ISSN 1755-5361

University of Essex

Department of Economics

Discussion Paper Series

No. 681 December 2009

Inflation dynamics under optimal discretionary fiscal and monetary policies

Stefan Niemann
Paul Pichler
Gerhard Sorger

Inflation dynamics under optimal discretionary fiscal and monetary policies*

Stefan NIEMANN[†] Paul PICHLER[‡] Gerhard SORGER[§]

December 17, 2009

Abstract

We examine the dynamic properties of inflation in a model of optimal discretionary fiscal and monetary policies. The lack of commitment and the presence of nominally risk-free debt provide the government with an incentive to implement policies which induce positive and persistent inflation rates. We show that this property obtains already in an environment with flexible prices and perfectly competitive product markets. Introducing nominal rigidities and imperfect competition has no qualitative but important quantitative implications. In particular, with a modest degree of price stickiness our model generates inflation dynamics very similar to those experienced in the U.S. since the Volcker disinflation of the early 1980s.

JEL *classification*: E31, E32, E61, E63
Keywords: Inflation dynamics; persistence; optimal fiscal and monetary policy; lack of commitment

*This paper has benefitted from conversations with Sanjay Chugh, Wouter Den Haan, Christian Ghiglino, Ken Judd, Eric Leeper, Fernando Martin, Monika Merz, Salvador Ortigueira, Michael Reiter, Stefanie Schmitt-Grohé, and Martin Uribe, among others. We further thank seminar and conference participants at the European Central Bank, the University of Osnabrueck, the University of Basel, the Verein für Socialpolitik Annual Meeting (2008, Graz), the 5th European Workshop in Macroeconomics (2009, Mannheim) and the North American Summer Meeting of the Econometric Society (2009, Boston) for valuable comments and suggestions. Financial support by the Austrian Science Fund (FWF) under project number P19686 is gratefully acknowledged.

[†]Corresponding author: Department of Economics, University of Essex, sniem@essex.ac.uk
[‡]Department of Economics, University of Vienna, paul.pichler@univie.ac.at
[§]Department of Economics, University of Vienna, gerhard.sorger@univie.ac.at

1 Introduction

Inflation rates in virtually all developed economies are positive on average, moderately volatile, and persistent. The present paper rationalizes these properties in the context of a dynamic stochastic general equilibrium model governed by optimal fiscal and monetary policies. The central feature of the model is a benevolent government without commitment power. The government has only access to distortionary instruments (a proportional income tax and the nominal interest rate) but can also issue nominal non-state contingent debt in order to shift distortions over time.

The lack of commitment power and the existence of nominal public debt create an incentive for the policy maker to use inflation as a tax on the financial wealth held by private households. This incentive increases with the amount of outstanding government debt and, in equilibrium, is balanced by the costs associated with such a policy. In our model, these costs arise because consumption purchases by the private sector cause transaction costs. Positive inflation increases these transaction costs and thus reduces current consumption possibilities. The trade-off between the ex-post incentive to inflate and the increased transaction costs due to realized inflation endogenously pins down the optimal issuance of debt together with taxes and interest rates.

Debt is used by the government to smooth tax distortions over time and, thus, displays a considerable degree of persistence. Taxes fall in response to productivity improvements and rise in response to an increase in government expenditures. Nominal interest rates do not follow the Friedman rule but are positive on average and larger than the rate of time preference. Hence, inflation is positive on average. Moreover, the nature of the trade-off faced by the policy maker renders optimal inflation rates highly correlated with the level of debt. Consequently, the persistence of debt carries over to inflation and it follows that optimal discretionary policies are characterized by positive and persistent inflation rates. It is important to emphasize that this property holds independently of nominal rigidities and the degree of competition in product markets.

With regard to its empirical predictions for the dynamics of inflation, our environment is distinct from the standard formulation of the New Keynesian model. In the latter model, nominal rigidities in the wage or price setting mechanism generate persistence in the price level but fail to produce persistence in inflation (Fuhrer and Moore, 1995). In an effort to formulate an empirically plausible model, the subsequent New Keynesian literature has therefore sought to augment the Phillips-curve by additional backward-looking elements: Fuhrer and Moore (1995) suggest backward-looking wage contracting;[1] Gali and Gertler (1999) and Steinsson (2003) postulate that a fraction of producers set their prices according to a rule of thumb; Christiano, Eichenbaum, and Evans (2005) propose a rich New Keynesian model featuring, among other things, partial indexation to past inflation. While these hybrid Phillips-curve approaches are successful in generating inflation persistence, they have been criticized for their lack of a convincing microeconomic foundation. Introducing backward-looking behavior or any departure from the assumption of rational expectations makes the theory susceptible to the Lucas critique (Rabanal and Rubio-Ramirez, 2003), whereas indexation is inconsistent with the evidence that many nominal prices remain constant for several periods; see, e.g., Bils and Klenow (2004), Bils, Klenow, and Malin (2009), and Nakamura and Steinsson (2008).[2]

In response to these criticisms, several recent papers have attempted to rationalize inflation persistence without resorting to assumptions that imply suboptimal or backward-looking behavior. These papers have identified persistent changes in monetary policy as driving forces behind persistent inflation rates. Cogley and Sbordone (2008) and Ireland (2007) consider shifts in the central bank's inflation target that translate into drifts in trend inflation and, thus, induce inflation persistence. Similarly, Erceg and Levin (2003) propose a model of incomplete information where inflation persistence is generated via the private sector's signal extraction problem in the face of uncertainty about the monetary policy rule. Common to these contributions is their emphasis on *exogenous* changes in the monetary policy regime. Our paper complements this recent literature by presenting a model in which optimal macroeconomic policies are determined *endogenously*. Our results point to the fact that monetary-fiscal interactions in general,

[1]Formalizing the wage contracting mechanism in terms of contemporaneous data instead, Holden and Driscoll (2003) show that inflation persistence no longer obtains.

[2]In addition, the hybrid Phillips-curve has recently also been questioned from an empirical angle, see e.g., Rudd and Whelan (2006).

and the dynamics of government debt in particular, can be an important determinant of shifts in monetary policy.[3]

Regarding its implications for inflation, our framework is fundamentally different from Ramsey models of optimal fiscal and monetary policy. In these models optimal inflation is typically negative on average and exhibits virtually zero persistence. This holds for environments in which the Friedman rule is optimal, e.g., Chari, Christiano, and Kehoe (1991), and carries over to environments with imperfect competition and nominal rigidities where the Friedman rule does not obtain, e.g., Schmitt-Grohe and Uribe (2004a, 2004b) or Siu (2004). In a recent paper, Chugh (2007) addresses this shortcoming and develops a Ramsey model with capital accumulation and habits in consumption in which inflation is persistent.[4] However, while Chugh (2007) is successful in generating persistence, his model implies an implausibly high volatility of inflation. Thus, his Ramsey approach, too, fails to fully account for the observed inflation dynamics.

Finally, from a methodological point of view the present paper contributes to a growing literature on time-consistent optimal policy. This literature formulates the policy problem as a game between successive governments, one for each time period, and analyzes Markov-perfect equilibria of this game. Klein, Krusell, and Rios-Rull (2008) and Ortigueira (2006) use this approach to study optimal fiscal policy, i.e., public expenditures and taxation, in models without money. Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008) and Martin (2009) consider optimal fiscal and monetary policies in deterministic economies with government debt. Assuming that only distortionary instruments are available, they examine the policy maker's time-inconsistency problem and its long-run implications for government debt. Adam and Billi

[3]Following contributions by, among others, Leeper (1991), **?** and **?**, the fiscal theory of the price level posits that fiscal policy can influence the price level. What separates our approach from the fiscal theory is that here government debt only affects monetary variables because it changes the inflation incentives faced by a government optimizing under discretion. In contrast, according to the fiscal theory, government debt affects the price level because it is assumed that the monetary authority commits to a (possibly suboptimal) interest rate rule while fiscal policy implements a (possibly suboptimal) exogenous path of real government surpluses; the price level must then adjust to satisfy the intertemporal government budget constraint.

[4]For the implications of habits in consumption in the context of monetary policy models see, e.g., Fuhrer (2000).

(2008) examine optimal discretionary policy in a monetary economy without debt where fiscal and monetary policies are implemented by separate authorities. They show that inflation conservatism remains desirable with endogenous fiscal policy. Finally, Niemann (2009) investigates monetary conservatism in a framework with nominal debt and finds that inflation conservatism has adverse welfare effects because of its implications for debt accumulation.

The remainder of the present paper is organized as follows. Section 2 describes a monetary economy with flexible prices. Section 3 formulates the optimal policy problem in this economy as a game between successive governments and defines Markov-perfect equilibria for this game. Section 4 contains the main results for the flexible price economy. We derive the equilibrium conditions, provide a number of analytical results about steady states in a non-stochastic version of the model, and finally present the quantitative properties of the equilibrium in a calibrated version of the model. We show in particular that optimal inflation rates are positive on average and persistent. Section 5 introduces nominal rigidity into the model and shows that even with a very small degree of price stickiness the model generates inflation dynamics that are very close to those observed in the United States since the Volcker disinflation of the early 1980s. Finally, section 6 concludes. Proofs and a brief evaluation of our computational algorithm are relegated to the Appendix.

2 A flexible-price economy

Time evolves in discrete periods $t \in \{0, 1, 2, \ldots\}$. We consider an infinite-horizon production economy populated by a large number (a continuum of measure 1) of identical private agents and a government. The private agents act both as consumers and as producers, prices are flexible, and a demand for money arises due to its role in facilitating consumption transactions.

2.1 The private sector

The preferences of the representative private agent are defined over sequences of consumption, $(c_t)_{t=0}^{\infty}$, and labor effort, $(h_t)_{t=0}^{\infty}$, and are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \alpha h_t], \tag{1}$$

where E_0 denotes the mathematical expectation operator conditional on information available in period 0, $\beta \in (0,1)$ is the time-preference factor, and $\alpha > 0$ is the constant marginal utility of leisure. We assume that the function u satisfies standard monotonicity, curvature, and smoothness properties.

The agent enters period t holding M_t units of money and B_t units of one-period risk-free bonds issued by the government. Each of these bonds pays one unit of money when it matures at the end of period t. The agent has two sources of income in period t. First, it supplies h_t units of labor to a perfectly competitive labor market earning the nominal after-tax wage income $(1-\tau_t)W_t h_t$, where τ_t and W_t denote the tax rate and the nominal wage rate, respectively, in period t. Second, it earns profits from producing a differentiated intermediate good, which forms an input for the production of the final consumption good. Each agent has access to a linear production technology $\tilde{y}_t = a_t \tilde{h}_t$, which takes labor $\tilde{h}_t$ as the only input and is subject to a stochastic productivity a_t. Notice that, while h_t is the agent's own labor supply, $\tilde{h}_t$ is the amount of labor it demands on the labor market to produce the intermediate good. Labor productivity a_t is the same for all agents and evolves according to

$$\log a_{t+1} = \rho_a \log a_t + \varepsilon_{t+1}^a,$$

where ρ_a measures the autocorrelation of labor productivity and $\varepsilon_{t+1}^a \sim N(0, \sigma_{\varepsilon^a}^2)$ denotes the period-$(t+1)$ innovation.

The final consumption good is a Dixit-Stiglitz aggregate of all intermediate goods. We denote by $\theta > 1$ the constant elasticity of substitution between any two intermediate inputs. When $\theta \to \infty$, the economy approaches the limiting case of perfectly competitive product markets. Denoting by $\tilde{P}_t$ the price of an intermediate good charged by its monopolistic producer and by P_t the aggregate price level, the demand for the intermediate good depends on aggregate output y_t and the relative price $\tilde{P}_t/P_t$ according to

$$d(\tilde{P}_t, P_t, y_t) = y_t \left(\tilde{P}_t/P_t\right)^{-\theta}.$$

When choosing its price $\tilde{P}_t$, the agent takes the demand function d together with the aggregate variables P_t and y_t as given.

Finally, we postulate that each agent has to pay a proportional transaction cost $s(v_t)$ when purchasing c_t units of the consumption good. Here, v_t is the agent's consumption-based money velocity defined by

$$v_t = P_t c_t / M_t. \tag{2}$$

Hence, as in Schmitt-Grohe and Uribe (2004a, 2004b), money is valued because it facilitates transactions. Notice that the timing assumption underlying the definition of velocity in (2) implies that the agents cannot reduce their transaction costs by rearranging their nominal asset portfolios at the start of a period, but that they are bound by their predetermined money holdings M_t. Thus, the velocity-based transaction cost $s(v_t)$ reflects a timing assumption corresponding to the cash-in-advance setting in Svensson (1985). As for the function s itself we follow Schmitt-Grohe and Uribe (2004a, 2004b) by assuming that (i) s takes non-negative values and is twice continuously differentiable with first and second derivative s_v and s_{vv}, (ii) there exists a satiation level $\underline{v} > 0$ such that $s(\underline{v}) = s_v(\underline{v}) = 0$, (iii) $(v - \underline{v})s_v(v) > 0$ for all $v \neq \underline{v}$, and (iv) $2s_v(v) + v s_{vv}(v) > 0$ for all $v \geq \underline{v}$. These assumptions guarantee that money demand is decreasing in the nominal interest rate and that the Friedman rule is not associated with an infinite money demand.

