Doctoral Thesis

Refining Value-at-Risk estimates: An Extreme Value Theory Approach

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Dedicated to

my parents in loving memory: Moses K Galabe and Emmerencia N Galabe. May you continue to rest in peace.

Abstract

This thesis proposes new approaches to Value-at-Risk estimation using (1) Multivariate GARCH Dynamic Conditional Correlation volatility model with skewed Student's-t distributions, (2) Bayesian GARCH model with Student's-t distribution, and (3) Bayesian Markov-Switching GJR-GARCH model with skewed Student's-t distributions, incorporating copula functions and extreme value theory. A new approach for selecting a proper threshold in the Peaks Over Threshold method for extreme value theory analysis called the *hybrid* method is also proposed. The proposed Value-at-Risk models are compared to the traditional Value-at-Risk models commonly used by banks. Back-testing results following Kupiec (1995) unconditional coverage test, Christoffersen (1998) independent and conditional coverage test, Basel *traffic light* test, Santos and Alves (2012) new independent test, Dowd (2002) bootstrap back-test, and Engle and Manganelli (2004) Dynamic Quantile test show that Value-at-Risk models constructed following extreme value theory produced reliable Value-at-Risk estimates. Furthermore, Value-at-Risk models incorporating the *hybrid* method for threshold selection produced more stable Value-at-Risk estimates compared to the traditional Value-at-Risk models.

Declaration

The work in this thesis titled “Refining Value-at-Risk estimates: An Extreme Value Theory Approach” is based on research carried out at the Department of Mathematical Sciences, University of Essex, England. No part of this thesis has been submitted elsewhere for any other degree or qualification and it is all my own work unless referenced to the contrary in the text and supervised by Dr Haslifah Mohamad Hasim.

Parts of this thesis have been published or submitted to journals for publication in the form of papers:

1. Estimating value-at-risk using a multivariate copula-based volatility model: Evidence from European banks. Sampid, M. G., Hasim, H. M., International Economics (2018). http://dx.doi.org/10.1016/j.inteco.2018.03.001.

2. Refining Value-at-Risk estimates using a Bayesian Markov-switching GJR-GARCH copula-EVT model. Sampid MG, Haslifah HM, Dai H, PLoS ONE Volume 13 (2018). https://doi.org/10.1371/journal.pone.0198753.

3. Forecasting robust Value-at-Risk estimates: Evidence from UK banks (Marius Galabe Sampid, Haslifah M. Hasim). Submitted to Quantitative Finance on 26 September 2017, conditional acceptance on 29 June 2018 and is currently under final review for publication.

Papers 1 and 2 above were presented at the Seventh International Conference of Mathematics and Statistical Methods for Actuarial Sciences and Finance MAF April 2016, University of Dauphine, Paris France, and the International Risk Management Conference (IRMC) June 2017, University of Florence, Italy, respectively.

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Chapter 1

Introduction

What is risk? And where does it come from? Risk is the probability that a chosen action or activity will lead to an undesirable outcome (Asbury, 2014). Risk can come as a result of (Jorion, 2007):

- unpredicted natural disaster; the 2011 Tsunami in Japan, 2005 Hurricane Katrina in New Orleans, USA,
- man-made; the September 11 2001 terrorists attack of the World Trade Center in New York, USA,
- from the primary source of long term economic growth namely, technological innovations, which can render existing technology obsolete and create dislocations in employment.

Financial institutions and firms are exposed daily to various categories of risks, which can be classified under business risk and financial risk. Business risk is the probability of a loss related with a given managerial decision. For example, decisions such as investment decisions, product-development choices, marketing strategies, the choice of the company's organizational structure and the business environment in which they choose to operate (Miles, 2011; Jorion, 2007). Financial risk is the risk that possible loses will occur due to various uncertain activities in the financial market (Wu, 2011). The identification, assessment, and prioritisation of financial risk followed by a well defined approach to minimise, monitor, and control the probability of such undesirable outcome from happening is referred to as financial risk management (Jorion, 2007).

1.1 Financial risk management an overview

The growth of the risk management industry has a long history as far back as the early 1970*s* following the increased instability of financial markets. For example: the fixed exchange rate system broke down in 1971, which led to flexible and volatile exchange rates. The Russian default in August 1998 sparked a global financial crisis that culminated in the near failure of a big hedge fund, Long Term Capital Asset Management. The September 11, 2001, terrorist attack, in the United States, destroyed the World Trade Center in New York, disrupting the financial markets for six days. In addition to the unspeakable human cost, the U.S. stock market lost $1.7 trillion in value (Jorion, 2007). These kind of events are very difficult to predict and plan for, however extremely destructive when they occur (Malz, 2011).

Financial risk management thus provides some limited protection against such sources of risk. Strategies of financial risk management usually involves transferring the risk to another party, avoiding the risk, reducing the negative effects or the probability of the risk from happening or even accepting some or all of the potential or actual consequences of a particular risk. Following the collapse of the Herstatt bank in Cologne in 1974, the central bank governors of the industrialise nations (the G-10) established the Basel Committee of Banking and Supervision (BCBS), which drafted rules and regulations for the banking industry so as to avoid future major bank collapses. This lead to the creation of the Basel

I accord in 1988, Basel II accord in 1996, and the Basel III accord in 2010.

In order to assess risk on financial assets, the variables of interest have to be clearly defined; portfolios values, earnings, capital, or any cash flows (Jorion, 2007), and also understand the various risk measures involved. An analysis of delta, gamma, and vega, which describes various aspects of risk in a portfolio of derivatives, produces very large number of different risk measures over the year. These risk measures gives valuable information for the financial institutions traders concern. Unfortunately, they do not provide a technique of measuring the total risk to which the financial institution is exposed to (Penza and Bansal, 2001). Value-at-Risk (VaR), a statistical technique used to measure and quantify the level of financial risk within a firm or investment portfolio over a specific time frame, provides an attempt to summarise the risk in a portfolio of financial assets to a single number. Investors do consider risk as the odds of losing money and VaR tries to calculate, in the worst case scenario, how much an investor can lose with a certain probability and within a certain time frame.

Despite all the technical tools put in place by financial institutions to calculate and monitor financial risk, there have been so many controversies. Though the BCBS, on the Basel II accord, recognised VaR models as the official risk management measure for measuring market risk, some researchers have criticised VaR models for a series of reasons. Most importantly is that VaR is calculated based on past events; because several assets in the past were negatively correlated do not necessary mean that these assets cannot be positively correlated in the future. Thus keeping the same portfolio of assets based on past outcomes will yield great losses in case some of the assets in the portfolio become positively correlated in the future. Furthermore, the calculations behind VaR tend to assume that markets follow a normal probability distribution. This can contribute to VaR having extremely "small values" while "big loses" are ruled out because extreme values are assumed to have very small chances of happening (Artzner et al., 1999).

1.2 Aim, Objectives, and Limitations

The aim of this research has emerged due to the fact that VaR, a very important risk measure in financial risk management, has been in the past years fiercely criticised for providing incorrect results of the level of risk in a portfolio of financial assets. Some researchers (Carmassi and Micossi, 2012; Rossignolo et al., 2012) have associated the 2008 global financial crisis to the failure of Basel II accord which was designed to ensure that banks have sufficient capital to provide a proper cushion capable to withstand sudden losses in periods of financial distress. (Turner et al., 2009) claimed that most VaR models were unable to capture fat-tailed risk. Thus, this research is a critical study of VaR models following the 2008 global financial crisis and the 2011 European financial crisis aimed at answering the following questions:

1. Can VaR be sufficiently improved to credibly communicate a banks risk?
2. Can Basel II be blamed for the 2008 crisis or poorly calibrated VaR models incapable to capture fat-tail risk?

In order to answer these questions, we develop our study along the following lines:

1. Understanding the VaR methods.
2. Reviewing and evaluating VaR models.
3. Propose alternative VaR models.

This research will focus solely on VaR models and *Generalized Autoregressive Conditional Heteroscedasticity* (GARCH(p,q)) volatility models as the underlying volatility and their implementations as risk measures in financial risk management and not a study of financial risk management as a whole. Many research works, for example, Sampid and Hasim (2018), Chen et al. (2017), Ardia et al. (2016), Ghalanos (2015), Soltane et al. (2012), Aas et al. (2009), Tsay et al. (2006), Haas et al. (2004), Bollerslev (1986), and many more, have shown that a simple GARCH(1,1) model is capable of modeling the serial correlation in the conditional mean and the conditional variance of a time series data. We therefore, follow the footsteps of these many research works and restrict the GARCH(p,q) volatility models implemented in this research to GARCH(1,1). Higher order GARCH(p,q) volatility models will be tested in subsequent future research works.

1.3 Research Design

This research work will be carried out using quantitative research methods of data gathering and analysis. This will involve analysis of historical financial time series data, official documents, journals, and books that discuses the perspectives of VaR.

This thesis is structured as follows:

- In Chapter 1, we give a brief introduction of the concept of financial risk management overall. We will also explain the aim, objectives and limitations of this research, as well as the research design.
- In Chapter 2, we introduce Value at Risk (VaR) including VaR definition, methods for estimating VaR and methods for validating VaR models.
- In Chapter 3, we present a novel approach to construct and investigate the reliability of a VaR model constructed using a multivariate GARCH dynamic conditional correlation volatility model with skewed student's-*t* distribution, copula functions and extreme value theory. We estimate VaR for banks of some selected European

countries, and thus offers a new contribution to the literature in this area of study.

- In Chapter 4, we present a novel approach to VaR estimation by combining a Bayesian GARCH(1,1) model with student's-t distribution, vine copula functions and extreme value theory. We propose a new method; an objective approach for selecting a proper threshold for extreme value theory analysis which we call the *hybrid* method for threshold selection and estimate VaR for some selected banks in the UK.

- In Chapter 5, we further test the *hybrid* method for threshold selection by applying extreme value theory directly to the exposures to risk factors and estimate VaR for some selected UK banks.

- In Chapter 6, we propose a model for forecasting Value-at-Risk (VaR) using Bayesian Markov Switching GJR-GARCH(1,1) model with skewed Student's-t distribution, copula functions and extreme value theory. Thus taking into account regime changes and time varying parameters.

- In Chapter 7, we present a summary of this thesis, conclusion and future work.

Chapter 2

Value at Risk

2.1 Introduction

Value at Risk (VaR) has, in the past decades, become very instrumental tool when it comes to measuring market risk as it provides risk managers with a quantitative measure of downside risk within a firm or investment of portfolio over a certain time frame. VaR provides an attempt to summarise the total risk in a portfolio of asset to a single number over a target horizon. The idea of VaR can be traced as far back as 1952. Harry Markowitz and Roy Arthur in 1952 both came out with mathematical models for VaR that produced very similar results (Holton, 2002). The idea was to create a mathematical model of portfolio selection that would optimise the expected return at a given level of risk. Markowitz explained that investors should be interested in risk as well as return. He studied the tradeoff between risk and return in the mean-variance framework, which is suitable when returns are normally distributed (Jorion, 2007). Roy's work was based on the "safety first" criteria for portfolio selection. It states that "the best portfolio is the one that has the smallest probability of producing a return bellow some specified level" (Elton et al., 2009). That is, *minimise* $(R_p < R_L)$, where R_p is the return on the portfolio and R_L is the level below which the investor does not wish returns to fall. With normally distributed returns, the optimum portfolio will be the one with R_L having the highest or maximum number of standard deviations away from the mean. The number of standard deviations R_L that lies below the mean is determine by subtracting the mean or expected return of the portfolio, $E(R_p)$, from R_L and divide by the portfolios standard deviation, σ_p. That is, *minimize* $\frac{R_L - E(R_p)}{\sigma_p}$. The origin of the name Value at Risk is "murky" because there were similar other names being used in the 1990′s such as "Dollar at Risk" (DaR), "Capital at Risk" (CaR), "Income at Risk" (IaR), "Earnings at Risk" (EaR), and "Value at Risk" (VaR). "It seems that users liked the *at Risk* [name], but were uncomfortable labeling exactly what was at risk" (Holton, 2002) .

European banks began adopting VaR in the early 1990s (Holton, 2002). International bank regulators also influenced the development and use of VaR when the Basel Committee of Banking and Supervision (BCBS) chose VaR as the international standard method for evaluating market risk of a portfolio of financial assets for regulatory purposes (Goodhart, 2011). Till Guldimann during his time as the head of global research at JP Morgan in the late 1980s can be viewed as the inventor of the name Value at Risk:

> The risk management group had to decide whether fully hedged meant investing in long-maturity bonds, thus generating stable earnings but fluctuations in market values, or investing in cash, thus keeping the market value constant. The bank decided that value risks were more important than earnings risks, paving the way for [VaR]. At that time, there was much concern about managing the risk of derivatives properly. The group of 30 (G-30), which had a representative from J.P. Morgan, provided a venue for discussing best risk management practices. The term found its way through the G-30 report published in July 1993. Apparently, this was the first widely publicised

appearance of the term Value at Risk (Jorion, 2007).

The rest of the Chapter is structured as follows: In Section 2.2 we define VaR, Section 2.3 presents various traditional methods to estimate VaR, Section 2.4 discuses some setbacks of VaR models. In Section 2.5, we present the expected tail loss i.e., loss beyond VaR estimates. In Section 2.6, we introduce the supervisory framework; the BCBS, followed by statistical approaches for validating VaR models in Section 2.7 and chapter summary in Section 2.8.

2.2 VaR Definition

VaR is a statistical quantity used to measure and quantify the level of market risk of financial assets within a firm or investment of portfolio over a specific time frame;

$$P(L_t > VaR_{q,t}) \leq 1 - q \tag{2.1}$$

that is, VaR is the smallest lost in absolute value, such that at a certain confidence level q and time t, the probability of experiencing a greater loss L is less than $1 - q$ (Jorion, 2007). Based on loss quantiles, VaR is defined as:

$$VaR_q(\mathbf{X}) = \inf\{x : \Pr(\mathbf{X} \leq x|\Omega) \geq q\}, \tag{2.2a}$$

$$= \inf\{x : \Pr(\mathbf{X} \leq VaR(q)|\Omega) \geq q\}, \tag{2.2b}$$

$$\Longrightarrow VaR_q(\mathbf{X}) = F_{\mathbf{X}}^{-1}(q) \tag{2.2c}$$

$$X = \begin{cases} -x & \text{for a long position} \\ x & \text{for a short position,} \end{cases} \tag{2.2d}$$

where **X** is the value of a portfolio or exposure to risk factor at time t, F_X is the distribution function of the loss random variable X, and Ω is the information available at time $t-1$. Eqn. (2.2b) can also be written as

$$VaR_q(\mathbf{X}) = \inf\{x : \Pr(\mathbf{X} + VaR < 0) \leq 1 - q\}, \tag{2.3}$$

which represents the smallest amount of money which when added to **X** keeps the probability of a negative outcome at any given time Δt below the level $1-q$ (Cherubini et al., 2011).

The calculations of VaR have three parts; time horizon ($\Bbbk$), the lost amount L, and the confidence level q. The time horizon (also known as the holding period) is usually between one day and one month. However, investors who invest in longer term investments such as pension funds will be much more interested in a longer time horizon of one year or more (Malz, 2011). Longer $\Bbbk$ will lead to increase in the level of VaR because volatility in financial markets increases approximately in proportion to the square root of time (Best, 2000). Banks usually calculate daily VaRs to compare with their daily profit and lost calculations; in case the VaR is not acceptable, the portfolio can be adjusted fairly quickly (Hull, 2009).

The confidence level can be between 90% and 99.99% depending on the preferences of the market participants using the VaR, the limitations of data and models used to calculate the VaR. VaR measures use the one tailed confidence level; concerned only with possible loses and not profit. Only the downward percentage of price changes not covered by the multiple of standard deviations are being used (Best, 2000). Just like with the holding period, VaR increases as the confidence level increases.

2.3 VaR Methods

There are many methods to forecast VaR (see Holton (2014); Malz (2011); Jorion (2007) and the references therein), which can be classified under:

- parametric methods; following analysis with econometric models, RiskMetrics, extreme value theory, and Monte Carlo simulation,
- nonparametric methods; historical simulation.

2.3.1 Parametric Methods

Consider the information set $\Omega_{t-1} = \{r_1, r_2, \ldots, r_{t-1}\}$ and $\mu_t = \mathbf{E}\left[r_t|\Omega_{t-1}\right]$; the expected return of r_t given some information at time $t-1$. Assume r_t is a stochastic process $\{r_t : t \in T\}$ given by

$$r_t = \mu_t + a_t, \quad a_t = z_t\sigma_t \tag{2.4a}$$

$$\mathbf{E}[a_t] = 0, \quad z_t \overset{iid}{\sim} N(0, 1; \theta),$$

$$\sigma_t^2 = g(\Omega_{t-1}; \omega). \tag{2.4b}$$

where θ and ω are vectors of unknown parameters of the mean (Eqn.(2.4a)) and variance (Eqn.(2.4b)) equations, and $g(.)$ is a time varying, positive and measurable function of information set Ω_{t-1} available at time $t-1$ (Angelidis and Skiadopoulos, 2008). VaR is then estimated as

$$VaR_{q,t} = \hat{\mu}_{t+1} + F_q^{-1}(X)\hat{\sigma}_{t+1}, \tag{2.5}$$

where $F_q^{-1}(X)$ is the q^{th} quantile of the distribution function of the loss random variable as defined in Eqn.(2.2d), $\hat{\mu}_{t+1}$ and $\hat{\sigma}_{t+1}$ are estimates of the conditional mean and conditional volatility at time $t+1$ respectively, given parameters θ and ω. For economic models, the mean and variance equations follows an *autoregressive moving average* (ARMA) process and a *generalized autoregressive conditional heteroscedasticity* (GARCH) process respectively.

RiskMetrics developed by J.P Morgan in the early 1990*s* assumes that asset returns in financial markets follows a conditional normal distribution i.e., $r_t[\Bbbk]|\Omega_{t-1} \approx N(0, \Bbbk\sigma^2_{t+1})$, where σ^2_t is the conditional variance of the return series r_t proportional to the time horizon $\Bbbk$. Under this condition, the mean equation is an *integrated* GARCH(1,1) (IGARCH(1,1)) process given by

$$r_t = a_t, \quad a_t = z_t\sigma_t \tag{2.6a}$$

$$\sigma^2_t = \alpha\sigma^2_{t-1} + (1-\alpha)a^2_{t-1}, \quad 0.9 < \alpha < 1. \tag{2.6b}$$

VaR is then estimated as

$$VaR_{q,t} = \Phi^{-1}_q(q)\sqrt{\Bbbk}\hat{\sigma}_{t+1}, \tag{2.7}$$

where $\Phi^{-1}_q(q)$ denotes the q^{th} quantile of the normal distribution and α is the parameter of the IGARCH(1,1) process.

Just like the RiskMetrics, the Variance-Covariance (VC) method also assumes normality of financial asset returns but rejects conditionality of the variance equation. Instead, the VC method uses the unconditional volatility σ of the return series as the underlying volatility model and VaR is estimated as

$$VaR_{q,t} = \Phi^{-1}_q(q)\sqrt{\Bbbk}\sigma. \tag{2.8}$$

Monte Carlo simulation can be use to understand the impact of risk of uncertainty of financial models. Running repeated trials of stochastic processes using random variables of financial data reconstructs the entire distribution of a portfolio, which can be use to forecast the level of risk on the portfolio. It also assumes normality of asset returns based on the strong law of large numbers and Central Limit Theorem (CLT). That is; let X be a real random variable whose mean $\mu = E[X_1]$ exist, and let $X_1, X_2, \ldots$ be an infinite sequence of *iid* replicas of X, the strong law of large numbers states: as n becomes sufficiently large, the sample $\bar{X} = \frac{1}{n}(X_1, X_2, \ldots X_n)$ obtained from a large number of trials converges to the unknown population mean $E[X_1]$ almost surely. The CLT states: with finite mean μ and finite variance $\sigma^2 > 0$ defined as

$$Z_n = \frac{X_1 + X_2 + ... + X_n}{\sigma\sqrt{n}}, \qquad n = 1, 2, ... \tag{2.9}$$

then the distribution of the sample mean is approximately normally distributed as n becomes large, i.e., $\lim_{n\to\infty} P\{Z_n \leq X\} = \Phi(X)$, where $\Phi(X)$ is a standard normal distribution function and $X \in \Re$ (real numbers) (Kijima, 2016). VaR is then estimated as

$$VaR_{q,t} = \hat{\mu}\delta t + \Phi_q^{-1}(q)\sqrt{\delta t}\hat{\sigma}, \tag{2.10}$$

for small changes in time δt.

Extreme value theory (EVT) is now widely used in financial risk management as it has been proven that majority of financial asset return distributions have heavy tails (see Berkowitz et al. (2011); Sheikh and Qiao (2010)). As VaR is concern with losses on the extreme left tail, EVT becomes a useful tool for statistical inference on the left tail. It is used to estimate extreme events with low frequency of happening but with high severity.

The EVT VaR is estimated as

$$VaR_{q,t} = \hat{\mu}_{t+1} + VaR_q(Z)\hat{\sigma}_{t+1}, \tag{2.11}$$

where $VaR_q(Z) = F_q^{-1}$ is the q^{th} quantile of the tail estimator defined as

$$VaR_q(Z) = \eta - \frac{\hat{\psi}(\eta)}{\hat{\xi}} \left\{ 1 - \left[\frac{T}{N_\eta}(1-q) \right]^{-\hat{\xi}} \right\}, \tag{2.12}$$

where η is the threshold, $\hat{\psi}(\eta)$ and $\hat{\xi}$ are the scale and shape parameters from EVT analysis respectively, T is the total number of observations, and N_η is the number of observations above the threshold (Tsay, 2014; Soltane et al., 2012; Bhattacharyya and Ritolia, 2008).

2.3.2 Nonparametric Method

Historical Simulation (HS) is a nonparametric method that uses the actual historical data for VaR estimation. It arranges historical data in order from worst to best case and tries to reproduce previous history on the existing position without any assumptions about the distribution of risk factors. The HS VaR is estimated as

$$VaR_{q,t} = F_q^{-1}(\mathbf{r}), \tag{2.13}$$

where F_q^{-1} represents the qth quantile of the historical profit and loss distribution $\mathbf{r} = \{r_1, \ldots, r_t\}$. This method captures the characteristics of the price change distribution of the portfolio, as VaR is calculated from the actual distribution of portfolio value changes. As a result, where a portfolio distribution has fat tails, it will turn to produce a slightly higher VaR estimates than the VaR calculated from the variance-covariance method (Best, 2000).

2.4 Draw Backs

VaR, though widely used by financial institutions as the major risk management measure, does face some potential drawbacks depending on the method. VC and and Monte Carlo simulation, for example, assume financial markets to be normally distributed, which imply that changes in asset prices are independent of each other. However, the distribution of asset returns do exhibit heavy tails with significant serial correlation. Because the risk on a portfolio is much dependent on the correlation between risk factors, the normality assumption becomes problematic in case of high positive correlation (Best, 2000). VC method does not capture the asymmetries in distributions of complex portfolios for nonlinear instruments whose payoffs changes with time while assuming normal distribution (Jorion, 2007). Volatility for example, depends on market movements and varies depending on the behavior of the asset and financial markets changes. Using stationary parameters to estimate VaR can be misleading because nonstationarities of the underlying return processes are also a source of risk as it imply that the distribution of risk factors can change over time (Tapiero, 2004). Monte Carlo simulations rely on stochastic processes. Wong (2013) has shown that stochastic volatility gives rise to a distribution with heavy tails. If normality is assumed, the VaR model will produce small values of VaR forecasts while extreme values are ruled out. If normality is not assumed and depending on how the VaR model is constructed (for example incorporating EVT), then there is high chance that the VaR model will capture extreme events.

HS method is quite simple but becomes very computational when the portfolio contains many assets with relatively longer lengths of historical data sets. However, for reliable VaR estimates, HS requires longer lengths of historical data. Jorion (2007) points out that a 99% daily VaR estimate for an observation period of 100 days only produces 1 observation in the tail on average, which necessarily leads to an unreliable VaR measure.

Also, the Var estimate might not be reliable because the sample data may also contain events such as the September 11, 2001 terrorist attack on the World Trade Center that may not or never appear in the future. Such events will inevitably lead to an overestimation of VaR.

Artzner et al. (1999) examined the credibility of VaR measures and suggested the following properties that a risk measure should satisfy:

- *Monotonicity*: If the return of portfolio 1 (P_1) is less than portfolio 2 (P_2), then P_1 has greater risk ($\rho(P_1)$) than P_2; $P_1 \leq P_2$ then $\rho(P_1) \geq \rho(P_2)$
- *Translation invariance*: Adding cash (C) to a portfolio reduces the risk by the amount of cash added; $\rho(P_1 + C) = \rho(P_1) - C$
- *Homogeneity*: Altering the size of a portfolio by a certain amount (γ) will simply scale the risk of the portfolio by γ; $\rho(\gamma P_1) = \gamma\rho(P_1)$
- *subadditivity*: Risk measure of two portfolios merged together cannot be greater than the sum of the risk measures of the individual portfolios; $\rho(P_1 + P_2) \leq \rho(P_1) + \rho(P_2)$

A risk measure that satisfies these four properties is said to be coherent. Quantile based VaR measures does not satisfy the subadditivity property unless the distribution is normal. For example, many short option positions with a low probability and hence low VaR merged together will create portfolios with increased risk rather than reducing risk (higher estimates of VaR) (Jorion, 2007).

VaR measures provides a rough estimate of potential future losses in normal market conditions but fails to provide any clue of what happens beyond VaR. In the case of buying and selling of investment instruments such as futures, commodities and foreign exchanges, etc., some portfolios might have losses very close to VaR while others might have losses several times greater than the calculated VaR (Jorion, 2007).

VaR benefits from the CLT. i.e, the size of the portfolio does contribute to the accuracy of VaR. This justifies the normality assumption of a portfolio spread across risk factors when the size is sufficiently large. If the size of the risk factors is not large enough, assuming normality will most probably lead to underestimation of risk (Jorion, 2007).

The most common drawback with EVT VaR is the lack of sufficient data on the left tail to do meaningful statistical inference. As seen later, EVT VaR suffers from the choice of threshold selection, which is very subjective. Based on their risk tolerance and preferences, different analyst might select different thresholds on the same data which will result to different VaR estimates. EVT also assumes extreme-events; events above the threshold, to be *iid* which might not hold in periods of severe crisis (Wong, 2013).

2.5 Expected Shortfall

One big disadvantages of VaR models is that it gives an estimate of a potential future loss and nothing about what happens beyond the estimated value. Expected Shortfall (ES) also known as Expected Tail Loss (ETL), Tail VaR (TVaR), Average VaR (AVaR), or Conditional VaR (CVaR) (Artzner et al., 1999) does provide an expectation of losses beyond VaR estimates. Besides, it can be shown that ES does satisfy all the conditions of the coherent property for risk measures. It can be seen as an extension of VaR to address the drawbacks of VaR measures. Denote the loss random variable X, as defined in Eqn.(2.2d), ES is defined as

$$E(X|X > VaR) = \frac{\int_{VaR}^{\infty} x f(x) dx}{Pr(X > VaR)} \tag{2.14}$$

that is, the expected loss of X given that the loss exceeds VaR.

$$Pr(X > VaR) = \int_{VaR}^{+\infty} f(x)dx. \tag{2.15}$$
$$= 1 - F(VaR)$$

The relationship between VaR and ES can be seen if we assume X to be continuous and let $u = F(x)$ for $VaR \leq x \leq \infty; \implies du = f(x)dx, F(VaR) = 1 - p, F(\infty) = 1$, $x = F^{-1}(u) = VaR_u$. Therefore, Eqn. (2.14) becomes

$$ES_{1-p} = \frac{\int_{1-p}^{1} VaR_u du}{p} \tag{2.16}$$

which is a simple average of all the points in the left tail of the VaR quantile and reflects the tail behavior of X better than VaR (Tsay, 2014; Wong, 2013). Thus, as VaR attempts to summarise the total risk in a portfolio of financial asset returns to a single number over a target horizon $\Bbbk$, ES summarises the expectation of losses beyond VaR estimates. Tsay (2014); Alexander (2009); Embrechts et al. (2005) have shown that for a normal distribution, the ES is given by

$$ES_{1-p} = \mu_t + \sigma_t \frac{\phi(\Phi^{-1}(1-p))}{p} \tag{2.17}$$

where ϕ is the probability density function of a standard normal random variable, $\Phi^{-1}(1-p)$ is the $(1-p)^{th}$ quantile of the inverse of the standard normal cumulative distribution function for a small tail probability p. For a standardised Student's-t distribution with $v > 2$ degrees of freedom, the ES for the random variable X is given by

$$ES_{1-p} = \mu_t + \sigma_t \sqrt{\frac{v}{v-2}} \left(\frac{t_v(t_v^{-1}(1-p))}{p} \right) \left(\frac{(v-2) + [t_v^{-1}]^2}{v-1} \right) \tag{2.18}$$

where t_v is the probability density function of a standardised sudent's-t distribution, $t_v^{-1}(1-p)$ is the inverse $(1-p)^{th}$ quantile of t_v.

2.6 Basel Committee of Banking and Supervision

Following the collapse of the Herstatt bank in Cologne in 1974, the G-10 central bank governors established the Basel Committee of Banking and Supervision (BCBS), which drafted rules and regulations for the banking industry so as to avoid future major bank collapses. In 1988, BCBS adopted the first Basel accord (Basel I) by introducing minimum capital requirements (MCR) of 8% of risk weighted assets (RWA) that must be held by banks. Basel I mainly addressed the problem of credit risk by raising deposits on lending to households and businesses (Carmassi and Micossi, 2012). The risk of collapse of a Banking system with insufficient capital that can provide proper cushion capable to withstand sudden losses in periods of distress was huge as a result of interest rate and market risks being totally neglected and no capital requirements defined.

Following critics from regulators and banks, Basel II was adopted in the late 1990s by amending Basel I to incorporate capital requirements for market risk. Designed to encourage banks in sensible risk taking, Basel II allows banks to calculate MCR for market risk based on their internal 99% VaR models using the Internal Model Approach (IMA):

$$MCR_t = max\left(\frac{\kappa}{60}\sum_{i=1}^{60} VaR_{t-i}, VaR_{t-1}\right) \tag{2.19a}$$

$$\kappa = \begin{cases} 3 & \text{if } \; T_1 \leq 4, \quad \text{green zone} \\ 3 + 0.2(T_1 - 4) & \text{if } \; 5 \leq T_1 \leq 9, \quad \text{yellow zone} \\ 4 & \text{if } \; T_1 \geq 10, \quad \text{red zone.} \end{cases} \tag{2.19b}$$

which is the maximum of the average of the last 60 days VaR or the most recent VaR. Basel also introduced a back-testing procedure to validate the reliability of the banks' internal 99% VaR model from which the supervisory multiplier κ is determined. Back-testing was designed to compare the subsequent VaR estimates with the actual returns and recording the number of days T_1 in which the realised losses exceeded the 99% VaR for a 250 days observation period. Basel also requires a liquidation period of 60 days, which they believe is sufficient enough for a financial institution in trouble to raise funds. Eqn.(2.19a) is design such that $VaR_{t-1} > \frac{\kappa}{60}\sum_{i=1}^{60} VaR_{t-i}$ will only occur in periods of extreme crisis such as a crash. The internal VaR model must be validated by supervisors.

2.7 Model Evaluation Framework

How reliable is the VaR model? The reliability of the VaR model i.e., the model does not overestimate or underestimate risk, is assessed by performing back-testing for some desired observation periods and confidence level. This involves comparing the VaR estimates for a given observation period to the subsequent returns. The number of days T_1 in which the loss on the portfolio exceeds VaR is recorded as the number of exceptions or failures. Too many exceptions implies the VaR model underestimates the level of risk, and too few exceptions implies the model overestimates risk. For the VaR model to be accepted as a reliable risk measure, the number of exceptions produced for any given observation period should satisfy the unconditional coverage (UC) and independent (IND) property. Consider an indicator function on the exceptions

$$\mathbb{I}_t(p) = \mathbb{I}_{\{L_t > VaR_{q,t}\}} = \begin{cases} 1, & \text{if } \; L_t > VaR_{q,t} \\ 0, & \text{otherwise,} \end{cases} \tag{2.20}$$

that is, $\mathbb{I}$ registers a 1 on day t if the loss on the portfolio L_t on day $t > VaR_{q,t}$ and 0 if the loss on day $t \leq VaR_{q,t}$. q is the choice of confidence level and $p = 1 - q$. For UC property, $\Pr[\mathbb{I}_t(1-q) = 1] \approx 1 - q, \forall_t$; i.e., the number of exceptions should be reasonably close to $T_w(1-q)\%$, depends on the choice of q, and should follow a binomial distribution

$$f(T_1|T_w, p) = \binom{T_w}{T_1} p^{T_1} q^{T_w - T_1}, \tag{2.21}$$

with mean pT_w and variance pqT_w. T_w is the size of the window over which back-testing is conducted. For IND property, the exceptions produced on day $t-1$ should be independent of exceptions produced on day t and evenly spread over time.

Several back-testing methods have since been proposed to test the UC and IND properties of reliable VaR models. In this research, the following back-testing methods are employed: (1) Kupiec (1995) "proportion of failures" (POF) test for UC, (2) Christoffersen (1998) test for IND and conditional coverage (CC), (3) the "traffic light" test proposed by the 1996 Basel Committee of Banking and Supervision (BCBS), (4) the new independence test for the tendency of clustering violations by Santos and Alves (2012), (5) the bootstrap back-test by Dowd (2002), and Engle and Manganelli (2004) Dynamic Quantile test.

2.7.1 Unconditional coverage (UC) test

Kupiec (1995) defined an approximate 95% confidence region whereby the number of exceptions produced by the VaR model must lie within this region for it to be considered a reliable risk measurement model. The test is based on the likelihood ratio

$$LR_{POF} = -2 \ln \frac{q^{T_0} p^{T_1}}{\left(1 - \frac{T_1}{T_w}\right)^{T_0} \left(\frac{T_1}{T_w}\right)^{T_1}} \approx \chi^2_1, \tag{2.22}$$

where $q = 1 - p$, $T_0 = T_w - T_1$, and the known VaR coverage p. Under the UC, the null hypothesis for LR_{POF} is $H_0 : E[\mathbb{I}_t(p)] = \frac{T_1}{T_w} = p$ against $H_a : E[I_t(p)] = \frac{T_1}{T_w} \neq p$. The VaR model is rejected if $LR_{POF} > \chi_1^2 = 3.841$. Note that Chi-Square distribution with k degrees of freedom (χ_k^2) is the sum of the squares of k independent samples from a normal distribution with a zero mean and unit variance (Lancaster and Seneta, 1969). We could also obtain a rejection region $[x_1, x_2]$ by equating Eqn.(2.22) to χ_1^2 and solving for T_1. The VaR model is rejected if $T_1 \notin [x_1, x_2]$ and accepted if $T_1 \in [x_1, x_2]$ (Holton, 2002).

2.7.2 Independent (IND) and conditional coverage (CC) test

Christoffersen (1998) extended Kupiec's POF test to test the independence of conditional coverage. Define T_{ij} the number of days that $\mathbb{I}_t = j|\mathbb{I}_{t-1} = i$, and $\pi_{ij} = \Pr(\mathbb{I}_t = j|\mathbb{I}_{t-1} = i)$, then we have:

	$\mathbb{I}_{t-1} = 0$	$\mathbb{I}_{t-1} = 1$	sum
$\mathbb{I}_t = 0$	T_{00}	T_{10}	$T_{00} + T_{10}$
$\mathbb{I}_t = 1$	T_{01}	T_{11}	$T_{01} + T_{11}$
sum	T_{00}+T_{01}	$T_{10} + T_{11}$	$T_{00} + T_{01} + T_{10} + T_{11}$

and probabilities

$$\pi_{01} = \frac{T_{01}}{T_{00} + T_{01}}, \quad \pi_{11} = \frac{T_{11}}{T_{10} + T_{11}} \quad \pi = \frac{T_{01} + T_{11}}{T_{00} + T_{01} + T_{10} + T_{11}}.$$

Under the null hypothesis that the number of exceptions produced are independent and evenly spread over time, $\pi_{01} = \pi_{11} = \pi$ with likelihood ratio

$$LR_{IND} = -2 \ln \frac{(1-\pi)^{(T_{00}+T_{10})} \pi^{(T_{01}+T_{11})}}{(1-\pi_{01})^{T_{00}} \pi_{01}^{T_{01}} (1-\pi_{11})^{T_{10}} \pi_{11}^{T_{11}}} \approx \chi_1^2. \tag{2.23}$$

The model is rejected for the independent property if $LR_{IND} > \chi_1^2 = 3.841$. Christoffersen (1998) conditional coverage test is a joint test of Kupiec's POF test and the IND test that test both properties of unconditional coverage and independence instantaneously. The conditional coverage test has likelihood ratio

$$LR_{CC} = LR_{POF} + LR_{IND} \approx \chi^2_2. \tag{2.24}$$

The hypothesis is $\Pr[\mathbb{I}_t(1-q) = 1|\Omega_{t-1}] = 1-q, \forall_t$ against $\Pr[\mathbb{I}_t(1-q) = 1|\Omega_{t-1}] \neq 1-q, \forall_t$, where Ω_{t-1} is the information available on day $t-1$. The model is rejected for the conditional coverage property if $LR_{CC} > \chi^2_2 = 5.99$.

2.7.3 Basel "Traffic Light" Test

The BCBS came up with a set of requirements that the VaR model must satisfy for it to be considered a reliable risk measure. That is, (1) VaR must be calculated with 99% confidence, (2) back-testing must be done using a minimum of one year observation period and must be tested over at least 250 days, (3) regulators should be 95% confident that they are not erroneously rejecting a valid VaR model, and (4) Basel specifies a one-tailed test —it is only interested in the underestimation of risk (Resti, 2008). For an unbiased VaR model, we will expect a maximum of 2.5 violations over a period of 250 days at p=1% confidence level. Depending on the number of exceptions produced, the financial institution is placed in a green, yellow, or red zone. Eqn.(2.19b) and Table 2.1 summarises the acceptance region for the Basel "traffic light" test for a 250 days observation period. In the red zone, the VaR model underestimates risk and is out-rightly rejected.

Zone	Number of Exceptions	Cumulative Probability
Green	≤ 4	89.22%
Yellow	5	95.88%
	6	98.63%
	7	99.60%
	8	99.89%
	9	99.97%
Red	≥ 10	99.99%

Table 2.1: *Acceptance region for Basel "traffic light" test for back-testing VaR models. CL = 99%, T = 250 (Jorion, 2007).*

2.7.4 The new independent test

The new independent test proposed by Santos and Alves (2012) is a test based on an exact distribution that does not depend on an unknown parameter. The test is used for identifying models with a tendency to generate violations that are clustered together. Unlike Kupiec (1995) POF test and Christoffersen (1998) CC test, the new independent test does not depend on an asymptotic distribution. The test statistics for the new independence test is defined as

$$T_{N,[N/2]} = \log 2 \frac{D_{N:N} - 1}{D_{[N/2]:N}} - logN \tag{2.25}$$

where $D_{1:N} \leq \ldots \leq D_{N:N}$ are the order statistics of durations $D_1, \ldots D_N$. $D_i = t_i - t_{t-1}$ is the duration between two consecutive exceptions and i is the time until the first exception. There is clustering of exceptions if *the median of* $D_{N:N}/D_{[N/2]:N}$ *is higher than the median under the IDN hypothesis.* (see Santos and Alves (2012); Araújo Santos (2010) for more details about the new independent test).

2.7.5 Dynamic Quantile test

Engle and Manganelli (2004) utilise the criterion that the number of exceptions produced on day t should be independent of the information available at day $t-1$ and introduced the out-of-sample dynamic quantile (DQ) test for model validation. Define the function

$$Hit_t = \mathbb{I}(L_t < -VaR^p_{q,t}) - (1-q) = \begin{cases} q, & \text{if } L_t < VaR^p_{q,t} \\ -(1-q), & otherwise, \end{cases} \tag{2.26}$$

where Hit_t assumes the value q when the loss on the portfolio at time t is less than $VaR^p_{q,t}$, and $-(1-q)$ otherwise. Note that $1-q$ is the probability associated with $VaR^p_{q,t}$, and the negative sign on $-VaR^{p}_{q,t}$ is to have the VaR be a positive number. The test statistics is given by

$$DQ = \frac{(Hit_t' X_t [X_t' X_t]^{-1} X_t' Hit_t)}{(1-q)q} \approx \chi^2_q, \tag{2.27}$$

where the vector X_t might include lags of Hit_t, $VaR^{p}_{q,t}$ and its lags. Under the null hypothesis $E[Hit_t] = 0$ and $E[Hit_t|\Omega_{t-1}] = 0$, Hit_t and X_t are orthogonal and Hit_t must be uncorrelated with its own lagged values Gaglianone et al. (2011); Engle and Manganelli (2004). The DQ test is easy to perform, and does not depend on the estimation procedure; all that is needed is a series of VaRs and the corresponding values of the portfolio returns Engle and Manganelli (2004).

2.7.6 Bootstrap Back-Test

The bootstrap back-test involves statistical bootstrapping using empirical observations to construct a $100(1-p)\%$ confidence interval wherein the VaR estimate must lie within this confidence interval for the VaR model to be considered a reliable risk measure.

T data points are drawn randomly from the new daily risk factor returns with replacement, and the desired q^{th} quantile (VaR) for the T data points is calculated. The process is repeated k times such that a sufficient distribution of VaRs is obtained. From the distribution of VaRs, the confidence interval and error band of the original VaR estimate are easily calculated. The error band is calculated as the upper bound minus the lower bound. The upper bound is the $100(1-p/2)\%$ quantile and the lower bound is the $100(p/2)\%$ quantile of the distribution. VaR estimates that fall outside the confidence interval are considered significantly different and thus the VaR model is rejected.

2.8 Chapter summary

In this chapter, we have discussed a brief history of VaR, traditional methods commonly used in constructing VaR models, and common methods for VaR models validation, which will be used through out this research to check the reliability of the proposed VaR models. In the next chapter, we present a novel approach to construct and investigate the reliability of a VaR model by incorporating multivariate GARCH Dynamic Conditional Correlation volatility models, copula functions and extreme value theory.

Chapter 3

Forecasting Value-at-Risk estimates using multivariate GARCH(1,1) Dynamic Conditional Correlation models, copula functions and Extreme Value Theory: Evidence from EU banks

This chapter presents a novel approach for forecasting value-at-risk estimates by combining multivariate GARCH(1,1) Dynamic Conditional Correlation (M-GARCH(1,1) DCC) models for modeling correlations, copula functions for modeling dependence, and extreme value theory for modeling the tails of the return distributions.

3.1 Introduction

Financial asset returns often demonstrate volatility clustering. Therefore, volatility plays an important role in VaR estimation. Many volatility models have been proposed, for example the generalized autoregressive conditional heteroskedasticity (GARCH) models and its extensions have been used to capture the effects of volatility clustering and asymmetry in VaR estimation. Many studies have applied a variety of univariate GARCH models in VaR estimation; see So and Philip (2006), Berkowitz and OBrien (2002), and McNeil and Frey (2000). In addition, Kuester et al. (2006) provides an extensive review of VaR estimation methods with a focus on univariate GARCH models. The results of all these studies suggest that GARCH models provide more accurate VaR estimates than traditional methods. Because financial applications typically deal with a portfolio of assets with several risk factors (as considered in this study), a multivariate GARCH(1,1) (M-GARCH(1,1)) model would be very useful for VaR estimation. Univariate VaR models focus on an individual portfolio, whereas the multivariate approach explicitly model the correlation structure of the covariance or volatility matrix of multiple asset returns over time. Bauwens and Laurent (2012) provides a comprehensive review of univariate volatility models and their applications.

Numerous M-GARCH models have since been developed, for example, Tsay (2013); Fengler and Herwartz (2008); Engle and Kroner (1995); Bollerslev et al. (1994) and the references therein. Bauwens et al. (2006) divides M-GARCH models into three categories: (1) direct generalization of univariate GARCH models (e.g., exponentially-weighted moving average (EWMA), vector error correction (VEC), BEKK, etc.), (2) linear combinations of univariate GARCH models (e.g., generalized orthogonal GARCH (GO-GARCH), principal component GARCH (PGARCH), etc.), and (3) nonlinear combinations of univariate GARCH models (e.g., dynamic conditional correlation (DCC) and constant conditional correlation (CCC) models). The article by Silvennoinen and Teräsvirta (2009) gives a concise review of most common M-GARCH models; parametric and semi-parametric models and their properties. See also Ghalanos (2015) for more details on M-GARCH models.

Most volatility models fail to satisfy the positive definite conditions of the covariance matrix of asset returns. M-GARCH DCC volatility model by Engle (2002) is employed in this study because of conditions (as seen later) that will guarantee the conditional volatility matrix to be positive-definite almost surely.

The rest of the chapter is structured as follows: In Section 3.2, we present the Dynamic Conditional Correlation (DCC) model, and Section 3.3 discuses the copula theory. In Section 3.4, we present extreme value theory (EVT) analysis. In Section 3.5, we discuss the data used to construct the VaR model. In Section 3.6, we present the results followed by conclusion in Section 3.7.

3.2 The Dynamic Conditional Correlation (DCC) model

Financial asset returns have been shown to be leptokurtic and heavy tailed with non constant volatility (Berkowitz et al., 2011; Sheikh and Qiao, 2010) and thus require conditional volatility models that will reflect the most current information in the market. The DCC model is motivated by the fact that it allows for the correlation matrix to be time varying, and hence reflects the current market condition.

To proper understand the volatility matrix of the DCC model, it is important to first understand the mean and variance equations of a univariate GARCH(1,1) model. GARCH(1,1) model first proposed by (Bollerslev, 1986) allows conditional variance to be dependent upon previous lags. The GARCH(1,1) model has the form

$$r_{i,t} = \mu_i + a_{i,t}, \quad a_{i,t} = \sigma_{i,t}\eta_{i,t} \tag{3.1a}$$

$$\sigma^2_{i,t} = \alpha_0 + \alpha_1 a^2_{i,t-1} + \beta_1 \sigma^2_{i,t-1}, \tag{3.1b}$$

$$\text{for} \quad i = 1, \ldots, N, \quad t = 1, \ldots, T$$

where $r_{i,t}$ are the log return series of daily stock prices, μ_i are the conditional means of the log returns, $a_{i,t}$ are the residuals of the mean equation (Eqn. (3.1a), $\eta_{i,t}$ represent white noise with zero mean and unit variance, $\sigma_{i,t}$ are the conditional volatility series from the variance equation (Eqn. (3.1b)), N represents the total number of stocks, T the sample size, and α_0, α_1 and β_1 are the GARCH(1,1) parameters.

