Bank liquidity, bank lending, and "bad bank" policies*

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Abstract

Why are bank deposits demandable when they are also negotiable? We present a General Equilibrium model in which demandable debt exposes banks to liquidity risk so that they can signal their types and ensure that their liabilities can circulate as a means of payment. Banks can manage their liquidity risk by altering their deposit rate and their lending scale. When banks are transparent, so that depositors have homogenous information about their assets, they use only the former tool: their lending scale is efficient, and they do not experience liquidity crisis. When banks are opaque, so that depositors receive private signals of their quality, they inefficiently shrink the scale of their lending. A bank's stock of liquid assets affects its capacity for risk taking. A "bad bank" policy can resolve liquidity crises by reducing the opacity of the bank's assets.

Keywords: Liquidity crises, demandable deposits, negotiable deposits, bad bank policies.

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1. Introduction

Notwithstanding several decades of research into commercial banking, no theory explains a fundamental fact about commercial banks: namely, that a significant fraction of their liabilities is both negotiable and demandable. Demandability places the bank in a precarious position, because it could at any time experience a large-scale withdrawal, which would cause a liquidity crisis. Why, then, do banks expose themselves to this risk? Following the seminal work of Diamond and Dybvig (1983), a vast literature argues that deposit demandability is a form of insurance that ensures that depositors will be able to satisfy an immediate consumption need, should it arise. But this explanation ignores the fact that bank liabilities are negotiable, so that a bank depositor could meet an immediate need for funds even if her deposit were not demandable: instead, she could use a bank card, make a bank transfer, or write a cheque on her account. Indeed, if depositors were to withdraw funds in response to every stochastic consumption need as Diamond and Dybvig (1983)-style models assume, then banks would experience a huge volume of daily withdrawals, with total value equal to depositors' aggregate demand for liquidity. But, in reality, the level of bank withdrawals in normal times is very small. [1] This fact pre-dates the emergence of modern, electronic payment systems:[2] Van Dillen (1964) reports that the Bank of Amsterdam experienced no withdrawals in the century before the Fourth Anglo-Dutch war of 1780–84.

The literature that follows Diamond and Dybvig (1983) ignores the negotiability of bank deposits and treats bank liquidity management as an optimal liquidity risk sharing problem. But we have already seen that this is an incorrect characterisation of real-world banking: in normal times, withdrawals are rare so that the liquidity needs of depositors do not have a significant effect upon bank liquidity. Our contribution is to investigate bank liquidity in a model that incorporates negotiable bank deposits. We start by explaining why banks offer the right to withdraw on demand when their depositors can use deposit negotiability to meet their liquidity needs. We then examine a series of questions of natural concern to banks in this case. First, under what circumstances will depositors exercise their rights and, hence, expose banks to substantial withdrawal demands? Second, when do banks fail to meet those withdrawal demands and, hence, experience liquidity crises? Third, how does the possibility of liquidity crises affect banks' lending decisions? Finally, we ask how regulators should respond to bank liquidity crises. We keep depositor liquidity shocks in our model in order to situate it in the existing literature, but liquidity risk sharing plays little role in our analysis,

[1] When depositors use their deposit as a means of payment, they transfer their claim to a seller of goods. If the seller has an account at the same bank, no cash is withdrawn and the transfer is accomplished through a series of bank ledger entries. If the seller's account is with a different bank, the payment creates an inter-bank liability. Even in this case, a transfer of bank cash reserves is very unusual: most inter-bank claims net out and the creditor bank seldom demands immediate settlement of the net position, given that the position earns inter-bank interest.

[2] The importance of negotiability for liquidity in general is discussed by Kiyotaki and Moore (2002).

and our results survive even when depositors do not experience liquidity shocks.

We solve a general equilibrium model in which, at time 0 banks use their liabilities to hire workers, who engage in long-term production. As in the classic Diamond and Dybvig set-up, workers face uncertainty over the timing of their consumption needs. Banks may give their liability holders the right to demand early repayment but, in contrast to Diamond and Dybvig, we allow liability holders to respond to consumption shocks by reselling their claims on the bank. Another difference between our analysis and Diamond and Dybvig's is that the banks in our model have time 0 private information about the quality of their long-term investments. Banks that face a run in our model suspend withdrawals and suffer a loss.

Our explanation for demandable deposits rests upon the information asymmetry between banks and depositors. We demonstrate that banks use demandable deposit contracts to signal their quality. High-quality banks promise lower long-term repayments than low-quality banks; a low-quality bank that mimics a high-quality bank experiences a run when its type is revealed. If the cost of liquidity crises is high enough, then in every equilibrium of our game all banks offer demandable deposits; nevertheless, there are no runs and there is no early withdrawal. Liquidity-shocked depositors use their bank claims to purchase consumption goods. Moreover, liquidity risk does not affect lending efficiency in this model: the second-best allocation is implemented in the competitive equilibrium despite the presence of asymmetric information. Banks have two techniques to control the scale of withdrawals and, hence, to avoid liquidity crises: they can reduce their lending scale, which, because they create deposits in order to lend, shrinks their depositor base; or they can promise higher depositor rates in order to discourage withdrawals. In our baseline model, banks use only the second technique so that they lend at the socially optimal level.

Our model therefore yields a novel explanation for deposit demandability. Depositor liquidity insurance derives from deposit negotiability and not from demandability. Demandability emerges because there is uncertainty over bank quality, and not because depositors face liquidity shocks: indeed, our results hold even when depositors face no liquidity risk. Demandability gives depositors the ability to inflict serious losses on their bank when they find evidence of unsoundness and, hence, can be used ex ante to demonstrate a bank's soundness.

In this version of our model banks experience no withdrawals and there are no bank runs. This is close to our experience of normal markets, in which banks seldom face mass withdrawals. When the cost of a liquidity crisis is lower, this model can deliver runs. With the lower cost, the model admits a partially separating equilibrium: the highest- and lowest-quality banks all offer the same contract, while banks of intermediate quality use their contracts to separate. The lowest quality banks all experience equilibrium runs when their types are revealed; the resultant costs are worth bearing because their are outweighed by the lower long-term payments that they achieve by pooling with highest quality banks.

The partially separating equilibrium of the previous paragraph supplies an explanation for equilibrium depositor withdrawals, and for the scale of those withdrawals. In the partially separating equilibrium, liquidity crises are a consequence of solvency problems, in the sense that the deposit rate is insufficient to compensate for default risk: either a bank is solvent, in which case none of its depositors withdraw early, or it is insolvent, in which case all of its depositors withdraw. It follows that, in this equilibrium, a bank's holdings of liquid assets have no part to play in its liquidity risk management strategy.

Our baseline model yields demand deposits and liquidity crises that derive from solvency risk. But it fails to address two important facts. First, solvent banks can experience liquidity crises. Second, liquid asset holdings are relevant to a bank's management of liquidity risk. An extension to our model addresses these points.

In this extension, we assume that depositors gather information about their banks from a variety of sources. Some banks experience an *opacity shock*: that is, their depositors receive private signals of their bank's quality. Information about *opaque banks* is therefore dispersed, so that no single person has a complete picture of their quality. Depositors with bad signals withdraw, irrespective of whether they have experienced a liquidity shock. The total scale of withdrawals depends, first, on the size of the bank and, second, on the distribution of the private signals.

Recall that banks in our baseline model use the deposit rate only to manage liquidity risk so that they lend at the socially optimal scale. In our extension, banks that anticipate opacity shocks manage liquidity risk by both raising the deposit rate and, in addition, reducing their lending scale. Four new insights emerge from our analysis. First, liquidity risk reduces bank efficiency: banks that anticipate opacity shocks respond by inefficiently reducing their lending scale. The scale of this effect depends upon the bank's stock of liquid assets. When the stock drops to zero, banks have no appetite for risk taking; this is very inefficient.

Second, liquidity risk causes more inefficiency in lower-quality banks. The reason is that depositors in higher-quality banks can be dissuaded from running by a smaller increase in depositor returns: consequently, lower-quality banks reduce their scale more in response to potential liquidity crises than do higher-quality banks.

Third, unanticipated opacity shocks cause liquidity crises. If banks do not expect their depositors to acquire idiosyncratic information, then they do not shrink. As a result, whatever the distribution of depositor signals, any bank whose depositors acquire dispersed information about its asset quality experiences a liquidity crisis. Therefore, even absent rumours and disinformation, banks can experience liquidity crises simply because their depositors start to gather information for themselves. Banks certainly cannot manage this type of liquidity risk by denying their depositors information, and the appropriate response may come from policy makers.

Fourth, policy makers can address liquidity crises using “Good Bank/Bad Bank” policies (“GB policies”), under which a troubled bank sells its impaired and non-performing assets to a “Bad Bank” in exchange for safer assets, which it combines with its performing assets to form a “Good Bank.” The Bad Bank could be a private business or a state-controlled business. The GB policy works by reducing the opacity of bank assets, which typically also reduces their perceived riskiness. GB policies are very widely used, but they have received almost no academic attention. Our paper is one of the first to model GB policies. Our analysis highlights the fact that GB policies are a response to liquidity, rather than solvency, crises. Indeed, the GB policy can resolve liquidity crises even when assets are sold to the Bad Bank at a discount to their fair value so that equity value is reduced.

Since Diamond and Dybvig (1983), a substantial literature studies bank contracts and bank liquidity as liquidity risk sharing problems. A strand of theoretical literature analyses the limitations of a Diamond and Dybvig-style contract for risk sharing when deposits are negotiable: see Jacklin (1987), Hellwig (1994), Allen and Gale (2004), and Farhi, Golosov, and Tsyvinski (2009). None of these papers endogenises deposit demandability. More recently, Andolfatto, Berentsen, and Martin (2019) present a model in which demand deposit contracts cannot improve liquidity sharing unless deposits are not completely negotiable. In contrast to all of these papers, our paper is not about liquidity risk sharing; as a result, we generate deposit demandability even when there is no impediment to negotiability.

Several authors analyse models in which bank liabilities serve as a means of payment in general equilibrium: see, for example, Stein (2012), Gu, Mattesini, Monnet, and Wright (2013), Jakab and Kumhof (2015), Donaldson, Piacentino, and Thakor (2018), and Wang (2018). While they deepen our understanding of money creation, these papers are not concerned with bank liquidity.

Calomiris and Kahn (1991) argue that demand deposits are a response to moral hazard problems between the bank and its depositors. Diamond and Rajan (2001) explain demandability as a commitment device that commits a relationship lender not to use its market power to renegotiate its claims ex post, rather than as a signalling device. Allen and Gale (1998) analyse a Diamond and Dybvig-style model in which runs allow for efficient risk sharing. But in neither Diamond and Rajan’s model nor Allen and Gale’s are deposits a form of money.

Bank runs are an equilibrium phenomenon in several papers. Jacklin and Bhattacharya (1988) study information-based runs, and Chen (1999) and Chari and Jagannathan (2012) examine panics that arise when some depositors interpret the behaviour of others as evidence of poor fundmanentals. Like us, Rochet and Vives (2004), Goldstein and Pauzner (2005), and Liu (2016) study models in which depositors receive private signals of fundamental information. But none of these papers incorporates the role of bank liabilities as a means of payment, or examines the implications of liquidity risk for banks' lending decisions.

2. Model

We consider an economy in which there are two types of agents: a continuum of workers $w \in [0,1] \times [0,1]$ and a continuum of banks $b \in [0,1]$. There is one consumption good, corn, in the economy. Corn can be costlessly stored.

The economy lasts for four dates: $t = 0, 1, 2, 3$. At time 0, each worker is endowed with one unit of labour and has a production technology that converts his time 0 labour endowment into one unit of corn at time 1. Each bank has a time 0 endowment of G units of corn and has a technology that converts h units of time 0 labour into Y units of corn at time 2, where

$$Y = Ah, \tag{1}$$

and $A \in \{\underline{A}, \bar{A}\}$. Banks are capacity-constrained: they can use the labour endowment of at most a measure 1 of workers, so that $h \leq 1$.

The productivity parameter A is drawn from $\{\underline{A}, \bar{A}\}$, where

$$\bar{A} > 1 > \underline{A} + G. \tag{2}$$

A realises only at time 2. The probability that a given bank has productivity $\bar{A}$ is given by its type, q.It is convenient to define $A_e(q)$ to be the expected productivity of a type q bank:

$$A_e(q) \triangleq q\bar{A} + (1-q)\underline{A}. \tag{3}$$

Bank types are distributed over $[0,1]$ with c.d.f. $F(\cdot)$ and realisations of A are independent across banks. Each bank knows its own type at time 0; bank types are revealed to workers only at time 1.

Workers can consume at time 1 or time 2. We follow Diamond and Dybvig (1983) and assume that each worker faces a privately observed and uninsurable risk of being an early consumer or a late consumer. Workers learn their type at time 1: the probability that any worker is an early consumer is θ, and workers' type realisations are independent. Early consumers only derive utility from time 1 consumption; late consumers are indifferent between time 1 and time 2 consumption. A worker who consumes levels of corn c_1 and c_2 at times 1 and 2 achieves the following utility:

$$U(c_1, c_2) = \begin{cases} c_1, & \text{if the worker is an early consumer;} \\ c_1 + c_2, & \text{if the worker is a late consumer,} \end{cases} \tag{4}$$

We say that early consumers have experienced a *liquidity shock*.

Banks can consume at time 0 or time 2; each bank derives the following utility from consumption levels b_0 and b_2 at times 0 and 2:

$$V_B(b_0, b_2) = b_0 + b_2. \tag{5}$$

At time 0, each worker can elect either to be *banked* or to be *unbanked*. An unbanked worker uses his own technology to turn his labour endowment into one unit of time 1 corn; a banked worker supplies his labour to a bank, which produces time 2 corn according to the production function of Equation (1).

The relationship between a banked worker and his bank is governed by a contract (R_1, R_2). The contract allows the worker to decide when he wishes to be paid by the bank: he can opt to demand payment R_1 at time 1, or R_2 at time 2. For convenience, we will sometimes refer to a single worker's contract (R_1, R_2) as a *unit* claim. We say that a worker who demands a time 1 payment of R_1 from his bank has performed an *early withdrawal*; we say that workers who do not withdraw early elect to *hold*.

The contract (R_1, R_2) establishes a tradeable claim on the bank: a banked worker who does not withdraw early can *resell* his claim to a time 2 payment. The claim trades at a price $1/R_{12}$ for each unit of time 2 corn promised under the contract. The return on corn that is stored from time 1 to time 2 is 1; the rate of return R_{12} exceeds 1 if there is excess time 1 demand for liquidity. We therefore refer to $R_{12} - 1$ as the *liquidity premium*. As there is no uncertainty when the time 1 market in bank claims clears (recall that q is revealed to all workers at time 1), R_{12} is the same for all bank claims. In summary, banked workers at time 1 can choose an action from the set $\{$*withdraw*, *hold*, *resell*$\}$.

Suppose that a bank has time 1 corn holding $H \leq G$ and that it experiences a total withdrawal demand of Q. If $Q > H$ then the bank is unable to meet its contractual obligations in full, and we say that it experiences a *liquidity crisis*. When a bank experiences a liquidity crisis, a fraction H/Q of its repayment demands are honoured. The bank's project then continues to time 2, when its outstanding liabilities are paid at the original contractual rate R_2. That is, the bank *suspends convertibility* until time 2. Banked workers who are not paid at time 1 can still sell their claims for $1/R_{12}$, and so need not wait to consume. We make the following assumption:

Assumption 1. *Banked workers choose to withdraw at time* 1 *precisely when the withdrawal value* R_1 *exceeds the resale value* p.

Assumption 1 implies that workers never withdraw when they are indifferent between withdrawal and resale. This reflects the fact that, in practice, banks can always increase time 2 promised repayments by an arbitrarily small $\varepsilon > 0$ and so discourage early withdrawal by late consumers. Assumption 1 also implies that workers attempt to withdraw when R_1 exceeds the resale value even in case $H = 0$, despite the fact that it is impossible actually to withdraw anything when $H = 0$. Without this assumption, banks would be able to avoid liquidity crises by retaining no liquid assets.

In reality, liquidity crises are very costly: they prevent socially valuable and potentially profitable lending from occurring. We capture this effect in our model by assuming that every bank that does not experience a liquidity crisis receives additional utility L at time 3. For convenience, we express bank utility gross of this figure: bank utility is therefore $V(b_0, b_2)$ if there is no liquidity crisis, and $V(b_0, b_2) - L$ if there is a crisis.

The timeline for our model is illustrated in Figure 1.

At time 0, banks learn their type q. Banks then decide whether or not to offer a contract to workers. We say that a bank is *active* if it offers a contract (R_1, R_2), and that it is *inactive* if it does not. Workers choose whether to be banked or unbanked. Banks then decide how much corn H to retain to time 1, and how much to consume immediately.

At time 1, bank types are revealed to workers, and liquidity shocks occur. The market for bank claims opens. Unbanked workers and banks may sell corn for bank claims; banked workers decide whether to demand early repayment, to resell their claim, or to hold. If a worker demands early repayment from a bank that experiences a liquidity crisis then the likelihood of payment reflects the bank's corn shortfall.

Banks settle outstanding claims at time 2 and banks that did not experience a time 1 liquidity crisis receive utility L.