To conclude, the agent's budget constraint in period t is given by

$$M_t + B_t + (1 - \tau_t)W_t h_t + \tilde{P}_t y_t \left(\tilde{P}_t/P_t\right)^{-\theta} - W_t \tilde{h}_t \geq P_t c_t[1 + s(v_t)] + M_{t+1} + q_t B_{t+1}, \tag{3}$$

where q_t denotes the price of bonds purchased in period t, i.e., q_t is the inverse of the gross nominal interest rate on these bonds.

2.2 The government

The government is benevolent and decides over monetary and fiscal policy instruments. It faces a stream of exogenous, stochastic, and unproductive expenditures $(g_t)_{t=0}^{\infty}$, which evolves according to

$$\log g_{t+1} = (1 - \rho_g) \log \bar{g} + \rho_g \log g_t + \varepsilon_{t+1}^g.$$

The parameter $\bar{g}$ denotes the steady state government expenditures, ρ_g is the autocorrelation coefficient, and $\varepsilon_{t+1}^g \sim N(0, \sigma_{\varepsilon^g}^2)$. To finance its expenditures, the government imposes a proportional labor income tax at rate τ_t, issues government bonds $\bar{B}_{t+1}$, and receives seignorage income $\bar{M}_{t+1} - \bar{M}_t$.[5] Monetary policy manages the supply of money $\bar{M}_{t+1}$ and sets the price of bonds q_t. The consolidated government budget constraint in nominal terms is thus given by

$$\tau_t W_t h_t + (\bar{M}_{t+1} - \bar{M}_t) + q_t \bar{B}_{t+1} \geq P_t g_t + \bar{B}_t. \tag{4}$$

The policy instruments τ_t, $\bar{B}_{t+1}$, q_t, and $\bar{M}_{t+1}$ must be chosen in such a way that (4) holds and that the markets for bonds and money clear.

3 Equilibrium definition

A convenient way to characterize optimal discretionary policies is to assume that the government actually consists of an infinite sequence of separate policy makers, one for each period. The policy maker who is in charge in period k will be referred to as the period-k government. This government seeks to maximize social welfare from period k onwards, whereby it takes the behavior both of its later incarnations and of the private sector as given. In other words, we consider an equilibrium in the game among the private sector and all period-k governments, where k ranges from 0 to $+\infty$. The private sector acts as a Stackelberg follower, whereas the governments play Nash among each other and act as Stackelberg leaders against the private sector. For simplicity, and following the dominant approach in the macroeconomic literature, we restrict attention to Markov-perfect equilibria.

[5]Where necessary, we use bars to distinguish aggregate variables from their individual counterparts.

3.1 Private-sector equilibrium

In this subsection we examine the properties of a private-sector equilibrium for given government policies. To this end, we first divide all nominal variables (except for q_t) by the aggregate money stock $\bar{M}_t$, as in Cooley and Hansen (1991). For notational simplicity, we eliminate $\tilde{h}_t$ and denote the normalized variables by lower case letters such as $p_t = P_t/\bar{M}_t$, $b_t = B_t/\bar{M}_t$, etc.. Equations (2), (3), and (4) can then be rewritten as

$$v_t = \frac{p_t c_t}{m_t}, \tag{5}$$

$$\frac{m_t + b_t + (1-\tau_t)w_t h_t}{p_t} + y_t \left(\frac{\tilde{p}_t}{p_t}\right)^{1-\theta} - \frac{w_t y_t}{a_t p_t}\left(\frac{\tilde{p}_t}{p_t}\right)^{-\theta} \geq c_t[1+s(v_t)] + \frac{(1+\mu_t)(m_{t+1} + q_t b_{t+1})}{p_t}, \tag{6}$$

$$\frac{\tau_t w_t h_t}{p_t} + \frac{(1+\mu_t)(1+q_t \bar{b}_{t+1})}{p_t} \geq g_t + \frac{1+\bar{b}_t}{p_t}, \tag{7}$$

where $\mu_t = \bar{M}_{t+1}/\bar{M}_t - 1$ denotes the growth rate of the aggregate money stock.

The representative agent takes aggregate variables and government policies as given and maximizes its objective functional (1) subject to obvious non-negativity constraints, a no-Ponzi game condition, the laws of motion for a_t and g_t, the identity (5), and the flow budget constraint (6). The solution to this problem is characterized by a set of standard first-order optimality conditions.[6] Imposing on these conditions that all private agents are identical and that markets clear, i.e., $b = \bar{b}$, $m = 1$, $\tilde{p} = p$, we obtain a set of conditions that characterize a symmetric

[6]See the Appendix for a formal derivation of these conditions.

private-sector equilibrium for given government policies. These conditions are[7]

$$0 = \frac{\alpha}{(1-\tau)w} - \frac{cu_c(c)}{v\gamma(v)}, \tag{8}$$

$$0 = \beta E\left\{\frac{c'u_c(c')}{v'\gamma(v')}\left[1 + s_v(v')v'^2\right]\right\} - \frac{cu_c(c)}{v\gamma(v)}(1+\mu), \tag{9}$$

$$0 = \beta E\left[\frac{c'u_c(c')}{v'\gamma(v')}\right] - \frac{cu_c(c)}{v\gamma(v)}(1+\mu)q, \tag{10}$$

$$0 = c\left[1 + s\left(v\right)\right] + \frac{(1+\mu)(1+qb')}{p} - \frac{1+b}{p} - \left(a - \frac{\tau w}{p}\right)h, \tag{11}$$

$$0 = v - cp, \tag{12}$$

$$0 = \frac{w}{p} - \frac{(\theta-1)a}{\theta}, \tag{13}$$

where $\gamma(v) = 1 + s(v) + s_v(v)v$.

Condition (8) equates the marginal disutility of supplying an additional unit of labor to the marginal benefit, taking into account labor taxation and transaction costs. Conditions (9) and (10) state that each household must be indifferent between consuming today and saving either via money or via bonds. Condition (11) is the agent's flow budget constraint, (12) defines the velocity, and (13) specifies the real wage. The latter condition follows from optimal price setting for intermediate goods. Notice that, since prices are flexible and the elasticity of substitution between intermediate inputs is constant, the real wage rate is a constant fraction $(\theta - 1)/\theta$ of the productivity a.

3.2 The optimal policy problem

In any given period k, the government maximizes the expected lifetime utility of the representative agent from period k onwards. Formally, the objective functional of the period-k government is

$$E_k \sum_{t=k}^{\infty} \beta^{t-k}[u(c_t) - \alpha h_t].$$

The government's payoff is thus determined both by its own actions in period k and by the actions of its future incarnations in periods $k+1$, $k+2$, etc.. While the period-k government

[7]Here and in what follows we employ recursive notation, i.e., time indices are omitted and a prime denotes evaluation at the next period.

cannot directly determine future policy decisions, it can influence these decisions indirectly via the state of the economy. This is the case because future governments respond to the aggregate state inherited from their respective predecessors. The aggregate state vector in the present model is (b, a, g).[8] When deciding upon the endogenous state b' passed on to the next government, the current government therefore needs to take into account how this affects future policy decisions which, in turn, affect current and future consumption and labor market decisions and, thus, welfare. Formally, this implies that each period-k government anticipates its future incarnations' policy rules together with the optimal response of the private sector to the chosen policies.

To describe the government's optimal policy problem, it is useful to introduce the continuation value function. Notice that the objective functional of the period-k government can be decomposed as

$$[u(c_k) - \alpha h_k] + E_k \sum_{t=k+1}^{\infty} \beta^{t-k}[u(c_t) - \alpha h_t], \tag{14}$$

where the first part gives the period-k government's instantaneous utility and the second part the present value of its payoff from future periods $t > k$.

Assume that all future governments use the policy rule $\Psi : \mathbb{S} \to \mathbb{D}$, where $\mathbb{S} \subseteq \mathbb{R}^3$ is the state space (spanned by the variables b, a, and g) and where $\mathbb{D} \subseteq \mathbb{R}^2$ is the policy decision space for the instruments τ and q.[9] Furthermore, let the private sector's equilibrium response to the policy rule Ψ be characterized by the rules $\mathcal{C} : \mathbb{S} \to \mathbb{R}_+$, $\mathcal{H} : \mathbb{S} \to \mathbb{R}_+$, $\mathcal{V} : \mathbb{S} \to \mathbb{R}_+$, $\mathcal{B}' : \mathbb{S} \to \mathbb{R}$, etc., which we collect in the multidimensional rule $\mathcal{D} = (\mathcal{C}, \mathcal{H}, \mathcal{P}, \mathcal{W}, \mathcal{V}, \mathcal{M}, \mathcal{B}')$. With this notation at hand and employing a recursive formulation, we can write (14) as

$$u(c) - \alpha h + \beta E \mathcal{U}(b', a', g'), \tag{15}$$

[8]Recall that because of symmetry and market clearing we have $m = 1$ and $b = \bar{b}$. That is, our normalization by the aggregate money stock implies that there is no need to keep track of $m = \bar{m} = 1$. In addition, individual and aggregate states must coincide. In the following, we will therefore drop the superscript bar and use b to denote the aggregate debt-to-money ratio.

[9]Notice that in our model there are four policy variables: τ, b', μ, and q. When choosing these variables, however, the policy maker has only two degrees of freedom. In what follows, we will assume that the policy authority chooses τ and q, while b' and μ are determined residually by money and bond market clearing conditions.

where $\mathcal{U} : \mathbb{S} \to \mathbb{R}$ is implicitly defined by the recursion

$$\mathcal{U}(b', a', g') = u(\mathcal{C}(b', a', g')) - \alpha\mathcal{H}(b', a', g') + \beta E\mathcal{U}(\mathcal{B}'(b', a', g'), a'', g'').$$

We will refer to $\mathcal{U}$ as the government's continuation value function.

For a given future policy rule Ψ and the corresponding continuation value function $\mathcal{U}$, the current government faces a static optimization problem. We adopt a primal approach to examine this problem. Hence, we postulate that the current government does not only decide upon its own policy instruments but that it also chooses the current private-sector allocation subject to the requirement that this allocation constitutes a symmetric private-sector equilibrium (given the government's policies). Formally, the optimal policy problem is given by

$$\max_{\tau,q,c,h,v,p,w,\mu,b'} \left[u(c) - \alpha h + \beta E\mathcal{U}(b', a', g') \right]$$

subject to the feasibility constraint (7) and the private-sector optimality conditions (8)-(13) with $c' = \mathcal{C}(b', a', g')$ and $v' = \mathcal{V}(b', a', g')$.[10]

3.3 Stationary Markov-perfect equilibrium

We now proceed to define the Markov-perfect equilibrium of the economy. To this end, we introduce some more notation. We denote by $\hat{\Psi}$ the policy rule for $\hat{\psi} = (\tau, q)$ employed by the current government, and we collect the current private-sector decision variables in a vector $\hat{d} = (c, h, p, w, v, \mu, b')$. The corresponding decision rule is denoted $\hat{\mathcal{D}} = (\hat{\mathcal{C}}, \hat{\mathcal{H}}, \hat{\mathcal{P}}, \hat{\mathcal{W}}, \hat{\mathcal{V}}, \hat{\mathcal{M}}, \hat{\mathcal{B}}')$.

Definition: A stationary Markov-perfect equilibrium is a set of functions $\{\hat{\Psi}, \Psi, \hat{\mathcal{D}}, \mathcal{D}, \mathcal{U}\}$ such that:

1. Given Ψ, $\mathcal{D}$, and $\mathcal{U}$,

$$\{\hat{\Psi}, \hat{\mathcal{D}}\} = \arg\max_{\hat{\psi},\hat{d}} [u(c) - \alpha h + \beta E\mathcal{U}(b', a', g')]$$

   subject to (7) and (8)-(13) with $c' = \mathcal{C}(b', a', g')$ and $v' = \mathcal{V}(b', a', g')$.

[10] The functions $\mathcal{C}$ and $\mathcal{V}$ are the private-sector optimal responses to future policies as defined above.

