annihilates the conflict of interests, yielding full revelation. Since the media outlet does not engage in costly misreporting, different costs k within $[0, \bar{k}/4]$ do not have different effects in determining which candidate wins and which policy is implemented. The voter is completely indifferent between any such costs. Figure 2.5 shows that welfare of the voter is weakly increasing with the costs k, but it is "flat" for every $k \in \left[0, \bar{k}/4\right]$.

Proposition 9. *The welfare of the voter is (weakly) increasing with the misreporting costs k. The welfare is constant for $k \in [0, \bar{k}/4]$.*

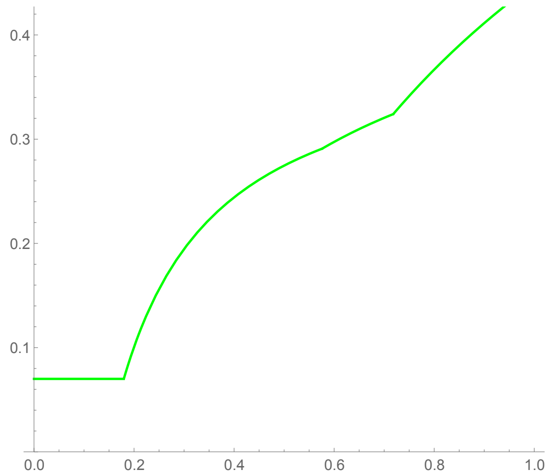


FIGURE 2.5: In green, the voter's welfare (vertical axis) as a function of the misreporting costs k (horizontal axis).

Proof. See proof in Appendix 2.6.3

This result suggests that a regulator concerned about the welfare of the voter should intervene by increasing the misreporting costs as much as possible. For instance, this could be done by the establishment of media watchdogs who can investigate and inform voters, consumers and governments about the activity of strategic news providers like press newspapers, social media, radio or broadcast television. This type of intervention would increase the probability of ex-post state verification. A news provider that is caught misreporting would then have to pay a fine and suffer from a loss of reputation, audience, and future profits. Alternatively, a regulator can simply increase the fine that a news provider has to pay when it is caught misreporting.

However, the flat portion in the welfare of the voter W_v suggests that such interventions could be fruitless: increasing the costs from k' to k'' when $k', k'' \in \left[0, \bar{k}/4\right]$ would have no effect at all, leaving the welfare unaltered. Furthermore, if interventions are costly, increasing k could also result in a welfare loss. Therefore, when misreporting costs are relatively low, a regulator should either enact a substantial intervention, or not intervene at all.[16] Indeed, ahead of the Germany's 2017 parliamentary elections, the German government has proposed to fine social media up

[16]Of course, another possible reason why governments and regulators can be lenient is that *fake news laws* are perceived to be inconsistent with the freedom of opinion and expression, and "stifle journalists from reporting in environments that are often contradictory and rapidly developing". See, e.g., `https://www.article19.org/pages/en/false-news.html`. In this paper, I provide a formal, parallel argument: in some circumstances, such kind of interventions might be fruitless.

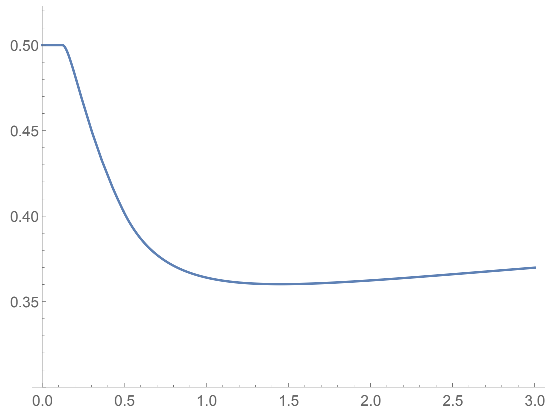


FIGURE 2.6: Incumbent disadvantage: in blue, the ex-ante probability of winning the election of the incumbent, as a function of the misreporting cost k.

to €500,000 for each fake news story that is not removed.[17] The main motivation for such proposal was the recent up-surging concern about the detrimental effects of misreported news on politics. Eventually, the proposal has been successively amended to fines up to €50 million.[18]

2.5.2 Incumbent Disadvantage

Incumbent candidates seeking re-election typically enjoy an electoral margin, the so-called "incumbency advantage". Such margin is one of the most well-documented phenomenon in elections, and has been studied both theoretically and empirically.[19] Among other advantages, incumbents can benefit from an easier access to campaign finance, a better name recognition, and control on the electoral agenda.

In particular, there is substantial evidence that incumbents enjoy a greater media coverage and exposure with respect to candidates in the opposition. This particular bias is known as the "incumbency bonus". However, Green-Pedersen, Mortensen, and Thesen, 2017 show that most news coverage of incumbents is actually negative. They find that "[...] for government parties, more media prominence is in fact followed by lower opinion ratings, whereas media prominence does not hurt the electoral support of the opposition. [...] In fact, our results indicate that the more government actors showed up in the news, the worse they did in opinion polls.".

[17] Retrieved from The Independent.

[18] Moreover, "individual members of staff responsible for handling complaints could also be fined up to €5 million for failing to comply with the regulations". Retrieved from The Telegraph and BBC.

[19] See references in Gordon and Landa, 2009 and Ansolabehere, Snowberg, and Snyder, 2006.

In this paper, the incumbent disadvantage arises endogenously throughout the procedures of policy-making and strategic news distortion. Candidates are ex-ante symmetrical: they have the same objective (maximizing the probability of winning) and the same quality distribution. The only element of differentiation is the timing: the incumbent makes a binding commitment to a policy proposal *before* the challenger. This is sufficient to create an unbalance that goes at the expense of the incumbent.

The tension faced by candidates between pandering to the voter or pleasing an influential media outlet generates a sort of "rock-paper-scissors" game: a populist policy is beaten by a moderate one; a moderate policy is beaten by a biased one; a biased policy is beaten by a populist one.[20] Whatever policy the incumbent proposes, the challenger can do better.

Lemma 17 (Incumbent Disadvantage)**.** *The ex-ante probability that the incumbent wins the election is strictly less than a half for any finite $k > \bar{k}/4$.*

Proof. See proof in Appendix 2.6.4

Only when the misreporting costs are low enough, $k \leq \bar{k}/4$, the incumbent has no ex-ante disadvantage over the challenger. This result suggests that the incumbent has an incentive to grant persuasive power to the media outlet before the election take place, so as to completely eliminate its disadvantage. This is best done by keeping the misreporting costs low, at the expense of the welfare of the voter. Therefore, such type of regulation should be enacted by a third-party that is independent of the incumbent government. Figure 2.5.2 shows the ex-ante probability of winning the election of the incumbent as a function of the cost of misreporting k.

[20]When the policy-making stage is with simultaneous moves, no candidate have an advantage over the other. However, some results would still carry on. See Appendix 2.6.5.

2.6 Appendix

2.6.1 Best Responses

Here I study the best response of the challenger given the proposal of the incumbent. Assume that the incumbent has committed to a policy proposal q_i. The expected utility of the challenger is $V_c(q_i, q_c) = \hat{l}(r^*(\mathbf{q}))$ if $q_c > q_i$ and $V_c(q_i, q_c) = \hat{h}(r^*(\mathbf{q}))$ otherwise. I define the "best response to the left" $BR_c^L(q_i)$ as the best response of the challenger subject to the constraint that $q_c \leq q_i$. That is, the BR_c^L is the policy that maximizes $V_c(q_i, q_c) = \hat{h}(r^*(\mathbf{q}))$ subject to $q_c \leq q_i$. Note that $\frac{\partial h(r^*(\mathbf{q}))}{\partial q_c} = 2\gamma(h_v - q_c) > 0$ and $\frac{\partial \gamma\tau_e}{\partial q_c} = 2\gamma(h_e - q_c) < 0$ for all $q_c \in [h_e, h_v]$, both functions are concave, and the equality $h(r^*(\mathbf{q})) = \gamma\tau_e$ is satisfied when,

$$q_c = q_i - \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)} =: \tilde{q}_c(q_i) \tag{2.26}$$

Therefore, $BR_c^L(q_i) = \tilde{q}_c$ when $\tilde{q}_c(q_i) \in [h_e, h_v]$. Otherwise, the maximum is $BR_c^L(q_i) = h_e$ if $\tilde{q}_c < h_e \leq q_i$, $BR_c^L(q_i) = q_i$ if $q_i < h_e$, and $BR_c^L(q_i) = h_v$ if $\tilde{q}_c(q_i) > h_v$. To sum up, the best response "to the left" for both players is,

$$BR_j^L(q_{-j}) = \begin{cases} q_{-j} & \text{if } q_{-j} \leq h_e \\ h_e & \text{if } q_{-j} \in \left[h_e, h_e + \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)}\right] \\ \tilde{q}_c(q_i) = q_{-j} - \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)} & \text{if } q_{-j} \in \left[h_e + \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)}, h_v + \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)}\right] \\ h_v & \text{if } q_{-j} \geq h_v + \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)} \end{cases}$$

Similarly, I define the "best reply to the right" $BR_c^R(q_i)$ as the best response of the challenger subject to the constraint that $q_c \geq q_i$. By playing these policies, the challenger seeks to maximize $V_c(q_i, q_c) = \hat{l}(r^*(\mathbf{q}))$. Since $\frac{\partial \hat{h}(r^*(\mathbf{q}))}{\partial q_c} = \frac{\partial \hat{l}(r^*(\mathbf{q}))}{\partial q_c}$, a similar procedure applies here: $\gamma\tau_e$ is decreasing in q_c, while $l(r^*(\mathbf{q}))$ is increasing in q_c, hence the policy that satisfies $l(r^*(\mathbf{q})) = \gamma\tau_e$ constitutes a minimum in the utility of c (with $q_c \geq q_i$). The best policy to the right of q_i is therefore $q_c = h_v$, as long as $l(r^*(q_i, h_v)) \geq 0$. Otherwise, imitating q_i is better as it gives $V(q_i, q_i) = 0$. Indeed, by definition $\gamma\tau_e(q_i, q_c = q_i) = 0$, and $l(\cdot)$ is maximized when $q_c = h_v$. The condition

$l(r^*(q_i, h_v)) \geq 0$ is satisfied whenever,

$$q_c \leq h_v - \sqrt{\frac{1}{2\gamma}\sqrt{\frac{\Delta}{k}}} \tag{2.27}$$

Therefore the "right" best response $BR_j^R(q_{-j})$ is,

$$BR_j^R(q_{-j}) = \begin{cases} h_v & \text{if } q_{-j} \leq h_v - \sqrt{\frac{1}{2\gamma}\sqrt{\frac{\Delta}{k}}} \\ q_{-j} & \text{otherwise} \end{cases}$$

In order to find the "overall" best response $BR_c(q_i)$, I need to compare the utility $V_c(\mathbf{q})$ obtained from best responding to the left and to the right of q_i. Because $\frac{\partial V_c(q_i, BR_c^R(q_i))}{\partial q_i} \leq 0$ and $\frac{\partial V_c(q_i, BR_c^L(q_i))}{\partial q_i} \geq 0$, there might exists a threshold $\bar{q}$ such that, if $q_i > \bar{q}$ ($q_i \leq \bar{q}$) then the best response coincide with the "left" ("right") best response $BR_c(q_i) = BR_c^L(q_i)$ ($BR_c(q_i) = BR_c^R(q_i)$). As a first step, I compare the $h(r^*(q_i, \tilde{q}_c))$ with $l(r^*(q_i, h_v)) = \gamma(h_v - q_i)^2 - \frac{1}{2}\sqrt{\frac{\Delta}{k}}$. Note that, by definition, $h(r^*(q_i, \tilde{q}_c)) = \gamma\tau_e(q_i, \tilde{q}_c)$. The utility from best replying to the left is,

$$h(r^*(q_i, \tilde{q}_c)) = \frac{1}{2}\sqrt{\frac{\Delta}{k}} - \gamma\left[2(h_v - q_i) + \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)}\right]\frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)} \tag{2.28}$$

The condition $h(r^*(q_i, \tilde{q}_c)) = l(r^*(q_i, h_v))$ can be rewritten as follows,

$$\gamma(h_v - q_i)^2 + 2\gamma\left(\frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)}\right)(h_v - q_i) + \gamma\left(\frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)}\right)^2 - \sqrt{\frac{\Delta}{k}} = 0 \tag{2.29}$$

By solving a quadratic equation in $(h_v - q_i)$, I obtain that the threshold is,

$$\bar{q}' = h_v + \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)} - \sqrt{\frac{1}{\gamma}\sqrt{\frac{\Delta}{k}}} \tag{2.30}$$

I do not need to consider the case where $BR_c^R(q_i) = q_i$ as such strategy would give a payoff of zero against a positive one from playing $BR_c^L(q_i)$. However, when q_i is low enough the best response to the left is to play either $BR_c^L(q_i) = h_e$. In this case, by equating $\gamma\tau_e(q_i, h_e) = l(r^*(q_i, h_v))$, I get,

$$\bar{q}'' = \frac{h_v + h_e}{2} - \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)} \tag{2.31}$$

Lastly, when $q_i < h_e$ the condition is $0 = l(r^*(q_i, h_v))$, which yields a threshold of $\bar{q}''' = h_v - \sqrt{\frac{1}{2\gamma}\sqrt{\frac{\Delta}{k}}}$. However, this last threshold is not really important as $q_i < h_e$ will never be played in equilibrium.

Importantly, the first threshold is valid as long as $\tilde{q}_c(\bar{q}') \geq h_e$. This is true if,

$$k \geq \frac{\Delta}{\gamma^2(h_v - h_e)^4} =: \bar{k}$$

When $\tilde{q}_c(\bar{q}') < h_e$, $BR_c^L(q_i) = h_e$ for $q_i \geq h_e$. Therefore, for lower misreporting costs, $k < \bar{k}$, the threshold is $\bar{q}''$. Also note that $\bar{q}' \geq \bar{q}''$ with $\bar{q}' = \bar{q}'' = h_e + \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)}$ at $k = \bar{k}$. To sum up, the threshold $\bar{q}$ is as follows,

$$\bar{q} = \begin{cases} \bar{q}' = h_v + \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)} - \sqrt{\frac{1}{\gamma}\sqrt{\frac{\Delta}{k}}} & \text{if } k \geq \bar{k} = \frac{\Delta}{\gamma^2(h_v - h_e)^4} \\ \bar{q}'' = \frac{h_v + h_e}{2} - \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)} & \text{otherwise} \end{cases}$$

When $k \geq \bar{k}$, $\bar{q}' \geq h_e + \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)}$. Therefore, with higher level of misreporting costs ($k \geq \bar{k}$), when q_i is close enough to the voter's bliss policy h_v ($q_i \geq \bar{q}'$), the best response is to the left of q_i but still higher than the media outlet's bliss, $BR_c = BR_c^L = \tilde{q}_c(q_i) > h_e$. However, for lower misreporting costs ($k < \bar{k}$), the threshold is $\bar{q}'' < h_e + \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)}$, therefore we have the flat portion at h_e. I shall define the threshold at which the challenger best replies with the media outlet's bliss h_e as follows,

$$\bar{q}''' = h_e + \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)} \tag{2.32}$$

Further, for $k > \bar{k}$, I have that $\bar{q}' < h_v - \sqrt{\frac{1}{2\gamma}\sqrt{\frac{\Delta}{k}}}$, which implies that, in the overall best response, whenever $q_i \leq \bar{q}'$), the challenger c always proposes $q_c = h_v$. Also, notice that since $\lim_{k\to\infty} \bar{q}' = h_v$ and $\frac{\partial \bar{q}'}{\partial k} > 0$, the threshold $\bar{q}'$ never reaches h_v for any finite $k > 0$, preventing the existence of equilibria where the challenger proposes the voter's bliss policy h_v. For all $k > 0$, $\bar{q}'' \leq h_v - \sqrt{\frac{1}{2\gamma}\sqrt{\frac{\Delta}{k}}}$, meaning that even when the misreporting costs are low enough ($k < \bar{k}$), whenever $q_i \leq \bar{q}''$), the challenger c always proposes $q_c = h_v$. To sum up, the best response depends on the level of misreporting costs k and is as follows,

$$BR_j^{k \geq \bar{k}}(q_{-j}) = \begin{cases} h_v & \text{if } q_{-j} < q' \\ \tilde{q}_j(q_{-j}) = q_{-j} - \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)} & \text{otherwise} \end{cases}$$

$$BR_j^{k < \bar{k}}(q_{-j}) = \begin{cases} h_v & \text{if } q_{-j} < \bar{q}'' \\ h_e & \text{if } q_{-j} \in [\bar{q}'', \bar{q}'''] \\ \tilde{q}_j(q_{-j}) = q_{-j} - \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v - h_e)} & \text{if } q_{-j} > \bar{q}''' \end{cases}$$

Where, in the case $k < \bar{k}$, the threshold is to the left of the media outlet's bliss policy ($\bar{q}'' \leq h_e$) when the misreporting costs are relatively very low $k \leq \tilde{k} := \frac{\Delta}{4\gamma^2(h_v - h_e)^4} = \frac{\bar{k}}{4}$.

2.6.2 Equilibrium Policy-Making

I denote with $V_i^k(q_i) \equiv V_i(q_i, BR_c(q_i))$ the utility of the incumbent given that the challenger will best respond.

$$V_i^{k \geq \bar{k}}(q_i) = \begin{cases} -\hat{l}(r^*(q_i, h_v)) & \text{if } q_i < \bar{q}' \\ -\hat{h}(r^*(q_i, \tilde{q}_c(q_i))) & \text{otherwise} \end{cases}$$

$$V_i^{k \leq \bar{k}}(q_i) = \begin{cases} -\hat{l}(r^*(q_i, h_v)) = -l(r^*(q_i, h_v)) & \text{if } q_i < \bar{q}'' \\ -\hat{h}(r^*(q_i, h_e)) = -\gamma\tau_e(q_i, h_e) & \text{if } \bar{q}'' \leq q_i \leq \bar{q}''' \\ -\hat{h}(r^*(q_i, \tilde{q}_c(q_i))) = -h(r^*(q_i, \tilde{q}_c(q_i))) & \text{if } q_i > \bar{q}''' \end{cases}$$

When the misreporting costs are relatively high, $k > \bar{k}$, the utility of the incumbent i is increasing in q_i until $q_i = \bar{q}'$, and decreasing afterwards. Because $-\hat{l}(r^*(\bar{q}', h_v)) = -\hat{h}(r^*(\bar{q}', \tilde{q}_c(q_i)))$, it follows that $q_i = \bar{q}'$ maximizes $V_i^{k \geq \bar{k}}(q_i)$. The challenger optimally replies to such policy with $q_c = BR_c^{k \geq \bar{k}}(\bar{q}') = \tilde{q}_c(\bar{q}')$.

