Detecting Bubbles in the US and UK Real Estate Markets

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Abstract

This study considers state of the art subset selection and shrinkage procedures − stepwise regression, ridge regression, lasso, bridge regression and the elastic net along with the commonly employed least squares regression − to detect bubbles in real estate markets. Our analysis of real estate indices representing the commercial, residential and equity real estate sectors in the United States and the United Kingdom finds evidence suggesting the existence of significant periods of *overvaluation* in residential real estate, as well as economically significant periods of *undervaluation* in equity real estate markets. The evolution of specific real estate indices in the United States is similar to the evolution of the corresponding indices in the United Kingdom. In order to determine whether the observed deviations of the actual price index from its fundamental value are due to the presence of bubbles, we use two complementary methodologies, the first based on right-side unit root tests for explosive behaviour and the second defined by regime switching models for bubbles. We show that employing an average of all complex models yields more robust forecasting over a seven years out-of-sample period.

Keywords: Bubbles identification, Fundamental value, Real estate index, Right-side unit root tests, Regime switching models

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1. Introduction

For many centuries, asset price bubbles have persistently impacted economies of countries. In modern times, real estate bubbles have dominated the ebbs and flows of illusion and disillusion in major economies, with devastating effects for society. Is it possible to signal the occurrence of a bubble in real estate markets, the largest spot markets in terms of intrinsic value?

Amongst the earliest bubble detection methods are the variance-bound tests proposed by Shiller (1981) and LeRoy and Porter (1981), who check the validity of the fundamental asset pricing equation by comparing the variance of the observed asset price with an upper bound limit given by the *ex post* rational price. Another method, proposed by Diba and Grossman (1984) and Hamilton and Whiteman (1985), uses stationarity tests to detect bubbles. Furthermore, Campbell and Shiller (1987) apply unit root and cointegration tests to examine the behaviour of the fundamental and bubble component of present value models. However, Evans (1991) shows that unit root and cointegration tests have limitations[5] because they are not capable of detecting the explosive patterns of periodically collapsing bubbles.

Although it has been proven that bubbles cannot exist in finite horizon rational expectation models (Tirole, 1982; Santos and Woodford, 1997), bubbles can appear in markets with some particular characteristics that can be also attributed to real estate markets, such as (1) when some particular traders behave myopically (Tirole, 1982), (2) in infinite horizon growing economies with rational traders (Tirole, 1985, Weil, 1990), (3) when there are irrational traders (De Long et al., 1990), (4) in economies where rational traders have differential beliefs and when arbitrageurs cannot synchronize trades (Abreu and Brunnermeier, 2003) or (5) when there are short

[5] Recently, a number of econometric methods have been developed that deal with Evans' critique and are capable of distinguishing between pure unit root processes and periodically collapsing bubbles.

sale/borrowing constraints (Scheinkman and Xiong, 2003). Applying the martingale theory of asset price bubbles in continuous time and continuous trading economies, Jarrow and Porter (2010) demonstrate that in the presence of bubbles, market price indices and fundamental values diverge and lead to serious errors in decision making by investors, financial institutions and regulators.

Debating the idea that the market cannot be efficient because it did not predict the 2008 subprime crisis, John Cochrane stated "crying 'bubble' is empty unless you have an operational procedure for identifying bubbles, in real time and not just after the fact, distinguishing them from rationally low-risk premiums, telling a 'bubble' from a justified 'boom,' and crying wolf too many years in a row", see Buckner (2017). In this paper we offer a procedure that can be used to timely detect bubbles in the real estate markets and we highlight the usefulness of our approach using an extended out-of-sample period 2009-2015.

The subprime mortgage crisis of 2007-2009 had its roots in a real estate bubble of gigantic proportions. There were clear signals (Case and Shiller, 2004, Belke and Wiedmann, 2005, Zhou and Sornette, 2006) that something was wrong with the residential real estate prices in the United States. There was evidence of real estate bubbles in the United Kingdom as well at the beginning of the 2000s (Zhou and Sornette, 2003, Black et al. 2006, Fraser et al. 2008). Nneji et al. (2013a, 2013b) examined the residential market in the United States between 1960 and 2011 and found evidence of an intrinsic bubble pre-2000 and, based on a regime-switching model, evidence of periodically rational bubbles in the post-2000 market. Even in real estate investment trusts (REITS) that behave more like an equity asset class, there was evidence of speculative bubbles (Brooks et al. 2001, Payne and Waters, 2005 and 2007 and Jirasakuldech et al., 2006). It is therefore highly desirable to have a mechanism for signalling the emergence of a bubble in the most valuable asset class of all, real estate.

In this paper, real estate price indices are decomposed into a fundamental and a non-fundamental component using a rich dataset of 19 variables covering financial indicators, price indicators, national income and business activity indicators, and employment and labour market indicators. Our study tries to cover exhaustively the real estate markets in the United States and the United Kingdom going back from the end of 2015 to the beginning of historical available data for real estate indices and their drivers in commercial, residential and REITs markets. We employ several subset selection and shrinkage procedures (stepwise regression, ridge regression, lasso, bridge regression and the elastic net along with the commonly employed least squares regression). In order to avoid model selection risk in extracting the fundamental value component of the real estate indices, we propose averaging the fundamental components of all models employed. Our findings suggest the existence of significant periods of *overvaluation* in real estate markets, particularly in residential real estate, as well as economically significant periods of *undervaluation*, particularly in equity real estate markets. The evolution of specific real estate indices in the United States is like the evolution of the corresponding indices in the United Kingdom.

In order to determine whether the observed deviations of the actual prices from their fundamental values are due to the presence of speculative bubbles, we use two complementary methodologies, both taking into account the information contained in the non-fundamental component of the asset price. To verify whether the deviation of the asset price from the fundamental value is due to the presence of speculative bubbles we employ the right-side augmented Dickey-Fuller test for explosive behaviour developed by Phillips et al. (2011) and Phillips et al. (2015) and the Van Norden and Schaller (1993, 1996) two-state regime switching model. The first methodology can also be used to date-stamp the periods of explosiveness in the real estate sectors. The second methodology is based on regime-switching models with two regimes: one where the bubble survives and continues to grow and the other where the bubble collapses. The findings from both methodologies provide significant in-sample evidence that the observed deviations of the actual price from the fundamental value were due to the presence of speculative bubbles. More importantly, our out-of-sample results show that in most cases the proposed regime-switching model for bubbles (averaged across all models employed) outperforms the historical average benchmark and the stylized alternative models.

The paper is organised as follows. Section 2 provides a description of the econometric methodology that we follow and Section 3 presents the data that are used. In Section 4 we present the in-sample bubble detection results, while in Section 5 we discuss the out-of-sample empirical results. Last section concludes our paper.

2. How to detect bubbles in asset markets?

Starting from Campbell, Lo and McKinlay (1997) and Cochrane (2005), the fundamental price of an asset is derived[6] as

$$P_t = E_t\left[\sum_{i=1}^{T}\left(\frac{1}{1+R}\right)^{i} D_{t+i}\right] + E_t\left[\left(\frac{1}{1+R}\right)^{T} P_{t+T}\right] \tag{1}$$

where the first term of the right-hand side of equation (1) represents the fundamental component, which is the expectation of all discounted cash flows, and the second term is the expectation of the discounted asset price T periods from time t, and P_t is the asset price at time t and D_{t+1} is the next period's cash flow.

In the case of real estate markets, expected cash flow payments are not directly available. One proxy widely used in the literature is the rent income stemming from holding the property, which is also not available for the majority of indices. To this end, we develop alternative models for the estimation of the fundamental component and consequently the bubble component of the real estate price indices. Specifically, we propose extracting it using subset selection and shrinkage procedures, such as stepwise regression, ridge regression, lasso, bridge regression and elastic net.[7] This is the first time these techniques have been employed in this context. The subsequent description of these methods is largely based on Hastie, Tibshirani, and Friedman (2009).

2.1. Model Selection Procedures for the Fundamentals

[6] Lai and van Order (2017) investigate U.S. house prices between 1980 and 2012 across 45 metropolitan areas, employing a version of the Gordon dividend discount model.

[7] In a recent paper, Shi (2017) employs a vector autoregressive (VAR) model and variables reflecting aggregate macroeconomic conditions in order to predict fundamental prices.

The *benchmark model* in our study is the classic normal linear regression model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon},$$

where $\mathbf{X} = \mathbf{x}_1, \dots, \mathbf{x}_p$ is the $T \times p$ matrix of predictors, $\boldsymbol{\beta} = (\beta_1, \dots, \beta_p)'$ is the coefficient vector and $\boldsymbol{\epsilon} \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_n)$ is the error vector. The ordinary least squares (OLS) estimator $\widehat{\boldsymbol{\beta}}_{OLS} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$ typically has poor predictive accuracy with low bias and high variance.

Ridge regression is a regression method estimating the coefficients subject to the l_2 penalty:

$$\underset{\boldsymbol{\beta}}{\operatorname{argmin}}[||\mathbf{y} - \mathbf{X}\boldsymbol{\beta}||^2 + \lambda||\boldsymbol{\beta}||^2] \quad (2)$$

where $\lambda \geq 0$ is a parameter for the amount of shrinkage. The second term of the equation is called the shrinkage penalty and in the case of the ridge regression it is based on l_2 regularization, where $\lambda||\boldsymbol{\beta}||^2 = \lambda \sum_{j=1}^{p} \beta_j^2$ and is small when $\beta_1, \dots, \beta_p$ are close to zero and has the effect of shrinking the coefficient estimates towards zero. When $\lambda = 0$ the penalty term has no effect and ridge regression will produce similar estimates to OLS. However, as $\lambda \to \infty$ the impact of the ridge penalty grows and the coefficient estimates will approach zero[8].

The *least absolute shrinkage and selection operator* (lasso) has a penalty term based on the l_1 norm, capable to yield sparse models. The lasso coefficient estimates are obtained by solving:

$$\underset{\boldsymbol{\beta}}{\operatorname{argmin}}[||\mathbf{y} - \mathbf{X}\boldsymbol{\beta}||^2 + \lambda||\boldsymbol{\beta}||_1] \quad (3)$$

where $\lambda \geq 0$ is the lasso tuning parameter. The regression penalty for the lasso is $\lambda||\boldsymbol{\beta}||_1 = \lambda \sum_{j=1}^{p} |\beta_j|$. The difference between this and ridge regression is that the lasso method imposes a penalty based on the l_1 norm instead of the l_2 norm, allowing for both shrinkage and variable selection, by setting some of the coefficients equal to zero.

[8] A disadvantage of ridge regression is that the penalty $\lambda||\boldsymbol{\beta}||^2$ will shrink all the coefficients towards zero, but it will never set them to zero. Having a model which uses all p predictors can be a problem for model interpretation.

Bridge regression has a penalty term which is based on the l_γ norm and the coefficients are estimated by minimizing:

$$\underset{\boldsymbol{\beta}}{\operatorname{argmin}}\left[\|\mathbf{y}-\mathbf{X}\boldsymbol{\beta}\|^2+\lambda\|\boldsymbol{\beta}\|_\gamma^\gamma\right] \quad (4)$$

subject to the constraint $\lambda \geq 0$ and $\gamma > 0$ are the two tuning parameters. The penalty term in the case of bridge regression is $\lambda\|\boldsymbol{\beta}\|_\gamma^\gamma = \lambda\sum_{j=1}^{p}\left|\beta_j\right|^\gamma$ and it is a generalization of the lasso ($\gamma = 1$) and ridge regression ($\gamma = 2$). The bridge regression ($1 < \gamma < 2$) performs shrinkage by keeping all predictors, similarly to ridge regression.

Finally, the *elastic net* (EN) method combines both l_1 and l_2 terms in the penalty, thus simultaneously performing continuous shrinkage and automatic variable selection, but it can also select groups of correlated variables. The elastic net coefficients are estimated by minimizing the following penalized residual sum of squares function:

$$\underset{\boldsymbol{\beta}}{\operatorname{argmin}}\left[\|\mathbf{y}-\mathbf{X}\boldsymbol{\beta}\|^2+\lambda\big((1-\alpha)\|\boldsymbol{\beta}\|_1+\alpha\|\boldsymbol{\beta}\|^2\big)\right] \quad (5)$$

where λ is the tuning parameter, $\|\boldsymbol{\beta}\|_1 = \sum_{j=1}^{p}\left|\beta_j\right|$ and $\|\boldsymbol{\beta}\|^2 = \sum_{j=1}^{p}\beta_j^2$. The term $(1-\alpha)\|\boldsymbol{\beta}\|_1 + \alpha\|\boldsymbol{\beta}\|^2$ with $\alpha \in [0,1]$ is called the elastic net penalty, which is a combination of the ridge regression and the lasso penalties. When $\alpha = 1$, the elastic net becomes a ridge regression; if $\alpha = 0$ it is the lasso, while if $\alpha \in (0,1)$ it has the properties of both methods.

2.3. Right-side Unit Root Tests and Date Stamping Procedure

The tests for speculative bubbles we employ in this study are based on right-side unit root tests implemented repeatedly on a forward expanding sample sequence to search for mildly explosive behaviour in the data. Those are the supremum augmented Dickey-Fuller (SADF) test and the generalized SADF (GSADF) test developed by Phillips et al. (2011, PWY)[9] and Phillips et al.

[9] Astill et al. (2017) propose tests that improve upon the detection of an end-of-sample asset price bubble of finite length and show that their tests detect several well-documented periods of exuberance earlier than existing methods. Fabozzi and Xiao (2018) propose a new recursive algorithm to deal with the inconsistency encountered when

(2015, PSY) respectively. The GSADF test has the advantage that it has an increased capacity to detect multiple bubbles in the data. To keep the analysis clear, we provide a detailed description of the tests in Appendix A.

2.4. The Regime-switching Bubble Model

Blanchard (1979) and Blanchard and Watson (1982) suggested a model for rational bubbles with two possible bubble states; one state is that the bubble survives and the other state is that the bubble collapses. The bubble process is then defined by:

$$B_{t+1}|S = \left(\frac{1+R}{q}\right) B_t + u_{t+1}, \qquad with\ probability\ q$$

and (6)

$$B_{t+1}|C = u_{t+1}, \qquad with\ probability\ 1-q$$

A rational bubble that has the above form obeys the restriction: $B_t = E_t\left[\frac{B_{t+1}}{1+R}\right]$, as long as the shock u_{t+1} satisfies $E(u_{t+1}) = 0$. Then

$$E_t(B_{t+1}|S) = \frac{(1+R)}{q} B_t, \qquad with\ probability\ q$$

and (7)

$$E_t(B_{t+1}|C) = 0, \qquad with\ probability\ 1-q$$

where *S* indicates the state that the bubble survives and *C* the state that it collapses. If the bubble survives in period $t+1$, it will grow at a rate $\left(\frac{1+R}{q}\right) - 1$, which is faster than R, in order to compensate the investors for the risk they take for the probability of a crash.

estimating the timeline of a bubble based on different samples. This method improves upon the PWY procedure by identifying more consistent starting points and by implementing a two-direction searching process for initialization.

The Blanchard and Watson model was generalized[10] by van Norden and Schaller (1993, 1999) in two ways. First, they allow the probability of the bubble being in the surviving state q to depend on the relative size of the bubble $q = q(b_t)$ where $b_t = B_t/P_t$ is the relative size of the bubble, which is the ratio of the non-fundamental component B_t to the actual price P_t. The absolute value of b_t is used since there can be positive or negative bubbles.

The second generalization allows for partial collapses, by permitting the expected value of the bubble conditional to the collapsing state to be non-zero. Van Norden and Schaller (1993, 1999) defined the expected size of a bubble in state *C* as $u_t P_t$ and assumed that it depends on the relative size of the bubble in a previous period:

$$E_t(B_{t+1}|C) = u(b_t)P_t \tag{8}$$

where $u(.)$ is a continuous and differentiable function such that $u(0) = 0$ and $0 \leq \frac{du(b_t)}{db_t} \leq 1$. The condition ensures that a collapsing bubble is smaller than the bubble in the previous period.

The two generalizations made by van Norden and Schaller lead to the following modified bubble model is obtained:

$$E_t(B_{t+1}|S) = \frac{(1+R)}{q(b_t)} B_t - \frac{1-q(b_t)}{q(b_t)} u(b_t)P_t, \qquad \textit{with probability } q(b_t)$$

and (9)

$$E_t(B_{t+1}|C) = u(b_t)P_t, \qquad \textit{with probability } 1-q(b_t)$$

The expected gross returns R^* for each regime are:

$$E_t(R^*_{t+1}|S) = (1+R) + \frac{1-q(b_t)}{q(b_t)}[(1+R)b_t - u(b_t)], \qquad \textit{with probability } q(b_t)$$

$$E_t(R^*_{t+1}|C) = (1+R)(1-b_t) + u(b_t), \ \textit{with probability } 1-q(b_t) \tag{10}$$

[10] Van Norden and Schaller (1993,1999) and Brooks and Katsaris (2005a,2005b) criticised Blanchard and Watson (1982) model because of lack of theoretical support and empirical evidence.