2. It holds that $\hat{\Psi} = \Psi$, $\hat{\mathcal{D}} = \mathcal{D}$, and

$$\mathcal{U}(b, a, g) = u(\mathcal{C}(b, a, g)) - \alpha\mathcal{H}(b, a, g) + \beta E\mathcal{U}(\mathcal{B}'(b, a, g), a', g').$$

The first condition states that the current policy rule $\hat{\Psi}$ for the policy variables τ and q is optimal for the current government, given that all future governments use the policy rule Ψ and given that these policies (of both current and future governments) induce a symmetric private-sector equilibrium. In other words, this is the Nash equilibrium condition for the game between the different incarnations of the government. The second condition imposes stationarity on this equilibrium. Note furthermore that the Markov-perfect equilibrium is time-consistent by construction.

4 Equilibrium characterization

In the present section we characterize the solution of the model. We start by deriving the equilibrium conditions. Then we consider steady states in a non-stochastic version of the model and prove analytically (i) that there is a non-distorted and a distorted steady state, (ii) that the Friedman rule is not optimal in the distorted steady state, and (iii) that the sign of the steady state level of the debt-to-money ratio does not only depend on the elasticity of intertemporal substitution. Finally, we return to the stochastic model and use a numerical approach to derive quantitative results for a calibrated version.

4.1 Equilibrium conditions

In order to derive conditions characterizing a Markov-perfect equilibrium, we first rewrite the government's optimization problem in a compact form. More specifically, we introduce the functions

$$\tilde{T}(c, v; a, g) = \left[\frac{\theta - 1}{\theta} - \frac{\alpha\gamma(v)}{au_c(c)}\right]\{c[1 + s(v)] + g\}, \tag{16}$$

$$\tilde{\mu}(c, v, b'; a, g) = \beta E\left\{\frac{\mathcal{C}u_c(\mathcal{C})v\gamma(v)}{cu_c(c)\mathcal{V}\gamma(\mathcal{V})}\left[1 + s_v(\mathcal{V})\mathcal{V}^2\right]\right\} - 1, \tag{17}$$

$$\tilde{r}(c, v, b'; a, g) = \beta E\left[\frac{\mathcal{C}u_c(\mathcal{C})v\gamma(v)}{cu_c(c)\mathcal{V}\gamma(\mathcal{V})}\right], \tag{18}$$

where, for the sake of better readability, we have omitted the arguments (b', a', g') from the future decision rules $\mathcal{V}$ and $\mathcal{C}$. The functions $\tilde{T}$, $\tilde{\mu}$, and $\tilde{r}$ are derived by solving (7)-(13) for $T = \tau wh/p$, μ, and $r = (1 + \mu)q$, respectively. Notice that these functions are not equilibrium policy functions, since they do not map the state space into the respective decisions. Rather, given a and g, they determine the real tax revenue, the money growth rate, and the (reciprocal value of the) gross real interest rate that are compatible with a private-sector equilibrium conditional on the choices for c, v, and b'.

Making use of (16)-(18), we can formulate the government's optimal policy problem as the maximization with respect to c, v, and b' of

$$u(c) - \frac{\alpha}{a}\{c[1 + s(v)] + g\} + \beta E\mathcal{U}(b', a', g')$$

subject to the implementability constraint

$$\mathcal{E}(c, v, b'; b, a, g) = \tilde{T}(c, v; a, g) + \frac{c}{v}\left[\tilde{\mu}(c, v, b'; a, g) + \tilde{r}(c, v, b'; a, g)b' - b\right] - g = 0.$$

The first-order conditions associated with this problem are[11]

$$u_c(c) - \frac{\alpha}{a}[1 + s(v)] = -\eta\frac{v}{c}\left\{\tilde{T}_c(\cdot) + \frac{1}{v}[\tilde{\mu}(\cdot) + \tilde{r}(\cdot)b' - b] + \frac{c}{v}[\tilde{\mu}_c(\cdot) + \tilde{r}_c(\cdot)b']\right\}, \tag{19}$$

$$\frac{\alpha c s_v(v)}{a} = \eta\frac{v}{c}\left\{\tilde{T}_v(\cdot) - \frac{c}{v^2}[\tilde{\mu}(\cdot) + \tilde{r}(\cdot)b' - b] + \frac{c}{v}[\tilde{\mu}_v(\cdot) + \tilde{r}_v(\cdot)b']\right\}, \tag{20}$$

$$\beta E\eta' = \eta\tilde{r}(\cdot) + \eta[\tilde{\mu}_{b'}(\cdot) + \tilde{r}_{b'}(\cdot)b'], \tag{21}$$

where $\eta v/c$ is the multiplier attached to the implementability constraint. Condition (19) characterizes the government's optimal choice of consumption. It equates the current utility gain of a change in c to the utility cost arising from a tightening of the budget constraint. Similarly,

[11] Arguments of the functions $\tilde{T}$, $\tilde{\mu}$, and $\tilde{r}$ are omitted for simplicity.

conditions (20) and (21) characterize the government's optimal choice of velocity v and the future debt-to-money ratio b', respectively. Note that condition (21) involves derivatives of the future policy functions $\mathcal{C}$ and $\mathcal{V}$ (which are contained in $\tilde{\mu}_{b'}(\cdot)$ and $\tilde{r}_{b'}(\cdot)$) and is usually referred to as the generalized Euler equation. The term $\beta E\eta'$ on the left-hand side of (21) is the discounted expected utility loss associated with a tightening of the future implementability constraint due to a marginal increase in b'. The term $\eta\tilde{r}(\cdot)$ on the right-hand side gives the *direct* utility gain obtained from relaxing the current budget constraint via a marginal increase in b'. The second term on the right-hand side, $\eta[\tilde{\mu}_{b'}(\cdot)+\tilde{r}_{b'}(\cdot)b']$, reflects the *indirect* utility gain caused by a marginal relaxation of the current budget constraint. This indirect effect is due to adjustments in the money growth rate and the interest rate that establish market clearing at b'. The adjustments are necessary because future governments will respond to variations in b', as prescribed by the rules $\mathcal{C}$ and $\mathcal{V}$. For a debt policy to be optimal, discounted expected future losses due to a larger b' must offset current (direct and indirect) gains, which is exactly what equation (21) establishes.

4.2 Non-stochastic steady states

Let us now examine optimal Markovian policies in the non-stochastic version of the model presented in Section 2. Restricting attention to the deterministic case allows us to obtain analytical results. Throughout this subsection, we therefore assume that $a = 1$ and $g = \bar{g}$ hold at all dates and states, and we will omit these functional arguments.

Inspection of the generalized Euler equation (21) suggests that there exist two non-stochastic steady states solutions: one of them is undistorted whereas the other one is distorted. In the undistorted steady state it holds that $\eta' = \eta = 0$ such that the implementability constraint is slack and

$$u_c(c) = \alpha.$$

At this steady state, the optimal (first-best) allocation is implemented, which is feasible if the government has sufficiently high claims on the private sector, allowing it to finance its expenditures without the need to resort to distortionary taxation. In particular, at this steady state the Friedman rule applies and the labor tax rate is negative with an absolute value high enough to undo the distortion due to monopolistic competition.[12]

The distorted steady state features $\eta' = \eta > 0$, which implies that the implementability constraint binds. In the distorted steady state the government must finance at least part of its expenditures via taxation. It is obvious from (18) that in the steady state we have $\tilde{r}(c^*, v^*, b^*) = \beta$, where b^* is the steady state debt level, $c^* = \mathcal{C}(b^*)$, and $v^* = \mathcal{V}(b^*)$. From (21) we therefore see that the debt level b^* at the distorted steady state is characterized by the condition

$$0 = \tilde{\mu}_{b'}(c^*, v^*, b^*) + \tilde{r}_{b'}(c^*, v^*, b^*)b^*. \tag{22}$$

Notice that equation (22) involves the derivatives of the unknown equilibrium policy functions $\mathcal{C}$ and $\mathcal{V}$ evaluated at the steady state; see also equations (17)-(18). In general, it is therefore impossible to solve for the steady state without solving for these functions. Nevertheless, one can prove two analytical results.

The first one shows that in a stationary Markov perfect equilibrium the Friedman rule cannot hold in a neighborhood around the distorted steady state b^* provided that the latter is dynamically stable. We prove this result for a utility function of the form

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \tag{23}$$

where σ is a positive parameter different from 1. Note that the Friedman rule can be formulated in two equivalent ways, namely either that the nominal interest rate is equal to 0, i.e., $q = \mathcal{Q}(b) = 1$, or that there are no transactions costs, i.e., $v = \mathcal{V}(b) = \underline{v}$.

Proposition 1 *Let the utility function be given by (23) with $0 < \sigma \neq 1$ and let b^* be the distorted steady state. If b^* is a dynamically stable fixed point of the equilibrium dynamics, then there does not exist $\delta > 0$ such that $\mathcal{V}(b) = \underline{v}$ holds for all $b \in (b^* - \delta, b^* + \delta)$.*

The formal proof is relegated to the Appendix. A brief sketch is as follows. Recall that the optimal choice of the debt-to-money ratio b' in a distorted Markovian steady state must

[12] See the working paper Niemann, Pichler, and Sorger (2008) for an analysis of the dynamic properties of optimal fiscal and monetary policy without commitment in the neighborhood of the first-best steady state within a similar economic environment.

eliminate the associated indirect utility effect, i.e., equation (22) must hold. As we show in the Appendix, if preferences are given by (23) and if the Friedman rule holds, this implies that $b^* = -1$. This means that the value of total government liabilities $\bar{M} + \bar{B}$ is equal to zero in the steady state. It is not difficult to verify that for $v = \underline{v}$ and $b = b' = b^* = -1$, the first-order effect of a marginal change in the velocity v on the implementability constraint, i.e., the right-hand side of (20), is non-zero. Due to the assumptions on the transaction cost function s (especially $s_v(\underline{v}) = 0$), the first-order effect on current utility is zero whenever $v = \underline{v}$; see the left-hand side of (20). Hence, the government has an incentive to deviate from $v = \underline{v}$.

When preferences are given by (23) with $\sigma = 1$, that is, under a logarithmic utility function, the above argument cannot be used to prove Proposition 1 as equation (22) holds identically for all debt-to-money ratios. Nevertheless, the first-order effect on transaction costs of deviating from $v = \underline{v}$ is still equal to zero, whereas the indirect effect via the implementability constraint is likely to be non-zero. Hence, we conjecture that Proposition 1 remains true, at least generically, also in the case $\sigma = 1$. Numerical computations support this conjecture.

Finally, notice that Proposition 1 holds irrespective of the degree of monopolistic competition in product markets. This is an important qualitative difference to optimal policy under commitment, where the Friedman rule typically turns out to be optimal in perfectly competitive environments; see, e.g., Chari, Christiano, and Kehoe (1991) or Schmitt-Grohe and Uribe (2004a).

Having established that the Friedman rule fails to hold in a distorted Markovian equilibrium, we now investigate the determinants of the non-stochastic steady state level of debt b^*. It is useful to begin this exercise by discussing the results obtained by Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008) and Martin (2009) in a related framework. These authors have shown that in a cash-in-advance economy with CES preferences for consumption, the sign and level of long-run debt are pinned down by the elasticity of intertemporal substitution. Specifically, b^* is increasing in σ with $b^* > 0$ if $\sigma > 1$, $b^* = 0$ if $\sigma = 1$, and $b^* < 0$ if $\sigma < 1$. This property can be explained as a result of the interaction of two principles of optimal taxation.

Whenever a cash-in-advance constraint binds, the price elasticity of current consumption is unitary. Therefore, if $\sigma > 1$, current consumption is relatively elastic as compared to future consumption. As a consequence, the government has a discretionary incentive to trade current distortions for future distortions, i.e., to increase current consumption and debt. Let us call this the *intertemporal elasticity effect*. If the level of debt is non-positive, this incentive triggers an increase of debt such that $b \leq 0$ cannot prevail in steady state. For positive levels of government debt, however, there is a counteracting effect since any increase in current consumption triggers a corresponding decrease in the price level due to the cash-in-advance constraint. Hence, if debt is positive, the deflation needed to increase current consumption hurts the government because it increases the real value of its liabilities. The magnitude of this *nominal debt effect* obviously increases with b. In a Markovian steady state the level of debt adjusts to that point where both effects exactly offset each other. This implies that $b^* > 0$ if $\sigma > 1$, $b^* = 0$ if $\sigma = 1$, and $b^* < 0$ if $\sigma < 1$.

The environment considered in the present paper does not include a cash-in-advance constraint but a velocity-based transaction cost function. The endogenous choice of velocity adds an additional degree of freedom to the government's optimal policy problem. Variations in consumption do not necessarily trigger adjustments in the price level, as they can be accommodated by variations in velocity. While the elasticity of intertemporal substitution still plays an important role in determining long-run debt, the clear-cut result of the cash-in-advance environment does not carry over to the transaction cost framework. The following proposition illustrates this by means of a simple example for which a closed form solution can be derived. This example is based on a logarithmic utility function, i.e., equation (23) holds with $\sigma = 1$, and shows that the level of debt need not be zero as in the cash-in-advance environment, but can be negative, zero, or positive depending on the parameters of the transaction cost function.

