# Labour market participation and frictional unemployment in stochastic equilibria

A thesis submitted for the degree of Doctor of Philosophy in Economics

by

Federico Sciacca

Department of Economics

University of Essex

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**Summary**

This thesis explores the role of inactivity in shaping unemployment fluctuations in frictional labour markets. In the First Chapter, I document several facts on the behaviour of inactive individuals in the United Kingdom using the Labour Force Survey, observing a high degree of heterogeneity within those that are not classified as being part of the labour force population. I analyse the behaviour of marginally attached individuals and those who do not desire to work and their role in explaining labour market fluctuations. Then, I use the results found in the First Chapter as a motivation for the rest of this thesis. In the Second Chapter, I consider a search and matching model where vacancies behave as a stock variable in the spirit of Coles and Moghaddasi (2017). Here, I include an exogenous participation margin and assume marginally attached search with a non-zero job finding probability. I calibrate and evaluate the performance of the model in generating the behaviour of unemployment and vacancies observed in the data for the UK. Finally, the Third Chapter introduces an endogenous participation decision: in every period non-employed individuals decide whether to look for a job or to be inactive according to the state of the economy. I numerically test how modelling the search choice affects the behaviour of individuals when the economy is hit by productivity and separation shocks.

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# Chapter 1

# Inactivity in the UK: Evidence from the Labour Force Survey

## 1.1 Introduction

Flows between employment and unemployment have a key role in the understanding of labour market dynamics. The analysis of the contribution of the job finding and separation rates to the fluctuations in the unemployment rate started with seminal papers by Clark and Summers (1979) and Blanchard and Diamond (1990), and has been recently revived by Shimer (2012). However, existing literature has neglected the importance of considering movements at the participation margin. Indeed, Elsby, Hobijn and Sahin (2015) show that a third of the variance in the US unemployment rate can be explained by movements between unemployment and inactivity. While the US has been at the centre of this line of research, recent studies have also started to analyse the UK labour market (Gomes, 2012 and Razzu and Singleton, 2016).

This Chapter aims at addressing the impact of inactivity on the unemployment fluctuations in the UK, as in Gomes (2012), but it also tries to investigate the heterogeneity in the labour force attachment of individuals who are classified as neither employed nor unemployed.

Flinn and Heckman (1983), by analysing employment hazard rates in the US, test whether unemployment and inactivity are two behaviourally distinct labour market states. They find that the two are significantly different as, in line with search theory, being unemployed gives a higher probability of transiting into employment. The availability of survey data gives the opportunity to inspect more thoroughly the composition of the inactivity pool. Using the Canadian Labour Force Survey, Jones and Riddell (1999, 2006) show that, within the inactives, there are certain categories that behave very differently and present transition probabilities similar to those of the unemployed. In particular, they distinguish between nonparticipants and marginally attached, where the latter present higher transition probabilities into employment.

Focusing on the UK labour market by using the Quarterly Labour Force Survey (LFS) for the period 2001-2017, I document some important facts about the role of inactivity. Following Gomes (2012), I first analyse the behaviour of labour market stocks. I observe a high degree of heterogeneity in the inactivity pool, suggesting the need to look closely at its composition and distinguishing between individuals who are seeking or would like to work (the *marginally attached* of Jones and Riddell, 1999) and those who can be defined as strictly nonparticipants. I also present some additional results on the determinants of individual propensity to move into and out of employment, unemployment, marginal attachment and nonparticipation based on probit estimates.

As Gomes (2012) for the UK and Elsby et al. (2015) for the US, I also present results on the cyclical behaviour of transition flows and rates between the labour market stocks. Indeed, I find that movements into and out of the labour market are significant and are able to explain a significant proportion of the variation of the unemployment rate. In particular, this Chapter tries to answer to the following question: within the inactive pool, are the marginally attached the main force behind the fluctuations in the unemployment rate? The variance decomposition analysis discussed below suggests this is the case.

The Chapter proceeds as follows: section 1.2 describes the LFS; section 1.3 provides some preliminary descriptive statistics; section 1.4 shows the determinants of each transition probability through a probit model that focuses on socio-economic characteristics of individuals; section 1.5 describes the cyclical behaviour of the labour market flows and transition probabilities; section 1.6 illustrates the role of labour market flows rate in explaining the variation in the unemployment rate; finally, section 1.7 concludes.

## 1.2 Data

I use data from the UK Quarterly Labour Survey (LFS) for the period 2001Q4-2017Q1[1]. The LFS has a rotating panel structure. Around 60,000 households are interviewed for five consecutive quarters, with 20% of the sample being replaced at each quarter by an incoming rotation group. Thus, in each quarter there are five rotation groups, four of which will be interviewed again in the following quarter. The respondents are asked information about their employment status, economic activity, education, as well as many other household's characteristics.

I consider only individuals up to their retirement age, i.e. females between 16 and 60 years of age and males between 16 and 65 years of age, as the main focus of this Chapter is to analyse labour market flows among working age individuals. Data are weighted in order to obtain estimates that are representative of the population.

I initially distinguish individuals based on their labour market status. In each quarter, those interviewed can be employed, unemployed or inactive. According

[1]LFS data has been retrieved through the UK Data Archive http://www.data-archive.ac.uk/. The quarterly survey starts in 1992. However, there are two one-quarter gaps, one in 1996 and another in 2001. As data are not available for those quarters, I use the sample starting from the last quarter of 2001 in order to have consecutive data points.

to the definition of the LFS, employed workers consist of people aged 16 and over who did one hour or more of paid work per week (as an employee or self-employed), those who had a job that they were temporarily away from, those on government-supported training and employment programmes, and those doing unpaid family work. On the other hand, unemployed are those without a job who have been actively seeking work in the past 4 weeks and are available to start work in the next 2 weeks. This definition also includes those who are out of work but have found a job and are waiting to start it in the next 2 weeks. Finally, inactives are those that do not belong either to the first, or to the second category.

## 1.3 Descriptive statistics

This section presents some descriptive statistics in order to illustrate the composition of labour market stocks. Let $E_t$, $U_t$ and $I_t$, be the number respectively of employed, unemployed and inactive individuals in quarter $t$.

Figure 1.1 shows the evolution of the unemployment, employment and inactivity rates in the United Kingdom during the period 2001-2017. Two well known facts are highlighted here: the unemployment rate is strongly countercyclical, while the employment rate is procyclical. In particular, the unemployment rate increased by almost 3 percentage points during the 2008-2009 recession. The inactivity rate, on the other hand, does not show a clear cyclical pattern: it first decreases at the onset of the recession, while rising at the end of it. Overall, however, it is possible to notice a downward trend in the inactivity rate.

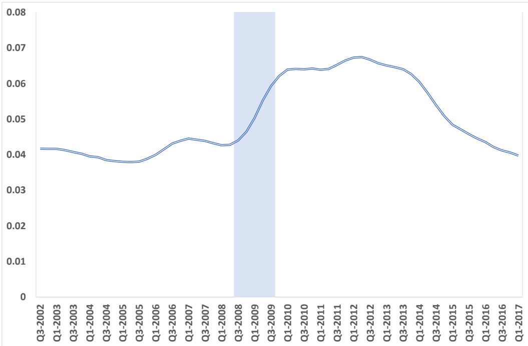
Table 1.1 also shows the employment, unemployment and inactivity rates as a percentage of the working-age population. When considering heterogeneity by sex, Table 1.1 indicates that males have a stronger labour force attachment than females as, on average, more than a quarter of women is out of the labour force.

Table 1.1: Labour market stocks

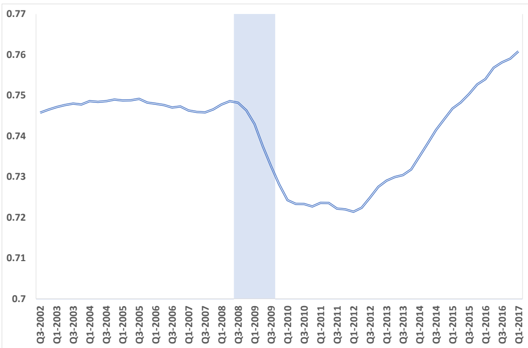
| | Total | Men | Women |
|---|---|---|---|
| Employed | 74.16 | 78.19 | 69.84 |
| Unemployed | 4.98 | 5.51 | 4.40 |
| Inactives | 20.86 | 16.30 | 25.76 |

*Note:* Average quarterly worker stocks, Labour Force Survey, 2001-2017. The stocks are cross-sectional averages expressed as a percentage of the working age population.

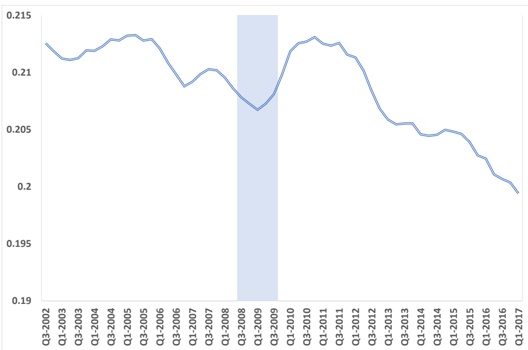
Figure 1.1: UK labour market stocks




(a) Unemployment rate




(b) Employment rate




(c) Inactivity rate

*Note:* The stock series are computed as percentages of working age population, and are four-quarter moving averages to remove seasonality and high frequency movements. Recession bars indicate UK recession dates defined by the ONS. Source: Labour Force Survey, 2001-2017.

Table 1.2 shows the composition of each employment status according to sex, age and level of education. 'Low' education represents all individuals with educational attainment below O-levels or GCSE; 'Medium' corresponds to those that achieved between a O-level or GCSE qualification to an A-level or equivalent; 'High' represents those who attained a post-school degree qualification. The result that women look less attached to the labour force is confirmed by Table 1.2. Indeed, in the UK, women represent almost 60% of the inactive population. As regards the age pattern, it is possible to observe a reversed U-shape in the employment that mirrors the U-shape in inactivity. Unemployment, on the other hand, falls with age. As expected, employed individuals are on average more educated.

Inactive individuals represent more than 20% of the total working age population, as previously highlighted in Table 1.1. Thus, it is important to further investigate who these individuals are and explore their characteristics. The LFS allows to break down the reasons for inactivity into three main groups: people who are searching but are not currently available for work; people who are not searching but would like to work; and people who are not searching and would not like to work. Each group can then be partitioned into several subgroups according to the reason behind their inactivity choice. Table 1.3 shows that there is a high degree of heterogeneity among inactive individuals. More than 70% of inactives are individuals that are not seeking, nor would like to work. These are mainly students, individuals looking after family or home, and long-term sick. However, there is a significant proportion (25%) of inactive individuals that would like to work if offered the opportunity. There are no significant differences in labour force attachment by sex, even if, on average, a higher proportion of women tend to look after their family or home, compared to men.

Table 1.2: Composition of labour market stocks

| | **Employed** | **Unemployed** | **Inactive** |
|---|---|---|---|
| *Sex:* | | | |
| Women | 45.35 | 42.60 | 59.48 |
| Men | 54.65 | 57.40 | 40.52 |
| *Age class:* | | | |
| 16-19 | 4.64 | 19.63 | 17.80 |
| 20-24 | 9.70 | 19.74 | 11.87 |
| 25-29 | 11.54 | 12.07 | 8.21 |
| 30-34 | 12.03 | 9.11 | 8.28 |
| 35-39 | 12.55 | 8.49 | 8.43 |
| 40-44 | 13.12 | 8.50 | 8.01 |
| 45-49 | 12.76 | 7.63 | 7.64 |
| 50-54 | 11.41 | 6.67 | 8.74 |
| 55-59 | 9.12 | 5.98 | 12.54 |
| 60-64 | 3.12 | 2.19 | 8.46 |
| *Education* | | | |
| Low | 17.39 | 30.69 | 37.48 |
| Middle | 40.53 | 45.93 | 43.29 |
| High | 42.08 | 23.38 | 19.23 |

*Note:* For each labour market status (i.e. column) the percentage by sex, age group and education level is displayed. Source: Labour Force Survey, 2001-2017.

Table 1.3: Inactivity by reason

| | Total | Men | Women |
|---|---|---|---|
| **Seeking, not available:** | | | |
| Student | 1.32 | 1.55 | 1.16 |
| Looking after family, home | 0.68 | 0.19 | 1.02 |
| Temporarily sick or injured | 0.16 | 0.23 | 0.11 |
| Long term sick or disabled | 0.14 | 0.23 | 0.07 |
| Other or no reason | 0.85 | 1.02 | 0.74 |
| *Total* | *3.15* | *3.22* | *3.10* |
| **Not seeking, would like to work:** | | | |
| Waiting results of job application | 0.13 | 0.16 | 0.10 |
| Student | 4.08 | 5.19 | 3.32 |
| Looking after family, home | 7.25 | 2.28 | 10.64 |
| Temporarily sick or injured | 1.12 | 1.46 | 0.88 |
| Long term sick or disabled | 8.32 | 12.15 | 5.72 |
| Believes no jobs available | 0.44 | 0.66 | 0.28 |
| Not started looking yet | 0.84 | 0.95 | 0.76 |
| Other or no reason | 2.22 | 2.78 | 1.84 |
| *Total* | *24.40* | *25.63* | *23.56* |
| **Not seeking, would not like to work:** | | | |
| Waiting results of job application | 0.06 | 0.08 | 0.05 |
| Student | 19.83 | 24.44 | 16.69 |
| Looking after family, home | 21.73 | 4.25 | 33.64 |
| Temporarily sick or injured | 0.94 | 1.03 | 0.88 |
| Long term sick or disabled | 17.60 | 23.16 | 13.82 |
| Does not need or want employment | 2.09 | 1.94 | 2.20 |
| Retired from paid work | 7.05 | 12.97 | 3.02 |
| Other or no reason | 3.14 | 3.24 | 3.05 |
| *Total* | *72.45* | *71.16* | *73.34* |

*Note:* For each column, individuals are classified according to their seeking behaviour, as a percentage of the total number of inactives. Source: Labour Force Survey, 2001-2017.

The results emerged from Table 1.3 suggest that there are different 'types' of inactives, depending on their seeking behaviour. A possible determinant of individuals' search intensity could be their age. Figure 1.2 looks at the behaviour of individuals that are out of the labour force by age. The U-shape behaviour of age suggested from Table 1.2 is clearly depicted here: inactive individuals are mostly young, with age below 25, or old, aged above 50. Figure 1.2c shows that the number of inactive women is more stable across age groups with respect to that of men. In particular, middle aged women have an inactivity rate that is more than double the one of males. On the other hand, the proportion of seeking individuals is monotonically decreasing with age, both for men and women. Thus, those who are seeking are generally young individuals.

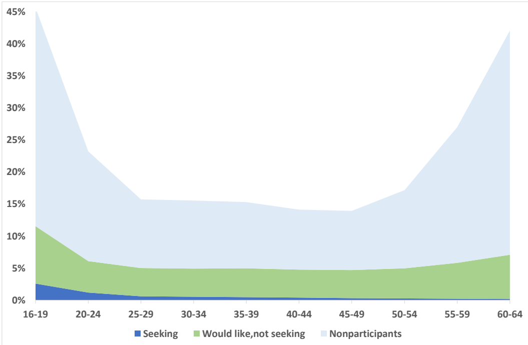
Figure 1.2 shows that there is a significant proportion of young individuals who are inactive. Thus, the next step is to investigate what is the proportion of students among individuals who are below 30 years old, by sex and reason for inactivity. Table 1.4 shows the corresponding statistics. As expected, the proportion of inactive students decreases with the age group, both for men and women and for all the categories considered. Of particular interest is that, while for the age group 16-19 there are no significant differences between males and females, from group 20-24 on, the proportion of female students drops more rapidly than the one of males. For example, for inactives that are not searching and would not like to work, and have age between 25 and 29 years old, the percentage of male students is 41% while the proportion of females is three times less. This result provides a possible explanation for why the U-shape in Figure 1.2 is less pronounced for female than for males: inactive young males are mainly students, who enter the labour force immediately after obtaining their degree, while inactive women who have not obtained their degree before 24 years old remain in the inactivity pool.

Table 1.4: Percentage of students by age class

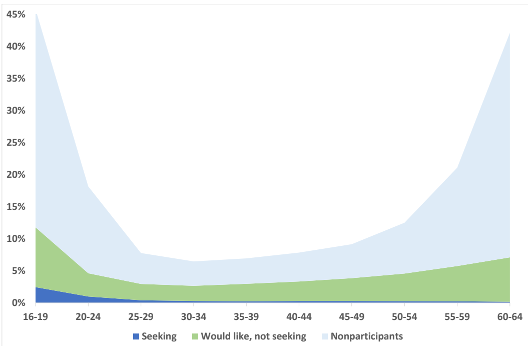
| | **Age Class** | | |
|---|---|---|---|
| | *16-19* | *20-24* | *25-29* |
| *Seeking, not available:* | 84.10 | 53.12 | 21.35 |
| Male | 83.23 | 61.43 | 32.40 |
| Female | 84.90 | 47.12 | 15.89 |
| *Not seeking, would like to work:* | 79.46 | 29.43 | 9.68 |
| Male | 81.81 | 41.72 | 15.51 |
| Female | 76.82 | 22.19 | 7.35 |
| *Not seeking, would not like to work:* | 91.10 | 60.00 | 19.63 |
| Male | 93.12 | 77.57 | 40.89 |
| Female | 89.02 | 48.56 | 13.56 |

*Note:* Each number represents the percentage of students for each age class, sex and seeking behaviour category. Source: Labour Force Survey, 2001-2017.

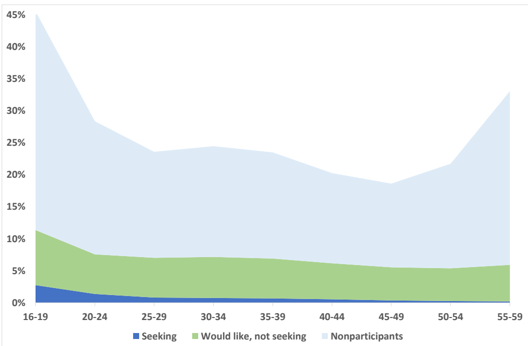
Figure 1.2: Inactivity by age




(a) Total




(b) Males




(c) Females

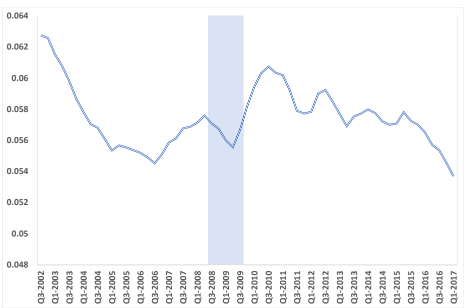
*Note:* Percentage of inactive individuals by age, decomposed by seeking behaviour. The blue area represents those seeking, but not available; the green one is for those not seeking, but who would like to work; the grey area represents those not seeking and not willing to work. Source: Labour Force Survey, 2001-2017.

As observed in this section, the inactivity group is heterogenous, ranging from those with a strong labour force attachment to those with no attachment at all. Thus, following Jones and Riddell (1999, 2006), I define as *marginally attached* ($M$) those inactive individuals who are seeking or would like to work, while *non-participants* ($N$) are those who are not seeking nor they desire to work. Therefore, in each quarter in the labour market, an individual can be in one of these four states: $E_t$, $U_t$, $M_t$ or $N_t$.

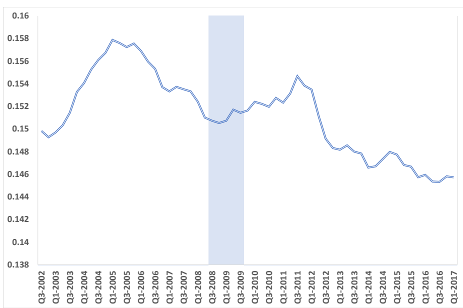
Figure 1.3 shows the behaviour of the stock of marginally attached individuals and nonparticipants for the time period considered here. Marginally attached individuals represent a smaller proportion of the total working age population. They account for less than one third of the total inactive pool, on average, as shown before in Table 1.3, and are about 6% of the total population, on average. The cyclical behaviour of these two time series is different: while marginally attachment presents some procyclicality, the nonparticipation, on the other hand, looks more acyclical and depicts a clear downward trend.

Based on these preliminary findings, next section investigates possible reasons behind movements between labour market states.

Figure 1.3: Labour market stocks: composition of inactive individuals



(a) Marginally attached



(b) Nonparticipants

*Note:* The stock series are computed as percentages of working age population, and are four-quarter moving averages to remove seasonality and high frequency movements. Marginally attached are those inactives who are seeking or would like to work; nonparticipants are those inactives who are neither seeking nor willing to work. Recession bars indicate UK recession dates defined by the ONS. Source: Labour Force Survey, 2001-2017.

## 1.4 Who is moving? Probit analysis

In this section I explore what are the determinants of moving from one labour market state to another, focusing in particular on the inactivity state. In doing so, I present probit estimates derived from a model where the dependent variable is whether or not the individual moves from a state to the other. For example, the dependent variable of column (1) in Table 1.5 is $UE$, which is equal to 1 when the individual transit from unemployment to employment, and 0 if the individual remains in the unemployment pool. The explanatory variables include a set of individual characteristics, such as age, sex, marital status, number of children and education. As regards marital status, individuals in the LFS can be generally classified into six main categories: (1) single, never married; (2) married, living with spouse; (3) married, separated from spouse; (4) divorced; (5) widowed and (6) currently or previously in civil partnership. Unfortunately, I cannot distinguish between currently or previously in civil partnership[2]. Thus, I define the explanatory variable *married* that takes value 1 if a respondent has marital status corresponding to category (2), and zero otherwise.

As in Carrillo-Tudela, Hobijn, She and Visschers (2016), I classify education into three groups: (1) high education; (2) middle education and (3) low education. Individuals belonging to the first group have post school degrees. Those in group (2) have a qualification ranging from O-level or GCSE to A-level or equivalent. Finally, low educated respondents achieved a qualification below O-level or GCSE.

There are also some variables that are specific to the origin market state the individual belongs to before moving. For example, for unemployed individuals I can observe both the duration of unemployment and methods of searching (the reference category for the seeking variable is going to a job centre). As regards the duration of unemployment, the LFS provides the following categorical variable:

[2]This information is available only for 2015Q4.

(1) less than 3 months; (2) $\geqslant$ 3 months, less that 6; (3) $\geqslant$ 6 months, less than 12; (4) $\geqslant$ 1 year, less than 2; (5) $\geqslant$ 2 years, less than 3; (6) $\geqslant$ 3 years, less than 4; (7) $\geqslant$ 4 years, less than 5 and (8) 5 years or more. Unemployed report how they are currently looking for a job. I define five broad search channels following Carrillo-Tudela et al. (2016): (1) job centre; (2) ads; (3) direct application; (4) ask friends or relatives (5) other channels. To the first category belong all workers who declared to "visit a job centre, job market or jobs and benefit centres", "visit a job club", "have your name on the books of a private employment agency", or "visit a careers office". Category (2) includes workers who "advertise for jobs in newspapers and journals", who "answer advertisements in newspapers and journals", or "study situations vacant in newspapers or journals". Workers belonging to (3) "apply directly to employers". Workers in category (4) are those who "ask friends, relatives, colleagues or trade unions about jobs". Finally, "other channels" encompasses all other methods of search.

For employed individuals, I can observe the duration of employment and whether the individual is working full-time. In particular, the employment duration is grouped in 8 categories: (1) less than 3 months; (2) $\geqslant$ 3 months, less that 6; (3) $\geqslant$ 6 months, less than 12; (4) $\geqslant$ 1 year, less than 2; (5) $\geqslant$ 2 years, less than 5; (6) $\geqslant$ 5 years, less than 10; (7) $\geqslant$ 10 years, less than 20 and (8) 20 years or more.