Thus, the returns in time $t+1$ depend on the regime of the previous period t. To estimate the model, the first-order Taylor series approximations of $E_t(R^*_{t+1}|S)$ and $E_t(R^*_{t+1}|C)$ with respect to b_t around some arbitrary value b_0 are taken, giving the linear regime switching model:

$$E_t(R^*_{t+1}|S) = \beta_{S0} + \beta_{S1} b_t$$

$$E_t(R^*_{t+1}|C) = \beta_{C0} + \beta_{C1} b_t \qquad (11)$$

where

$\beta_{S1} = \left(-\frac{1}{q(b_0)^2}\frac{dq(b_0)}{db_t}[(1+R)b_0 - u(b_0)] + \frac{1-q(b_0)}{q(b_0)}\left[1+R-\frac{du(b_0)}{db_t}\right]\right)$ and

$$\beta_{C1} = \left(\frac{du(b_0)}{db_t} - (1+R)\right). \qquad (12)$$

The regime switching model can be rewritten as:

$$R^*_{S,t+1} = \beta_{S0} + \beta_{S1} b_t + \varepsilon_{S,t+1}, \qquad \varepsilon_{S,t+1} \sim N(0, \sigma_S^2)$$

$$R^*_{C,t+1} = \beta_{C0} + \beta_{C1} b_t + \varepsilon_{C,t+1}, \qquad \varepsilon_{C,t+1} \sim N(0, \sigma_C^2) \qquad (13)$$

where σ_S, σ_C are the standard deviations of the error terms of $\varepsilon_{S,t+1}$ and $\varepsilon_{C,t+1}$ respectively. The parameters β_{S0} and β_{C0} represent the mean returns for the surviving and the collapsing state respectively and the coefficients β_{S1} and β_{C1} show how changes in the relative size of the bubble affect the returns in each state.

For the functional form of $q(b_t)$, van Norden and Schaller use a probit model:

$$P(R^*_{t+1}|S) = q(b_t) = \Phi(\beta_{q0} + \beta_{q1}|b_t|)$$

$$P(R^*_{t+1}|C) = 1 - q(b_t) = 1 - \Phi(\beta_{q0} + \beta_{q1}|b_t|) \qquad (14)$$

where Φ is the standard normal cumulative density function and β_{q1} describes the effect that the absolute value of the relative size of the bubble has on the probability of being in the surviving state. Van Norden (1996) uses both b_t and b_t^2 instead of $|b_t|$, van Norden and Vigfusson (1998) employ b_t, while Schaller and van Norden (2002) use b_t^2. The model considers the restriction $\beta_{q1} < 0$, since as the deviation from the fundamentals grows, so does the probability of collapse. Furthermore, assuming $R > 0$, then $\beta_{C1} < 0$, because as the relative size of the bubble grows, it leads to greater capital losses when the bubble collapses and $\beta_{S1} > \beta_{C1}$, since a large relative size of the bubble means that the difference between the returns of the surviving and collapsing state will be greater.

The parameters estimates are found by maximizing the log-likelihood function:

$$\sum_{t=1}^{T} ln\left[q(b_t)\varphi\left(\frac{R_{S,t+1}^{*}-\beta_{S0}+\beta_{S1}b_t}{\sigma_S}\right)\sigma_S^{-1} + \left(1-q(b_t)\right)\varphi\left(\frac{R_{C,t+1}^{*}-\beta_{C0}+\beta_{C1}b_t}{\sigma_C}\right)\sigma_C^{-1}\right] \quad (15)$$

where φ is the standard normal probability density function and the parameters to be estimated are $\beta_{S0}, \beta_{S1}, \beta_{C0}, \beta_{C1}, \beta_{q0}, \beta_{q1}, \sigma_S$ and σ_C. The probability of being in regime $i = S, C$ in period $t+1$ depends on the relative size of the bubble b_t and is given by the formula: $\Phi\left(l(i)\left(\beta_{q0} + \beta_{q1}|b_t|\right)\right)$, where $l(i) = 1$ in the surviving state and $l(i) = -1$ in the collapsing state.

3. Data Description

3.1. Real Estate Data

We analyse the main real estate indices in each real estate market from each country. For commercial, residential and equity real estate sectors, the indices for the United States are NCREIF, S&P/Case-Shiller and US FTSE EPRA/NAREIT, while for the United Kingdom the indices in this study are IPD UK All Property, UK House Price Index and UK FTSE EPRA/NAREIT.[11]

All real estate indices price levels are retrieved from Bloomberg. For the US, data on quarterly frequency is available for the NCREIF from the fourth quarter of 1977 to the fourth quarter of 2015, while commercial real estate data for the UK are available on a monthly frequency

[11] The properties of these indices are described in Chapter 2 of Tunaru (2017).

for the IPD index for the period from December 1986 to December 2015, providing a total of 153 quarterly and 349 monthly observations respectively. For equity real estate monthly data on the transactions-based FTSE EPRA/NAREIT indices for the US and the UK are available for the period from December 1989 to December 2015, with a total number of 313 monthly observations for each index. Finally, for the residential real estate market, monthly data is available on the S&P/Case-Shiller home price index for the US from January 1987 to December 2015 and on the UK House Price Index from January 1995 to December 2015, totalling 348 and 252 monthly observations for each time series respectively. The real estate indices are adjusted for inflation using the Consumer Price Index (CPI) for the US and the Retail Price Index (RPI) for the UK. The augmented Dickey-Fuller tests (ADF tests) for all indices indicate[12] that the level series are non-stationary.

3.2. Economic Data

We are guided by the extant literature in selecting the economic variables employed to construct the fundamental value models for the real estate indices. Ghysels et al. (2013) provide an extensive review of the literature on real estate forecasting based on the type of predictive information used. We extracted the potential drivers of the fundamental value of real estate markets from previous studies.[13]

For both economies, we employ a set of 19 explanatory variables, which are classified into four broad categories: financial indicators, price indicators, national income and business activity indicators, and employment and labour market indicators. Specifically, the financial variables are a stock price index, the US/UK exchange rate, the money supply M2, the central bank rate, the 5-year and 10-year government bond yields and a mortgage rate. The price indicators include the

[12] All tests for stationarity are presented in Table B1 in the Appendix.

[13] See Case and Shiller (1990), Dobson and Goddard (1992), Liu and Mei (1992), Mei and Liu (1994), Ling and Naranjo (1997), Ling, Naranjo and Ryngaert (2000), De Wit and Van Dijk (2003), Himmelberg et al. (2005), Clayton, Ling and Naranjo (2009), MacKinnon and Al Zaman (2009), Plazzi et al. (2010). The list is by no means exhaustive and there is a very long list of articles in this area.

inflation rate, gold price, oil price and the rent price index. The national income and business activity indicators are the real GDP, real personal disposable income, industrial production and housing starts. Finally, the labour market indicators are the unemployment rate, labour cost and labour productivity. Variable definitions are presented in Table 1, while data sources are outlined in Table B2 in the Appendix.

[Insert Table 1 Here]

4. In-Sample Empirical Analysis

4.1. Results for the Fundamental Value

In order to apply the right-side unit root tests and the regime switching model on the real estate indices, the fundamental and bubble components must first be retrieved. This is usually done by constructing a supply and demand model, through which the price index is regressed on various economic variables using OLS. The fitted value of the regression model represents the fundamental value of the index, which is determined by the economic variables. The error term of the model is the part of the index, which is not explained by the model predictors and represents the non-fundamental or bubble component of the index price.

Due to the large number of predictors, we employ several shrinkage model selection procedures along with the OLS to create alternative measures for the fundamental and bubble component. The SADF and GSADF tests are applied to the non-fundamental component. Furthermore, in order to estimate the regime switching model the relative size of the bubble is required, which is constructed using the actual price and fundamental price. Specifically, to extract the fundamental price from the regressions the following formula is used:

$$p_t^f = \left(1 - r_t^f\right) p_{t-1}^f, \qquad where\ p_0^f = p_0 \tag{16}$$

where p_t^f is the fundamental price at time t, r_t^f is the fitted value of the regression of the index returns on the stationary predictors[14] at time t and p_0 is the actual price of the index at $t = 0$. Subsequently, the relative size of the bubble is computed using the following formula:

$$b_t = \frac{p_t - p_t^f}{p_t} \tag{17}$$

where b_t is the relative size of the bubble and p_t is the actual price of the index at time t.

Figure 1 plots the actual price, the average fundamental value and the average relative size of the bubble, for all six real estate indices.[15] The average fundamental value or bubble size is simply computed by taking the average of the fundamental value or relative bubble size of all fitting procedures for each market. In this way, we overcome the model risk associated with the employment of one particular model for bubble estimation. The left-hand scale of Figure 1 plots the actual index price against the average fundamental price and on the right-hand scale the extent of under- or overvaluation is depicted. There have been periods of overvaluation and undervaluation in all six markets across our sample. Those periods of overvaluation and undervaluation could be associated with market sentiment of illusion and disillusion, respectively.

For the US, the commercial real estate as reflected by the NCREIF index was often undervalued, from the end of the 1980s right to the eruption of the subprime crisis in 2007. There were short periods of overvaluation between 1982 and 1986 and between 2007 and 2008. A similar picture is portrayed for the US FTSE EPRA/NAREIT Index with long periods of undervaluation around the dot.com crisis of 2000-2002 and in the aftermath of the subprime crisis.

[14] The predictors are the 19 economic variables listed in Table 1. The OLS, stepwise regression, ridge regression, the lasso, bridge regression and the elastic net were applied and the tuning parameters were selected using tenfold cross validation. For the lambda tuning parameters, a grid of 100 values between 10^{-2} and 10^{2} was chosen. The bridge regression tuning parameter, gamma, is given a grid of values between 1.1 and 1.9 with step 0.1, while for the elastic net alpha tuning parameter a grid of values between 0 and 1 with step 0.1 is chosen.

[15] Tables B3-B5 in the Appendix present the coefficient estimates from the fundamental models

[Insert Figure 1 Here]

The residential real estate evolution in the United States paints a different picture, with a long undervaluation period between 1991 and 2002, followed by an economically significant overvaluation period ending in 2009 and followed by undervaluation that peaked in 2012.

The IPD index in the UK seems to be closer to the fundamental value. There are short periods of overvaluation, the most notable one being the period before the start of the subprime crisis, and likewise short periods of undervaluation, the only economically significant one being the period 2009-2015. Similar to the US, the equity index for the UK indicates that this market was generally characterised by undervaluation. Mei and Saunders (1997) found evidence of a trend-chasing strategy of buying high and selling low followed by commercial banks and thrifts on their real estate investments. Their conclusion is in line with our results on REITS markets in the US and the UK reaching an overall judgement that undervaluation was omnipresent.

The residential real estate in the United Kingdom had a similar evolution for the residential real estate in the United States with the only difference that the period 2002-2003 indicates the start of a bubble in the United Kingdom that ended only in 2012. In both countries there has been a long period of significant overvaluation of house prices that started after 2002 and ended in 2009 in the United States and in 2011 in the United Kingdom. Holly et al. (2011) argued that there is a direct link between London house prices and New York houses prices and also suggested that economic shocks to the metropolis prices propagated contemporaneously and spatially to other regions in the same country. Their argument may explain our evidence on the similarity of overvaluation and undervaluation periods in the two countries.

4.2. Results of the Right-side ADF Tests for Explosive Behaviour

Table 5 summarizes the results for the SADF and GSADF tests on the real estate indices for the US and the UK. In the interest of saving space, we report the tests based on the average of the non-fundamental components derived from the alternative proposed models described in

Section 2.2.[16] Following the rule suggested by PSY, the minimum window size is set to $0.01 + 1.8/\sqrt{T}$ of the total sample size for each index. The finite sample critical value sequences are obtained by Monte Carlo simulation with 2000 replications, while the ADF lag is chosen to minimize the Schwarz Information Criterion.

[Insert Table 2 Here]

Overall, the SADF and GSADF tests provide evidence of bubble formation for all real estate indices. Specifically, both tests find evidence of explosive behaviour for all U.S. real estate indices at the 1% significance level. According to the SADF test, all U.K. real estate indices exhibit explosive behaviour at a 1% significance level, with the exception of the U.K. equity real estate index, where the null hypothesis that there is a unit root is rejected at a 10% significance level. The results of the GSADF tests for the U.K. and the U.S. reveal evidence that multiple bubbles are present in the commercial, equity and residential real estate indices of both countries.

Our tests point to strong evidence of exuberance in all real estate indices and we employ the BSADF test in order to identify the origin and collapse date of the bubble periods for each index. Similarly to the GSADF test, the minimum window is set to $0.01 + 1.8/\sqrt{T}$ of the total sample observations and the ADF lag is chosen to minimize the Schwarz Information Criterion. Figure 2 illustrates that for the NCREIF Property Index the two major bubble periods occur in the late 80s to early 90s and from 2005 to 2008, while the bubble period with the greatest duration for the IPD UK Property Index is from 2005 to 2008, with shorter periods appearing in the late 90s and in 2013-1014. For the US and the UK real estate indices the bubbles with the longer duration occur in the late 90s and early 2000s, with shorter bubble periods appearing between 2006 and 2007. For the S&P/Case-Shiller Index the two major bubbles are observed for the period 1990-

[16] Individual model results are available from the authors upon request.

1998 and another one in the period 2000 to 2007, while for the UK residential real estate the bubble with the longest duration is between 2001 and 2007, with smaller bubble periods after 2009.

[Insert Figure 2 Here]

4.3. Results for the Regime Switching Models for Bubbles

To determine whether the deviations of the actual prices from their fundamentals were due to the presence of periodically collapsing bubbles, we apply the van Norden and Schaller (vNS) regime switching model to the returns of the real estate indices. Tables 6-8 present the results of the regime switching model based on both the average bubble size and the model specific ones for the commercial, equity and residential real estate markets, respectively. The regime switching model we apply has two regimes. In the first regime the bubble survives and continues to grow yielding a positive return, while in the second regime the bubble collapses and prices fall. According to the bubble theory realised returns should be higher in the surviving regime, while volatility should be higher in the collapsing regime. We first focus on the findings with respect to the average bubble size and then we compare it to the individual model ones.

The coefficient of the bubble term for the surviving regime (β_{S1}) is statistically significant at the 5% level only for the S&P/Case-Shiller Home Price Index. In this case, all individual bubble models provide positive statistically significant results. For the IPD UK Index, β_{S1} is positive and statistically significant for the average bubble size at the 10% level, while the results of individual models is mixed with only bridge and elastic net pointing to the same direction. Furthermore, the coefficient of the bubble term when the bubble collapses, β_{C1}, is statistically significant for the all indices with the exception of the UK commercial and equity index. For these indices, only bridge supports the theoretical negative coefficient. Overall, the coefficients in the surviving regime are greater than those in the collapsing regime, which suggests that the bubble in the collapsing regime leads to more negative returns than in the surviving regime.

The coefficient β_{q1} is negative, in the case of the U.S. residential and the U.K. equity and residential indices, which signifies that the larger the bubble size, the higher the probability of the bubble collapsing in the next period. The estimates for β_{q1} are statistically significant at the 5% level for the US and the UK equity and the UK residential real estate indices. For the equity indices, both OLS and stepwise point to non-statistically significant coefficients, while for the UK residential index, all models agree.

The estimates for the mean returns in the surviving regime are 1.75%, 0.40%, 0.44%, 0.70%, -0.29% and 0.57%, while in the collapsing regime they are -6.10%, -18.98%, -0.44%, -0.20%, -25.93% and -0.06% for the commercial, equity and residential real estate markets for the US and the UK respectively. These represent the expected yields when there is no bubble and are quite similar across models.

[Insert Tables 3-5 Here]

Turning to coefficient restriction tests and the results based on the average bubble, we note that the restriction $\beta_{S0} \neq \beta_{C0}$ holds for all sectors (at the 10% level) except for the UK equity real estate sectors (marginally), while the restriction $\beta_{S1} \neq \beta_{C1}$ holds for all indices except for the IPD UK Property Index and the UK FTSE EPRA/NAREIT index. It is interesting to note, tough, that we observe considerable heterogeneity among individual bubble specifications. More in detail, for the NCREIF index, both restrictions are rejected when the bridge bubble is employed and for IPD UK, the restriction $\beta_{S1} \neq \beta_{C1}$ holds for the bridge and elastic net specification. In a similar vein, OLS rejects both restrictions and stepwise only the second one for the US equity real estate index. On the other hand, both restrictions hold based on the bridge bubble specification and the UK FTSE index.