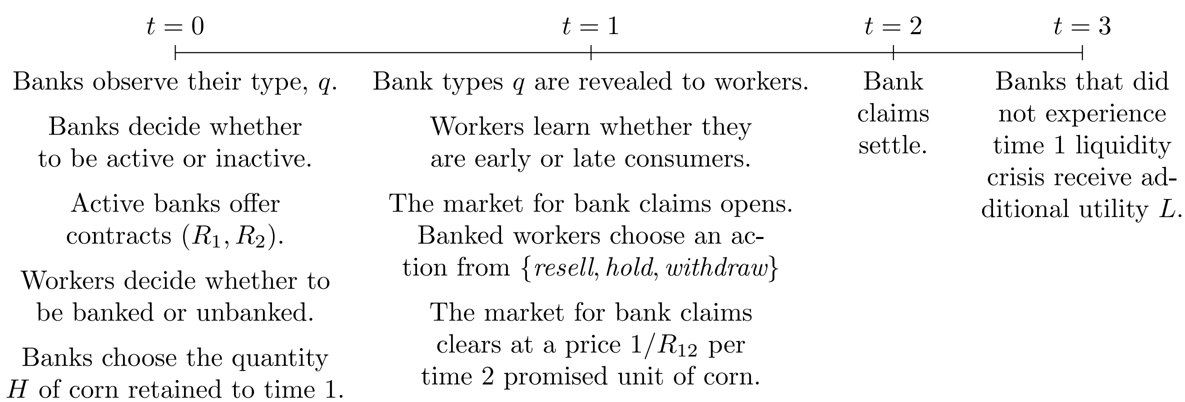

Figure 1. **Model timings.** At time 0, banks learn their types and select contracts; workers decide whether to be banked and banks decide how much corn to retain. At time 1, workers learn their types and that of bankers; withdrawal decisions are made and the market for bank claims clears. Bank claims settle at time 2 and banks then experience the disutility associated with liquidity crises.

2.1 *Discussion*

In practice, corporations acquire deposits, that is, bank liabilities, as a result of the double ledger entries that occur when their banks lend them money. Corporate deposits are then used to pay workers, who create the output that is ultimately used to repay the corporation's bank loans. In the interests of tractability, we fold the corporate sector into the banking sector in our model. Hence, at time 0, banks in our model create a liability contract (R_1, R_2) in order to hire workers; this liability contract can therefore be viewed as a deposit contract and, hence, we will sometimes refer to banked workers as *depositors*; accordingly, if $R_1 > 0$, then we will refer to $R_2/R_1 - 1$ as the bank's *deposit rate*.

Our model is closely related to Diamond and Dybvig's (1983) classic analysis. Agents in the Diamond and Dybvig setup are endowed with real consumption goods, while our agents have only a labour endowment at time 0. This difference is superficial: an unbanked worker in our model produces one unit corn at time 1 which corresponds to the output from short-term production in the Diamond and Dybvig model; similarly, banked workers in our model correspond to long-term production in the Diamond and Dybvig setup. Early withdrawal is a potential solution to short-term consumption shocks in both models. The key difference between our set-up and the Diamond and Dybvig model is that, in our model, workers can respond to a liquidity shock by reselling their deposit contract. That is, our workers choose a time 1 action from the set $\{$*withdraw*, *hold*, *resell*$\}$, while Diamond and Dybvig's depositors choose an action from the set $\{$*withdraw*, *hold*$\}$.

Our model therefore captures an important real-life fact that is absent in many models of demand deposits. In practice, people seldom withdraw cash to meet an unexpected liquidity demand; rather, they use a cheque or a bank card. In other words, they use their deposit claims as a means of payment. Hence, if banks were to set $R_1 = 0$, they could avoid the risk of a costly liquidity crisis without preventing depositors from satisfying early consumption needs. In the next Section, we show that banks nevertheless offer full demandability: that is, that the withdrawal value of equilibrium deposit contracts is equal to their resale value. In this case, we show that, although banks are exposed to the risk of liquidity crises, they can manage this risk by setting deposit rates at an appropriate level, so that, despite their liquidity concerns, banks lend at an efficient scale.

3. Full demandability, lending efficiency, and liquidity concerns

We solve the model under the assumption that every active bank has scale $h = 1$; we demonstrate below that this is indeed the optimal scale for active banks. We start establishing two results that we will use in our subsequent analysis.

Lemma 1. *If a bank's deposit contract induces time* 1 *depositor withdrawal, then the bank* *faces a liquidity crisis. That is, if the depositors' withdrawal right R_1 exceeds the resale value p of deposit contracts, then R_1 also exceeds the bank's time 1 liquidity stock H.*

At time 1, depositors with a contract (R_1, R_2) receive either the contract's resale value p or its withdrawal value R_1. If the contract is acceptable to depositors and it also induces withdrawal, then the withdrawal value R_1 must be sufficient to pay workers' wages. But the bank's liquidity stock H must be no greater than G, which is less than 1, the sum required to pay workers. A contract that induces withdrawal therefore promises more than the banks can afford and, hence, causes a liquidity crisis.

Lemma 2. *Banks default at time 2 if $A = \underline{A}$.*

We now proceed to an analysis of the time 1 and time 0 decisions of banks and their depositors.

We write $p(R_2, H, q)$ for the time 1 resale value of a deposit contract (R_1, R_2) issued by a type q bank that retains corn holding H and that does not experience time 1 withdrawal. If $A = \underline{A}$, then, by Lemma 2, the bank defaults; in this case, its depositors receive the entirety of the bank's corn production, $\underline{A}$, plus the return AR_{12} that the bank earns by investing its corn stock H at time 1. It follows that p is given by Equation (6):

$$p(R_2, H, q) = \frac{qR_2 + (1-q)(\underline{A} + R_{12}H)}{R_{12}}. \tag{6}$$

A q-bank does not experience a run if and only if the following liquidity constraint is satisfied:

$$R_1 \leq p(R_2, H, q). \tag{7}$$

When Condition (7) is satisfied, the time 1 expected income of a worker with that bank is $p(R_2, H, q)$.

If $R_1 > p(R_2, H, q)$, then a q-bank experiences time 1 withdrawal and, by Lemma 1, suffers a liquidity crisis. In this case, a banked worker attempts to withdraw R_1 at time 1. With probability H/R_1, her withdrawal demand is satisfied; with probability $(1 - H/R_1)$, she is entitled to the time 2 payment of the contract (R_1, R_2) (which she may sell). That payment is R_2 with probability q; with probability $1-q$, by Lemma 2 the bank defaults and depositors receive a pro-rata share of the $\underline{A}$ return from the project, divided amongst the $1 - H/R_1$ remaining claimants. The time 1 value of the banked worker's claim is therefore given by Equation (8):

$$V_W^r(R_1, R_2, q) = H + \frac{1}{R_{12}}\left(1 - \frac{H}{R_1}\right)\left(qR_2 + (1-q)\left(\frac{\underline{A}}{1 - H/R_1}\right)\right). \tag{8}$$

A banked worker therefore earns the following time 1 expected value from a q-bank with contract (R_1, R_2) that retains corn holding H:

$$V_W(R_1, R_2, H, q) = \begin{cases} V_W^r(R_1, R_2, H, q), & \text{if } R_1 > p(R_2, H, q); \\ p(R_2, H, q), & \text{if } R_1 \leq p(R_2, H, q). \end{cases} \tag{9}$$

At time 0, workers choose between accepting a bank contract and being unbanked. Unbanked workers have a time 0 income of 1. A banked worker with bank contract (R_1, R_2) assesses posterior $F_{(R_1,R_2)}$ for the distribution of q and expects a type q bank to retain corn holding $H(q)$. She therefore accepts any contract in the set Θ of contracts that yields a time 0 expected payout of at least 1:

$$\Theta = \left\{ (R_1, R_2) \mid \int_0^1 V_W(R_1, R_2, H(q), q) \, \mathrm{d}F_{(R_1,R_2)}(q) \geq 1 \right\}. \tag{10}$$

If a q-bank's contract (R_1, R_2) is accepted and it retains corn holding H, then its time 1 expected income is

$$V_B(R_1, R_2, H, q) = \begin{cases} V_B^{nr}(R_1, R_2, H, q), & \text{if } R_1 \geq p(R_2, H, q); \\ V_B^r(R_1, R_2, H, q), & \text{if } R_1 < p(R_2, H, q). \end{cases} \tag{11}$$

where, using Equation (5) and the fact that the banks defaults in the bad state,

$$V_B^{nr}(R_2, H, q) = (G - H) + q(\bar{A} + R_{12}H - R_2) \tag{12}$$

is the value of the bank if it does not experience a run, and

$$V_B^r(R_2, H, q) = (G - H) + q\left(\bar{A} - \left(1 - \frac{H}{R_1}\right) R_2\right) - L \tag{13}$$

is the value of the bank if it experiences a run.

A contract is accepted precisely when it is in the set Θ. A q-bank therefore solves the following maximisation problem:

$$\max_{H \in [0,G], \ (R_1,R_2) \in \Theta} V_B(R_1, R_2, H, q). \tag{14}$$

An inactive q-bank earns utility $\max_H (G - H) + HR_{12} = GR_{12}$. The q-bank therefore elects to be active if and only if the solution $(R_1(q), R_2(q), H(q))$ to Problem (14) exceeds GR_{12}. This requirement is equivalent to the following condition:

$$V_B(q) = V_B(R_1(q), R_2(q), H(q), q) \geq GR_{12}. \tag{15}$$

We write Q for the set of q that satisfy Condition (15). We need Q to identify the clearing conditions for the time 1 market for bank deposits and, hence, the price R_{12} of liquidity.

At time 1, the demand $D(R_{12})$ for corn by liquidity-shocked banked workers is given by Equation (16):

$$D(R_{12}) = \theta \int_{q \in Q} V_W(R_1(q), R_2(q), H(q), q) \,\mathrm{d}F(q). \tag{16}$$

Let

$$\mu(Q) = \int_{q \in Q} 1 \,\mathrm{d}F(q)$$

be the number of banked workers. Then the mass of unbanked workers that have not experienced a liquidity shock at time 1, and so can supply corn to the market, is $(1-\theta)(1-\mu(Q))$; the potential supply of time 1 corn by banks is H. We write

$$S(R_{12}) = (1-\theta)(1-\mu(Q)) + H. \tag{17}$$

Lemma 3 (Market clearing condition).

1. *If $R_{12} > 1$ then the clearing condition for the time 1 corn market is that $S(R_{12}) = D(R_{12})$;*
2. *If $R_{12} = 1$ then the clearing condition for the time 1 corn market implies that $S(R_{12}) \geq D(R_{12})$.*

Proof. If $R_{12} > 1$, then non-liquidity shocked banked workers never consume at time 1 and all holders of corn prefer to buy a claim to time 2 corn rather than to consume immediately or to store their corn. In this case, the market clearing condition is that $S(R_{12}) = D(R_{12})$.

If $R_{12} = 1$, then non-liquidity shocked banked workers may opt to consume at time 1 so that the total demand D for corn in the time 1 market may exceed $D(R_{12})$; moreover, corn holders may opt to consume or to store their corn rather than to supply it to the time 1 market so that the supply S of corn in the time 1 market may be less than $S(R_{12})$. In this case, the market clearing condition $S = D$ implies that $D(R_{12}) \leq S(R_{12})$.

We will search for Perfect Bayesian Equilibria of the game of this Section. As usual in this type of game, there are many possible equilibria, and we must therefore supplement our equilibrium definition with a condition that rules out "unreasonable" beliefs. Our task is complicated by the fact that, when workers form a poor opinion of a bank, they simply refuse to accept its contract; when that happens, the bank earns precisely the return achieved by any bank that does not issue a contract and, hence, is no worse off. Standard refinements, such as the Cho and Kreps (1987) Intuitive Criteria, therefore do not restrict the set of possibile equilibria.

We adopt instead a slight modification of the Intuitive Criterion. Faced with an off-equilibrium path contract (R_1, R_2), we assume that workers assign zero probability to any bank type q that would be worse off *if the contract were to be accepted.* More formally, in equilibrium, a type q bank earns $V_B(q)$ (Equation (15)). If the off-equilibrium contract (R_1, R_2) is accepted, the type q bank selects its corn holding $H(R_1, R_2, q)$ to maximise $V_B(R_1, R_2, H, q)$ (see Equation (11)). With this notation, our refinement reduces to a worker belief that an off-equilibrium contract (R_1, R_2) was offered by a bank of type q, where

$$q \in \Psi(R_1, R_2) \triangleq \{q \mid V_B(q) < V_B(R_1, R_2, H(R_1, R_2, q), q)\}\,. \tag{18}$$

Condition (18) boils down to a worker belief that, if a bank that offers an off-equilibrium-path contract (R_1, R_2), it sincerely wishes the contract to be accepted. We therefore say that beliefs that satisfy our refinement satisfy the *sincerity criterion.* Under the sincerity criterion, workers accept an off-equilibrium-path contract (R_1, R_2) if and only if $\Psi(R_1, R_2)$ is non-empty and

$$\min_{q \in \Psi(R_1, R_2)} V_W(R_1, R_2, H(R_1, R_2, q), q) \geq 1. \tag{19}$$

We are now able to define an equilibrium for the game of this Section.

Definition 1. *An equilibrium for the game with q drawn from $[0,1]$ comprises:*

1. *A set $Q \subseteq [0,1]$ and a mapping $(\sigma, H) : Q \to \Re^2 \times [0, G]$ such that bank types with $q \notin Q$ are inactive, and other banks offer the contract $\sigma(q) = (R_1(q), R_2(q))$ and retain corn holding $H(q)$;*
2. *Worker beliefs form the distribution $F_{(R_1,R_2)}$ over q given a contract offer (R_1, R_2);*
3. *A set Θ of acceptable contracts;*
4. *A time 1 price $1/R_{12}$ for time 2 bank claims,*

such that Q, σ, H, $F_{(R_1,R_2)}$ and R_{12} satisfy the following conditions:

i. *For each $q \in Q$, $(R_1(q), R_2(q), H(q))$ solve the q-bank's maximisation problem (14);*

ii. *Q is the set of q values for which Condition (15) for banks to be active is satisfied;*

iii. *$F_{(R_1,R_2)}$ is derived from F and σ using Bayes' Rule where possible;*

iv. *Θ is derived from $F_{(R_1,R_2)}$ using Equation (10) for the set of individually rational worker contracts;*

v. *The equilibrium is robust to the sincerity criterion: for every off-equilibrium-path contract (R_1, R_2), either*

   (a) *No bank would be better off if its offer of (R_1, R_2) were accepted:*

$$\Psi(R_1, R_2) = \emptyset; \tag{20}$$

   or

(b) *Workers strictly prefer not banking to banking with at least one bank that would be better off if it offered* (R_1, R_2)*:*

$$\min_{q \in \Psi(R_1,R_2)} V_W(R_1, R_2, H(R_1, R_2, q), q) < 1; \tag{21}$$

vi. *The time* 1 *price* R_{12} *satisfies the market clearing conditions of Lemma 3.*

Before searching for equilibria of our game, we establish a benchmark second-best allocation, in which a social planner faces no information friction but is subject to a "free trade" constraint. As Hellwig (1994), Allen and Gale (2004), and Farhi, Golosov, and Tsyvinski (2009) show in a setting without adverse selection, the early consumers in a competitive equilibrium with negotiable bank deposits should receive the payment that they would receive if they used their endowment for short-term production. The social planner in our model should therefore pay each liquidity-shocked worker one unit of corn. The planner must determine which banks should run projects, and, hence, the allocation of workers.

Clearly, if the social planner allows a bank of type q to run projects, then it must also allow banks of type $q' > q$ to do so. The planner's problem therefore reduces to the selection of the entry threshold q_c that maximises time 2 output:

$$\max_{q_c} \int_{q_c}^{1} A_e(q)\,\mathrm{d}F(q) + G + F(q_c) - \theta \tag{22}$$

$$s.t. \quad \theta \le G + F(q_c). \tag{23}$$

Condition (23) is the resource constraint that the time 1 output be sufficient to give one unit of corn to each worker who experiences a liquidity shock.

Let

$$q^* = \frac{1 - \underline{A}}{\bar{A} - \underline{A}}. \tag{24}$$

A bank of type q^* generates a return $A_e(q^*) = 1$ from its workers and, hence, is the lowest quality bank that meets the opportunity cost of a worker. The social planner should therefore set q_c equal to q^* if this value satisfies the resource constraint (23); if it does not, the constraint must bind, so that $q_c = F^{-1}(\theta - G)$. This intuitive argument can be confirmed by solving the social planner's problem directly. We therefore have:

$$q_c = \begin{cases} F^{-1}(\theta - G), & \text{if } \theta > G + F(q^*); \\ q^*, & \text{if } \theta \le G + F(q^*). \end{cases} \tag{25}$$

We now demonstrate that, provided the cost L of a liquidity crisis is high enough, in every equilibrium, the second-best allocation of Equation (25) is achieved, and all active banks offer full withdrawal rights.

Proposition 1 (Separating market equilibrium). *Assume*

$$L > \hat{L} \triangleq \max((1-q_c)(R_{12}(1-G) - \underline{A}), (1-q_c)(R_{12} - \underline{A}) - (R_{12}-1)G). \tag{26}$$

Then banks are active if and only if they have type $q \geq q_c$ so that the second best allocation of Equation (25) is achieved. All active banks offer contract $\sigma(q) = (1, R_2(q))$, where

$$R_2(q) = \frac{R_{12} - (1-q)(\underline{A} + R_{12}H(q))}{q}, \tag{27}$$

so that their liquidity constraint (7) binds, and

$$R_{12} = A_e(q_c). \tag{28}$$

There is no withdrawal at time 1; liquidity-shocked workers use their bank contracts to buy corn.