Let $\mathbf{x}_t$ ($t = 1, 2, \ldots, T$) be a stochastic vector of financial time series data with dimension $k \times 1$ and conditional mean μ_t given some market information Ω observed until at time $t-1$, then the mean equation for a general M-GARCH model is defined as

$$\boldsymbol{x}_t|\Omega_{t-1} = \mu_t + a_t, \quad a_t = \Sigma_t^{1/2}\boldsymbol{\eta}_t, \tag{3.2}$$

where $\boldsymbol{\eta}_t = (\eta_{1t}, \ldots, \eta_{kt})'$ are *iid* random vectors such that $E[\boldsymbol{\eta}_t] = 0$ and $cov[\boldsymbol{\eta}_t] = \mathbf{I}_k$; an identity matrix of order K. $\Sigma_t^{1/2}$ is a $k \times k$ positive definite matrix of the conditional volatility matrix Σ_t. The conditional correlation matrix ρ_t has the form

$$\rho_t = D_t^{-1}\Sigma_t D_t^{-1}, \tag{3.3}$$

where $D = diag\left\{\sqrt{\sigma_{11,t}}, \ldots, \sqrt{\sigma_{kk,t}}\right\}$; is the diagonal matrix of the conditional volatilities $\sigma_{ii,t}$.

Engle (2002) proposed a DCC model where the conditional correlation matrix is dependent on two parameters θ_1 and θ_2. Let $\eta_{1t} = a_{1,t}/\sqrt{\sigma_{11,t}}$, and $\boldsymbol{\eta}_{it} = (\eta_{1t}, \ldots, \eta_{kt})'$ be vectors of marginal standardised residuals, where $\{\sigma_{ii,t}\}_{i=1}^{K}$ are conditional volatilities obtained following GARCH(*1,1*) volatility model, then the DCC model by Engle (2002) is defined as

$$Q_t = (1 - \theta_1 - \theta_2)\bar{Q} + \theta_1 Q_{t-1} + \theta_2 \eta_{t-1}\eta'_{t-1}, \tag{3.4a}$$

$$\rho_t = \Lambda_t Q_t \Lambda_t, \tag{3.4b}$$

where ρ_t is the correlation matrix of the vectors of standardised residuals $\boldsymbol{\eta}_{it}$, $\bar{Q}$ is the unconditional covariance matrix of $\boldsymbol{\eta}_{it}$, $\theta_i \in \Re^+$, $0 < \theta_1 + \theta_2 < 1$ for $i = 1, 2$ and controlled by Q_t; a positive definite matrix. $\Lambda_t = diag\left\{q_{11,t}^{-1/2}, \ldots, q_{kk,t}^{-1/2}\right\}$, where $q_{ii,t}$ are the $(i,i)^{th}$ element of Q_t (Ghalanos, 2015; Tsay, 2013). The constraints on θ_1 and θ_2 guarantees ρ_t to be positive definite almost surely.

The DCC model of Engle (2002) however, will not capture the asymmetric response of volatility commonly displayed by financial asset returns. Therefore, we also employ the asymmetric DCC (aDCC) model proposed by Cappiello et al. (2006). They investigated if the signs of profit and loss (*P&L*) distributions of financial asset at time $t-1$ has any influence on the current conditional variances, covariances, and correlations. They found evidence of asymmetries in conditional covariance of both equity and bond returns, evidence of asymmetry in conditional volatility of equities, evidence of asymmetry in the conditional correlation of both bonds and equities. The aDCC model has the form

$$Q_t = (\bar{Q} - A'\bar{Q}A - B'\bar{Q}B - G'\bar{N}G) + A'\eta_{t-1}\eta'_{t-1}A + B'Q_{t-1}B + G'\epsilon_{t-1}\epsilon'_{t-1}G, \tag{3.5}$$

$$\rho_t = \Lambda_t Q_t \Lambda_t,$$

where A, B, and G are diagonal parameter matrices; asymmetry is captured in G, $\bar{Q} =$

$E(\eta_t\eta_t')$, $\epsilon_t = I(\eta_t < 0) \circ \eta_t$, where $\circ$ represents the Hadamard product, and $\bar{N} = E(\epsilon_t'\epsilon_t)$ (see Cappiello et al. (2006) and the references therein).

3.3 Copula theory

Copula theory was first developed by Sklar (1959) to describe the dependence structure between random variables. It was later introduced to the finance literature by Frey et al. (2001); Embrechts and McNeil (1999); and Li (2000). Consequently, Embrechts et al. (2002) introduced the application of copula theory to financial asset returns, and Patton (2004) expanded the framework of copula theory with respect to the time-varying nature of financial dependence schemes. Copula theory has also been extensively used in risk management to measure VaR of portfolios, including both unconditional (Cherubini et al., 2004; Cherubini and Luciano, 2001; Embrechts et al., 2001) and conditional distributions (Silva Filho et al., 2014; Huang et al., 2009; Fantazzini, 2008).

Copula functions enables the construction of a flexible multivariate distribution with varying margins and dependence structures that are free from assumptions of normality or linear correlation. In addition, copulas can easily capture the tail dependence of asset returns, i.e., the joint probability of large market movements (Cherubini et al., 2004) or joint distribution modeling. They are used as a modeling tool for modeling non-linear correlations and not an assessment tool such as the Pearson's or Spearman's correlations, or Kendall's τ.

In multivariate settings, we use the following version of Sklar's theorem as given by Cherubini et al. (2004) for the purpose of VaR estimation.

Theorem 1 ***Sklar's theorem:*** *Consider an n-dimensional joint distributional function* $F(x)$*, with uniform margins* $F_1(x_1), \ldots, F_n(x_n)$*;* $x = (x_1, \ldots, x_n)$*, with* $-\infty \leq x_i \leq \infty$*, then there exists* *a copula* $C : [0,1]^n \to [0,1]$ *such that*

$$F(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n)), \tag{3.6}$$

determined under absolute continuous margins as

$$C(u_1, \ldots, u_n) = F(F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n)), \tag{3.7}$$

otherwise, C is uniquely determined on the range $R(F_1) \times R(F_1) \times \ldots \times R(F_n)$*. Equally, if C is a copula and* $F_1, \ldots, F_n$ *are univariate distribution functions, then* $C(F_1(x_1), \ldots, F_n(x_n))$ *is a joint distribution function with margins* $F_1, \ldots, F_n$ *(Tsay, 2013).*

The copula $C(u_1, \ldots, u_n)$ has density $c(u_1, \ldots, u_n)$ associated to it and defined as

$$c(u_1, \ldots, u_n) = \frac{\partial_n C(u_1, \ldots, u_n)}{\partial u_1, \ldots, \partial u_n} \tag{3.8}$$

and is related to the density function F for continuous random variables denoted as f, by the canonical copula representation

$$f(x_1, \ldots, x_n) = c(F_1(x_1), \ldots, F_n(x_n)) \prod_{i=1}^{n} f_i(x_i), \tag{3.9}$$

where f_i are the marginal densities that can be different from each other (Ghalanos, 2015; Bob, 2013; Tsay, 2013; Huang et al., 2009; Cherubini et al., 2004).

Bob (2013) and Cherubini et al. (2011) discuss two commonly used families of copulas in financial applications: the elliptical and the Archimedean copulas.

The most common elliptical copulas used in financial applications are the Gaussian and the Student's t copulas, which are symmetric. Their dependence structure is determined by the standardised correlation or dispersion matrix

$$\rho_t = \begin{pmatrix} 1 & \dots & \rho_{1,n} \\ \vdots & \ddots & \vdots \\ \rho_{n,1} & \dots & 1 \end{pmatrix} \tag{3.10}$$

because of the invariant property of copulas. $\rho_{i,j}$ is the dispersion parameter, which can be set to either Kendall's tau or Spearman's rho, as discussed later.

Consider a symmetric positive definite matrix ρ_t, (Eq.3.10) with $diag(\rho_t) = (1, 1, \dots, 1)^T$; where T is the sample size. We can represent the multivariate Gaussian copula (MGC) as

$$C_{\rho_t}^{G_a} = P(\Phi(X_1) \leq u_1, \dots, \Phi(X_n) \leq u_n) = \Phi_{\rho_t}(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_n)), \tag{3.11}$$

where Φ_{ρ_t} is the standardised multivariate normal distribution and $\Phi_{\rho_t}^{-1}$ is the inverse standard univariate normal distribution function of u with correlation matrix ρ_t. If the margins are normal, then the Gaussian copula will generate the standard Gaussian joint distribution function with density function

$$c_{\rho_t}^{G_a}(u_1, u_2, \dots, u_n) = \frac{1}{|\rho_t|^{\frac{1}{2}}} exp\left(-\frac{1}{2}\varsigma'(\rho_t^{-1} - \boldsymbol{I})\varsigma\right), \tag{3.12}$$

where $\varsigma = (\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_n))^T$, and $\boldsymbol{I}$ represents the identity matrix.

On the other hand, a multivariate Student's t copula (MTC) has the form

$$T_{\rho_t,v}(u_1, \dots, u_n) = t_{\rho_t,v}(t_v^{-1}(u_1), \dots, t_v^{-1}(u_n)), \tag{3.13}$$

with density function

$$c_{\rho_t,v}(u_1,\ldots,u_n) = |\rho_t|^{-\frac{1}{2}} \frac{\Gamma(\frac{v+n}{2})}{\Gamma(\frac{v}{2})} \left(\frac{\Gamma(\frac{v}{2})}{\Gamma(\frac{v+1}{2})} \right)^n \frac{(1 + \frac{1}{v}\varsigma' \rho_t^{-1} \varsigma)^{-\frac{v+n}{2}}}{\prod_{j=1}^n \left(1 + \frac{\varsigma_j^2}{v}\right)^{-\frac{v+1}{2}}}, \tag{3.14}$$

where $t_{\rho_t,v}$ is the standardised Student's t distribution with correlation matrix ρ_t and v degrees of freedom.

Archimedean copulas are built via a generator as

$$C(u_1,\ldots,u_n) = \varphi^{-1}(\varphi(u_1) + \ldots + \varphi(u_n)) \tag{3.15}$$

with density function

$$c(u_1,\ldots,u_n) = \varphi^{-1}(\varphi(u_1) + \ldots + \varphi(u_n)) \prod_{i=1}^n \varphi^{'}(u_i), \tag{3.16}$$

where φ is the copula generator and φ^{-1} is completely monotonic on $[0,\infty]$. That is, φ must be infinitely differentiable with derivatives of ascending order and alternative sign such that $\varphi^{-1}(0) = 1$ and $\lim_{x\to+\infty} \varphi(x) = 0$ (Cherubini et al., 2011). Thus, $\varphi^{'}(u) < 0$ (i.e., φ is strictly decreasing) and $\varphi^{''}(u) > 0$ (i.e., φ is strictly convex).

Archimedean copulas are very useful in risk management analysis because they capture an asymmetric tail dependence between financial asset returns. The most commonly used Archimedean copulas in financial applications are the Gumbel (1960), Clayton (1978) and Frank (1979) copulas (Yan et al., 2007).

The Gumbel copula captures upper tail dependence, is limited to positive dependence, and has generator function $\varphi(u) = (-\ln(u))^\alpha$ and generator inverse $\varphi^{-1}(x) = \exp(-x^{\frac{1}{\alpha}})$.

This will generate a Gumbel n-copula represented by

$$C(u_1, \ldots, u_n) = \exp\left\{-\left[\sum_{i=1}^{n}(-\ln u_i)^{\alpha}\right]^{\frac{1}{\alpha}}\right\} \qquad \alpha > 1. \tag{3.17}$$

The generator function for the Clayton copula is given by $\varphi(u) = u^{-\alpha} - 1$ and generator inverse $\varphi^{-1}(x) = (x+1)^{-\frac{1}{\alpha}}$, which yields a Clayton n-copula represented by

$$C(u_1, \ldots, u_n) = \left[\sum_{i=1}^{n}(u_i^{-\alpha} - n + 1)\right]^{-\frac{1}{\alpha}} \qquad \alpha > 0. \tag{3.18}$$

The Frank copula has generator function $\varphi(u) = \ln\left(\frac{\exp(-\alpha u)-1}{\exp(-\alpha)-1}\right)$ and generator inverse

$$\varphi^{-1}(x) = -\frac{1}{\alpha}\ln\left(1 + e^{x}(e^{-\alpha} - 1)\right), \tag{3.19}$$

which will result in a Frank n-copula represented by

$$C(u_1, \ldots, u_n) = -\frac{1}{\alpha}\ln\left\{1 + \frac{\prod_{i=1}^{n}(e^{-\alpha u_i} - 1)}{(e^{-\alpha} - 1)^{n-1}}\right\} \qquad \alpha > 0, \tag{3.20}$$

(Cherubini et al., 2004). We follow Bob (2013); Breymann et al. (2003) and employ Gaussian, t, Gumbel, Frank and Clayton copulas in this study.

3.3.1 Measuring Dependence

The traditional way to measure the relationship between markets and risk factors is to look at their linear correlations, which depend both on the marginal and joint distributions of the risk factors. In the case of non-normality, the results might be misleading (Cherubini et al., 2011). Kendall's τ or Spearman's ρ; nonparametric invariant measures that are not dependent on marginal probability distributions are more suitable to use.

Copulas measure a form of dependence between pairs of risk factors (i.e., asset returns) known as concordance using these invariant measures. Two observations (x_i, y_i) and (x_j, y_j) from a vector (X, Y) of continuous random variables are concordant if $(x_i - x_j)(y_i - y_j) > 0$ and discordant if $(x_i - x_j)(y_i - y_j) < 0$. Large values of X are paired with large values of Y and small values of X are paired with small values of Y as the proportion of concordant pairs in the sample increases. On the other hand, the proportion of concordant pairs decreases as large values of X are paired with small values of Y and small values of X are paired with large values of Y (Alexander, 2008).

Consider n paired continuous observations (x_i, y_i) ranked from smallest to largest, with the smallest ranked 1, the second smallest ranked 2, and so on. Then, Kendall's τ is defined as the sum of the number of concordant pairs minus the sum of the number of discordant pairs divided by the total number of pairs, i.e., the probability of concordance minus the probability of discordance:

$$\tau_{X,Y} = \Pr[(x_i - x_j)(y_i - y_j) > 0] - \Pr[(x_i - x_j)(y_i - y_j) < 0] = \frac{\mathbf{C} - \mathbf{D}}{\mathbf{C} + \mathbf{D}}, \tag{3.21}$$

where **C** is the number of concordant pairs below a particular rank that are larger in value than that particular rank, and **D** is the number of discordant pairs below a particular rank that are smaller in value than that particular rank.

Spearman's ρ, on the other hand, is defined as the probability of concordance minus the probability of discordance of the pair of vectors (x_1, y_1) and (x_2, y_3) with the same margins. That is,

$$\rho_{X,Y} = 3(\Pr[(x_1 - x_2)(y_1 - y_3) > 0] - \Pr[(x_1 - x_2)(y_1 - y_3)] < 0).$$

The joint distribution function of (x_1, y_1) is $H(x, y)$, while the joint distribution function of

(x_2, y_3) is $F(x)G(y)$ because x_2 and y_3 are independent (Nelsen, 2007). Alternatively,

$$\rho_{X,Y} = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)},$$

where d is the difference between the ranked samples. Nelsen (2007) has shown that Kendall's τ and Spearman's ρ depend on the vectors (x_1, y_1), (x_2, y_2) and (x_1, y_1), (x_2, y_3), respectively, through theirs copulas C, and that the following relationship holds:

$$\tau_{X,Y} = 4\int_0^1\int_0^1 C(u,v)dC(u,v) - 1$$

$$\rho_{X,Y} = 12\int_0^1\int_0^1 C(u,v)dudv - 3.$$

3.4 Extreme Value Theory

EVT is a statistical approach for estimating extreme events with low frequency but high severity. This technique is widely used in financial risk management since empirical evidence from various studies (see Sheikh and Qiao (2010); Berkowitz et al. (2011)) have shown that in majority of cases, financial asset return distributions are heavy-tailed, especially in times of financial instability.

There are two methods for modeling extreme events with low frequency but high severity; the block maxima method and the Peaks Over Threshold (POT) method. For financial time series data, the POT method is often used to model extreme events. The block maxima method is not commonly used to do statistical inference with financial time series data because (1) the method does not make sufficient use of data as it uses only data from sub-period maxima, (2) the choice of sub-period length is not clearly defined, (3) the method is unconditional and does not take into account the effects of other explanatory variables (Tsay, 2014). We used the POT method based on the generalized Pareto distribution (GPD). The POT method focuses on modeling exceedances of losses above a certain threshold ϑ and the time of occurrence. The threshold is selected such that there are enough data points to do meaningful statistical analysis. Let $\{x_i\}_{i=1}^T$ represent the loss variables of an asset returns, then as $T \to \infty$, $\{x_i\}_{i=1}^T$ is assumed to be independent and identically distributed, and $(x-\mu)/\sigma$ follows a generalized extreme value (GEV) distribution:

$$F_{\xi,\mu,\sigma}(x) = \begin{cases} exp[-(1+\xi x)^{-1/\xi}] & \text{for} \quad \xi \neq 0, \\ exp[-e^{-x}] & \text{for} \quad \xi = 0, \end{cases} \tag{3.22}$$

where ξ is the shape parameter and $1/\xi$ is the tail index of the GEV distribution. $x < -1/\xi$ if $\xi < 0$ and $x > -1/\xi$ if $\xi > 0$. Also, let the probability of the conditional distribution of the excesses over the threshold ϑ, i.e., $x_i - \vartheta = y | x_i > \vartheta$, be given by

$$Pr(x-\vartheta \leq y | x > \vartheta) = \frac{Pr(\vartheta \leq x \leq y+\vartheta)}{Pr(x > \vartheta)} = \frac{Pr(x \leq y+\vartheta) - Pr(x \leq \vartheta)}{1 - Pr(x \leq \vartheta)} \tag{3.23a}$$

$$= \frac{F(y+\vartheta) - F(\vartheta)}{1 - F(\vartheta)} = F_\vartheta(y). \tag{3.23b}$$

Again, as $T \to \infty$, $(y+\vartheta-\mu)/\sigma$ follows a GEV distribution (Eqn.3.22). Therefore,

$$\begin{aligned} Pr(x-\vartheta \leq y | x > \vartheta) &= \frac{F(y+\vartheta) - F(\vartheta)}{1 - F(\vartheta)} \\ &= \frac{exp\left[-(1+\frac{\xi(y+\vartheta-\mu)}{\sigma})^{-1/\xi}\right] - exp\left[-(1+\frac{\xi(\vartheta-\mu)}{\sigma})^{-1/\xi}\right]}{1 - exp\left[-(1+\frac{\xi(\vartheta-\mu)}{\sigma})^{-1/\xi}\right]} \\ &\approx 1 - \left(1 + \frac{\xi y}{\sigma + \xi(\vartheta-\mu)}\right)^{-1/\xi} \end{aligned} \tag{3.24}$$

where $y > 0$ and $\sigma + \xi(\vartheta-\mu) > 0$. If we let $\psi(\vartheta) = \sigma + \xi(\vartheta-\mu)$, and as $\vartheta \to \infty$, Eqn.(3.24) is approximated by the generalized Pareto distribution (GPD)

$$G_{\xi,\psi(\vartheta)}(y) = \begin{cases} 1 - \left[1 + \frac{\xi y}{\psi(\vartheta)}\right]^{-1/\xi} & \text{for} \quad \xi \neq 0, \\ 1 - exp[\frac{-y}{\psi(\vartheta)}] & \text{for} \quad \xi = 0, \end{cases} \tag{3.25}$$

with shape parameter ξ and scale parameter $\psi(\vartheta)$. $\psi(\vartheta) > 0, y \in [0, x - \vartheta]$ when $\xi \geq 0$, and $y \in [0, -\frac{\psi(\vartheta)}{\xi}]$ when $\xi < 0$. If $\xi = 0$, then Eqn.(3.25) becomes an exponential distribution with parameter $1/\sigma$ (Tsay, 2014). We know from Eqn.(3.23a) that $y = x - \vartheta$. Therefore,

$$\frac{F(y + \vartheta) - F(\vartheta)}{1 - F(\vartheta)} = \frac{F(x) - F(\vartheta)}{1 - F(\vartheta)} \approx G_{\xi,\psi(\vartheta)}(x - \vartheta)$$

$$\implies F(x) = F(\vartheta) + [1 - F(\vartheta)]\, G_{\xi,\psi(\vartheta)}(x - \vartheta). \tag{3.26a}$$

We can now state the tail estimator for the underlying distribution $F(x|\xi, \psi(\vartheta))$ using the empirical estimate of $F(\vartheta)$. i.e., $\hat{F}(\vartheta) = (T - N_\vartheta)/T$ as

$$\hat{F}(x|\xi, \psi(\vartheta)) \approx \frac{T - N_\vartheta}{T}\left[1 + \frac{\hat{\xi}(x - \vartheta)}{\hat{\psi}(\vartheta)}\right]^{-1/\hat{\xi}}, \tag{3.27}$$

where N_ϑ is the number of observations above the threshold (Tsay, 2014; Soltane et al., 2012). The mean excess function plot proposed by Davison and Smith (1990) is used to help identify the threshold value. A mean excess function of x over a certain threshold ϑ is defined as

$$e(\vartheta) = E(x - \vartheta | x > \vartheta) = \frac{\sigma + \xi\vartheta}{1 - \xi}. \tag{3.28}$$

A property of the GPD states that if the excess distribution of x given a threshold ϑ_0 is a GPD with shape parameter ξ and scale parameter $\psi(\vartheta_0)$, then for any random threshold $\vartheta > \vartheta_0$, the excess distribution over the threshold ϑ has a GPD with shape parameter ξ

and scale parameter $\psi(\vartheta) = \psi(\vartheta_0) + \xi(\vartheta - \vartheta_0)$, where $0 < \xi < 1$ (Tsay, 2014). Then

$$e(\vartheta) = E(x - \vartheta | x > \vartheta) = \frac{\psi(\vartheta_0) + \xi(\vartheta - \vartheta_0)}{1 - \xi}, \tag{3.29}$$

which is a linear function of $\vartheta - \vartheta_0$ with slope $\xi/(1 - \xi)$ for $\vartheta > \vartheta_0$. From the ordered sample of losses, $\{x_i\}$, we can calculate and plot the mean excess function (Eqn.(3.29)) against each chosen ϑ_i for $\vartheta_i > \vartheta_0$. The threshold ϑ is then identified as the lowest point on the mean excess plot above which the graph appears to be approximately linear.

Inverting Eqn.(3.27) gives the q^{th} quantile $F_q^{-1} = VaR_q$, for any given small upper tail probability p for VaR estimation as

$$VaR_q = \vartheta - \frac{\hat{\psi}(\vartheta)}{\hat{\xi}} \left\{ 1 - \left[\frac{T}{N_\vartheta}(1 - q) \right]^{-\hat{\xi}} \right\}, \tag{3.30}$$

where $q = 1 - p$ (Tsay, 2014; Soltane et al., 2012; Bhattacharyya and Ritolia, 2008). Assuming that N_ϑ are independent and identically distributed, then the parameters $\psi(\vartheta)$ and ξ can be estimated by means of maximum likelihood estimation with likelihood function

$$l(x_i, \ldots, x_{N_\vartheta} | \xi, \sigma, \mu) = \prod_{i=1}^{N_\vartheta} f(x_i) \quad \text{for} \quad x_i > \vartheta. \tag{3.31}$$

3.5 Data

The data employed consist of daily closing prices of 26 stocks from five different European countries —France, Greece, UK, Spain, and Sweden as seen in Table 3.1. Our aim is to investigate the risk of collapse in the banking system with insufficient capital to provide proper cushions capable to withstand sudden losses in periods of distress. Thus, the stocks belong to the banking sectors and within the top ten banks for each country. The choice of countries is based on how much impact the 2008 and 2011 financial crisis had on its banking system and the risk of collapse. For example, UK was in recession for 15 months from the second quarter of 2008 to the second quarter of 2009, Sweden was in recession for 15 months from the first quarter of 2008 to the first quarter of 2009, France was in recession for 15 months from the second quarter of 2008 to the second quarter of 2009 and 6 months from the fourth quarter of 2012 to the first quarter of 2013, Spain was in recession for 21 months from the second quarter of 2008 to the fourth quarter of 2009 and 27 months from the second quarter of 2011 to the second quarter of 2013, and finally the most affected country Greece was in recession for 63 months from the third quarter of 2008 to the second quarter of 2014 and 27 months from the first quarter of 2015 to the first quarter of 2017 (DAmuri and Peri, 2014; Jenkins et al., 2012; Lin et al., 2012). Therefore, we select our data starting from 31^{st} of December 2004 to 31^{st} of December 2015 to contain the 2008 global financial crisis and 2011 European financial crisis periods. All data are from DataStream and each stock consist of 2870 daily observations.

Country	**Stocks from various banks**					
France	F.BNP	F.SGE	F.CRDA	F.KNF	F.CC	F.CAI
Greece	G.PIST	G.PEIR	G.EFG	G.ETE	G.ATT	G.ELL
UK	UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN	
Spain	E.SCH	E.BBVA	E.BSAB	E.BKT	E.POP	
Sweden	W.NDA	W.SVK	W.SWED	W.SEA		

Table 3.1: *Stocks from the banking sector belonging to the top ten banks of each country.*

We used an out-of-sample data of $m = T - n$ observations for back-testing. This means that we have $n = 1869$ sample-in return observations for VaR estimation procedure containing the 2008 crisis, and $m = 1000$ return observations for back-testing. VaR is estimated for day $t = n + 1$ using data from day $t = 1$ to day $t = n$, VaR for day $t = n + 2$ is estimated using data from day $t = 2$ to day $t = n + 1$, and so on until the out-of-sample data are all used up.

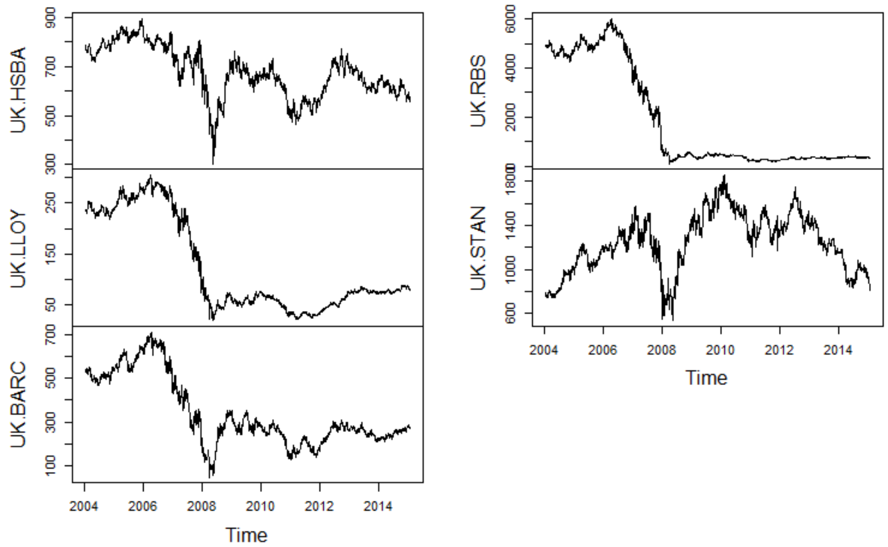
The daily log return series of the stocks are calculated as

$$r_t = \left[log\left(\frac{S_{1,t+\tau}}{S_{1,t}}\right), \ldots, log\left(\frac{S_{N,t+\tau}}{S_{N,t}}\right) \right] = (r_{1t}, \ldots, r_{Nt}), \tag{3.32}$$

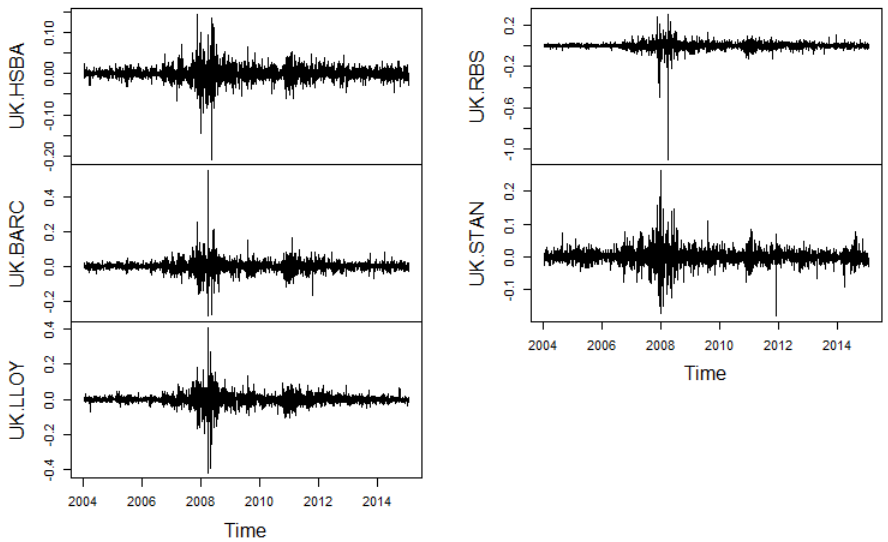
where N represents the number of stocks in the sample. Figures 3.1, 3.2, 3.3, 3.4, and 3.5 show trends in the stock prices and time series plots of the daily log returns for the different countries. The trends clearly show the effects of the 2008 global financial crisis and the 2011 European financial crisis. The daily log return series plots show evidence of volatility clustering. Basic statistics of the stock returns are reported in percentages in Table 3.2. From the table, we see that the stock returns are far from being normally distributed as indicated by their high excess kurtosis and skewness. We confirm this by running a multivariate autoregressive conditional heteroscedasticity (ARCH) test based on Ljung-Box test statistics $Q_k(m)$ and its modification known as robust $Q_k^r(m)$ test on the log returns at 5% significance, where m is the number of lags of cross-correlation matrices used in the tests. The modification involves discarding those observations from the return series whose corresponding standardised residuals exceed 95^{th} quantile in order to reduce the effect of heavy tails. The motivation for $Q_k^r(m)$ test is because $Q_k(m)$ may fare poorly in finite samples when the residuals of the time series, $a_t = \Sigma_t^{1/2}\boldsymbol{\eta}_t$, have heavy tails (Tsay, 2013). The test statistics is given by

$$Q_k(m) = T^2 \sum_{i=1}^{m} \frac{1}{T-i} \mathbf{b}_i'(\hat{\boldsymbol{\rho}}_0^{-1} \otimes \hat{\boldsymbol{\rho}}_0^{-1})\mathbf{b}_i \approx \chi^2_{k^2}(m), \tag{3.33}$$

which is asymptotically equivalent to the multivariate Lagrange multiplier (LM) test for conditional heteroscedasticity by Engle (1982). k is the dimension of a_t, T is the sample size, $\mathbf{b}_i = vec(\hat{\rho}_i')$ with $\hat{\boldsymbol{\rho}}_j$ being the lag -j cross-correlation matrix of a_t^2. The test tests if the current cross correlation matrix of a_t^2 does not depend on the lag cross correlation matrix of a_{t-1}^2. Multivariate ARCH test, also shown on Table 3.2, indicates the presence of conditional heteroscedasticity as p-values are all equals to zero.

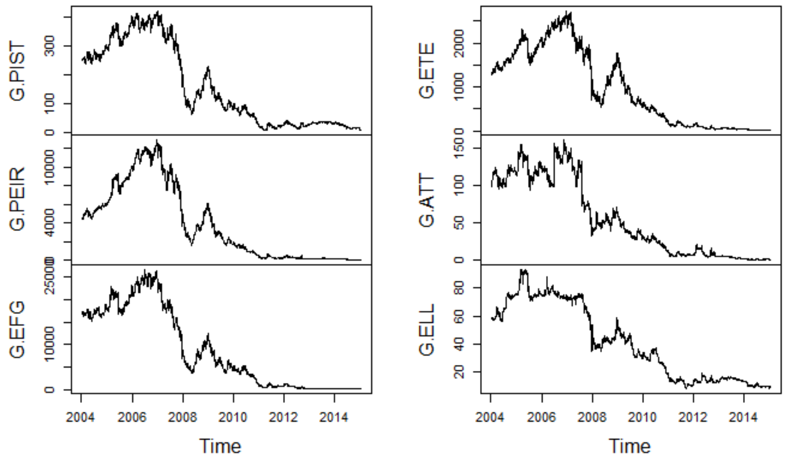


(a)

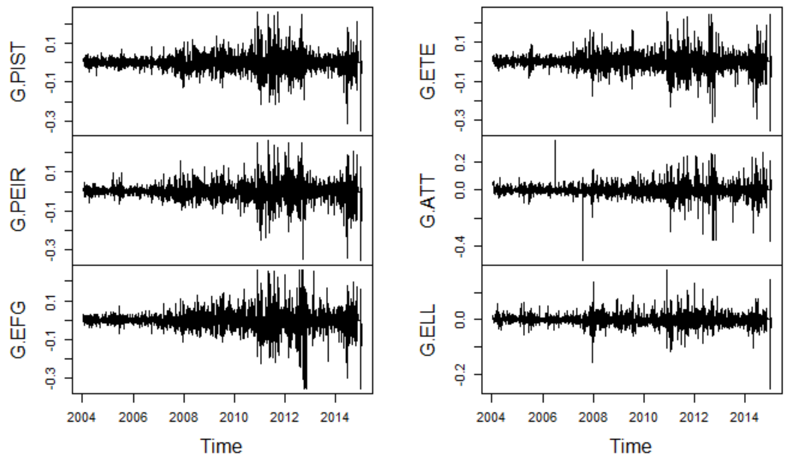


(b)

Figure 3.1: *Trends of UK stock prices (a) and time plots of log-return series (b) for the period of 31st of December 2004 to the 31st December 2015.*

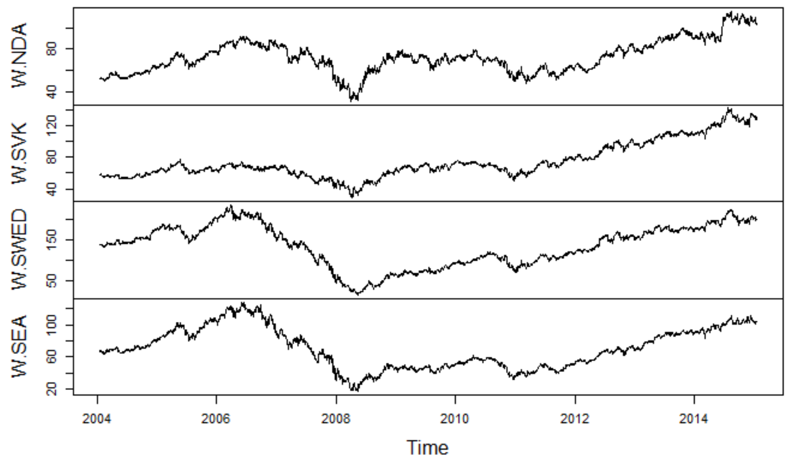


(a)

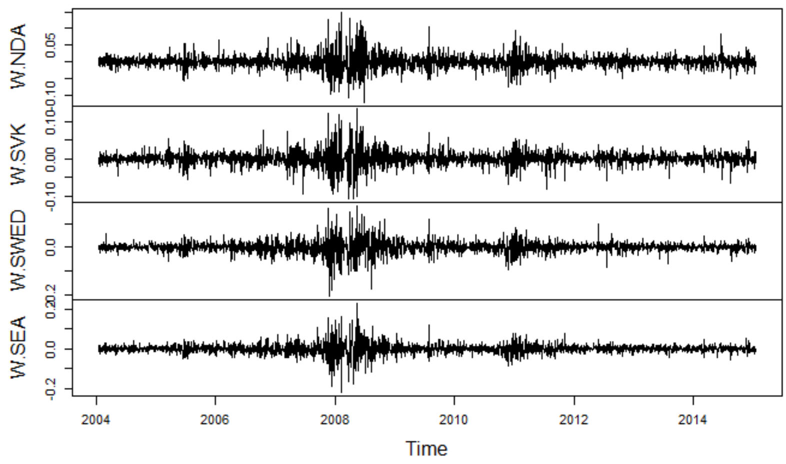


(b)

Figure 3.2: *Trends of Greek stock prices (a) and time plots of log-return series (b) for the period of 31st of December 2004 to the 31st December 2015.*

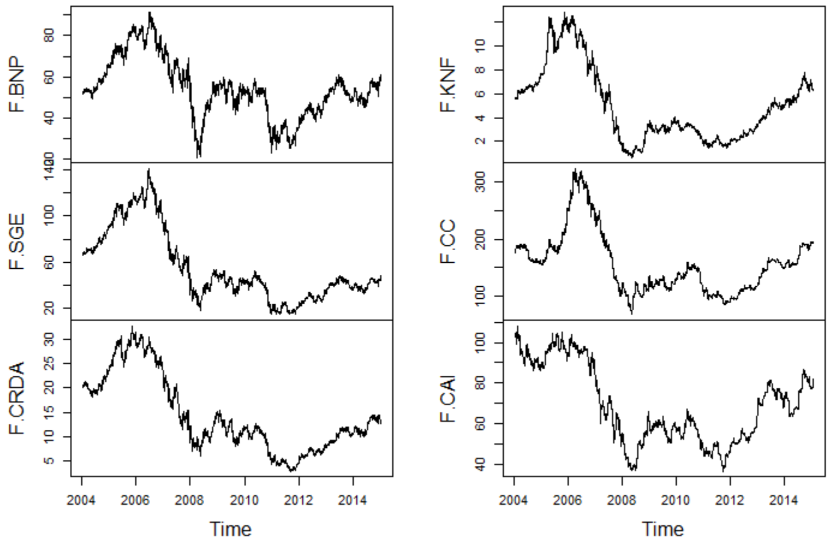


(a)

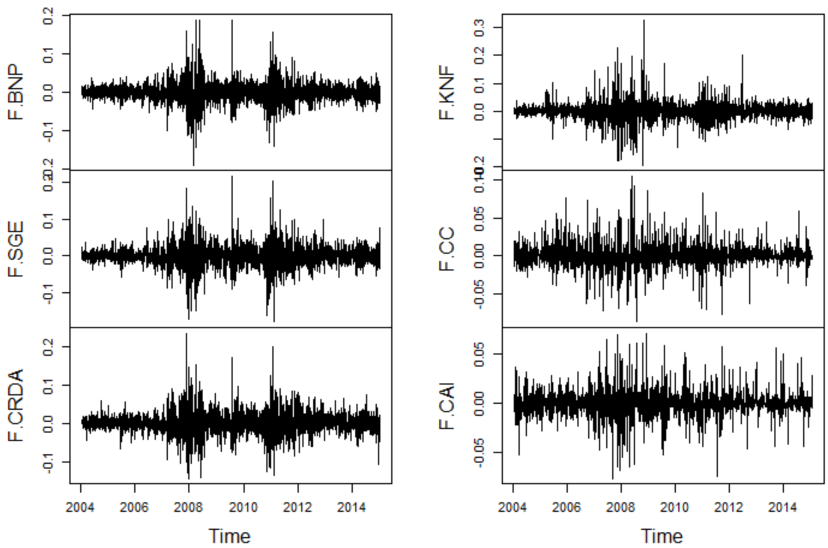


(b)

Figure 3.3: *Trends of Swedish stock prices (a) and time plots of log-return series (b) for the period of 31st of December 2004 to the 31st December 2015.*

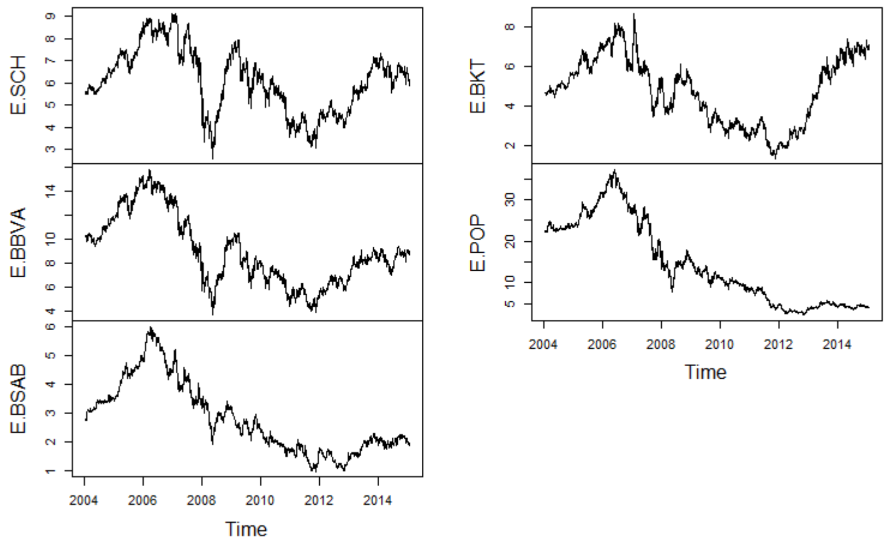


(a)

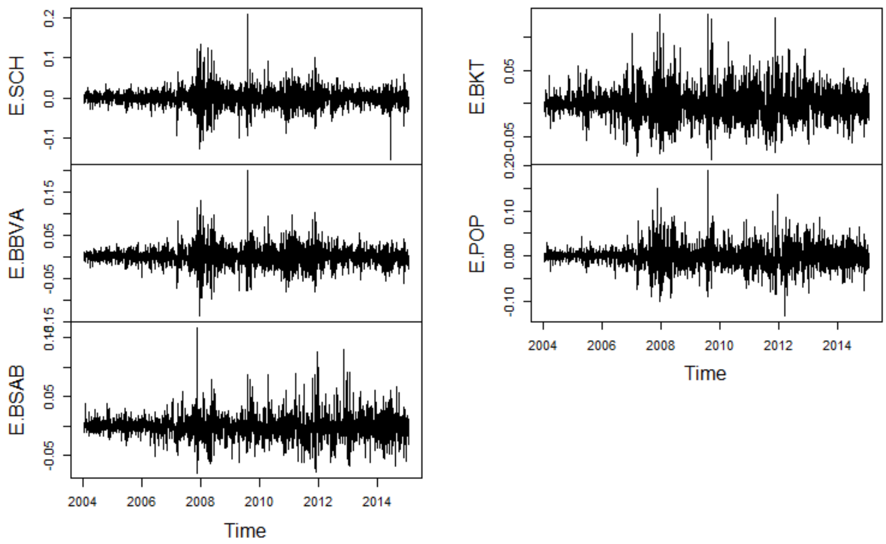


(b)

Figure 3.4: *Trends of French stock prices (a) and time plots of log-return series (b) for the period of 31st of December 2004 to the 31st December 2015.*



(a)



(b)

Figure 3.5: *Trends of Spanish stock prices (a) and time plots of log-return series (b) for the period of 31st of December 2004 to the 31st December 2015.*

Country	Stocks from various banks							
France		F.BNP	F.SGE	F.CRDA	F.KNF	F.CC	F.CAI	M-ARCH test
	Mean	0.0006	-0.0151	-0.0215	-0.0019	0.0006	-0.0108	$Q_k(10) = 2209$, *p-value* $= 0$
	Variance	0.0647	0.0807	0.0764	0.0959	0.0197	0.0145	$Q_k^r(10) = 2606$, *p-value* $= 0$
	Stdev	2.5427	2.8399	2.7645	3.0969	1.4027	1.2029	
	Skewness	34.4305	6.9738	27.7890	60.0710	62.1017	7.6754	
	Excess Kurtosis	867.8928	680.0429	624.0289	1209.1365	821.6399	678.3979	
Greece		G.PIST	G.PEIR	G.EFG	G.ETE	G.ATT	G.ELL	
	Mean	-0.1603	-0.3371	-0.3377	-0.2872	-0.2045	-0.0642	$Q_k(10) = 3587$, *p-value* $= 0$
	Variance	0.2287	0.2876	0.3162	0.2487	0.2818	0.0448	$Q_k^r(10) = 3180$, *p-value* $= 0$
	Stdev	4.7826	5.3624	5.6234	4.9866	5.3087	2.1157	
	Skewness	-12.5182	-105.8321	-61.0233	-103.6301	-59.4886	-7.5496	
	Excess Kurtosis	827.1241	1063.2666	865.8341	1044.0274	1317.2709	1519.9341	
UK		UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN		
	Mean	-0.0124	-0.0306	-0.0407	-0.0971	-0.0112		$Q_k(10) = 1321$, *p-value* $= 0$
	Variance	0.0293	0.1030	0.1077	0.1504	0.0594		$Q_k^r(10) = 2840$, *p-value* $= 0$
	Stdev	1.7122	3.2098	3.2823	3.8785	2.4376		
	Skewness	-33.6697	143.8658	-105.4936	-840.1325	31.6077		
	Excess Kurtosis	1690.7965	4021.7880	3727.5413	23552.6326	1308.5010		
Spain		E.SCH	E.BBVA	E.BSAB	E.BKT	E.POP		
	Mean	-0.0070	-0.0154	-0.0199	0.0117	-0.0702		$Q_k(10) = 808$, *p-value* $= 0$
	Variance	0.0463	0.0444	0.0359	0.0512	0.0529		$Q_k^r(10) = 1547$, *p-value* $= 0$
	Stdev	2.1519	2.1078	1.8940	2.2617	2.2991		
	Skewness	20.4736	32.7311	71.6545	49.2691	43.5654		
	Excess Kurtosis	828.4460	668.4718	623.2511	304.9592	494.7029		
Sweden		W.NDA	W.SVK	W.SWED	W.SEA			
	Mean	0.0202	0.0234	0.0109	0.0104			$Q_k(10) = 5747$, *p-value* $= 0$
	Variance	0.0419	0.0347	0.0631	0.0643			$Q_k^r(10) = 1832$, *p-value* $= 0$
	Stdev	2.0471	1.8633	2.5116	2.5361			
	Skewness	52.6835	12.2476	-20.9301	5.3782			
	Excess Kurtosis	657.9097	690.7580	906.9637	1278.0779			

Table 3.2: *Summary statistics of daily log-returns series reported in percentages. High excess kurtosis and skewness suggest stock returns are not normally distributed. A normal distribution has kurtosis of* 3 *and excess kurtosis of* 0*, is symmetric around the mean with* 0 *skewness. As shown on the table, the return distributions are highly skewed; some highly positively skewed and others highly negatively skewed indicating that the distribution of the data sets has heavier tails than the normal distribution. Multivariate ARCH (M-ARCH) tests at* 5% *significance level on the log-returns for each country's stock returns at* $m = 10$ *are also reported. The test rejects the null hypothesis of no conditional heteroscedasticity in the log return series.*

3.6 Results

3.6.1 Modelling the volatility matrix and copula parameters

We obtained the volatility matrix Σ_t, which consists of the marginal standardised residuals $\{\eta_{i,t}\}$, for $i = 1, \ldots, N, t = 1, \ldots, T$ by applying the M-GARCH(1,1) DCC model to the log return series. For the conditional distributions of r_t, we employ a skew and fat tail error distribution; the skewed Student's-t error distribution, to account for the heavy tails and skewness. Recent studies by Chen et al. (2017, 2012) have shown that skewed Student's-t errors distribution is a worthy choice, when compared to a range of existing alternatives. The skewed Student's-t distribution has the form

$$E(|z_t|) = \frac{2\gamma^2}{\gamma + \frac{1}{\gamma}} \frac{\Gamma(\frac{1+\nu}{2}) 2\sqrt{\nu - 2}}{\sqrt{\pi}(\nu - 1)\Gamma(\frac{\nu}{2})} \tag{3.34}$$

where the constraint on the degrees of freedom parameter $\nu > 2$ is imposed to guarantee that the second order moment exist. $\Gamma(.)$ is the Gamma function and $\gamma > 0$ is the asymmetry parameter. For symmetric Student-t, $\gamma = 1$ (Peters, 2001).