Proposition 2 *Assume that $u(c) = \log c$ and $s(v) = v/(4A) + A/v - 1$, where A is a positive parameter. The steady-state level of debt in a distorted Markovian equilibrium is given by $b^* = A - 1$.*

The proof is again relegated to the Appendix. Even if this result is stated only for a very special case, numerical calculations show that the basic message holds much more generally: both the sign and the size of long-run debt are not pinned down by the elasticity parameter σ alone. The intuition behind this property is that an increase of the parameter A reduces both the level and the slope of the transaction cost function $s(v)$ for any fixed value of v. This means that it becomes less costly for the policy maker to accommodate an increase of current consumption by a corresponding increase of velocity. In other words, the nominal debt effect mentioned above becomes weaker for any given value of debt. In order for it to still compensate the (unchanged) intertemporal substitution effect, the steady state level of debt must therefore increase. This illustrates why the steady state indebtedness b^* is increasing with respect to A.

4.3 Quantitative results

We now return to the stochastic economy and explore the quantitative properties of optimal policies under discretion. Since the stochastic model lacks analytical tractability, we solve it numerically. For this task we use projection methods as described in Judd (1992) and compute fourth-order accurate polynomial approximations to the equilibrium policy functions.[13] We then simulate artificial time series and study the equilibrium dynamics by means of summary statistics and impulse responses.

Parameterization

In a first step we assign values to the model parameters. We set $\beta = 1/1.04$ which is a standard calibration for models with annual data. The utility function is specified as in (23) with $\sigma = 2$, a value in the middle of the parameter range typically considered in the literature. As for the elasticity of substitution between intermediate goods we choose $\theta = 20$; this implies a monopolistic mark-up of 5.26%, similar to Siu (2004). The technology parameters are set to $\rho_a = 0.82$ and $\sigma_a = 0.023$ as in Schmitt-Grohe and Uribe (2004a). The preference parameter α is selected such that labor supply in steady state is roughly equal to one third of the time endowment; this yields $\alpha = 10.4$. The remaining parameters are chosen in line with U.S. data available from Martin (2009). Government expenditure in steady state is set to $\bar{g} = 0.06$ corresponding to roughly 18% of output, $\rho_g = 0.8$ matching the autocorrelation coefficient of government expenditures in the data, and $\sigma_g = 0.04$ such that government spending differs by roughly four percentage points from its average. Finally, the transaction cost function is chosen

[13] A brief description and evaluation of our algorithm is available in the Appendix.

to be $s(v) = A_1 v + A_2/v - 2\sqrt{A_1 A_2}$, where $A_1 = 0.137$ and $A_2 = 2.3$. The calibration of A_1 and A_2 ensures that the model generates the steady state velocity $v^* = 4.37$, which is in line with the average velocity in the data, and the government debt-to-GDP ratio equal to 30.8%.[14]

Non-optimality of the Friedman rule and inflation persistence

Having assigned values to the parameters we can now present our results. Table 1 contains several summary statistics for the key policy and real variables: inflation, the net nominal interest rate $R = q^{-1} - 1$, taxes, velocity, hours worked, consumption, the debt-to-money ratio b, and real debt b/p. The first panel considers perfectly competitive product markets and the second one our baseline calibration with monopolistic competition and mark-ups of 5.26%. The numbers reported are computed as averages over $N = 500$ simulations, each simulation of length $T = 1000$ periods.

The first panel in Table 1 demonstrates that, on average, annual inflation is around 15% with a first-order autocorrelation coefficient exceeding 0.75. Hence, optimal inflation rates are positive on average, even when product markets are perfectly competitive. Moreover, inflation displays substantial persistence. The high inflation rates are associated with high nominal interest rates. For example, under perfect competition, the nominal interest rates are close to 20%, which demonstrates that optimal monetary policy is far from implementing the Friedman rule. These observations contrast sharply with findings in the Ramsey literature (e.g., Schmitt-Grohe and Uribe, 2004a).

The intuition behind this striking difference in optimal policy prescriptions is best understood as follows. First, to see why optimal inflation rates are positive, recall the arguments used in the context of Proposition 1. In particular, we established that, when $\sigma > 1$, future consumption is inelastic relative to current consumption, which gives the government a discretionary incentive to increase current consumption at the expense of higher future indebtedness. In equilibrium,

[14]While our choice of functional form for the transaction cost function $s(v)$ is identical to Schmitt-Grohe and Uribe (2004a, 2004b), our values for A_1 and A_2 differ considerably from the corresponding values used there. This is because our values are calibrated rather than estimated from money demand regressions. We prefer the former approach as we found the parameter estimates obtained from money demand regressions to be extremely sensitive to the data sample employed, as also reported by Cooley and Hansen (1991).

Table 1: Dynamics under flexible prices

x	mean(x)	std(x)	corr(x', x)	corr(x, y)	corr(x, a)	corr(x, g)
Perfect competition ($\theta = \infty$)						
π	15.1443	3.5622	0.7655	-0.0105	-0.3101	0.7019
R	19.7405	3.7042	0.9434	0.0245	-0.3331	0.8298
τ	16.1550	0.7889	0.7382	0.0759	-0.3558	0.9184
v	4.2694	0.0323	0.9673	0.0120	-0.3058	0.7594
y	0.3380	0.0070	0.8106	1.0000	0.9040	0.4246
h	0.3380	0.0077	0.7978	-0.6668	-0.9212	0.3794
c	0.2776	0.0067	0.8298	0.7281	0.9470	-0.3073
b/p	0.1098	0.0032	0.8133	0.5820	0.5492	0.3138
b	1.6904	0.0628	0.9693	0.0089	-0.2474	0.7549
Imperfect competition ($\theta = 20$)						
π	26.8860	4.1799	0.7822	0.0087	-0.3204	0.7577
R	31.9527	4.3741	0.9218	0.0425	-0.3395	0.8737
τ	15.2300	0.8307	0.7241	0.0712	-0.3526	0.9177
v	4.3725	0.0377	0.9533	0.0342	-0.3173	0.8197
y	0.3293	0.0068	0.8109	1.0000	0.9071	0.4195
h	0.3293	0.0075	0.8000	-0.6758	-0.9230	0.3763
c	0.2684	0.0066	0.8284	0.7100	0.9374	-0.3371
b/p	0.1016	0.0025	0.7266	0.7090	0.7602	0.1194
b	1.6561	0.0477	0.9598	0.0244	-0.2571	0.8015

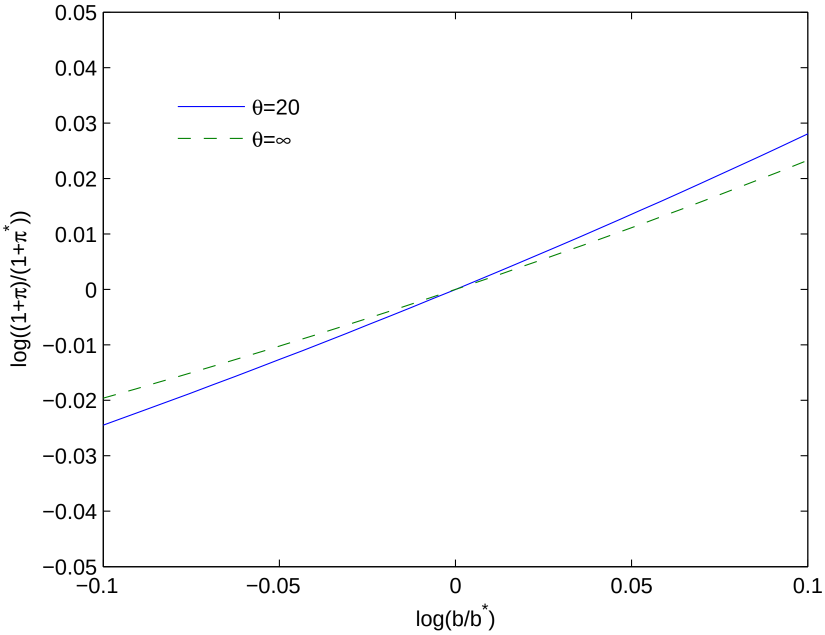
this incentive must be balanced by economic costs arising from increasing current consumption. An increase in consumption can be accommodated via an increase in velocity and/or a decrease in the price level. Hence, costs from higher consumption arise whenever these channels are costly to exercise. For increases in velocity to be costly, v must be above the satiation level $\underline{v}$, driving monetary policy away from the Friedman rule. As a result, we observe positive nominal interest rates (and inflation rates).

A similar argument explains why optimal inflation rates under discretion are persistent. Recall that, in an equilibrium with $\sigma > 1$, it is not sufficient that increases in velocity are costly, but decreases in the price level must be costly, too. Put differently, the government generally has an incentive to create inflation in order to monetize nominal debt. Notice that this incentive depends strongly on the level of debt. In particular, the government's inflation incentives and, thus, the realized inflation rates are increasing in the level of debt. This property is illustrated in Figure 1, which visualizes the equilibrium inflation policy as a function of the endogenous state variable b. Moreover, the desire to smooth consumption makes the government implement a relatively smooth path for debt such that the variable b is highly persistent. Taken together, these properties generate persistence in the incentive to inflate and consequently in realized inflation rates.

Taxes, inflation, and debt as shock absorbers

We next discuss the shock-absorbing role of labor income taxes, inflation, and debt. The statistics presented in the first panel of Table 1 show that both the labor tax and inflation are persistent and relatively volatile. Moreover, both instruments are negatively correlated with productivity and positively correlated with government expenditure, suggesting that both play a similar role in absorbing macroeconomic shocks. To investigate this property more thoroughly, we examine the impulse response to an uncorrelated government purchases shock. The dynamic adjustments displayed in Figure 2 confirm that, under optimal discretionary policy, taxes, inflation, and debt are jointly used as shock absorbers. In particular, the government responds to an expenditure shock in period one by simultaneously raising the tax rate, printing money,

Figure 1: Inflation policy under flexible prices

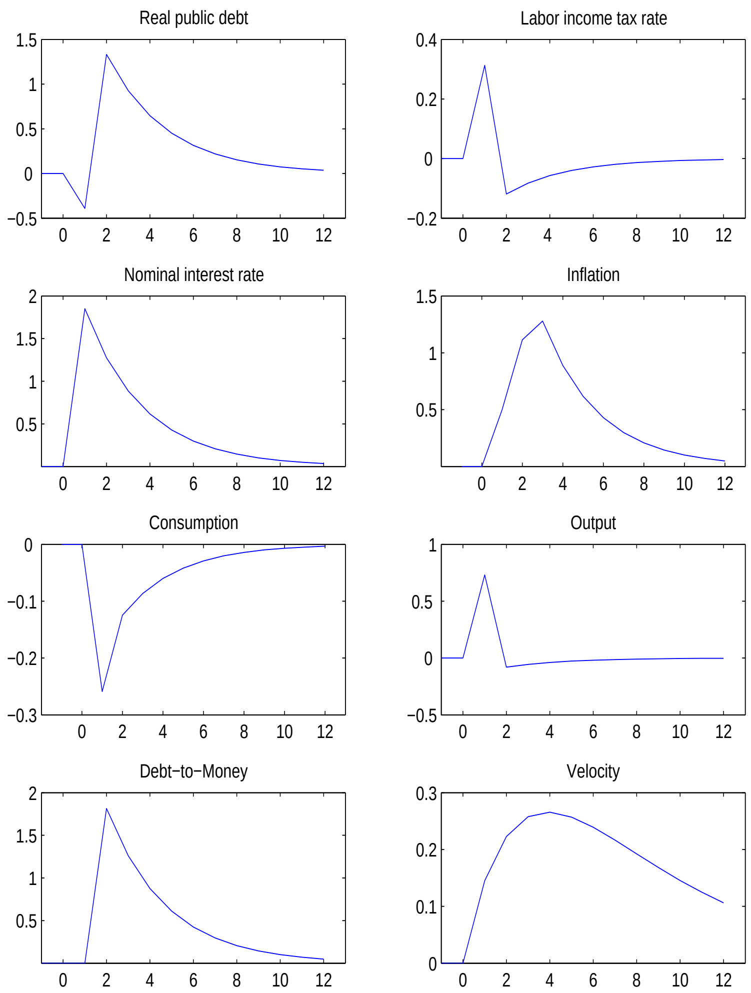


Note: On the horizontal axis we plot the percentage deviation of the debt-to-money ratio from its steady state level, and on the vertical axis the percentage deviation of inflation from its steady state level. The technology and government expenditure levels are fixed at their steady state levels $a = 1$ and $\bar{g}$, respectively.

and issuing debt.[15] This policy reduces private consumption and stimulates economic activity, such that the aggregate resource constraint holds at the higher level of public expenditure. In period two, government expenditure returns to its pre-shock level but the government has a higher amount of debt outstanding, which must be serviced in the following periods. As revealed by the impulse responses shown in Figure 2, the government essentially achieves this by means of a monetary expansion. It increases the inflation rate, which reaches a maximum two years after the shock and then gradually returns to its steady state level. Observe that several years after the shock has occurred, inflation is still noticeably above its pre-shock level. On the other hand, the government in period two finds it optimal to reduce the labor tax, even slightly below the pre-shock level, and to raise taxes slightly but gradually later. This policy stimulates labor effort and helps to maintain a smooth consumption path, a feature of optimal

[15]Notice that nominal debt is initially inelastic, such that the instantaneous rise in the price level suppresses the real value of debt in period one.

