Asset Pricing in Monetary Economies*

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September 10, 2020

Abstract

This paper reviews and extends recent research on liquidity and asset pricing. We start by asking how can intrinsically-worthless fiat money be valued in equilibrium? The literature on which we build formalizes how money is valued for its liquidity when exchange is hindered by various frictions. Once one sees how money can be priced above its fundamental value, it is clear that many other assets can be, too, if they also convey liquidity. We study under which conditions money can be valued if assets have fundamental value, how the liquidity values of money and assets interact, and how they are affected by changes in parameters such as acceptability, pledgeability, or the type of the asset.

*For comments we thank Athanasios Geromichalos, David Andolfatto, Alberto Trejos, Steve Williamson and Yu Zhu. Wright acknowledges the Ray Zemon Chair in Liquid Assets at the Wisconsin School of Business for research support. The usual disclaimers apply.

1 Introduction

This paper reviews and extends recent research on markets where liquidity plays an explicit role and discusses the implications for asset pricing. The research on which we build can be described as a quest for micro foundations in macro, financial and monetary economics, and has been labeled the New Monetarist approach.[1] Early papers in this literature studied primitive models, although they were theoretically pure, in the sense that there were few ad hoc assumptions (like imposing a cash-in-advance constraint or putting assets in utility functions with a vague appeal to "convenience" as in Krishnamurthy and Vissing-Jorgensen 2012). That made sense to the extent that those papers were designed to make conceptual points by addressing basic questions such as these: When is the institution of monetary exchange (as opposed to, e.g., pure barter or credit) an equilibrium or an efficient arrangement? What properties of an object make it more likely or more desirable that it will serve as a medium of exchange? How can fiat currency that is intrinsically worthless be valued in equilibrium?

This last question about fiat money is especially important for our purposes, even though in this paper we follow much of the literature and go well beyond the early primitive models. The reason is this: While we all under-

[1] See Williamson and Wright (2010a,b) for extended discussions of methods in this research program, including the logic behind the label (in brief, many who we call New Monetarists find appealing some things, but not everything, in Old Monetarist writings, and are skeptical of the Old or New Keynesian approach). See Lagos et al. (2017) or Nosal and Rocheteau (2017) for more recent surveys; see Kareken and Wallace (1980) for older work in a similar spirit. As an aside, to avoid confusion, note that New Monetarist Economics is distinct from Modern Monetary Theory, which has been in the news recently. Indeed, the two are polar opposites in many ways – e.g., the former strives for theoretical rigor and is quite technical, while the latter seems based more on intuition and preferences for certain policies that we find hard to justify. In particular, New Monetarists do not advocate printing money as a way for government to give us a cornucopia of goods and services, mainly because we do not share their faith that printing money does not cause inflation, a faith that is not supported by theory or historical data.

stand loosely that money is valued for its liquidity, it is important to formalize this idea in a rigorous way, because once one understands how the most rudimentary asset – fiat currency – can be priced above its fundamental value of 0, it is easier to understand how the price of any asset can involve a liquidity component.[2] To pursue these ideas, one obviously must go beyond classical GE (general equilibrium) theory. The framework analyzed below captures liquidity by taking seriously the process of exchange in the presence of explicit frictions. The object of this paper is to show how money and other assets interact in the presence of these frictions, and we think one can learn a lot from this exercise.[3]

Section 2 describes the basic features of our framework. Section 3 analyzes the decision problems of the individuals in the model and defines equilibrium. While in general equilibrium entails a dynamic path, here we focus on stationary equilibria, or steady states, and Section 4 discusses existence as well as the effects of changes in monetary policy and asset market conditions on liquidity. In particular, in stationary equilibrium it is equivalent for policy to set the growth rate of the money supply, the inflation rate, or the nominal interest rate (although it is important to note that there are many interest rates determined by the theory, not just one). While most of the analysis focuses on assets that can be interpreted as equity in a technology that produces a

[2] Moreover, once one understands this, it becomes apparent that liquidity is to some extent a self-fulfilling prophecy. As a rudimentary example, the reason you value currency, and hence accept it in exchange for your labor time or other things of value, is that it facilitates future trade as long as others accept it; but if no one believes others accept it then they do not value it. From this it follows that the monetary exchange can be unstable, in the sense that there can be equilibria where the value of money can oscillate or fluctuate randomly over time. And from this it is a small step to see how asset prices, in general, can oscillate or fluctuate even when fundamentals are constant. Having said this, we do not focus on multiplicity or dynamics in this paper, but relegated that to future work.

[3] This approach has successfully been used to rationalize a number of asset pricing puzzles. Examples are Lagos (2010) for the equity premium puzzle; Geromichalos et al (2016) for the term premium puzzle; Geromichalos and Simonovska (2014) for the home asset bias puzzle; and Jung and Lee (2015) for the uncovered interest parity puzzle.

dividend each period, Section 5 takes up three alternative specifications, where assets can be interpreted as bonds, as productive capital (like in neoclassical growth theory), and as housing. Section 6 concludes.

2 Environment

Although one can choose from several specifications in the New Monetarist literature that differ in their details (see the surveys mentioned in footnote 1), we build on one that has become a workhorse in monetary economics over the last several years, based on Lagos and Wright (2005). This is a discrete-time environment where every period has two subperiods: first agents interact in a decentralized market (DM) with various frictions described below; then they interact in a frictionless centralized market (CM). This framework is natural for our purposes because at its core is an asynchronization of expenditures and receipts that is key to any analysis of money, credit or liquidity. Namely, some agents desire expenditures in the DM while their receipts accrue in the CM, which means they must either bring in purchasing power from a previous CM, or (credibly) promise to make payments in a future CM. Moreover, agents in the DM trade with each other – not merely against their budget lines as in traditional GE theory – making it natural to incorporate frictions like limited commitment and private information.

There are two types of infinite-lived agents, called buyers and sellers, based on their roles in the DM.[4] The measure of buyers is normalized to 1 and the measure of sellers is n. They meet bilaterally and randomly in the DM, where α is the probability a buyer meets a seller and α/n is the probability a seller

[4]In the original version agents are homogeneous when they enter the DM, but end up being buyers or sellers, depending on who meets whom; here, following Rocheteau and Wright (2005), they know in the CM whether they will be buyers or sellers in the next DM. Most results are qualitatively similar in the two versions.

meets a buyer. One can interpret buyers as households trying to acquire consumption goods, firms trying to acquire inputs, or financial institutions trying to acquire assets in the DM, as all have been deployed to good effect in the literature (again see the afore-mentioned surveys). Here the interpretation does not matter much, but for concreteness, let's say the object buyers want in the DM is a good q. A different good x is traded in the CM, and it is numeraire in a standard CM budget equation. Labor ℓ is also traded in the CM at real wage $w = 1$, because we assume 1 unit of ℓ makes 1 unit of x, although that is easy to generalize.

Goods are nonstorable, ruling out both direct barter and commodity money in the DM. There are other storable objects. One asset is, following Lucas (1978), in fixed supply A, and pays a dividend $\rho > 0$ in CM numeraire each period.[5] The other is an intrinsically-worthless fiat money with supply growing at rate μ, so $M_{t+1} = (1 + \mu)M_t$. While M can be a store of value and A can be a medium of exchange, it facilitates the presentation to call A the (real) asset and M the (fiat) money. Their CM prices are ϕ_a and ϕ_m. Newly printed money is distributed to buyers in the CM through lump-sum transfers, $T = \phi_m \mu M$, although the main results are the same if this seigniorage is used by the government to buy x. Now, if any asset is to have liquidity value, credit must be less than perfect: if it were perfect, buyers could trade in the DM with promises to pay in the next CM. To hinder credit, therefore, assume there is no commitment so buyers can renege on such promises, and they are anonymous so it is not possible to punish renegers by taking away future credit.[6]

[5] The case $\rho < 0$ is actually interesting, but to conserve space we only mention it in footnotes. For now, note that it is possible here to have an asset valued even if $\rho < 0$, as long as $|\rho|$ is not too big, due to liquidity considerations that are missing in standard finance or GE theory.