1. *If $\theta > G + F(q^*)$ then active banks set $H(q) = G$ and the equilibrium is unique;*
2. *If $\theta \leq G + F(q^*)$ then active banks are indifferent over corn holdings $H \in [0, G]$.*

A formal proof of Proposition 1 appears in the Appendix. To understand the intuition behind the result, suppose at first that there is a sufficiently high demand θ for liquidity to ensure there is a positive liquidity premium $R_{12} - 1$. When this is the case, banks retain all of their corn so as to profit from the time 1 purchase of bank claims: that is, $H(q) \equiv G$. The argument then proceeds in stages as follows:

First, because the cost L of a liquidity crisis is high enough, the equilibrium contract (R_1, R_2) selected by a bank of type q must satisfy the time 1 liquidity constraint $R_1 \leq p(R_2, G, q)$. Hence, for any equilibrium contract $(\hat{R}_1(q), \hat{R}_2(q))_{q \in Q}$, workers obtain the resale value: that is, $V_W = p(\hat{R}_2(q), G, q)$ (see Equation (9)).

Second, the value to workers of a given contract (R_1, R_2) is increasing in the quality q of the bank that offers the contact.

Third, note that, if $R_2(q)$ is given by Equation (27), the resale value p of the contract $(1, R_2(q))$ is equal to 1. It follows from point two that the resale value of the same contract would be less than 1 if it were offered by a bank of type $q' < q$. Hence, no q'-bank would offer contract $(1, R_2(q))$ because, if it did so, it would experience a costly liquidity crisis.

Fourth, for any equilibrium contract $(\hat{R}_1(q), \hat{R}_2(q))_{q \in Q}$, we must have

$$V_W(\hat{R}_1(q), \hat{R}_2(q), G, q) = 1$$

for all $q \in Q$. Suppose that there were an equilibrium for which this was not the case. If there was a q^l with $V_W(\hat{R}_1(q^l), \hat{R}_2(q^l), G, q^l) < 1$ then the contract $(\hat{R}_1(q^l), \hat{R}_2(q^l))$ would be acceptable to depositors only if it was also offered by a type q^h for which V_W exceeded one. The q^h-bank could deviate from its equilibrium strategy by offering contract $(1, R_2(q^h))$. Point three implies that no bank with type below q^h would ever offer this contract if it believed that the contract would be accepted. Hence, by the sincerity criterion (part (iv) of Definition 1), workers would believe that the contract $(1, R_2(q^h))$ had been offered by a bank of type at least q^h and, because $V_W(1, R_2(q^h), q^h) = 1$, they would accept the contract. This would make the q^h-bank strictly better off, which cannot be possible. Hence, $V_W \equiv 1$ in any equilibrium.

Fifth, any equilibrium must be separating. For, if two types $q^l < q^h$ pool on (R_1, R_2) in equilibrium, point two implies that $V_W(R_1, R_2, G, q^l) < V_W(R_1, R_2, G, q^l)$, and this contradicts point four.

Sixth, the liquidity constraint must bind for every active bank in equilibrium since, if it did not for some $\hat{q}$, then some banks with marginally lower type than $\hat{q}$ could profitably pool with the $\hat{q}$-bank without experiencing a run. This is ruled out by point five.

Seventh, let $(\hat{R}_1(q), \hat{R}_2(q))_{q \in Q}$ be an equilibrium contract. Then by point six, $\hat{R}_1(q) = p(\hat{R}_2(q), G, q)$ for any $q \in Q$ and, using points one and four,

$$p(\hat{R}_2(q), G, p) = V_W(\hat{R}_1(q), \hat{R}_2(q), G, q) = 1.$$

That is, for every $q \in Q$, $\hat{R}_1(q) = 1$ and, re-arranging $p(\hat{R}_2(q), G, q) = 1$, we have $\hat{R}_2(q) = R_2(q)$, as in Equation (27).

This argument establishes that, for high enough θ, the equilibrium is unique and that it is separating. Hence, there is a bank quality cutoff $\hat{q}_c$. The marginal banker's time 1 income is $A_e(\hat{q}_c)/R_{12} - 1 + G$. Equation (28) must therefore be satisfied to render the marginal banker indifferent between being active and not. Because $R_{12} \geq 1$, we must have $\hat{q}_c \geq q^*$. If $\hat{q}_c > q^*$, then Equation (28) implies that $R_{12} > 1$. In this case, all of the available corn is consumed at time 1 by early consumers and the resource constraint Equation (23) is binding: that is, $\theta = G + F(\hat{q}_c) > G + F(q^*)$. If $\hat{q}_c = q^*$, we have $R_{12} = 1$ so that the resource constraint (23) implies that $\theta \leq G + F(\hat{q}_c) = G + F(q^*)$. This argument establishes that $R_{12} > 1$ if and only if $\theta > G + F(q^*)$ so that steps one to seven above yield the unique equilibrium of Part 1 of the Proposition; we have also demonstrated that $\hat{q}_c = q_c$ so that the second-best allocation is achieved.

Now suppose that $\theta \leq G + F(q^*)$. In this case, the previous paragraph's argument implies that $R_{12} = 1$. Banks are indifferent between corn holdings $H \leq G$ so that, in this case, we do not have uniqueness of equilibrium. All other properties of the equilibrium go through as before. In particular, every active bank's liquidity constraint binds.

Finally, the minimum cost $\hat{L}$ that supports a separating equilibrium is the one that renders the marginal banker indifferent between offering $(1, R_2(q_c))$ and offering the contract

$(1, R_2(1) = R_{12})$ that the best banker offers. That requirement yields Condition (26).

The resale value of real-life deposits is equal to the value of the depositor's withdrawal rights: that is, a depositor can buy the same things by writing a cheque or using a bank card as she can if she withdraws her cash and then spends it. It is therefore desirable that a model of bank liquidity explain this fact: it is true in our analysis because the bank's liquidity constraint binds. In real life, this exposes the bank to the risk of a liquidity crisis: anything that causes a fall in the resale value of its deposits precipitates a run. Our model suggests that banks expose themselves to this risk so as to convince depositors of their quality and so have their liabilities accepted at time 0.

Proposition 1 demonstrates that the contract $(1, R_2(q))$ is the unique equilibrium outcome. We interpret this contract as a demand deposit contract, but it could be interpreted in another way: that is, $(1, R_2(q_c))$ could be implemented in a different way. Under the alternative arrangement, banks would issue only short-term debt one unit of which promised a time 1 repayment $R_1 = 1$. At time 1, they would sell a claim $R_2(q)$ to time 2 corn, either in exchange for corn that could redeem existing short-term debt, or in exchange for that debt. The latter exchange would then correspond to a debt roll-over by existing short-term depositors. The choice to roll over or to redeem in this set-up is precisely equivalent to the choice to hold on or to withdraw when the contract is interpreted as a demand deposit. In both cases, our central point holds: banks have to expose themselves to liquidity in order to sell their liabilities at time 0. In the absence of any time 1 security issuance costs, the two arrangements are equivalent; with such costs, the demand deposit contact must be superior to short-term debt issuance with roll over.

When $L \geq \hat{L}$, there are no equilibrium bank runs in Proposition 1. We now analyse the equilibria for lower values of L. In this case, the threat of a liquidity crisis is insufficient to dissuade low-quality banks from pooling with the highest quality banks. As a result, low-quality banks experience runs when their types are revealed at time 1.

Proposition 2 (Partial pooling market equilibrium). *Let* $q_c = F^{-1}(\theta - G)$*. Suppose that* $0 < L < \hat{L}$ *and that* $q_c A_e(q_c) \geq 1$*. Then* $R_{12} > 1$ *and banks are active if and only if* $q \geq q_c$*. Active banks set* $H(q) = G$*. There exist* R_2^**,* q_m*, and* $\bar{q}$ *satisfying* $R_2^* > R_{12}$ *and* $q_c < q_m < \bar{q}$ *such that:*

1. *Banks with* $\bar{q} \leq q$ *offer contract* $(R_1 = 1, R_2 = R_2^*)$ *and do not experience time* 1 *liquidity crises;*
2. *Banks with* $q_m \leq q < \bar{q}$ *offer the separating contract* $(R_1 = 1, R_2 = R_2(q))$ *of equation* (27) *and do not experience time* 1 *liquidity crises;*
3. *Banks with* $q_c \leq q < q_m$ *offer contract* $(R_1 = 1, R_2 = R_2^*)$ *and experience time* 1 *liquidity crises.*

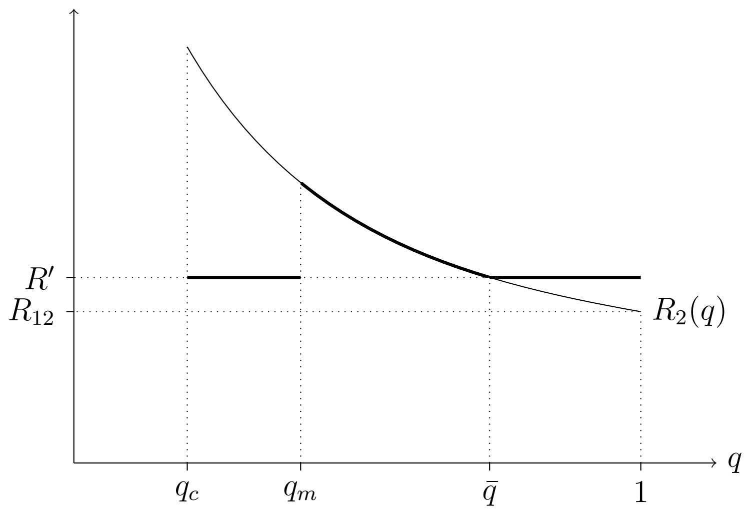

Figure 2. **Partial pooling market equilibrium**. For Ł $\geq \hat{L}$, active banks offer the separating contract $(R_1 = 1, R_2 = R_2(q))$. For $L < \hat{L}$, equilibrium contracts are illustrated in bold: banks with $q \in [q_c, q_m)$ and $q \in [\bar{q}, 1]$ offer the contract $(R_1, = 1, R_2 = R_2(\bar{q})$; banks with $q \in [q_m, \bar{q})$ offer the separating contract. The cutoff $\bar{q}$ is selected to render depositors' participation constraint binding.

To build intuition for Proposition 2, suppose that $L = \hat{L}$ so that, as proved in Proposition 1, every bank type $q \geq q_c$ offers the separating contract $(1, R_2(q))$. $R_2(q)$ is a declining function of q, as illustrated in Figure 2. If L falls fractionally below $\hat{L}$, then banks in a right-neighbourhood of q_c find it worthwhile to pool with the $q = 1$ bank. When that happens, the $q = 1$ bank contract no longer satisfies the workers' individual rationality constraint. The bank therefore raises its contractual payment to workers slightly to a new value R_2'.

The increase in the $q = 1$ bank's time 2 contractual payment has two effects. First, it renders pooling for low-quality banks less attractive, and so shrinks the measure of low-quality banks that do so. This serves to increase the value of the $q = 1$ bank's contract to workers and, hence, slackens their participation constraint. Second, it induces banks for which $R_2(q) \leq R_2'$ to pool with the $q = 1$ bank; if they did not do so, then they would experience pooling with low-quality banks that would violate their workers' participation constraints. Provided the $q = 1$ bank's contract continues to set $R_1 = 1$, any banks in a left neighbourhood of $q = 1$ that pool with the $q = 1$ bank satisfy their time 1 liquidity constraint. They therefore increase the value of the $q = 1$ bank's contract to workers and, once again, slacken their workers' participation constraint. The $q = 1$ bank continues to raise R_2 until its workers' participation constraint is again binding. At this point, the equilibrium comprises a region $[\bar{q}, 1]$ of high-quality banks that offer contract $(1, R_2^* = R_2(\bar{q}))$, a region $[0, q_m)$ of low quality banks that offer the same contract and experience a time 1 liquidity crisis, and an intermediate region $[q_m, \bar{q})$ of banks that offer the separating contract $(1, R_2(q))$ of Proposition 1.

Provided $\theta > G + F(q^*)$, bank entry is driven in general equilibrium by the binding resource constraint that there be exactly enough corn to meet time 1 liquidity needs and, hence, the threshold value q_c is the same in Propositions 1 and 2. In particular, the threshold is independent of the cost L of liquidity crises. Any incentive that a lower L gives banks with $q < q_c$ to become active must therefore be offset by a corresponding cost experienced by active banks. That cost must be an increase in the cost R_{12} of hiring workers. It follows that R_{12} increases as L decreases.

This observation has policy relevance. Because banks experience costly liquidity crises on the equilibrium path in Proposition 2, the government has an incentive to bail out failing banks. In our model, the effect of doing so would be to set $L = 0$. One might expect this policy to induce lower-quality banks to enter the market. Proposition 2 demonstrates that this is not the case: bank entry is driven in general equilibrium by the binding resource constraint that the the time 1 supply of corn $F(q_c) + G$ be equal to the demand θ. Hence, setting $L = 0$ does not give types $q < q_c$ an incentive to enter. The only consequence of a bailout policy is an interesting distributional effect: as we have already established, lowering the cost L of liquidity crises serves to increase the cost R_{12} of hiring workers. The government's bailout policy therefore serves to re-distribute wealth from banks to workers.

Liquidity crises are an equilibrium phenomenon in Proposition 2. The same quantity of resources is assigned to the banking sector as in the equilibrium of Proposition 1 but, because banks with $q_c \leq q < q_m$ experience the costs of liquidity crises, welfare is lower.

We will that, as promised at the start of this Section, every active bank in Propositions 1 and 2 has optimal scale $h = 1$. The profit margin of a bank that does not experience a liquidity crisis is independent of its scale; it therefore operates at full scale. The low-q banks that experience a liquidity crisis in Proposition 2 could avoid the crisis by operating at a lower scale h but, if they were to do so, they would make expected payment 1 to workers: those banks would therefore earn no more profit than they would by offering a separating contract and, by Proposition 2, separation is dominated by pooling and experiencing equilibrium liquidity crises.

Proposition 2 has the advantage that it does not explain bank runs as self-fulfilling prophecies. But the model still cannot address some important real-world problems. First, the time 1 bank runs that Proposition 2 studies occur when a bank's deposit rate $R_2/R_1 - 1$ is insufficient to cover the risk that it defaults, so that the bank is insolvent. This is a reasonable model of banking in normal times, when banks experience runs only if adverse information about their solvency becomes public. The Proposition therefore cannot provide guidance for policy makers who wish to design the best support packages for solvent banks that are faced with liquidity crises. Second, banks do not hold liquid assets in order to manage their liquidity risk; the only reason that they retain liquid assets is in order to exploit the liquidity premium. Third, liquidity risk has no effect upon the efficiency of banks' lending scale. Section 4 extends our model to address these lacunae.

4. Bank opacity and inefficiency caused by liquidity concerns

We now present an extension of our model in which solvent banks can experience runs. We have in mind situations in which workers become concerned about the solvency of their bank. Concerned workers gather information about their bank from a variety of sources. As a result, they form heterogeneous expectations about the bank's prospects. Some workers have such a poor opinion of their bank that they withdraw early, and their actions have a detrimental effect upon the bank's prospects.

4.1 *Model extension*

To capture our intuition, we allow banks to have two *informational types*, $\tau \in \{O, T\}$. A bank's informational type captures the likelihood that a concerned depositor will find additional information about the bank's solvency if she goes looking for it. *Transparent banks* ($\tau = T$) are identical to the banks modeled in Section 3; nothing other than q can be discovered about their asset values. In contrast, each of the workers that banks with an *opaque bank* ($\tau = O$) receives a signal s at time 1 that is drawn from an atomless distribution on $[0, \infty)$. s has distribution function $\bar{F}$ if $A = \bar{A}$ and $\underline{F}$ if $A = \underline{A}$; the corresponding density functions are $\bar{f}$ and $\underline{f}$. The likelihood ratio

$$m(s) = \frac{\underline{f}(s)}{\bar{f}(s)} \tag{29}$$

is monotonically decreasing from ∞ to 0 over $[0, \infty)$, so that a smaller s corresponds to a higher likelihood that $A = \underline{A}$ and a lower asset value.

To summarise, at time 1, all information about the quality of transparent banks is common knowledge, while some information about opaque banks is private. Each depositor in an opaque banks is therefore worried about what other depositors know, and about how they will act given their information.

The informational and quality types τ and q are iid across banks. Ex ante, any bank is transparent with probability ζ.

A bank knows its informational type at time 0; it is revealed to workers at time 1. Section 6.1 considers an extension in which no bank believes at time 0 that it might be opaque at time 1, so that opacity shocks are unanticipated.

After signals are revealed, workers decide whether to withdraw their corn, to resell their deposit, or to hold their deposit. We make one further modification to our model: after workers make their withdrawal/resale/hold decision and any trading orders have been placed, immediate revelation of each bank's productivity shock $A \in \{\underline{A}, \bar{A}\}$ occurs with probability e before the market for deposits opens and clears.

We assume throughout this Section that the cost L of a liquidity crisis so high that no bank wishes to incur it. One of the major consequences of opacity is that opaque banks might elect to reduce their investment scale in order to manage their liquidity risk. We write $h \in [0, 1]$ for a bank's investment scale. In previous sections, because the unique equilibrium implemented the second best, each active bank found it optimal to set $h = 1$; in constrast, we will exhibit equilibria in this Section for which $h < 1$ for opaque banks. We rule out the possibility that banks could use h to signal their type q by assuming that h is not observed by workers at time 0, although they learn it a time 1.

In summary, the game of this Section has the following time $t = 1$ stages:

1. Banked workers observe their bank's quality q and scale h. Every bank's informational type τ is publicly revealed;
2. A fraction θ of workers experience a liquidity shock and so must consume early;
3. Depositors in opaque banks receive independent draws s from the appropriate signal distribution for their bank;
4. Bank depositors decide whether to withdraw, resell, or hold their contracts;
5. Each bank's productivity shock $A \in \{\underline{A}, \bar{A}\}$ is publicly revealed with probability e;
6. The market for bank deposits clears.

Our analysis rests upon the assumption that depositors have heterogenous beliefs. We therefore require that information contained in private signals not be completely shared before the investors make decisions and trade their bank claims. Hence, we assume that the aggregate quantity of bank contracts that are offered for sale at time 1 is not publicly observed by buyers.