The mean equation is modeled by an ARMA(1,1) model; see Appendix A.1 and Tables A.T1 and A.T2 for the estimated GARCH(1,1) parameters with skewed-t distribution. Table 3.3 shows parameter estimates of the fitted DCC and aDCC models respectively for each country. Based on the log-likelihood ratios and AIC values, aDCC model is preferred for France, Greece, UK and Sweden while for Spain DCC model is preferred. The difference in the log-likelihood ratios for both models are quite small. For example, the difference in the log-likelihood ratios for aDCC and DCC models for Greece is 0.06, and exactly the same AIC values. This suggest that the signs of the $P\&L$ distributions of the data used at time $t - 1$ has very little or no influence on the conditional variances, covariances and correlations at time t, i.e., the current conditional variances, covariances and correlations.

	Parameters	France	Greece	UK	Spain	Sweden
DCC	θ_1	0.0144 (0.0037)	0.0296 (0.0038)	0.0213 (0.0043)	0.01733 (0.0030)	0.0180 (0.0032)
	θ_2	0.9645 (0.0142)	0.9225 (0.0125)	0.9442 (0.0148)	0.9742 (0.0055)	0.9584 (0.0090)
	$Log-Likelihood$	50283.53	40506.21	41169.48	44019.77	35196.64
	AIC	-35.0110	-28.195	-28.6760	-30.6530	-24.5100
	ν	4.5547	4.8815	4.6583	4.4662	4.0000
aDCC	A	0.0121 (0.0032)	0.0296 (0.0032)	0.0124 (0.0034)	0.0159 (0.0030)	0.0143 (0.0029)
	B	0.9649 (0.0139)	0.9225 (0.0294)	0.9496 (0.0134)	0.9747 (0.0054)	0.9562 (0.0108)
	G	0.0058 (0.0030)	0.0000 (0.0194)	0.0194 (0.0053)	0.0032 (0.0025)	0.0095 (0.0050)
	$Log-Likelihood$	50285.31	40506.27	41176.50	44019.10	35199.38
	AIC	-35.0120	-28.195	-28.6800	-30.6520	-24.5110
	ν	4.6307	4.8814	4.8813	4.5391	4.0894

Table 3.3: *Parameter estimates of fitted DCC and aDCC models; standard errors in parenthesis. Based on the log-likelihood ratios and AIC values, aDCC model is preferred for France, Greece, UK and Sweden, while for Spain normal DCC model is preferred. However, the difference in the log-likelihood ratios and AIC values between the two models are quite minimal. See Tables A.T1 and A.T2 for the estimated GARCH(1,1) parameters with skewed-t distribution.*

We use canonical maximum likelihood (CML) method to estimate the copula parameters (Cherubini et al., 2004). That is, we use pseudo-observations of the standardised residuals from the fitted M-GARCH(1,1) aDCC models for France, Greece, UK and Sweden, and M-GARCH(1,1) DCC model for Spain to estimate the marginals and then estimated the copula parameters by "inversion of Kendall's τ". Kendall's τ is one of the most commonly used invariant measures and has been proven to provide more efficient way of estimating correlations (Howell, 2012; Croux and Dehon, 2010).

The copula that fits the data best is selected by maximum likelihood estimation (MLE) method by maximising the likelihood function

$$\hat{\Theta}_2 = ArgMax_{\Theta_2} \sum_{t=1}^{T} \ln c(\hat{F}_1(x_{1t}), \ldots, \hat{F}_n(x_{nt}); \Theta_2). \tag{3.35}$$

and then compare their log-likelihood ratios, where $\hat{\Theta}_2$ are estimates of the copula parameters. The copula with the highest likelihood ratios is selected as the best fit. Table 3.4 presents the estimated copula parameters based on “inversion of Kendall’s tau” (standard errors in parenthesis), log-likelihood ratios and *Akaike information criterion* (AIC) values. We see from the table the selected copulas in bold; i.e., for the Archimedean copula family, Clayton copula is selected for France, UK and Spain, while Gumbel copula is selected for Greece and Sweden as the best fit. For the elliptical copula family, only the Student-*t* copula is selected as the best fit. The same copula types are selected based on the smallest AIC value. Chollete et al. (2009) argues that by selecting the best model based on MLE method or a related criterion such as the *Akaike information criterion* (AIC) or *Bayesian information criterion* (BIC) (See Appendix A.2), the restriction does not matter since copulas, which allow for negative dependence, will still be chosen if the data set contains periods with negative dependence of the data. We select two models, one from each copula family; Archimedean and elliptical copulas. Note that Gaussian copula gives higher log-likelihood values for the different countries compared to the Archimedean copulas but also higher AIC values. Thus, the Gaussian copula is not a good fit for the data when compared to the Archimedean copulas and elliptical *t*-copula.

		Copula type				
		Gumbel	Clayton	Frank	Gaussian	student-t
France (aDCC model)	*Log-Likelihood*	5.2150	**192.3000**	5.8360	1999.0000	**2530.0000**
Copula parameter	Kendall's τ	1.1456 (0.0050)	0.2074 (0.0090)	0.4160 (0.0000)	$\rho_{\tau_{FRAN}}(\rho_{SE})$	$\rho_{\tau_{FRAN}}(\rho_{SE})$
	AIC	-1.3031	**-8.5181**	-1.5281	14.7992	**14.3281**
Greece (aDCC model)	*Log-Likelihood*	**3400.0000**	3155.0000	3180.0000	4186.0000	**5387.000**
Copula parameter	Kendall's τ	1.6176 (0.0020)	1.2353 (0.0040)	3.88613 (0.0290)	$\rho_{\tau_{GREC}}(\rho_{SE})$	$\rho_{\tau_{GREC}}(\rho_{SE})$
	AIC	**-14.2631**	-14.1135	-14.1293	13.3210	**12.8165**
UK (aDCC model)	*Log-Likelihood*	443.3000	**471.1000**	376.6000	2992.000	**3699.0000**
Copula parameter	Kendall's τ	1.3622 (0.0120)	0.6290 (0.0230)	1.6419 (0.0050)	$\rho_{\tau_{UK}}(\rho_{SE})$	$\rho_{\tau_{UK}}(\rho_{SE})$
	AIC	-10.1930	**-10.3101**	-9.8624	4.2039	**3.5684**
Spain (DCC model)	*Log-Likelihood*	314.1000	**568.3000**	333.9000	3684.0000	**4178.0000**
Copula parameter	Kendall's τ	1.5015 (0.0110)	0.8416 (0.0190)	1.8738 (0.0040)	$\rho_{\tau_{SPN}}(\rho_{SE})$	$\rho_{\tau_{SPN}}(\rho_{SE})$
	AIC	-9.4994	**-10.6853**	-9.6217	3.5765	**3.3248**
Sweden (aDCC model)	*Log-Likelihood*	**2905.0000**	2173.000	2591.0000	3212.0000	**4048.0000**
Copula parameter	Kendall's τ	1.8646 (0.0300)	1.7291 (0.0590)	5.0580 (0.0560)	$\rho_{\tau_{SWE}}(\rho_{SE})$	$\rho_{\tau_{SWE}}(\rho_{SE})$
	AIC	**-13.9484**	-13.3677	-13.7196	-4.1493	**-4.6120**

Table 3.4: *Log-likelihood ratios, copula parameters based on "inversion of Kendall's tau" (standard errors in parenthesis), and AIC values. The best copula for dependance modeling is selected based on the highest log-likelihood ratio (in bold). If the selection criterion is based on AIC, the same copula types are selected (in bold) as with the MLE method.*

$\rho_{\tau_{FRAN}}(\rho_{SE})$		F.BNP	F.SGE	F.CRDA	F.KNF	F.CC	F.CAI
	F.BNP	1					
	F.SGE	-0.3355 (0.0200)	1				
	F.CRDA	-0.2994 (0.0210)	0.8185 (0.0080)	1			
	F.KNF	-0.0658 (0.0230)	0.2543 (0.0210)	0.2632 (0.0210)	1		
	F.CC	-0.1270 (0.0210)	0.0212 (0.0210)	0.0380 (0.0210)	0.0087 (0.0210)	1	
	F.CAI	-0.3224 (0.0190)	0.0542 (0.0210)	0.0468 (0.0210)	0.0091 (0.0210)	0.1441 (0.0210)	1

$\rho_{\tau_{GREC}}(\rho_{SE})$		G.PIST	G.PEIR	G.EFG	G.ETE	G.ATT	G.ELL
	G.PIST	1					
	G.PEIR	0.5830 (0.0150)	1				
	G.EFG	0.6593 (0.0140)	0.7009 (0.0130)	1			
	G.ETE	0.6024 (0.0150)	0.7101 (0.0130)	0.6912 (0.0130)	1		
	G.ATT	0.4920 (0.0180)	0.5280 (0.0170)	0.5852 (0.0160)	0.5278 (0.0170)	1	
	G.ELL	0.3212 (0.0200)	0.4899 (0.0180)	0.4250 (0.0190)	0.4601 (0.0180)	0.3661 (0.0190)	1

$\rho_{\tau_{UK}}(\rho_{SE})$		UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN
	UK.HSBA	1				
	UK.BARC	-0.1165 (0.0230)	1			
	UK.LLOY	-0.0871 (0.0230)	0.7067 (0.0130)	1		
	UK.RBS	-0.1067 (0.0230)	0.7051 (0.0130)	0.7117 (0.0130)	1	
	UK.STAN	0.5623 (0.0160)	-0.1882 (0.0210)	-0.1553 (0.0220)	-0.1614 (0.0210)	1

$\rho_{\tau_{SPN}}(\rho_{SE})$		E.SCH	E.BBVA	E.BSAB	E.BKT	E.POP
	E.SCH	1				
	E.BBVA	-0.4184 (0.0190)	1			
	E.BSAB	-0.5534 (0.0150)	0.6021 (0.0140)	1		
	E.BKT	-0.2890 (0.0200)	0.7008 (0.0120)	0.5152 (0.0170)	1	
	E.POP	-0.4818 (0.0170)	0.7122 (0.0120)	0.7514 (0.0100)	0.6004 (0.0150)	1

$\rho_{\tau_{SWE}}(\rho_{SE})$		S.NDA	S.SVK	S.SWED	S.SEA
	W.NDA	1			
	W.SVK	0.7213 (0.0120)	1		
	W.SWED	0.4509 (0.0180)	0.5984 (0.0160)	1	
	W.SEA	0.5972 (0.0150)	0.6974 (0.0120)	0.7877 (0.0100)	1

Table 3.5: *Kendall's τ; $\rho_\tau(\rho_{SE})$ for Gaussian and student's-t copulas, standard errors in parenthesis.*

The next step is to specify the desired marginal distributions, which we set to a Student's t distribution. The choice of Student's t distributions for the margins is because multivariate ARCH test on the standardised residuals, $\{\eta_{i,t}\}_{t=1}^{T}$, i.e., after fitting the DCC models fail to reject the null hypothesis of no conditional heteroscedasticity; Table A.T3. Finally, the estimated copula parameters are then used to generate a new matrix of 10000 simulations

$$\hat{\Sigma} = \{\zeta_{i,t}\}, \quad i = 1, \ldots, N, \ t = 1, \ldots, 10000 \tag{3.36}$$

with margins that are totally free from assumptions of normality or linear correlations. As seen on Table A.T4, multivariate Arch test on $\{\zeta_{i,t}\}$ show no evidence of serial correlation or conditional heteroscedasticity for $m = 10$. However, for the non-robust test at $m = 5$, the null hypothesis of no conditional heteroscedasticity is rejected for UK at 5% significance level after modeling dependence with t-copula. This is an indication that the residuals have heavy tails.

We follow the approach by McNeil and Frey (2000) and apply POT method of EVT to each of the marginal distributions of $\{\zeta_{i,t}\}$ i.e., Eqn.(3.36) to obtain the q^{th} quantile; $VaR_q(Z)$, of the noise variables for VaR estimation. Let $\{x_{i,\iota}\}$ be the negative variables of the marginal distributions of $\{\zeta_{i,t}\}$ such that $\{x_{i,\iota}\} \subseteq \{\zeta_{i,t}\}$. Then from the ordered sample of $\{x_{i,\iota}\}$, the mean excesses x_i are plotted against each calculated threshold ϑ_i for $i = 1, \ldots, \iota$. ϑ_i are calculated from the distribution of x_i given a series of selected quantiles. As an example, Figure 3.6 shows the mean excess plots for the various distributions of $\{x_{i,\iota}\}$ for UK.HSBA. Subjective cut-off points from the mean excess plots as threshold values, POT parameter estimates, $VaR_q(Z)$ and $ESF_q(Z)$ at $q = (99\%, 95\%, 90\%)$ are presented on Tables 3.6 and 3.7 following the selected Clayton and Student's-t copulas, respectively.

From these tables, we can see that Sweden produced the highest $VaR_q(Z)$ followed by Spain and Greece with Archimedean copulas. With t-copulas, Sweden still produced the highest $VaR_q(Z)$ followed by Spain and UK. The results for Greece and Spain are expected as these are the countries that were greatly affected by the 2008 and 2011 financial crisis. It can also be noted that the number of exceedances above the threshold compared to the size of the data (i.e., T=10,000), seems to lie towards the body of the distribution which might result to poor approximation of GPD parameters and $VaR_q(Z)$.

Now that we have $VaR_q(Z)$ for the individual banks, we can now proceed to compute the portfolio quantile VaRs; $VaR_q^p(Z)$, for investing in multiple positions by applying the risk formula as follows: let **Inv** be the total amount of money invested in the portfolio, x_i be the fraction of the total investment invested in stock i, and w_i the weights such that $w_i = \frac{x_i}{\mathbf{Inv}}$. Then we have

$$VaR_q^p(Z) = \left(\sum_{i=1}^{N} w_i^2 VaR_{q,i}^2(Z) + 2w_i w_j \sum_{i<j}^{N} \rho_{ij} VaR_{q,i}(Z) VaR_{q,j}(Z) \right)^{1/2}, \quad \sum_{i=1}^{N} w_i = 1, \quad (3.37)$$

where $\rho_{i,j}$ is the Pearsons cross-correlation coefficient between the returns of the ith and jth stocks. Since the stocks are all from banks of almost the same strength and ratings (i.e., the top 10% from each country) , we assume equal weights and compute $VaR_q^p(Z)$ for an investment in all the banks involved for each country at $q = (99\%, 95\%, 90\%)$. Results are presented on Table 3.8. Using Eqn.(3.37), we can now define the portfolio VaR estimate for a single period as

$$VaR_{q,t}^p = \hat{\mu}_{t+1} + VaR_q^p(Z)\hat{h}_{t+1}^{1/2} \quad (3.38)$$

where $\hat{h}_{t+1}$ are estimates of conditional variances and $\hat{\mu}_{t+1}$ is an estimate of the conditional mean obtained by fitting a univariate ARMA(1,1)-GARCH(1,1) model with skewed

Student's-t distribution to the portfolio returns constructed from the original return series. Note that the portfolios consisting of each country's stocks are constructed using the simple returns of each of the stocks, and then converted to log returns for further analysis. The reason for this is because log-returns can not be used for portfolios as they are not additive across assets whereas simple returns are additive across assets. The portfolio simple returns are converted to log-returns because we want to calculate the portfolio VaR for different time horizons.

The data used are of daily stock returns, thus these are daily VaR estimates referred to as *robust* because they incorporates volatility clustering and are free from any normality assumptions. To obtain VaR estimates for any desired time horizon $\Bbbk$ in relation to a 1-day horizon (Eqn.(3.38)), we employ the $\alpha - root$ of time rule as discussed by Danielsson and De Vries (2000) and Tsay (2014). Thus, the relationship between Eqn.(3.38) and $\Bbbk$-day horizon is defined as

$$VaR_{q,t}(\Bbbk) = \Bbbk^{1/\alpha} VaR_{q,t}, \quad 1/\alpha = \xi, \tag{3.39}$$

where ξ is the shape parameter from the POT method of EVT and α is the tail index (Tsay, 2014).

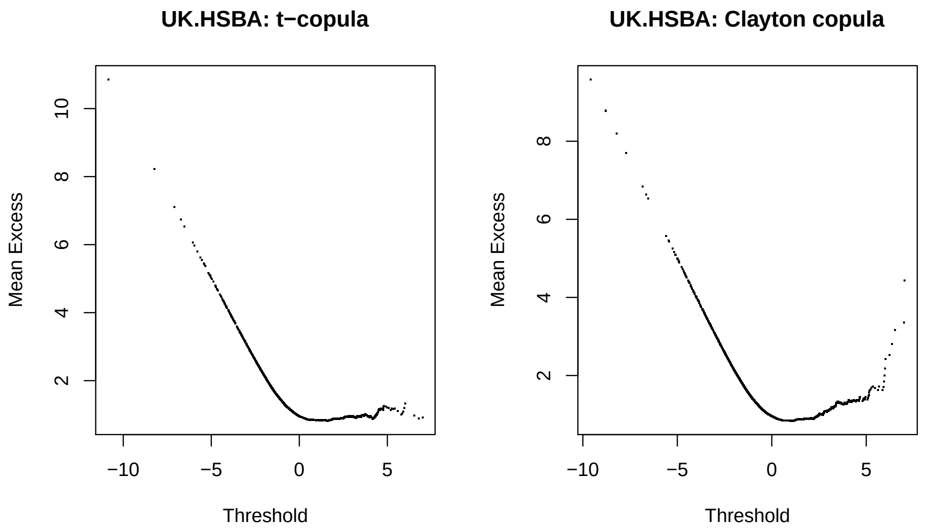


Figure 3.6: *Mean excess plots of the noise variables for UK.HSBA, generated following aDCC Archimedean and elliptical copula models.*

		POT Parameter estimates						$VaR_q(Z)$			$ESF_q(Z)$		
		$\hat{\xi}$	$\hat{\psi}(\vartheta)$	ϑ	N_ϑ	μ	σ	99%	95%	90%	99%	95%	90%
UK	UK.HSBA	0.0759	0.7905	1.4683	1037	-0.1775	0.6656	3.4914	2.0612	1.4971	4.5128	2.9653	2.3548
	UK.BARC	0.1078	0.7383	1.4360	1020	-0.0582	0.5773	3.3842	1.9831	1.4506	4.4469	2.8766	2.2798
	UK.LLOY	0.1371	0.7085	1.6673	789	0.1480	0.5002	3.3592	2.0008	1.5021	4.4492	2.8750	2.2970
	UK.RBS	0.1380	0.7105	1.4803	1031	0.0946	0.5193	3.4360	2.0211	1.5021	4.5733	2.9319	2.3298
	UK.STAN	0.1267	0.7208	1.5527	926	0.0721	0.5331	3.4060	2.0147	1.4975	4.5004	2.9071	2.3149
France	F.BNP	0.0200	0.8611	1.4109	1151	-0.4110	0.8247	3.5671	2.1349	1.5322	4.4899	3.0284	2.4134
	F.SGE	0.1349	0.7334	1.3694	1107	-0.0272	0.5449	3.4522	1.9847	1.4444	4.6249	2.9284	2.3039
	F.CRDA	0.1214	0.7744	1.8975	622	0.0718	0.5528	3.4822	2.0688	1.5402	4.5825	2.9739	2.3722
	F.KNF	0.1472	0.7530	1.6126	912	0.0926	0.5293	3.5794	2.0854	1.5433	4.8021	3.0501	2.4144
	F.CC	0.0840	0.7387	1.3348	1232	-0.0835	0.6196	3.4000	2.0268	1.4903	4.4081	2.8968	2.3110
	F.CAI	0.0876	0.7618	1.2518	1396	-0.1257	0.6412	3.5107	2.0702	1.5097	4.5623	2.9836	2.3693
Greece	G.PIST	0.0473	0.8333	1.4323	1091	-0.3204	0.7504	3.5404	2.0946	1.5050	4.5197	3.0021	2.3832
	G.PEIR	0.0886	0.8330	1.7678	701	-0.2044	0.6582	3.5382	2.0535	1.4765	4.6243	2.9952	2.3621
	G.EFG	0.0871	0.8100	1.6472	845	-0.1536	0.6531	3.5470	2.0820	1.5117	4.6156	3.0109	2.3861
	G.ETE	0.1335	0.7899	1.5378	952	-0.0565	0.5771	3.6146	2.0690	1.4991	4.8462	3.0625	2.4047
	G.ATT	0.1078	0.7804	1.3010	1285	-0.1354	0.6255	3.5950	2.0765	1.4994	4.7470	3.0449	2.3981
	G.ELL	0.1757	0.7142	1.4470	1075	0.1343	0.4808	3.5435	2.0299	1.4988	4.8533	3.0171	2.3728
Spain	E.SCH	0.0722	0.8028	1.4322	1154	-0.1729	0.6869	3.5797	2.1243	1.5478	4.6121	3.0435	2.4220
	E.BBVA	0.0954	0.7656	1.1929	1481	-0.1437	0.6380	3.5461	2.0688	1.4993	4.6407	3.0076	2.3780
	E.BSAB	0.1050	0.8367	1.5798	918	-0.1872	0.6511	3.6686	2.1048	1.5086	4.8486	3.1013	2.4351
	E.BKT	0.0708	0.8390	1.5170	1078	-0.2118	0.7166	3.6897	2.1795	1.5802	4.7581	3.1329	2.4880
	E.POP	0.0673	0.7969	1.0753	1743	-0.2380	0.7085	3.5867	2.1134	1.5264	4.6223	3.0427	2.4134
Sweden	W.NDA	0.0772	0.9266	1.3434	1245	-0.4397	0.7889	3.9231	2.2193	1.5482	5.1432	3.2967	2.5695
	W.SVK	0.1698	0.8150	1.5766	986	0.0156	0.5500	3.8560	2.1632	1.5651	5.3039	3.2649	2.5445
	W.SWED	0.1288	0.9130	1.6940	834	-0.2471	0.6631	3.9208	2.1768	1.5302	5.2979	3.2962	2.5539
	W.SEA	0.2086	0.8212	1.5122	1037	0.0294	0.5118	3.9881	2.1592	1.5421	5.6785	3.3675	2.5877

Table 3.6: *POT parameter estimates, $VaR_q(Z)$ and $ESF_q(Z)$ following Archimedean copulas.*

		POT Parameter estimates						$VaR_q(Z)$			$ESF_q(Z)$		
		$\hat{\xi}$	$\hat{\psi}(\vartheta)$	ϑ	N_ϑ	μ	σ	99%	95%	90%	99%	95%	90%
UK	UK.HSBA	0.0583	0.8074	1.4671	939	-0.3168	0.7034	3.3988	1.9854	1.4164	4.3759	2.8749	2.2707
	UK.BARC	0.0977	0.7758	1.4013	1107	-0.1349	0.6257	3.5038	2.0425	1.4806	4.5912	2.9717	2.3489
	UK.LLOY	0.1476	0.7617	1.6365	881	0.0816	0.5322	3.5911	2.0866	1.5409	4.8232	3.0581	2.4180
	UK.RBS	0.1104	0.7406	1.3650	1249	-0.0116	0.5887	3.5215	2.0785	1.5317	4.6215	3.9994	2.3849
	UK.STAN	0.1862	0.6975	1.6170	834	0.2301	0.4392	3.4312	1.9914	1.4926	4.7034	2.9342	2.3211
France	F.BNP	0.1245	0.7172	1.4750	1040	0.0603	0.5411	3.4250	2.0249	1.5032	4.5214	2.9223	2.3264
	F.SGE	0.1213	0.7409	1.4878	977	-0.0134	0.5587	3.4331	2.0048	1.4706	4.5449	2.9194	2.3114
	F.CRDA	0.1359	0.7341	1.4497	1030	0.0143	0.5390	3.4642	2.0072	1.4715	4.6307	2.9445	2.3245
	F.KNF	0.1521	0.7264	1.7381	794	0.2109	0.4942	3.5072	2.0862	1.5735	4.6812	3.0052	2.4006
	F.CC	0.0537	0.7672	1.4585	1041	-0.1758	0.6794	3.3737	2.0323	1.4893	4.2930	2.8755	2.3018
	F.CAI	0.1339	0.7289	2.0750	483	0.2594	0.4857	3.3529	2.0498	1.5695	4.3921	2.8875	2.3330
Greece	G.PIST	0.0829	0.7225	1.3146	1255	-0.0630	0.6083	3.3483	2.0056	1.4803	4.3199	2.8559	2.2831
	G.PEIR	0.0859	0.7394	1.4903	981	-0.0661	0.6057	3.3555	2.0033	1.4761	4.3396	2.8603	2.2836
	G.EFG	0.0750	0.7485	1.3078	1248	-0.1343	0.6403	3.3879	2.0165	1.4750	4.3659	2.8823	2.2978
	G.ETE	0.0906	0.7247	1.4108	1129	-0.0237	0.5948	3.3751	2.0233	1.4992	4.3676	2.8812	2.3048
	G.ATT	0.0628	0.7170	1.3041	1282	-0.0776	0.6302	3.2878	2.0000	1.4836	4.1858	2.8112	2.2607
	G.ELL	0.0997	0.7396	1.5908	856	-0.0215	0.5788	3.3614	1.9992	1.4767	4.3788	2.8659	2.2855
Spain	E.SCH	0.0510	0.8139	1.1949	1443	-0.3053	0.7373	3.5223	2.0812	1.4961	4.5051	2.9866	2.3700
	E.BBVA	0.1302	0.7535	1.3630	1223	-0.0221	0.5732	3.5933	2.0778	1.5167	4.7932	3.0509	2.4059
	E.BSAB	0.1128	0.8109	1.1197	1646	-0.2040	0.6615	3.7909	2.1538	1.5354	5.0448	3.1994	2.5023
	E.BKT	0.1045	0.7659	1.0766	1761	-0.1399	0.6388	3.6385	2.1073	1.5231	4.7929	3.0829	2.4306
	E.POP	0.0991	0.8212	1.4562	1087	-0.1798	0.6592	3.6667	2.1191	1.5250	4.8212	3.1036	2.4441
Sweden	W.NDA	0.0740	0.8400	1.2674	1357	-0.2923	0.7246	3.6836	2.1378	1.5267	4.7838	3.1144	2.4545
	W.SVK	0.0808	0.8457	1.3835	1179	-0.2769	0.7116	3.6925	2.1346	1.5236	4.8155	3.1207	2.4560
	W.SWED	0.1540	0.7927	1.3401	1266	-0.0632	0.5765	3.8025	2.1318	1.5305	5.1879	3.2131	2.5022
	W.SEA	0.1251	0.7886	1.3597	1286	-0.0669	0.6101	3.7330	2.1505	1.5612	4.9739	3.1650	2.4915

Table 3.7: *POT parameter estimates, $VaR_q(Z)$ and $ESF_q(Z)$ following elliptical Student's-t copulas.*

	Archimedean copulas			Elliptical copulas		
	99%	95%	90%	99%	95%	90%
UK	2.7818	1.6422	1.2135	2.8449	1.6601	1.2160
France	2.5243	1.5946	1.0887	2.4758	1.4640	1.0865
Greece	2.9077	1.6881	1.2228	2.7372	1.6390	1.2094
Spain	3.1830	1.8657	1.3499	3.1852	1.8646	1.3818
Sweden	3.5063	1.9482	1.3822	3.3322	1.9118	1.3727

Table 3.8: *Portfolio quantile VaR estimates;* $VaR_q^p(Z)$*, at* $q = (99\%, 95\%, 90\%)$ *for an investment in multiple positions. i.e., an investment in all banks involved for each country while assuming equal weights.*

3.6.2 Model validation

The reliability of the VaR model is assessed following the techniques discussed in Section 2.7 of Chapter 2. That is, we employ the unconditional coverage (UC) test, the independent (IND) and conditional coverage (CC) test, Basel "traffic light" test, DQ test, and the new independent test. The out of sample data is divided into blocks of 250, 500, and 1000 trading days to see how the model behaves for longer and shorter observation periods and to also meet the Basel requirements for back-testing. The observed number of exceptions produced for the different observation periods are presented on Table 3.9. Following the Basel regulations of back-testing (the Basel "traffic light" test), the VaR model is accepted or rejected if the number of exceptions produced at 99% confidence level fall within the following category:

Observation period	Green zone	Yellow zone	Red zone
250	≤ 4	≤ 9	≥ 10
500	≤ 8	≤ 14	≥ 15
1000	≤ 13	≤ 22	≥ 23

Based on the number of exceptions produced at 99% confidence level and 250 observation period, the VaR model passed the "traffic light" test; see Table 3.10. The fewer the number of exceptions the better the model according to the Basel rules. The model fall in the green zone in all cases and is thus deemed reliable. Back-testing results for LR_{UC},

LR_{IND}, LR_{CC}, DQ, and $T_{N,[N/2]}$ test are presented in Tables 3.11, 3.12 and 3.13. Kupiec's UC test rejects the VaR model that produces zero or very few exceptions further away from the expected for being too conservative (see Tables 3.11).

The overall test results suggest that the VaR models captures VaR quite well at different observation periods and confidence levels. The model is thus reliable and can be used as a measure of risk in these countries.

Window		250			500			1000	
Probability	1%	5%	10%	1%	5%	10%	1%	5%	10%
Expected exceptions	2.5	12.5	25	5	25	50	10	50	100
Observed exceptions E-Cop									
UK	2	2	4	10	24	49	26	49	107
coverage rate	0.008	0.004	0.004	0.040	0.048	0.049	0.104	0.098	0.107
France	4	5	13	20	31	69	21	57	123
coverage rate	0.016	0.010	0.013	0.080	0.062	0.069	0.116	0.114	0.123
Greece	4	5	13	13	33	69	20	53	122
coverage rate	0.016	0.010	0.113	0.052	0.066	0.069	0.080	0.106	0.122
Spain	1	1	3	11	19	40	25	40	89
coverage rate	0.004	0.002	0.003	0.044	0.038	0.040	0.100	0.080	0.089
Sweden	0	1	2	7	18	38	18	34	76
coverage rate	0.000	0.002	0.002	0.028	0.036	0.038	0.072	0.068	0.076
Observed exceptions A-Cop									
UK	2	2	5	11	25	52	27	50	108
coverage rate	0.008	0.004	0.005	0.044	0.050	0.052	0.108	0.010	0.108
France	3	4	12	14	23	51	29	57	123
coverage rate	0.012	0.008	0.112	0.056	0.046	0.051	0.116	0.114	0.123
Greece	2	3	10	13	32	66	20	52	119
coverage rate	0.008	0.006	0.010	0.052	0.064	0.066	0.080	0.104	0.119
Spain	1	1	3	11	19	40	25	40	91
coverage rate	0.004	0.002	0.003	0.044	0.038	0.040	0.010	0.080	0.091
Sweden	0	0	1	7	18	37	18	34	76
coverage rate	0.000	0.000	0.001	0.028	0.036	0.037	0.072	0.068	0.076

Table 3.9: *Observed number of exceptions following M-GARCH(1,1) aDCC copula EVT VaR model for UK, France, Greece, Sweden and M-GARCH(1,1) DCC copula EVT models for Spain. Out-of-sample data after* 2011 *European financial crisis is divided into windows of* 250, 500, *and* 1000 *observation periods. At* 250, 500, *and* 1000 *observation periods and time horizons of* 1 *day, we expect to have at* $p = 1\%$*:* 3, 5, *and* 10 *exceptions, at* $p = 5\%$*:* 13, 25, *and* 50 *exceptions, and at* $p = 10\%$*:* 25, 50, *and* 100 *exceptions, respectively. coverage rate =* $\frac{T_1}{T_w}$*, A-cop = Archimedean copula and E-cop = Elliptical copula.*

	Window			Zone			Test Result		
	250	500	1000	250	500	1000	250	500	1000
Observed exceptions E-Cop									
UK	2	10	26	Green	Yellow	Red	A	A	R
France	4	20	21	Green	Red	Yellow	A	R	A
Greece	4	13	13	Green	Yellow	Yellow	A	A	A
Spain	1	11	25	Green	Yellow	Red	A	A	R
Sweden	0	7	18	Green	Green	Yellow	A	A	A
Observed exceptions A-Cop									
UK	2	11	27	Green	Yellow	Red	A	A	R
France	3	14	29	Green	Yellow	Red	A	R	R
Greece	2	13	20	Green	Yellow	Yellow	A	A	A
Spain	1	11	25	Green	Yellow	Red	A	A	R
Sweden	0	7	18	Green	Green	Yellow	A	A	A

Table 3.10: *Basel "traffic light" test results. Following Basel rules of back-testing, the VaR model fall in the green zone and is therefore deemed reliable. A = accept, R = reject.*

p = 1%
E-cop

	Window	Exceptions	Back-test type LR_{UC}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$	Test results
UK	250	2	0.108 (0.742)	0.123 (0.726)	0.231 (0.891)	1.000 (0.986)	0.746 (0.649)	(A A A A A)
	500	2	2.353 (0.125)	0.035 (0.852)	2.388 (0.303)	1.826 (0.935)	0.746 (0.650)	(A A A A A)
	1000	5	3.094 (0.079)	0.066 (0.797)	3.160 (0.206)	3.930 (0.686)	5.619 (0.126)	(A A A A A)
France	250	3	0.095 (0.758)	0.288 (0.592)	0.383 (0.826)	25.907 (0.000)	2.842 (0.264)	(A A A R A)
	500	4	0.217 (0.641)	0.226 (0.635)	0.443 (0.801)	14.719 (0.023)	2.619 (0.425)	(A A A R A)
	1000	12	0.380 (0.538)	0.809 (0.368)	1.189 (0.552)	8.876 (0.181)	0.109 (0.742)	(A A A A A)
Greece	250	2	0.108 (0.742)	0.258 (0.611)	0.366 (0.833)	2.726 (0.842)	-0.667 (0.999)	(A A A A A)
	500	3	0.943 (0.332)	0.200 (0.655)	1.143 (0.565)	6.566 (0.363)	-0.608 (0.988)	(A A A A A)
	1000	10	0.000 (1.000)	0.691 (0.691)	0.691 (0.708)	6.372 (0.383)	0.406 (0.667)	(A A A A A)
Spain	250	1	1.176 (0.278)	0.026 (0.872)	1.202 (0.548)	0.418 (0.999)	*NaN*	(A A A A -)
	500	1	4.813 (0.028)	0.009 (0.924)	4.822 (0.090)	1.925 (0.926)	*NaN*	(R A A A -)
	1000	3	6.826 (0.009)	0.037 (0.874)	6.863 (0.033)	4.933 (0.552)	9.530 (0.252)	(R A R A A)
Sweden	250	0	*NaN*	-	-	2.157 (0.905)	-	(- - - A -)
	500	0	*NaN*	-	-	3.494 (0.745)	-	(- - - A -)
	1000	1	13.476 (0.000)	0.018 (0.893)	13.494 (0.000)	6.659 (0.354)	0.607 (0.670)	(R A R A A)

p = 1%
A-cop

	Window	Exceptions	LR_{UC}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$	Test results
UK	250	2	0.108 (0.742)	0.074 (0.786)	0.182 (0.913)	79.862 (0.000)	0.746 (0.649)	(A A A R A)
	500	2	2.353 (0.125)	0.035 (0.852)	2.388 (0.303)	38.635 (0.000)	0.746 (0.652)	(A A A R A)
	1000	4	4.706 (0.030)	4.429 (0.035)	9.135 (0.010)	22.806 (0.001)	13.480 (0.070)	(R R R R A)
France	250	4	0.769 (0.381)	0.161 (0.688)	0.930 (0.628)	25.907 (0.000)	33.559 (0.085)	(R A A R A)
	500	5	0.000 (1.000)	0.115 (0.734)	0.115 (0.944)	14.719 (0.023)	5.545 (0.129)	(A A A R A)
	1000	13	0.831 (0.362)	1.039 (0.308)	1.870 (0.393)	10.333 (0.111)	0.816 (0.480)	(A A A A A)
Greece	250	4	0.769 (0.381)	0.063 (0.802)	0.832 (0.660)	2.856 (0.827)	0.770 (0.644)	(A A A A A)
	500	5	0.000 (1.000)	0.071 (0.790)	0.071 (0.965)	6.917 (0.329)	0.416 (0.876)	(A A A A A)
	1000	13	0.831 (0.362)	0.407 (0.525)	1.309 (0.520)	5.438 (0.489)	0.668 (0.527)	(A A A A A)
Spain	250	1	1.176 (0.278)	0.016 (0.899)	1.192 (0.551)	0.418 (0.999)	*NaN*	(A A A A -)
	500	1	4.813 (0.028)	0.009 (0.924)	4.822 (0.090)	1.925 (0.926)	*NaN*	(R A A A -)
	1000	3	6.826 (0.009)	0.037 (0.847)	6.863 (0.032)	4.933 (0.552)	9.530 (0.251)	(R A R A A)
Sweden	250	0	*NaN*	-	-	2.157 (0.905)	-	(- - - A -)
	500	1	4.813 (0.028)	-	-	3.494 (0.745)	-	(R - - A -)
	1000	2	9.627 (0.002)	0.005 (0.944)	9.632 (0.008)	8.297 (0.217)	*NaN*	(R A R A -)

Table 3.11: *Back-testing results at 99% confidence level. A-cop = Archimedean copula and E-cop = Elliptical copula.*

p = 5%
E-cop

	Window	Exceptions	LR_{UC}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$	Test results
					Back-test type			
UK	250	11	0.197 (0.657)	2.003 (0.157)	2.200 (0.332)	3.544 (0.738)	1.163 (0.414)	(A A A A A)
	500	25	0.000 (1.000)	1.343 (0.247)	1.343 (0.511)	1.978 (0.922)	0.865 (0.432)	(A A A A A)
	1000	52	0.083 (0.773)	2.670 (0.102)	2.753 (0.252)	5.123 (0.528)	0.152 (0.644)	(A A A A A)
France	250	14	0.183 (0.669)	0.111 (0.739)	0.294 (0.863)	10.683 (0.099)	-0.050 (0.692)	(A A A A A)
	500	23	0.173 (0.677)	0.161 (0.688)	0.334 (0.846)	10.022 (0.124)	-0.939 (0.936)	(A A A A A)
	1000	51	0.021 (0.885)	0.064 (0.800)	0.085 (0.958)	8.308 (0.216)	0.445 (0.541)	(A A A A A)
Greece	250	13	0.021 (0.885)	0.387 (0.534)	0.408 (0.815)	25.557 (0.000)	2.287 (0.299)	(A A A R A)
	500	32	1.903 (0.168)	0.023 (0.879)	1.926 (0.382)	33.888 (0.000)	1.356 (0.357)	(A A A R A)
	1000	66	4.918 (0.027)	0.848 (0.357)	5.766 (0.056)	24.848 (0.000)	-0.999 (0.930)	(R A A R A)
Spain	250	11	0.197 (0.657)	1.669 (0.196)	2.232 (0.328)	5.943 (0.430)	1.933 (0.287)	(A A A A A)
	500	19	1.647 (0.199)	0.071 (0.790)	2.348 (0.309)	4.772 (0.573)	1.346 (0.345)	(A A A A A)
	1000	40	2.253 (0.133)	0.624 (0.430)	2.877 (0.237)	6.267 (0.394)	1.077 (0.363)	(A A A A A)
Sweden	250	7	3.009 (0.083)	0.981 (0.322)	3.990 (0.136)	5.655 (0.423)	1.025 (0.603)	(A A A A A)
	500	18	2.277 (0.131)	0.215 (0.643)	2.492 (0.288)	7.946 (0.242)	0.267 (0.602)	(A A A A A)
	1000	37	3.895 (0.048)	0.023 (0.879)	3.918 (0.141)	9.103 (0.168)	-0.480 (0.801)	(R A A A A)

p = 5%
A-cop

	Window	Exceptions	LR_{UC}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$	Test results
UK	250	10	0.563 (0.453)	0.011 (0.916)	0.574 (0.751)	3.712 (0.716)	0.367 (0.187)	(A A A A A)
	500	24	0.043 (0.836)	0.234 (0.629)	0.277 (0.871)	3.193 (0.784)	0.824 (0.500)	(A A A A A)
	1000	49	0.021 (0.885)	0.598 (0.439)	0.619 (0.734)	5.085 (0.533)	0.092 (0.623)	(A A A A A)
France	250	20	4.040 (0.235)	1.103 (0.294)	5.143 (0.076)	7.987 (0.239)	2.213 (0.227)	(R A A A A)
	500	31	1.413 (0.044)	1.239 (0.266)	2.652 (0.266)	4.112 (0.661)	-1.056 (0.961)	(A A A A A)
	1000	69	6.830 (0.009)	0.398 (0.528)	7.228 (0.027)	2.687 (0.847)	-0.813 (0.901)	(R A R A A)
Greece	250	13	0.021 (0.885)	0.387 (0.534)	0.408 (0.815)	17.497 (0.008)	2.287 (0.301)	(A A A R A)
	500	33	2.459 (0.117)	0.000 (1.000)	2.459 (0.292)	27.163 (0.000)	-0.231 (0.734)	(A A A R A)
	1000	69	6.830 (0.009)	0.778 (0.378)	7.608 (0.022)	21.159 (0.001)	-0.955 (0.910)	(R A R R A)
Spain	250	11	0.197 (0.657)	2.037 (0.154)	2.234 (0.327)	5.943 (0.430)	3.048 (0.749)	(A A A A A)
	500	19	1.647 (0.199)	0.156 (0.693)	1.803 (0.406)	4.772 (0.573)	1.821 (0.323)	(A A A A A)
	1000	40	2.253 (0.133)	0.624 (0.430)	2.877 (0.237)	6.267 (0.394)	1.077 (0.363)	(A A A A A)
Sweden	250	7	3.009 (0.083)	0.981 (0.322)	3.990 (0.136)	5.518 (0.479)	1.025 (0.606)	(A A A A A)
	500	18	2.277 (0.131)	0.215 (0.643)	2.492 (0.288)	8.272 (0.219)	0.267 (0.602)	(A A A A A)
	1000	38	3.294 (0.070)	0.002 (0.964)	3.296 (0.192)	9.521 (0.146)	-0.453 (0.827)	(A A A A A)

Table 3.12: *Back-testing results at 95% confidence levels. A-cop = Archimedean copula and E-cop = Elliptical copula.*

p = 10%
E-cop

	Window	Exceptions	Back-test type LR_{UC}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$	Test results
UK	250	27	0.174 (0.677)	0.006 (0.938)	0.180 (0.914)	3.524 (0.741)	1.941 (0.227)	(A A A A A)
	500	50	0.000 (1.000)	0.271 (0.603)	0.271 (0.873)	5.478 (0.484)	-0.599 (0.854)	(A A A A A)
	1000	108	0.695 (0.404)	1.505 (0.220)	2.185 (0.335)	6.278 (0.393)	-0.283 (0.751)	(A A A A A)
France	250	29	0.680 (0.410)	0.676 (0.411)	1.356 (0.508)	6.120 (0.410)	2.524 (0.188)	(A A A A A)
	500	57	1.047 (0.306)	0.795 (0.373)	1.842 (0.398)	12.804 (0.046)	1.849 (0.232)	(A A A R A)
	1000	123	5.518 (0.019)	0.433 (0.511)	5.951 (0.051)	16.605 (0.011)	-0.884 (0.905)	(R A A R A)
Greece	250	20	1.185 (0.276)	0.000 (1.000)	1.185 (0.553)	18.718 (0.004)	-0.916 (0.923)	(A A A R A)
	500	52	0.088 (0.767)	0.005 (0.944)	1.278 (0.528)	24.227 (0.000)	-1.371 (0.979)	(A A A R A)
	1000	119	3.805 (0.051)	0.007 (0.933)	3.812 (0.149)	21.071 (0.002)	-1.338 (0.968)	(A A A R A)
Spain	250	25	0.000 (1.000)	0.012 (0.913)	0.012 (0.994)	0.634 (0.996)	-0.100 (0.750)	(A A A A A)
	500	40	2.369 (0.124)	0.632 (0.427)	3.001 (0.223)	3.574 (0.734)	2.030 (0.196)	(A A A A A)
	1000	91	0.925 (0.336)	0.855 (0.355)	1.780 (0.411)	6.208 (0.400)	1.230 (0.324)	(A A A A A)
Sweden	250	18	2.389 (0.122)	0.549 (0.459)	2.938 (0.230)	4.746 (0.577)	0.575 (0.519)	(A A A A A)
	500	34	6.337 (0.012)	1.612 (0.204)	7.949 (0.019)	6.780 (0.342)	-1.447 (0.983)	(R A R A A)
	1000	76	6.920 (0.009)	0.109 (0.741)	7.029 (0.030)	10.227 (0.006)	-0.518 (0.805)	(R A R R A)

p = 10%
A-cop

	Window	Exceptions	LR_{UC}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$	Test results
UK	250	26	0.044 (0.834)	0.106 (0.745)	0.150 (0.928)	3.524 (0.741)	1.903 (0.276)	(A A A A A)
	500	49	0.022 (0.882)	0.466 (0.495)	0.488 (0.783)	5.478 (0.484)	-0.620 (0.836)	(A A A A A)
	1000	107	0.534 (0.465)	1.792 (0.181)	2.326 (0.313)	6.079 (0.414)	-0.292 (0.737)	(A A A A A)
France	250	29	0.680 (0.410)	0.676 (0.244)	1.356 (0.508)	6.120 (0.410)	2.524 (0.187)	(A A A A A)
	500	57	1.047 (0.306)	0.795 (0.175)	1.842 (0.398)	12.805 (0.046)	1.849 (0.231)	(A A A R A)
	1000	123	5.518 (0.018)	0.433 (0.015)	5.951 (0.051)	16.604 (0.011)	-0.884 (0.905)	(R A A R A)
Greece	250	20	1.185 (0.276)	0.000 (1.000)	1.185 (0.553)	17.262 (0.008)	-0.916 (0.924)	(A A A R A)
	500	53	0.197 (0.657)	0.304 (0.581)	0.501 (0.778)	23.196 (0.001)	-1.352 (0.970)	(A A A R A)
	1000	122	5.062 (0.024)	0.090 (0.764)	5.152 (0.076)	18.679 (0.000)	-1.314 (0.969)	(R A A R A)
Spain	250	25	0.000 (1.000)	0.012 (0.913)	0.012 (0.994)	0.634 (0.996)	-0.100 (0.750)	(A A A - A)
	500	40	2.369 (0.124)	0.632 (0.427)	3.001 (0.223)	4.628 (0.592)	2.030 (0.197)	(A A A A A)
	1000	89	1.391 (0.238)	0.676 (0.411)	2.067 (0.356)	6.144 (0.407)	1.208 (0.329)	(A A A A A)
Sweden	250	18	2.389 (0.122)	0.549 (0.459)	2.938 (0.230)	4.746 (0.577)	0.575 (0.517)	(A A A A A)
	500	34	6.337 (0.012)	1.612 (0.204)	7.949 (0.019)	6.780 (0.342)	-1.445 (0.983)	(R A R A A)
	1000	76	6.920 (0.009)	0.109 (0.741)	7.029 (0.030)	12.271 (0.006)	-0.518 (0.803)	(R A R R A)

Table 3.13: *Back-testing results at 90% confidence levels. A-cop = Archimedean copula and E-cop = Elliptical copula.*

3.7 Conclusion

In this chapter, we presented a theoretical review of M-GARCH(1,1) DCC model incorporating copula functions and extreme value theory (EVT) analysis for forecasting VaR estimates. Financial asset return distributions have been proven to be leptokurtic, exhibits volatility clustering, leverage effects, and auto-correlations of squared returns. To capture these features, we have used M-GARCH(1,1) DCC volatility models to model the correlation structure amongst the asset returns which allows the correlation matrix to be time varying and thus reflects the current market conditions.

