

Bankruptcy: Is It Enough to Forgive or Must We Also Forget?*

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Abstract

In many countries, lenders are restricted in their access to information about borrowers' past defaults. We study this provision in a model of repeated borrowing and lending with moral hazard and adverse selection. We analyze its effects on borrowers' incentives and credit access, and identify conditions under which it is welfare improving. Our model's predictions are consistent with the evidence on the impact of these credit bureau regulations on borrowers' and lenders' behavior as well as on credit provision. We also show that "forgetting" must be the outcome of a regulatory intervention.

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JEL Classification Numbers: D86, G33, K35.

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I Introduction

In studying the “fresh start” provisions of personal bankruptcy law, economists typically focus on the *forgiveness* of debts, that is, on the effects of allowing borrowers to forego repaying past debts that they have incurred. However, another important feature is the *forgetting* of past defaults. In many countries, lenders are not provided with information about past defaults after a specified period of time has elapsed.¹

In the United States, the Fair Credit Reporting Act (FCRA) prescribes that a personal bankruptcy filing may be reported by credit bureaus for up to 10 years, after which it must be removed from the records made available to lenders.² Musto (2004) studies the effect on lenders and individual borrowers of these restrictions on the reporting of past defaults, using U.S. data. He shows that such restrictions are binding: (i) both borrowers’ credit scores and their access to credit increase significantly when the bankruptcy “flag” is dropped from their credit files (in particular, scores rise by 19% and credit card limits increase by \$1,800);³ (ii) when these individuals obtain new credit, after the flag is dropped, they are more likely to default than those with similar credit scores who did not default in the past. Further supporting the notion that negative information limits access to credit, Jagtiani and Li (2014) show that the amount of credit (as measured by credit limits on bank cards) drops significantly following a bankruptcy filing.⁴

Using Swedish data, Bos and Nakamura (2013) study the effect of a change in credit bureau reporting policies, in particular of a shortening of the time period during which defaults may be reported. They find that this change is associated with (i) tighter credit standards for those borrowers with poor credit histories, (ii) higher default rate, and (iii) higher overall total credit provision.

Analogous provisions limiting information on past defaults also exist in most other countries. In Figure 1 we summarize the distribution *across countries* of credit bureau regulations

¹This paper is concerned with the effects of restrictions on the information provided by *consumer* credit bureaus. Another literature studies the information content of securities ratings. For example, Skreta and Veldkamp (2009) show that when the assets being rated are less transparent, then there is greater scope for ratings shopping, which leads to ratings inflation.

²Other derogatory information can be reported for a maximum of seven years; see Hunt (2006) for a discussion of the history and regulation of consumer credit bureaus in the United States. This time period is often even shorter in other countries; Jappelli and Pagano (2004) report several specific examples.

³Musto (2004) obtains these results for borrowers who did not experience many other adverse events in the ten-year period following their bankruptcy filing. For the other borrowers, with worse credit histories, he finds a smaller effect.

⁴Although late-night television commercials suggest that some post-bankruptcy credit is available, the quantity actually extended is very small.

governing the length of time this information is kept on the records.⁵ Of the 113 countries with credit bureaus as of January 2007, over 90 percent of them had provisions for restricting the reporting of adverse information after a certain period of time.⁶ Differences in information-sharing regimes across countries — whether a credit-reporting system exists, and whether there are time limits on reporting past defaults — are also associated with differences in the provision of credit. Figure 1 also graphs the average ratio of *total* private credit to GDP according to whether or not the country restricts the time period of information sharing. It is interesting to note that countries in which defaults are always reported tend to have *lower* provision of credit than those countries in which defaults are not reported (“erased”) after a certain period of time.⁷

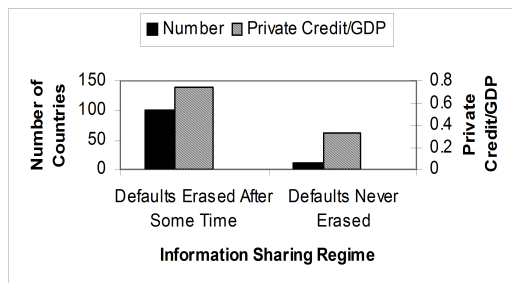


Figure 1: Information-Sharing Regime and the Provision of Credit

In this paper we analyze the effects of these *information* restrictions in a model of repeated borrowing and lending, and determine conditions under which they are welfare improving. In particular, we study an environment where entrepreneurs must repeatedly seek external funds to finance a sequence of risky projects under conditions of both adverse selection, as the entrepreneur’s type is privately observed, and moral hazard, as the project’s outcome also depends on the effort exerted by the entrepreneur. We have in mind a world of small entrepreneurs who finance their business ventures with loans for which they are personally liable.⁸ In this setup, an entrepreneur’s *credit history*, as captured by the past history of

⁵The credit bureau regulations are current as of January 2007. Throughout, we use the term “credit bureau” to refer to both private credit bureaus and public credit registries. Source: Doing Business Database, World Bank, 2008.

⁶See also Jappelli and Pagano (2006).

⁷Private credit/GDP is constructed from the IMF International Financial Statistics for year-end 2006. As in Djankov, McLiesh, and Shleifer (2009), private credit is given by lines 22d and 42d (claims on the private sector by commercial banks and other financial institutions).

⁸And indeed, Avery, Bostic, and Samolyk (1998) use the NSSBF and SCF to show that “[l]oans with personal commitments comprise a majority of small business loans.”

repayments and defaults on his loans, determines the entrepreneur's reputation, that is, the probability assessment by lenders that he is a safer type of borrower, and this in turn affects the terms at which he can get credit and his incentives to exert effort.

We show that in equilibrium entrepreneurs who default because their project fails will see a significant deterioration in their reputation (i.e., they will be perceived as much less likely to be safe) and, consequently, for reasons to be explained below, incentives will also be much worse; they will thus no longer be able to obtain financing. On the other hand, the success of a project and repayment of his loan improves an entrepreneur's reputation and allows him to get credit at a lower interest rate.

We then consider the impact of restricting the availability to lenders of information on entrepreneurs' past defaults. Forgetting a default improves an entrepreneur's reputation, as he is pooled again with those entrepreneurs who have not defaulted; this allows him to obtain financing when he otherwise would not be able to and to get it at a lower rate.⁹ Such a restriction thus leads to a tradeoff in the environment we consider. On the one hand, forgetting defaults weakens incentives, *ex-ante*, because by reducing the impact of this failure on his reputation the punishment incurred after a failure is going to be weaker. On the other hand, forgetting may strengthen the entrepreneur's incentives *ex-post*, once a failure has occurred, for two reasons. First, he now has more to lose (viz., his improved reputation) from another project failure in the future. In addition, with a better reputation the borrower obtains a lower interest rate in the current period, which also helps his incentives. This improvement in incentives and access to credit captures the benefits of the fresh start engendered by forgetting.¹⁰

In particular, we show that forgetting, that is restricting the information available to lenders on borrowers' credit history, can be beneficial in some cases, primarily when the severity of the incentive problem, as captured by the cost of effort, is not too high, and the share of risky entrepreneurs in the economy is not too high (so the adverse selection problem is not too severe). In these cases the welfare loss due to the decline in *ex-ante* incentives is more than compensated by a welfare gain due to the improvement in *ex-post* incentives. On the other hand, we find that complete forgetting — that is never reporting a default —

⁹The beneficial effects of facilitating pooling by restricting the information that could be used to screen borrowers, also appears in other environments. For example, Mayer, Piskorski, and Tchistyi (2013) show that mortgage prepayment penalties can enhance social welfare because they prevent borrowers whose credit quality improves from refinancing, thus keeping interest rates low for riskier ones, and thereby lowering default rates.

¹⁰We can also understand why forgetting *successes* would not be beneficial in our context, because it would not improve an entrepreneur's reputation, and thus only the negative effect would be present.

is only optimal in some rather extreme circumstances, where the severity of the incentive problem is very low. Thus another implication of our findings is that, outside this case, having a credit bureau that reports *some* information on credit histories is optimal.

A key feature for our finding that some forgetting may be beneficial is the property that incentives are stronger the higher an entrepreneur’s reputation; in the next sections we discuss the elements of the model which generate this property. We also argue that forgetting must be the outcome of a regulatory intervention by the government — no lender would willingly agree to ignore the information available to him.

The effects of “forgetting” on lenders’ and individual borrowers’ behavior in our model are consistent with the empirical evidence presented by Musto (2004). However, while Musto interprets this evidence as an indication that laws imposing restrictions on memory are suboptimal, we argue that these restrictions may actually improve social welfare. In addition, the findings of Bos and Nakamura (2013) on the costs and benefits of shortening the period for which defaults are reported provide empirical support for the predictions of our model. Indeed, in line with our results, they interpret their evidence as suggesting that this policy change was beneficial. Finally, our model also helps to explain the international evidence reported above.

In the congressional debate surrounding the adoption of the FCRA (U.S. House, 1970, and U.S. Senate, 1969), the following arguments were put forward in favor of forgetting past defaults: (i) if information was not erased, the stigmatized individual would not obtain a “fresh start” and so would be unable to continue as a productive member of society, (ii) old information might be less reliable or salient, and (iii) there is limited computer storage capacity. On the other hand, the arguments raised against forgetting this information were that (i) it discourages borrowers from repaying their debts by reducing the penalty for failure, (ii) it increases the chance of costly fraud or other crimes by making it harder to identify seriously bad risks, (iii) it could lead to a tightening of credit policies (which would affect the worst risks disproportionately), and finally, (iv) it forces honest borrowers to subsidize the dishonest ones. The same arguments have also been made in recent testimony in front of the U.S. House of Representatives (2014), in support of efforts to further limit the reporting of information on past defaults. We will show that our model, while admittedly quite stylized, allows us to capture many of these arguments and will use it to assess the trade-offs between the positive and negative effects of forgetting.

The paper is organized as follows. In section II we present the model and the strategy sets of entrepreneurs and lenders. In the following section we show that a Markov Perfect

Equilibrium (MPE) always exists and characterize the equilibrium strategies at the MPE where social welfare is highest. In section IV we study the effects of introducing a forgetting clause on equilibrium outcomes and welfare, and derive conditions under which forgetting defaults is socially optimal. We relate then these findings to the empirical evidence and the policy debate surrounding the adoption of the FCRA. Section V discusses the robustness of the results to some key features of the model. Section VI concludes, and the proofs are in the Appendix.

Related Literature

Our model features the presence of principals and agents interacting repeatedly under conditions of both adverse selection and moral hazard, like other papers in the literature on reputation, such as Diamond (1989), Mailath and Samuelson (2001), Fishman and Rob (2005), among others. The equilibrium in our model shares many similarities with the ones in these papers, in that agents value their reputation and try to build it over time. There are nevertheless some key differences between our model and theirs — in both the setting and in the structure of markets and information. Here we mention one in particular.¹¹ In our setup, riskier borrowers are unable to perfectly imitate safer ones; that is, whatever effort they choose, the probability of success of their projects will be lower than for the safer types. This plays an important role, as we will see, in generating the property that incentives are always stronger the higher an agent’s reputation and hence in generating the benefits of a fresh start. By contrast, many of the reputation models previously studied in the literature do not have this property, and instead the strategic type of agent can always choose an action which allows him to perfectly replicate the observed outcome of the action of the other (‘commitment’) type; thus incentives worsen when reputation becomes too good.

This is also the case for the few papers that explore the effects on agents’ behavior of limiting information about past outcomes. Ekmekci (2011) shows that the presence of a central authority using a random, finite rating system to summarize the publicly available information over past play of the game can help to preserve the agents’ incentives to maintain a reputation, except at the highest possible rating. Similarly, Liu and Skrzypacz (2013) show that restricting the public history of past play to be finite in length can encourage players to exert high effort, for a finite number of periods, until they reach their maximum reputation level, which is then exploited, restarting the process again.¹² Also, unlike ours, neither of

¹¹See also Remark 3 and the example in section IV.

¹²Analogously, Jehiel and Samuelson (2012) show that incentives can be preserved if the full information

these papers has as its focus the effect of these restrictions on social welfare.

Along these lines, we should also cite Vercammen (1995), which is the closest in spirit to ours. Like us, he studies the effect on incentives of restricting the information available on borrowers' credit histories within a model of repeated lending under moral hazard and adverse selection. His setup is similar to those cited in the previous paragraph, and thus the primary benefit of forgetting in his model is to prevent the negative effect on incentives arising from reputation becoming *too good*. Finally, note that Vercammen's conclusions rely on an approximated solution of a numerical example (based on a few, rather strong simplifications). By contrast, in our paper a stronger reputation never has a negative effect on incentives, and the beneficial effects of forgetting are quite distinct from those in the papers above. In our analysis, forgetting helps the agents who have failed (those with the worst reputation), who have difficulty being financed, by giving them the chance for a "fresh start", thus capturing a central point in the policy debate surrounding this issue. Also, our characterization of forgetting — in which only failures are erased — seems to be closer to the institutional details of credit bureau regulation in the United States, in which failures are indeed erased, while successes may be reported forever.

The benefits of limiting the availability of information on borrowers' past histories have also been explored in a few other papers using different arguments and models, as Padilla and Pagano (2000) and Pagano and Jappelli (2003). For instance, in the second paper the authors argue that if the credit bureau retains information indefinitely, the potentially harsh consequences of failing might deter risk-averse borrowers from ever undertaking entrepreneurial projects. Our paper, by contrast, does not rely on borrowers' risk aversion, and entrepreneurs always find financing desirable.

We should also recall that the positive effects that a credit bureau can have by increasing the information publicly available on borrowers' histories have been widely discussed. Pagano and Jappelli (1993) focus on lenders' incentives to voluntarily share information, while Padilla and Pagano (1997) show that a credit bureau may benefit borrowers by making it more difficult for lenders to exert monopoly power. Empirical work by Djankov, McLiesh, and Shleifer (2009), Brown, Jappelli, and Pagano (2009), and Jappelli and Pagano (2002) has also found that credit bureaus are positively associated with increased credit.

As discussed above, the beneficial effects of credit bureaus also arise in our model. However, we also address the question of when social welfare may be improved by *limiting* the information such a bureau can report. Also, in our set-up lenders are short-lived and have no

on agents past actions is not used when calculating an agent's reputation.

direct information over borrowers' credit histories, so that the information publicly provided by a credit bureau has no effect on competition among lenders. With long-lived lenders some of them may have superior information about a borrower, due to a previous experience with this agent, and this could be a source of market power to be taken into account when evaluating the effects of limiting the information publicly provided by credit bureaus.¹³

Finally, while our paper and the ones mentioned above consider the effect of restricting credit histories on entrepreneurs' incentives and access to credit in a production economy, Chatterjee, Corbae, and Rios-Rull (2007) study a calibrated version of a model in which risk-averse consumers borrow in order to insure themselves against income risk and weigh the benefits of defaulting against its reputational costs.

II The Model

Our analysis features a model with three key ingredients.

First, information about borrowers' financing history (their record of repayments and defaults on past loans) matters because it affects their reputation, their incentives and hence their access to credit. In our environment entrepreneurs are potentially infinitely lived, and need to borrow in each period to finance their project, whose probability of success depends both on their type and their effort.