4.2 *Worker and banker decisions and equilibrium definition*

We now define an equilibrium for the game of this Section. For a given contract (R_1, R_2), workers assess a distribution over bank types q and τ. We define Θ to be the set of contracts that satisfy the worker's participation constraint in equilibrium.

Throughout this Section, we assume that all banks retain corn holding $H(q) = G$. This assumption significantly reduces the notational overhead in our presentation, and it is harmless: as we demonstrate below, opaque banks strictly prefer to hold corn at time 0, and transparent banks are at least indifferent between holding and consuming corn.

4.2.1 Transparent bank's decision

We first consider the decision problem for a transparent bank. If the bank defaults at time 2, each of its workers receives

$$\underline{R}_2^T = \frac{h\underline{A} + R_{12}G}{h}. \tag{30}$$

Hence, similarly to Equation (6), the resale value of a worker's claim on a transparent bank is

$$p^T(R_2, h, q) = \frac{1}{R_{12}}\left[qR_2 + (1-q)\min(R_2, \underline{R}_2^T)\right]. \tag{31}$$

By assumption, L is so large that the bank will always select a contract to ensure that the following time 1 liquidity constraint is satisfied:

$$p^T(R_2, h, q) \geq R_1. \tag{32}$$

Because the liquidity constraint (32) is always satisfied, the time 1 expected income that a banked worker derives from a contract with a transparent bank is

$$V_W^T(R_1, R_2, h, q) = p^T(R_2, h, q). \tag{33}$$

If a bank offers contract (R_1, R_2) and the offer is accepted, then the banker earns the following expected income:

$$V_B^T(R_1, R_2, h, q) = h\left(\frac{A_e(q)}{R_{12}} - V_W^T(R_1, R_2, h, q)\right). \tag{34}$$

The bracketed term in this expression is the banker's profit margin. Multiplying this by the banker's operating scale h yields V_B^T. Given a set Θ of contracts that workers will accept, the banker therefore finds $(R_1^T(q), R_2^T(q))$ and $h^T(q)$ to solve the following optimisation problem:

$$(R_1^T(q), R_2^T(q), h^T(q)) \in \underset{(R_1,R_2)\in\Theta,\ h\in[0,1]}{\arg\max} V_B^T(R_1, R_2, h, q), \tag{35}$$

subject to the liquidity constraint Equation (32). We write

$$V_B^T(q) = V_B^T(R_1^T(q), R_2^T(q), h^T(q), q) \tag{36}$$

for the optimal value of a transparent bank of type q.

Note that no active transparent bank will choose h so as to make $\underline{R}_2^T > R_2$, because, by Equation (31), the workers would then earn a payoff R_2/R_{12}, which does not depend on h; the bank's profit margin would therefore be independent of h, and, if that margin were positive, it would therefore increase h until it ceased to be risk-free.

4.2.2 Opaque bank's workers

We now consider workers with opaque banks. By Bayes' Law, an opaque bank depositor with signal s assesses the following success probability for a type q bank:

$$\eta(s,q) = \frac{q}{q + (1-q)m(s)}. \tag{37}$$

By our assumptions on the likelihood ratio $m(s)$, $\eta(s,q)$ increases from 0 to 1 as s increases from 0 to ∞.

With probability $1-e$, the quality of a bank's assets is not publicly revealed at time 1; let p^O be the price at which an opaque bank worker can then sell his contract.

With probability e, the quality of the bank's assets is revealed at time 1. Should this happen, the time 2 payoff of a worker's bank contract is R_2 in case $A = \bar{A}$. In case $A = \underline{A}$, we write $\underline{R}_2^O$ for a worker's expected payoff from a unit of deposit. A contract-holder in an opaque type q bank who decides to resell his deposit after receiving a signal s therefore earns the following expected income:

$$V_r^O(s,q) = e\frac{\eta(s,q)R_2 + (1-\eta(s,q))\underline{R}_2^O}{R_{12}} + (1-e)p^O. \tag{38}$$

Note that each banked worker takes $\underline{R}_2^O$ and p^O as given and, hence, views V_r^O as a strictly increasing function of his signal s. It follows that there exists a signal $s_r \geq 0$ such that $v_r^O(s,q) < R_1$, and, hence, workers prefer withdrawal to resale, if and only if $s < s_r$.

We can characterise $\underline{R}_2^O$ in terms of the threshold s_r. Similarly to the transparent bank case, no opaque bank will ever select its scale h so small that it has any surplus funds in the bad state. At time 2, there are $h(1-\underline{F}(s_r))$ contracts outstanding and the value of the bank's assets is $h\underline{A} + (G - h\underline{F}(s_r)R_1)R_{12}$. It follows that

$$\begin{aligned}\underline{R}_2^O &= \frac{h\underline{A} + (G - h\underline{F}(s_r)R_1)R_{12}}{h(1-\underline{F}(s_r))} \\ &= \frac{\underline{R}_2^T - \underline{F}(s_r)R_1R_{12}}{1-\underline{F}(s_r)},\end{aligned} \tag{39}$$

where the second line follows from Equation (30). Note that $\underline{R}_2^O$ is decreasing in s_r if $R_1R_{12} > \underline{R}_2^T$. For a given success state repayment R_2, the value of the opaque bank contract is therefore lower when more depositors withdraw at time 1. Time 1 withdrawal therefore imposes a negative externality upon workers that do not withdraw.

If the contract-holder opts to hold his deposit to time 2 then his expected income is

$$V_h^O(s,q) = \frac{\eta(s,q)R_2 + (1-\eta(s,q))\underline{R}_2^O}{R_{12}}. \tag{40}$$

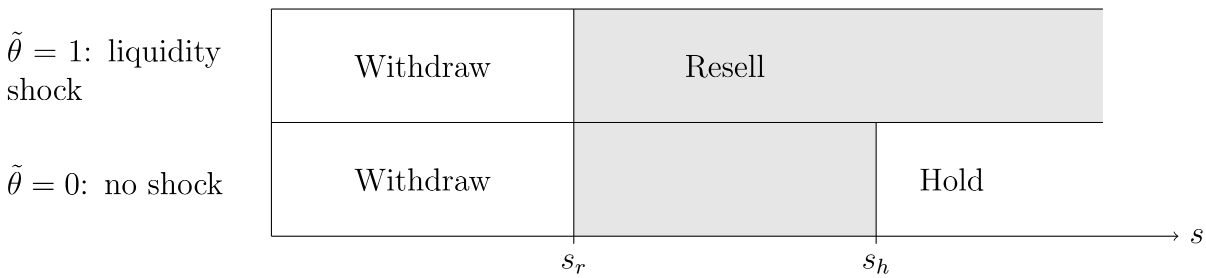

Figure 3. **State space partition for opaque bank depositors**. Depositors that experience a liquidity schock withdraw their funds when $s < s_r$ and otherwise resell their contracts. Depositors with no liquidity shock withdraw if $s < s_r$ and resell if $s_r \leq s \leq s_h$; they hold their deposits if $s > s_h$.

The depositor prefers holding his contract to resale precisely when $V_h^O(s,q) \geq V_r^O(s,q)$. This requirement is equivalent to the requirement that $s \geq s_h$, where

$$V_h^O(s_h, q) = p^O. \tag{41}$$

Note that s_h must exceed s_r: a depositors who wishes to hold his contract clearly prefers not to withdraw early.

Lemma 4 summarises the above discussion:

Lemma 4 (State space partition). *There exist $s_r \geq 0$ and $s_h > s_r$ such that:*

1. *Depositors that experience a liquidity shock withdraw their funds when $s < s_r$ and otherwise resell their contracts;*
2. *Depositors with no liquidity shock withdraw if $s < s_r$ and resell if $s_r \leq s \leq s_h$; they hold their deposits if $s > s_h$.*

Figure 3 illustrates the state space partition described in this Lemma.

The time 1 price of deposits is determined by the size of the shaded re-sale region. We denote by Ω the event that a particular depositor finds himself in this region:

$$\Omega = \left\{\omega | \tilde{\theta} = 1 \wedge s > s_r\right\} \cup \left\{\omega | \tilde{\theta} = 0 \wedge s \in [s_r, s_h]\right\}. \tag{42}$$

We write

$$q_\Omega = \mathbb{P}\left[A = \bar{A} | \omega \in \Omega\right]. \tag{43}$$

Then

$$p = \frac{q_\Omega R_2 + (1 - q_\Omega)\underline{R}_2^O}{R_{12}}. \tag{44}$$

Equations (41) and (44) yield Equation (45):

$$q_\Omega = \eta(s_h, q). \tag{45}$$

Equation (45) has a simple intuition. The right hand side is an average success probability given that the signal falls in the Resell region of Figure 3; the left hand side is the success probability given that $s = s_h$. In other words, the line $s = s_h$ can be thought of as passing through the "centre of gravity" for the probability mass in the Resell region.

Using Bayes' Law,

$$\begin{aligned} q_\Omega &= \mathbb{P}\left[A = \bar{A} | \omega \in \Omega\right] \\ &= \frac{q}{q + (1-q)\frac{\mathbb{P}[\omega \in \Omega | A = \underline{A}]}{\mathbb{P}[\omega \in \Omega | A = \bar{A}]}}. \end{aligned} \tag{46}$$

Hence, using Equations (37), (45) and (46), we have

$$\begin{aligned} m(s_h) &= \frac{\mathbb{P}[\omega \in \Omega | A = \underline{A}]}{\mathbb{P}[\omega \in \Omega | A = \bar{A}]} \\ &= \frac{\theta(1 - \underline{F}(s_r)) + (1-\theta)(\underline{F}(s_h) - \underline{F}(s_r))}{\theta(1 - \bar{F}(s_r)) + (1-\theta)(\bar{F}(s_h) - \bar{F}(s_r))}. \end{aligned} \tag{47}$$

Equation (47) defines a function $s_h(s_r)$ that relates the thresholds s_r and s_h. The intuitive interpretation of $s = s_h$ as a line passing through the centre of gravity for the Resell region of Figure 3 suggests that s_h is increasing in s_r; it is easy to use the Implicit Function Theorem to demonstrate that this is the case.

This discussion enables us to characterise the withdrawal threshold in the subgame that starts at time $t = 1$:

Lemma 5 (Characterisation of the withdrawal threshold). *The withdrawal threshold in the subgame that starts at time $t = 1$ has a functional form $s_r(R_1, R_2, h, q)$ that is determined as follows:*

1. *Suppose that*

$$R_1 > e\frac{\underline{R}_2^T}{R_{12}} + (1-e)V_h^O(s_h(0), q), \tag{48}$$

where $\underline{R}_2^T$ is given by Equation (30). Then $s_r(R_1, R_2, h, q) > 0$; it is implicitly determined by the following equation:

$$R_1 = eV_h^O(s_r, q) + (1-e)V_h^O(s_h(s_r), q). \tag{49}$$

2. *If Condition (48) is violated then $s_r(R_1, R_2, h, q) = 0$.*

Proof. We prove the result by demonstrating that $s_r > 0$ if and only if Equation (48) is satisfied. $s_r > 0$ if and only if depositors who receive the signal $s = s_r$ are indifferent between withdrawing and holding. It follows that, at a positive threshold signal s_r, $V_r^O(s_r, q) = R_1$:

that is,

$$R_1 = eV_h^O(s_r, q) + (1-e)p^O. \tag{50}$$

Combining Equations (41) and (50) gives us Equation (49). Substituting $s_h(s_r)$ into Equation (49) yields the equilibrium value $s_r(R_1, R_2, h, q)$. Because $s_h'(s_r) > 0$, the right hand side of Equation (49) must exceed its value when $s_r = 0$; that value is given by the right hand side of Equation (48). That is, Equation (49) can hold when $s_r > 0$ if and only if Equation (48) holds.

4.2.3 Opaque bank's decision

We now present the opaque bank's optimisation problem. Recall that any depositor in a type q opaque bank who receives a signal $s < s_r(R_1, R_1, h, q)$ withdraws her funds. The type q opaque bank's time 1 liquidity constraint is therefore as follows:[3]

$$h\underline{F}(s_r(R_1, R_2, h, q))R_1 \leq G. \tag{51}$$

Now let $V_{W,s}^O(R_1, R_2, h, q)$ and $V_{W,ns}^O(R_1, R_2, h, q)$ be the expected value to a worker of a contract (R_1, R_2) with an opaque bank of scale h and type q in the respective cases where the worker is, and is not, liquidity shocked. Both expectations are assessed at time 1, before the signal s is revealed. Lemma 4 implies that

$$V_{W,s}^O = \int_0^{s_r} R_1 d\tilde{F}(q) + \int_{s_r}^1 V_r^O d\tilde{F}(q); \tag{52}$$

$$V_{W,ns}^O = \int_0^{s_r} R_1 d\tilde{F}(q) + \int_{s_r}^{s_h} V_r^O d\tilde{F}(q) + \int_{s_r}^1 V_h^O d\tilde{F}(q). \tag{53}$$

The worker will be liquidity shocked with probability θ and she therefore assesses expected value $V_W^O(R_1, R_2, h, q)$ for her contract, where

$$V_W^O(R_1, R_2, h, q) = \theta V_{W,s}^O(R_1, R_2, h, q) + (1-\theta)V_{W,ns}^O(R_1, R_2, h, q). \tag{54}$$

An opaque type q bank whose contract offer (R_1, R_2) is accepted earns the following expected income from operating at scale h:

$$V_B^O(R_1, R_2, h, q) = h\left(\frac{A_e(q)}{R_{12}} - V_W^O(R_1, R_2, h, q)\right). \tag{55}$$

Given a set Θ of contracts that workers will accept, the banker therefore selects $(R_1^O(q), R_2^O(q))$

[3] For any s, $\underline{F}(s) > \bar{F}(s)$, so that the liquidity constraint in the good state is slacker than in the bad state.

and $h^O(q)$ to solve the following optimisation problem:

$$(R_1^O(q), R_2^O(q), h^O(q)) \in \underset{(R_1,R_2)\in\Theta,\ h\in[0,1]}{\arg\max} V_B^O(R_1, R_2, h, q), \tag{56}$$

subject to the liquidity constraint Equation (51). We write

$$V_B^O(q) = V_B^O(R_1^O(q), R_2^O(q), h^O(q), q) \tag{57}$$

for the optimal value of an opaque bank of type q.

4.2.4 Market clearing conditions

Finally, we discuss market clearing conditions. At time 1, a measure ζ of banks is transparent. Each transparent q-bank has $h^T(q)$ banked workers. A fraction θ of those workers is liquidity shocked; those workers sell their contracts for corn. The total demand for liquidity from workers with transparent banks therefore amounts to

$$\zeta\theta \int_{Q^T} V_W^T(R_1^T(q), R_2^T(q), h^T(q), q) h^T(q)\ dF(q).$$

Similarly, the expected liquidity demand over all possible signals s of a liquidity-shocked worker with an opaque bank is given by $V_{W,s}^O$, defined in Equation (52). The total demand for liquidity from workers with opaque banks therefore amounts to

$$(1-\zeta)\theta \int_{Q^O} V_{W,s}^O(R_1^O(q), R_2^O(q), h^O(q), q) h^O(q)\ dF(q).$$

The total time 1 demand for liquidity is therefore

$$D(R_{12}) = \theta \left(\zeta \int_{Q^T} V_W^T h^T\ dF(q) + (1-\zeta) \int_{Q^O} V_{W,s}^O h^O\ dF(q) \right), \tag{58}$$

where we suppress dependencies in the interests of clarity.

Let

$$\mu = \zeta \int_{q\in Q^T} h^T(q) dF(q) + (1-\zeta) \int_{q\in Q^O} h^O(q) dF(q) \tag{59}$$

be the total mass of banked workers. Then there is a mass $(1-\theta)(1-\mu)$ of unbanked workers at time 1 who have not experienced a liquidity shock and, hence, can supply corn to the market. The potential supply of time 1 corn by banks amounts to G. Hence, as in Equation (17), the time 1 supply of liquidity is

$$S(R_{12}) = (1-\theta)(1-\mu) + G. \tag{60}$$

4.2.5 Equilibrium definition

Finally, we have the following analog to Lemma 3:

Lemma 6 (Market clearing condition).

1. *If* $R_{12} > 1$ *then the clearing condition for the time* 1 *corn market is that* $S(R_{12}) = D(R_{12})$*;*
2. *If* $R_{12} = 1$ *then the clearing condition for the time* 1 *corn market implies that* $S(R_{12}) \geq D(R_{12})$*.*

We can now present the formal definition of equilibrium for the game of this Section. Once again, we examine equilibria that are robust to our sincerity criterion. The criterion is slightly harder to apply, because workers form beliefs over a bank's type q and also over its information type τ; we explain its meaning in the following Definition.