There are three different configurations to consider when the misreporting costs are lower than $\bar{k}$: when $\frac{\bar{k}}{4} < k < \bar{k}$, the relevant thresholds are contained within the bliss policies of the voter and the media outlet, $h_e < \bar{q}'' < \bar{q}''' < h_v$. When $\frac{\bar{k}}{16} < k < \frac{\bar{k}}{4}$, the threshold $\bar{q}''$ becomes lower than the media outlet's bliss h_e, and we have $\bar{q}'' < h_e < \bar{q}''' < h_v$. Finally, for lower costs of misreporting, $0 < k < \frac{\bar{k}}{16}$, both thresholds are beyond the bliss policies, $\bar{q}'' < h_e < h_v < \bar{q}'''$.

Consider first the case where $k \in (\bar{k}/4, \bar{k})$. Note that for such k, $\bar{q}'' < \bar{q}' < \bar{q}'''$, $\frac{\partial -l(r^*(q_i,h_v))}{\partial q_i} = 2\gamma(h_v - q_i) > 0$ for $q_i < h_v$, $\frac{\partial -\gamma\tau_e(q_i,h_e)}{\partial q_i} = 2\gamma(h_e - q_i) > 0$ for $q_i > h_e$, while $\frac{\partial -h(r^*(q_i,\tilde{q}_c(q_i)))}{\partial q_i} = -\frac{\sqrt{\frac{\Delta}{k}}}{2(h_v-h_e)} < 0$. At $\bar{q}''$, the two utilities coincide, $-l(r^*(\bar{q}'', h_v)) = -\gamma\tau_e(\bar{q}'', h_e)$. Further, since $\tilde{q}_c(q_i)$ undercuts q_i just enough to grant the media outlet full persuasive power, we have that $h(r^*(q_i, \tilde{q}_c(q_i))) = -\gamma\tau_e(q_i, \tilde{q}_c(q_i))$. Hence, when $q_i = \bar{q}'''$, $-h(r^*(q_i, \tilde{q}_c(q_i))) = -\gamma\tau_e(\bar{q}''', h_e)$. Therefore, the incumbent maximizes her utility by proposing $q_i = \bar{q}''$, and the challenger best respond by proposing $q_c = BR_c^{k<\bar{k}}(\bar{q}'') = h_e$.

The same line of reasoning applies to the other two cases where $k < \bar{k}$. The incumbent proposes $q_i = \bar{q}''$ when $k < \bar{k}/4$ and $q_i = h_e$ in case $\bar{q}'' < h_e$, which happens when $k < \bar{k}/4$. The last $\bar{k}/16$ is useless as the equilibrium is the same.

Therefore, the equilibrium policies are,

$$q_i^*(k) = \begin{cases} \bar{q}' = h_v + \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v-h_e)} - \sqrt{\frac{1}{\gamma}\sqrt{\frac{\Delta}{k}}} & \text{if } k > \bar{k} \\ \bar{q}'' = \frac{h_v+h_e}{2} - \frac{\sqrt{\frac{\Delta}{k}}}{4\gamma(h_v-h_e)} & \text{if } k \in \left[\bar{k}/4, \bar{k}\right] \\ h_e & \text{if } k < \bar{k}/4 \end{cases}$$

$$q_c^*(k) = \begin{cases} \tilde{q}_c(\bar{q}') = h_v - \sqrt{\frac{1}{\gamma}\sqrt{\frac{\Delta}{k}}} & \text{if } k > \bar{k} \\ h_e & \text{if } k \leq \bar{k} \end{cases}$$

2.6.3 Voter's Welfare

Equilibrium policies are such that $q_i^*(k) \geq q_c^*(k)$ for every finite $k \geq 0$. Therefore, the incumbent wins if $\theta \geq \hat{h}(r^*(q_i^*(k), q_c^*(k)))$. Since valence is uniformly distributed, this event occurs with ex-ante probability $P(\theta \geq \hat{h}(\cdot)) = \frac{\phi - \hat{h}(\cdot)}{2\phi}$ and grants the voter a utility of $-\gamma(h_v - q_i^*(k))^2 + \mathbb{E}[\theta | \theta > \hat{h}(\cdot)]$. When $\theta < \hat{h}(r^*(q_i^*(k), q_c^*(k)))$, the voter elects the challenger c and enjoys a utility of $-\gamma(h_v - q_c^*(k))^2$. Therefore, the voter's welfare is,

$$\begin{aligned} W_v(k) = &\left(\frac{\hat{h}(r^*(q_i^*(k), q_c^*(k))) + \phi}{2\phi}\right)[-\gamma(h_v - q_c^*(k))^2] \\ &+ \left(\frac{\phi - \hat{h}(r^*(q_i^*(k), q_c^*(k)))}{2\phi}\right)\left[-\gamma(h_v - q_i^*(k))^2 + \frac{1}{2}\left(\phi + \hat{h}(r^*(q_i^*(k), q_c^*(k)))\right)\right] \end{aligned} \tag{2.33}$$

In order to simplify the notation for the derivative of the voter's welfare with respect to k, I shall write $\hat{h}_k^* = \frac{\partial \hat{h}(r^*(q_i^*(k), q_c^*(k)))}{\partial k}$, $\hat{h}^* = \hat{h}(r^*(q_i^*(k), q_c^*(k))) \geq 0$, and rewrite $(h_v - q)^2 = \tau_v(q, h_v)$. Therefore,

$$\begin{aligned} \frac{\partial W_v(k)}{\partial k} = \frac{1}{2\phi}\Bigg\{ &\hat{h}_k^* \times \left[\gamma\left(\tau_v(q_i^*(k), h_v) - \tau_v(q_c^*(k), h_v)\right) - \frac{\phi + \hat{h}^*}{2}\right] \\ &+ \hat{h}^* \times \left[\gamma\left(\frac{\partial \tau_v(q_i^*(k), h_v)}{\partial k} - \frac{\partial \tau_v(q_c^*(k), h_v)}{\partial k}\right) - \frac{\hat{h}_k^*}{2}\right] \\ &- \phi \times \left[\gamma\left(\frac{\partial \tau_v(q_i^*(k), h_v)}{\partial k} + \frac{\partial \tau_v(q_c^*(k), h_v)}{\partial k}\right) - \frac{\hat{h}_k^*}{2}\right]\Bigg\} \geq 0 \end{aligned} \tag{2.34}$$

Note that $\frac{\partial q_i^*(k)}{\partial k} < \frac{\partial q_i^*(k)}{\partial k}$ for $k > \bar{k}$, $\frac{\partial q_i^*(k)}{\partial k} > \frac{\partial q_i^*(k)}{\partial k}$ for $k >\in [\bar{k}/4, \bar{k}]$, and $\frac{\partial q_i^*(k)}{\partial k} = \frac{\partial q_i^*(k)}{\partial k} = 0$ for all non-negative $k < \bar{k}/4$. Also, $\frac{\partial \tau_v(q_j^*(k), h_v)}{\partial k} = -2(h_v - q_j^*(k))\frac{\partial q_j^*(k)}{\partial k} \leq 0$ for $j \in \{i, c\}$. Therefore $\frac{\partial W_v(k)}{\partial k} \geq 0$, and the voter is always better off with higher misreporting costs. In particular, for all $k \in [0, \bar{k}/4]$ the derivative is always zero, $\frac{\partial W_v(k)}{\partial k} = 0$. To see it, notice that since $q_i^*(k) = q_c^*(k) = h_e$, then $\frac{\partial \tau_v(q_j^*(k), h_v)}{\partial k} = 0$ and $\hat{h}_k^* = 0$.

2.6.4 Incumbent Disadvantage

In equilibrium, the ex-ante probability that the incumbent wins the election is,

$$P(i \text{ wins}|k) = P\left(\theta \geq \hat{h}(r^*(q_i^*(k), q_c^*(k)))\right) = \frac{1}{2} - \frac{\hat{h}(r^*(q_i^*(k), q_c^*(k)))}{2\phi} \tag{2.35}$$

Given that for all $k \geq 0$, $q_c^*(k) \leq q_i^*(k)$, then $\hat{h}(r^*(q_i^*(k), q_c^*(k))) \geq 0$. The probability of winning of the incumbent is, at best, a half. In particular, the (ex-ante) probability that the incumbent wins is exactly zero for all $k \leq \bar{k}/4$. This is because when $q_i = q_c$ there is no conflict of interest between the media outlet and the voter as

$\tau_e = \tau_v$. Without conflict, the media outlet fully reveals its private information, and since by assumption $\mathbb{E}_p[\theta] = 0$, the incumbent wins half of the time, $P(\theta \geq 0) = 1/2$. However, for every finite $k > \bar{k}/4$, the challenger gathers the media outlet's support by proposing policies that are closer to h_e with respect to the proposal of the incumbent. In these cases, $\hat{h}(r^*(q_i^*(k), q_c^*(k))) > 0$, and the incumbent is expected to win less than 50% of the times. Only at the limit when $k \to +\infty$, $q_i^* = q_c^* = h_v$, $\hat{h}(r^*(h_v, h_v)) = 0$, and $P(i \text{ wins}|k) = 1/2$.

2.6.5 Extension: Simultaneous Policy-Making

Proposition 10 (Simultaneous Media Convergence). *If $k \leq \frac{\Delta}{4\gamma^2(h_v - h_e)^4} = \frac{\bar{k}}{4}$, then there is an equilibrium where both candidates simultaneously offer the media outlet's bliss policy h_e.*

Proof. Assume $\mathbf{q} = (h_e, h_e)$ is an equilibrium, with $h_e < h_v$ being respectively the media outlet and voter's bliss policies. Then there is no profitable deviation of candidate c if, for every q_c,

$$P(0) \geq P\left(\hat{l}(r^*(h_e, q_c))\right)$$

Under full convergence to h_e, candidate c wins whenever $\theta < 0$. So we have that $0 \geq \hat{l}(r^*(h_e, q_c))$ if $0 \geq \min\left\{\gamma\tau_v(h_e, q_c) - \frac{1}{2}\sqrt{\frac{\Delta}{k}}, \gamma\tau_e(h_e, q_c)\right\}$. However, when $q_i < q_c$, it is always the case that $0 \leq \gamma\tau_e$. Therefore, the condition leads to $k \leq \frac{\Delta}{4(\gamma\tau_v(h_e, q_c))^2}$ for every $q_c \in \mathbb{R}$. The most profitable deviation from $q_c = h_e$ (that is not h_e itself) is $q_c = h_v$. Indeed, $\frac{\partial \tau_v}{dq_c} = 2[h_v - q_c] = (>)0$ when $q_c = (<)h_v$. I can rewrite the condition as follows,

$$k \leq \frac{\Delta}{4\gamma^2(h_v - h_e)^4} = \frac{\bar{k}}{4}$$

The proof is similar and leads to the same condition for a deviation of i. □

Proposition 11 (Median Voter Theorem breaks down). *The Median Voter Theorem always breaks down for every finite $k \geq 0$.*

Proof. Assume $\mathbf{q} = (h_v, h_v)$ is an equilibrium. Consider a deviation by i to $q_i < h_v$. Such deviation is profitable if $\hat{l}(r^*(q_A, h_v)) < 0$. The condition is $\gamma\tau_v(q_i, h_v) - \sqrt{\frac{\Delta}{k}} < 0$, where $\tau_v(q_i, h_v) = (h_v - q_i)^2$. Therefore, any $q_i > h_v - \sqrt{\frac{1}{\gamma}\sqrt{\frac{\Delta}{k}}}$ satisfies such condition. A profitable deviation is always possible for any finite γ and k, and $\Delta > 0$. This contradicts $\mathbf{q} = (h_v, h_v)$ being an equilibrium. $\square$

Chapter 3

Competition and Misreporting in the Market for News

3.1 Introduction

A central tenet of regulatory policies asserts that "competition in the news market promotes truth".[1] However, due to strategic reasons, more competition can yield more information distortions: each news provider might misreport to persuade consumers *and* to prevent competitors from persuading consumers, and so on. Therefore, arguments in favour of competition in the news market should not be based solely on the availability of more sources, but on how the strategic interaction among media outlets affects the way they report news. Competing forces can indeed facilitate the dissemination of conflicting and inaccurate stories, which might result in more confusion rather than better information. For instance, according to a poll conducted roughly one year after the 2016 Brexit referendum, about a quarter of "Leave voters" said they were misled during the Brexit campaign. Moreover, the poll shows that the result of the referendum would be overturned because a relevant number of UK voters has changed their mind.[2]

[1]The concept is also central in arguments for a free press, legal doctrine and political traditions. The idea on which the tenet rests is that a consumer with access to different and independent pieces of evidence would eventually understand the truth. Yet, the presence of more competitors does not necessarily guarantee more evidence. This is typical in the news market, where information is rebroadcast and media outlets might possess the same evidence. See Gentzkow and Shapiro, 2008.

[2]For instance, the Leave campaign repeatedly claimed that "The EU costs the UK over £350 million every week", while the Labour's Alan Johnson said that "Two thirds of British jobs in manufacturing are dependent on demand from Europe". The independent fact-checking charity FullFact found both claims to be misleading. The poll is by Opinium Research.

On the other hand, competition can be effective in preventing distorted news to have detrimental influence. This was the case during the "Killian documents controversy", where CBS aired unauthenticated documents critical of the incumbent President George W. Bush less than two months before the 2004 US Presidential Election. Shortly thereafter, conservative blogs flagged such documents as fraudulent, and accusations of forgeries were soon rebroadcast by major media outlets. CBS itself admitted its failure to authenticate the documents and broadcast an apology. Even those who viewed only CBS would have learned about the blogs' accusations.

These contexts are best described by a market for news with the following characteristics: (i) news providers compete for influence by providing alternative views of the same stories, and (ii) information spreads quickly, and eventually the consumers listen to all the viewpoints. When consumers multihome, even small media outlets like blogs can play a prominent role in the news market. This is what I call *direct competition for news*, as opposed to indirect competition where providers maximise profits and consumers do not have access to all available sources.

Nevertheless, direct competition has been highly understudied, and to date there is no formal model exploring to what extent it is beneficial or detrimental. When two media firms compete for influence, two contrasting scenarios might occur. News distortion is mitigated when one firm credibly reports truthfully, forcing its opponent to chase after. Alternatively, news distortion is enhanced and it propagates when one firm misreports to sway consumers, prompting its competitor to react by misreporting as well. It is not clear which effect is more likely to dominate and how the strategic interplay between media outlets affects consumers' beliefs. Can full information revelation be achieved? Can the escalation of news distortion jeopardize the additional value of consulting multiple sources with "diversity of opinions"? Would consumers benefit from competition at all? The fact that media bias is still a major source of concern suggests that competition in the market for news does not yield truth after all.[3]

I address these questions with a model in which a fully rational decision maker (e.g. the median voter) gathers relevant information from two experts (the media

[3]Empirical evidence suggests that, while voters are able to discount biased news, they are still influenced at the ballot box. See, e.g., DellaVigna and Kaplan, 2007, Gerber, Karlan, and Bergan, 2009, Chiang and Knight, 2011 and Martin and Yurukoglu, 2017.

outlets) before selecting an alternative (voting for a candidate). The main innovation with respect to the literature is the focus on direct competition. Accordingly, I assume that the two media outlets cover the same stories and the voter multihomes, obtaining information from both. I abstract from aggregation problems by assuming that both media outlets have the same information, and allow them to misreport it at a cost.[4] Media outlets' news is about a relative characteristic of the candidates (e.g. quality), over which the voter and the media outlets agree upon. I shall refer to this characteristic as "valence" (Stokes, 1963).[5]

The main result of this study is that competition always outperforms monopoly independently of the bias of the additional media outlet, and even if it triggers more news distortions. While full revelation cannot be achieved, the voter can gather better information on average by cross validating and rationally accounting for the credibility of news reports.

When competing media outlets are biased in the same direction with respect to the voter, they coordinate in delivering the news. In these cases, competition has limited benefits because cross validation of reports is not possible and the voter is stuck in a monopolistic-like scenario. The situation is different when competing media outlets have opposing interests. With a very strong candidate, misreports that sway the voter are too expensive, hence both media outlets report truthfully. Instead, when there is no sufficiently strong candidate, equilibrium behaviour is more nuanced: with some positive probability both media outlets report truthfully. Otherwise, they misreport with positive and independent probability by exaggerating the quality of their favourite candidate. In order to successfully persuade the median voter, the media outlet supporting the weakest candidate has to exaggerate relatively more her quality to overcome the strong endorsement of the competing media outlet.

In addition to the equilibrium reporting behaviour, my results provide several empirical implications regarding the nature of media bias. The media outlet endorsing

[4]This is a natural modelling strategy in news markets: going back to the example, several news executives connected with the CBS controversy were asked to resign, and a $70 million lawsuit was filed against the network. Misreporting cost can arise for a variety of reasons. For example, probabilistic ex-post state verifications could result in a fine, loss of reputation, circulation and profits.

[5]For instance, valence can be candidates' relative intelligence, ability, appearance, campaign performance, or fit with the current state of the world. In the CBS example, the controversial documents claimed that external pressure had been exercised to improve President George W. Bush's records after he had disobeyed orders during his military service.

the strongest candidate misrepresent news more frequently, but the one supporting the weakest candidate delivers more blatant misreports. I show that as one of the two media outlets becomes increasingly biased, both invest more and more frequently in distorting news. The same holds when the misreporting cost of one media outlet decreases. In these cases, the voter is less likely to elect the strongest candidate.

Mass media often report different accounts of the same underlying fact even though consumers can freely consult and compare news from different sources. Moreover, consumers are aware of the potential conflict of interest and that misreporting is a costly activity. Although direct competition is fierce, inefficiencies and information distortions persist in the news market: the voter cannot undo media outlets' reporting strategies to fully acquire their private information. As I show in the paper, such effect is captured in a class of mixed-strategy equilibria.