Second, lenders are short lived and can only obtain such information from an external agency (a *credit bureau*) that can suppress part of this information (in particular, negative information) without lenders being able to infer it in other ways. This ensures that forgetting policies which limit the information available to lenders on borrowers' past defaults actually matter.

Third, entrepreneurs' incentives become stronger as their reputation improves. This will mean, as we will see, that the removal of bankruptcy filing from an entrepreneur's credit record, by improving his reputation, can also help his incentives.

These ingredients capture some key features of the empirical evidence. In particular, as already discussed in the introduction, both Musto (2004) and Bos and Nakamura (2013) show that when negative information is removed from an individual's credit record, he is able to significantly expand his borrowing, suggesting that the information provided by the credit bureau is salient, and that forgetting policies do indeed affect access to credit. Finally, while various specific features of our model ensure the presence of these ingredients, as we

¹³See also the discussion of long term contracts in Section V.C.

explain in what follows, quite a few of them could be dispensed in a richer model (where, however, the analysis would be less transparent).

Entrepreneurs

We consider an economy in which a continuum (of unit mass) of risk-neutral *entrepreneurs* is born in each period $z \in \mathbb{Z}$ (that is, time in the economy runs from $-\infty$ to $+\infty$). The entrepreneurs born at date z form generation z , and generations are all identical. Any entrepreneur of generation z has a constant probability $(1 - \delta) \in (0, 1)$ of dying at the end of each period, whatever the date z of his birth. This is a discrete-time perpetual youth environment along the lines of Frenkel and Razin (1986), into which we embed a dynamic model of reputation and incentives.

At the beginning of each period in which he is alive an entrepreneur is endowed with a new project, which requires one unit of financing in order to be undertaken. This project yields either R (success) or 0 (failure) at the end of the same period. Entrepreneurs have no resources of their own and output is non-storable, so they must seek external financing in each period. Entrepreneurs discount the future at the rate $\beta \leq 1$. Hence their “effective discount rate,” which also takes into account the probability δ of survival, is $\tilde{\beta} = \delta \cdot \beta$.

We assume that there are two types of entrepreneurs. In each generation z there is a fraction $s_0 \in (0, 1)$ of *safe* agents whose projects always succeed (i.e., their return is R with probability one), and the remaining fraction $1 - s_0$ of *risky* agents, for whom the project may fail with some positive probability.¹⁴ The returns on the risky agents’ projects are independently and identically distributed among them and the success probability of a risky agent depends on his effort choice. If he chooses to exert high (h) effort, incurring a utility cost $c > 0$, the success probability will be $\pi_h \in (0, 1)$. Hence his utility within a period, when his net revenue is x , is given by $x - c$. Alternatively, if he chooses to exert low (l) effort, this is costless, but the success probability under low effort is only $\pi_l \in (0, \pi_h)$. Both an entrepreneur’s type, as well as his age¹⁵ and the effort he undertakes, are his private information. We make the following assumption:

Assumption 1. $\pi_h R - 1 > c$, $\pi_l R < 1$;

i.e., the project has a positive NPV if high effort is exerted (even when the cost of exerting high effort is taken into account), while it has a negative NPV under low effort.

¹⁴The property that safe types never fail is adopted here for simplicity and will be relaxed in the example in Section IV.

¹⁵The assumption that age is not observable by lenders is in line with the provisions of the Equal Credit Opportunity Act (ECOA). While age can be used for marketing purposes, the ECOA prevents its use as a criterion for actually granting credit.

In addition, we require the cost of effort c to be sufficiently high that reputation matters for incentives. The following condition implies, as we will see, that when the entrepreneur is known for certain to be a risky type, high effort cannot be implemented in a static framework and hence, given Assumption 1, it is not profitable to finance him.

Assumption 2. $\frac{c}{\pi_h - \pi_l} > R - 1/\pi_h$

Lenders

Each period there are also $N \geq 2$ risk-neutral lenders, each of whom has access to an unlimited amount of funding at an intra-period gross interest rate of 1, lives only a single period and is replaced by a new lender in the following period. Thus he can only offer funding to entrepreneurs via short-term contracts, simply described by the gross interest rate r at which one unit of financing is offered at the beginning of a period, with repayment due at the end of the same period. The fact that borrowers face a different lender in each period is consistent with actual practice in U.S. credit markets (particularly for consumer credit), where borrowers often switch lenders. Furthermore, as we discuss below (see Section V.C), allowing long-term contracts would result in outcomes that are more extreme and less realistic, and also yield a lower level of total surplus, than those we obtain in the situation considered here.

If the entrepreneur accepts financing and the project succeeds, the entrepreneur makes the required interest payment r — up to his revenue R — to the lender. On the other hand, if the project fails, the entrepreneur is unable to make any payment and, therefore, defaults on the loan. We assume that there is limited liability, and the debt is forgiven in case of default (i.e., discharged). So with no loss of generality, r can be taken to lie in $[0, R] \cup \emptyset$ (with \emptyset indicating that no contract is offered).

The Credit Bureau

The loan market is characterized by the presence of both adverse selection and moral hazard. The history of past financing decisions and hence of project outcomes and loan repayments of an entrepreneur may convey some information regarding his type and may, therefore, affect the contracts he will receive in the future. Since lenders do not live beyond the current period, we assume that there is a *credit bureau* that records this information in every period and makes a *credit history* available to future lenders. A credit history specifies the number t of periods in which an entrepreneur is known to have been financed in the past, as well as the ordered sequence of outcomes (given by success or failure) of the entrepreneur's

projects in those periods. Hence a credit history of length t is denoted by $\sigma_t \in \Sigma_t \equiv \{S, F\}^t$. Observe that the bureau does not keep a record of periods in which the borrower is not financed, nor of his age.

In addition to entrepreneurs' credit histories, lenders also have access to information on the set of contracts offered in the past. More precisely, they know the set of contracts that were *offered* to borrowers in the past but not the particular contracts that were *chosen* by an *individual* borrower. This is in line with actual practice; while credit bureaus do not report the actual contracts adopted by individual borrowers, the set of contracts generally offered in the past to borrowers with different credit histories is available from databases such as "Comperemedia."

In the environment described, at any given date all lenders face entrepreneurs of different types and ages, who can be distinguished only by the information reported by the credit bureau.¹⁶ Hence the information provided by the credit bureau matters, and information suppressed from an entrepreneur's credit history cannot be inferred in other ways, in line with the first two key ingredients outlined at the beginning of this section.

Forgetting Policy

The focus of our paper is on the effect of restrictions on the information transmitted by credit bureau records. We refer to such a restriction as a *forgetting policy* and model it as follows. Consider an entrepreneur i , with credit history σ_t^i at the beginning of some period z , and whose project has failed at the end of the period. With probability q , the failure of this project is not recorded and hence the borrower proceeds to the following period with an unchanged credit history σ_t^i , just as if the loan never took place. Since his credit history is now shorter than that of agents who had his same credit history at the beginning of period z , but whose projects did not fail, the borrower is now pooled with entrepreneurs that at the beginning of period z had a shorter credit history — for instance, those belonging to the next generation. Figure 2 illustrates the evolution of credit histories under this model of forgetting.

The forgetting policy in the economy is then described by the parameter $q \in [0, 1]$. Note that we take q as being fixed over time, which is in line with existing laws. As we will see in what follows, by pooling together entrepreneurs with different histories of successes and failures, the forgetting policy affects the terms of credit and hence the incentives of

¹⁶Since output is not storable, at the beginning of each period all entrepreneurs have the same initial wealth. The non-storability of output and the unobservability of the entrepreneur's age could be dropped in a richer environment, where other sources of uncertainty are present, so that wealth and age would not suffice to make a perfect inference over a borrower's past history.

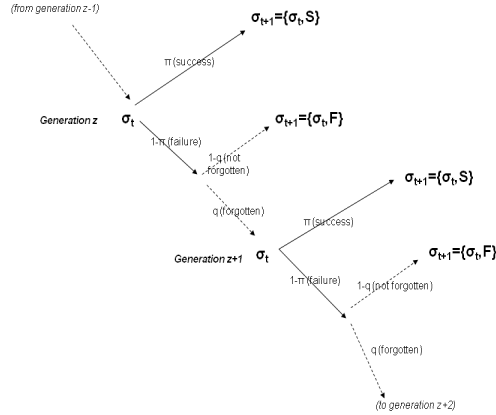


Figure 2: The Evolution of Credit Histories

entrepreneurs and contract offers by lenders. The main objective of our analysis is to study and evaluate these effects.

Our representation of forgetting is clearly stylized, but we believe that it captures the essential feature of such policies as implemented in the United States. In particular, credit bureaus do indeed erase the entire record of a bad account when the statute dictates that such negative information can no longer be reported — exactly as in our paper. Also, just as in this paper, only negative information is erased; positive information is reported indefinitely.¹⁷ The main difference between our formulation and actual practice is that, in the latter case, defaults are erased with the passage of time, rather than probabilistically.¹⁸ However, the consequences of higher values of the forgetting probability q are analogous to those of allowing for a shorter period until negative information is forgotten,¹⁹ using q makes the analysis more tractable and provides us with a continuous parameterization of the forgetting policy.

Timeline

The timeline of a single period is then as follows. At the beginning of each period lenders simultaneously post the rate at which they are willing to lend 1 unit to entrepreneurs with credit history σ_t , and do so for all possible credit histories, $\sigma_t \in \Sigma_t$, for all t . At the same

¹⁷For example, the FICO scoring algorithm places considerable positive weight on the age of the oldest account still open. Both in practice, and in our model, this is because having an account open for a long time suggests many periods of financing without a default.

¹⁸A similar, probabilistic approach to credit bureau regulation is also taken by Padilla and Pagano (2000).

¹⁹See Remark 4 for further discussion of this point.

time, the risky entrepreneurs choose their effort level — and incur the associated effort cost — basing their choice on the contracts they anticipate will be offered that period. Next, each entrepreneur — both safe and risky — after observing the loans offered to him, chooses one of them (or none). If an entrepreneur is offered financing, and chooses one of the loans he is offered, he undertakes the project (funds lent cannot be diverted to consumption). The outcome of the project is realized at the end of the same period: if the project succeeds, the entrepreneur uses the revenue R to make the required payment r to the lender, while if the project fails, the entrepreneur defaults and makes no payment. The credit history of the entrepreneur is then updated. If the project was financed, a S is added to his credit history when the project succeeded and the loan was repaid, a F is added when it failed and the borrower defaulted, and this default was not forgotten. If the project was not financed, or it failed and that failure was forgotten (which occurs with probability q), his credit history is left unchanged.

Next period, the same sequence is repeated: for each updated credit history, lenders choose the contracts they will offer and the risky entrepreneurs make their effort choice, then each entrepreneur freely chooses which contract to accept among the ones he is offered, if any, and so on for every z .

Strategies

Given the agents' information and the timing of events described above, strategies are as follows. The public history at the beginning of each period z is given by the set of contracts $\mathcal{C}_{z'}(\sigma_t)$ offered at any previous date $z' < z$, by the N lenders, to entrepreneurs with credit history σ_t , for all σ_t, t , as well as by the credit history σ_t^i of each entrepreneur i present at that date. At any given date and for each possible history, a lender's strategy consists in the choice of the contracts offered to entrepreneurs, while the strategy of an entrepreneur specifies, for any set of contracts presently offered by the lenders, the contract — if any — that is chosen. If the entrepreneur is risky, his strategy also specifies his choice of effort level.

To evaluate the expected profit of a loan offered by a lender to an entrepreneur with credit history σ_t , at date z , an important role is played by the lenders' belief, $p_z(\sigma_t)$, that the entrepreneur is a *safe* type. We term this the *credit score* of the entrepreneur. This belief is computed by lenders on the basis of the public history and the entrepreneurs' effort strategies, as we describe in the next section.

Remark 1. (Which Markets?) The market for consumer credit (particularly credit cards) is probably the one whose features are the closest to the environment described. It is characterized by many anonymous borrowers entering short-term contracts with

various lenders, and information about potential borrowers is primarily obtained from credit bureaus. Also, the empirical evidence suggests the presence of agents with different default probabilities. As mentioned in the Introduction, Musto (2004), looking at the market for consumer credit, finds that those agents who default and have then this information erased from their credit record are more likely to default again.

To indicate possible values for the fraction of safe and risky entrepreneurs and the probability of default in our model in line with the evidence, we can use data from the FRBNY/Equifax Consumer Credit Panel, restricting attention to those consumers who were between the ages of 20 and 25 in 2003, and had no credit cards (i.e., with a “blank” credit history). In 2004, 19 percent of those who obtained a credit card were in default. Of those who did not default in 2004, the subsequent default rate in 2005 fell to 13.69 percent. Fitting this data to our model, we obtain an initial fraction of safe entrepreneurs of 0.48, and a default probability of 37 percent for the risky entrepreneurs.²⁰

Another setting in which there appears to be some evidence of the presence of safe and risky borrowers is entrepreneurship. Gompers et al (2010) show that entrepreneurs who have a past history of successes are more likely to pick the best time and industry in which to start new ventures, and have a 20 percent higher success rate in the future.

Remark 2. (Timing of Effort Choice) We assumed that risky entrepreneurs choose their effort level at the beginning of each period. Since this is before the contracts are actually offered by lenders in that period, this effort choice is made on the basis of their expectation over the contract that will be offered. This assumption implies that entrepreneurs cannot instantaneously adjust their effort in response to a deviation by a lender. It means that the mapping from credit histories to lenders’ beliefs regarding the type of entrepreneur they face is not affected by the deviation, and allows to keep the analysis tractable. Otherwise, if the entrepreneurs who faced this deviation could modify their effort, they would still be pooled with entrepreneurs from different generations who end up with the same credit history but did not face this deviation and hence exerted a different effort level; this would make lenders’ inference more complex and expand the dimensionality of the problem considerably.

We should point out that this assumption on the timing of effort is not needed if one uses a simpler, but also somewhat less “realistic” specification of forgetting, where a forgotten failure is recorded as a success (rather than as a non-event as in the model in

²⁰These are the probabilities of default for borrowers with histories \emptyset and $\{S\}$, respectively, calculated using equations (5) and (6) below, and assuming $q = 0$. For background on the FRBNY/Equifax Consumer Credit Panel, see Lee and van der Klaauw (2010).

the current paper). With this simpler specification the length of the credit history of an entrepreneur who received financing every period is always equal to the entrepreneur’s age. This case has been considered in an earlier version of the paper (Elul and Gottardi, 2009) using a simpler environment where all entrepreneurs are born at the same time. We find that an equilibrium with analogous properties to those obtained in Proposition 1 in the next section exists, and that the main qualitative results concerning the optimal forgetting policy also remain valid.