Because VaR models often focus on the behavior of asset returns in the left tail, we used copula functions to model the dependence structure between the asset returns and EVT to model the left tail of the distribution of the noise variables to obtain the q^{th} quantile for VaR estimation. Results from back-testing suggest that the M-GARCH(1,1) DCC copula-EVT model captures VaR quite well at shorter and longer observation periods. However, following the traditional method to obtain a cut off point known as the threshold on the left tail of the distribution for GPD parameter estimation, the number of points beyond the threshold sometimes lie towards the center of the distribution. The threshold selection process is also very subjective. GPD is not a good approximation for the the center of a sample data and might result in poor approximation of parameter estimates and hence incorrect VaR estimates. Based on this fact, though back-testing results deemed the model to be reliable, the forecast VaR estimates might be inaccurate. In the next chapter, we introduce a more objective method for threshold selection that avoids the body of the distribution and restricts inferences only on the tails.

Chapter 4

Forecasting robust Value-at-Risk estimates using Bayesian GARCH(1,1) model, vine-copula functions and Extreme Value Theory: Evidence from UK banks

This chapter proposes an objective approach for threshold selection, which we term the *hybrid* method that will restrict inferences to the tails of asset return distributions when Extreme Value Theory (EVT) is employed in estimating VaR. Thus, our main goal is to improve the threshold selection method used in the POT method. As already seen in Chapter 3.7, ARCH LM test fail to reject the null hypothesis of no conditional heteroscedasticity in the standardised residuals; $\{\eta_{i,t}\}_{t=1}^{T}$, after the fitted DCC model. This is a weakness of DCC models because it is not quite easy to ascertain that all correlations evolve in the same manner regardless of the assets involved, and diagnostic checks often rejects fitted DCC models (Tsay, 2013). Therefore, we employ a Bayesian GARCH(1,1) model with student's-t distribution as the underlying volatility model. Many researchers, for example Aas et al. (2009) and Ardia and Hoogerheide (2010) have shown that a simple GARCH(1,1) model is able to capture the dynamics of changes in asset returns. The motivation of using Bayesian-GARCH(1,1) model is because Bayesian estimation methods provides reliable results even for finite samples, and are usually straightforward to obtain the posterior distributions of any non-linear function of the model parameters whereas for the classical maximum likelihood method, it is not easy to perform inferences on non-linear function of the model parameters, the convergence rate is slow, and presents limitations when the residuals are heavy tailed. The constraints on the GARCH parameters to guarantee a positive variance can be incorporated via priors whereas the classical maximum likelihood method may impede some optimization procedures (Virbickaite et al., 2015; Hall and Yao, 2003). The motivation of Student's-t distribution is because it is able to account for the excess kurtosis in the conditional distribution common with financial time series processes (Ardia and Hoogerheide, 2010).

4.1 Introduction

Traditional VaR models such as the commonly used variance-covariance method and Monte Carlo simulation often assume asset returns in financial markets to be normally distributed. Numerous studies (see for example Berkowitz et al. (2011); Sheikh and Qiao (2010)) have shown that financial asset returns are in fact leptokurtic and heavy tailed with non-constant volatility. Normality assumptions in situations of non-normality will without doubt lead to inaccurate estimates of the probability of extreme events and hence wrong estimates of VaR. This is because a normal distribution has light tails, and VaR attempts to capture the behaviour of the portfolio return in the left tail. A model based on normal distribution where data is not will underestimates the frequency of the outliers and hence the true VaR (Jorion, 2007). Normality assumption also implies volatility is constant over time, and recent price changes which are based on current market information will be assigned weights in equal proportion to older ones. If the dependence characteristics of the extreme realisations differ from all others in the sample, the consequences might be dire (Poon et al., 2003). To avoid the normality assumption, most analysts now turn to use EVT to model the tail behaviour of asset returns. However, as stated earlier, EVT also assumes extreme events to be normally distributed which will probably not be the case in stressed periods (Wong, 2013).

Thus, this work is motivated by the work of McNeil and Frey (2000) who suggested applying EVT to the noise variable of the return series which are normally distributed to obtain the q^{th} quantile used to estimate conditional robust VaR estimates. By doing so, the problem of volatility clustering and other related effects such as excess kurtosis are accounted for. This approach was further investigated by Soltane et al. (2012) where they combined GARCH(1,1) model as the underlying volatility model with EVT to estimate VaR and showed that the GARCH-EVT-based VaR approach appears to be effective and realistic than the traditional VaR methods. Bob (2013); Hsu et al. (2012) also combined GARCH-EVT and copula functions (to model dependence) in estimating VaR. Their findings showed better performance compared to traditional VaR estimation methods, and also better estimates of VaR than copulas with conventional employed empirical distributions.

We construct and investigate the reliability of our VaR model, in line with Basel II and Basel III, and estimate VaR and minimum capital requirements (MCR) in some selected banks in the United Kingdom (UK) using actively traded stocks in the London Stock

Exchange.

The rest of the Chapter is structured as follows: Section 4.2 gives a brief discussion of Basel Committee of Banking and Supervision (BCBS) post 2008 global financial crisis. In Section 4.3, we present the Bayesian GARCH(1,1) model with Student's-*t* distribution and discuss the Bayesian approach to estimate the GARCH(1,1) parameters. In Section 4.4, we introduce vine copula functions. In Section 4.5 we discuss the data used. Section 4.6 presents the results; the *hybrid* method for threshold selection and back-testing results followed by conclusion in Section 4.7.

4.2 BCBS post 2008 Global Financial Crisis

Some researchers, after the 2008 global financial crisis, have demonstrated that Basel II failed to provide proper cushions against banks' actual losses on their market risk. For example, in the Global Financial Stability Report published by International Monetary Fund (IMF) in April 2009, IMF stated that during the 2008-9 financial crisis, the risk weighted capital ratios were unable to distinguish between banks that were in distress or bailed out with tax payers money and banks that were able to cope on their own (Carmassi and Micossi, 2012). The Financial Services Authority of the United Kingdom stated that the assumption of normal distribution with short term observations can lead to huge underestimation of probability of extreme loss events (Turner et al., 2009). Banks that were bailed out or collapsed showed higher and improving solvency ratios in the months preceding their collapse (Rossignolo et al., 2012; Carmassi and Micossi, 2012). However, McAleer et al. (2011) points out that Basel II was operational in Europe only from 2008 and the effects of the global financial crisis of 2008 cannot be associated to any failings of Basel II because it was never implemented in the United States of America (USA), which was the epicentre of the crisis.

4.2.1 Basel III

In late 2010, the BCBS adopted a more practical framework, Basel III, with stricter measures to strengthen regulation, supervision, risk management, transparency and disclosures in the banking sector. Under this framework, banks are still allowed to calculate their MCR for market risk using the IMA as required in Basel II, but with an introduction of a stressed VaR (sVaR) metric that automatically increases MCR for market risk. The sVaR must be calculated from a data set of continuous 12-months period of substantial financial stress (Rossignolo et al., 2012):

$$MCR_t = max\left(\frac{k}{60}\sum_{i=1}^{60} VaR_{t-i}, VaR_{t-1}\right) + max\left(\frac{k}{60}\sum_{i=1}^{60} sVaR_{t-i}, sVaR_{t-1}\right). \tag{4.1}$$

Following evidence that capital ratios higher than up to 2.5% of risk weighted assets would have been needed to correct internal model errors for market risk that resulted to underestimation of losses (Carmassi and Micossi, 2012), Basel III introduced: (1) a "micro-prudential 2.5% conservation buffer"; a cushion to protect MCR against falling below its minimum during financial distress or a crash. This buffer will force the restriction of dividend payouts once it fall below its minimum and reversed only when restored to its original value, (2) a "macro-prudential countercyclical buffer" ranging from 0-2.5% and applied by the national authorities depending on the banks' credit-to-GDP ratio to protect the banking system against potential losses insofar as these are related to an increase in risks in the system as a result of excessive growth in lending.

4.3 Bayesian GARCH(1,1) model with Student's-t distribution

The GARCH(1,1) model following Student's-t distribution has the form

$$r_t = \mu_t + a_t, \quad a_t = \eta_t \left(\frac{v-2}{v} \omega_t h_t \right)^{1/2} \tag{4.2a}$$

$$h_t = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 h_{t-1}, \tag{4.2b}$$

$$\eta_t \overset{iid}{\sim} \Phi(0,1); \quad \omega_t \overset{iid}{\sim} IG\left(\frac{v}{2}, \frac{v}{2}\right); \quad t = 1, \ldots, T$$

where r_t, same as in Eqn.(3.1), are the log-returns and μ_t is simply the unconditional mean of the log-returns. IG and $\Phi(0,1)$ symbolises the inverted gamma and standard normal distributions, respectively. The degrees of freedom parameter $v > 2$ guarantees finite conditional variance (Ardia, 2008).

We use Bayesian statistics, following the procedures delineated in Ardia (2015); Ardia and Hoogerheide (2010) to estimate the parameter values of the variance equation. Let $\boldsymbol{a} = (a_1, \ldots, a_T)'$, $\omega = (\omega_1, \ldots, \omega_T)'$ and $\alpha = (\alpha_0, \alpha_1)'$, a diagonal matrix is defined by

$$\Sigma = \Sigma(\psi, \omega) = diag\left\{ \left(\omega_t \frac{v-2}{v} h_t(\alpha, \beta_1) \right)_{t=1}^{T} \right\}, \tag{4.3}$$

where $\psi = (\alpha, \beta_1, v)$ for

$$h_t(\alpha, \beta_1) = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 h_{t-1}(\alpha, \beta_1). \tag{4.4}$$

Because the data samples are independent and drawn from a normal distribution, the likelihood function can be written as

$$L(\psi, \omega | \boldsymbol{a}) \propto (det\Sigma)^{-1/2} exp\left[-\frac{1}{2} \boldsymbol{a}' \Sigma^{-1} \boldsymbol{a} \right]. \tag{4.5}$$

The prior distribution of ω_t given v is given by

$$p(\omega|v) = \left(\frac{v}{2}\right)^{\frac{Tv}{2}} \left[\Gamma\left(\frac{v}{2}\right)\right]^{-T} \left(\prod_{t=1}^{T} \omega_t\right)^{-\frac{v}{2}-1} exp\left[-\frac{1}{2}\sum_{t=1}^{T}\frac{v}{\omega_t}\right] \tag{4.6}$$

where $\omega_t \overset{iid}{\sim} IG(\cdot)$ (e.g. Geweke (1993)). For the degrees of freedom, the prior distribution is a translated exponential distribution with parameters $\lambda > 0$ and $\delta \geq 2$ (see Deschamps (2006)) represented as

$$p(v) = \lambda \exp[-\lambda(v - \delta)]1\{v > \delta\}. \tag{4.7}$$

Ardia and Hoogerheide (2010) and Deschamps (2006) point out two important considerations for the prior density $p(v)$: (i) It is useful to guarantee that $v \gg 2$ so the conditional variance will be finite. (ii) The error term can be assumed to be normally distributed when δ is chosen to be large. This is possible while still maintaining reasonably tight priors, which can lead to better convergence of the sampler. Assuming independence among the model parameters of the joint prior distribution (i.e. $p(\psi, \omega) = p(\alpha)p(\beta)p(\omega|v)p(v)$), the likelihood function of the model parameters is combined with the prior density to obtain the posterior density as

$$p(\psi, \omega|\boldsymbol{a}) \propto L(\psi, \omega|\boldsymbol{a})p(\psi, \omega). \tag{4.8}$$

We employ the Metropolis Hastings (MH) algorithm of Markov Chain Monte Carlo (MCMC) simulation to estimate parameter values from the posterior distribution of the variance equation. Because of the recursive nature of the variance equation of GARCH(1,1) model, the prior density and the posterior density does not belong to the same distributional family. MH algorithm allows draws to be generated from any density, even if the normalising constant is unknown (Greenberg, 2012).

In the MH algorithm, (ψ, ω) is a random variable with Markov chains constructed as $(\psi^{[0]}, \omega^{[0]}), \ldots, (\psi^{[j]}, \omega^{[j]}), \ldots$ in the parameter space. As the number of realised chains goes to infinity, $p(\psi, \omega|\boldsymbol{a})$ tends to a normalised probability distribution with a random variable $(\psi^{[j]}, \omega^{[j]})$ (Ardia and Hoogerheide, 2010). The chain converges to its stationary distribution and the optimal mean values of the posterior distribution parameters are realised. More details on M-H algorithms can be found in Greenberg (2013); Ardia (2008); Tierney (1994); Roberts and Smith (1994); Casella and George (1992) and the references therein.

4.4 Vine copulas

For higher dimensions, standard multivariate copulas can become inflexible and do not allow for different dependent structures between pairs of variables (Krämer and Schepsmeier, 2011). Vine copulas are special cases of bivariate copulas and a more flexible tool to model dependence for higher dimensional distribution, and will allow for different dependance structures between pairs of variables.

For a bivariate case, we use the following version of Sklar's theorem (Ghalanos, 2015; Tsay, 2013; Krämer and Schepsmeier, 2011; Cherubini et al., 2004):

Definition 1 *A* $2-$*dimensional copula* $C(u, v)$ *is a distribution function on* $\boldsymbol{I}^2$ *with standard uniform margins.*

Let F be a joint distribution function with margins F_1 and F_2, then there exist a copula C such that

$$\forall(x_1, x_2) \in \mathbf{I}^2; F(x_1, x_2) = C(F_1(x_1), F_2(x_2)). \tag{4.9}$$

If F_1 and F_2 are continuous, then C is unique; otherwise, C is uniquely determined on the range: $RanF_1 \times RanF_2$,

$$C(u,v) = F(F_1^{-1}(u), F_2^{-1}(v)) \tag{4.10}$$

$$c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v} \tag{4.11}$$

$$f(x_1, x_2) = c_{uv}(F_1(x_1), F_2(x_2)) f_1(x_1).f_2(x_1) \tag{4.12}$$

Each copula has a density Eqn.(4.11), joint density Eqn.(4.12) and conditional densities

$$f(x_1|x_2) = c_{uv}(F_1(x_1), F_2(x_2)) f_1(x_1) \tag{4.13a}$$

$$f(x_2|x_1) = c_{uv}(F_1(x_1), F_2(x_2)) f_2(x_2) \tag{4.13b}$$

where c_{uv} is the paired-copula density for $F_1(x_1)$ and $F_2(x_2)$. For a $d-$dimensional vector $\mathbf{v}$, we have

$$f(x_1, \ldots, x_d) = c_{1 \ldots d}(F_1(x_1), \ldots, F_d(x_d)).f_1(x_1) \ldots f_d(x_d) \tag{4.14}$$

$$f(x|\mathbf{v}) = c_{xv_j|\mathbf{v}_{-j}}(F(x|\mathbf{v}_{-j}), F(v_j|\mathbf{v}_{-j})).f(x|\mathbf{v}_{-j}), \tag{4.15}$$

where Eqn.(4.14) is the joint density function, and Eqn.(4.15) is a general formula for the pair-copula multiplied by their conditional marginal densities (Aas et al., 2009). Bedford and Cooke (2001) presented a tree diagram for selecting the possible pair-copula constructions referred to as the *regular vine structure*. This includes the *canonical vines* (C-vines), where each tree has a unique node connected to all other nodes, and the *drawable vines* (D-vines), where each tree is a path (Krämer and Schepsmeier, 2011). In particular, for a

D-vine, the density $f(x_1,\dots,x_d)$, is given by

$$f(x_1,\dots,x_d)=\prod_{k=1}^{d}f(x_k)\prod_{j=1}^{d-1}\prod_{i=1}^{d-j}c_{i,i+j|i+1,\dots,i+j-1}$$
$$\times\,(F(x_i|x_{i+1},\dots,x_{i+j-1}),F(x_{i+j}|x_{i+1},\dots,x_{i+j-1})), \tag{4.16a}$$

and for a C-vine, the density is given by

$$f(x_1,\dots,x_d)=\prod_{k=1}^{d}f(x_k)\prod_{j=1}^{d-1}\prod_{i=1}^{d-j}c_{j,j+i|1,\dots,j-1}$$
$$\times\,(F(x_j|x_1,\dots,x_{j-1}),F(x_{j+i}|x_1,\dots,x_{j-1})), \tag{4.17a}$$

where index j identifies the trees, and i denoted the edges in each tree. (Aas et al., 2009).

4.5 Data

We employ the same data set as in Chapter 3 to test the reliability of the Bayesian-GARCH(1,1) Vine-Copula EVT VaR model. For simplicity, we use data from UK only, since our aim is to improve the threshold selection procedure for EVT analysis. That is, 2870 observations of daily stock prices actively traded on the London Stock Exchange. The stocks belong to the banking sector and of the top five banks in UK: UK.HSBA from HSBC bank, UK.LLOYDS from LLOYDS Banking Group, UK.BARC from Barclays bank, UK.RBS from Royal Bank of Scotland, and UK.STAN from Standard Chartered PLC Bank. The motivation for choosing these banks is because we want to test the reliability of the VaR model in banks with relatively high ratings in periods of distress and also investigate the risk of collapse in banks with insufficient capital to provide proper cushions during crisis periods. If the top banks are not able to cope during financial distress, then the risk of collapse in the country's banking system is large. Therefore, our data covers the period from 31^{st} of December 2004 to 31^{st} of December 2015 to include the 2008 global financial crisis and 2011 European financial crisis.

We use same out-of-sample data for back-testing and sample-in data for VaR estimation as in Chapter 3. As already seen in Chapter 3 (summary statistics and multivariate ARCH test; Table 3.2, time plots of the log return series; Figure 3.1), the stock returns are leptokurtic, show evidence of volatility clustering and conditional heteroscedasticity.

4.6 Results

To capture the tail distribution and the dynamics of fluctuations in the time series data, we fit a GARCH(1,1) model with a Student's-*t* distribution to the time series data to accommodate the heavy tails and estimate the GARCH parameters using Bayesian statistics as follows: (i) We assign a prior distribution with initial hyperparameters following GARCH specifications (i.e., $\alpha_0 > 0, \alpha_1, \beta_1 \geq 0$ and $\alpha_1 + \beta_1 < 1$) and generate two Markov Chain Monte Carlo (MCMC) simulations of $100,000$ draws each; (ii) if convergence is attained, we discard the first $50,000$ draws and select only the $50th$ draw from each chain such that auto-correlation between draws is reduced to almost zero. We then merge the two chains together to obtain a sample data set of 2000 observations. (iii) If convergence is not attained, repeat (i) using parameter estimates from the previous draw as the hyperparameters to increase the chance of convergence. The mean value of each parameter with respect to its respective posterior distribution is the optimal parameter estimate of the Bayesian-GARCH(1,1) model with Student's-*t* distribution. We test for convergence of the sampler with the help of diagnostic test by Gelman and Rubin (1992) (i.e., Figures 4.1, 4.2, 4.2, and 4.4 in the case of the portfolio return series, and Table 4.1 shows no evidence against convergence), for example, the acceptance rate of the GARCH parameters are determined by the potential scale reduction factor (psrf) and should be < 1.2. The trace plots; Figure 4.1, are smooth meaning the chain does not get stuck in certain areas indicating good mixing. The K*th* lag autocorrelation becomes smaller as the number of iterations in the chain increases; see Figures 4.2 and 4.3, also indicating good mixing. Figure 4.5 is the posterior density of $\alpha_1 + \beta_1$, which controls the power of the clustering in the variance process. A value closer to one implies that past shocks and variances will have longer impact on future conditional variance (Ardia and Hoogerheide, 2010). Estimation results are presented in Table 4.1 with standard errors in parenthesis.

Employing Eqn.(4.2), we obtain a matrix Σ_t, that consists of the marginal standardised residuals $\{\eta_{i,t}\}_{t=1}^{T}$. That is

$$\eta_{i,t} = (r_{i,t} - \mu_i)\left(\frac{v-2}{v}\omega_{i,t}h_{i,t}(\alpha_j, \beta_1)\right)^{-1/2}, \quad j = 0, 1, i = 1, \ldots, N;\ t = 1, \ldots, T \tag{4.18}$$

where the vectors $(r_{i,t} - \mu_i)^{'}$ are the residuals of the mean equation. Multivariate ARCH test (Eqn.(3.33)) on the standardised residuals at 95% significance level show evidence of conditional heteroscedasticity with $Q_k(10) = 287.2526$; p-value = 0 and $Q_k^r(10) = 273.0017$; p-value = 0.1519 (robust test after discarding those observations whose standardised residuals exceeds the 95^{th} quantile).

	Parameter				Diagnostic check		
	α_0	α_1	β_1	ν	a.rate (α)	a.rate (β)	psrf
UK.HSBA	2.078e-06(1.355e-08)	0.0934(3.105e-4)	0.9057(2.781e-4)	4.9740(1.097e-4)	92.72%	97.63%	1.01
UK.BARC	2.526e-06(1.733e-08)	0.0827(3.014e-04)	0.9171(2.663e-04)	5.200(1.138e-02)	89.59%	97.63%	1.00
UK.LLOY	6.375e-06(3.827e-08)	0.1022(3.532e-04)	0.8951(3.259e-04)	5.351(1.196e-02)	92.66%	97.71%	1.00
UK.RBS	3.446e-06(2.477e-08)	0.0876(4.046e-04)	0.9077(3.386e-04)	4.862(1.328e-02)	90.98%	97.64%	1.00
UK.STAN	1.106e-05(7.034e-08)	0.1290(5.029e-04)	0.8558(5.383e-04)	5.392(1.253e-02)	94.20%	97.70%	1.00
Portfolio	2.379e-06(1.397e-08)	0.0976(2.988e-04)	0.9005(2.765e-04)	7.599(2.403e-02)	90.16%	97.70%	1.00

Table 4.1: *Parameter estimates of Bayesian-GARCH(1,1) model with student-t distribution (Note: standard errors in parenthesis). a.rate = parameter acceptance rate, which is the proportion of the total number of single values in the MCMC chain to the total number of values in the chain. A high acceptance rate tells us that the chain does not get stuck in certain areas in the parameter space, thus producing good mixing as seen in the example of Figure 4.1. psrf =* $\sqrt{\frac{\hat{Var}(x)}{W}}$*; the potential scale reduction factor, and should be* < 1.2*, where* $\hat{Var}(x)$ *is a weighted average of the average of the m within-sequence variance,* s_j^2*, each based on* $n-1$ *degrees of freedom, and the variance between the m sequence means,* $\bar{x}_j$*, each based on n values of x:* $\hat{Var}(x) = (\frac{n-1}{n})W + \frac{1}{n}B$*;* $W = \frac{1}{m}\sum_{j=1}^{m} s_j^2$*,* $B = \frac{n}{m-1}\sum_{j=1}^{m}(\bar{x}_j - \bar{x}_{..})^2$*,* $s_j^2 = \frac{1}{n-1}\sum_{j=1}^{n}(x_{ij} - \bar{x}_j)^2$*. If psrf* > 1.2*, then the length of the chain should be increased to improve convergence to a stationary distribution (see Gelman and Rubin (1992) for more details).*

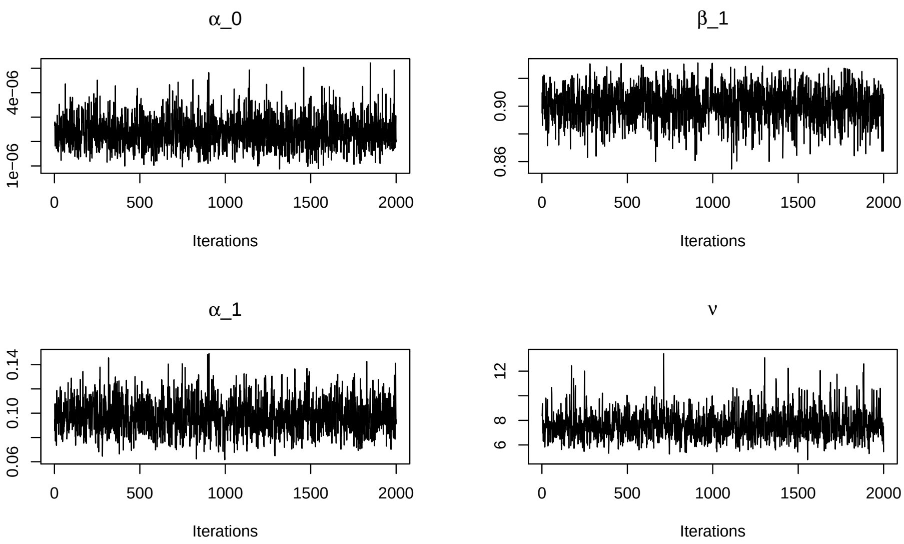
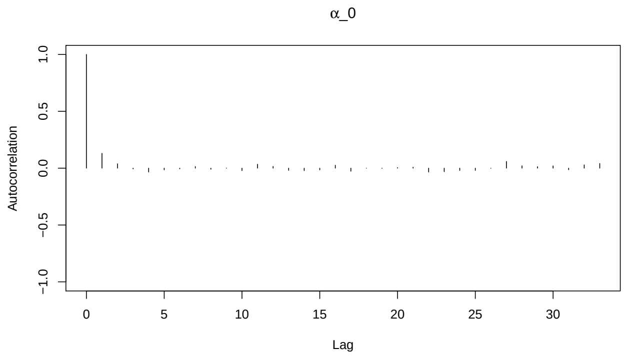
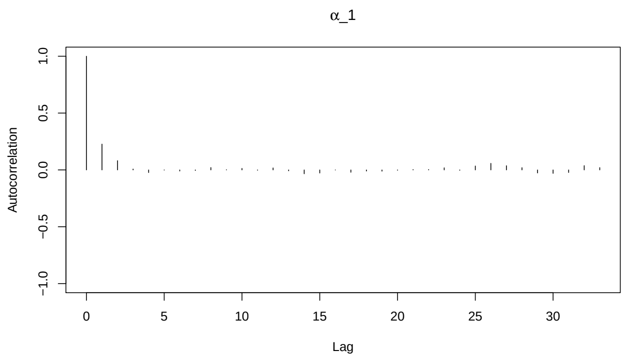


Figure 4.1: *Trace plots of* 2000 *iterations against the values of the draws of the parameters at each iteration after merging the two chains. The plots shows no evidence against convergence. The chain does not get stuck in certain areas, indicating good mixing.* ν *is the distribution of the degree of freedom parameter.*

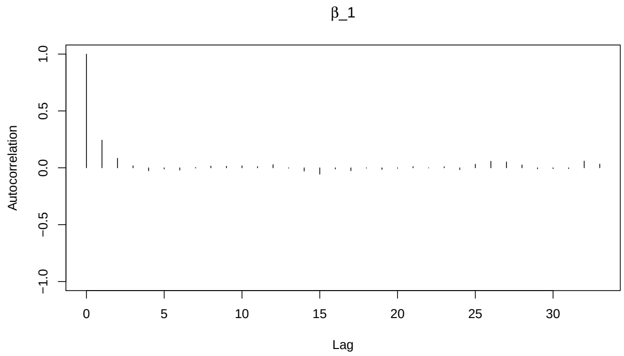


(a)

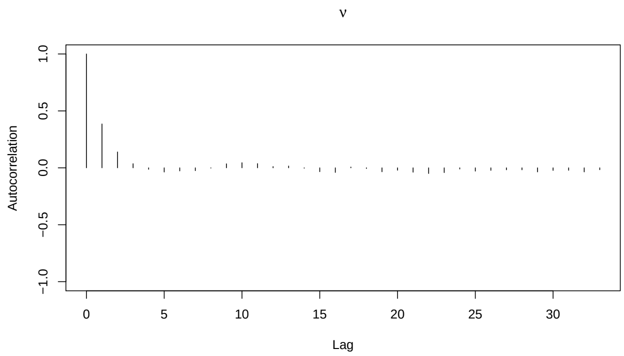


(b)

Figure 4.2: *Autocorrelation plots of* 2000 *samples for* α_0 *and* α_1 *after merging the two chains. That is, we use the first* 50, 000 *draws from the full Markov chain as the burn in period for each chain and select only every 50th draw to get rid of autocorrelation. As the number of iterations increases, the Kth lag autocorrelation becomes smaller indicating good mixing.*



(a)



(b)

Figure 4.3: *Autocorrelation plots of* 2000 *samples for* β_1 *and* ν *after merging the two chains.*

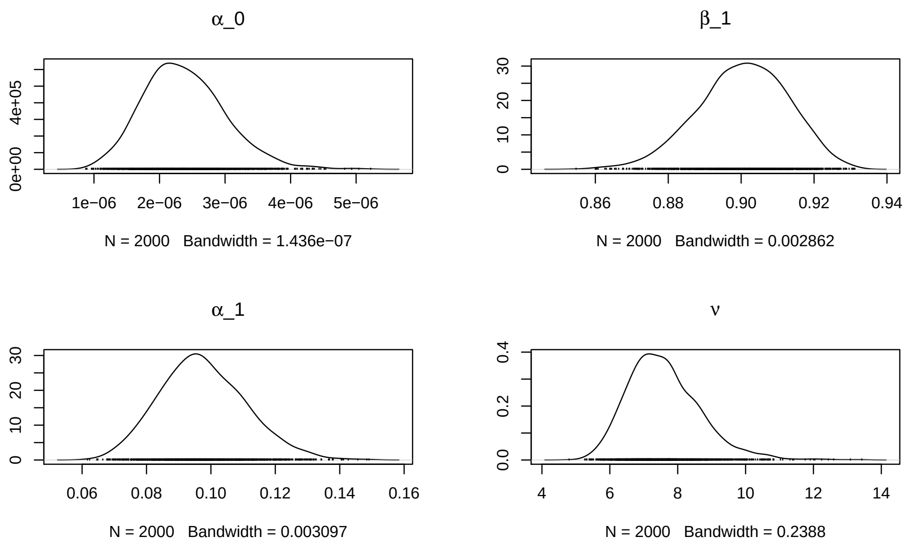


Figure 4.4: *Density plots of the posterior distributions of the model parameters based on* 2000 *draws. Density plots are used to test the covariance stationarity condition. For GARCH(1,1) model,* $\alpha_1 + \beta_1 < 1$ *(see Figure 4.5).*

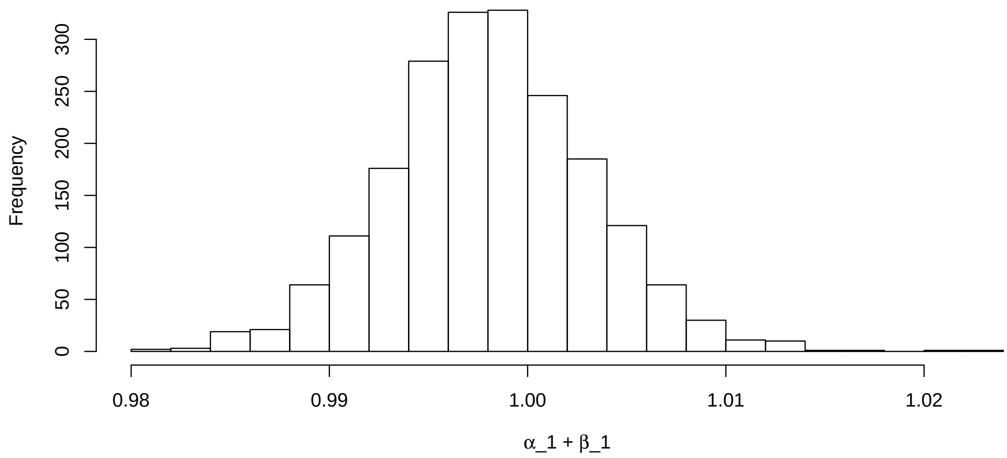


Figure 4.5: *Posterior density of $\alpha_1 + \beta_1$; the degree of persistence controlling the power of the clustering in the variance process. A value closer to one implies that past shocks and variances will have longer impact on future conditional variance (Ardia and Hoogerheide, 2010)*

4.6.1 Modeling dependence

We model the dependence structure among the stock returns using vine copula functions. The C- and D-vine copula parameters are estimated by maximising the likelihood function:

$$
\begin{aligned}
l_{C-vine}(\Theta|\mathbf{x}) = & \sum_{j=1}^{d-1}\sum_{i=1}^{d-j}\sum_{t=1}^{T} \log[c_{j,j+i|1,\ldots,j-1}(F(x_{j,t}|x_{1,t},\ldots,x_{j-1,t}), \\
& F(x_{j+i,t}|x_{1,t},\ldots,x_{j-1,t})|\Theta_{j,j+i|1,\ldots,j-1})]; \qquad (4.19a) \\
l_{D-vine}(\Theta|\mathbf{x}) = & \sum_{j=1}^{d-1}\sum_{i=1}^{d-j}\sum_{t=1}^{T} \log[c_{i,i+j|i+1,\ldots,i+j}(F(x_{i,t}|x_{i+1,t},\ldots,x_{i+j-1,t}), \\
& F(x_{i+j,t}|x_{i+1,t},\ldots,x_{i+j-1,t})|\Theta_{i,i+j|i+1,\ldots,i+j-1})], \qquad (4.19b)
\end{aligned}
$$

where $x_i \in [0,1]$ are pseudo observations of the standardised residuals and, Θ_i and Θ_j are the pair-copula parameters of the joint distribution function (Schepsmeier and Brechmann, 2015; Aas et al., 2009).

The conditional distribution functions in C-vine (i.e., Eqn.(4.19a)) and D-vine (i.e., Eqn. (4.19b)) copulas are obtained from the conditional distribution

$$h(\mathbf{x}, \mathbf{v}, \ominus) = F(\mathbf{x}|\mathbf{v}) = \frac{\partial C_{xv_j|\mathbf{v}_{-j}}(F(x|\mathbf{v}_{-j}), F(v_j|\mathbf{v}_{-j}))}{\partial F(v_j|\mathbf{v}_{-j})}, \tag{4.20}$$

where $C_{xv_j|\mathbf{v}_{-j}}$ is a bivariate copula distribution function, and v is a d-dimensional vector Allen et al. (2017); Aas et al. (2009).

We select the best pair-copula for the decomposition of the n-variate copula densities based on the paired copula with the smallest AIC value from a range of copula families (Table 4.2). To select which of the vine copulas is the best to model the dependence among the risk factors, we follow the procedure of Vuong (1989). That is, we use a likelihood-ratio based test to compare non-nested models. We cannot select between the two vine copulas based on their likelihoods because the two copulas are non-nested (Aas et al., 2009). The test statistics is given by

$$\begin{aligned} \boldsymbol{v} &= \frac{\frac{1}{N}\sum_{i=1}^{N} m_i}{\sqrt{\sum_{i=1}^{N}(m_i - \bar{m})^2}} \\ m_i &= log\left[\frac{c_1(x_i|\hat{\ominus}_1)}{c_2(x_i|\hat{\ominus}_2)}\right], \quad x_i \in [0,1], \quad i = 1, \ldots, N \end{aligned} \tag{4.21}$$

where $\boldsymbol{v}$ is the standardised sum of the log differences of the pointwise likelihoods m_i between two competing vine copulas c_1 and c_2 and the estimated parameters $\hat{\ominus}_1$ and $\hat{\ominus}_2$. Vine model 1 is selected in favor of vine model 2 at a certain level of confidence α if and only if $\boldsymbol{v} > \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)$, and vine model 2 is selected in favor of vine model 1 if and only if $\boldsymbol{v} < -\Phi^{-1}\left(1 - \frac{\alpha}{2}\right)$. If $|\boldsymbol{v}| \leq \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)$ then no decision among the models is possible (Schepsmeier and Brechmann, 2015). Based on the likelihood ratio test, C-vine copula is selected in favor of D-vine with $\boldsymbol{v} = 2.4989$ and $\Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 1.96$ at $\alpha = 5\%$.

From now on, all analysis are based on the selected C-vine copula. Estimated copula parameters are used to simulate 10000 pairs of (u_i, u_j) observations of $[0, 1]$ uniformly distributed random variables with joint distribution function $C(u_i, u_j)$ (see Cherubini et al. (2004) for a detailed simulation technique). Figures 4.6 and 4.7 show density plots for the selected n-variate bivariate copulas and Figures 4.8 and 4.9 show scatter plots of $c(u_i, u_j)$ copulas after 10000 simulations. The Student's-t copula appears to have more data points concentrated around the corners than the Frank copula. This is because of the non-negative tail dependence of Student's-t copulas (Tsay, 2013).

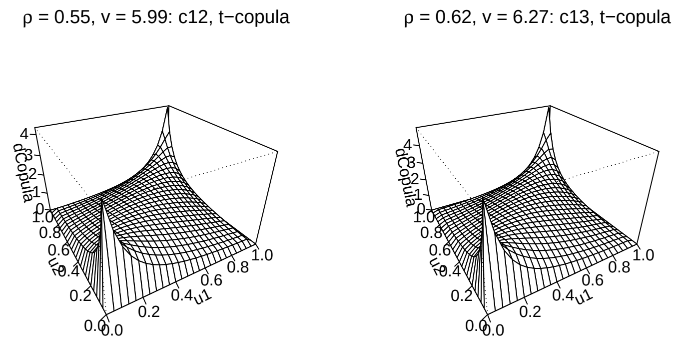
The simulated data is then transformed to the original scales of the noise variables using the inverse quantile function $F_i^{-1}(u_i)$, of the desired marginal distributions to obtain a new matrix

$$\hat{\Sigma} = \{\zeta_{i,t}\}, \quad i = 1, \ldots, N, \, t = 1, \ldots, 10000 \tag{4.22}$$

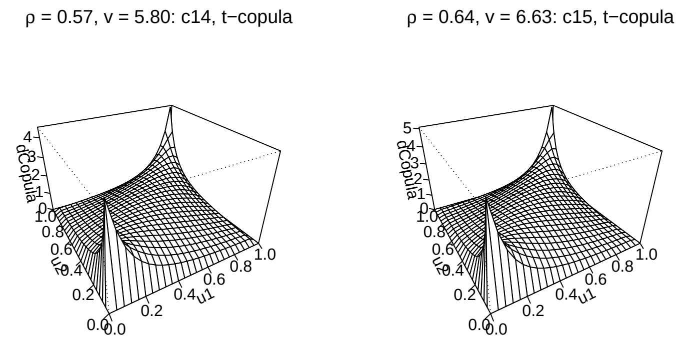
free from any normality assumptions and linear correlations. Here, we compare between Student's-t and normal marginal distributions by keeping the copula fixed and changing the marginals. A multivariate ARCH test on $\{\zeta_{i,t}\}$ at 95% significance level show no evidence of conditional heteroscedasticity (see Tables A.T5). Therefore, Bayesian-GARCH(1,1) C-vine copula model is a better model in describing the conditional heteroscedasticity in the log return series and to model dependence as opposed to Bayesian-GARCH(1,1) model without copula functions where there is evidence of ARCH effect in the standardised residuals.