[6] There are many details that we cannot go into here concerning anonymity and the viability of credit without commitment; see Kehoe and Levine (1993) and Kocherlakota (1998) for important contributions to theories of credit and money, and Gu et al. (2015) for

This allows assets to have a role in facilitating DM trade. To make things more interesting, suppose sellers may differ in terms of what they accept in the DM: a buyer meets a seller that only accepts money with probability $\alpha_m \geq 0$; he meets one that only accepts assets with probability $\alpha_a \geq 0$; he meets one that accepts either with probability $\alpha_e \geq 0$; where $\alpha_m + \alpha_a + \alpha_e \leq 1$ (the inequality may be strict since he may meet no one). Special cases discussed below include $\alpha_a = \alpha_m = 0 < \alpha_e$, where assets are perfect substitutes in terms of acceptability, and $\alpha_a = \alpha_e = 0 < \alpha_m$ which resembles a cash-in-advance model, although that is superficial, as those models do not have agents trading bilaterally. Also, sellers who accept something may not accept an arbitrarily large amount. Thus, let χ_a or χ_m be the fractions of a buyer's real asset or money holdings that a seller accepts, given that he accepts it at all. The α's capture liquidity along the extensive margin (whether a counterparty accepts something) while the χ's capture liquidity on the intensive margin (how much he accepts).[7]

Although we take them as parameters, the α's and χ's can be endogenized using information theory. Lester et al. (2012) model the α's by making sellers pay a cost to recognize high quality vs low quality (e.g., counterfeit) versions of assets, and the fraction that pay this cost is endogenous. Since they also assume low quality versions can be produced on the spot for free, a seller that does not recognize asset quality simply rejects it. In Rocheteau (2011) buyers

an integrated version.

[7] We also mention that one can incorporate many real assets, say a_h, where $h = 1, 2, ...H$, and all can be valued as long as $\rho_h > 0\ \forall h$ or $\rho_h < 0\ \forall h$ (recall from footnote 5 that an asset can be valued even with a negative dividend if it conveys liquidity). In fact, all that matters is $\Sigma_h \rho_h A_h$, at least as long as $\alpha_h = \alpha$ and $\chi_h = \chi\ \forall h$. In this case the assets are perfect substitues in liquidity provision and must have the same return r_h, defined as in (9) below, which can happen by having ϕ_h adjust iff all the ρ_h's have the same sign. Relatedly, one can incorporate many types of fiat currencies, but their relative prices are not pinned down (exchange rate indeterminacy as first pointed out by Kareken and Wallace 1981) unless their α's or χ's are different (see Gomis-Porqueras et al. 2017).

have risky assets and riskless bonds, and by choosing which assets to use as a means of payment they can signal the quality of their risky assets. Li et al. (2012) model the χ's by making buyers pay a cost to produce low quality (e.g., counterfeit) assets. Then a seller seeing a buyer with, say, \$100, can accept up to, say, \$50 if it costs buyers \$50 to produce \$100 in low quality, because the buyer can credibly argue that he would be happy to give up \$50 of high quality rather than pay \$50 to produce \$100 worth of low quality and only pass \$50 (assuming for the sake of example that low quality is worth 0). Among other reasons to think that it is important to endogenize the α's and χ's, note that when one does there can be multiple equilibria, with different α's or χ's, and the impact of policy can differ across these equilibria (Rocheteau et al. 2018). Still, in this exercise they are fixed exogenously.

We also emphasize that there are different ways for assets to facilitate trade[8] . The first and most obvious way involves immediate settlement: a DM buyer hands over assets as a medium of exchange. The second and more subtle way involves deferred settlement: a DM buyer promises payment in the next CM using assets as collateral (if he reneges some assets can be seized). As in the literature following Kiyotaki and Moore (2005), suppose only a fraction χ of assets can be pledged as collateral, often rationalized by saying that if a debtor reneges we can only seize a fraction of his assets. However, we can also rationalize this specification using private information, as discussed above, and that does not depend on whether the assets are used as collateral or media of exchange. Now notice this: in the collateral story, a debtor will honor an

[8] In all the interpretations discussed here, asset liquidity is 'direct': buyers use assets as a medium of exchange or collateral in a bilateral meeting with a seller. Geromichalos and Herrenbrueck (2016) develop the idea of 'indirect liquidity' by introducing into the model a third market, namely, a secondary OTC asset market that opens before the DM. Buyers who have an opportunity to consume cannot use assets as medium of exchange/collateral in the DM, but they can visit the OTC market to sell assets for cash. For more on this, see also Geromichalos and Herrenbrueck (2017).

obligation iff it is worth no more than the pledged assets. So he can only pledge χ times the value of his assets. But then, rather than using them as collateral, he may as well hand them over as a medium of exchange – the implications are the same.[9]

What keeps the analysis tractable is the assumption of quasi-linear payoffs.[10] For buyers, the life-time payoff is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(q_t) + U(x_t) - \ell_t], \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, $u(q_t)$ is the payoff from DM trade, and $U(x_t) - \ell_t$ is the payoff from CM trade. If one interprets buyers as households and q as a consumption good $u(\cdot)$ is a standard utility function; if one interprets them as firms and q as a productive input $u(\cdot)$ is a production function for x; and a similar story applies if one interprets them as investors or financial institutions and q as an asset. Again, we are agnostic about the interpretation because the formal results do not depend on this.

As usual, we assume $u(0) = 0$, $u'(q) > 0$, $u''(q) < 0$ and $u'(0) = \infty$, plus $U(0) = 0$, $U'(x) > 0$, $U''(x) < 0$ and $U'(0) = \infty$. Similarly, sellers have quasi-linear payoffs, but all we need to specify for them is their cost $c(q)$ from trading q to buyers, which can be a disutility cost of producing q or an opportunity

[9] These observations, about the equivalence of using assets as a medium of exchange or as collateral, are made in several places, including Lagos (2010,2011). Another equivalent scenario is to assets used in repos: a seller takes possession of assets in the DM with a commitment to sell them back to the buyer in the next CM at a prearranged price. To be clear, this is not meant to be a deep theory of alternative payment arangements; it is meant to argue that the same equations apply to different institutional details. Now one can come up with circumstances where these arrangements are not equivalent, but one has to be somewhat clever; see Madison (2018) and Loberto (2018). Also, some papers in the literature have, e.g., old or new capital used as collateral for buying new capital, and there it is obviously not equivalent to use it as a medium of exchange – you get nowhere trading k for k. However, other papers have a different asset a used as collateral to buy k, similar to our model, and then it is equivalent to use a as a medium of exchange.

[10] Wong (2016) shows that quasilinearity is not a necessary condition for tractability in this class of models, but we will use it here to keep the math simple.

cost of giving it up from an inventory. Assume $c(0) = 0$, $c'(q) > 0$, $c''(q) \geq 0$, $c'(0) = 0$ and $c(\bar{q}) = u(\bar{q})$ for some $\bar{q} > 0$. Let x^* and q^* denote the efficient CM and DM trades, defined by $U'(x^*) = 1$ and $u'(q^*) = c'(q^*)$.[11]

3 Equilibrium

Denote by $W(m, a)$ and $V(m, a)$ a buyer's value functions in the CM and DM, respectively. Then his CM problem is

$$W(m,a) = \max_{x,\ell,\hat{m},\hat{a}} \{U(x) - \ell + \beta V(\hat{m}, \hat{a})\} \tag{2}$$
$$\text{st } x = \ell + \rho a + T + \phi_m(m - \hat{m}) + \phi_a(a - \hat{a})$$

where $(\hat{m}, \hat{a})$ is the portfolio he takes to the next DM. The problem for a seller is similar and hence omitted. Assuming an interior solution for labor, $0 < \ell < 1$, where 1 is the time endowment, we can eliminate ℓ and rewrite (2) as

$$W(m,a) = \Omega + \max_x\{U(x) - x\} + \max_{\hat{m},\hat{a}}\{-\phi_m\hat{m} - \phi_a\hat{a} + \beta V(\hat{m}, \hat{a})\}, \tag{3}$$

where $\Omega = \phi_m m + (\phi_a + \rho)a + T$ is wealth. The first-order condition for x is $U'(x) = 1$, which means $x = x^*$, while portfolio choice solves

$$\phi_m = \beta V_1(\hat{m}, \hat{a}) \tag{4}$$
$$\phi_a = \beta V_2(\hat{m}, \hat{a}). \tag{5}$$

These imply portfolio choice $(\hat{m}, \hat{a})$ is independent of (m, a). Moreover, the envelope conditions $W_1(m, a) = \partial\Omega/\partial m = \phi_m$ and $W_2(m, a) = \partial\Omega/\partial a = \phi_a + \rho$ imply W is linear in (m, a).

[11] For some applications one might prefer limiting special cases – e.g., $U(x) = x$ may be natural if one thinks firms' payoffs are linear in profit, and if $U(x) = x$ we do not need them to supply labor.