Finally, we perform likelihood ratio tests to determine whether the vNS bubble model can explain returns better than alternative models such as volatility regimes, fads and mixture-normal models. Our results, based on the average bubble specification, indicate that the vNS model is more efficient in capturing return dynamics for all indices, except for the two commercial real estate indices. For the NPI the volatility regimes and the mixture-normal models outperform the bubble model, while for the IPD the mixture-normal model is better at describing the returns. For these indices, all bubble specifications point to the same direction with the exception of the bridge bubble that points to superiority of the vNS model over the mixture-normal model. With respect to US FTSE index, contrary to the average bubble and the majority of fundamental models, stepwise and OLS reject the superiority of the vNS models versus all alternative stylised models (OLS at the 5% level for the fads model). On the other hand, for the UK FTSE index, only lasso and bridge (along with the average) are in favour of the vNS model. Similarly to the coefficient restriction tests, all fundamental model specifications agree on the superiority of the vNS model for the US and UK residential indices.

Figure 3 illustrates the evolution of the probability of collapse for each specific real estate sector (based on the average bubble size) in both the US and the UK. The only indication of a possible crash in the commercial real estate market in the US is for 1992-1993 and 2009. The equity market in the US was close to a crash in 2004, 2009 and 2012. For the residential real estate in the US as reflected by the Case-Shiller index, clear problems related to the collapse of the market were in 1990-1991, 2006-2011, 2014 and 2015. The situation in the United Kingdom was slightly different. The probability of collapse attached to the IPD index was very high between 1990-1994 and 2007-2010. The equity market in the United Kingdom was only ever close to a crash around 2009. The residential market as represented by the UK House Price index was close to a collapse between 2008 and 2009 and the probability of collapse even reached zero in the period 2002-2008.

[Insert Figure 3 Here]

In the next section, we assess the out-of-sample forecasting ability of the vNS regime switching model relative to the stylised bubble models and the historical average model (random walk with drift). We also scrutinise the forecasting ability of the proposed fundamental models employed for the relative bubble calculation and check whether employing the average relative bubble offers a hedge against model uncertainty.

5. Out-of-Sample Empirical Analysis

This section examines whether the van Norden-Schaller regime-switching model can be used to generate reliable out-of-sample forecasts. We consider 1-month, 3-month and 6-month forecasting horizons (the analysis for the NCREIF Property Index is for only 1-quarter and 2-quarters ahead). Given the total number of T observations of each index, the sample is split to an out-of-sample part, Q and an in-sample part, $P = T - Q$. In our experiment, the out-of-sample window is set to eight years for all indices (32 observations for the NCREIF Property Index and 96 observations for the rest of the indices). In this respect, the out-of-sample period starts at 2008 and coincides with the global financial crisis creating considerable challenges for our forecasting experiment. The h-period ahead forecasts (h=1, 3, 6 months) of the regime switching model are generated by estimating the van Norden-Schaller model recursively increasing the initial window, P, with one observation at a time. The average relative and individual fundamental bubble sizes, which are used as an input in the model, are also constructed recursively from the estimates of all the fundamental models at each iteration.

The forecasting performance of the van Norden-Schaller model and the alternative nested regime switching specifications are evaluated using the mean square forecast error (MSFE) criterion, which is given by:

$$MSFE_i = \frac{1}{Q}\sum_{t=1}^{Q}\left(r_{P+t} - \hat{r}_{i,P+t}\right)^2 \qquad (18)$$

where $\hat{r}_{i,P+t}$ denotes the forecast from model i. In order to evaluate the forecasting accuracy of the regime switching models, we compare them with the historical average benchmark model (random walk with drift). We compute the MSFE ratios of the regime switching models relative to the benchmark and alternative nested regime specifications. A ratio below unity implies that the regime switching model forecast is more accurate than the benchmark and alternative models in terms of MSFE. Additionally, to test whether the improvement in MSFE for the regime switching models against the historical average (and the nested regime switching specifications) is statistically significant, we employ the Clark and West (2007) test that utilises the MSFE-adjusted statistic, which is approximately normally distributed when comparing forecasts from nested models. The MSFE-adjusted statistic is computed by first defining:

$$f_{i,t} = (r_{P+t} - \bar{r}_{P+t})^2 - \left(r_{P+t} - \hat{r}_{i,P+t}\right)^2 + \left(\bar{r}_{P+t} - \hat{r}_{i,P+t}\right)^2 \quad (19)$$

where $\bar{r}_{P+t}$, is the forecast of r_{P+t}, using the historical average benchmark. The Clark-West t-statistic is compared to the critical value of 1.282 corresponding to the 10% significance level. The null-hypothesis is that the MSFE of the benchmark is less or equal to the MSFE of model i, while the alternative is that MSFE of the benchmark is greater than the MSFE of model i.

5.1. One-month ahead forecasts

Tables 6-8 detail the MSFE ratios of the various models relative to the benchmark for the 1-month ahead horizon, while the Clark-West t-statistics are reported below in parenthesis. Overall, our 1-month out-of-sample findings suggest that the van Norden and Schaller model is more accurate than the benchmark in all the indices considered while it beats the alternative regime switching models in four of the indices under consideration.

More in detail, the top panel of Table 6 compares the performance of the forecasts with the historical average for the UK commercial index, while the bottom panel compares the out-of-sample performance of the van Norden and Schaller model with each of the stylized alternative models. Our findings suggest that both the normal-mixture model and the bubble model have statistically significant better out-of-sample performance compared to the historical average. The fundamental bubble calculated via the bridge regression attains the lowest MSFE (0.7898) among the alternative fundamental models and the average bubble. Comparing the performance of the vNS model to the stylized alternative models, we note that the vNS model beats both the volatility regimes and the fads model (but not the normal-mixture one).

Turning to equity real estate indices, our findings, reported in Table 7, suggest that for the US, the forecasts generated by the vNS model and the average bubble are the most accurate (MSFE=0.9069) albeit non-significant. However, stepwise vNS model forecasts are statistically significantly lower that the historical average benchmark. We should also note that the elastic net, the bridge and average bubble fads model attain superior forecasts. For the UK equity index, all normal mixture and vNS models (with the exception of Lasso) achieve lower forecast errors than the historical average benchmark. Stepwise vNS delivers more accurate forecasts among the alternative bubble models followed by the average bubble. With respect to the residential real estate indices, the vNS model achieves superior forecasting performance irrespective of the fundamental bubble employed for both the US and US markets (Table 8). Specifically, for the S&P/Case-Shiller Home Price Index, the average bubble delivers the lowest MSFE (0.6079) followed by ridge (0.6176) and lasso (0.6354). As expected, the vNS model outperforms all stylised nested specifications (Panel B) by a wide margin. Similar findings pertain for the UK House Price Index. In this case, the lowest MSFE is achieved by lasso vNS (0.6852) followed by the average bubble vNS (0.6958). As expected, Panel B of Table 8 verifies the forecasting superiority of vNS relative to the volatility regimes, the fads and the mixture of normal model.

[Insert Tables 6-8 Here]

To gain a visual understanding of the accuracy of our models, the cumulative difference between forecast errors for the historical average against each of the alternative bubble vNS models for all real estate markets are plotted in Figure 4. These graphs can be used to assess whether the alternative models consistently outperform the historical average benchmark for any particular out-of-sample period. To determine this, the height of the curve at the beginning and end points of the period of interest are compared. If the curve is higher at the end of the segment compared to the beginning then the forecast based on the regime switching model has a lower MSFE than the historical average benchmark during that period. For a model to always outperform the historical average, the slope should be positive for the whole out-of-sample period.

Overall, the path of the cumulative forecast error differences are quite diverse for the indices considered. More specifically, for the IPD, we observe all fundamental bubble models along with the average being in the positive territory for the whole out-of-sample period, experiencing small losses in the aftermath of the financial crisis. They then stabilise and retain their ranking position up to the end of the sample period. For the US FTSE real estate index, the financial crisis period is marked with losses for all the models followed by a quick recovery in 2009. Beyond 2009, all models move similarly with the average ranking higher and on the other hand, the bridge model deteriorating to rank lowest at the end of 2015. Turning to the UK FTSE real estate index, all models behave similarly during the financial crisis showing divergent patterns in the aftermath. Specifically, stepwise vNS followed by the average quickly gain ground and retain their superiority up to the end of the sample, while lasso vNS is for the majority of the out-of-sample period in negative territory showing worse forecasting performance that the historical average. The superior forecasting performance of all fundamental models is apparent in the case of the US house price index, as all models exhibit quick gains during the financial crisis which they manage to retain and increase (upward sloping curve) up to the end of the sample. Although all models move close together, the average bubble ranks first while the bridge one takes the lowest position. Finally, the UK house price index paints a different picture. Similarly, to the US house price index, the financial turmoil benefits all specifications, but soon after our fundamental models form three groups. In the best performing one, associated with consistent forecasting gains over the out-of-sample period, we see lasso and the average bubble vNs model, while elastic net and bridge form the group of worst performing models.

[Insert Figure 4 Here]

5.2. Longer forecasting horizons

Since most investors except portfolio investors need more lead time than one month, we also consider 3-month and 6-month forecasting horizons. Table 9 reports the related findings for all the indices at hand. Overall, the majority of alternative vNS bubble models are superior to the random walk in all cases. More in detail, for the commercial real estate indices, all fundamental vNS models achieve superior forecasting ability relative to the historical average with the bridge vNS achieving the lowest MSFE (0.7181 and 0.6347, for the US and UK respectively). This performance is closely followed by the average fundamental vNS model, which ranks second for the NCREIF index. The normal mixture model also outperforms the historical average, but is associated with inferior forecasts relative to the vNS. Turning to equity real estate indices, we to note that the best model in terms of MSFE is the elastic net fads model for the US (0.9349) and the Bridge fads model for the UK (0.8378). The vNS model ranks second with the average bubble and Bridge bubble model performing best for the US and UK, respectively. Finally, for the residential real estate indices, all vNS specifications rank first and succeed in reducing the random walk MSFE by almost half both for the US and UK. For example, for the Case-Shiller index, the best forecasting model is the elastic net vNS that achieves an MSFE of 0.5989 closely followed by all models with the average just a little over 0.6077. For the UK house price index, the best performing model is the stepwise vNS model (MSFE= 0.6175) followed by ridge, lasso and the average fundamental vNS model.

[Insert Table 9 Here]

Figure 5 plots the cumulative difference between forecast errors for the historical average against each of the alternative bubble vNS models for all real estate markets and the 3-month horizon. In both the US and UK commercial real estate indices, all vNS models appear successful in improving forecasts in the aftermath of the financial crisis. However, for NCREIF all specifications underperform in 2008 followed by sharp gains after 2009 and small losses afterwards. These movements are rather muted for the elastic net bubble model. The best performance is attained by the bridge vNS model followed by the average bubble one. For IPD, all specifications move quite similarly experiencing sharp gains during the financial crisis followed by stabilisation in the aftermath. In contrast, performance of the equity real estate indices is quite diverse among fundamental specifications. Specifically, for the US only the average and elastic net manage to retain gains at the end of the out-of-sample period with bridge showing the worst performance. However, bridge and the average are consistently superior and rank first for the UK FTSE index. Finally, all fundamental models move closely together in the case of the US residential index, while for the UK one fundamental models form two groups with elastic net and bridge in belonging in the worst performing one.

[Insert Figure 5 Here]

Turning to the 6-month horizon, our findings reported in Table 10, suggest that the best forecasting performance is attained for the Case-Shiller Index followed by the IPD UK index. For the Case Shiller index, all fads, normal mixture and vNs models appear significantly more accurate than the random walk, with the vNS ranking first. In this set of models, the one with the lowest MSFE is the Stepwise (MSFE=0.4951), while the average bubble model also proved accurate with a statistically significant MSFE of 0.5488. In the case of IPD, the most accurate model is the Bridge vNS (MSFE=0.5689) followed by the elastic net (MSFE=0.7132). The average bubble vNS fares well with a statistically significant MSFE of 0.7658. On the other hand, all vNS specifications fail to improve upon the historical average model for both the US and UK FTSE indices. In these cases, the fads model beats the historical average model with the average bubble and the ridge bubble fads model ranking first for the US and UK, respectively. Finally, the NCREIF and the UK house price index provide mixed evidence. For the NCREIF, the bridge and elastic net fads model are the best followed by stepwise OLS. For this index, all fundamental bubble vNS models (with the exception of the OLS) offer improvements over the historical average as judged by the Clark-West test.

[Insert Table 13 Here]

The pattern of the forecasting ability of the various bubble vNS models for the 6-month horizon is graphically shown in Figure 6. For this forecasting horizon, overall we get diminished forecasting power for the majority of indices and more divergent behaviour across specifications with the exception of the US residential index. For the US commercial index, all models experience some gains in 2009, followed by sharp losses in 2010, which for the case of stepwise and lasso are smaller and lead to significant improvements over the out-of-sample period. For the UK IPD index, bridge and average are the models benefiting more from the financial crisis compared to the remaining specifications. Finally, two groups of forecasting models can be identified for the UK residential index. In the group of best performing specifications are OLS, stepwise, ridge and lasso. This group experiences gains in 2008 that are mostly retained in the ot-of-sample period, while the worst performing group quickly loses any benefits and continues to underperform the historical average model until the end of the out-of-sample period.

[Insert Figure 6 Here]

6. Conclusions

In this research we confirm the existence of bubbles in real estate markets in the United States and the United Kingdom using advanced statistical models for extracting the fundamental component underpinning these markets. To investigate the bubble dynamics in real estate markets, fundamental models were constructed using several fitting procedures and a wide range of economic variables. For the first time in the literature on real estate bubbles, we examined extracting the fundamental value underpinning commercial, residential and equity real estate markets using stepwise regression, ridge regression, lasso, bridge regression, elastic net and a model averaging those. In all real estate markets, the actual price diverges from the respective fundamental value. The right-side unit root tests showed significant evidence of the presence of periodically collapsing bubbles in all indices. The regime switching model for bubbles was compared to alternative models and the results showed that for the United States and the United Kingdom equity and residential real estate the bubble model is preferable to the alternatives. The out-of-sample analysis reveals that for one period ahead, the van Norden and Schaller model has an excellent forecasting performance for residential real estate markets both in the United Kingdom and the United States, at one, three and six month performance.

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Table 1: Predictors

	United States	United Kingdom
Financial indicators	• S&P 500 Index • US/UK exchange rate • M2 • Effective federal funds rate • 3-month Treasury bill: (secondary market rate) • 5-year Treasury constant maturity rate • 10-year Treasury constant maturity rate • 30-year fixed rate mortgage average	• FTSE All-Share Index • US/UK exchange rate • Retail M4 (or M2) • Official bank rate • 3-month Treasury bill • Generic government 5-year yield • Generic government 10-year yield • Mortgage rate
Price indicators	• Inflation rate (CPI) • London Bullion Market Association (LBMA) gold price • WTI crude oil price • US rent price index	• Inflation rate (RPI) • London Bullion Market Association (LBMA) gold price • IMF Brent crude oil price • UK rent price index
National income and business activity indicators	• Real GDP • Dallas Fed US real personal disposable income index • Industrial production • Housing starts	• Real GDP • Dallas Fed UK real personal disposable income index • Industrial production • Housing starts
Employment and labour market indicators	• Unemployment rate • OECD Labour cost • OECD Labour productivity	• Unemployment rate • OECD Labour cost • OECD Labour productivity

Table 2 The SADF and GSADF test results on the non-fundamental component.

The null hypothesis is that there is a unit root and the alternative that there is explosive behavior. Figures in bold indicate the rejection of the null hypothesis at the respective significance level. The critical values for the SADF and GSADF tests were computed from Monte Carlo simulations with 2000 replications, with the minimum window set to $0.01+1.8/\sqrt{T}$ of the total sample observations. The ADF lag is chosen to minimize the Schwarz Information Criterion with the maximum lag length set to 4 quarters for the NCREIF Property Index and to 12 months for the remaining indices. Sample size: 151 for the NCREIF Property Index, 311 for the US FTSE EPRA/NAREIT Index, 346 for the S&P/Case-Shiller Home Price Index, 348 for the IPD UK Property Index, 312 for the UK FTSE EPRA/NAREIT Index and 251 for the UK House Price Index.

	NCREIF Property Index		US EPRA/NAREIT Index		S&P/Case-Shiller Index	
	SADF	GSADF	SADF	GSADF	SADF	GSADF
Test statistic	5.3090	6.4302	2.9105	2.9986	2.7291	6.9542
90% Critical Value	**1.0845**	**1.7792**	**1.1439**	**1.9219**	**1.1442**	**1.9424**
95% Critical Value	**1.3843**	**2.0663**	**1.4257**	**2.1340**	**1.4350**	**2.1843**
99% Critical Value	**1.9300**	**2.7919**	**1.9585**	**2.6837**	**1.9417**	**2.8751**
	IPD UK Property Index		UK EPRA/NAREIT Index		UK House Price Index	
	SADF	GSADF	SADF	GSADF	SADF	GSADF
Test statistic	3.1161	4.5725	1.2414	2.9815	3.9483	7.0706
90% Critical Value	**1.1471**	**1.9431**	**1.1416**	**1.9179**	**1.1664**	**1.9113**
95% Critical Value	**1.4620**	**2.1877**	1.4027	**2.1460**	**1.4760**	**2.1935**
99% Critical Value	**2.0303**	**2.7542**	2.0189	**2.8789**	**2.0308**	**2.8802**

Table 3. Results from the van Norden and Schaller speculative bubble model for the US and the UK commercial real estate indices.
The coefficients of the van Norden and Schaller model are presented along with p-values in parenthesis.