	Unconditional and conditional pairs copulas	Selected copula family	Parameter	Log-likelihood	paired AIC
C-vine Copula					
Tree$_1$	$C_{1,2}$	*student*-t *copula*	ρ = 0.5536, v = 5.98	545.2690	-1086.54
	$C_{1,3}$	*student*-t *copula*	ρ = 0.6222, v = 6.27	716.3512	-1428.70
	$C_{1,4}$	*student*-t *copula*	ρ = 0.5701, v = 5.80	573.0684	-1142.14
	$C_{1,5}$	*student*-t *copula*	ρ = 0.6384, v = 6.63	758.9981	-1513.10
Tree$_2$	$C_{2,3\|1}$	*Frank copula*	λ = 4.0043	497.7722	-993.54
	$C_{2,4\|1}$	*student*-t *copula*	ρ = 0.5870, v = 7.94	602.6068	-1201.21
	$C_{2,5\|1}$	*Frank copula*	λ = 1.9893	140.8872	-279.77
Tree$_3$	$C_{3,4\|1,2}$	*student*-t *copula*	ρ = 0.3687, v = 8.77	222.7078	-441.42
	$C_{3,5\|1,2}$	*student*-t *copula*	ρ = 0.2124, v = 10.83	74.4631	-144.93
Tree$_3$	$C_{4,5\|1,2,3}$	*Frank copula*	λ = 0.5513	10.8833	-19.77
Log-likelihood				4143.0070	
DVine Copula					
Tree$_1$	$C_{1,2}$	*student*-t *copula*	ρ = 0.5536, v = 5.98	545.2690	-1086.54
	$C_{2,3}$	*student*-t *copula*	ρ = 0.6969, v = 6.19	688.4580	-1893.67
	$C_{3,4}$	*student*-t *copula*	ρ = 0.7197, v = 4.91	464.5665	-2106.17
	$C_{4,5}$	*student*-t *copula*	ρ = 0.5385, v = 6.11	726.9948	-972.03
Tree$_2$	$C_{1,3\|2}$	*student*-t *copula*	ρ = 0.3915, v = 8.26	440.2784	-493.94
	$C_{2,4\|3}$	*student*-t *copula*	ρ = 0.4342, v = 8.50	548.5746	-620.12
	$C_{3,5\|4}$	*student*-t *copula*	ρ = 0.3534, v = 9.67	115.0825	-396.34
Tree$_3$	$C_{1,4\|2,3}$	*student*-t *copula*	ρ = 0.1499, v = 12.11	184.4686	-76.18
	$C_{2,5\|3,4}$	*student*-t *copula*	ρ = 0.1434, v = 12.83	50.6013	-71.75
Tree$_4$	$C_{15\|234}$	*student*-t *copula*	ρ = 0.3787, v = 12.44	-189.6890	-447.73
Log-likelihood				3574.6050	

Table 4.2: *C-vine and D-vine copula parameter estimates. The copula types for the decomposition of n-variate bivariate copulas for unconditional and conditional pairs are selected based on AIC values. That is, the paired copula with the smallest AIC value.* $1, 2, \ldots, 5$ *represents the stocks.*

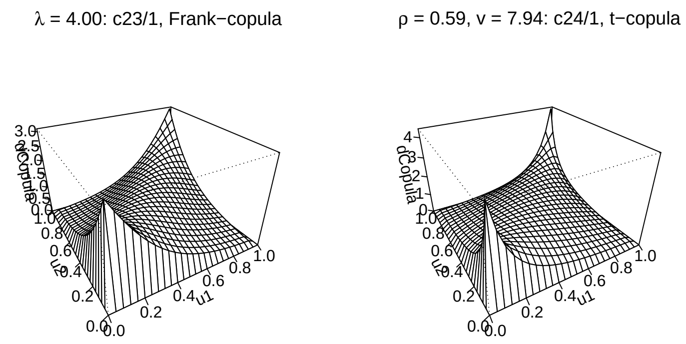


(a)

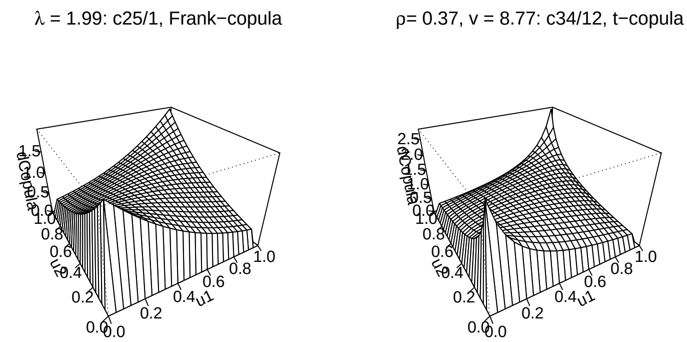


(b)

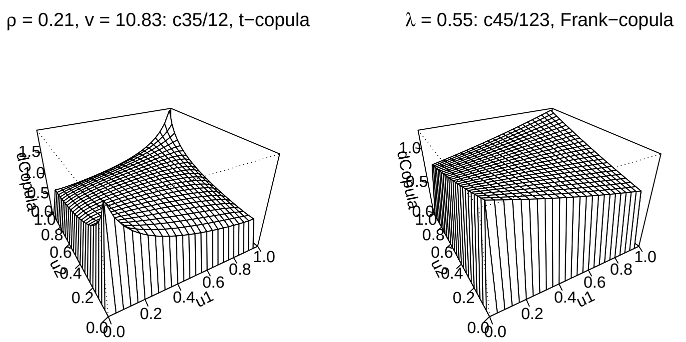
Figure 4.6: *Density plots of n-variate bivariate unconditional pair-copula decomposition.*



(a)

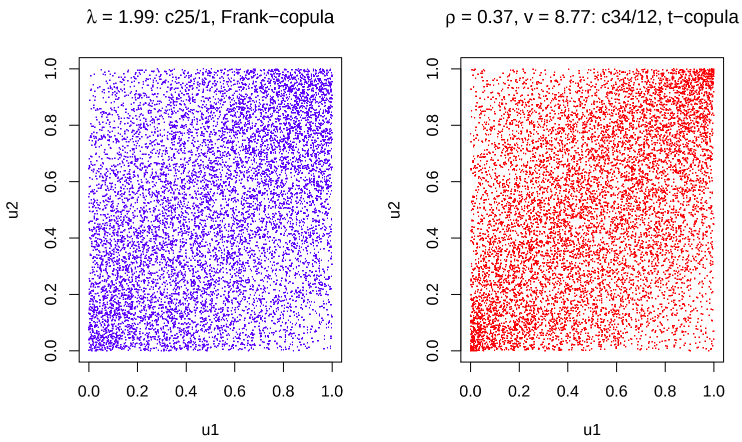


(b)

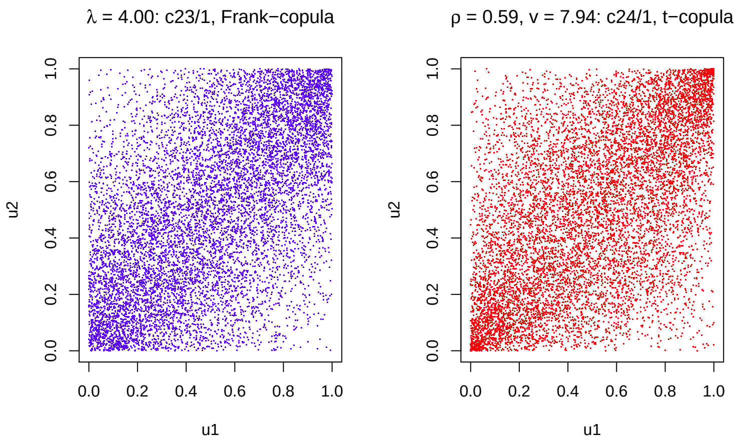


(c)

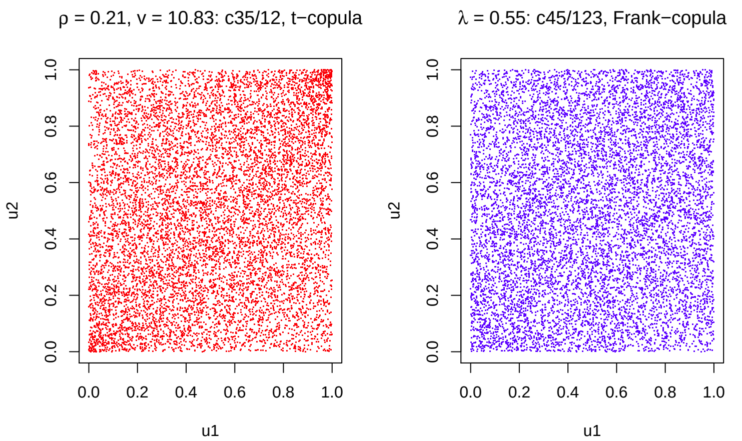
Figure 4.7: *Density plots of n-variate bivariate conditional pair-copula decomposition.*



(a)

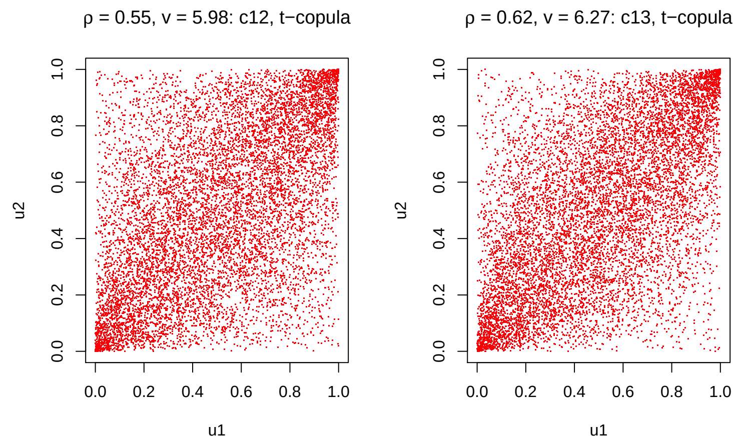


(b)

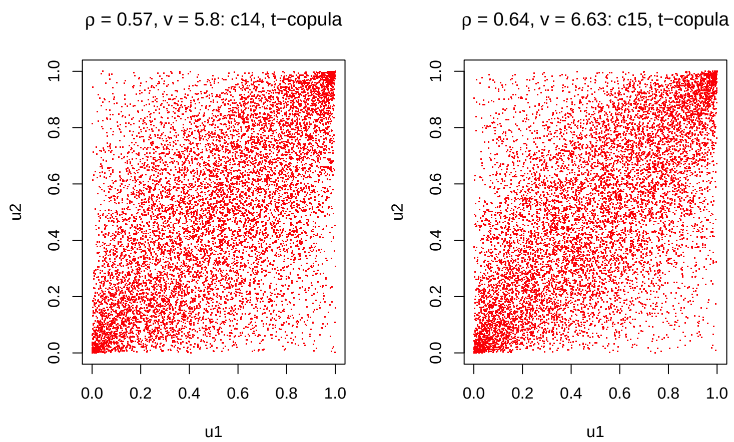


(c)

Figure 4.8: *Scatter plots of bivariate copulas for conditional pairs based on 10000 draws.*



(a)



(b)

Figure 4.9: *Scatter plots of bivariate copulas for unconditional pairs based on 10000 draws.*

4.6.2 Threshold selection and robust VaR estimates

As in Chapter 3, we apply the POT method of EVT to the marginal distributions of $\{\zeta_{i,t}\}$ (Eqn.(4.22)) to obtain the q^{th} quantile $VaR_q(Z)$ of the noise variable for VaR estimation. The mean excess function plot for UK.RBS; Figure 4.10, following Bayesian GARCH(1,1) model with t-distribution and C-vine copula functions with t-margins is used as an example to demonstrate the threshold selection method. According to this plot, we should select a subjective threshold value of about 1.2. That is, $\vartheta_0 = 1.2$ is the lowest point on the graph above which the graph appears to be approximately linear. However, if we select this point as the threshold value, we will have 1470 exceedances which are too many compared to the size of the data ($T = 10,000$); the number of exceedances will lie towards the body of the data and will inevitably result in a poor approximation of GPD parameters and hence the correct VaR estimate.

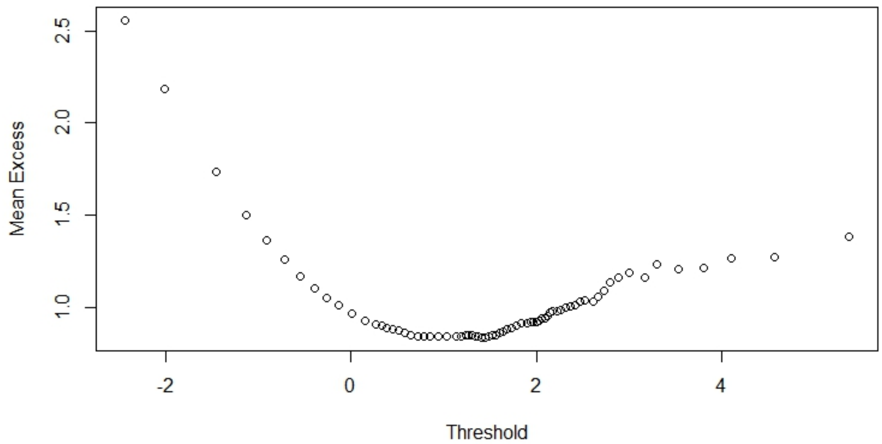


Figure 4.10: *Mean excess function plot of the standardised residuals for UK.RBS following Bayesian GARCH(1,1) model with t-distribution and C-vine copula functions with t-margins. A subjective* $\vartheta_0 = 1.2$ *is identified as the threshold value.*

The choice of the threshold is an important step in the POT method because Eqn.(3.30) is dependent on ϑ and the number of points (i.e., exceedances) above N_ϑ since the parameters are estimated based on the exceedances. Thus, it is very important to find the proper threshold value. There is no clear-cut or wholly satisfactory method in determining a proper threshold value so far. Danielsson and De Vries (1998) developed a semi-parametric estimator for the tails of the distribution and estimated the threshold through a bootstrap of the Mean Square Error (MSE) of the tail index. Danielsson et al. (2001) further used a two-step subsample bootstrap method to determine the threshold that minimises the asymptotic MSE. Hill et al. (1975) and Davison and Smith (1990) proposed graphical tools to help identify the proper threshold known as the Hill plot and the mean excess plot respectively. We propose an extension to the mean excess plot for an objective threshold selection known as the *hybrid* method.

From the mean excess plot; Figure 4.10, we identify the lowest point, making the graph appears to be approximately linear, a point ϑ_0, then insert a tangent line from ϑ_0 through the rest of the points ϑ_i where $\vartheta_i > \vartheta_0$; see Figure 4.11. Since the tangent to a linear curve is the tangent itself and the mean excess function is a linear function of the threshold, we take an average of the set of points that lie on the tangent line as the threshold value. We call this point ϑ^*. ϑ^* will lead to a better approximation of VaR estimates than ϑ_0 because the inference is restricted to the left tail. Apart from better approximation of VaR estimates, this method significantly reduces the probability of having different VaR estimates on the same data and also the probability of selecting a very low or very high threshold value. Let $\vartheta_i = \vartheta_1, \ldots, \vartheta_\hbar$ be a set of points that lie on the straight line, then we obtain the value of ϑ^* as

$$\vartheta^* = \frac{1}{\hbar}\sum_{i=1}^{\hbar} \vartheta_i, \qquad \vartheta_i \geq \vartheta_0, \tag{4.23}$$

where $\hbar$ is the number of points in the set. As can also be seen in Figure 4.11, the points from the beginning of the fitted line, i.e., ϑ_0 are too compact and might lead to missing some important points. A better way to obtain the value of ϑ^* is by fitting a regression line

$$\hat{y} = b_0 + b_1 x \tag{4.24}$$

based on the least square method to the points $\{\vartheta_i\}_{i=1}^{\hbar}$, where $\hat{y}$ is the estimate of the dependent variable, x the independent variable with intercept b_0 and slope b_1. In the presence of heteroscedasticity and outliers, it may be advantageous to consider fitting a robust regression line. Robust regression methods are not influenced by outliers, and are also very useful when there are problems with heteroscedasticity in the data set. Figure 4.12 show a comparison of the simple linear and the robust regression methods. We can see that the effect of the outliers are very minimal. The data also has no problem with heteroscedasticity hence a simple linear regression method is reliable. Following this method, we obtain a threshold value of $\vartheta^* = 2.1146$ and 465 exceedances which are sufficient to allow reasonable statistical inference with EVT, and will give far better results of POT parameter estimates and hence better forecast for quantile VaR estimates compared to $\vartheta_0 = 1.2$ and 1470 exceedances. If we take a look at the threshold range plot of $e(\vartheta)$; Figure 4.13, which is a plot of the reparameterised scale of the shape parameter, $\vartheta^* = 2.1146$ appears to be a reasonable choice to use as the threshold value because 2.1146 seems to yield POT parameter estimates that will not change significantly within uncertainty bounds.

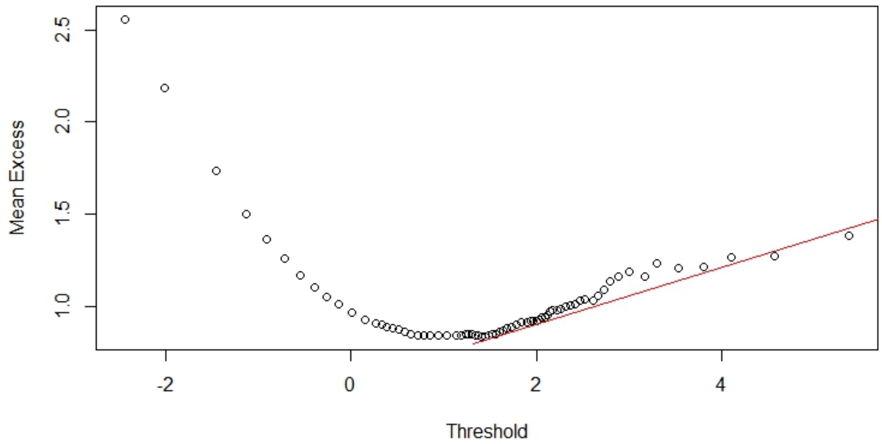


Figure 4.11: *Mean excess function plot of standardised residuals following Bayesian GARCH(1,1) model with t-distribution and C-vine copula with t-margins demonstrating the* **hybrid** *method of threshold selection. The threshold is the average of the points that lie on the straight line.*

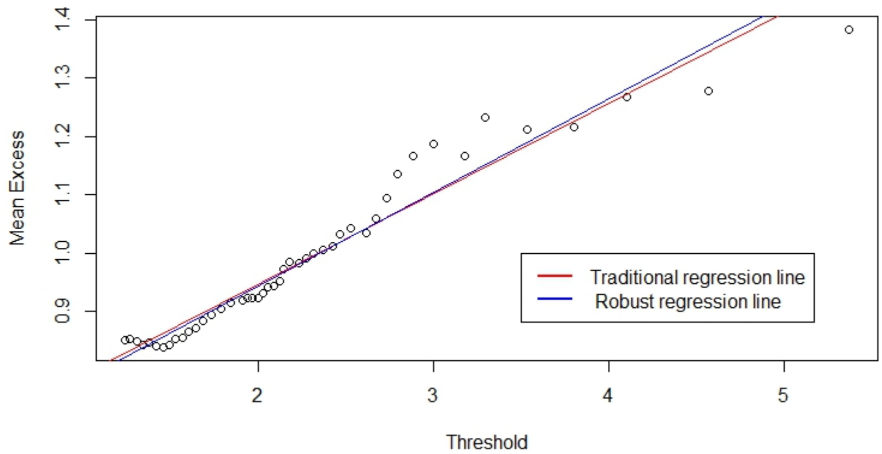


Figure 4.12: *Robust regression line and simple linear regression line fitted to the points* $\{\vartheta_i\}_{i=1}^{\hbar}$, $\vartheta_i \geq \vartheta_0$ *of the mean excess plot. The effect of the outliers on the simple regression line is minimal. A proper threshold value is obtained by taking the average of the set of points that lie on the regression line.*

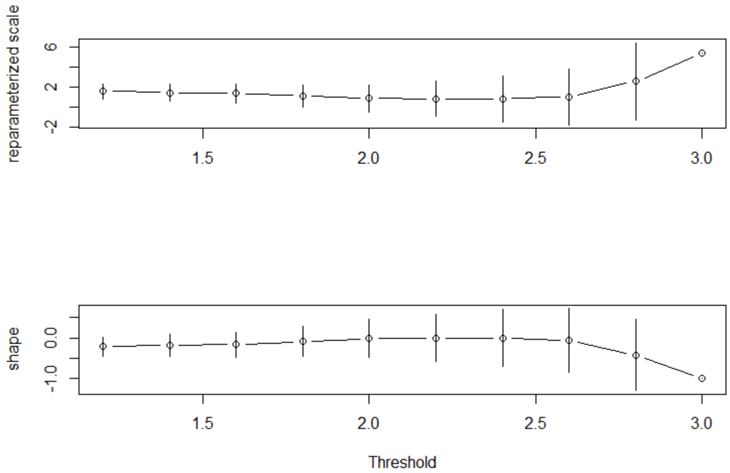


Figure 4.13: *The threshold range plot show that* $\vartheta^* = 2.1146$ *is appropriate to use as threshold value as it seems to yield POT parameter estimates that will not change much within uncertainty bounds.*

We further perform a simulation study on $\{\vartheta_i\}_{i=1}^{\hbar}$ to check the reliability of the *hybrid* method. This entails the use of empirical distributions of $\{\vartheta_i\}_{i=1}^{\hbar}$ for bootstrapping a $100(1-\alpha)\%$ confidence interval for ϑ^*, where α is the level of significance. Simulation studies lay a powerful framework for answering novel questions of the statistical accuracy of a model. For example, the use of bootstrapping methods, which uses re-sampling techniques to find empirical estimates of sampling distributions and confidence intervals when the parameter sampling distribution is unknown or non normal (Hallgren, 2013). We use bootstrapping to generate data sets that are in conformity to the mean and standard deviation of $\{\vartheta_i\}_{i=1}^{\hbar}$, which is then used to construct a $100(1-\alpha)\%$ confidence interval for ϑ^*. The algorithm is a follows:

1. Obtain an empirical distribution of $T = 10000$ observation with approximately the same mean and standard deviation as $\{\vartheta_i\}_{i=1}^{\hbar}$ from a normal distribution.

2. $N = 1000$ data points are drawn randomly from T with replacement, and then calculate ϑ^*.

3. Repeated (2) 1000 times to obtain sufficient distribution of ϑ^*, which is then used to

calculate a $100(1-\alpha)\%$ confidence interval for ϑ^*.

Figure 4.14 presents a 95% confidence interval for ϑ^*; upper bound (Ub) = 2.123, lower bound (Lb) = 2.0942, and standard error (SE) = 0.74%. This means that for 95% of the time that the experiment is conducted, ϑ^* will be within the confidence interval $(2.123, 2.0942)$, with an error band of approximately 0.0288 (see Appendix A.3 for bootstrapping of 95% confidence interval).

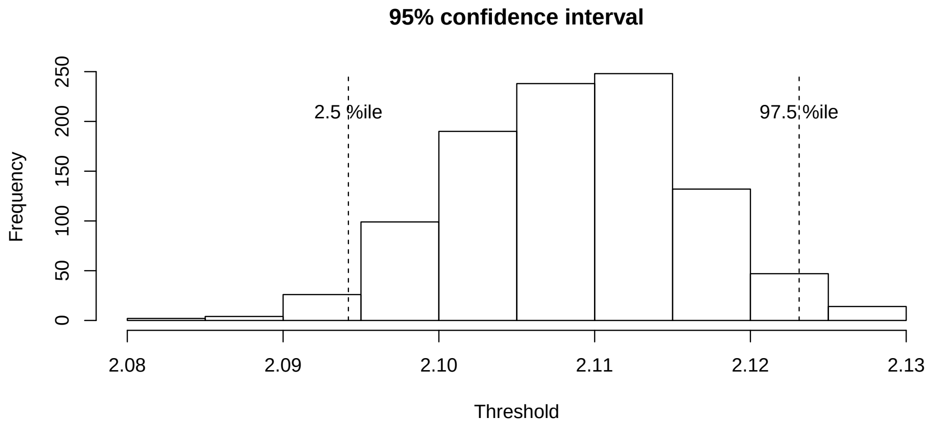


Figure 4.14: *Empirical distribution of* $\{\vartheta_i\}_{i=1}^{\hbar}$ *used for bootstrapping a 95% confidence interval for* ϑ^*. *Upper bound (Ub)* = 2.123, *lower bound (Lb)* = 2.0942, *and standard error (SE)* = 0.74%.

Table 4.3 shows the POT parameter estimates and the q^{th} quantile VaR estimates; $VaR_q(Z)$, of the noise variables at $q = (99\%, 95\%, 90\%)$. Visual observation of the quantiles suggest that Bayesian GARCH(1,1) vine copula-EVT model with t margins and v degrees of freedom outperforms both the traditional GARCH(1,1) model with student's-t distribution and v degrees of freedom and Bayesian-GARCH(1,1) model with Student's-t distribution and v degrees of freedom without copula functions and EVT at 99% confidence level (see Table 4.4 and 4.3). However, the validity of the model needs to be checked by conducting back-testing on the portfolio VaR. An important point to note is that the shape parameters of the POT method with normal margins are all less than zero. Looking at the conditions for the GPD (Eqn(3.25)), $\psi(\vartheta) > 0, y \in [0, x-\vartheta]$ when $\xi \geq 0$, and

$y \in [0, -\frac{\psi(\vartheta)}{\xi}]$ when $\xi < 0$, the appropriate value for the shape parameter ξ for a financial time series data must be greater than zero since the upper bound of financial losses cannot be fixed (Soltane et al., 2012; Gilli et al., 2006; Bhattacharyya and Ritolia, 2008). Therefore, the GARCH(1,1) C-vine copula-EVT model with normal margins is not a reliable model in this case as the shape parameters are all less than zero. The shape parameter gives an indication of the thickness of the tail of the distribution. $\xi < 0$ imply lighter tail.

Employing the risk formula (Eqn.(3.37)), we obtain the portfolio quantile VaRs as $VaR^{p}_{99\%}(Z) = 2.7891$, $VaR^{p}_{95\%}(Z) = 1.6363$, and $VaR^{p}_{90\%}(Z) = 1.1978$. Figure 4.15, which is a time plot of profit and loss ($P\&L$) of the portfolio return series and portfolio VaR estimates suggests that the VaR model performs quite well in capturing the dynamics in the portfolio return series. Again, the model have to be validated through back-testing.

		Bayesian GARCH(1,1) C-vine copula EVT with t-margins				
		UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN
Parameters	$\psi(\vartheta^*)$	0.9593	0.9077	0.7017	0.7941	0.7696
	ξ	0.0400	0.0889	0.1765	0.1655	0.0736
	ϑ^*	2.5481	2.7751	2.0632	2.1146	1.9858
	N_{ϑ^*}	270	200	501	465	495
	μ	-0.6778	-0.2242	0.4316	0.2040	-0.0890
	σ	0.8301	0.6411	0.4137	0.4780	0.6166
$\boldsymbol{VaR_q(Z)}$	99%	3.5200	3.4240	3.3712	3.5043	3.2922
	95%	1.9642	1.9764	2.0646	2.0573	1.9781
	90%	1.3244	1.4139	1.6066	1.5435	1.4584
$\boldsymbol{ES_q(Z)}$	99%	4.5598	4.4836	4.5037	4.7314	4.2271
	95%	2.9391	2.8947	2.9170	2.9975	2.8083
	90%	2.2726	2.2774	2.3609	2.3818	2.2473

		Bayesian GARCH(1,1) C-vine copula EVT with normal margins				
		UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN
Parameters	$\psi(\vartheta^*)$	0.4265	0.4202	0.3749	0.3880	0.3274
	ξ	-0.1523	-0.1784	-0.0771	-0.1326	-0.0665
	ϑ^*	1.4231	1.5786	1.5762	1.6775	1.6628
	N_{ϑ^*}	560	379	380	260	280
	μ	-0.1204	-0.2893	0.1820	-0.1437	0.3407
	σ	0.6616	0.1705	0.4824	0.6294	0.4154
$\boldsymbol{VaR_q(Z)}$	99%	2.0694	2.0770	2.0517	2.0257	1.9886
	95%	1.4710	1.4593	1.4722	1.4124	1.4692
	90%	1.1645	1.1335	1.1996	1.1052	1.2278
$\boldsymbol{ES_q(Z)}$	99%	2.3542	2.3581	2.3657	2.3276	2.2753
	95%	1.8349	1.8340	1.8277	1.7860	1.7883
	90%	1.5688	1.5575	1.5746	1.5148	1.5619

Table 4.3: *POT parameter estimates and q^{th} quantile VaRs; $VaR_q(Z)$, and Expected Shortfalls; $ES_q(Z)$, of the noise variables at $q = (99\%, 95\%, 90\%)$. For normal margins $\xi < 0$ and hence not appropriate to use for VaR estimation in this case.*

	GARCH(1,1)				Byesian-GARCH(1,1)			
	99%	95%	90%	v	99%	95%	90%	v
UK.HSBA	3.3318	2.0045	1.4706	5.1214	3.3723	2.0174	1.4770	4.9740
UK.BARC	3.3428	2.0080	1.4724	5.0804	3.2746	1.9862	1.4615	5.3510
UK.LLOY	3.3089	1.9972	1.4670	5.2103	3.3115	1.9981	1.4674	5.2000
UK.RBS	3.4631	2.0458	1.4912	4.6806	3.4052	2.0278	1.4822	4.8620
UK.STAN	3.2882	1.9906	1.4637	5.2940	3.2650	1.9831	1.4599	5.3920

Table 4.4: *Quantile VaR estimates for traditional GARCH(1,1) model with Student's-t and Bayesian-GARCH(1,1) model with Student's-t distributions at* $q = (99\%, 95\%, 90\%)$*. Comparing theses results to Table 4.3 suggests that Bayesian-GARCH(1,1) C-vine copula-EVT model outperforms at 99% confidence level.*

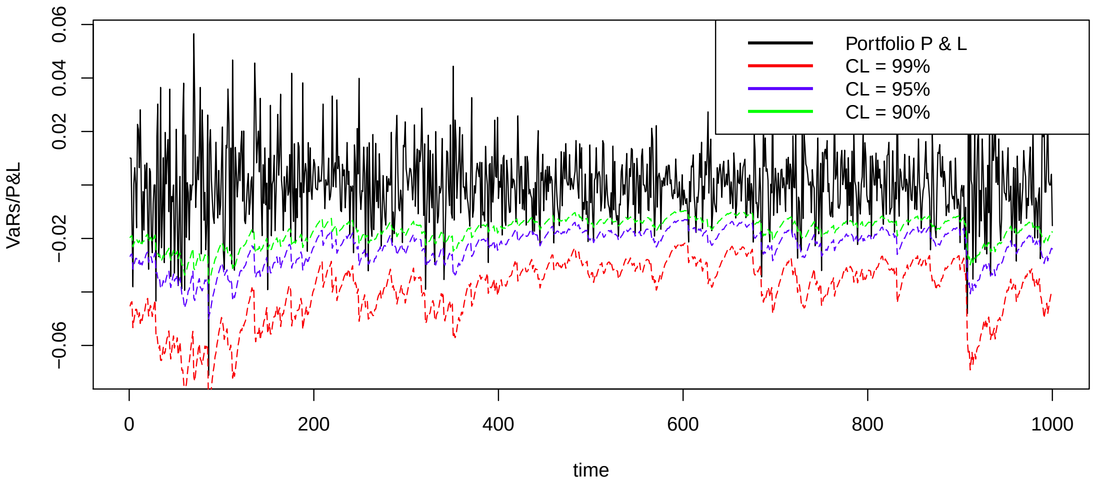


Figure 4.15: *Estimated daily VaRs and profit and loss (p&L) plot following Bayesian GARCH(1,1) with Student's-t distribution, C-vine copula functions and EVT.*

4.6.3 Back-testing

For model validation, we employ the Kupiec (1995) unconditional coverage test, Christoffersen (1998) independent and conditional coverage test, Basel *traffic light* test, Santos and Alves (2012) new independent test, and Engle and Manganelli (2004) DQ test. The out-of-sample data, just as in the previous chapter, is divided into blocks of 250, 500, and 1000 trading days.

At 99% confidence level, i.e., $p = 1\%$, we observe zero exceptions; Table 4.5. The model does not underestimate risk but rather assumed to be too *conservative*. With zero exceptions, the model passed the "traffic light" test and is placed in the green zone (see Table 2.1). Most financial institutions will prefer this model at 99% confidence level; ". . . Financial institutions routinely produce plots of *P&L* that show no violation of their 99% confidence VaR over long periods, proclaiming that this supports their risk model . . . The amount of economic capital banks currently hold is in excess of their regulatory capital. As a result, banks may prefer to report higher VaR numbers to avoid the possibility of regulatory intrusion" (Jorion, 2007).

Back-testing results are presented in Table 4.6. The VaR model perform better at lower observation periods compared to longer observation periods at $p = 95\%$ and $p = 90\%$ confidence levels. This is because at longer observation periods, the model will have longer term memory and is not easily affected by sudden changes in the underlying volatility. Volatility in financial markets fluctuates as time passes, and there exist volatility clustering with most recent price changes providing more information with regards to the current volatility compared to older price changes. Thus, shorter observation periods will be more responsive to changes in volatility than longer observation periods. This results confirm the findings of Best (2000) where he showed that VaR at 95% is more effective at lower observation periods than longer observation periods.

The out of sample data was taken immediately after the 2011 financial crisis. We therefore use VaR estimates during this period to calculate MCR for market risk in accordance with Basel II (Eqn.(2.19a)). To comply with Basel III rules, we consider back-testing for the periods of January 2008 to December 2008 and January 2011 to December 2011. This constitutes a continuous 12-months crisis period - the 2008 and 2011 global and European financial crisis, respectively for an observation period of 262 trading days for 2008 and 260 trading days for 2011. VaR estimates during these periods are referred to as stress VaR estimates (sVaR) because they are calculated in a period of significant financial distress.

We used the sVaR estimates to calculate MCR for market risk in accordance with Basel III rules (Eqn.(4.1)). Back-testing results during these periods; Tables 4.7 and 4.8, suggest that the VaR measure is able to capture the dynamics of volatility in periods of severe crisis.

We also consider back-testing to include the two crisis period. That is, incorporating the 2008 crisis and transitioning into and including the 2011 crisis, consisting of 1000 trading days. We observe an increase in the number of exceptions produced as shown in Table 4.9. The lower number of exceptions produced immediately after the 2011 financial crisis as seen in Table 4.5, is an indication that the market was calm and in a recovery state.

As seen in Table 4.11, the MCR for market risk in relation to Basel II is almost three times the maximum loss per day and much higher in relation to Basel III. This result suggests that with the correct VaR model, Basel III is not needed. Moreover, these results also confirm the previous findings by McAleer et al. (2011) as stated earlier that the global financial crisis cannot be associated to the failure of Basel II as it was implemented in Europe only from 2008, and never in the USA. Banks that displayed higher solvency ratios and higher credit-to-GDP ratios prior to their collapse or bailouts probably had their internal risk models for market risk manipulated or as a result of poor VaR models that were unable to capture fat-tail risk. However, this claim is not 100% certain as the model needs to be tested in those countries whose banks were severely affected during the crisis period.

	Expected number of exceptions			Observed number of exceptions		
p	250	500	1000	250	500	1000
1%	3	5	10	0	0	0
5%	13	25	50	7	14	28
10%	25	50	100	24	47	94

Table 4.5: *Observed number of exceptions versus expected number of exceptions following Bayesian GARCH(1,1) vine copula-EVT VaR model. Out-of-sample data after* 2011 *financial crisis is divided into blocks of* 250, 500 *and* 1000 *trading days (observation periods); time horizon* = 1 *day.*

p = 1%,

				Back-test type			
Window	Exceptions	LR_{POF}	LR_{IND}	LR_{CC}	$T_{N,[N/2]}$	DQ	Test results
250	0	*NaN*	-	-	-	2.485 (0.870)	(- - - - A)
500	0	*NaN*	-	-	-	5.010 (0.543)	(- - - - A)
1000	0	*NaN*	-	-	-	10.061 (0.122)	(- - - - A)

p = 5%,

Window	Exceptions	LR_{POF}	LR_{IND}	LR_{CC}	$T_{N,[N/2]}$	DQ	Test results
250	7	3.009 (0.083)	0.952 (0.329)	3.961 (0.138)	1.916 (0.448)	5.963 (0.427)	(A A A A A)
500	14	6.018 (0.014)	1.791 (0.181)	7.809 (0.020)	-0.560 (0.855)	8.404 (0.210)	(R A R A A)
1000	28	12.036 (0.001)	0.302 (0.583)	12.338 (0.002)	-0.744 (0.876)	1.445 (0.025)	(R A R A R)

p = 10%,

Window	Exceptions	LR_{POF}	LR_{IND}	LR_{CC}	$T_{N,[N/2]}$	DQ	Test results
250	24	0.045 (0.832)	0.018 (0.893)	0.063 (0.969)	0.288 (0.587)	9.473 (0.149)	(A A A A A)
500	47	0.204 (0.652)	0.374 (0.541)	0.579 (0.749)	-0.558 (0.845)	15.994 (0.014)	(A A A A R)
1000	94	0.403 (0.526)	4.837 (0.028)	5.240 (0.073)	-1.251 (0.957)	16.735 (0.010)	(A R A A R)

Table 4.6: *Back-testing results immediately after* 2011 *financial crisis, p-values in parenthesis. The VaR model performs better at shorter observation periods compared to longer observation periods. A = Accept, R = Reject.*

			p = 1%	p = 5%	p = 10%
2008 crisis:	262 trading days	Expected number of exceptions	3	13	26
		Observed number of exceptions	0	16	27
2011 crisis:	260 trading days	Expected number of exceptions	3	13	26
		Observed number of exceptions	0	10	31

Table 4.7: *Observed number of exceptions versus expected number of exceptions following Bayesian GARCH(1,1) vine copula-EVT VaR model during periods of financial distress; January* 2008 *to December* 2008 *and January* 2011 *to December* 2011.

		Back-test type					
Window		LR_{POF}	LR_{IND}	LR_{CC}	$T_{N,[N/2]}$	DQ	Test results
262 trading days (2008)	p = 5%	0.657 (0.418)	0.356 (0.551)	1.013 (0.603)	2.946 (0.149)	0.061 (0.837)	(AAAAA)
	p = 10%	0.034 (0.854)	0.210 (0.647)	0.244 (0.885)	0.343 (0.622)	0.103 (0.623)	(AAAAA)
260 trading days (2011)	p = 5%	0.034 (0.854)	1.726 (0.189)	1.760 (0.415)	-0.322 (0.807)	0.039 (0.267)	(AAAAA)
	p = 10%	1.242 (0.265)	0.102 (0.749)	1.344 (0.511)	-0.661 (0.881)	0.122 (0.892)	(AAAAA)

Table 4.8: *Back-testing results based on* 5% *and* 10% *significance levels conducted separately for the* 2008 *and* 2011 *financial crisis periods. The VaR model shows reliability in periods of severe crisis. A = Accept, R = Reject.*

	Expected number of exceptions			Observed number of exceptions		
p	250	500	1000	250	500	1000
1%	3	5	10	0	1	1
5%	13	25	50	16	27	48
10%	25	50	100	27	49	105

Table 4.9: *Observed number of exceptions versus expected number of exceptions following Bayesian GARCH(1,1) vine copula-EVT VaR model for the period of January* 2008 *to December* 2011.

p = 5%,

		Back-test type					
Window	Exceptions	LR_{POF}	LR_{IND}	LR_{CC}	$T_{N,[N/2]}$	DQ	Test results
250	16	0.951 (0.329)	0.379 (0.538)	1.330 (0.514)	2.946 (0.149)	3.478 (0.747)	(A A A A A)
500	27	0.164 (0.686)	1.916 (0.166)	2.080 (0.353)	5.542 (0.034)	4.413 (0.621)	(A A A R A)
1000	48	0.085 (0.771)	0.078 (0.780)	0.163 (0.922)	1.179 (0.334)	9.978 (0.126)	(A A A A A)

p = 10%,

Window	Exceptions	LR_{POF}	LR_{IND}	LR_{CC}	$T_{N,[N/2]}$	DQ	Test results
250	27	0.174 (0.677)	0.245 (0.621)	0.419 (0.928)	0.343 (0.621)	13.597 (0.349)	(A A A A A)
500	49	0.022 (0.882)	0.089 (0.765)	0.111 (0.946)	-0.253 (0.762)	8.180 (0.225)	(A A A A A)
1000	105	0.274 (0.601)	0.142 (0.706)	0.416 (0.812)	-1.015 (0.930)	6.447 (0.375)	(A A A A A)

Table 4.10: *Back-testing results covering the period from January* 2008 *to December* 2011*; incorporating the crisis period, based on* 5% *and* 10%*s significance levels with p-values in parenthesis. The VaR model is reliable in periods of calm and severe crisis. A = Accept, R = Reject.*

				MCR	
VaR estimation period	$VaR(99\%)$	$\frac{1}{60}\sum_{i=1}^{60}(VaR(99\%)$	$\frac{k}{60}\sum_{i=1}^{60}(VaR(99\%)$	**Basel II**	**Basel III**
Feb.2012 to Dec.2015 (C.VaR)	3.89%	4.03%	12.10%	12.10%	-
sVaR: Jan.2008 to Dec.2008	9.19%	15.69%	47.06%	47.06%	59.15%
sVaR: Jan.2011 to Dec.2011	5.67%	9.04%	27.11%	27.11%	39.20%

Table 4.11: *Minimum Capital Requirements (MCR) for market risk in accordance with Basel II rules is calculated from current VaR estimates (C.VaR; from February* 2012 *to December* 2015*); i.e., covering the out of sample data. MCR for market risk in accordance with Basel III rules is calculated from C.VaR estimates and continuous* 12*-month period of significant financial stress i.e.,* 2008 *and* 2011 *crisis. Regulatory multiplier* $k = 3$ *in all cases because at* 99% *confidence level we observe* ≤ 4 *exceptions (see Eqn.*(2.19b)*)*

4.7 Conclusion

In this chapter, we constructed a VaR model by combining a Bayesian GARCH(1,1) model with Student's-t distributions as the underlying volatility model, vine copula functions to model dependence, and EVT to model the left tail, hence the name Bayesian GARCH(1,1) vine copula-EVT model. Back-testing results show that the model is reliable in forecasting risk on financial assets in periods when the market is relatively calm and in periods of severe crisis. Comparing the quantile VaR estimates for each bank used for calculating the portfolio VaR estimates following EVT show that Bayesian GARCH(1,1) vine copula-EVT model outperforms both the traditional GARCH(1,1) models with student's-*t* distribution and Bayesian GARCH(1,1) with student's-*t* distribution without copula functions and EVT at 99% confidence level.

Chapter 5

Applying copula functions and EVT to the exposures to risk factor returns to forecast VaR estimates

In this Chapter, we further test the *hybrid* method for threshold selection in forecasting VaR by applying EVT directly to the exposures to risk factors. We further compare results to the VaR models of Chapters 3 and 4, and traditional VaR methods: variance-covariance and historical simulation methods commonly used by banks. Back-testing results show that VaR models constructed using conditional volatility models, copula functions and the peaks over threshold (POT) method of EVT incorporating the *hybrid* method for threshold selection performs better than the other methods.

5.1 Introduction

We investigate the performance of the VaR model when the POT method of EVT is applied directly to the risk factors incorporating the *hybrid* method to forecast VaR estimates.

We combine Bayesian-GARCH(1,1) model with a Student's-*t* distribution discussed in Chapter 4, commonly used Archimedean and elliptical copula functions in financial risk management and the POT method of EVT incorporating the *hybrid* method for threshold selection discussed in Chapters 3 and 4 respectively, to forecast VaR estimates in top UK banks.

The rest of the chapter is structured as follows: In Section 5.2 we discuss modeling the marginal distributions and dependance. Results of the forecast VaR estimates and model validation are presented in Section 5.3. Section 5.4 compares the various VaR methods discussed in this thesis to the traditional VaR methods commonly used by financial institutions, followed by the conclusion in Section 5.5.

5.2 Modelling the marginal distributions

We employ the same time series data as in Chapter 4 and modeled the marginal distributions by fitting a Bayesian GARCH(1,1) model with Student's-*t* distribution since the log return distributions are leptokurtic (see Table 3.2). QQ-plots; Figure 5.1, which are plots of the theoretical quantiles of the marginal standardised residuals against the empirical quantiles of a studentised residuals from a linear model suggest that the marginal standardised residuals from the Bayesian GARCH(1,1) model with Student's-*t* distributions are normally distributed. However, univariate ARCH LM test and Ljung-Box test suggest there still exist some serial correlation in the standardised residuals for UK.RBS; see Table 5.1. Multivariate ARCH test reported in Chapter 4 also rejected the presence of no conditional heteroscedasticity in the standardised residuals.

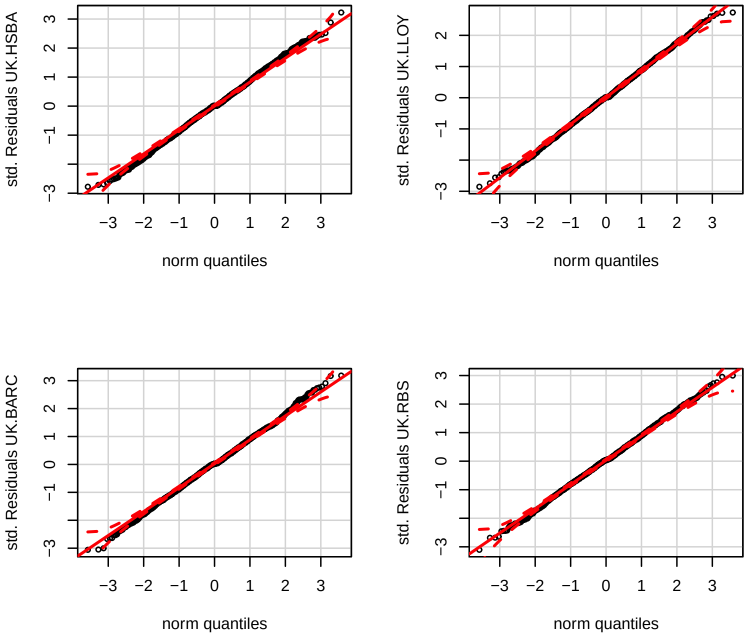


Figure 5.1: *Theoretical quantile plots of the marginal standardised residuals against empirical quantiles from a linear model suggesting that the standardised residuals are normally distributed.*

			ARCH LM Test		
	UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN
LM(5)	3.68	3.34	6.29	21.59	5.57
p-value	0.5964	0.6479	0.2792	0.0006	0.3500
LM(10)	5.94	6.97	8.99	24.56	7.56
p-value	0.8204	0.7279	0.5328	0.0063	0.6713
			Ljung-Box Test		
	UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN
Q(5)	3.62	4.39	5.63	23.416	5.54
p-value	0.6059	0.4945	0.3442	0.0003	0.3540
Q(10)	5.94	7.64	8.50	26.76	7.56
p-value	0.8204	0.6637	0.5802	0.0028	0.6718

Table 5.1: *Univariate ARCH LM test on the standardised residuals and Ljung-Box test on the standardised squared residuals. The null hypothesis of no ARCH effect and no serial correlation is rejected at 5% significance level for UK.RBS.*

5.2.1 Modelling dependence

Dependence structure among the risk factors are modeled using copula functions. The copula parameters are estimated by the CML method as explained in Chapter 3. Estimated copula parameters and Kendall's τ are reported in Table 5.2 alongside the AIC values.

Frank and Student's-t copulas are selected from each copula family based on the highest maximum likelihood estimation values. From now on, all analysis are based on the selected copulas. Next, we specify the desired marginal distributions, which we set to Student's-t distribution, and generate $j = 1, \ldots, T$ simulations from the fitted copulas to obtain a new matrix of marginal standardised residuals

$$\hat{\Sigma} = \left\{\zeta_{i,j}\right\}, \quad j = 1, \ldots, T, i = 1, \ldots, N \tag{5.1}$$

which is free from any normality assumptions and linear correlations. Multivariate ARCH test on $\left\{\zeta_{i,j}\right\}$ shows no evidence of conditional heteroscedasticity with $Q_k(10) = 7.8434$: p-value $= 0.6441$, $Q_k^r(10) = 229.4318$: p-value $= 0.8202$ for Frank copula functions, and $Q_k(10) = 5.1843$: p-value $= 0.8785$, $Q_k^r(10) = 223.5385$: p-value $= 0.8845$ for Student's-t copula functions. Therefore, Bayesian-GARCH(1,1) copula model is a better model in describing the conditional heteroscedasticity in the log return series as opposed to Bayesian-GARCH(1,1) model without copula functions where we have evidence of ARCH effect in the standardised residuals.

Finally, we reintroduce the GARCH model of Eqn. (4.2) and convert the daily simulated data with t-margins to daily risk factor returns as

$$\mathbf{r}_{i,t} = \zeta_{i,t}\left(\frac{v-2}{v}\omega_{i,t}h_{i,t}(\alpha_i, \beta_i)\right)^{1/2} + \mu_i, \quad i = 1, \ldots, N; t = 1, \ldots, T. \tag{5.2}$$

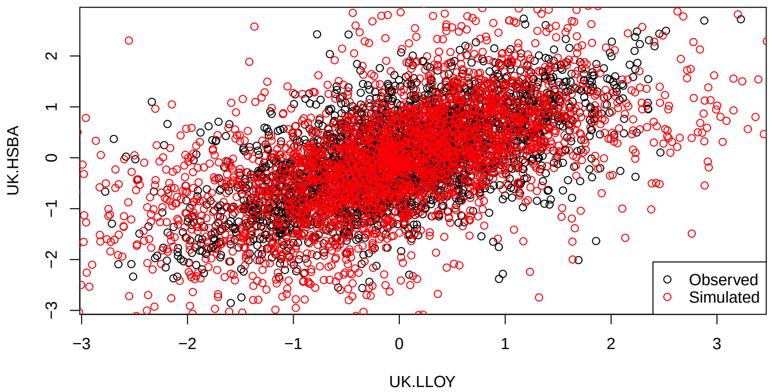
Note that $\{\zeta_{i,t}\}_{t=1}^{T}$ of Eqn.(5.2) are the marginal standardised residuals after modeling the dependence structure among the asset returns with copula functions, and $(.)^{1/2}$ are estimates of the conditional volatilities of the risk factors from the Bayesian-GARCH(1,1) model with Student's-t distribution. As an illustration, Figures 5.2(a) and 5.2(b) are scatter plots of the simulated standardised residuals and the new return distribution, respectively, following Frank copula with t-marginals, and plotted together with the original standardised residuals and the original return distribution between UK.HSBA and UK.LLOY. As can be seen, the Frank copula produce results that captures the extreme observations (See Figures A.F1, A.F2, A.F3, and A.F4).

