The Politico-Economic Dynamics of China’s Growth

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Abstract

China's rapid growth has been driven by policy reforms that significantly reduce market frictions. Policy reforms are determined by the government according to its own politico-economic considerations. This paper embeds these politico-economic considerations in a macro model of China to endogenously study government policies, market frictions, and economic growth. In the model, an elite runs the government and maximizes its own incomes, facing a political constraint: getting enough supporters. The government provides high enough incomes to state workers in order to gain their support. It also controls capital allocations in the state and the private sector to balance between keeping enough supporters and extracting more taxes from the private sector. These policies initially generate rapid growth accompanied by declining labor and capital market frictions but in the long run, keep the frictions persistent, which are harmful to growth. The calibrated model can quantitatively account for salient aspects of China’s recent development and provide predictions for future dynamics.

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1 Introduction

China has been growing at a stellar rate for about 40 years. During this era, several policy reforms have been taken to reduce market frictions. The reduction of frictions has resulted in rapid economic growth. For example, in 1994, the government initiated the so-called “grasping the large and letting the small go” policies on state-owned enterprises (SOEs), and in late 1997, the 15th Congress of the Communist Party of China officially acknowledged the private sector as an important pillar of the economy, and these reforms led to a rapid state-to-private transition. Song et al. (2011) show that the labor and capital reallocations to the more efficient private sector have contributed significantly to the rapid growth since 1998. It is generally acknowledged that policy reforms have been crucial for the great transformation of China (see Chow (2015)) and that they have been determined by the government according to its own politico-economic considerations (see Shirk (1993) for the political logic of economic reforms). Understanding these considerations and policy reforms is crucial for understanding China’s development. This paper takes an important step toward this direction by incorporating, into a macro model, the government’s objective function and political constraint: The government is run by the elite who maximizes its own incomes, subject to the constraint that enough workers support the government. Policy reforms and dynamics of market frictions can be endogenously generated by the model. Providing the microfoundation for policies and frictions is useful not only for understanding the dynamics of frictions and growth in the past, but also for predicting future frictions and growth. Otherwise, the government policies and the dynamics of frictions must be assumed exogenously, usually following past trends. This assumption can be largely counterfactual. For example, the recent trends of labor and capital wedges and allocations significantly deviate from the trends before 2008. Hsieh and Song (2015) document that after 2007, the capital wedge between state and private firms stopped declining, suggesting that it becomes more difficult for private firms to borrow. Storesletten and Zilibotti (2014) show that the reallocation of labor from state to private firms has
stopped since 2008. Therefore, a model endogenously generating changes in frictions and policies is necessary for predicting future economic dynamics.

How does the government set policies to satisfy the political constraint - getting enough supporters? First, it provides state sector workers high incomes, using both wages and transfers, in order to gain their support. This policy creates labor market distortions. For example, Ge and Yang (2014) find that China’s state sector workers have been enjoying a wage premium between 20% and 30%, and this labor market wedge has been persistent and even increasing. State sector workers, not surprisingly, are found to be more supportive of the current regime and less supportive of democracy. Chen and Lu (2011) study survey data of Chinese individuals and show that state sector workers and the middle class are less supportive of democratic values, for example, multiparty competition, freedom of demonstrations, etc. Second, the government balances the capital allocations in state and private firms to maintain enough workers in the state sector. As shown in Storesletten and Zilibotti (2014) and reproduced here in Figure 1, China’s private employment share in manufacturing stopped increasing when it reached about 60% in 2008, after ten years of rapid growth. In other words, state sector employment has stopped declining and has stayed around 40% since 2008. How does the state maintain this significant fraction of workers in the state sector, while state firms are less productive than private firms (see
Brandt and Zhu (2010))? It does so through the capital allocation in favor of state firms. State firms get more financial resources, e.g., bank loans, so they can keep investing and hiring a large fraction of workers. Brandt and Zhu (2010) document that the state sector’s investment share stays around 60% though their employment share has been declining from 1998 to 2007.

In this paper, I embed the government’s political constraint in a two-sector growth model to study how political considerations shape government policies and market frictions and how policies and frictions affect growth. In the model, a political elite runs the government, extracts surpluses from state firms, and taxes the private sector. However, it faces a political constraint, i.e., it must gain support from a sufficient number of workers. The elite uses the following policy tools to maintain support and to maximize its incomes: in the labor market, setting wages and transfers for state sector workers, and in the capital market, controlling capital allocations to state and private firms. First, the government sets state workers’ incomes sufficiently high - as high as their expected incomes in the democracy - so that state workers support the current regime instead of democratization. The government sets state workers’ incomes using wages and transfers, at two costs: labor distortion from regulating wages and the direct cost of using transfers. If transfers paid by the government are large (i.e., wage burdens on state firms are low, given the targeted incomes of state workers), then state firms tend to hire more workers than efficiency would dictate. The redundant labor generates a labor wedge: The labor productivity in the state sector is lower than in the private sector. Second, the government can also control capital allocations in the state and private sectors when it needs to keep enough workers in the state sector. When the private sector is small and its capital level is low, maintaining enough political support is not costly, for two reasons: The number of state workers is large and expected wages and incomes in the democracy are low. As private sector capital grows, a trade-off for the government emerges: A larger private sector contributes more

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1 In this context, we can use “elite” and “government” interchangeably. Only when we discuss democracy, these two terms are not equivalent.
tax revenues, but also results in higher costs for maintaining the regime, because a growing private sector increases wages and competition for workers. When private sector capital grows to a high level such that state employment declines to the critical level for sustaining the regime, the government must maintain high enough investments in the state sector to hire enough workers there. If the private sector capital level becomes too high, then the government may even prefer to limit the size of the private sector, e.g., by reducing loans to private firms.

The government changes capital market policies as private sector capital grows, therefore the economy’s growth pattern also changes accordingly. The economy develops along a three-stage politico-economic transition. The first stage is rapid growth, during which the private sector grows rapidly, capital and labor are reallocated to the private sector, and the productivity gain from the reallocation contributes to the rapid growth. This stage, according to the quantitative model, corresponds to 1998–2007 in China. As privatization continues, the state employment share declines to the critical level, and then the economy enters the second stage, state capitalism. In this stage - 2008 to 2036 - the government overinvests in the state sector to keep its employment share sufficiently high. This stage features a halt to privatization and an increase of investment in the state sector. Though the relative size of the private sector does not change, its absolute level of capital still grows, while state sector capital grows proportionally. The economy still grows rapidly, mainly driven by the large investment instead of efficient reallocation of capital and labor. As private sector capital keeps growing, the economy enters the third stage after 2036, and there are two possible scenarios. The first is middle-income trap. The government chooses to maintain the existing political system and the frictions. It creates barriers to private sector capital growth, e.g., by tightening lending to private firms. Private sector capital growth slows and the economy stops growing before reaching efficient levels of capital and output. This case happens if the cost of sustaining the regime is low, e.g., if costs of transfers and investments are low. The other possible case is sustained growth. As
private sector capital grows, it becomes too costly to continue investing in the less efficient state sector and therefore the government chooses to stop maintaining the regime and to democratize. Frictions in the labor and the capital market vanish, and the economy keeps growing until the output reaches the efficient level.

The model is calibrated to China’s manufacturing sector since 1998 and can account for salient aspects of China’s recent growth experience, especially the following three: the state-to-private transition, the dynamics of labor and capital market wedges between state and private firms, and output growth. The first stage - rapid growth - corresponds to the Chinese economy from the 1990s until 2007. During this period, the private sector employment share increases rapidly, the productivity of labor and capital in state firms relative to that in private firms increases, and efficiency and output grow rapidly. See private sector employment growth and output growth before 2007 in Figure 1: the private employment share in manufacturing (left panel) increases from 15% in 1998 to 60% in 2007, and real output growth in manufacturing (blue solid line in the right panel) is high and increases from 8% to 15%. Notice that in that period, output growth in the aggregate economy (real GDP growth shown by the red dashed line) follows a very similar pattern as that in manufacturing, suggesting that the economic dynamics in the manufacturing sector is representative of the aggregate economy. The model in this paper, focusing on the manufacturing sector, can deliver this rapid labor reallocation from state to private firms, with the help of the rapid increase of private sector capital, which benefits from the capital reallocation from state firms and also from the capital accumulation of private firms. Moreover, the model can also generate the rapid output growth from 1998 to 2007, given TFP growth estimated by Brandt and Zhu (2010). Notice that TFP growth is not the only reason for the output growth, and efficiency gain from reallocation is the other important factor, both in the data and in the model. Hsieh and Song (2015) document an increase of the labor productivity of state firms in manufacturing relative to labor productivity of private firms from 60% in 1998 to 75% in 2007, and an increase of relative
capital productivity from 34% to 46%. Using the total factor productivity (TFP) growth of state firms relative to that of private firms implied by Hsieh and Song (2015) as the input, the model can successfully account for the dynamics of labor and capital wedges through the reallocation of capital and labor.

The state capitalism stage started when the employment share of the state sector reached the critical level in 2008. The state employment share stays around 40% afterward. The model delivers the halt of labor reallocation through the political constraint: The government needs to keep sufficiently many state sector workers. The model also generates a continuous increase of the labor productivity of state firms relative to that of private firms, and a much slower increase of the capital productivity of state firms, consistent with the findings in Hsieh and Song (2015). Capital productivity grows slowly for two reasons: overinvestment in the state sector to maintain workers there and decline of transfers relative to incomes.\(^2\) Stop of state-to-private transition and favor of state investments are consistent with the phenomenon “the state advances the private sector retreats” (guojin mingtui) and the increasing difficulty of financing private-sector investments in recent years. Without state-to-private reallocation and the efficiency gain, the growth rate declines in this stage, as we can see in Figure 1. However, because of the large state investment, the absolute level of growth is still reasonably high, e.g., stays above 5% in 2017, both in the data and in the model.

The model also provides predictions for future dynamics of wedges and growth, and for potential development paths. The third stage in the calibrated economy is the middle-income trap. The government directly restrict growth of private sector capital by further tightening private sector borrowing constraint.\(^3\) Labor market frictions persist and capital market frictions even increase, at a faster pace than in the second stage. The frictions

\(^2\)As incomes increase, costs of using transfers increase, and the size of transfers relative to incomes declines. The decline of transfers leads to increases of wages, labor productivity, and capital-labor ratio in the state sector, and consequently, results in slow growth of capital productivity.

\(^3\)In the state capitalism stage, the costs and difficulty of private sector borrowing already increase, which is an indirect consequence of increasing state investments.
slows down growth and the economy stops growing before its income converges to the high level of developed economies. The government chooses to maintain the frictions and the middle income level in order to sustain the regime. The other possible development path is sustained growth. This path emerges if costs of state investments and transfers are sufficiently large - much larger than in the benchmark calibration. Then the elite finds maintaining a large enough state sector too costly and chooses to democratize. Capital and labor market frictions vanish and growth continues until the income reaches the high level.

This paper contributes to three strands of literature. The first is on China’s economic growth with labor and capital market frictions. The contribution of this paper is providing a microfoundation for capital and labor market frictions, studying their endogenous evolution and illustrating their importance for growth. Brandt and Zhu (2010) and Brandt et al. (2012) use a three-sector growth model with capital wedges between state and private firms to account for China’s growth since 1978. Song et al. (2011) construct a two-sector growth model to study China’s growth in the transition from the state to the private sector, given exogenously the financial constraints on private firms, focusing on the manufacturing sector from 1998 on. Cheremukhin et al. (2015) study China’s structural changes and the dynamics of wedges in the labor and the capital market and their contributions to growth from 1953. This paper offers explanations for the dynamics of labor and capital market wedges and frictions in the past, which are taken given in the above papers, and moreover, this paper provides predictions for future dynamics of frictions and studies the implications for future growth.4

The second strand of literature is the study of the interaction between political development and economic development. One aspect of the interaction is that political institutions affect economic development. For example, Acemoglu (2008) studies the economic performance in oligarchic societies, in comparison with that in democratic societies. He shows

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Notice that in this paper, frictions and the state-to-private transition are important but not the only determinant of growth. TFP growths of state and private firms are also important for growth.
that an oligarchic society may achieve higher efficiency at first because the elite protects its property better and invests more than in a democracy. However, in the long run, the elite blocks new entrepreneurs from entering, and economic growth becomes slower than in the democracy. This paper’s analysis of growth in the oligarchy is in the spirit of Acemoglu (2008), but with an important difference regarding efficiency. In this paper, the higher short-run growth in the oligarchy is not due to higher efficiency but is precisely because of its distortions in the labor and the capital market: The labor market distortion leads to a low initial output such that growth can be high initially; and investment in the state sector in the state capitalism stage is inefficiently high but can keep the growth rate high. The mechanism generating slower long-run growth in the oligarchy is similar to that in Acemoglu (2008): In the long run, the elite creates barriers for entrepreneurs. The other aspect of the interaction, i.e., how economic development affects political development, is also studied in my paper. Economic progress may lead to political progress, e.g., democratization, but the latter happens only under the right conditions. The impact of economic development on political progress described in this paper is related to but different from the modernization theory started by Lipset (1959), which suggests that economic progress is sufficient for political progress such as democratization.

Finally, this paper also contributes to the literature of the middle-income trap. There are many important factors determining the middle-income trap, such as the income threshold, labor costs, population dynamics, etc. This paper shows that government policies is one of the important factors. Why do some countries successfully adopt good policies and rapidly grow out of poverty but then suddenly fail to implement appropriate policies and to become rich countries? This paper offers a theoretical explanation: The interest of the elite aligns with growth in early stages of development but not in later stages. It also studies various government policies that could help middle-income countries to grow further and the conditions that induce governments to implement these policies.

The rest of the paper is organized as follows. Section 2 presents the model. In Section
3, the model is calibrated to the manufacturing sector in China from 1998 to present. The quantitative model accounts for the time series in China’s recent development in the manufacturing sector and offers predictions for the future development. Extensions of the model, including an calibration to China’s urban economy since 1994 and the alternative development path are also studied. Section 4 concludes. Proofs and more details of the model and data are in the Online Appendix.

2 The Model

This section presents a two-sector growth model, in which the dynamic general equilibrium is deterministic and agents have perfect foresight. The two sectors are the state (S) and the private (P) sector. There is a continuum of state firms and private firms. Firms are neoclassical: They produce the same final goods using capital and labor and maximize profits while taking prices as given. There are three groups of infinitely many agents: the elite (e), private entrepreneurs (p), and workers (w). The population size of workers is normalized to 1, while the sizes of the elite and entrepreneurs are both infinitesimal and denoted as $\epsilon$. The elite supplies capital to state firms, and entrepreneurs supply capital to private firms. They receive capital incomes, consume, and save. Workers supply labor and receive wages. A worker may work in an S firm or a P firm. Representative agents are used to describe the behavior of some groups of agents, i.e., “the elite,” “the entrepreneur,” “the S firm,” and “the P firm”. Differently, we still refer to workers as “S workers” and “P workers,” respectively.

Political economy is incorporated into the growth model. The economy starts in an oligarchic regime, where the elite controls the government but faces a political constraint: It needs sufficiently many workers to support the regime. If the number of supporters is

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5 The population size of the elite and entrepreneurs don’t matter in the model as long as they are small enough such that in the democracy the median voter is a worker. We will see more details in Subsection 2.5.

6 Population sizes of state sector workers and private sector workers change over time, so it is better to discuss them as groups instead of as two representative agents.
large enough, the regime survives; otherwise it collapses, and democratization occurs. Each worker decides whether to support the regime by comparing her incomes in the current regime with her expected incomes in the democracy. To maintain enough supporters, the government can strategically increase state workers’ incomes using the following three policies: (1) setting state sector wages, (2) giving transfers to state workers, and (3) allocating capital in the state and the private sector.

In this section, I first describe the model setup in Subsection 2.1 to 2.4, and then solve for the model and characterize some analytical properties of the solution in subsections 2.5 to 2.7.

## 2.1 Preferences, Technology, and Markets

Agents live for infinite periods. The lifetime utility of the elite is the discounted sum of utility in all future periods:

$$U = \sum_{t=0}^{\infty} \beta^t \log(c_{et}).$$

The elite discounts the future at the rate $\beta$ and the coefficient of relative risk aversion is 1. The representative entrepreneur has the same utility function as that of the elite and her consumption is denoted as $c_{pt}$. In each period, the entrepreneur supplies capital to the P sector and receives capital income. The elite supplies capital to the S sector and receives capital income, and additionally, it obtains another income in the oligarchy: the government’s budget surplus. After the government redistributes using taxes and transfers, the elite and the entrepreneur consume and save for the next period. Savings can affect capital supply and capital incomes in the next period. A worker supplies one unit of labor to a firm in each period. A worker is assumed to live hand to mouth and to be myopic: She consumes all her current-period income and cares only about current-period income, even when she makes political decisions.