The analysis here shows how and to what extent competition in news markets impact on information distortion, transmission and revelation. My findings illustrate the working mechanism of competing forces in natural scenarios like in the the above-mentioned Brexit campaign or CBS controversy, and provide a strong case for competition between antagonistic news providers.

3.2 Related Literature

This paper contributes primarily to the literature on the political economy of media bias and its market determinants. Studies within this literature explore the biased reporting of media firms and its effects on political outcomes.[6] There are two approaches to study the market for political news, which can be broadly categorized as demand side and supply side.

Papers emphasizing the demand side of the market for news assume profit-maximising media firms that either conform to their audience's preconceptions or cater to more profitable segments of voters. In general, media outlets' preferences over candidates are second-order. Within this strand, media bias can emerge because news firms or journalists care about reputation for accurate reporting (Gentzkow and

[6] Extensive surveys on the topic are provided by, among others, Gentzkow, Shapiro, and Stone, 2014 and Gentzkow and Shapiro, 2008.

Shapiro, 2006; Shapiro, 2016) or because readers have preferences for confirmatory news (Mullainathan and Shleifer, 2005; Bernhardt, Krasa, and Polborn, 2008).

Previous works emphasizing the supply side of the market for news assume that media outlets have intrinsic preferences over outcomes and candidates. For instance, media bias can originate because of ideological motivations and career concerns of journalists (Baron, 2006). In electoral competitions, a news firm maximizes the probability of election of its favourite candidate by filtering information (Duggan and Martinelli, 2011) or through the design of a public signal (Alonso and Camara, 2016).

I also focus on the supply side of the market by assuming that media outlets have a preferred candidate. Unlike these papers, in the present model the consumer multihomes and media outlets can misreport at a cost. These features allow me to study how competing forces in the market for news affect media bias and the welfare of the voter.

Hence, this paper is closely related to the literature studying how competition among media outlets affects bias and political outcomes. More competition deters governments from capturing the media (Besley and Prat, 2006) and increases the probability that distorted news will be exposed ex-post, therefore reducing bias (Gentzkow and Shapiro, 2006). On the other hand, more competition allows confirmatory consumers to better self-segregate, hence heightening bias (Mullainathan and Shleifer, 2005). My analysis departs from these works because here media outlets compete against each other for persuading a fully rational voter. Importantly, no market segmentation occurs as the voter can access multiple sources.

The most closely related paper to my analysis is Perego and Yuksel, 2015. They study a model of competition between profit-maximizing media outlets. Increasing the level of competition brings more information on ideological issues, as there is more disagreement and more room for differentiation. However, this implies less information on the relative quality of politicians. Therefore, the equilibrium share of votes going to the socially optimal candidate decreases. My paper differs from this work in the type of competition that is explored: I focus on a market for news where media outlets *directly* compete to wittingly influence a multihoming decision maker via the strategic delivery of potentially distorted news reports.

A salient modelling strategy of my paper is that news can be fabricated at a cost.

The above papers either assume that misreporting is completely costless (Chakraborty and Ghosh, 2016; Gul and Pesendorfer, 2012; Baron, 2006; Bernhardt, Krasa, and Polborn, 2008) or prohibitively expensive (Besley and Prat, 2006; Strömberg, 2004; Duggan and Martinelli, 2011; Alonso and Camara, 2016). Introducing costs of misreporting allows me to perform interesting comparative statics. In particular, I can study what type of interventions improve the welfare of voters. As mentioned above, costly misreporting is a natural characteristic in this context and has crucial regulatory implications.

My paper touches upon the literature of strategic communication with lying costs (Kartik, 2009; Kartik, Ottaviani, and Squintani, 2007; Chen, 2011; Ottaviani and Squintani, 2006). To the best of my knowledge, this is the first paper studying multiple senders in that literature.[7]

3.3 The Model

Consider a decision maker (DM), player 0, who has to choose one alternative $a \in A = \{L, R\}$. The utility from each alternative depends on an unknown state of nature $\theta \in \Theta \subseteq \mathbb{R}$ according to some continuous cumulative function $P(\cdot)$ with density $p(\cdot)$. The DM has no further knowledge of θ. There are two experts in the set $E = \{1, 2\}$ who perfectly observe the state of nature and then simultaneously or privately deliver to the DM a report $r_e \in \mathbb{R}$ for $e \in \{1, 2\}$. I shall also refer to the DM as "the voter", to each expert as "the media outlet" and to the state of nature as "valence".

Each player has a vNM utility function over alternatives $u : A \times \Theta \to \mathbb{R}$. Let $\tau_i \in \mathbb{R}$ be an individual threshold parameter of player $i \in \{0, 1, 2\}$, and normalize $\tau_0 = 0$. Utilities over alternatives are of the form $u(a, \theta, \tau_i)$, where I indicate $u(a, \theta) \equiv u(a, \theta, \tau_0)$.

In addition, expert e bears a cost $k_e C(r, \theta)$ for delivering a report r when the state of nature is θ, where $k_e \geq 0$ is a scalar parameter. $C(r, \theta)$ is twice continuously differentiable on $\mathbb{R} \times \Theta$, with $C_{rr} > 0 > C_{r\theta}$. Truthful reporting is assumed to be costless, i.e. $C(\theta, \theta) = 0$ for all $\theta \in \Theta$, and $C(r, \theta) > 0$ for every $r \neq \theta$. Experts' overall

[7] A strand of literature studies cheap talk communication with multiple senders. Papers belonging to this body are Battaglini, 2002, Krishna and Morgan, 2001b, Krishna and Morgan, 2001a, Gilligan and Krehbiel, 1989, among others.

utility is $v(a, \theta, \tau_e, r_e) = u(a, \theta, \tau_e) - k_e C(r_e, \theta)$, where $v : A \times \Theta \times \mathbb{R} \to \mathbb{R}$. DM's utility is not *directly* affected by expert's reports, while experts' utility $v(\cdot)$ depends on their own report when $k_e > 0$. Hereafter I will assume $C(r, \theta) = (r - \theta)^2$, that is, the misreporting costs are represented by a quadratic loss function.[8]

For some states of nature players might disagree on which alternative is best, but there is a consensus that higher realizations of θ make alternative L relatively more appealing than alternative R. That is, an increase in the valence score yields a higher marginal utility $u_\theta(\cdot)$ if the implemented alternative is L rather than R. Formally, for each player, $u_\theta(R, \cdot) = 0 < u_\theta(L, \cdot)$. Therefore, valence is an element of vertical differentiation and players have common values on such dimension. I shall assume the following utility functions $u(\cdot)$ and $v(\cdot)$ for, respectively, DM and experts $e \in \{1, 2\}$:[9]

$$u(a, \theta) = \mathbb{1}_{\{a=L\}}\theta \tag{3.1}$$

$$v(a, \theta, \tau_e, r_e) = \mathbb{1}_{\{a=L\}}(\theta - \tau_e) - k_e(r_e - \theta)^2 \tag{3.2}$$

The threshold τ_i determines for every player $i \in \{0, 1, 2\}$ and at each state θ the set of ideal alternatives $A_i(\theta)$ as follows: $A_i(\theta) = \{L\}$ if $\theta > \tau_i$, $A_i(\theta) = \{R\}$ if $\theta < \tau_i$ and $A_i(\theta) = \{L, R\}$ if $\theta = \tau_i$. The ideal alternative(s) for the DM in state θ is $a^*(\theta) \in A_0(\theta)$. Similarly, $a^*(\theta, \tau_e) \in A_e(\theta)$ is the ideal alternative of expert $e \in \{1, 2\}$. Since in general $\tau_e \neq 0$, $|\tau_e|$ indicates the policy-bias of expert e relatively to the DM. The media outlet i is said to be *more biased* than media outlet j if $|\tau_i| > |\tau_j|$.

Following the definitions in Krishna and Morgan, 2001a, I divide the problem in two cases: when either $\tau_1, \tau_2 < 0$ or $\tau_1, \tau_2 > 0$ experts are said to have *like biases*; if instead $\tau_1 < 0 < \tau_2$, they are said having *opposing biases*. When $\tau_e < 0$, for all $\theta \in (\tau_e, 0)$, $a^*(\theta, \tau_e) = L \neq a^*(\theta) = R$. I refer to such expert as being *left biased*, since it prefers alternative L for more states of nature compared to the DM. Similarly, an expert e such that $\tau_e > 0$ is said to be *right biased*.

A (pure) strategy for the DM is a function $a : \mathbb{R} \times \mathbb{R} \to A$ which associates each pair of reports to an alternative $a(r_1, r_2) \in A$. A belief function for DM is a function

[8] The introduction of misreporting costs transforms cheap talk communication into a costly signalling game. I model such costs with the square loss function because of its mathematical tractability and appealing qualitative properties for this context. However, all results carry on with the strict convexity of $C(r, \theta)$, which guarantees the single-cross property to be satisfied.

[9] $\mathbb{1}_{\{\cdot\}}$ is the indicator function, $\mathbb{1}_{\{x=y\}} = 1$ if $x = y$ and 0 otherwise.

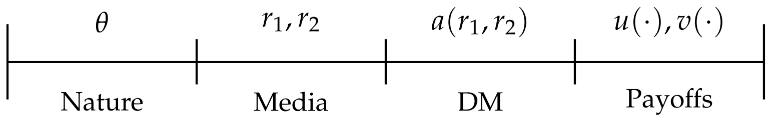


FIGURE 3.1: Timing structure.

$p : \mathbb{R} \times \mathbb{R} \to \Delta(\Theta)$ which, given any pair of reports, yields a posterior belief $p(\cdot|r_1, r_2)$. I indicate the expected valence given the reports with $\bar{\mu}(r_1, r_2) = \mathbb{E}_p[\theta|r_1, r_2]$. A (pure) strategy for expert e is a function $r_e : \Theta \to \mathbb{R}$ such that, for each realisation of the state of nature, associates a report $r_e \in \mathbb{R}$.

Figure 3.1 illustrates the timing of the game: (i) nature selects θ according to $F(\theta)$, which is privately observed by both experts only; (ii) experts report simultaneously a report – a literal statement about the state θ – to the DM; (iii) the DM chooses and implements one alternative $a \in A$ and payoffs are realised.

An equilibrium is a weak Perfect Bayesian Equilibrium (wPBE). I shall restrict focus on monotone reporting strategies, namely, if $a^*(\theta, \tau_e) = L$ then $r_e(\theta) \geq \theta$ and vice-versa. I will hereafter refer to wPBE that satisfy monotone reporting just as "equilibria".

Apart from the state of nature θ, which is private information of the experts and unknown to the DM, every other aspect of the model, including utility functions and biases, is common knowledge.

3.3.1 A Monopolistic Market for News

Consider the case where in the economy there is only one strategic media outlet with private information about valence. The media outlet is left-biased, $\tau_1 < \tau_0$, and can misreport at a cost $k_1 C(r, \theta)$. Apart from considering only one news firm, the model is as described above.

The media outlet is a monopolistic owner of information about valence and delivers a report $r(\theta)$ to the DM (or, equivalently, the electorate). In equilibrium, upon receiving a report r, the DM makes her assessment about valence and selects one alternative accordingly.

Given that misreporting is costly, in any equilibrium the reported valence r cannot be too far from the true realized valence θ. Otherwise, the cost suffered by the media outlet from misreporting would certainly outweigh the gain from persuading the voter. Therefore, the voter expects the realized valence to lie within a certain range from the report. I define the expected valence given a single report r as $\bar{\mu}^m(r) = \mathbb{E}_p[\theta|r]$ and the "lowest" (highest) misreporting type as in the following Definition 5.

Definition 5 (Lowest and Highest misreporting types). *The following are, respectively, the lowest and the highest misreporting type.*

$$l(r_e, \tau_e) = r_e + \frac{1}{2k_e}\left[1 - \sqrt{1 + 4k_e(r_e - \tau_e)}\right] \text{ if } r_e > \tau_e \tag{3.3}$$

$$h(r_e, \tau_e) = r_e - \frac{1}{2k_e}\left[1 - \sqrt{1 + 4k_e(\tau_e - r_e)}\right] \text{ if } r_e < \tau_e \tag{3.4}$$

A natural question is whether the media outlet, given DM's full awareness of its bias and costs, is ever going to misreport information. Definition 6 illustrates what is meant for the media outlet to "report truthfully" and to play a "truthful strategy".

Definition 6 (Truthful reporting and strategy). *Media outlet e is said to report truthfully in state θ when its message matches its private information, $r_e(\theta) = \theta$. It is said to play a truthful strategy if it reports truthfully every state, $r_e(\theta) = \theta \ \forall \theta \in \Theta$.*

If in equilibrium the DM can neutralize any attempts of persuasion, then the expert would economize by unravelling its private information. At this point we still do not know if an equilibrium exists at all, but the following proposition shows that if it exists, then it must entail misreporting behaviour.

Lemma 18 (Equilibrium misreporting). *With a monopolistic media outlet, any wPBE exhibits some misreporting behaviour.*

Proof. Given a report r', the lowest type $\theta \leq r'$ of media outlet e that could have profitably reported $r(\theta) = r'$ is $l_e(r') \equiv l(r', \tau_e)$, where $l(r_e, \tau_e)$ is the lowest misreporting type as described in Definition 5. Suppose $(r(\theta) = \theta, \bar{\mu}^m(r) = r)$ for all $r \in \mathbb{R}$, $\theta \in \Theta$ is a wPBE. All types $\theta \in (l_1(\tau_0), \tau_0)$ have a strict incentive to deviate from the truthful reporting rule by delivering $r = \tau_0$. □

This game admits infinitely many wPBE, mainly sustained by nonsensical off path beliefs. In order to prune out pathological equilibria, I focus on those that survive Cho and Kreps, 1987's Intuitive Criterion refinement. It turns out that there exists only one unique "hybrid" equilibrium reporting rule that passes the Intuitive Criterion test, and it always exists. In such equilibrium extreme types separate by reporting truthfully, while central types pool by inflating their private information. Types in a convex set pool to the same counterfeited report $r = \theta^*$ in a way that just sways the decision of the median voter ($\bar{\mu}(\theta^*) = \tau_0$).[10]

Proposition 12 (Robust Monopolistic wPBE)**.** *The equilibrium is a pair of reporting rule – system of beliefs $(r(\theta), p(r))$ such that,*[11]

$$r(\theta) = \begin{cases} \theta^* & \textit{if } \theta \in (l_1(\theta^*), \theta^*) \\ \theta & \textit{otherwise} \end{cases} \tag{3.5}$$

$$\bar{\mu}^m(r) = \begin{cases} \mathbb{E}_p\left[\theta | l_1(\theta^*) \leq \theta \leq \theta^*\right] = \tau_0 & \textit{if } r = \theta^* \\ \mathbb{E}_p\left[\theta | l_1(\theta') \leq \theta \leq \frac{\theta' + \theta^*}{2}\right] < \tau_0 & \textit{if } r = \theta' \in (l_1(\theta^*), \theta^*) \\ \theta & \textit{otherwise} \end{cases} \tag{3.6}$$

where θ^* is the unique $r \in \mathbb{R}$ such that $\mathbb{E}_p\left[\theta | l_1(\theta^*) \leq \theta \leq \theta^*\right] = \tau_0$ and voters' decision is sequentially rational. Such equilibrium always exists, is unique and robust to NWBR.

Proof. The proof is similar to the one for Proposition 1 in Chapter 1.

In the unique robust equilibrium in Proposition 12, the expert disagrees with the voter when $\theta \in (\tau_1, \tau_0)$. For greater conflicts of interest, viz. when $\theta \in (\tau_1, l_1(\theta^*)]$, it is too expensive to misreport and therefore truthful reporting occurs. Instead, when $\theta \in (l_1(\theta^*), \tau_0)$ the expert delivers $r(\theta) = \theta^*$, which is effective in that it induces the implementation of its favourite alternative. Under this latter contingency, the phenomenon which I refer to as "election overthrowing" takes place: the expert's preferred alternative is implemented even though, under complete information, the DM (or a majority of voters) would select the other one.

[10] It is widely known that for more than two types the Intuitive Criterion might fail to prune unreasonable pooling equilibria. A natural question is whether the equilibrium in Proposition 12 survives stronger refinements. In Chapter 1, I show that the equilibrium is robust to Cho and Kreps, 1987's D1 and D2, Banks and Sobel, 1987's Divinity and Universal Divinity, and Kohlberg and Mertens, 1986's Never a Weak Best Response (NWBR).

[11] In this specification, out of equilibrium beliefs are drawn from the idea that voters avail themselves of a refinement-like procedure upon observing an unexpected report. However, this characterization would work for any $\mu'(\theta|\theta') < \tau_0$ for $\theta' \in (l_1(\theta^*), \theta^*)$.

When $\theta \in [\tau_0, \theta^*)$, the expert engages in costly misreporting even though it agrees with the voter. The intuition is that once the voter is aware the expert can fabricate information, she becomes skeptical. In these cases, if the expert were to reveal truthfully its private information, its report would be discounted to a point where its less desirable alternative is implemented. Because of the innocuous nature of these falsehoods in terms of the implemented alternative, I refer to this phenomenon as "white lies". Overall, information is jammed for $\theta \in (l_1(\theta^*), \theta^*)$. In contrast, for all other valence realisations, unravelling is discriminating enough that counterfeiting is not necessary. Figure 1.6 illustrates the above behaviours.

3.4 Competition in the Market for News

3.4.1 Robustness

In the following sections I depart from monopoly to explore how strategic communication takes place between two informed mass media organizations and a voter (or voters). The focus on a duopolistic media market is a first step and an approximation for studying how competition impacts on information transmission and supply side media bias.

The following argument seems to undermine the interest on studying strategic communication under competing senders: in a market with three (or more) media outlets, no matter their conflict of interest with voters, full information revelation at every state of nature is always an equilibrium. To see this, consider an equilibrium where, given the state of nature $\theta \in \Theta$, all media outlets deliver the same report $r(\theta) = \theta$ and voters take the message as truthful as it is. Suppose a media outlet wants to deviate and deliver $r' \neq r$. Such deviation can be immediately detected and neutralized by voters as it has always to be the case that the deviating sender is the only one which report does not match with those of the other senders. This very simple fully revealing equilibrium naturally holds true for both cheap talk and costly communication models.