	NCREIF Property Index							IPD UK Property Index						
Parameters	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net	Average	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net	Average
β_{S0}	1.0188	1.0185	1.0181	1.0182	1.0167	1.0165	1.0175	1.0069	1.0070	1.0069	1.0069	1.0068	1.0072	1.0070
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_{S1}	0.0089	0.0070	0.0027	0.0051	-0.0056	-0.0110	-0.0003	-0.0026	-0.0029	0.0031	0.0052	0.0121	0.0148	0.0097
	(0.3844)	(0.4695)	(0.7632)	(0.5839)	(0.4589)	(0.3842)	(0.9800)	(0.7355)	(0.6625)	(0.6648)	(0.4548)	(0.0002)	(0.0043)	(0.0962)
β_{C0}	0.9459	0.9449	0.9393	0.9433	0.9962	0.9340	0.9390	0.9982	0.9973	0.9981	0.9981	0.9986	0.9977	0.9980
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_{C1}	-0.1604	-0.1622	-0.1611	-0.1586	0.0510	-0.1746	-0.1668	0.0321	0.0411	0.0136	0.0135	-0.0375	-0.0288	-0.0127
	(0.0005)	(0.0003)	(0.0000)	(0.0002)	(0.5702)	(0.0000)	(0.0000)	(0.2424)	(0.1734)	(0.5998)	(0.6000)	(0.0145)	(0.1523)	(0.5815)
β_{q0}	-2.8408	-2.8196	-2.9355	-2.8555	-1.0180	-3.4508	-3.0083	-1.4803	1.1517	-1.4112	1.3888	1.2678	-1.2781	-1.2584
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.2767)	(0.0001)	(0.0000)	(0.0003)	(0.0051)	(0.0004)	(0.0006)	(0.0017)	(0.0007)	(0.0012)
β_{q1}	3.4307	2.9801	3.8308	2.8063	-5.4348	6.5097	3.5989	7.2884	-0.9526	6.1666	-5.4300	-1.8742	3.1424	3.0181
	(0.4451)	(0.4902)	(0.2954)	(0.5233)	(0.2249)	(0.2046)	(0.4573)	(0.0842)	(0.8222)	(0.1166)	(0.1553)	(0.4339)	(0.3184)	(0.3781)
σ_S	0.0150	0.0151	0.0151	0.0151	0.0140	0.0154	0.0152	0.0062	0.0061	0.0061	0.0061	0.0058	0.0059	0.0060
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
σ_C	0.0202	0.0196	0.0184	0.0194	0.0360	0.0124	0.0174	0.0173	0.0171	0.0174	0.0174	0.0162	0.0169	0.0173
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Tests of coefficient restrictions														
$\beta_{S0}\neq\beta_{C0}$	37.0124	41.8808	51.0395	44.0316	1.8871	125.0816	56.3834	10.6412	11.6538	10.5023	10.3259	10.4075	12.3070	10.7871
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.1695)	(0.0000)	(0.0000)	(0.0011)	(0.0006)	(0.0012)	(0.0013)	(0.0013)	(0.0005)	(0.0010)
$\beta_{S1}\neq\beta_{C1}$	12.5608	13.4290	20.0220	13.3886	0.3955	45.5019	19.1911	1.3235	1.8596	0.1340	0.0856	10.1999	4.3316	0.8217
	(0.0004)	(0.0002)	(0.0000)	(0.0003)	(0.5294)	(0.0000)	(0.0000)	(0.2500)	(0.1727)	(0.7143)	(0.7698)	(0.0014)	(0.0374)	(0.3647)
Bubble model specification test against alternative models														
Volatility regime	6.0245	5.8360	5.9050	5.3145	3.4929	5.9956	5.1383	13.1066	10.1463	11.1716	11.1679	28.2891	18.4964	11.3828
	(0.1973)	(0.2117)	(0.2064)	(0.2565)	(0.4790)	(0.1995)	(0.2734)	(0.0108)	(0.0380)	(0.0247)	(0.0247)	(0.0000)	(0.0010)	(0.0226)
Fads	10.2856	10.4791	11.1973	10.2752	8.0119	11.3421	10.6333	20.6783	17.6434	18.4485	17.7409	29.6868	21.4947	17.0178
	(0.0163)	(0.0149)	(0.0107)	(0.0164)	(0.0458)	(0.0100)	(0.0139)	(0.0001)	(0.0005)	(0.0004)	(0.0005)	(0.0000)	(0.0001)	(0.0007)
Mixture-normal	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	12.8417	3.0491	0.0100
	(0.9997)	(0.9997)	(0.9997)	(0.9997)	(0.9997)	(0.9997)	(0.9997)	(0.9997)	(0.9997)	(0.9997)	(0.9997)	(0.0050)	(0.3841)	(0.9997)

Table 4. Results from the van Norden and Schaller speculative bubble model for the US and the UK equity real estate indices.
The coefficients of the van Norden and Schaller model are presented along with p-values in parenthesis.

	US FTSE EPRA/NAREIT Index							UK FTSE EPRA/NAREIT Index						
Parameters	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net	Average	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net	Average
β_{S0}	1.0064	1.0073	1.0051	1.0039	1.0026	1.0034	1.0040	0.9997	0.9998	0.9983	0.9969	0.9896	0.9971	0.9971
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_{S1}	-0.0049	-0.0018	-0.0073	-0.0114	-0.0177	-0.0128	-0.0115	-0.0048	-0.0048	-0.0064	-0.0081	-0.0191	-0.0083	-0.0085
	(0.6552)	(0.8518)	(0.4771)	(0.2826)	(0.0467)	(0.2264)	(0.2977)	(0.5504)	(0.5405)	(0.4163)	(0.3437)	(0.0222)	(0.3264)	(0.3287)
β_{C0}	0.8892	0.9129	0.8002	0.7968	0.8100	0.7900	0.8102	0.8582	0.8887	0.7326	0.7954	0.7540	0.7782	0.7407
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0014)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_{C1}	-0.2197	-0.2032	-0.2710	-0.2910	-0.1464	-0.2864	-0.2705	-0.1708	-0.1462	-0.2616	-0.1990	-0.1418	-0.2202	-0.2430
	(0.1113)	(0.1414)	(0.0333)	(0.0407)	(0.0411)	(0.0240)	(0.0283)	(0.4181)	(0.3583)	(0.2344)	(0.1370)	(0.0078)	(0.1518)	(0.0984)
β_{q0}	3.0088	2.5812	-4.2995	-4.3159	-6.6387	-4.6459	-4.4448	4.0527	3.5410	5.2037	4.5707	-7.2487	4.7347	5.2640
	(0.0000)	(0.0000)	(0.0000)	(0.0001)	(0.0009)	(0.0001)	(0.0000)	(0.0201)	(0.0112)	(0.0014)	(0.0010)	(0.0000)	(0.0011)	(0.0002)
β_{q1}	-1.8062	-0.5839	3.6171	3.7348	6.0346	4.2569	4.1916	-2.1116	-1.5229	-3.0491	-2.6715	4.3614	-2.7937	-3.3972
	(0.1128)	(0.5938)	(0.0038)	(0.0077)	(0.0095)	(0.0047)	(0.0050)	(0.1663)	(0.2075)	(0.0473)	(0.0341)	(0.0006)	(0.0367)	(0.0163)
σ_S	0.0407	0.0407	0.0418	0.0420	0.0427	0.0421	0.0419	0.0503	0.0499	0.0506	0.0493	0.0477	0.0498	0.0497
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
σ_C	0.1247	0.1274	0.1196	0.1184	0.1355	0.1189	0.1199	0.1097	0.1091	0.1012	0.0934	0.0874	0.0956	0.0916
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Tests of coefficient restrictions														
$\beta_{S0} \neq \beta_{C0}$	2.6195	2.8041	3.9578	3.2508	3.1973	3.9697	3.7989	0.6008	0.8302	1.3514	2.1014	5.8611	1.9121	2.5970
	(0.1056)	(0.0940)	(0.0467)	(0.0714)	(0.0738)	(0.0463)	(0.0513)	(0.4383)	(0.3622)	(0.2450)	(0.1472)	(0.0155)	(0.1667)	(0.1071)
$\beta_{S1} \neq \beta_{C1}$	2.4327	2.1340	4.3270	3.9251	3.2147	4.7053	4.4589	0.6213	0.7927	1.3455	2.0333	5.0551	1.9103	2.5266
	(0.1188)	(0.1441)	(0.0375)	(0.0476)	(0.0730)	(0.0301)	(0.0347)	(0.4306)	(0.3733)	(0.2461)	(0.1539)	(0.0246)	(0.1669)	(0.1119)
Bubble model specification test against alternative models														
Volatility regime	4.5484	1.8080	11.6828	12.5982	30.1942	15.3703	14.7110	3.2372	2.9189	5.8300	8.4368	25.6241	7.6889	10.1248
	(0.3368)	(0.7710)	(0.0199)	(0.0134)	(0.0000)	(0.0040)	(0.0053)	(0.5189)	(0.5715)	(0.2122)	(0.0768)	(0.0000)	(0.1037)	(0.0384)
Fads	7.1879	5.0067	13.5061	13.6537	26.5998	15.8503	15.4786	4.0729	3.6249	6.2864	7.8439	22.9221	7.3379	9.8032
	(0.0661)	(0.1713)	(0.0037)	(0.0034)	(0.0000)	(0.0012)	(0.0015)	(0.2537)	(0.3049)	(0.0985)	(0.0494)	(0.0000)	(0.0619)	(0.0203)
Mixture-normal	3.5658	0.8254	10.7002	11.6156	29.2116	14.3877	13.7284	1.4639	1.1456	4.0567	6.6635	23.8508	5.9156	8.3515
	(0.3123)	(0.8434)	(0.0135)	(0.0088)	(0.0000)	(0.0024)	(0.0033)	(0.6906)	(0.7661)	(0.2554)	(0.0834)	(0.0000)	(0.1158)	(0.0393)

Table 5. Results from the van Norden and Schaller speculative bubble model for the US and the UK residential real estate indices.

The coefficients of the van Norden and Schaller model are presented along with p-values in parenthesis.

	S&P/Case-Shiller Home Price Index							UK House Price Index						
Parameters	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net	Average	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net	Average
β_{S0}	1.0042	1.0042	1.0042	1.0042	1.0038	1.0044	1.0039	1.0051	1.0055	1.0052	1.0053	1.0059	1.0056	1.0057
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_{S1}	0.0106	0.0113	0.0118	0.0112	0.0166	0.0139	0.0149	-0.0132	-0.0125	-0.0077	-0.0085	-0.0035	-0.0046	-0.0078
	(0.0021)	(0.0013)	(0.0003)	(0.0009)	(0.0000)	(0.0000)	(0.0000)	(0.0379)	(0.0686)	(0.2019)	(0.1813)	(0.4841)	(0.4002)	(0.2403)
β_{C0}	0.9942	0.9942	0.9949	0.9944	0.9967	0.9956	0.9960	0.9991	0.9991	0.9996	0.9996	0.9975	0.9985	0.9994
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_{C1}	-0.0346	-0.0349	-0.0318	-0.0322	-0.0228	-0.0275	-0.0243	-0.1268	-0.1295	-0.1294	-0.1338	-0.0773	-0.1013	-0.1140
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_{q0}	-0.5549	-0.6088	-0.3381	-0.5343	-0.4052	-0.0426	-0.4632	-0.7556	-0.1297	0.0777	0.0652	0.2084	0.0178	0.2309
	(0.2096)	(0.1710)	(0.4430)	(0.2403)	(0.4049)	(0.9331)	(0.3317)	(0.2728)	(0.8636)	(0.9043)	(0.9276)	(0.7623)	(0.9793)	(0.7339)
β_{q1}	2.6717	2.7929	-0.1604	1.6158	0.0758	-2.3939	0.1247	-12.1367	-14.9164	14.9327	15.2631	9.2006	12.1565	-14.7843
	(0.5244)	(0.5138)	(0.9655)	(0.6969)	(0.9775)	(0.5039)	(0.9651)	(0.0152)	(0.0051)	(0.0011)	(0.0015)	(0.0036)	(0.0017)	(0.0005)
σ_S	0.0042	0.0042	0.0041	0.0042	0.0039	0.0040	0.0040	0.0091	0.0088	0.0089	0.0089	0.0088	0.0089	0.0090
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
σ_C	0.0042	0.0044	0.0046	0.0045	0.0056	0.0050	0.0054	0.0027	0.0045	0.0042	0.0042	0.0047	0.0049	0.0049
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Tests of coefficient restrictions														
$\beta_{S0} \neq \beta_{C0}$	218.7708	190.0803	156.4087	170.2617	48.2574	92.9463	64.2196	21.0044	13.4726	10.7906	10.1537	16.5139	12.3667	9.6589
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0002)	(0.0010)	(0.0014)	(0.0000)	(0.0004)	(0.0019)
$\beta_{S1} \neq \beta_{C1}$	76.9262	68.2720	64.0140	64.0555	50.8936	53.3688	49.7649	75.9723	25.7643	31.7482	26.4138	31.5721	29.3276	22.4570
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Bubble model specification test against alternative models														
Volatility regime	51.8753	47.9361	48.5529	45.4679	46.3767	45.4770	44.5277	21.7529	25.6001	25.1875	25.8914	18.5668	21.3688	25.3115
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0002)	(0.0000)	(0.0000)	(0.0000)	(0.0010)	(0.0003)	(0.0000)
Fads	58.6704	55.5599	56.1470	53.1805	60.1836	53.7911	53.0791	17.6257	21.5197	24.6443	22.5199	23.8871	25.3749	26.8535
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0005)	(0.0001)	(0.0000)	(0.0001)	(0.0000)	(0.0000)	(0.0000)
Mixture-normal	38.0458	34.1067	34.7235	31.6385	32.5473	31.6476	30.6983	10.7870	14.6343	14.2216	14.9256	7.6009	10.4029	14.3457
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0129)	(0.0022)	(0.0026)	(0.0019)	(0.0550)	(0.0154)	(0.0025)

Table 6. MSFE ratios and Clark and West (2007) t-statistics for the commercial real estate indices -1 month horizon.
The first panel reports the MSFE ratios between the historical average benchmark and the volatility regimes, fads, mixture-normal and the vNS bubble models respectively. A below unity ratio indicates that the respective model outperforms the historical average. The second panel reports the MSFE ratios between the van Norden and Schaller model and the alternative regime switching models. A below unity ratio indicates that the bubble model outperforms the respective regime switching model. The figures in parentheses are the t-statistics from the Clark and West (2007) test. Figures in bold indicate the rejection of the null hypothesis at the 10% significance level. The out-of-sample period is set to eight years.

A. Historical average set as the benchmark.

	IPD UK Property Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
Historical Average	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
Volatility regimes model	1.0462	1.0462	1.0462	1.0462	1.0462	1.0462	1.0462
	(-1.2571)	(-1.2571)	(-1.2571)	(-1.2571)	(-1.2571)	(-1.2571)	(-1.2571)
Fads model	1.0321	0.9992	1.0324	1.0202	1.0493	1.0698	1.0319
	(-1.7195)	(0.5254)	(-1.9392)	(-1.5285)	(-2.6045)	(-3.2751)	(-2.4887)
Normal-mixture model	**0.7606**	**0.7606**	**0.7606**	**0.7606**	**0.7606**	**0.7606**	**0.7606**
	(4.1167)	(4.1167)	(4.1167)	(4.1167)	(4.1167)	(4.1167)	(4.1167)
vNS bubble model	**0.7984**	**0.7991**	**0.8382**	**0.8173**	**0.8401**	**0.7898**	**0.8672**
	(3.6898)	(3.5236)	(3.5104)	(3.4117)	(3.9675)	(4.2887)	(3.6808)

B. Van Norden and Schaller model set as the benchmark.

	IPD UK Property Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
vNS bubble model	0.0001	0.0001	0.0002	0.0002	0.0002	0.0001	0.0002
Volatility regimes model	**0.7632**	**0.7638**	**0.8012**	**0.7812**	**0.8031**	**0.7549**	**0.8289**
	(3.4830)	(3.4269)	(3.2986)	(3.3190)	(3.5447)	(3.9338)	(3.3448)
Fads model	**0.7736**	**0.7997**	**0.8119**	**0.8010**	**0.8007**	**0.7383**	**0.8403**
	(3.5773)	(3.4211)	(3.4100)	(3.3577)	(3.7697)	(4.0934)	(3.5798)
Normal-mixture model	1.0497	1.0506	1.1020	1.0744	1.1045	1.0383	1.1401
	(-0.7878)	(-0.7175)	(-2.3504)	(-1.2532)	(-2.2685)	(-0.1573)	(-2.8420)

Table 7. MSFE ratios and Clark and West (2007) t-statistics for the equity real estate indices 1 month horizon.
The first panel reports the MSFE ratios between the historical average benchmark and the volatility regimes, fads, mixture-normal and the vNS bubble models respectively. A below unity ratio indicates that the respective model outperforms the historical average. The second panel reports the MSFE ratios between the van Norden and Schaller model and the alternative regime switching models. A below unity ratio indicates that the bubble model outperforms the respective regime switching model. The figures in parentheses are the t-statistics from the Clark and West (2007) test. Figures in bold indicate the rejection of the null hypothesis at the 10% significance level. The out-of-sample period is set to eight years.