Definition 2 (Equilibrium with opaque banks). *An equilibrium for the game with* q *drawn from* $[0, 1]$ *comprises:*

1. *Sets* $Q^\tau \subseteq [0, 1]$ *for* $\tau \in \{T, O\}$ *and mappings* $\sigma^\tau : Q^\tau \to \Re^2$ *and* $h^\tau : Q^\tau \to [0, 1]$ *such that* τ *banks with type* $q \notin Q^\tau$ *are inactive; other* τ *banks are active and offer the contract* $\sigma^\tau(q) = (R_1^\tau(q), R_2^\tau(q))$ *and operate at scale* $h^\tau(q)$*;*
2. *Worker beliefs for the distribution* $F_{(R_1,R_2)}$ *over* (q, τ) *given a contract offer* (R_1, R_2)*;*
3. *A set* Θ *of acceptable contracts;*
4. *A time* 1 *price* $1/R_{12}$ *for time* 2 *bank claims,*

such that σ^τ*,* $F_{(R_1,R_2)}$ *and* R_{12} *satisfy the following conditions:*

i. *For each* $q \in Q^T$*,* $(R_1^T(q), R_2^T(q))$ *and* $h^T(q)$ *solve problem* (35) *and, for each* $q \in Q^O$*,* $(R_1^O(q), R_2^O(q))$ *and* $h^O(q)$ *solve problem* (56)*;*

ii. Q^τ *is the set of* q *values for which* $V_B^\tau(q) \geq G$*;*

iii. $F_{(R_1,R_2)}$ *is derived from* F *and* σ *using Bayes' Rule where possible;*

iv. Θ *is the set of all contracts whose expected value to workers using* $F_{(R_1,R_2)}$ *is at least* 1*;*

v. *The equilibrium is robust to the sincerity criterion: for every off-equilibrium-path contract* (R_1, R_2)*, either*
   - *(a) No bank could increase its expected income by offering* (R_1, R_2)*; or*
   - *(b) If there are banks that could increase their expected income by offering* (R_1, R_2)*, then workers accepting that contract from at least one of those banks would be strictly worse off than an unbanked worker;*

vi. *The time* 1 *price* R_{12} *satisfies the market clearing conditions of Lemma 6.*

4.3 *Withdrawal externalities*

Proposition 3 (Information harms depositors). *Let* (R_1, R_2) *and* h *be a bank contract and operating scale that satisfy Equation* (48). *Then* $V^T(R_1, R_2, h, q) > V^O(R_1, R_2, h, q)$*: that is, the value to depositors of a transparent bank with contract* (R_1, R_2) *and scale* h *exceeds the value to depositors of the corresponding opaque bank.*

The proof of Proposition 3 appears in the Appendix. Ceteris paribus, each depositor benefits from having more information. But the equilibrium effects of information revelation are damaging. When all depositors receive a signal and Equation (48) is satisfied, Lemma 5 implies that $s_r > 0$, so that some depositors withdraw their funds early. As a result, the value of the bank's time 2 claim in the bad state of the world is lower. Prices reflect this fact and all depositors lose out. We therefore identify a negative pecuniary externality that one depositor's withdrawal imposes on others. This effect is in the spirit of Diamond and Dybvig (1983), where anticipation of this type of effect can result in equilibrium bank runs. However, in Diamond and Dybvig's analysis, the withdrawal externality need not occur in equilibrium, and it is triggered only when depositor beliefs are coordinated by a sunspot. The withdrawal externalities of Proposition 3 are not sunspot phenomena, and, when they occur, they are a deliberate consequence of each bank's contract choices.

Proposition 3 has a counter-intuitive implication. It implies that the value of a bank's contracts could be strictly decreasing in the amount that the contract promises. Specifically:

Corollary 1. V_W^O *is strictly decreasing in* R_1 *at* $R_1 = e\frac{R_2^T}{R_{12}} + (1-e)V_h^O(s_h(0), q)$.

Corollary 1 has a simple intuition. Increasing the promised time 1 repayment can make early withdrawal more attractive for workers and, hence, can trigger the externality of Proposition 3.

4.4 *Liquidity and efficiency*

In our set-up, banks do not retain liquidity reserves in order to insure their worker's liquidity shocks. As we demonstrated in Section 3, banks use demand deposits in order to signal their types. Depositors use the time 1 market for bank liabilities to satisfy their liquidity needs and, to the extent that banks retain liquid reserves, they do so in order to exploit the time 1 liquidity premium by buying other banks' liabilities.

We now adopt the following assumption:

Assumption 2 (Depositors do not experience liquidity shocks).

$$\theta \equiv 0. \tag{61}$$

Assumption 2 renders the complex equilibrium of this Section tractable. In light of the above observations, this simplification, in ruling out depositor liquidity shocks, does not affect the economic rationale for demand depositors or for bank liquidity reserves.

Lemma 7 presents two properties of equilibria under Assumption 2:

Lemma 7 (Equilibrium properties without liquidity shocks).

1. $R_{12} = 1$, *so that there is no time* 1 *liquidity premium;*
2. $s_h = s_r$, *so that no worker re-sells an opaque bank contract at time* 1.

The intuition for part 1 of the Lemma is straightforward: when $\theta \equiv 0$ there is no time 1 demand for liquidity and, hence, the liquidity premium is zero. The intuition for part 2 can be understood by considering Figure 3, which illustrates the case where $\theta = 1$ with positive probability. In this case, the time 1 value of a bank contract reflects the fact that every banked worker in the shaded region sells his contract. Non liquidity-shocked workers with $s = s_h$ are indifferent between selling at this price and holding their contract. When $\theta \equiv 0$, the shaded region now includes only workers for whom $s \in [s_r, s_h]$. Bank contracts are priced at the average signal $s \in [s_r, s_h]$. Workers at the higher end of this range believe that their contracts have a higher value and are therefore unwilling to sell. A standard unravelling argument implies that *all* workers with $s \in [s_r, s_h]$ also hold and, hence, that $s_r = s_h$.

The fact that $s_h = s_r$ implies that the right hand side of Equation (48) is equal to $\underline{R}_2^T$. Hence, $s_r > 0$ if and only if $R_1 > \underline{R}_2^T$.

Lemma 8 (Opaque contract value). *If a type-q opaque bank's liquidity constraint is satisfied, then*

1. *If* $R_1 \geq \underline{R}_2^T$, *then*
$$R_1 = \eta(s_r, q)R_2 + (1 - \eta(s_r, q))\underline{R}_2^O; \tag{62}$$

2. *For all values of* R_1*:*
$$V_W^O(R_1, R_2, h, q) = R_1 + (1 - q)\underline{f}(s_r)(R_1 - \underline{R}_2^O)H(s_r); \tag{63}$$
$$= R_1 + q\bar{f}(s_r)(R_2 - R_1)H(s_r), \tag{64}$$

where the net chance of gain $H(s_r)$ *of the contract* (R_1, R_2) *is as follows:*

$$H(s_r) = \frac{1 - \bar{F}(s_r)}{\bar{f}(s_r)} - \frac{1 - \underline{F}(s_r)}{\underline{f}(s_r)} > 0. \tag{65}$$

Equations (63) and (64) each expresses the value V_W^O of an opaque bank's contract to its workers as the sum of two terms: the base value R_1 that the workers extract from their contract through early withdrawal, and the additional sum they earn in case they do not withdraw. A worker who holds onto his contract earns an expected income $q(R_2 - R_1)$ from doing so in case $\tilde{A} = \bar{A}$, and an expected loss $(1-q)(R_2 - R_1)$ in case $\tilde{A} = \underline{A}$. A worker with signal s_r must be indifferent between withdrawing and not withdrawing, and this fact implies that the expected gain and loss have a fixed ratio, $\underline{f}(s_r)/\bar{f}(s_r)$. We therefore define the *normalised gain* from holding onto an opaque bank contract to be $q(R_2 - R_1)\bar{f}(s_r) = (1-q)(R_1 - \underline{R}_2^O)\underline{f}(s_r)$: that is, the normalised gain is obtained by multiplying the expected profit or loss by the appropriate signal pdf, evaluated at the threshold s_r.

Equations (63) and (64) state that the expected value of holding onto an opaque bank's contract is equal to the normalised gain from doing so, multiplied by the factor $H(s_r)$. $H(s_r)$ is the difference between the probabilities that holding onto the contract generates a gain and a loss for the worker, scaled by the normalisation factor $\underline{f}(s_r)$ or $\bar{f}(s_r)$; we refer to it as the contract's *net chance of gain.* Note that the monotone likelihood ratio property guarantees that the net chance of gain, which is equal to the difference between two inverse harzard rate ratios, is positive. Lemma 8 therefore states that the expected value of an opaque bank's contract to its workers is equal to the withdrawal value R_1 plus the product of the normalised gain from retaining the contract and their net change of gain when they do so.

Lemma 9 (Withdrawal threshold). *Let*

$$\chi(s) \triangleq \frac{1 - \underline{F}(s)}{m(s)} \tag{66}$$

Then, in any equilibrium in which $R_1 > \underline{R}_2^T$, s_r is determined by Condition (67):

$$\chi(s_r) = \frac{1-q}{q} \frac{R_1 - \underline{R}_2^T}{R_2 - R_1}. \tag{67}$$

Obviously, $\chi(0) = 0$ and $\chi'(0) > 0$. Moreover, we have $\lim_{s\to\infty} \chi(s) = 0$: to see this, note that $\chi(s) \geq 0$ and that the inequality in Equation (65) can be written as $1 - \bar{F}(s) - \chi(s) > 0$, so that $\lim_{s\to\infty} \chi(s) \leq 0$. It follows that $\chi(s)$ is not monotonic. These remarks motivate the following definition:

Definition 3. *The* stable region *for $\chi(s_r)$ is the right neighbourhood of* 0 *in which $\chi(s_r)$ is increasing in s_r.*

For the remainder of this Section, we restrict our attention to equilibria in the stable region; within that region, there can be at most one equilibrium for any R_1, R_2, and h. In general, Equation (67) has multiple solutions s_r. The multiplicity arises because of the withdrawal externality identified in Proposition 3: when a small number of withdrawals is expected, the value of holding a contract is high so that the number of withdrawals is indeed

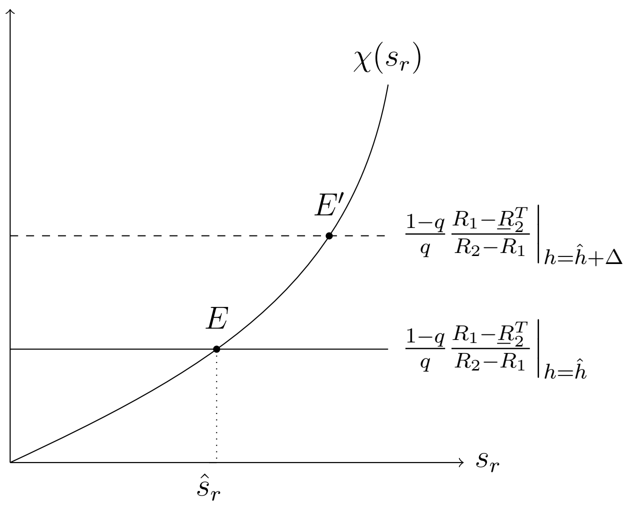

Figure 4. **The stable region for $\chi(s_r)$.** Condition (67) states that the equilibrium E is at the intersection of $\chi(s_r)$ and $\frac{1-q}{q}\frac{R_1-R_2^T}{R_2-R_1}$. Increasing h to $h+\Delta$ moves the latter curve up to the dashed curve and the new equilibrium, with a correspondingly higher withdrawal threshold s_r, is at point E'.

small; in contrast, when a high number of withdrawals is expected, the value of holding a contract is lower and a high number of withdrawals therefore occurs.

The stable region is illustrated in Figure 4. Condition (67) is satisfied at the equilibrium point E, where $h = \hat{h}$ and $s_r = \hat{s}_r$. Increasing h raises the right hand side of the expression and, hence, increases the cutoff point s_r, as illustrated in the Figure. Consider an agent with signal $\hat{s}_r$ when h is $\hat{h}$, and suppose that h increases to $\hat{h}+\Delta$ while R_1 and R_2 remain unchanged. If every agent continues to believe that $s_r = \hat{s}_r$, then the agent with $s = \hat{s}_r$ expects to earn less in case his project fails. He therefore strictly prefers to withdraw. Agents with lower signals similarly continue to prefer to withdraw and, hence, the threshold value for withdrawal increases. That is, dynamic adaptation moves the cutoff point s_r in the right direction. In this sense, equilibria are stable when $\chi(s_r)$ is increasing in s_r. If $\chi(s_r)$ were decreasing in s_r, then equilibria would be dynamically unstable in that adaptation of this type could never converge to the equilibrium s_r.

Lemma 10 (Acceptable contracts). *In any equilibrium, any contract with $R_1 \geq 1$ is acceptable to workers. That is,*

$$\{(R_1, R_2)|R_1 \geq 1\} \subseteq \Theta. \tag{68}$$

Proof. By the Sincerity Criterion, when L is high no bank offers a contract that would induce a liquidity crisis if it were accepted. Hence, workers will always be able to withdraw at time 1 if they choose to. It follows that the value to workers of any contract is no less than R_1. When $R_1 \geq 1$, Equation (68) follows.

For the remainder of this Section, we restrict ourselves to equilibria in which

$$\Theta = \{(R_1, R_2)|R_1 \geq 1\}. \tag{69}$$

We determine bank strategies in any equilibrium that satisfies Equation (69) and then demonstrate that the equilibrium is robust to the Sincerity Criterion. First, note that, when any $R_1 \geq 1$ is acceptable, no bank offers a contract with $R_1 > 1$. That is:

$$R_1 = 1. \tag{70}$$

Second, we can characterise the set of active bank types:

Lemma 11 (Active bank types). *In equilibria that satisfy Equation* (69)*, the set Q^τ of active bank types is equal to $[q^*, 1]$ for both $\tau = T$ and $\tau = O$.*

Proof. We first show that $Q^\tau \subseteq [q^*, 1]$. This is true because, for a given τ, a type q bank is active only if it earns a non-negative profit margin from hiring workers: this statement is equivalent to the requirement that $q\bar{A} + (1-q)\underline{A} \geq V_W^\tau \geq 1$. That is, the bank is active only if $q \geq q^*$.

We now show that $[q^*, 1] \subseteq Q^\tau$. To show this, we demonstrate that banks with $q \geq q^*$ are willing to offer the risk-free contract $R_1 = R_2 = 1$. A bank can offer this contract if its scale h is sufficiently small to guarantee that $(\underline{A}h + G)/h \geq R_1$. This requirement is equivalent to Condition (71):

$$h \leq h_f \triangleq \frac{G}{1 - \underline{A}}. \tag{71}$$

The type q bank earns profit margin $q\bar{A} + (1-q)\underline{A} - 1$ from the risk-free contract, which is positive if $q > q^*$, as required.

Consider the contract choice of a transparent bank. It follows from the proof of Lemma 10 that the value to workers of any acceptable contract is at least 1. Hence, the best that any bank can do is to offer a contract with value 1 to workers and to run at full scale: $h = 1$. A transparent bank can achieve this by setting $h = 1$ and offering the contract $\sigma(q)$ identified in Proposition 1 and operating at full scale:

$$(R_1, R_2) = (1, R_2(q)) = \left(1, \frac{1 - (1-q)(\underline{A} + G)}{q}\right). \tag{72}$$

An opaque bank cannot offer the contract in Equation (72): its liquidity constraint would be violated if it did so. To see this, observe that, by Equation (64), the liquidity constraint implies $V_W^O > 1$. We have $V_W^T = 1$, so that $V_W^O > V_W^T$ if the opaque bank's liquidity constraint is satisfied; by Proposition 3, this is impossible. This argument suggests that the liquidity constraint for opaque banks binds; Lemma 12 confirms that this is the case.

Lemma 12 (Opaque bank liquidity constraint binds). *If $h > h_f$ then, in any equilibrium for which $R_1 = 1$, the liquidity constraint Equation* (51) *binds:*

$$h\underline{F}(s_r) = G. \tag{73}$$

We can now characterise the signal threshold s_r below which depositors withdraw as a function of the equilibrium project scale h:

Lemma 13 (Equilibrium withdrawal threshold). *The equilibrium withdrawal threshold s_r is given by Equation* (74)*:*

$$s_r(h) = \begin{cases} 0, & \text{if } h \leq h_f; \\ \underline{F}^{-1}(G/h), & \text{if } h > h_f. \end{cases} \tag{74}$$

Proof. Recall from the proof of Lemma 11 that h_f is the maximum bank scale at which banks can offer a riskless contract to workers. Hence, when $h \leq h_f$, $R_1 = R_2 = 1$ and we have $R_1 \leq \underline{R}_2^T/R_{12}$. Then, by Lemma 5, we must have $s_r = 0$.

If a bank sets $h > h_f$ then, by Lemma 12, the bank's liquidity constraint must bind. Hence, $s_r = \underline{F}^{-1}(G/h)$.

Lemma 14 (Opaque bank value). *The value of an opaque q-bank of scale h to its workers is given by Equation* (75)*:*

$$V_W^O(h, q) = \begin{cases} 1, & \text{if } h \leq h_f; \\ 1 + (1-q)\left(1 - \frac{\underline{A}}{1-\underline{F}(s_r(h))}\right)\gamma(s_r(h)), & \text{if } h > h_f. \end{cases} \tag{75}$$

where

$$\gamma(s) \triangleq m(s)(1 - \bar{F}(s)) - (1 - \underline{F}(s)). \tag{76}$$

Proof. If $h \leq h_f$ then the bank offers the risk-free contract $R_1 = R_2 = 1$ and $V_W^O = 1$. The result when $h > h_f$ follows by substituting $s_r = F^{-1}(G/h)$ into Equation (63).

Note that s_r is discontinuous at h_f, because $\lim_{h \downarrow h_f} s_r = F^{-1}(1 - \underline{A}) > 0 = \lim_{h \uparrow h_f} s_r$. Nevertheless, V_W^O is continuous at h_f, because $\lim_{h \downarrow h_f} V_W^O(h) = 1$.

We write

$$\pi^O(h, q) \triangleq A_e(q) - V_W^O(h, q) \tag{77}$$

for the profit margin of an opaque q-bank with scale h. Observe that $A_e(q)$ is increasing in q and that, by Equation (75), V_W^O is decreasing in q; hence, for a given scale h, the profit margin is increasing in q.

The opaque q-bank's problem is therefore

$$\max_{h\in[0,1]} V_B^O(h,q) \triangleq h \times \pi^O(h,q). \tag{78}$$

The profit margin $\pi^O(h,q)$ is equal to $A_e(q)-1$ for any $h \le h_f$ and, hence, an opaque bank never selects $h < h_f$. The optimal size h therefore lies between h_f and 1. Lemma 15 identifies the tradeoff that an opaque bank faces within this range.