For inactive individuals, I define the variable 'Want a job', that takes value 1 if the individual is seeking but is not available, or not seeking but would like to work. Thus, this variable takes value 1 if the individual is a marginally attached worker and 0 if he is nonparticipant. Finally, I also include the unemployment rate as a proxy for the business cycle.

Table 1.5 shows estimates of the probit specification outlined above. A 1% rise in the unemployment rate is associated with a reduction in the probability of transiting from unemployment to employment by 2.09%. Similarly, in a recession also the probability of moving from employment to inactivity strongly decreases. On the other hand, as expected, periods with a higher unemployment rate are associated with an increase in the probability of moving from E to U. Interestingly, I can observe that it is more likely to move to unemployment if inactive following the increase in the unemployment rate. In addition, during a recession the probability of movements between employment and inactivity dampens in both directions, with a stronger effect in IE movements.

As regards the effect of individual characteristics on the transition probabilities, I find that age is generally associated with a reduction in the probability of moving between market states. Moreover, individuals with a higher education are more likely to move into employment, regardless of their origin state. Column (6) shows also that inactive individuals are more likely to move to unemployment if highly educated. A possible explanation is that those individuals are mainly students who just obtained their degree and are starting to look more actively for a job. As already established in the literature, the longer the unemployment duration, the lower the probability of finding a job. However, also the probability of moving into inactivity decreases. On the other hand, the longer the duration of employment, the lower the probability of leaving that state. Finally, a marginally attached is more likely to leave the inactivity state compared to an individual that does not want a job.

Table 1.5: Probit analysis: U-E-I

| | (1)<br>UE | (2)<br>UI | (3)<br>EU | (4)<br>EI | (5)<br>IE | (6)<br>IU |
|---|---|---|---|---|---|---|
| Unemployment rate | -2.091*** | -1.825*** | 0.077*** | -0.031*** | -0.453*** | 0.257*** |
| | (0.132) | (0.126) | (0.006) | (0.007) | (0.026) | (0.023) |
| Age | 0.001 | -0.018*** | -0.001*** | -0.003*** | -0.006*** | -0.002*** |
| | (0.001) | (0.001) | (0.000) | (0.000) | (0.000) | (0.000) |
| Age$^2$ | -0.045*** | 0.235*** | 0.009*** | 0.041*** | 0.043*** | 0.001*** |
| | (0.012) | (0.011) | (0.000) | (0.001) | (0.002) | (0.000) |
| Married | 0.060*** | 0.051*** | -0.004*** | -0.000 | 0.012*** | -0.009*** |
| | (0.005) | (0.005) | (0.000) | (0.000) | (0.001) | (0.000) |
| Number of children | -0.021*** | 0.020*** | -0.000 | 0.000*** | -0.005*** | -0.000 |
| | (0.002) | (0.002) | (0.000) | (0.000) | (0.000) | (0.000) |
| Female | 0.010*** | 0.109*** | -0.004*** | 0.003*** | -0.001 | -0.010*** |
| | (0.004) | (0.004) | (0.000) | (0.000) | (0.000) | (0.000) |
| High education | 0.123*** | -0.014** | -0.004*** | -0.002*** | 0.068*** | 0.021*** |
| | (0.006) | (0.05) | (0.000) | (0.000) | (0.002) | (0.001) |
| Middle education | 0.068*** | 0.019*** | -0.002*** | -0.001*** | 0.029*** | 0.010*** |
| | (0.004) | (0.004) | (0.000) | (0.000) | (0.001) | (0.001) |
| Duration unemp | -0.059*** | -0.014*** | | | | |
| | (0.001) | (0.001) | | | | |
| Seek: ads | 0.023*** | 0.036*** | | | | |
| | (0.005) | (0.004) | | | | |
| Seek: direct application | 0.063*** | 0.116*** | | | | |
| | (0.008) | (0.008) | | | | |
| Seek: ask friends/relatives | 0.052*** | 0.135*** | | | | |
| | (0.009) | (0.009) | | | | |
| Seek: other channels | 0.106*** | 0.154*** | | | | |
| | (0.010) | (0.011) | | | | |
| Duration emp | | | -0.004*** | -0.002*** | | |
| | | | (0.000) | (0.000) | | |
| Full-time | | | -0.003*** | -0.018*** | | |
| | | | (0.000) | (0.000) | | |
| Want a job | | | | | 0.029*** | 0.070*** |
| | | | | | (0.001) | (0.001) |
| $N$ | 74,666 | 66,586 | 1,592,340 | 1,599,877 | 436,757 | 436,515 |
| Region Dummies | Yes | Yes | Yes | Yes | Yes | Yes |
| pseudo-$R^2$ | 0.084 | 0.064 | 0.094 | 0.102 | 0.095 | 0.109 |

*Note*: Marginal effects from a probit specification. The dependent variable in each column takes value 1 if the individual moves between the two market states, and 0 if he remains in the same state. For example, in column (1), UE=1 if the individual moves from unemployment to employment, while UE=0 if the individual remains unemployed. Standard errors are reported in parentheses. *, **, *** indicates statistical significance at the 10, 5, and 1 % levels, respectively. Source: Labour Force Survey, 2001-2017.

More interesting for the main purpose of this Chapter are the probit models presented in Tables 1.6 and 1.7. In Table 1.6 the dependent variable $UM$ is equal to 1 if the individual is unemployed in quarter $t-1$ and marginally attached in quarter $t$, but it is equal to 0 if the individual is again unemployed in quarter $t-1$ and nonparticipant in quarter $t$. In this way, I am able to understand what are the determinants for an unemployed individual who moves into incactivity of choosing between marginal attachment and nonparticipation. The same holds for $EM$. Estimates show that, during a recession, both unemployed and employed individuals are more likely to switch to marginal attachment than nonparticipation, if they become inactive. Overall, by comparing column (1) with (2), it is possible to see that the two regressions present similar results. The effect of age is U-shaped: when age increases, individuals prefer to move to the marginal attachment group compared to the nonparticipation. However, after a certain level of age, they prefer the nonparticipation state, probably due to retirement decisions. Women have a higher probability of entering into nonparticipation with respect to males. The longer the duration of unemployment, the lower the probability of becoming nonparticipant, capturing the attachment to the labour market due to having spent time searching for a job as an unemployed individual. The longer the duration of employment, on the other hand, the higher the probability of becoming nonparticipant, again probably due to retirement decisions.

On the other hand, Table 1.7 focuses only on those individuals who remain in the inactivity pool for two consecutive quarters. For example, the dependent variable in column (1) is $MN$, which is equal to 1 if the individual moves from $M$ to $N$ and to 0 if he remains marginally attached. As expected, the marginal effects present opposite signs mirroring each other: for example, following a recession, a nonparticipant is more likely to become marginally attached, but the opposite is not true. When age increases, individuals are more likely either to move from nonparticipation to marginal attachment, or to stay in the marginal attachment state, if they were already there in the previous quarter. The opposite holds when age increases after a certain threshold: the individual either moves to nonparticipation from marginal attachment, or remains nonparticipant.

Table 1.6: Probit analysis: U-E-M

| | (1) UM | (2) EM |
|---|---|---|
| Unemployment rate | 0.687** | 1.204*** |
| | (0.306) | (0.265) |
| Age | 0.012*** | 0.025*** |
| | (0.002) | (0.002) |
| Age$^2$ | -0.142*** | -0.313*** |
| | (0.025) | (0.021) |
| Married | -0.042*** | -0.075*** |
| | (0.011) | (0.009) |
| Number of children | 0.013*** | 0.016*** |
| | (0.004) | (0.003) |
| Female | -0.046*** | -0.071*** |
| | (0.009) | (0.008) |
| High education | -0.010 | -0.110*** |
| | (0.013) | (0.009) |
| Middle education | 0.016 | -0.056*** |
| | (0.010) | (0.009) |
| Duration unemp | 0.017*** | |
| | (0.003) | |
| Seek: ads | 0.033*** | |
| | (0.011) | |
| Seek: direct application | 0.033** | |
| | (0.015) | |
| Seek: ask friends/relatives | 0.047*** | |
| | (0.016) | |
| Seek: other reasons | -0.013 | |
| | (0.019) | |
| Duration emp | | -0.020*** |
| | | (0.002) |
| Full-time | | 0.047*** |
| | | (0.008) |
| $N$ | 15,546 | 24,766 |
| Region Dummies | Yes | Yes |
| pseudo-$R^2$ | 0.016 | 0.030 |

*Note*: Marginal effects from a probit specification. The dependent variable in each column takes value 1 if the individual moves between the two market states, and 0 if he moves into inactivity. For example, in column (1), UM=1 if the individual moves from unemployment to marginal attachment, while UM=0 if the individual moves from unemployment to inactivity. Standard errors are reported in parentheses. *, **, *** indicates statistical significance at the 10, 5, and 1 % levels, respectively. Source: Labour Force Survey, 2001-2017.

Table 1.7: Probit analysis: M-N

| | (1) MN | (2) NM |
|---|---|---|
| Unemployment rate | -0.385*** | 0.358*** |
| | (0.111) | (0.048) |
| Age | -0.015*** | 0.005*** |
| | (0.001) | (0.000) |
| Age$^2$ | 0.190*** | -0.088*** |
| | (0.009) | (0.004) |
| Married | 0.063*** | -0.036*** |
| | (0.004) | (0.002) |
| Number of children | -0.001 | 0.003*** |
| | (0.001) | (0.001) |
| Female | 0.054*** | -0.014*** |
| | (0.003) | (0.002) |
| High education | 0.023*** | -0.005*** |
| | (0.005) | (0.002) |
| Middle education | 0.011*** | 0.014*** |
| | (0.003) | (0.001) |
| $N$ | 100,072 | 308,147 |
| Region Dummies | Yes | Yes |
| pseudo-$R^2$ | 0.019 | 0.034 |

*Note*: Marginal effects from a probit specification. The dependent variable in each column takes value 1 if the individual moves between the two market states, and 0 if he remains in the same state. For example, in column (1), MN=1 if the individual moves from marginal attachment to inactivity, while MN=0 if the individual remains marginally attached. Standard errors are reported in parentheses. *, **, *** indicates statistical significance at the 10, 5, and 1 % levels, respectively. Source: Labour Force Survey, 2001-2017.

## 1.5 Labour market flows and transition probabilities

The rotating panel structure of the LFS allows to compute worker gross flows, i.e. the number of individuals who transits from a labour market state to another in a given quarter. In every quarter, for around 80% of the interviewed, the LFS provides information on the labour market status in the current quarter and in the following one. Thus, I can, for example, calculate the marginal attachment to unemployment gross flows, defined as $MU_t$, that is the number of workers who are inactive and marginally attached in quarter $t-1$ and unemployed in quarter $t$. Then, from these gross flows, it is straightforward to estimate the associated transition probability, i.e. the probability that an individual transits from a state to another in a given quarter. For example, the probability that a marginally attached inactive moves into unemployment is $p_{MU_t} = \frac{MU_t}{M_{t-1}}$.

Figure 1.4 displays the evolution of the flows between one labour market state and another as defined in section 1.3 in the UK for the period 2001-2017. Figure 1.4a shows that $UM$ and $MU$ flows present a similar pattern. In addition, both series increase during a recession and they almost overlap during the entire time period considered here. The same joint behaviour is not depicted in Figure 1.4b for the flows between unemployment and nonparticipation. The number of nonparticipants flowing into unemployment is always larger than the inflow $UN$. In addition, Figure 1.4d also shows that the largest flows in every quarter are those happening in the inactivity pool (the magnitudes of $MN$ and $NM$ are both very large). Moreover, there is a net outflow of nonparticipants towards the marginal attachment group. The fact that individuals seem to flow out of nonparticipation, both towards marginal attachment and towards unemployment could possibly explain the downward trend in nonparticipation observed in Figure 1.3b. Finally, Figures 1.4e and 1.4f depict movements between the inactivity pool and

Table 1.8: Transition probability matrix

| | $E_t$ | $U_t$ | $M_t$ | $N_t$ |
|---|---|---|---|---|
| $E_{t-1}$ | 0.9675 | 0.0135 | 0.0060 | 0.0130 |
| $U_{t-1}$ | 0.2560 | 0.5667 | 0.1064 | 0.0710 |
| $M_{t-1}$ | 0.0783 | 0.1123 | 0.5722 | 0.2371 |
| $N_{t-1}$ | 0.0478 | 0.0321 | 0.0973 | 0.8228 |

*Note:* Transition probabilities between labour market states from $t-1$ to $t$. The original flow series are four-quarter moving averages. Source: Labour Force Survey, 2001-2017.

employment that are quantitatively not negligible: when summing up the nonparticipation with the marginal attachment, the resulting flows seem quantitatively as large as those between employment and unemployment. Both $ME$ and $NE$ flows show a procyclical pattern.

Table 1.8 is a transition matrix between the four labour market groups described above. The main diagonal shows significant persistence, i.e. individuals tend to stay in the same labour market state for two consecutive quarters. This is particularly evident for employed workers[3]. Also nonparticipation shows a high degree of persistence. However, both unemployed and marginally attached individual have less propensity to stay in their category, compared to $E$ and $N$ groups. The quarterly job finding rate for unemployed individuals is 25%, while for marginally attached is almost 8%. The marginally attached individuals are twice as likely to join the labour force, and in particular almost four times more likely to become unemployed than nonparticipants. Around 11% of unemployed workers move into marginal attachment, while only 7% move into nonparticipation. At the same time, in every quarter it is more likely for a marginally attached worker to enter into unemployment than to directly find a job. Finally, there are not negligible movements also between the two inactivity groups: about 24% of marginally attached individuals decide to lower their search intensity and move to the nonparticipation group in every quarter.

Section 1.3 has shown that there is considerable heterogeneity inside the inactivity

[3]This analysis does not distinguish between an individual who stays in the same job or experiences job-to-job reallocations as it would be beyond the scope of this Chapter.

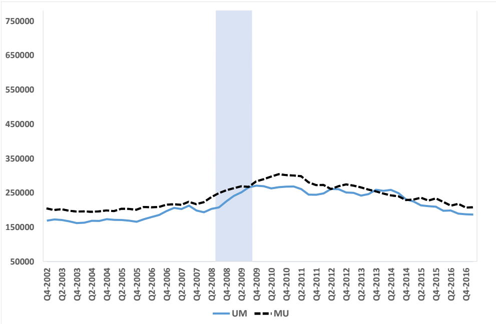
state. Based on this evidence, I compute the transition probabilities towards employment and unemployment only for each sub-category of inactivity presented in Table 1.3. From Table 1.9 it is possible to observe that individuals who are seeking, but not available, present a probability of finding a job that is very similar to that of the unemployed (Table 1.8 shows an $UE$ probability of almost 26%). Thus, according to this decomposition, the seeking but not available category could potentially be included in the unemployment pool, instead of belonging to the inactivity one, a claim also proposed by Moffat and Yoo (2015). In addition, inside the category "Not seeking, would like to work", there are also those who are waiting for the result of their job applications. They present a job finding rate that is even higher than that observed for unemployed workers. On average, marginally attached individuals tend to present a higher probability of moving into unemployment than employment. The opposite is true for those who are not seeking, nor willing to work (i.e. the nonparticipants): if they enter into the labour force, it is more likely that they do so without undergoing any unemployment spell. Nevertheless, the probability of moving into the labour force is always lower for nonparticipants than for marginally attached individuals.

Table 1.9: Transition rates, by reason

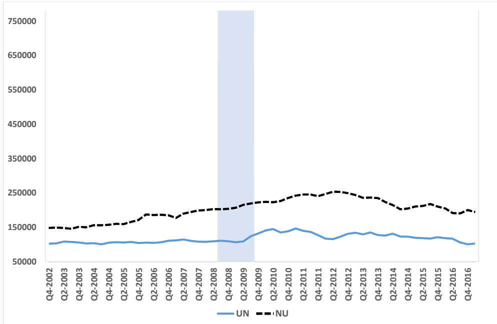
| Transition from | Transition to | |
|---|---|---|
| | Employment | Unemployment |
| **Seeking, not available:** | | |
| Student | 0.246 | 0.237 |
| Looking after family, home | 0.138 | 0.238 |
| Temporarily sick or injured | 0.128 | 0.282 |
| Long term sick or disabled | 0.066 | 0.184 |
| Other or no reason | 0.301 | 0.300 |
| *Total* | *0.224* | *0.254* |
| **Not seeking, would like to work:** | | |
| Waiting results of job application | 0.284 | 0.264 |
| Student | 0.132 | 0.159 |
| Looking after family, home | 0.043 | 0.078 |
| Temporarily sick or injured | 0.071 | 0.149 |
| Long term sick or disabled | 0.014 | 0.029 |
| Believes no jobs available | 0.052 | 0.186 |
| Not started looking yet | 0.154 | 0.209 |
| Other or no reason | 0.115 | 0.131 |
| *Total* | *0.061* | *0.090* |
| **Not seeking, would not like to work:** | | |
| Waiting results of job application | 0.166 | 0.238 |
| Student | 0.115 | 0.073 |
| Looking after family, home | 0.030 | 0.024 |
| Temporarily sick or injured | 0.057 | 0.071 |
| Long term sick or disabled | 0.007 | 0.009 |
| Does not need or want employment | 0.047 | 0.015 |
| Retired from paid work | 0.024 | 0.006 |
| Other or no reason | 0.185 | 0.081 |
| *Total* | *0.054* | *0.035* |

*Note:* Transition probabilities between inactivity and employment/unemployment from $t-1$ to $t$. Inactivity is divided by search behaviour. The original flow series are four-quarter moving averages. Source: Labour Force Survey, 2001-2017.

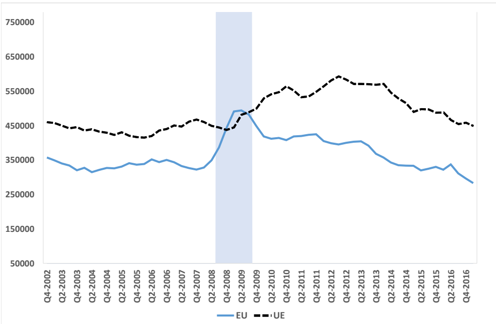
Figure 1.4: Worker Flows



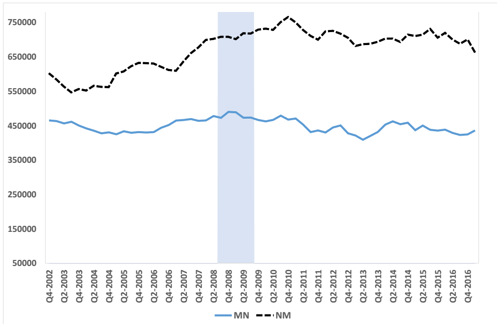

(a) UM-MU



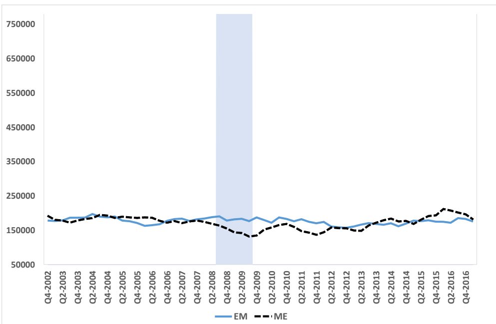

(b) UN-NU



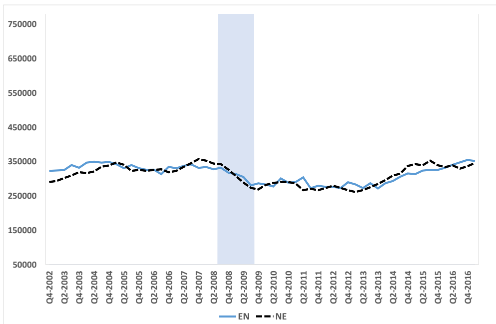

(c) UE-EU




(d) MN-NM




(e) EM-ME



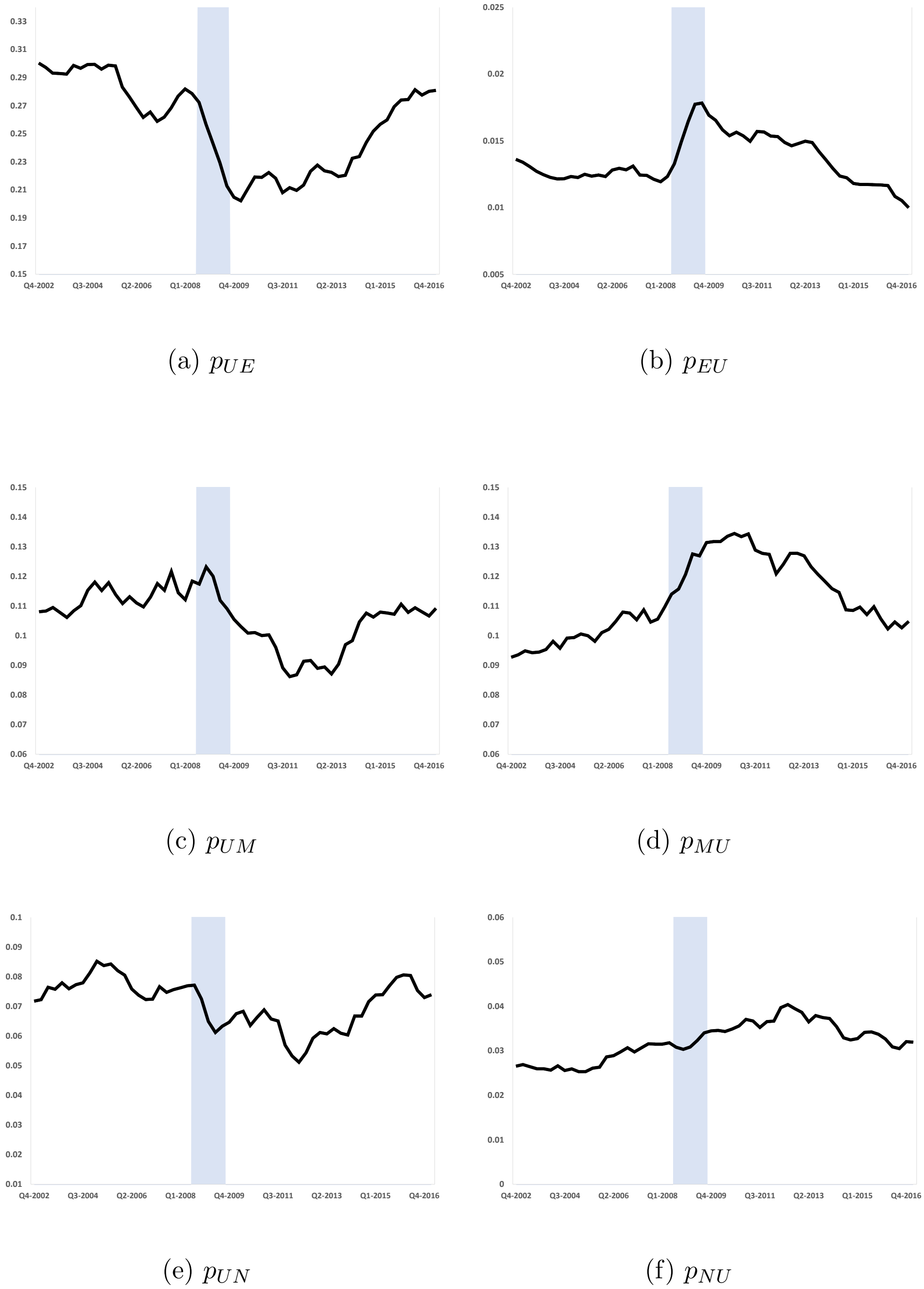

(f) EN-NE

*Note:* The flow series are four-quarter moving averages to remove seasonality and high frequency movements. Recession bars indicate UK recession dates defined by the ONS. Source: Labour Force Survey, 2001-2017.