While W depends only on m and a through their impact on $\phi_m m+(\phi_a+\rho)a$, V generally depends on the portfolio $(\hat{m},\hat{a})$. By the linearity of W, it is given by

$$V(m,a)=W(m,a)+\alpha_m[u(q_m)-p_m]+\alpha_a[u(q_a)-p_a]+\alpha_e[u(q_e)-p_e] \quad (6)$$

where q_j is the quantity the buyer gets and p_j is the payment he makes in a type j meeting, $j=m,a,e$, both of which can depend on (m,a). Note that payments are denominated in numeraire in the next CM, and are constrained by the buyers liquidity position in each type of meeting: $p_m \leq z_m \equiv \chi_m \phi_m m$, $p_a \leq z_a \equiv \chi_a(\phi_a+\rho)a$ and $p_e \leq z_e \equiv z_m + z_a$. Note that we only count the pledgeable value of assets in defining the liquidity position.

As for determining of DM terms of trade (p,q), we are agnostic and use a generic mechanism described as follows: to get quantity q requires payment $p=v(q)$, where v is some strictly increasing function with $v(0)=0$. The only other requirement is the following: let $p^*=v(q^*)$; then we impose:

$$p^* \leq z \Rightarrow p=p^* \text{ and } q=q^*;\ p^*>z \Rightarrow p=z \text{ and } q=v^{-1}(z). \quad (7)$$

This is satisfied for many common trading protocols, including Nash, Kalai and other standard bargaining solutions, which are natural when trade is bilateral, as well as perfectly or imperfectly competitive price taking, which may make more sense if it is multilateral.[12] Also, while in principle $v(\cdot)$ can be different in each type of meeting, for simplicity we take it to be the same.

Before defining equilibrium, we first note the standard result that buyers hold all the money, because sellers have no need of a payment instrument and

[12] It also holds for exotic mechanisms like the one in Hu et al. (2009), and can be derived axiomatically as in Gu and Wright (2016). As an example, Kalai bargaining leads to $v(q)=\theta c(q)+(1-\theta)u(q)$, where θ is buyers' bargaining power. Nash baragining is similar but more complicated in models with liquidity constraints. For perfectly competitive (Walrasian) markets, $v(q)=Pq$, where agents take P parametrically but it is determined in equilibrium by market clearing (see Rocheteau and Wright 2005). Note that sometimes the mechanism matters for substantive conclusions, but that is not really the case here.

cash is not a good saving vehicle, except possibly in an exceptional case where a certain nominal interest rate is at its lower bound. To see what this means in a simple context, consider the stationary case where real money balances are constant, which means the nominal price level $1/\phi_{m,t}$ rises at the same rate as monetary expansion, so inflation is given by $1 + \pi \equiv \phi_{m,t}/\phi_{m,t+1} = 1 + \mu$. Then define a nominal interest rate i by the following thought experiment: ask an agent how many dollars he would require in the next CM to give up a dollar in this CM; the answer will be $1+i$, where i satisfies the Fisher equation $1 + i = (1 + \pi)/\beta$. The lower bound for i consistent with the existence of a stationary monetary equilibrium (as defined below) is $i \geq 0$.

We focus on $i > 0$, but also consider the limit $i \to 0$, which is called the Friedman rule. Except in the limiting case where $i \to 0$ sellers strictly prefer to hold no cash, so money market clearing in the CM is $\hat{m}_t = (1 + \pi) M_t$. The situation is slightly different for the other asset, because it may be a good saving vehicle.[13] Hence, while buyers hold $\hat{a}_t$, sellers may also hold some, say $\tilde{a}_t$. Given the measures of buyers and sellers, asset market clearing in the CM is $\hat{a}_t + n\tilde{a}_t = A$. Finally to define equilibrium we need as initial conditions the endowments of (m, a) for all agents in the DM at the beginning of time. Then we have the following:

Definition 1: An equilibrium is a list of nonnegative, bounded CM asset prices and DM trades, $\{\phi_{m,t}, \phi_{a,t}, q_{j,t}, p_{j,t}\}$ for all t, such that: (i) portfolios demanded in the CM satisfy market clearing; (ii) exchanges in the DM satisfy (7) in each type of meeting; and (iii) initial conditions hold. It is a monetary equilibrium if $\phi_{m,t} > 0$ for all t.

[13] Due to quasilinear preferences, agents are only willing to save if assets are priced at their fundamental value in this model - and then they are indifferent about the quantity of savings. Altermatt (2019) develops a model in which agents value assets both for liquidity and savings purposes by combining the framework presented here with an OLG structure.

As is standard, when we say asset prices are bounded here we mean to rule out explosive paths where $\beta^t \phi_{j,t} \to \infty$, a condition that can be derived from the transversality condition to the CM problem (e.g., Rocheteau and Wright 2013). This holds trivially in the following:

Definition 2: A stationary equilibrium is a list of the same objects, where $\phi_{m,t}$ falls at rate μ, so $\phi_{m,t} M_t$ is constant, as are $\phi_{a,t}$, $q_{j,t}$ and $p_{j,t}$, satisfying (i) and (ii) in Definition 1. It is a stationary monetary equilibrium if $z_m > 0$.

It is useful to work with real money balances $z_m = \chi_m \phi_m M$, rather than ϕ_m, since z_m is constant in stationary equilibrium. Then in the interests of symmetry we work with $z_a = \chi_a(\phi_a + \rho)a$. Thus z_m and z_a are liquidity embodied in currency and real assets, respectively. To pursue symmetry further, define the real returns between the CM at t and the CM at $t+1$ by

$$1 + r_{m,t} = \frac{\phi_{m,t+1}}{\phi_{m,t}} \tag{8}$$

$$1 + r_{a,t} = \frac{\phi_{a,t+1} + \rho}{\phi_{a,t}}. \tag{9}$$

Given this, rewrite (3) and (6) as

$$W(z_m, z_a) = \Omega + U(x^*) - x^* + \max_{\hat{z}_m, \hat{z}_a} \left\{ -\frac{\hat{z}_m}{\chi_m(1+r_m)} - \frac{\hat{z}_a}{\chi_a(1+r_a)} + \beta V(\hat{z}_m, \hat{z}_a) \right\}$$

$$V(z_m, z_a) = W(z_m, z_a) + \alpha_m[u(q_m) - p_m] + \alpha_a[u(q_a) - p_a] + \alpha_e[u(q_e) - p_e],$$

where $\Omega = z_m/\chi_m + z_a/\chi_a + T$ is still wealth, and it as well as (p_j, q_j) depend on (z_m, z_a).

It is straightforward to differentiate $V(\cdot)$ to get

$$V_1(z_m, z_a) = \frac{1}{\chi_m} + \alpha_m[u'(q_m)q_m'(z_m) - p_m'(z_m)] + \alpha_e[u'(q_e)q_e'(z_e) - p_e'(z_e)]$$

$$V_2(z_m, z_a) = \frac{1}{\chi_a} + \alpha_a[u'(q_a)q_a'(z_a) - p_a'(z_a)] + \alpha_e[u'(q_e)q_e'(z_e) - p_e'(z_e)].$$

From (7), if the constraint $p_j \leq z_j$ is slack then p'_j and q'_j vanish, while if it binds then $p'_j = 1$ and $q'_j = 1/v'(q_j)$, where the latter follows from $z_j = v(q_j)$. Therefore,

$$V_1(z_m, z_a) = \frac{1}{\chi_m} + \alpha_m \lambda(q_m) + \alpha_e \lambda(q_e) \tag{10}$$

$$V_2(z_m, z_a) = \frac{1}{\chi_a} + \alpha_a \lambda(q_a) + \alpha_e \lambda(q_e), \tag{11}$$

where $\lambda(q) \equiv u'(q)/v'(q) - 1$ is called the liquidity premium, equal to the Lagrange multiplier on the constraint $p \leq z$. Inserting (10)-(11) into the first-order conditions from the portfolio problem in (3) we get the Euler equations

$$1 = \beta(1 + r_m)\left[1 + \alpha_m \chi_m \lambda(q_m) + \alpha_e \chi_m \lambda(q_e)\right] \tag{12}$$

$$1 = \beta(1 + r_a)\left[1 + \alpha_a \chi_a \lambda(q_a) + \alpha_e \chi_a \lambda(q_e)\right]. \tag{13}$$

While it may not be obvious from (12)-(13), these equations are dynamic because r_j includes ϕ_j in this period and next period. Indeed, using (8)-(9), we can rewrite them as

$$\phi_{m,t} = \beta \phi_{m,t+1}\left[1 + \alpha_m \chi_m \lambda(q_{m,t+1}) + \alpha_e \chi_m \lambda(q_{e,t+1})\right] \tag{14}$$

$$\phi_{a,t} = \beta\left(\rho + \phi_{a,t+1}\right)\left[1 + \alpha_a \chi_a \lambda(q_{a,t+1}) + \alpha_e \chi_a \lambda(q_{e,t+1})\right], \tag{15}$$

which is a two-dimensional system in asset prices, because the q's can be eliminated using $q_j = v^{-1}(z_j)$, where z_j is a simple function of ϕ_j.[14] These alternative ways of expressing the equilibrium conditions convey different economic intuition, and can be more or less useful, depending on the application.