Lemma 15 (Liquidity risk management tradeoff). *For any $h \in (h_f, 1]$,*

$$\frac{\partial}{\partial h}\pi^O(h,q) < 0; \tag{79}$$

$$\frac{\partial^2}{\partial h \partial q}\pi^O(h,q) > 0. \tag{80}$$

Opaque banks can manage liquidity risk in two ways: they can reduce their lending scale (and, hence, the scale of their liabilities), or they can increase their deposit rate. The first result in Lemma 15 demonstrates that opaque banks face a trade-off between these approaches. For a given profit margin π^O, profit is increasing in scale. But, by Lemma 15, the profit margin is a decreasing function of bank scale. This result is driven by the fact that, as banks become larger, the threshold s_r must drop to prevent liquidity crises. The bank lowers s_r by persuading depositors with weak signals not to withdraw; the only way to accomplish this is by increasing the deposit rate and, hence, lowering the bank's profit margin.

Lemma 15 also demonstrates that $\frac{\partial^2}{\partial h \partial q}\pi^O > 0$, so that increased scale harms profit margins less for higher quality banks. This result suggest that higher types should operate at a greater scale. This intuition is confirmed by Proposition 4:

Proposition 4 (Opaque bank scale). *The equilibrium scale at which an opaque bank operates is a weakly increasing function of its quality:*

$$\frac{dh^O}{dq} \ge 0.$$

There exist $\hat{q}$, $\tilde{q}$ with $1 > \hat{q} > \tilde{q} > q^*$ such that $h^O(q) = 1$ for $q \in [\hat{q}, 1]$ and $h^O(q) = h_f$ for $q \in [q^*, \tilde{q}]$.

For q close to 1, opaque banks have a high profit margin and the marginal cost that they experience from an increase in h is low. They therefore operate at full scale, so that $h^O(q) = 1$ when $q > \hat{q}$. For q close to q^*, the marginal cost of increasing h is high and profit margins are low so that, as in the statement of the Proposition, banks elect not to increase their scale beyond the risk-free scale h_f when $q \in [q^*, \tilde{q}]$.

The first best allocation is for each type with $q > q^*$ to run its project to full scale with $h = 1$. Proposition 4 shows that liquidity concerns lead some banks to operate at an inefficiently low scale. Moreover, the level of inefficiency induced by liquidity problems is greater the lower the bank's type and the lowest quality banks shrink to the risk-free scale h_f.

The risk-free bank scale $h_f = G/(1 - \underline{A})$ is proportional to G, which suggests that the quantity of liquid assets is a key determinant of the efficiency of the equilibrium. Proposition confirms this intuition.

Proposition 5 (Liquid assets necessary). *For any $q < 1$, the scale of an opaque q-bank shrinks to zero as the bank's liquidity stock G tends to zero: $\lim_{G \downarrow 0} h^O(q) = 0$.*

The intuition behind Proposition 5 is straightforward. If a bank has a very small endowment G of liquid assets, then it can accommodate very little withdrawal. It therefore avoids taking any risks at all. As a result, banks with $q < 1$ become vanishingly small as G shrinks to zero. Banks for which $q = 1$ assume no risk and, hence, do not alter their size in response to changes in G.

Note that G has two roles in our model. It represents both the bank's stock of liquid assets, and also its stock of safe assets. This is the reason that the safe bank size h_f tends to zero as G tends to zero. In a more general model that separated safe assets from liquid assets, bank size would shrink to the risk-free scale (which could exceed zero) as the stock of liquid assets dropped to zero.

The opaque bankers' total loss of risk appetite when the stock of liquid assets drops to zero is inefficient. That inefficiency is entirely a consequence of the heterogeneous beliefs of opaque bank workers, which separate the bank's liquidity concerns from its solvency concerns. In contrast, transparent banks of any type $q > q^*$ operate at full scale: i.e., with $h = 1$. That is, transparent banks do not experience the efficiency loss that affects opaque banks. This is because transparent bank illiquidity is entirely a consequence of insolvency: that is, transparent banks experience a mass withdrawal only if they are insolvent.

Our analysis thus far describes bank strategies and their properties. To demonstrate that these strategies form an equilibrium, it remains to show that they are robust to the Sincerity Criterion. Any equilibrium contract with $R_1 \geq 1$ is acceptable, so we need only consider off-equilibrium contracts with $R_1 < 1$. Lemma 16 demonstrates that all such contracts are ruled out by the Sincerity Criterion.

Lemma 16 (Sincerity criterion with opaque bank types). *Let (R_1^*, R_2^*) be a contract with $R_1^* < 1$. Either*

1. *No bank would earn a higher-than-equilibrium profit if its workers accepted contract (R_1^*, R_2^*); or*

2. *There exists a bank type for which (R_1^*, R_2^*) generates a higher-than-equilibrium profit, and the value to whose workers of (R_1^*, R_2^*) is less than one.*

The first condition in Lemma 16 would ensure that no bank offered (R_1^*, R_2^*); the second, that workers can legitimately form beliefs under which they never accept an offer of (R_1^*, R_2^*).

5. Interbank liquidity provision

The threat of withdrawals in excess of their liquidity endowment leads opaque banks to reduce their lending scale. In this Section, we consider the effectiveness of interbank liquidity markets in addressing this problem. An opaque bank experiences a time 1 liquidity demand of $h\bar{F}(s_r)R_1$ in good states, and of $h\underline{F}(s_r)R_1$ in bad states. If it were to cover the shortfall between this figure and its corn endowment G, then its total demand for interbank liquidity would reveal its type. Banks with low productivity would then fail. Note that it cannot be optimal for good state banks to borrow the same amount as bad state banks. The reason is that, were they to do so, they would be valued at the average of good and bad state banks; a good state bank will therefore borrow only what it needs, which is $h\bar{F}(s_r)R_1 - G$. It follows that, in our model, interbank lending cannot help banks to address liquidity problems. The reasoning suggests the more general result that interbank markets can help to resolve bank liquidity problems precisely when, as in the Diamond and Dybvig (1983) model, liquidity demands occur in response to random events that reveal nothing about bank quality.

6. Policy: Good-Bank/Bad-bank

Policy interventions that are designed to help banks with severe liquidity or solvency problems to operate effectively fall into one of two categories. The simplest is an equity injection, typically funded by taxpayers. An alternative approach is a Good Bank/Bad Bank policy (hereinafter, "GB policy"). According to Faucette, Cunningham, and Loegering (2009), the aim of this policy is to replace some of a troubled bank's assets with assets that are in some sense of a higher quality. More precisely, the troubled bank first separates its balance sheet into a "Bad Bank," whose assets are the troubled bank's non-performing and impaired assets; and a "Good Bank, that has the remaining assets. The Bad Bank is then sold: the buyer could be another private business, an entity created and managed by the state, or a firm created and owned by the original shareholders. The Bad Bank is usually sold at a discount. The consideration paid for the Bad Bank's assets replaces those assets on the Good Bank's balance sheet; that consideration is usually cash but it could be another type of safe asset. In probably the largest scale of application of the GB policy, the Chinese prime minister Zhu Rongji in 1999 set up four asset management companies to be paired with the four largest commercial banks of China[4]. Each of the four banks replaced 1.4 trillion RMB of impaired assets with a profile of 580 billion RMB of cash and 820 billion RMB of the long term debt of the paired asset management company.[5]

If the GB policy replaces risky impaired assets with safe assets, then one might ask if a GB policy succeeds for the same reason that deposit insurance does. However, the GB policy is distinguished from deposit insurance because, as we will show, the GB policy's success rests not upon the fact that the replacement assets are safe but, rather, because they are less opaque than the assets that they replace. Moreover, a GB policy is substantially distinct from an equity injection, because GB policies frequently involve below-market price asset sales and, hence, impose losses upon bank shareholders. This fact suggests that GB policies are intended to resolve *liquidity*, rather than *solvency* problems.

We develop these ideas in two ways. First, we consider unforeseen opacity shocks; second, we examine the use of GB policies in situations where opacity shocks arise with a rationally anticipated probability.

6.1 *Unforeseen opacity shocks*

In this Section, we consider a situation in which all banks believe at time 0 that liquidity crises are a measure zero event, and then consider the appropriate policy response when a crisis occurs. This is arguably a reasonable way to think about the 2007–08 financial crisis, which appears to have come as a complete surprise to the vast majority of market participants.

We model this situation by assuming that all banks assign zero probability at time 0 to the event that they are opaque. When these beliefs obtain, Proposition 1 implies that every bank of type $q \geq q^*$ writes contract $(1, R_2(q))$ with its depositors and operates at scale $h = 1$. Assume now that a fraction $1 - \zeta$ of banks experience an opacity shock at time 1: that is, each of their workers receives a signal s drawn from distribution $\bar{F}$ or $\underline{F}$, as discussed in Section 4.1. That is, while some workers remain comfortable that their banks have high-quality investments, others experience unanticipated doubts.

As in Section 4.2.2, every worker whose signal s is below $s_r^*(q) \triangleq s_r(R_1 = 1, R_2(q), h = 1, q)$ withdraws from its bank at time 1. Provided the bank satisfies its liquidity constraint, s_r is determined by Lemma 9. Our first result is that every risky bank that experiences an opacity shock then experiences a liquidity crisis.

Proposition 6 (Opacity shocks cause liquidity crises). *Any bank with $q < 1$ that experiences an unanticipated opacity shock also experiences a liquidity crisis: that is, for every*

[4]Those banks are Bank of China, China Construction Bank (in combination with China Development Bank), Industrial and Commercial Bank of China and Agricultural Bank of China.

[5]See https://www.zhihu.com/question/21501374/answer/18458290. The source is in Chinese, but can be read with the aid of Google Translate.

$q < 1$, $G < \underline{F}(s_r^*(q))$.

Workers with a signal s of the quality of their bank form a posterior assessment $\eta(q, s)$ of its success probability. This is lower than their prior q if and only if $s < m^{-1}(1)$. The fraction $\underline{F}(m^{-1}(1))$ of depositors whose updated opinion is pessimistic could be very small. But early withdrawals by those depositors imposes a withdrawal externality on every other depositor, as in Proposition 3. Proposition 6 demonstrates that the effect of this externality is to cause a liquidity crisis for all banks with $q < 1$ that experience an unanticipated opacity shock, no matter how small is the fraction $m^{-1}(1)$ of pessimistic depositors.

The runs identified in Proposition 6 occur because the opacity shock is unforeseen. When bankers believe themselves to be transparent, they operate at the maximum scale $h = 1$. Operating at this scale renders any risky bank fragile: if only a small fraction of its workers are concerned about the soundness of their bank, their withdrawal is sufficient to trigger a liquidity crisis. This mechanism is consistent with the experience of the 2007–08 Financial Crisis, which had its genesis in relatively small numbers of liquidity demands on highly levered banks.

We now show how banks that experience an unanticipated liquidity shock can be helped by a GB policy that replaces some of their opaque assets with assets of the same quality q. Critically, depositors have no heterogenous signals regarding the replacement assets (that is, the replacement assets are transparent) and, hence, they will not trigger withdrawals. It follows that, if every asset in the liquidity-shocked bank were replaced, the liquidity crisis would be prevented and, hence, that there is some minimum scale of asset replacement that ensures that the scale of depositor withdrawal is no larger than G.

The GB policy diversifies the liquidity-shocked bank's assets and so reduces the risk to which its depositors are exposed. But there would be no run if every asset were replaced with transparent assets of the same quality and, in this case, there would be no diversification and no risk reduction. It follows that the GB policy does not work because it reduces risk, but because it reduces opacity. In other words, the GB policy relies upon a different mechanism than deposit insurance. However, as a practical matter, it is probably impossible to replace opaque assets with transparent assets whose quality q is the same. Consequently, the replacement assets are typically safer than the opaque assets and, in practice, implementing a GB policy serves to reduce the riskiness of the opacity-shocked bank.

In light of this argument, we analyse a GB policy that replaces opaque assets with safe assets, such as government debt, that a government can easily create. Under the policy, the government is willing to acquire the type q bank's assets, paying for each unit of assets with a safe asset with payout $\delta A_e(q)$, for some $\delta \leq 1$. The government's GB policy reduces the bank's risk and, hence, even when $\delta = 1$, it shifts value from the bank, which is a residual claimant, to its depositors, who hold a concave claim on the bank's assets. If the GB policy is effective, it cannot be because it enhances the bank's equity value. Transparent banks, which do not face liquidity crises, will therefore never avail themselves of the government's offer.

A type q opaque bank will swap some of its assets only if it is able to avoid a liquidity crisis. We write s^* for the maximum withdrawal that a bank can accommodate:

$$s^* \triangleq F^{-1}(G). \tag{81}$$

The type q opaque bank therefore opts to exchange the minimum volume of assets $x(q)$ that reduces the threshold s_r at least as far as the cutoff s^*. If no $x(q) \leq 1$ lowers the threshold far enough, then the GB policy does not prevent runs in type q banks.

Lemma 17 establishes the optimal exchange policy for opaque banks under the GB policy.

Lemma 17 (Optimal opaque bank exchange policy). *The minimum volume of assets $x(q)$ that an opaque bank can exchange under the GB policy in order to avoid a liquidity crisis is given by Equation (82):*

$$x(q) = (1 - \chi(s^*))\frac{1 - G - A}{\delta A_e(q) - \underline{A}}. \tag{82}$$

A type q opaque bank can use the GB policy to avoid a crisis if and only if $x(q) < 1$, in which case it exchanges $x(q)$ of its assets for safe assets. Every opaque bank can avoid a liquidity crisis if and only if Condition (83) is satisfied:

$$\delta \geq \underline{\delta} \triangleq (1 - \chi(s^*))(1 - G) + \chi(s^*)\underline{A} > \underline{A}. \tag{83}$$

As $\underline{\delta} < 1 - G < 1$, a GB policy can prevent bank runs from occurring even when it reduces bank equity. It does so by resolving depositors' liquidity concerns. After swapping x units of its project, a type q bank earns $(\underline{A} + G) + x(\delta A_e(q) - \underline{A})$ in the bad state. This expression is increasing in x, and higher bad state payments reduce the workers' incentive to withdraw early. Depositors with higher quality bankers require a smaller transfer to maintain their confidence and, hence, $x'(q) < 0$.

6.2 *Anticipated opacity*

We now return to the version of our model introduced in Section 4.1, in which every bank knows at time 0 whether or not it will be opaque at time 1, although neither this fact nor the bank's type is revealed to workers until time 1. In this case, Propositions 4 and 5 demonstrate that opaque banks operate at an inefficiently low scale in order to avoid bearing any risk when the stock G of liquid assets is small enough. We demonstrate that a GB policy can address this inefficiency.

Under the GB policy of this Section, the government allows type q banks to swap their projects at time 1 for $A_e(q)$ units of safe assets. The existence of this scheme is common knowledge at time 0. If a risk-free bank swaps x units of its project for the safe asset, then its scale h_f solves $\underline{A}(h_f - x) + xA_e(q) + G = h_f$. That is, the risk-free scale after swapping x units is

$$h_f(x) = \frac{G + x(A_e(q) - \underline{A})}{1 - \underline{A}}. \tag{84}$$

Note that $h_f(x)$ is increasing in x.

When a bank exchanges x units of its project for safe government assets, the value to workers of an opaque type q bank changes from the expression in Equation (75) to the following value:

$$V_O^W(h, x, q) \triangleq \begin{cases} 1, & \text{if } h \leq h_f(x); \\ 1 + (1-q)\left(1 - \frac{\underline{A}h + x(A_e(q) - \underline{A})}{h - G}\right)\gamma(s_r), & \text{if } h > h_f(x). \end{cases} \tag{85}$$

A type q opaque bank's profit margin when it exchanges x units of its project is

$$\pi^O(h, x, q) \triangleq A_e(q) - V_O^W(h, x, q), \tag{86}$$

and the bank maximizes $h\pi^O(h, x, q)$. Lemma 18 identifies conditions under which the GB policy increases opaque bank scale and, hence, increases efficiency.

Lemma 18 (Efficiency conditions for GB policy). *In the stable region where $\chi(s)$ is increasing, $\frac{\partial h}{\partial x} > 0$ whenever $h(x, q) < 1$.*

Suppose that a type q opaque bank swaps a quantity

$$x_{\text{safe}}(q) \triangleq \frac{1 - G - \underline{A}}{A_e(q) - \underline{A}}$$

of its assets. Then, because $\underline{A}(1 - x_{\text{safe}}(q)) + x_{\text{safe}}(q)A_e(q) + G = 1$, the bank can operate at full scale without assuming any risk and, as a result, all inefficiency is eliminated. Hence, if the supplyS_{safe} of safe assets is large enough, the GB policy can eliminate all inefficiency. If, instead,

$$S_{\text{safe}} < (1 - \zeta)\int_{q^*}^{1} x_{\text{safe}}(q)\, dF(q),$$

then access to the GB policy must be rationed. We now investigate the efficiency of using competitive bids to allocate access to the policy.

Let $h^O(x, q)$ be the scale chosen by a type q opaque bank that expects to swap x units of its project under the GB policy. The associated efficiency improvement is $(h(x, q) -$

$h(0,q))(A_e(q)-1)$. The optimal allocation $x^*(q)$ of safe assets therefore solves the following problem

$$\max_{x:[q^*,1]\to[0,1]} \int_{q^*}^{1} (h(x(q),q)-h(0,q))(A_e(q)-1)\ dq \tag{87}$$
$$\text{s.t. } (1-\zeta)\int_{q^*}^{1} x(q)\ dq \le S_{\text{safe}}.$$

It is obvious that, if $h(0,1)=1$ so that the type q opaque bank operates at the efficient scale without any intervention, then the optimal allocation $x^*(q)$ to the q bank under the GB policy is zero.