The above argument requires that voters can consult privately or simultaneously at least three different news organizations, all perfectly informed about the state of nature. If obtaining information comes at no or little cost, as it is largely common nowadays, voters could gain perfect information and absence of media bias almost for free. Nevertheless, all such fully revealing equilibria are not robust to the idea that each media outlet might have an arbitrarily small probability to make mistakes or not being perfectly informed. If this is the case, full revelation is never an equilibrium, and the question of how communication takes place and how much information can be aggregated by the decision maker is still open when there are multiple senders with misreporting costs.

It is therefore necessary to focus on equilibria that are robust to the following refinement firstly proposed in Battaglini, 2002: each media outlet i independently observes the true state of nature with probability $1 - \varepsilon_i$ and a random state with probability ε_i. The latter has a continuous distribution $G_i(\cdot)$ with density $g_i(\cdot)$ and has the same support as valence, Θ. An equilibrium is robust to the refinement if there exists a pair of distribution (G_1, G_2) and a sequence $\varepsilon^n = (\varepsilon_1^n, \varepsilon_2^n)$ converging to zero such that the prescribed out of equilibrium beliefs of the original equilibrium are the limit as $\varepsilon^n \to 0^+$ of the ε-perturbed game.[12]

In particular, even in a ε-perturbed game where both media outlets play truthful strategies there is a positive probability that the voter would observe different reports. This is because, by construction, the random state has the same support as valence. Hence, every tuple of reports has a positive probability to be observed, and there is no real need to account for out of equilibrium beliefs. Further, with ε-perturbation it is not possible to achieve full revelation as both media outlets might observe a random state rather than the realized valence score. Upon receiving messages that are inconsistent, the decision maker understands that at least one media outlet has committed a mistake or has no private information. This refinement imposes consistency in the construction of posterior beliefs of the original game.

I shall show the importance of considering robust equilibria only. In the like bias case there exists equilibria that are fully revealing, but they are not robust. All robust equilibria display misreporting behaviour of both media outlets and voters' persuasion. In contrast, in the opposing bias case, full revelation is not attainable even without imposing robustness.

[12]Trembling hand like refinement cannot be used as there is a continuous set of type. Note that the ε-refinement is weak in the sense that it allows for any continuous distribution for the wrong observations.

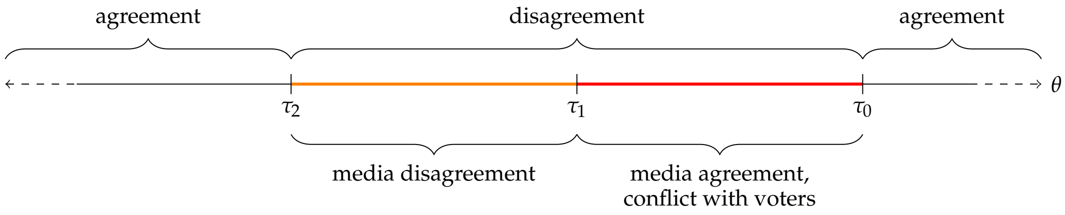


FIGURE 3.2: Like biased media outlets.

3.4.2 Like Biased Competition

I argued above that with a single monopolistic media outlet, the unique equilibrium displays misreporting behaviour and voter persuasion. A natural question is whether the increase in discipline from adding a second media outlet coupled with costly misreporting could yield equilibria where the voter has enough information to always make optimal decisions. In this section I study the case where both media outlets are ex-ante more likely to endorse one specific alternative with respect to the median voter. This is the case where, given the available alternatives, the political predispositions τ_e of the two media outlets are both either to the left or to the right of the median voter's predisposition τ_0.

For extreme realizations of valence, namely high or low enough scores, all players agree on which alternative is better. For intermediate values instead, disagreement can take two forms: either the two media outlets are in agreement between each other but disagree with the median voter or the two media outlets are in disagreement and one of the two agrees with the median voter. In Figure 3.2 the former type of disagreement arises when $\theta \in (\tau_2, \tau_1)$ and the latter when $\theta \in (\tau_1, \tau_0)$. When $\theta \in (\tau_2, \tau_0)$, there is always disagreement between two players. Since there are only two alternatives, it cannot be the case that all players disagree with each other.

In a Fully Revealing Equilibrium (FRE), given the media outlets' reports, the voters always understand what is the state of the world, and therefore the best alternative to vote for. However, this canonical notion of full revelation is too strong for the current setting: in order to take an optimal decision, the voter needs to know only whether the relative quality is higher or lower than her partisan threshold τ_0. In order to accommodate for such a weaker requirement, I introduce in Definition 7 the notion of Voter Efficient Equilibrium (VEE).

Definition 7 (Voter Efficient Equilibrium)**.** *A Voter Efficient Equilibrium (VEE) is an equilibrium where, at every* $\theta \in \Theta$, $\bar{\mu}(r_1(\theta), r_2(\theta)) \geq \tau_0$ *when* $\theta \geq \tau_0$ *and* $\bar{\mu}(r_1(\theta), r_2(\theta)) < \tau_0$ *when* $\theta < \tau_0$.

A VEE does not require that voters understand precisely what is the state of nature, therefore we might have a VEE without complete information on the voters' side. However, since misreporting information is costly, any VEE has to be in truthful strategies; that is, it should not involve information fabrication. Otherwise, one of the two media outlets would be fruitlessly paying a cost to misreport, and would be better off by reporting truthfully. Therefore, every VEE must also be a FRE in truthful strategies. [13]

In a similar scenario Krishna and Morgan, 2001a (p. 756) construct a FRE with two like biased experts, simultaneous reporting, and cheap talk communication ($k = 0$).[14] To see how full revelation can be obtained in the present setting, consider two like biased media outlets with $\tau_2 \leq \tau_1 < \tau_0$. Both media outlets are ex-ante more likely to support the left alternative than the median voter, so they are both "left biased". A FRE exists where both media outlets report truthfully their private information at every state of nature, $r_e(\theta) = \theta$, $e \in \{1, 2\}$, for every $\theta \in \Theta$. Voters' posterior beliefs are such that the expected valence realization is below the median voter's threshold if the lowest of the two report is below as well, $\bar{\mu}(r_1, r_2) < \tau_0$ if $\min\{r_1, r_2\} < \tau_0$.

There are no profitable individual deviations from the above equilibrium. Even when valence takes values where both media outlets disagree with the median voter, $\theta \in (\tau_1, \tau_0)$, language inflation would be costly but without any effect on voters' beliefs. This equilibrium features a complete lack of coordination in misreporting, as there are contingencies where both media outlets would like to persuade the voter to take the same action but are not able to because the other media outlet is known to reveal its type.

However, this fully revealing equilibrium is not robust to the ε-refinement proposed in Battaglini, 2002, as out of equilibrium beliefs are not smooth enough. Indeed, it requires that, whatever is the profile of reports, voters' action depends only on

[13]This argument holds for both the like bias and opposing bias cases, and it is formally illustrated in the first part of the proof of Lemma 20 in Appendix 3.6.2.

[14]More precisely, Krishna and Morgan, 2001a show that full revelation is a PBE if messages are delivered simultaneously, but not if they are delivered sequentially, as it would require non-optimizing behaviour off equilibrium path.

the lowest report. If we allow for the possibility that media outlets have imperfect information or commit mistakes with some small probability, beliefs have to be smoother. This has to hold also at the limit as these probabilities shrink to zero, which is not the case in the above FRE. The idea is that the voter, upon observing two contrasting reports, has to believe that at least one of the two, if not both, is simply wrong. Therefore, the expected valence has to be somewhere in between of the two reports.

It is however possible that while the above FRE is not ε-robust, there exist other fully revealing equilibria that survive the refinement. As Lemma 19 shows, this is not the case.

Lemma 19. *There is no ε-robust Fully Revealing Equilibrium with like bias competition.*

Proof. See the proof in Appendix 3.6.1.

If "Voter Efficient" and "Fully Revealing" robust equilibria do not exist, the question of how communication takes place is still open. The imposed smoothness consistency over out of equilibrium beliefs breaks down any possibility of full revelation and efficiency. Since the expected valence has to be somewhere in between the two reports, there always exists a state of nature $\theta < \tau_0$ such that one of the two media outlets can unilaterally and profitably misreport by inflating the valence score. Such deviation would convince voters to expect the relative quality θ to be above the median voter's threshold τ_0.

In order to understand equilibrium communication with two like biased media outlets, it is useful to compare their persuasive power. I say that media outlet i is *more persuasive* than media outlet j if in the monopolistic case it is ex-ante more likely to misreport, and therefore to induce an election overthrowing.[15] However, in the current setting it does not have to be the case that a *more biased* media outlet also misreports and persuades more frequently. Indeed, the extent to which a media outlet fabricates information and induces election overthrowing in the unique monopolistic equilibrium is a function of both its relative bias and its misreporting cost k_e. A less

[15]See the first Chapter for an extensive analysis of the monopolistic case. Note that misreporting does not necessarily lead to an election overthrowing. However, the more one media outlet is (ex-ante) likely to engage in information fabrication, the more is (ex-ante) likely to induce the implementation of the alternative that is less preferred by a majority of voters.

biased media outlet has less incentives to misreport, because its interests are more aligned with the median voter. Yet, if it faces a very low cost for misreporting, it can be the case that it persuades voters more frequently than a relatively more biased media outlet. Therefore, a less biased media outlet can be more persuasive at the same time. Definition 8 formally describes what is meant for a media outlet to be more persuasive.[16]

Definition 8 (More persuasive media outlet)**.** *Suppose two media outlets are both left biased. Media outlet i is more persuasive than media outlet j if, in the monopolistic equilibrium reporting rule, $M_i \supseteq M_j$, where $M_e = (l_e(\theta_e^*), \theta_e^*)$ is the set of states of nature for which misreporting occurs. The definition is similar for two right biased media outlets.*

A robust equilibrium exists, where both media outlets coordinate in reporting the same message at every state of nature, $r_1(\theta) = r_2(\theta)$ for all $\theta \in \Theta$. In such equilibrium, the reporting rule is exactly the same as in the monopolistic case of the *less persuasive* media outlet, except for the fact that now the voter receives two messages at the same time. Because the less persuasive outlet drives the way communication takes place, introducing a second expert can only mitigate voters' persuasion. However, like biased competition has no effect at all if, according to Definition 8, the second outlet is more persuasive than the first one.

In this scenario, voters never observe contrasting reports. As it happens in the monopolistic equilibrium, information is pooled around the median voter's threshold τ_0. Upon observing the reports $r_1 = r_2 = \theta_j^*$, voters' assessment is such that the median voter is just indifferent between the two alternatives. For more extreme states, even though there is some disagreement between the players, truthful reporting occurs. Proposition 13 shows the characterization of the robust wPBE with like bias.

Proposition 13 (Like Biased Misreporting Equilibrium)**.** *With two like biased media outlets there exists an ε-robust equilibrium where, for $e = \{1, 2\}$, $r_e(\theta) = \theta_j^*$ for all $\theta \in M_j$ and $r_e(\theta) = \theta$ otherwise, where $j := \{e \in E : M_e \subseteq M_{-e}\}$, $\theta_j^* := \{\theta \in \Theta : \mathbb{E}_p[\theta | \theta \in M_j] = \tau_0\}$. The median voter selects $a(r_1, r_2) = L$ if $\bar{\mu}(r_1, r_2) \geq \tau_0$ and $a(r_1, r_2) = R$ otherwise. Posterior beliefs $p(\theta | r_1, r_2)$ are according to Bayes' rule whenever possible and*

[16] Note that Definition 8 works only for two (or more) like biased media outlets.

such that $\bar{\mu}(r_1, r_2) < \tau_0$ *if* $\min\{r_1, r_2\} < \tau_0$*. This is the unique ε-robust equilibrium with like bias.*

Proof. See proof in Appendix 3.6.1.

The above equilibrium, which requires perfect coordination in reporting between the media outlets, is ε-robust, while the Fully Revealing Equilibrium, which entails coordination failure, is not. For any finite cost of misreporting $k_j \geq 0$, the media outlets manage to persuade the voter with ex-ante probability $p(\theta \in [l_j(\theta_j^*), \tau_0]) > 0$.

One might wonder if there are robust equilibria entailing less misreporting activity, say with $M' \subset M_j$. If this is the case, it might be possible to construct an M' arbitrarily small and therefore attain robust *almost full revelation*. However, this is not possible as the smoothness of beliefs induced by the refinement implies it is always the case that there exists some state of nature at which one media outlet can profitably deviate by inflating information about the relative quality of candidates.

Although Krishna and Morgan, 2001a study a model of cheap talk communication where experts deliver messages sequentially, it is interesting to make a comparison with their like bias case. They find that the decision maker can do no better than to consult only the *more loyal* expert, which, in their setting, is both the less biased *and* the less persuasive. Hence, there is no (monotonic) PBE with two experts that is informationally superior to the most informative PBE with a single expert.[17]

The same conclusion applies here even though reports are delivered simultaneously. However, in Krishna and Morgan, 2001a adding a second like biased expert leads generally to a loss of information transmission, and it is *at best* redundant. In contrast, Proposition 13 shows that the amount of information gathered by the voter in the like biased equilibrium is never lower than when consulting only the less persuasive media outlet.

In the present paper, it is not the sequentiality of messages or the cheap nature of communication that drives the redundancy of the more persuasive (or, less loyal) expert as the same phenomenon occurs with simultaneous messages and costly misreporting.

[17]Krishna and Morgan, 2001a, p. 759, Proposition 2.

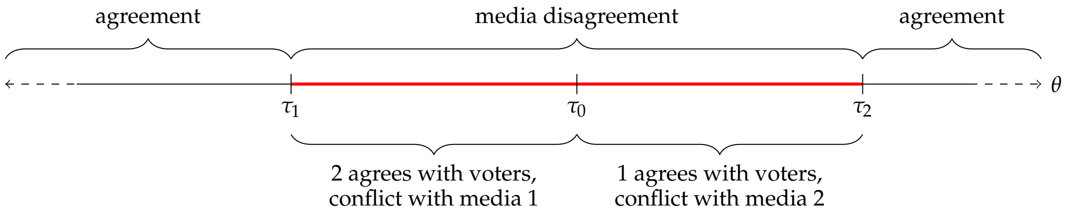


FIGURE 3.3: Opposing biased media outlets.

Rather, when messages are simultaneous, the best media outlets can do is to coordinate in misreporting as the less persuasive would do if it were to be alone. Misreporting more often would not be convenient for the less biased outlet, and misreporting less often would make room for profitable deviation. Pure coordination in reporting deprives the voter from the possibility of cross validating the messages. Hence, the introduction of any additional outlet that does not qualify as being the less persuasive is irrelevant in determining the equilibrium reporting strategy and voter welfare.

3.4.3 Opposing Biased Competition

In this section I explore the scenario with two opposing biased media outlets. Consider without loss of generality the case where $\tau_1 < 0 = \tau_0 < \tau_2$. Media outlet 1 is ex-ante more likely to support alternative L than both the median voter and media outlet 2. Similarly, the latter is ex-ante more likely to support alternative R than the median voter and media outlet 1.

As before, for extreme realizations of valence all players agree on which alternative is best. In contrast with the like bias case, for intermediate realizations, there is always a conflict of interest between the two media outlets. When valence takes values around the median voter's threshold, media outlet 1 is supportive of alternative L while media outlet 2 supports alternative R. In the lower range of states where there is disagreement, (τ_1, τ_0), media outlet 2 agrees with a majority of voters that valence is not high enough for alternative L to be better than R. Instead, in the top range (τ_0, τ_2), media outlet 1 agrees with a majority that the best alternative is L, but disagrees on this with media outlet 2.

Differently from the like bias case, there are no contingencies where media outlets agree with each other but disagree with the median voter. This situation rules out the possibility that they want to coordinate in misreporting to persuade voters and influence the political outcome as they do in the equilibrium in Proposition 13. On the contrary, competition is stiffer and a natural question is whether voters can take advantage of this situation to gather more information with respect to the like bias case.

A first step in this direction is to check whether increased competition and conflict between the media outlets can potentially yield full information revelation. I showed before that in the like biased case there exists a FRE, but it is not ε-robust. Surprisingly, in the opposing bias case fully revealing equilibria, even non-ε-robust, do not exist at all.

Lemma 20. *There are no Voter Efficient and Fully Revealing Equilibria with opposing bias competition.*

Proof. See the proof in Appendix 3.6.2.

As described in Definition 7, in any Voter Efficient Equilibrium (VEE), after observing both reports, the DM optimally takes an action *as if* she knows perfectly the state of nature. This weaker condition of informational efficiency requires the DM to behave as if she knows whether $\theta \geq \tau_0$ or $\theta < \tau_0$, but not to perfectly know its realization as it would happen in a FRE. In particular, neither the VEE nor the FRE require the media outlets to report truthfully their private information. On the contrary, in a "truthful" FRE, both outlets always reveal their private information, $r_e(\theta) = \theta, \forall \theta \in \Theta$. The first part of the proof of Lemma 20 shows that, if there exists a VEE, then there must exists a FRE with truthful reporting (as in Definition 6). In the second part of the proof, I show that no truthful FRE exists, and therefore no FRE or VEE exist at all. In order to reach this conclusion, I do not need to make use of the ε-robustness criterion.

Another salient feature of the like biased misreporting equilibrium in Proposition 13 is that it is in pure strategies. In contrast, in the opposing biased case, the antagonistic nature of the conflict of interest between media outlets undermines the possibility of having equilibria with pure strategies. This is formally showed in Lemma 21.

Lemma 21. *There is no pure strategy equilibrium with opposing bias competition.*

Proof. See the proof in Appendix 3.6.2.

In order to see the intuition behind Lemma 21, suppose instead that there exists a pure strategy equilibrium. Because of Lemma 20, we know that an equilibrium in pure strategies cannot be voter efficient or fully revealing. Therefore, in some state of nature the DM takes the wrong action after observing the pair of reports. For this to happen there must be some pooling, that is, there must be some states of nature for which the same reports are delivered, but where instead the DM chooses the correct alternative. However, this would imply that one media outlet is investing in costly misreporting even if it knows that its worst alternative will be implemented, contradicting that to be an equilibrium.