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7We will discuss the government’s surplus, together with its redistribution later in Subsection 2.3.
The S firm and the P firm are different in two aspects: access to the capital market and productivity. First, the S firm rents capital from the S-sector capital market, where capital is supplied by the elite, and the P firm rents capital from the P sector capital market, where capital is supplied by the entrepreneur. Both firms hire from the same pool of workers. Second, the S firm is less productive than the P firm. The technologies of the S and the P firms are described by the following Cobb–Douglas production functions:

\[ Y_{St} = z_{St} K_{St}^{\alpha} L_{St}^{1-\alpha}, \]
\[ Y_{Pt} = z_{Pt} K_{Pt}^{\alpha} L_{Pt}^{1-\alpha}, \]

where \( z_{j,t}, K_{j,t}, \) and \( L_{j,t}, \ j \in \{S, P\} \) stand for TFP, capital, and labor in sector \( j \) at time \( t \), respectively. TFP sequences are dynamic and deterministic. Capital depreciates at the rate \( \delta \). Firms’ profit maximization problems imply that wages and capital returns are equal to the marginal productivity:

\[ w_{St} = (1 - \alpha) z_{St} K_{St}^{\alpha} L_{St}^{1-\alpha}, \quad (1) \]
\[ w_{Pt} = (1 - \alpha) z_{Pt} K_{Pt}^{\alpha} L_{Pt}^{1-\alpha}. \quad (2) \]

where \( w_{St} \) and \( w_{Pt} \) represent wages, and

\[ r_{St} = \alpha z_{St} K_{St}^{\alpha-1} L_{St}^{1-\alpha}, \quad (3) \]
\[ r_{Pt} = \alpha z_{Pt} K_{Pt}^{\alpha-1} L_{Pt}^{1-\alpha}, \quad (4) \]

where \( r_{St} \) and \( r_{Pt} \) stand for gross returns of capital (before depreciation and taxes).

The labor market clearing condition is

\[ L_{St} + L_{Pt} = 1. \quad (5) \]

\[ ^{8}\text{In the equilibrium, all workers prefer to work in the S sector because S workers’ incomes are higher, and then the S firm randomly draw a fraction of workers according to its labor demand.} \]
In the financial market, there is a representative competitive bank. It has access to the international bond market where the interest rate $r$ is exogenously given. Let us denote $R = 1 + r$. So the bank serves as an intermediary which allows domestic agents to save and borrow at interest rate $r$. The elite can borrow from the bank to invest in the S firm, together with its own assets. The elite has deep pockets, i.e., there is no constraint on how much it can borrow. The entrepreneur can also borrow from the bank, but she faces a borrowing constraint: bank loans cannot exceed $\eta_t - 1$ fraction of her assets. In other words, the maximal capital-to-asset ratio in the P sector is $\eta_t$:

$$K_{Pt} \leq \eta_t a_{pt},$$

(6)

where $a_{pt}$ and $K_{Pt}$ represent the entrepreneur's assets and the P firm's capital, respectively. $\eta_t$ may vary over time, depending on the government's policy, which will be described later in subsection 2.3.

2.2 The Political Constraint

In the oligarchy, the elite runs the government and sets policies to maximize its lifetime utility. However, it faces the following political constraint: It must obtain political support from sufficiently many workers. Otherwise, a revolution occurs, and the regime switches to democracy. \(^9\) In each period, workers in the S sector decide whether to support the oligarchic regime. If the number of supporters in the S sector is larger than a critical level $L$, the regime survives. \(^10\) Let us denote a worker's expected income in the democracy as

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\(^9\) Expected incomes in the democracy matter for the oligarchy, and they can be taken as exogenously given and being determined by the equilibrium in the democracy, when we focus on the equilibrium in the oligarchy. Later in Subsection 2.5, we will study the equilibrium in the democracy.

\(^10\) This setting, focusing on S workers' political support, implies that in the equilibrium path, having a large enough fraction of supporters in the S sector is equivalent to getting enough support for the oligarchy and we don’t have to calculate P workers’ political support. This setting is without loss of generality: In the equilibrium path, P workers always support democratization, as government policies are in favor of the state sector and P workers’ incomes are always lower than their expected incomes in the democracy. This political process can be formalized as a global game according to Morris and Shin (2000). Details of the game are in the Online Appendix.
\( y_{wt}^D \) and take it as given here. The expression of \( y_{wt}^D \), depending on state and private sector capital, will be derived in Subsection 2.5. Now the political constraint is equivalent to the following two economic constraints. First, the high-income constraint, i.e., an S worker's income is high enough:

\[
y_{St} = w_{St} + T_T \geq y_{wt}^D,
\]

where \( y_{St} \) stands for the S worker's income in period \( t \), which includes the wage paid by the S firm \( w_{St} \) and transfers from the government \( T_T \). Second, the large-state-employment constraint, i.e., the S sector's employment is high enough:

\[
L_{St} \geq L.
\]

2.3 The Government

In the oligarchy, the elite controls the government and decides government policies.\(^{11}\)

In the beginning of each period, the elite can choose to democratize voluntarily or to stay in the oligarchy. If the first is chosen, the regime switches to democracy forever. If the latter is chosen, then the elite sets the following policies: S sector wage, transfers to each S worker, S sector capital, and credit constraint of the P firm.\(^{12}\) Moreover, the government taxes P sector workers and the entrepreneur at an exogenous rate \( \tau > 0. \)

To simplify the expressions, I assume that the tax is imposed on the gross return on the

\(^{11}\) We can use "the elite" and "the government" interchangeably in this context.

\(^{12}\) S sector capital is set optimally for the elite as a group. This model setup implies that the collective action problem for the elite is assumed away: every individual elite member supplies capital according to the decision made by the elite as a group, though she may have incentives to choose a different level of capital that she supplies.

\(^{13}\) The tax rate can be endogenized. I also extend the model and allow the government to choose the tax rate of the entrepreneur given an upper bound \( \bar{\tau} \), which is exogenously determined by the state capacity described in Besley and Persson (2009). Then the government can potentially choose \( \tau < \bar{\tau} \). However, given the calibration in this paper and the initial assets of the entrepreneur, I find that the government always sets \( \tau = \bar{\tau} \) along the transition. So in this paper, I do not need to consider the endogenous tax rate. The case \( \tau < \bar{\tau} \) happens only in the event that the entrepreneur's asset level is extremely small but this case is not relevant for the years studied in the model. Moreover, in reality, once a tax rate is set, it lasts for a relatively long time and it is not easy to adjust it annually. So, even in a year that the optimal tax rate can be lower, the government, knowing the optimal tax rate will be at \( \bar{\tau} \) soon, should set the tax rate according to the optimal level in the medium and long run, instead of on an annual basis.
entrepreneur’s investment in the P firm, i.e., \( r_{Pt} K_{Pt} \), so that taxes from P sector workers and the entrepreneur together equal \( \tau \) fraction of the P sector output.

The cost of transfers to each S worker is a convex function of the transfers: \( b_t T_t^2 \), where \( T_t \) represents transfers to each S worker, and the cost parameter \( b_t \) can change over time. Given S workers’ incomes and the S firm’s capital, higher transfers imply lower wages paid by the S firm, higher labor demand, and lower labor productivity of the S firm. In the model, we refer to the variable \( T \) as transfers to state workers, while it should be interpreted more generally as all real-world policies which increase political support of state workers and are financed by the government. Those policies include not only cash transfers, but also non-cash benefits, subsidies and government expenditures on social stability, and all policies that increase state workers’ expected incomes in the current regime and reduce their expected incomes after democratization. For example, housing, education, and tax benefits for state workers increase state workers’ factual incomes. Employment subsidies for state firms is equivalent to direct transfers to state workers from the government. Government expenditures on social stability increase workers’ incomes and utility in the current regime and reduce their expected incomes after democratization.\(^{14}\) Spending on political campaign and propaganda can change workers’ belief on the outcomes of democratization and their corresponding expected incomes.\(^{15}\)

The government can set S sector capital without any financial constraint, because it has deep pockets and can borrow as much as it wishes from the bank at the interest rate \( r \). The government can also influence the P sector capital: It can set the P firm financial constraint \( \eta_t \) in a bounded region \([\eta, \tilde{\eta}]\). The government can create barriers to entrepreneurs’ access to the financial market, or give administrative instructions to state banks regarding how much lending is allocated to entrepreneurs (see Brandt and Zhu (2000)). \( \tilde{\eta} \) represents the

\(^{14}\)Increasing expenditures on courts, police and procuratorate can reduce crimes and social instability, while it can also increase the cost of revolution and democratization, and therefore can reduce workers’ expected incomes after democratization.

\(^{15}\)Non-monetary benefits of democracy affect workers’ political preference. Those factors and policies that alter those factors can also be captured by the model variable \( T \). See more details about the general definition of transfers in the Online Appendix.
natural borrowing constraint on entrepreneurs if the government does not put additional restrictions on private sector borrowing.\textsuperscript{16} \( \eta \) represents the leverage of the private firm given the highest level of restriction that the government can set.\textsuperscript{17} The setting on the financial market is similar to that in Song et al. (2011), but with a crucial difference: Capital in the state sector and credit constraints in the private sector are endogenously determined by the government in this paper. Therefore, this model can endogenously account for past dynamics of capital market frictions and can be used to predict the future trend of capital market frictions.

Revenues in the government budget include the profit of the S firm and taxes from the P sector, and the expenditures are costs of transfers to S workers and payments to the elite. We assume that payments to the elite are equal to revenues minus transfers to S workers, i.e., the government’s budget is be balanced in every period.\textsuperscript{18} The government’s budget constraint can be written as

\[
\pi_{St} + \tau Y_{Pt} = b_i T_i^2 L_{St} + y_{et},
\]

where \( \pi_{St} \) is the profit of the S firm and \( y_{et} \) is the elite’s non-asset income which equals the government revenues minus expenditures. The government’s budget constraint can be

\textsuperscript{16}In this case, there is still a natural limit on the borrowing of private entrepreneurs, due to the following moral hazard problem: If a private entrepreneur gets a loan that is too large to be supported by her assets, she may choose to default on the loan and leave with the money.

\textsuperscript{17}For example, if the most stringent policy the government can set is to let the bank lend nothing to the entrepreneur, then the entrepreneur has to finance her investment using her own savings. This implies \( \eta = 1 \).

\textsuperscript{18}We can also allow for budget surpluses and deficits, and the equilibrium stays the same. We can let the government pay more to the elite than the difference between revenues and expenditures in a certain period, and the deficit is financed by a debt. Eventually, the debt will be paid back in the future by reducing payments to the elite, given the no-Ponzi condition. As long as the interest rate for the government debt and the interest rate for the elite’s savings and bank loans are equal to the international interest rate \( r \), the government’s debt is essentially equivalent to the elite’s debt. What matters is simply the elite’s assets minus the government debt, which equals the elite’s assets when we assume the government budget is balanced in every period.
used to derive the expression for the elite’s non-asset income $y_{et}$:

$$y_{et} = \pi_{St} + \tau Y_{Pt} - b_t T_t^2 L_{St}$$

$$= \alpha z_{St} K_{St}^{\alpha} L_{St}^{1-\alpha} - r K_{St} + \tau z_{Pt} K_{Pt}^{\alpha} L_{St}^{1-\alpha} - b_t T_t^2 L_{St}.$$ 

In later analysis, we will substituted this expression into the elite’s problem, and the government’s budget constraint will be automatically satisfied in every period.

### 2.4 Timing of Events

The events in each period are the following:

1. In the beginning of period $t$, the elite decides whether to voluntarily democratize. If so, the political system switches to democracy forever; if not, the following events occur sequentially.

2. Capital allocation: The elite sets $K_{St}$ and $\eta_t$, and the entrepreneur chooses $K_{Pt}$.

3. The elite sets the S sector’s wage $w_{St}$ and transfers $T_t$.

4. The S firm and the P firm hire workers. The S firm randomly select workers according to its labor demand.

5. S workers decide whether to support the regime. If there are not enough supporters, the political system switches to the democracy. If there are enough supporters, the oligarchy survives.

6. Firms produce. Labor and capital incomes are distributed. Taxes are collected and transfers are made.

7. The elite and the entrepreneur consume and save. The economy enters the next period.

Notice that the game in the oligarchy may end in step 1 or step 5 if democratization happens. If so, the exit payoffs of the elite and workers are determined by the equilibrium in the democracy, discussed in the next subsection.
2.5 Democracy

In the oligarchy, when workers decide whether to support democratization, they consider their expected incomes in the democracy. So does the elite when it decides whether to voluntarily democratize. So the outcomes in the democracy are useful for pinning down the equilibrium in the oligarchy. Here I summarize the model setup and some main results in the democracy, especially the outcomes relevant for the equilibrium in the oligarchy, i.e., (1) workers’ incomes $y_{wt}^D$; and (2) the elite’s non-asset income $y_{et}^D$. The other equilibrium outcomes and conditions are in the Online Appendix.

The major difference is that in the democracy, the government is run by the representative worker. The government taxes the elite and the entrepreneur at the exogenous rate $\tau > 0$ and then transfers tax revenues to all workers. The economy is simply a competitive equilibrium given exogenous tax rates. The economy is similar to the one in Song et al. (2011).

The competitive labor market implies that wages in the S and the P sector are the same, determined by the marginal productivity of labor:

$$w_t^D = (1 - \alpha) z_{St} (K_{St})^\alpha (L_{St}^D)^{-\alpha} = (1 - \alpha) z_{Pt} (K_{Pt})^\alpha (L_{Pt}^D)^{-\alpha} = (1 - \alpha) \left( \frac{1}{z_{St}^\alpha K_{St}} + \frac{1}{z_{Pt}^\alpha K_{Pt}} \right)^\alpha,$$

where $w_t^D$ is the wage and $L_{St}^D$ and $L_{Pt}^D$ represent the labor allocation in the democracy. The representative worker’s final income includes the wage and transfers:

$$y_{wt}^D = w_t^D + \tau \left( \alpha z_{St} K_{St}^\alpha (L_{St}^D)^{1-\alpha} + \alpha z_{Pt} K_{Pt}^\alpha (L_{Pt}^D)^{1-\alpha} \right) = \left( 1 + \frac{\tau \alpha}{1 - \alpha} \right) w_t^D = (1 - \alpha + \tau \alpha) \left( \frac{1}{z_{St}^\alpha K_{St}} + \frac{1}{z_{Pt}^\alpha K_{Pt}} \right)^\alpha.$$  

19The workers’ dominant population size, i.e., $1 > 2\epsilon$, guarantees that they win the majority voting. In this sense, partial democracies where political voting systems exist but are manipulated by the elite do not correspond to the democracy in this model but rather to the oligarchy.
The capital market in the democracy is also competitive. Each elite member supplies capital to the S sector, taking prices as given. The major difference from in the oligarchy is that in the competitive equilibrium, S sector capital supply is no longer collectedly decided by the elite as a group but by the market interest rate \( r \): marginal return of investment in the S firm equals interest rate \( r \), if the S sector exists.\(^\text{20}\) If the S sector does not exist because P sector capital is large enough and the return to S sector capital is lower than \( r \), then the elite saves its assets in the bank and also receives the return at the rate \( r \). In either case, the elite’s income simply equals the return from assets at the rate \( r \), implying that the elite’s non-asset income is 0. Basically, the elite “retires” after democratization:

\[
y^D_{et} = 0, \quad (12)
\]

where \( y^D_{et} \) is the elite’s non-asset income in the democracy.

The entrepreneur faces the borrowing constraint. If the marginal rate of return on her investment in the P firm is higher than \( r \), she borrows as much as possible given the constraint \( K_{pt} \leq \eta_t a_{pt} \). The government always prefers a higher \( \eta_t \), which implies a higher \( K_{pt}, \) a higher \( w_t \), and a higher worker income \( y^D_{w} \), so it sets \( \eta_t \) to its upper bound:

\[
\eta^D_t = \bar{\eta}. \quad (13)
\]

The dynamic equilibrium in the democracy is similar to that in Song et al. (2011). If the S sector exists, then the P sector’s capital return equals the world interest rate; otherwise it can be lower:

\[
r^D_{St} = \begin{cases} 
  r & \text{if } L^D_{St} > 0, \\
  \leq r & \text{if } L^D_{St} \leq 0.
\end{cases} \quad (14)
\]

If the S sector exists, then the rate of return determines the capital-labor ratio in the S

\(^{20}\)In the oligarchy, when the elite controls the government to set S sector capital, it internalizes the impact of capital supply on capital return.
sector as in equation 3, and the latter determines the wage as in equation 1. The wage in the P sector is the same as that in the S sector, and therefore, the P sector’s wage $w_{Pt}$ and rate of return to capital $r_{Pt}$ are determined by equations 2 and 4, respectively. If $r_{Pt}$ is high enough, the entrepreneur’s assets and the P sector capital grow over time. Eventually, all workers move to the P sector and the S sector disappears. In this case, the P sector labor equals one, and then the P sector’s capital-labor ratio determines the wage and the return to capital. The main results in the democracy are summarized in the proposition below.

**Proposition 1.** In the democracy, the representative worker’s incomes equal the wage determined by the competitive equilibrium given S and P sector capital $w^D_t$, plus transfers which equal $\tau \alpha / (1 - \alpha)$ fraction of the wage: $y^D_{Dt} = (1 + \tau \alpha / (1 - \alpha)) w^D_t$.

The elite “retires”: it receives only the return on its assets but zero non-asset incomes, i.e., $y^D_{et} = 0$.

The government sets $\eta^D_t = \bar{\eta}$.