In any case, the important result is that the price of a_j exceeds its fundamental value iff liquidity considerations are operative, meaning that there is a positive probability of being in a situation where a binding constraint $p_j \leq z_j$

[14]Note that (14)-(15) would look more like standard Euler equations for investments if $U'(x_t)$ appeared on the LHS and $U'(x_{t+1})$ on the RHS, but in this simple version of the framework (not all versions), these terms cancel, because $x_t = x^* \; \forall t$.

is relaxed by having more of the asset. First consider solutions to (14) with $\alpha_m \chi_m \lambda(q_m) = \alpha_e \chi_m \lambda(q_e) = 0$. The only solution consistent with equilibrium is $\phi_{m,t} = \phi_m^* = 0\ \forall t$, because any other solution path explodes, and hence any equilibrium is nonmonetary: fiat currency must be valued at its fundamental price of 0. However, $\alpha_m \chi_m \lambda(q_m) > 0$ or $\alpha_e \chi_m \lambda(q_e) > 0$ implies there can be equilibria with $\phi_{m,t} > 0$, which means there can be monetary equilibria. Similarly, consider solutions to (15) with $\alpha_a \chi_a \lambda(q_a) = \alpha_e \chi_a \lambda(q_e) = 0$. The only solution consistent with equilibrium is $\phi_{a,t} = \phi_a^* \equiv \beta\rho/(1-\beta)\ \forall t$, because any other solution path explodes, and in the same sense that $\phi_m = 0$ is the fundamental value of cash $\phi_a = \phi_a^*$ is the fundamental value of the real asset. However, $\alpha_a \chi_a \lambda(q_a) > 0$ or $\alpha_e \chi_a \lambda(q_e) > 0$ implies there can be equilibria with $\phi_{a,t} > \phi_a^*$, which means there can be situations with asset prices above their fundamental values.[15]

For many applications it is convenient to write the Euler equations as a forward-looking dynamical system in (z_m, z_t),

$$\begin{bmatrix} z_{m,t} \\ z_{a,t} \end{bmatrix} = \begin{bmatrix} F_m(z_{m,t+1}, z_{a,t+1}) \\ F_a(z_{m,t+1}, z_{a,t+1}) \end{bmatrix}, \tag{16}$$

where

$$\begin{aligned} F_m(z_m, z_a) &\equiv \frac{\beta z_m}{1+\mu}\left[1 + \alpha_m \chi_m L(z_m) + \alpha_e \chi_m L(z_m + z_a)\right] \\ F_a(z_m, z_a) &\equiv \chi_a \rho A + \beta z_a \left[1 + \alpha_a \chi_a L(z_a) + \alpha_e \chi_a L(z_m + z_a)\right] \end{aligned}$$

and we define $L(z) = \lambda \circ v^{-1}(z)$ to express the liquidity premium in terms of z rather than q. Notice that $z_{m,t} = 0\ \forall t$ always satisfies $z_{m,t} = F_m(z_{m,t+1}, z_{a,t+1})$, so the economy can always be nonmonetary; in that case we effectively have

[15] This situation (asset prices above their fundamental values) is commonly called a bubble; see Awaya et al. (2019) for a recent discussion of the issues and literature related to asset price bubbles.

a one-asset economy, and a one-dimensional dynamical system, which is interesting in its own right, but not as interesting as monetary equilibria.[16]

4 Stationary Equilibria

While the dynamic implications of this system are interesting, for now, let us discuss stationary monetary equilibria, or SME for short. Starting in the DM with an initial distribution (z_m, z_a), if after both the DM and CM convene the distribution is the same as the initial conditions, we are in steady state, and that constitutes a stationary equilibrium. If we start with an arbitrary distribution, there will in general be a transition, but this economy gets to steady state in one period as long as agents have interior solutions for the object that enters payoffs linearly, which here means $0 < \ell < 1$. In Appendix A we show that if a stationary monetary equilibrium exists it is unique.[17] We now discuss existence.

After a little algebra, in general a steady state of (16) with strictly positive (z_m, z_a) can be described as solving

$$i = \alpha_m \chi_m L(z_m) + \alpha_e \chi_m L\left(z_m + z_a\right) \tag{17}$$

$$r = \frac{\chi_a \rho A\left(1+r\right)}{z_a} + \alpha_a \chi_a L(z_a) + \alpha_e \chi_a L\left(z_m + z_a\right). \tag{18}$$

The first condition defines the M-curve and the second defines the A-curve in (z_m, z_a) space, which cannot cross more than once (Appendix A). To proceed,

[16] Also notice that fiat currency and the real asset are not quite symmetric: a change in the return on the asset in terms of ρ affects the intercept of F_a, while a change in the return on currency in terms of μ affects the slope of F_m.

[17] Note that uniqueness requires $\rho > 0$; if $\rho < 0$ the asset can still be valued for its liquidity, but then there will be multiple steady states. Also, note that typically $\bar{\imath} > 0$, except for extreme specifications for $v(\cdot)$ (e.g., bargaining when the buyer has bargaining power $\theta = 0$ implies monetary equilibrium cannot exist for any $i > 0$). Also note that $\bar{\imath} = \infty$ for some $v(\cdot)$ (e.g., Nash bargaining with $\theta > 0$) while $\bar{\imath} < \infty$ for others (e.g., Kalai bargaining with $\theta > 0$).

assume $v(\cdot)$ is such that $L'(z) < 0$.[18] Then we consider several different cases depending on whether $\alpha_j > 0$ or $\alpha_j = 0$, always assuming $\chi_j > 0$ (which is without loss of generality since $\chi_j = 0$ is the same as $\alpha_j = 0$). All possibilities are listed in Table 1.

case	α_m	α_a	α_e	description
1	0	0	0	not interesting (no trade)
2	+	0	0	only m accepted in DM
3	0	+	0	only a accepted in DM
4	+	+	0	m or a accepted, not both
5	0	0	+	m accepted iff a accepted
6	+	0	+	m more often accepted
7	0	+	+	a more often accepted
8	+	+	+	various possibilities

Table 1: Possible SME

Since Case 1 is trivial, we start with Case 2, where the DM is basically a pure currency market. This case is easy because the system dichotomizes: z_m solves (17) and z_a solves (18), where the equations are independent. Starting with z_a, since no one accepts a in the DM it has no liquidity value, from (18) we get $z_a = \chi_a \rho A (1+r)/r$.[19] In this case $\phi_a = \phi_a^* = \rho/r$. Moving to z_m, it solves

$$i = \alpha_m \chi_m L(z_m), \tag{19}$$

which is the same as a pure currency economy (i.e., one with $A = 0$). Given $L'(z) < 0$ plus $L(z) = 0$ for $z \geq z^*$, for any $i > 0$ $\exists z_m \in (0, z^*)$ solving (17) iff $i < \bar{\imath} \equiv \alpha_m \chi_m L(0)$. This is a standard result that emerges because in Case 2 a provides no liquidity.

[18] We also use this in the uniqueness proof. This is automatic for many mechanisms, including Kalai bargaining and Walrasian price taking, but not for others, including Nash bargaining; however, in most of the cases discussed here, one can show $L'(z) < 0$ at an equilibrum value of z, even if this cannot be guaranteed globally following Gu and Wright (2016).

[19] Notice that χ_a appears even though no one actually accepts a in DM trade, because we can still ask how much liquidity a provides if (out of equilibrium) someone did accept it.

Next consider Case 3, which is in a sense the mirror image of Case 2, since a is and m is not accepted in the DM. Now monetary equilibrium does not exist, but for the sake of symmetry, consider (18) with $\alpha_e = 0$. The solution $z_a > 0$ exists and can be characterized as follows: one possibility is $z_a \geq z^*$, so again a conveys no liquidity at the margin, and then, as in Case 2, $z_a = \chi_a \rho A (1+r)/r$, and if $\chi_a \rho A (1+r)/r > v(q^*)$ we get the fundamental price $\phi_a = \phi_a^*$; but if $\chi_a \rho A (1+r)/r < v(q^*)$ then the liquidity embodied in a is scarce and in equilibrium z_a satisfies $\chi_a \rho A (1+r)/r < z_a < v(q^*)$ and we get $\phi_a > \phi_a^*$. To be clear, while $\phi_a < \phi_a^*$ is impossible we can have $\phi_a > \phi_a^*$, i.e., the asset priced above its fundamental value because it conveys liquidity.