Therefore, I need to introduce further notation to allow for mixed strategies. In order to accommodate for the presence of mass points or atoms, I shall employ a framework that allows me to incorporate probability distributions over reports that are partly discrete and partly continuous. This is best done by describing the reporting strategies through probability density functions rather than cumulative distribution functions. In particular, mixed type distributions, or mixed probability measures, allow for a partial probability density over a discrete as well as a continuous component of the distribution.[18]

A strategy for a media outlet e in state θ is a mixed probability measure $f_e^\theta : \Theta \to \Delta(\mathbb{R})$ over the report space. Hence, $f_e^\theta(r) \geq 0$ is the density of sending the report r when the state of nature is θ, and $\int f_e^\theta(r)dr = 1$. The support of f_e^θ for media outlet e in state θ is $\text{supp}\{f_e^\theta\} = \mathcal{S}_e^\theta$. By report monotonicity, if $a^*(\theta, \tau_e) = L$ then $\mathcal{S}_e^\theta \subseteq [\theta, +\infty)$, and $\mathcal{S}_e^\theta \subseteq (-\infty, \theta]$ otherwise.

In order to take into account for potential atoms, I partition the support $\mathcal{S}_e^\theta$ into two subsets: $\mathcal{C}_e^\theta$ is for the continuous part of the distribution and $\mathcal{D}_e^\theta$ is for the discrete part, which contains the atoms. I define a partial probability density function $\alpha_e^\theta(\cdot)$ on $\mathcal{D}_e^\theta$ for the discrete part of the distribution, such that $0 \leq \alpha_e^\theta(r_e) \leq 1$ for $r_e \in \mathcal{D}_e^\theta$ and $\sum_{r\in\mathcal{D}_e^\theta} \alpha_e^\theta(r) =: \bar{\alpha}_e^\theta$. The continuous part of the distribution for media outlet e in state θ is also described by a partial probability density function $\psi_e^\theta(\cdot)$ such that

[18] The existence of a probability density function for the continuous part of a mixed distribution is not guaranteed. However, in equilibrium we do not have this kind of problem as the density is well defined.

$\int_{r \in C_e^\theta} \psi_e^\theta(r)dr = 1 - \bar{\alpha}_e^\theta$. I set $\alpha_e^\theta(\theta') = 0$ for $\theta' \notin \mathcal{D}_e^\theta$ and $\psi_e^\theta(\theta'') = 0$ for $\theta'' \notin C_e^\theta$. Since in equilibrium $\mathcal{D}_e^\theta = \{\theta\}$, I shall simply write $\alpha_e^\theta = \alpha_e^\theta(\theta) = \bar{\alpha}_e^\theta$. The "generalized" probability density function of reports by each media outlet e in every state θ is determined by the partial probabilities α_e^θ and $\psi_e^\theta(\cdot)$ through the well defined mixed distribution $f_e^\theta(x) = \delta(x-\theta)\alpha_e^\theta + \psi_e^\theta(x)$, where $\delta(\cdot)$ is the Dirac delta "function".[19]

After observing the pair of reports $(r_1, r_2) \in \mathbb{R}^2$ the voter updates her belief to $p(\theta|r_1, r_2)$, which is a probability function over states such that $\int_\Theta p(\theta|r_1, r_2)d\theta = 1$. In equilibrium, beliefs are updated according to Bayes' rule whenever possible. Given posterior beliefs $p(\theta|r_1, r_2)$ the voter selects an alternative based on its expectation about valence, $\bar{\mu}(r_1, r_2)$. Given that voters' optimal decision depends solely on their assessment about the expected valence, I shall often refer to voters' posterior belief directly to as their expectation about the valence score.

Intuitively, reports indicating a higher (lower) valence score should induce the voter to believe that the true realized valence is higher (lower). There are two main forces at work here: on the one hand, media outlets are willing to misreport in order to pull the voter's posterior $\bar{\mu}(\cdot)$ to their advantage; on the other hand, they do not want to pull too much more than what is necessary because misreporting is costly. If profitable, each outlet would like to just offset its opponent's report so as to sway the voter's decision while economizing on misreporting costs. The following Definition 9 introduces the "swing report". Given a fixed report r and posterior beliefs $\bar{\mu}(\cdot)$, there might exists (even though not necessarily) a second report that makes the voter indifferent between the two alternatives.

Definition 9 (Swing report)**.** *Given an equilibrium, voter's belief $\bar{\mu}(\cdot)$ and a report r, the swing report is defined as a report that makes the voter indifferent between the two alternatives,*

$$s(r) := \left\{ r' \in \mathbb{R} : \overline{\mu}\left(r', r\right) = \tau_0 \equiv 0 \right\} \tag{3.7}$$

Assume that, given the profile (r_1, r_2), the voter is indeed indifferent between the two alternatives L and R. According to the above definition, it means r_1 is the swing report of r_2, and also that r_2 is the swing of r_1. If one of the two outlets would

[19]The dirac delta $\delta(x)$ is a generalized function such that $\delta(x) = 0$ for all $x \neq 0$, $\delta(0) = \infty$ and $\int_{-\epsilon}^{\epsilon} \delta(x)dx = 1$ for any $\epsilon > 0$.

deliver a report that is higher than the swing report, then we should expect the voter to (weakly) prefer alternative L to R. Indeed, her assessment of valence cannot be lower than before, when she was indifferent.

If beliefs are smooth in reports, as the concept of ε-robustness implies, beliefs should not be discontinuous when varying reports. Therefore, if given a certain report r_i by media outlet i and if voters do not always take the same action, then there should exists a swing report. While this does not have to be the case, I shall put structure on voters' beliefs and restrict attention to those that satisfy the following assumptions.

Assumption 1. *The swing report $s(r) : \mathbb{R} \to \mathbb{R}$, exists for every $r \in \mathbb{R}$ and is monotonically decreasing, $\frac{ds(r)}{dr} < 0$.*

Assumption 1 introduces a reasonable structure on voters' beliefs: (i) in order to keep the voter indifferent, reports signalling that valence is higher must be balanced by reports saying that valence is lower, and (ii) for any report delivered by one outlet, there always exists a second report that makes the voter indifferent.

Before proceeding with the equilibrium analysis, I introduce some further useful definitions. For any realization of valence θ, each media outlet can misreport information at a cost that is a function of the state of nature and the delivered message. When there is a conflict of interest, media outlet 1 might want to "inflate" the realized valence score in order to endorse its favourite alternative L, while for the same reason media outlet 2 might want to "deflate" information about valence.

However, given that misreporting is a costly activity, there is a natural limit to the gains from information fabrication. Such limit depends both on how expensive is to misreport and, importantly, on the intensity of media outlets' endorsements. For instance, the higher the valence, the more media outlet 1 prefers alternative L to R. Therefore, it is more willing to invest in information fabrication in order to induce the implementation of L. Similarly, media outlet 2 would be less willing to misreport in favour of R as with an increase of the valence score L gives a relatively higher pay-off.

Given a state of nature θ, the set of reports that are potentially feasible is therefore finite. Some reports cannot be delivered in any situation because, even if in equilibrium they would yield the desired outcome with certainty, it would be too costly to deliver them. Definition 10 shows what is the most expensive report that can be delivered by each media outlet. Suppose truth-telling would yield the worst outcome with certainty while any other report would yield the best outcome with certainty. The reach is the report that makes a media outlet indifferent between reporting truthfully and reporting the reach. No report beyond the reach can be feasible in any equilibrium.

Definition 10 (Reach)**.** *The reach $\bar{r}(\theta, \tau_e) \equiv \bar{r}_e^\theta$ is the most expensive monotone and potentially incentive compatible report that media outlet e can deliver in state θ,*

$$\bar{r}_e^\theta = \begin{cases} \theta + \sqrt{\frac{\theta - \tau_e}{k_e}} & \text{if } \theta \geq \tau_e \\ \theta - \sqrt{\frac{\tau_e - \theta}{k_e}} & \text{if } \theta \leq \tau_e \end{cases} \tag{3.8}$$

If there is a limit to the profitability of misreporting, there might be cases where valence is actually revealed to the voter. To see this, consider a state of nature in the top range of the area of conflict, $\theta \in (0, \tau_2)$, such that the reach of media outlet 2 (endorsing candidate R) is not enough to make the median voter indifferent given that media outlet 1 is reporting truthfully. More formally, in order to swing $r_1 = \theta$, media 2 should deliver a report that is beyond its reach, $s(\theta) < \bar{r}_2^\theta$. There might be situations where outlet 2 would like to deflate its private information, but in order to misreport effectively (that is, to induce the implementation of its favourite alternative, R) it has to pay an excessively high cost even if its opponent were to report truthfully. In such cases, media outlet 1 understands that it is not possible for outlet 2 to persuade voters without incurring in a loss, and therefore would just report truthfully. In turn, media outlet 2 would economize in misreporting costs by reporting truthfully as well. Definition 11 illustrates cut-offs in the state space beyond which, in equilibrium, truthful reporting must occur.

Definition 11 (Truthful Cut-offs)**.** *The "truthful cut-offs" (θ_1, θ_2) are states of nature such that, for any $\theta \notin (\theta_1, \theta_2)$, only truthful reporting can be an equilibrium strategy,*

$$\theta_2 := \{\theta \in (0, \tau_2) : \bar{r}_2^\theta = s(\theta)\} \tag{3.9}$$

$$\theta_1 := \{\theta \in (\tau_1, 0) : \bar{r}_1^\theta = s(\theta)\} \tag{3.10}$$

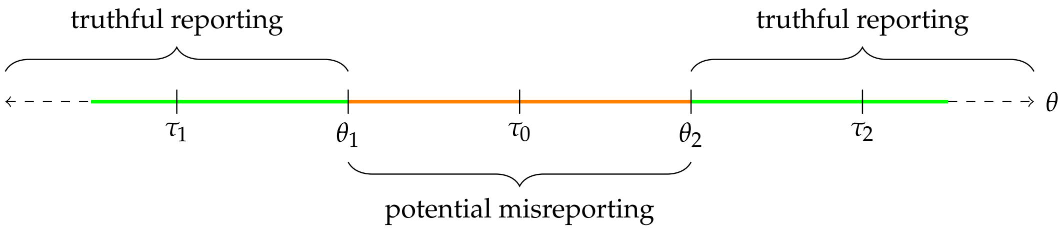


FIGURE 3.4: Cut-offs for truthful reporting in equilibrium. For realizations of quality in the green area, both outlets report truthfully. In contrast, for realizations $\theta \in (\theta_1, \theta_2)$, each outlet misreports with positive probability.

However, it is possible that the truthful cut-offs are outside the area of conflict of interest. This is the case when $\theta_1 < \tau_1$ or $\theta_2 > \tau_2$. In such cases, truthful reporting has to occur in equilibrium when there is agreement, that is for all $\theta \notin (\tau_1, \tau_2)$.

The achievement of full revelation under agreement is in contrast with the like bias case, where there are states of nature at which both media outlets misreport even if all players would agree on which alternative is best. On the contrary, in the opposing bias case, when both media outlets deliver the same report it has to be that they are revealing their private information.

3.4.4 Equilibrium

In equilibirum, posterior beliefs $p(\theta|r_1, r_2)$ determine the voter's expectation $\bar{\mu}(r_1, r_2)$, which in turn determines the swing report function $s(r)$. This has to hold also off-path. In particular, Assumption 1 has straightforward implications in equilibrium. Because of report monotonicity, the conditional expectation about valence has to be between the two reports, $\bar{\mu}(r_1, r_2) \in [r_2, r_1]$. Hence, given a positive (negative) report r, it must be that the swing $s(r)$ is negative (positive) in order to make the voter indifferent, $\bar{\mu}(r, s(r)) = 0$. Formally, $s(r) < 0$ for $r > 0$ and $s(r) > 0$ for $r < 0$. For the same reason, it must be that the swing of zero is exactly zero, $s(0) = 0$. Further, the report that swings "the report that swings the report r", is the report r itself. This is better understood formally by $s(s(r)) = r$. This equality naturally holds because, given equilibrium beliefs, $\bar{\mu}(s(s(r)), s(r)) = 0$ only if $s(s(r)) = r$, as $\bar{\mu}(r, s(r)) = 0$ is true by definition. If a report is the swing of another one, then it has to hold the opposite.

Furthermore, in any equilibrium the voter cannot believe that the realized valence is negative (positive) upon observing a pair of reports that are with certainty within

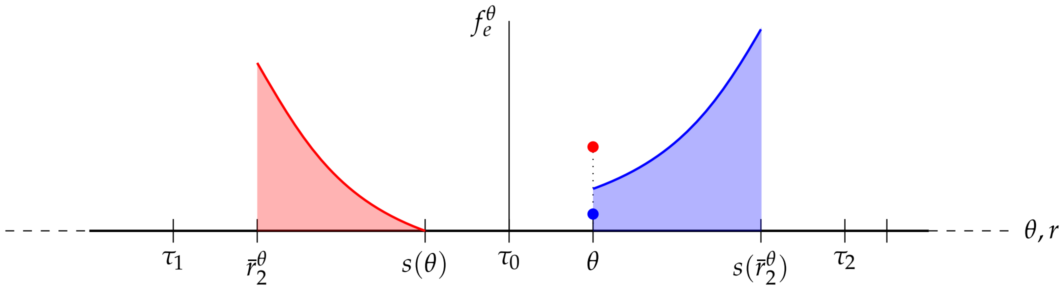


FIGURE 3.5: Misreporting equilibrium with opposing bias competition. The reporting strategies of media outlet 1 and 2 are, respectively, in blue and red. Dots represent atoms over truthful reporting.

the reach of only positive (negative) realization of valence. For example, when the lowest misreporting type of a report is positive, $l_1(r_1) > 0$, it must be impossible for the voter to expect valence to be negative in equilibrium. Similarly, if a report r_1 is close enough to the reach $\bar{r}_1^\theta$ of some strictly positive $\theta > 0$, then in any equilibrium it must be that the voter understands that valence is positive. In particular, this holds true for every feasible report by the competing outlet, $r_2 \geq \bar{r}_2^\theta$. Therefore, in equilibrium, for every $\theta \geq 0$, the reach of outlet 1 can swing the reach of outlet 2, $\bar{r}_1^\theta \geq s(\bar{r}_2^\theta)$. For the same reason, we must have that for every $\theta \leq 0$, $\bar{r}_2^\theta \leq s(\bar{r}_1^\theta)$.

The above depicted belief structure defines a *class* of possible swing reports, and therefore a class of posterior beliefs that can be sustained in equilibrium. Given such beliefs, the voter's optimal decision rule is,

$$a(r_1, r_2) = \begin{cases} L & \text{if } \bar{\mu}(r_1, r_2) \geq 0 \\ R & \text{if } \bar{\mu}(r_1, r_2) < 0 \end{cases} \tag{3.11}$$

Where, without loss of generality, ties are solved in favour of alternative L. Every time a media outlet swings the report of its opponent, it persuades the voter to elect its favourite candidate. However, given that reporting occurs simultaneously, it is not possible to simply "best reply" to the competitor's report. Each outlet has to guess what the opponent is likely to deliver, and then try to swing its report while minimizing the costs.

In equilibrium, when valence takes values in the area of disagreement $(\theta_1, \theta_2) \subseteq (\tau_1, \tau_2)$, both media outlets report truthfully with a state-contingent probability α_i^θ for $i \in \{1, 2\}$ and misreport otherwise. Truthful reporting occurs more frequently in extreme states and less frequently as the state of nature is closer to the voter's threshold τ_0. Therefore, for almost every realization of valence, there is a positive probability that both media outlets report truthfully their private information and the state is fully revealed to voters. Again, this is in stark contrast with the like bias case where, for every finite misreporting costs, there exists a non-empty and convex set of states where both media outlets always misreport.

When a media outlet misreports, more fabricated messages (i.e. more distant to the true realization of valence) are delivered more frequently. Figure 3.5 shows the mixed (density) strategies for the case where valence is positive, but not exceeding the truthful cut-off. Such strategies are formalized in Definition 12, while their equilibrium supports are in Definition 13.

Definition 12 (Mixed strategies). *Consider for media outlet $e \in \{1,2\}$ the probability density over reports $f_e^\theta(x) = \delta(x-\theta)\alpha_e^\theta + \psi_e^\theta(x)$ such that,*

$$\psi_1^\theta(x) = \frac{2k_2\,(s(x)-\theta)}{\tau_2-\theta}\frac{ds(x)}{dx} \quad \textit{and} \quad \psi_2^\theta(x) = -\frac{2k_1(s(x)-\theta)}{\theta-\tau_1}\frac{ds(x)}{dx} \tag{3.12}$$

and atoms are,

$$\alpha_1^\theta = \begin{cases} \frac{k_2(s(\theta)-\theta)^2}{\tau_2-\theta} & \textit{if } \theta \geq 0 \\ 1-\frac{k_2\left(s\left(\bar{r}_1^\theta\right)-\theta\right)^2}{\tau_2-\theta} & \textit{if } \theta < 0 \end{cases} \tag{3.13}$$

$$\alpha_2^\theta = \begin{cases} 1-\frac{k_1\left(s\left(\bar{r}_2^\theta\right)-\theta\right)^2}{\theta-\tau_1} & \textit{if } \theta > 0 \\ \frac{k_1(s(\theta)-\theta)^2}{\theta-\tau_1} & \textit{if } \theta \leq 0 \end{cases} \tag{3.14}$$

Definition 13 (Supports). *Consider the following supports, where $\mathcal{S}_e^\theta = \mathcal{D}_e^\theta \cup \mathcal{C}_e^\theta$,*

$$\mathcal{D}_e^\theta = \{\theta\} \tag{3.15}$$

$$\mathcal{C}_1^\theta = \left[\max\{s(\theta),\theta\}, \min\left\{\bar{r}_1^\theta, s\left(\bar{r}_2^\theta\right)\right\}\right] \tag{3.16}$$

$$\mathcal{C}_2^\theta = \left[\max\left\{\bar{r}_2^\theta, s\left(\bar{r}_1^\theta\right)\right\}, \min\{s(\theta),\theta\}\right] \tag{3.17}$$

I identify a class of equilibria, where each equilibrium belonging to this class depends on the swing report function $s(\cdot)$ and has the same qualitative features outlined above. In the following Proposition 14, I show that in all such equilibria, both media outlets play a mixed strategy featuring probabilistic misreporting when $\theta \in (\theta_1, \theta_2)$, and always report truthfully otherwise. In all equilibria the mixed strategies are described in Definition 12 and their supports are in Definition 13.