Figure 2: Impulse responses to an i.i.d. government purchases shock



Note: The size of innovation in government purchases is one standard deviation. The shock takes place in period one. Public debt, consumption, output, the debt-to-money ratio, and velocity are measured in percent deviations from their pre-shock values. The labor income tax rate, the nominal interest rate, and the inflation rate are measured in percentage points.

policy that is familiar from the Ramsey literature.

The effects of imperfect competition

Finally, we briefly examine the role of imperfect competition. A comparison of the two panels of Table 1 reveals that, unlike in an environment of full commitment, imperfect competition has no *qualitative* effect on optimal policy under discretion. In particular, as we have argued before, the non-optimality of the Friedman rule does not hinge on this feature. However, the statistics in Table 1 show that there are noticeable *quantitative* effects of imperfect competition. For example, with $\theta = 20$ the government relies more on the inflation tax and less on the labor tax to finance its outlays compared to the perfectly competitive case $\theta = \infty$. The mechanism driving this result is the same that breaks the optimality of the Friedman rule under commitment. When markets are imperfectly competitive, households earn positive profits, which are pure rents from monopoly power. The government would like to confiscate these profits but cannot do so because it lacks the proper tax instrument. However, it can use inflation as an indirect tax on profits because the transactions technology requires households to hold money in order to make consumption purchases. This property makes inflation relatively more attractive as a tax instrument compared to an environment with perfect competition.[16]

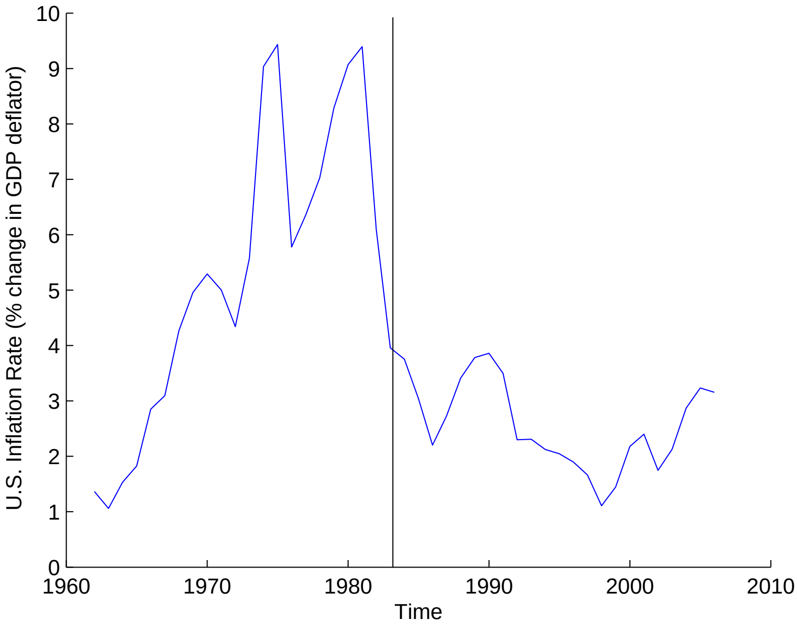
5 Sticky prices

The results of the previous section have demonstrated that inflation rates under optimal discretionary fiscal and monetary policies are positive and persistent, two features in line with the actual time series evidence. As visualized in Figure 3, annual inflation rates in the U.S. averaged at roughly 4% over the time period 1962-2006 with a standard deviation of about 2% and a first-order autocorrelation coefficient around 0.8. Considering only the period after the Volcker disinflation, 1983-2006, inflation was on average around 2.6% with a standard deviation of 0.8% and an autocorrelation coefficient of slightly below 0.8. These numbers demonstrate that the degree of persistence in our flexible price model is broadly in line with reality, whereas

[16]For further details see Schmitt-Grohe and Uribe (2004b).

the level of inflation clearly is not. Hence, the question arises whether the introduction of some form of nominal rigidity into the model is helpful in generating inflation dynamics resembling those observed in actual data. In what follows, we explore this question.

Figure 3: The U.S. GDP deflator 1962-2006



To introduce nominal rigidities into the model, we assume that price changes induce resource costs. Specifically, following Rotemberg (1982) and Schmitt-Grohe and Uribe (2004b), we postulate that households face quadratic price adjustment costs given by

$$(\kappa/2)\left(\tilde{P}_t/\tilde{P}_{t-1}-1\right)^2,$$

where κ is a non-negative parameter. Incorporating price adjustment costs complicates the model along two important dimensions. First, the price level of the preceding period, P_{t-1}, becomes a second endogenous state variable in the recursive formulation of the government's optimization problem. This is the case because the government, when choosing optimal policies, needs to know the previous period's price level in order to infer the magnitude of the price adjustment cost that is incurred. Second, in the model with costly price adjustment, the real wage rate is no longer a constant fraction of marginal productivity. Rather, real wages are determined by the forward-looking Phillips curve relation

$$\frac{w}{p} = \frac{(\theta-1)a}{\theta} + \frac{\kappa}{\theta h}\left\{\pi(1+\pi) - \beta E\left[\frac{u_c(c')\gamma(v)}{u_c(c)\gamma(v')}\pi'(1+\pi')\right]\right\}, \tag{24}$$

where π denotes the net inflation rate. The real wage thus generally differs from the desired mark-up over the marginal product of labor. Equation (24) is well-known in the macroeconomic literature, and we thus relegate its formal derivation, together with a description of the Markov-perfect equilibrium conditions in the sticky-price environment, to the Appendix.

Dynamics under sticky prices

We proceed by discussing the quantitative predictions of the sticky price model as summarized in Table 2. The most striking observation is that already a modest degree of rigidity has substantial effects on the inflation dynamics. With $\kappa = 0.5$, average annual inflation is down to below 3% compared to approximately 26% under flexible prices. With $\kappa = 1$, average inflation is close to 1%.[17] Similarly, the volatility of inflation as measured by its standard deviation declines substantially. On the other hand, both the level and the volatility of labor income taxes rise. This reflects the fact that, under sticky prices, the resource costs associated with inflation induce the government to rely more on labor taxes rather than on inflation to finance its outlays.

We observe furthermore that price stickiness slightly but noticeably affects the persistence of optimal inflation rates as measured by the first-order autocorrelation. Moreover, this effect is non-monotone. The intuition behind this property is the following. As previously explained, the level of government debt is the principal determinant of inflation. Specifically, the government's inflation incentives and, thus, equilibrium inflation are increasing in debt. Under sticky prices, the convexity of the price adjustment costs implies that the reduction of the government's inflation incentive is stronger at high levels of debt (or inflation). This property reduces the correlation between inflation and the level of debt, as confirmed by the policy functions depicted in Figure 4. Consequently, this effect leads to a lower degree of inflation persistence. On the

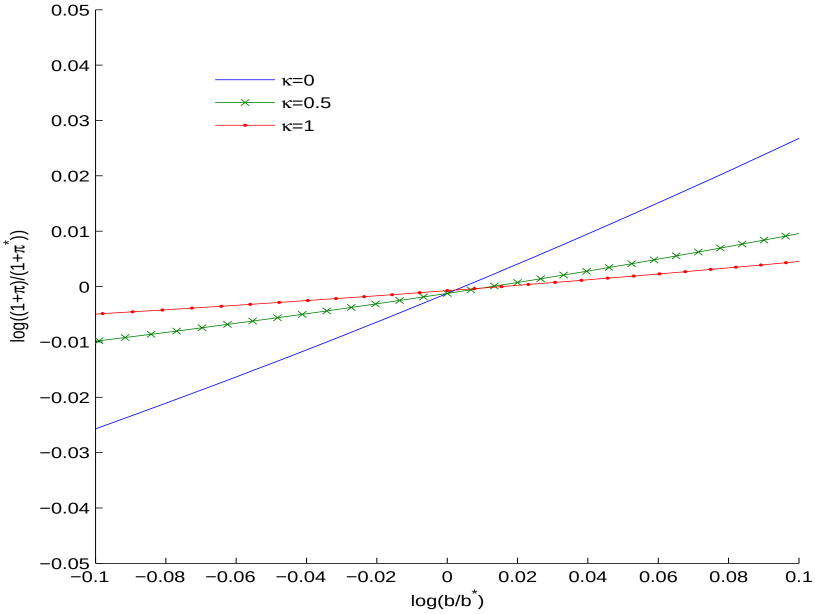
[17] Notice that our value for κ is several magnitudes smaller than values typically considered in the macroeconomics literature. For example, Schmitt-Grohe and Uribe (2004b) employ the value $\kappa = 4.375$ which, according to them, represents a "*miniscule* degree of price stickiness" (emphasis added).

Table 2: Dynamics under sticky prices

x	mean(x)	std(x)	corr(x', x)	corr(x, y)	corr(x, a)	corr(x, g)
			$\kappa = 0.5$			
π	2.5744	0.9648	0.7129	-0.1997	-0.5443	0.6171
R	6.6534	1.5874	0.8354	-0.4512	-0.7766	0.5456
τ	19.8609	1.2751	0.7851	0.1216	-0.3396	0.9247
v	4.1567	0.0271	-0.0024	0.0593	0.1528	0.1153
y	0.3289	0.0066	0.8447	1.0000	0.8822	0.4525
h	0.3289	0.0079	0.7437	-0.6246	-0.9184	0.3735
c	0.2685	0.0063	0.8768	0.6925	0.9323	-0.3253
b/p	0.0919	0.0048	0.9691	0.0190	-0.1167	0.4484
b	1.4252	0.0960	0.9665	-0.2194	-0.4024	0.6010
			$\kappa = 1$			
π	1.1429	0.6677	0.7879	-0.1845	-0.4758	0.5243
R	5.1588	1.3653	0.7990	-0.5025	-0.8152	0.4578
τ	19.9525	1.2531	0.7782	0.1488	-0.3310	0.9181
v	4.1437	0.0313	-0.0828	0.0754	0.2541	-0.0031
y	0.3291	0.0066	0.8461	1.0000	0.8674	0.4735
h	0.3291	0.0081	0.7359	-0.5916	-0.9140	0.3794
c	0.2688	0.0061	0.8848	0.6878	0.9331	-0.3090
b/p	0.0762	0.0089	0.9889	-0.0578	-0.1670	0.3774
b	1.1766	0.1525	0.9837	-0.1612	-0.3014	0.5026

other hand, introducing sticky prices increases the persistence of debt since it limits the shock-absorbing role of inflation (compare Schmitt-Grohe and Uribe, 2004b). This latter effect leads to a higher degree of inflation persistence. Whether inflation persistence increases or decreases due to price stickiness depends therefore on which of the two effects dominates.

Figure 4: Inflation policy under sticky prices



Note: On the horizontal axis we plot the percentage deviation of the debt-to-money ratio from its steady state level, and on the vertical axis the percentage deviation of inflation from its steady state level. The technology and government expenditure levels are fixed at the steady state levels $a = 1$ and $\bar{g}$, respectively.