III Equilibrium

III.A Markov Perfect Equilibrium

In what follows we will focus on stationary *Markov Perfect Equilibria* (MPE) in which players’ strategies optimally depend on history only through the entrepreneurs’ credit scores, and do not depend on the date z . A key appeal of such equilibria is not only that players’ strategies are simpler, but also that they resemble actual practice in consumer credit markets, where lending decisions are primarily conditioned on credit scores, most notably the “FICO score” developed by Fair Isaac and Company. In addition, we will show in what follows that the credit score $p(\sigma_t^i)$ of an arbitrary entrepreneur i , is a sufficient statistic for his credit history σ_t^i , at all nodes in which this history has no defaults; hence it summarizes the payoff-relevant component of an entrepreneur’s credit history. The differences between MPE and other equilibria are discussed in section V.B, where we will argue that the welfare effects of forgetting are similar for those equilibria.

In particular, we will establish the existence and analyze the properties of *stationary, symmetric, sequential* MPE, where (i) all agents of a given type (i.e., all lenders, or all safe entrepreneurs, or all risky entrepreneurs with the same credit score) optimally choose the same strategy, at any date z , (ii) beliefs are determined by Bayes’ Rule whenever possible and, when this is not possible, they must be consistent in the Sequential Perfect Equilibrium sense. We now more formally describe the agents’ strategies and choice problems for the stationary MPE we consider.

The strategy of a lender specifying which contract, if any, is offered to an entrepreneur with credit score p is denoted by $r(p) \in [0, R] \cup \emptyset$. Along the equilibrium path the set of contracts offered by lenders to borrowers with credit score p is then simply given by $r(p)$.

A safe entrepreneur’s strategy $r^s(p, \mathcal{C}') \in \mathcal{C}' \cup \emptyset$ indicates, for any credit score p and any set of contracts \mathcal{C}' currently offered to entrepreneurs with such credit score, which contract is chosen ($r^s = \emptyset$ indicates that no contract is chosen). It is obtained as a solution to the

following problem:

$$v^s(p, \mathcal{C}') = \max_{r \in \mathcal{C}' \cup \emptyset} \begin{cases} R - r + \tilde{\beta}v^s(p^S(p)), & \text{if } r \neq \emptyset; \\ \tilde{\beta}v^s(p^\emptyset(p)), & \text{if } r = \emptyset. \end{cases} \quad (1a)$$

where $p^S(p)$ specifies the updated belief of lenders in case of success of the project at the end of the period and $p^\emptyset(p)$ the updated belief if the agent was not financed, or failed and had this failure forgotten. The value at the solution, $v^s(p, \mathcal{C}')$, is the maximal discounted expected utility attainable by the entrepreneur starting from a date where his credit score is p and the contracts offered are \mathcal{C}' . The term $R - r$ on the first line of (1a) represents the payoff from the current project if a loan is accepted (recall that the project succeeds for sure for this type of entrepreneur) while the second term is the discounted continuation utility in this case, obtained by solving problem (1a) recursively (when contract offers are as in the equilibrium path). In the second line of (1a) we have the discounted continuation utility if either no loan is made or accepted. Note that the difference between these two continuation utilities reflects the fact that current financing decision and the outcome of the project affect how lenders update their beliefs concerning the entrepreneur's type, and hence the contracts the entrepreneur will be offered in the future.

A strategy of a risky entrepreneur with credit score p regarding the choice of the contract $r^r(p, \mathcal{C}')$ solves the analogous problem:

$$v^r(p, \mathcal{C}') = \max_{r \in \mathcal{C}' \cup \emptyset} \begin{cases} [e^r(p)\pi_h + (1 - e^r(p))\pi_l] [(R - r) + \tilde{\beta}v^r(p^S(p))] - e^r(p)c \\ + \tilde{\beta}[e^r(p)(1 - \pi_h) + (1 - e^r(p))(1 - \pi_l)] [qv^r(p^\emptyset(p)) + (1 - q)v^r(p^F(p))], & \text{if } r \neq \emptyset; \\ \tilde{\beta}v^r(p^\emptyset(p)) - e^r(p)c, & \text{if } r = \emptyset. \end{cases} \quad (1b)$$

where $p^F(p)$ denotes the updated belief of lenders in case of a failure (that is not forgotten) of an entrepreneur's project and $e^r(p)$ the strategy specifying the effort exerted by the entrepreneur. We allow for mixed strategies with regard to the effort choice, hence $e^r(p) \in [0, 1]$ indicates the probability with which high effort is exerted. Given the timing assumption made in the previous section $e^r(\cdot)$ only depends on p and is obtained — together with $r^r(p)$ — by maximizing with respect to e and r an expression analogous to (1b) with $\mathcal{C}' = r(p)$, that is, when the contracts offered are as anticipated on the equilibrium path. The term $e^r(p)\pi_h + (1 - e^r(p))\pi_l$ in the first line of (1b) gives the success probability of a risky entrepreneur with effort strategy $e^r(p)$. In the same line the utility cost of effort appears with a minus sign. Since the project can now fail, in the second line of (1b) we

also have a term giving the expected utility in that case (when the project fails the payoff of the current project is zero and the continuation utility depends on whether the failure is forgotten ($v^r(p^\emptyset(p))$) or not ($v^r(p^F(p))$)).

To derive some properties of the solutions of problems (1a) and (1b), note first that the posterior beliefs in the events of a recorded failure and non-financing are easily determined:

Observation 1. Since only risky entrepreneurs can fail, $p^F(p) = 0$ for all p . By the Markov property of agents' strategies, we also have $p^\emptyset(p) = p$ for all p .

This property — that an entrepreneur who fails is unambiguously identified as risky — plays an important role in determining the third key ingredient of our model, that incentives get stronger as reputation improves, because it implies that a risky entrepreneur with a better reputation has more to lose from a default. The other important aspect of the model in this regard is the fact that risky entrepreneurs always fail with higher probability than the safe agents. This implies that the better their reputation, the lower the interest rate they face.

In addition, since lenders cannot observe the specific contract chosen by an individual borrower in any past round of financing, but only the sequence of past successes and (non forgotten) failures, the entrepreneurs' contract choice problem has a simple solution:

Observation 2. Whenever an entrepreneur accepts financing, he will choose the contract with the lowest interest rate: i.e., for all p, C' we have $r^j(p, C') \in \{\min r \in C'\} \cup \emptyset$, for $j = s, r$. Also, since $p^\emptyset(p) = p$ entrepreneurs never refuse financing on the equilibrium path: $r^j(p) \neq \emptyset$ for $j = s, r$, whenever $r(p) \neq \emptyset$.

Next, we examine the problem of an arbitrary lender n who chooses the interest rate r he offers to entrepreneurs with credit score p , so as to maximize his profit, given the entrepreneurs' strategies, $r^s(\cdot)$, $r^r(\cdot)$, and $e^r(\cdot)$, and the strategies of the other lenders. Given our focus on stationary symmetric MPE, the contracts offered by the other lenders consist of the single contract $r(p)$.

As established in Observation 2, all entrepreneurs with credit score p accept the lowest rate offered. So if lender n 's offer is lower than that of the other lenders ($r < r(p)$) he gains the entire market and his profits per unit offered are:

$$\Pi(r, p, r(p), r^s(\cdot), r^r(\cdot)) = [p + (1 - p)\pi_{e^r(p)}] r - 1, \quad (2)$$

where $\pi_e^r(p) \equiv e^r(p)\pi_h + (1 - e^r(p))\pi_l$. On the other hand, if he offers the same rate as all of the other lenders ($r = r(p)$) he shares the market with the other $N - 1$ lenders and n 's

profits are given by the expression on the right hand side of (2) divided by the number of lenders N . If $r > r(p)$ then n 's offer is not accepted and his profits are zero. Since a lender lives only a single period, his objective is to choose r so as to maximize his expected profits as described above.

It remains to specify the initial belief p_0 , corresponding to an entrepreneur with an empty credit history. We say that p_0 is *correctly specified* if it is equal to the proportion $p_0(s_0, q)$ of safe agents amongst all those with empty credit history. While in the absence of forgetting ($q = 0$) this is equal to the fraction s_0 of safe agents born in any generation, with forgetting this is endogenously determined in equilibrium (since some of those agents with empty credit history are risky entrepreneurs who were born in previous periods, failed in all of their projects, but had these failures forgotten). The exact expression for $p_0(s_0, q)$ is given in equation (5) below.

We are now ready to give a formal definition of the equilibrium we consider:

Definition 1. *A symmetric, stationary sequential Markov Perfect Equilibrium is a collection of lenders' and borrowers' strategies $(r(\cdot), r^s(\cdot), r^r(\cdot), e^r(\cdot))$ and beliefs $(p_0, p^S(\cdot), p^F(\cdot), p^\emptyset(\cdot))$, such that:*

- *Lenders maximize their total expected net revenue: for every p , $r = r(p)$ maximizes a lender's profits, when the other lenders also offer $r(p)$;*
- *Entrepreneurs' strategies are sequentially rational. That is,*
 - *for all p, C' , $r^r(p, C')$ solves (1b) and $r^s(p, C')$ solves (1a).*
 - *for all p , $(e^r(p), r^r(p))$ maximize (1b) when $C' = r(p)$.*
- *Initial beliefs are correctly specified, and the updated beliefs are computed via Bayes' Rule whenever possible and are consistent otherwise.*

Notice that all entrepreneurs with the same credit history cannot be distinguished and are offered the same contract. Since each of them always chooses the lowest rate offered (and never refuses financing), the risky entrepreneurs, in particular, are thus pooled with the safe ones until they experience a failure that is not forgotten. Thus we never have separation in equilibrium.

III.B Existence and Characterization of Equilibrium

The following proposition establishes that a Markov Perfect Equilibrium exists. The proof is constructive, and so we also derive the properties of an MPE. In Proposition 2 we show that the equilibrium we characterize is the MPE that maximizes total welfare.²¹

Let $r_{zp}(p, e)$ denote the lowest interest rate consistent with lenders' expected profits being non-negative on a loan to entrepreneurs with credit score p , when all agents accept financing at this rate and risky entrepreneurs exert effort e :

$$r_{zp}(p, e) \equiv \frac{1}{p + (1-p)(e\pi_h + (1-e)\pi_l)}. \quad (3)$$

Note that $r_{zp}(p, e)$ is decreasing in both p and e and is larger than R when p and e are sufficiently close to zero (by Assumption 1). The lowest value of p for which this break-even rate is admissible when the risky entrepreneurs exert low effort ($e = 0$) is $p_{NF} \equiv \frac{1-\pi_l R}{(1-\pi_l)R}$, such that $r_{zp}(p_{NF}, 0) = R$. By contrast, when the risky entrepreneurs exert high effort ($e = 1$), $r_{zp}(p, 1) \leq R$ for all p ; i.e., lenders always break even.

Proposition 1. *Under assumptions 1-3, a (symmetric, stationary, sequential) Markov Perfect Equilibrium always exists with the following properties:*

- i. *Lenders make zero profits in equilibrium: either $r(p) = r_{zp}(p, e^r(p))$, or $r(p) = \emptyset$.*
- ii. *Lenders never offer financing to entrepreneurs known to be risky with probability 1: $r(0) = \emptyset$, and so $v^r(0) = 0$.*
- iii. *Lending and effort strategies are as follows:*
 - a. *When the cost of effort c is high, that is if $\frac{(R-1)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)} \leq \frac{c}{\pi_h-\pi_l}$, an entrepreneur is financed if, and only if, $p \geq p_{NF}$ and if risky exerts low effort ($e^r(p) = 0$)*
 - b. *For intermediate values of the cost of effort, $\frac{(R-1/\pi_h)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)} < \frac{c}{\pi_h-\pi_l} < \frac{(R-1)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)}$, there exists $0 < p_l \leq p_m \leq p_h < 1$ (with $p_l \leq p_{NF}$) such that:*
 - *there is financing if and only if $p \geq p_l$*
 - *risky entrepreneurs exert high effort if $p \geq p_h$, low effort if $p \in [p_l, p_m)$, and mix between high and low effort for $p \in [p_m, p_h)$ (with $e^r(p)$ strictly increasing for $p \in [p_m, p_h)$).*

²¹We discuss other MPE in section V.A below.

c. When the cost of effort is low, $\frac{c}{\pi_h - \pi_l} \leq \frac{(R-1/\pi_h)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)}$, there is financing for all $p > 0$, and risky entrepreneurs exert high effort ($e^r(p) = 1$).

Note that we do not restrict players' strategies to be Markov. Rather, we show in the proof that if all other players adopt a strategy in which history enters only through the credit score p , the optimal response of any player is to also adopt a Markov strategy.

Recall that the third key ingredient of our model is that incentives become stronger in the MPE we consider as p increases, i.e., $e^r(p)$ is weakly increasing in p , for all q . To get some intuition for this, consider the incentive compatibility condition that must be satisfied for the risky entrepreneurs to exert high effort in the equilibrium of Proposition 1:²²

$$\frac{c}{\pi_h - \pi_l} \leq R - r_{zp}(p, 1) + \tilde{\beta} [v^r(p^S(p)) - qv^r(p)], \quad (4)$$

where we use the property, also established in the Proposition, that $v^r(0) = 0$. Recall that $r_{zp}(p, e)$ is decreasing in p (holding e fixed). Thus the higher is p , the lower the interest rate offered to the entrepreneur and hence the higher the current payment he receives in case of success. We show in the proof of Proposition 1 that this implies that the righthand side of (4) is also increasing in p . Hence the cross-subsidization from safe to risky entrepreneurs that occurs in equilibrium, where safe and risky entrepreneurs with the same credit score are pooled together, has a beneficial effect on incentives, since the higher p , the larger is this cross-subsidy to any single risky entrepreneur who is financed.

The characterization of the MPE provided in part *iii.* of the Proposition shows how its properties vary with the severity of the incentive problem, as captured by the effort cost c . This is illustrated in Figure 3.²³ When c is high (region a.), incentives are weak, and risky entrepreneurs exert low effort whenever they are financed. Nevertheless, financing is still profitable for the lenders and occurs as long as p is not too low ($p > p_{NF}$). By contrast, when c is low (region c.), incentives are strong enough that the risky entrepreneurs exert high effort for all $p > 0$ and hence financing is profitable for all $p > 0$. The more interesting case occurs for intermediate values of c (region b.), where incentives depend on p . When p is sufficiently high ($p \geq p_m$), interest rates (both current and future) are low, which makes incentives strong enough that high effort can be sustained. By contrast, when $p < p_m$,

²²This expression is obtained by requiring that the utility the risky entrepreneur receives from high effort be weakly greater than that from low effort, with the risky entrepreneur's utility obtained from (1b), using the values of $r(p)$ and $v^r(p)$ at the MPE described. For further details see also the proof of Lemma 1 below.

²³More precisely, the bounds on the different regions in this figure are specified in terms of the normalized effort cost $c/(\pi_h - \pi_l)$. The lower bound on $c/(\pi_h - \pi_l)$ in the figure follows from Assumption 2.

interest rates are not sufficiently low to sustain high effort. Moreover, when p is particularly low ($p < p_l$), it is not feasible for lenders to break even, just as in region a., and therefore no financing is granted.