The competitive equilibrium determines $K_{St}$ and $w_{St}$.

### 2.6 Equilibrium in the Oligarchy

Given the model setup described above, now we can set up the dynamic problems of the two major agents (the entrepreneur and the elite), define the equilibrium, and solve for it. Let us first formalize the entrepreneur’s problem. She maximizes her lifetime utility by choosing the P sector capital supply subject to the borrowing constraint and the deterministic sequences of consumption and savings, taking the return to P sector capital $r_{Pt}$ as given:

$$\max_{\{K_{Pt}, a_{Pt}, c_{Pt}\}} \sum_{t=0}^{\infty} \beta^t \log c_{Pt}$$

s.t. $c_{Pt} = Ra_{Pt} + ((1 - \tau) r_{Pt} - \delta - \alpha) K_{Pt} - a_{p,t+1}$.  

(15)
The no-Ponzi condition is imposed. Each entrepreneur takes \( r_{pt} \) as given because each entrepreneur’s choice cannot affect aggregate variables and the political system.

The elite chooses the political system, government policies in the oligarchy, consumption, and savings to maximize its lifetime utility. The elite’s problem can be decomposed into three parts: (1) choosing the sequence of political systems \( M_t \in \{O, D\} \); (2) setting government policies in the oligarchy, subject to the political constraints; and (3) choosing consumption and savings throughout the lifetime. First, because once democratization happens, democracy is consolidated, choosing the sequence of the political systems \( M_t \) is simply equivalent to choosing the period to democratize \( T^D \), which is an extended natural number, including \(+\infty\), which represents staying in oligarchy forever. We can write \( T^D \in \mathbb{Z}_{\geq 0} = \mathbb{Z}_{\geq 0} \cup \{+\infty\} \). Second, for \( t < T^D \), i.e., in the oligarchy, the elite chooses government policies \( \{K_{St}, w_{St}, T_t, \eta_t\} \), given the political constraint. The elite takes into account that these government policies affect the aggregate prices and the entrepreneur’s choices on assets and capital. After democratization happens, i.e., for each \( t \geq T^D \), the elite “retires” and basically receives only the return on its assets, as discussed in Proposition 1. Third, the elite chooses consumption and saving subject to the budget constraint, as we often see in standard growth models. To sum up, we can form the elite’s problem as:

\[
\max_{T^D, \{K_{St}, w_{St}, T_t, \eta_t\}^{T^D-1}_t, \{c_e, a_e\}^{\infty}_t} \sum_{t=0}^{\infty} \beta^t \log c_e
\]

subject to the following sets of constraints: (1) the budget constraint

\[
c_e = R a_e + y_e - a_{e,t+1}, \quad \text{(18)}
\]

where

\[
y_e = \begin{cases} 
\alpha z_{St} K_{St}^\alpha L_{St}^{1-\alpha} - r K_{St} + \tau z_{pt} K_{pt}^\alpha L_{St}^{1-\alpha} - b_{t} T_t^2 L_{St} & \text{if } t < T^D \\
0 & \text{if } t \geq T^D,
\end{cases} \quad \text{(19)}
\]
(2) the political support constraint for all $t < T^D$, or equivalently, two economic constraints described by equation 8 and equation

$$w_{St} + T_t \geq (1 - \alpha + \tau \alpha) \left( z_{St}^\frac{1}{\alpha} K_{St} + z_{Pt}^\frac{1}{\alpha} K_{Pt} \right)^\alpha,$$

which is obtained by substituting the expression of workers’ incomes in the democracy (equation 11) into the high-income constraint in the oligarchy (equation 7), (3) optimization of firms and prices determination, described in equations 1 to 4, (4) optimization of the entrepreneur: the sequence of allocations $\{K_{Pt}, a_{Pt}\}_{t=0}^\infty$ is the solution of the entrepreneur’s problem given the sequence $\{r_{Pt}\}_{t=0}^\infty$, as described by equations 15, 16 and 6, and (5) market clearing conditions. Moreover, the no-Ponzi condition is imposed to the elite’s problem.

It is useful to compare the elite’s problem with a problem seen in many economic models: a household’s problem with endogenous retirement decision. The elite chooses to “retire,” or more precisely, to democratize, in period $T^D$. Before “retirement,” it makes “efforts” - government policies - and obtains income $y_{et}$, in addition to the return from its assets, while after that, it lives on the return of assets but has no additional income: $y_{e,t\geq T^D} = y_{e}^D = 0$. Of course, there are also important differences, including the time horizon, the political constraint, and that the elite takes into account how its choices affect aggregate variables and prices. However, the core concepts of the problem and the solution are similar. Notice that zero non-asset income after democratization, i.e., $y_{e}^D = 0$ implies that other equilibrium outcomes after $T^D$ do not matter for the elite’s problem.

The equilibrium in the oligarchy is not a competitive equilibrium, because the elite can directly choose aggregate variables and prices, e.g., $K_{St}$ and $w_{St}$. So the equilibrium can be considered as the solution of a planner’s problem, and the planner is the elite who chooses the sequence of political systems, government policies, prices, and allocations to maximize its own utility, subject to the budget constraint, the political constraint, and
incentive constraints of other agents - firms and the entrepreneur. In this problem, the equilibrium outcomes in the democracy, including the elite’s income $y_e^D$, are taken as exogenously given. Moreover, some other model parameters are also taken as exogenously given, including firm productivities, cost parameter of transfers, and other parameters including the tax rate and the minimal number of supporters in the oligarchy. These exogenous sequences are dynamic, but deterministic. So the equilibrium concept is perfect foresight. We can formally define the dynamic politico-economic equilibrium starting in the oligarchy as follows.

**Definition 1.** A sequence of policies, prices, and allocations

$$\{M_t, \eta_t, T_t, w_{St}, w_{Pt}, r_{St}, r_{Pt}, K_{St}, K_{Pt}, a_{et}, a_{pt}, c_{et}, c_{pt}\}_{t=0}^{\infty}$$

is a dynamic politico-economic equilibrium starting in the oligarchy if

1. The sequence of the political systems $\{M_t\}_{t=0}^{\infty}$ can be partitioned into two sub-sequences by an integer $T^D$: $M_{t<T^D} = O$ and $M_{t \geq T^D} = D$.
2. Firms maximize their profits taking prices as given, implying that prices satisfy equations 1 to 4.
3. The entrepreneur chooses $\{K_{Pt}, a_{pt}, c_{pt}\}_{t=0}^{\infty}$ to maximize her lifetime utility described by equation 15, subject to constraints described by equations 6 and 16.
4. The elite chooses $T^D, \{K_{St}, w_{St}, T_t, \eta_t\}_{t=0}^{T^D-1}$, and $\{c_{et}, a_{et}\}_{t=0}^{\infty}$ to maximize its lifetime utility, subject to its budget constraint described by 18 and 19, the politico-economic constraints 7 and 8, and the incentive constraints that firms and the entrepreneur optimize.
5. The market clears, as described by equation 5.
6. Aggregate variables in the democracy are given by the competitive equilibrium characterized by Proposition 1. In particular, $\{y_{et}, \eta_t, r_{St}\}_{t=T^D}^{\infty}$ are determined by equations 12, 13, and 14.

Notice that two simplifications are made, in order to focus on the equilibrium in the oligarchy and keep the definition of the equilibrium succinct. First, description of the equi-
librium after democratization in item 6 is kept simple: only variables that are important and different from those in the oligarchy, i.e., \( \{y_{et}, \eta_t, r_{St}\}_{t=T}^\infty \), are described. Some other equilibrium conditions in the democracy, e.g., firms’ and the entrepreneur’s optimization problems, are standard and the same as in the oligarchy, so they are not added into item 6.\(^{21}\) Second, some intermediate variables, which can be expressed as functions of other variables defined in the equilibrium, are not included in the definition. For example, workers’ income in democracy \( y_{wt}^D \) can be expressed as a function of capital, as in equation 11, and we can substitute this expression into the elite’s problem, as in equation 17. Then we don’t have to include \( y_{wt}^D \) in the definition of the equilibrium. Similarly, the government budget constraint is in the expression of \( y_{et} \), so it is automatically satisfied and is not included in the definition.

### 2.7 Analytical Properties of the Equilibrium

The equilibrium need to be solved numerically, but we can analytically characterize two important static properties of the equilibrium: allocations and wedges in the labor and the capital market.

Let us first focus on how S sector labor is determined, given S sector and P sector capital. The S sector wage determines the marginal productivity of labor in the S firm, and thereby the labor demand of the S firm conditional on capital, as captured by equation 1. Moreover, given the constraint that S workers’ incomes need to be high enough, described by equation 7, S workers’ wages are then determined by their expected incomes in the democracy and transfers, if the constraint is binding. In this case, we can express S sector...
labor as

\[ L_{St} = \left( (1 - \alpha) z_{St} \right)^{1/\alpha} w_{St}^{-1/\alpha} K_{St} \]
\[ = \left( (1 - \alpha) z_{St} \right)^{1/\alpha} \left( y_{wt}^{D} - T_t \right)^{-1/\alpha} K_{St}. \]  

(21)

We can see that \( L_{St} \) is positively related to \( T_t \), i.e., more transfers lead to lower wages, and then higher labor demand of the S firm. Intuitively, when S workers are directly paid with large transfers from the government, the labor cost for the S firm is low, so the S firm hire many workers, resulting in redundant labor and low labor productivity in the S sector. We can define the redundant labor as the gap between the S sector labor in the oligarchy and the counterpart in a competitive equilibrium, which is the same as in the democracy and denoted as \( L_{DSt} = z_{St}^{1/a} K_{St} / \left( z_{St}^{1/a} K_{St} + z_{Pt}^{1/a} K_{Pt} \right) \). Then we can formally state the results above as follows:

**Proposition 2.** If \( T_t \geq \tau \alpha \left( 1 - \alpha \right) w_{St} \), then \( L_{St} \geq L_{DSt} \), and \( Y_{St} / L_{St} \leq Y_{Pt} / L_{Pt} \).

If transfers are larger than a certain fraction of the wage, then there is redundant labor in the S sector: S sector labor is larger than the efficient level \( L_{DSt} \), given capital in the S and the P sector. Moreover, labor productivity in the S sector is lower than the counterpart in the P sector.

The proof of the proposition is in the Online Appendix.\(^{22}\) This proposition also shows \( Y_{St} / L_{St} \leq Y_{Pt} / L_{Pt} \), i.e., transfers create a labor wedge: labor productivity in the S sector can be lower than that in the P sector if transfers are large enough. The government chooses transfers \( T_t \) given the cost parameter \( b_t \). If \( b_t \) is low, transfers are large, and S sector labor productivity is low. This relation between \( b_t \) and labor productivity in the S sector allows us to set \( b_t \) to match the labor wedge, as we will see later in the calibration of the model.

\(^{22}\) This simple expression for the lower bound of \( T_t \) comes from the fact that a worker’s expected income in the democracy is a simple function of the S sector and the P sector capital. It includes the wage in the democracy and the tax revenue, which equals \( \tau \alpha / (1 - \alpha) \) fraction of the wage. So if transfers in the oligarchy are larger than in the democracy, then the S sector wage is lower than the efficient wage, resulting in the redundant labor and lower labor productivity.
State capital $K_S$ is another important determinant of state labor $L_{St}$, as we see from equation 21. Higher state capital results in higher state labor, and makes it easier to satisfy the large state labor constraint represented by equation 8. Given certain level of $T_t$, a large enough $K_S$ is necessary for keeping $L_{St}$ greater than $L$. This result can be formally stated as follows.

**Proposition 3.** Given any $\mu > 0$, if $T_t \leq \mu w_{St}$, then the two constraints $L_{St} \geq L$ and $T_t + w_{St} \geq y_{wt}^D$ together imply that $K_{St} \geq \kappa K_{Pt}$, where

$$\kappa = \left(\frac{1 + \tau}{1 - \alpha}\right)^{\frac{1}{\alpha}} L \left(\frac{z_{Pt}}{z_{St}}\right)^{\frac{1}{\alpha}}.$$  

If transfers are bounded from above, then large enough state sector capital relative to private sector capital is necessary for sustaining the oligarchy.

Details of the proof is in the Online Appendix. The intuition is the following: If transfers relative to wages are lower, then high incomes of S workers imply that S wages cannot be too low. Given S wages and the corresponding marginal productivity of labor, keeping enough workers in the S sector requires high enough capital in the S sector, as we see from equation 1. This requirement for S capital may result in misallocation of capital and overinvestment in the S sector. Notice that this result is quite different from the results implied by a competitive equilibrium, in which more P sector capital leads to less S sector capital because with more capital, the P firm hires more workers, and then the S firm hires fewer workers and needs less capital. In this model, if the large-state-employment constraint is binding, S sector capital grows as the P sector capital grows, in order to guarantee enough well-paid workers in the S sector. Moreover, overinvestment in the S sector can result in a capital wedge: lower S sector capital productivity than the counterpart in the P sector.

The two propositions above together imply that if transfers are bounded from above and below, then the political constraint generates labor and capital wedges in the S and the P sector. As we will see in the quantitative exercise studied below, this is indeed the case: transfers vary in a bounded region, and consequently, labor and capital wedges exist and vary over time.
3 Quantitative Analysis

In this section, the model is calibrated to the Chinese economy and the quantitative model can deliver a successful account of China’s growth experience since the 1990s, including the private sector growth, the labor and capital market frictions, and the economic growth. More specifically, the benchmark model is calibrated to the manufacturing sector in China since 1998. In an extension, the model is also calibrated to urban China since 1994, including both the manufacturing and the service sectors. The model does not apply to rural China, where the government uses different strategies to maintain political support, because politico-economic conditions in the rural area are quite different. The government does not use the divide-and-rule strategy or keep many workers in state farms; Instead, it maintains support from the rural population as a whole group using other policies, e.g., propaganda, which is more effective given the lower educational level of the rural population.

Exogenous inputs of the model include TFP growth rates in the S and the P sector, upper bound of the borrowing constraint $\bar{\eta}$, and the interest rate $r$. Other major parameters are endogenously backed out in the calibration: For example, the discount factor of the entrepreneur $\beta$ is chosen to match the growth of the private employment share, and the cost parameter of transfers $b$ is set to match the labor productivity of state firms relative to that of private firms. The untargeted moments generated by the model, including

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23 The calibration to the manufacturing sector is more precise: Many crucial statistics, e.g., labor and capital productivity gaps between state firms and private firms, are estimated and well understood for the manufacturing sector, due to the better data availability, while they are less accurate in the service sector. The model can be calibrated to the service sector and the urban economy, while the calibration to manufacturing is still useful and necessary: some crucial statistics and model parameters, e.g., the labor market wedge and the cost parameter of transfers, need to be first backed out in the latter, and then can be used in the calibration to the urban economy, with the assumption that these statistics and parameters are similar in the urban economy. For example, the NBS data of manufacturing firms allow Hsieh and Song (2015) to back out the labor and the capital market wedge in the state and the private sector, while this dataset is not available for the service sector.

24 Propaganda makes rural workers believe that their incomes are high in the current regime and will be low after democratization. Difficulties and Costs of starting revolution in the rural area are also large, because of low population density, weak civil society and low education. Though the rural population are quite poor, demand for democracy in the rural population is low and challenges to the regime are weak.
output growth since 1998 and state invest share, are used to check the validity of the model. Moreover, the model provides predictions of future growth, market frictions and development paths.

3.1 Calibration

The model is calibrated to the Chinese manufacturing sector since 1998, following Song et al. (2011). Table 1 summarizes the parameters that are obtained exogenously from the literature. The production function is Cobb–Douglas with the capital share $\alpha = 0.5$ (Bai et al. (2006)). Following Song et al. (2011), the annual depreciation rate is set to $\delta = 0.1$ and the world interest rate $r = 5\%$. The leverage of private firms in China in 2002 is 22%, according to Li et al. (2008). We set $\bar{\eta}$ such that the leverage generated by the model is consistent with the empirical finding in 2002. In the calibrated model, before 2008, the economy is in the rapid growth stage and it is always the case that $\eta_t = \bar{\eta}$. So setting $\bar{\eta} = 1.22$ gives the correct leverage in 2002. $\eta$ is set to 1, implying that in the extreme case, the government can order banks to lend only to state firms but not to any private firms.

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Table 1: Exogenous Parameters

<table>
<thead>
<tr>
<th>Para.</th>
<th>Value</th>
<th>Description</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
<td>Capital share</td>
<td>Bai et al. (2006)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.1</td>
<td>Depreciation rate</td>
<td>Song et al. (2011)</td>
</tr>
<tr>
<td>$r$</td>
<td>0.05</td>
<td>World interest rate</td>
<td>Song et al. (2011)</td>
</tr>
<tr>
<td>$\bar{\eta}$</td>
<td>1.22</td>
<td>Max. leverage of the P firm</td>
<td>Li et al. (2008)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1</td>
<td>Min. leverage of the P firm</td>
<td></td>
</tr>
<tr>
<td>$\bar{g}_z$</td>
<td>1%</td>
<td>Long-run TFP growth</td>
<td>2% long-run output growth</td>
</tr>
<tr>
<td>$z_S/z_P$</td>
<td>0.45, 0.58</td>
<td>TFP ratio between the S and P firm</td>
<td>Hsieh and Song (2015)</td>
</tr>
</tbody>
</table>

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25 The world interest rate in Song et al. (2011) corresponds to the rate of return of capital in financially integrated firms when the iceberg cost is zero. Their Figure 8 shows that as the iceberg cost declines, the rate of return converges to about 5%.