A. Historical average set as the benchmark.

	US FTSE EPRA/NAREIT Index							UK FTSE EPRA/NAREIT Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
Historical Average	0.0062	0.0062	0.0062	0.0062	0.0062	0.0062	0.0062	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044
Volatility regimes model	1.0014	1.0014	1.0014	1.0014	1.0014	1.0014	1.0014	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
	(-0.0590)	(-0.0590)	(-0.0590)	(-0.0590)	(-0.0590)	(-0.0590)	(-0.0590)	(0.1402)	(0.1402)	(0.1402)	(0.1402)	(0.1402)	(0.1402)	(0.1402)
Fads model	0.9969	0.9953	0.9872	0.9879	**0.9846**	**0.9557**	**0.9815**	1.0015	0.9992	1.0032	**0.9862**	**0.9883**	**0.9495**	**0.9917**
	(0.5345)	(0.6754)	(1.2187)	(1.2597)	(1.3823)	(1.9051)	(1.5898)	(-0.1741)	(0.3484)	(-0.5220)	(2.0726)	(1.6339)	(2.8627)	(2.0140)
Normal-mixture model	1.0239	1.0239	1.0239	1.0239	1.0239	1.0239	1.0239	**0.9178**	**0.9178**	**0.9178**	**0.9178**	**0.9178**	**0.9178**	**0.9178**
	(0.0182)	(0.0182)	(0.0182)	(0.0182)	(0.0182)	(0.0182)	(0.0182)	(1.3715)	(1.3715)	(1.3715)	(1.3715)	(1.3715)	(1.3715)	(1.3715)
vNS bubble model	0.9317	**0.9176**	0.9249	0.9291	0.9169	0.9426	0.9069	**0.9825**	**0.9390**	**0.9814**	1.0022	**0.9931**	**0.9751**	**0.9669**
	(1.2486)	(1.2919)	(1.0732)	(1.0534)	(1.1150)	(1.2132)	(1.1902)	(1.5844)	(1.4917)	(1.6810)	(0.2171)	(2.0022)	(2.0128)	(1.4041)

B. Van Norden and Schaller model set as the benchmark.

	US FTSE EPRA/NAREIT Index							UK FTSE EPRA/NAREIT Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
vNS bubble model	0.0058	0.0057	0.0058	0.0058	0.0057	0.0059	0.0056	0.0044	0.0042	0.0043	0.0044	0.0044	0.0043	0.0043
Volatility regimes model	0.9305	**0.9164**	0.9236	0.9278	0.9156	0.9413	0.9057	**0.9827**	**0.9392**	**0.9816**	1.0024	**0.9933**	**0.9753**	**0.9671**
	(1.2705)	(1.3299)	(1.1204)	(1.0976)	(1.1547)	(1.2088)	(1.2334)	(1.4787)	(1.4559)	(1.6237)	(0.2033)	(1.7987)	(2.0614)	(1.3528)
Fads model	0.9346	**0.9220**	0.9369	0.9404	0.9312	0.9864	0.9241	**0.9809**	**0.9397**	**0.9782**	1.0163	1.0049	1.0269	0.9750
	(1.2320)	(1.2828)	(1.0307)	(0.9984)	(1.0619)	(0.8839)	(1.1217)	(1.4027)	(1.4737)	(1.7131)	(-1.5630)	(0.1118)	(-1.5260)	(1.1540)
Normal-mixture model	0.9100	0.8963	0.9033	0.9074	0.8955	0.9206	0.8858	1.0705	1.0231	1.0693	1.0920	1.0821	**1.0624**	1.0535
	(1.1370)	(1.1449)	(0.9934)	(0.9948)	(1.0476)	(1.1165)	(1.0820)	(1.1895)	(1.2049)	(1.2184)	(1.1077)	(1.1574)	(1.2953)	(1.1594)

Table 8. MSFE ratios and Clark and West (2007) t-statistics for the residential real estate indices 1 month horizon.
The first panel reports the MSFE ratios between the historical average benchmark and the volatility regimes, fads, mixture-normal and the vNS bubble models respectively. A below unity ratio indicates that the respective model outperforms the historical average. The second panel reports the MSFE ratios between the van Norden and Schaller model and the alternative regime switching models. A below unity ratio indicates that the bubble model outperforms the respective regime switching model. The figures in parentheses are the t-statistics from the Clark and West (2007) test. Figures in bold indicate the rejection of the null hypothesis at the 10% significance level. The out-of-sample period is set to eight years.

A. Historical average set as the benchmark.

	S&P/Case-Shiller Home Price Index							UK House Price Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
Historical Average	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Volatility regimes model	1.0205	1.0205	1.0205	1.0205	1.0205	1.0205	1.0205	**0.9961**	**0.9961**	**0.9961**	**0.9961**	**0.9961**	**0.9961**	**0.9961**
	(-2.2443)	(-2.2443)	(-2.2443)	(-2.2443)	(-2.2443)	(-2.2443)	(-2.2443)	(2.2354)	(2.2354)	(2.2354)	(2.2354)	(2.2354)	(2.2354)	(2.2354)
Fads model	1.0294	1.0215	1.0184	1.0184	1.0445	1.0569	1.0328	1.0156	1.0268	1.0311	1.0347	1.0129	1.0172	1.0387
	(-3.4020)	(-2.5539)	(-2.4455)	(-2.4782)	(-3.1169)	(-3.4699)	(-3.0841)	(-0.5857)	(-1.2351)	(-1.9521)	(-2.2302)	(-1.2342)	(-1.7342)	(-2.4942)
Normal-mixture model	**0.8184**	**0.8184**	**0.8184**	**0.8184**	**0.8184**	**0.8184**	**0.8184**	1.0050	1.0050	1.0050	1.0050	1.0050	1.0050	1.0050
	(3.3062)	(3.3062)	(3.3062)	(3.3062)	(3.3062)	(3.3062)	(3.3062)	(-0.9123)	(-0.9123)	(-0.9123)	(-0.9123)	(-0.9123)	(-0.9123)	(-0.9123)
vNS bubble model	**0.6424**	**0.6409**	**0.6176**	**0.6354**	**0.6423**	**0.6626**	**0.6079**	**0.7416**	**0.7503**	**0.7312**	**0.6852**	**0.8540**	**0.8351**	**0.6958**
	(5.3701)	(5.4697)	(5.3216)	(5.3637)	(4.9644)	(4.6450)	(5.4104)	(4.2838)	(3.8842)	(4.0720)	(4.5234)	(2.8027)	(2.6092)	(3.8015)

B. Van Norden and Schaller model set as the benchmark.

	S&P/Case-Shiller Home Price Index							UK House Price Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
vNS bubble model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Volatility regimes model	**0.6295**	**0.6280**	**0.6052**	**0.6226**	**0.6294**	**0.6493**	**0.5957**	**0.7445**	**0.7532**	**0.7341**	**0.6879**	**0.8574**	**0.8384**	**0.6986**
	(5.4441)	(5.5419)	(5.3984)	(5.4359)	(5.0473)	(4.7386)	(5.4824)	(4.2610)	(3.8628)	(4.0459)	(4.5029)	(2.7834)	(2.5843)	(3.7871)
Fads model	**0.6241**	**0.6274**	**0.6065**	**0.6239**	**0.6149**	**0.6269**	**0.5885**	**0.7302**	**0.7306**	**0.7091**	**0.6623**	**0.8431**	**0.8210**	**0.6699**
	(5.3565)	(5.3800)	(5.3239)	(5.3571)	(5.1148)	(4.9509)	(5.4607)	(4.4499)	(4.2550)	(4.2805)	(4.7128)	(2.9297)	(2.7303)	(3.9995)
Normal-mixture model	**0.7850**	**0.7831**	**0.7547**	**0.7764**	**0.7848**	**0.8097**	**0.7428**	**0.7378**	**0.7465**	**0.7275**	**0.6818**	**0.8498**	**0.8309**	**0.6923**
	(4.6070)	(4.6536)	(4.6515)	(4.6372)	(4.0624)	(3.7924)	(4.5175)	(4.2777)	(3.8755)	(4.0480)	(4.4926)	(2.8539)	(2.6678)	(3.8103)

Table 9. MSFE ratios and Clark and West (2007) t-statistics for the real estate indices- 3 month horizon.

	NCREIF Property Index							IPD UK Property Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
HistoricalAverage	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014
Volatility regimes model	0.9930	0.9930	0.9930	0.9930	0.9930	0.9930	0.9930	1.0635	1.0635	1.0635	1.0635	1.0635	1.0635	1.0635
	(0.5526)	(0.5526)	(0.5526)	(0.5526)	(0.5526)	(0.5526)	(0.5526)	(-1.2038)	(-1.2038)	(-1.2038)	(-1.2038)	(-1.2038)	(-1.2038)	(-1.2038)
Fads model	1.0264	1.0165	0.9966	1.0059	0.9845	0.9811	0.9993	1.0641	1.0488	1.0614	1.0581	1.0955	1.1330	1.0701
	(-0.3252)	(-0.1479)	(0.3920)	(0.0616)	(1.1374)	(1.2366)	(0.2823)	(-1.3750)	(-1.5190)	(-1.7616)	(-1.8475)	(-2.8762)	(-3.4383)	(-2.6131)
Normal-mixture model	**0.9175**	**0.9175**	**0.9175**	**0.9175**	**0.9175**	**0.9175**	**0.9175**	**0.7675**	**0.7675**	**0.7675**	**0.7675**	**0.7675**	**0.7675**	**0.7675**
	(1.5778)	(1.5778)	(1.5778)	(1.5778)	(1.5778)	(1.5778)	(1.5778)	(3.4933)	(3.4933)	(3.4933)	(3.4933)	(3.4933)	(3.4933)	(3.4933)
vNS bubble model	**0.7414**	**0.7797**	**0.7674**	**0.7657**	**0.9209**	**0.7181**	**0.7259**	**0.6733**	**0.7420**	**0.7569**	**0.6983**	**0.7132**	**0.6347**	**0.7354**
	(1.7668)	(1.7229)	(1.7421)	(1.7539)	(1.6654)	(1.7507)	(1.8234)	(4.1215)	(4.4968)	(4.1575)	(3.7781)	(4.1421)	(4.3549)	(4.0974)
	US FTSE EPRA/NAREIT Index							UK FTSE EPRA/NAREIT Index						
HistoricalAverage	0.0160	0.0160	0.0160	0.0160	0.0160	0.0160	0.0160	0.0139	0.0139	0.0139	0.0139	0.0139	0.0139	0.0139
Volatility regimes model	0.9974	0.9974	0.9974	0.9974	0.9974	0.9974	0.9974	0.9927	0.9927	0.9927	0.9927	0.9927	0.9927	0.9927
	(0.4594)	(0.4594)	(0.4594)	(0.4594)	(0.4594)	(0.4594)	(0.4594)	(1.2321)	(1.2321)	(1.2321)	(1.2321)	(1.2321)	(1.2321)	(1.2321)
Fads model	**0.9834**	**0.9766**	**0.9605**	**0.9472**	**0.9349**	**0.9476**	**0.9424**	**0.9597**	**0.9514**	**0.9648**	**0.9436**	**0.9428**	**0.8378**	**0.9683**
	(1.4476)	(1.9775)	(2.3340)	(2.8474)	(2.7515)	(2.6859)	(2.6620)	(2.8172)	(2.9397)	(2.0129)	(2.8863)	(2.7143)	(4.0156)	(1.8745)
Normal-mixture model	1.1394	1.1394	1.1394	1.1394	1.1394	1.1394	1.1394	**0.9802**	**0.9802**	**0.9802**	**0.9802**	**0.9802**	**0.9802**	**0.9802**
	(-0.3977)	(-0.3977)	(-0.3977)	(-0.3977)	(-0.3977)	(-0.3977)	(-0.3977)	(3.8694)	(3.8694)	(3.8694)	(3.8694)	(3.8694)	(3.8694)	(3.8694)
vNS bubble model	**1.0052**	1.0573	1.0244	1.0126	**0.9886**	1.2666	**0.9820**	**0.9806**	1.0320	1.1126	**0.9104**	**0.9365**	**0.8758**	**0.9023**
	(1.2847)	(1.0491)	(1.2778)	(1.2560)	(1.4228)	(1.1984)	(1.4384)	(1.4512)	(1.0737)	(0.4425)	(2.0172)	(3.9340)	(2.6308)	(3.2333)
	S&P/Case-Shiller Home Price Index							UK House Price Index						
HistoricalAverage	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008
Volatility regimes model	1.0343	1.0343	1.0343	1.0343	1.0343	1.0343	1.0343	1.0319	1.0319	1.0319	1.0319	1.0319	1.0319	1.0319
	(-3.153)	(-3.153)	(-3.1533)	(-3.153)	(-3.153)	(-3.153)	(-3.153)	(-3.3564)	(-3.3564)	(-3.3564)	(-3.3564)	(-3.3564)	(-3.3564)	(-3.3564)
Fads model	1.0303	1.0241	1.0156	1.0135	1.0566	1.0884	1.0307	1.0239	1.0363	1.0253	1.0407	1.0219	1.0193	1.0224
	(-1.8896)	(-1.4370)	(-1.1528)	(-0.8687)	(-3.0618)	(-3.6629)	(-1.9311)	(-1.4583)	(-2.1629)	(-1.6776)	(-2.9089)	(-1.7182)	(-1.9625)	(-1.5865)
Normal-mixture model	**0.7397**	**0.7397**	**0.7397**	**0.7397**	**0.7397**	**0.7397**	**0.7397**	1.0046	1.0046	1.0046	1.0046	1.0046	1.0046	1.0046
	(4.1302)	(4.1302)	(4.1302)	(4.1302)	(4.1302)	(4.1302)	(4.1302)	(-2.0022)	(-2.0022)	(-2.0022)	(-2.0022)	(-2.0022)	(-2.0022)	(-2.0022)
vNS bubble model	**0.6025**	**0.6087**	**0.6048**	**0.6020**	**0.5989**	**0.6150**	**0.6077**	**0.7482**	**0.6175**	**0.6575**	**0.6761**	**0.9284**	**0.9996**	**0.7439**
	(5.6693)	(5.5950)	(5.3509)	(5.4391)	(5.2564)	(5.2443)	(5.3781)	(4.0465)	(4.4968)	(4.7331)	(4.6012)	(3.1015)	(2.4182)	(4.0023)

Table 10. MSFE ratios and Clark and West (2007) t-statistics for the real estate indices- 6 month horizon.