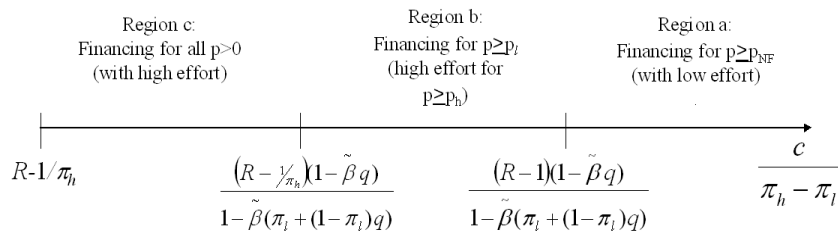


Figure 3: Equilibrium regions

Figure 4 illustrates the equilibrium outcomes obtained in region b., for different values of the credit score p . Recall that $0 < p_l \leq p_m \leq p_h < 1$, so the low-effort and mixing regions may be empty, while the high-effort and no-financing regions always exist.

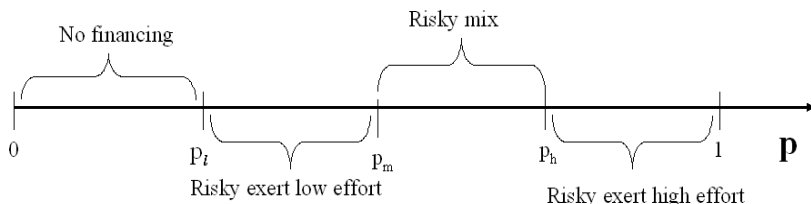


Figure 4: Financing pattern in region b.

A Markov Perfect Equilibrium requires that lenders use Bayes' Rule to update their beliefs whenever possible. The updated beliefs in case of failure, p^F , and no financing, p^0 , have already been determined in Observation 1. We now specify the expression for $p_0(s_0, q)$, the initial belief for an entrepreneur with an empty credit history,²⁴ and $p^S(p)$, the beliefs in case of success, along the equilibrium path.²⁵

²⁴This expression applies when such entrepreneurs are financed in equilibrium; when they are not financed, we clearly have $p_0 = s_0$ instead.

²⁵The numerator in each expression corresponds to the fraction of agents with a given credit history who are safe types, whereas the denominator includes all the agents with this credit history (some of whom may be risky entrepreneurs from other generations who failed in the past, but had this failure erased from their record). The precise derivation of these expressions is somewhat complex, and can be found in Appendix B (available at http://www.elul.org/papers/forget/appendix_b.pdf).

$$p_0(s_0, q) = s_0 \left[\frac{1 - (1 - \pi_{e^r(p_0)})\delta q}{1 - s_0(1 - \pi_{e^r(p_0)})\delta q} \right], \text{ and} \quad (5)$$

$$p^S(p) = \frac{p}{p + (1 - p) \frac{\pi_{e^r(p)}}{1 - (1 - \pi_{e^r(p^S(p))})\delta q}}. \quad (6)$$

We see from (5) that p_0 is increasing in the measure of safe entrepreneurs s_0 born in each period. Also, both p_0 and $p^S(p)$ are decreasing in the probability of forgetting q , since with more forgetting the likelihood that a risky entrepreneur, with the same credit history but from a different generation, fails in any given period and has this failure forgotten increases. We will also show in the proof of Proposition 1 that $e^r(p^S(p)) \geq e^r(p)$, and so (6) implies that $p^S(p) > p$. That is, the credit score p is strictly increasing in the length of the string of successes and is thus a sufficient statistic for all credit histories σ_t with no failures, as claimed.

We are now ready to prove Proposition 1. We first establish property ii. — that entrepreneurs who are known to be risky are never financed — and show that this is actually a general property of Markov equilibria.

Lemma 1. *Under Assumptions 1 and 2, any Markov Perfect Equilibrium is characterized by no financing when $p = 0$: i.e., $r(0) = \emptyset$ and hence $v^r(0) = 0$.*

The basic intuition is that once $p = 0$, an entrepreneur's credit score remains the same regardless of the outcome of any future project. Hence in any Markov Perfect Equilibrium his continuation utility is also the same, which reduces his incentive problem to a static one, for which we showed that financing cannot occur (Assumption 2). This result implies that, in equilibrium, any entrepreneur who fails is excluded forever from financing (unless this failure is “forgotten”). Hence the discharge of the liability of the borrower in bankruptcy is not sufficient to restore access to credit for him, as the weakening in his incentives due to the fall in his reputation is too severe. To give a defaulting entrepreneur a real fresh start forgetting is also needed.

The rest of the proof of Proposition 1 is in the Appendix.

Next, we show that the equilibrium characterized in Proposition 1 is the MPE yielding the highest welfare, provided δq is not too large (hence in particular when q is close to 0). The welfare criterion we consider is the total surplus generated by the entrepreneurs' projects that are financed; given agents' risk-neutrality, this is equivalent to the sum of the discounted expected utilities of all agents in the economy, including lenders.

Proposition 2. *When $\delta q < \frac{1-\pi_h}{1-\pi_l}$, the equilibrium constructed in Proposition 1 maximizes total surplus amongst all MPE.*

To prove the result, we first show that the equilibrium as constructed in the proof of Proposition 1 implements the highest possible effort at any p . This is clearly true for credit scores $p \geq p_h$, since high effort will be exerted in the current period, as well as in any future round of financing. The same is also true for $p < p_m$, as the risky entrepreneurs exert low effort if financed, and this is the maximal effort level that can be sustained. The result is completed by showing this is true even when $p \in [p_m, p_h)$, i.e., in the mixing region of Proposition 1.²⁶

IV Optimal Forgetting

In this section we derive conditions under which forgetting entrepreneurs' failures is a socially optimal policy. That is when, in the equilibrium characterized in Proposition 1, total surplus is higher when $q > 0$ than with $q = 0$.

What are the effects of the forgetting policy on the equilibrium properties? A first effect is to make the exclusion process of the risky types slower; hence risky entrepreneurs with initial credit scores sufficiently high that they are financed in the first period of their life will be financed for more periods. This is welfare improving when the risky types exert high effort (in region c.), since the welfare generated from each period of financing of a risky entrepreneur is $G \equiv \pi_h R - 1 - c > 0$. By contrast, in region a. it is welfare decreasing since they exert low effort, and the welfare generated in each period is $B \equiv \pi_l R - 1 < 0$.

A second effect of forgetting is the weakening of incentives, since the punishment following a default is lower. This can be seen from the fact that the term $qv^r(p)$ (corresponding to the expected continuation utility following a default), which appears with a minus sign on the righthand side of the incentive constraint for high effort, (4), goes up as q is increased. This is then reflected in the fact that the boundaries of regions a., b., and c. all shift to the left when q is increased (see Figure 3). This has a negative effect on welfare, since for the same credit score p , a risky entrepreneur may switch from high to low effort and the no-financing region expands.²⁷

²⁶The upper bound on δq ensures that p is increasing along the equilibrium path for *any* MPE (this is needed because the proof of Proposition 2 proceeds via backward induction on p).

²⁷For example, raising q can move the equilibrium from region c., where there is financing (with high effort) for all $p > 0$, to region b., where there is only financing for $p > p_l$ (and low effort for $p < p_m$).

Taken together, the above observations imply that in region a. forgetting is always welfare decreasing, since both of these effects on welfare are negative. As for region c., as long as the level of q is not too large (so that it does not induce a shift out of this region), forgetting will be welfare increasing because the first effect is positive while the second, negative one, is not present.²⁸

Let $q(s_0)$ denote the welfare maximizing level of q (which clearly depends on the proportion s_0 of safe types born in each period, as the equilibrium depends on it). Formally, we obtain:

Proposition 3. *The welfare maximizing forgetting policy for high and low values of c , respectively, is as follows:*

1. If $\frac{c}{\pi_h - \pi_l} \geq \frac{R-1}{1-\beta\pi_l}$, no forgetting is optimal for all s_0 : $q(s_0) = 0$.
2. If $\frac{c}{\pi_h - \pi_l} < \frac{R-1/\pi_h}{1-\beta\pi_l}$, for all $s_0 > 0$, some degree of forgetting is strictly optimal: $q(s_0) > 0$.

We now turn our attention to region b., that is, to intermediate values of c . An important feature of region b. is that the level of effort varies along the equilibrium path (switching at some point, after a sequence of successes, from low to high). The weakening of incentives due to forgetting now manifests itself not only in the change of the boundaries of this region, which again shift to the left as q increases, but also possibly in the change of the values of p along the equilibrium path at which the switch from low to high effort takes place, i.e., $p_m(q)$ and $p_h(q)$, and so there may be more periods of financing with low effort.²⁹ These switching points are key to the analysis of the welfare impact of raising q , since an extra period of financing with high effort makes a positive contribution to the social surplus, while one with low effort makes a negative contribution.

Notice first that when $p_0(s_0, 0) = s_0 > p_h(0)$, high effort is always exerted by a risky entrepreneur when financed. The situation is then similar to the one in region c., and an analogous argument to that used to prove part 2. of Proposition 3 establishes that the socially optimal level of q is above 0 in this case.

On the other hand, when $s_0 \in [p_{NF}, p_h(0)]$ raising q above 0, while leading to a lower probability of exclusion, does not necessarily increase welfare.³⁰ There is in fact a trade-off

²⁸Note that if positive — instead of negative — information were suppressed, this first, positive effect, would not be present, and hence forgetting would never be beneficial.

²⁹To highlight the dependence of these points (introduced in Proposition 1) on q , they are now written as functions of q .

³⁰When $s_0 < p_{NF}$ the borrower is not necessarily financed, in which case raising q will not affect welfare.

between the positive effect of forgetting when high effort is exerted (which happens after a sufficiently long string of successes allows the borrower's credit score p to exceed $p_h(q)$), and the negative effect when low effort is exerted (when $p < p_h(q)$). There are two facets to the negative effect just mentioned when $p < p_h(q)$. As discussed above, an agent whose failure is forgotten has an opportunity to exert low effort once again, which lowers the social surplus. In addition, a longer string of successes will be required until a risky entrepreneur switches to high effort, for several reasons. First, "the updating will be slower," that is, $p^S(p)$ will be closer to p . Also, $p_0(s_0, q)$ will be lower. Finally, failures are less costly with a higher level of q , and so $p_h(q)$ may also increase³¹, thus generating an additional welfare loss.

In the second part of the next Proposition, we establish that in region b. the positive effect of raising q prevails over the negative one when $s_0 \in [p_{NF}, p_h(0)]$, provided (i) $p_0(s_0, 1) \geq p_{NF}$, (ii) agents are sufficiently patient ($\tilde{\beta}$ close to 1), (iii) $|B|$ is sufficiently small relative to G , (iv) s_0 is sufficiently high, and (v) $p_h(0)$ is not too high. The first condition ensures that raising q (even up to $q = 1$) never pushes the initial credit score into the no-financing region, which would clearly be suboptimal. The remaining conditions are needed because the positive effect follows the negative one along the equilibrium path. In particular, (ii) and (iii) ensure that the welfare loss from the extra periods of low effort will be modest relative to the future benefit from more periods of financing under high effort, and (iv) and (v) limit the number of periods for which low effort will be exerted before switching to high effort. Thus we have:³²

Proposition 4. *For intermediate values of c , $\frac{R-1/\pi_h}{1-\tilde{\beta}\pi_l} \leq \frac{c}{\pi_h-\pi_l} < \frac{R-1}{1-\tilde{\beta}\pi_l}$, the optimal policy may also exhibit forgetting. More precisely:*

1. *If $s_0 > p_h(0)$, welfare is maximized at $q(s_0) > 0$.*
2. *If $s_0 \in [p_{NF}, p_h(0)]$, when $p_0(s_0, 1) \geq p_{NF}$, $\frac{p_h(0)(1-\pi_h^2)+\pi_h^2}{[1-\pi_h(1-\pi_h)(1-p_h(0))]} < \frac{\pi_l s_0((1-\pi_l)-(1-\pi_h)B/G)}{\pi_l s_0(1-\pi_l)-(1-\pi_h)(1-(1-\pi_l)s_0)B/G}$ and $\tilde{\beta}$ is sufficiently close to 1, we also have $q(s_0) > 0$.*

Figure 5 illustrates the welfare-maximizing forgetting policy, as derived in Propositions 3 and 4, as a function of the cost of effort c .

³¹This increase in p_h is indeed typically, although not always the case, because a higher value of q also increases the continuation utility upon success.

³²While the second inequality in part 2. of the Proposition is stated in terms of $p_h(0)$, an endogenous variable, we can show that the condition is satisfied for an open set of parameter values (see also the example below in the text). Let $\pi_l \rightarrow 1/R$. Then $B \rightarrow 0$, so that the term on the right-hand side of the second inequality approaches 1, and also $p_{NF} \rightarrow 0$, implying that both inequalities will be satisfied.

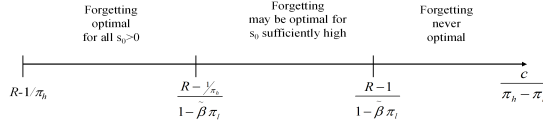


Figure 5: Welfare-maximizing forgetting policy, as a function of c

Remark 3. (Forgetting and the Fresh Start) The previous analysis shows that forgetting provides a benefit by strengthening entrepreneurs’ incentives following a default, allowing additional periods of financing with high effort, and so capturing the idea of a fresh start. Two elements of the model that play an important role in this regard are that the incentive problem only concerns the risky entrepreneurs, and these entrepreneurs have a higher probability of failure than the safe types, even when they exert high effort (i.e. $\pi_h < 1$). As a consequence, forgetting allows risky entrepreneurs who fail to pool anew with the safe entrepreneurs, giving them a lower interest rate than the one corresponding to their individual probability of default, and thus enhancing their incentives. Also recall that, as we have discussed above, simply discharging the liability of a defaulting borrower (“forgiving”) would not be sufficient to give this entrepreneur a fresh start.

This suggests that the benefits of forgetting would be less clear if the safe entrepreneurs also had an incentive problem, since for them pooling would imply a higher interest rate, which would hurt their incentives. Also, if high effort ensured instead certain success for the risky entrepreneurs ($\pi_h = 1$, just as for the safe types), as, for example, in Diamond (1989), default would only be due to misbehavior, whereas in our set-where it may also be ascribed to bad luck. It is then easy to verify that in such case forgetting would never be beneficial.³³

Remark 4. (Forgetting Policies) Consider an alternative specification of the forgetting policy, where a default is erased from the credit record τ periods after it occurs (rather than immediately with probability q). In such case the condition for high effort to be

³³When $\pi_h = 1$ it is easy to see that region b. is empty. If the cost of effort is high (i.e., we are in region a.), we already know from Proposition 3 that forgetting is not optimal. By contrast, for low values of c (belonging to region c.), risky entrepreneurs always exert high effort, hence never fail when $\pi_h = 1$, so in this case the forgetting policy is irrelevant.

incentive compatible would be analogous to (4), except for the fact that q would be replaced by $\tilde{\beta}^\tau$. The effect of forgetting on the updating of lenders' beliefs would also be similar: it would slow down the updating in case of success and boost beliefs upwards when the information about a default is eventually removed from a borrower's record (with analogous effects on incentives). Thus lenders' behavior would also be similar, and in this sense we can argue that the differences between these specifications of the forgetting rule lie primarily in different normalization choices.