Furthermore, all such equilibria are robust to the Intuitive Criterion test (Cho and Kreps, 1987) and the ε-robustness (Battaglini, 2002).

Proposition 14 (Opposing bias equilibrium)**.** *Given any posterior belief $p(\theta|\cdot)$ satisfying the above conditions, there exists a class of robust equilibria $(\{f_e^{\theta}(\cdot)\}_E, s(\cdot), a(\cdot))$ where, for every realisation $\theta \in (\theta_1, \theta_2)$ strategies, beliefs, and supports are as in Definitions 12, 13. For $\theta \notin (\theta_1, \theta_2)$ there is truthful reporting.*

Proof. The proof is in Appendix 3.6.3.

3.4.5 Competition Versus Monopoly

In canonical product markets, competition usually limits firms' ability to raise their price above marginal costs. When "news" are modelled as standard products, consumers are for the most part bounded to gather information only from one source. Therefore, they ignore the information delivered by other news firms. Further, if information is regarded as being a final good, then it is not exploited as a mean to affect the beliefs of consumers. In such cases, any distortion in news provision originates from the need of profit-maximizing firms to please the demand side.

However information has very peculiar properties that makes it different from most goods: it is non-rivalrous and hardly excludable. The same news can be gathered and rebroadcast by many consumers at almost no cost. As shown in the CBS's "Killian documents" example outlined in the Introduction, even those who watched only CBS have eventually learnt the accusation of an online blog.

If we let consumers to get news from multiple sources, and these sources have antagonistic interests over the consumers' actions, then we obtain what I call a "direct competition in the market for news". Going back to the example, CBS and the blog were not competing in the product market, but they were in the news market. Within this type of competition, the object of news providers is not to maximize profits. Rather, they compete for influence over consumers' choices via the strategic distortion of information. In these circumstances, the usual market analysis can be inadequate.

In the present paper, I study to what extent direct competition limits the ability of news providers to influence the consumers' beliefs. As argued in Gentzkow and Shapiro, 2008, "two firms compete in this sense if 1) they cover the same events and 2) at least some consumers will learn the facts reported by both".

The overall effect of competition is not straightforward. The class of equilibria in Proposition 14 shows that the introduction in a monopoly market of an additional media outlet with opposed but greater bias, has the two following effects: (i) both media outlets misreport in more states of nature; (ii) the same report is sometimes fully revealing, but in other occasions it is even more fabricated than before. It is precisely the emergence of direct competition that drives the intensification of misreporting behaviour: a media outlet has to deliver stronger evidence to balance potentially fabricated news delivered by the competitor. However, this eventually backfires as it gives to the competitor a further incentive to misreport, and so on and so forth.

Thus it is not clear whether competition, and in particular diversity of opinions, is always beneficial for voters. Extremely biased media outlets might bring more confusion than information as they also prompt the less biased media outlet to engage more in news misrepresentation. A natural question is whether diversity of opinion has always a value with respect to monopoly, even when the additional source of information has an extreme conflict of interest with the voter.

Clearly, the introduction in a monopoly market of a media outlet with opposing interests that is *less biased* can only improve the voters' welfare: a lower conflict of interest makes any of its endorsements more credible. This has a disciplinary effect on the incumbent media outlet. In the extreme case where the entrant media outlet is completely unbiased, full revelation in every state of nature is an equilibrium. The intuition is straightforward: in absence of conflict, the voter completely relies on media outlet 2's reports and neglects whatever media outlet 1 says. In order to economize in useless but yet costly news fabrication, media outlet 1 reports truthfully in every contingency. Formally, as τ_2 tends to zero, meaning absence of conflict with the decision maker, both truthful cut-offs θ_1 and θ_2 tend to zero as well. By

Proposition 1 and 14, truthful reporting occurs at every state of nature. Therefore, with an unbiased media outlet ($\tau_2 = \tau_0$), there is full revelation in every state of nature.

In contrast, as the second media outlet become more biased the voter gathers less information despite the possible cross validation of reports. A more biased media outlet is not only ex-ante more likely to endorse one particular alternative with respect to the voter, but it has also more incentives to misreport as its stake in the voter's decisions are higher. Hence, it has more room for investing resources in news fabrication. As argued before, this prompts the first media outlet to react by misreporting even in contingencies where it used to report truthfully in the first place. As a consequence, both media outlets misreport news in more states, at every state they invest more resources in misreporting, and they are less likely to report truthfully.

Proposition 15 (More bias)**.** *As the second media outlet become more biased, the ex-ante probability that the voter selects the wrong alternative increases.*

Proof. The proof is in Appendix 3.6.4.

The above proposition confirms the intuition that a more biased media outlet generates more confusion and a loss of information on the voter's side. The remaining question is whether there is a point where the propagation of misreporting behaviour jeopardizes the benefits of diversity of opinions, even though the voter is fully aware of the conflict of interests and can cross validate news reports. If this were to be the case, the voter would be better by getting rid of the noise generated by the presence of an extremely biased media outlet and gathering news in a monopolistic market. The following proposition states that, independently of the additional media outlet's bias, competition performs better than monopoly.

Proposition 16. *Under competition between two opposing biased media outlets, the ex-ante probability of selecting the wrong alternative is strictly lower than in monopoly.*

Proof. The proof is in Appendix 3.6.4.

Even if the second media outlet is extremely biased, its presence moderates the *effects* of fabricated news. This is because the voter makes her assessments based on the pair of reports, not on each single report separately. In equilibrium, she is always able to rationally account for the credibility of each report given its source's bias. The voter severely discounts the extreme media outlet's news and asks for overwhelming evidence in support of its claims. This latter can be used to cross validate the first media outlet's reports, diminishing its profitability of misreporting.

Competition also impacts on the way outlets misreport information. The set of states where misreporting occurs increases as one of the two outlets becomes increasingly biased. However, this effect is compensated by the fact that, at every state, full revelation occurs with positive probability. In contrast, some states are never revealed in a monopolistic market. In addition, full revelation occurs more frequently in more extreme states, when the voter cares more about taking a correct decision.

Proposition 1 shows that, with respect to monopoly, any additional like-biased media outlets can only make the voter better but never worse. Proposition 16 goes further, providing an argument in favour of diversity of opinion in "the marketplace of ideas": even if the second media outlet is extremely biased, the voter is always strictly better than in a monopoly.

3.5 Conclusion

News suppliers have often a stake in consumers' decisions. This is the case, for example, when they deliver relevant political news, like in the Brexit campaign and CBS controversy examples. These scenarios make room for an agency problem: media outlets might affect political outcomes through the provision of distorted and biased news. Relevant information is quickly rebroadcast and consumers eventually learn all the viewpoints. Therefore, media outlets covering the same stories *directly* compete for influence. Forces that characterize direct competition in the market for news are very different from those described in traditional models. Increasing competing pressure might generate the dissemination of more slanted, fabricated, and misrepresented news, which might result in more confused consumers making mistakes. However, the idea that that truth prevails in a competitive news market is widespread and central for regulatory policies.[20] Nevertheless, to date there is no formal model addressing to what extent direct competition effectively promotes truth.

In this paper I explore this question with a model of strategic communication between an uninformed voter (or an electorate) and two informed media organizations, with the following innovations: all media outlets cover the same events, consumers learn all the news, and misreporting information is possible but costly.

In equilibrium, both media outlets report truthfully with positive probability and misreport otherwise. A pure strategy equilibrium does not exist: when misreporting, media outlets randomize over reports exaggerating the quality of their favourite candidate. In this class of mixed strategy equilibria, highly fabricated news are more likely to be delivered than small distortions. Because both media outlets play a mixed strategy, the voter cannot undo the misrepresentations and fully recover their private information by cross validating the two reports. However, if candidates' relative quality is large enough, both media outlets always report truthfully, as effective misreporting would be too expensive.

My results provide numerous direct empirical implications. I show that full information revelation is never possible, and that is a direct consequence of media outlets' ability to fabricate information. Further, competition is substantially different if it is between two like biased or two opposing biased media outlets. In the former case, increased competition might not reveal the truth more than a monopolistic market. Even the introduction of a less biased media outlet can be futile in promoting truth if its cost of misreporting is low enough. Finally, I confirm the traditional perspective on the matter: competition between two opposing biased media outlets always outperforms monopoly from the voter's viewpoint. This is true independently of how biased is the second media outlet. The propagation of fabricated news due to increased competition does not jeopardize the value of consulting antagonistic sources.

While most of the media industry is highly concentrated, competition in the information market is fiercer than in the product market. To what extent direct

[20]"The widest possible dissemination of information from diverse and antagonistic sources is essential to the welfare of the public" (Associated Press v. United States, 1945).

competition, even between biased media outlets, increases the voters' welfare? The present paper focuses on a duopolistic market. The equilibrium I found is connected to a particular type of all-pay contest. This suggests that techniques developed in the all-pay contest literature can potentially be adapted to study large information markets. Technically the task is not simple since in equilibrium players' strategies assign a mixed discrete/continuous probability distribution over reports, with non-connected and idiosyncratic supports. This is an interesting avenue of research that would enable the full understanding of direct competition and the type of interventions that make the median voter better off as inefficiencies persist despite the pressure of competitive forces. I leave these questions for future research.

3.6 Appendix

3.6.1 Like Biased Competition

The following is the proof of Lemma 19.

Proof. From Battaglini, 2002, an ε-perturbed equilibrium is a game in which each media outlet i perfectly and independently observes the true state of nature with probability $(1-\varepsilon_i)$, but with probability ε_i observes a random realization θ' from a random variable with the same support as θ and density distribution $g_i(\cdot)$. Consider a FRE with like bias where $\tau_2 \leq \tau_1 < \tau_0 = 0$. The posterior belief p_ε of the DM is,

$$p_\varepsilon(\theta|r_1,r_2) = p(\theta)\frac{\varepsilon_1\varepsilon_2 g_1(r_1)g_2(r_2) + \varepsilon_1(1-\varepsilon_2)g_1(r_1)\mathbb{1}_{\{\theta=r_2\}} + (1-\varepsilon_1)\varepsilon_2 g_2(r_2)\mathbb{1}_{\{\theta=r_1\}}}{\varepsilon_1\varepsilon_2 g_1(r_1)g_2(r_2) + p(r_2)\varepsilon_1(1-\varepsilon_2)g_1(r_1) + p(r_1)(1-\varepsilon_1)\varepsilon_2 g_2(r_2)} \tag{3.18}$$

The expected valence upon observing the two reports is $\bar{\mu}_\varepsilon(r_1,r_2) = \int_\Theta \theta p_\varepsilon(\theta|r_1,r_2)d\theta$. Given the FRE, say media outlet 2 wants to deviate by inflating information, $r_2 > \theta$, while media outlet 1 sticks with equilibrium truthful reporting. In order to persuade the voter, media outlet 2 has to induce $\bar{\mu}_\varepsilon(\cdot) \geq \tau_0 = 0$, which gives the condition,

$$\varepsilon_1\varepsilon_2 g_1(r_1)g_2(r_2)\int_\Theta \theta p(\theta)d\theta + r_1 p(r_1)(1-\varepsilon_1)\varepsilon_2 g_2(r_2) + r_2 p(r_2)\varepsilon_1(1-\varepsilon_2)g_1(r_1) \geq 0 \tag{3.19}$$

As $\varepsilon_i \to 0$ for $i = 1,2$, $\varepsilon_1\varepsilon_2$ shrinks quickly to zero and we can neglect the first term of the condition.[21] I shall consider $\varepsilon_1\varepsilon_2 \approx 0$ and set $c(\varepsilon_1,\varepsilon_2) = \frac{p(r_1)g_2(r_2)\varepsilon_2}{p(r_2)g_1(r_1)\varepsilon_1}$. At the limit the condition can be restated as $r_2 \geq -c(\varepsilon_1,\varepsilon_2)\cdot r_1$. Since in equilibrium media outlet 1 reports trutfully, $r_1 = \theta$ and media outlet 2 can persuade the voter with an unilateral deviation from the FRE whenever $\bar{r}_2(\theta) \geq -c(\varepsilon_1,\varepsilon_2)\cdot\theta$. A profitable deviation from the FRE in the perturbed game always exists when,

$$0 > \theta > \frac{1}{2(1+c)^2 k_2}\left[1 - \sqrt{1-4(1+c)^2\tau_2 k_2}\right] = g_2(c) \tag{3.20}$$

While the sequence $\varepsilon^n = (\varepsilon_1^n, \varepsilon_2^n)$ goes to zero, $c(\cdot)$ can be any finite positive number depending mostly on the ratio $\frac{\varepsilon_2}{\varepsilon_1}$. For any such $c(\cdot)$, there is always a state θ close enough to zero such that a profitable deviation exists, e.g. $\theta = \frac{g_2(c)}{2}$. The proof is

[21] That is of course also true if prior expectation $\int_\Theta \theta p(\theta)d\theta = 0$.

similar for deviations of media outlet 1. Hence, there is no ε-robust FRE with like bias. $\square$

The following is the first part of the proof of Proposition 13.

Proof. Define $\bar{\theta}_i := \{\theta' \in \Theta : \mathbb{E}_p \left[\theta | l_i(\bar{\theta}_i) \leq \theta \leq \bar{\theta}_i\right] = \tau_0\}$, the set $M_i = [l_i(\bar{\theta}_i), \bar{\theta}_i]$ and media outlet $j \in \{1, 2\}$ as the media outlet such that $l_j(\bar{\theta}_j) \geq l_{-j}(\bar{\theta}_{-j})$. Note also that $\bar{\theta}_j \leq \bar{\theta}_{-j}$. If media outlet i were to be alone in the market, i.e. a monopolist owner of the information about valence θ, the unique equilibrium surviving the Intuitive Criterion refinement would prescribe to report $r_i(\theta) = \bar{\theta}_i$ for $\theta \in M_i$ and $r_i(\theta) = \theta$ otherwise. See the first Chapter for the proof and extensive discussion.

As shown in Proposition 13, with two like biased media outlets the equilibrium reporting rule is similar, $r_e(\theta) = \bar{\theta}_j$ for $\theta \in M_j$ and $r_e(\theta) = \theta$ otherwise for $e = \{1, 2\}$, where posterior beliefs $p(\theta | r_1, r_2)$ are such that $\bar{\mu}(r_1, r_2) < \tau_0$ if $\min\{r_1, r_2\} < \tau_0$. Importantly, note such beliefs do not necessarily make the equilibrium ε-robust, but I will show later specific out of equilibrium beliefs that make it robust.

Both media outlets agree and find profitable to deliver $\bar{\theta}_j$ for every $\theta \in M_j$, which induces, given Bayes' rule, $\bar{\mu}(\cdot) = \tau_0$ and the implementation of alternative L. It is straight forward that any individual deviation is not profitable as it is either not effective (that is, cannot induce the implementation of the desired political outcome) or not incentive compatible (due to prohibitively misreporting costs). In this equilibrium the two media outlets always deliver the same report and coordinate on j's monopolistic equilibrium strategy.

The voter is persuaded (i.e. selects the wrong alternative) with ex ante probability $p(\theta \in [l_j(\bar{\theta}_j), \tau_0])$. Voters, given the reporting rule and beliefs, cast a ballot according to the unique sequentially rational voting rule. Beliefs are updated according Bayes' rule whenever possible and refined according to the Intuitive Criterion and, as I shall show below, ε-robustness. $\square$

The following proof shows that the Misreporting like bias equilibrium of Proposition 13 is robust to the ε-refinement.

Proof. Say, without loss of generality, that the less persuasive media outlet is $j = 1$. The only possible interesting profitable deviation is considering media outlet 2 inflating information in $(\tau_2, l_1(\theta^*))$, trying to sway the voter when media outlet 1 does not have an incentive to do so (maybe because of its stance or due to excessively high costs from misreporting). In that set, the equilibrium prescribes truthful revelation.

The perturbed posterior upon observing two report is as above, $p_\varepsilon(\theta|r_1, r_2)$. The condition for misreporting to be profitable is also similar: we need $r_2 > -c(\cdot) \cdot r_1$, where $r_1 = \theta$. Following the above proof, this might be profitable whenever $l_1(\theta^*) > \theta > \frac{1}{2(1+c)^2 k_2}\left[1 - \sqrt{1 - 4(1+c)^2 k_2 \tau_2}\right]$. Remember that $c = \frac{p(r_1)g_2(r_2)\varepsilon_2}{p(r_2)g_1(r_1)\varepsilon_1}$. The condition for that to be a robust equilibrium is to find a sequence of $\varepsilon^n = (\varepsilon_1^n, \varepsilon_2^n)$ going to zero such that the previous condition is violated. If k_2 is large enough, $k_2 > \frac{l_1(\theta^*) - \tau_2}{l_1(\theta^*)^2}$, then it is always true that $l_1(\theta^*) < \frac{1}{2(1+c)^2 k_2}\left[1 - \sqrt{1 - 4(1+c)^2 \tau_2 k_2}\right]$ for any $c > 0$. However, while the argument holds, this cannot be the case. Indeed if k_2 is that large, the reach of 2 will be 0 or less. But then it contradicts (i) 2 being $-j$ (ii) the above being an equilibrium.

Instead for $0 < k_2 \leq \frac{l_1(\theta^*) - \tau_2}{l_1(\theta^*)^2}$, it is sufficient to set $c > \left(\sqrt{\frac{l_1(\theta^*) - \tau_2}{k_2 l_1(\theta^*)^2}} - 1\right)$. This is possible by letting the sequence ε^n going to zero keeping a specific ratio of $\frac{\varepsilon_2}{\varepsilon_1}$. This is always possible and therefore the equilibrium is robust. Note that other deviations (e.g. misreporting less in $[l_1(\theta^*), \theta^*]$) are not interesting (in that case you would decrease the expectation by the intuitive criterion). □

The following proof shows that the like bias equilibrium of Proposition 13 is unique.