	NCREIF Property Index							IPD UK Property Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
HistoricalAverage	0.0033	0.0033	0.0033	0.0033	0.0033	0.0033	0.0033	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044
Volatility regimes model	0.9927	0.9927	0.9927	0.9927	0.9927	0.9927	0.9927	1.0551	1.0551	1.0551	1.0551	1.0551	1.0551	1.0551
	(0.5630)	(0.5630)	(0.5630)	(0.5630)	(0.5630)	(0.5630)	(0.5630)	(-0.7260)	(-0.7260)	(-0.7260)	(-0.7260)	(-0.7260)	(-0.7260)	(-0.7260)
Fads model	1.0268	1.0124	0.9989	1.0061	**0.9422**	**0.9374**	1.0000	1.0548	1.0414	1.0257	1.0452	1.0379	1.1543	1.0029
	(-0.0856)	(0.1530)	(0.4131)	(0.2148)	(2.2199)	(2.6104)	(0.3692)	(-0.0457)	(-0.2217)	(0.1048)	(-0.6090)	(-0.3326)	(-2.7337)	(1.0107)
Normal-mixture model	**0.9609**	**0.9609**	**0.9609**	**0.9609**	**0.9609**	**0.9609**	**0.9609**	**0.9120**	**0.9120**	**0.9120**	**0.9120**	**0.9120**	**0.9120**	**0.9120**
	(1.3390)	(1.3390)	(1.3390)	(1.3390)	(1.3390)	(1.3390)	(1.3390)	(2.2890)	(2.2890)	(2.2890)	(2.2890)	(2.2890)	(2.2890)	(2.2890)
vNS bubble model	1.0643	**0.9550**	**1.0985**	**0.9805**	**1.1076**	**1.1186**	**1.0502**	**0.7840**	**0.7815**	**0.7530**	**0.7546**	**0.7132**	**0.5689**	**0.7658**
	(1.0718)	(1.5859)	(1.5097)	(1.4848)	(1.7334)	(1.6191)	(1.6353)	(2.9586)	(3.5745)	(3.8847)	(4.0102)	(3.5106)	(3.9689)	(4.5953)
	US FTSE EPRA/NAREIT Index							UK FTSE EPRA/NAREIT Index						
HistoricalAverage	0.0372	0.0372	0.0372	0.0372	0.0372	0.0372	0.0372	0.0297	0.0297	0.0297	0.0297	0.0297	0.0297	0.0297
Volatility regimes model	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0011	1.0011	1.0011	1.0011	1.0011	1.0011	1.0011
	(0.0882)	(0.0882)	(0.0882)	(0.0882)	(0.0882)	(0.0882)	(0.0882)	(-0.1547)	(-0.1547)	(-0.1547)	(-0.1547)	(-0.1547)	(-0.1547)	(-0.1547)
Fads model	0.9907	**0.9885**	**0.9762**	**0.9724**	**0.9709**	**1.1553**	**0.9660**	**0.9722**	**0.9731**	**0.9719**	**0.9817**	**0.9677**	**0.9490**	**0.9896**
	(1.2491)	(1.5471)	(2.1842)	(2.3441)	(2.3337)	(2.7924)	(2.4995)	(2.5971)	(2.6878)	(2.4150)	(1.5209)	(2.3381)	(2.2633)	(1.8501)
Normal-mixture model	1.3093	1.3093	1.3093	1.3093	1.3093	1.3093	1.3093	**0.9907**	**0.9907**	**0.9907**	**0.9907**	**0.9907**	**0.9907**	**0.9907**
	(-1.5283)	(-1.5283)	(-1.5283)	(-1.5283)	(-1.5283)	(-1.5283)	(-1.5283)	(2.0716)	(2.0716)	(2.0716)	(2.0716)	(2.0716)	(2.0716)	(2.0716)
vNS bubble model	1.8717	1.7965	1.8768	1.7959	1.7922	1.8334	1.7534	1.0630	1.1848	1.0416	1.0188	1.0247	1.2123	1.1353
	(-1.1075)	(-0.9840)	(-0.9747)	(-0.6848)	(-0.8389)	(-0.5541)	(-0.8372)	(0.6969)	(0.5675)	(0.9367)	(0.6848)	(0.4449)	(0.5747)	(-0.7401)
	S&P/Case-Shiller Home Price Index							UK House Price Index						
HistoricalAverage	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020
Volatility regimes model	1.0034	1.0034	1.0034	1.0034	1.0034	1.0034	1.0034	1.0366	1.0366	1.0366	1.0366	1.0366	1.0366	1.0366
	(0.2716)	(0.2716)	(0.2716)	(0.2716)	(0.2716)	(0.2716)	(0.2716)	(-5.3407)	(-5.3407)	(-5.3407)	(-5.3407)	(-5.3407)	(-5.3407)	(-5.3407)
Fads model	**0.9180**	**0.9101**	**0.8980**	**0.9096**	**0.9059**	**0.9683**	**0.9039**	**0.9866**	0.9982	1.0084	1.0003	1.0172	1.0183	1.0124
	(5.4353)	(5.4666)	(7.1082)	(5.5138)	(4.6663)	(2.1240)	(5.5595)	(1.7551)	(0.6764)	(-0.2153)	(0.4340)	(-0.7367)	(-1.0012)	(-0.3375)
Normal-mixture model	**0.7381**	**0.7381**	**0.7381**	**0.7381**	**0.7381**	**0.7381**	**0.7381**	1.3690	1.3690	1.3690	1.3690	1.3690	1.3690	1.3690
	(4.2411)	(4.2411)	(4.2411)	(4.2411)	(4.2411)	(4.2411)	(4.2411)	(1.2160)	(1.2160)	(1.2160)	(1.2160)	(1.2160)	(1.2160)	(1.2160)
vNS bubble model	**0.5036**	**0.4951**	**0.5388**	**0.5136**	**0.5506**	**0.5545**	**0.5488**	**0.5977**	**0.7013**	**0.7736**	**0.8164**	**1.3968**	**1.3352**	**1.1736**
	(7.5880)	(7.1996)	(7.0621)	(7.2244)	(7.2499)	(7.2342)	(7.0663)	(4.2701)	(3.9288)	(3.5832)	(3.6796)	(2.1236)	(2.2183)	(2.9174)

Figure 1. Actual price, average fundamental value and the average relative bubble size of the real estate indices.

The shaded areas indicate the periods of under- or overvaluation, the dashed line is the fundamental price and the full line is the actual price.

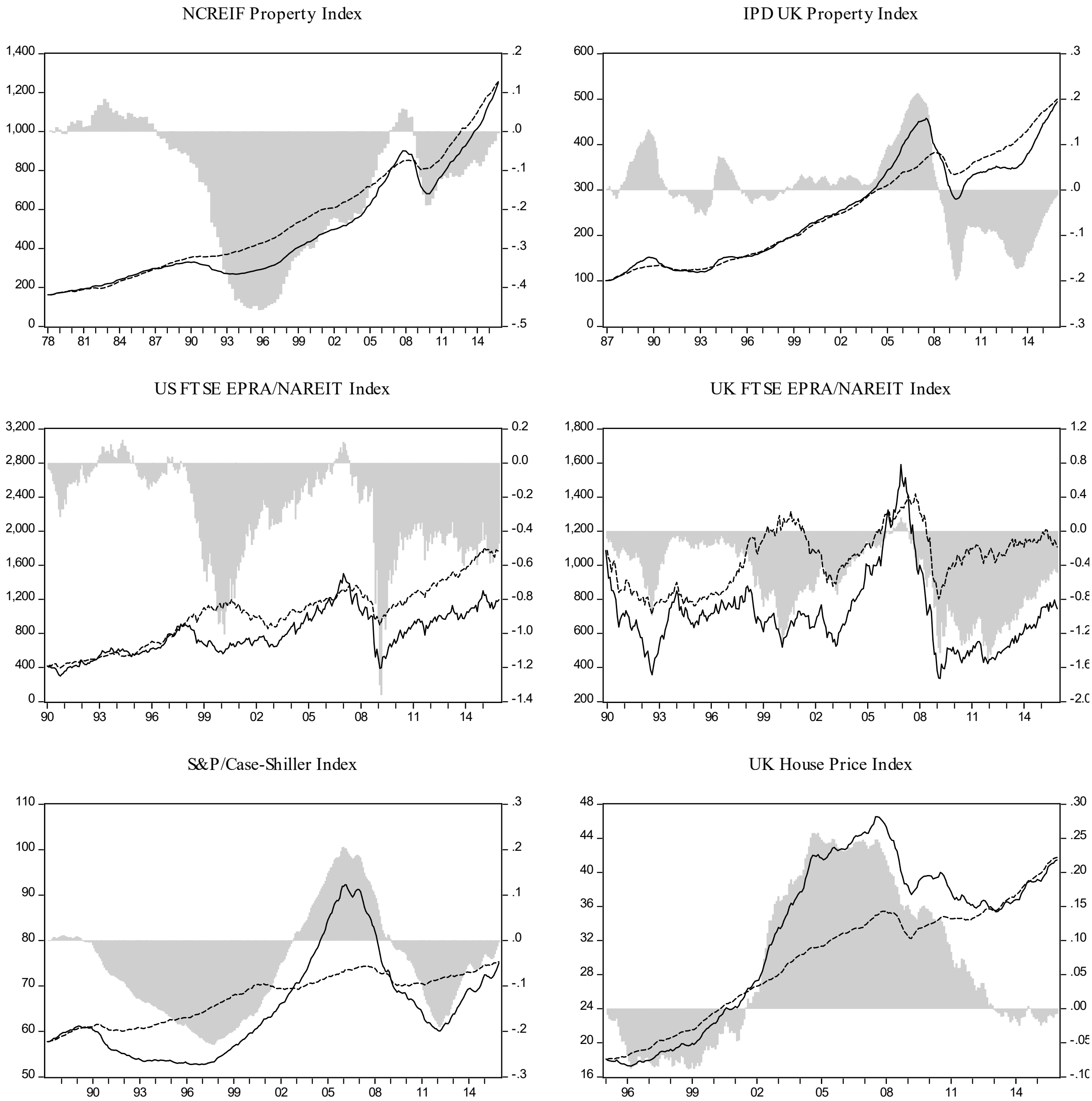


Figure 2. Date stamping the periods of explosiveness in the non-fundamental component of real estate indices.

The shaded areas indicate the bubble periods, the dashed line is the 95% critical value sequence and the full line is the BSADF test statistic sequence. The bubble periods are identified using the BSADF test based on Monte Carlo simulations with 2000 replications, with the minimum window set to $0.01+1.8/\sqrt{T}$ of the total sample observations. Sample size: 151 for the NCREIF Property Index, 311 for the US FTSE EPRA/NAREIT Index, 346 for the S&P/Case-Shiller Home Price Index, 348 for the IPD UK Property Index, 312 for the UK FTSE EPRA/NAREIT Index and 251 for the UK House Price Index.

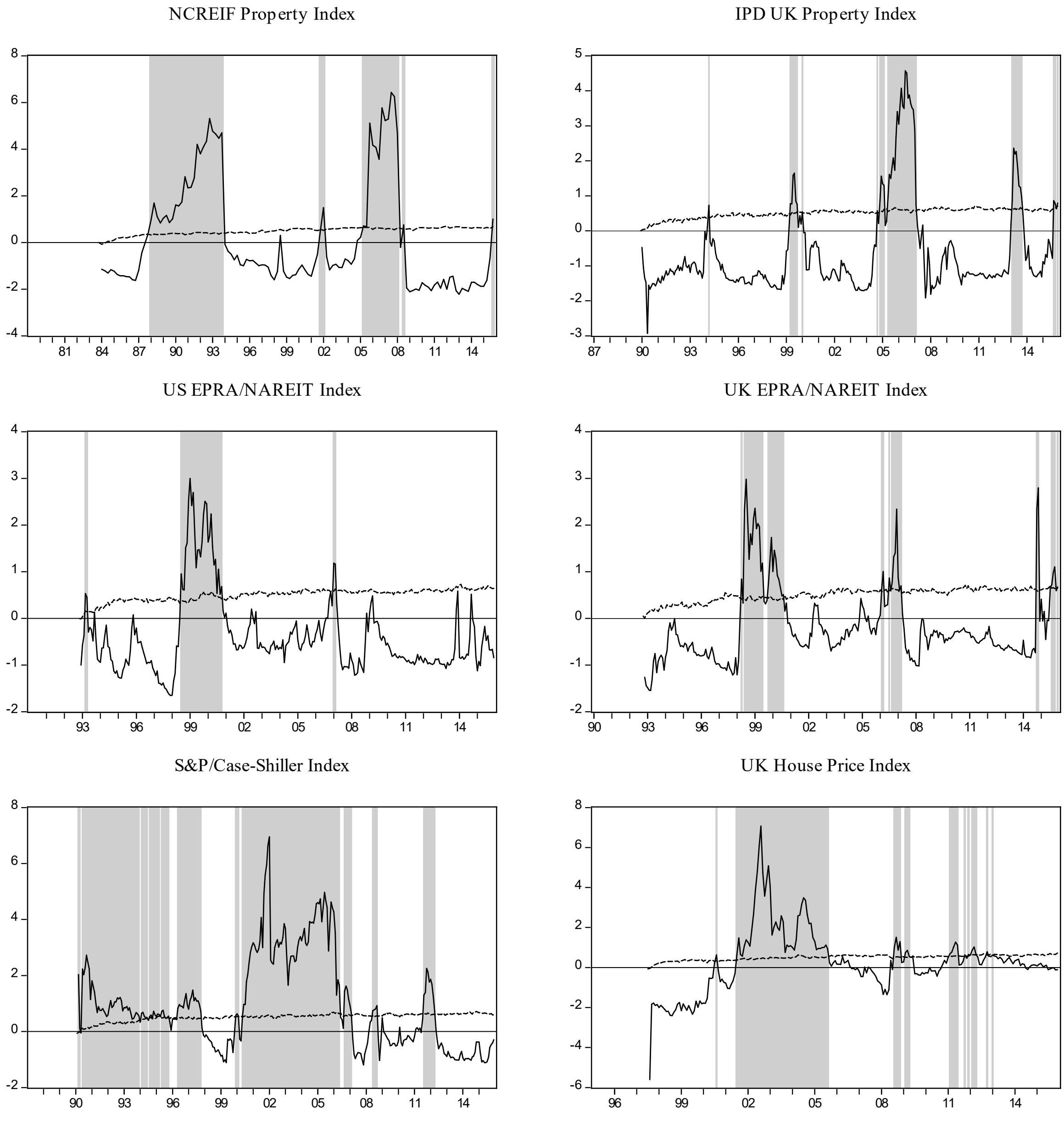


Figure 3. Estimated probability of collapse for all real estate sectors based on the average bubble size.

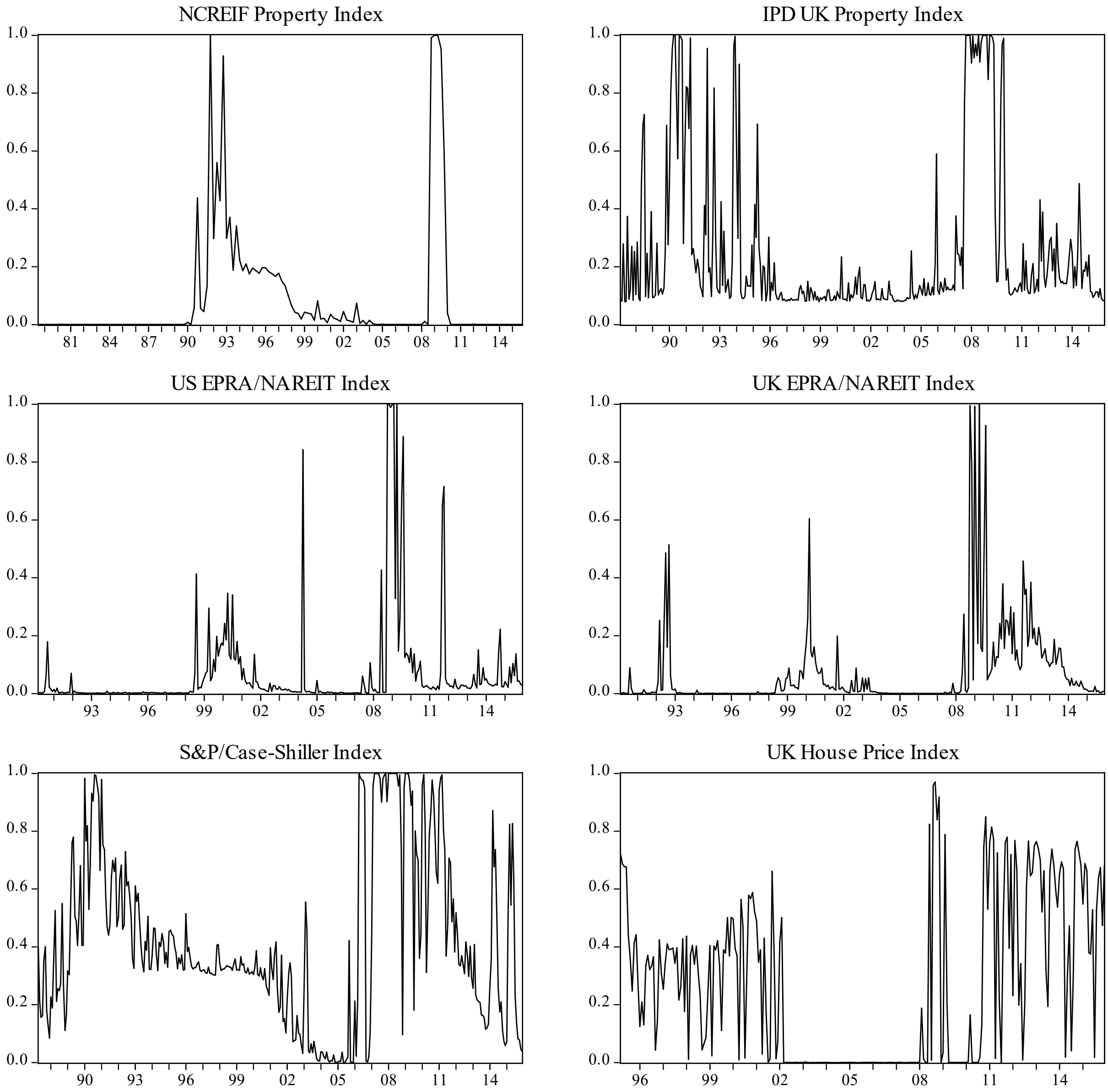


Figure 4. The cumulative difference between forecast errors for the historical average against the vNS regime switching model based on different fundamental specifications, 1 month ahead.