As an alternative to forgetting, one could also consider allowing for "re-aging." That is, a borrower who defaults could be given the possibility of repaying his debt in full from his future income (possibly at a penalty interest rate). In that case the lender would in turn agree not to report the default. This would provide some of the benefits of forgetting, although its effectiveness would be limited by the future ability to pay of the borrower (and could also negatively impact his future incentives). Note that this would still have the effect of maintaining safe and risky entrepreneurs in the same pool of borrowers in the eyes of future lenders.³⁴

While the previous results give conditions under which some $q > 0$ maximizes total welfare, we can also determine when $q(s_0) = 1$, i.e., when welfare is maximized by keeping no record of any failure. This will be the case when the risky entrepreneurs exert high effort when financed even when $q = 1$. From Proposition 1 and Assumption 2 it is easy to see that region c. becomes empty as $q \rightarrow 1$, and so a sufficient set of conditions for $q = 1$ to be optimal is that we remain in region b. even when $q = 1$ (i.e., $\frac{c}{\pi_h - \pi_l} < R - 1$), and that we are in the high-effort portion of this region ($p_0(s_0, 1) \geq p_h(1)$), which will be the case when s_0 is sufficiently high. More precisely:

Proposition 5. *When $\frac{c}{\pi_h - \pi_l} < R - 1$, then $q = 1$ maximizes total welfare for $s_0 \geq \frac{\frac{c\pi_h}{\pi_h - \pi_l} - (\pi_h R - 1)}{(1 - \pi_h)\left((1 - \delta)\left(R - \frac{c}{\pi_h - \pi_l}\right) + \delta\right)}$.*³⁵

The following is also immediate from the proof of the Proposition:

Corollary 1. *Having no credit bureau is optimal if and only if $\frac{c}{\pi_h - \pi_l} < R - 1$ and $s_0 \geq \frac{1 - \pi_h\left(R - \frac{c}{\pi_h - \pi_l}\right)}{(1 - \pi_h)\left(R - \frac{c}{\pi_h - \pi_l}\right)}$.*

³⁴We are ignoring here the possible inference that other lenders could draw from the existence of such debt. (Also note that lenders, though short-lived, agree to receive a deferred payment.)

³⁵It is not hard to see that the righthand side of the second inequality is less than 1 when the first inequality holds, and thus can be satisfied for s_0 sufficiently close to 1.

This result shows that high effort can sometimes be implemented even when the forgetting policy is so extreme that defaults are never reported, and a defaulting borrower never faces a loss of reputation. It highlights the role of the other element that affects borrowers' incentives in our model, namely the fact that risky borrowers with credit score p face an interest rate which is lower than the level corresponding to their actual default probability, because they are pooled with safe entrepreneurs. In particular, if we were to generalize the incentive constraint for high effort (4) to a case in which there were no credit bureau, the second term on the right-hand side of this inequality, capturing the effect of reputation, would not be present, but the first term, where the interest rate appears, would still vary significantly with p . On the other hand, the conditions stated in the Corollary are quite stringent, and when they do not hold the presence of a credit bureau, reporting *some* information on borrowers' past defaults, is optimal.

An Example Here we consider a numerical example to illustrate the results obtained. Let $R = 3$, $\pi_h = 0.43$, $\pi_l = 0.3333$, $c = 0.15$, $\tilde{\beta} = 0.95$, and $\delta = 0.999$. For these values, Assumptions 1 and 2 are satisfied and we are in region b. of Proposition 1, for which high effort is implemented in equilibrium when $p \geq p_h(q)$. The threshold $p_h(0)$ above which high effort is exerted when $q = 0$ can be computed from equation (12) in the Appendix, which yields: $p_h(0) = 0.084$.

When s_0 is above this threshold ($s_0 > 0.084$), from Proposition 4, part 1., we know that $q(s_0) > 0$ is always optimal. In Figure 7 below we report the optimal forgetting policy $q(s_0)$, which in this region is given by high values of q (close to 1).

When $s_0 \in [p_{NF}, p_h(0)) = [0.00005, 0.084)$ low effort is exerted with $q = 0$ in the initial round(s) of financing.³⁶ For the parameters of this example, the inequality stated in part 2. of Proposition 4 is satisfied if $s_0 > 0.002$ and in this case we also have $p_0(s_0, 1) > p_{NF}$. Hence by part 2. of Proposition 4 some degree of forgetting will be optimal for $\tilde{\beta}$ sufficiently close to 1 (for instance, when $\tilde{\beta} = 0.95$).

Consider in particular $s_0 = 0.05$. When $q = 0$, we have $p^S(p_0) = 0.136 > p_h(0)$, and so low effort is exerted for the first round of financing along the equilibrium path, and high effort forever after the first success, as long as the project succeeds. On the other hand, when $q > 0$, more rounds of financing with low effort may be needed before risky entrepreneurs begin to exert high effort. In Figure 6 we plot the number of successes that are required under low effort, starting from the credit score $p_0(s_0, q)$, corresponding to an empty credit

³⁶In this example, the risky entrepreneurs never randomize in their effort choice along the equilibrium path.

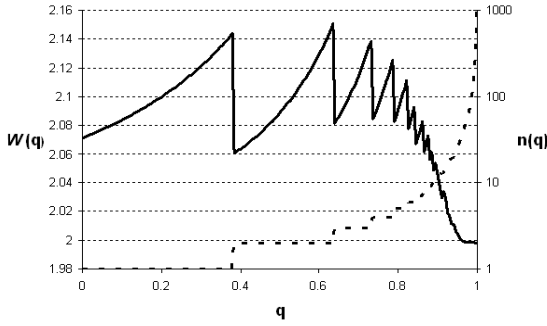


Figure 6: Total welfare (solid line), and number of successes required to reach high-effort region (dashed line), as a function of q ; when $s_0 = 0.05$

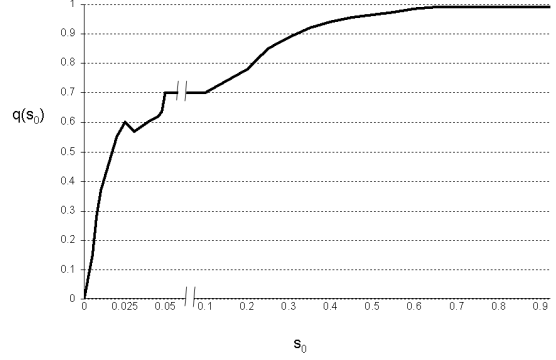


Figure 7: Welfare-maximizing value of q , as a function of s_0

history, until $p_h(q)$ is reached (where high effort starts being exerted), for different values of q . Figure 6 also plots the welfare associated with these different specifications of the forgetting policy. We see from this figure that – when $s_0 = 0.05$ – the optimum is $q = 0.635$.³⁷ Figure 7 then plots the optimal policy for all values of $s_0 \in (0, 1)$.³⁸

In the context of this example we also examine the consequences of relaxing the assumption that the safe entrepreneurs’ projects always succeed. Suppose that the success probability of safe entrepreneurs is now $\pi = 0.95$, while all other parameters are unchanged. In this case, the posterior following the observation of a failure is no longer zero and hence the entrepreneur may be able to still get financing.³⁹ However, a sufficient number of failures will still lead to exclusion.

We find in particular that entrepreneurs who fail will continue to be financed as long as their credit score remains above $p_{NF} = 0.00005$. For $q = 0$ and $s_0 = 0.05$, this means that an entrepreneur can experience two consecutive failures before being excluded from further financing.⁴⁰ The optimal forgetting policy is now $q = 0.69$; that is, forgetting is still optimal and the optimal q is actually higher than when the safe entrepreneurs never fail (we saw that when $\pi = 1$ the optimal policy is $q = 0.635$). The reason is that forgetting now also benefits the safe entrepreneurs, as they may also be excluded from financing after

³⁷For such value of q two successes under low effort are required, starting from $p_0(0.05, 0.635) = 0.029$, until reaching $p_h(0.635) = 0.091$.

³⁸Although the condition in 2. of Proposition 4 is violated for $s_0 \leq 0.0009$, we can nevertheless still have $q(s_0) > 0$, since the condition is only sufficient, not necessary.

³⁹For the parameter values considered in this example it is still true that high effort is implemented in equilibrium for all $p \geq p_h$, in contrast to standard reputation models, such as Mailath and Samuelson (2001).

⁴⁰Starting from a credit score higher than 0.05, more failures are possible before exclusion occurs.

experiencing sufficiently many failures.

Empirical Evidence and Policy Implications

Our model captures many of the effects of forgetting shown in the empirical literature, as well as the key arguments made in the policy debates on the reporting of negative information on borrowers by credit bureaus. This provides some validation of our model. Our analysis then allows one to evaluate forgetting policies in terms of their welfare consequences, and to identify conditions under they are beneficial.

We begin by noting that Jagtiani and Li (2014) find that credit drops sharply following a bankruptcy. This is also a feature of our model — those entrepreneurs who default are identified as risky and receive no further financing. Furthermore, Musto (2004) shows that when the bankruptcy flag drops off a borrower’s credit bureau record, then his borrowing expands dramatically.⁴¹ This is also a feature of our model. Taken together, these results suggest that — both in the data as well as our model — forgetting policies matter and that, after a default is suppressed, lenders are not able to fully separate borrowers on the basis of the information reported by the credit bureau.

Our model also captures the key positive and negative effects of forgetting that were put forward in the Congressional debate around the adoption of the FCRA. The main argument that was put forward at that time in favor of forgetting — that it allows individuals to obtain a true fresh start and hence to continue being productive members of society⁴² — also plays a central role in our model. In particular, the improvement in reputation induced by forgetting may induce them to exert high effort and hence increase aggregate surplus. More recent testimony in front of the U.S. House of Representatives (2014) has also highlighted yet another reason forgetting may provide a benefit, which also plays a role in our model, namely that defaults may occur solely due to chance, such as job loss, falling house prices, or predatory lending (in our model, this is reflected in the fact that risky entrepreneurs are always at risk of failing, even when they exert maximal effort).

Furthermore, most of the arguments made at that time *against* forgetting also operate in our model: (i) forgetting weakens incentives by reducing the penalty for failure — in

⁴¹Musto finds that this increase in credit is concentrated amongst borrowers with relatively good credit histories — this would also arise in the extension of our model presented above, in which both types can fail.

⁴²Two other arguments were also made — that old information may be less relevant, and that there is limited storage space; these do not have a role in our model. Furthermore, even if old information were less relevant (as would be the case if the type of an entrepreneur could change over time), lenders would take this into account and give less weight to past information anyway.

our set-up, as we raise q , region c. shrinks, and region a. increases in size; (ii) by erasing the records of those who defaulted in the past, there is an increased risk that frauds will be committed in the future — the analog in our model is that forgetting “slows down” the weeding out of risky entrepreneurs, hence the average quality of borrowers is lower; and (iii) forgetting can lead to tighter lending standards — in our model this may be seen most sharply in the fact that raising q can shift us from region c., where there is financing for all $p > 0$, to region b., where financing may not occur (for $p < p_l$),⁴³ or even if it does, it is at a higher interest rate (for $p \in (p_l, p_h)$). Finally, while the policy debate suggested that another negative effect of forgetting is that it forces safe agents to subsidize the risky ones, this is in fact socially optimal in our environment because it thereby improves the risky entrepreneurs’ incentives.⁴⁴

Bos and Nakamura (2013) have studied the effect of a change in credit bureau policy in Sweden — a shortening of the time period for which negative information may be reported. Their findings provide empirical support for our model. First, they show that this policy change led to tighter credit standards for those with poor credit histories; as discussed above the negative effect on incentives of a higher level of q could also generate this effect in our model. In addition, they find that default rates rose overall, which is consistent with the fact that such a policy change would allow more risky entrepreneurs to obtain financing. Finally, despite the tighter standards, they find higher overall credit provision; in our model this is indeed the key source of the welfare benefits of forgetting.

The international evidence summarized in Figure 1 is also consistent with our finding that (under appropriate conditions) forgetting policies may increase credit volumes and thus be welfare improving. In that figure, those countries in which information is only reported for a limited period of time have higher provision of credit than the ones that never forget defaults. An implication of our model is indeed that, if the forgetting clause is optimally specified and economies only differ with regard to the strength of the incentive problems (as captured by c), there will indeed be a positive relationship between credit volume and the degree of forgetting (as measured by q); those countries for which forgetting is optimal will be precisely those in which it leads to more financing with high effort.

A final policy implication of our analysis is that even when forgetting past defaults is welfare-improving, it would never arise independently in equilibrium as the outcome of the choice of lenders. As shown in Lemma 1, there exist no Markov Perfect Equilibrium in which

⁴³See Proposition 1: just as suggested in the policy debate, the cohorts who are excluded from financing as a result of the introduction of such a policy are those with a low credit score p_0 — the worst risks.

⁴⁴Since only they face a moral hazard problem.

agents who are known to be risky (as is the case for those who failed) obtain financing. Thus forgetting can only occur through government regulation of the credit bureau’s disclosure policies.

V Robustness

We conclude with a discussion of the robustness of our results with respect to the consideration of other types of equilibria and to some of the assumptions.

V.A Other MPE

We showed in Proposition 2 that the equilibrium characterized in Proposition 1 is the MPE yielding the highest surplus (if $\delta q < \frac{1-\pi_h}{1-\pi_l}$). We argue here that other MPE, as per Definition 1, may exist, but our conclusions on the welfare benefits of forgetting policies extend to them. Note first that in parameter region a. of Proposition 1 all MPE must implement low effort above p_{NF} and no financing below; thus it is still optimal not to forget failures. By contrast, in regions b. and c. other MPE may exist, characterized by higher values of the threshold of p above which higher effort is implemented. In these equilibria low effort is implemented for a wider range of values of the credit score p , as lenders anticipate that the risky entrepreneurs exert low effort also for higher values of p , charge higher interest rates accordingly, which in turn makes low effort incentive compatible. In any case, as long as the equilibrium effort choice is increasing in the number of successes a risky entrepreneurs has experienced, the same argument as in the previous section can be used to show that forgetting is beneficial, under appropriate conditions.

V.B Non-Markov Equilibria

At the MPE we characterized, the property that players’ strategies depend on entrepreneurs’ past histories through the credit score p only binds at nodes where the entrepreneur is not financed, that is when $p = 0$ after a failure. This is because, as observed in Section III.B, when an agent with $p > 0$ is financed the updated belief in case of success will always be higher than the prior one ($p^S(p) > p$). Thus p never hits the same value twice and so is a sufficient statistic for the credit history of an entrepreneur with all successes. But once a failure (that is not forgotten) occurs, we have $p = 0$, whatever the entrepreneur’s past history, and in a Markov equilibrium he never gets further financing.