Case 4 in a sense combines 2 and 3 and also dichotomizes. While Case 4 can have both m and a valued, they do not interact because of the dichotomy and so, e.g., changes in i affect z_m but not z_a, while changes in ρA affect z_a but not z_m. Therefore the conditions for money to be valued are exactly the same as in Case 2, while the conditions for assets to be priced above their fundamental value are the same as in Case 3.

Case 5 is a situation where a DM seller accepts m iff he accepts a, making them perfect substitutes, up to any difference in χ_m and χ_a. While the system does not dichotomize, it is recursive: (17)-(18) reduce to

$$i = \alpha_e \chi_m L(z_m + z_a) \tag{20}$$

$$r = \frac{\chi_a \rho A (1+r)}{z_a} + \alpha_e \chi_a L(z_m + z_a). \tag{21}$$

Notice that (20) determines $z_e = z_m + z_a$, and as usual $\exists z_e > 0$ solving this iff $i < \bar{\imath}$ for some $\bar{\imath}$; then (21) determines z_a. In fact, eliminating $L(z_m + z_a)$ from (21) using (20), we get

$$r = \frac{\chi_a \rho A (1+r)}{z_a} + i \frac{\chi_a}{\chi_m}, \tag{22}$$

which easily solves for z_a.

We must check $z_a \geq \chi_a (\rho + \phi_a^*) A$, because the asset price cannot be below its fundamental value. If the solution z_a to (22) satisfies this, we can compute $z_m = z_e - z_a$, and if $z_m > 0$ we have a SME, while if $z_m < 0$ we do not. Intuitively, if the liquidity embodied by a is plentiful then m cannot be valued. Notice that $u'(0) = \infty$ is not relevant for this because a buyer can always get some amount of q, the amount solving $v(q) = z_a$, using no cash. Still, the bottom line is that for some parameters, e.g., low values of ρA, the liquidity embodied in a is not sufficient to get q^*, and then money can be valued if i is low. In this situation, a change in, e.g., i affects z_m directly and that changes z_a, while a change in, e.g., ρA affects z_a directly and that changes z_m.

In Case 5, where a and m are perfect substitutes in DM trade, their rates of return must be equal. One implication, first noted by Geromichalos et al. (2007), in a specification with $\chi_m = \chi_a$, is the following: since $r_a = \rho/\phi_a > 0$, this can only be the case if there is deflation, $\mu = \pi < 0$. Now, this can be fixed as in Lagos and Zhang (2017), by having a depreciate to some degree (i.e., vanish with some probability), although that's only because the generalized point is that the expected rates of return of m and a must be equal. In any event, relaxing $\chi_m = \chi_a$, the general result is that we need

$$\mu < \bar{\mu} \equiv \frac{(\chi_m - \chi_a) r}{\chi_a (1 + r)},$$

so we can have SME with $\mu > 0$ in Case 5 as long as $\chi_m > \chi_a$.

Another way for m and a to both be valued is to make them imperfect substitutes in terms of acceptability. Case 6 is similar to Case 5 in that there is no dichotomy, so z_m and z_a are not independent, but it is easier to get SME because in some meetings m is the only object accepted. So again there is a SME if $i < \bar{\imath}$, but the threshold $\bar{\imath}$ can be higher because m can provide liquidity in both type m and type b meetings. In fact, m provides liquidity in type e meetings when z_a is low, as in Case 3, which is again what gives $\phi_a > \phi_a^*$. So giving m an advantage in terms of α or χ is a formal way to capture the classic rate of return dominance observation emphasized in monetary economics at least since Hicks (1935).

Case 7 is similar, but now it is harder to get SME because in no meetings is m the only object accepted by the seller, and in some a is the only object accepted. Thus m must pay a higher return than a to compensate for its lower liquidity. With $\chi_m = \chi_a$, this implies that $\bar{\mu} < 0$, meaning that even slight deflation might not be enough for an SME to exist. Case 8 is again similar in the sense that z_m and z_a are determined interdependently, but because for any χ_m and χ_a, neither m nor a is strictly dominated as a means of payment, there are less restrictions on the existence of SME in this case compared to cases 5-7. Heuristically, m can be valued iff $i < \bar{\imath}$ for some $\bar{\imath}$, and a bears a liquidity premium iff ρA is low. We do not dwell on getting exact bounds in order to proceed to more substantive applications.

In stationary monetary equilibrium, policy can set the money growth rate μ, which equals inflation π, and that determines i through the Fisher equation. Hence, targeting μ, π and i are all possible within certain bounds (e.g., $0 \leq i < \bar{\imath}$) and all achieve the same outcome. It is also possible to target some other interest rates, like the real return on a, given by $1 + \rho/\phi_a$ in stationary equilibrium, but policy cannot select the real rate defined by analogy to the way we defined i except using numeraire rather than dollars: ask an agent how much x he would require in the next CM to give up a unit of x in this CM; the answer will be $1 + r = 1/\beta$. Hence, r is independent of inflation, a version of Irving Fisher's theory, but it is important to understand that his theory does not apply to all interest rates, and, in particular, it does not apply to liquid assets.[20]

Figure 1: The three regimes for relatively low inflation rates.

In general, if $i > 0$, the constraint $p_m \leq z_m$ binds in type m meetings with many standard mechanisms (although not necessarily exotic mechanisms like the one in Hu et al. 2009). Hence we focus on the case $p_m = z_m$. The other constraints may or may not bind, but clearly $p_e \leq z_e$ is slack if $p_a \leq z_a$ is slack. Ergo three distinct regimes are possible: (i) $p_a \leq z_a$ and $p_e \leq z_e$ are both slack; (ii) $p_a \leq z_a$ binds but $p_e \leq z_e$ is slack; and (iii) $p_a \leq z_a$ and $p_e \leq z_e$ both bind. Intuitively, regime (i) obtains when the asset supply is big, $A \geq A^*$; (ii) obtains when it is medium, $A^* > A \geq \underline{A}$; (iii) obtains when it is small, $A < \underline{A}$; and the thresholds A^* and $\underline{A}$ can depend on i, ρ and other parameters.

[20] This should be obvious from the case of fiat currency: Fisher's theory (that the real return on an asset is independent of inflation) cleary cannot hold for that asset. But because many modern models do not have fiat currency that obvious point seems to be missed by some practitioners.

Figure 2: The three regimes for relatively high inflation rates.

Figure 1 shows the three regimes for relatively low inflation rates, and Figure 2 shows them for relatively high inflation rates. Higher inflation has three main effects: It decreases q_m, it decreases q_e for any value of A in regime (iii), and it increases the threshold value $\underline{A}$. The first effect occurs because higher inflation makes it more costly to consume in type m meetings, so buyers bring less real money balances; the second and third effect occur because due to holding less money for any A, buyers hold less total real balances for any supply of assets, and they need more assets to be unconstrained in type e meetings.

One can study all three regimes, but (i) is less interesting for asset pricing, because then A does not bear a liquidity premium, the way it does in (ii) and (iii). Indeed, (iii) is perhaps the most interesting for reasons discussed below.

In monetary steady state, in regime (iii) liquidity is scarce even in type e

meetings. Then it is routine to show

$$
\begin{aligned}
\frac{\partial z_m}{\partial i} &= \frac{1}{\chi_m D_1}\left[\frac{\rho A(1+r)}{z_a^2} - \alpha_a L'(z_a) - \alpha_e L'(z_e)\right] < 0 \\
\frac{\partial z_a}{\partial i} &= \frac{\alpha_e L'(z_e)}{\chi_m D_1} > 0 \\
\frac{\partial z_e}{\partial i} &= \frac{1}{\chi_m D_1}\left[\frac{\rho A(1+r)}{z_a^2} - \alpha_a L'(z_a)\right] < 0,
\end{aligned}
$$

where

$$
D_1 \equiv [\alpha_m L'(z_m) + \alpha_e L'(z_e)]\left[\frac{\rho A(1+r)}{z_a^2} - \alpha_a L'(z_a)\right] - \alpha_m \alpha_e L'(z_m) L'(z_e) < 0
$$

because $L'(z_j) < 0$ when the constraint $p_j \leq z_j$ binds (see Gu and Wright 2016 for details). Thus, higher i (or π or μ) reduces z_m and increases z_a because when nominal interest (or inflation or money growth) rates rise buyers try to substitute out of cash and into other assets, which on net reduces total liquidity z_e. This identifies the direct impact of monetary policy on liquidity.