Suppose that the state uses competitive bids to set the unit price μ for safe assets under the GB policy. The opaque bank's choice $x(\mu,q)$ solves the following problem:

$$\arg\max_{h,\ x} h\left(A_e(q) - V_O^W(h,x,q)\right)_{s_r=\underline{F}^{-1}(G/h)} - x\mu.$$

The equilibrium premium μ^* is determined by the clearing condition (88):

$$(1-\zeta)\int_{q^*}^{1} x(\mu,q)\ dF(q) = S_{\text{safe}}. \tag{88}$$

Lemma 19 establishes that it may be inefficient to allocate access to the GB policy in this way:

Lemma 19 (Competitive access to safe assets may be inefficient). *Let*

$$\hat{\mu} \triangleq \max_{q\in[\hat{q},1]} \frac{\partial \pi^O}{\partial x}(1,0,q) = \max_{q\in[\hat{q},1]} (1-q)\gamma\left(\underline{F}^{-1}(G)\right)\frac{A_e(q)-\underline{A}}{1-G}. \tag{89}$$

Then, if $\mu^* < \hat{\mu}$, under a competitive market allocation of safe assets under the GB policy, $x(\mu^*,q) > 0$ for q in a non-empty interval $(q_1,q_2) \subset [\hat{q},1]$. As $x^*(q)=0$ for all $q \ge \hat{q}$, it follows that a competitive market allocation of the GB policy is inefficient when $\mu^* < \hat{\mu}$.

Proof. A type q opaque bank swaps $x>0$ units of its assets for the safe asset if the resultant increase in its profit margin exceeds the marginal cost. That is, if $h^O(q)\frac{\partial \pi^O}{\partial x}(h^O(q),0,q) > \mu^*$. For $q > \hat{q}$, we have $h^O(q)=1$; this is equivalent to the requirement that $\frac{\partial \pi^O}{\partial x}(1,0,q) > \mu^*$. If $\hat{\mu} > \mu^*$, then the result follows immediately.

7. Conclusion

This paper addresses three important questions related to bank liquidity. First, why do banks expose themselves to the risk of costly liquidity crises when their depositors can respond to consumption shocks by selling their claims? Second, given the negotiability of bank deposits, why do banks face withdrawal demands? Third, how should banks manage this liquidity risk, and how could regulators help them?

We present a general equilibrium model in which bank liabilities operate as a means of payment, and in which banks nevertheless fund themselves using demand deposits. Our work deviates from previous treatments of this topic, because demand deposits are not necessary for depositor liquidity insurance in our model. Banks are of uncertain quality in our model and, as a result, workers may be unwilling to accept bank liabilities as wage payments. Banks resolve this problem by granting the right to withdraw on demand. In short, banks elect to expose themselves to the risk of a costly liquidity crisis in order to signal their types and, hence, to ensure that their liabilities circulate.

Our analysis of withdrawal volumes considers first normal times, when depositors all have the same information about their banks. In this case, banks can manage their liquidity risk by selecting an appropriate deposit rate; there are few withdrawals, and banks do not experience liquidity crises. In normal times, then, liquidity concerns have minimal implications for the operation and efficiency of banks.

We also consider a scenario in which depositors in some banks receive heterogenous signals of bank type. Depositors with a bad signal withdraw from the bank even if they have not experienced a consumption shock. If banks anticipate this effect, then they adopt two techniques to manage liquidity risk: as they do in normal times, they alter their deposit rate; and, in addition, they may lower their investment level. Liquidity risk in this scenario therefore has a real effect upon bank efficiency; the effect is most pronounced for the lowest quality banks, which experience the largest scale reduction. This analysis demonstrates the importance of each bank’s stock of liquid assets: if that stock drops to zero, then the bank loses its capacity to take any risks. If, on the other hand, banks fail to anticipate that their depositors will receive heterogeneous signals of bank quality, then they do not make the necessary operational adjustments to their deposit rates and investment scale; as a result, they experience costly equilibrium liquidity crises.

When liquidity crises occur in our model, they can be resolved using a Good Bank/Bad Bank policy (“GB Policy”), under which risky assets are shifted from troubled banks into a new “Bad Bank” and are replaced with safe assets; the restructured troubled bank is referred to as a “Good Bank.” GB Policies are commonly used in practice, but ours is the first theoretical explanation of their effectiveness. GB policies work by reducing the opacity of the Good Bank, and so ameliorating the depositors’ incentive to run.

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ON LINE APPENDIX

Proof of Lemma 1

A deposit contract (R_1, R_2) is acceptable to workers if and only if

$$\max(R_1, R_2/R_{12}) \geq 1. \tag{90}$$

If $R_2/R_{12} < 1$, then $R_1 \geq 1 > H$ and the result is proved. Now suppose that $R_2/R_{12} \geq 1$. Withdrawal occurs if and only if R_1 exceeds the price p that would obtain conditional upon no withdrawal. That price is as follows:

$$\begin{aligned} p &= \min\left(\frac{R_2}{R_{12}}, \frac{qR_2 + (1-q)(\underline{A} + R_{12}H)}{R_{12}}\right) \\ &= \min\left(\frac{R_2}{R_{12}}, q \times \frac{R_2}{R_{12}} + (1-q) \times \frac{\underline{A} + R_{12}H}{R_{12}}\right) \\ &= \min(1, q + (1-q)H) \\ &\geq H. \end{aligned}$$

Hence, if $R_1 > p$, so that withdrawal occurs, we must have $R_1 > H$.

Proof of Lemma 2

First, consider the case where there is no time 1 withdrawal and, hence, no time 1 liquidity crisis. The bank's time 2 asset value in this case is $\underline{A} + HR_{12}$. The bank therefore defaults if and only if Condition (91) is satisfied:

$$\underline{A} + HR_{12} < R_2. \tag{91}$$

The resale value p of the contract is no greater than R_2/R_{12} and the no withdrawal condition is that $R_1 \leq p$, so that $R_1 \leq R_2/R_{12}$. The contract (R_1, R_2) is acceptable if and only if $R_2/R_{12} = \max(R_1, R_2/R_{12}) \geq 1$. Then, because $R_{12} \geq 1$ and, by Assumption (2), $\underline{A}+G < 1$, we must have

$$\underline{A} + GR_{12} \leq (\underline{A} + G)R_{12} < R_{12} \leq R_2.$$

It follows that

$$\underline{A} + HR_{12} \leq \underline{A} + GR_{12} < R_2,$$

which implies Condition (91).

Now consider the case where depositors withdraw at time 1, so that the bank experiences a liquidity crisis. A fraction H/R_1 of depositors successfully withdraw in this case, so that the bank's total time 2 obligation is $(1 - H/R_1)R_2$. We prove that

$$\underline{A} < (1 - H/R_1)R_2.$$

Because the time 2 contract payoff is no larger than R_2, the time 1 contract value can be no larger than

$$(H/R_1) \times R_1 + (1 - H/R_1) \times R_2/R_{12}.$$

If the contract is acceptable, then this value cannot be less than 1. It follows that

$$(1 - H/R_1)R_2 \geq (1 - H)R_{12} \geq 1 - H \geq 1 - G > \underline{A}.$$

Proof of of Proposition 1

We exhibit the elements defined in parts 1–4 of Definition 1 and then demonstrate that they satisfy conditions i–vi of the Definition. We then demonstrate that the equilibrium is unique.

The set Q of part (1) of the Definition is $[q_c, 1]$. Workers form beliefs for the worker type q that offered a contract (R_1, R_2) as follows. First, if $(R_1, R_2) = (1, R_2(q))$ for some q, then workers believe that the contract was offered by a q-bank. Second, if the off-equilibrium contract has $R_1 \geq 1$ then the workers assume that $q = 0$. Third, if $R_1 < 1$ then the workers assume that q satisfies $p(q) = R_1$.

The set Θ of acceptable contracts is the set of contracts that yields a time 0 expected payout of at least 1 using the beliefs of the previous paragraph. The liquidity premium is given by Equation (28).

Part (i) of the equilibrium definition requires that equilibrium contracts maximize banker payoffs given Θ. Because $p(q, R_2(q)) = 1$, the q-banker's equilibrium strategy does not induce a run. If the q-bank deviates to the equilibrium strategy of a $q^h > q$ bank, then, because $p(q, R_2(q^h)) < 1$, the q-bank experiences a time 1 run. The most profitfable way for a banker to induce a run is by offering the contract $(1, R_2(1)) = (1, R_{12})$, and the q_c-bank has the most to gain by doing so. But, by Equation (26), L exceeds the value $\hat{L}$ that renders the q_c-bank indifferent between offering $(1, R_2(1)) = (1, R_{12})$ and offering $(1, R_2(q_c))$. Hence, no bank profits by mimicking a higher type. The q-bank would not experience a run if it offered the contract for a lower type, but it would thereby earn a lower expected income. If the bank deviated to any contract with $R_1 \neq 1$ then our assumptions on worker beliefs guarantee that it would receive no deposits and, hence would be worse off.

For part (ii), a q-bank elects to be active precisely when $A_e(q)/R_{12} + G - 1 \geq G$. This is true precisely when $q \geq q_c$, where q_c satisfies Equation (28).

Part (iii) of the definition is obviously satisfied. For part (iv), $\Theta = \{(R_1, R_2) \,|\, R_1 > 1 \;\&\; R_1 R_{12} \leq R_2\} \cup \{(1, R_2(q))\}$. Part (v) requires the equilibrium to be robust to the Sincerity

Criterion. For any contract with $R_1 > 1$, $\Psi(R_1, R_2) = \emptyset$. Recall that, for $R_1 < 1$, workers believe that $R_2(q) = R_1$; this q is in $\Psi(R_1, R_2)$ and, with this bank, workers earn expected income $R_1 < 1$. Finally, q_c is defined to ensure that the market clears.

Uniqueness is established using the argument in the body of the text.

Proof of Proposition 2

Suppose that $L < \hat{L}$ and that the $q = 1$ bank offers a contract with $R_2 = R_{12}$. Then, because $L < \hat{L}$, a bank with q very close to q_c offers to pool with the $q = 1$ bank. As a result, the depositor IR constraint is violated. The $q = 1$ bank must therefore offer a contract (R_1, R_2^*) for some $R_2^* > R_{12}$.

Suppose now that a bank with $q < 1$ can pool with the $q = 1$ bank without experiencing a time 1 liquidity crisis. Such a bank will always opt to pool. It is identified by the condition $R_1 < p(q, R_2^*)$, which reduces to the following condition:

$$q \geq \hat{q}(R_1) = \frac{R_1 R_{12} - (\underline{A} + R_{12} G)}{R_2^* - (\underline{A} + R_{12})}. \tag{92}$$

Banks for which $q < \hat{q}(R_1)$ experience a time 1 liquidity crisis if they pool with the $q = 1$ bank. They must therefore choose between separation without a liquidity crisis and pooling with a liquidity crisis. They elect to separate if and only if $V_B^r(R_2^*, q) \leq V_B^{nr}(R_2(q), q)$; this reduces to the following condition on q:

$$q \geq q^m = \frac{R_{12}(1 - G) - \underline{A} - L}{R_2^*(1 - G) - \underline{A}}. \tag{93}$$

For $q < q^m$ banks prefer pooling with high type banks to separation. In turn, they prefer pooling to withdrawal from the banking market precisely when $V_B^r(R_2*, q) > R_{12}G$; this is equivalent to the following condition on q:

$$q \geq q^c = \frac{L + R_{12}G}{\bar{A} - (1 - G)R_2^*}. \tag{94}$$

Note now that, if $L = \hat{L}$ and $R_2^* = R_{12}$, banks at $q = q_c$ are indifferent between banking and not banking, and are also indifferent between separating and pooling, so that $q_m = q_c$. Increasing R_2^* above R_{12} increases q^c and lowers q^m, and so reduces the volume of low types that pool with the high type bank.

Lemma 20. *R_2^* is never raised so high that $q_m = q_c$.*

Proof. If this were the case then the expected depositor income from investment in a high-type bank would be $(\theta/R_{12} + 1 - \theta)\, R_2^*$, which, because $R_2^* > R_{12}$, exceeds the income $\theta +$

$(1-\theta)R_{12}$ that a worker earns from being unmonitored. This contradicts the requirement that depositor IR constraints bind in equilibrium.

Lemma 21. $R_1 = 1$ *for high bank types.*

Proof. Suppose that the depositor IR constraint binds for some $R_1 > 1$. Lowering R_1 slightly reduces $\hat{q}(R_1)$ without altering q_c or q_m. This serves to raise the average quality of banks in the pool and so slackens the IR constraint; the high type bank can then increase its welfare slightly by selecting a lower R_1^*.

Proof of Proposition 3

We start by establishing an expression for V_W^O. We can write

$$vV_W^O = q\bar{V}_W^O + (1-q)\underline{V}_W^O, \tag{95}$$

where $\bar{v}^o$ is the value of the opaque bank conditional upon $A = \bar{A}$ and $\underline{v}^O$ is its value conditional upon $A = \underline{A}$. We can further break these expressions down as follows:

$$\bar{v}^o = \theta\bar{v}_s^o + (1-\theta)\bar{v}_{ns}^o; \tag{96}$$
$$\underline{v}^o = \underline{v}_s^0 + (1-\theta)\underline{v}_{ns}^o, \tag{97}$$

where the s and ns subscripts denote the respective events that the depositor experiences, and does not experience, a liquidity shock. We have

$$\bar{v}_s^o = \bar{F}(s_r)R_1 + (1-\bar{F}(s_r))\left(e\frac{R_2}{R_{12}} + (1-ep\right); \tag{98}$$
$$\bar{v}_{ns}^o = \bar{F}(s_r)R_1 + (\bar{F}(s_h) - \bar{F}(s_r))\left(e\frac{R_2}{R_{12}} + (1-e)p\right) + (1-\bar{F}(s_h))\frac{R_2}{R_{12}}. \tag{99}$$
$$\underline{v}_s^o = \underline{F}(s_r)R_1 + (1-\underline{F}(s_r))\left(e\frac{\underline{R}_2^O}{R_{12}} + (1-e)p\right) + (1-\underline{F}(s_h))\frac{\underline{R}_2^O}{R_{12}}; \tag{100}$$
$$\underline{v}_{ns}^o = \underline{F}(s_r)R_1 + \underline{F}(s_h) - \underline{F}(s_r))\left(e\frac{\underline{R}_2^O}{R_{12}} + (1-e)p\right) + (1-\underline{F}(s_h))\frac{\underline{R}_2^O}{R_{12}}. \tag{101}$$

To avoid notational clutter, we write η_h for $\eta(s_h, q)$ so that, using Equation (41), we have

$$p = \frac{R_2 - (1-\eta_h)(R_2 - \underline{R}_2^O)}{R_{12}}. \tag{102}$$

Substituting Equation (102) into Equations (98) and (99) yields the Equations (103) and

(104):

$$\bar{v}_s^o = \frac{R_1}{R_{12}} - \bar{F}(s_r)\left(\frac{R_2}{R_{12}} - R_1\right) - (1-\bar{F}(s_r))(1-e)(1-\eta)\frac{(R_2 - \underline{R}_2^O)}{R_{12}}; \tag{103}$$

$$\bar{v}_{ns}^o = \frac{R_1}{R_{12}} - \bar{F}(s_r)\left(\frac{R_2}{R_{12}} - R_1\right) - (\bar{F}(s_h) - \bar{F}(s_r))(1-e)(1-\eta)\frac{(R_2 - \underline{R}_2^O)}{R_{12}}. \tag{104}$$

Similarly, substituting Equation (39) for $\underline{R}_2^O$ and Equation (102) for p into Equations (100) and (101) yields the following expressions for $\underline{v}_s^0$ and $\underline{v}_{ns}^o$:

$$\underline{v}_s^o = \frac{h\underline{A} + GR_{12}}{hR_{12}} + \eta(1-e)(1-\underline{F}(s_r))\frac{R_2 - \underline{R}_2^O}{R_{12}}; \tag{105}$$

$$\underline{v}_{ns}^o = \frac{h\underline{A} + GR_{12}}{hR_{12}} + \eta(1-e)(\underline{F}(s_h) - \underline{F}(s_r))\frac{R_2 - \underline{R}_2^O}{R_{12}}. \tag{106}$$

Combining Equations (103), (104), (105) and (106) with Equations (96) and (97) yields Equations (107) and (108):

$$\begin{aligned}\bar{v}^o =\ & \frac{R_2}{R_{12}} - \bar{F}(s_r)\left(\frac{R_2}{R_{12}} - R_1\right) \\ & - (1-e)(1-\eta)\frac{R_2 - \underline{R}_2^O}{R_{12}}\left[\theta(1-\bar{F}(s_r)) + (1-\theta)(\bar{F}(s_h) - \bar{F}(s_r))\right]; \end{aligned} \tag{107}$$

$$\begin{aligned}\underline{v}^o =\ & \frac{h\underline{A} + GR_{12}}{hR_{12}} \\ & \eta(1-e)\frac{R_2 - \underline{R}_2^O}{R_{12}}\left[\theta(1-\underline{F}(s_r)) + (1-\theta)(\underline{F}(s_h) - \underline{F}(s_r))\right]. \end{aligned} \tag{108}$$

Substitute Equations (107) and (108) into Equation (95) to get

$$\begin{aligned}v^o =\ & \frac{1}{R_{12}}\left(qR_2 + (1-q)\left(\frac{h\underline{A} + GR_{12}}{h}\right)\right) - q\bar{F}(s_r)\left(\frac{R_2}{R_{12}} - R_1\right) \\ & + (1-e)\frac{R_2 - \underline{R}_2^O}{R_{12}}\left[\theta(\eta - q) + q(1-\eta)(\bar{F}(s_r) - (1-\theta)\bar{F}(s_h)\right) \\ & -\eta(1-q)(\underline{F}(s_r) - (1-\theta)\underline{F}(s_h)] \end{aligned} \tag{109}$$

Substituting Equations (36), (37) and (47) into this expression and performing lengthy manipulations yields the following expression:

$$\begin{aligned}v^o =\ & v^t - q\bar{F}(s_r)\left(\frac{R_2}{R_{12}} - R_1\right) \\ & - (1-e)\theta\frac{q(1-q)}{q + (1-q)m(s_h)}\frac{2(\underline{F}(s_r) - \bar{F}(s_r))}{\theta(1-\bar{F}(s_r)) + (1-\theta)(\bar{F}(s_h) - \bar{F}(s_r))}. \end{aligned} \tag{110}$$

The second term in this expression is at most zero; the third is negative for $\theta > 0$ because the monotonicity of m implies that $\underline{F} > \bar{F}$.