26 This result - $\eta_t = \bar{\eta}$ in 2002 - is also robust to different parameters, as long as we target finishing state to private labor reallocation in 2008. Theoretically, it is possible that $\bar{\eta} > 1.22$ and in 2002 the government optimally chooses $\eta_t = 1.22 < \bar{\eta}$, but it does not happen in the calibrated economy because as long as the economy is still in the rapid growth stage in 2002, the government does not want to create additional barriers to slow down P firm growth.
The steady-state TFP growth is set to 1% for both the S and the P firms, targeting for 2% output growth given that $\alpha = 0.5$. TFP sequences of the S firm and the P firm are also taken exogenously from the literature. Brandt and Zhu (2010) estimate the yearly TFP levels of state and private firms. The implied TFP growth rate shows an increasing trend from 1999 to 2007. I use the linear interpolation of their estimated TFP growth rates between 1999 and 2007 as inputs to the model, so that the trend in this period is kept while the yearly fluctuation around the trend is removed. The P firm growth rate increases from 1.3% in 1999 to 4.7% in 2007. After 2007, the P-firm TFP growth rate is assumed to gradually decline to 1% in 2017, and to stay at that steady-state level forever. TFP levels of the S firm before 2007 are set according to Brandt and Zhu (2010) and Hsieh and Song (2015): The former paper provides the increasing trend of TFP growth rate, and the latter provides the TFP gap between state and private firms. Those two papers’ findings are qualitatively consistent with each other, both suggesting that state-firm TFP grows faster than private-firm TFP during this period. Hsieh and Song (2015) find state firms’ TFP is 45% of private firms’ in 1998 and that number grows to 58% in 2007. Hsieh and Song (2015)’s estimation of state firms’ TFP in 1998 and 2007, relative to private firms’ TFP, tells us about the average growth rate of state firms’ TFP, while Brandt and Zhu (2010) provide the trend of TFP growth rate during that period. To combine these two sources, I use state firms’ TFP growth rates estimated by Brandt and Zhu (2010)

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27 This growth model focuses on growth trend in the median- and long-run but does not study yearly fluctuations.

28 Hsieh and Song (2015) report in Table 5 that state firms’ labor productivity is 60% of private firms’ labor productivity in 1998 and 75% in 2007. They also report the counterparts for capital productivity: 35% in 1998 and 47% in 2007. In the neoclassical framework, the Cobb–Douglas production function implies that the TFP gap can be backed out by combining the labor and capital productivity gaps, as follows:

$$
\frac{z_{St}}{z_{Pt}} = \left( \frac{Y_{St}/L_{St}}{Y_{Pt}/L_{Pt}} \right)^{1-\alpha} \left( \frac{Y_{St}/K_{St}}{Y_{Pt}/K_{Pt}} \right)^{\alpha},
$$

where $Y_{St}/L_{St}$ stands for the labor productivity in the S sector, etc. Brandt and Zhu (2010)’s estimation is similar in 1998: 45% and that number grows to 50% in 2007.

29 Hsieh and Song (2015)’s estimation suggests a faster average growth rate of state firms’ TFP than Brandt and Zhu (2010)’s. We choose to use the former, because it is necessary for getting labor and capital productivity gaps between state and private firms in 1998 and 2007, which are two important targeted moments, correct.
and adjust them up so that the average TFP growth rate is in line with Hsieh and Song (2015)'s estimation, i.e., state firms’ TFP grows to 58% of private firms' in 2007. More specifically, I first take the linear interpolation of state firms’ TFP growth rates from 1999 to 2007 in Brandt and Zhu (2010), as I do for private firms’ TFP, and then adjust state firms’ TFP growth rates before 2007 up, equally for each year. Eventually, as the inputs to the model, the growth rate of state firms’ TFP is 2% in 1999 and increases to 10% in 2007. Afterwards, the ratio of TFP between state firms and private firms is assumed to continue to linearly increase, but at a lower speed until period 50 in the model. We set the ratio between the S-firm and the P-firm TFP to grow at 1/4 of the speed before 2007, to be consistent with Hsieh and Song (2015)'s findings on the slower decline of productivity gaps between state and private firms. TFP levels of the S and the P firm are shown in the left panel of Figure 2 and the ratio in the middle panel. The rest of model parameters are set endogenously to match corresponding moments in the data, as summarized in Table 2. Storesletten and Zilibotti (2014) document that the private employment share increases from 15% in 1998 to above 60% in 2008 and afterward, stays around 61.5% - the average private employment share between 2008 and 2012. The discount factor of the entrepreneur is set as $\beta = 0.82$ to match the increasing trend of private employment share from 1998 to 2008. The final stable level of private employment share pins down

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30 Hsieh and Song (2015) find that after 2007, labor productivity of state firms relative to private firms grows at a slower rate than before, and capital productivity of state firms relative to private firms does not grow anymore. If we set speed of the relative labor productivity growth to be 1/2 of the speed before 2007, and counterpart for capital productivity to be 0, then the Cobb-Douglas production function with $\alpha = 0.5$ implies that the TFP of state firms as a ratio of private firms grows at 1/4 of the speed before 2007.
Table 2: Endogenous Parameters

<table>
<thead>
<tr>
<th>Para.</th>
<th>Value</th>
<th>Description</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>0.82</td>
<td>Discount factor of the entrepreneur</td>
<td>(L_{P,98-08})</td>
</tr>
<tr>
<td>(L)</td>
<td>0.385</td>
<td>Min. state employment share</td>
<td>(L_{P,08-12} = 0.615)</td>
</tr>
<tr>
<td>(\tau)</td>
<td>0.2</td>
<td>Tax rate</td>
<td>(y_S/y_P = 1.2)</td>
</tr>
<tr>
<td>(b_{98,b_{07}})</td>
<td>0.45,0.58</td>
<td>Cost Parameter of transfers</td>
<td>(MPL_S/MPL_P : 0.6,0.75)</td>
</tr>
</tbody>
</table>

the minimal state employment share needed for sustaining the regime as \(L = 38.5\%\). The private employment share in 1998 pins down the initial assets of the entrepreneur. The tax rate is set to \(\tau = 20\%\), generating an average of 20% income premium for state workers between 1998 and 2007, which is consistent with Ge and Yang (2014). The cost parameter of transfers \(b_t\) is chosen to match the labor productivity gap between state and private firms: A higher cost parameter implies less transfers, lower state sector wages, and lower state labor productivity. As I discussed above, Hsieh and Song (2015) document that state sector’s labor productivity is 60% of the private sector’s in 1998, and 75% in 2007. Setting \(b_t\) to 0.45 in 1998 and 0.58 in 2007 matches the corresponding labor wedges. After 2007, \(b_t\) is assumed to keep growing at the same rate as before - 0.015 per period - for 150 periods. In other words, all model parameters reach the steady state after period 150. The right panel of Figure 2 shows the sequence of \(b_t\). The calibration gives an increasing sequence of \(b_t\) because the labor productivity gap between state and private firms decreases over time at a faster rate than capital productivity gap, as documented by Hsieh and Song (2015). \(b_t\) is negatively associated with transfers, positively with wages paid by state firms, negatively with redundant labor and positively with labor productivity. So the increasing \(b_t\) helps to match the decreasing labor productivity gap between state and private firms from 1998 to 2007. It is reasonable to expect that using transfers becomes less efficient and more costly over time. As we discussed above in Subsection 2.3, transfers in the model is more than just cash transfers, but more generally, include non-cash benefits, subsidies, and political propaganda, which change workers’ expected incomes in the current regime and in the democracy. Over time, as the population become richer and more educated,
those policies, e.g., propaganda, become less efficient in changing workers’ expectations and therefore become more costly.  

3.2 The Solution

First, we solve the representative entrepreneur’s problem, and then we can solve for the elite’s problem, who takes the optimal choices of the entrepreneur as a given. The solution of the entrepreneur’s problem is very simple and is the same as in Moll (2014): Given that the entrepreneur’s income is proportional to her assets and the log utility, the entrepreneur saves $\beta$ fractions of her total resources, including assets and their returns, to the next period and consumes the rest, independent of the rate of return on her assets. See more details in the Online Appendix.

Given the entrepreneur’s decision rules, we can solve the elite’s problem. The elite acts like a social planner but cares only about its own utility; So after solving for the elite’s problem, the equilibrium of the economy is known. The elite’s problem cannot be solved sequentially but recursively. The reason is that the elite’s decision on whether to democratize is discrete and also depends on the lifetime utility in the oligarchy; Therefore, it cannot be characterized by a first-order condition. Instead, we can write down the elite’s problem recursively and solve it backward. First, we can form the elite’s problem at the steady state recursively and solve it using value function iterations. Let us denote the

31 Though costs of raise funding for cash transfers may not increase after the 1994 SOE reform, as Brandt and Zhu (2000) point out, the increasing costs of other transfers may dominate. First, subsidies and in-kind wages enjoyed by state workers become less efficient and less popular, as the market develops and supplies more variety of goods and services. For example, clothes, education, and housing provided by the government were very valuable for workers 20 years ago, because these goods and services supplied by the market are similar and limited. It is quite different now: Supplies from the market are abundant, high quality and with more varieties, so subsidies and in-kind wages become less efficient in increasing state workers’ real incomes and utility. For example, a state worker may not like the clothes or the apartment provided by the government because she prefers some other types supplied by the market, so she does not find the government subsidies valuable. Brandt and Zhu (2000) document that those non-cash benefits become less important and less popular over time. Second, political propaganda may also become less efficient over time. As people become richer and more educated, their utility in the democracy may increase, and it is also harder to change their expected incomes and utility. Changes in efficiency of these policies can be modeled as the increasing $b_t$. 

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elite’s value function choosing oligarchy as $V^O$ and choosing democracy $V^D$. By comparing $V^O$ and $V^D$, we can obtain the elite’s optimal political choice and the corresponding value function $V = \max\{V^O, V^D\}$. $V^D$ is simple and obtained exogenously from the equilibrium in the democracy. $V^O$ is solved by finding the optimal economic policies $\{\eta, T, w_S, K_S\}$ and private consumption $c_e$ given the continuation value $V$ for the next period. Then, knowing the steady-state solution, say, after period $t_{ss}$, we can solve for the elite’s problem in period $t_{ss} - 1$. We can iterate this process backward and obtain the elite’s value functions and policy functions in all periods $t = t_{ss} - 2, t_{ss} - 3, ..., 1, 0$.

Below, I discuss the solution of the calibrated model and the interpretations. In the balanced growth path, many variables, including assets, capital, incomes, and transfers, grow at 2% given that TFP grows at 1% and $\alpha = 0.5$. We can normalize these variables using the long-run growth rate so that the normalized variables stay constant at the steady state. Using the normalized variables, Figure 3 shows the solution of the elite’s problem and equilibrium outcomes at the steady state, i.e., how policies and equilibrium variables depend on the state variable - the entrepreneur’s assets $a_p$. A noticeable feature of the politico-economic equilibrium in the oligarchy, as we can see from Figure 3, is that policy functions, e.g., how $K_S$ responds to $a_p$, are not monotonic. Instead, whether $K_S$ and other variables increase or decrease as $a_p$ increases depends on the size of $a_p$. If $a_p$ is small, the equilibrium is still similar to the competitive equilibrium in the democracy, e.g., $K_S$ and $L_S$ are negatively related to $a_p$; If $a_p$ is large, the equilibrium looks very different from the competitive one, because the political constraint becomes binding. According to how equilibrium variables, especially $L_S$ and $\eta$, depend on $a_p$, we can split $a_p$ into three regions by $a_p = 1$ and 1.5, corresponding to the dashed vertical lines in Figure 3. If $a_p$ grows from

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32 In the original formation of the elite’s problem, there are two state variables: assets of the elite and the entrepreneur. The elite’s assets affect only its consumption and saving, but not other equilibrium variables, including S sector capital and labor, because the elite has deep pockets: No matter how low or high its own asset level is, it can always set S sector capital to the level that maximize its lifetime income. More generally, the elite’s choice set of policies and corresponding gains in the lifetime income are independent of its assets. So the real relevant state variable for the equilibrium is only the entrepreneur’s assets. See more details in the Online Appendix.
Figure 3: Solution of the Equilibrium at the Steady State

a low level from region 1, to region 2 and finally region 3, government policies including \( L_S \) and \( \eta \) and other equilibrium outcomes including output growth will be very different in those three regions. The development of the economy, across these three regions, can be interpreted as the three-stage politico-economic transition, which will be discussed in detail when we simulate the economy and study its dynamics in Subsection 3.3.

In region 1, where the entrepreneur’s assets are low, \( a_p < 1 \), increasing \( a_p \) leads to increasing \( K_P \) and decreasing \( K_S \) and \( L_S \). This pattern - the negative relation of the S and the P sector sizes - is the same as in the competitive equilibrium in the democracy: As the entrepreneur’s assets grow, the P sector grows and employs more workers and the S sector shrinks. However, if the entrepreneur’s assets are high enough, i.e., \( a_p \geq 1 \), the equilibrium pattern in region 2 changes completely: S sector labor no longer decreases as the entrepreneur’s assets increase and S sector capital increases. The reason is that the political constraint becomes binding. \( L_S \) needs to stays constant at the critical level \( L \) to keep enough workers in the S sector. Moreover, S sector workers’ incomes and wages need to be high enough, so S sector capital needs to be high enough, as stated in Proposition 3.

In region 3, where the entrepreneur’s assets are too high for the elite, i.e., \( a_p > 1.5 \), as \( a_p \)
increases, the elite prefers to reduces P sector capital by reducing leverage \( \eta \). Though a larger P sector contributes more tax revenues, it also makes maintaining enough workers in the S sector more costly: wages are higher, capital levels are higher, and capital return in the S sector becomes lower. Now the cost of P sector growth exceeds the benefit for the elite, whose interests are no longer in line with growth. The elite reduces the lending to the P sector and slows down growth of the P sector. Consequently, aggregate output growth also slows down. The elite wants to keep the P sector size at a median level: As we can see from the lower-left panel of Figure 3, the elite’s lifetime income is maximized at \( a_p = 1.5 \). If \( a_p > 1.5 \), then the elite prefers to reduce lending to the P firm \( \eta \) and lower the size of P sector capital and assets. However, there is a limit on the elite’s ability in reducing lending to the P firm and slowing down P sector growth, i.e., \( \eta \) cannot be lower than \( \eta_l \). Therefore, after \( \eta \) drops to its lowest level, growing \( a_p \) can still lead to growing \( K_P \): The P sector can grow again.

Transfers and wages for state workers follow private sector capital, as shown in the lower middle panel of Figure 3: As private sector capital becomes higher, the government increases both state workers’ wages and transfers, to maintain their incomes high enough. Finally, the entrepreneur’s asset level tomorrow is a concave function of the asset level today, as shown in the lower right panel. This function intersects with the 45 degree line at \( a_p = 1.8 \), and the entrepreneur’s asset level converges to that level in the long run.

Above is the solution at the steady state. In other periods, given different parameters, the policy functions are qualitatively similar with some quantitative differences. One difference is that in the first few periods, the cost parameter \( b \) is lower than in later periods, so transfers are cheaper and larger, the S sector wage lower, and labor productivity gap between the S and the P sector larger. In later periods, costs of transfers become larger, and wages and labor productivity in the S sector also become higher.
3.3 Dynamics of The Economy

We can simulate the economy starting from 1998 by setting the initial level of the entrepreneur’s assets such that the initial P-sector employment share equals the 1998 private sector employment share in the data - 15% (see Storesletten and Zilibotti (2014)), and then use the policy functions and equilibrium conditions solved above to find out other equilibrium variables from 1998 on. Now we can see how well the dynamics generated by the model match with historical data it was calibrated to and those it was not calibrated to.

Targeted Moments  Figures 4 shows the targeted moments and the corresponding sequences generated by the model from 1998 to 2017. The dashed lines plot the data series, while the solid lines show the series generated by the model economy. The targeted moments are the state employment share from 1998 to 2012, labor and capital productivity gaps between state firms and private firms in 1998 and in 2007. In the left panel of Figure 4, the dashed line represents the state employment share in each year between 1998 and 2012, documented in Storesletten and Zilibotti (2014). In the calibration, the entrepreneur’s initial assets are chosen to match the state employment share in 1998, and the time discount factor is chosen to match the declining trend of the state employment from 1998 to 2008. Notice that the declining trend in the data is approximately linear but in the model a bit different: The state employment decline slows down around 2005, because the S-firm TFP growth computed based on Brandt and Zhu (2010) accelerates in later years (2% in 1999, 8% in 2005, 10% in 2007) and consequently, the decline of state
employment share slows down from 2005. To minimize the deviation of state employment in the model and in the data, I choose $\beta$ such that in the beginning, the state employment share declines faster than in the data, and later from 2005, slower than in the data. Eventually, the average difference between the model and the data is small: -0.0067 and so is the average absolute deviation: 0.0313. Notice that the dynamics of the state employment is quite different from that in a competitive equilibrium, where private-sector capital and labor grow exponentially, and the decline of the state employment speeds up over time until it reaches 0. In this model, the political constraint makes the government willing to take actions to slow and stop growth of the private sector.