Similarly it is routine to show

$$
\begin{aligned}
\frac{\partial z_m}{\partial A} &= -\frac{\rho(1+r)}{D_1 z_a}\alpha_e L'(z_e) < 0 \\
\frac{\partial z_a}{\partial A} &= \frac{\rho(1+r)}{D_1 z_a}[\alpha_m L'(z_m) + \alpha_e L'(z_e)] > 0 \\
\frac{\partial z_e}{\partial A} &= \frac{\rho(1+r)}{D_1 z_a}\alpha_m L'(z_m) > 0.
\end{aligned}
$$

Thus, higher A increases z_a but lowers z_m. The results for other regimes can be seen as limiting cases – e.g., in regimes (i) and (ii) $L'(z_e) = 0$, and hence $\partial z_a/\partial i = \partial z_m/\partial A = 0$, which shows how the impact of one asset's return on demand for the other asset depends on them being substitutes as sources of scarce liquidity in at least some situations, and which is why regime (iii) is interesting. Also note that the effects of changes in A are the same as changes in ρ, because only ρA matters, so one can interpret the above results in terms of improvements in asset markets conditions – i.e., a rise in the dividend ρ.[21]

The real asset price is bounded by its fundamental value, $\phi_a \geq \phi_a^*$, with strict inequality – i.e., with a liquidity premium – in two situations: $\alpha_a > 0$ and $p_a \leq z_a$ binds at $\phi_a = \phi_a^*$; or $\alpha_a = 0 < \alpha_e$ and $p_e \leq z_e$ binds at $\phi_a = \phi_a^*$. Similarly, its return is bounded by $0 \leq r_a \leq r$, where again r is defined by the amount of x agents would require in the next CM to give up a unit in this CM, which can be interpreted as the return on an asset that is illiquid in the sense that it can never be used in the DM, in the same two situations. Thus, the liquidity premium shows up as $\phi_a > \phi_a^*$ and $r_a < r$, i.e., as a higher price and lower return. Moreover, it follows from (12)-(13) that

$$r_m < r_a \iff \alpha_m \chi_m \lambda(q_m) + \alpha_e \chi_m \lambda(q_e) > \alpha_a \chi_a \lambda(q_a) + \alpha_e \chi_a \lambda(q_e).$$

For instance, $\chi_m = \chi_a$ implies m can have a lower return than a if $\alpha_m > \alpha_a$ (it is more acceptable) and $\lambda(q_m) > \lambda(q_a)$ (the constraint is tighter in type m meetings).[22]

5 Extensions

Above we analyze how different assumptions about acceptability, pledgeability etc. affect asset pricing, liquidity premia and the existence of SME when the asset is equity in a technology that produces dividend ρ each period. This section considers three different types of assets, namely, bonds, capital and housing. For bonds, the key difference is maturity: while there are bonds with infinite maturity (consols), in the typical case it is finite. For capital, there

[21] One can similarly derive the effects of the α_j's and χ_j's, and the effects on the ϕ_j's and r_j's. We leave these as exercises.

[22] This discussion assumes $\rho > 0$. For an asset with $\rho < 0$, which can be valued if its liquidity premium is big, the return is bounded by $r_a = \rho/\phi_a \leq 0$ since $\phi_a \geq 0$. One can also check that $\rho > 0$ implies a fall in A or ρ decreases r_a, while $\rho < 0$ implies a fall in A or $|\rho|$ increases r_a. The intuition is straightforward: if $\rho < 0$, decreases in A or $|\rho|$ increase ρA, which increases the price of the asset and thereby brings its return closer to zero.

are several differences: the supply of it is endogenous; a higher supply lowers the marginal return; and its price in terms of x is 1 (it is the same physical object as numeraire). For housing, the difference is that it is not only an asset, it enters the utility function directly, and one can assume either the supply is fixed or endogenous; and its price if not fixed at 1.

5.1 Bonds

Here we consider bonds.[23] For simplicity, we study one-period bonds B that mature in the CM by paying off their face value in numeraire[24]. Further, when B matures, it is replaced by an equal issuance of new bonds in the CM, assumed to be endowed to sellers (but that is inconsequential for the main results). With this modification, the buyer's CM problem (again, we omit the similar problem for the seller) is

$$W(m,b) = \max_{x,\ell,\hat{m},\hat{b}} \{U(x) - \ell + \beta V(\hat{m},\hat{b})\}$$
$$\text{st } x = \ell - \phi_m(\hat{m} - m) - \phi_b \hat{b} + b + T.$$

The change from the baseline model is the absence of a resale value for the asset in the budget constraint. Assuming an interior solution, we eliminate ℓ and rewrite this as

$$W(m,b) = \Omega + \max_x \{U(x) - x\} + \max_{\hat{m},\hat{b}} \{-\phi_m \hat{m} - \phi_b \hat{b} + \beta V(\hat{m},\hat{b})\}, \tag{23}$$

where $\Omega = \phi_m m + b + T$. The first-order conditions for portfolio choice are

$$\phi_m = \beta V_1(\hat{m},\hat{b}) \tag{24}$$

$$\phi_b = \beta V_2(\hat{m},\hat{b}), \tag{25}$$

[23] See Wallace (1981), Berentsen and Waller (2001), Boel and Camera (2006) and Rocheteau et al. (2018) for related analyses.

[24] We could include a coupon γ, so that an agent holding b bonds receives $(1+\gamma)b$ units of x, but for one-period bonds $\gamma = 0$ is a normalization and without loss of generality.

whereas the envelope conditions are $W_1(m,b) = \partial\Omega/\partial m = \phi_m$ and $W_2(m,b) = \partial\Omega/\partial b = 1$.

In the DM,

$$V(m,b) = W(m,b) + \alpha_m[u(q_m) - p_m] + \alpha_b[u(q_b) - p_b] + \alpha_e[u(q_e) - p_e], \quad (26)$$

where the liquidity constraints are now $p_m \leq z_m \equiv \chi_m \phi_m m$, $p_b \leq z_b \equiv \chi_b b$ and $p_e \leq z_e \equiv z_m + z_b$. Emulating the above procedures, the Euler equations are

$$1 = \beta(1 + r_{m,t})\left[1 + \alpha_m \chi_m \lambda(q_{m,t+1}) + \alpha_e \chi_m \lambda\left(q_{e,t+1}\right)\right] \quad (27)$$

$$\phi_{b,t} = \beta\left[1 + \alpha_b \chi_b \lambda\left(q_{b,t+1}\right) + \alpha_e \chi_a \lambda\left(q_{e,t+1}\right)\right]. \quad (28)$$

From (28), one can immediately notice a difference from the baseline model: the bond price in period t is independent of future bond prices. This has implications for steady-state bounds on bond returns and for the SME existence conditions (as well as dynamics, but we do not analyze these here).

Equilibrium is defined the same as in the baseline model. By defining $1 + r_{b,t} = 1/\phi_{b,t}$, we can write the above equations in stationary equilibrium as

$$i = \alpha_m \chi_m L(z_m) + \alpha_e \chi_m L\left(z_m + z_b\right) \quad (29)$$

$$r = r_b + (1 + r_b)\left[\alpha_b \chi_b L(z_b) + \alpha_e \chi_b L\left(z_m + z_b\right)\right]. \quad (30)$$

From (30), $r_b = r$ in regime (i), which implies $\phi_b = \phi_b^* \equiv \beta$. In regimes (ii) and (iii), $r_b < r$ and $\phi_b > \phi_b^*$. Moreover, r_b is now bounded by $-1 \leq r_b \leq r$, while in the baseline model the lower bound is 0. While the conditions for existence of SME are similar to the baseline model, the different lower bound on r_b has implications in cases 5 and 7: for sufficiently low B, an SME can now exist in these cases even at positive inflation. In monetary steady state, we have

$$\begin{aligned}
\frac{\partial z_m}{\partial i} &= \frac{1}{\chi_m\left[\alpha_m L'(z_m)+\alpha_e L'(z_e)\right]} < 0 \\
\frac{\partial z_b}{\partial i} &= 0 \\
\frac{\partial z_e}{\partial i} &= \frac{1}{\chi_m\left[\alpha_m L'(z_m)+\alpha_e L'(z_e)\right]} < 0.
\end{aligned}$$

As in the baseline model, an increase in i leads to a decrease in z_m. However, since $z_b = \chi_b B$ for any i, changes in i do not have any effects on z_b.