Proof of Corollary 1

When $R_1 = \underline{R}_2^T$, $s_r = 0$ so that there is no equilibrium withdrawal and $V_W^T = V_W^O$. If R_1 increases marginally then the value V_W^T of a transparent bank does not change; by Proposition 3 V_W^O must decrease.

Proof of Lemma 7

For part 1, note from Equation (16) that $D(R_{12}) = 0$ when $\theta = 0$ so that, using Equation (17), $S(R_{12}) \geq G > D(R_{12})$. By part 1 of Lemma 6, R_{12} cannot be greater than 1 and, hence $R_{12} = 1$.

For part 2, when $\theta = 0$, by Equation (47) we have

$$m(s_h) = \frac{\underline{F}(s_h) - \underline{F}(s_r)}{\bar{F}(s_h) - \bar{F}(s_r)}. \tag{111}$$

If $s_h > s_r$, then, by Cauchy's Mean Value Theorem, there exists $\xi \in (s_r, s_h)$ such that $m(s_h) = \underline{f}(\xi)/\bar{f}(\xi) = m(\xi)$. But m is strictly monotonically decreasing and, hence, $\xi = s_h$, which contradicts the assumption that $s_h > s_r$. It follows that $s_h = s_r$.

Proof of Lemma 8

The first part follows from Lemma 7 and Equation (49) when $\theta \equiv 0$.

For the second part, note that we can write

$$V_W^O = R_1 + q(1 - \bar{F}(s_r))(R_2 - R_1) - (1-q)(1 - \underline{F}(s_r))(R_1 - \underline{R}_2^O). \tag{112}$$

That is, at time 1, a worker with an opaque bank will earn R_1 from withdrawing his income, plus any additional income he earns in case he does not withdraw (recall from Lemma 7 that he never sells his contract when $\theta \equiv 0$). With probability q, $\tilde{A} = \bar{A}$, in which case his probability of non-withdrawal is $1 - \bar{F}(s_r)$, he earns an additional income of $R_2 - R_1$ relative to the case where he withdraws; with probability $1 - q$, $\tilde{A} = \underline{A}$, in which case his probability of non-withdrawal is $1 - \underline{F}(s_r)$, he loses $R_1 - \underline{R}_2^O$ relative to the case where he withdraws.

Now note from Equation (62) that $\eta(s_r, q)R_1 = (1 - \eta(s_r, q))\underline{R}_2^O$, and, using Equation (37) to substitute for η,

$$\frac{\underline{f}(s_r)}{\bar{f}(s_r)} = \frac{q(R_2 - R_1)}{(1-q)(R_1 - \underline{R}_2^O)}. \tag{113}$$

Substituting Equation (113) into Equation (112) yields Equations (63) and (64).

Proof of Lemma 9

Equations (30), (39), and (62) yield

$$R_1 = \frac{(1-\underline{F}(s_r))\eta(s_r,q)}{1-\eta(s_r,q)\underline{F}(s_r)}R_2 + \frac{1-\eta(s_r,q)}{1-\eta(s_r,q)\underline{F}(s_r)}\underline{R}_2^T, \tag{114}$$

from which the following equation follows:

$$\frac{1-\eta(s_r,q)}{1-\eta(s_r,q)\underline{F}(s_r)} = \frac{R_2-R_1}{R_2-\underline{R}_2^T}. \tag{115}$$

Substituting for $\eta(s_r,q)$ in Equation (115) yields the following expression:

$$\frac{1-\eta(s_r,q)}{1-\eta(s_r,q)\underline{F}(s_r)} = \frac{(1-q)m(s_r)}{q(1-\underline{F}(s_r))+(1-q)m(s_r)}. \tag{116}$$

Using Equation (116) and setting $R_1 = 1$ yields Equation (67).

Proof of Corollary 15

Because $h > h_f$,

$$\frac{\partial}{\partial h}\pi^O(h,q) = -\frac{\partial}{\partial h}V_W^O(s_r,q) = \frac{\partial}{\partial s_r}V_W^O(s_r,q)\times\frac{ds_r}{dh}.$$

We have

$$\frac{\partial V_W^O}{\partial s_r} = (1-q)\frac{-\underline{A}\underline{f}(s_r)}{(1-\underline{F}(s_r))^2}\gamma(s_r) + (1-q)\left(1-\frac{\underline{A}}{1-\underline{F}(s_r)}\right)\gamma'(s_r);$$

this expression is negative, because $\gamma'(s_r) = m'(s_r)(1-\underline{F}(s_r)) < 0$. Moreover, $\frac{ds_r}{dh} = -\frac{1}{\underline{f}(s_r)}\frac{G}{h^2} < 0$. It follows that $\frac{\partial}{\partial h}V_W^O(h,q) > 0$ and, hence, that $\frac{\partial}{\partial h}\pi^O(h,q) < 0$.

Finally,

$$\frac{\partial^2}{\partial h\partial q}\pi^O(h,q) = \left(\frac{-\underline{A}\underline{f}(s_r)}{(1-\underline{F}(s_r))^2}\times\gamma(s_r) + \left(1-\frac{\underline{A}}{1-\underline{F}(s_r)}\right)\times\gamma'(s_r)\right)\times\frac{ds_r}{dh}.$$

This expression is positive because each of its factors is negative.

Proof of Proposition 4

We first prove that the optimal size an opaque bank is weakly increasing in its type q. Suppose that $q' < q$ and that the corresponding optimal opaque bank sizes h' and h satisfy $h < h'$. We derive a contraction. By Lemma 15, $\pi^O(h,q)$ is supermodular, from which Expression (117) follows:

$$\pi^O(h,q') \times \pi^O(h',q) > \pi^O(h,q) \times \pi^O(h',q). \tag{117}$$

However, because h and h' are optimal,

$$h\pi^O(h,q) \geq h'\pi^O(h',q)$$
$$h'\pi^O(h'q') \geq h\pi^O(h,q').$$

It follows that

$$\pi^O(h,q)\pi^O(h',q') \geq \pi^O(h,q')\pi^O(h',q).$$

which contradicts Expression (117).

To prove the rest of the proposition, it suffices to show that $\hat{q} \triangleq \inf\{q|h^O(q) = 1\} < 1$ and that $\tilde{q} \triangleq \sup\{q|h^O(q)(q) = h_f\} > q^*$. To that end, observe that, with $V_W^O(h,q)$ given by Equation (75) and $\pi^O(h,q)$ given Equation and (77), the derivative of the bank's profit V_B^O with respect to h is $\frac{\partial}{\partial h}V_B^O(h,q) = A_e(q) - 1 - (1-q)\tilde{\phi}(h)$, where

$$\begin{aligned}\tilde{\phi}(h) \triangleq &\left(1 - \frac{\underline{A}}{1-\underline{F}(s_r)}\right) \times \gamma(s_r) \\ &+ h\left(\left(1 - \frac{\underline{A}}{1-\underline{F}(s_r)}\right) \times (-\gamma'(s_r)) + \frac{\underline{A}}{(1-\underline{F}(s_r))^2}\underline{f}(s_r)\gamma(s_r)\right)\frac{\underline{F}(s_r)}{\underline{f}(s_r)},\end{aligned}$$

where s_r depends upon h as in Equation (74) and, because $s_r \to \underline{F}^{-1}(1-\underline{A})$ as $h \downarrow h_f$, $s_r(h_f) = \underline{F}^{-1}(1-\underline{A})$. It is easy to see that $\tilde{\phi}(h)$ is continuous in $[h_f, 1]$ and, hence, both $\phi^{\min} \triangleq \min_{h\in[h_f,1]} \tilde{\phi}(h)$ and $\phi^{\max} \triangleq \max_{h\in[h_f,1]} \tilde{\phi}(h)$ exist, and are finite.

If $A_e(q) - 1 - (1-q)\phi^{\max} > 0$, then $\frac{\partial}{\partial h}V_B^O(h,q) > 0$ for all $h \in [h_f, 1]$ and, hence, the optimal size for this type is $h = 1$. It follows that $\inf\{q|h^O(q) = 1\} \leq \inf\{q|A_e(q) - 1 - (1-q)\phi^{\max} > 0\} < 1$.

If $A_e(q) - 1 - (1-q)\phi^{\min} < 0$, then $\frac{\partial}{\partial h}V_B^O(q) < 0$ for all $h \in (h_f, 1]$ and, hence, the optimal size for this type is $h = h_f$. Because $A_e(q^*) = 1$, it follows that $\sup\{q|h^O(q) = h_f\} \geq \sup\{q|A_e(q) - 1 - (1-q)\phi^{\min} < 0\} > q^*$.

Proof of Proposition 5

Suppose that, for some $q' < 1$, $\lim_{G \downarrow 0} h^O(q') = h' > 0$. As $h_f = G/(1 - \underline{A})$, we must have $h^O(q') > h_f$ for small G and, hence, by Lemma 13, $s_r(h^O(q')) = \underline{F}^{-1}(G/h)$. It follows that $\lim_{G \to 0} s_r(h^O(q')) = 0$. Hence, using the equilibrium condition Equation (67), $\lim_{G \to 0}(R_1 - \underline{R}_2^T)/(R_2 - R_1) = \lim_{G \to 0}(R_1 - (\underline{A} + G/h^O(q)))/(R_2 - R_1) = 0$. Because R_2 is bounded, we must have $\lim_{G \to 0}(\bar{A} + G/h^O(q')) = R_1 = 1$; this contradicts the fact that, if $\lim_{G \to 0} h^O(q) = h' > 0$, then $\lim_{G \to 0}(\underline{A} + G/h^O(q)) = \underline{A} < 1$.

Proof of Lemma 16

Note that, because no worker can generate higher product than $\bar{A}$, (R_1^*, R_2^*) can generate higher-than-equilibrium profits for a bank only if $R_2^* < \bar{A}$. Moreover, no worker would accept contract (R_1^*, R_2^*) if $R_2^* < 1$. We can therefore restrict our attention to contracts (R_1^*, R_2^*) with $R_1^* < 1$ and $1 \le R_2^* < \bar{A}$.

Given any contract (R_1^*, R_2^*) that satisfies these conditions, let

$$q' \triangleq \begin{cases} \frac{R_1^* - \underline{A}}{R_2^* - \underline{A}}, & \text{if } R_1^* > \underline{A}; \\ q^*/2, & \text{otherwise.} \end{cases}$$

We show that type q' transparent banks earn a higher-than-equilibrium profit from contract (R_1^*, R_2^*), and that their workers earn less than 1 from this contract.

By definition, $q' R_2^* + (1 - q')\underline{A} \ge R_1^*$ if $R_1^* \le \underline{A}$ and otherwise $q'\bar{A} + (1 - q')\underline{A} = R_1^*$, so that a type q' transparent bank's liquidity constraint is satisfied if it offers contract (R_1^*, R_2^*) and consumes its corn stock G at time $t = 0$. The value of (R_1^*, R_2^*) to the workers in this case is $V_W^T = q' R_2^* + (1 - q')\underline{A} = \max(R_1^*, 0.5 \times q^* R_2^* + (1 - 0.5 \times q^*)\underline{A}) < 1$. The result is therefore proved if the type q' transparent bank's payoff from (R_1^*, R_2^*) exceeds its equilibrium payoff.

The q'-bank's payoff if it offers contract (R_1^*, R_2^*) and consumes its corn stock G at time 0 is $\tilde{V}_B^T = A_e(q') - V_W^T + G$. Its equilibrium profit is $V_B^T(q') = \max(A_e(q) - 1, 0) + G$. $\tilde{V}_B^T(q') - V_B^T(q') = \min(1 - V_W^T, A_e(q') - V_W^T) = \min(1 - V_W^T, q'(\bar{A} - R_2^*) > 0$.

Proof of Proposition 6

Suppose that there exists a $q < 1$ with $G \ge \underline{F}(s_r^*(q))$ so that opaque type q banks do not experience a liquidity crisis. When $\theta \equiv 0$, Lemma 9 implies that $s_r^*(q)$ is determined by the following equation:

$$\frac{1 - q}{q} \frac{1 - \underline{R}_2^T}{R_2 - 1} = \frac{1 - \underline{F}(s_r)}{m(s_r)}.$$

In this case, we have $R_2 = (1-(1-q)(\underline{A}+G))/q$ and $\underline{R}_2^T = \underline{A}+G$, so that $(1-\underline{R}_2^T)/(R_2-1) = q/(1-q)$, so that

$$\frac{1-\underline{F}(s_r)}{m(s_r)} = 1. \tag{118}$$

Recall from Lemma 8 that the net chance of gain $H(s) = \frac{1-\bar{F}(s)}{\bar{f}(s)} - \frac{1-\underline{F}(s)}{\underline{f}(s)} > 0$. Hence $(1-\underline{F}(s))/m(s) < 1-\bar{F}(s) \leq 1$, which contradicts Equation (118).

Proof of Lemma 17

Suppose that a type q bank swaps x units of its project under the GB policy and so avoids a run. Its bad state repayment absent withdrawal is then $\underline{R}_2^{GB} \triangleq (1-x)\underline{A} + x\delta A_e(q) + G$. We can substitute $\underline{R}_2^{GB}$ for the bad state payment absent withdrawal $\underline{R}_2^T$ in Equation (67) for the withdrawal threshold s_r as follows:

$$\begin{aligned}\chi(s^*) &= \frac{1-q}{q}\frac{1-\underline{R}_2^{GB}}{R_2(q)-1} \\ &= \frac{1-(\underline{A}+G)-x(\delta A_e(q)-\underline{A})}{1-(\underline{A}+G)}.\end{aligned}$$

Equation (82) follows immediately. Opaque type q banks can avoid a liquidity crisis if and only if $x(q) \leq 1$. It is easy to see that $x'(q) < 0$; all banks can therefore be saved precisely when $x(q^*) \leq 1$, which is true if and only if Condition (83) is satisfied.

Proof of Lemma 18

$\pi^O = A_e(q) - V_W^O(h,x,q)$. A type q opaque bank therefore solves the following problem:

$$\max_{h\in[0,1]} V_B^O(h,x,q) \triangleq h \times \pi^O(h,q). \tag{119}$$

The type q opaque bank's first order condition is

$$\pi^O(h,x,q) + h\frac{\partial \pi^O}{\partial h}(h,x,q) = 0. \tag{120}$$

Differentiating with respect to x yields

$$\frac{\partial h}{\partial x} = -\frac{\frac{\partial \pi^O}{\partial x} + h\frac{\partial^2 \pi^O}{\partial h \partial x}}{2\frac{\partial \pi^O}{\partial h} + h\frac{\partial^2 \pi^O}{\partial h^2}}. \tag{121}$$

The second order condition for the bank's maximisation problem is that the numerator of this expression be negative. Hence, $\frac{\partial h}{\partial x} > 0$ if the numerator is positive.

We have

$$\frac{\partial \pi^O}{\partial x} = (1-q)\frac{A_e(q) - \underline{A}}{h - G}\gamma(s_r) > 0.$$

so that

$$\frac{\partial^2 \pi^O}{\partial h \partial x} = -(1-q)\frac{A_e(q) - \underline{A}}{h - G}\left(\frac{\gamma(s_r)}{h-G} + \gamma'(s_r)\frac{\underline{F}(s_r)}{h\underline{f}(s_r)}\right)$$

Substituting $G = h\underline{F}(s_r)$ gives us

$$\frac{\partial^2 \pi^O}{\partial h \partial x} = -(1-q)\frac{A_e(q) - \underline{A}}{h - G}\left((-\gamma'(s_r))\frac{\underline{F}(s_r)}{\underline{f}(s_r)} - \frac{\gamma(s_r)}{1-\underline{F}(s_r)}\right).$$

Substituting this expression gives us the following value for the numerator of Equation (121):

$$\frac{\partial \pi^O}{\partial x} + h\frac{\partial^2 \pi^O}{\partial h \partial x} = (1-q)\frac{A_e(q) - \underline{A}}{h - G}\underline{F}(s_r)\left[(-\gamma'(s_r))\frac{1}{\underline{f}(s_r)} - \gamma(s_r)\frac{1}{1-\underline{F}(s_r)}\right].$$

This expression is positive, and, hence, $\frac{\partial h}{\partial x}$, precisely when the square bracketed term is positive. That is, precisely when

$$-\frac{d}{ds_r}\left[\log(s_r) - \log(1-\underline{F}(s_r))\right] > 0.$$

This is equivalent to the requirement that $\log(\gamma(s_r)/(1-\underline{F}(s_r)))$ be decreasing in s_r, or that $m(s_r)\frac{1-\bar{F}(s_r)}{1-\underline{F}(s_r)} - 1 = \frac{1-\bar{F}(s_r)}{\chi(s_r)} - 1$ be decreasing in s_r. That condition is satisfied throughout the stable region, in which $\chi(s)$ is increasing.