	Archimedean copula			**Elliptical copula**	
	Gumbel	Clayton	Frank	Gaussian	Student's-t
Kendall's τ	1.778 (0.023)	1.556 (0.046)	4.681 (0.041)	$\rho_G = \rho_\tau(\rho_{SE})$	$\rho_t = \rho_\tau(\rho_{SE})$
MLE	3234	2759	**3239**	3839	**4098**
AIC	-14.163	-13.845	**-14.166**	3.494	**3.363**

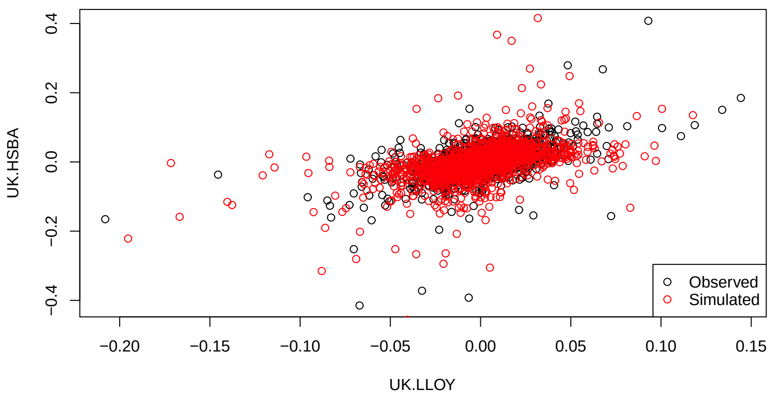
Table 5.2: *Copula parameter estimates based on "inversion of Kendall's τ" and MLEs following CML estimation method. Standard errors in parentheses. The best copula for modeling dependence among the risk factors is that with the highest MLE value or smallest AIC value (in bold). Frank copula is selected from the Archimedean copula family and t-copula is selected from the elliptical copula family.*

	UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN
UK.HSBA	1				
UK.BARC	0.5539 (0.015)	1			
UK.LLOY	0.6249 (0.013)	0.7076 (0.011)	1		
UK.RBS	0.5751 (0.015)	0.7204 (0.011)	0.7268 (0.011)	1	
UK.STAN	0.6425 (0.012)	0.5441 (0.015)	0.5465 (0.014)	0.6056 (0.015)	1

Table 5.3: *Kendall's τ; $\rho_\tau(\rho_{SE})$ for Gaussian and Student's-t copula parameter estimates (standard errors in parenthesis).*



(a)



(b)

Figure 5.2: *Scatter plots of simulated standardised residuals; Figure 5.2(a) and the new return distribution; Figure 5.2(b), following Frank copula with t-marginals, plotted together with the original standardised residuals and original return distribution between UK.HSBA and UK.RBS. As can be seen, the Frank copula produces results that captures the extreme observations.*

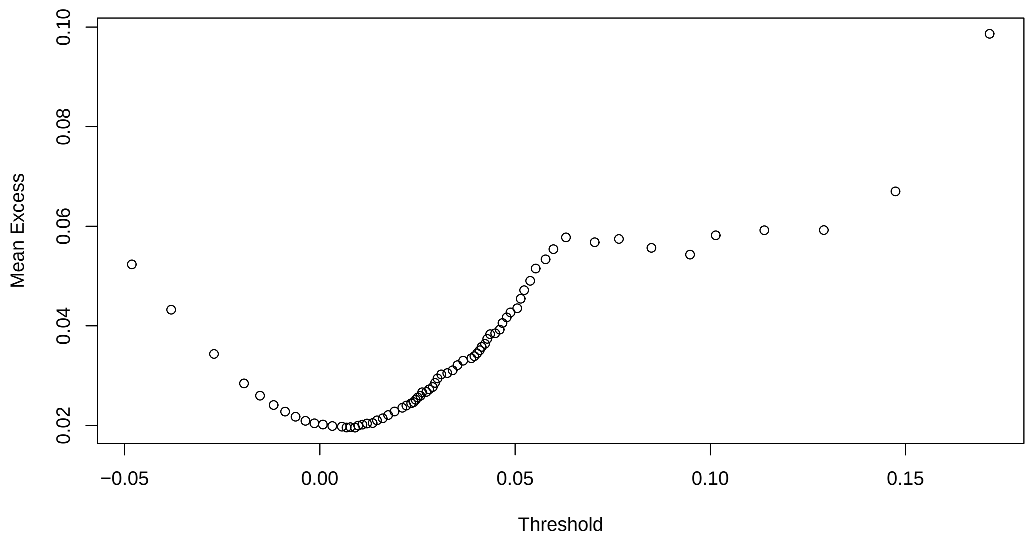
5.3 Results

5.3.1 Forecasting VaR estimates

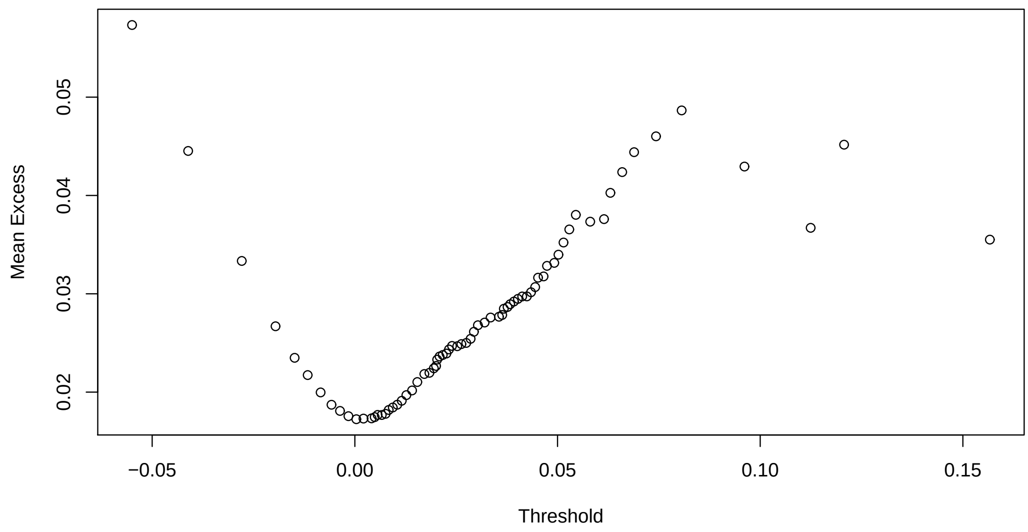
We now apply the risk factor mappings to Eqn.(5.2) and construct a portfolio that constitutes all five banks as

$$\bar{R}_{p,t} = E\left(R_{p,t}\right) = \sum_{t=1}^{N} w_i E\left(\mathbf{r}_{i,t}\right), \quad \sum_{i=1}^{N} w_i = 1, \tag{5.3}$$

where $w_i = \frac{x_i}{\mathbf{Inv}}$, and x_i is the fraction of the total investment; **Inv**, invested in stock i, and w_i the weights. We assume equal weights. Thus, the expected return on the portfolio, $E\left(R_{p,t}\right)$, on day t is a weighted average of the returns on the individual stocks. Note that the portfolio is constructed using the simple returns of Eqn.(5.2) and then converted back to log-returns for further analysis. Depending on the desired confidence level, the q^{th} quantile VaR; $VaR_q(Z)$, of the portfolio is obtained by applying the POT method of EVT and the *hybrid* method for threshold selection to the portfolio return distribution $\left\{\bar{R}_p\right\}_{t=1}^{T}$. $VaR_q(Z)$ is the VaR of the portfolio for day T to day $T+1$ since we are dealing with daily returns and applying EVT directly to the portfolio returns and not to the standardised residuals. Figure 5.3 shows the mean excess plots drawn using the portfolio returns from the Bayesian Frank and Student's-t copula models for the lower tail losses, while Figures 5.4 and 5.5 demonstrates the *hybrid* method for the threshold selection. It can be seen on Figures 5.4(b) and 5.5(b) that the regression lines for standard regression models are affected by outliers in the left tails of the mean excess plots, hence a robust regression model is more reliable. In Table 5.4, we present the POT parameter estimates, VaR and ESF estimates for the Bayesian GARCH(1,1) Frank copula-EVT and Bayesian GARCH(1,1) Student's-t copula-EVT models. From the table we notice that the Bayesian GARCH(1,1) Frank copula-EVT model produce a slightly higher portfolio VaR estimates and lower exceedances compared to the Bayesian GARCH(1,1) Student's-*t* copula-EVT model. We also compute the portfolio VaR estimates; $VaR_q^p(Z)$, based on the individual bank's VaR estimates and confidence level. i.e., using the risk formula; Eqn.(3.37). As noted, the overall risk measures are quite stable for both models and different thresholds indicating that the model has effectively captured the dynamics of fluctuations in the left tails of the return distributions. This claim must be validated through back-testing the model. We can also see the effect of diversification on the risk of the individual banks on the portfolio VaR.

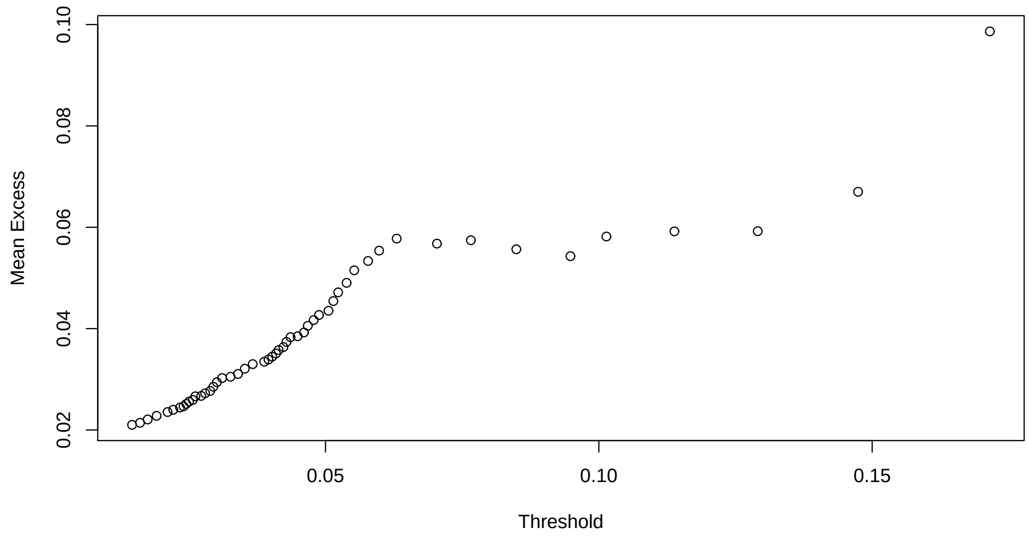


(a)

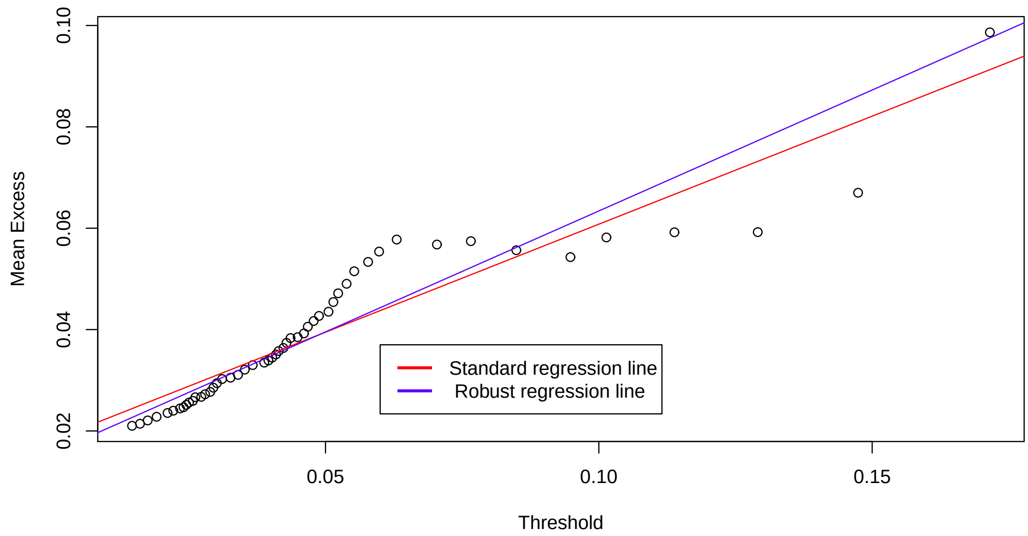


(b)

Figure 5.3: *Mean excess function plots drawn using the portfolio returns following a Bayesian GARCH(1,1) Frank copula model; Figures 5.3(a), and a Bayesian GARCH(1,1) Student's-t copula model; Figure 5.3(b), for the lower tail loses.*

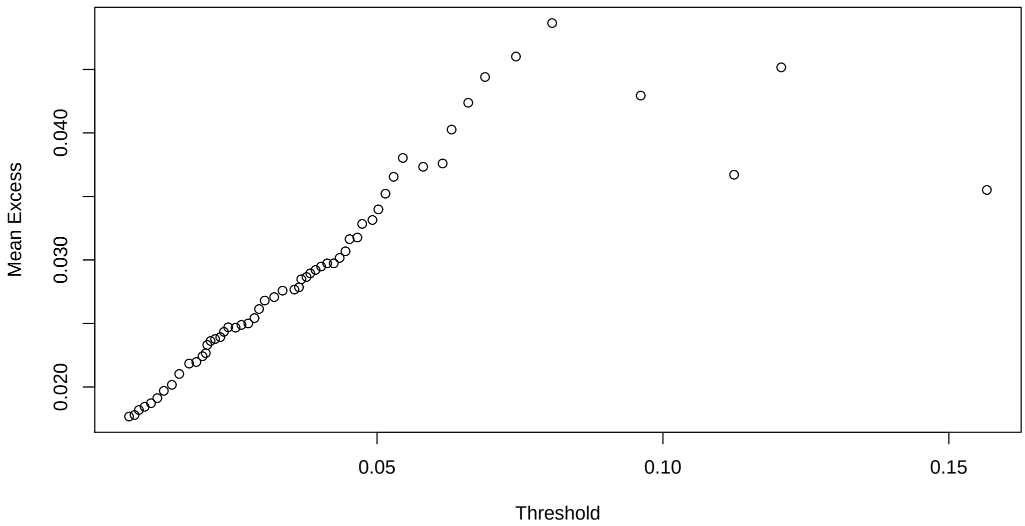


(a)

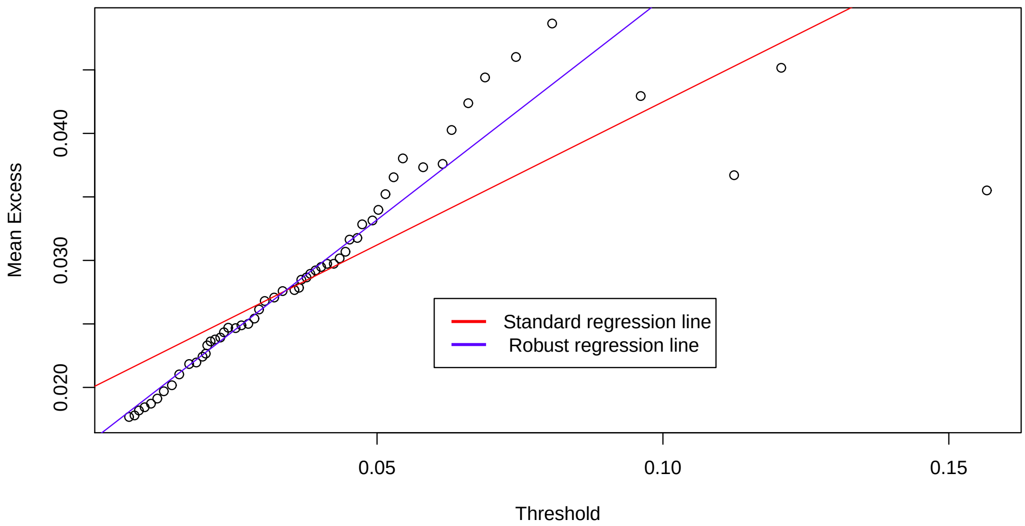


(b)

Figure 5.4: *Mean excess function plots of the portfolio return following a Bayesian GARCH(1,1) Frank copula-EVT model for the number of exceedances above* ϑ_0*; Figures 5.4(a), and a demonstration of the hybrid method for threshold selection; Figure 5.4(b). A reliable threshold is calculated by taking an average of the set of points that lie on the robust regression line.*



(a)



(b)

Figure 5.5: *Mean excess function plots of the portfolio return following Bayesian GARCH(1,1) Student's-t copula-EVT model for the number of exceedances above ϑ_0; Figures 5.5(a), and a demonstration of the hybrid method for threshold selection; Figure 5.5(b). A reliable threshold is calculated by taking an average of the set of points that lie on the robust regression line.*

		UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN	Portfolio	$VaR_q^p(Z)$
Bayesian GARCH(1,1) Frank copula-EVT model								
Parameters	$\psi(\vartheta^*)$	0.2594	0.0531	0.0437	0.0582	0.0214	0.0180	
	ξ	0.0164	0.4861	0.2682	0.3859	0.4063	0.4080	
	ϑ^*	0.0348	0.0969	0.0741	0.0904	0.0430	0.0303	
	N_{ϑ^*}	171	57	89	78	150	257	
	μ	-0.0006	0.0032	-0.0247	-0.0230	0.0062	0.0026	
	σ	0.0072	0.0076	0.0172	0.0145	0.0064	0.0067	
$\boldsymbol{VaR_q(Z)}$	99%	6.35%	13.36%	13.20%	16.15%	9.34%	9.42%	9.62%
	95%	3.21%	5.44%	5.45%	5.88%	4.39%	4.22%	3.99%
	90%	2.22%	3.53%	3.02%	3.08%	3.08%	2.83%	2.44%
$\boldsymbol{ES_q(Z)}$	99%	9.56%	27.17%	21.29%	30.10%	16.39%	16.88%	17.30%
	95%	5.33%	11.75%	10.71%	13.38%	8.06%	8.08%	8.11%
	90%	3.99%	8.04%	7.39%	8.82%	5.85%	5.75%	5.60%
Bayesian GARCH(1,1) Student's*t* copula-EVT model								
		UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN	Portfolio	$VaR_q^p(Z)$
Parameters	$\psi(\vartheta^*)$	0.0142	0.0364	0.0297	0.0437	0.0193	0.0209	
	ξ	0.1963	0.2893	0.2462	0.2883	0.1474	0.2482	
	ϑ^*	0.0277	0.0640	0.0441	0.0774	0.0303	0.0324	
	N_{ϑ^*}	172	105	198	90	255	195	
	μ	-0.0029	-0.0136	-0.0141	-0.0184	-0.0089	-0.0085	
	σ	0.0082	0.0140	0.0154	0.0161	0.0135	0.0107	
$\boldsymbol{VaR_q(Z)}$	99%	5.82%	12.14%	11.76%	13.67%	7.99%	8.36%	8.47%
	95%	3.04%	5.31%	5.41%	5.84%	4.18%	3.90%	3.90%
	90%	2.08%	3.22%	3.36%	3.43%	2.80%	2.47%	2.44%
$\boldsymbol{ES_q(Z)}$	99%	8.33%	19.60%	18.10%	22.21%	11.10%	12.82%	13.14%
	95%	4.86%	10.00%	9.67%	11.21%	6.64%	6.90%	6.99%
	90%	3.68%	7.06%	6.96%	7.83%	5.02%	4.99%	5.02%

Table 5.4: *VaR estimates following Bayesian GARCH(1,1) Frank copula-EVT and Bayesian GARCH(1,1) Student's-t copula-EVT models for a time horizon of* 1 *day at* $q = (99\%, 95\%, 90\%)$ *confidence levels. The risk measures are quite stable for different thresholds and copula functions indicating that the VaR models have successfully capture the dynamics of fluctuations in the left tails.*

5.3.2 Reliability of the VaR model

In Chapters 3 and 4, we check the reliability of the VaR model by employing Kupiec (1995) unconditional coverage test, Christoffersen (1998) independent and conditional coverage test, Basel *traffic light* test, the new independent test by Santos and Alves (2012), and the DQ test by Engle and Manganelli (2004). Since we apply EVT directly to the exposures to risk factors, we will obtain for each quantile an estimate of VaR for a desired window of the distribution of risk factors. Therefore, we need a different approach to back-testing. The most commonly used methods are the *Rosenblatt transformation* by Rosenblatt (1952) and the bootstrap back-test by Dowd (2002). We employ the bootstrap back-test discussed in Chapter 2 because this method will extract more information from the original sample data compared to the *Rosenblatt transformation* method. Also, bootstrap can be used to generate for a single parameter as many estimates as desired, thus generating a 'sample of sample estimates', which can be used to estimate a confidence interval for that particular parameter (Dowd, 2002).

Table 5.5 presents 95% confidence intervals for VaR estimates at $q = (99\%, 95\%, 90\%)$, $k = 2500$ repetitions, and $T = 1000$ data points for the Bayesian GARCH(1,1) Frank copula-EVT and Bayesian GARCH(1,1) Student's-t copula-EVT VaR models. All VaR estimates of the top five banks in the UK and a portfolio consisting of all these banks fall within the confidence interval and therefore the proposed VaR models are suitable to be used as a measure of risk. Notice from the table that the VaR estimates of the re-samples are very close to the Bayesian-GARCH(1,1) copula-EVT VaR estimates, especially for the portfolio. Thus, we are 95% confidence that the calculated VaR estimates will lie within the proposed interval. If the VaR estimates fall outside of these intervals, the model should be rejected.

		Bayesian GARCH(1,1) Frank copula EVT					
Confidence level		UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN	Portfolio
99%	Cl	[4.86%, 8.13%]	[9.60%, 20.30%]	[9.97%, 14.96%]	[11.36%, 20.62%]	[7.11%, 11.18%]	[6.74%, 13.36%]
	Error band	3.27%	10.70%	4.98%	9.27%	4.08%	6.62%
	VaR of re-samples	6.13%	13.23%	12.53%	15.67%	9.08%	10.00%
	BG Copula-EVT VaR	6.35%	13.36%	13.20%	16.15%	9.34%	9.42%
95%	Cl	[2.72%, 3.74%]	[4.60%, 6.51%]	[4.81%, 6.61%]	[5.08%, 7.39%]	[3.82%, 5.01%]	[3.65%, 4.78%]
	Error band	1.02%	1.91%	1.79%	2.31%	1.20%	1.13%
	VaR of re-samples	3.18%	5.50%	5.62%	6.15%	4.36%	4.21%
	BG Copula-EVT VaR	3.21%	5.44%	5.45%	5.88%	4.39%	4.22%
90%	Cl	[1.92%, 2.42%]	[3.05%, 3.93%]	[3.31%, 4.22%]	[3.33%, 4.31%]	[2.58%, 3.38%]	[2.56%, 3.16%]
	Error band	0.50%	0.89%	0.91%	0.98%	0.80%	0.60%
	VaR of re-samples	2.17%	3.52%	3.77%	3.82%	2.95%	2.86%
	BG Copula-EVT VaR	2.22%	3.53%	3.02%	3.08%	3.08%	2.83%

		Bayesian GARCH(1,1) Student's−t copula EVT					
Confidence level		UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN	Portfolio
99%	Cl	[4.81%, 6.94%]	[8.91%, 15.37%]	[8.79%, 15.32%]	[10.12%, 18.36%]	[6.46%, 9.35%]	[6.31%, 11.51%]
	Error band	2.14%	6.46%	6.52%	8.23%	2.89%	5.19%
	VaR of re-samples	5.92%	12.40%	11.74%	13.88%	7.87%	18.40%
	BG Copula-EVT VaR	5.82%	12.14%	11.76%	13.67%	7.99%	8.36%
95%	Cl	[2.65%, 3.41%]	[4.29%, 6.26%]	[4.54%, 6.29%]	[5.23%, 7.11%]	[3.55%, 4.71%]	[3.32%, 4.56%]
	Error band	0.75%	1.97%	1.74%	1.88%	1.16%	1.24%
	VaR of re-samples	3.01%	5.25%	5.47%	6.09%	4.15%	3.93%
	BG Copula-EVT VaR	3.04%	5.31%	5.41%	5.84%	4.18%	3.90%
90%	Cl	[1.78%, 2.31%]	[2.70%, 3.75%]	[2.98%, 3.88%]	[3.20%, 4.32%]	[2.43%, 3.17%]	[2.18%, 2.91%]
	Error band	0.53%	1.05%	0.90%	1.13%	0.74%	0.73%
	VaR of re-samples	2.00%	3.12%	3.38%	3.65%	2.77%	2.53%
	BG Copula-EVT VaR	2.08%	3.22%	3.36%	3.43%	2.80%	2.47%

Table 5.5: *Bootstrap back-test based on 95% confidence interval (CI). The VaR model is not rejected at 99%, 95%, and 90% confidence level. BG = Bayesian GARCH(1,1).*

5.4 Model comparison

In Figures 5.6 and 5.7, we plot the forecasts portfolio VaR estimates against several quantiles and compare the results with commonly used variance-covariance and historical simulation VaR models. We see from the plots that VaR estimates from Bayesian GARCH(1,1) Frank copula-EVT and Bayesian GARCH(1,1) Student's-*t* copula-EVT models perform better (i.e., grow faster) than traditional variance-covariance and historical simulation VaR estimates at higher quantiles. This is because EVT captures the left tail data better than the normal distribution. The poor performance of the variance-covariance method is as a result of the heavy tailed distribution while assuming normality. VaR estimates will be too conservative and inaccurate by assuming a normal distribution. Better performance of VaR estimates from Bayesian GARCH(1,1) copulas with historical simulation is due to the fact that copula functions enable the construction of malleable multivariate distributions with different margins and dependence structures that are free from any normality assumptions and linear correlations. The shape of the distribution is determined by the historical data and takes care of heavy tails and skewness. At lower quantiles, the performance of the Bayesian GARCH(1,1) Frank copula-EVT and Student's-*t* copula-EVT models are almost identical but overlap each other slightly at higher quantiles; Figure 5.8. Thus, an indication that we could either choose the Frank copula or the Student's-*t* copula to model dependence between the asset returns without compromising the accuracy of VaR estimates.

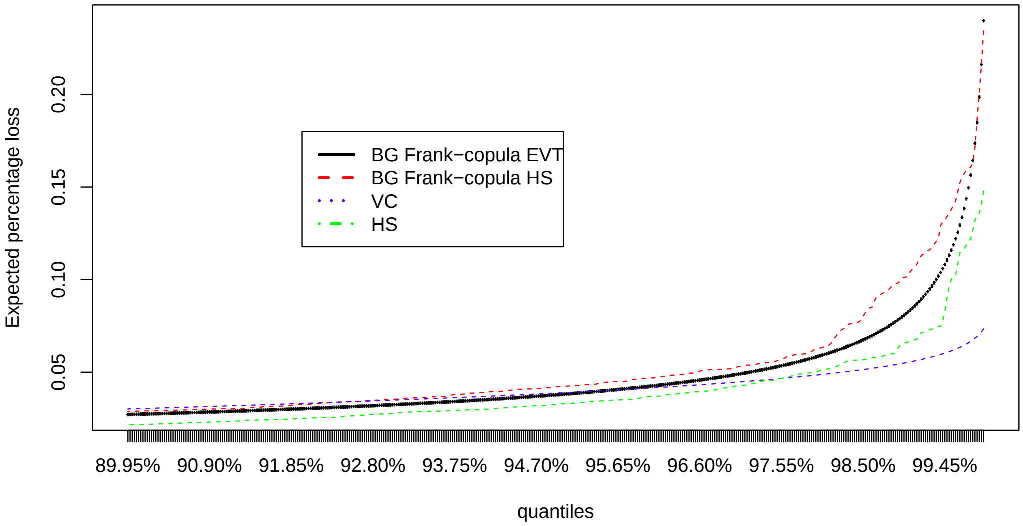


Figure 5.6: *Comparison of VaR estimates for different quantiles.The Bayesian GARCH(1,1) Frank copula EVT model and the Bayesian GARCH(1,1) Frank copula historical simulation model gives better VaR estimates than the traditional parametric variance-covariance and historical simulation methods.*

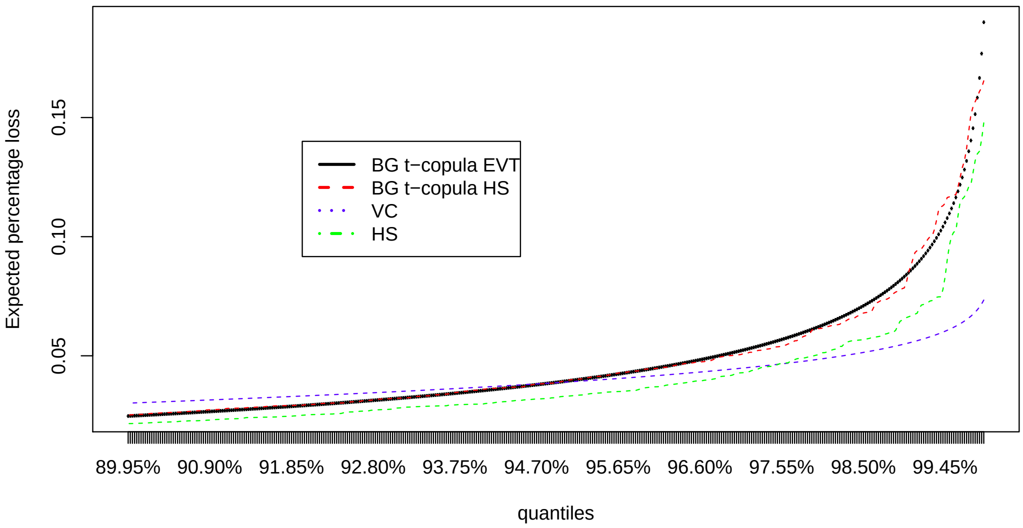


Figure 5.7: *Comparison of VaR estimates for different quantiles.The Bayesian GARCH(1,1) Student's-t copula EVT model and the Bayesian GARCH(1,1) Student's-t copula with historical simulation model gives better VaR estimates than the traditional parametric variance-covariance and historical simulation methods.*

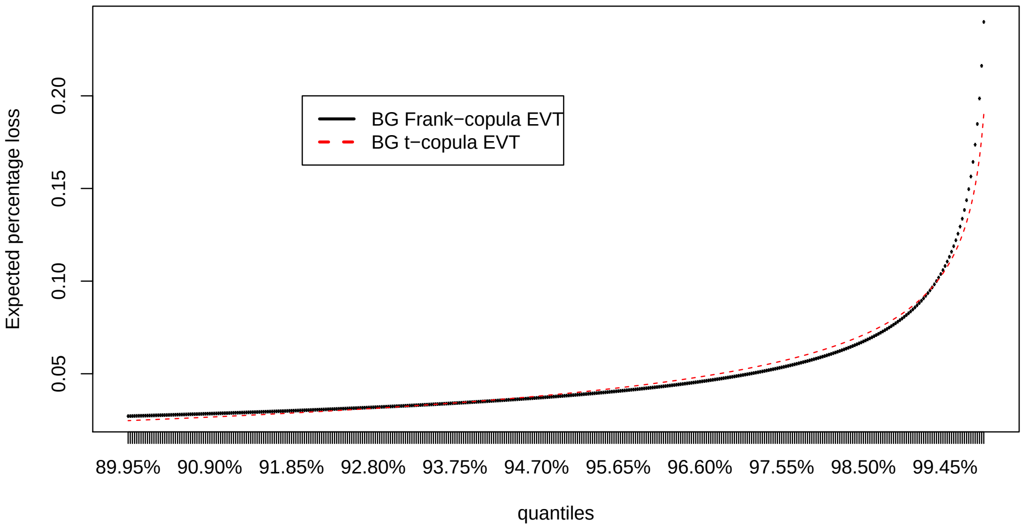


Figure 5.8: *Comparison of Bayesian GARCH(1,1) Student's-t copula EVT and Bayesian GARCH(1,1) Frank copula EVT VaR estimation models for different quantiles. Both models produce stable VaR estimates.*

In Tables 5.6, 5.7, 5.8, and 5.9, we compare the Bayesian GARCH(1,1) Frank copula-EVT and Bayesian GARCH(1,1) Student's-*t* copula-EVT models to the VaR models discussed in Chapters 3 and 4 based on the back-testing results. At higher confidence levels, EVT VaR models based on the *hybrid* method for threshold selection (EVT*) produced more stable VaR estimates compared to the traditional EVT VaR models; Table 5.6. One reason for this observation is because EVT* VaR models restricts inferences only on the tails of the risk factors distributions. This restriction reduces the margin of errors for unreliable VaR estimates. Whereas with the traditional EVT VaR models, inferences are not resticted on the left tail and sometimes moves towards the center of the distribution.

The best performance was recorded by VaR models with EVT and the worst performance was the HS and variance-covariance VAR models. VaR models following Bayesian GARCH(1,1) Frank and Student's-*t* copula functions also performed quite well at all levels but not as good as the models with EVT.

Kupiec's unconditional coverage test assumes the Bayesian GARCH(1,1) vine-copula

EVT VaR model to be too conservative because of very few or zero exceptions thus, the rejection at 95% confidence level for 500 and 1000 observation periods. This pattern can be clearly seen at 99% confidence level with zero exceptions; see Tables 5.7 and 5.10. Following the Basel rules, i.e., the Basel "traffic light" test, the Bayesian GARCH(1,1) vine-copula EVT VaR model with zero or very few exceptions will fall in the green zone. However, one should be extremely careful when very few or zero exceptions are observed. For example, as with the variance-covariance method, the fewer number of exceptions produced is most often as the result of the model ignoring extreme events because the risk factors distributions are assumed to have lighter tails.

The results of Tables 5.7, 5.8, and 5.9 also suggest that when we incorporate extreme value theory and employ the suggested *hybrid* method for extreme value analysis to forecast VaR, the type of underlying conditional volatility model used does not really matter. However, without EVT the type of volatility model does play a significant role in forecasting VaR estimates.

We further test the models for the periods of 2008 and 2011 financial crisis and present the results in Table 5.10. Based on the back-testing results of Table 4.8 in Chapter 4, i.e., during the 2008 and 2011 financial crisis periods, and the results in the previous three tables (Tables 5.7, 5.8, and 5.9), the VaR models will pass the independence test except for the historical simulation and variance covariance methods. Thus, the results in Table 5.10 are based only on the Kupiec's unconditional coverage (UC) test. The two periods constitute 262 and 260 observation periods for 2008 and 2011, respectively. Therefore, at confidence levels of 99%, 95%, and 90%, the number of exceptions T_1 must fall within the intervals $1 \leq T_1 \leq 6$, $7 \leq T_1 \leq 20$, and $18 \leq T_1 \leq 36$, respectively for the 2008 crisis period and $1 \leq T_1 \leq 6$, $7 \leq T_1 \leq 20$, and $17 \leq T_1 \leq 36$, respectively for the 2011 crisis period. We see from Table 5.10 that only the copula-EVT based VaR models are able to capture the dynamics of fluctuations in the left tails of the portfolio return distributions in both periods of severe financial distress. This findings thus show that EVT is very important in order to forecasting reliable and stable VaR estimates. We consider Kupiec's UC test and Christoffersen's CC test as the most powerful test. For example, Kupiec's UC test will reject a model that produces very few exceptions. Christoffersen's CC test will also reject a VaR model that produces very few exceptions, while at the same time testing for independence of the exceptions. The other back-testing methods are mostly concerned with testing the independence property.

EVT VaR model	$VaR^{p}_{99\%}(Z)$	$VaR^{p}_{95\%}(Z)$	$VaR^{p}_{90\%}(Z)$
Bayesian GARCH(1,1) Vine copula-EVT⋆	2.7891	1.6363	1.1979
Bayesian GARCH(1,1) Frank copola-EVT ⋆	2.7565	1.6629	1.2542
Bayesian GARCH(1,1) Student's-*t* copula-EVT⋆	2.7518	1.6495	1.2638
DCC-GARCH(1,1) Clayton copila-EVT	2.7818	1.6422	1.2135
DCC-GARCH(1,1) Student's-*t*-copula-EVT	2.8449	1.6601	1.2160

Table 5.6: *Comparison of portfolio quantile VaR estimates for UK stocks. EVT⋆ implies $VaR^{p}_{q}(Z)$ was estimated incorporating the hybrid method for threshold selection. EVT⋆ produced stable $VaR^{p}_{q}(Z)$ estimates at higher confidence level (i.e., at 99%) relative to the traditional EVT VaR models.*

p = 1%

VaR model	Window	Exceptions	LR_{POF}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$	Test results
			Type of back-test					
DCC-G Clayton cop-EVT	250	2	0.108 (0.742)	0.074 (0.786)	0.182 (0.913)	79.862 (0.000)	0.746 (0.649)	(A A A R A)
	500	2	2.353 (0.125)	0.035 (0.852)	2.388 (0.303)	38.635 (0.000)	0.746 (0.652)	(A A A R A)
	1000	4	4.706 (0.030)	4.429 (0.035)	9.135 (0.010)	22.806 (0.001)	13.480 (0.070)	(R R R R A)
DCC-G *t*-cop-EVT	250	2	0.108 (0.742)	0.123 (0.726)	0.231 (0.891)	1.000 (0.986)	0.746 (0.649)	(A A A A A)
	500	2	2.353 (0.125)	0.035 (0.852)	2.388 (0.303)	1.826 (0.935)	0.746 (0.650)	(A A A A A)
	1000	5	3.094 (0.079)	0.066 (0.797)	3.160 (0.206)	3.930 (0.686)	5.619 (0.126)	(A A A A A)
BG Vine cop-EVT⋆	250	0	*NaN*	-	-	2.485 (0.870)	-	(- - - A -)
	500	0	*NaN*	-	-	5.010 (0.543)	-	(- - - A -)
	1000	0	*NaN*	-	-	10.061 (0.122)	-	(- - - A -)
BG Student's-*t* cop-EVT⋆	250	2	0.108 (0.742)	0.075 (0.784)	0.183 (0.913)	2.484 (0.871)	0.746 (0.650)	(A A A A A)
	500	2	2.353 (0.125)	0.035 (0.852)	2.570 (0.277)	5.010 (0.543)	0.746 (0.650)	(A A A A A)
	1000	5	3.094 (0.079)	4.425 (0.035)	7.519 (0.023)	10.061 (0.122)	13.480 (0.070)	(A R R A A)
BG Frank cop-EVT ⋆	250	2	0.108 (0.742)	0.075 (0.784)	0.183 (0.913)	2.485 (0.870)	0.746 (0.651)	(A A A A A)
	500	2	2.353 (0.125)	0.035 (0.852)	2.570 (0.277)	5.010 (0.543)	0.746 (0.650)	(A A A A A)
	1000	5	3.094 (0.079)	4.421 (0.035)	7.515 (0.023)	10.061 (0.122)	13.480 (0.071)	(A R R A A)
BG Student's-*t* cop-HS	250	4	0.769 (0.381)	0.278 (0.598)	1.047 (0.592)	184.723 (0.000)	0.243 (0.716)	(A A A R A)
	500	8	1.538 (0.215)	0.540 (0.462)	2.078 (0.354)	163.430 (0.000)	0.635 (0.547)	(A A A R A)
	1000	17	4.091 (0.043)	0.344 (0.558)	4.435 (0.109)	143.716 (0.000)	8.604 (0.016)	(R A A R R)
BG Frank cop-HS	250	8	7.734 (0.005)	0.394 (0.530)	8.128 (0.017)	78.531 (0.000)	0.924 (0.478)	(R A R R A)
	500	9	2.613 (0.106)	0.995 (0.319)	3.608 (0.165)	80.367 (0.000)	8.200 (0.056)	(A A A R A)
	1000	18	5.225 (0.022)	2.300 (0.129)	7.525 (0.023)	63.787 (0.000)	8.354 (0.010)	(R A R R R)
HS	250	8	7.734 (0.005)	2.674 (0.102)	10.408 (0.005)	13.839 (0.032)	6.931 (0.049)	(R A R R R)
	500	12	7.111 (0.008)	2.263 (0.132)	9.404 (0.009)	9.675 (0.139)	0.414 (0.581)	(R A R A A)
	1000	20	7.827 (0.005)	3.921 (0.048)	11.748 (0.003)	40.497 (0.000)	5.952 (0.027)	(R A R R R)
VC	250	1	1.177 (0.278)	-	-		-	(A - - - -)
	500	1	4.813 (0.028)	-	-		-	(R - - - -)
	1000	1	13.476 (0.000)	-	-		-	(R - - - -)

Table 5.7: *Comparison of back-testing results for the various VaR models at 99% confidence level. BG = Bayesian-GARCH(1,1) model, cop = copula, HS = Historical simulation, and VC = Variance-Covariance.*

p = 5%

			Type of back-test					
VaR model	Window	Exceptions	LR_{POF}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$	Test results
DCC-G Clayton cop-EVT	250	10	0.563 (0.453)	0.011 (0.916)	0.574 (0.751)	3.712 (0.716)	0.367 (0.187)	(A A A A A)
	500	24	0.043 (0.836)	0.234 (0.629)	0.277 (0.871)	3.193 (0.784)	0.824 (0.500)	(A A A A A)
	1000	49	0.021 (0.885)	0.598 (0.439)	0.619 (0.734)	5.085 (0.533)	0.092 (0.623)	(A A A A A)
DCC-G *t*-cop-EVT	250	11	0.197 (0.657)	2.003 (0.157)	2.200 (0.332)	3.544 (0.738)	1.163 (0.414)	(A A A A A)
	500	25	0.000 (1.000)	1.343 (0.247)	1.343 (0.511)	1.978 (0.922)	0.865 (0.432)	(A A A A A)
	1000	52	0.083 (0.773)	2.670 (0.102)	2.753 (0.252)	5.123 (0.528)	0.152 (0.644)	(A A A A A)
BG Vine cop-EVT⋆	250	7	3.009 (0.083)	0.952 (0.329)	3.961 (0.138)	1.916 (0.448)	5.963 (0.427)	(A A A A A)
	500	14	6.018 (0.014)	1.791 (0.181)	7.809 (0.020)	-0.560 (0.855)	8.404 (0.210)	(R A R A A)
	1000	28	12.036 (0.001)	0.302 (0.583)	12.338 (0.002)	-0.744 (0.876)	1.445 (0.025)	(R A R A R)
BG Student's-*t* cop-EVT⋆	250	12	0.021 (0.885)	0.106 (0.745)	0.127 (0.722)	5.963 (0.427)	3.580 (0.124)	(A A A A A)
	500	26	0.042 (0.838)	0.388 (0.533)	0.430 (0.512)	5.876 (0.437)	0.785 (0.449)	(A A A A A)
	1000	52	0.083 (0.773)	1.730 (0.188)	1.813 (0.178)	10.849 (0.093)	0.092 (0.625)	(A A A A A)
BG Frank cop-EVT⋆	250	11	0.197 (0.657)	0.017 (0.896)	0.214 (0.899)	5.963 (0.427)	3.667 (0.188)	(A A A A A)
	500	23	0.173 (0.677)	0.041 (0.840)	0.214 (0.899)	5.875 (0.437)	0.908 (0.487)	(A A A A A)
	1000	48	0.085 (0.771)	0.891 (0.345)	0.976 (0.614)	10.849 (0.093)	0.172 (0.602)	(A A A A A)
BG Student's-*t* cop-HS	250	18	2.256 (0.133)	0.762 (0.383)	3.018 (0.221)	41.452 (0.000)	1.962 (0.246)	(A A A R A)
	500	32	1.903 (0.168)	0.204 (0.652)	2.107 (0.349)	45.328 (0.000)	-0.231 (0.733)	(A A A R A)
	1000	59	1.616 (0.204)	0.277 (0.599)	1.893 (0.388)	56.580 (0.000)	0.890 (0.426)	(A A A R A)
BG Frank cop-HS	250	19	3.091 (0.079)	0.456 (0.499)	3.547 (0.170)	27.231 (0.000)	1.677 (0.347)	(A A A R A)
	500	32	1.903 (0.168)	0.004 (0.950)	1.907 (0.385)	31.772 (0.000)	0.725 (0.509)	(A A A R A)
	1000	54	0.329 (0.566)	0.093 (0.760)	0.422 (0.810)	38.944 (0.000)	-0.424 (0.782)	(A A A R A)
HS	250	26	11.865 (0.001)	2.674 (0.102)	14.539 (0.001)	3.454 (0.750)	6.931 (0.049)	(R A R A R)
	500	39	7.102 (0.008)	2.263 (0.132)	9.365 (0.009)	8.912 (0.179)	0.414 (0.581)	(R A R A A)
	1000	56	0.731 (0.393)	3.921 (0.048)	4.652 (0.098)	27.387 (0.000)	5.952 (0.027)	(A R A R R)
VC	250	4	8.185 (0.004)	0.305 (0.581)	8.490 (0.014)	-	0.000 (0.801)	(R A R - A)
	500	5	24.736 (0.000)	0.219 (0.640)	24.955 (0.000)	-	2.389 (0.451)	(R A R - A)
	1000	6	64.564 (0.000)	0.150 (0.699)	64.714 (0.000)	-	10.073 (0.031)	(R A R - R)

Table 5.8: *Comparison of back-testing results for the various VaR models at 95% confidence level.*

$p = 10\%$

VaR model	Window	Exceptions	Type of back-test LR_{POF}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$	Test results
DCC-G Clayton cop-EVT	250	26	0.044 (0.834)	0.106 (0.745)	0.150 (0.928)	3.524 (0.741)	1.903 (0.276)	(A A A A A)
	500	49	0.022 (0.882)	0.466 (0.495)	0.488 (0.783)	5.478 (0.484)	-0.620 (0.836)	(A A A A A)
	1000	107	0.534 (0.465)	1.792 (0.181)	2.326 (0.313)	6.079 (0.414)	-0.292 (0.737)	(A A A A A)
DCC-G *t*-cop-EVT	250	27	0.174 (0.677)	0.006 (0.938)	0.180 (0.914)	3.524 (0.741)	1.941 (0.227)	(A A A A A)
	500	50	0.000 (1.000)	0.271 (0.603)	0.271 (0.873)	5.478 (0.484)	-0.599 (0.854)	(A A A A A)
	1000	108	0.695 (0.404)	1.505 (0.220)	2.185 (0.335)	6.278 (0.393)	-0.283 (0.751)	(A A A A A)
BG Vine cop-EVT⋆	250	24	0.045 (0.832)	0.018 (0.893)	0.063 (0.969)	0.288 (0.587)	9.473 (0.149)	(A A A A A)
	500	47	0.204 (0.652)	0.374 (0.541)	0.579 (0.749)	-0.558 (0.845)	15.994 (0.014)	(A A A A R)
	1000	94	0.403 (0.526)	4.837 (0.028)	5.240 (0.073)	-1.251 (0.957)	16.735 (0.010)	(A R A A R)
BG Student's-*t* cop-EVT⋆	250	25	0.000 (1.000)	0.021 (0.885)	0.021 (0.990)	9.473 (0.149)	1.980 (0.272)	(A A A A A)
	500	48	0.090 (0.764)	0.129 (0.719)	0.219 (0.896)	13.407 (0.037)	-0.579 (0.824)	(A A A R A)
	1000	100	0.000 (1.000)	1.750 (0.186)	1.750 (0.417)	13.610 (0.034)	-1.313 (0.965)	(A A A R A)
BG Frank cop-EVT⋆	250	25	0.000 (1.000)	0.021 (0.885)	0.021 (0.990)	9.473 (0.149)	1.980 (0.271)	(A A A A A)
	500	48	0.090 (0.764)	0.129 (0.719)	0.219 (0.896)	13.407 (0.037)	-0.579 (0.827)	(A A A R A)
	1000	101	0.011 (0.916)	1.309 (0.251)	1.320 (0.517)	12.984 (0.043)	-1.323 (0.970)	(A A A R A)
BG Student's-*t* cop-HS	250	34	3.274 (0.070)	0.165 (0.685)	3.439 (0.179)	31.455 (0.000)	1.009 (0.433)	(A A A R A)
	500	66	5.223 (0.022)	1.555 (0.212)	6.779 (0.034)	33.545 (0.000)	-0.186 (0.949)	(R A R R A)
	1000	119	3.805 (0.051)	0.473 (0.492)	4.277 (0.118)	29.898 (0.000)	-0.620 (0.845)	(A A A R A)
BG Frank cop-HS	250	33	2.612 (0.106)	0.820 (0.365)	3.432 (0.180)	21.665 (0.001)	1.356 (0.358)	(A A A R A)
	500	56	0.773 (0.379)	1.168 (0.280)	1.941 (0.379)	36.016 (0.000)	0.382 (0.569)	(A A A R A)
	1000	104	0.176 (0.675)	1.856 (0.173)	2.032 (0.362)	58.452 (0.000)	0.208 (0.579)	(A A A R A)
HS	250	40	8.623 (0.003)	2.674 (0.102)	11.297 (0.004)	11.128 (0.084)	6.931 (0.049)	(R A R A R)
	500	68	6.548 (0.011)	2.263 (0.132)	8.811 (0.012)	9.564 (0.144)	0.414 (0.581)	(R A R A A)
	1000	107	0.534 (0.465)	3.921 (0.048)	4.455 (0.108)	11.829 (0.066)	5.952 (0.027)	(R A R A A)
VC	250	13	7.627 (0.006)	0.157 (0.692)	7.784 (0.020)	-	0.208 (0.719)	(R A R - A)
	500	16	34.045 (0.000)	0.709 (0.340)	34.754 (0.000)	-	5.199 (0.048)	(R A R - R)
	1000	22	95.951 (0.000)	3.003 (0.083)	95.954 (0.000)	-	13.445 (0.001)	(R A R - R)

Table 5.9: *Comparison of back-testing results for the various VaR models at* 90% *confidence level.*

VaR model		2008 Financial Crisis $p = 1\%$	$p = 5\%$	$p = 10\%$	2011 Financial Crisis $p = 1\%$	$p = 5\%$	$p = 10\%$	Test results 2008	2011
DCC-G Clayton cop-EVT	Exceptions	4	18	30	1	12	35	(A A A)	(A A A)
DCC-G *t*-cop-EVT	Exceptions	3	17	30	1	11	35	(A A A)	(A A A)
BG Frank cop-EVT*	Exceptions	4	18	29	3	14	33	(A A A)	(A A A)
BG *t*-cop-EVT*	Exceptions	4	18	28	3	15	33	(A A A)	(A A A)
BG Vine cop-EVT*	Exceptions	0	16	27	0	10	31	(- A A)	(- A A)
BG Student's-*t* cop-HS	Exceptions	8	33	59	9	29	43	(R R R)	(R R R)
BG Frank cop-HS	Exceptions	8	37	59	8	23	43	(R R R)	(R R R)
HS	Exceptions	12	37	62	17	33	52	(R R R)	(R R R)
VC	Exceptions	22	33	47	7	19	26	(R R R)	(R A A)

Table 5.10: *Reliability of the VaR models in periods of financial distress based on Kupiec's unconditional coverage back-test. The VaR model is reliable if the number of exceptions fall within the intervals:* $1 \leq T_1 \leq 6$ *for* 99% *confidence level,* $7 \leq T_1 \leq 20$ *for* 95% *confidence level,* $18 \leq T_1 \leq 36$ *for* 90% *confidence level for the* 2008 *crisis period and* $1 \leq T_1 \leq 6$ *for* 99% *confidence level,* $7 \leq T_1 \leq 20$ *for* 95% *confidence level,* $17 \leq T_1 \leq 36$ *for* 90% *confidence level for the* 2011 *crisis period.*

5.5 Conclusion

In this chapter, we have constructed a VaR model for VaR estimation by applying EVT directly to the exposures to risk factors and incorporating the proposed *hybrid* method for threshold selection. We construct the VaR model by combining a Bayesian GARCH(1,1) model with Student's-*t* distributions as the underlying volatility model, copula functions to model dependence among risk factors, and EVT to model the left tail. We compare the different VaR models to the traditional historical simulation and variance-covariance methods commonly used by banks. Back-testing results suggest that EVT based VaR models and EVT based VaR models incorporating the *hybrid* method for threshold selection ($EVT^{\star}$) produced more reliable estimates of VaR. At higher confidence levels, EVT VaR estimates based on the *hybrid* method for threshold selection are more stable compared to the traditional EVT VaR models.