The middle panel of Figure 4 shows the labor productivity of state firms relative to private firms in the model and in the data. The two dots in 1998 and 2007 represent ratio of state firms’ labor productivity and private firms’ estimated by Hsieh and Song (2015), and the dotted line connecting these two data points shows the increasing trend. The ratios of state firms’ labor productivity and private firms’ in 1998 and 2007 are the targeted moments of the model, and as we can see from the blue solid line, which represents the labor productivity ratio generated by the model, these two targets are matched well, by choosing the cost parameter of transfers $b$ in 1998 and 2007.\(^{33}\) In the right panel, ratios of capital productivity are shown.\(^{34}\)

**Untargeted Moments** The time series of labor and capital productivities after 2007 are not targeted. Moreover, other variables, e.g., output growth, investment share of state firms, and growth of transfers, from 1998 to 2017 are not targeted. We can compare these untargeted time series generated by the model with the historical data to discuss the validity of the model.

Let us first look at the labor productivity and capital productivity after 2007. In the

\(^{33}\)The dot in 2012 represents the labor productivity ratio in 2012, which is an untargeted moment and will be discussed in the next paragraph.

\(^{34}\)The two dots in 1998 and 2007 are automatically matched as long as the labor productivity ratios are matched, given the right TFP ratios.
model, labor productivity of state firms keeps increasing after 2007 but at a lower speed, because TFP of state firms grows at a lower speed. Still, state firms’ labor productivity grows faster than their capital productivity and TFP, because costs of transfers increase. Growing incomes and cost parameter $b$ lead to increasing costs of transfers, decreasing transfers as a share of incomes. Consequently, redundant labor in state firms decrease, which acts as an additional force of increasing state labor productivity. Since the labor productivity grows faster than TFP in the state sector, capital productivity grows slower. Those patterns, i.e., slower growth of state labor productivity after 2007 than before and even slower growth state capital productivity, are qualitatively consistent with Hsieh and Song (2015), who state that “… average labor productivity of the state-owned firms continued to increase from 2007 to 2012 relative to incumbent private firms, albeit at a lower rate than in the 1998 to 2007 period. … there is little convergence in capital productivity after 2007.” In Figure 4, I plot labor productivity and capital productivity of state firms in 2012 implied by Hsieh and Song (2015): in the middle panel, the dot in 2012 represents labor productivity of state firms relative to that of private firms, assuming that growth speed of labor productivity is half of that before 2007; in the right panel, the dot in 2012 represents state firms’ capital productivity relative to private firms’, assuming that it does not grow after 2007. We can see that the model’s implications for labor and capital productivity, as the solid lines show, are consistent with the counterparts in the data in 2012. Generally speaking, because of increasing costs of transfers, labor productivity in state firms grows faster than TFP and capital productivity, not only between 2007 and 2012, but also in the long run.

Now we can look into other variables from 1998 to 2017. The most important one is output growth. The solid line in panel 1 of Figure 5 shows the output growth rate in the model: It starts from around 8% in 1999, increases to a high level - more than 18% - in 2007, declines dramatically in 2008 and then gradually decreases toward 5% in 2017. The pattern of output growth - high and increasing growth before 2007, decline of growth
after 2008 is qualitatively consistent with the pattern in the data. The dashed line in the figure plots real output growth in manufacturing, while the dash-dotted line shows real GDP growth in the aggregate economy.\footnote{Output growth per capita in manufacturing and aggregate economy show similar pattern but are slightly slower by about 0.5 percentage point.} As we can see, both in the model and in the data, output growth rate increases before 2007 and decreases after 2008, but still stays at a relatively high level. A noticeable difference is that in the model, given the estimated super high state-sector TFP growth in 2007, output growth around 2007 is higher than in the data. It does not mean that the model cannot generate the right output growth around 2007: If we use a lower (still reasonably high) TFP growth rate in 2007, e.g., 6%, which is between the value estimated by Brandt and Zhu (2010) and the value implied by Hsieh and Song (2015), then the output growth before 2007, plotted as the dotted line in the left panel of Figure 5, becomes very close to the counterpart in the data. Growth after 2008 generated by the benchmark model is very close to the counterpart in the data: It drops to around 10% in 2008 and steadily declines to above 5% in 2017. One reason for slower output growth after 2008 is the stop of state-to-private labor reallocation and efficiency gain. Though lower than before, the growth rate after 2008 is still reasonably high, and it is mainly driven by large state investment. In panel 2 of Figure 5, the solid line shows state investment share in the model. The dash-dotted line shows the state investment share in the urban China computed by Brandt and Zhu (2010) while the dashed line shows the counterpart in the aggregate economy computed by Knight and Ding (2010).
model, the investment share of state sector decreases from about 65% in 1998 to about 45% in 2004, and afterward, it stops declining and fluctuates around that level, which is higher than the state employment share. The pattern is similar in the data: Brandt and Zhu (2010) document that state investment share drops from about 63% in 1998 to 50% in 2005 and fluctuates around that level. They argue that the state sector’s investment share declines as its employment share declines, but the investment share stays higher than the employment share, which is consistent with the implication of the model in this paper. Moreover, the pattern that the state investment share stays high and even slightly increase after the state-private labor reallocation ends is consistent with the recent discussion on the phenomenon that “the state advances as the private sector retreats.”

It is also interesting to look at transfers from the government implied by the model. As the right panel of Figure 5 shows, in the first a few years, transfers grow at a very high rate of around 20% - even higher than output growth. Gradually, transfer growth rate decreases and converges to output growth rate, e.g., at around 10% in 2013. Remember that transfers in the model should be interpreted more generally, to include all government expenditures that help to maintain political stability of the regime, e.g., by increasing workers’ current incomes in comparison with the net gains of democratization. Those expenditures include not only direct transfers, but also expenditures on public goods and security, welfare, and subsidies. The Statistical Yearbook and Finance Yearbook of China reports government expenditures, and the main relevant items include expenditure for (domestic) public security, and expenditure for social safety net and employment. China's expenditure for public security on police, armed police, procuratorate, etc. improves public security and reduces social instability. Improving public security corresponds to, in the model, increasing welfare of workers, as transfers do. Reducing social instability corresponds to reducing workers’ expected incomes after democratization by increasing the control and costs of revolution, which is also equivalent to the generally interpreted transfers in the model. Reducing social instability is also directly related to stabilizing the
political system, as widely discussed in the media, e.g., Financial Times and Reuters. For this reason, I use expenditure on public security as the proxy for transfers in the model. In the right panel of Figure 5, the dash-dotted line shows growth of expenditure on public security, computed using data from Finance Yearbook of China, while the dashed line shows the growth, adjusted using nominal GDP growth. Growth of public-security expenditure follows a similar pattern of transfer growth in the model: the growth rate is around 20% in 1999, much higher than output growth rate, and then it gradually declines and becomes similar to output growth rate, e.g., 10% in 2013. In the model, transfers grow fast initially, because when the private sector is small, transfers are set at a relatively low level, in order to make the private sector grow fast. Lower transfers result in less redundant labor in the state sector, more labor in the private sector, and faster private sector growth. As the private sector grows fast, transfers also grow fast and quickly reach a relatively high level, to balance the cost of maintaining the political support and growth of the private sector. After the state-private transition finishes, transfers grow at a lower rate, converging to the level of output growth. In the long run, transfers become more costly as incomes and the cost parameter $b$ increases over time.

**Three-Stage Politico-Economic Transition** Now we can extend the focus to the longer term and analyze the whole picture of the transition. Figure 6 extend the time horizon of variables studied above, i.e., state employment share, labor and capital productivity, output growth, etc. until 2050. The economy develops along a three-stage politico-

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36 Other government expenditures, such as expenditure for social safety net, subsidy for employment and housing, are also partly related to transfers in the model, because they increase incomes and welfare of citizens, but they are also partly affected by other economic factors, e.g., pension and housing reforms. Those expenditures follow a qualitatively similar pattern but are much noisier. For example, growth rate of expenditure on social safety net and employment is very high in 2000 because the pension reform in 1999 requires more and growing pension expenditure.

37 The original series of expenditure on public security is measured in RMB, i.e., the dash-dotted line shows nominal expenditure growth. To make it directly comparable with growth of transfers in the unit of real output generated by the model, i.e., the solid line, one can adjust the data series by deducting nominal GDP growth in the data and then adding real output growth in the model, which gives us the dashed line.

38 Growth rate of expenditure on social safety net is also very high initially - higher than GDP growth rate and gradually declines towards GDP growth rate.
Figure 6: Dynamics of The Economy: 1998 – 2050

The economic transition, separated by 2010 when state employment stops declining and 2036 when private firm leverage starts to decline fast, represented by the two vertical dashed lines. Economic variables, including labor, capital and output evolve in different patterns in different stages. The first stage starts in 1998 and ends in 2010, during which the state employment share declines from 85% to 38.5%. Capital and labor in the private sector grow very rapidly and output also grows fast (see Figure 6). This stage is called rapid growth, whose main feature is rapid growth of the private sector and output. The private sector grows in both the relative size (employment share) and the absolute size (capital level). Output growth is fast: above 10%, while private-sector growth contributes significantly. Though the state sector TFP grows faster, capital accumulation in the private sector is even faster, so the relative size of the state sector declines, and the contribution of the private sector TFP to output growth becomes relatively more important. The labor productivity of state firms relative to that of private firms grows from 60% to 75%, while the capital productivity grows from 35% to 47%, as documented in Hsieh and Song (2015).

39 In the data, state employment share stops declining in 2008, while in the calibration, to balance the speed of state employment decline before and after 2005, we choose \( \beta \) such that the state employment share stops declining in 2010. So in the model, the second stage starts in year 2010, but when we discuss the real economy, we should use the year that the state employment share stops declining as the beginning of the second stage.
The labor and capital productivity of state firms grows following the growth of the state sector TFP. In addition to the TFP growth, which increases both productivity of labor and capital, transfers also affect the labor and the capital productivity differently: It directly reduces wages and labor productivity of state firms and indirectly increases the capital productivity through the lower capital-labor ratio. In this stage, transfers grows very fast, as we can see from the third panel of Figure 6; So state firms’ capital productivity grows faster than their labor productivity and TFP. In later stages, when growth of transfers slows down, labor productivity growth starts to catch up with capital productivity growth.

After state employment stops declining and stays around 38.5%, the economy enters the second stage of development: state capitalism. In this stage, private-sector capital keeps growing, but at a slower speed. State-sector capital grows proportionally to private-sector capital, in order to maintain state employment share. Private sector’s capital growth slows because its employment share does not grow anymore and wages are pushed up by the large investment in the state sector. Private-sector growth is still an important direct driving force of output growth, but now the private sector’s share in the economy does not grow anymore. Private sector capital still keep growing, while state sector capital grows proportionally, and the over-investment in the state sector keeps the aggregate economy’s growth rate still high: above 5% until 2017, though it declines from the high level in 2008. Another important implication of state over-investment is that borrowing becomes more costly and difficult for private firms, which is consistent with what has happened in China recently (see e.g., Brandt et al. (2012) ). The second feature in this stage is that growth of capital productivity in the state sector becomes slower than growth of labor productivity, because transfer growth declines. The cost of transfers grows as both the cost parameter $b$ and incomes grow over time, so transfers from the government grow slow, wages paid by state firms grow fast, and redundant labor in state firms decline. Growing TFP, together with declining transfers and redundant labor, generate fast growth of labor productivity in the state sector. This is also one reason for capital productivity of state
firms to grow slow: less redundant labor implies higher capital-labor ration and lower capital productivity. The other reason is the over-investment in the state sector. As we can see in the upper-middle panel of Figure 6, state firms’ labor productivity keeps growing towards the counterpart of private firms while their capital productivity does not increase much and stays below 50% of private firms’. This prediction is qualitatively consistent with the finding in Hsieh and Song (2015). They show that from 2007 to 2012, the labor productivity of state firms relative to that of private firms keeps growing but at a slower rate than before, while the capital productivity stops increasing.\(^{40}\)

As private-sector capital keeps growing, the elite’s revenues from the private sector - taxes - can become lower than cost of maintaining the regime. Then the elite starts reducing lending to private firms and directly slowing down their capital growth. In the model, this reduction happens in 2036, and then private sector capital growth significantly slows down. The economy enters the third stage: the middle-income trap. The credit constraint on private firms becomes even tighter and private firms find it even harder to grow. Output growth quickly drops when credit constraint becomes tighter: As we can see from the right-lower panel of Figure 6, in the first two years of the third stage, private firms’ leverage drops to the lower bound, and output growth drops rapidly. Afterwards, since the leverage cannot drop anymore, output growth recovers.\(^{41}\) In the long run, private sector output level is lower than the efficient level in the democracy, for two reasons: First, private firms’ capital and loans are reduced by the government; Second, private sector employment is lower than in the democracy because of the political constraint. Eventually, output growth slows down before output reaches the level in democracy, and growth stays at the steady-state level: 2%.

\(^{40}\)There are some qualitative differences from the findings in Hsieh and Song (2015): Growth of labor and capital productivity after 2007 generated by my model is faster than that documented by Hsieh and Song (2015). The main reason is that TFP growth after 2007 used in my paper is faster. The quite slow growth of productivity after 2007 found by Hsieh and Song (2015) implies a large drop of the TFP growth rate after 2007 to about one fourth of the rate before 2007. In my paper, the state sector TFP growth is assumed to decline smoothly, so the changes in state firms’ productivity growth are not that dramatic.

\(^{41}\)It may take more than two years for the leverage to drop to the lower bound, in different calibration of the model and perhaps also in reality. In those cases, low output growth era can last longer.
To sum up, the model generates a three-stage politico-economic transition. The first two stages are consistent with China’s growth experience since 1998. The model accounts for the rapid privatization that started in 1998 and stopped around 2008. It can also account for dynamics of the labor and the capital market frictions: the declining labor productivity gap between state firms and private firms, and the larger capital productivity gap which initially declined but recently stopped decreasing. It also generates reasonable output growth until today. The model offers the following explanation for the dynamics of frictions and the growth since 1998: the political constraint - buying support from state sector workers using wages and transfers - creates redundant labor and over-investment in the state sector, but the private sector still grows and contributes to the rapid growth. Finally, the model predicts that frictions will persist and that private sector capital growth will slow. The rapid growth in the current regime is not sustainable and the long-run output will be lower than that in the democracy.

3.4 Alternative Development Path: Sustained Growth

In the benchmark calibration, capital market frictions persist, because they help the elite to sustain the regime and maximize its incomes. These frictions harm growth of the private sector and the economy, which eventually enters the middle-income trap. The other possible scenario of the politico-economic transition is sustained growth: the elite implements political reforms, including democratization and liberalization of the financial market. In this case, private firms can access the capital market better and grow faster. Political reforms do not happen in the benchmark calibration because costs of sustaining the regime are low: The government can invest in the state sector at low costs as it can borrow from the international market at a low interest rate, and it can use transfers to buy support also at low costs. If those costs become high enough, the elite may prefer democratization to paying costs to maintain oligarchy. For example, if the interest rate \( r \) becomes high enough or the elite cannot borrow from the international market, then investing in the state sector
can be very expensive and the elite may stop supporting the oligarchic regime. If the cost of transfers is large enough, democratization may also happen.\textsuperscript{42}

To study consequences of democratization, I consider the case that in 2030, the government chooses to democratize and remove all labor and capital market frictions instead of choosing to restrict private sector growth, which can happen if $b$ and $L$ jump to high enough levels in 2030. Figure 7 shows the dynamics if democratization happens in 2030 in solid lines, and in comparison, the dynamics in the benchmark model in dash-dotted lines. As we can see, right after democratization, capital and labor market frictions disappear. First, leverage of private firms stays at the highest level. Second, the state sector’s labor productivity immediately becomes as high as the private sector’s labor productivity. Third, the employment share of the state sector immediately drops and then starts declining. As we can see from the lower-left and lower-middle panels, private sector capital grows faster than in the benchmark, and state sector capital quickly decreases. The private sector grows

\textsuperscript{42}As we discussed above, transfers in the model can be more generally interpreted as all policies that increase workers’ expected incomes in the oligarchy relative to the counterparts in the democracy, e.g., political propaganda and control. If economic development results in better education, higher demand for civil rights, and better organized civil society, then costs of transfers and similar policies that prevent democratization become higher. Eventually, if those costs grow to a large enough level, democratization happens. This case is what modernization theory (see Lipset (1959)) focuses on.
faster because now lending to private firms and private employment are not restricted by the government. State sector capital decreases as private sector capital grows, as the outcome of the competitive equilibrium. As shown in the lower-right panel, output after democratization grows faster than in the benchmark, especially around 2036 when in the benchmark the elite starts to reduce leverage of private firms. In the long run, output in the democracy converges to a higher level than that in the oligarchy because of the removal of capital and labor market frictions. Notice that this result is also true even if state firms do not disappear in the democracy. In this extension, because state firms’ productivity keeps increasing, and saving rate of entrepreneurs is not high, in the long run, state firms survive, as efficiency would dictate. In the long run, state sector size can be higher in the democracy than in the oligarchy, as we can see from the upper-right panel, but output and efficiency are higher in the democracy.