5.2 Capital

Now suppose the asset is productive capital as in neoclassical growth theory.[25] Thus, investing k units of CM numeraire in period t yields $f(k)$ in the next CM of period $t+1$, with $f(0)=0$, $f'(k)>0$, $f'(0)=\infty$ and $f''(k)<0$. Capital depreciates at rate δ, so investing k units of capital at t returns $f(k)+(1-\delta)k$ units of x at $t+1$. While in the baseline model, the supply is fixed but the price ϕ_a is endogenous, with capital the price is fixed at 1 while the supply is endogenous. The buyer's CM problem is

$$\begin{aligned}
W(m,k) &= \max_{x,\ell,\hat{m},\hat{k}} \{U(x)-\ell+\beta V(\hat{m},\hat{k})\} \\
&\text{st } x = \ell - \phi_m(\hat{m}-m) - \hat{k} + f(k) + (1-\delta)k + T.
\end{aligned}$$

As usual, we rewrite this as

$$W(m,k) = \Omega + \max_x\{U(x)-x\} + \max_{\hat{m},\hat{k}}\{-\phi_m\hat{m} - \hat{k} + \beta V(\hat{m},\hat{k})\}, \tag{31}$$

where $\Omega = \phi_m m + f(k) + (1-\delta)k + T$. The portfolio choice now solves

$$\phi_m = \beta V_1(\hat{m},\hat{k}) \tag{32}$$

$$1 = \beta V_2(\hat{m},\hat{k}), \tag{33}$$

[25] This extension is related to work by Lagos and Rocheteau (2008), Aruoba and Wright (2003), Aruoba et al. (2011), and Venkateswaran and Wright (2014). A different approach is used in Wright et al. (2018,2019), where capital (instead of consumption) goods are traded in the DM.

so $(\hat{m}, \hat{k})$ is still independent of (m, k), while the envelope conditions are $W_1(m,k) = \partial\Omega/\partial m = \phi_m$ and $W_2(m,k) = \partial\Omega/\partial k = f'(k) + 1 - \delta$. In the DM,

$$V(m,k) = W(m,k) + \alpha_m[u(q_m) - p_m] + \alpha_k[u(q_k) - p_k] + \alpha_e[u(q_e) - p_e], \quad (34)$$

with the constraints $p_m \leq z_m \equiv \chi_m \phi_m m$, $p_k \leq z_k \equiv \chi_k (f(k) + (1-\delta)k)$ and $p_e \leq z_e \equiv z_m + z_k$.

The Euler equations are

$$1 = \beta(1 + r_{m,t}) [1 + \alpha_m \chi_m \lambda(q_{m,t+1}) + \alpha_e \chi_m \lambda (q_{e,t+1})] \quad (35)$$

$$1 = \beta[f'(k_{t+1}) + 1 - \delta] [1 + \alpha_k \chi_k \lambda (q_{k,t+1}) + \alpha_e \chi_k \lambda (q_{e,t+1})] . \quad (36)$$

Notice that k_{t+1} is determined from these equations independent of k_t, which means the economy jumps to the steady state in one period, at least if we avoid corner solutions for ℓ.[26] In any case, defining

$$1 + r_k = f'(k) + 1 - \delta,$$

we write the equations for steady state as

$$i = \alpha_m \chi_m L(z_m) + \alpha_e \chi_m L (z_m + z_k) \quad (37)$$

$$r = r_k + (1 + r_k) [\alpha_k \chi_k L(z_k) + \alpha_e \chi_k L (z_m + z_k)] . \quad (38)$$

From (38), in regime (i) $r_k = r$ and thus the capital stock is determined by $f'(k^*) = r + \delta$, which is what a social planner would choose. In regimes (ii) and (iii), $r_k < r$ and therefore $k > k^*$, which means capital is over accumulated (as in Lagos and Rocheteau 2008). Compared to the baseline model, where scarce liquidity increases the asset price above its fundamental level,

[26] For a discussion of what happens in similar models when conrner solutions can occur, see Wright et al. (2019), footnote 15.

here scarce liquidity increases the supply of capital above its efficient level. For the existence of SME, again the bounds on r_k are relevant, which are now given by $-\delta \leq r_k \leq r$. For $\delta = 1$ the lower bound coincides with that on bonds, while for $\delta = 0$ it coincides with that on assets in the baseline model. Thus the upper bound on inflation that allows an SME to exist in cases 5 and 7 is increasing in δ.

Focusing on regime (iii), where liquidity is scarce even in type e meetings, it is straightforward to derive

$$\begin{aligned}
\frac{\partial z_m}{\partial i} &= \frac{1}{\chi_m D_2}\left\{\frac{f''(k)}{\beta\chi_k[f'(k)+1-\delta]^3} + \alpha_k\chi_k L'(z_k) + \alpha_e\chi_k L'(z_e)\right\} < 0 \\
\frac{\partial z_k}{\partial i} &= -\frac{1}{\chi_m D_2}\alpha_e\chi_k L'(z_e) > 0 \\
\frac{\partial z_e}{\partial i} &= \frac{1}{\chi_m D_2}\left\{\frac{f''(k)}{\beta\chi_k[f'(k)+1-\delta]^3} + \alpha_k\chi_k L'(z_k)\right\} < 0,
\end{aligned}$$

where

$$\begin{aligned}
D_2 \equiv &\left\{\frac{f''(k)}{\beta\chi_k[f'(k)+1-\delta]^3} + \alpha_k\chi_a L'(z_k)\right\}[\alpha_m L'(z_m) + \alpha_e L'(z_e)] \\
&+ \alpha_m\alpha_e\chi_k L'(z_m)L'(z_e) > 0.
\end{aligned}$$

In particular, consistent with the so-called Mundell-Tobin effect, an increase in i leads to an increase in capital investment. Moreover, we have

$$\begin{aligned}
\frac{\partial z_m}{\partial \delta} &= -\frac{1}{\beta[f'(k)+1-\delta]^2 D_2}\alpha_e L'(z_e) > 0 \\
\frac{\partial z_k}{\partial \delta} &= \frac{1}{\beta[f'(k)+1-\delta]^2 D_2}[\alpha_m L'(z_m) + \alpha_e L'(z_e)] < 0 \\
\frac{\partial z_e}{\partial \delta} &= \frac{1}{\beta[f'(k)+1-\delta]^2 D_2}\alpha_m L'(z_m) < 0,
\end{aligned}$$

and if $f(k) = \varepsilon \tilde{f}(k)$ for each k,

$$
\begin{aligned}
\frac{\partial z_m}{\partial \varepsilon} &= \frac{\tilde{f}'(k)}{\beta[f'(k)+1-\delta]^2 D_2}\alpha_e L'(z_e) < 0 \\
\frac{\partial z_k}{\partial \varepsilon} &= -\frac{\tilde{f}'(k)}{\beta[f'(k)+1-\delta]^2 D_2}[\alpha_m L'(z_m) + \alpha_e L'(z_e)] > 0 \\
\frac{\partial z_e}{\partial \varepsilon} &= -\frac{\tilde{f}'(k)}{\beta[f'(k)+1-\delta]^2 D_2}\alpha_m L'(z_m) > 0.
\end{aligned}
$$

Hence, an increase in δ leads to a decrease in z_k and z_e and an increase in z_m, while an increase in productivity ε has the opposite effects.

5.3 Housing

In the baseline model, the asset pays a dividend ρ but otherwise has no intrinsic utility; with housing, agents do not obtain dividends but enjoy utility from the asset.[27] Assume that buyers enjoy utility $\tilde{U}(x, h)$ from consuming x and holding h units of housing, while for simplicity sellers do not obtain any utility h, and impose the usual assumptions on $\tilde{U}$. The reason housing provides liquidity is this: with limited commitment, buyers cannot get unsecured loans, so there is room for home equity to play a role as collateral. Therefore, buyers may be willing to pay more for a house than its fundamental value, similar to other assets.

In this section we explicitly assume that buyers use housing as collateral in the DM, buy DM goods using credit secured by h, and pay off any debt d in the CM. This is a case where using housing as collateral is more realistic – it is not reasonable to hand over part of your house in the DM, but it is reasonable to make a promise collateralized by your house. Also, since housing enters utility directly, using it as a medium of exchange is not equivalent to using it as collateral. If a buyer hands over part h' of his housing h in the DM,

[27] This presentation is based on He et al. (2015).

he gets utility only from the net $h - h'$, but if he uses h as collateral he still gets utility from h units.