### 1.5.1 Cyclical fluctuations

Figures 1.5 and 1.6 show the cyclical behaviour of the transition probabilities $p_{i,j_t}$, for $i, j \in \{E, U, M, N\}$. Following Elsby et al. (2015), I implement for all the plotted series a correction for margin of error that restricts the estimates of individuals' transition rates to be consistent with the evolution of the corresponding labour market stocks shown in Figures 1.1 and 1.3. There are clear regularities in the behaviour of these transition rates over the business cycle in the UK. Among these, we can observe the strong countercyclicality of the employment-to-unemployment probability and the procyclicality of the unemployment-to-employment rate. Recent emphasis has been given to the role of inactivity. However, generally inactivity is considered as a single, homogeneous group. Here I distinguish between marginally attached and nonparticipants. The transition probabilities $UM$ and $MU$ present both a cyclical pattern, that contributes to the cyclicality of the unemployment rate. Indeed, $UM$ is procyclical, thus reducing the unemployment during a recession, while $MU$ is countercyclical, increasing the unemployment rate during recessionary periods. I don't observe the same clear pattern between unemployment and nonparticipation. Figure 1.6 show that the job finding rates of marginally attached individuals and nonparticipants are both highly procyclical. Movements between the two inactivity states do not seem to show any clear cyclicality.

Figure 1.5: Transition probabilities (1)




(a) $p_{UE}$

(b) $p_{EU}$

(c) $p_{UM}$

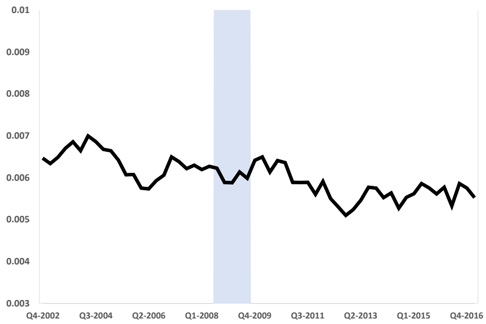
(d) $p_{MU}$

(e) $p_{UN}$

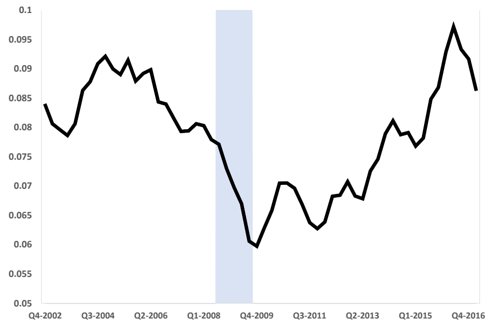
(f) $p_{NU}$

*Note:* Quarterly flow transition probabilities corrected for margin of error. Recession bars indicate UK recession dates defined by the ONS. Source: Labour Force Survey, 2001-2017.

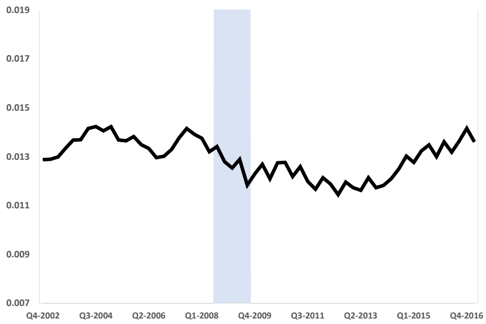
Figure 1.6: Transition probabilities (2)



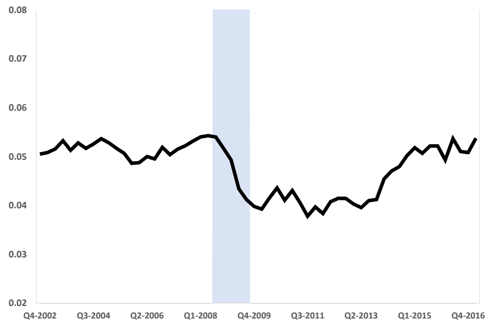

(a) $p_{EM}$



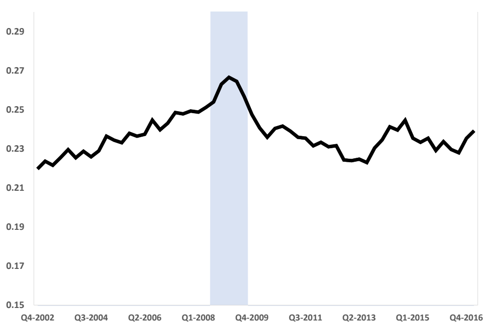

(b) $p_{ME}$



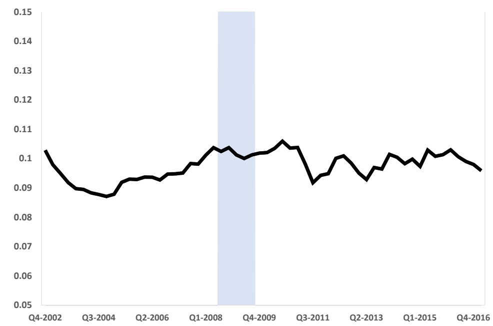

(c) $p_{EN}$




(d) $p_{NE}$




(e) $p_{MN}$




(f) $p_{NM}$

*Note:* Quarterly flow transition probabilities corrected for margin of error. Recession bars indicate UK recession dates defined by the ONS. Source: Labour Force Survey, 2001-2017.

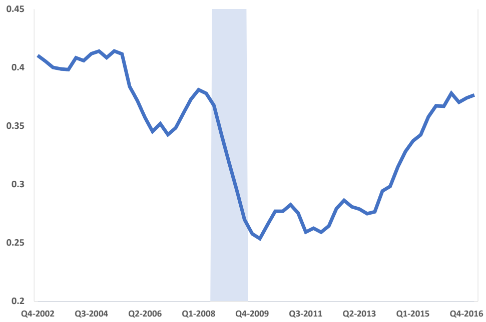
The LFS, as all survey data, is subject to measurement error. A first issue is the classification error: interviewed individuals may systematically report to belong to the wrong labour market status, leading to spurious measured transitions. An example is an individual who is unemployed for three consecutive quarters, but is recorded as marginally attached in the second quarter. This leads to two wrongly recorded transitions, one from unemployment to marginal attachment and the other one back to unemployment in the following quarter. This problem is particularly evident in transitions between unemployment and inactivity. For the US there is the possibility to correct for this type of error, using re-interviews from a sub-sample of the CPS. This correction, originally provided by Abowd and Zellner (1985), shows that almost 10% of those who are recorded as unemployed were in fact initially mistakenly classified as inactive. Unfortunately, the LFS does not have a sub-sample of re-interviewed individuals as the CPS, thus it is not possible to apply the correction as in Abowd and Zellner (1985). Therefore, the analysis hereby presented does not use any type of correction for classification error. Elsby et al. (2015) perform an additional correction, as a robustness check for the US: they recode the unemployment-inactivity flows that appear within three or four months. For example, every $IUI$ flow is directly treated as a $III$, calling this flow 'deNUNified'. They find that both the Abowd and Zellner (1985) correction and the 'deNUNified' approach do not have a significant impact on the cyclicality of the time series considered[4].

Another important bias is due to time aggregation. Data are recorded at quarterly frequencies. However, an individual can make several transitions between consecutive interviews. For example, consider a person that is recorded as nonparticipant in a quarter and employed in the following one. I record this movement as a $NE$ transition. However, there could have been several movements between the two surveys, that are not recorded. For example, this individual could have moved initially to marginal attachment and/or to unemployment, before ending up in

[4]A possible extension could be the implementation of the 'deNUNified' approach of Elsby et al. (2015) to the LFS.

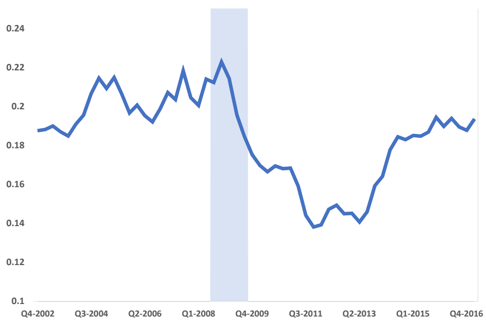
the employment pool. I apply the correction provided by Shimer (2012). The instantaneous flow hazard rates $f_{ij}$ is the continuous time equivalent of each discrete transition probability $p_{ij}$. Figures 1.7 and 1.8 present the implied transition probabilities over the time period here considered, when both time aggregation and margin of error corrections have been applied. We observe that even if the magnitude of the implied probabilities is affected by this correction, the series show the same cyclicality observed in Figures 1.5 and 1.6.

Figure 1.7: Transition probabilities, time aggregated (1)



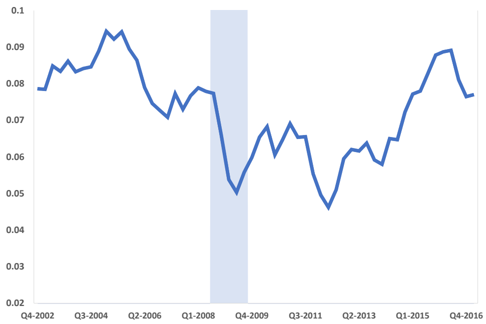

(a) $f_{UE}$


(b) $f_{EU}$

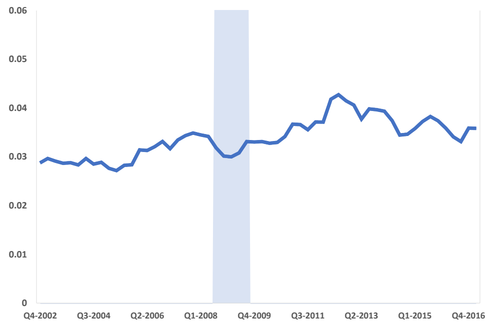



(c) $f_{UM}$


(d) $f_{MU}$




(e) $f_{UN}$



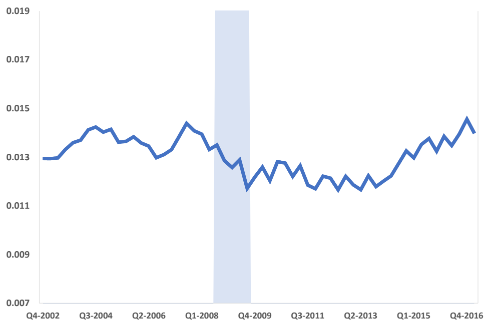

(f) $f_{NU}$

*Note:* Implied quarterly flow transition probabilities corrected for margin of error and time aggregation. Recession bars indicate UK recession dates defined by the ONS. Source: Labour Force Survey, 2001-2017.

Figure 1.8: Transition probabilities, time aggregated (2)

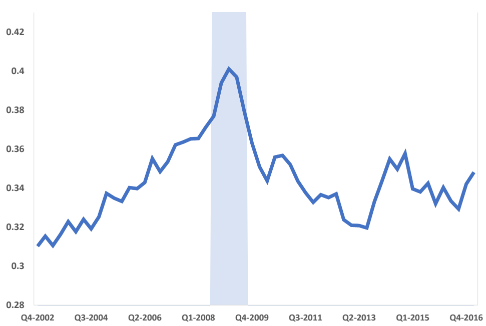

(a) $f_{EM}$


(b) $f_{ME}$




(c) $f_{EN}$


(d) $f_{NE}$




(e) $f_{MN}$


(f) $f_{NM}$

*Note:* Implied quarterly flow transition probabilities corrected for margin of error and time aggregation. Recession bars indicate UK recession dates defined by the ONS. Source: Labour Force Survey, 2001-2017.

## 1.6 Determinants of unemployment fluctuations

The previous sections have explored flows between market states and highlighted the importance of movements into and out of the labour force. This section aims at analysing whether these movements explain the volatility of the unemployment rate. In doing so, I perform a similar exercise as in Elsby et al. (2015). Using CPS monthly data, they decompose the variance of each labour market stock. The aim of their decomposition is to identify the share of the variance of the unemployment rate accounted for by each flow hazard $f_{ij}$. Differently from Elsby et al. (2015), I consider a four-state environment: $E_t$, $U_t$, $M_t$ and $N_t$ are the shares of the population of, respectively, employed, unemployed, marginally attached and nonparticipants in period $t$[5]. The following system describes the evolution of these state variables:

$$\begin{aligned} E_t &= (1-p_{EU}-p_{EM}-p_{EN})E_{t-1}+p_{UE}U_{t-1}+p_{ME}M_{t-1}+p_{NE}N_{t-1} \\ U_t &= p_{EU}E_{t-1}+(1-p_{UE}-p_{UM}-p_{UN})U_{t-1}+p_{MU}M_{t-1}+p_{NU}N_{t-1} \\ M_t &= p_{EM}E_{t-1}+p_{UM}U_{t-1}+(1-p_{ME}-p_{MU}-p_{MN})M_{t-1}+p_{NM}N_{t-1} \\ N_t &= p_{EN}E_{t-1}+p_{UN}U_{t-1}+p_{MN}M_{t-1}+(1-p_{NE}-p_{NU}-p_{NM})N_{t-1}. \end{aligned} \tag{1.1}$$

As $E_t+U_t+M_t+N_t=1$, the Markov process governing the labour market system in (1.1) can be written as a reduced three-dimensional system of the form:

$$\begin{bmatrix} E \\ U \\ M \end{bmatrix}_t = \begin{bmatrix} 1-p_{EU}-p_{EM}-p_{EN}-p_{NE} & p_{UE}-p_{NE} & p_{ME}-p_{NE} \\ p_{EU}-p_{NU} & 1-p_{UE}-p_{UM}-p_{UN}-p_{NU} & p_{MU}-p_{NU} \\ p_{EM}-p_{NM} & p_{UM}-p_{NM} & 1-p_{ME}-p_{MU}-p_{MN}-p_{NM} \end{bmatrix}_t \begin{bmatrix} E \\ U \\ M \end{bmatrix}_{t-1} + \begin{bmatrix} p_{NE} \\ p_{NU} \\ p_{NM} \end{bmatrix}_t \tag{1.2}$$

[5]I use the same definition of $M$ and $N$ as above.

that can be written as $\boldsymbol{s}_t = \boldsymbol{P}_t \boldsymbol{s}_{t-1} + \boldsymbol{q}_t$. Thus, the steady state of this system is given by

$$\bar{\boldsymbol{s}}_t = (\boldsymbol{I} - \boldsymbol{P}_t)^{-1} \boldsymbol{q}_t \tag{1.3}$$

Following Elsby et al. (2015), the change in the labour market stock is given by:

$$\boldsymbol{\Delta s}_t = (\boldsymbol{I} - \boldsymbol{P}_t)\boldsymbol{\Delta}\bar{\boldsymbol{s}}_t + (\boldsymbol{I} - \boldsymbol{P}_t)\boldsymbol{P}_{t-1}(\boldsymbol{I} - \boldsymbol{P}_{t-1})^{-1}\boldsymbol{\Delta s}_{t-1}. \tag{1.4}$$

The first term on the RHS of Equation (1.4) captures all the current changes in the transition rates that affect the steady state $\bar{\boldsymbol{s}}_t$. The second term, on the other hand, includes all the past changes in the transition rates that affect the current labour market state. By iterating Equation (1.4) backward, I can write the present change in labour market stocks as a distributed lag function of the change in steady state values and some initial value for the labour market stocks:

$$\boldsymbol{\Delta s}_t = \sum_{k=0}^{t-1} \boldsymbol{C}_{k,t} \boldsymbol{\Delta}\bar{\boldsymbol{s}}_{t-k} + \boldsymbol{D}_t \boldsymbol{\Delta}\bar{\boldsymbol{s}}_0 \tag{1.5}$$

where $\boldsymbol{C}_{k,t} = [\prod_{n=0}^{s-1}(\boldsymbol{I} - \boldsymbol{P}_{t-n})\boldsymbol{P}_{t-n-1}(\boldsymbol{I} - \boldsymbol{P}_{t-n-1})^{-1}](\boldsymbol{I} - \boldsymbol{P}_{t-k})$ and $\boldsymbol{D}_t = \prod_{k=0}^{t-1}(\boldsymbol{I} - \boldsymbol{P}_{t-k})\boldsymbol{P}_{t-k-1}(\boldsymbol{I} - \boldsymbol{P}_{t-k-1})^{-1}$.

When replacing the transition probabilities with the flow hazards, Elsby et al. (2015) take a first-order Taylor approximation to the change in the steady state $\boldsymbol{\Delta}\bar{\boldsymbol{s}}_t$. Basically, the change in the steady state labour market stocks is given by the sum of the changes in each flow hazard rate $\Delta f_{ij_t}$ multiplied by its effect $\frac{\partial \bar{\boldsymbol{s}}_t}{\partial f_{ij_t}}$:

$$\boldsymbol{\Delta}\bar{\boldsymbol{s}}_t \approx \sum_{i \neq j} \frac{\partial \bar{\boldsymbol{s}}_t}{\partial f_{ij_t}} \Delta f_{ij_t}. \tag{1.6}$$

When using flow hazard rates, the flow steady state can be rewritten as $\bar{\boldsymbol{s}}_t =$

$-\boldsymbol{F}_t^{-1}\boldsymbol{g}_t$, where

$$\boldsymbol{F}_t = \begin{bmatrix} -f_{EU} - f_{EM} - f_{EN} - f_{NE} & f_{UE} - f_{NE} & f_{ME} - f_{NE} \\ f_{EU} - f_{NU} & -f_{UE} - f_{UM} - f_{UN} - f_{NU} & f_{MU} - f_{NU} \\ f_{EM} - f_{NM} & f_{UM} - f_{NM} & -f_{ME} - f_{MU} - f_{MN} - f_{NM} \end{bmatrix}_t$$

and

$$\boldsymbol{g}_t = \begin{bmatrix} f_{NE} \\ f_{NU} \\ f_{NM} \end{bmatrix}_t .$$

The resulting variance decomposition is given by:

$$var(\boldsymbol{\Delta s}_t) \approx \sum_{i \neq j} cov(\boldsymbol{\Delta s}_t, \sum_{k=0}^{t-1} \boldsymbol{C}_{k,t} \frac{\partial \bar{\boldsymbol{s}}_{t-k}}{\partial f_{ij_{t-k}}} \Delta f_{ij_{t-k}}) \tag{1.7}$$

Equation (1.7) implies that it is possible to compute the proportion of the variance of the quarterly changes in any labour market stock explained by the variation in each transition hazard rate $f_{ij}$[6]:

$$\beta_{MU}^{U} = \frac{cov(\Delta U_t, [\sum_{k=0}^{t-1} \boldsymbol{C}_{k,t} \frac{\partial \bar{\boldsymbol{s}}_{t-k}}{\partial f_{MU_{t-k}}} \Delta f_{MU_{t-k}}]_{2,3})}{var(\Delta U_t)}. \tag{1.8}$$

Equation (1.8) for example shows the contribution of changes in the flow hazard rate $f_{MU}$ corresponding to a transition from marginal attachment to unemployment, in explaining the variation in the unemployment stock $U_t$.

If one wants to analyse the effect on the unemployment rate instead of the unemployment stock, Elsby et al. (2015) use the following transformation:

$$\Delta u_t \approx (1 - u_{t-1}) \frac{\Delta U_t}{E_{t-1} + U_{t-1}} - u_{t-1} \frac{\Delta E_t}{E_{t-1} + U_{t-1}}, \tag{1.9}$$

where $u_t = U_t/(E_t + U_t)$.

Table 1.10 summarises the results for the above decomposition for the UK unem-

[6] I performed the same exercise using also the discrete transition probabilities $p_{ij}$.

ployment rate, in the period 2001-2017, using both flow transition probabilities $p_{ij}$ and flow hazard rates $f_{ij}$. Cyclically, the time aggregation bias tends to lead to a substantial underestimation of the relative importance of flows from unemployment, while the reverse flows are overestimated. In both cases, we observe that movements between employment and unemployment account for two thirds of total variation in the unemployment rate. This result is similar to the one obtained by Elsby et al. (2015) for the US with a three-state decomposition. As expected, movements between unemployment and marginal attachment are able to explain more of the variation in the unemployment rate than movements between unemployment and nonparticipation. This is particularly important because the marginal attachment pool represents only one third of the total inactivity group. Thus, it is important to distinguish between the two categories in order to have a better understanding of the determinants of the fluctuations in the unemployment rate.

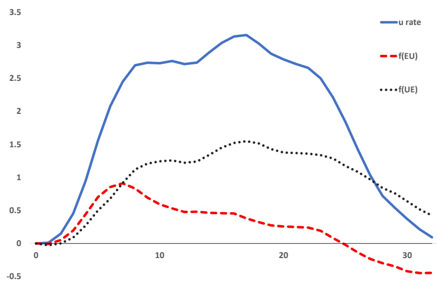
Table 1.10: Variance Decomposition

| | $p_{ij}$ | $f_{ij}$ |
|---|---|---|
| Share of Variance | | |
| EU | 0.36 | 0.29 |
| UE | 0.31 | 0.40 |
| UM | 0.07 | 0.13 |
| MU | 0.08 | 0.03 |
| UN | 0.06 | 0.06 |
| NU | 0.05 | 0.01 |
| EM | 0.00 | 0.00 |
| ME | 0.03 | 0.04 |
| EN | -0.01 | -0.01 |
| NE | 0.01 | 0.01 |
| MN | 0.00 | 0.01 |
| NM | 0.01 | 0.01 |
| Residual | 0.03 | 0.04 |
| | | |
| Total Between: | | |
| U and E | 0.67 | 0.69 |
| U and M | 0.15 | 0.16 |
| U and N | 0.11 | 0.07 |
| E and M | 0.03 | 0.04 |
| E and N | 0.00 | 0.00 |
| M and N | 0.01 | 0.02 |

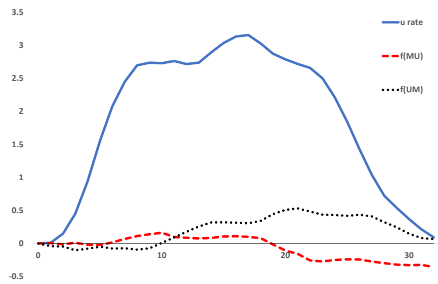
*Note:* Variance decomposition of the change in quarterly average unemployment rate. Both discrete probabilities $p_{ij}$ and flow hazard rates $f_{ij}$ are presented. "Total Between U and E" indicates, for example, the share of variance of the sum of EU and UE. Source: Labour Force Survey, 2001-2017.

Using the stock decomposition, I can then focus on how the evolution of the unemployment rate between the 2008 recession and 2015 was determined by changes in the hazard rates. Figure 1.9 gives the cumulative contribution of changes in each of the hazard rates to the percentage point change in the unemployment rate. It is possible to observe that the role of flows between employment and unemployment is the main determinant of the spike in the unemployment rate during the last recession, as expected. However, there is a significant role explained by the marginally attachment movements into and out of unemployment.