Finally, we compare the dynamics of inflation in our sticky price model to inflation rates in the United States. The first panel in Table 2 shows that all three summary statistics (mean, standard deviation, autocorrelation coefficient) are somewhat smaller in the model than their counterparts in the data over the entire sample period 1962-2006. However, if we consider only the period after the Volcker disinflation (1983-2006), we find that the inflation dynamics in the model are well in line with the data. With $\kappa = 0.5$, inflation is approximately 2.6% on average in the model which is very close to the data; its standard deviation is roughly 1% in the model and 0.8% in the data; the first-order autocorrelation coefficient is 0.72 in the model compared to 0.78 in the data. With $\kappa = 1$, the simulations almost exactly reproduce the persistence of inflation in the data sample 1983-2006, while the average level and volatility of inflation are somewhat lower than their empirical counterparts. These results show that the predictions of the sticky price model are broadly in line with the inflation dynamics in the U.S. observed over the past 25 years. Regarding this positive implication, our framework improves upon Ramsey models of optimal policy which typically fail to rationalize observed inflation dynamics.[18]

Simple monetary policy rules

We conclude our analysis of optimal fiscal and monetary policy under sticky prices by examining how well these policies can be approximated by simple rules. In a seminal paper, Taylor (1993) has argued that U.S. monetary policy can be characterized in terms of a feedback rule according to which the nominal interest rate responds to changes in inflation and the output gap:

$$R_t = \rho_0 + \rho_\pi \pi_t + \rho_y \hat{y}_t + u_t. \tag{25}$$

In the above expression, ρ_0, ρ_π, and ρ_y are coefficients chosen by the monetary authority, $\hat{y}$ denotes the percentage deviation of output from its long-term trend (the output gap), and u_t is a residual. Taylor argues for $\rho_\pi = 1.5$ and $\rho_y = 0.5$, such that monetary policy is active in the sense that it responds to inflation more than one-by-one. In other words, the monetary authority raises the *real* interest rate in response to an increase in inflation. This property, known as the Taylor principle, is appealing also from a normative perspective because it stabilizes the economy by ensuring uniqueness of the rational expectations equilibrium.[19]

In what follows, we examine how well optimal monetary policy in the sticky price model introduced above can be represented by a Taylor-type policy rule. To this end, we generate simulated time series for the nominal interest rate, inflation, and output to estimate (25) by

[18] In this context, notice that by choosing a particular value of κ, one can effectively pin down the average inflation rate in the sticky price model. However, this has effects on the volatility and persistence of inflation. We believe that our model is in line with observed inflation dynamics because it simultaneously matches *all three* statistics (mean, standard deviation, autocorrelation coefficient). In addition, independent of their calibration, Ramsey models of optimal fiscal and monetary policy typically cannot generate any inflation persistence.

[19] This result has been derived by Leeper (1991) in a setting with passive fiscal policy, i.e., a fiscal policy rule which guarantees that the government budget constraint is satisfied for all possible paths of the price level.

means of OLS. Reporting the average coefficients across a sample of $N = 500$ simulations with a period length of $T = 1000$, the estimated equation is given by

$$R_t = 2.74 + 1.52\pi_t + 0.00\hat{y}_t + u_t, \qquad \mathrm{R}^2 = 0.85.$$

Accordingly, optimal monetary policy responds strongly to inflation and satisfies the Taylor principle.[20] It does not respond at all to the output gap, which is in line with Schmitt-Grohe and Uribe (2007) who argue that interest rate rules featuring a positive response to output can lead to significant welfare losses.

It is widely argued that monetary authorities display a tendency to adjust nominal interest rates only gradually in response to changes in economic conditions. Moreover, such a policy may be desirable from a welfare perspective (Woodford, 2003). In order to assess our model's implications in this light, we further introduce an interest-rate smoothing term in the estimated equation. Our main results are qualitatively insensitive to this modification. In particular, adding the lagged interest rate as explanatory variable we obtain

$$R_t = 1.29 + 1.09\pi_t + 0.00\hat{y}_t + 0.39R_{t-1} + u_t, \qquad \mathrm{R}^2 = 0.93.$$

Optimal policy still satisfies the Taylor principle and does not respond to output, while the interest-rate smoothing term is significant with a coefficient close to 0.4.[21] Finally, notice that the R^2 measures of the above regressions are 0.85 and 0.93, respectively. Thus, these simple Taylor-type rules seem to do a reasonably good job in describing optimal monetary policy.[22]

6 Conclusion

This paper has examined the dynamic properties of inflation in a model of optimal fiscal and monetary policy under discretion. In this model, there is a single benevolent government that

[20] The 99% confidence interval for the inflation coefficient is $[1.4067, 1.6352]$.

[21] The 99% confidence interval for the inflation coefficient is now $[1.0057, 1.1842]$, while for the interest rate coefficient it is $[0.3413, 0.4283]$.

[22] To examine the quality of approximation more thoroughly, we could follow Schmitt-Grohe and Uribe (2007) and compare welfare under both Taylor-type rules with welfare under optimal policy. This exercise is, however, beyond the scope of the present analysis.

can only use distortionary tax instruments, but can issue nominal state-noncontingent debt to shift distortions over time. Under lack of commitment and with nominal public debt, the government's problem is to optimally trade off the benefits and costs of inflation. On the one hand, unanticipated inflation in our model is attractive since it reduces the real value of outstanding liabilities. On the other hand, inflation is costly because it reduces current consumption possibilities by increasing transaction costs. This critical trade-off generates a rationale for fiscal and monetary policies that lead to positive and persistent inflation rates in equilibrium. This is true already for an economy with perfectly competitive product markets and flexible prices. Thus, our main results hold in a neo-classical environment. The introduction of New-Keynesian elements, i.e., imperfect competition and price stickiness, is found to have no qualitative effects on inflation dynamics but important quantitative implications. In particular, we show that, with a very modest degree of price rigidity, the dynamics of optimal inflation rates implied by our model closely resemble the dynamics observed for actual inflation rates in the U.S. since the Volcker disinflation of the early 1980s.

References

Adam, K., and R. M. Billi (2008): "Monetary conservatism and fiscal policy," *Journal of Monetary Economics*, 55(8), 1376–1388.

Bils, M., and P. J. Klenow (2004): "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, 112(5), 947–985.

Bils, M., P. J. Klenow, and B. A. Malin (2009): "Reset Price Inflation and the Impact of Monetary Policy Shocks," NBER Working Papers 14787, National Bureau of Economic Research, Inc.

Chari, V. V., L. J. Christiano, and P. J. Kehoe (1991): "Optimal Fiscal and Monetary Policy: Some Recent Results," *Journal of Money, Credit and Banking*, 23(3), 519–39.

Christiano, L., M. Eichenbaum, and C. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

Chugh, S. K. (2007): "Optimal inflation persistence: Ramsey taxation with capital and habits," *Journal of Monetary Economics*, 54(6), 1809–1836.

Cogley, T., and A. M. Sbordone (2008): "Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve," *American Economic Review*, 98(5), 2101–26.

Cooley, T. F., and G. D. Hansen (1991): "The Welfare Costs of Moderate Inflations," *Journal of Money, Credit and Banking*, 23(3), 483–503.

Diaz-Gimenez, J., G. Giovannetti, R. Marimon, and P. Teles (2008): "Nominal Debt as a Burden on Monetary Policy," *Review of Economic Dynamics*, 11(3), 493–514.

Erceg, C. J., and A. T. Levin (2003): "Imperfect credibility and inflation persistence," *Journal of Monetary Economics*, 50(4), 915–944.

Fuhrer, J., and G. Moore (1995): "Inflation Persistence," *The Quarterly Journal of Economics*, 110(1), 127–59.

Fuhrer, J. C. (2000): "Habit Formation in Consumption and Its Implications for Monetary-Policy Models," *American Economic Review*, 90(3), 367–390.

Gali, J., and M. Gertler (1999): "Inflation dynamics: A structural econometric analysis," *Journal of Monetary Economics*, 44(2), 195–222.

Heer, B., and A. Maussner (2005): *Dynamic general equilibrium modelling: computational methods and applications*. Springer.

Holden, S., and J. C. Driscoll (2003): "Inflation Persistence and Relative Contracting," *American Economic Review*, 93(4), 1369–1372.

Ireland, P. N. (2007): "Changes in the Federal Reserve's Inflation Target: Causes and Consequences," *Journal of Money, Credit and Banking*, 39(8), 1851–1882.

Judd, K. L. (1992): "Projection methods for solving aggregate growth models," *Journal of Economic Theory*, 58(2), 410–452.

——— (1998): *Numerical Methods in Economics*, vol. 1 of *MIT Press Books*. The MIT Press.

Klein, P., P. Krusell, and J.-V. Rios-Rull (2008): "Time-Consistent Public Policy," *Review of Economic Studies*, 75(3), 789–808.

Leeper, E. M. (1991): "Equilibria under 'active' and 'passive' monetary and fiscal policies," *Journal of Monetary Economics*, 27(1), 129–147.

Marimon, R., and A. Scott (1999): *Computational Methods for the Study of Dynamic Economies*. Oxford University Press.

Martin, F. (2009): "A Positive Theory of Government Debt," *Review of Economic Dynamics*, 12(4), 608–631.

Nakamura, E., and J. Steinsson (2008): "Five Facts about Prices: A Reevaluation of Menu Cost Models," *The Quarterly Journal of Economics*, 123(4), 1415–1464.

Niemann, S. (2009): "Dynamic Monetary-Fiscal Interactions and the Role of Monetary Conservatism," Economics Discussion Papers 667, University of Essex, Department of Economics.

Niemann, S., P. Pichler, and G. Sorger (2008): "Optimal Fiscal and Monetary Policy Without Commitment," Economics Discussion Papers 654, University of Essex, Department of Economics.

Ortigueira, S. (2006): "Markov-Perfect Optimal Taxation," *Review of Economic Dynamics*, 9(1), 153–178.

Rabanal, P., and J. F. Rubio-Ramirez (2003): "Inflation persistence: how much can we explain?," *Economic Review*, (Q2), 43–55.

Rotemberg, J. J. (1982): "Sticky Prices in the United States," *Journal of Political Economy*, 90(6), 1187–1211.

Rudd, J., and K. Whelan (2006): "Can Rational Expectations Sticky-Price Models Explain Inflation Dynamics?," *American Economic Review*, 96(1), 303–320.

Schmitt-Grohe, S., and M. Uribe (2004a): "Optimal fiscal and monetary policy under imperfect competition," *Journal of Macroeconomics*, 26(2), 183–209.

——— (2004b): "Optimal fiscal and monetary policy under sticky prices," *Journal of Economic Theory*, 114(2), 198–230.

——— (2007): "Optimal simple and implementable monetary and fiscal rules," *Journal of Monetary Economics*, 54(6), 1702–1725.

Siu, H. E. (2004): "Optimal fiscal and monetary policy with sticky prices," *Journal of Monetary Economics*, 51(3), 575–607.

Steinsson, J. (2003): "Optimal monetary policy in an economy with inflation persistence," *Journal of Monetary Economics*, 50(7), 1425–1456.

Svensson, L. E. O. (1985): "Money and Asset Prices in a Cash-in-Advance Economy," *Journal of Political Economy*, 93(5), 919–44.

Taylor, J. B. (1993): “Discretion versus policy rules in practice,” *Carnegie-Rochester Conference Series on Public Policy*, 39(1), 195–214.

Woodford, M. (2003): “Optimal Interest-Rate Smoothing,” *Review of Economic Studies*, 70(4), 861–886.

Appendix

A1. The private agent's optimality conditions

In this appendix we derive the optimality conditions associated with the representative private agent's optimization problem. We employ a recursive formulation of this problem.

Inspection of (5) and (6) reveals that the set of *individual* endogenous payoff-relevant state variables in the representative agent's problem is given by $\{m, b\}$. Furthermore, (7) shows that there is a single *aggregate* endogenous state variable, the government debt-to-money ratio $\bar{b}$. The latter variable becomes an aggregate state variable, because the government bases its policy decisions on it so that it enters the private agent's problem via policy variables and market prices (such as the interest rate or the after-tax wage).

Taking these consideration into account and using recursive notation (where primes indicate next-period variables), the optimization problem of the representative agent is given by

$$V(m, b, \bar{b}; a, g) = \max_{c,h,m',b',\tilde{p}} \left[u(c) - \alpha h + \beta E V(m', b', \mathcal{B}'(\bar{b}, a, g); a', g') \right]$$

subject to obvious non-negativity constraints, a no-Ponzi constraint, the laws of motion for a and g, and the flow budget constraint

$$c\left[1 + s\left(\frac{pc}{m}\right)\right] + \frac{(1+\mu)(m' + qb')}{p} \leq \frac{m + b + (1-\tau)wh}{p} + y\left(\frac{\tilde{p}}{p}\right)^{1-\theta} - \frac{wy}{ap}\left(\frac{\tilde{p}}{p}\right)^{-\theta}.$$

The latter is a combination of (5) and (6) and is repeated here for convenience. Notice that $\mathcal{B}'(\bar{b}, a, g)$ is the private agent's perceived law of motion for the endogenous aggregate state variable $\bar{b}$. Denoting the Lagrangian multiplier attached to the budget constraint by λ, the first-order optimality conditions are given by

$$\begin{aligned}
0 &= u_c(c) - \lambda\left[1 + s\left(\frac{pc}{m}\right) + cs_v\left(\frac{pc}{m}\right)\frac{p}{m}\right], \\
0 &= -\alpha + \frac{\lambda}{p}(1-\tau)w, \\
0 &= \beta E\left\{\frac{\lambda'}{p'}\left[1 + s_v\left(\frac{p'c'}{m'}\right)\left(\frac{p'c'}{m'}\right)^2\right]\right\} - \frac{\lambda}{p}(1+\mu), \\
0 &= \beta E\frac{\lambda'}{p'} - \frac{\lambda}{p}(1+\mu)q,
\end{aligned}$$

$$0 = \lambda \left[\frac{(1-\theta)y}{p} \left(\frac{\tilde{p}}{p} \right)^{-\theta} + \frac{\theta w y}{a p^2} \left(\frac{\tilde{p}}{p} \right)^{-\theta-1} \right],$$

where s_v and u_c denote the derivatives of the functions s and u, respectively. Notice that we have used the envelope conditions

$$V_m = \frac{\lambda}{p} \left[1 + s_v \left(\frac{pc}{m} \right) \left(\frac{pc}{m} \right)^2 \right] \qquad \text{and} \qquad V_b = \frac{\lambda}{p}$$

to eliminate the derivatives of the value function. Imposing symmetry and market clearing, we obtain $m = 1$, $b = \bar{b}$, and $\tilde{p} = p$. The optimality conditions of the agent can then be written as

$$\begin{aligned}
0 &= u_c(c) - (\lambda/p)\left[1 + s(v) + s_v(v)v\right] p, \\
0 &= -\alpha + (\lambda/p)(1-\tau)w, \\
0 &= \beta E \left\{ \frac{\lambda'}{p'} \left[1 + s_v(v')(v')^2 \right] \right\} - \frac{\lambda}{p}(1+\mu), \\
0 &= \beta E(\lambda'/p') - (\lambda/p)(1+\mu)q, \\
0 &= c\left[1 + s(v)\right] + \frac{(1+\mu)(1+qb')}{p} - \frac{1+b}{p} - \left(1 - \frac{\tau w}{ap}\right) ah, \\
0 &= v - pc, \\
0 &= \frac{w}{p} - \frac{(\theta-1)a}{\theta}.
\end{aligned}$$

Eliminating λ/p and introducing the function $\gamma(v) = 1 + s(v) + s_v(v)v$, one arrives at the system (8)-(13).