In contrast, at non-Markov equilibria lenders' strategies may differ for different histories such that p equals 0. For instance, we may have further financing being granted after the first failure but not after two or more. As a consequence, the entrepreneur may be able to sustain more than one failure before being permanently excluded from financing, even though after the very first failure he is known to be risky ($p = 0$). This threat of exclusion after finitely many failures may be enough to induce high effort and hence to make financing profitable for lenders.⁴⁵

These non-Markov equilibria have some similarities with the MPE with forgetting we considered, in that a risky entrepreneur is not necessarily excluded immediately after his first failure. Still, we argue in what follows that forgetting does more than that, and hence allows us to attain a higher level of total surplus than a non-Markov equilibrium (when the latter exists). The reason is that forgetting lowers the interest rate faced by risky agents who have failed, by pooling them together with safe entrepreneurs, and this further improves their incentives.

To see this, notice first that these non-Markov equilibria with financing after some failures cannot exist in parameter regions a. and b. In these regions the risky entrepreneurs' incentives are too weak to allow even a single round of financing once they have been identified as risky, even with the threat of permanent exclusion after one or more failures.⁴⁶ By contrast, we have shown in the previous section that in region b. forgetting may be welfare improving at the MPE we consider.

Consider next those parameter values for which we are in region c. when $q = 0$ and in region b. for $q = 1$ (that is, $\frac{R-1/\pi_h}{1-\beta\pi_l} \leq \frac{c}{\pi_h-\pi_l} \leq R-1$). In Proposition 5 we showed that, for these parameter values, keeping no record of failure (i.e., $q = 1$) is optimal for s_0 close to 1, so that an entrepreneur is never excluded following a failure. By contrast, at non-Markov equilibria (without forgetting) permanent exclusion would be required following some finite number k of failures in order to sustain incentives (precisely because the interest rate this risky agent faces is higher than in the MPE with $q = 1$), so that social welfare is lower.

⁴⁵Since these strategies imply that an entrepreneur is not treated identically at different nodes where $p = 0$, they require some coordination among current and future lenders. One may then argue that such non-Markov equilibria are rather fragile.

⁴⁶This is immediate for region a., where only low effort can be implemented in any equilibrium, even with $q = 0$. Similarly, for region b., if the agent is known to be risky ($p = 0$) only low effort can be implemented in any equilibrium.

V.C Long-term Contracts

It is also useful to compare the MPE we considered with the equilibria we would obtain if long term contracts were feasible; that is, if lenders lived forever, rather than a single period as assumed. In that case lenders would only need to break even over their infinite lifetime, and not period-by-period. Therefore, they could use the time profile of the payment required from borrowers to screen safe from risky entrepreneurs. This would lead to rather extreme contracts being offered in equilibrium, where any net revenue to borrowers from the projects financed is postponed as far into the future as possible: that is, the interest payments would equal R in the initial periods, and subsequently be zero. Contracts yielding a net revenue to borrowers only after an uninterrupted string of successes of their projects are less attractive to the risky entrepreneurs, who face the risk of a failure in any period, and more attractive to the safe ones.

Nevertheless, in regions a. and b., the effort cost is high enough that an entrepreneur identified as risky cannot obtain financing even with long-term contracts, by a similar argument to that given above for non-Markov equilibria (see footnote 46). Hence, a separating equilibrium with long-term contracts does not exist, and in these regions the only equilibria still exhibit pooling. Moreover, the total surplus at these pooling equilibria will be lower than at the MPE with short-term contracts (and forgetting) characterized in Proposition 1. The reason is that the postponement of payments that occurs with long-term contracts decreases the cross-subsidy from safe to risky entrepreneurs, and this will have a negative impact on the risky entrepreneurs' incentives; overall, there will be fewer periods of financing where high effort is exerted.

In region c., by contrast, risky entrepreneurs would be able to obtain financing on their own. Hence a separating equilibrium may exist, but total welfare at such an equilibrium would be lower than at the MPE with forgetting, as the lack of any cross-subsidy means again that incentives are weaker and financing to risky entrepreneurs will be more limited.

We should add that, when lenders are long-lived, credit bureaus are not the only source of information on borrowers' repayment histories (this is true even when parties cannot commit to long-term relationships). In particular, a lender may have some direct information on a borrower to which he lent in the past. As a consequence, if a credit bureau were to impose a restriction on the information available on borrowers' credit history, this would create an informational asymmetry in favor of the lenders who traded in the past with these borrowers, for whom this restriction would have less effect. This asymmetry would be a source of market power for these lenders and would make achieving a real fresh start for defaulting borrowers

much more difficult. In an environment of this kind, Padilla and Pagano (1997) show that it may indeed be beneficial for lenders to be able to commit to share information about past loans with other lenders, as this allows borrowers to move freely between lenders and hence limits the surplus that can be extracted from borrowers.

VI Conclusion

In this paper we have investigated the effects of restrictions on the information available to lenders on borrowers' past performance. We have considered an environment where lending markets feature both moral hazard and adverse selection, this information matters (because it affects borrowers' reputation and their incentive to exert effort), and it can only be obtained from an external agency. We have shown that these restrictions on information may facilitate a "fresh start" for borrowers in distress, but also affect borrowers' incentives and hence their access to credit. Our model captures many of the effects found in previous empirical studies, and also captures many of the arguments made in the Congressional debate surrounding the adoption of the FCRA. As such, it provides a framework within which we can identify conditions under which the positive arguments prevail over the negative ones.

In particular, the central tradeoff in our model is between the negative effect of forgetting on ex-ante incentives (because it lowers the cost of failing) and the potential positive effects ex-post (because by pooling entrepreneurs who have failed afresh with those who have not, their reputation improves and hence their incentives). We show that forgetting is beneficial when the severity of the incentive problem, as captured by the cost of exerting effort, is not too great, and the adverse selection problem is also not too severe. In these cases the welfare loss due to the decline in ex-ante incentives is more than compensated by a welfare gain due to the improvement in ex-post incentives.

Recall that the three key features of the model that deliver this tradeoff between the ex-ante and ex-post effects of forgetting, and the resulting welfare implications are (i) that information affect incentives, (ii) that this information must be provided by credit bureaus, and cannot be inferred through other means when they are directed not to report it, and (iii) that increasing an entrepreneur's reputation improves his incentives. The short-lived nature of lending relationships is important for (i) and (ii). The main assumptions needed with regard to the last point are that the incentive problem falls primarily on the riskier types, and that even when they exert the highest effort they fail with higher probability than the safer types. The other assumptions were introduced primarily for convenience, and could be

relaxed at the cost of complicating the analysis.

Finally, our model has some empirical implications that can be tested. First, when considering the effect of a change in credit bureau reporting policy, our model predicts that the impact of the policy — be it on the availability of credit, on default rates, or on the pricing of credit — will differ depending on where an agent falls in the credit score distribution, with the biggest effects on the former being in the bottom of the score distribution, and the latter two occurring in the middle of this distribution. In addition, when considering the introduction of a credit bureau in a country that did not formerly have one, our model predicts that the volume of credit will increase in markets where the adverse selection and moral hazard problems are severe.

VII Appendix — Proofs

Proof of Lemma 1 — No financing when known to be risky

If $p = 0$, we must have $p^S(p, \mathcal{C}') = 0 = p^F(p, \mathcal{C}')$ whatever \mathcal{C}' , i.e., the agent will be known to be risky in the future as well. Furthermore, under Assumption 1, if the agent is known to be the risky type, he can only be financed in a given period if he exerts high effort with some probability, as otherwise lenders cannot break even. But for high effort (or mixing) to be incentive compatible, the utility from high effort must be no less than that from low effort, i.e., the interest rate r offered must be such that:

$$\begin{aligned} \pi_h(R - r) - c + \pi_h\tilde{\beta}v^r(p^S(0)) + (1 - \pi_h)q\tilde{\beta}v^r(0) + (1 - \pi_h)(1 - q)\tilde{\beta}v^r(p^F(0)) \geq \\ \pi_l(R - r) + \pi_l\beta v^r(p^S(0)) + (1 - \pi_l)q\tilde{\beta}v^r(0) + (1 - \pi_l)(1 - q)\tilde{\beta}v^r(p^F(0)), \end{aligned}$$

which simplifies to the static incentive compatibility condition:

$$\frac{c}{\pi_h - \pi_l} \leq R - r, \tag{7}$$

since when $p = 0$ we have $p^S = p^F = 0$.

By Assumption 2, this can be satisfied only if $r < 1/\pi_h$, in which case lenders cannot break even. Thus the agent cannot be financed in equilibrium if he is known to be risky. Finally, since this agent is never financed, it is immediate that $v^r(0) = 0$.

Proof of Proposition 1 — Characterization of the Equilibrium

To complete the proof of Proposition 1, we establish the remaining properties of the MPE and

the specific features of this equilibrium for parameter regions a., b., and c., by verifying that in each case there are no profitable deviations by either borrowers or lenders. a. To show that the strategies specified in the Proposition constitute an MPE when $\frac{c}{\pi_h - \pi_l} \geq \frac{(R-1)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)}$, we need to demonstrate that (a-i) low effort is incentive compatible for $p \geq p_{\text{NF}}$; (a-ii) $r(p) = r_{zp}(p, 0) \leq R$ for $p \geq p_{\text{NF}}$, i.e., it is admissible; and (a-iii) there are no profitable deviations by lenders.

a-i. By an analogous argument to that used to derive (7) above, for low effort to be incentive compatible we need to show that:

$$\frac{c}{\pi_h - \pi_l} \geq R - r_{zp}(p, 0) + \tilde{\beta}[v^r(p^S(p)) - qv^r(p)].$$

Now, given the above strategies and implied beliefs, from (1b) we have:

$$v^r(p) = \pi_l(R - r_{zp}(p, 0)) + \pi_l\tilde{\beta}v^r(p^S(p)) + (1 - \pi_l)q\tilde{\beta}v^r(p), \quad (8)$$

since from Lemma 1, $v^r(p^F(p)) = v^r(0) = 0$. Given our conjecture that risky entrepreneurs exert low effort for all $p \geq p_{\text{NF}}$, $p^S(p)$ is uniquely determined by equation (6), with $p^S(p) > p$.

Solving (8) for $v^r(p^S(p))$ in terms of $v^r(p)$ and substituting into the low-effort incentive compatibility condition above, we obtain the following equivalent incentive compatibility condition:

$$\frac{c\pi_l}{\pi_h - \pi_l} \geq v^r(p)(1 - \tilde{\beta}q), \quad (9)$$

But since $r_{zp}(p, 0) > r_{zp}(1, 0) = 1$ for all $p < 1$, this incentive compatibility condition becomes:

$$v^r(p) < \frac{\pi_l(R - 1)}{1 - \tilde{\beta}(\pi_l + (1 - \pi_l)q)},$$

where the term on the right-hand side is the present discounted utility of a risky entrepreneur who is financed in every period at the rate $r = 1$ (until he has a failure that is not forgotten) and exerts low effort. Using this, it is immediate to verify that (9) holds for the values of c in this region.

a-ii. $r_{zp}(p, 0) \leq R$ if and only if $\frac{1}{p+(1-p)\pi_l} \leq R$, or equivalently $p \geq p_{\text{NF}}$.

a-iii. Consider a deviation by a lender. First note that lenders make zero profits in equilibrium, so refusing to offer a contract would never be profitable.

Nor can a lender profit by offering a different interest rate for $p \geq p_{\text{NF}}$. To see this, first note that a higher rate than $r(p)$ would not be accepted by any borrower. What if a lender offers a lower rate r' , so that the set of contracts offered is $\mathcal{C}' = \{r(p), r'\}$? We show next that a sequentially rational strategy for all entrepreneurs is to never refuse financing when it is offered (just as on the equilibrium path). If all entrepreneurs accept financing, also off the equilibrium path, the updated belief would be the same as on the equilibrium path: $p^S(p, \mathcal{C}') = p^S(p)$ (again, since effort is chosen before observing the deviation). In this situation, if a (single) entrepreneur were to refuse financing, his updated credit score would then be $p^\theta(p, \mathcal{C}') = p$, and hence his utility would be lower. It is then optimal for all entrepreneurs to accept financing (at the lowest rate offered, by Observation 2).

As a consequence, the lender would earn a negative profit from this deviation, since he is offering a rate below $r_{zp}(p, 0)$, all entrepreneurs accept the offer and the risky entrepreneurs continue to exert low effort, since effort is chosen before observing the lender's deviation. This implies that the lender would earn a negative profit from this deviation.

A similar argument shows that a lender cannot profit by offering financing at $p < p_{\text{NF}}$.

b. Next, we show that for intermediate values of c , $\frac{(R-1/\pi_h)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)} < \frac{c}{\pi_h-\pi_l} < \frac{(R-1)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)}$, an MPE exists characterized by $0 < p_l \leq p_m \leq p_h < 1$ such that: for $p \geq p_l$ entrepreneurs are always financed, $e^r(p) = 1$ for $p \geq p_h$, $e^r(p) \in (0, 1)$ and is (strictly) increasing in p for $p \in [p_m, p_h)$, $e^r(p) = 0$ for $p \in [p_l, p_m)$ and $r(p) = r_{zp}(p, e^r(p))$.

We begin by characterizing the values of (b-i) p_h , (b-ii) p_m and (b-iii) p_l , showing that the effort choices specified above for the risky entrepreneurs are optimal. In (b-iv) we demonstrate that there are no profitable deviations for lenders.

b-i. We will construct p_h to be the lowest value of p for which high effort is incentive compatible, given that high effort is also exerted for all $p' > p_h$. That is p_h is the value of p that satisfies the following high-effort incentive compatibility condition with equality:

$$\frac{c}{\pi_h - \pi_l} = R - r_{zp}(p_h, 1) + \tilde{\beta}\hat{v}^r(\hat{p}^S(p_h)) - \tilde{\beta}q\hat{v}^r(p_h) \quad (10)$$

In this equation, $\hat{v}^r(p)$ is the analog of the equilibrium utility $v^r(p)$, computed in the case where the risky entrepreneurs exert high effort for all $p > 0$ (and are not financed

when $p = 0$), and are offered the break-even interest rate . That is,

$$\hat{v}^r(p) = \pi_h(R - r_{zp}(p, 1)) - c + \tilde{\beta}\pi_h\hat{v}^r(\hat{p}^S(p)) + \tilde{\beta}(1 - \pi_h)q\hat{v}^r(p). \quad (11)$$

where $\hat{p}^S(p) = \frac{p}{p+(1-p)\frac{\pi_h}{1-(1-\pi_h)q\delta}}$ similarly denotes the posterior belief, following a success, that an entrepreneur is safe, when the prior belief is $p \in (0, 1)$ and the risky entrepreneurs undertake high effort at both p and $\hat{p}^S(p)$, obtained from the analog to (6). Also note that $\hat{p}^S(p) > p$. $p' \geq p$.⁴⁷ Using (11) to simplify (10):

$$\frac{c\pi_l}{\pi_h - \pi_l} = \hat{v}^r(p_h)(1 - \tilde{\beta}q) \quad (12)$$

Observe that, since $\hat{p}^S(p)$ is strictly increasing in p , and $r_{zp}(p, 1)$ is strictly decreasing, $\hat{v}^r(p)$ is strictly increasing in p . Thus the term on the right-hand side of (12) is increasing in p , and so (12) has at most one solution.