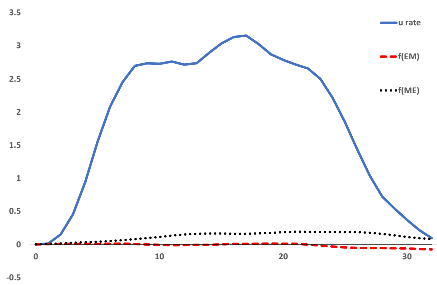
Figure 1.9: Contributions of the hazard rates to the unemployment



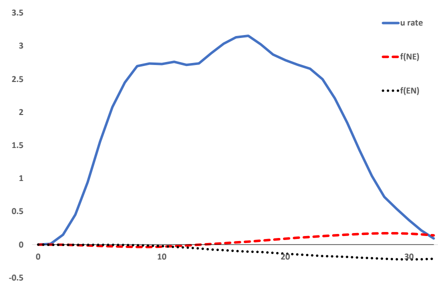

(a) $f_{EU}$ and $f_{UE}$



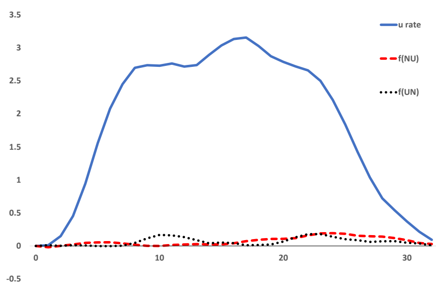

(b) $f_{MU}$ and $f_{UM}$



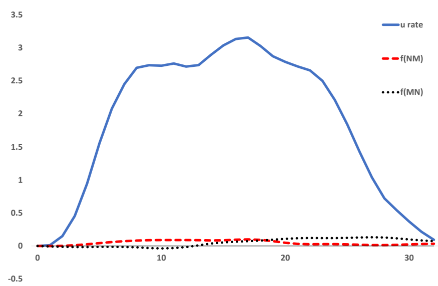

(c) $f_{EM}$ and $f_{ME}$




(d) $f_{EN}$ and $f_{NE}$




(e) $f_{NU}$ and $f_{UN}$




(f) $f_{MN}$ and $f_{NM}$

*Note:* Cumulative percentage point contributions from changes in hazard rates to the unemployment rate change. Points on the horizontal axis represent the number of quarters from 2008Q1 (point 0). Source: Labour Force Survey, 2008-2015.

### 1.6.1 Variance decomposition with a broader definition of unemployment

In the previous section, I have presented a variance decomposition analysis in a labour market where individuals can be classified in four different market states. As Table 1.9 shows, there are categories of individuals belonging to the marginal attachment pool that behave similarly to unemployed individuals as they present similar transition probabilities towards employment. As suggested by Jones and Riddell (1999) using data for Canada, some inactives can be classified as unemployed.

I perform a similar exercise as in the previous section, but I decide to *merge* unemployed workers and marginally attached in a single state $\tilde{U}_t = U_t + M_t$. Thus, in every period $t$, there is a three-states environment where $E_t + \tilde{U}_t + N_t = 1$. Following the same structure as above, Table 1.11 presents the variance decomposition of unemployment showing that more than 90% of it is due to movements occurring between employment and $\tilde{U}$. If I consider this broader definition, movements into and out of the labour force do not play a major role in explaining variation in the unemployment.

Figure 1.10 describes the contribution of movements between employment and unemployment to the increase in unemployment observed during the Great Recession. In particular, it is possible to observe from Figure 1.10b that the sum of the movements between unemployment and employment mirrors exactly the increase in unemployment rate. Thus, nonparticipation do not have an important role in explaining variation in the unemployment rate. As the previous section showed, this is not true for the marginally attached.

Table 1.11: Variance Decomposition: a broader definition of unemployment rate

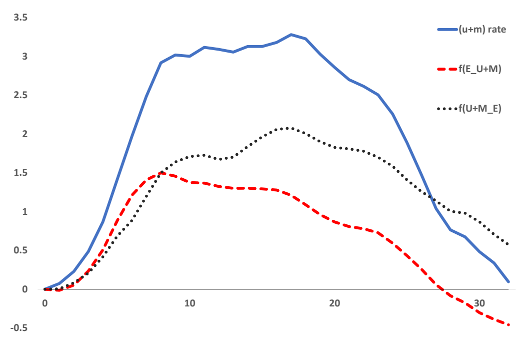
| | $f_{ij}$ |
|---|---|
| Share of Variance | |
| $E\tilde{U}$ | 0.45 |
| $\tilde{U}E$ | 0.47 |
| $\tilde{U}N$ | 0.03 |
| $N\tilde{U}$ | 0.05 |
| $EN$ | -0.05 |
| $NE$ | 0.02 |
| Residual | 0.03 |
| Total Between: | |
| $\tilde{U}$ and $E$ | 0.92 |
| $\tilde{U}$ and $N$ | 0.08 |
| $E$ and $N$ | -0.03 |

*Note:* Variance decomposition of the change in quarterly average unemployment rate, where unemployment is calculated as the sum of $U$ and $M$. Only flow hazard rates $f_{ij}$ are presented. "Total Between $\tilde{U}$ and $E$" indicates, for example, the share of variance of the sum of $E\tilde{U}$ and $\tilde{U}E$. Source: Labour Force Survey, 2001-2017.

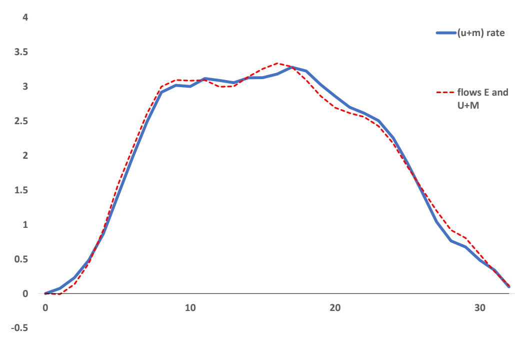
## 1.7 Conclusions

This Chapter describes some key facts and statistics about the UK labour market using the Labour Force Survey for the period 2001-2017. In doing so, it provides information about the composition of the labour market stocks, focusing in particular on the heterogeneity observed in the inactivity pool. As I find different level of labour market attachment, I distinguish the inactives between *marginally attached* and *nonparticipants*. After some preliminary descriptive statistics, I investigate further the determinants of labour market transition probabilities by using a probit model. Results suggest that there is a different effect of individual characteristics on the transition to and from nonparticipation and marginal attachment. Then, I described the cyclical behaviour of the gross flows and the transition probabilities between employment, unemployment, marginal attachment and nonparticipation, for the period 2001-2017. Finally, I performed a vari-

Figure 1.10: Contributions of the hazard rates to a broader definition of unemployment




(a) $f_{E\tilde{U}}$ and $f_{\tilde{U}E}$




(b) $f_{E\tilde{U}} + f_{\tilde{U}E}$

*Note:* Cumulative percentage point contributions from changes in hazard rates to the unemployment rate change. Points on the horizontal axis represent the number of quarters from 2008Q1 (point 0). Source: Labour Force Survey, 2008-2015.

ance decomposition of the unemployment rate, aimed at understanding the role that movements into and out of the four labour market states have on changes in the unemployment rate. In particular, while flows between employment and unemployment explain two thirds of the variation in unemployment, I find that movements between unemployment and marginal attachment are important and should not be neglected. This analysis provides the motivation for the introduction of an inactivity state in a search and matching framework that will be explained in the next Chapter.

# Chapter 2

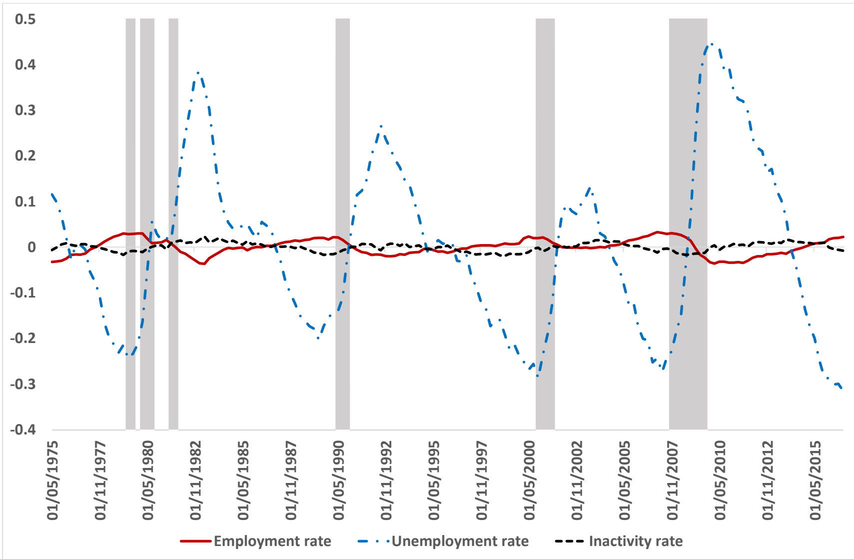
# Inactivity in a Search and Matching Model with Diamond Entry

## 2.1 Introduction and Literature Review

One of the central objectives of macroeconomic research is to understand the forces that shape the business cycle fluctuations in the labour market. Much of the recent work on labor markets is based on the matching model of Mortensen and Pissarides (1994) and emphasizes the role of fluctuations in job finding and job loss rates as the driving forces behind movements in employment and unemployment.

Existing literature generally considers all agents in the economy as part of the labour force. Indeed, these studies assume workers can be either employed or unemployed. This assumption is justified by the fact that inactivity does not need to be included due to its acyclicality over the business cycle, as pointed out by Shimer (2005) and Hall and Milgrom (2008). Figure 2.1 shows that in the US, for the period 1975-2016, unemployment is the most volatile series and it

Figure 2.1: Employment, Unemployment and Inactivity Volatilities: US 1976-2016




Note: All series are quarterly averages of the seasonally adjusted monthly series constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). They are reported in logs as deviations from an HP trend with smoothing parameter $10^5$. Recession bars indicate US recession dates defined by NBER.

is strongly countercyclical, while employment is procyclical. On the other hand, inactivity does not have a clear pattern over the cycle[1].

Nonetheless, recent empirical evidence shows that movements into and out of the labor force are quantitatively as important as those between employment and unemployment in the US labor market. The rotating panel structure of the CPS allows to compute worker gross flows, i.e. the number of individuals who transit from a labour market state to another in a given month. For example, I can calculate the unemployment to employment gross flows, defined as $UE_t$, that is the number of workers who are unemployed in month $t-1$ and employment in month $t$. From these gross flows, it is straightforward to estimate the associated

[1]According to the BLS, people are considered employed if they did any work at all for pay or profit during the survey reference week. This includes all part-time and temporary work, as well as regular full-time, year-round employment. Unemployed are those who do not have a job, have actively looked for work in the prior 4 weeks, and are currently available for work. People who are neither employed nor unemployed are not in the labor force.

transition probability, i.e. the probability that an individual transits from a state to another in a given month. Then, the probability that an unemployed worker finds a job is $p_{UE_t} = \frac{UE_t}{U_{t-1}}$.

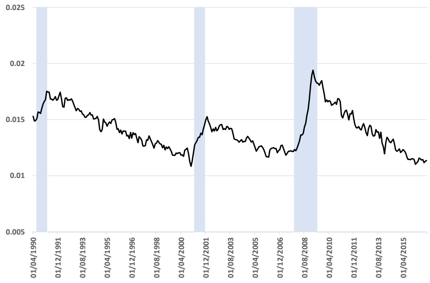
Table 2.1: Labour Market Transition Probabilities: US 1976-2016

| From | To | | |
|---|---|---|---|
| | E | U | I |
| E | 0.958 | 0.014 | 0.027 |
| U | 0.250 | 0.529 | 0.220 |
| I | 0.047 | 0.027 | 0.926 |

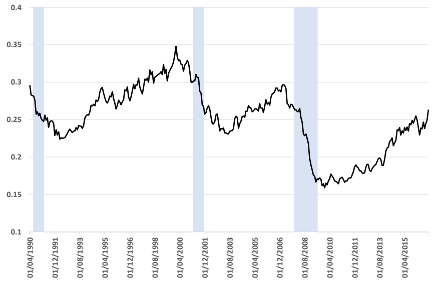
Note: Data compiled from two sources. From January 1976 through January 1990, data on flows are made available from Shimer (2012). From February 1990 on, they are available from the BLS gross flows statistics.

As Table 2.1 suggests, from 1976 to 2016, about 4.7% of inactives move directly to employment in a given month and 2.7% seek work and become unemployed on average. Of particular importance is that, on average, 22% of unemployed workers quit searching in a given month. Given that the number of inactives in the US is almost ten times that of unemployed workers, the effect of individuals moving from inactivity to unemployment is a large source of the fluctuations we observe in the unemployment rate, and it should not be neglected. By constructing transition rates between employment, unemployment and inactivity, it is possible to document the importance of the movements between unemployment and inactivity in accounting for changes in the aggregate unemployment rate as shown in Figure 2.2. In particular, it is possible to observe that the rate at which inactives enter into unemployment increases in downturns, while the rate at which unemployed workers exit the labor force falls in times of recession. Elsby, Hobijn and Sahin (2015) show that fluctuations at the participation margin contribute towards increasing unemployment during a recession.

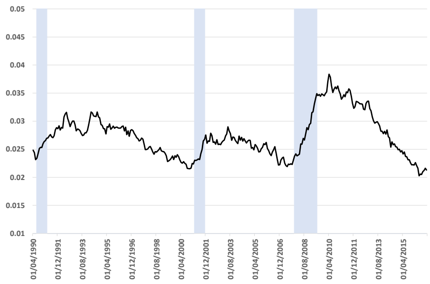
Figure 2.2: Worker flows transition rates



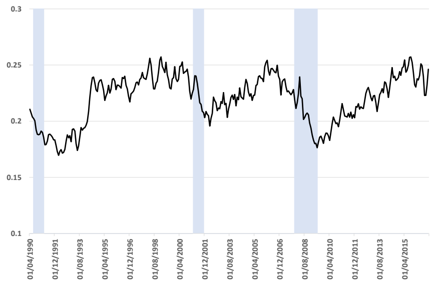

(a) $p_{EU}$



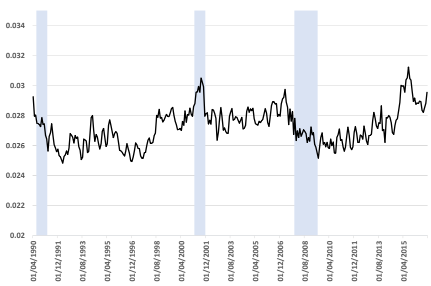

(b) $p_{UE}$



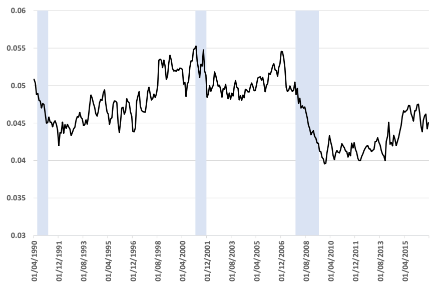

(c) $p_{IU}$




(d) $p_{UI}$




(e) $p_{EI}$




(f) $p_{IE}$

Note: US monthly transition probabilities for the period 1990-2016. Data are from the BLS. Recession bars indicate US recession dates defined by NBER.

Chapter 1 provides an empirical analysis of the UK labour market and in particular of the role of inactivity in explaining unemployment volatility. A major advantage of using the UK Quarterly Labour Force Survey (LFS)[2] is that it allows to distinguish flows by reason for moving. Chapter 1 finds significant heterogeneity outside of the labour force. Thus, I distinguished between *marginally attached* individuals and *nonparticipants*. Following Jones and Riddell (1999, 2006), I defined as marginally attached ($M$) those inactive individuals who are either seeking or would like to work, while nonparticipants ($N$) are those who are not seeking nor they desire to work. Therefore, in each quarter in the labour market, an individual can be in one of these four states: $E$, $U$, $M$ or $N$. Table 2.2 shows that nonparticipants tend to stay within their class significantly more than marginally attached. In particular, marginally attached individuals have a probability of moving into the labour force which is more than twice the one observed for nonparticipants. Chapter 1 also shows that movements into and out of nonparticipation do not have a strong impact on unemployment fluctuations, differently from marginal attachment flows. Indeed, while flows from and to nonparticipation explain just 9% of the variance of the unemployment rate, marginal attachment is able to explain 22%.

[2] A detailed description of the LFS is provided in Chapter 1.

Table 2.2: Labour Market Transition Probabilities: UK 2001-2017

| | $E_t$ | $U_t$ | $M_t$ | $N_t$ |
|---|---|---|---|---|
| $E_{t-1}$ | 0.9675 | 0.0135 | 0.0060 | 0.0130 |
| $U_{t-1}$ | 0.2560 | 0.5667 | 0.1064 | 0.0710 |
| $M_{t-1}$ | 0.0783 | 0.1123 | 0.5722 | 0.2371 |
| $N_{t-1}$ | 0.0478 | 0.0321 | 0.0973 | 0.8228 |

*Note:* Transition probabilities between labour market states from $t-1$ to $t$. The original flow series are four-quarter moving averages. Source: Labour Force Survey, 2001-2017.

In light of this empirical evidence, it seems appropriate to consider a theoretical framework which, based on the differences between marginally attached and nonparticipants, introduces a third state that includes only marginally attached. The choice of exluding nonparticipants form the third state is driven by the absence of large movements from and into that state.

Tripier (2004) considers a Mortensen and Pissarides search and matching model with three states in order to investigate the business cycle properties of the major labor market variables. While his model is able to match the behaviour of employment, it fails in matching the empirical properties of unemployment and labor force participation. In particular, when the economy is subject to aggregate technology shocks, the model fails to generate the observed strong countercyclicality of unemployment and the strong negative relationship between unemployment and vacancies, i.e. the Beveridge curve. Similarly, Veracierto (2008) extends the Lucas and Prescott (1974) islands model by adding an inactive sector with endogenous job acceptance and job separation decisions. By investigating the dynamic properties of the labor market variables, Veracierto (2008) finds, as Tripier (2004), that the model does not perform well when the third state is introduced. Indeed, he finds that the volatility of unemployment is much lower than the one observed in the data. Moreover, unemployment becomes weakly procyclical, while labor force participation becomes strongly procyclical and turns out to be as volatile as employment.

However, existing models that include inactivity do not allow for agents out of the labour force to transit directly into employment. Inactives do not search for employment opportunities and have to pass through a spell of unemployment in order to become employed. In addition, all these models consider the totality of inactive individuals without extracting the component represented by the marginally attached. As shown in Chapter 1, marginally attached are different from nonparticipants and they directly move into employment. Thus, a model which encompasses direct flows from marginal attachment to employment is needed. One exception in the literature is represented by Pries and Rogerson (2009) where, in a partial equilibrium framework, inactives engage in passive search associated with a lower but non-zero job finding probability.

While I introduce this third state, I do not consider the standard Mortensen Pissarides (1994) model. Though the qualitative implications of their framework resemble the features observed in the data, this model has been largely criticized for its inability to quantitatively match the right amplitude, co-movement and persistence of unemployment and vacancy fluctuations[3]. Coles and Moghaddasi (2018) find that a model with inertial job creation is able to generate not only the observed volatility and persistence in unemployment and vacancies, but also the negative relationship between these two variables, i.e. the Beveridge curve.

Thus, I extend the Coles and Moghaddasi (2018) model by adding an exogenous participation margin where also marginally attached are allowed to search. The calibrated model succeeds at generating the right persistence in unemployment and vacancies and the right negatively-sloped Beveridge curve. However, the model explains half of the observed volatility in unemployment and vacancies observed in the UK.

This paper is organized as follows: section 2.2 introduces the model; section 2.3 describes the calibration strategy while section 2.4 presents the numerical results. Finally, section 2.5 concludes

## 2.2 The Model

The purpose of the present paper is to compare the dynamic properties of the standard matching model with free entry of vacancies as in Pissarides (2000) against a model of Diamond entry where vacancies evolve as a stock variable as in Coles and Moghaddasi (2018), under the assumption that the economy is

[3]See Mortensen and Nagypal (2007) and Elsby, Michaels and Ratner (2015) for a review of the elements of the model that may account for the failures found in matching the data.

characterized by three labour market states: employment, unemployment and marginal attachment.

Time is discrete with an infinite time horizon. There is a fixed measure $N$ of firms who search for profitable business projects. Each firm has one independent business idea in each period. Given that idea, the firm compares its investment cost $x$ with its expected return. The expected return of the business concept depends on the state of the aggregate economy at time $t$, denoted by $\Omega_t$. Let $J_t^V = J^V(\Omega_t)$ denote the expected return of a business concept in state $\Omega_t$. The investment cost $x$ is considered as an idiosyncratic random draw from an exogenous cost distribution $H$. By assumption, the investment cost $x$ captures all of the idiosyncratic features associated with any given business project. Thus, highly profitable concepts correspond to low realised values of $x$. If the firm decides to adopt a business concept, it pays the upfront investment cost $x$ and then holds a project with expected value $J_t^V$. As in Coles and Moghaddasi (2018), each firm invests in their business concept if and only if it has a positive value, i.e. when $J_t^V - x \geq 0$. Since investment occurs if $x \leq J_t^V$ then, at the aggregate level, $i_t = NH(J_t^V)$ describes total period $t$ investment in new projects. This implies that a higher aggregate return $J_t^V$ yields greater vacancy creation rate $i_t$. As in Coles and Moghaddasi (2018), I refer to this investment process as Diamond-entry.

Here, differently from Coles and Moghaddasi (2018), there is an unit mass of equally productive and infinitely lived individuals that switch between employment, unemployment and marginal attachment. All are risk neutral and have the same discount factor $0 < \beta < 1$. Let $u_t$ describe the number of unemployed workers and $n_t$ the number of marginally attached in period $t$[4]. An unemployed worker enjoys per period payoff $b > 0$, while a marginally attached receives per period payoff $z > 0$. Firms are either matched with a worker or unfilled. Let $v_t$ describes the number of vacancies opened in period $t$, posted at per period cost

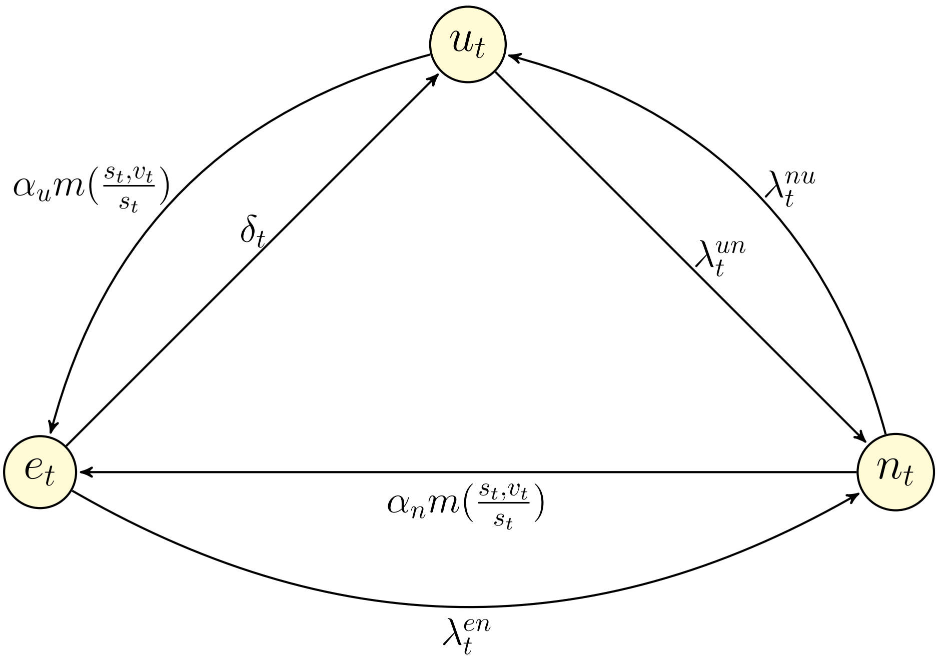
[4]In the theoretical model marginally attached individuals are defined by $n$ and are often referred to as inactives. In the model inactives and marginally attached have the same meaning.

*c*. Each job-worker match produces the same market output $p = p_t$, where aggregate productivity $p_t$ evolves according to an exogenous AR(1) process. Employed workers receive wage $w_t$ determined by Nash bargaining.