A2. Proof of Proposition 1

The proof is by contradiction. Let us therefore assume that the Friedman rule holds close to the steady state b^*, and that b^* is a stable fixed point of the equilibrium dynamics. The Friedman rule implies that $v = \mathcal{V}(b) = \underline{v}$ and $q = \mathcal{Q}(b) = 1$ whenever $b \in (b^* - \delta, b^* + \delta)$. This shows that, close to the steady state, the functions $\tilde{\mu}(c, v, b')$ and $\tilde{r}(c, v, b')$ defined in (17)-(18) depend on b' only via the function $\mathcal{C}(b')$. Moreover, because the utility function has the form (23) we know that $\mathcal{C}(b')u_c(\mathcal{C}(b')) = \mathcal{C}(b')^{1-\sigma}$. Taking these observations together, it follows from (17)-(18) that equation (22) is equivalent to

$$0 = \beta(1-\sigma)(1+b^*)\mathcal{C}_b(b^*)/\mathcal{C}(b^*). \qquad (26)$$

Now consider the government's budget constraint (7) and the implementability constraints (8)-(13). Using the assumptions that we have a deterministic model with $g = \bar{g}$ and $a = 1$ and that the Friedman rule holds around the steady state, i.e., $q = 1$ and $v = \underline{v}$, we can write these conditions as

$$
\begin{aligned}
&\frac{\tau w h}{p} + \frac{(1+\mu)(1+b')}{p} - \frac{1+b}{p} = \bar{g},\\
&\frac{\alpha p}{(1-\tau)w} = \frac{p c u_c(c)}{\underline{v}},\\
&\beta f(c') = f(c)(1+\mu),\\
&c + \frac{(1+\mu)(1+b')}{p} - \frac{1+b}{p} = \left(1 - \frac{\tau w}{p}\right) h,\\
&\underline{v} = cp,\\
&\frac{w}{p} = \frac{\theta - 1}{\theta},
\end{aligned}
$$

where $f(c) = c u_c(c)$. Using the first equation as well as the last three equations to eliminate the variables w, p, h, and τ, one obtains

$$[\mathcal{C}(b) + \bar{g}] \left[\frac{\theta - 1}{\theta} - \frac{\alpha}{u_c(\mathcal{C}(b))}\right] + \frac{[1 + \mathcal{M}(b)][1 + \mathcal{B}'(b)]\mathcal{C}(b)}{\underline{v}} - \frac{(1+b)\mathcal{C}(b)}{\underline{v}} = \bar{g}, \tag{27}$$

$$\beta f(\mathcal{C}(\mathcal{B}'(b)) = f(\mathcal{C}(b))[1 + \mathcal{M}(b)] \tag{28}$$

for all b sufficiently close to b^*. Because $\mathcal{B}'(b^*) = b^*$ holds by the definition of a steady state, it follows immediately from (28) that $\mathcal{M}(b^*) = \beta - 1$.

In the next step, we prove that $\mathcal{C}_b(b^*) \neq 0$. Suppose to the contrary that $\mathcal{C}_b(b^*) = 0$ holds. In that case it follows from (28) that $\mathcal{M}_b(b^*) = 0$ must hold as well. Differentiating (27) with respect to b, evaluating at $b = b^*$, and using $\mathcal{B}'(b^*) = b^*$, $\mathcal{M}(b^*) = \beta - 1$, and $\mathcal{M}_b(b^*) = 0$, one obtains

$$\mathcal{C}_b(b^*) \left\{\frac{\theta - 1}{\theta} - \frac{\alpha}{u_c(\mathcal{C}(b^*))} + \frac{\alpha[\mathcal{C}(b^*) + \bar{g}]u_{cc}(\mathcal{C}(b^*))}{u_c(\mathcal{C}(b^*))^2} + \frac{(\beta - 1)(1 + b^*)}{\underline{v}}\right\} = \frac{\mathcal{C}(b^*)}{\underline{v}}[1 - \beta \mathcal{B}'_b(b^*)].$$

Since $\mathcal{C}_b(b^*) = 0$ has been assumed, the left-hand side of this equation is equal to zero. But this implies that the right-hand side must also be equal to zero which, in turn, requires $\mathcal{B}'_b(b^*) = 1/\beta$. On the other hand, stability of b^* in the equilibrium dynamics requires that $|\mathcal{B}'_b(b^*)| < 1$. This contradiction completes the proof of $\mathcal{C}_b(b^*) \neq 0$.

For the final step of the proof, we note that $\sigma \neq 1$ and $\mathcal{C}_b(b^*) \neq 0$ together with (26) imply that $1 + b^* = 0$. Now consider the optimality condition with respect to velocity, (20). Evaluating this condition at a distorted steady state where $\eta \neq 0$ and noting that $v = \underline{v}$ due to the Friedman rule, it follows that

$$0 = \underline{v}\tilde{T}_v(\underline{v}, c^*)/c^* - (\beta - 1)(1 + b^*)/\underline{v} + \beta[1 + \underline{v}\gamma_v(\underline{v})]\,(1 + b^*)\,/\underline{v}.$$

Since we know that $1 + b^* = 0$ must hold, this condition can be written as $\underline{v}\tilde{T}_v(\underline{v}, c^*)/c^* = 0$. On the other hand, from assumptions (ii) and (iv) on the properties of the transaction cost function $s(v)$ it follows that $\underline{v}\tilde{T}_v(\underline{v}, c^*)/c^* < 0$. Hence, we have a contradiction and the proof of the proposition is complete.

A3. Proof of Proposition 2

PROOF: Since $u(c) = \log c$ we have $cu_c(c) = 1$ such that the functions $\tilde{\mu}$ and $\tilde{r}$ in (17)-(18) do not depend on c and that they depend on b' only via the function $\mathcal{V}(b')$. Moreover, because of $s(v) = v/(4A) + A/v - 1$ we have $s_v(v) = 1/(4A) - A/v^2$ and $\gamma(v) = v/(2A)$. Using these observations, one easily finds that

$$\begin{aligned}\tilde{\mu}_{b'}(c^*, v^*, b^*) &= \frac{2\beta(A-1)\mathcal{V}_{b'}(b^*)}{v^*},\\ \tilde{r}_{b'}(c^*, v^*, b^*) &= -\frac{2\beta\mathcal{V}_{b'}(b^*)}{v^*}.\end{aligned}$$

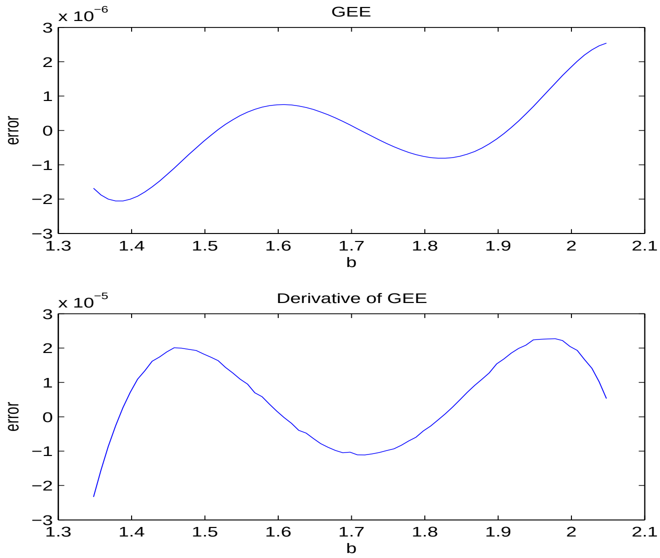
Substituting this into condition (22) it follows that $b^* = A - 1$ is a steady state. □

A4. The numerical algorithm

Our numerical solution for the Markov-perfect equilibrium employs a Galerkin projection method along the lines of Judd (1992). This method is well-known among economists and discussed in several textbooks such as Judd (1998), Marimon and Scott (1999), or Heer and Maussner (2005). We therefore refer the interested reader to these textbooks for a general description of the methodology, and to our computer code for details about the particular implementation.

We do present, however, a brief evaluation of the numerical accuracy of the algorithm. Figure 5 plots the approximation error in the generalized Euler equation (21) as well as its partial

Figure 5: Approximation error



derivative (wrt. the endogenous state variable b) as a function of b, holding the exogenous state variables fixed at their respective steady state levels. Notice that the inspection of the error in the derivative is important because the generalized Euler equation establishes a *functional* relationship such that optimal policy rules must guarantee not only that the equation itself holds (approximately), but also that its derivative is approximately equal to zero at all values of b. For our fourth-order polynomial approximation, the errors in the generalized Euler equation and its derivative are roughly of the order of magnitude 10^{-5}, which we view as a reasonable level of accuracy. Numerical experiments using a fifth-order approximation have shown that the approximation error could be further reduced. However, small accuracy improvements come at the cost of a dramatically increased computational burden since the model has several state variables. More importantly, refining the approximation turned out to have only negligible effects on the dynamic properties of the model as captured by summary statistics or impulse responses. We thus conclude that the fourth-order approximation is sufficiently accurate for our purposes.

A5. Equilibrium conditions in the sticky price model

In this appendix we derive the equilibrium conditions for the sticky price model from Section 5. First, notice that with quadratic price adjustment costs, the private agent's budget constraint reads

$$
\begin{aligned}
&M_t + B_t + (1-\tau_t)W_t h_t + \tilde{P}_t y_t \left(\tilde{P}_t/P_t\right)^{-\theta} - W_t \tilde{h}_t - (\kappa/2)\left(\tilde{P}_t/\tilde{P}_{t-1} - 1\right)^2 P_t \\
&\qquad \geq P_t c_t[1+s(v_t)] + M_{t+1} + q_t B_{t+1}.
\end{aligned}
$$

Dividing this equation by P_t and using $a_t \tilde{h}_t = y_t(\tilde{P}_t/P_t)^{-\theta}$, we obtain

$$
\begin{aligned}
&\frac{m_t + b_t}{p_t} + \frac{(1-\tau_t)w_t h_t}{p_t} + y_t \left(\frac{\tilde{p}_t}{p_t}\right)^{1-\theta} - \frac{w_t y_t}{p_t a_t}\left(\frac{\tilde{p}_t}{p_t}\right)^{-\theta} - \frac{\kappa}{2}\left[\frac{\tilde{p}_t(1+\mu_{t-1})}{\tilde{p}_{t-1}} - 1\right]^2 \\
&\geq c_t[1+s(v_t)] + \frac{(1+\mu_t)(m_{t+1} + q_t b_{t+1})}{p_t},
\end{aligned}
$$

where lower case letters indicate normalized variables of the kind $x_t = X_t/\bar{M}_t$ and where $1+\mu_t$ is the rate of money growth between (the beginning of) periods t and $t+1$. Introducing $\tilde{z}_t = (1+\mu_{t-1})/\tilde{p}_{t-1}$ and using recursive notation, the optimization problem of a representative agent is given by

$$
V(m, b, \bar{b}, \tilde{z}; a, g) = \max_{c,h,m',b',\tilde{p}} \left[u(c) - \alpha h + \beta E V(m', b', \mathcal{B}'(\bar{b}), \tilde{z}'; a', g')\right]
$$

subject to obvious non-negativity constraints, the laws of motion for a and g, and the flow budget constraint

$$
\begin{aligned}
&c\left[1 + s\left(\frac{pc}{m}\right)\right] + \frac{(1+\mu)(m' + qb')}{p} \\
&\quad \leq \frac{m+b}{p} + \frac{(1-\tau)wh}{p} + y\left(\frac{\tilde{p}}{p}\right)^{1-\theta} - \frac{wy}{pa}\left(\frac{\tilde{p}}{p}\right)^{-\theta} - \frac{\kappa}{2}\left(\tilde{p}\tilde{z} - 1\right)^2.
\end{aligned}
$$