Chapter 6

Forecasting Value-at-Risk estimates using Bayesian Markov-Switching GJR-GARCH(1,1) copula-EVT model: Evidence from UK banks

This chapter propose a model for forecasting Value-at-Risk (VaR) using a Bayesian Markov Switching GJR-GARCH(1,1) model with skewed Student's-t innovations, copula functions and extreme value theory.

6.1 Introduction

The previous three chapters uses GARCH(1,1) models in which the parameters are not time varying. A study by Bauwens et al. (2014) have shown that volatility predictions following econometric models that ignore regime changes and time varying parameters can result to several drawbacks. For example, they may fail to capture the dynamics of fluctuations in the time series data. Ignoring regime changes and time varying parameters in high-volatility periods causes significant upwards bias in estimating the GARCH parameters, which impairs volatility forecasts Haas et al. (2004). Markov-switching GARCH model, first developed by Gray (1996) and later improved by Haas et al. (2004); Klaassen (2002), helps address the issues since it allows the parameters of GARCH models to vary over time according to a latent discrete Markov process, which leads to volatility forecasts that can rapidly adapt to variations Ardia et al. (2016).

We combine Bayesian Markov-switching GJR-GARCH(1,1) model that identifies non-constant volatility over time and allows the GARCH parameters to vary over time following a Markov process with copula functions and EVT. The *hybrid* method for threshold selection is also employed to formulate the Bayesian Markov-switching GJR-GARCH(1,1) copula-EVT VaR model which is then used to forecast the level of risk on financial asset returns.

The rest of the chapter is structured as follows: Section 6.2 presents the Bayesian Markov-switching GJR-GARCH methodology, Section 6.3 presents data and results. In Section 6.4, we check the reliability of the VaR model followed by conclusion in Section 6.5.

6.2 Methodology

6.2.1 Markov-switching GJR-GARCH model

Let r_t represent a time series, then a general Markov-switching GARCH specification can be represented as

$$r_t|(\Delta_t = k, \Omega_{t-1}) \sim D(0, h_{k,t}, \Theta_k), \tag{6.1a}$$

$$r_t = \epsilon_t(h_{\Delta_t,t}^{1/2}), \tag{6.1b}$$

where Δ_t is a Markov chain (a stochastic variable) defined on the parameter space $S = \{1, \ldots, K\}$ that symbolises the model, ϵ_t is the noise, which assumes a skewed Student's-t distribution, $D(0, h_{k,t}, \Theta_k)$ is a continuous distribution with zero mean and conditional variance $h_{k,t}$, Ω_{t-1} is the information set observed up to time $t-1$, and Θ_k is a vector of the shape parameters. We define a $K \times K$ transition probability matrix $\mathbf{P}$, with distinctive elements

$$p_{ij} = \mathbf{P}[\Delta_t = j|\Delta_{t-1} = i] \quad \forall i, j \in \{1, \ldots, K\}, \quad 0 < p_{ij} < 1, \quad \Sigma_{j=1}^{K} p_{ij} = 1, \tag{6.2}$$

where p_{ij} is the probability of transition from state $\Delta_{t-1} = i$ to state $\Delta_t = j$. The conditional variance $h_{k,t}$ for $k = 1, \ldots K$ are assumed to follow GARCH type volatility models Ardia et al. (2016); Haas et al. (2004). k represents each regime in the Markov chain.

Volatility reacts differently with large negative returns as compared to positive returns reflecting leverage effects (Jorion, 2007); a condition commonly referred to as the asymmetric response of volatility. It is well known that traditional GARCH models cannot capture the asymmetric response of volatility. Several extensions of GARCH models have since been developed as possible solutions to these drawbacks. The most common of these are the exponential generalized ARCH (EGARCH) model of Nelson (1991), the threshold GARCH (TGARCH) model of Zakoian (1994), and the GJR-GARCH model of Glosten et al. (1993). The only significant, albeit minor, difference between TGARCH and GJR-GARCH models is that TGARCH uses standard deviation instead of variance in its specifications (Ali et al., 2013). We employ the Markov-switching GARCH model of Haas et al. (2004) to capture the differences in the variance dynamics of high and low volatility periods (Ardia et al., 2016), and use the GJR-GARCH model to capture the asymmetry response in the conditional volatility process, hence the Markov-switching GJR-GARCH (MS-GJR-GARCH) model.

The conditional variance of a MS-GJR-GARCH model is defined as

$$h_{k,t} = \alpha_{0,k} + (\alpha_{1,k} + \alpha_{2,k}\mathbb{I}_{\{r_{t-1}<0\}})r_{t-1}^2 + \beta_k h_{k,t-1}, \qquad k = 1, \ldots K, \tag{6.3}$$

where $\mathbb{I}_{\{\cdot\}}$ is an indicator function introduced to capture the leverage effect such that

$$\mathbb{I}_{t-1} = \begin{cases} 1, & \text{if} \quad r_{t-1}<0, \\ 0, & \text{if} \quad r_{t-1}\geq 0. \end{cases} \tag{6.4}$$

$\alpha_{2,k}$ controls the degree of asymmetry in the conditional volatility to the past shock in regime k (Ardia et al., 2016). Thus, $\alpha_{2,k} > 0$ indicates the presence of leverage effect which implies previous negative returns have higher influence on the volatility. The constraints $\alpha_{0,k} > 0$, $\alpha_{1,k} + \alpha_{2,k} \geq 0$ and $\beta_k \geq 0$ ensures a positive variance while covariance stationary is achieved by ensuring that

$$\alpha_{1,k} + \alpha_{2,k} E[\epsilon_{k,t}^2 \mathbb{I}_{\{\epsilon_{k,t}<0\}}] + \beta_k < 1, \tag{6.5}$$

where $\mathbb{I}_{\{.\}} = 1$ if the condition holds and 0 otherwise. Note that $\mathrm{E}[\epsilon_{k,t}^2 \mathbb{I}_{\{\epsilon_{k,t}<0\}}] = \frac{1}{2}$ when ϵ_k

is symmetrically distributed.

For the conditional distribution of r_t in each regime of the Markov chain, we employ a skew and fat tail error probability distribution; the skewed Student's-*t* distribution. We use the skewed Student's-*t* distribution because it is able to account for the excess kurtosis in the conditional distribution that is common with financial time series processes (Ardia, 2008). Moreover, recent studies by Chen et al. (2017, 2012) have shown that skewed Student's-*t* errors distribution is a good choice, when compared to a range of existing alternatives. The probability density function (PDF) of a Student's-*t* distribution is defined as

$$f_s(\epsilon, \nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{(\nu-2)\pi\Gamma(\frac{\nu}{2})}}\left(1 + \frac{\epsilon^2}{\nu-2}\right)^{-\frac{\nu+1}{2}}, \quad \epsilon \in \mathbb{R}, \tag{6.6}$$

where the constraint on the degrees of freedom parameter $\nu > 2$ is imposed to guarantee that the second order moment exist, and $\Gamma(\cdot)$ is the Gamma function. Skewness is introduced by an additional parameter $\gamma_k > 0$ as defined in Fernández and Steel (1998); that is

$$p(\epsilon_k|v, \gamma_k) = \frac{2}{\gamma_k + \frac{1}{\gamma_k}}\left\{f_s\left(\frac{\epsilon_k}{\gamma_k}\right)\mathbb{I}_{[0,\infty)}(\epsilon_k) + f_s(\gamma_k\epsilon_k)\mathbb{I}_{(-\infty,0)}(\epsilon_k)\right\}. \tag{6.7}$$

When $\gamma_k \neq 1$, the posterior distribution, $p(\epsilon_k|v, \gamma_k)$, loses symmetry (see Trottier and Ardia (2016); Ardia et al. (2016); Fernández and Steel (1998) for more details on skewed Student's-*t* probability distribution).

We define a vector of the risk factor returns as $\mathbf{r} = (r_1, \ldots, r_T)'$, $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \beta_k, \mathbf{P})'$, and a vector of the model parameters as $\Lambda = (\theta_1, \Theta_1, \ldots, \theta_K, \Theta_K)$; $\Theta_K = (\nu_K, \gamma_K)$. Then, from Bayes theorem and prior distribution of the model parameters $p(\Lambda)$, we have

$$p_{ij} = \mathbf{P}[\Delta_t = j|\Delta_{t-1} = i] = \frac{f(r_t|\Delta_t = j, \Omega_{t-1}; \Lambda) \Pr(\Delta_t = j|\Omega_{t-1})}{\Sigma_{i=1}^{k} f(r_t|\Delta_t = i, \Omega_{t-1}; \Lambda) \Pr(\Delta = i|\Omega_{t-1})}, \tag{6.8}$$

where $f(r_t|\Delta_t = j, \Omega_{t-1}; \Lambda)$ is the conditional probability density of r_t at time t restrictive on Ω_{t-1} and regime j. Therefore, we have

$$f(r_t|\Lambda, \Omega_{t-1}) = \sum_{i=1}^{k} \sum_{j=1}^{k} \Pr[\Delta_t = j|\Delta_{t-1} = i] f_D(r_t|\Delta_t = j, \Omega_{t-1}; \Lambda) \tag{6.9}$$

and a likelihood function

$$L(\Lambda|\mathbf{r}) = \prod_{t=1}^{T} f(r_t|\Lambda, \Omega_{t-1}). \tag{6.10}$$

The Metropolis Hasting (MH) algorithm of Markov Chain Monte Carlo (MCMC) is then employed to estimate the parameter values of the posterior distribution, where Λ is a random variable with Markov chains generated as $(\Lambda^{[0]}), \ldots, (\Lambda^{[j]}), \ldots$ in a parameter space. As $(\Lambda^{[0]}), \ldots, (\Lambda^{[j]}) \ldots \to \infty$, the posterior distribution; $p(\mathbf{r}|\Lambda)$, converges to its stationary distribution from which the optimal mean parameters are calculated as an average of the posterior distribution.

6.3 Data and Results

We employ same data as in Chapters 4 and 5, and same in-sample and out-of-sample data for VaR forecasting and back-testing. To capture the tail distribution and the dynamics of fluctuations in the time series data, we consider a single-state, $k = 1$ and two-state, $k = \{1, 2\}$ Markov Switching GARCH specifications. The underlying volatility model is a GJR-GARCH(1,1) model with skewed Student's-t distribution. Since we use just one variance specification (i.e., GJR-GARCH), the two-state Markov Switching GARCH is generated by setting the number of regimes in the conditional distribution to 2. For the single-state, the length of the variance specification is equal to the length of the conditional distribution, which is 1 (see Ardia et al. (2016)). Also note that the single-state Markov Switching GJR-GARCH(1,1) model corresponds to GJR-GARCH(1,1) model without regime change. Therefore, we simply refer to the single-state and two-state Markov Switching GJR-GARCH(1,1) models as GJR-GARCH(1,1) and MS-GJR-GARCH(1,1) models, respectively (see Ardia et al. (2016)). GARCH parameters are estimated using Bayesian statistics as discussed in Chapter 4. Here, we assign a prior distribution with initial hyperparameters and generate for each state, two MCMC simulations of 20000 draws each. If convergence is attained, discard the first 10000 draws and select only the 10*th* draw from each chain such that auto-correlation between draws is reduced to almost zero. The two chains are then merged together to obtain a sample of 2000 observations from which the mean values of each parameter with respect to its posterior distribution is calculated as the optimal parameter estimate of the Bayesian GJR-GARCH(1,1) and Bayesian MS-GJR-GARCH(1,1) models with a skewed Student's-*t* distributions. Estimation results are presented in Tables 6.1 and 6.2 with standard errors in parenthesis. For MS-GJR-GARCH(1,1) model, the degrees of freedom parameter, ν, is fixed across the regimes.

	α_0	α_1	α_2	β_1	ν	γ
UK.HSBA	7.0531e-06(0.0000)	0.0508(0.0010)	0.1001(0.0000)	0.8488(0.0002)	5.8153 (0.0114)	1.0067(0.005)
UK.BARC	1.4764e-6(0.0000)	0.0509(0.0000)	0.1001(0.0000)	0.8570(0.0001)	6.4085 (0.0133)	1.0014 (0.0005)
UK.LLOY	7.0777e-06(0.0000)	0.0511(0.0000)	0.1001(0.0000)	0.8716(0.0001)	6.1691 (0.0118)	1.0009 (0.0005)
UK.RBS	9.4281e-06(0.0000)	0.0511(0.0000)	0.1002(0.0000)	0.8688(0.0001)	5.9166 (0.0111)	1.0160 (0.0005)
UK.STAN	2.0683e-05(0.0000)	0.0508(0.0000)	0.1002(0.0000)	0.8321(0.0002)	6.3657 (0.0138)	1.0266 (0.0005)
Portfolio	5.4112e-06(0.0000)	0.0510(0.0000)	0.1002(0.0000)	0.8670(0.0001)	9.4379 (0.0298)	0.9936 (0.0005)

Table 6.1: *Parameter estimates following Bayesian GJR-GARCH(1,1) model with skewed Student's-t distribution. Standard errors in parentheses.*

Two-state (k=1,2)	$k = 1$					
	α_{0_1}	α_{1_1}	α_{2_1}	β_{1_1}	ν	$\gamma_{\_1}$
UK.HSBA	2.9335e-07(0.0000)	0.0270 (0.0010)	0.0121 (0.0004)	0.9612 (0.0005)	6.2679 (0.0159)	1.0380 (0.0009)
UK.BARC	1.9132e-06(0.0000)	0.0302 (0.0004)	0.0811 (0.0014)	0.9208 (0.0011)	7.9120 (0.0270)	1.0115 (0.0010)
UK.LLOY	2.7159e-07(0.0000)	0.0109 (0.0002)	0.0253 (0.0003)	0.9729 (0.0002)	5.6388 (0.0142)	0.9516 (0.0007)
UK.RBS	1.0034e-07(0.0000)	0.0367 (0.0002)	0.0030 (0.0001)	0.9595 (0.0002)	7.4127 (0.0220)	1.0146 (0.0009)
UK.STAN	3.8283e-06(0.0000)	0.0338 (0.0008)	0.0932 (0.0032)	0.9083 (0.0027)	7.2792 (0.0186)	1.0462 (0.0011)
Portfolio	1.5441e-05(0.0000)	0.0341 (0.0004)	0.1694 (0.0040)	0.8665 (0.0025)	14.1506 (0.0621)	0.9954 (0.0013)
	$k = 2$					
	α_{0_2}	α_{1_2}	α_{2_2}	β_{1_2}	ν	$\gamma_{\_1}$
UK.HSBA	1.0589e-05(0.0000)	0.0412 (0.0005)	0.1568 (0.0011)	0.8566 (0.0007)	6.2679 (0.0159)	0.9496 (0.0012)
UK.BARC	1.8650e-05(0.0000)	0.0056 (0.0002)	0.2322 (0.0027)	0.8586 (0.0015)	7.9120 (0.0270)	0.9782 (0.0018)
UK.LLOY	1.5749e-05(0.0000)	0.0558 (0.0004)	0.0776 (0.0014)	0.9045 (0.0006)	5.6388 (0.0142)	1.1826 (0.0026)
UK.RBS	6.8672e-05(0.0000)	0.0683 (0.0013)	0.8202 (0.0043)	0.4771 (0.0022)	7.4127 (0.0220)	1.0138 (0.0023)
UK.STAN	1.8827e-05(0.0000)	0.1016 (0.0011)	0.3991 (0.0040)	0.6778 (0.0028)	7.2792 (0.0186)	1.0085 (0.0015)
Portfolio	2.2462e-06(0.0000)	0.0334 (0.0004)	0.2052 (0.0040)	0.8543 (0.0023)	14.1506 (0.0298)	0.9936 (0.0005)

Table 6.2: *Parameter estimates for two-state MS-GJR-GARCH(1,1) model with skewed Student's-t distribution. Standard errors in parentheses. Degrees of freedom parameter, ν is fixed across the regimes*

Applying Eqs (6.1a) and (6.1b), we then obtain a matrix Σ, which consists of the filtered marginal standardised residuals, $\{\epsilon_{i,t}\}_{t=1}^{T}$, of the overall process. That is

$$\{\epsilon_{i,t}\} = \Sigma_{i,t} = (r_{i,t})h_{\Delta_{i,t},i,t}^{-1/2}, \qquad i = 1, \ldots, N;\ t = 1, \ldots, T. \tag{6.11}$$

ARCH LM test and Ljung-Box test on the standardised residuals and standardised squared residuals, respectively, for lags 5 and 10 are presented in Table 6.3. For GJR-GARCH(1,1) model, there still exist some serial correlation in the standardized residuals of UK.RBS stock. For MS-GJR-GARCH(1,1) model, there is no evidence of an ARCH effect or serial correlations in the standardized residuals.

		UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN
	ARCH LM test					
GJR-GARCH(1,1) model	LM(5)	2.21	2.35	2.96	17.78	4.01
	p-value	0.820	0.800	0.706	0.003	0.548
	LM(10)	10.26	4.13	7.04	18.62	6.79
	p-value	0.820	0.942	0.722	0.045	0.745
	Ljung-Box test					
		UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN
	LM(5)	2.24	2.35	2.95	17.65	4.11
	p-value	0.815	0.799	0.707	0.003	0.534
	LM(10)	9.92	4.14	7.17	18.57	6.89
	p-value	0.447	0.941	0.710	0.046	0.736

		UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN
	ARCH LM test					
MS-GJR-GARCH(1,1) model	LM(5)	2.164	2.29	8.74	5.31	3.06
	p-value	0.826	0.807	0.120	0.379	0.690
	LM(10)	6.01	3.75	13.39	5.83	5.30
	p-value	0.815	0.958	0.203	0.829	0.870
	Ljung-Box test					
		UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN
	Q(5)	2.13	2.22	8.60	5.30	10.56
	p-value	0.831	0.818	0.126	0.380	0.061
	Q(10)	5.98	3.70	13.27	5.88	12.71
	p-value	0.817	0.960	0.209	0.826	0.240

Table 6.3: *ARCH LM test on the standardised residuals and Ljung-Box test on the standardised squared residuals.*

We now apply copula functions discussed in Chapter 3 to model dependence. Copula parameters are estimated by the CML estimation method and inversion of Kendall's τ. Table 6.4 presents results of the estimated copula parameters, MLE and AIC values. Based on the MLE value and AIC value, Frank and Student's-t copulas are selected as the best fit to model dependence. With the marginal distributions set to Student's-t distributions, the copula parameters are used to generate a new matrix of size $10000 \times N$:

$$\hat{\Sigma} = \left\{\zeta_{i,j}\right\}, \quad j = 1, \ldots, 10000,\ i = 1, \ldots, N, \tag{6.12}$$

assumed to be free from assumptions of normality and linear correlations. Multivariate ARCH test on $\hat{\Sigma}$ at 5% significance level show no evidence of conditional heteroscedasticity or serial correlation; see Table 6.6.

GJR-GARCH(1,1)	Archimedean copulas			Elliptical copulas	
	Gumbel	Clayton	Frank	Gaussian	Student's-t
Kendall's τ	1.782 (0.023)	1.563 (0.046)	4.697 (0.042)	$\rho_G = \rho_\tau(\rho_{SE})$	$\rho_t = \rho_\tau(\rho_{SE})$
MLE	3226	2745	**3250**	3846	**4108**
AIC	-14.158	-13.835	**-14.173**	3.491	**3.359**
MS-GJR-GARCH(1,1)					
	Gumbel	Clayton	Frank	Gaussian	Student's-t
Kendall's τ	1.773 (0.023)	1.546 (0.046)	4.657 (0.041)	$\rho_G = \rho_\tau(\rho_{SE})$	$\rho_t = \rho_\tau(\rho_{SE})$
MLE	3163	2705	**3206**	3773	**4013**
AIC	-14.119	-13.806	**-14.146**	3.529	**3.405**

Table 6.4: *Copula parameter estimates are based on inversion of Kendall's τ following CML estimation method; standard errors in parentheses. The best copula for modeling dependence among the risk factors is that with the highest MLE value or smallest AIC value (in bold). Frank copula is selected from the Archimedean copula family and Student's-t-copula is selected from the elliptical copula family.*

		UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN
GJR-GARCH(1,1)						
	UK.HSBA	1				
	UK.BARC	0.6230 (0.013)	1			
	UK.LLOY	0.5521 (0.015)	0.7054 (0.011)	1		
	UK.RBS	0.5741 (0.014)	0.7262 (0.011)	0.7176 (0.011)	1	
	UK.STAN	0.6383 (0.013)	0.6027 (0.014)	0.5437 (0.015)	0.5460 (0.015)	1
MS-GJR-GARCH(1,1)						
	UK.STAN	1				
	UK.BARC	0.6257 (0.013)	1			
	UK.LLOY	0.5544 (0.015)	0.7074 (0.011)	1		
	UK.RBS	0.5779 (0.015)	0.7282 (0.011)	0.7225 (0.011)	1	
	UK.STAN	0.6437 (0.012)	0.6075 (0.014)	0.5439 (0.015)	0.5483 (0.015)	1

Table 6.5: *Kendall's* τ*;* $\rho_\tau(\rho_{SE})$ *for Gaussian and Student's-t copula parameter estimates (standard errors in parenthesis).*

Copula type	GJR-GARCH(1,1)		MS-GJR-GARCH(1,1)	
Frank	$Q_k(10) = 11.413$	$Q_k^r(10) = 267.925$	$Q_k(10) = 5.288$	$Q_k^r(10) = 245.072$
	p-value = 0.326	p-value = 0.208	p-value = 0.871	p-value = 0.576
Student's-t	$Q_k(10) = 5.507$	$Q_k^r(10) = 235.133$	$Q_k(10) = 2.554$	$Q_k^r(10) = 249.171$
	p-value = 0.855	p-value = 0.742	p-value = 0.990	p-value = 0.503

Table 6.6: *Multivariate ARCH test on* $\left\{\zeta_{i,j}\right\}$ *show no evidence of conditional heteroscedasticity.*

We now apply the POT method of EVT and the proposed *hybrid* method for threshold selection to obtain $VaR_q(Z)$ for the individual banks. Using $VaR_q(Z)$, we employ Eqn.(3.37) to obtain $VaR_q^p(Z)$; the portfolio quantile VaR, used to forecast daily VaR estimates. Results are presented in Tables 6.7 and 6.8. The one day ahead VaR is then calculated as

$$VaR_{q,t}^p = VaR_q^p(Z)\hat{h}_{\Delta_t,t+1}^{1/2}, \tag{6.13}$$

where $\hat{h}_{\Delta_t,t+1}^{\frac{1}{2}}$ is the one-step-ahead conditional volatility forecast of the overall conditional variance for the portfolio at time $t+1$ for state k, Δ_t is a Markov chain as defined in 6.1a and 6.1b but for the portfolio. That is, $\bar{R}_{p,t}|(\Delta_t = k, \Omega_{t-1})$, and the parameters are sampled from the posterior distribution using MH algorithm. Figs 6.1 and 6.2 show time plots of profit and loss (P&L) of the portfolio return series and forecasts portfolio VaR estimates at 99% and 95% confidence levels. A visual observation of the plots suggests that the

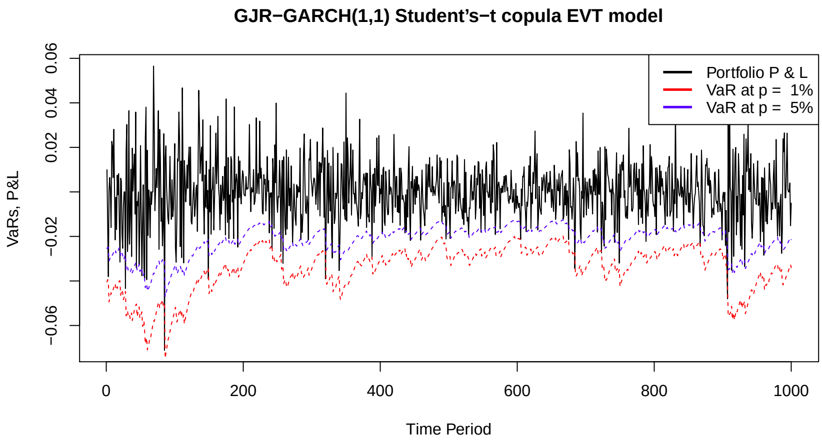
VaR models performs quite well in capturing the dynamics in the portfolio return series. However, the model needs to be validated through back-testing.

				Parameters				$VaR_{q,i}(Z)$	
		ξ	$\psi(\vartheta^*)$	ϑ^*	N_{ϑ^*}	μ	σ	99%	95%
Student's-*t* copula:	UK.HSBA	0.1660	0.7143	2.2448	333	0.3881	0.4061	3.1958	1.9640
	UK.BARC	0.2239	0.6479	2.4624	218	0.7974	0.2750	3.0141	1.9716
	UK.LLOY	0.1838	0.6353	2.2321	356	0.6481	0.3441	3.1407	2.0229
	UK.RBS	0.1273	0.7301	2.4687	271	0.3567	0.4612	3.2448	2.0385
	UK.STAN	0.1465	0.7135	2.3293	287	0.3538	0.4242	3.1428	1.9489
$VaR_q^p(Z)$								2.5862	1.6282
Frank copula:	UK.HSBA	0.1019	0.7239	2.6476	176	0.2503	0.4796	3.0688	1.9305
	UK.BARC	0.0497	0.7489	2.4331	235	-0.1297	0.6216	3.0867	1.8781
	UK.LLOY	0.0390	0.7892	2.6040	223	-0.1855	0.6804	3.2469	1.9767
	UK.RBS	0.2073	0.6862	2.5407	217	0.7266	0.3102	3.1174	2.0147
	UK.STAN	0.1062	0.6892	3.1337	105	0.6440	0.4249	3.1674	2.1425
$VaR_q^p(Z)$								2.5410	1.6459

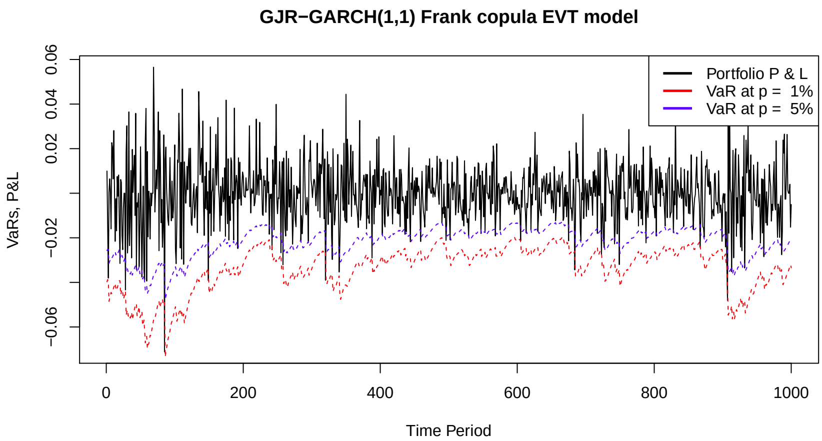
Table 6.7: *POT parameter estimates, $VaR_{q,i}(Z)$ and $VaR_q^p(Z)$ following Bayesian GJR-GARCH(1,1) Frank and Student's-t copula-EVT models.*

				Parameters				$VaR_{q,i}(Z)$	
		ξ	$\psi(\vartheta^*)$	ϑ^*	N_{ϑ^*}	μ	σ	99%	95%
Student's-*t* copula:	UK.HSBA	0.0194	0.8795	1.8425	565	-0.4893	0.8343	3.1862	1.9389
	UK.BARC	0.0239	0.8003	1.9903	409	-0.3040	0.7455	2.9471	1.8512
	UK.LLOY	0.0462	0.7642	2.0585	453	-0.1450	0.6624	3.2542	1.9832
	UK.RBS	0.0686	0.6675	2.3155	260	0.7943	0.5632	3.0091	1.8140
	UK.STAN	0.0038	0.6891	1.9164	493	-0.1460	0.6814	3.0191	1.9067
$VaR_q^p(Z)$								2.5127	1.5471
Frank copula:	UK.HSBA	0.0868	0.8295	2.5454	201	-0.2030	0.5910	3.1457	1.9804
	UK.BARC	0.0970	0.7780	1.9443	440	-0.7856	0.5132	2.9769	1.8480
	UK.LLOY	0.0819	0.7132	2.1074	358	0.0287	0.5430	3.0662	1.8724
	UK.RBS	0.0806	0.6030	2.5029	200	0.4796	0.4399	2.9327	1.9703
	UK.STAN	0.0924	0.6688	2.3509	230	0.2210	0.4719	2.9300	1.8498
$VaR_q^p(Z)$								2.4535	1.5513

Table 6.8: *POT parameter estimates, $VaR_q(Z)$ and $VaR_q^p(Z)$ following Bayesian MS-GJR-GARCH(1,1) Frank and Student's-t copula-EVT models.*

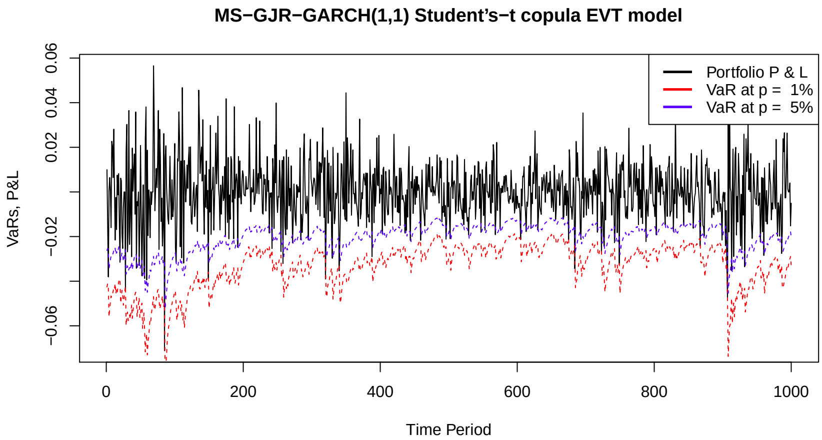


(a)

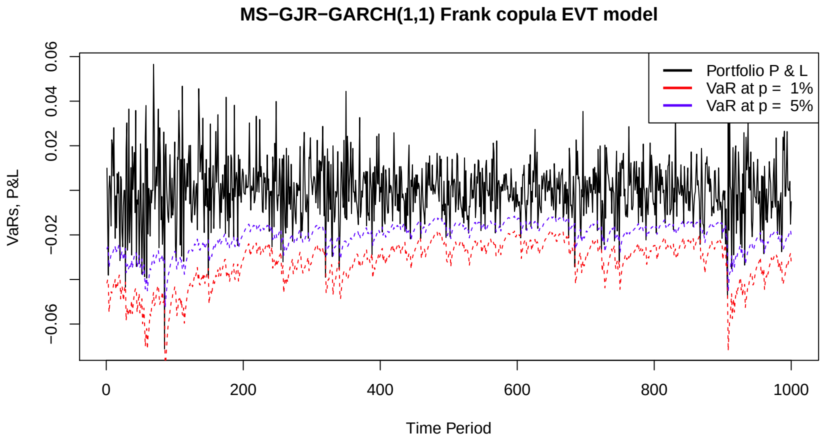


(b)

Figure 6.1: *Forecasts daily VaRs estimates and daily profit and loss (P&L) plots for an investment in a portfolio consisting of all banks following Bayesian GJR-GARCH(1,1) copula EVT model.*



(a)



(b)

Figure 6.2: *Forecasts daily VaRs estimates and daily profit and loss (P&L) plots for an investment in a portfolio consisting of all banks following Bayesian MS-GJR-GARCH(1,1) copula EVT model.*

6.4 Model validation

In this section, we present back-testing results for the VaR model validation for an out-of-sample data of $m = T - n$ observations. Table 6.9 presents the expected and observed number of exceptions produced following each model for a portfolio consisting of all five banks. At 99% confidence level and 250 days, the GJR-GARCH(1,1) copula EVT VaR model registered 3 exceptions whereas the MS-GJR-GARCH(1,1) VaR model registered

0 exceptions. Thus, following Basel rules for back-testing, the VaR models passed the reliability test and are placed in the green zone. Back-testing results based on LR_{UC}, LR_{IND}, LR_{CC}, DQ, and $T_{N,[N/2]}$ tests are presented in Tables 6.10 and 6.11. For the DQ test, we use a lagged value of 4. In Tables 6.12 and 6.13 we present, as a benchmark for our VaR models, back-testing results of well known standard GJR-GARCH(1,1) (sGJR-GARCH(1,1)) and standard GARCH(1,1) (sGARCH(1,1)) with skewed Student's-*t* distributions VaR models. It can be seen from the number of exceptions recorded that the MS-GJR-GARCH(1,1) copula EVT VaR model and the benchmark VaR models does not underestimate risk but rather too "conservative" at 99% and 95% confidence levels and thus preferred by most financial institutions. GJR-GARCH(1,1) copula EVT VaR model captures VaR quite well in periods of calm and in periods of crisis for short and long observation periods. It does not overestimate or underestimate the level of risk on the portfolio and should be considered reliable as a measure of risk. Performance evaluation for rejection or acceptance of the VaR models, based on 5% significance level, are presented in Tables 6.14 and 6.15. Furthermore, comparing the results of Tables 6.12, 6.13, 6.14, and 6.15 to Tables 5.7, 5.8 and 5.10, it is evident that the robust EVT VaR models perform better than the sGARCH(1,1), sGJR-GARCH(1,1), HS, and VC VaR models.

VaR model		250 1%	250 5%	500 1%	500 5%	1000 1%	1000 5%
	Expected exceptions:	2.5	12.5	5	25	10	50
Bayesian GJR-GARCH(1,1) Student's-*t* copula EVT	Observed:	3	11	4	26	8	57
	Coverage rate	0.012	0.044	0.008	0.052	0.008	0.057
Bayesian GJR-GARCH(1,1) Frank copula EVT	Observed:	3	11	5	24	9	55
	Coverage rate	0.012	0.044	0.010	0.048	0.009	0.055
Bayesian MS-GJR-GARCH(1,1) Student's-*t* copula EVT	Observed:	0	6	0	15	0	33
	Coverage rate	0.000	0.024	0.000	0.030	0.000	0.033
Bayesian MS-GJR-GARCH(1,1) Frank copula EVT	Observed:	0	6	0	14	0	32
	Coverage rate	0.000	0.024	0.000	0.028	0.000	0.032

Table 6.9: *Expected versus observed number of exceptions. Out-of-sample data is divided into blocks of* 250, 500, *and* 1000 *observation periods, time horizon of* 1 *day. The coverage rate* $\frac{T_1}{T_w} \approx 1 - q$.

Student's-*t* copula			Back-test type				
Prob	Window	Exceptions	LR_{UC}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$
1%:	250	3	0.095 (0.758)	0.170 (0.680)	0.265 (0.876)	0.213 (0.999)	-0.072 (0.966)
	500	4	0.217 (0.641)	0.140 (0.708)	0.357 (0.837)	0.415 (0.998)	-0.616 (0.994)
	1000	8	0.434 (0.510)	0.268 (0.605)	0.702 (0.704)	1.057 (0.983)	1.386 (0.385)
5%:	250	11	0.197 (0.657)	2.475 (0.116)	2.672 (0.263)	2.057 (0.914)	2.108 (0.348)
	500	26	0.042 (0.838)	1.997 (0.158)	2.039 (0.361)	2.962 (0.814)	-0.254 (0.742)
	1000	57	0.989 (0.320)	3.145 (0.076)	4.134 (0.127)	7.494 (0.278)	-1.039 (0.939)

Frank copula			Back-test type				
Prob	Window	Exceptions	LR_{UC}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$
1%:	250	3	0.095 (0.758)	0.168 (0.682)	0.263 (0.877)	0.213 (0.999)	-0.072 (0.967)
	500	5	0.000 (1.000)	0.219 (0.640)	0.219 (0.896)	0.415 (0.998)	-0.839 (0.999)
	1000	9	0.105 (0.746)	0.340 (0.560)	0.445 (0.801)	1.066 (0.983)	0.537 (0.676)
5%:	250	11	0.197 (0.657)	1.887 (0.170)	2.084 (0.353)	2.057 (0.914)	2.426 (0.244)
	500	24	0.043 (0.836)	1.343 (0.247)	1.386 (0.500)	2.305 (0.890)	-0.174 (0.723)
	1000	55	0.510 (0.475)	2.531 (0.112)	3.041 (0.219)	6.335 (0.387)	-1.004 (0.934)

Table 6.10: *Back-testing results following Bayesian GJR-GARCH(1,1) Student's-t and Frank copula-EVT VaR models. p-values in parenthesis. For DQ test, we use a lagged value of 4.*

Student's-t copula			**Back-test type**				
Prob	Window	Exceptions	LR_{UC}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$
1%:	250	0	*NaN*	-	-	0.213 (0.999)	-
	500	0	*NaN*	-	-	0.415 (0.998)	-
	1000	0	*NaN*	-	-	1.057 (0.983)	-
5%:	250	6	4.369 (0.037)	0.641 (0.423)	5.010 (0.082)	1.527 (0.958)	3.069 (0.379)
	500	15	4.884 (0.027)	0.001 (0.975)	4.885 (0.087)	2.226 (0.898)	1.154 (0.474)
	1000	33	6.878 (0.009)	0.032 (0.858)	6.910 (0.032)	0.697 (0.995)	-0.793 (0.908)

Frank copula			Back-test type				
Prob	Window	Exceptions	LR_{UC}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$
1%:	250	0	*NaN*	-	-	0.213 (0.999)	-
	500	0	*NaN*	-	-	0.415 (0.998)	-
	1000	0	*NaN*	-	-	1.057 (0.983)	-
5%:	250	6	4.369 (0.037)	0.699 (0.403)	5.068 (0.079)	30.724 (0.000)	2.070 (0.307)
	500	14	6.018 (0.014)	0.032 (0.858)	6.050 (0.049)	13.819 (0.032)	1.223 (0.382)
	1000	32	7.777 (0.005)	0.007 (0.933)	7.784 (0.020)	7.321 (0.292)	-1.386 (0.979)

Table 6.11: *Back-testing results following Bayesian MS-GJR-GARCH(1,1) Student's-t and Frank copula-EVT VaR models. p-values in parenthesis. For DQ test, we use a lagged value of 4.*

	250		500		1000	
	1%	5%	1%	5%	1%	5%
Expected exceptions	2.5	12.5	5	25	10	50
Observed exceptions sGARCH(1,1)	0	7	0	16	0	32
Coverage rate	0.000	0.028	0.000	0.032	0.000	0.032
Observed exceptions sGJR-GARCH(1,1)	0	7	0	14	0	26
Coverage rate	0.000	0.028	0.000	0.028	0.000	0.026

Table 6.12: *Expected versus observed number of exceptions following sGARCH(1,1) and sGJR-GARCH(1,1) models with skewed Student's-t distributions. The coverage rate* $\frac{T_1}{T_w} \approx 1 - q$

sGARCH(1,1)					**Back-test type**		
Prob	Window	Exceptions	LR_{UC}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$
1%:	250	0	*NaN*	-	-	37.933 (0.000)	-
	500	0	*NaN*	-	-	18.530 (0.005)	-
	1000	0	*NaN*	-	-	11.132 (0.084)	-
5%:	250	7	3.009 (0.083)	0.962 (0.327)	3.971 (0.137)	3.888 (0.692)	1.916 (0.448)
	500	16	3.888 (0.049)	0.011 (0.916)	3.899 (0.142)	2.409 (0.879)	-0.069 (0.708)
	1000	32	7.777 (0.005)	0.007 (0.933)	7.784 (0.020)	5.486 (0.483)	0.416 (0.541)

sGJR-GARCH(1,1)					Back-test type		
Prob	Window	Exceptions	LR_{UC}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$
1%:	250	0	*NaN*	-	-	23.077 (0.001)	-
	500	0	*NaN*	-	-	48.651 (0.000)	-
	1000	0	*NaN*	-	-	95.686 (0.000)	-
5%:	250	7	3.009 (0.083)	0.962 (0.327)	3.971 (0.137)	57.759 (0.000)	4.119 (0.228)
	500	14	6.018 (0.014)	0.032 (0.858)	6.050 (0.049)	85.830 (0.000)	-0.773 (0.908)
	1000	26	14.597 (0.000)	0.263 (0.608)	14.860 (0.001)	169.533 (0.000)	-0.975 (0.928)

Table 6.13: *Back-testing results following sGARCH(1,1) and sGJR-GARCH(1,1) models with skewed Student's-t distributions. p-values in parenthesis. For DQ test, we use a lagged value of 4.*

P = 1%			Back-test type			
VaR model	Window	LR_{UC}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$
GJR-GARCH(1,1) Student's-*t* copula-EVT	250	A (0.758)	A (0.680)	A (0.876)	A (0.999)	A (0.966)
	500	A (0.641)	A (0.708)	A (0.837)	A (0.998)	A (0.994)
	1000	A (0.510)	A (0.605)	A (0.704)	A (0.983)	A (0.385)
GJR-GARCH(1,1) Frank copula-EVT	250	A (0.758)	A (0.682)	A (0.877)	A (0.999)	A (0.967)
	500	A (1.000)	A (0.640)	A (0.896)	A (0.998)	A (0.999)
	1000	A (0.746)	A (0.560)	A (0.801)	A (0.983)	A (0.676)
MS-GJR-GARCH (1,1) Student's-*t* copula-EVT	250	R (*NaN*)	R (-)	R (-)	A (0.999)	R (-)
	500	R (*NaN*)	R (-)	R (-)	A (0.998)	R (-)
	1000	R (*NaN*)	R (-)	R (-)	A (0.983)	R (-)
MS-GJR-GARCH(1,1) Frank copula-EVT	250	R (*NaN*)	R (-)	R (-)	A (0.999)	R (-)
	500	R (*NaN*)	R (-)	R (-)	A (0.998)	R (-)
	1000	R (*NaN*)	R (-)	R (-)	A (0.983)	R (-)
sGARCH(1,1)	250	R (*NaN*)	R (-)	R (-)	R (0.000)	R (-)
	500	R (*NaN*)	R (-)	R (-)	R (0.005)	R (-)
	1000	R (*NaN*)	R (-)	R (-)	A (0.084)	R (-)
sGJR-GARCH(1,1)	250	R (*NaN*)	R (-)	R (-)	R (0.001)	R (-)
	500	R (*NaN*)	R (-)	R (-)	R (0.000)	R (-)
	1000	R (*NaN*)	R (-)	R (-)	R (0.000)	R (-)

Table 6.14: *Performance Evaluation of the VaR models at* 1% *significance level. The best performance is registered by GJR-GARCH(1,1) Student's-t copula-EVT and GJR-GARCH(1,1) Frank copula-EVT VaR models. A = Accept, R = Reject.*

P = 5%		Back-test type				
VaR model	Window	LR_{UC}	LR_{IND}	LR_{CC}	DQ	$T_{N,[N/2]}$
GJR-GARCH(1,1) Student's-t copula-EVT	250	A(0.657)	A (0.116)	A (0.263)	A (0.914)	A (0.348)
	500	A (0.838)	A (0.158)	A (0.361)	A (0.814)	A (0.742)
	1000	A (0.320)	A (0.076)	A (0.127)	A (0.278)	A (0.939)
GJR-GARCH(1,1) Frank copula-EVT	250	A (0.657)	A (0.170)	A (0.353)	A (0.914)	A (0.244)
	500	A (0.836)	A (0.247)	A (0.500)	A (0.890)	A (0.723)
	1000	A (0.475)	A (0.112)	A (0.219)	A (0.387)	A (0.934)
MS-GJR-GARCH(1,1) Student's-t copula-EVT	250	R (0.037)	A (0.423)	A (0.082)	A (0.958)	A (0.379)
	500	R (0.027)	A (0.975)	A (0.087)	A (0.898)	A (0.474)
	1000	R (0.009)	A (0.858)	R (0.032)	A (0.995)	A (0.908)
MS-GJR-GARCH(1,1) Frank copula-EVT	250	R (0.037)	A (0.403)	A (0.079)	R (0.000)	A (0.307)
	500	R (0.014)	A (0.858)	R (0.049)	R (0.032)	A (0.382)
	1000	R (0.005)	A (0.933)	R (0.020)	A (0.292)	A (0.979)
sGARCH(1,1)	250	A (0.083)	A (0.327)	A (0.137)	A (0.692)	A (0.448)
	500	R (0.049)	A (0.916)	A (0.142)	A (0.879)	A (0.708)
	1000	R (0.005)	A (0.933)	R (0.020)	A (0.483)	A (0.541)
sGJR-GARCH(1,1)	250	A (0.083)	A (0.327)	A (0.137)	R (0.000)	A (0.228)
	500	R (0.014)	A (0.858)	R (0.049)	R (0.000)	A (0.908)
	1000	R (0.000)	A (0.608)	R (0.001)	R (0.000)	A (0.928)

Table 6.15: *Performance Evaluation of the VaR models at* 5% *significance level. GJR-GARCH(1,1) and MS-GJR-GARCH(1,1) copula EVT VaR models performs better than sGARCH(1,1) and sGJR-GARCH(1,1) VaR models. A = Accept, R = Reject.*

6.5 Conclusion

In this chapter, we constructed VaR models by combining a single-state and a two-state Bayesian MS-GJR-GARCH(1,1) models as the underlying volatility models with skewed Student's-*t* distributions, copula functions to model dependence, and EVT to model the left tail. The single-state MS-GJR-GARCH(1,1) volatility model is a GJR-GARCH(1,1) volatility model without regime change, hence the names: Bayesian GJR-GARCH(1,1) copula-EVT VaR model for the single-state MS-GJR-GARCH(1,1) and Bayesian MS-GJR-GARCH(1,1) copula-EVT VaR model for the two-state MS-GJR-GARCH(1,1). It can be seen that the proposed *hybrid* method for threshold selection restricts inferences to the left tail; Tables 6.7 and 6.8, and diminishes the possibility of selecting a less suitable threshold value.