Not only unemployed workers come into contact with potential offers, but here also a marginally attached can find a job. However, they do so at a lower rate than unemployed workers as inactives engage in what Pries and Rogerson (2009) define as *passive search*. In order to capture this search behaviour, let $\alpha_u$ and $\alpha_n$ represent respectively the exogenous search effectiveness of the unemployed and marginally attached, with the assumption that $\alpha_u > \alpha_n$. Thus, $\alpha_u u_t$ ($\alpha_n n_t$) describes the total search units of unemployed workers (marginally attached). Define $s_t = \alpha_u u_t + \alpha_n n_t$ as the total number of effective search units. As hiring is frictional, the number $m_t$ of new job-worker matches in period $t$ is described by a matching function $m_t = m(s_t, v_t)$, where $m(.)$ is positive, increasing, concave and homogeneous of degree one. An unemployed worker finds a job in period $t$ with probability $\alpha_u m(\frac{s_t,v_t}{s_t})$, while a marginally attached becomes employed with probability $\alpha_n m(\frac{s_t,v_t}{s_t})$. Thus, in this model hires from inactivity comes at the expense of the unemployment pool, creating a congestion effect. With probability $m(\frac{s_t,v_t}{v_t})$ a vacancy is filled in period $t$.

Job destruction is an exogenous, stochastic process, where $\delta_t$ describes the probability that any given job-worker match is destroyed. In the event of such a job destruction shock, the worker becomes unemployed and the job's continuation value is zero. The job destruction parameter, $\delta_t$, evolves according to an AR(1) process. With probability $\lambda_t^{en}$ an employed worker becomes inactive and the job becomes a vacancy. With probability $\lambda_t^{un}$ unemployed workers become marginally attached. In period $t$, an inactive enters into the labour force as an unemployed worker at constant rate $\lambda_t^{nu}$. In this model I assume transition probabilities $\lambda_t^{en}$, $\lambda_t^{un}$ and $\lambda_t^{nu}$ to be time-invariant. Figure 2.3 illustrates these flows.

Figure 2.3: Worker Flows




### 2.2.1 Timing

Each period has 6 stages:

**Stage I (new realizations)**: given $(p_{t-1}, \delta_{t-1})$ from the previous period, new values of $p_t$, $\delta_t$ are realised according to

$$\ln p_t = \rho_p \ln p_{t-1} + \epsilon_t \tag{2.1}$$

$$\ln \delta_t = \rho_\delta \ln \delta_{t-1} + (1 - \rho_\delta) \ln \bar{\delta} + \eta_t \tag{2.2}$$

where $(\epsilon_t, \eta_t)$ are white noise innovations drawn from a normal distribution with mean zero, covariance matrix $\Sigma$, and $\bar{\delta} > 0$ is the long run average destruction rate.

**Stage II (bargaining and production)**: the wage $w_t$ is determined by Nash bargaining. Production takes place so that a firm with a filled job enjoys one period profit $p_t - w_t$ while an employed worker gets wage $w_t$. Each unemployed worker enjoys payoff $b$ while each marginally attached enjoys payoff $z$.

**Stage III (vacancy investment)**: firms invest in $i_t = NH(J_t^V)$ new vacancies

**Stage IV (matching)**: let $s_t$ and $v_t$ denote the stock of effective search units and vacancies at the start of this stage. Matching takes place so that $m_t = m(s_t, v_t)$ describes the total number of new hires.

**Stage V (job destruction)**: each vacancy and each filled job is independently destroyed with probability $\delta_t$.

**Stage VI (marginal attachment)**: each employed and unemployed worker becomes marginally attached at constant rates $\lambda_t^{en}$ and $\lambda_t^{un}$, respectively. At the same time, a marginally attached becomes unemployed with probability $\lambda_t^{nu}$.

### 2.2.2 Dynamics and equilibrium

Let $u_t$ be the number of unemployed workers in period $t$ immediately prior to the matching stage IV. $n_t$ is the number of marginally attached before the matching takes place. Thus $u_t$ evolves according to:

$$u_t = (1-\lambda_{t-1}^{un})u_{t-1} + \delta_{t-1}(1-u_{t-1}-n_{t-1}) - (1-\delta_{t-1})\alpha_u(\frac{s_{t-1}, v_{t-1}}{s_{t-1}})u_{t-1} + \lambda_{t-1}^{nu}n_{t-1} \tag{2.3}$$

So the stock of unemployed workers in period $t$ is given by

- those unemployed workers that did not become inactives at the end of period $t-1$
- the employed workers in period $t-1$ that were hit by a job destruction shock
- the outflow from unemployment given by the matches formed in period $t-1$ that were not hit by the destruction shock

- the inflow into unemployment from inactivity at the last stage of period $t-1$.

The measure of marginally attached $n_t$ instead evolves according to:

$$n_t = (1-\lambda^{nu}_{t-1})n_{t-1}+\lambda^{en}_{t-1}(1-u_{t-1}-n_{t-1})+\lambda^{un}_{t-1}u_{t-1}-(1-\lambda^{en}_{t-1})\alpha_n m(\frac{s_{t-1}, v_{t-1}}{s_{t-1}})n_{t-1} \tag{2.4}$$

So the stock of inactives in period $t$ is given by

- those marginally attached that did not enter into unemployment at the end of period $t-1$
- the inflow into no-participation from employment at the end of period $t-1$
- the inflow into marginal attachment from unemployment at the end of period $t-1$
- the outflow from marginal attachment that is given by the matches formed in period $t-1$.

Adding up determines employment:

$$e_t = 1 - u_t - n_t \tag{2.5}$$

Hence, (2.3) and (2.4) give a complete description of workers side dynamics.

The vacancy dynamics are described by

$$v_t = (1-\delta_t)[v_{t-1} - (1-\lambda^{en}_{t-1})m_{t-1} + \lambda^{en}_{t-1}e_{t-1}] + i_t \tag{2.6}$$

The first term indicates not only those vacancies that remained unfilled in period $t-1$ and there were not destroyed, but also those jobs whose workers left the labour force in period $t-1$. The second term describes new vacancy creation.

Equations (2.3),(2.4), (2.5) and (2.6) describe the dynamic evolution of the state variables $\{u_t, n_t, e_t, v_t\}$ which are driven by new project investments $i_t$. In order to determine the equilibrium $i_t$, I restrict attention to equilibria where all use Markov strategies.

Immediately after Stage I, the intermediate stock of vacancies is defines as

$$\tilde{v_t} = (1 - \delta_t)[v_{t-1} - (1 - \lambda^{en}_{t-1})m_{t-1} + \lambda^{en}_{t-1}e_{t-1}] \tag{2.7}$$

which is the number of surviving vacancies carried over from the previous period. Define the stage II state vector $\Omega_t = \{p_t, \delta_t, u_t, n_t, \tilde{v_t}\}$.

Stage II determines wages according to a standard Nash bargaining procedure, yielding a wage rule of the form $w_t = w(\Omega_t)$. The Nash bargaining wage depends on the outside option of the worker. In this model both unemployed workers and marginally attached can receive job offers. Thus, there are potentially two wages that can be determined in equilibrium: one for workers coming from unemployment and one for workers coming from inactivity. However, for simplicity I assume that a marginally attached who gets a job offer is considered as an unemployed worker during the bargaining process. This ensures that there is only one wage in equilibrium.

Stage III determines the optimal investment in new vacancies, taking the form $i_t = i(\Omega_t)$. As the matching and separation dynamics ensure $\Omega_t$ evolves as first order Markov process, then $\Omega_t$ is a sufficient statistic for optimal decision making in period $t$.

In order to determine equilibrium wage formation, I characterises the Bellman equations describing optimal behaviour. In period $t$ and at the start of stage II with state vector $\Omega_t$ let:

- $J^V_t = J^V(\Omega_t)$ denote the expected value of a vacancy

- $J_t^F = J^F(\Omega_t)$ denote the expected value of a filled job
- $J_t^U = J^U(\Omega_t)$ denote the worker's expected value of unemployment
- $J_t^E = J^E(\Omega_t)$ denote the worker's expected value of employment
- $J_t^N = J^N(\Omega_t)$ denote the individual's expected value of non-participation

Let $E[.|\Omega_t]$ denote the expectations operator in period t with current state vector $\Omega_t$.

Then, the firms' value functions are defined recursively by the following Bellman equations. The expected value of a vacancy is given by:

$$\begin{aligned} J_t^V =& - c + \beta(1-\delta_t)E\Big\{m(\frac{s_t, v_t}{v_t})[\lambda_t^{en} J_{t+1}^V + (1-\lambda_t^{en})J_{t+1}^F] \\ &+ (1 - m(\frac{s_t, v_t}{v_t}))J_{t+1}^V|\Omega_t\Big\} \end{aligned} \tag{2.8}$$

From the firm's perspective, a vacancy is subject to a flow cost $c$ for the period that it remains unfilled. A vacancy is filled with probability $\frac{m(s_t, v_t)}{v_t}$. In period $t+1$ the entrepreneur enjoys the value of a filled job $J_{t+1}^F$ if the filled vacancy is not destroyed and the worker does not leave the labour force at the end of period $t$. If the vacancy remains unfilled and it is not destroyed, the entrepreneur will still have an open vacancy at the beginning of period $t+1$. The expected value of a filled job is given by:

$$J_t^F = p_t - w_t + \beta(1-\delta_t)E\left\{(1-\lambda_t^{en})J_{t+1}^F + \lambda_t^{en} J_{t+1}^V|\Omega_t\right\} \tag{2.9}$$

A filled job produces a flow output $p_t$ and pays the worker a wage $w_t$. With probability $(1-\delta_t)(1-\lambda_t^{en})$, the joint probability that the job is not destroyed and the worker does become marginally attached, the entrepreneur has a filled job in period $t+1$. If the worker enters into marginal attachment the job becomes a vacancy.

An individual's value functions are also defined recursively. In particular, the worker's expected value of unemployment is given by:

$$\begin{aligned} V_t^U =& b + \beta E\Big\{[\alpha_u m(\frac{s_t, v_t}{s_t})[\delta_t[\lambda_t^{un} V_{t+1}^N + (1-\lambda_t^{un})V_{t+1}^U] + (1-\delta_t)(\lambda_t^{en} V_{t+1}^N + (1-\lambda_t^{en})V_{t+1}^E)] \\ &+ (1-\alpha_u m(\frac{s_t, v_t}{s_t}))[\lambda_t^{un} V_{t+1}^N + (1-\lambda_t^{un})V_{t+1}^U]]|\Omega_t\Big\} \end{aligned} \tag{2.10}$$

Unemployed workers receive a flow payoff $b$. They find a job with probability $\alpha_u m(\frac{s_t, v_t}{s_t})$. Once matched, if the job is hit by a job destruction shock they re-enter into unemployment. If a match is not found, unemployed individuals become inactive with probability $\lambda_t^{un}$.

The expected value of an employed worker is given by:

$$V_t^E = w_t + \beta E\left[\delta_t[\lambda_t^{un} V_{t+1}^N + (1-\lambda_t^{un})V_{t+1}^U] + (1-\delta_t)[\lambda_t^{en} V_{t+1}^N + (1-\lambda_t^{en})V_{t+1}^E)]|\Omega_t\right] \tag{2.11}$$

An employed worker receives a flow wage $w_t$. With probability $\delta_t$, the job is destroyed and the worker suffers a capital loss by entering either into unemployment or marginal attachment in period $t+1$. If the job is not destroyed, at rate $\lambda_t^{en}$, the employed worker becomes marginally attached.

Finally, the individual's expected value of non-participation is given by:

$$\begin{aligned} V_t^N =& z + \beta E\Big\{\alpha_n m(\frac{s_t, v_t}{s_t})[\delta_t[\lambda_t^{un} V_{t+1}^N + (1-\lambda_t^{un})V_{t+1}^U] + (1-\delta_t)[\lambda_t^{en} V_{t+1}^N + (1-\lambda_t^{en})V_{t+1}^E]] \\ &+ (1-\alpha_n m(\frac{s_t, v_t}{s_t}))[\lambda_t^{nu} V_{t+1}^U + (1-\lambda_t^{nu})V_{t+1}^N]|\Omega_t\Big\} \end{aligned} \tag{2.12}$$

A marginally attached receives a flow payoff $z$. He receives a job offer with probability $\alpha_n m(\frac{s_t, v_t}{s_t})$. Once employed, he can either enter into unemployment as his job is destroyed with probability $\delta_t$ or he can re-enter immediately into marginal attachment with probability $\lambda_t^{un}$. If not matched, the individual becomes unemployed in period $t+1$ with probability $\lambda_t^{nu}$.

In order to find the equilibrium wages, I use the standard Nash bargaining approach. As in this model the decision of a worker of being out of the labour force is treated as exogenous, I assume that the only threat point of an employed worker is his value of being unemployed. Thus, as workers have bargaining power $\phi \in [0, 1]$, the axiomatic Nash bargaining approach closes the model with

$$(1-\phi)[V_t^E - V_t^U] = \phi[J_t^F - J_t^V]. \tag{2.13}$$

In order to close the model, as in Coles and Kelishomi (2018), I need to determine the equilibrium investment and wage outcomes. As Diamond entry implies that an entrepreneur will invest if and only if $x \leq J_t^P$, then equilibrium investment $i_t = i(\Omega_t)$ is given by

$$i_t = NH(J_t) \tag{2.14}$$

where $J_t = J(\Omega_t)$.

Thus, the dynamic paths of the economy are determined by equations (2.8)-(2.14) under the laws of motion for unemployment (2.3), marginal attachment (2.4) and for vacancies (2.6), and the exogenous productivity and separation processes (2.1)-(2.2).

## 2.3 Calibration

The purpose of the paper is to compare the behaviour of the model presented in section 2.2 with that of a standard free entry framework, with inactivity as a third market state.

The theoretical framework illustrated above is calibrated at quarterly frequency by matching steady-state properties of the model to UK data (2002Q4-2017Q1).

There are 19 parameters to pin down and their relative choices for both models are summarized in column $A$ and $B$ of Table (3.1).

Table 2.3: Parameter Values

| Symbol | Description | *(A)* Free Entry | *(B)* Diamond Entry |
|---|---|---|---|
| $\beta$ | Quarterly discount factor | 0.9901 | 0.9901 |
| $\gamma$ | Elasticity parameter of the matching function | 0.6 | 0.6 |
| $\phi$ | Worker bargaining power | 0.6 | 0.6 |
| $\alpha_u$ | Search effectiveness of the unemployed | 1 | 1 |
| $\alpha_n$ | Search effectiveness of the marginally attached | 0.330 | 0.330 |
| $\mu$ | Scale parameter of the matching function | 0.417 | 0.417 |
| $\lambda_t^{nu}$ | Quarterly transition probability from N to U | 0.134 | 0.134 |
| $\lambda_t^{un}$ | Quarterly transition probability from U to N | 0.133 | 0.133 |
| $\lambda_t^{en}$ | Quarterly transition probability from E to N | 0.008 | 0.008 |
| $\bar{\delta}$ | Mean quarterly job separation probability | 0.015 | 0.015 |
| $\rho_\delta$ | Separation autocorrelation | 0.849 | 0.849 |
| $\sigma_\delta$ | Standard deviation of separation shocks | 0.059 | 0.059 |
| $\rho_p$ | Productivity autocorrelation | 0.818 | 0.818 |
| $\sigma_p$ | Standard deviation of productivity shocks | 0.010 | 0.010 |
| $\rho_{p\delta}$ | Cross correlation | -0.74 | -0.74 |
| $b$ | Outside value of leisure of the unemployed | 0.7 | 0.7 |
| $z$ | Outside value of leisure of the marginally attached | 0.7 | 0.7 |
| $c$ | Per period vacancy posting cost | 0.073 | 0 |
| $N$ | Entrepreneurial activity | - | 0.09 |

In order to have an annual interest rate of 4 percent, the quarterly discount factor $\beta$ is set to 0.9901.

As common in the literature, the matching function is assumed to be Cobb-Douglas:

$$m(s, v) = \mu s^\gamma v^{1-\gamma}. \tag{2.15}$$

Based on estimates of Petrongolo and Pissarides (2001), the matching elasticity $\gamma$ is set to be 0.6. Similarly, I fix the worker bargaining power $\phi$ to 0.6. Thus, given market tightness $\theta = \frac{v}{s}$, unemployed workers get a job with probability $\alpha_u m(\frac{s,v}{s}) = \alpha_u \mu \theta^{1-\gamma}$. Similarly, inactives become employed with probability $\alpha_n m(\frac{s,v}{s}) = \alpha_n \mu \theta^{1-\gamma}$.

Table 2.4: Labour Market Transition Probabilities: UK 2001-2017

| From | To | | |
|---|---|---|---|
| | E | U | M |
| E | 0.977 | 0.015 | 0.008 |
| U | 0.250 | 0.617 | 0.133 |
| M | 0.082 | 0.134 | 0.784 |

Transition probabilities between labour market states from $t-1$ to $t$. The original flow series are four-quarter moving averages. Source: Labour Force Survey, 2001-2017.

While Table 2.2 describes the transition probabilities when the inactive population is distinguished between marginally attached and nonparticipants, Table 2.4 depicts the quarterly rates at which individuals move in a world where there are only employed, unemployed and marginally attached individuals in the UK. Quarterly transition data from Table 2.4 are used to calibrate the job finding probability for the unemployed and the marginally attached. Thus, I target $\alpha_u \mu \theta^{1-\gamma} = 0.250$, which is the mean quarterly transition rate from unemployment to employment from 2001 to 2017 in the UK. Likewise, the target is $\alpha_n \mu \theta^{1-\gamma} = 0.082$, representing the transition rate from marginal attachment to employment in the same period. As by definition all unemployed workers search, their search effectiveness $\alpha_u$ is normalized to 1. It follows that $\alpha_n = 0.330$. As in Burgess and Turon (2010), the probability of a vacancy being filled in the UK, $m(\frac{s,v}{v}) = \mu\theta^{-\gamma}$, is set to 0.90. Thus, the resulting labour market tightness is $\theta = 0.278$, implying that the scale of the matching function, $\mu$, is equal to 0.417.

The transition probability from marginal attachment to unemployment, $\lambda_t^{nu} = 0.134$, corresponding to the quarterly transition rate found in the data. Similarly, from those quarterly transitions, I choose the rates at which employed and unemployed workers move to non-participation, $\lambda_t^{en} = 0.008$ and $\lambda_t^{un} = 0.133$. The mean quarterly job separation probability $\bar{\delta}$ is set to 0.015 to match the quarterly transition rate from employment to unemployment in the data. This choice of parameters allows to replicate both the marginal attachment and the unemployment rates as found in the UK in the period 2001-2012, that are equal to 0.068 and 0.059, respectively.

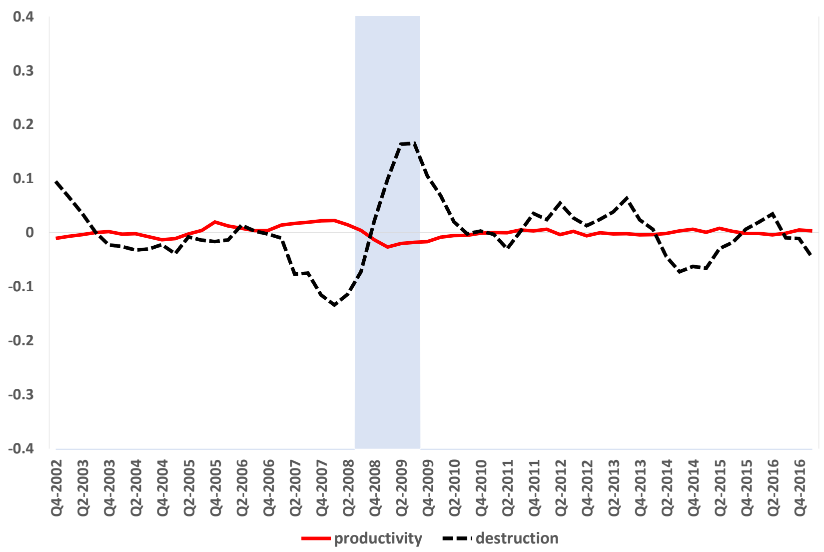
Following Hall and Milgrom (2008), I set $b$, the flow value of unemployment, equal to 0.7. From equation (2.1), the long run mean of productivity $\bar{p}$ is equal to one, with corresponding large surplus $(\bar{p} - b)/b = 0.43$. As there is no direct empirical evidence on the flow value on inactivity, I assume $z = b = 0.7$. Changing the value of $z$ has no effect on the dynamics of the model and the relationships between the variables, because this model does not include a participation decision as movements between unemployment and inactivity are exogenous.

I now specify the stochastic process for $\{p_t, \delta_t\}$. Data for the separation rates and for aggregate productivity are both recorded quarterly. I first derive the autocorrelation $\rho_\delta$ directly from quarterly transition rate from employment to unemployment constructed from the LFS. Modelling this series as an AR(1) process leads to $\rho_\delta = 0.849$ and standard deviation $\sigma_\delta = 0.059$. As regards the aggregate productivity, I used quarterly data from the Office of National Statistics (ONS)[5]. Thus, I estimate the quarterly autocorrelation $\rho_p = 0.818$ with standard deviation $\sigma_p = 0.010$. The cross correlation between productivity and separation is $\rho_{p\delta} = -0.74$. Figure 2.4 shows the behaviour of these two time series.

In order to compare the two models, most of the parameters will remain unchanged in the two versions. However, as turnover is different when free entry is relaxed, I will assume that all job creation costs are attributed to the investment process $x \sim H(.)$. Thus, as in Coles and Moghaddasi (2018) there are no advertising costs of a vacancy and, therefore, $c$ is set to zero. Consider now the investment rate

[5]I used output per worker for the whole economy from 2001 to 2017 as a measure of labour productivity.

Figure 2.4: UK Labour Productivity and Separation Rates




Note: Both quarterly series are in logs as deviations from a HP trend with smoothing parameter 1,600.

$i_t = NH(J_t^V)$. Following Fujita and Ramey (2005), I assume $H$ is uniform, so that vacancy creation is neither elastic nor inelastic, i.e. $i_t = NJ_t^V$. With $c = 0$, the entrepreneurial activity $N$ is set to 0.09 in order to fit the long run turnover means discussed above. In the specification of the model with free entry $c$ is equal to 0.073.

## 2.4 Numerical Simulations

In order to investigate the quantitative predictions of the two different model specifications, I first log-linearize the system around the steady state. I simulate the economy to obtain $30,300$ observations at quarterly frequencies and I discard the first 300 periods. Finally, following Hagedorn and Manovskii (2008), I Hodrick-Prescott filter the logged series with smoothing parameter $1,600$ to obtain second moments. The statistical properties of these simulated time series are then compared to the statistical properties of the corresponding data generated by the UK economy in the period 2001-2017. Data for the UK are transformed in a manner analogous to the transformation undertaken on the simulated data.

Table 2.5 reports the standard deviation of unemployment $\sigma_u$, of vacancies $\sigma_v$, of the vacancy/unemployment ratio $\sigma_{v/u}$, of employment $\sigma_e$ and inactivity $\sigma_n$ from trend as a measure of business cycle volatility[6].