3.5 Calibration to China’s Urban Economy

The theory also applies to China’s urban economy including both the manufacturing and the service sector. Because of data availability and accuracy, the benchmark model is calibrated to the manufacturing sector. Now given statistics and parameters backed out from the benchmark, I can extend the model to study the urban economy since 1994 under the assumption that missing statistics and parameters in the urban economy are in the same pattern of those in manufacturing.

The time series of state employment share in the urban economy is taken from Song et al. (2011). The pattern of the state employment share is similar to that in manufacturing, but with some quantitative difference: The decline started earlier than the counterpart in manufacturing, became slower than the latter after 2001, and stayed parallel to the latter after 2004. The labor and capital productivity gaps between state and private firms after 1998 are assumed to be the same as the counterparts in the manufacturing sector, estimated by Hsieh and Song (2015). TFP series of state firms and private firms before
1998 are obtained by extrapolating the series after 1998. In the calibration, the endogenous parameters - $L$, $\beta$ - are reset to match the changes in the above targeted moments, and other parameters are kept the same as those in the benchmark.

The model calibrated to the urban economy produces a qualitatively similar three-stage transition, and prediction for future political development is the same: the third stage of the transition is the middle-income trap. See more detailed discussion of the results and figures in the Online Appendix. There are some quantitative differences from the benchmark. The most important one is that in the calibration to the urban economy, the state employment share is higher. First, the lower bound of state employment share $L$ is higher: 43.5%. This number requires setting $\beta$ lower in the calibration to slow down decline of state employment share so that it drops to 43.5% instead of 38.5% in 10 years. Second, the lower $\beta$ implies that the entrepreneur’s assets will be lower in the long run, and if state firms’ productivity keep catching up for enough years, state employment share may go up to a level above $L$, i.e., the minimal state employment constraint may not be binding if state firms become productive enough, which does not happen in the benchmark, but happens in the calibration to the urban economy. This scenario, i.e., the state sector revives again in the future is possible in reality. Another quantitative difference is that state employment share decline in the urban economy is slower than that in the manufacturing sector, especially after 2001, suggesting that the state-to-private transition is slower in the service sector. The slower transition is partly generated by the different model parameters, including the lower discount factor of entrepreneurs, and partly by endogenously changed policies, including higher state investment and less lending to private firms. Reducing leverage of private firms happens earlier in this calibration than in the benchmark. The leverage of private firms decreases dramatically in 2011 and triggers a decline of output. In reality, decline of leverage is less dramatic and so is decline of output growth rate.


3.6 Discussion

In this subsection, I discuss the model’s mechanics, extensions, and implications in greater detail. I first discuss the core mechanism that generates the rapid growth by analyzing efficiency and redistribution in the oligarchy. Then I discuss an extension of the model which assumes that the economy is closed. Finally, I study the implications of the model for the middle-income trap and for middle-class activism.

Efficiency vs. Redistribution  In this model, initial growth in the oligarchy is faster than in democracy, while the long-run growth is slower. This result and the intuition behind it are similar to those in Acemoglu (2008): In the early stage of development, the elite's interests are in line with growth so it implements growth-enhancing policies, but in the long run, the elite’s interests conflict with growth of entrepreneurs so it sets policies to restrict growth. However, the implications for efficiency and redistribution are different: In Acemoglu (2008), the oligarchic society produces more than the democratic society in the early stage because the former achieves higher efficiency given that the elite - the main producer then - can protect its property better than it can in the democracy; In my paper, initial growth is rapid because of inefficiencies, i.e., labor and capital market distortions. First, labor market distortion results in an initially low output level, which makes output growth faster, as long as the distortion declines. In this sense, output efficiency in the oligarchy is lower, though growth is faster. Figure 8 compares the dynamics in the democracy, starting from the first period (blue lines), with the dynamics in the benchmark (red dashed lines). From the figure, we can see that in the democracy, labor productivity of state firms is always equal to that of private firms (upper-left panel). This higher efficiency in labor allocation implies that the initial output level is higher than in the oligarchy (lower-middle panel), though initial output growth is lower (lower-right panel). In the medium run, output in the oligarchy catches up fast with output in democracy, and becomes about the same as the latter around 2018, because the over-investment in the state
sector keeps growth fast: As shown in the lower-left panel, while in the democracy, state capital should decline to zero in 2018, in the oligarchy, it keeps increasing. However, this high output is mainly due to the very high level of state sector capital, but not high productivity of capital and labor. It is true that the oligarchy can be more efficient in promoting output growth than the democracy does, because institutions and policies in the oligarchy are suitable for generating fast capital accumulation. However, if efficiency is measured by the productivity of resources such as capital and labor, then efficiency in the oligarchy is still lower.

If we focus on efficiency in achieving high social welfare, how aggregate output is distributed matters. In the oligarchy, the large state capital intended for sustaining the regime, so it generates a high income for the elite. Private sector workers’ incomes are lower in the oligarchy; and state sector workers’ incomes are not higher in the oligarchy.

\[\text{In alternative calibrations, output in the oligarchy can be even higher than than in the democracy in the state capitalism stage, if state sector TFP is high enough. Also notice that capital is initially lower in the oligarchy than that in the democracy. The reason is that the government is the monopolizer of state sector capital; So it prefers to set state sector capital at the low monopolistic level to obtain a higher return to capital. However, when the private sector’s employment share reaches the critical level, the government must set state sector capital high enough to maintain enough supporters. Afterward, state sector capital in the oligarchy exceeds the level in the democracy.}\]
either. Population size of workers is much larger than that of the elite and entrepreneurs, so the utilitarian social welfare in the oligarchy is lower because of lower incomes for the majority. In other words, inequality is larger and social welfare is generally lower.

The Agricultural Sector  In the benchmark and the extensions, the model is used to study the manufacturing sector and the urban economy which includes also the service sector. The rural economy is not in the model because the divide-and-rule strategy - buying support from state sector workers by providing them high incomes - is not the main policy for maintaining political stability in the rural area. Instead, other types policies are more important for maintaining the support among the rural population as a group, e.g., propaganda.

The agricultural sector matters for the model because the structural transformation from agriculture to manufacturing and service increases the workforce in the latter two. In the model, this transformation can be captured by introducing population growth of workers. Suppose that population of workers increases at the rate $g_w$, then as in a neoclassic growth model with population growth, we can normalize variables by population size. We can use capital per capita, and study variables in per capita term, which converge to a steady state. Population growth is similar to productivity growth, in neoclassic growth models and also in this model. Then the agriculture to manufacturing transformation and the growth of workers’ population are equivalent to adding the growth rate of labor to the productivity growth rate, i.e., $g = \hat{g} + g_w$, where $\hat{g}$ is the original TFP growth rate. We can still solve the model like we did above, while the interpretations of $g$ and employment are different. One key difference is that with population growth, state employment share can decline even if state employment does not. For example, if new workers in the economy mostly enter the private sector, then state employment share declines faster than state employment level. In the model, reallocating old workers from the state to the private sector and allocating more new workers in the private sector than in the state sector are
The Closed Economy  If we assume that the elite cannot borrow from the international market and can use only domestic savings to invest in the state sector, then the cost of sustaining the regime changes and the elite may want to influence the domestic savings. Let us first consider the case in which the elite does not manipulate domestic savings and the corresponding interest rate. Then the elite has to use its own savings to finance state sector investment, and the elite’s assets becomes a state variable of the equilibrium. As entrepreneurs’ assets grow and the economy enters the state capitalism stage, the elite must increase its savings to maintain enough capital in the state sector. If time discount factor of the elite is small, the elite’s savings can be low in the open economy, but in the closed economy, the elite is forced to save more and consume less. The cost of maintaining the oligarchy increases. Two possible outcomes may emerge: One is that the elite finds it too costly to sustain the oligarchy and chooses to democratize; the other is that the elite chooses to sustain the oligarchy and adopts even harsher policies to restrict private sector growth. The two outcomes imply that international sanctions on non-democratic countries, which forbid the governments from accessing the international financial market, may create two opposite results. In some countries, governments stop maintaining the regime, e.g., the Soviet Union; In some other countries, governments respond by tightening their control over their economies and then the societies become even less democratic, e.g., Iran. This outcome is the opposite of what the sanctions aim to achieve. The different outcomes of sanctions are consistent with the empirical findings in Grauvogel and von Soest (2013) and von Soest and Wahman (2015).

If we consider the case that the government can influence domestic savings and the interest rate, then the government wants to increase savings in the domestic banking system and also wants to reduce the corresponding interest rate, which is the cost of using the savings to sustain the regime. Compulsory saving, for example through the
pension system, can help to increase control over the politico-economic system, while regulated low interest rates for household savings, can reduce the cost of maintaining the regime. If we consider a half-closed economy where the population can have access to the international financial market, under the regulation of the government, then capital control policies in China and the recent tightening of capital outflows are the policies which help the government to control more financial resources and to maintain the regime.

**Middle-Income Trap** Government policies in the oligarchy, endogenously generated by the model, result in rapid growth in the beginning, but then become detrimental to growth when the elite's interests conflict with private-sector growth. The model explains why some governments adopt the right policies to achieve rapid growth out of the poverty but then suddenly implement the “wrong” policies that stop the convergence to rich countries: Those policies are actually not wrong for the elite but only wrong for growth. In the real world, the negative impacts of government policies can be more detrimental than described above in my model. In the benchmark calibration, the restriction that the government can put on private firms is moderate: At the maximal, it can set lending to private firms at zero, thus reducing their capital by 22%. The long-run output level is lower than but not far from the level in the democracy. In reality, the government may implement much harsher policies to restrict private firms and these policies may be much more harmful for growth. The government can directly confiscate capital of some private firms. The government can also forbid private firms from entering or staying in certain industries, which they may have invested in before. If the government implements these two policies, there are two consequences. First, capital of private firms can be even lower than entrepreneurs’ assets, if private firms are not allowed to increase their investments in certain industries or if capital of some private firms is confiscated by the government. Second, profits and the productivity of private firms can decline when private firms are excluded from some profitable industries. These policies can be modeled as allowing the
government to set $\eta_t < 1$ or to directly reduce $z_{Pt}$. If these two setups are allowed in the model, long-run output in the oligarchy can be even lower than that in the benchmark. If I extend the benchmark model to allow the government to set $\eta_t < 1$, $\eta_t$ in the long run is set to 0.9 by the government, and this lower $\eta_t$ in turn reduces the long-run output by another 10%. In the real world, many governments can be much more afraid of private sector growth than the government in my model. For example, if the productivity of state firms or elite-controlled firms in an economy is lower than that in my model, then the government implements harsher policies in controlling private firms, and the long-run output becomes even lower. Such an economy can stop growing when its income is still far away from the income of the United States, and may stay as a lower middle-income economy for very long.

Middle-class Activism An important political implication of this model is that the middle class do not necessarily support democracy. What we learn from European history and also from modernization theory is that as the middle class become richer and receive more education, their demand for democracy grows and they become more supportive of democratization. A similar mechanism exists in this paper: As incomes of state workers increase, cost of transfers rises, which can be interpreted as that efficiency of propaganda declines and demand of democracy increases. However, this paper shows that, first, the middle class may not be more supportive for democracy than the lower class, and second, the demand for democracy may not grow to a high enough level such that democratization happens. The government can reduce demand for democracy from the middle class by providing them well-paid state sector jobs and benefits. Then the middle class, being afraid of losing their decent incomes and benefits, become less supportive of democratization.

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44In my model, the implications of these two policies are similar, for the following reason: the P firm output is $z_{Pt} (\eta_t a_{Pt})^\alpha L_{Pt}^{1-\alpha - \beta}$, and decreasing $z_{Pt}$ by $x\%$ is equivalent to decreasing $\eta$ by $1\%$ in the impacts on output. They are different only in their respective impacts on the cost of the P firm, because reducing $\eta$ reduces the borrowing of the P firm and the related cost, while reducing $z_{Pt}$ does not. However, given the relatively low interest rate for borrowing compared to the high rate of return to P firms’ capital, this difference is small.
than the lower class are. In fact, a large fraction of the middle class is created by the state. The creation of the middle class relying on the state is especially common in countries with historically large state sectors. For example, in countries which transformed from communist states, many people still rely on the state but not the market to receive incomes, goods, and services including education, health, and housing, so it is natural and not costly for the government to create and maintain a large fraction of the middle class relying on the state. As discussed above, China is an example. Rosenfeld (2017) finds a similar pattern for Russian middle class. He reveals that the middle class from the state sector in Russia are less likely to mobilize against electoral fraud, and argues that potential coalitions in support of democratization are weakened by middle-class growth in state-dependent sectors. Moreover, even if the demand for democracy by the middle class is large and grows over time, it is not always the case that this demand will be sufficient to pressure the non-democratic government to democratize. This paper shows the possibility that the government, at least for very long time, can maintain the non-democratic regime, as long as it is willing to pay the costs, even if incomes and demand for democracy have been growing. The “China model” can successfully promote economic development and lead to growing incomes, education, and middle-class population, but it does not necessarily lead to political development at the same speed of economic development.

4 Conclusion

This paper proposes a politico-economic theory to understand China’s recent growth experience and to predict developments in China’s economic and political transition in the future. The political constraint - maintaining enough supporters in the state sector - creates labor and capital market frictions and results in a three-stage transition. The first two stages are rapid growth and state capitalism, which are consistent with several salient aspects of China’s recent development, including the growth of the private sector,
persistent labor and capital market wedges, and rapid output growth. In the future, China is likely to enter a *middle-income trap* with persistent capital market frictions, given the economically and politically powerful state. To switch to the development path that leads to *sustained growth*, economic and political reforms are necessary, though such reforms may endanger the current regime.

Although this paper focuses on China, it is also useful for understanding the development of many other emerging countries and some developed countries having similar patterns. The political constraint in the theory also exists in some other countries such as Kuwait, where the political elite need to gain support from public workers, and Korea before 1980, when politicians needed support from workers in industries tightly connected to the government. In Korea before 1980, large conglomerates (chaebol) were granted privileged access to low-cost credit and the employment share of small and medium enterprises (SMEs) had stagnated. The difference is that after 1980, democratic changes and financial reforms happened together, and the employment share of SMEs increased from 50% to 68% in 1990 and continued in the early 1990s after democracy was consolidated.45 This political and economic development path is consistent with the second development path: *sustained growth*.

Moreover, the theory is also useful in pondering an important question in development: Should other developing countries apply the “China model,” i.e., the combination of authoritarian politics and state-guided capitalism, to promote economic growth? Some in favor of adopting this model cite its recent success, but the long-run implications should be carefully examined and distinguished from the short-run implications. This paper provides a quantitative framework to evaluate the economic and political consequences of the “China model,” which under the right conditions, may be followed by democratization and sustained growth, but under other conditions, may lead to slow growth in the long run.

45See Song et al. (2011) for more details. They also discuss the case of Taiwan and the drop of SMEs’ employment share before 1971 and the increase until 1991.
References


5 Appendix (Available Online)

In the Online Appendix, I first provide proofs of the propositions. Then Subsection 5.3 presents more details of the model, including the equilibrium in the democracy. How to solve the equilibrium in the oligarchy, including how to solve the elite’s problem, is discussed in Subsection 5.4, in two steps: First, the elite’s problem is transformed into the recursive form; Second, the algorithm for solving the recursive problem is presented. The solution to the elite’s problem essentially gives us the solution of the equilibrium. In Subsection 5.5, an extension of the benchmark model to study the whole urban China is discussed. Finally, I discuss more details of the model, including the micro-foundation of the political constraint, the general interpretation of transfers, and the decomposition of the elite’s problem.