By a continuity argument, it can then be verified that:

Claim 1. *A solution $p_h \in (0, 1)$ to (12) always exists.*⁴⁸

Given the monotonicity of the term on the right-hand side of (12), it is immediate that the incentive compatibility constraint for high effort is satisfied for all $p \geq p_h$. So we let $e^r(p) = 1$ for $p \geq p_h$, and thus $v^r(p) = \hat{v}^r(p)$ in this region.

b-ii. Next, we find p_m , the lower bound of the region where risky agents mix over effort.

Let $\tilde{p}^S(p, e)$ denote the posterior belief, following a success, that an entrepreneur is safe, when the prior belief is $p \in (0, 1)$, the effort undertaken at p if risky is e , and we follow the equilibrium path for $p' > p$. From equation (6), this solves:

$$\tilde{p}^S(p, e) = \frac{p}{p + (1 - p)\frac{\pi_e}{1-(1-\pi_{e'})q\delta}},$$

where recall that, for any effort level e' , $\pi_{e'} \equiv \pi_h e' + \pi_l(1 - e')$ was defined to be the probability that the risky entrepreneurs' project succeeds. We show in what follows that, for the equilibrium we construct, $e^r(\tilde{p}^S(p, e)) \geq e$, and so there is a unique solution

⁴⁷Note that while $\hat{v}^r(p)$ and $\hat{p}^S(p)$ are well defined for all $p \in (0, 1)$, they only coincide with the equilibrium values $v^r(p)$ and $p^S(p)$ when both $p \geq p_h$ and $e^r(p) = 1$.

⁴⁸ The proofs of claims 1-5 can be found in appendix B (available at http://www.elul.org/papers/forget/appendix_b.pdf).

to this equation, with $\tilde{p}^S(p, e) > p$. Similarly, let $\tilde{v}^r(p, e)$ denote the utility that a risky entrepreneur obtains when the effort undertaken at p is e and it then follows the equilibrium path for $p' > p$. Then $\tilde{v}^r(p, e)$ satisfies the following equation:

$$\tilde{v}^r(p, e) = \pi_e(R - r_{zp}(p, e)) - c \cdot e + \tilde{\beta}\pi_e v^r(\tilde{p}^S(p, e)) + \tilde{\beta}(1 - \pi_e)q\tilde{v}^r(p, e). \quad (13)$$

For mixing to be an equilibrium strategy at p , risky entrepreneurs must be indifferent between high and low effort, i.e.,

$$\frac{c}{\pi_h - \pi_l} = R - r_{zp}(p, e) + \tilde{\beta}v^r(\tilde{p}^S(p, e)) - \tilde{\beta}q\tilde{v}^r(p, e) \quad (14)$$

for some $e \in [0, 1]$. By using (13) to substitute for $v^r(\tilde{p}^S(p, e))$ in (14), we get the following equivalent expression:

$$\frac{c\pi_l}{\pi_h - \pi_l} = \tilde{v}^r(p, e)(1 - \tilde{\beta}q). \quad (15)$$

To conclude the proof, we demonstrate that:

Claim 2. *There exists a lowest value $p_m \leq p_h$ such that (15) admits a solution $e^r(p)$ for all $p \in [p_m, p_h]$, with $e^r(p)$ increasing in p . Moreover, there is only a single period of mixing along the equilibrium path.*

For each $p \in [p_m, p_h]$, $v^r(p) = \tilde{v}^r(p, e^r(p))$. Observe that (15) implies that $v^r(p)$ is constant in the mixing region, and from (12) it follows that it is equal to $v^r(p_h)$.

b-iii. We determine p_l , the lower bound of the financing region, and demonstrate that low effort is incentive compatible in $[p_l, p_m)$.

◆ If $p_m \geq p_{NF}$, set $p_l = p_{NF}$. By construction, $r_{zp}(p, 0) \leq R$ for all $p \geq p_{NF}$; hence the contract $r_{zp}(p, e^r(p))$ is admissible for all $p \geq p_{NF}$.

Alternatively, if $p_m < p_{NF}$ set p_l to be the lowest value of $p \geq p_m$ such that the contract $r_{zp}(p, e^r(p))$ is admissible (i.e., not greater than R). Since $r_{zp}(p, e)$ is decreasing in e , $r_{zp}(p, e^r(p)) \leq r_{zp}(p, 0)$ for all $p \in [p_m, p_{NF}]$, so $p_l \leq p_{NF}$. In this case we also redefine p_m , with some abuse of notation, to be equal to p_l ; following this redefinition the low effort region $[p_l, p_m)$ is then empty in this case.

Observe that in either case we have $p_l > 0$. Furthermore, $p_l \leq p_{NF}$, which implies that $r_{zp}(p, 0) > R$ for $p < p_l$. Finally, $p_l \leq p_m$, with p_m as defined above.

◆ To conclude, we show that low effort is incentive compatible for $p \in [p_l, p_m)$. It suffices to consider the case $p_l = p_{NF}$ since when $p_l < p_{NF}$, we showed immediately above that $p_l = p_m$, in which case there is no low-effort region.

Claim 3. *The contract $r_{zp}(p, 0)$ satisfies the low-effort IC constraint for $p \in [p_{NF}, p_m)$.*

The argument is a little lengthier in this case and proceeds by induction. We first establish the property for values of p for which the updated posterior following a success is above p_m . We then show that this property also holds for all values of p such that the posterior belief upon success falls in the interval obtained in the first step, and so on.

b-iv. By the same argument as in a-iii. above, there can be no profitable lender deviations.

c. Finally, consider the low values of c : $\frac{c}{\pi_h - \pi_l} \leq \frac{(R-1/\pi_h)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)}$. Note first that, by Assumption 1, $r_{zp}(p, 1) \leq R$ for all $p > 0$, so $r(p) = r_{zp}(p, 1)$ is always admissible. Also, the argument that there are no profitable deviations for lenders is again the same as the one in a-iii. So it remains only to verify that risky entrepreneurs indeed prefer to exert high rather than low effort for all $p > 0$.

For high effort to be incentive compatible for all $p > 0$, we need to show that

$$\frac{c\pi_l}{\pi_h - \pi_l} \leq \hat{v}^r(p)(1 - \tilde{\beta}q), \quad (16)$$

where recall that $\hat{v}^r(p)$ is the utility to the risky entrepreneur when he exerts high effort for all $p' \geq p$, defined in (11) above.

Notice that, for any $p > 0$, a lower bound for $\hat{v}^r(p)$ is given by $\frac{\pi_h(R-1/\pi_h)-c}{1-\tilde{\beta}(\pi_h+q(1-\pi_h))}$, which is the present discounted utility for a risky entrepreneur who is financed in every period (until a failure that is not forgotten) at $r = 1/\pi_h$ and exerts high effort.⁴⁹ But then the upper bound on c that defines region c. immediately implies (16). This completes the proof of Proposition 1.

Proof of Proposition 2 — Efficiency of Equilibrium

We begin by showing that the equilibrium constructed in Proposition 1 maximizes the effort exerted by the risky entrepreneurs, for any p ; this will play an important role in the proof of the proposition. This result is intuitive, as the equilibrium of Proposition 1 was constructed recursively, with effort chosen to be maximal at each stage.

⁴⁹This follows immediately from the fact that $\hat{v}^r(p)$ is the present discounted utility under the same circumstances except that the interest rate is $r(p) = r_{zp}(p, 1) < 1/\pi_h$ for all $p > 0$.

Claim 4. When $\delta q < \frac{1-\pi_h}{1-\pi_l}$, the equilibrium constructed in Proposition 1 is such that the risky entrepreneurs' effort $e^r(p)$ is higher, at any p , than at any other symmetric sequential MPE.

The following corollary is immediate, since for lenders to break even when $p < p_l$ a higher level of effort is needed than in the equilibrium of Proposition 1, contradicting Claim 4.

Corollary 2. *No MPE can implement financing when $p < p_l$.*

Recall that welfare is given by the total surplus accruing from the agents' projects that are financed. Let $W(s_0, q)$ denote the total surplus at the MPE of Proposition 1 when the measure of safe entrepreneurs born into every generation is s_0 , and let $\overline{W}(s_0, q)$ denote the total surplus at a different MPE. We then conclude by showing that:

Claim 5. $W(s_0, q) \geq \overline{W}(s_0, q)$

The proof of this claim is by induction on p , relying at each stage on the fact that surplus will be higher whenever effort is higher. The result then follows from Claim 4 above.

Proof of Proposition 3 – Optimal Forgetting (regions a. and c.)

1. When $\frac{c}{\pi_h - \pi_l} \geq \frac{R-1}{1-\pi_l\tilde{\beta}}$, since $\frac{(R-1)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)}$ is decreasing in q , the condition defining region a. in Proposition 1 is satisfied for all q . At the MPE, there is financing only when $p_0 \geq p_{\text{NF}}$ and risky entrepreneurs never exert high effort, regardless of the value of q . Hence if $p_0 \geq p_{\text{NF}}$, the total surplus generated in equilibrium by the loans to risky entrepreneurs is $\frac{B}{1-(\pi_l+(1-\pi_l)q)\tilde{\beta}}$, which is strictly decreasing in q since $B < 0$. Thus $q = 0$ is optimal. If on the other hand $p_0 < p_{\text{NF}}$, such surplus is zero for all q , and so $q = 0$ is also (weakly) optimal.

2. Again notice that $\frac{(R-1/\pi_h)(1-\tilde{\beta}q)}{1-\tilde{\beta}(\pi_l+(1-\pi_l)q)}$ is decreasing in q . Thus when $\frac{c}{\pi_h - \pi_l} < \frac{R-1/\pi_h}{1-\tilde{\beta}\pi_l}$, the condition defining region c. of Proposition 1 is satisfied for all $q \in [0, q^*]$, where $q^* = \frac{(R-1/\pi_h) - \frac{c}{\pi_h - \pi_l}(1-\tilde{\beta}\pi_l)}{\tilde{\beta}((R-1/\pi_h) - \frac{c}{\pi_h - \pi_l}(1-\pi_l))} > 0$. Hence at the MPE there is always financing whatever p_0 is, and for all $q \in [0, q^*]$, and risky entrepreneurs always exert high effort. That is, for $q \in [0, q^*]$, the total surplus generated in equilibrium by the loans to risky entrepreneurs is

$$\frac{G}{1 - (\pi_h + (1 - \pi_h)q)\tilde{\beta}}.$$

Now this is increasing in q since $G > 0$. Thus any $q \in (0, q^*]$ dominates $q = 0$ and the optimal value will be $q(p_0) \geq q^*$.⁵⁰

⁵⁰The optimal value of q could be higher than q^* , which would push us out of region c. and into region b.

Proof of Proposition 4 – Optimal Forgetting (region b.)

1. When $s_0 > p_h(0)$ the proof is an immediate corollary of part 2. of Proposition 3.
2. Since $p_0(s_0, 1) \geq p_{NF}$ (condition i.), the agents will always be financed at the initial date, irrespective of q . Thus, by the argument used to prove Proposition 3, it suffices to show that we can increase the surplus generated by the risky entrepreneurs' projects. Recall that $W^r(s_0, q)$ denotes the surplus from the risky agents' projects, when the forgetting policy is q and the measure of safe types born into each generation is s_0 . We will show that under the conditions stated in the proposition, we can find some $\bar{q} > 0$ such that $W^r(s_0, \bar{q}) > W^r(s_0, 0)$.

We proceed as follows. For any $q > 0$ we first find a threshold $\tilde{p}_h(q)$ such that if $p_h(q) < \tilde{p}_h(q)$ then the surplus from risky entrepreneurs' projects is higher at q than at 0. We then show that the parameter restrictions stated in the Proposition indeed ensure the existence of $\bar{q} > 0$ such that $p_h(\bar{q}) \leq \tilde{p}_h(\bar{q})$.

Before proceeding with the proof, recall first that $p_0(q)$ satisfies equation (5). When $s_0 < p_m(0)$ then it is in fact uniquely determined by this equation for any q . Otherwise, we take the highest value that solves (5), which corresponds to the maximal equilibrium effort.

Let $n(s_0, q)$ denote the number of successes (or forgotten failures), starting from the prior $p_0(s_0, q)$, until the risky entrepreneurs first exert high effort with probability 1, when the forgetting policy is q . In the simple case in which there is no mixing in equilibrium, the surplus $W^r(n(s_0, q), q)$ from the risky entrepreneur's projects can be defined recursively as follows:

$$\begin{aligned} W^r(n, q) &= (\pi_l R - 1) + \pi_l \tilde{\beta} W^r(n-1, q) + (1 - \pi_l) q \tilde{\beta} W^r(n, q) \\ W^r(0, q) &= \frac{\pi_h R - 1 - c}{1 - (\pi_h + (1 - \pi_h)q)\tilde{\beta}}, \end{aligned} \tag{17}$$

where $W^r(0, q)$ is the surplus generated by the risky entrepreneur's projects once he is in the high-effort region. When there is mixing in equilibrium, the exact expression for W^r depends on the equilibrium level of effort exerted in the mixing region. However, since there can only be a single period of mixing (in the period before high effort is exerted with probability 1), we can bound the surplus generated by lending to the risky entrepreneurs.

In particular, an upper bound on the surplus $W^r(n(s_0, 0), 0)$ generated with the forgetting policy $q = 0$ can be obtained by assuming that high effort is exerted in the mixing region with probability 1.⁵¹ In this case we have:

$$W^r(s_0, 0) \leq \frac{B(1 - (\pi_l \tilde{\beta})^{n(s_0, 0) - 1})}{1 - \pi_l \tilde{\beta}} + \frac{G(\pi_l \tilde{\beta})^{n(s_0, 0) - 1}}{1 - \pi_h \tilde{\beta}}. \tag{18}$$

⁵¹That is, so there are only $n_0(s_0, 0) - 1$ periods in which low effort is exerted with positive probability.