Proof. Consider a like biased misreporting equilibrium with $[x, y] = M' \subset M_j$, $l_j(\bar{\theta}_j) < x < y < \bar{\theta}_j$, such that $\mathbb{E}_p[\theta|\theta \in M'] = \tau_0$. This latter condition is necessary to achieve persuasion to the median voters' indifference and preserve robustness to the Intuitive Criterion refinement. To check ε-robustness, I look for individual profitable deviations. Whenever there is no available profitable deviations for media outlet i and for some $c(\varepsilon_1, \varepsilon_2)$ (which depends on the ratio $\frac{\varepsilon_2}{\varepsilon_1}$), then it is easy to see that given $c(\cdot)$ it is profitable for media outlet $-i$ to inflate language to some report above τ_0 (and vice versa). □

3.6.2 Opposing Biased Competition

The following is the proof of Lemma 20.

Proof. Assume $p(\theta|r_1, r_2)$, $a(r_1, r_2)$ and $f_e(r_e|\theta)$ for $e \in \{1, 2\}$ is a VEE (or FRE). For any $\theta \in \Theta$ and for any (r_1, r_2) such that $r_e \in supp\{f_e(\cdot|\theta)\}$, the DM takes an action such that $a(r_1, r_2) = a^*(\theta)$. Because different reports have different costs, and due to monotonicity, in equilibrium the support of f_e must be a singleton for every $\theta \in \Theta$. Therefore, in any VEE and FRE, the experts play pure strategies only, that is $r_e(\theta)$. The following step of the proof follows the argument of Lemma 1 in Battaglini, 2002 and a similar logic as the revelation principle. It shows that, if there is a VEE (or FRE), then there exists also a FRE in truthful strategies.

Define $\tilde{p}(\theta|x', x) = p(\theta|r_1(x'), r_2(x))$ and $\tilde{a}(x', x) = a(r_1(x'), r_2(x))$. Then, for $\tilde{r}_e = \theta$ for $e \in \{1, 2\}$, $\tilde{p}(\theta|\tilde{r}_1, \tilde{r}_2)$ and $\tilde{a}(\tilde{r}_1, \tilde{r}_2)$ is a truthful FRE. If not, it must be a player has a strictly profitable deviation. Say in state θ expert e deviates by reporting $\theta' \neq \theta$. If $r_e(\theta') = r_e(\theta)$, the decision maker would not change action and then it cannot be strictly preferred. But if $r_e(\theta') \neq r_e(\theta)$, then this contradicts the first to be an equilibrium as it would be a profitable deviation there as well. A similar argument applies for deviations by the DM.

For the second part, I focus on the truthful FRE where $r_e(\theta) = \theta$. Consider $\theta < 0$ close enough to zero. In such equilibrium, the DM would select alternative R after observing the two reports. Consider the following deviation of expert 1 which, at that state, endorses alternative L: $0 < r_1' < \bar{r}_1(\theta)$. Whatever out of equilibrium beliefs are, if $\bar{\mu}(r_1', r_2) > 0$ then that is a strictly profitable deviation because it would induce, under sequential rationality, $a(r_1', r_2) = L$. If instead $\bar{\mu}(r_1', r_2) < 0$, consider the instance where the state of nature is $\theta' = r_1'$. In this latter case, expert 2 can profitably deviate by delivering $r_2 = \theta < 0$ while expert 1 reports truthfully $r_1(\theta') = \theta'$. As long as $r_2 > \bar{r}_2(\theta')$, this is a strictly profitable deviation because it would induce $\bar{\mu}(r_1', r_2) < 0$ and $a(r_1', r_2) = R$. It is always possible to pick a θ and r_1' such that this is possible. Therefore there can not be a truthful FRE as deviation are always possible in some state. Given the first part, there are no FRE in general. □

The following is the proof of Lemma 21

Proof. Suppose $r_1(\theta), r_2(\theta)$, $\bar{\mu}(r_1(\theta), r_2(\theta))$ and $a(r_1(\theta), r_2(\theta))$ is an equilibrium, and remember it cannot be fully revealing. That is, there exists a $\theta' \in \Theta$ such that $a(r_1(\theta'), r_2(\theta')) \neq a^*(\theta')$. For this to be an equilibrium, the "losing" expert must be saving on misreporting costs by reporting truthfully. Say $\theta' < 0$, then $r_2(\theta') = \theta'$, $\bar{\mu}(\theta', r_1(\theta')) \geq 0$ and $a(\theta', r_2(\theta')) = L$. In order for the DM not to understand $\theta' < 0$ despite 2 is reporting truthfully, there must be some $\theta'' > 0$ such that $r_1(\theta'') = r_1(\theta')$, $r_2(\theta'') = \theta' < 0$. This would require 2 to misreport at a cost when L is implemented anyway. Therefore, it cannot be an equilibrium. The case $\theta' > 0$ is similar. Note I did not use report monotonicity or assumed a specific tie rule. □

3.6.3 Misreporting Equilibrium with Opposing Biases

In this section I check that the strategies and supports of Proposition 14 constitute an equilibrium. At the moment, I am not imposing any robustness criteria. Firstly, I check the equilibrium mixed strategies by using the method of pay-off equation. Secondly, I shall find the atoms, namely reports that are delivered with positive probability. Finally, I will describe the strategies as mixed partial probability distributions. I will show and check the strategies' supports in equilibrium for every realisation of valence. Lastly, I will check all the equilibria in the class described by the above strategies and supports are indeed robust to the ε-refinement.

Payoffs and strategies

Define the expected utility of media outlet e from playing a pure strategy r_e as,

$$U_e(r) = \mathbb{E}\left[v\left(a(r_e, r_{-e}), \theta, \tau_e, r_e)|f^{\theta}_{-e}(\cdot)\right)\right] \tag{3.21}$$

Given the strategies f^{θ}_e we obtain the following,

$$U_1(r_1) = \begin{cases} \mathbb{1}_{\{\theta>0\}}\alpha^{\theta}_2[\theta - \tau_1] & \text{if } r_1 = \theta \\ \left(1 - \Psi^{\theta}_2(s(r_1))\right)[\theta - \tau_1] - k_1(r_1 - \theta)^2 & \text{if } r_1 \in \mathcal{S}^{\theta}_1 \backslash \{\theta, \tilde{r}^{\theta}_1\} \\ [\theta - \tau_1] - k_1(\tilde{r}^{\theta}_1 - \theta)^2 & \text{if } r_1 = \tilde{r}^{\theta}_1 \end{cases} \tag{3.22}$$

$$U_2(r_2) = \begin{cases} \left(1 - \mathbb{1}_{\{\theta<0\}}\alpha^{\theta}_1\right)[\theta - \tau_2] & \text{if } r_2 = \theta \\ \left(1 - \Psi^{\theta}_1(s(r_2))\right)[\theta - \tau_2] - k_2(r_2 - \theta)^2 & \text{if } r_2 \in \mathcal{S}^{\theta}_2 \backslash \{\theta, \tilde{r}^{\theta}_2\} \\ -k_2(\tilde{r}^{\theta}_2 - \theta)^2 & \text{if } r_2 = \tilde{r}^{\theta}_2 \end{cases} \tag{3.23}$$

By using the method of payoff-equation, I can find the cumulative distributions $\Psi_e^\theta(\cdot)$ and atoms α_e^θ. As an instance, for $\theta > 0$ equate $U_2(\theta) = U_2(r_2)$ for $r_2 \in \mathcal{S}_2^\theta \backslash \{\theta, \tilde{r}_2^\theta\}$. By rearranging we can find $\Psi_1^\theta(s(r_2))$. Denote $x = s(r_2)$ and note that $s(x) = r_2$. Therefore, $\Psi_1^\theta(x) = \frac{k_2(s(x)-\theta)^2}{\tau_2-\theta}$. Equivalently, we can set $\frac{dU_2(r_2)}{dr_2} = 0$ and directly find the density $\psi_1^\theta(x) = \frac{2k_2(s(x)-\theta)}{\tau_2-\theta}\frac{ds(x)}{dx}$. As follows, the partial cumulative distributions for $x \in \mathcal{S}_e^\theta$,

$$\Psi_1^\theta(x) = \begin{cases} \frac{k_2(s(x)-\theta)^2}{\tau_2-\theta} & \text{if } \theta \geq 0 \\ 1 - \frac{k_2[(\bar{r}_2^\theta-\theta)^2-(s(x)-\theta)^2]}{\tau_2-\theta} & \text{if } \theta < 0 \end{cases} \tag{3.24}$$

$$\Psi_2^\theta(x) = \begin{cases} \frac{k_1[(\bar{r}_1^\theta-\theta)^2-(s(x)-\theta)^2]}{\theta-\tau_1} & \text{if } \theta > 0 \\ 1 - \frac{k_1(s(x)-\theta)^2}{\theta-\tau_1} & \text{if } \theta \leq 0 \end{cases} \tag{3.25}$$

Where $\Psi_e^\theta(x) = 1$ for $x > \max \mathcal{S}_e^\theta$ and $\Psi_e^\theta(x) = 0$ for $x < \min \mathcal{S}_e^\theta$. Taking derivatives $\frac{d\Psi_e^\theta(x)}{dx}$ we can find the partial densities $\psi_e^\theta(x)$. These are partial densities because they do not integrate to 1.

Atoms

Note that, for $\theta \geq 0$, $\lim_{x\to\theta} \Psi_1^\theta(x) = \frac{k_2(s(\theta)-\theta)^2}{\tau_2-\theta}$ is positive whenever $\theta > 0$, despite $\theta = \min \mathcal{S}_1^\theta$ in this case. Therefore there must be an atom at θ of size $\alpha_1^\theta = \frac{k_2(s(\theta)-\theta)^2}{\tau_2-\theta}$. Note that when $r_1 = r_2 = r$, $p(\theta = r|r_1, r_2) = 1$. Hence, $s(0) = 0$ because, by definition of swing report, $p(\theta = 0|0,0) = 1$ and $\bar{\mu}(0,0) = 0$. Therefore $\alpha_1^\theta = 0$ when $\theta = 0$ and $\alpha_1^\theta > 0$ for $0 < \theta < \theta_2$.

For $\theta \geq 0$ the upper bound of the support of 1's strategy is $\tilde{r}_1^\theta = \min\{\bar{r}_1^\theta, s(\bar{r}_2^\theta)\}$. But $\Psi_1^\theta(\tilde{r}_1^\theta) = 1$ only if $s(\tilde{r}_1^\theta) = \bar{r}_2^\theta$, or equivalently $\tilde{r}_1^\theta = s(\bar{r}_2^\theta) \leq \bar{r}_1^\theta$. However, note that $\Psi_1^\theta(\tilde{r}_1^\theta) = \frac{k_2(s(\tilde{r}_1^\theta)-\theta)^2}{\tau_2-\theta} < 1$ if $s(\tilde{r}_1^\theta) > \bar{r}_2^\theta$. This might suggests the possible existence of a positive atom at $\tilde{r}_1^\theta$. However, that would imply $\tilde{r}_1^\theta = \bar{r}_1^\theta < s(\bar{r}_2^\theta)$. But then, if $\bar{r}_2^\theta < s(\bar{r}_1^\theta)$, media outlet 2 would profitably gain by moving probability mass to $r_2 = (s(\bar{r}_1^\theta) - \epsilon)$ a positive $\epsilon > 0$ small enough. That cannot constitute an equilibrium. The case $\Psi_1^\theta(\tilde{r}_1^\theta) > 1$ leads to a contradiction as it would imply $\tilde{r}_1^\theta = \min\{\bar{r}_1^\theta, s(\bar{r}_2^\theta)\} > s(\bar{r}_2^\theta)$. In equilibrium it must be that, for $\theta > 0$, $\mathcal{S}_1^\theta = [\theta, s(\bar{r}_2^\theta)]$ and DM's beliefs are such that for all $\theta \in (0, \theta_2)$, $s(\bar{r}_2^\theta) \leq \bar{r}_1^\theta$. Since $\bar{r}_2^\theta \geq s(\bar{r}_1^\theta)$, it also has to be $\mathcal{S}_2^\theta = [\bar{r}_2^\theta, s(\theta)] \cup \{\theta\}$, since any $r_2 \in (s(\theta), \theta)$ is strictly dominated by truthful reporting, while by construction any other report in the support yields the same expected pay-off. Lastly, note that given the support, $\Psi_2^\theta(\bar{r}_2^\theta) = 0$ always.

Similarly, $\Psi_2^\theta(s(\theta)) = \frac{k_1(s(\bar{r}_2^\theta)-\theta)^2}{\theta-\tau_1} < 1$ when $\bar{r}_1^\theta > s(\bar{r}_2^\theta)$. There cannot be an atom at $r_2 = s(\theta)$ as that report is strictly dominated by truthful reporting (positive misreporting cost but losing with certainty). Therefore the atom must be at $r_2 = \theta$ and of size $\alpha_2^\theta = 1 - \frac{k_1(s(\bar{r}_2^\theta)-\theta)^2}{\theta-\tau_1}$. Again, $\alpha_2^\theta = 0$ when $\theta = 0$.

Consider now the case $\theta \leq 0$. By using a similar line of reasoning it can be checked that $\Psi_2^\theta(\theta) < 1$, with an atom $\alpha_2^\theta = \frac{k_1(s(\theta)-\theta)^2}{\theta-\tau_1} > 0$ when $0 > \theta > \theta_1$ and $\alpha_2^\theta = 0$ when $\theta = 0$. $\Psi_2^\theta(\tilde{r}_2^\theta) > 0$ if $\tilde{r}_2^\theta = \max\{\bar{r}_2^\theta, s(\bar{r}_1^\theta)\} > s(\bar{r}_1^\theta)$, which implies $\bar{r}_2^\theta > s(\bar{r}_1^\theta)$. But as before, media outlet 1 can profitably deviate by moving mass to $r_1 = s(\bar{r}_2^\theta) < \bar{r}_1^\theta$. Hence, in this equilibrium it has to be, for all $\theta_1 < \theta < 0$, $\mathcal{S}_2^\theta = [s(\bar{r}_1^\theta), \theta]$ and beliefs such that $\bar{r}_2^\theta \leq s(\bar{r}_1^\theta)$. Given $\tilde{r}_2^\theta = s(\bar{r}_1^\theta) \geq \bar{r}_2^\theta$, and thus $\bar{r}_1^\theta \leq s(r_2^\theta)$, $\Psi_1^\theta(\tilde{r}_1^\theta) = 1$. Lastly, $\Psi_1^\theta(s(\theta)) > 0$ if $\bar{r}_2^\theta < s(\bar{r}_1^\theta)$. But the atom can not be at $s(\theta)$, otherwise media outlet 2 would never put positive probability over truthful reporting. Hence $\alpha_1^\theta = 1 - \frac{k_2(s(\bar{r}_1^\theta)-\theta)^2}{\tau_2-\theta} > 0$ for $\theta < 0$ and $\alpha_1^\theta = 0$ for $\theta = 0$.

The following are the atoms:

$$\alpha_1^\theta = \begin{cases} \frac{k_2(s(\theta)-\theta)^2}{\tau_2-\theta} & \text{if } \theta \geq 0 \\ 1 - \frac{k_2(s(\bar{r}_1^\theta)-\theta)^2}{\tau_2-\theta} & \text{if } \theta < 0 \end{cases} \tag{3.26}$$

$$\alpha_2^\theta = \begin{cases} 1 - \frac{k_1(s(\bar{r}_2^\theta)-\theta)^2}{\theta-\tau_1} & \text{if } \theta > 0 \\ \frac{k_1(s(\theta)-\theta)^2}{\theta-\tau_1} & \text{if } \theta \leq 0 \end{cases} \tag{3.27}$$

Strategies and partial probability distributions

The above densities ψ_e^θ are "partial densities" as they do not integrate to 1. Indeed, as an instance, for $0 < \theta < \theta_2$,

$$\int_{-\infty}^{+\infty} \psi_1^\theta(x)dx = \Psi_1^\theta\left(s(\bar{r}_2^\theta)\right) - \Psi_1^\theta(\theta) = 1 - \alpha_1^\theta < 1 \tag{3.28}$$

Where $\alpha_1^\theta \in (0,1)$. This is because the equilibrium strategies f_e^θ are mixed random variables with a discrete part (the atom α_e^θ at truthful reporting θ) and a continuous part (misreporting). Define the discrete part of the support of media outlet e at state θ as $\mathcal{D}_e^\theta = \{\theta\}$. Similarly, define the continuous part of the support as $\mathcal{C}_e^\theta$. The total support is $\mathcal{S}_e^\theta = \mathcal{C}_e^\theta \cup \mathcal{D}_e^\theta$. The atom α_e^θ is a partial probability density function on $\mathcal{D}_e^\theta$ and $\psi_e^\theta(x)$ is a partial probability function on $\mathcal{C}_e^\theta$ such that $\int_{\mathcal{C}_e^\theta} \psi_e^\theta(x)dx = 1 - \alpha_e^\theta$. Moreover, $\alpha_e^\theta = 0$ for $\theta' \notin \mathcal{D}_e^\theta$ and $\psi_e^\theta(x) = 0$ for $\theta' \notin \mathcal{C}_e^\theta$. The distribution of equilibrium reports r_e by each media outlet in state θ is determined by the partial probabilities α_e^θ and ψ_e^θ through the well defined mixed distribution $f_e^\theta(x) = \delta(x - \theta)\alpha_e^\theta + \mathbb{1}_{\{x \in \mathcal{C}_e(\theta)\}}\psi_e^\theta(x)$, where $\delta(\cdot)$ is the Dirac delta "function". Indeed,

$$\int_{-\infty}^{+\infty} \left[\delta(x-\theta)\alpha_e^\theta + \mathbb{1}_{\{x\in\mathcal{C}_e(\theta)\}}\psi_e^\theta(x)\right] dx = \alpha_e^\theta + 1 - \alpha_e^\theta = 1 \tag{3.29}$$

Beliefs

Given equilibrium reporting strategies f_e^θ, beliefs are according to Bayes'rule whenever possible, $p(\theta|r_1, r_2) = \frac{p(\theta)p(r_1,r_2|\theta)}{p(r_1,r_2)}$, where $p(r_1, r_2|\theta)$ is,

$$\begin{aligned} p(r_1, r_2|\theta) = &\ \mathbb{1}_{\{r_1=r_2=\theta\}}\alpha_1^\theta\alpha_2^\theta \\ &+ \mathbb{1}_{\{r_1=\theta; r_2\in\mathcal{C}_2^\theta\}}\alpha_1^\theta\psi_2^\theta(r_2) \\ &+ \mathbb{1}_{\{r_1\in\mathcal{C}_1^\theta; r_2=\theta\}}\psi_1^\theta(r_1)\alpha_2^\theta \\ &+ \mathbb{1}_{\{r_1\in\mathcal{C}_1^\theta\}}\mathbb{1}_{\{r_2\in\mathcal{C}_2^\theta\}}\psi_1^\theta(r_1)\psi_2^\theta(r_2) \end{aligned} \tag{3.30}$$

Out of equilibrium, it is sufficient that, given $p(\theta|r_1, r_2)$, $\bar{\mu}(r_1, r_2) > 0$ for every $r_2 > s(r_1)$ and $r_1 > s(r_2)$, $\bar{\mu}(r_1, r_2) < 0$ for every $r_2 < s(r_1)$ and $r_1 < s(r_2)$. Any out of equilibrium belief sustaining that condition is fine, but notice this might not imply ε-robustness which I shall discuss later. Given the voter's beliefs, media outlets do not have profitable deviation from equilibrium strategies by construction.