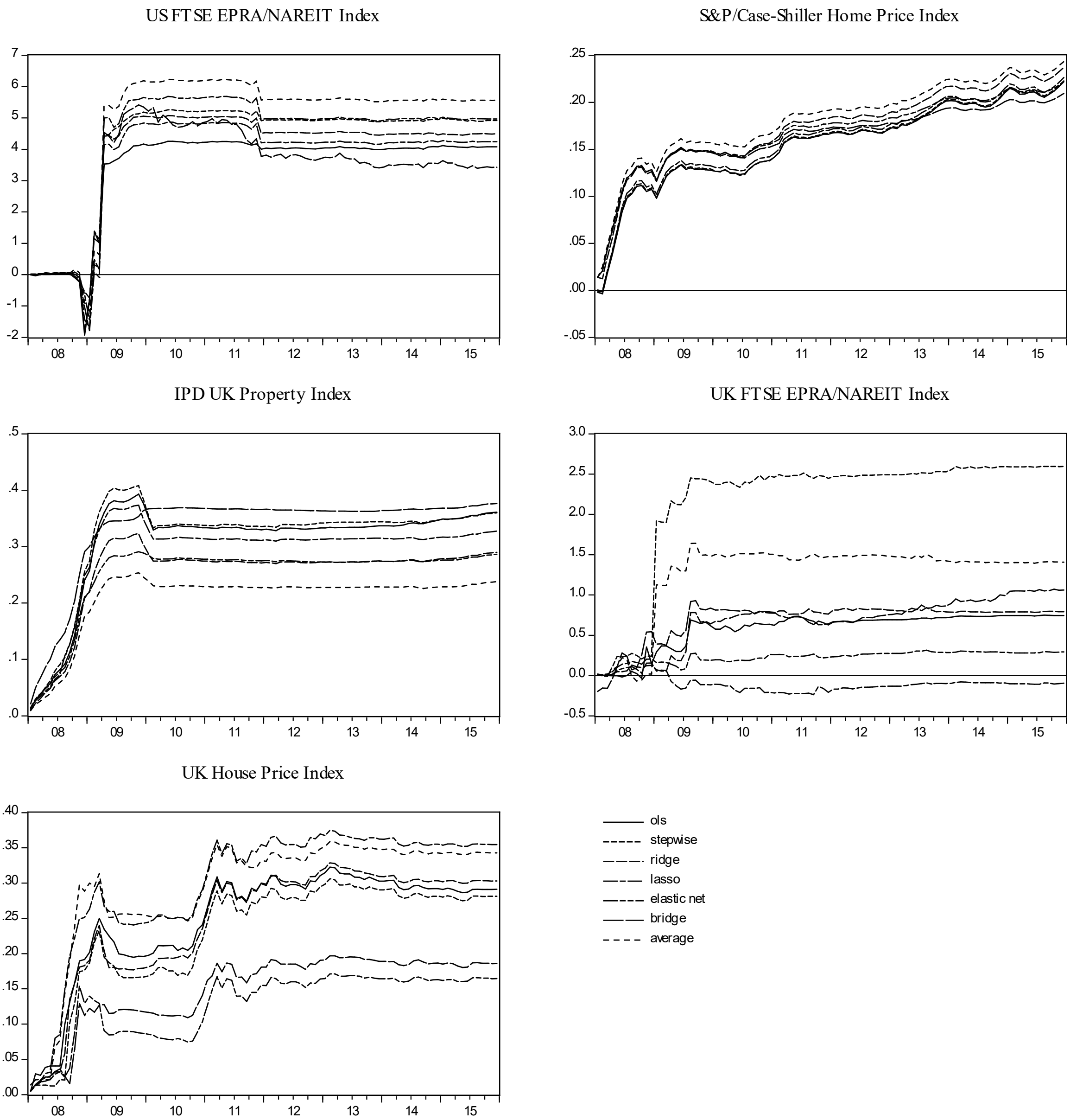


Figure 5. The cumulative difference between forecast errors for the historical average against the vNS regime switching model based on different fundamental specifications, 3 months ahead.

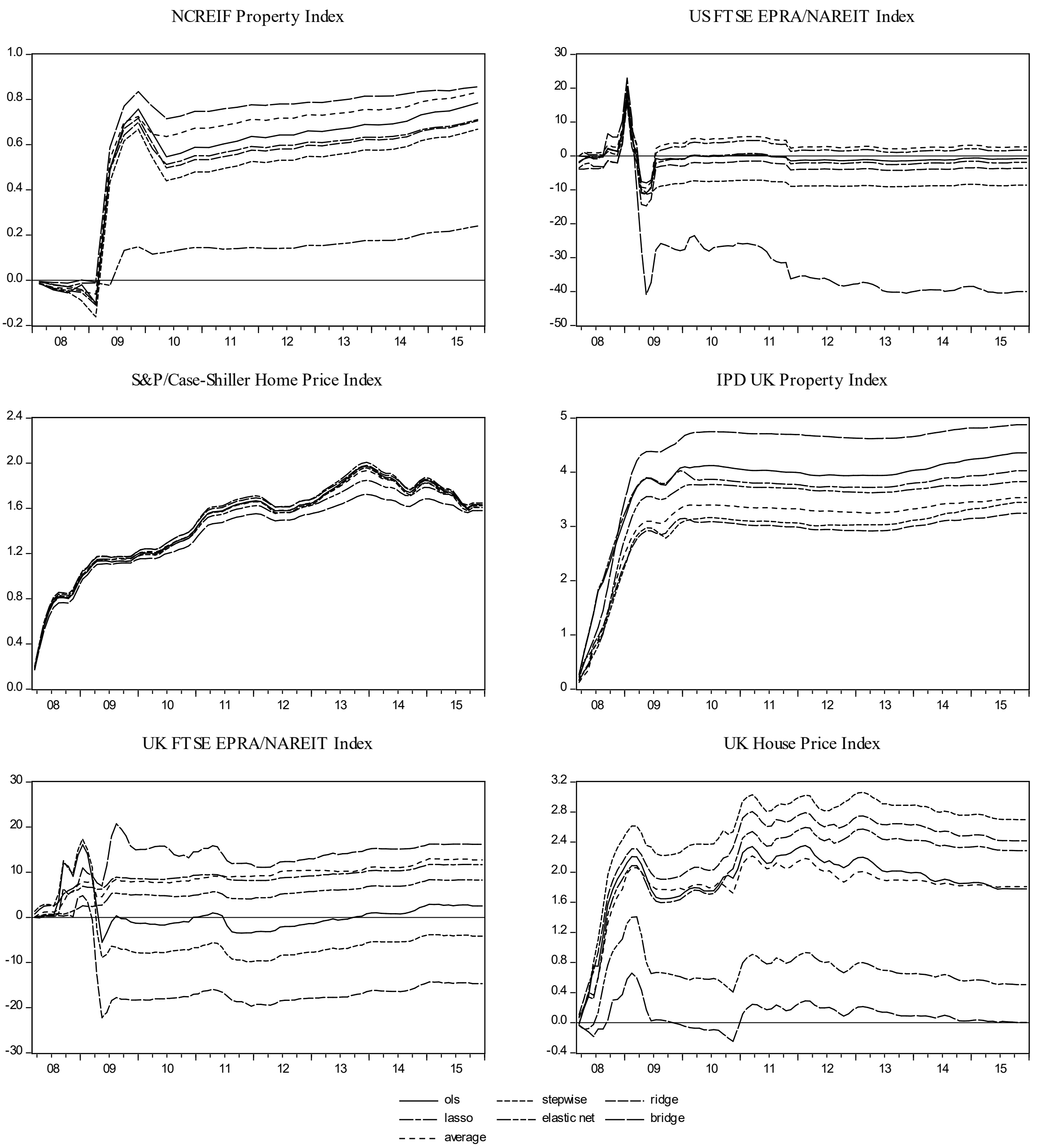
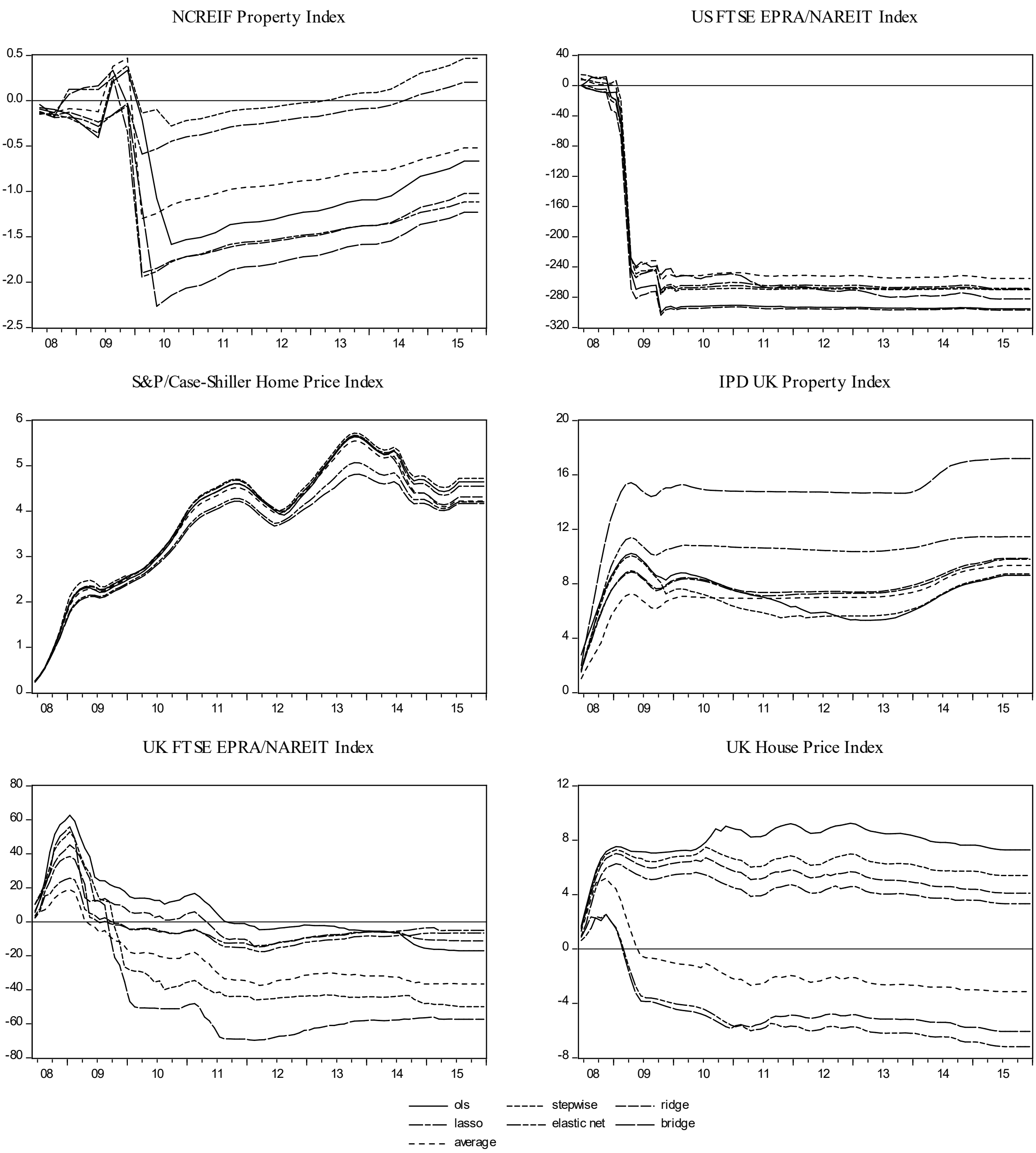


Figure 6. The cumulative difference between forecast errors for the historical average against the vNS regime switching model based on different fundamental specifications, 6 months ahead.



Appendices

Appendix A: The PWY (Phillips et al., 2011) and the PSY (Phillips et al., 2015) tests

The PWY and PSY tests are based on the assumption that asset prices follow a random walk process with an asymptotically negligible drift:

$$y_t = dT^{-\eta} + \theta y_{t-1} + e_t, \qquad e_t \sim iid\ N(0, \sigma^2) \tag{1}$$

where d is a constant, T is the sample size, $\eta > 1/2$ is a localizing coefficient that controls for the magnitude of the drift as the sample size approaches infinity and e_t is the error term. PWY sets $\eta \rightarrow \infty$ and PSY set d, η and θ to unity. Strong upward departures from fundamental values lead the asset price time series to follow an explosive process.

The econometric implementation is based on the ADF test and the use of recursive regressions with variable window widths. This test is applied to each time series y_t to test for a unit root against the alternative of an explosive root. By defining the window's start and end points as r_1 and r_2 respectively, the empirical regression model is specified as:

$$\Delta y_t = a_{r_1,r_2} + \beta_{r_1,r_2} y_{t-1} + \sum_{i=1}^{k} \psi_{r_1,r_2}^{i} \Delta y_{t-i} + \varepsilon_t \tag{2}$$

where y_t can be either a price-to-income ratio or the non-fundamental component, a_{r_1,r_2} is the intercept, k is the maximum number of lags and $\varepsilon_t \sim iid\ N(0, \sigma_{r_1,r_2}^2)$. The sample interval is $[0,1]$ after normalizing the original sample by T and the number of observations in each recursive regression is $T_w = \lfloor T r_w \rfloor$, where $r_w = r_2 - r_1$ is the fractional window size of the regression. The ADF t-statistic that is used is: $ADF_{r_1}^{r_2} = \frac{\beta_{r_1,r_2}}{SE(\beta_{r_1,r_2})}$, where β_{r_1,r_2} and $ADF_{r_1}^{r_2}$ are the regression coefficient and its corresponding ADF t-statistic over the sample $[r_1, r_2]$,respectively.

The SADF test is based on calculating the ADF statistic in each recursive regression performed on a forward expanding sample window. The starting point r_1 of the estimation window remains fixed for all recursive regressions and is the first observation of the sample. The end point r_2 of the first estimation window is set according to some choice of minimum window size r_0 required for the adequate initial estimation of equation (2). Therefore, the first regression involves $T_0 = \lfloor Tr_0 \rfloor$ observations for a minimum fraction, r_0, of the total sample. Each subsequent regression increments the initial fraction of the sample by one observation, giving a forward expanding window size $r_2 \in [r_0, 1]$. The ADF statistic is calculated for each recursive regression and is denoted by ADF_{r_2}. The SADF test statistic is defined as the supremum value of ADF_{r_2} for $r_2 \in [r_0, 1]$:

$$SADF(r_0) = \sup_{r_2 \in [r_0,1]} \left\{ADF_{r_2}\right\} \tag{3}$$

The GSADF test generalizes the SADF test by having more flexible estimation window widths and by allowing the starting point r_1 to change within the range $[0, r_2 - r_0]$ for each regression. The GSADF test statistic is defined as the supremum value of $ADF_{r_1}^{r_2}$ for $r_1 \in [0, r_2 - r_0]$ and $r_2 \in [r_0, 1]$:

$$GSADF(r_0) = \sup_{\substack{r_2 \in [r_0,1] \\ r_1 \in [0, r_2 - r_0]}} \left\{ADF_{r_1}^{r_2}\right\} \tag{4}$$

The SADF and GSADF tests can also be used to date stamp the origination and collapse of the bubbles in a time series. The date stamping procedure of the SADF test compares each ADF_{r_2} statistic to each respective right-side critical value of the standard ADF statistic to identify whether a bubble exists at time Tr_2. The origination date of a bubble, Tr_e, where r_e is the fractional estimate of the beginning of the bubble period, is determined as the time point when the ADF_{r_2} sequence crosses its respective critical value sequence from below. The collapse date of the bubble, Tr_f, where r_f is the fractional estimate of the end of the bubble period, is marked when the ADF_{r_2} sequence crosses its respective critical value sequence from above. The fractional origin and collapse points of the bubble for the SADF test are denoted as:

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \left\{ r_2 : ADF_{r_2} > cv_{r_2}^{\beta_T} \right\} ,$$

$$\hat{r}_f = \inf_{r_2 \in [\hat{r}_e, 1]} \left\{ r_2 : ADF_{r_2} < cv_{r_2}^{\beta_T} \right\} \tag{5}$$

where $cv_{r_2}^{\beta_T}$ is the $100(1-\beta_T)\%$ critical value of the limit distribution of the standard ADF statistic based on $\lfloor Tr_2 \rfloor$ sample observations and β_T is the size of the one sided test.

The date stamping procedure for the GSADF test is based on calculating a sup ADF statistic on backward expanding samples, with fixed ending points at r_2 and varying starting points $r_1 = [0, r_2 - r_0]$. The backward SADF statistic is defined as:

$$BSADF_{r_2}(r_0) = \sup_{r_1 = [0, r_2 - r_0]} \left\{ ADF_{r_1}^{r_2} \right\} \tag{6}$$

Similarly to the SADF date stamping procedure, the fractional origin and collapse points of the bubble for the GSADF test are denoted as:

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \left\{ r_2 : BSADF_{r_2}(r_0) > cv_{r_2}^{\beta_T} \right\}$$

$$\hat{r}_f = \inf_{r_2 \in [\hat{r}_e, 1]} \left\{ r_2 : BSADF_{r_2}(r_0) < cv_{r_2}^{\beta_T} \right\} \tag{7}$$

where $cv_{r_2}^{\beta_T}$ is the $100(1-\beta_T)\%$ critical value of the limit distribution of the standard ADF statistic based on $\lfloor Tr_2 \rfloor$ sample observations.