We use, as a benchmark, VaR models constructed using sGJR-GARCH(1,1) and sGARCH(1,1) volatility models with skewed Student's-*t* distributions, but without copula functions and EVT to compare the performance of our VaR models. Back-testing results show that the single-state Bayesian MS-GJR-GARCH(1,1) copula-EVT VaR model is more reliable than the two-state Bayesian MS-GJR-GARCH(1,1) copula EVT VaR model and the benchmark VaR models. The single-state Bayesian MS-GJR-GARCH(1,1) copula EVT VaR model does not overestimate or underestimate the level of risk on the portfolio and is thus reliable as a measure of risk, whereas the two-state MS-GJR-GARCH(1,1) copula EVT VaR model and the benchmark VaR models seems to overestimate the level of risk; Tables 6.14 and 6.15.

Chapter 7

Conclusion and proposed future work

In recent decades, VaR has become the most common risk management measure used by financial institutions to assess the market risk of financial assets. VaR models often focus on the behavior of asset returns in the left tail and is therefore important that the models are calibrated such that they do not underestimate or overestimate the proportion of outliers, as this will have significant effects on the allocation of economic capital for investments. Due to the "Extremistan" (Taleb, 2017) nature of financial asset returns and volatility, the real tail risk of a financial asset is not stable as time passes, and the maximum loss is difficult to predict. Thus, to implement a reliable VaR model, the time horizon and type of volatility model used is very important. However, we have demonstrated through a variety of different VaR models that when EVT is employed for VaR estimations, the type of volatility model used is not very important as long as the volatility model is conditional on the previous days information.

It is important to draw attention to the fact that when employing EVT, the q^{th} quantile of the estimator; $VaR_q(Z)$ (Eqn.(3.30)) is a point estimate with an error band that gets bigger as we move to more extreme quantiles. It is concerned only with the number of exceedances above a certain threshold and is not affected by data outside the tail of the distribution (Wong, 2013). This can be problematic in some cases due to limited data points in the tail, which can inhibit proper analysis. $VaR_q(Z)$ depends on the threshold and the number of points (i.e. exceedances) above the threshold because the parameters are estimated based on the exceedances. Thus, it is logical to say that the reliability of Eqn.(3.30) rests solely on the choice of the thresholds, which is very subjective.

In Chapter 3, we presented a novel approach for estimating VaR using multivariate GARCH models, copula functions and EVT. We estimated VaR using multivariate Dynamic Conditional Correlation (DCC) GARCH models as the underlying volatility model to model the correlation structure of the covariance matrix of multiple asset returns, copula functions to model dependence among the asset returns and EVT to model the tail behavior.

In Chapter 4 we introduced a novel approach for VaR estimation by combining Bayesian GARCH(1,1) model with Student's-t distributions as the underlying volatility model, vine copula functions and EVT. We proposed a new method for threshold selection; an objective *hybrid* method that restricts inferences on the tails of the distributions and diminishes the possibility of selecting a threshold that can compromise VaR estimates when employing EVT.

In Chapter 5, we applied EVT directly to the exposures to risk factors to further test the *hybrid* method of threshold selection. The underlying volatility model is a Bayesian GARCH(1,1) model with Student's-t distribution. This chapter also compared results of the various VaR models proposed in this research to the traditional VaR models commonly used by financial institutions.

Chapter 6 proposed a model for forecasting Value-at-Risk (VaR) using a Bayesian

Markov Switching GJR-GARCH(1,1) model with skewed Student's-t innovations, copula functions and extreme value theory, taking into account regime changes and time varying parameters.

The overall test suggest that VaR models based on EVT produced more reliable results and VaR models based on EVT incorporating the *hybrid* method for threshold selection ($EVT^{\star}$) produced VaR estimates that are more stable at higher confidence levels.

This research, based on the calculated MCR for market risk in relation to Basel II and Basel III (4.11), also adds evidence to the previous findings by McAleer et al. (2011) as stated earlier that the global financial crisis cannot be associated to the failure of Basel II as it was implemented in Europe only from 2008, and never in the USA. Banks that were presumed to be in good shape with higher solvency ratios and higher credit-to-GDP ratios before their collapse or bailouts probably tampered with their internal risk models for market risk or as a result of poor VaR models that were unable to capture fat-tail risk. This claim is evident in the banking system of Greece which was one of the countries that was gravely affected from the end of 2008 to around early 2014. However, this claim is not 100% certain as the proposed VaR models need to be tested in other countries whose banks were severely affected during the crisis period and needed to be bailed out.

In this research, we have used stock prices in the banking sector to build the proposed VaR models. It would be interesting to see how the model behaves when applied to stock indices and high frequency financial data.

We conclude by saying that this approach can be implemented with other conditional multivariate volatility models providing positive-definite volatility matrices such as the exponential weighted moving average (EWMA) model, BEKK model by Engle and Kroner (1995) and more.

Chapter 8

Appendix

A.1 Autoregressive Moving Average Model

A time series $\{X_t\}$ is an ARMA(1,1) model if

$$X_t = \phi_1 X_{t-1} + \phi_0 + Z_t + \Theta Z_{t-1} \tag{A.1}$$

for every t, where ϕ_1, ϕ_0, and Θ are constant parameters, and Z_t is a random Gaussian variable with zero mean and standard deviation σ_t (Boone, 2005). The model is stationary if $|\phi_1| < 1$ and $E(X_t) = \frac{\phi_0}{1-\phi_1}$. When $\phi = 0$, then X_t is a MA(1)=ARMA(0,1) model defined as

$$MA(1) : X_t = Z_t + \Theta Z_{t-1}. \tag{A.2}$$

When $\Theta = 0$, then X_t is an AR(1)=ARMA(1,0) model defined as

$$AR(1) : X_t = \phi X_{t-1} + \phi_0 + Z_t \tag{A.3}$$

A.2 *Akaike information criterion* (AIC) and *Bayesian information criterion*

Akaike information criterion (AIC) and *Bayesian information criterion* (BIC) are used for model selection and defined as

$$AIC = 2k - 2ln(L) \tag{A.4}$$

$$BIC = ln(n)k - 2ln(L) \tag{A.5}$$

where k is the number of estimated parameters, n is the sample size, and L is the maximum value of the likelihood function for the model. The best model selected from a set of models is that with the smallest AIC or BIC value.

A.3 R-code syntax for bootstrapping of 95% confidence interval

$$\boldsymbol{\vartheta} = c(\vartheta_1, \vartheta_2, \ldots, \vartheta_\hbar)$$

$$set.seed(20)$$

$$empDist_{\boldsymbol{\vartheta}} = rnorm(10000, mean = mean(\boldsymbol{\vartheta}), sd = sd(\boldsymbol{\vartheta}))$$

$$\boldsymbol{\vartheta} = NULL$$

$$\text{for (i in } 1:1000)\{$$

$$S = sample(1 : length(empDist_{\boldsymbol{\vartheta}}), replace = TRUE)$$

$$empDist_{\boldsymbol{\vartheta}}S = empDist_{\boldsymbol{\vartheta}}[S]$$

$$\boldsymbol{\vartheta}^* = mean(empDist_{\boldsymbol{\vartheta}}S)$$

$$\vartheta = c(\boldsymbol{\vartheta}, \boldsymbol{\vartheta}^*)$$

$$\}$$

$$q_{upper} = 0.975$$

$$q_{lower} = 0.025$$

$$CI = c(quantile(\boldsymbol{\vartheta}, q_{lower}), quantile(\boldsymbol{\vartheta}, q_{upper}))$$

$$CI$$

$$error = quantile(eta, q_upper) - quantile(eta, q_lower)$$

$$par(mfcol = c(1, 1))$$

$$hist(\boldsymbol{\vartheta}, main = \text{"95\% confidence interval"}, xlab = \text{"}Threshold\text{"})$$

$$SE = abs((quantile(\boldsymbol{\vartheta}, q_{upper}) - quantile(\boldsymbol{\vartheta}, q_{lower}))/(2 * qnorm(q_{lower})))$$

$$segments(CI, y0 = 0, y1 = 250, lty = 2)$$

$$\text{text}(CI, y = 210, labels = c(\text{"2.5\%ile"}, \text{"97.5\%ile"}))$$

Country	Parameters	Stocks from various banks					
France	ARMA(1,1)-GARCH(1,1)	F.BNP	F.SGE	F.CRDA	F.KNF	F.CC	F.CAI
	μ	0.0003 (0.0003)	0.0004 (0.0003)	0.0004 (0.0004)	0.0009 (0.0004)	0.0002 (0.0000)	-0.0001 (0.0002)
	ar1	0.8233 (0.0471)	-0.2975 (0.1619)	-0.3488 (0.6253)	-0.9447 (0.0067)	0.9925 (0.0018)	0.7760 (0.0423)
	ma1	-0.8598 (0.0413)	0.3571 (0.1569)	0.3711 (0.1569)	0.9521 (0.0061)	-0.9872 (0.0000)	-0.6907 (0.0502)
	α_0	6e-6 (0.0000)	6e-6 (0.0000)	5e-6 (0.0000)	8e-6 (0.0000)	3e-6 (0.0000)	1.2e-5 (0.0000)
	α_1	0.0877 (0.0122)	0.0896 (0.0123)	0.0731 (0.0216)	0.1131 (0.0242)	0.1219 (0.1264)	0.3267 (0.0449)
	β_1	0.9113 (0.0153)	0.9094 (0.0158)	0.9259 (0.0267)	0.8859 (0.0228)	0.8386 (0.1746)	0.5879 (0.0533)
	Θ	0.9683 (0.0234)	1.0195 (0.0242)	1.0328 (0.0257)	1.0471 (0.0241)	1.1086 (0.0242)	1.0039 (0.0205)
Greece		G.PIST	G.PEIR	G.EFG	G.ETE	G.ATT	G.ELL
	μ	0.0004 (0.0005)	0.0007 (0.0005) -0.0002 (0.0005)	0.0001 (0.0005)	-0.0010 (0.0005)	-0.0002 (0.0003)	
	ar1	-0.6948 (0.1440)	-0.1810 (0.2638) -0.4041 (0.2476)	-0.2960 (0.7176)	-0.5073 (0.1328)	0.0304 (0.5809)	
	ma1	0.7342 (0.1350)	0.2556 (0.2574)	0.4705 (0.2381)	0.3476 (0.7050)	0.5544 (0.1263)	-0.0158 (0.5801)
	α_0	1.1e-5 (0.0000)	6e-6 (0.0000)	1.5e-5 (0.0000)	1.9e-5 (0.0000)	4.6e-5 (0.0000)	8e-6 (0.0000)
	α_1	0.1067 (0.0158)	0.1011 (0.0215)	0.1473 (0.0191)	0.1490 (0.0299)	0.2260 (0.0372)	0.1963 (0.0296)
	β_1	0.8923 (0.0172)	0.8979 (0.0242)	0.8517 (0.0214)	0.8500 (0.0338)	0.7628 (0.0335)	0.7991 (0.0285)
	Θ	1.0270 (0.0203)	1.0258 (0.0183)	1.0179 (0.0196)	1.0010 (0.0205)	1.0539 (0.0222)	1.0877 (0.0244)
UK		UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN	
	μ	1.6e-5 (0.0002)	0.0002 (0.0003)	0.0002 (0.0003)	9.7e-5 (0.0003)	0.0002 (0.0003)	
	ar1	0.9295 (0.0255)	-0.4523 (0.2222)	-0.4647 (0.1583)	-0.5187 (0.1638)	0.8675 (0.0348)	
	ma1	-0.9470 (0.0172)	0.5032 (0.2141)	0.5070 (0.1444)	0.5750 (0.1558)	-0.8900 (0.0312)	
	α_0	2e-6 (0.0000)	6e-6 (0.0000)	2e-6 (0.0000)	4e-6 (0.0000)	8e-6 (0.0000)	
	α_1	0.0895 (0.2937)	0.0990 (0.0171)	0.0753 (0.1924)	0.0998 (0.0267)	0.1151 (0.0213)	
	β_1	0.9095 (0.2991)	0.9000 (0.0181)	0.9237 (0.2046)	0.8992 (0.0304)	0.8810 (0.0210)	
	Θ	1.0022 (0.0392)	1.0097 (0.0231)	1.0086 (0.0257)	1.0194 (0.0227)	1.0308 (0.0233)	
Spain		E.SCH	E.BBVA	E.BSAB	E.BKT	E.POP	
	μ	0.0005 (0.0002)	0.0003 (0.0003)	0.0000 (0.0003)	0.0004 (0.0003)	-0.0001 (0.0003)	
	ar1	0.7919 (0.0715)	-0.2010 (0.2426)	0.3666 (0.5211)	-0.3081 (0.9202)	0.1617 (0.2908)	
	ma1	-0.8227 (0.0666)	0.2565 (0.2383)	-0.2901 (0.5395)	0.3308 (0.9114)	-0.1045 (0.2937)	
	α_0	5e-6 (0.0000)	4e-6 (0.0000)	2e-6 (0.0000)	6e-6 (0.0000)	2e-6 (0.0000)	
	α_1	0.0934 (0.0187)	0.0843 (0.0187)	0.0469 (0.0162)	0.0994 (0.0242)	0.0588 (0.0346)	
	β_1	0.9056 (0.0212)	0.9147 (0.0223)	0.9476 (0.0189)	0.8996 (0.0365)	0.9402 (0.0392)	
	Θ	0.9515 (0.0220)	0.9925 (0.0234)	0.9878 (0.0205)	1.0359 (0.0226)	1.0271 (0.0205)	
Sweden		W.NDA	W.SVK	W.SWED	W.SEA		
	μ	0.0005 (0.0003)	0.0004 (0.0002)	0.0007 (0.0002)	0.0008 (0.00022		
	ar1	0.2752 (0.2913)	0.4971 (0.4552)	0.8133 (0.0553)	0.7318 (0.1505)		
	ma1	-0.3478 (0.2865)	-0.5455 (0.4397)	-0.8490 (0.0498)	-0.7758 (0.1405)		
	α_0	5e-6 (0.0000)	4e-6 (0.0000)	3e-6 (0.0000)	6e-6 (0.0000)		
	α_1	0.0777 (0.0806)	0.0924 (0.0505)	0.0640 (0.0225)	0.0869 (0.0212)		
	β_1	0.9213 (0.0979)	0.9066 (0.0536)	0.9350 (0.0255)	0.9121 (0.0271)		
	Θ	1.0083 (0.0239)	0.9734 (0.0221)	0.9510 (0.0225)	1.0055 (0.0227)		

Table A.T1: *ARMA(1,1)-GARCH(1,1) parameter estimates following M-GARCH(1,1) DCC model, standard errors in parenthesis.*

Country	Parameters	Stocks from various banks					
France	ARMA(1,1)-GARCH(1,1)	F.BNP	F.SGE	F.CRDA	F.KNF	F.CC	F.CAI
	μ	0.0003 (0.0003)	0.0004 (0.0004)	0.0004 (0.0004)	0.0009 (0.0004)	0.0002 (0.0000)	-0.0001 (0.0002)
	ar1	0.8233 (0.0469)	-0.2969 (0.1616)	-0.3480 (0.6203)	-0.9448 (0.0070)	0.9925 (0.0018)	0.7758 (0.0424)
	ma1	-0.8599 (0.0412)	0.3566 (0.1566)	0.3704 (0.6139)	0.9522 (0.0066)	-0.9872 (0.0000)	-0.6903 (0.0503)
	α_0	6e-6 (0.0000)	6e-6 (0.0000)	5e-6 (0.0000)	8e-6 (0.0000)	3e-6 (0.0000)	1.2e-5 (0.0000)
	α_1	0.0876 (0.0127)	0.0896 (0.0126)	0.0729 (0.0232)	0.1124 (0.0244)	0.1213 (0.1269)	0.3251 (0.0445)
	β_1	0.9114 (0.0157)	0.9094 (0.0160)	0.9261 (0.0282)	0.8862 (0.0231)	0.8386 (0.1765)	0.5882 (0.0529)
	Θ	0.9681 (0.0234)	1.0197 (0.0243)	1.0328 (0.0257)	1.0469 (0.0241)	1.1090 (0.0244)	1.0039 (0.0204)
Greece		G.PIST	G.PEIR	G.EFG	G.ETE	G.ATT	G.ELL
	μ	0.0004 (0.0005)	0.0007 (0.0005)	-0.0002 (0.0005)	0.0001 (0.0005)	-0.0010 (0.0005)	-0.0002 (0.0003)
	ar1	-0.6948 (0.1442)	-0.1811 (0.2637)	-0.4041 (0.2472)	-0.2962 (0.7179)	-0.5076 (0.1324)	0.0309 (0.5625)
	ma1	0.7343 (0.1352)	0.2557 (0.2574)	0.4704 (0.2377)	0.3478 (0.7053)	0.5546 (0.1260)	-0.0162 (0.5617)
	α_0	1.1e-5 (0.0000)	6e-6 (0.0000)	1.5e-5 (0.0000)	1.9e-5 (0.0000)	4.6e-5 (0.0000)	8e-6 (0.0000)
	α_1	0.1067 (0.0157)	0.1011 (0.0211)	0.1473 (0.0200)	0.1490 (0.0311)	0.2258 (0.0372)	0.1963 (0.0296)
	β_1	0.8923 (0.0170)	0.8979 (0.0238)	0.8517 (0.0222)	0.8500 (0.0350)	0.7628 (0.0335)	0.7992 (0.0286)
	Θ	1.0270 (0.0202)	1.0258 (0.0183)	1.0179 (0.0197)	1.0011 (0.0206)	1.0539 (0.0222)	1.0877 (0.0244)
UK		UK.HSBA	UK.BARC	UK.LLOY	UK.RBS	UK.STAN	
	μ	1.7e-5 (0.0007)	0.0002 (0.0003)	0.0002 (0.0003)	9.9e-5 (0.0003)	0.0002 (0.0003)	
	ar1	0.9304 (0.1149)	-0.4508 (0.2167)	-0.4645 (0.1742)	-0.5177 (0.1633)	0.8677 (0.0348)	
	ma1	-0.9477 (0.0793)	0.5018 (0.2087)	0.5069 (0.1565)	0.5740 (0.1553)	-0.8901 (0.0312)	
	α_0	2e-6 (0.0001)	5e-6 (0.0000)	2e-6 (0.0000)	4e-6 (0.0000)	8e-6 (0.0000)	
	α_1	0.0891 (1.4566)	0.0988 (0.0185)	0.0757 (0.2489)	0.0999 (0.0281)	0.1137 (0.0209)	
	β_1	0.9099 (1.4662)	0.9002 (0.0193)	0.9233 (0.2607)	0.8991 (0.0312)	0.8808 (0.0211)	
	Θ	1.0020 (0.1686)	1.0092 (0.0232)	1.0086 (0.0278)	1.0190 (0.0228)	1.0307 (0.0233)	
Spain		E.SCH	E.BBVA	E.BSAB	E.BKT	E.POP	
	μ	0.0005 (0.0002)	0.0003 (0.0003)	0.0000 (0.0003)	0.0004 (0.0003)	0.0001 (0.0003)	
	ar1	0.7919 (0.0712)	-0.2004 (0.2411)	0.3778 (0.5023)	-0.3066 (0.8087)	0.1618 (0.2887)	
	ma1	-0.8228 (0.0663)	0.2560 (0.2369)	-0.3000 (0.5217)	0.3295 (0.8009)	-0.1045 (0.2915)	
	α_0	5e-6 (0.0000)	4e-6 (0.0000)	2e-6 (0.0000)	6e-6 (0.0000)	2e-6 (0.0000)	
	α_1	0.0934 (0.0194)	0.0842 (0.0196)	0.0460 (0.0161)	0.0990 (0.0253)	0.0589 (0.0353)	
	β_1	0.9056 (0.0219)	0.9146 (0.0229)	0.9484 (0.0190)	0.9000 (0.0370)	0.9401 (0.0398)	
	Θ	0.9512 (0.0221)	0.9923 (0.0235)	0.9878 (0.0204)	1.0361 (0.0227)	1.0273 (0.0206)	
Sweden		W.NDA	W.SVK	W.SWED	W.SEA		
	μ	0.0005 (0.0003)	0.0004 (0.0002)	0.0007 (0.002)	0.0008 (0.0002)		
	ar1	0.2776 (0.3282)	0.4994 (0.4733)	0.8135 (0.0550)	0.7337 (0.1483)		
	ma1	-0.3502 (0.3240)	-0.5479 (0.4570)	-0.8491 (0.0495)	-0.7774 (0.1384)		
	α_0	4e-6 (0.0000)	4e-6 (0.0000)	3e-6 (0.0000)	5e-6 (0.0000)		
	α_1	0.0772 (0.1521)	0.0917 (0.0498)	0.0639 (0.0219)	0.0868 (0.0237)		
	β_1	0.9218 (0.1801)	0.9065 (0.0532)	0.9351 (0.0246)	0.9122 (0.0295)		
	Θ	1.0083 (0.0244)	0.9730 (0.0222)	0.9508 (0.0226)	1.0053 (0.0228)		

Table A.T2: *ARMA(1,1)-GARCH(1,1) parameter estimates following M-GARCH(1,1) aDCC model, standard errors in parenthesis.*

	France	Greece	UK	Spain	Sweden
$Q_k(5)$	5726.8990	2598.1180	1938.3120	1772.3850	4590.3380
p-value	0.0000	0.0000	0.0000	0.0000v	0.0000
$Q_k^r(5)$	18150.1700	3055.1820	10519.9600	3182.9470	4545.424
p-value	0.0000	0.0000	0.0000	0.0000	0.0000
$Q_k(10)$	8873.0210	4062.3610	2678.1480	2722.5010	7513.1970
p-value	0.0000	0.0000	0.0000	0.0000	0.0000
$Q_k^r(10)$	28610.3900	4440.2240	14539.9800	4072.3640	5502.2000
p-value	0.0000	0.0000	0.0000	0.0000	0.0000

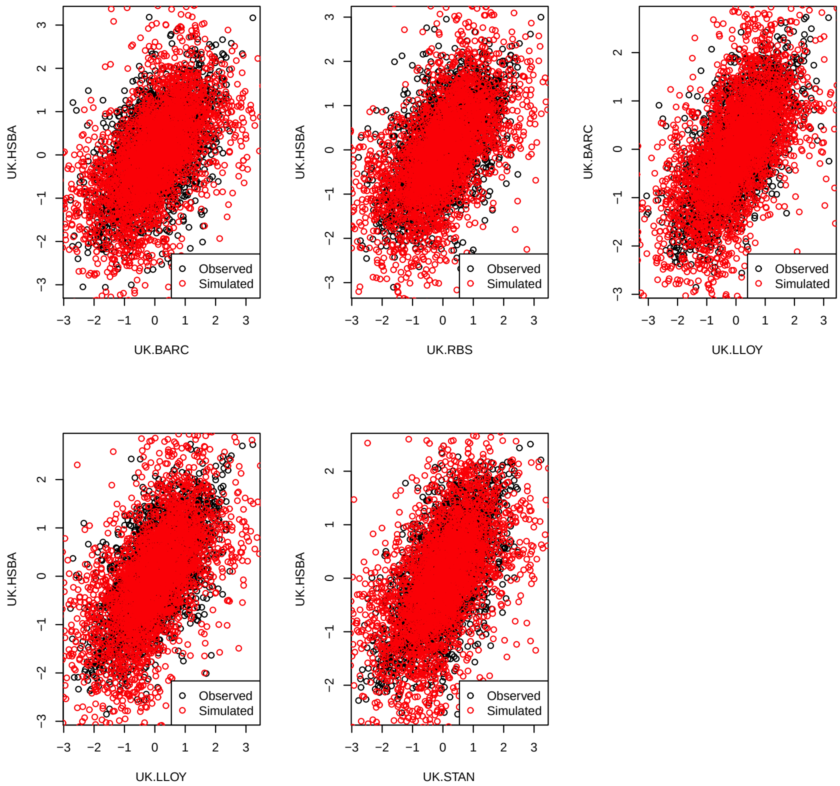
Table A.T3: *Multivariate ARCH test on the standardised residuals after the fitted M-GARCH(1,1) aDCC model for France, Greece, UK, Sweden, and M-GARCH(1,1) DCC model for Spain shows evidence of ARCH effect or conditional heteroscedasticity.*

	Archimedean Copula family					Elliptical t-Copula				
	France	Greece	UK	Spain	Sweden	France	Greece	UK	Spain	Sweden
$Q_k(5)$	3.9751	3.2266	4.7525	3.6042	4.5163	7.7360	6.8554	12.4768	10.7047	2.4232
p-value	0.5530	0.6651	0.4468	0.6077	0.4778	0.1714	0.2316	0.0288	0.0691	0.7880
$Q_k^r(5)$	165.7847	174.2798	113.2169	134.8293	74.1949	168.4713	166.4156	142.2465	120.6231	87.2783
p-value	0.7687	0.6062	0.7665	0.2585	0.6618	0.7210	0.7579	0.1387	0.5940	0.2705
$Q_k(10)$	8.3966	5.1023	9.6294	7.8085	6.1371	11.6012	10.1747	17.4906	16.2716	11.3141
p-value	0.5902	0.8842	0.4736	0.6475	0.8036	0.3126	0.4253	0.0642	0.0921	0.3336
$Q_k^r(10)$	366.2421	329.8300	227.0368	266.6985	147.5931	347.5079	366.3677	283.3849	221.8118	186.1114
p-value	0.3989	0.8713	0.8485	0.2235	0.7501	0.6222	0.3971	0.0720	0.8999	0.0772

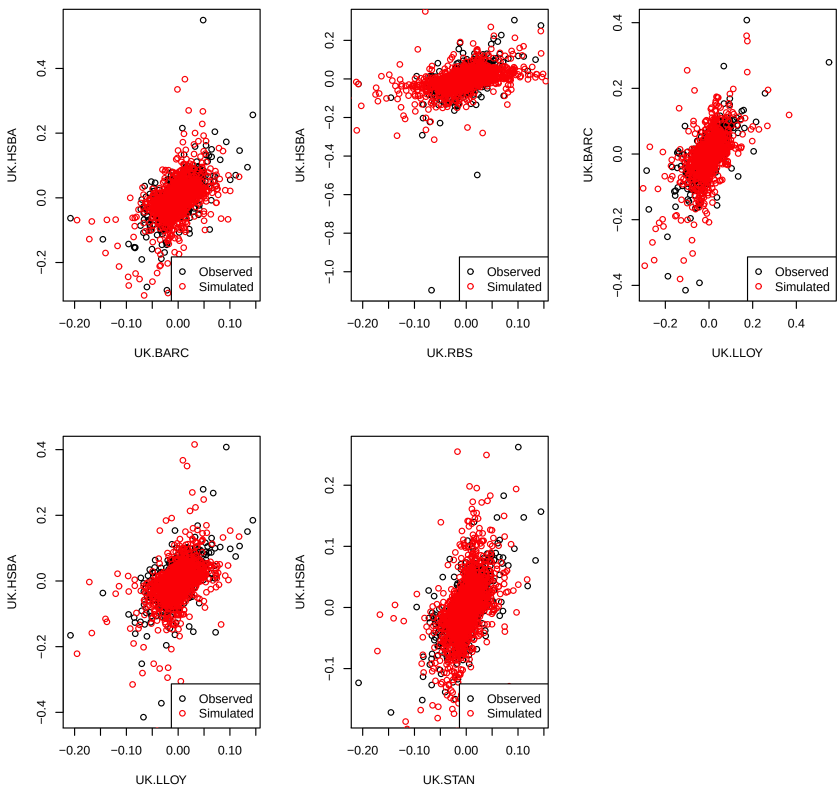
Table A.T4: *Multivariate ARCH test on the standardised residuals after modeling dependence with copulas. We see that the null hypothesis of no conditional heteroscedasticity is rejected for UK after modeling dependence with t-copula for $m = 5$ at 5% significance level for the non-robust test.*

		$m = 5$	$m = 10$	$m = 15$	$m = 20$	$m = 30$
t-distribution	$Q_k(m)$	1.8923	4.9683	7.1327	9.3807	19.0364
	p-value	0.8638	0.8933	0.9539	0.9781	0.9392
	$Q_k^r(m)$	117.9122	251.0490	371.3950	519.748	764.4626
	p-value	0.6608	0.4694	0.5429	0.2619	0.3489
normal distribution	$Q_k(m)$	2.6596	5.1806	10.9645	15.7037	28.9405
	p-value	0.7523	0.8788	0.7551	0.7348	0.5207
	$Q_k^r(m)$	123.6943	245.9011	348.5009	491.0212	705.8559
	p-value	0.5162	0.5614	0.8332	0.6043	0.8740

Table A.T5: *Multivariate ARCH test on the standardised residuals after modeling dependence with C-vine copulas. There is no evidence of ARCH effect or conditional heteroscedasticity.*

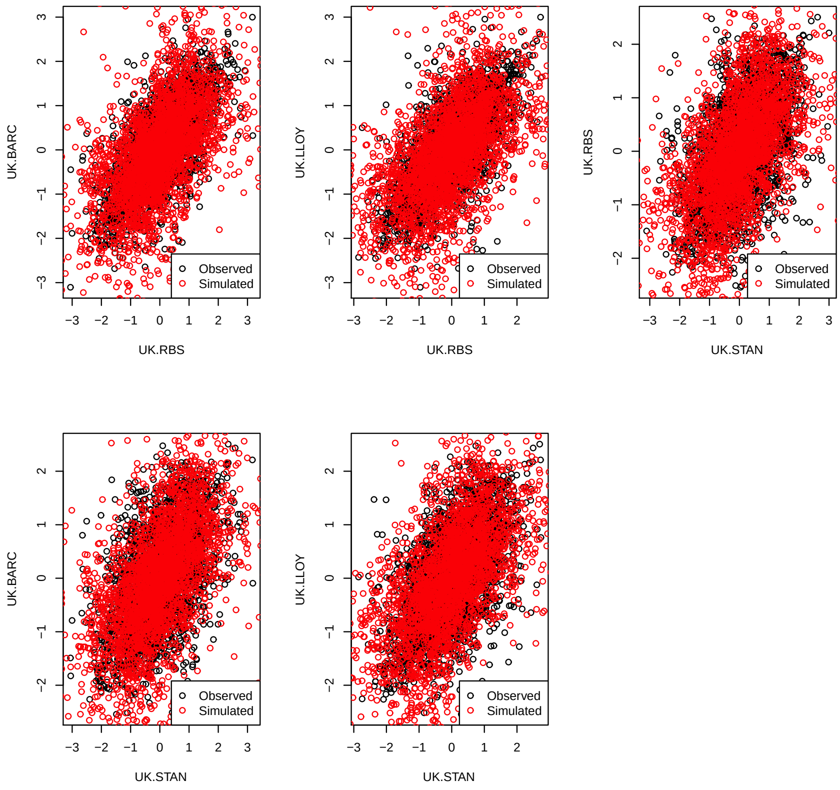


(a)

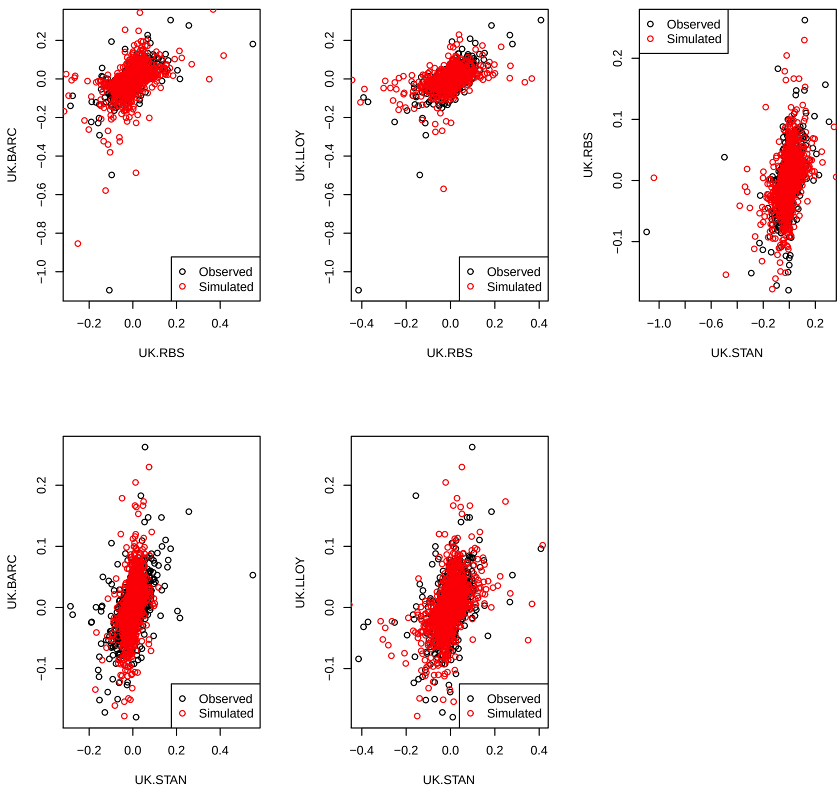


(b)

Figure A.F1: *Scatter plots of simulated standardised residuals; Figure 8.1(a), and the new return distribution; Figure 8.1(b), following Frank copula with t-marginals, plotted together with the original standardised residuals and original return distribution between banks. As can be seen, the Frank copula produce results that captures the extreme observations.*

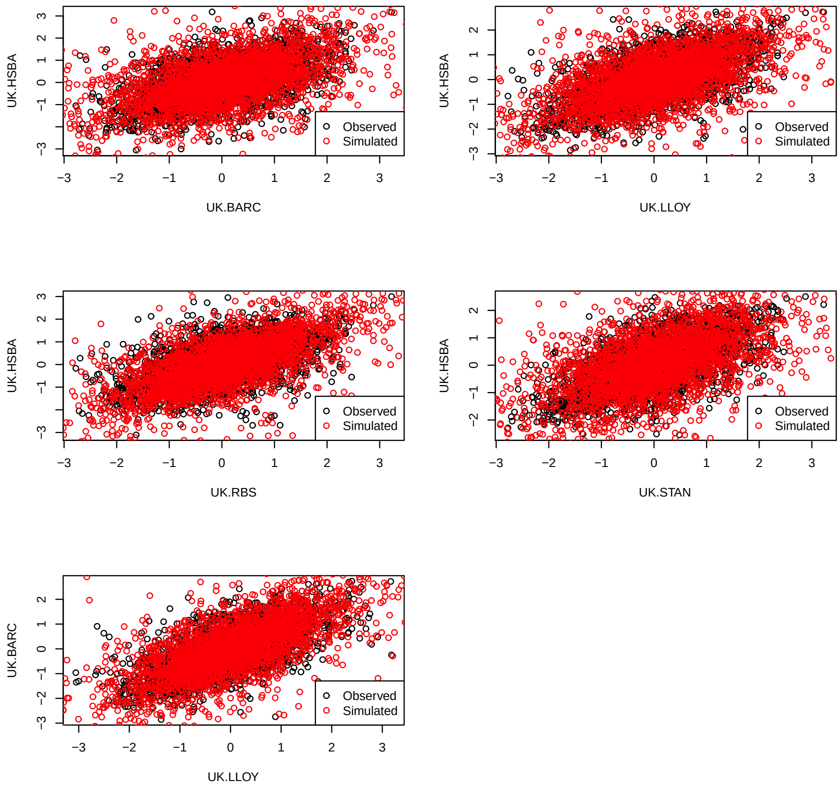


(a)

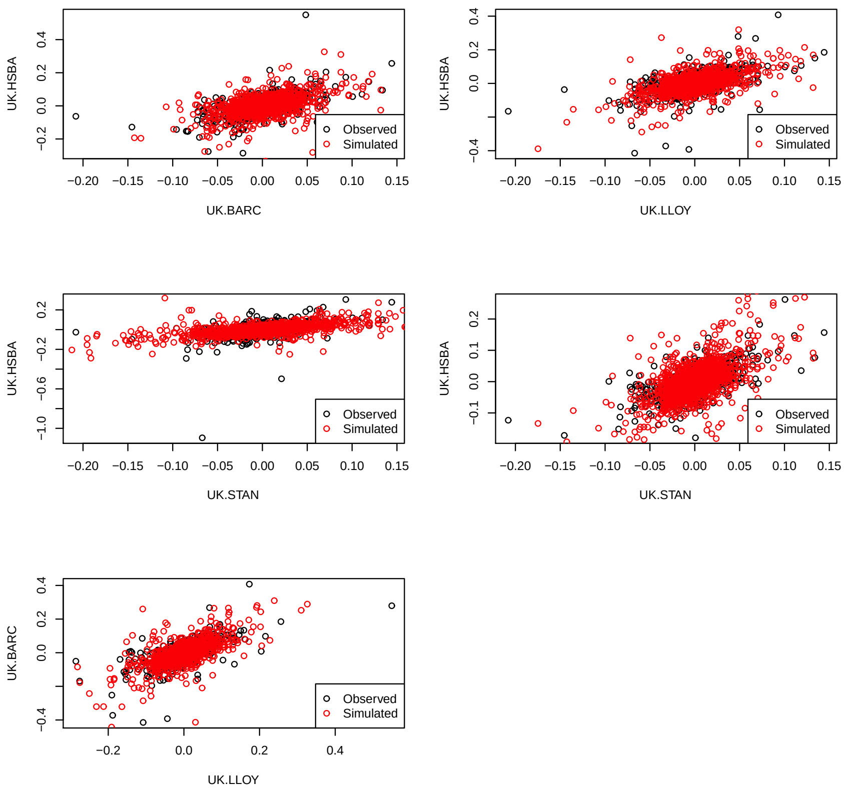


(b)

Figure A.F2: *see Figure A.F1.*

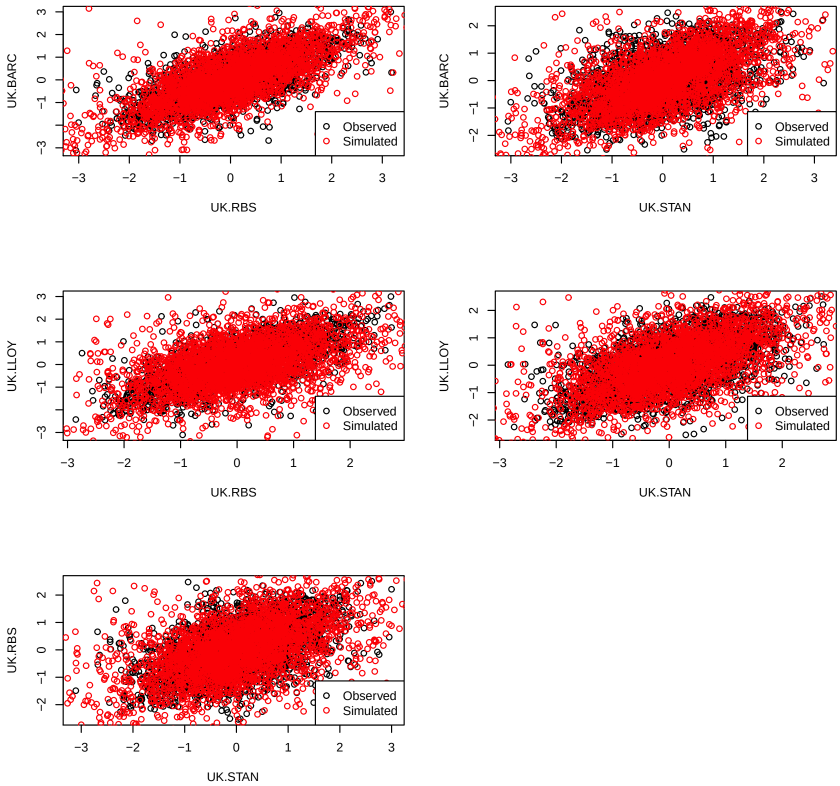


(a)

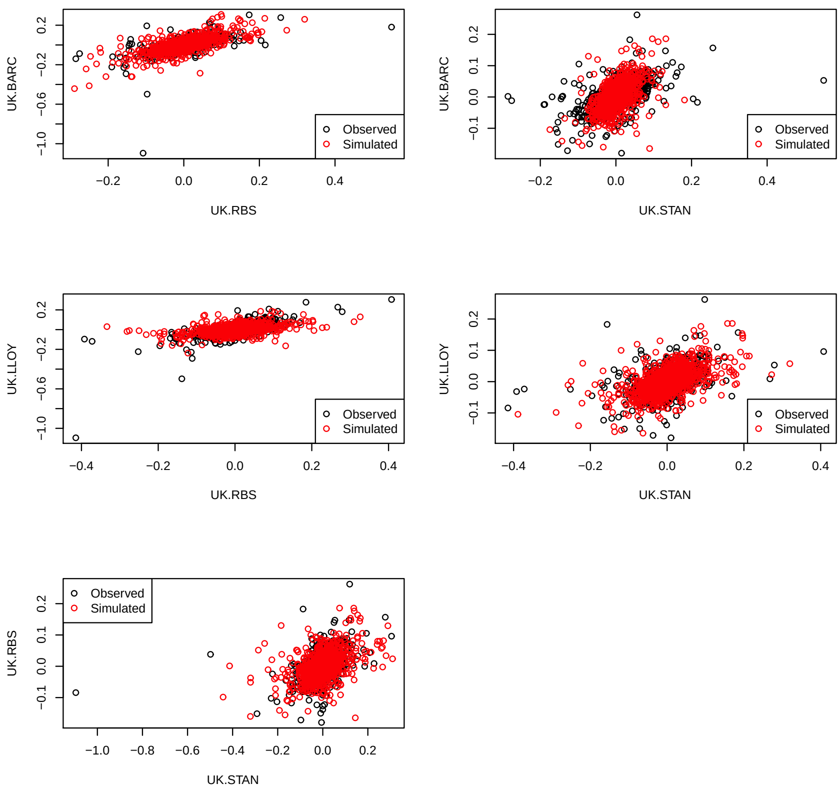


(b)

Figure A.F3: *Scatter plots of simulated standardised residuals; Figure 8.3(a) and the new return distribution; Figure 8.3(b), following t-copula with t-marginals, plotted together with the original standardised residuals and original return distribution between banks the various banks. As can be seen, the t-copula produce results that captures the extreme observations.*



(a)



(b)

Figure A.F4: *See Figure A.F3.*

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