Table 2.5: Volatility of Labour Market Variables

| **Volatility** | **UK Data** | **Free Entry** | **Diamond Entry** |
|---|---|---|---|
| $\sigma_u$ | 0.1521 | 0.0527 | 0.0765 |
| $\sigma_v$ | 0.1435 | 0.0671 | 0.0802 |
| $\sigma_{v/u}$ | 0.3123 | 0.1077 | 0.1501 |
| $\sigma_e$ | 0.0172 | 0.0199 | 0.0216 |
| $\sigma_n$ | 0.0531 | 0.0320 | 0.0494 |

The model with free entry explains only around one third of the observed volatility of unemployment, vacancies and the $v/u$ ratio. These findings are similar to those in Shimer (2005) for the standard matching model without inactivity when calibrated to the US. Thus, the introduction of inactives in the model does not alter the implications of the standard model for its main variables as regards the volatility. The introduction of vacancy stock dynamics slightly increases volatility in all variables. In particular, labour market tightness $v/u$ increases by almost 50% than in the free entry case. Both models are able to match only the volatility of employment. On the other hand, both specifications overestimate the volatility of employment and, in particular, inactivity.

Figures (2.5) and (2.6) describe the impulse response functions of the endogenous variables of the model to a single productivity shock at date zero, holding separation constant in the case of free entry (FE) and Diamond entry (DE), respectively.

With free entry of vacancies, the positive productivity shock induces a sharp upward jump in vacancies, and consequently in market tightness, followed by a

[6]Time series for unemployment, employment and inactives are from the LFS. The series for vacancy is from ONS and cover the same period considered in this Chapter, i.e. 2001-2017.

Figure 2.5: Impulse Responses to a Productivity Shock: Free Entry

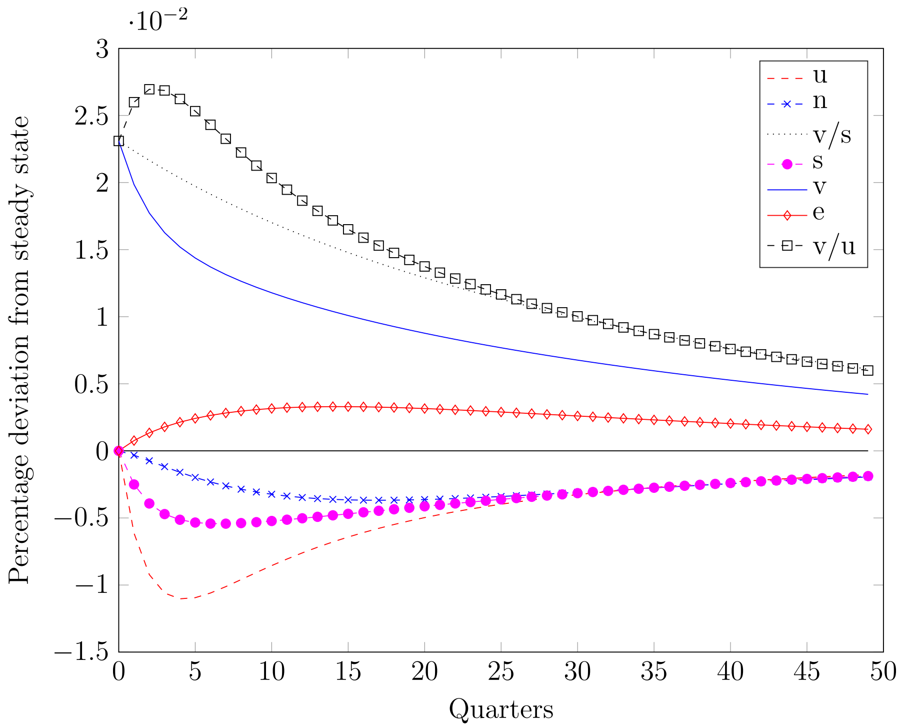



Figure 2.6: Impulse Responses to a Productivity Shock: Diamond Entry

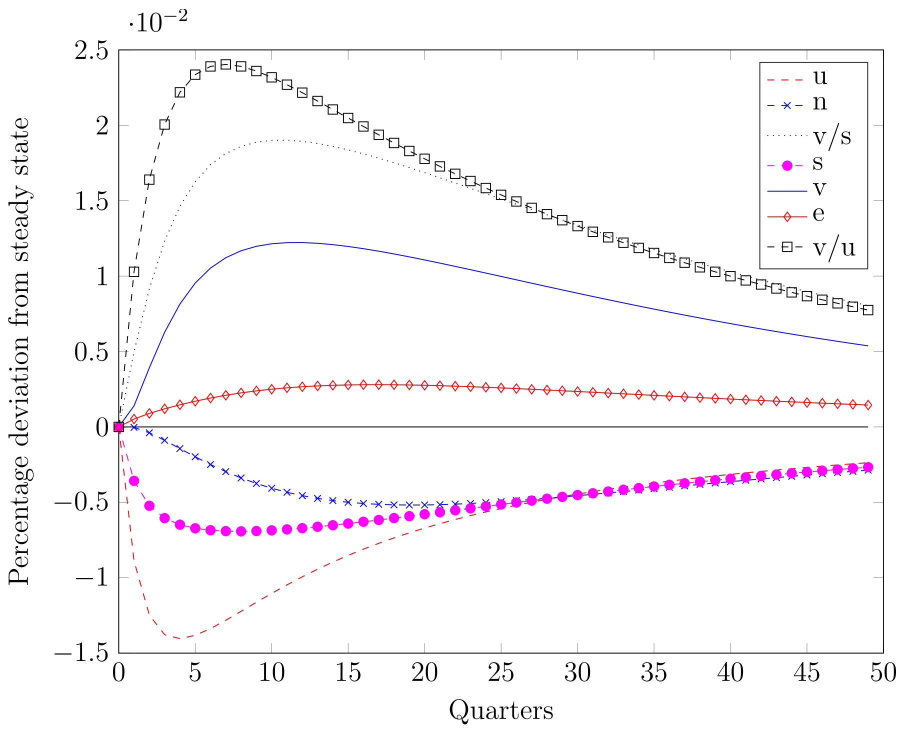
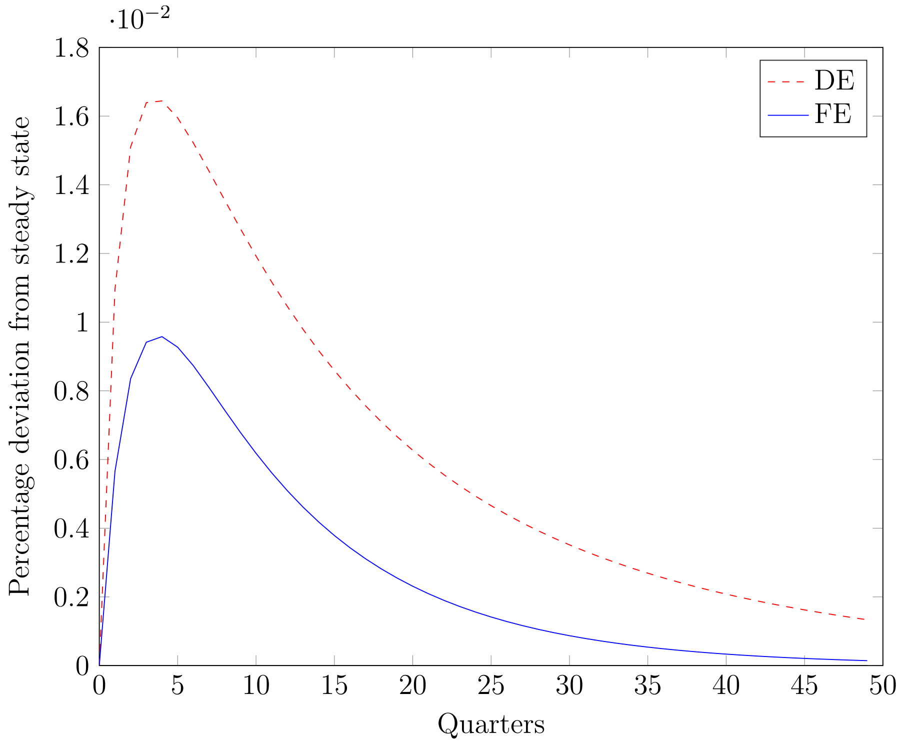


Figure 2.7: Impulse Response of Unemployment to a Separation Shock

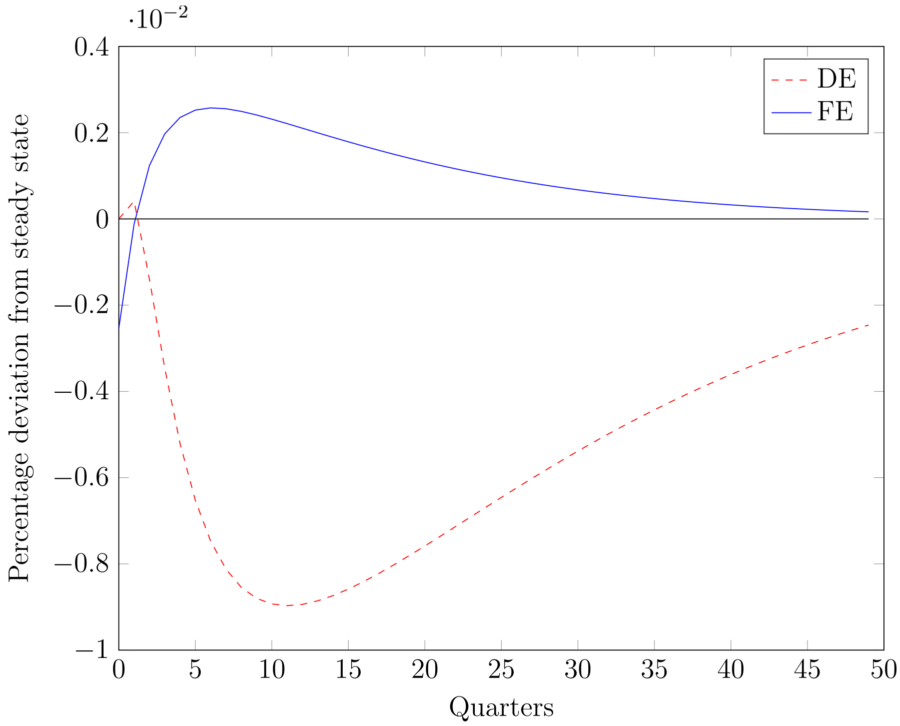



monotonic decline. Thus, there is an immediate increase in both job finding rates for both marginally attached and unemployed workers, leading to a reduction in the unemployment and marginal attachment rates and an increase in the number of employed individuals. The model with Diamond entry exhibits qualitatively similar results but generate much stronger propagation dynamics. For example, the simulated response of the $v/u$ ratio builds in magnitude for almost 10 quarters.

Figures (2.7) and (2.8) describe the impulse response function of unemployment and vacancies following a separation shock at time zero, keeping productivity fixed. In Figure (2.7) we can note that in the free entry case unemployment is slightly less persistent. It peaks immediately and quickly falls to its steady state level. The increase in unemployment is larger when I consider the effect of a job destruction shock in the Diamond entry setting.

As in Coles and Moghaddasi (2018), the larger persistence in the unemployment rate is due to the behaviour of vacancies. In the free entry case, as shown in Figure (2.8), in response to the increase in unemployment, vacancies immediately

Figure 2.8: Impulse Response of Vacancies to a Separation Shock




jump. Consequently, unemployment recovers quickly to its steady state value. On the other hand, when Diamond entry is considered, the stock of vacancies falls as the number of unemployed workers increases. Indeed, with sluggish vacancies, new vacancy creation reacts only partially to job creation incentives driven by the increase in the pool of searchers. As a consequence, the rise in unemployed workers looking for a job does not produce a quick entry of vacancies but causes the partial depletion of the outstanding stock of vacancies, depressing even further the job finding rate of the unemployed workers.

Table (2.6) describes the persistence of unemployment, vacancies, employment and inactivity.

Table 2.6: Persistence in Labour Market Variables

| **Serial Persistence** | **UK Data** | **Free Entry** | **Diamond Entry** |
|---|---|---|---|
| $corr(u_t, u_{t-1})$ | 0.9470 | 0.9147 | 0.9219 |
| $corr(v_t, v_{t-1})$ | 0.9317 | 0.9029 | 0.9787 |
| $corr(e_t, e_{t-1})$ | 0.9249 | 0.9826 | 0.9857 |
| $corr(n_t, n_{t-1})$ | 0.8524 | 0.8939 | 0.9771 |

The free entry model seems to slightly underestimate the serial correlation of $u$ and $v$ with their lagged values. On the other hand, Diamond entry generates a higher persistence more similar to the data. As regards $e$, both models produce a higher persistence than the one empirically observed. The free entry specification is able to match the right persistence of inactivity.

Table 2.7: The Beveridge Curve

| **Correlation** | **UK Data** | **Free Entry** | **Diamond Entry** | **FE, shocks to $\delta_t$ only** | **DE, Shocks to $\delta_t$ only** |
|---|---|---|---|---|---|
| $corr(u_t, v_t)$ | -0.8617 | -0.7131 | -0.8547 | 0.9923 | -0.9944 |

Next I look at the correlation of unemployment and vacancies. From Table (2.7) we observe the large and negative correlation between the two variables, the so-called Beveridge curve. Both models show the right negative relation, but only the Diamond entry specification is quantitatively consistent with the data. However, if only separation shocks are considered the free entry model generates a counterfactually positive correlation.

### 2.4.1 The role of marginal attachment

This section investigates what are the implications of a simple model which restricts attention to a labour market where there are only employed and unemployed workers, comparing it to the benchmark three-states model illustrated above. In addition, I compare the impact of a Diamond entry specification versus a free entry specification, on both the benchmark and the simplified two-states model. For the simplified model, I set $\lambda^{un}$, the probability at which an unemployed exits the labour force, and $\lambda^{en}$, the probability at which an employed becomes inactive, equal to zero. This results in having a marginal attachment rate $n = 0$ in steady state. Table (2.8) focuses only on the simplified two-states model. I use the same calibration parameters as in the benchmark model with three-states. The table shows that the introduction of Diamond entry does a better job in replicating the empirical volatilities. This result highlights the robustness of the results of Coles and Moghaddasi (2018) to a different calibration strategy.

Table 2.8: Volatility of Labour Market Variables: Two States

| **Volatility** | **UK Data** | **Free Entry** | **Diamond Entry** |
|---|---|---|---|
| $\sigma_u$ | 0.1521 | 0.0958 | 0.2132 |
| $\sigma_v$ | 0.1435 | 0.0623 | 0.2253 |
| $\sigma_{v/u}$ | 0.3123 | 0.1039 | 0.3721 |
| $\sigma_e$ | 0.0172 | 0.0072 | 0.0120 |

By comparing Tables (2.8) and (2.5), it is possible to observe that, in the Diamond entry case, the two-states model behaves much better than the three-states one in matching the standard deviations of unemployment, employment, vacancies and tightness. The same is not true for the free entry model. Indeed, while the two-states specification produces a higher volatility in the unemployment rate, the volatility of vacancies and market tightness remain unchanged when inactivity is included. At the same time, we can observe that the Diamond entry model with three-states produces slightly higher standard deviations than the two-states free entry model as regards vacancies and tightness. To understand why including Diamond entry yields a better result even in a three-states environment, Figures (2.9) and (2.10) show the impulse response functions of unemployment and vacancies following a shock to the separation rate only.

I focus first on the behaviour of the free entry model with both two and three states. Following a job separation shock, in the three-states specification the impulse response function of unemployment produces a smaller increase in unemployment than in the two-states scenario. In the three-state model newly unemployed workers become inactive with probability $\lambda^{nu} > 0$, thus slightly reducing the unemployment pool. In both cases the unemployment rate quickly recovers to its steady state. The reason lies in the reaction of vacancies to an increase in the unemployment rate. In both cases, vacancies increase as it easy to match with a larger pool of searchers. However, when only unemployed workers are

Figure 2.9: Impulse Response of Unemployment to a Separation Shock: shutting down inactivity

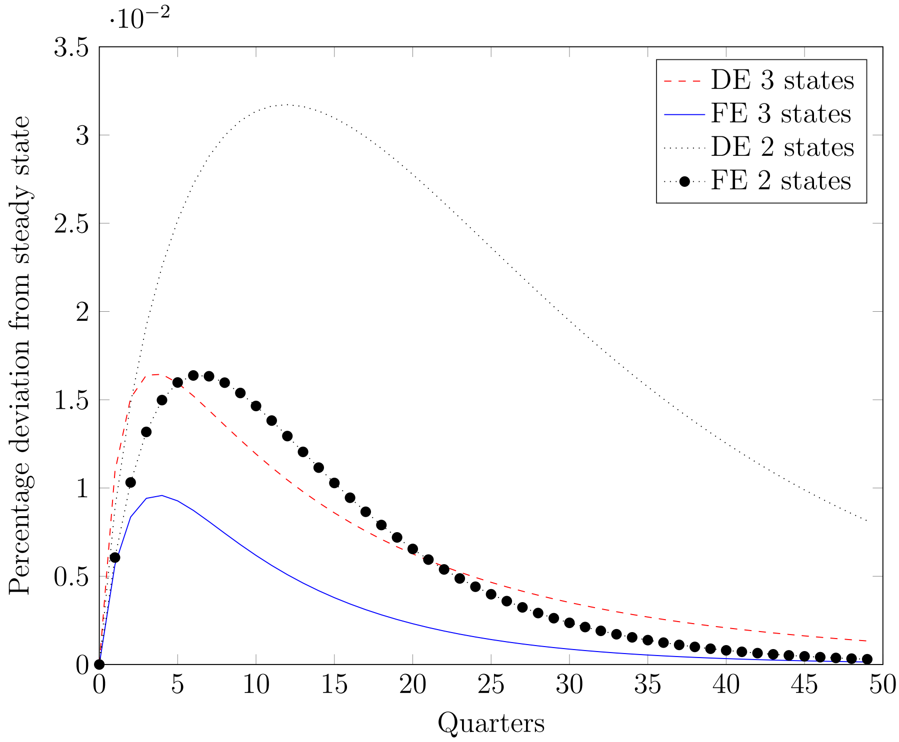



Figure 2.10: Impulse Response of Vacancies to a Separation Shock: shutting down inactivity

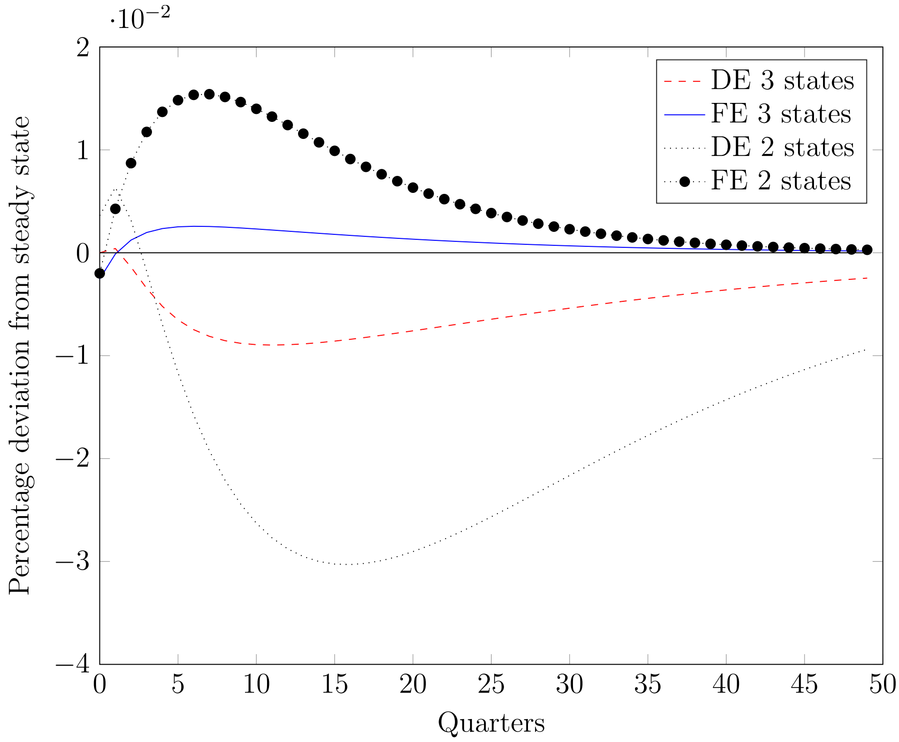
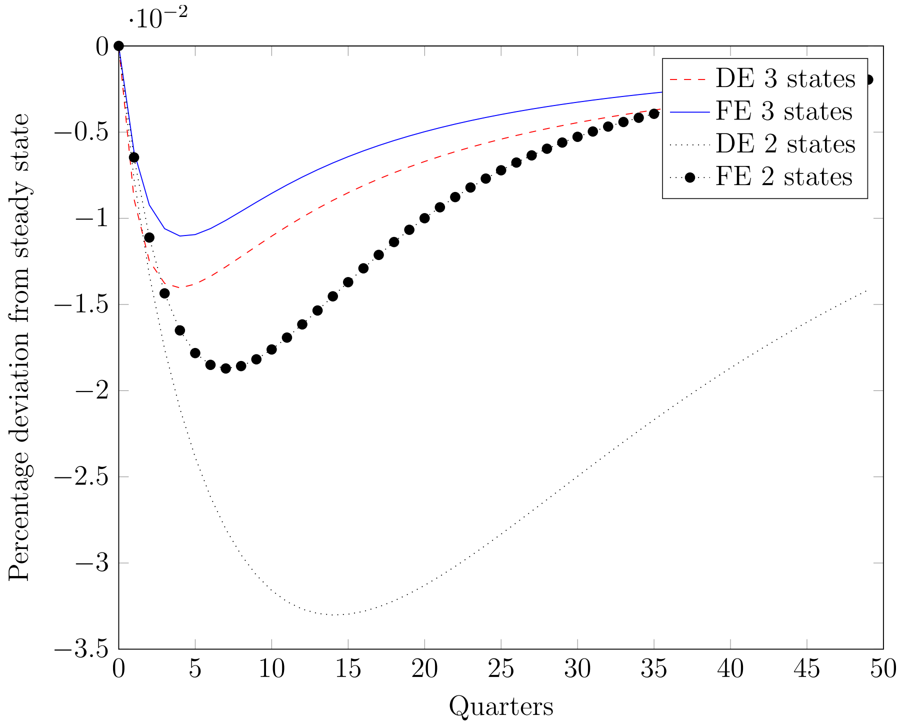


Figure 2.11: Impulse Response of Unemployment to a Productivity Shock: shutting down inactivity

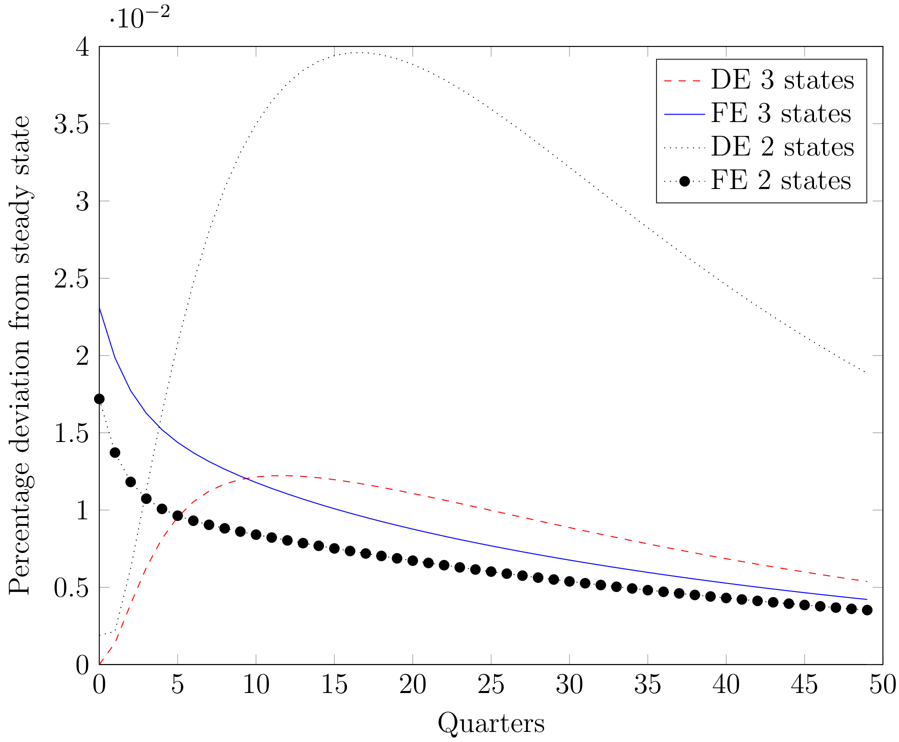



looking for a job (two-state model) even a small increase in the unemployment has a large impact on vacancy creation. On the other hand, when there are also marginally attached, an increase in unemployment leads to a lower increase in the number of searchers because unemployed workers represent only a fraction of the searchers pool. Thus, the job filling rate does not move as much as in the two-state model leading to a lower increase in vacancies. Both specifications yield a counterfactually positive correlation between unemployment and vacancies.