5.1 Proof of Proposition 2

First, workers’ expected incomes in the democracy $y_{wt}^D$ can be written as a function of capital allocations $K_{St}$ and $K_{Pt}$, combining equations 10 and 11:

$$y_{wt}^D = \left(1 + \frac{\tau \alpha}{1 - \alpha}\right)(1 - \alpha) \left(z_{St} K_{St} + z_{Pt} K_{Pt}\right)^{\alpha}.$$ 

So the S sector labor in the oligarchy $L_{St}$ can be expressed as a function of $y_{wt}^D, T_t, K_{St}$ and $K_{Pt}$, while $L_{St}^D$ as a function of $K_{St}$ and $K_{Pt}$:

$$L_{St} = ((1 - \alpha) \, z_{St})^{\frac{1}{\alpha}} \, w_{St}^{-\frac{1}{\alpha}} \, K_{St}$$

$$= ((1 - \alpha) \, z_{St})^{\frac{1}{\alpha}} \left(y_{wt}^D - T_t\right)^{-\frac{1}{\alpha}} \, K_{St},$$

$$L_{St}^D = \frac{\frac{1}{\alpha} K_{St} z_{St}^\alpha}{z_{St}^\alpha K_{St} + z_{Pt}^\alpha K_{Pt}}.$$
Finally, we can compare $L_{St}$ and $L_{DSt}$ and check for which levels of the transfer, we get $L_{St} \geq L_{DSt}$. Let us define $d_t = \frac{T_t}{w_{St}}$, then we have:

\[
(1 - \alpha)^{\frac{1}{\alpha}} \left( \frac{1}{1 + d_t} \left( 1 + \frac{\tau \alpha}{1 - \alpha} \right) \right)^{\frac{1}{\alpha}} \geq \frac{1}{z_{St}^{\alpha} K_{St}^{\frac{1}{\alpha}} + z_{Pt}^{\alpha} K_{Pt}^{\frac{1}{\alpha}}} \geq \frac{1}{z_{St}^{\alpha} K_{St}^{\frac{1}{\alpha}} + z_{Pt}^{\alpha} K_{Pt}^{\frac{1}{\alpha}}},
\]

\[
\left( \frac{1}{1 + d_t} \left( 1 + \frac{\tau \alpha}{1 - \alpha} \right) \right)^{\frac{1}{\alpha}} \geq 1,
\]

\[
1 + d_t \geq 1 + \frac{\tau \alpha}{1 - \alpha},
\]

\[
d_t \geq \frac{\tau \alpha}{1 - \alpha}.
\]

So if the transfer is large enough, i.e., $\frac{T_t}{w_{St}} \geq \frac{\tau \alpha}{1 - \alpha}$, then there is redundant labor in the S sector compared to the efficient labor allocation in the democracy: $L_{St} \geq L_{DSt}$, given the capital allocations $K_{St}$ and $K_{Pt}$. Moreover, we know that the labor allocation in the democracy is determined by the competitive equilibrium and efficient, so the labor productivity of the S and P sector in the democracy are the same: $\frac{Y_{St}^{D}}{L_{St}^{D}} = \frac{Y_{Pt}^{D}}{L_{Pt}^{D}}$. Then $L_{St} \geq L_{DSt}^{D}$ implies a labor productivity gap between the S and the P sector: $\frac{Y_{St}^{D}}{L_{St}^{D}} \geq \frac{Y_{Pt}^{D}}{L_{Pt}^{D}} \geq \frac{Y_{Pt}}{L_{Pt}}$, given the decreasing productivity of labor.

### 5.2 Proof of Proposition 3

Given $\mu$ as an upper bound of $T_t / w_{St}$, the constraint $T_t + w_{St} \geq y_{wl}^{D}$ implies that $w_{St}$ cannot be too low:

\[
w_{St} \geq \frac{y_{wl}^{D}}{1 + \mu}.
\]

Then given the not too low $w_{St}$, the enough state labor constraint $L_{St} \geq L_{DSt}$ implies that the state sector capital $K_{St}$ cannot be too low.
More precisely, the state sector capital cannot be too low compared to the private sector capital, if the political constraint, or equivalently, the two economic constraints, are to be satisfied.

5.3 More Details on Democracy

The equilibrium in the democracy is a decentralized competitive equilibrium given taxes, similar to Song et al. (2011). The competitive labor market implies that wages are the same in the S and the P sector. The competitive capital market and the elite’s infinity borrowing capacity imply that the return of S sector capital to the elite is equal to the interest rate $r$ if the S sector exist. In this case, the raw rate of return to S sector capital, i.e., the marginal productivity of S sector capital, is simply $r / (1 - \tau)$, where $\tau$ is the tax rate on the elite’s income. This helps to pin down the S sector capital labor ratio. The higher productivity of the P firm imply that the capital return in the P sector is higher than in the S sector, while the credit constraint of the entrepreneur implies that the entrepreneur’s capital supply is limited by the credit constraint. Given a large enough time preference parameter $\beta$ and

\[
\left(\frac{1}{1 + \mu} \left(1 + \frac{\tau \alpha}{1 - \alpha}\right)\right)^{-\frac{1}{\alpha}} \frac{z_{St}^{\frac{1}{\alpha}} K_{St}}{z_{St}^{\frac{1}{\alpha}} K_{St} + z_{Pt}^{\frac{1}{\alpha}} K_{Pt}} \geq L,
\]

\[
\frac{K_{St}}{K_{St} + (z_{Pt} / z_{St})^{\frac{1}{\alpha}} K_{Pt}} \geq \left(1 + \mu \left(1 + \frac{\tau \alpha}{1 - \alpha}\right)\right)^{\frac{1}{\alpha}} L,
\]

\[
\frac{K_{Pt}}{(z_{Pt} / z_{St})^{\frac{1}{\alpha}} K_{Pt}} \geq \left(1 + \mu \left(1 + \frac{\tau \alpha}{1 - \alpha}\right)\right)^{\frac{1}{\alpha}} L,
\]

\[
K_{St} \geq \frac{(1 + \frac{\tau \alpha}{1 - \alpha})^{\frac{1}{\alpha}} L}{(1 + \mu)^{\frac{1}{\alpha}} - (1 + \frac{\tau \alpha}{1 - \alpha})^{\frac{1}{\alpha}} L} \left(z_{Pt}^{\frac{1}{\alpha}} K_{Pt}\right) \approx \kappa K_{Pt}.
\]
the corresponding high enough saving rate, the entrepreneur keeps accumulating assets and the P sector keeps growing until all workers move to the P sector while S firms no longer produce. In this case, the elite does not supply capital to the S sector anymore but simply saves its assets in the bank and get the return at the rate \( r \). More details are discussed below.

First, in the democracy, if the S firm still exists, then the rate of return of S firm capital to the elite has to be \( r \). If it is greater than \( r \), each elite member wants to supply infinite capital and it is not an equilibrium; if it is lower than \( r \), the elite does not want to get any loan or supply any capital to S sector, but prefers to put all its assets in the bank and get the return at rate \( r \). In other words, competition of S sector capital supply implies that the net rate of return to the elite, denoted as \( \rho_{et}^D \), equals the marginal cost:

\[
\rho_{et}^D = (1 - \tau) \alpha z_{St} K_{St}^{\alpha-1} \left( L_{St}^D \right)^{1-\alpha} - \delta = r.
\]

This determines S sector capital labor ratio and the wage:

\[
\frac{K_{St}}{L_{St}^D} = \left( \frac{r + \delta}{(1 - \tau) \alpha z_{St}} \right)^{\frac{1}{\alpha-1}} \Rightarrow \\
\frac{K_{St}}{L_{St}^D} = (1 - \alpha) z_{St} \left( \frac{K_{St}}{L_{St}^D} \right)^{\frac{1}{1-\alpha}} = (1 - \alpha) z_{St}^{-1} \left( \frac{1 - \tau}{r + \delta} \right)^{\frac{\alpha}{1-\alpha}}.
\]

The wage pins down the private sector capital labor ratio, the labor given capital and the rate of return to capital:

\[
\frac{K_{Pt}}{L_{Pt}^D} = \left( \frac{w_i^D}{(1 - \alpha) z_{Pt}} \right)^{\frac{1}{\beta}} \Rightarrow \\
\frac{K_{Pt}}{L_{Pt}^D} = \left( \frac{w_i^D}{(1 - \alpha) z_{Pt}} \right)^{-\frac{1}{\alpha}} K_{Pt}, \quad (22) \\
r_{Pt}^D = z_{Pt}^{\frac{1}{\alpha}} \alpha \left( \frac{w_i^D}{1 - \alpha} \right)^{\frac{\alpha-1}{\alpha}}. \quad (23)
\]
The elite in the democracy receives no transfers from the government, so it only replies on the return to the assets at the rate \( r \), and its income from other sources is simply 0.

The entrepreneur maximizes her lifetime utility by optimally choose every period capital supply and saving, taking rates of returns to P sector capital and the borrowing limits as given, as follows:

\[
\max_{\{K_{pt}, a_{pt+1}\}} \sum_{t=0}^{\infty} \beta^t \log c_{pt}
\]

\[\text{s.t. } K_{pt} \leq \eta_t a_{pt},\]

\[y_{pt} = \rho^D_{pt} K_{pt} - rK_{pt}\]

\[a_{pt+1} = Ra_{pt} + y_{pt} - c_{pt},\]

where \( \rho^D_{pt} \) is the net rate of return to capital for the entrepreneur, which is the rate of return to P sector capital to the P firm minus the tax on the entrepreneur, i.e., \( \rho^D_{pt} = (1 - \tau) r^D_{pt} \). The entrepreneur’s problem in the oligarchy is in fact the same problem, except that \( \rho^D_{pt} \) is generally different from the level in the democracy. The reason is that in both regimes, the representative entrepreneur takes prices as given. Moll (2014) solves a similar entrepreneur’s problem. Because the utility function is logarithm and the income is proportional to the assets, the solution of the entrepreneur’s lifetime problem is quite simple: the entrepreneur maximizes each period’s income and saves a constant fraction of the income to the next period. In the following we formally prove that the solution described above is optimal, or the readers can look into Moll (2014), who solves the problem in the recursive form.

First, we show that the entrepreneur maximizes income in every period. Suppose that the sequence \( \{a^*_t, K^*_t\}_{t=0}^{\infty} \) is the optimal solution to the sequential problem. Then \( \forall t, K^*_t \) must maximize \( y_{pt} \), given \( a^*_pt \). Otherwise, \( \exists \hat{K}_pt \) such that \( \rho^D_{pt} \hat{K}_pt - r \hat{K}_pt > \rho^D_{pt} K^*_pt - rK^*_pt \). Then we can simply construct a new sequence of \( \{a_{pt}, K_{pt}\} \) by replacing \( K^*_pt \) by \( \hat{K}_pt \) while keeping all other \( K^*_pt \) for all \( t' \neq t \) and all other \( a^*_pt' + 1 \). The new sequence is feasible and
implies $c_{p_t} > c_{p_t}'$, and $\forall t', t, c_{p_t'} = c_{p_t}'$. The lifetime utility of the new sequence is higher. By contradiction, we know that $K^*_{p_t}$ must maximize $y_{p_t}$. The logic behind this proof is the following: the current period capital decision only has a direct effect on current period income, but no direct effect on future variables, so there is no dynamic tradeoff in choosing $K_{p_t}$ and no reason for not maximizing $y_{p_t}$. If the return to capital for the entrepreneur is higher than the cost, i.e., $\rho_{p_t}^D > r$, it is optimal for the entrepreneur to borrow as much as possible and set $K_{p_t}$ as high as possible. However, the choice of $K_{p_t}$ is subject to the borrowing constraint, so the optimal $K^*_{p_t}$ to maximize $y_{p_t}$ given $a^*_{p_t}$ is simple:

$$K^*_{p_t} = \begin{cases} 
\eta_t a^*_{p_t} & \text{if } \rho_{p_t}^D > r, \\
[0, \eta_t a^*_{p_t}] & \text{if } \rho_{p_t}^D = r, \\
0 & \text{if } \rho_{p_t}^D < r.
\end{cases}$$

In the first case, which happens when $a_{p_t}$ is small and $r_{p_t}$ and $\rho_{p_t}^D$ are large, $K^*_{p_t} = \eta_t a^*_{p_t}$ and $y^*_{p_t} = (\rho_{p_t}^D - r) \eta a^*_{p_t}$. Adding $r a_{p_t}$, we get the total income, which all come from the return on assets: $y^*_{p_t} = Ra^*_{p_t} + y^*_{p_t} = (R + (\rho_{p_t}^D - r) \eta_t) a^*_{p_t}$. So $\rho_{p_t}^{tot,D} = R + (\rho_{p_t}^D - r) \eta_t$ can be considered as the total rate of return to the entrepreneur’s assets and the asset return is the entrepreneur’s only income source. In the other two cases, where $\rho_{p_t}^D = r$ or $\rho_{p_t}^D < r$, the total return to the entrepreneur’s assets is simply $r$. Finally, given the return to assets, the entrepreneur optimally chooses next-period assets, i.e., $a_{p_t+1}$, taking the total return to assets $\rho_{p_t}^{tot,D}$ as given:

$$\max_{\{a_{p_t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log c_{p_t}$$

s.t. $a_{p_t+1} = \rho_{p_t}^{tot,D} a_{p_t} - c_{p_t}$.

Given the log-utility, the substitution effect of the return to saving exactly cancels the wealth effect, so in each period, the agent saves $\beta$ fraction of the total resource to the next
period, i.e., $a_{pt+1}^* = \beta \rho_{pt+1}^{tot,D} a_{pt}^*$, no matter how high or low $\rho_{pt+1}^{tot,D}$ is.

Given the entrepreneur’s optimal choices on asset accumulation, the dynamics in the democracy is the following, similar to Song et al. (2011). Starting from a low $a_{pt}$ and related high $r_{pt}$ and $\rho_{pt}^{tot,D}$, we have $\beta \rho_{pt}^{tot,D} > 1$, so $a_{pt}$ and $K_{pt}$ keep growing. Meanwhile, $L_{Pt}$ keeps growing according to (22), and $L_{Dt}$ declines gradually. When $a_{pt}$ is large enough - $a_{pt} \geq 1/\bar{\epsilon} \left( w_{Dt}^D / \left( (1-\alpha) z_{pt} \right) \right)^{\frac{1}{\alpha}}$, $L_{Pt}$ reaches 1, and the $S$ sector vanishes. Afterwards, the entrepreneur keeps accumulating assets, and the economy behaves like a neoclassic growth model, until $\beta \rho_{pt}^{tot,D} = 1$ and the economy reaches the steady state.

5.4 Solving the Equilibrium in the Oligarchy

First, the entrepreneur’s problem in the oligarchy is basically the same as the entrepreneur’s problem in the democracy, as she takes prices, i.e., the returns to P sector capital and her assets, and policies - the borrowing limit - as given. Though the returns are different in the democracy and in the oligarchy, the basic structure of the entrepreneur’s problem and the solution is the same: the entrepreneur borrows as much as possible subject to the borrowing constraint if the return to P sector capital is higher than $r$ to maximize her income in each period, and then same $\beta$ fraction of her total resource in hand - assets and the return - to the next period.

Second, the elite takes the entrepreneur’s optimal choices into account and solves the problem described by equation 15 and the associated constraints. The elite’s problem is essentially a planner’s problem, though the planner only cares about her own utility. In a standard neoclassic growth model, the planner’s problem can be solved sequentially by transforming the maximization of the lifetime utility into sequences of first order conditions in which the planner’s optimal choice in certain period can be characterized by the relation of variables in nearby periods. For example, the Euler equation characterizes the optimal choices of saving by the relation of consumptions in consecutive two periods. However, in this model, this approach does not work, because the elite’s choice of the polit-
ical system is discrete, and it depends on the lifetime utility in different political systems. So we need to know the elite’s lifetime utility in the oligarchy and in the democracy to characterize its political decision, and the problem can be solve backwards and recursively.

In the following, we write down the recursive form of the elite’s problem and then we can discuss the algorithm for solving the recursive problem. Generally speaking, the value functions and policy functions in the recursive problem can depend on time, because parameters, e.g., TFP levels of the S firm and the P firm, vary over time. So we need to keep $t$ as the subscript for the value functions and policy functions. Once all parameters reach their steady state levels, the value functions and policy functions do not depend on time anymore. In this case, we can simply drop the $t$ subscript. Let us first write down the elite’s problem when all parameters are at the steady state levels. If the elite chooses oligarchy, the lifetime utility is denoted as the value function $W^O(a_e, a_p)$; if democracy, $W^D(a_e, a_p)$; and before making the political choice, $W(a_e, a_p)$.