Similarly, considering instead the case in which low effort is exerted with probability 1 in the mixing region, we obtain a lower bound on $\mathcal{W}^r(n(s_0, q), q)$:

$$\mathcal{W}^r(s_0, q) \geq B \frac{1 - \left(\frac{\pi_l \tilde{\beta}}{1 - (1 - \pi_l) q \tilde{\beta}} \right)^{n(s_0, q)}}{1 - \left(\frac{\pi_l \tilde{\beta}}{1 - (1 - \pi_l) q \tilde{\beta}} \right)} + G \frac{\left(\frac{\pi_l \tilde{\beta}}{1 - (1 - \pi_l) q \tilde{\beta}} \right)^{n(s_0, q)}}{1 - (\pi_h + (1 - \pi_h) q) \tilde{\beta}}. \quad (19)$$

So to show that $\mathcal{W}^r(s_0, q) > \mathcal{W}^r(s_0, 0)$, it suffices to show that we can find $q > 0$ such that:

$$\frac{B(1 - (\pi_l \tilde{\beta})^{n(s_0, 0) - 1})}{1 - \pi_l \tilde{\beta}} + \frac{G(\pi_l \tilde{\beta})^{n(s_0, 0) - 1}}{1 - \pi_h \tilde{\beta}} < B \frac{1 - \left(\frac{\pi_l \tilde{\beta}}{1 - (1 - \pi_l) q \tilde{\beta}} \right)^{n(s_0, q)}}{1 - \left(\frac{\pi_l \tilde{\beta}}{1 - (1 - \pi_l) q \tilde{\beta}} \right)} + G \frac{\left(\frac{\pi_l \tilde{\beta}}{1 - (1 - \pi_l) q \tilde{\beta}} \right)^{n(s_0, q)}}{1 - (\pi_h + (1 - \pi_h) q) \tilde{\beta}}.$$

Letting $\tilde{\beta} \rightarrow 1$ and simplifying, the above expression reduces to:

$$\frac{B}{G} \frac{1 - \pi_l^{n(s_0, 0) - 1}}{1 - \pi_l} + \frac{\pi_l^{n(s_0, 0) - 1}}{1 - \pi_h} < \frac{B}{G} \frac{1 - \left(\frac{\pi_l}{1 - (1 - \pi_l) q} \right)^{n(s_0, q)}}{(1 - \pi_l)(1 - q)} + \frac{\left(\frac{\pi_l}{1 - (1 - \pi_l) q} \right)^{n(s_0, q)}}{(1 - \pi_h)(1 - q)},$$

since $1 - (\pi_l + (1 - \pi_l) q) = (1 - \pi_l)(1 - q)$ and $1 - (\pi_h + (1 - \pi_h) q) = (1 - \pi_h)(1 - q)$. This is equivalent to

$$\frac{B/G}{1 - \pi_l} + \pi_l^{n(s_0, 0) - 1} \left(\frac{1}{1 - \pi_h} - \frac{B/G}{1 - \pi_l} \right) < \frac{B/G}{(1 - \pi_l)(1 - q)} + \frac{1}{1 - q} \left(\frac{\pi_l}{1 - (1 - \pi_l) q} \right)^{n(s_0, q)} \left(\frac{1}{1 - \pi_h} - \frac{B/G}{1 - \pi_l} \right) \quad (20)$$

It will be useful to rewrite (20) in terms of a condition on $p_h(q)$ and $p_h(0)$. To this end, notice that $p_h(0)$ and $n(s_0, 0)$ are related by the following expression: $n(s_0, 0)$ is the smallest integer for which⁵²

$$\frac{p_0(s_0, 0)}{p_0(s_0, 0) + (1 - p_0(s_0, 0)) \pi_l^{n(s_0, 0)}} \geq p_h(0), \quad (21)$$

so that when $\tilde{\beta}$ is close to 1 we have $\pi_l^{n(s_0, 0) - 1} \leq \frac{1}{\pi_l} \frac{s_0}{1 - s_0} \left(\frac{1 - p_h(0)}{p_h(0)} \right)$.⁵³

⁵²When there is no mixing in equilibrium, i.e., $p_m(0) = p_h(0)$, the validity of (21) follows immediately from the definition of $p_h(0)$ and $n(s_0, 0)$. The fact that it also holds with mixing can be seen by noticing that in such case the probability of success is greater or equal than when low effort is exerted, and so the posterior is $\tilde{p}^S(p, e^r(p)) \leq \tilde{p}^S(p, 0)$. Hence $n(s_0, 0)$ will be greater or equal than the term satisfying (21). But $n(s_0, 0)$ cannot be strictly greater, as this would imply that we mix for more than a single period, which we have shown (in the proof of Proposition 1) cannot happen.

⁵³We use equations (5) and (6) for $q = 0$.

Similarly, as $\delta \rightarrow 1$, an upper bound for $n(s_0, q)$ ⁵⁴ is given by the lowest integer that satisfies

$$\frac{p_0(s_0, q)}{p_0(s_0, q) + (1 - p_0(s_0, q)) \left(\frac{\pi_l}{1 - (1 - \pi_l)q} \right)^{n(s_0, q)}} \geq p_h(q). \quad (22)$$

This implies that $\left(\frac{\pi_l}{1 - (1 - \pi_l)q} \right)^{n(s_0, q) - 1} \geq \frac{p_0(s_0, q)}{1 - p_0(s_0, q)} \left(\frac{1 - p_h(q)}{p_h(q)} \right)$, or

$$\left(\frac{\pi_l}{1 - (1 - \pi_l)q} \right)^{n(s_0, q)} \geq \left(\frac{\pi_l}{1 - (1 - \pi_l)q} \right) \frac{p_0(s_0, q)}{1 - p_0(s_0, q)} \left(\frac{1 - p_h(q)}{p_h(q)} \right).$$

If we now substitute the expression for $p_0(s_0, q)$ from (5) as $\delta \rightarrow 1$, we then obtain:

$$\left(\frac{\pi_l}{1 - (1 - \pi_l)q} \right)^{n(s_0, q)} \geq \pi_l \frac{s_0}{1 - s_0} \left(\frac{1 - p_h(q)}{p_h(q)} \right).$$

Thus to satisfy (20) it suffices to show that:

$$\frac{B/G}{1 - \pi_l} + \frac{1}{\pi_l} \frac{s_0}{1 - s_0} \left(\frac{1 - p_h(0)}{p_h(0)} \right) \left(\frac{1}{1 - \pi_h} - \frac{B/G}{1 - \pi_l} \right) < \frac{1}{1 - q} \left[\frac{B/G}{1 - \pi_l} + \pi_l \frac{s_0}{1 - s_0} \left(\frac{1 - p_h(q)}{p_h(q)} \right) \left(\frac{1}{1 - \pi_h} - \frac{B/G}{1 - \pi_l} \right) \right] \quad (23)$$

We will obtain a welfare improvement by taking $q \rightarrow 1$. In order for us to satisfy (23) as $q \rightarrow 1$, however, the right-hand side of the inequality above must be positive. This will be the case if:

$$p_h(q) < \tilde{p}_h(q) \equiv \frac{\pi_l s_0 ((1 - \pi_l) - (1 - \pi_h)B/G)}{\pi_l s_0 (1 - \pi_l) - (1 - \pi_h)(1 - (1 - \pi_l)s_0)B/G}. \quad (24)$$

We now show that the joint condition on $s_0, p_h(0)$, and B/G , stated in the proposition ensures that we can find \bar{q} close to 1 such that $p_h(\bar{q})$ satisfies (24) and so we can achieve a welfare improvement. We begin by providing a convenient upper bound for $p_h(q)$.

For intermediate values of c , lying in the region where type b. equilibria obtain when $q = 0$, $p_h(0)$ belongs to $(0, 1)$ and satisfies equation (12) above. It is then easy to see from the definition of this region in Proposition 1 that, when $\tilde{\beta}$ is sufficiently close to 1, c will remain in the same region for any $q > 0$.⁵⁵ So for $\tilde{\beta}$ close to 1, $p_h(q)$ also lies in $(0, 1)$ and so equation (12) relates $p_h(0)$ and $p_h(q)$:

⁵⁴Since from equation (6) we know that $p^S(p)$ is increasing in $e^r(p^S(p))$, the upper bound is obtained by assuming that low effort is exerted even in the final period when $p_h(q)$ is reached.

⁵⁵For $\tilde{\beta}$ close to 1, the boundaries of the region are approximately equal to $\frac{(R-1/\pi_h)}{1-\pi_l}$ and $\frac{(R-1)}{1-\pi_l}$, both independent of q .

$$\hat{v}^r(p_h(0); 0) = \hat{v}^r(p_h(q); q)(1 - \tilde{\beta}q). \quad (25)$$

Recall that $\hat{v}^r(p; q)$, derived in (11), denotes the discounted expected utility of a risky entrepreneur with credit score p , when he exerts high effort for all $p' \geq p$ and the contracts offered are $r_{zp}(p, 1)$, now highlighting its dependence on the forgetting policy q .

By a similar argument to that in the proof of parts a. and c. of Proposition 1, a (strict) upper bound for $\hat{v}^r(p_h(0); 0)$ is given by the utility of being financed in the current period at the rate $r_{zp}(p_h(0), 1)$, and in future periods at the rate $r = 1$, until a failure occurs, all the while exerting high effort, i.e., by $\pi_h(1 - r_{zp}(p_h(0), 1)) + \frac{\pi_h(R-1)-c}{1-\beta\pi_h}$. Conversely, when the forgetting policy is q , a (strict) lower bound for $\hat{v}^r(p_h(q); q)$ is given by $\frac{\pi_h(R-r_{zp}(p_h(q), 1))-c}{1-\tilde{\beta}(\pi_h+(1-\pi_h)q)}$, that is, the utility of a risky agent when financed at the constant rate $r_{zp}(p_h(q), 1)$ until he experiences a failure that is not forgotten, still exerting high effort. Together with (25) this implies that:

$$\pi_h(1 - r_{zp}(p_h(0), 1)) + \frac{\pi_h(R-1)-c}{1-\tilde{\beta}\pi_h} > (1 - \tilde{\beta}q) \frac{\pi_h(R - r_{zp}(p_h(q), 1)) - c}{1 - \tilde{\beta}(\pi_h + (1 - \pi_h)q)}.$$

When $\tilde{\beta} \rightarrow 1$, the above inequality becomes

$$\pi_h(1 - r_{zp}(p_h(0), 1)) + \frac{\pi_h(R-1)-c}{1-\pi_h} > \frac{\pi_h(R - r_{zp}(p_h(q), 1)) - c}{1-\pi_h},$$

or, simplifying: $r_{zp}(p_h(q), 1) > (1 - \pi_h)r_{zp}(p_h(0), 1) + \pi_h$.

Using the definition of $r_{zp}(\cdot, \cdot)$ in (3), the previous expression can be rewritten as follows:

$$\frac{1}{p_h(q) + (1 - p_h(q))\pi_h} > (1 - \pi_h) \frac{1}{p_h(0) + (1 - p_h(0))\pi_h} + \pi_h,$$

or

$$\begin{aligned} p_h(0) + (1 - p_h(0))\pi_h &> (1 - \pi_h)[p_h(q) + (1 - p_h(q))\pi_h] + \pi_h[p_h(q) + (1 - p_h(q))\pi_h][p_h(0) + (1 - p_h(0))\pi_h] \\ &= [p_h(q) + (1 - p_h(q))\pi_h][1 - \pi_h(1 - \pi_h)(1 - p_h(0))], \end{aligned} \quad (26)$$

which is in turn equivalent to:

$$p_h(0)(1 - \pi_h) + \pi_h > [p_h(q)(1 - \pi_h) + \pi_h][1 - \pi_h(1 - \pi_h)(1 - p_h(0))],$$

i.e.,

$$\frac{p_h(0)(1 - \pi_h) + \pi_h}{1 - \pi_h(1 - \pi_h)(1 - p_h(0))} > p_h(q)(1 - \pi_h) + \pi_h.$$

The above inequality implies that when $\tilde{\beta}$ is close to 1 the following upper bound on the level of $p_h(q)$ must hold, for all q :

$$p_h(q) < \bar{p}_h \equiv \frac{p_h(0)(1 - \pi_h^2) + \pi_h^2}{1 - \pi_h(1 - \pi_h)(1 - p_h(0))}. \quad (27)$$

Finally, recall that for q close to 1, a sufficient condition for q to implement a welfare improvement over $q = 0$ is that $p_h(q) < \tilde{p}_h(q)$, which is given in (24) by $\tilde{p}_h(q) = p_h(q) < \tilde{p}_h(q) \equiv \frac{\pi_l s_0((1 - \pi_l) - (1 - \pi_h)B/G)}{\pi_l s_0(1 - \pi_l) - (1 - \pi_h)(1 - (1 - \pi_l)s_0)B/G}$.

Hence, the condition in the proposition implies that for q close to 1 we have $\bar{p}_h < \tilde{p}_h(q)$.

Thus, on the basis of the previous discussion, we can conclude that there exists \bar{q} yielding a welfare improvement over $q = 0$.

Proof of Proposition 5 — $q = 1$ optimal

From the description of the equilibrium in Proposition 1, it is clear that region c. is empty when $q = 1$, and that the upper bound of region b. is $\frac{c}{\pi_h - \pi_l} \leq R - 1$. Hence a necessary condition for $q = 1$ to be optimal is that c lies in region b. Also recall that whenever we are in the interior of region b. (i.e. $\frac{c}{\pi_h - \pi_l} < R - 1$), we have $p_h < 1$. So a sufficient condition for $q = 1$ to be optimal in this case is $p_0(s_0, 1) \geq p_h(1)$.

We derive next a condition on the fraction s_0 of safe entrepreneurs that are born in each period that ensures that this is the case. We first find an upper bound for $p_h(1)$. Recall that from equation (12), $p_h(1)$ must satisfy:

$$\frac{c\pi_l}{\pi_h - \pi_l} = \hat{v}^r(p_h(1); 1)(1 - \tilde{\beta}). \quad (28)$$

Note that when $q = 1$, $\hat{v}^r(p; 1) \geq \frac{\pi_h(R - r_{zp}(p, 1)) - c}{1 - \tilde{\beta}}$ (the righthand side of this inequality is the payoff from being financed forever at the current interest rate). Substituting this into (28) above, we obtain $\pi_h(R - r_{zp}(p_h(1), 1)) - c \leq \frac{c\pi_l}{\pi_h - \pi_l}$, or, simplifying,

$$R - r_{zp}(p_h(1), 1) \leq \frac{c}{\pi_h - \pi_l} \quad (29)$$

If we now substitute $r_{zp}(p_h(1), 1) = \frac{1}{p_h(1) + (1 - p_h(1))\pi_h}$ into (29) and solve for $p_h(1)$, we obtain:

$$p_h(1) \leq \frac{1 - \pi_h(R - \frac{c}{\pi_h - \pi_l})}{(1 - \pi_h)(R - \frac{c}{\pi_h - \pi_l})}. \quad (30)$$

So a sufficient condition for $q = 1$ to be optimal is that $p_0(s_0, 1)$ is greater than or equal

to the term on the righthand side of (30). Using the expression for $p_0(s_0, q)$ given in (5), we obtain the following condition:

$$s_0 \geq \frac{\frac{c\pi_h}{\pi_h - \pi_l} - (\pi_h R - 1)}{(1 - \pi_h) \left((1 - \delta) \left(R - \frac{c}{\pi_h - \pi_l} \right) + \delta \right)}. \quad (31)$$

Finally, note that the righthand side of (31) is always less than 1, since by the stated assumptions of the Proposition $\frac{c}{\pi_h - \pi_l} < R - 1$, so that (31) can be satisfied for s_0 is sufficiently high.

Proof of Corollary — No Credit Bureau Optimal

The corollary follows by first noting that when there is no credit bureau, credit scores are constant over time (at the level $p = s_0$). This then implies that $\hat{v}^r(p; 1) = \frac{\pi_h(R - r_{zp}(p, 1)) - c}{1 - \beta}$. Hence for high effort to be implemented in equilibrium, s_0 must be (weakly) greater than the term on the righthand side of (30) above, which verifies that the condition in the Corollary is both necessary and sufficient.

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