With Diamond entry a rise in unemployment does not produce an increase in vacancies, but it partially depletes the existing vacancy stock as explained above for the benchmark three-state model.

Figures (2.11) and (2.12) document the behaviour of unemployment and vacancies following a productivity shock. In free entry, vacancies immediately jump up as market conditions are better and quickly fall to the steady state. This effect is larger in the three-state model because vacancies can sample not only from the unemployed workers but also from inactives. The possibility that hires

Figure 2.12: Impulse Response of Vacancies to a Productivity Shock: shutting down inactivity




from marginal attachment come at the expenses of hires from the unemployed workers creates a congestion externality, thus reducing the job finding rate of the unemployed. This results in a lower reduction of unemployment in the three-state model.

This congestion externality is present also in the Diamond entry model. Vacancies do not immediately jump following the productivity shock as their creation is not perfectly elastic. In a three-state world, vacancies increase less than in the two-state as, when there are also inactives looking for jobs, there are more searchers. Thus, vacancies fall to the steady state as they are filled faster.

## 2.5 Conclusions

In Chapter 1, I documented the importance of considering inactivity as a major determinant of unemployment fluctuations in the UK. In particular, marginally attached individuals behave very differently from those individuals that are neither looking for or wanting a job, i.e. nonparticipants. Thus, in this Chapter, I have included an inactivity margin in a search and matching model where vacancies behave as a stock variable. In contrast with previous literature, inactives engage in search associated with a lower but non-zero job finding probability than unemployed worker. I calibrate the model to match moments from UK data and the inactive pool resembles the behaviour of marginally attached individuals. The calibrated model succeeds at generating the right persistence in the endogenous variables and the right negatively-sloped Beveridge curve. However, the model is able to explain only half of the observed volatility in unemployment and vacancies in UK data. For tractability, the decision to search is not modelled here. Thus, in Chapter 3 of this dissertation, I extend the model by including an endogenous labour supply decision.

# Chapter 3

# Endogenous Participation Decision in a Search and Matching Model

## 3.1 Introduction and Literature Review

Recent empirical studies find that not only movements between employment and unemployment, but also flows into and out of the labour force are crucial for understanding labour market dynamics. Indeed, transitions into and out of the labour force display significant levels of volatility over the business cycle and they play a major role in determining unemployment, particularly during recessions as documented by Elsby, Hobijn and Sahin (2015) for the US. Moreover, flows into and out of the labour force, which are an order of magnitude larger than the flows from employment to unemployment, are key for explaining long-run differences in employment rates across countries (Pries and Rogerson, 2009).

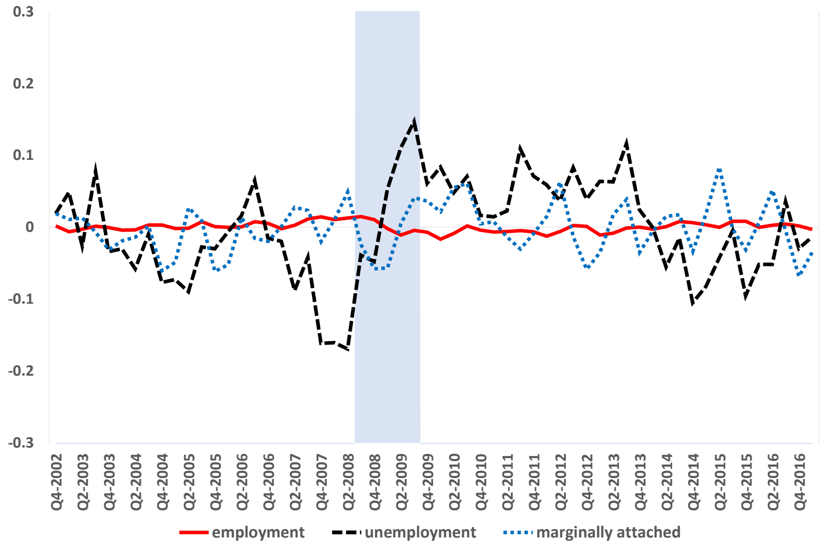
In Chapter 1, I have documented the behaviour of movements in the UK labour market. In particular, the Quarterly Labour Force Survey (LFS) allows me to distinguish between different categories of inactive individuals according to their attachment to the labour market. I defined as “marginally attached” those individuals classified as out of the labour force who are either looking for a job or they would like to work if offered the opportunity. On the other hand, “nonparticipants” are those individuals who are neither seeking, nor would like to work. Then, I showed that marginally attached individuals, though respresenting less than a third of the total inactive population in the UK for the period 2001-2017, are responsible for a significant proportion of the variation observed in the unemployment rate. Nonparticipants, on the contrary, do not play a significant role.

Theoretical literature that tries to model the labour market do not consider inactivity as a major source of fluctuations of the unemployment rate. This is mainly due to the fact that inactivity is mostly acyclical and does not present a large volatility. However, by decomposing the inactivity state into marginal attachment and nonparticipation, it is possible to observe that marginal attachment presents a significant degree of cyclicality. Figure 3.1 shows that marginal attachment is a more volatile state than the employment one.

On the basis of the results highlighted in Chapter 1, I introduced in Chapter 2 a third state in a search and matching model where vacancies behave as in Coles and Moghaddasi (2018). As the empirical evidence suggests, I considered only those individuals belonging to the inactivity state who are marginally attached. I calibrated the model to capture features of the UK labour market and, finally, I simulated it. However, the decision to search has not been modelled there as individuals were moving between unemployment and marginal attachment at a constant exogenous rate.

However, in the UK we observed large movements between unemployment and marginal attachment that needs to be considered, as suggested by Figure 3.2. The series seem to follow the same pattern and magnitude, presenting a countercyclical behaviour. When I restrict the attention to the flow transition probabilities

Figure 3.1: Employment, Unemployment and Marginal Attachment Volatilities

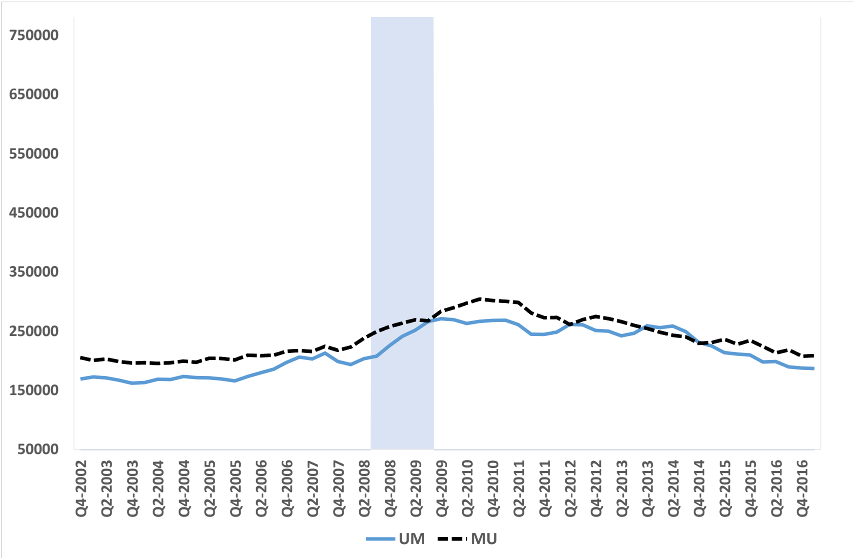



Note: All series are quarterly series constructed by LFS Data. They are reported in logs as deviations from an HP trend with smoothing parameter 1,600. Recession bars indicate UK recession dates defined by ONS.

depicted in Figure 3.3, the behaviour of the movements between these two series changes. While it is more likely to become unemployed for a marginally attached during a recession, the opposite is not true. In a downturn, the probability of moving into marginal attachment decreases for an unemployed individual.

A possible explanation for the behaviour of Figure 3.3a can be found in the compositional changes observed in the unemployment pool when the economy is hit by a recession. For example, Mueller (2017) finds that in downturns the pool of unemployed shifts towards workers with high wages in their previous job in the US. Thus, these workers are more attached to the labour market and they face lower transition rates into inactivity. On the other hand, Mankart and Oikonomou (2017) attribute the countercyclical pattern observed in Figure 3.3b to the so-called “added worker effect”. Indeed, the inactive individual within the household could enter into unemployment during a recession in order to replace the lost income the follows a job loss faced by the other member of the household. However,

Figure 3.2: Flows between unemployment and marginal attachment

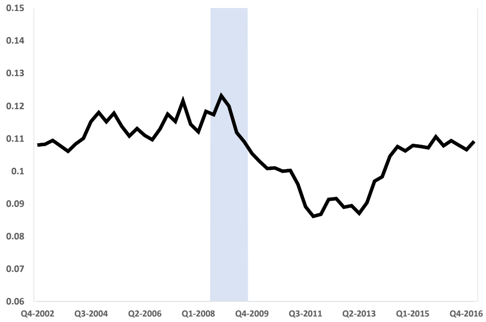



Note: The flow series are four-quarter moving averages to remove seasonality and high frequency movements. Recession bars indicate UK recession dates defined by the ONS. Source: Labour Force Survey, 2001-2017.

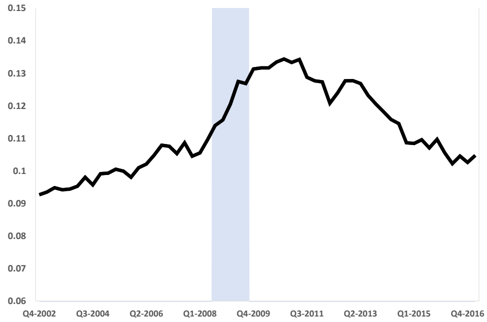
the purpose of this paper is not to investigate the causes of these two phenomena, but only to introduce a participation decision in a search and matching model. Thus, it seems natural to include an endogenous participation decision in the model which explores the choices faced by non-employed individuals. Therefore, I introduce a free entry condition between unemployment and marginal attachment. As search is costly unemployed workers receive a lower benefit from home production but they face a higher transition probability into employment with respect to inactive individuals. Thus, in every period non-employed face a trade-off between a higher home benefit and a lower probability of finding a job. I then calibrate the model to match some feature of the UK labour market following a calibration strategy similar to the one used in Chapter 2 and explore the features of the numerically simulated model.

The Chapter proceeds as follows: section 3.2 illustrates the theoretical model; section 3.3 presents the calibration strategy; section 3.4 shows the numerical results;

Figure 3.3: Transition probabilities between unemployment and marginal attachment




(a) $p_{UM}$




(b) $p_{MU}$

Note: Quarterly flow transition probabilities corrected for margin of error. Recession bars indicate UK recession dates defined by the ONS. Source: Labour Force Survey, 2001-2017.

finally, section 3.5 concludes.

## 3.2 The Model

This section describes the labour market search model with endogenous participation decision. I adopt a discrete time version of the standard model of Pissarides (2000) where the matching process between non-employed workers and vacancies is subject to a search friction.

Vacancies exhibit stock dynamics as in Coles and Moghaddasi (2018). As in Chapter 2, there is a fixed measure $N$ of firms who search for profitable business projects. Each firm has one independent idea in each period. Given that business idea, the firm compares its investment cost $x$ with its expected return. The expected return of the business concept depends on the state of the aggregate economy at time $t$, denoted by $\Omega_t$. Let $J_t^V = J^V(\Omega_t)$ denote the expected return of a business concept in state $\Omega_t$. The investment cost $x$ is considered as an idiosyncratic random draw from an exogenous cost distribution $H$. By assumption, the investment cost $x$ captures all of the idiosyncratic features associated with any given business project. Thus, highly profitable concepts correspond to low realised values of $x$. If the firm decides to adopt a business concept, it pays the upfront investment cost $x$ and then holds a project with expected value $J_t^V$. As in Diamond (1982), each firm invests in their business concept if and only if it has a positive value, i.e. when $J_t^V - x \geq 0$. Since investment occurs if $x \leq J_t^V$ then, at the aggregate level, $i_t = NH(J_t^V)$ describes total period $t$ investment in new projects. This implies that a higher aggregate return $J_t^V$ yields greater vacancy creation rate $i_t$. As in Coles and Moghaddasi (2018), I refer to this investment process as Diamond-entry.

Here the framework is modified in order to account not only for transitions between employment and non-employment but also for changes in the composition of the pool of non-employed individuals. There is a unit measure of equally productive and infinitely lived individuals who can be employed or non-employed. Individuals in the non-employment state can be either unemployed or marginally attached. Agents in the economy are risk neutral and have discount factor $0 < \beta < 1$. Let $e_t$, $u_t$ and $n_t$ be the number of employed workers, unemployed and marginally attached at time $t$.

In each period, a workers can be matched with a firm. Every worker-firm match produces the same output level $p = p_t$ in $t$, where productivity $p_t$ evolves according to an AR(1) process. Define $v_t$ as the number of open vacancies at time $t$; firms posting vacancies pay a cost $c$ in every period. Every worker employed in period $t$ receives the same wage $w_t$ negotiated with the firm according to Nash bargaining. Any given job-worker match is destroyed with probability $\delta_t$, evolving as an AR(1) process. In the event of such a shock, the worker becomes non-employed. For those who are not employed, the decision is whether to search for a job or not. Those who decide to search for a job are called *unemployed*. Those who do not search are called *marginally attached*. I assume there is no cost of switching between marginal attachment and the unemployment pool. However, I assume that marginally attached can directly move into the employment state without becoming unemployed first. Define $s_t = u_t + \lambda n_t$ as the total number of effective search units, where $\lambda < 1$ represents the search effectiveness of the marginally attached relative to unemployed workers. Let $\alpha_t = \frac{n_t}{u_t+m_t}$, then the number of effective search units in period $t$ can be written as

$$s_t = (1 - e_t)[(1 - \alpha_t) + \lambda\alpha_t]. \tag{3.1}$$

Due to search frictions, non-employed individuals and open vacancies coexist in any period $t$. The number $m_t$ of new job-worker matches in period $t$ is described by a matching function $m_t = m(s_t, v_t)$, where $m(.)$ is positive, increasing, concave and homogeneous of degree one. An unemployed enjoys per period payoff $b > 0$ and finds a job in period $t$ with probability $m(\frac{s_t,v_t}{s_t})$. If inactive, he gets payoff $z$ and becomes employed at rate $\lambda m(\frac{s_t,v_t}{s_t})$. I assume $z > b$ as unemployed, looking more intensively for jobs, forego a higher share of utility from leisure or home production than marginally attached. This assumption is crucial for the model as it determines the number of unemployed and marginal attached workers in equilibrium.

Each period $t$ has 5 stages:

**Stage I (new realizations)**: given $(p_{t-1}, \delta_{t-1})$ from the previous period, new values of $p_t$, $\delta_t$ are realised according to

$$\ln p_t = \rho_p \ln p_{t-1} + \epsilon_t \tag{3.2}$$

$$\ln \delta_t = \rho_\delta \ln \delta_{t-1} + (1 - \rho_\delta) \ln \bar{\delta} + \eta_t \tag{3.3}$$

where $(\epsilon_t, \eta_t)$ are white noise innovations drawn from a normal distribution with mean zero, covariance matrix $\Sigma$, and $\bar{\delta} > 0$ is the long run average destruction rate.

**Stage II (bargaining and production)**: the wage $w_t$ is determined by Nash bargaining. Production takes place so that a firm with a filled job enjoys one period profit $p_t - w_t$ while an employed worker gets payoff $w_t$. Each unemployed worker enjoys payoff $b$ while each marginally attached enjoys payoff $z$.

**Stage III (matching)**: let $s_t$ and $v_t$ denote the stock of effective search units and vacancies at the start of this stage. Matching takes place so that $m_t = m(s_t, v_t)$ describes the total number of new hires.

**Stage IV (job destruction)**: each vacancy and each filled job is independently destroyed with probability $\delta_t$.

**Stage V (in and out of labour force)**: non-employed workers choose whether to be into unemployment or marginal attachment.

The number of employed workers in period $t$ immediately prior to the matching stage III is defined by $e_t$ that evolves according to

$$e_t = (1 - \delta_{t-1})(e_{t-1} + m_{t-1}) \tag{3.4}$$

Thus, the stock of employed workers in period $t$ is given by

- those workers who were employed in $t-1$ and whose job was not destroyed;
- those matches formed in period $t-1$ that were not hit by the destruction shock.

The vacancy dynamics are described by

$$v_t = (1 - \delta_t)[v_{t-1} - m_{t-1}] + i_t \tag{3.5}$$

The first term indicates those vacancies that remained unfilled in period $t-1$ and were not destroyed. The second term describes new vacancy creation. Differently from the standard free entry model, here vacancies become a predetermined variable.

Equations (3.4)and (3.5) describe the dynamic evolution of the state variables $\{e_t, v_t\}$ which are driven by new project investments $i_t$. In order to determine the equilibrium $i_t$, I restrict attention to equilibria where all use Markov strategies. Immediately after Stage I, the intermediate stock of vacancies is defined as

$$\tilde{v_t} = (1 - \delta_t)[v_{t-1} - m_{t-1}] \tag{3.6}$$

which is the number of surviving vacancies carried over from the previous period.

Stage II determines wages according to a standard Nash bargaining procedure, yielding a wage rule of the form $w_t = w(\Omega_t)$. Stage III determines the optimal investment in new vacancies, taking the form $i_t = i(\Omega_t)$. As the matching and separation dynamics ensure $\Omega_t$ evolves as first order Markov process, then $\Omega_t$ is a sufficient statistic for optimal decision making in period $t$.

In order to determine equilibrium wage formation, I characterise the Bellman equations describing optimal behaviour. In period $t$ and at the start of stage II with state vector $\Omega_t = \{p_t, \delta_t, e_t, \tilde{v}_t\}$ let:

- $J_t^V = J^V(\Omega_t)$ denote the expected value of a vacancy;
- $J_t^F = J^F(\Omega_t)$ denote the expected value of a filled job;
- $V_t^U = V^U(\Omega_t)$ denote the worker's expected value of unemployment;
- $V_t^E = V^E(\Omega_t)$ denote the worker's expected value of employment;
- $V_t^N = V^N(\Omega_t)$ denote the individual's expected value of inactivity.

The expected value of an employed worker is given by:

$$V_t^E = w_t + \beta E\Big\{\delta_t \max\left\{V_{t+1}^U, V_{t+1}^N\right\} + (1-\delta_t)V_{t+1}^E|\Omega_t\Big\}. \tag{3.7}$$

An employed worker receives a flow wage $w_t$. With probability $\delta_t$, the job is destroyed, and the worker suffers a capital loss by entering into non-employment in period $t+1$. Then, he will choose whether to become either unemployed or marginally attached. The value of being employed is always higher than the value of being either unemployed or marginally attached. Thus, a worker always prefers to work if given the opportunity.

The expected present value of current and future payoffs for an unemployed worker is

$$\begin{aligned} V_t^U =& b + \beta E\Big\{m(\frac{s_t, v_t}{s_t})[\delta_t \max\left\{V_{t+1}^U, V_{t+1}^N\right\} + (1-\delta_t)V_{t+1}^E] \\ &+ (1 - m(\frac{s_t, v_t}{s_t}))\max\left\{V_{t+1}^U, V_{t+1}^N\right\}|\Omega_t\Big\}. \end{aligned} \tag{3.8}$$

Unemployed receive a flow payoff $b$. They find a job with probability $m(\frac{s_t, v_t}{s_t})$.

Once matched, if the job is hit by a destruction shock they re-enter into non-employment where they decide if they want to be unemployed or inactives. Finally, the individual's expected value of marginal attachment is given by:

$$\begin{aligned} V_t^N =& z + \beta E\Big\{\lambda m(\frac{s_t, v_t}{s_t})[\delta_t \max\left\{V_{t+1}^U, V_{t+1}^N\right\} + (1-\delta_t)V_{t+1}^E] \\ &+ (1-\lambda m(\frac{s_t, v_t}{s_t}))\max\left\{V_{t+1}^U, V_{t+1}^N\right\}|\Omega_t\Big\}. \end{aligned} \tag{3.9}$$

An individual out of the labour force receives a flow payoff $z$ reflecting the value of leisure and home production. He receives a job offer with probability $\lambda m(\frac{s_t, v_t}{s_t})$. In each period $t$, an individual can freely move between the two non-employment states. The assumption of free entry implies that the individual is indifferent between unemployed and inactivity. Let $V_t^{NE}$ be the value of being non-employed in period $t$, such that $V_t^U = V_t^N \equiv V_t^{NE}$. Thus, Equations (3.7)-(3.9) can be written respectively as:

$$V_t^E = w_t + \beta E\Big\{\delta_t V_{t+1}^{NE} + (1-\delta_t)V_{t+1}^E|\Omega_t\Big\}, \tag{3.10}$$

$$\begin{aligned} V_t^{NE} =& b + \beta E\Big\{m(\frac{s_t, v_t}{s_t})[\delta_t V_{t+1}^{NE} + (1-\delta_t)V_{t+1}^E] \\ &+ (1-m(\frac{s_t, v_t}{s_t}))V_{t+1}^{NE}|\Omega_t\Big\}, \end{aligned} \tag{3.11}$$

$$\begin{aligned} V_t^{NE} =& z + \beta E\Big\{\lambda m(\frac{s_t, v_t}{s_t})[\delta_t V_{t+1}^{NE} + (1-\delta_t)V_{t+1}^E] \\ &+ (1-\lambda m(\frac{s_t, v_t}{s_t}))V_{t+1}^{NE}|\Omega_t\Big\}. \end{aligned} \tag{3.12}$$

The assumption of no cost of switching between unemployment and inactivity implies the following arbitrage condition:

$$z - b = \beta E\Big\{(1-\lambda)m(\frac{s_t, v_t}{s_t})[(1-\delta_t)(V^E_{t+1} - V^{NE}_{t+1})]|\Omega_t\Big\} \tag{3.13}$$

In equilibrium all opportunities from moving between unemployment and inactivity are exploited and there are no incentives to change between the two market states.

The expected value of a vacancy is given by:

$$\begin{aligned} J^V_t =& - c + \beta(1-\delta_t)E\Big\{m(\frac{s_t, v_t}{v_t})J^F_{t+1} \\ &+ (1 - m(\frac{s_t, v_t}{v_t}))J^V_{t+1}|\Omega_t\Big\} \end{aligned} \tag{3.14}$$

From the firm's perspective, a vacancy is subject to a flow cost $c$ for the period that it remains unfilled. A vacancy is filled with probability $m(\frac{s_t, v_t}{v_t})$. In period $t+1$ the entrepreneur enjoys the value of a filled job $J^F_{t+1}$ if the filled vacancy is not destroyed . If the vacancy remains unfilled and it is not destroyed, the entrepreneur will still have an open vacancy at the beginning of period $t+1$.