Denoting the Lagrangian multiplier attached to the budget constraint by λ, the first-order conditions associated with the agent's problem are given by

$$
\begin{aligned}
0 &= u_c(c) - \lambda\left[1 + s\left(\frac{pc}{m}\right) + cs_v\left(\frac{pc}{m}\right)\frac{p}{m}\right],\\
0 &= -\alpha + \frac{\lambda}{p}(1-\tau)w,\\
0 &= \beta E\left\{\frac{\lambda'}{p'}\left[1 + s_v\left(\frac{p'c'}{m'}\right)\left(\frac{p'c'}{m'}\right)^2\right]\right\} - \frac{\lambda}{p}(1+\mu),\\
0 &= \beta E\frac{\lambda'}{p'} - \frac{\lambda}{p}(1+\mu)q,\\
0 &= \lambda\left[\frac{(1-\theta)y}{p}\left(\frac{\tilde{p}}{p}\right)^{-\theta} + \frac{\theta w y}{ap^2}\left(\frac{\tilde{p}}{p}\right)^{-\theta-1} - \kappa\left(\tilde{p}\tilde{z}-1\right)\tilde{z}\right] + \beta E\left[\lambda'\kappa\left(\tilde{p}'\tilde{z}'-1\right)\frac{\tilde{p}'\tilde{z}'}{\tilde{p}}\right],\\
0 &= \tilde{z}' - (1+\mu)/\tilde{p},
\end{aligned}
$$

where we have used $\partial\tilde{z}'/\partial\tilde{p} = -\tilde{z}'/\tilde{p}$ as well as the envelope conditions

$$
\begin{aligned}
V_m &= \frac{\lambda}{p}\left[1 + s_v\left(\frac{pc}{m}\right)\left(\frac{pc}{m}\right)^2\right],\\
V_b &= \lambda/p,\\
V_z &= -\lambda\kappa\left(\tilde{p}\tilde{z}-1\right)\tilde{p}.
\end{aligned}
$$

Our focus is on a symmetric private-sector equilibrium. In such an equilibrium, it must hold that $m = 1$, $b = \bar{b}$, $\tilde{p} = p$, and $\tilde{z} = z$. Substituting this into the optimality conditions from above, we obtain

$$
\begin{aligned}
0 &= u_c(c) - (\lambda/p)\left[1 + s(v) + s_v(v)\,v\right]p,\\
0 &= -\alpha + (\lambda/p)(1-\tau)w,\\
0 &= \beta E\left\{\frac{\lambda'}{p'}\left[1 + s_v(v')\,(v')^2\right]\right\} - \frac{\lambda}{p}(1+\mu),\\
0 &= \beta E(\lambda'/p') - (\lambda/p)(1+\mu)q,\\
0 &= \lambda(pz-1)pz - \beta E\left[\lambda'(p'z'-1)p'z'\right] - \frac{\theta\lambda h}{\kappa p}\left(w - \frac{\theta-1}{\theta}ap\right), \qquad (29)\\
0 &= z' - (1+\mu)/p,\\
0 &= v - pc.
\end{aligned}
$$

Noting that pz is the gross inflation rate, one can see that (29) is the forward-looking New-Keynesian Philips curve (24). Moreover, in a private-sector equilibrium the agent's and the government's budget constraints must be satisfied. These conditions read

$$0 = c\left[1 + s\left(v\right)\right] + \frac{(1+\mu)(1+qb')}{p} - \frac{1+b}{p} - (1 - \frac{\tau w}{ap})ah + \frac{\kappa}{2}\left(pz - 1\right)^2,$$
$$0 = \frac{(1+\mu)(1+qb')}{p} - \frac{1+b}{p} + \frac{\tau wh}{p} - g,$$

and can be combined to the aggregate resource constraint

$$0 = ah - c\left[1 + s\left(v\right)\right] - g - (\kappa/2)\left(pz - 1\right)^2.$$

Let us now recall the definition of the function $\gamma(v) = 1 + s(v) + s_v(v)v$. By Walras' law we can replace the private agent's budget constraint with the aggregate resource constraint. After elimination of λ/p this gives us the following system of equations:

$$0 = \frac{\alpha}{(1-\tau)w} - \frac{cu_c(c)}{v\gamma(v)}, \tag{30}$$
$$0 = \beta E\left\{\frac{c'u_c(c')}{v'\gamma(v')}\left[1 + s_v(v')v'^2\right]\right\} - \frac{cu_c(c)}{v\gamma(v)}(1+\mu), \tag{31}$$
$$0 = \beta E\left[\frac{c'u_c(c')}{v'\gamma(v')}\right] - \frac{cu_c(c)}{v\gamma(v)}(1+\mu)q, \tag{32}$$
$$0 = (pz-1)pz - \beta E\left[\frac{u_c(c')\gamma(v)}{u_c(c)\gamma(v')}(p'z'-1)p'z'\right] - \frac{\theta h}{\kappa}\left[\frac{w}{p} - \frac{(\theta-1)a}{\theta}\right], \tag{33}$$
$$0 = c[1+s(v)] + g + (\kappa/2)\left(pz-1\right)^2 - ah, \tag{34}$$
$$0 = \frac{(1+\mu)(1+qb')}{p} - \frac{1+b}{p} + \frac{\tau wh}{p} - g, \tag{35}$$
$$0 = z' - (1+\mu)/p, \tag{36}$$
$$0 = v - pc. \tag{37}$$

We can now define the functions $\tilde{\mu}$ and $\tilde{r}$ as

$$\tilde{\mu}(v, c, b'; a, g) = \beta E\left\{\frac{\mathcal{C}u_c(\mathcal{C})v\gamma(v)}{cu_c(c)\mathcal{V}\gamma(\mathcal{V})}\left[1 + s_v(\mathcal{V})\mathcal{V}^2\right]\right\} - 1, \tag{38}$$
$$\tilde{r}(v, c, b'; a, g) = \beta E\left\{\frac{\mathcal{C}u_c(\mathcal{C})v\gamma(v)}{cu_c(c)\mathcal{V}\gamma(\mathcal{V})}\right\}, \tag{39}$$

where the functions $\mathcal{C}$ and $\mathcal{V}$ are evaluated at (b', z', a', g') with $z' = \left[1 + \tilde{\mu}(v, c, b'; a, g)\right]c/v$. Notice that (38) and (39) define $\tilde{\mu}$ and $\tilde{r}$ only implicitly. Furthermore, using (37) and the aggregate resource constraint (34), we can write labor supply in a private-sector equilibrium as

$$\tilde{h}(v, c; z, a, g) = \frac{1}{a}\left\{c[1 + s(v)] + g + \frac{\kappa}{2}\left(\frac{vz}{c} - 1\right)^2\right\}. \tag{40}$$

Notice further that equation (33) determines the real wage as

$$\frac{w}{p} = \frac{(\theta - 1)a}{\theta} + \frac{\kappa}{\theta h}\left\{(pz - 1)pz - \beta E\left[\frac{u_c(c')\gamma(v)}{u_c(c)\gamma(v')}(p'z' - 1)p'z'\right]\right\},$$

such that we can write the equilibrium wage as a function

$$\begin{aligned}\tilde{w}(v, c, b', z'; z, a, g) = &\frac{(\theta - 1)va}{\theta c}\\ &+ \frac{\kappa v}{\theta c}\left\{\left(\frac{vz}{c} - 1\right)\frac{vz}{c} - \beta E\left[\frac{u_c(\mathcal{C})\gamma(v)}{u_c(c)\gamma(\mathcal{V})}\left(\frac{\mathcal{V}c}{v\mathcal{C}}[1 + \tilde{\mu}(\cdot)] - 1\right)\frac{\mathcal{V}c}{v\mathcal{C}}[1 + \tilde{\mu}(\cdot)]\right]\right\}\tilde{h}(\cdot)^{-1}.\end{aligned} \tag{41}$$

Finally, from (30) we have

$$\tilde{\tau}(v, c, b', z'; z, a, g) = 1 - \frac{\alpha v \gamma(v)}{c u_c(c)\tilde{w}(\cdot)}. \tag{42}$$

Combining (40), (41), and (42), we can construct a function $\tilde{T}$, which summarizes the government's equilibrium real labor tax revenue, given choices for v, c, b', z', and given the states z, a, and g. This function is given by

$$\tilde{T}(v, c, b', z'; z, a, g) = \frac{c\tilde{\tau}(\cdot)\tilde{w}(\cdot)\tilde{h}(\cdot)}{v}.$$

In the next step, we introduce again the government's continuation value function. In analogy to the flexible price environment, this function is recursively defined by

$$\begin{aligned}\mathcal{U}(b', z', a', g') = &\, u(\mathcal{C}(b', z', a', g')) - \alpha\mathcal{H}(b', z', a', g')\\ &+ \beta E\mathcal{U}(\mathcal{B}'(b', z', a', g'), \mathcal{Z}'(b', z', a', g'), a'', g''),\end{aligned}$$

where $\mathcal{C}$, $\mathcal{H}$, $\mathcal{B}'$, and $\mathcal{Z}'$ are the solution to the private-sector equilibrium conditions under sticky prices and for given policies.

Having introduced the function $\tilde{T}$ and the continuation value function $\mathcal{U}$, the current government seeks to maximize the objective function

$$u(c) - \frac{\alpha}{a}\left\{c[1 + s(v)] + g + \frac{\kappa}{2}\left(\frac{vz}{c} - 1\right)^2\right] + \beta E\mathcal{U}(b', c[1 + \tilde{\mu}(\cdot)]/v, a', g')\right\}$$

with respect to c, v, and b and subject to

$$\tilde{T}(\cdot) + \frac{c}{v}[\tilde{\mu}(\cdot) + \tilde{r}(\cdot)b' - b] - g = 0. \tag{43}$$

The first-order conditions associated with this problem are

$$\begin{aligned}
&- u_c(c) + \frac{\alpha}{a}\left[1 + s(v) - \kappa\left(\frac{vz}{c} - 1\right)\frac{vz}{c^2}\right] = \frac{\eta v}{c}\left\{\tilde{T}_c(\cdot) + \frac{\tilde{\mu}(\cdot) + \tilde{r}(\cdot)b' - b}{v}\right. \\
&\qquad \left. + \frac{c\left[\tilde{\mu}_c(\cdot) + \tilde{r}_c(\cdot)b'\right]}{v}\right\} - \beta\kappa E\left[\frac{\alpha}{a'}\left(\frac{v'z'}{c'} - 1\right)\frac{v'}{c'}\right]\frac{1 + \tilde{\mu}(\cdot) + c\tilde{\mu}_c(\cdot)}{v}, \\
&\frac{\alpha}{a}\left[cs_v(v) + \kappa\left(\frac{vz}{c} - 1\right)\frac{z}{c}\right] = \frac{\eta v}{c}\left\{\tilde{T}_v(\cdot) - \frac{c\left[\tilde{\mu}(\cdot) + \tilde{r}(\cdot)b' - b\right]}{v^2} + \frac{c\left[\tilde{\mu}_v(\cdot) + \tilde{r}_v(\cdot)b'\right]}{v}\right\} \\
&\qquad + \beta\kappa E\left[\frac{\alpha}{a'}\left(\frac{v'z'}{c'} - 1\right)\frac{v'}{c'}\right]\frac{c}{v}\left[\frac{1 + \tilde{\mu}(\cdot)}{v} - \tilde{\mu}_v(\cdot)\right], \\
&\beta E\eta' = \frac{\eta v}{c}\tilde{T}_{b'}(\cdot) + \eta\left[\tilde{r}(\cdot) + \tilde{\mu}_{b'}(\cdot) + \tilde{r}_{b'}(\cdot)b'\right] - \beta\kappa E\left[\frac{\alpha}{a'}\left(\frac{v'z'}{c'} - 1\right)\frac{v'}{c'}\right]\frac{c\tilde{\mu}_{b'}(\cdot)}{v},
\end{aligned}$$

where $\eta v/c$ is the multiplier attached to the government's implementability constraint. Together with (43), these first-order conditions fully characterize the Markov-perfect equilibrium. Finally, notice that when $\kappa = 0$, then $\tilde{T}_{b'} = 0$ and the equilibrium conditions boil down to the flexible price conditions (19)-(21).