Appendix B: Tables

Table B1. The ADF test results for the log real price of the real estate indices.

The null hypothesis is that there is a unit root and the alternative that the series are stationary. Figures in bold indicate the rejection of the null hypothesis at the respective significance level. The ADF lag is chosen to minimize the Schwarz Information Criterion with the maximum lag length set to 4 quarters for the NCREIF Property Index and to 12 months for the US FTSE EPRA/NAREIT Index, the S&P/Case-Shiller Home Price Index, the IPD UK Property Index, the UK FTSE EPRA/NAREIT Index and the UK House Price Index.

	NCREIF Property Index	U.S. EPRA/NAREIT Index	Case-Shiller Index
ADF statistic	-0.0072	-2.2872	-0.8251
90% Critical Value	**-2.5771**	**-2.5717**	**-2.5712**
95% Critical Value	**-2.8807**	**-2.8707**	**-2.8697**
99% Critical Value	**-3.4743**	**-3.4514**	**-3.4491**
	IPD UK Index	UK EPRA/NAREIT Index	UK House Price Index
ADF statistic	-0.9260	-2.3257	-2.8640
90% Critical Value	**-2.5712**	**-2.5717**	**-2.5732**
95% Critical Value	**-2.8697**	**-2.8706**	**-2.8734**
99% Critical Value	**-3.4491**	**-3.4512**	**-3.4576**

Table B2. Sources of the potential determinant variables

Variable name	US variables	UK variables
Stock market index	Bloomberg (SPX Index)	Bloomberg (ASX Index)
Exchange rate	FRED (EXUSUK)	BoE (XUMAGBD)
Money supply	FRED (M2SL)	BoE (LPMVQWK)
Central bank rate	FRED (FEDFUNDS)	BoE (IUMABEDR)
3-month Treasury Bill	FRED (TB3MS)	BoE (IUMAAJNB)
5-year Govt bond yield	FRED (GS5)	BoE (IUMASNZC)
10-year Govt bond yield	FRED (GS10)	BoE (IUMAMNZC)
Mortgage rate	FRED (MORTGAGE30US)	BoE (CFMHSDE) and Three centuries of data version 2.3 dataset.
Inflation rate	Bloomberg (CPURNSA% Index)	Bloomberg (UKRPMOM Index)
Gold price	Bloomberg (GOLDLNAM Index)	Bloomberg (GOLDBPAM Index)
Oil price	FRED (OILPRICE and POILWTIUSDM)	Bloomberg (WRCOBREN Index)
Rent price index	OECD (2016), "Prices: Analytical house price indicators".	OECD (2016), "Prices: Analytical house price indicators".
GDP	Bloomberg (GDP CHWG Index)	Bloomberg (UKGRABMI Index)
Disposable income	Bloomberg (DDIRU.S. Index)	Bloomberg (DDIRGB Index)
Industrial production	Bloomberg (IP Index)	Bloomberg (UKIPI Index)
Housing starts	FRED (HOUST)	Department for Communities and Local Government
Unemployment rate	Bloomberg (USURTOT Index)	Bloomberg (UKUEILOR Index)
Labor cost	Bloomberg (EOUSU001 Index)	Bloomberg (EOUKU001 Index)
Labor productivity	Bloomberg (EOUSD007 Index)	Bloomberg (EOUKD007 Index)

Table B3. Fundamental model estimation results for the US and the UK commercial real estate indices.
In-sample fitting results for our six models, using the set of economic covariates and the indicative commercial property indexes, NCREIF for US, and IPD for UK.

	NCREIF Property Index						IPD UK Property Index					
	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net
Intercept	-2.25E-03	-7.06E-04	5.13E-04	3.99E-04	1.38E-02	6.60E-03	2.14E-03	2.49E-03	2.50E-03	2.11E-03	4.65E-03	2.97E-03
Stock market index	1.46E-02	-	1.49E-02	8.94E-03	1.15E-07	2.76E-03	1.23E-02	-	1.27E-02	5.26E-03	1.21E-07	0
Exchange rate	4.97E-02	4.95E-02	3.80E-02	3.65E-02	2.12E-07	1.31E-02	7.91E-03	-	4.53E-03	0	-2.28E-07	0
Money supply	9.20E-01	8.38E-01	6.64E-01	7.23E-01	1.69E-06	3.31E-01	3.79E-02	-	4.56E-02	0	5.90E-07	0
Central bank rate	4.74E-03	4.01E-03	2.11E-03	2.44E-03	1.46E-08	3.13E-04	2.66E-04	-	2.70E-03	5.41E-04	5.93E-08	2.11E-03
3-month Treasury Bill	-2.64E-03	-	9.48E-04	0	1.73E-08	0	7.73E-03	6.46E-03	4.74E-03	5.14E-03	5.47E-08	2.27E-03
5-year Govt bond yield	1.17E-02	-	-2.12E-03	0	2.40E-09	0	-2.48E-03	-	-4.34E-05	0	3.39E-08	0
10-year Govt bond yield	-2.28E-02	-1.37E-02	-5.65E-03	-7.64E-03	-4.58E-09	-1.54E-03	2.51E-04	-	-1.10E-03	0	2.02E-08	0
Mortgage rate	6.56E-03	9.56E-03	4.05E-03	3.48E-03	6.69E-09	0	-1.14E-03	-	-9.18E-04	0	2.76E-08	0
Inflation rate	-1.14E-02	-1.08E-02	-9.91E-03	-1.09E-02	-4.38E-08	-6.70E-03	-4.55E-03	-4.79E-03	-3.90E-03	-3.85E-03	-1.75E-08	-1.36E-03
Gold price	4.15E-03	-	4.23E-03	0	-6.20E-08	0	-7.14E-03	-	-5.72E-03	0	-1.21E-07	0
Oil price	-5.82E-04	-	-4.36E-03	0	-6.48E-08	-2.58E-04	-2.26E-03	-	-2.71E-03	0	-3.36E-08	0
Rent price index	-6.71E-01	-6.76E-01	-4.53E-01	-4.54E-01	-7.41E-07	-1.26E-01	-5.90E-01	-5.28E-01	-5.29E-01	-3.92E-01	-3.34E-06	-1.48E-01
GDP	-3.19E-01	-	8.40E-02	0	3.09E-06	1.91E-02	1.96E+00	2.24E+00	1.51E+00	2.17E+00	1.18E-05	9.65E-01
Disposable income	3.36E-01	3.75E-01	3.33E-01	2.61E-01	4.06E-06	2.27E-01	1.19E-01	-	1.10E-01	0.0080219	1.61E-06	0
Industrial production	1.45E-01	-	1.73E-01	1.84E-02	2.28E-06	1.40E-01	1.07E-01	1.15E-01	9.71E-02	6.93E-02	1.03E-06	2.40E-02
Housing starts	-1.76E-02	-	-9.89E-03	-1.11E-02	1.40E-07	0	-2.85E-03	-	-1.84E-03	0	6.00E-08	0
Unemployment rate	-3.65E-02	-3.56E-02	-2.46E-02	-3.42E-02	-1.26E-07	-1.72E-02	-1.37E-02	-1.16E-02	-1.53E-02	-9.25E-03	-1.61E-07	-9.10E-03
Labor cost	1.72E-01	-	1.52E-01	1.59E-01	1.22E-06	8.78E-02	9.89E-02	-	4.86E-02	0	-1.76E-06	0
Labor productivity	2.18E-01	-	-2.99E-01	0	-3.82E-07	-7.15E-02	4.13E-01	-	6.71E-01	0	1.00E-05	4.49E-01
Lambda	-	-	0.0031	0.0004	100.0000	0.0100	-	-	0.0016	0.0004	100.0000	0.0100
Alpha	-	-	-	-	-	0.1000	-	-	-	-	-	0.1000
Gamma	-	-	-	-	1.1000	-	-	-	-	-	1.1000	-

Table B4. Fundamental model estimation results for the US and the UK equity real estate indices.
In-sample fitting results for our six models, using the set of economic covariates and the indicative equity property indexes FTSE EPRA/NAREIT for US, and for UK.

	U.S. FTSE EPRA/NAREIT Index						UK FTSE EPRA/NAREIT Index					
	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net
Constant	-9.82E-04	-9.70E-04	-2.80E-04	-5.05E-07	5.02E-03	3.86E-04	-1.37E-03	9.89E-04	-2.81E-03	-6.25E-04	4.50E-04	-6.53E-04
Stock market index	7.28E-01	7.56E-01	6.27E-01	6.48E-01	2.65E-06	5.97E-01	8.17E-01	8.18E-01	7.13E-01	7.58E-01	2.94E-06	6.90E-01
Exchange rate	3.91E-02	-	3.17E-02	0	1.01E-06	0	-8.09E-02	-	-5.58E-02	0	1.06E-07	0
Money supply	-4.62E-01	-	-4.61E-01	0	-5.09E-06	0	5.80E-01	-	4.46E-01	0	2.45E-06	0.0111648
Central bank rate	3.84E-02	4.25E-02	2.39E-02	7.717E-05	1.02E-07	1.37E-04	8.26E-03	-	2.88E-03	0	2.51E-08	0
3-month Treasury Bill	-4.32E-02	-4.10E-02	-5.00E-03	0	2.53E-08	0	-1.99E-02	-	-7.08E-03	0	-1.47E-08	0
5-year Govt bond yield	1.45E-01	1.37E-01	1.79E-02	0	-1.04E-07	0	9.34E-03	-	-1.13E-02	0	-1.58E-07	-4.87E-03
10-year Govt bond yield	-1.37E-01	-1.26E-01	-3.04E-02	0	-1.66E-07	-3.42E-03	-4.90E-02	-4.25E-02	-2.94E-02	-1.81E-02	-2.07E-07	-2.42E-02
Mortgage rate	-6.75E-02	-7.22E-02	-4.08E-02	-3.30E-02	-2.42E-07	-2.75E-02	3.39E-02	3.04E-02	2.47E-02	6.15E-03	9.85E-08	1.22E-02
Inflation rate	-2.14E-03	-	1.42E-03	0	1.87E-09	0	-4.13E-03	-	-3.22E-03	0	-2.60E-09	0
Gold price	6.40E-02	-	4.19E-02	0	2.69E-07	0	4.22E-02	-	2.81E-02	0	1.34E-07	0
Oil price	-2.09E-02	-	-3.06E-02	0	-1.56E-07	0	1.63E-02	-	1.23E-02	0	2.31E-08	0
Rent price index	8.17E+00	-	6.96E+00	0	5.92E-05	0	-2.76E+00	-2.56E+00	-2.32E+00	-1.06E+00	-1.09E-05	-1.51E+00
GDP	-8.20E-02	-	6.62E-01	0	1.43E-05	0	-1.05E+00	-	4.15E-01	7.09E-01	1.86E-05	8.57E-01
Disposable income	-3.68E-01	-	2.27E-01	0	3.30E-06	0	2.22E-01	-	3.10E-01	0	1.68E-06	0
Industrial production	-6.23E-02	-	-2.65E-01	0	-1.15E-06	0	6.79E-01	6.30E-01	6.24E-01	3.14E-01	4.44E-06	4.50E-01
Housing starts	-4.82E-02	-	-4.63E-02	0	-2.47E-07	0	7.68E-02	7.18E-02	6.67E-02	4.05E-04	3.47E-07	3.15E-02
Unemployment rate	-8.23E-03	-	-7.15E-03	0	-8.50E-08	0	-3.61E-02	-	-2.47E-02	0.00E+00	-1.79E-07	-8.01E-03
Labor cost	7.32E-02	-	-1.51E-01	0	-5.55E-06	0	1.25E+00	1.41E+00	1.12E+00	0.00E+00	2.90E-06	4.37E-01
Labor productivity	2.61E+00	-	1.67E+00	0	1.70E-05	0	2.88E+00	2.21E+00	1.77E+00	0.00E+00	1.83E-05	6.53E-01
Lambda	-	-	0.0099	0.0050	100.0000	0.0100	-	-	0.0083	0.0038	100.0000	0.0100
Alpha	-	-	-	-	-	0.5000	-	-	-	-	-	0.200
Gamma	-	-	-	-	1.1000	-	-	-	-	-	1.1000	-

Table B5. Fundamental model estimation results for the US and the UK residential real estate indices.
In-sample fitting results for our six models, using the set of economic covariates and the indicative commercial property indexes, S&P/Case-Shiller for US, and House Price for UK.

	S&P/Case-Shiller Home Price Index						UK House Price Index					
	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net
Constant	-2.79E-03	-2.90E-03	-1.83E-03	-1.80E-03	7.75E-04	4.08E-04	1.02E-03	1.06E-03	1.26E-03	1.10E-03	3.35E-03	1.91E-03
Stock market index	2.96E-03	-	4.27E-03	0	5.52E-08	0	6.88E-03	-	7.34E-03	2.03E-03	1.26E-07	0
Exchange rate	-2.58E-03	-	-2.20E-03	0	-3.22E-08	0	-5.48E-02	-6.60E-02	-4.65E-02	-5.12E-02	-5.43E-07	-2.20E-02
Money supply	2.59E-01	2.42E-01	1.76E-01	1.22E-01	2.51E-07	0	3.75E-01	3.66E-01	3.01E-01	2.88E-01	2.21E-06	8.82E-02
Central bank rate	3.84E-03	4.88E-03	3.04E-03	2.33E-03	2.75E-08	4.66E-06	2.87E-03	-	1.30E-03	0	5.53E-08	0
3-month Treasury Bill	1.18E-03	-	2.23E-03	1.49E-03	3.01E-08	1.68E-04	-4.30E-04	-	1.44E-03	0.00E+00	5.70E-08	7.86E-04
5-year Govt bond yield	3.86E-03	4.13E-03	5.97E-04	0	6.34E-09	0	5.56E-03	-	2.65E-03	2.03E-03	4.72E-08	8.93E-04
10-year Govt bond yield	-3.95E-04	-	-2.18E-04	0	1.52E-09	0	-3.12E-03	-	-2.52E-04	0	3.12E-08	0
Mortgage rate	-5.97E-03	-6.39E-03	-1.92E-03	-4.59E-04	4.69E-09	0	-9.87E-03	-7.49E-03	-6.58E-03	-2.60E-03	2.39E-08	0
Inflation rate	-3.99E-03	-4.14E-03	-3.24E-03	-3.25E-03	-2.23E-08	-6.74E-04	-3.48E-03	-3.61E-03	-2.79E-03	-2.92E-03	-1.49E-08	-7.38E-04
Gold price	-2.32E-03	-	-2.27E-03	0	-5.28E-08	0	-1.46E-03	-	-4.55E-03	0	-1.46E-07	0
Oil price	-1.33E-02	-1.40E-02	-1.22E-02	-1.26E-02	-7.85E-08	-1.79E-03	-3.01E-03	-	-2.56E-03	0	1.43E-08	0
Rent price index	-3.08E-01	-	-1.52E-01	0	1.63E-06	0	-1.36E+00	-1.50E+00	-1.02E+00	-1.13E+00	-6.86E-06	-2.52E-01
GDP	1.17E+00	1.19E+00	7.16E-01	8.72E-01	4.22E-06	1.78E-01	1.03E+00	1.27E+00	7.36E-01	1.18E+00	7.73E-06	4.71E-01
Disposable income	-2.49E-01	-2.45E-01	-5.89E-02	0	1.91E-06	0	1.06E+00	1.10E+00	8.34E-01	9.86E-01	5.41E-06	4.19E-01
Industrial production	2.62E-02	-	4.18E-02	0	9.36E-07	1.23E-03	8.98E-03	-	1.14E-02	0	4.94E-07	0
Housing starts	2.75E-03	-	2.39E-03	0	2.53E-08	0	1.80E-02	1.85E-02	1.48E-02	1.33E-02	1.47E-07	3.81E-03
Unemployment rate	-1.41E-03	-	-2.47E-03	-1.70E-03	-3.92E-08	-4.08E-04	-1.04E-02	-9.87E-03	-9.37E-03	-6.03E-03	-9.04E-08	-2.41E-03
Labor cost	3.50E-01	3.87E-01	2.46E-01	2.24E-01	7.53E-07	0	2.41E-02	-	6.05E-02	0	7.51E-07	0
Labor productivity	-1.53E-01	-	7.53E-02	0	2.25E-06	0	1.94E-01	-	4.07E-01	0	7.23E-06	2.59E-01
Lambda	-	-	0.0011	0.0002	100.0000	0.0100	-	-	0.0026	0.0003	100.0000	0.0100
Alpha	-	-	-	-	-	0.1000	-	-	-	-	-	0.1000
Gamma	-	-	-	-	1.1000	-	-	-	-	-	1.1000	-