The expected value of a filled job is given by:

$$J^F_t = p_t - w_t + \beta(1-\delta_t)E\left\{J^F_{t+1}|\Omega_t\right\} \tag{3.15}$$

A filled job produces a flow output $p_t$ and pays the worker a wage $w_t$.

In order to find the equilibrium wages, I use the standard Nash bargaining approach. With the assumption of free entry between unemployment and marginal attachment, the threat point of an employed worker is his value of being non-employed. Thus, as workers have bargaining power $\phi \in [0, 1]$, the axiomatic Nash bargaining approach closes the model with

$$(1-\phi)[V_t^E - V_t^{NE}] = \phi[J_t^F - J_t^V]. \tag{3.16}$$

In order to close the model, as in Coles and Kelishomi (2018), I need to determine the equilibrium investment and wage outcomes. As Diamond entry implies that an entrepreneur will invest if and only if $x \leq J_t^P$, then equilibrium investment $i_t = i(\Omega_t)$ is given by

$$i_t = NH(J_t) \tag{3.17}$$

where $J_t = J(\Omega_t)$.

Thus, the dynamic paths of the economy are determined by equations (3.10), (3.11), (3.14), (3.15) , (3.16), (3.17) and the arbitrage condition (3.13) under the laws of motion for employment (3.4) and for vacancies (3.5), and the exogenous productivity and separation processes (3.2)-(3.3).

## 3.3 Calibration

As for Chapter 2, the theoretical framework illustrated above is calibrated at quarterly frequency by matching steady-state properties of the model to UK data for the period 2001-2017. The quarterly discount factor $\beta$ is set to 0.9901, which implies an annual interest rate of 4 percent. The matching elasticity $\gamma$ is set to be 0.6, within the range of estimates of Petrongolo and Pissarides (2001). Similarly, I fix the worker bargaining power $\phi$ to 0.6. Thus, given market tightness $\theta = \frac{v}{s}$, unemployed get a job with probability $m(\frac{s,v}{s}) = \mu\theta^{1-\gamma}$. Similarly, marginally attached become employed with probability $\lambda m(\frac{s,v}{s}) = \lambda\mu\theta^{1-\gamma}$. In order to calibrate the job finding probabilities of non-employed individuals I use labour market transition probabilities constructed from the LFS, as shown in Chapter 2. Thus, I target for the job finding probability of unemployed workers $\mu\theta^{1-\gamma}$ the mean quarterly transition probability from unemployment to employment from 2001 to 2017 in the UK that is equal to 0.250. Likewise, the target is $\lambda\mu\theta^{1-\gamma} = 0.082$, representing the transition probability from marginal attachment to employment in the same period. It follows that $\lambda = 0.330$. The probability of a vacancy being filled, $m(\frac{s,v}{v}) = \mu\theta^{-\gamma}$, is set to 0.90 as in Burgess and Turon (2010). Thus, the resulting labour market tightness is $\theta = 0.278$, implying that the scale of the matching function, $\mu$, is equal to 0.417. Differently from Chapter 2, the mean quarterly job separation probability $\bar{\delta}$ is set to 0.023 to match the sum of the quarterly transition rates from employment to unemployment and from employment to inactivity in the data. I do this as once a worker loses his job he becomes non-employed and later he will decide whether to be unemployed or marginally attached. This pins down an employment rate of 87.3% as observed in the data. Following Hall and Milgrom (2008), I set $b$, the flow value of unemployment, equal to 0.7. From equation (3.2), the long run mean of productivity $\bar{p}$ is equal to one, with corresponding large surplus $(\bar{p} - b)/b = 0.43$. I choose $z = 0.78$ in order to obtain the average number of unemployed and marginally attached individuals in the UK, i.e. $u = 0.059$ and the number of inactives $n$ equal to 0.068. As in Chapter 2, I will assume that all job creation costs are attributed to the investment process $x \sim H(.)$. Thus, as in Coles and Mogghaddasi (2018) there are no advertising costs of a vacancy and, therefore, $c$ is set to zero. Consider now the investment rate $i_t = NH(J_t^V)$. Following Fujita and Ramey (2005), I assume $H$ is uniform, so that vacancy creation is neither elastic nor inelastic, i.e. $i_t = NJ_t^V$. With $c = 0$, the entrepreneurial activity $N$ is set to 0.006 in order to fit the long run turnover means discussed above.

I now specify the stochastic process for $\{p_t, \delta_t\}$. Data for the separation rates and for aggregate productivity are both recorded quarterly. I first derive the autocorrelation $\rho_\delta$ directly from the sum of the quarterly transition rates from employment to unemployment and employment to inactivity constructed from the LFS. Modelling this series as an AR(1) process leads to $\rho_\delta = 0.833$ and standard deviation $\sigma_\delta = 0.056$. As regards the aggregate productivity, I used quarterly data from the Office of National Statistics (ONS)[1]. Thus, I estimate the quarterly autocorrelation $\rho_p = 0.818$ with standard deviation $\sigma_p = 0.010$. The cross correlation between productivity and separation is $\rho_{p\delta} = -0.72$.

Table 3.1: Parameter Values

| **Symbol** | **Description** | **Value** |
|---|---|---|
| $\beta$ | Quarterly discount factor | 0.9901 |
| $\gamma$ | Elasticity parameter of the matching function | 0.6 |
| $\phi$ | Worker bargaining power | 0.6 |
| $\lambda$ | Search effectiveness of marginally attached | 0.330 |
| $\mu$ | Scale parameter of the matching function | 0.417 |
| $\bar{\delta}$ | Mean quarterly job separation probability | 0.023 |
| $\rho_\delta$ | Separation autocorrelation | 0.833 |
| $\sigma_\delta$ | Standard deviation of separation shocks | 0.056 |
| $\rho_p$ | Productivity autocorrelation | 0.818 |
| $\sigma_p$ | Standard deviation of productivity shocks | 0.010 |
| $\rho_{p\delta}$ | Cross correlation | -0.72 |
| $b$ | Unemployment value of leisure | 0.7 |
| $z$ | Marginal attachment value of leisure | 0.78 |
| $c$ | Per period vacancy posting cost | 0 |
| $N$ | Entrepreneurial activity | 0.006 |

## 3.4 Numerical Simulation

In order to investigate the quantitative predictions of the two different model specifications, I first log-linearize the system around the state state. I simulate the economy to obtain $30,300$ observations at quarterly frequencies and I discard the first 300 periods. Finally, following Hagedorn and Manovskii (2008), I Hodrick-Prescott filter the logged series with smoothing parameter $1,600$ to obtain second moments. The statistical properties of these simulated time series are then compared to the statistical properties of the corresponding data generated by the UK

[1] I used output per worker for the whole economy from 2001 to 2017 as a measure of labour productivity.

economy in the period 2001-2017. Data for the UK are transformed in a manner analogous to the transformation undertaken on the simulated data.

Table 3.2 reports the standard deviation of employment $\sigma_e$, of non-employment $\sigma_{ne}$, of vacancies $\sigma_v$, of the vacancy/non-employment ratio $\sigma_{v/ne}$ as a measure of business cycle volatility. The model underestimates the volatility of vacancies while the standard deviation of employment and non-employment are largely overestimated.

Table 3.2: Volatility of Labour Market Variables

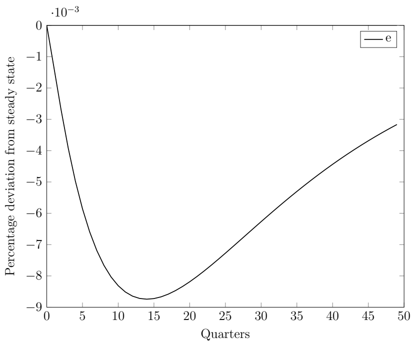
| **Volatility** | **UK Data** | **Model** |
|---|---|---|
| $\sigma_e$ | 0.0172 | 0.1035 |
| $\sigma_{ne}$ | 0.0755 | 0.1603 |
| $\sigma_v$ | 0.1435 | 0.0714 |
| $\sigma_{v/ne}$ | 0.2863 | 0.2186 |

I do not present the results of the standard deviation of unemployment and inactivity as, differently from Chapter 2, here they are jump variables. Figure 3.4 describes the impulse response functions of the endogenous variables of the model to a single job destruction shock at date zero, holding productivity constant. The result is similar to Coles and Moghaddasi (2018): the shock increases the number of non-employed individuals and the number of vacancies decreases. Thus, the model is able to replicate the negative relationship between vacancies and non-employment observed in the data. The calibrated model captures the right fall in tightness $\frac{v}{ne}$, too.

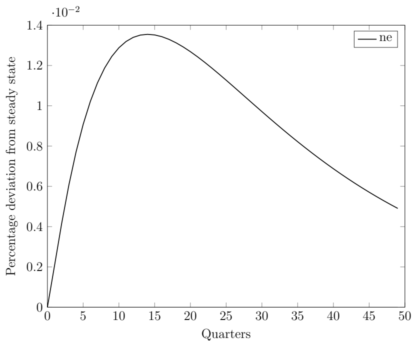
An interesting feature of this model is that it allows to analyse composition effects following a one-period shock, illustrated in Figure 3.5. Following a job destruction shock, the number of non-employed individuals increases, as shown in Figure (3.4). However, it is possible to disentangle the individual effect on unemployed and marginally attached: while marginally attached individuals rapidly increase, I observe a strong fall in the number of unemployed workers. As the fall in the number of unemployed workers dominates the rise in the inactivity pool, the number of total searchers (i.e. the total number of effective search units $s_t$ explained in the theoretical model above) declines, even in a recession. The relevant labour market tightness for the agents in the model is the tightness $v/s$, that determines the job finding rates. From Figure 3.5 it seems that during a recession it is easier to find a job, because the drop in the number of total searchers is larger than the fall in the proportion of vacancies. Thus, we should expect a rise in the number of unemployed workers. On the contrary, the opposite is true.

Why does the number of unemployed decline following a job destruction shock? A possible explanation is given by the behaviour of wages: following a destruction shock, wages slightly fall, thus reducing the incentive to look for a job. The drop in wages dominates the increase in labour market tightness. As the returns from searching are lower (i.e. lower salaries), non-employed individuals prefer to move into inactivity and not to look more intensively for a job, thus increasing the proportion of marginally attached out of the non-employed. This is represented by the increase in $\alpha_t$.

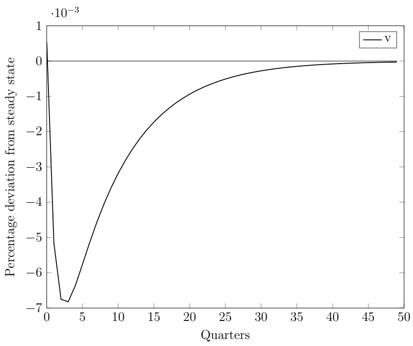
Figure 3.4: Impulse response functions to a job destruction shock



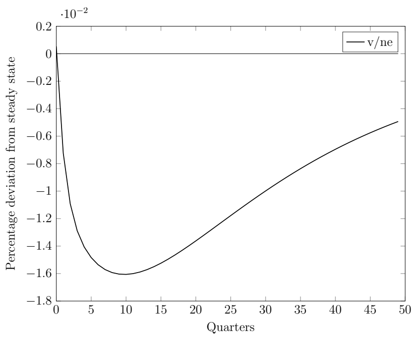

(a) Employment




(b) Non-employment

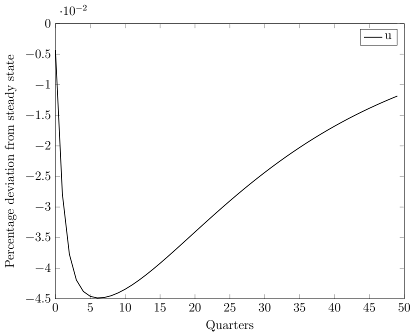



(c) Vacancies

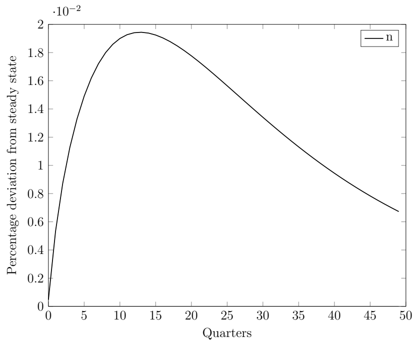



(d) Tightness $\frac{v}{ne}$

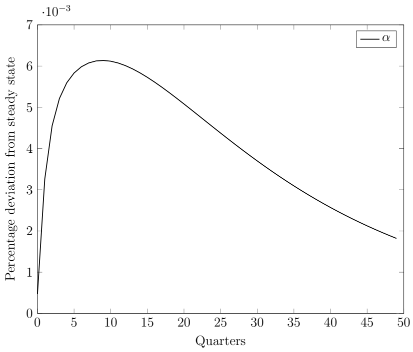
Figure 3.5: Impulse response functions to a job destruction shock: composition effects



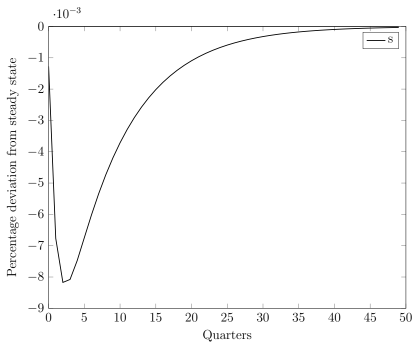

(a) Unemployed



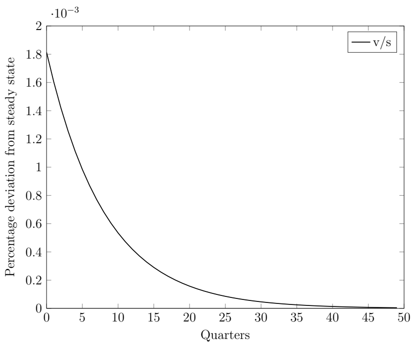

(b) Marginally attached



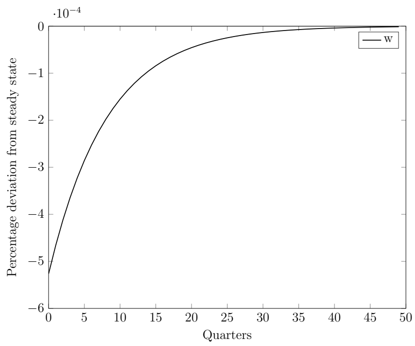

(c) $\alpha$




(d) Total searchers




(e) Tightness $\frac{v}{s}$

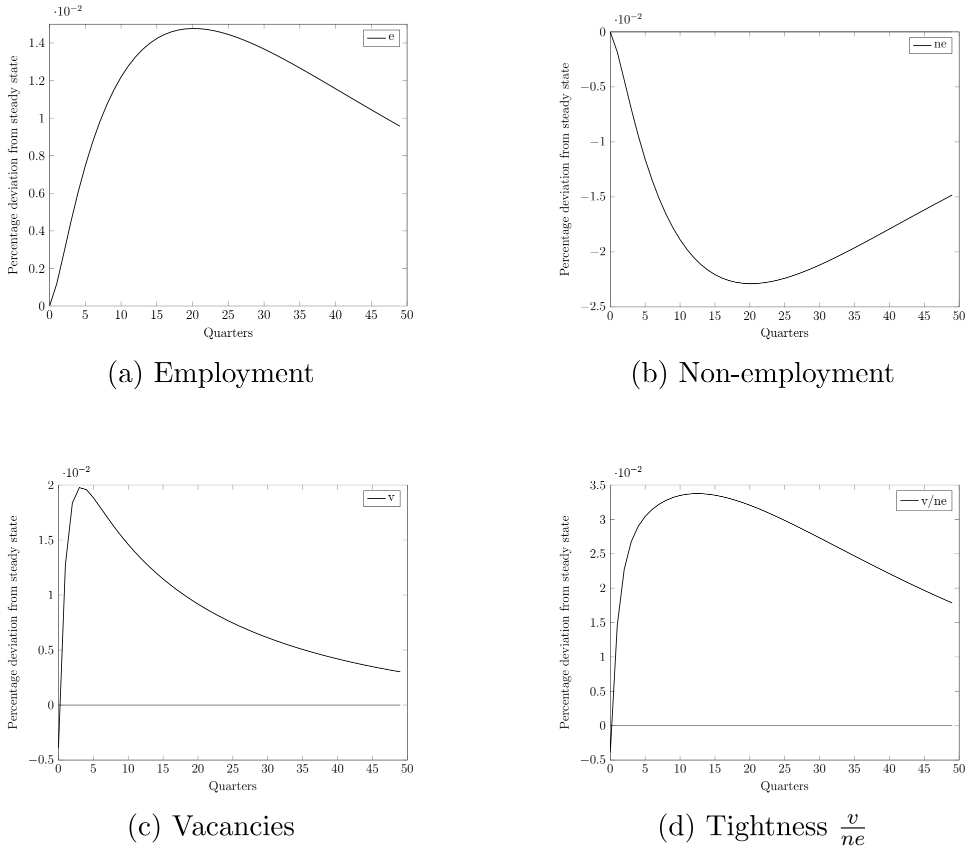



(f) Wage

Figure 3.6 describes the impulse response functions of the endogenous variables of the model to a single productivity shock at date zero, holding separation constant ($\delta_t = 0.023$). The positive productivity shock induces a sharp upward jump in vacancies, an increase in employed workers, and consequently in market tightness $v/ne$, followed by a monotonic decline. When looking at the composition effects, Figure 3.7 mirrors Figure 3.5 for a destruction shock. Indeed, a large proportion of inactives decides to look for a job in a boom, hence the number of unemployed workers rapidly increases. Thus, the number of total searchers increases too. As the increase in total searchers more than offsets the jump in vacancies, the relevant labour market tightness $v/s$ immediately falls. It is now more difficult to find a job. However, as the wage slightly increases, the returns to search are larger and the proportion of inactives $\alpha$ falls down.

The model produces a counterfactually procyclical unemployment. This result is consistent with Tripier (2004) and Veracierto (2008). Indeed, as the economy is hit by a positive productivity shock, for example, more individuals quit inactivity. Thus, more workers begin to search for jobs since it is a bad time to be out of the labour force, as the opportunity cost from not searching is higher in a boom. As it takes time to form a match, not all unemployed workers searching for jobs meet a vacancy. As a consequence, unemployment increases at first. As firms open more vacancies, employment increases, and unemployment starts to decrease. As unemployment decreases, there are less incentives for firms to open vacancies. Thus, vacancy creation falls as well. As both vacancies and unemployment increase at the time the economy is hit by the positive technology shock, the model cannot generate the downward sloping Beveridge curve.

Figure 3.6: Impulse response functions to a productivity shock



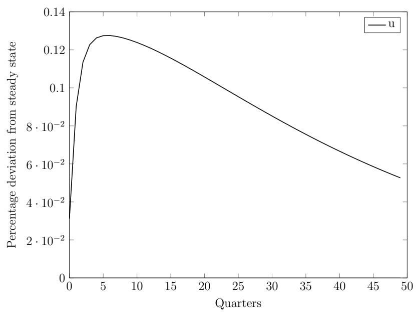

(a) Employment

(b) Non-employment

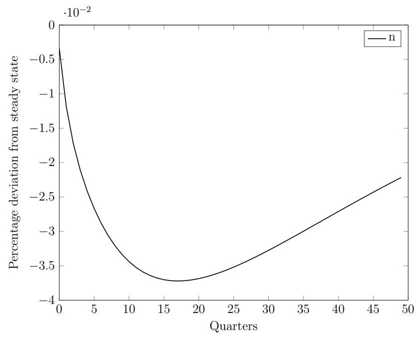
(c) Vacancies

(d) Tightness $\frac{v}{ne}$

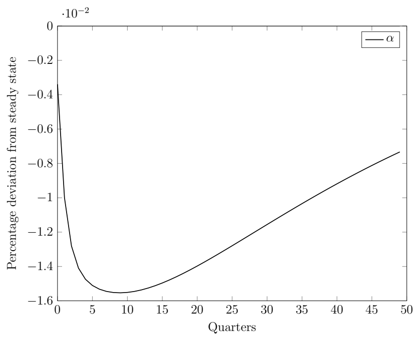
Figure 3.7: Impulse response functions to a productivity shock: composition effects



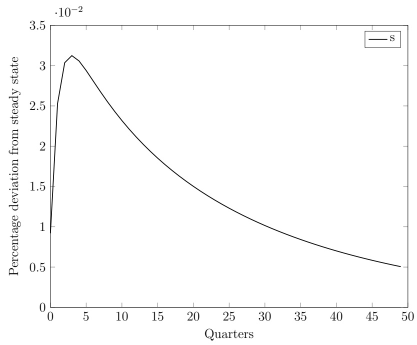

(a) Unemployed



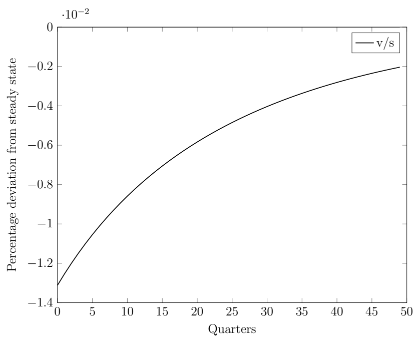

(b) Marginally attached



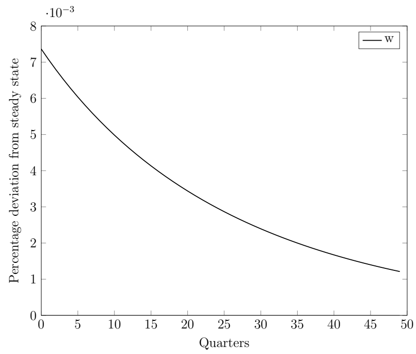

(c) $\alpha$




(d) Total searchers




(e) Tightness $\frac{v}{s}$




(f) Wage

### 3.4.1 The role of the arbitrage condition

The arbitrage condition in Equation 3.13 is key to the results presented in this Chapter. Non-employed workers face a trade-off between being unemployed (i.e. a lower flow benefit $b$ and a higher job finding rate) and marginally attached (i.e. a higher flow benefit $z$ and a lower job finding rate). Following a positive productivity shock, the value of being unemployed increases. This is because, while the flow benefits of both unemployed and marginally attached individuals remain the same, the probability of finding a job increases. Thus, we observe a inflow of inactives towards the unemployment state. However, as the number of unemployed workers increases, the value of being unemployed starts to fall as there are more people looking for jobs. The arbitrage condition ensures that this process will continue until the value of being unemployed equals the value of being marginally attached.

Crucial in determining the equilibrium number of unemployed and marginally attached individuals is the calibration of $b$ and $z$. A small increase in $b$ would determine a surge in unemployment and an immediate fall in inactive individuals. For example, this would follow after an increase in the unemployment benefit (UI). Therefore, in the context of this model, an increase in UI would lead to a zero inactivity rate in steady state. The same is true for an increase in $z$. If the value of being marginally attached slightly increases, unemployment will be equal to zero in equilibrium.

## 3.5 Conclusions

Following the empirical results shown in Chapter 1, I introduced an exogenous participation margin in a search and matching model in Chapter 2. Differently from the latter, here I have introduced an endogenous decision: non-employed individuals can freely decide whether to be unemployed or marginally attached in every period. In doing so, I introduce a free entry condition between the two non-employment labour market states that allows to determine the proportion of unemployed and marginally attached workers in equilibrium. The calibrated model matches qualitatively the behaviour of vacancies, employment and non-employment. In addition, it allows to detect composition effects, distinguishing between the behaviour of unemployed workers and inactives, following either a productivity or a destruction shock. However, in line with recent literature, the model produces a counterfactually procyclical unemployment rate.

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