Robustness with Opposing biases

Here I check the robustness of the equilibrium with opposing biases and that there are no individual profitable deviations from the equilibrium. Consider the case where $\theta > 0$. The other case is similar. If $\theta < \theta_2$, then equilibrium reporting strategies are described by the mixed probability distributions f_e^θ. Given the equilibrium strategies and beliefs, media outlet 1 would never deviate by reporting $r_1 > s(\bar{r}_2^\theta)$. Even if that would result in an out of equilibrium pair of reports, 1 would win with probability 1 by reporting $s(\bar{r}_2^\theta)$ and economise in misreporting costs. Similarly, media outlet 2 would never misreport below $\bar{r}_2^\theta$.

Consider a deviation by media outlet 2 misreporting in $r_2' \in [s(\theta), \theta)$. If $r_2' \in [0, \theta)$ then the pair (r_2', r_1) would be out of equilibrium. However, because I am focusing in monotone reporting strategies, that would lead the voter to believe $p(\theta \in [r_2', r_1]) = 1$ which results in $\bar{\mu}(r_1, r_2') \geq 0$. No such deviation is therefore profitable as would be strictly dominated by truthful reporting. If instead 2 deviates by misreporting $r_2' \in [s(\theta), 0)$, the pair (r_2', r_1) would never be out of equilibrium. Indeed, for any such r_2', there exists a $0 < \theta' < \theta$ such that $r_2' = s(\theta')$. At $\theta' < \theta$, $\mathcal{S}_1^{\theta'} = [\theta', s(\bar{r}_2^{\theta'})] \supset [\theta, s(\bar{r}_2^{\theta})] = \mathcal{S}_1^{\theta}$ because $\bar{r}_2^{\theta} > \bar{r}_2^{\theta'}$ and therefore $s(\bar{r}_2^{\theta}) < s(\bar{r}_2^{\theta'})$. Hence, for any such deviation r_2', there exists a state $\theta'' = \theta' - \epsilon_\theta > 0$ for $\epsilon_\theta > 0$ small enough such that any pair (r_1, r_2') is observed in equilibrium when the state is θ''. Note that I need θ'' because $\psi_2^{\theta'}(s(\theta')) = 0$. Hence such kind of deviation does never result in observing an out of equilibrium pair of reports and is always strictly dominated by truthful reporting since $\min \mathcal{S}_1^{\theta} = \theta > s(r_2')$.

Consider now a $\theta \in [\theta_2, \tau_2]$. The equilibrium prescribes both media outlets to report the state of nature truthfully. Any deviation by media outlet 2 in $r_2' \in [0, \theta)$ cannot be profitable even if it would result in an out of equilibrium pair for the same reason as above. Hence we need to check for $\bar{r}_2^{\theta} \leq r_2' < 0$. In order to have the reach of media outlet 2 below zero, the state of nature has to be low enough, $\theta < \frac{1}{2k_2}[\sqrt{1 + 4k_2\tau_2} - 1]$. However, any deviation $r_2' \in [\bar{r}_2^{\theta}, 0)$ can be observed in equilibrium when $r_1 = \theta$ if the state of nature is too low. Indeed, in equilibrium, media outlet 1 can potentially deliver the highest misreport when the state is zero, $\tilde{r}_1^0 = \bar{r}_1^0$. For $\theta < 0$, $\bar{r}_1^{\theta} < \bar{r}_1^0$, while for $\theta > 0$, $s(\bar{r}_2^{\theta}) < \bar{r}_1^0$. Given that, for $\theta > 0$ $\bar{r}_2^{\theta} > \bar{r}_2^0$, any pair $(r_1 = \theta, r_2' \in [\bar{r}_2^0, 0))$ is observable in equilibrium if $\bar{r}_1^0 \geq \theta$, and would again be strictly dominated by truthful reporting. This last condition implies $\theta > \sqrt{\frac{-\tau_1}{k_1}}$.

Deviations from media outlet 2 that lead to out of equilibrium reports are possible if $\theta \in \left(\max\left\{\theta_2, \sqrt{\frac{-\tau_1}{k_1}}\right\}, \min\left\{\tau_2, \frac{1}{2k_2}\left[\sqrt{1 + 4k_2\tau_2} - 1\right]\right\}\right)$. Following the previous robustness check, since at such state the equilibrium prescribes $r_1 = \theta$ always, the condition for the deviation of media outlet 2 to be profitable is $\bar{r}_2^{\theta} \leq r_2' < -c(\varepsilon_1, \varepsilon_2)\theta$, so that it can induce $\bar{\mu}(r_1, r_2') < 0$. It is always possible to pick a $c(\varepsilon_1, \varepsilon_2)$ high enough such that $\bar{r}_2^{\theta} > -c(\varepsilon_1, \varepsilon_2)\theta$. The equilibrium is therefore always robust. Moreover, even without considering ε-robustness, such type of deviation would violate another mild robustness test, namely the Intuitive Criterion. Indeed, for any such θ it has to be $l_1(\theta) > 0$ because $\theta > \sqrt{\frac{-\tau_1}{k_1}}$. Following the Intuitive Criterion refinement, the voter upon observing $r_1 = \theta$ must infer that $\theta > 0$.

3.6.4 Comparative Statics

Probability of Selecting the Wrong Alternative

The (ex-ante) probability that the median voters selects the wrong alternative, that is $W = Pr\left(a(r_1, r_2) \neq \bar{a}(\theta)\right)$, is,

$$\begin{aligned} W = &\int_{\theta=0}^{\theta_2} \alpha_1^\theta(1-\alpha_2^\theta)p(\theta)d\theta + \int_{\theta=0}^{\theta_2} p(\theta)\int_{r=\theta}^{s(\bar{r}_2^\theta)} \Psi_2^\theta(s(r))\psi_1^\theta(r)drd\theta \\ &+ \int_{\theta=\theta_1}^{0} \alpha_2^\theta(1-\alpha_1^\theta)p(\theta)d\theta + \int_{\theta=\theta_1}^{0} p(\theta)\int_{r=s(\bar{r}_1^\theta)}^{\theta} (1-\Psi_1^\theta(s(r)))\psi_2^\theta(r)drd\theta \end{aligned} \tag{3.31}$$

Notice that, using integration by parts,

$$\begin{aligned} \int_{r=\theta}^{s(\bar{r}_2^\theta)} \Psi_2^\theta(s(r))\psi_1^\theta(r)dr &= \left[\Psi_2^\theta(s(r))\Psi_1^\theta(r)\right]_{r=\theta}^{s(\bar{r}_2^\theta)} - \int_{r=\theta}^{s(\bar{r}_2^\theta)} \psi_2^\theta(s(r))\Psi_1^\theta(r)dr \\ &= -\alpha_1^\theta(1-\alpha_2^\theta) + \frac{2k_1k_2}{(\theta-\tau_1)(\tau_2-\theta)}\int_{r=\theta}^{s(\bar{r}_2^\theta)} (r-\theta)(s(r)-\theta)^2 dr \end{aligned} \tag{3.32}$$

Therefore the first two integrals of W can be reduced to the following,

$$2k_1k_2\int_{\theta=0}^{\theta_2} \frac{p(\theta)}{(\theta-\tau_1)(\tau_2-\theta)}\int_{r=\theta}^{s(\bar{r}_2^\theta)} (r-\theta)(s(r)-\theta)^2 drd\theta \tag{3.33}$$

The second part can be reduced similarly. The overall ex-ante probability of selecting the wrong alternative, W, can be rewritten as follows,

$$\begin{aligned} W =&2k_1k_2\Bigg[\int_{\theta=0}^{\theta_2} \frac{p(\theta)}{(\theta-\tau_1)(\tau_2-\theta)}\int_{r=\theta}^{s(\bar{r}_2^\theta)} (r-\theta)(s(r)-\theta)^2 drd\theta \\ &+ \int_{\theta=\theta_1}^{0} \frac{p(\theta)}{(\theta-\tau_1)(\tau_2-\theta)}\int_{r=s(\bar{r}_1^\theta)}^{\theta} (\theta-r)(s(r)-\theta)^2 drd\theta\Bigg] \end{aligned} \tag{3.34}$$

The first part of W is the probability that $a(r_1, r_2) = R$ when $\bar{a}(\theta) = L$ (and vice versa for the second part). Hereafter I normalize media outlet 1's policy-bias, $\tau_1 = -1$ and both media outlets' costs of misreporting, $k_1 = k_2 = 1$. I also assume valence is uniformly distributed in $[-\phi, \phi]$, with density $p(\theta) = \frac{1}{2\phi}$.

Proofs

For ease of exposition I hereafter assume that (i) valence is uniformly distributed in $[-\phi, \phi]$ and (ii) the swing report function $s(\cdot)$ is linear in r. This latter assumption helps to nail down a unique swing function. Suppose $s(r) = -\beta r$ if $r \geq 0$ and $s(r) = -\frac{r}{\beta}$ if $r < 0$ with $\beta > 0$. The condition $s(\bar{r}_2^\theta) \leq \bar{r}_1^\theta$ is then $-\frac{\bar{r}_2^\theta}{\beta} \geq \bar{r}_1^\theta$ for all $\theta > 0$, that is $\beta \geq -\frac{\bar{r}_2^\theta}{\bar{r}_1^\theta}$. Note this ratio is maximised at $\theta = 0$, as both $\bar{r}_2^\theta$ and $\bar{r}_1^\theta$ increase with θ. A necessary condition is therefore $\beta \geq -\frac{\bar{r}_2^0}{\bar{r}_1^0}$. For $\theta < 0$ is similar and I can look at $\beta \leq -\frac{\bar{r}_2^0}{\bar{r}_1^0}$. The only possible linear swing report function $s(\cdot)$ has $\beta = -\frac{\bar{r}_2^0}{\bar{r}_1^0} = \sqrt{\frac{\tau_2}{-\tau_1}\frac{k_1}{k_2}} > 0$.

The following is the proof of Proposition 15.

Proof. The derivative of W with respect to τ_2 is,

$$\begin{aligned}\frac{\partial W}{\partial \tau_2} =& \frac{1}{\phi}\Bigg[-\int_{\theta=0}^{\theta_2}\int_{r=\theta}^{\frac{-\bar{r}_2^\theta}{\sqrt{\tau_2}}} \frac{(r-\theta)(-\sqrt{\tau_2}r-\theta)^2}{(1+\theta)(\tau_2-\theta)^2} dr d\theta + \int_{\theta=0}^{\theta_2} \frac{\frac{\partial \frac{-\bar{r}_2^\theta}{\sqrt{\tau_2}}}{\partial \tau_2}\left(\frac{-\bar{r}_2^\theta}{\sqrt{\tau_2}}-\theta\right)\left(\bar{r}_2^\theta-\theta\right)^2}{(1+\theta)(\tau_2-\theta)} d\theta \\ &+ \int_{\theta=0}^{\theta_2}\int_{r=\theta}^{\frac{-\bar{r}_2^\theta}{\sqrt{\tau_2}}} \frac{r(r-\theta)(\sqrt{\tau_2}r+\theta)}{\sqrt{\tau_2}(1+\theta)(\tau_2-\theta)} dr d\theta - \int_{\theta=\theta_1}^{0}\int_{r=-\sqrt{\tau_2}\bar{r}_1^\theta}^{\theta} \frac{(\theta-r)\left(-\frac{r}{\sqrt{\tau_2}}-\theta\right)^2}{(1+\theta)(\tau_2-\theta)^2} dr d\theta \\ &+ \int_{\theta=\theta_1}^{0} \frac{\frac{\partial\sqrt{\tau_2}\bar{r}_1^\theta}{\partial\tau_2}\left(\theta+\sqrt{\tau_2}\bar{r}_1^\theta\right)\left(\bar{r}_1^\theta-\theta\right)^2}{(1+\theta)(\tau_2-\theta)} d\theta + \int_{\theta=\theta_1}^{0}\int_{r=-\sqrt{\tau_2}\bar{r}_1^\theta}^{\theta} \frac{r(\theta-r)\left(-\frac{r}{\sqrt{\tau_2}}-\theta\right)}{\tau_2^{\frac{3}{2}}(1+\theta)(\tau_2-\theta)} dr d\theta\Bigg]\end{aligned} \tag{3.35}$$

Consider the first three elements of the derivative. Only the first term is negative, while the other two are positive. In particular, when $\theta = 0$, the positive terms equal the negative one,

$$\frac{\frac{\partial \frac{-\bar{r}_2^\theta}{\sqrt{\tau_2}}}{\partial \tau_2}\left(\frac{-\bar{r}_2^\theta}{\sqrt{\tau_2}}-\theta\right)\left(\bar{r}_2^\theta-\theta\right)^2}{(1+\theta)(\tau_2-\theta)} + \int_{r=\theta}^{\frac{-\bar{r}_2^\theta}{\sqrt{\tau_2}}} \frac{r(r-\theta)(\sqrt{\tau_2}r+\theta)}{(1+\theta)(\tau_2-\theta)\sqrt{\tau_2}} dr = \int_{r=\theta}^{\frac{-\bar{r}_2^\theta}{\sqrt{\tau_2}}} \frac{(r-\theta)(-\sqrt{\tau_2}r-\theta)^2}{(1+\theta)(\tau_2-\theta)^2} dr \tag{3.36}$$

Both sides of the above equation are equal to $\frac{1}{4\tau_2}$. I show now that as θ increases within the interval of integration, the sum of the positive terms exceeds the negative one. Since the function $(r-\theta)(-\sqrt{\tau_2}r-\theta)^2$ is continuous, monotone and convex in

r, I have the following relation,

$$\int_{r=\theta}^{\frac{-\bar{r}_2^{\theta}}{\sqrt{\tau_2}}} (r-\theta)(-\sqrt{\tau_2}r-\theta)^2 dr \leq \frac{1}{2}\left(\frac{-\bar{r}_2^{\theta}}{\sqrt{\tau_2}}-\theta\right)^2 \left(\bar{r}_2^{\theta}-\theta\right)^2 \tag{3.37}$$

Namely, the negative term is always bounded by the function on the right hand side in the above equation. The integral $\int_{r=\theta}^{\frac{-\bar{r}_2^{\theta}}{\sqrt{\tau_2}}} \frac{r(r-\theta)(\sqrt{\tau_2}r+\theta)}{(1+\theta)(\tau_2-\theta)\sqrt{\tau_2}} dr$ is always positive for any $\theta \geq 0$. The difference between the second term and the bound of the negative term is $\frac{\partial \frac{-\bar{r}_2^{\theta}}{\sqrt{\tau_2}}}{\partial \tau_2} - \frac{\left(\frac{-\bar{r}_2^{\theta}}{\sqrt{\tau_2}}-\theta\right)}{2}$, which is increasing in θ for every $\tau_2 > \theta > 0$. The proof works similarly for the last three terms.

□

The following is the proof of Proposition 16.

Proof. I consider now the case of two competing opposing biased media outlets, where I normalise $\tau_1 = -1$ and the costs $k_1 = k_2 = 1$. Proposition 15 says that the higher is the policy-bias of the second media outlet, the higher is W. Therefore, the probability of selecting the wrong alternative is maximised when $\tau_2 \to +\infty$. As follows I calculate W at such limit. The truthful cutoffs when media outlet 2's bias tends to infinity are $\lim_{\tau_2\to\infty} \theta_1 = \frac{1-\sqrt{5}}{2}$ and $\lim_{\tau_2\to\infty} \theta_2 = 1$.

$$\lim_{\tau_2\to\infty} \int_0^{\theta_2} \int_{r=\theta}^{-\frac{\bar{r}_2^{\theta}}{\beta}} \frac{(r-\theta)(-\beta r-\theta)^2}{(\theta+1)(\tau_2-\theta)} dr d\theta \approx 0.08 \tag{3.38}$$

$$\lim_{\tau_2\to\infty} \int_{\theta_1}^{0} \int_{r=-\beta\bar{r}_1^{\theta}}^{\theta} \frac{(\theta-r)(-\frac{r}{\beta}-\theta)^2}{(\theta+1)(\tau_2-\theta)} dr d\theta \approx 0.06 \tag{3.39}$$

That is, the probability of selecting the wrong alternative as the second media outlet becomes infinitely biased is about $\frac{0.14}{\phi}$. Consider now the case where media outlet 1 is the solo monopolistic owner of information about valence. In equilibrium the media outlet misreports when $\theta \in (l_1(\theta^*), \theta^*)$, where $l_1(\theta^*) = -\theta^*$ and $l_1(r) = r + \frac{1}{2}\left[1-\sqrt{1+4(r+1)}\right]$. However, the DM selects the wrong alternative only when $\theta \in (l_1(\theta^*), 0)$. This generates a probability of selecting the wrong alternative of $\frac{\sqrt{17}-1}{16\phi} \approx \frac{0.195}{\phi} > \frac{0.14}{\phi}$. The introduction of a second opposed-biased media outlet is always beneficial from the DM's viewpoint, no matter its conflict of interest. □

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