First, the Bellman equation given that the elite chooses oligarchy is:

$$W^O(a_e, a_p) = \max_{K_S, w_S, T, \eta, c_e, a_e} \log c_e + \beta W(a'_e, a'_p), \quad (24)$$

s.t. $c_e = Ra_e + y_e - a'_e$,

$$y_e = \alpha z_S K_S^\alpha L_S^{1-\alpha} - r K_S + \tau z_P K_P^\alpha L_S^{1-\alpha} - b T^2 L_S,$$

$$w_S + T \geq y^D_w = (1 - \alpha + \tau \alpha) \left( z_S^\pi K_S + z_P^\pi K_P \right)^{\frac{\alpha}{1-\alpha}},$$

$$L_S \geq L,$$

$$K_p = \eta a_p,$$

$$a'_p = \beta \left( r a_p + (1 - \tau) r_p (K_p - a_p) \right).$$

\[46\] There is in fact a third choice for the elite, i.e., staying in the oligarchy while not respecting the political constraint. In this case, revolution happens and the regime switches to the democracy. Because the revolution happens after the capital allocation is determined by the elite, while democratization allows the capital to be determined by a competitive equilibrium, the equilibrium outcomes and the elite’s income in the case of revolution can be different from the voluntary democratization. However, in the calibrated model, the lifetime utility of the elite in the revolution is always dominated by democratization or sustained oligarchy, we can simplify from this case here. In the quantitative model, I do keep track of this case.
where $L_S = (1 - \alpha) z_S^{\frac{1}{\alpha}} (w_S)^{-\frac{1}{\alpha}} K_S$, $L_P = 1 - L_S$, and $r_P = (1 - \tau) \alpha K_P^{\alpha - 1} L_P^{1 - \alpha}$. Notice that I do not include the expressions of $L_S, L_P$ and $r_P$, or equivalently, the firms’ maximization problems and market clearing conditions, into the constraints but treat them as known functions of other variables, to simplify the expressions of the elite’s problem. I also simply write the entrepreneur’s capital decision as $K_P = \eta a_P$. Precisely speaking, one can include the complete expression of $K_P$ depending on whether $r_P$ is greater or smaller than $r$. In the calibrated model, it is always the case that $r_P > r$ in the oligarchy, so it is without loss of generality to simplify the expression of $K_P$ here. Second, the lifetime utility if the elite chooses to democratize - $W^D(a_e, a_p)$ - is given by the equilibrium in the democracy and can be taken as given here. \(^{47}\) Finally, we can write the elite’s political decision as:

$$W(a_e, a_p) = \max \{ W^O(a_e', a_p'), W^D(a_e', a_p') \}.$$ 

There are two state variables in the elite’s problem, the elite’s assets $a_e$ and the entrepreneur’s assets $a_p$. The entrepreneur’s assets matter for the elite because it affects the P sector capital and labor, and eventually the elite’s policies and the equilibrium outcomes. The elite’s assets matter for its consumption sequence for sure. Does it also matter for the policies and the equilibrium outcomes other than the elite’s consumption sequence? The answer is no, because the elite has deep pockets. Consider two cases with the same level of the entrepreneur’s assets, but in the first one the elite has a high level of assets, and in the second case a low level of assets. In the second case the elite can copy the policies in the first case and achieve the same level of lifetime income, and achieve a sequence of consumption that is lower than the consumption sequence of in the first case reflecting only the difference in the initial assets of the elite. There is no reason for the elite in the second case to obtain a lower lifetime income than the first elite. Formally speaking, the elite’s

\(^{47}\)Later, when we transform the elite’s problem from lifetime utility maximization to its equivalent form of lifetime income maximization, we can see that the lifetime income of the elite in democracy is in fact very simple and we even do not need to compute the elite’s lifetime income or utility in the democracy.
problem can be decomposed into two subproblems: (1) conditional on \( a_p \), maximization of the lifetime income; and (2) given \( a_e \), maximization of the lifetime utility. The solution from the decomposed problems gives the same solution of the combined problem. I leave the formal proof to Subsection 5.8. The elite’s lifetime income maximization problem, again, when all parameters are at their steady state levels, can be written as:

\[
V(a_p) = \max \left\{ V^O(a_p), V^D(a_p) \right\},
\]

\[
V^O(a_p) = \max_{\eta, K_S, w_S, T} y_e + \frac{1}{R} V(a'_p) \tag{25}
\]

s.t. \( y_e = \alpha z S K_S L_S^{1-\alpha} - r K_S + \tau z p K_P L_S^{1-\alpha} - b T^2 L_S \),

\[
w_S + T \geq y^D_w = (1 - \alpha + \tau \alpha) \left( z_S^{\alpha} K_S + z_p^{\alpha} K_P \right)^{\alpha},
\]

\[
L_S \geq L,
\]

\[
a'_p = \beta \left( r a_p + (1 - \tau) r p \left( K_p - a_p \right) \right),
\]

where \( V \) is the value function representing the discounted lifetime income of the elite excluding the return to its initial assets \( R a_e \), and \( V^O \) and \( V^D \) stand for the value functions when the elite chooses sustaining oligarchy and democratization. In fact, \( V^D = 0 \) because the elite has no other income except the return from its assets at the rate \( r \), as we discussed in the solution in the democracy. Given the maximized lifetime income, we can recover the elite’s lifetime utility by solving the second subproblem:

\[
W(a_e, a_p) = \max \left\{ c_{et} \right\}_{t=0}^{\infty} \sum_{t=0}^{\infty} \beta^t \log c_{et} \tag{26}
\]

s.t. \( \sum_{t=0}^{\infty} c_{et} R_t = R a_e + V(a_p) \),

where \( V(a_p) \) is the maximized lifetime income calculated from the first subproblem.

For periods before parameters reach their steady state values, the logic is the same and the only difference is that we need to keep subscript \( t \) for time-varying parameters, value
functions and policy functions. For example, we need to write the elite’s lifetime income maximization problem as

\[ V_t(a_p) = \max \left\{ V_t^O(a_p), V_t^D = 0 \right\}, \]

\[ V_t^O(a_p) = \max_{\eta, K_S, w_s, T} y_{et} + \frac{1}{R} V_{t+1}(a_p') \]

s.t. \[ y_{et} = a z_{St} K_S^{\alpha} L_{St}^{1-\alpha} - r K_S + \tau z_{Pt} K_P^{\alpha} L_{Pt}^{1-\alpha} - b_t T^2 L, \]

\[ w_S + T \geq y_{wt}^D = (1 - \alpha + \tau \alpha) \left( \frac{1}{z_{St} K_S} + \frac{1}{z_{Pt} K_P} \right)^{\alpha}, \]

\[ L_S \geq L, \]

\[ a_p' = \beta \left( r a_p + (1 - \tau) r_{Pt} \left( K_P - a_p \right) \right). \]

As we can see, though some variables are not time varying, e.g., \( V^D \), some other parameters such as \( z_{St}, z_{Pt} \), and \( b_t \) are time varying, and consequentially, related variables \( y_{et} \) and \( r_{Pt} \) depend also on time. We keep the subscript \( t \) for these variables and the value functions because they can be different in different periods even given the same state variable. For example, in an early period where the S firm TFP relative to P firm TFP is lower than the counterpart in the steady state, even given the same level of entrepreneur’s assets, so the elite may need to invest more in the S sector to sustain the oligarchy, and the elite’s lifetime utility can be different.

Given the recursive formation of the elite’s problem, we can use value function iterations from backwards to numerically solve the elite’s problem and eventually the equilibrium in the oligarchy. First, we solve for the elite’s problem when all parameters have reached the steady state values, i.e., the value functions \( V^O \) and \( V \) and the Bellman equation 25. This recursive problem is solved using the standard value function iteration method with grid search. Starting from an initial guess of \( V^O \) and \( V \), we iterate using the Bellman equation until the value functions converge. All parameters converge to the steady state in period \( T \), or year 2040 in the calibrated model, so we know the value
functions and policy functions for \( t \geq T \). The second step is to solve for the value functions in period \( t = T - 1 \) to \( t = 0 \) from backwards. Since we know \( V_T(a_p) = V(a_p) \), we can solve for \( V_T^{O}(a_p) \) from equation 27, and also \( V_{T-1}(a_p) \). Then we can continue on backward induction until we get \( V_0^{O}(a_p) \) and the associated policy functions in all periods. Finally, now that we know what the elite would choose given any level of \( a_p \) in period 0, we can simulate the economy starting from any \( a_{p0} \). The policy functions in period 0 gives us corresponding \( K_{S0}, w_{S0} \) and \( \eta_0 \), which allows us to compute \( L_{P0} \) and \( a_{p1} \). Then the policy functions in period 1 allows us to obtain variables in that period and also \( a_{p2} \). We can continue this process until all variables converge. Then an equilibrium path given a \( a_{p0} \) is obtained. There is one level of \( a_{p0} \) that gives us \( L_{P0} = 15\% \), corresponding to the initial private sector employment share in 1998, which is the starting year of the benchmark model. Starting from this level of \( a_{p0} \), we can compute the equilibrium path that correspond to our benchmark economy.

Notice that we even do not need to compute the elite’s consumption and saving to back out other equilibrium variables that we care about, e.g., \( L_{Pt}, \eta_t \), etc. Again, this is because that the elite has a deep pocket and its policies can be independent of its assets. Still, if one is interested, one can back out the elite’s consumption and saving by solving the second subproblem in equation 26 and associated constraints.

5.5 More Details on the Calibration to China’s Urban Economy

The model is calibrated to China’s urban economy since 1994. The state employment share in the urban economy is documented by Song et al. (2011). The decline of the state employment share is in a similar pattern as that in the manufacturing sector, with some quantitative differences: It started earlier, and became slower than that in the manufacturing sector after 2001 and parallel to the latter after 2004. I extend the state employment share in the urban economy to 2012, assuming that it stays parallel to that in manufacturing which is available until 2012 and documented in Storesletten and Zilibotti.
Figure 9: Sequences of Parameters in The Calibration to the Urban Economy

(2014). Since the state employment share in manufacturing does not change much after 2008, the extended state employment share in the urban economy also does not change much after 2008. This assumption used to extend the state employment share data is essentially assuming that the state employment share in the urban economy stays constant after 2008, which is reasonable and consistent with the policies and discussions of “advance of the state and retreat of the private”. The labor and capital productivity gaps between state and private firms after 1998 are assumed to be the same as the counterparts in the manufacturing sector, estimated by Hsieh and Song (2015). I extrapolate TFP growth of private firms and state firms to the years before 1998. Now given all the exogenous variables from 1994, we can set the endogenous parameters - $L, \beta, b_{98}, b_{33}$ - to match the changes in the above targeted moments. The time series of inputs are shown in Figure 9. I keep other parameters the same as those in the benchmark. Results from the newly calibrated model are similar to those from the benchmark: There is a three-stage transition, whose third stage is middle-income trap, as shown in Figure 10.

5.6 General Interpretation of Transfers

In the model, variable $T$ is referred as transfers while it can actually represent a broader set of government policies that increase state workers’ incomes in the oligarchy relative to their incomes in the democracy but not the wages. These policies can be classified as three groups: cash compensation, non-cash benefits, and policies that change workers’
expectations. Here we formally illustrate how these policies affect the equilibrium and why they are equivalent to transfers.

The first set of policies includes cash compensation to state workers paid by the government, including not only direct cash transfers but also subsidies paid by the government to state firms. The former is the narrow interpretation of variable $T$ and is denoted as $T^{1c}$. The latter is denoted as $T^{1s}$. The second set includes non-cash benefits, including housing, education and tax benefits that state sector workers enjoy. We can model them as increasing state workers’ final income by $T^2$. Third, political propaganda can change workers’ expected income in the oligarchy if they support the regime and their income in the democracy. Let us denote the policies that increase workers’ expected income in the oligarchy if they support the regime, e.g., the reward of loyalty, as $T^{3+}$, policies that decrease their expected income if they do not support the regime, e.g., cost of revolution, as $T^{3-}$, and policies that decrease their expected income in democracy, e.g., cost of transition to democracy, as $T^D$. Then a state sector worker supports the regime if

$$w_s + T^{1c} + T^{1s} + T^2 + T^{3+} \geq y_w^D - T^{3-} - T^D. \quad (28)$$
We can then denote $T = T^1c + T^1s + T^2 + T^{3+} + T^{3-} + T^D$ and rearrange the constraint as

$$w_s + T \geq y^D_w.$$ 

Factors that do not affect the income but the expected utility can also be represented by $T$, if their impacts on utility are additive to income. This can include other non-monetary benefits of democracy such as more civil rights, for example.

### 5.7 Microfoundation of the Political Constraint and the Equilibrium in Oligarchy

In the main text, to keep the model simple and to focus on the quantitative exercise, the political constraint - enough supporters from the state sector - is given as a feature of the oligarchic regime. It can be rigorously modeled as a political game of all workers and the elite. Below, I extend the model to incorporate the microfoundation of the political constraint.

The government faces the political constraint: it needs support from sufficiently many workers to keep the regime stable. In each period, each worker $i$, decides whether to support the oligarchic regime ($m_i = 1$) or not ($m_i = 0$). The aggregate mass of supportive workers is $M_w = \int_0^1 m_idi$. If it is larger than a crucial threshold $L$, the regime survives this period, otherwise democratization occurs.\(^{48}\)

The political decision of a worker is made after she gets employed - either by the S firm or the P firm - and before she receives her wage and final income. The expected

\(^{48}\)The setting that only the support from workers counts is without loss of generality, for two reasons. First, the population size of the workers are much larger than the other two groups, so they should count. Second, the elite and the entrepreneur’s welfare is generally higher in oligarchy in all most all cases, so the model is robust to whether considering their political support or not.

In the extreme case that the elite holds dominating political power, and needs little support from workers - e.g., it uses mostly military force to control the citizens - $L$ can be close to 0. In the other case that the elite needs to win a majority voting, $L$ can be 50% if the voting system is fair, or smaller than 50% if the voting system is manipulated in favor of the elite. A regime with a voting system may not necessarily be a democracy. It can still be an oligarchy, and the government serves the interests of the elite.
Table 3: Payoffs of Workers

<table>
<thead>
<tr>
<th></th>
<th>$M_w \geq L$</th>
<th>$M_w &lt; L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j = S$</td>
<td>$y_{wS}$</td>
<td>$y_{wS}^D$</td>
</tr>
<tr>
<td>$j = P$</td>
<td>$y_{wP}$</td>
<td>$y_{wP}^D$</td>
</tr>
</tbody>
</table>

final income of a worker from sector $j \in \{S, P\}$ in the oligarchy is denoted as $y_{wj}$, and after democratization $y_{wj}^D$. They are endogenously determined by economic factors, which will be explained later in this subsection. 49 So the payoffs can be summarized as in Table 3. Obviously, for a myopic worker $i$ in sector $S$, one (weakly) dominating pure strategy is to support oligarchy if and only if the expected income is higher than that in the democracy, i.e., $m_i = 1$ if $y_{wS} \geq y_{wS}^D$ and $m_i = 0$ if $y_{wS} < y_{wS}^D$. Same for a $P$ sector worker. This strategy expresses voters’ sincere preferences. Without loss of generality, I assume that workers use this strategy, similar to the sincere voting assumption in the literature. 50 This political game is a simple global game of regime switching, in the spirit of Morris and Shin (2000). The game here is simple because there is no heterogeneous information, no cost of being against the regime, and no punishment for the supporters of the regime after the regime collapses.

5.8 Decomposition of the Elite’s Problem

In Subsection 5.4, we claim that the elite’s lifetime utility maximization problem can be equivalently written as two subproblems: lifetime income maximization and consumption smoothing, because the elite has a deep pocket and can set policies to maximize its lifetime income independent of its assets. This logic can be easily seen and formally proved in the sequential problem. The proof in the recursive form is also possible but more complicated. Let us prove this claim using the sequential problem first.

49 Workers are ex-ante identical, so there is no need for subscript $i$ to denote worker $i$.
50 Of course, given that there are a continuum of workers, worker $i$ knows that her action does not affect the aggregate political outcome and feels indifferent about what she does. There are other dominating strategies and equilibria with pure or mixed strategies. However, if there are finite workers and there is some small probability that worker $i$’s choice can be pivotal, then it is wise to follow the sincere strategy described above.
Denote the lifetime utility achieved by solving the two sub-problems - first maximizing lifetime income and then maximizing lifetime utility - as $U$. Remember that the solution to the original one complete problem gives lifetime utility $W$. First, $U \leq W$. Let us look at the two subproblems. The solution to the first subproblem achieving $V$ can be denoted as $\{\hat{M}_t, \hat{w}_{St}, \hat{K}_{St}, \hat{\eta}_t\}_{t=0}^\infty$ in the sequential form. The corresponding consumption and saving decisions obtaining $U$ are denoted as $\{\hat{c}_{et}, \hat{a}_{et+1}\}_{t=0}^\infty$. Combined together, the choice $\{\hat{M}_t, \hat{w}_{St}, \hat{K}_{St}, \hat{\eta}_t, \hat{c}_{et}, \hat{a}_{et+1}\}_{t=0}^\infty$ achieving $U$ is a feasible choice of the original problem, given that in every period the choice set for $M_t, w_{St}, K_{St}, \eta_t$ is independent of $a_{et}$. So the optimal solution for the original problem should be at least as good as this candidate choice, i.e., $W \geq U$.

Second, $U \geq W$. Denote the choice that solves the original problem and achieves $W$ with stars, as $\{M_t^*, w_{St}^*, K_{St}^*, \eta_t^*, c_{et}^*, a_{et}^*\}_{t=0}^\infty$. Let us compare the lifetime income given $\{M_t^*, w_{St}^*, K_{St}^*, \eta_t^*\}_{t=0}^\infty$ with the counterpart given the solution of the first subproblem achieving $V$: $\{\hat{M}_t, \hat{w}_{St}, \hat{K}_{St}, \hat{\eta}_t\}_{t=0}^\infty$. Obviously, the latter is at least as high as the former: $\hat{V} = \sum \hat{y}_{et} / R_t \geq \sum y_{et}^* / R_t = V^*$, given that $\{M_t^*, w_{St}^*, K_{St}^*, \eta_t^*\}_{t=0}^\infty$ is also a candidate solution to the first subproblem, again because of the independence of policies from the elite's assets. Then, in the second sub-problem, choosing $\hat{c}_{e0} = c_{e0}^* + \hat{V} - V^*$ and $\{\hat{c}_{et}, \hat{a}_{et}\}_{t=1}^\infty = \{c_{et}^*, a_{et}^*\}_{t=1}^\infty$ gives at least as high lifetime utility as $\{c_{et}^*, a_{et}^*\}_{t=0}^\infty$. In other words, if the solution from the subproblem gives higher lifetime income, then consuming the extra lifetime income in the first period and following the same consumption path of the solution to the direct lifetime utility maximization problem afterwards result in at least as high lifetime utility as $W$.

Combing these two results, we have $U = W$, i.e., solving the original lifetime utility maximization problem is the same as solving the two sub-problems.