To begin, suppose the supply of housing is fixed at H. Buyers' CM problem is

$$W(m,h,d) = \max_{x,\ell,\hat{m},\hat{h}} \{\tilde{U}(x,h) - \ell + \beta V(\hat{m},\hat{h})\} \tag{39}$$
$$\text{st } x = \ell + T - d + \phi_m(m - \hat{m}) + \phi_h(h - \hat{h}).$$

Assuming an interior solution, as usual, we have

$$W(m,h,d) = \Omega + \max_x\{\tilde{U}(x,h) - x\} + \max_{\hat{m},\hat{h}}\{-\phi_m\hat{m} - \phi_h\hat{h} + \beta V(\hat{m},\hat{h})\},$$

where $\Omega = \phi_m m + \phi_h h + T - d$. The first-order conditions for portfolio choice are

$$\phi_m = \beta V_1(\hat{m},\hat{h})$$
$$\phi_h = \beta V_2(\hat{m},\hat{h}).$$

Again $(\hat{m},\hat{h})$ is independent of (m,h,d). Moreover, $W_1(m,h,d) = \phi_m$, $W_2(m,h,d) = \phi_a + \tilde{U}_2(x,h)$ and $W_3(m,h,d) = -1$.

In the CM,

$$V(m,h) = W(m,h,0) + \alpha_m[u(q_m) - p_m] + \alpha_h[u(q_h) - p_h] + \alpha_e[u(q_e) - p_e],$$

while $p_m \le z_m \equiv \chi_m\phi_m m$, $p_h \le z_h \equiv \chi_h\phi_h h$ and $p_e \le z_e \equiv z_m + z_h$. The Euler equations are

$$\begin{aligned}
\phi_{m,t} &= \beta\phi_{m,t+1}\left[1 + \alpha_m\chi_m\lambda(q_{m,t+1}) + \alpha_e\chi_m\lambda\left(q_{e,t+1}\right)\right] \\
\phi_{h,t} &= \beta\tilde{U}_2[X(H),H] + \beta\phi_{h,t+1}\left[1 + \alpha_h\chi_h\lambda(q_{h,t+1}) + \alpha_e\chi_h\lambda\left(q_{e,t+1}\right)\right]
\end{aligned}$$

where $X(H)$ is defined by the first-order condition: $\tilde{U}_1[X(H),H] = 1$.

Equilibrium is defined similar to the baseline model. In stationary equilibrium, we have

$$\begin{aligned} i &= \alpha_m \chi_m L(z_m) + \alpha_e \chi_m L\left(z_m + z_h\right), \\ r &= \frac{\chi_h H \tilde{U}_2[X(H), H]}{z_h} + \alpha_h \chi_h L(z_h) + \alpha_e \chi_h L\left(z_m + z_h\right). \end{aligned}$$

In regime (iii), where $p_h \leq z_h$ and $p_e \leq z_e$ both bind, it is routine to show

$$\begin{aligned} \frac{\partial z_m}{\partial i} &= \frac{1}{\chi_m D_3}\left[\frac{H\tilde{U}_2}{z_h^2} - \alpha_h L'(z_h) - \alpha_e L'(z_e)\right] < 0 \\ \frac{\partial z_h}{\partial i} &= \frac{\alpha_e L'(z_e)}{\chi_m D_3} > 0 \\ \frac{\partial z_e}{\partial i} &= \frac{1}{\chi_m D_3}\left[\frac{H\tilde{U}_2}{z_h^2} - \alpha_h L'(z_h)\right] < 0, \end{aligned}$$

where

$$D_3 \equiv [\alpha_m L'(z_m) + \alpha_e L'(z_e)]\left[\frac{H\tilde{U}_2}{z_h^2} - \alpha_h L'(z_h)\right] - \alpha_m \alpha_e L'(z_m) L'(z_e) < 0.$$

Thus, as in the baseline model, higher i (or π or μ) reduces z_m and increases z_h because when nominal interest (or inflation or money growth) rates rise buyers try to substitute out of cash and into other assets, which on net reduces total liquidity z_e.

Similarly,

$$\begin{aligned} \frac{\partial z_m}{\partial H} &= -\frac{\tilde{U}_2 + H\left[\tilde{U}_{22} - \frac{(\tilde{U}_{12})^2}{\tilde{U}_{11}}\right]}{D_3 z_a} \alpha_e L'(z_e) \\ \frac{\partial z_h}{\partial H} &= \frac{\tilde{U}_2 + H\left[\tilde{U}_{22} - \frac{(\tilde{U}_{12})^2}{\tilde{U}_{11}}\right]}{D_3 z_a}[\alpha_m L'(z_m) + \alpha_e L'(z_e)] \\ \frac{\partial z_e}{\partial H} &= \frac{\tilde{U}_2 + H\left[\tilde{U}_{22} - \frac{(\tilde{U}_{12})^2}{\tilde{U}_{11}}\right]}{D_3 z_a} \alpha_m L'(z_m). \end{aligned}$$

Now the effects of changes in H are ambiguous, although we know $\partial z_m / \partial H > 0$ iff $\partial z_h / \partial H < 0$. Intuitively, increasing the quantity H lowers the price ϕ_h, which on net can raise or lower $\phi_h H$.

Now suppose the supply of housing is endogenously determined. The technology of home building is summarized by a cost function $\tilde{c}$: increasing the stock of housing by Δh_t requires an input of $\tilde{c}(\Delta h_t)$ in numeraire, with the usual monotonicity and curvature assumptions imposed on $\tilde{c}$. Moreover, assume houses depreciate at the rate of δ. Houses are constructed by a representative price-taking firm in the CM, and hence the supply of housing is determined by

$$\phi_{h,t} = \tilde{c}'[h_{t+1} - (1-\delta)h_t].$$

The buyer's problem is the same, except for the depreciation, which implies $z_h = \chi_h(1-\delta)\phi_h h$. The Euler equations are

$$\begin{aligned}
\phi_{m,t} &= \beta\phi_{m,t+1}\left[1 + \alpha_m\chi_m\lambda(q_{m,t+1}) + \alpha_e\chi_m\lambda\left(q_{e,t+1}\right)\right], \\
\phi_{h,t} &= \beta\tilde{U}_2[X(h_{t+1}), h_{t+1}] \\
&\quad +\beta(1-\delta)\phi_{h,t+1}\left[1 + \alpha_h\chi_h\lambda(q_{h,t+1}) + \alpha_e\chi_h\lambda\left(q_{e,t+1}\right)\right].
\end{aligned}$$

Equilibrium can be defined in the natural way, given an given initial supply H_0. In stationary equilibrium,

$$\begin{aligned}
i &= \alpha_m\chi_m L(z_m) + \alpha_e\chi_m L\left(z_m + z_h\right), \\
\frac{r+\delta}{1-\delta} &= \frac{\chi_h H(z_h)\tilde{U}_2\{X[H(z_h)], H(z_h)\}}{z_h} + \alpha_h\chi_h L(z_h) + \alpha_e\chi_h L\left(z_m + z_h\right),
\end{aligned}$$

where $H(z_h)$ satisfies

$$z_h = \chi_h(1-\delta)H(z_h)\tilde{c}'[\delta H(z_h)].$$

In SME, in regime (iii) where $p_h \le z_h$ and $p_e \le z_e$ both bind, liquidity is scarce even in type e meetings. Then it is routine to show

$$\begin{aligned}
\frac{\partial z_m}{\partial i} &= \frac{1}{\chi_m D_4}\left[N_4 + \frac{H(z_h)\tilde{U}_2}{z_h^2} - \alpha_h L'(z_h) - \alpha_e L'(z_e)\right] \\
\frac{\partial z_h}{\partial i} &= \frac{\alpha_e L'(z_e)}{\chi_m D_4} \\
\frac{\partial z_e}{\partial i} &= \frac{1}{\chi_m D_4}\left[N_4 + \frac{H(z_h)\tilde{U}_2}{z_h^2} - \alpha_h L'(z_h)\right],
\end{aligned}$$

where

$$N_4 \equiv -\frac{\tilde{U}_2 + H(z_h)\left[\tilde{U}_{22} - \frac{(\tilde{U}_{12})^2}{\tilde{U}_{11}}\right]}{z_h \chi_h (1-\delta)\{\tilde{c}'[\delta H(z_h)] + \delta H(z_h)\tilde{c}''[\delta H(z_h)]\}}$$

and

$$D_4 \equiv [\alpha_m L'(z_m) + \alpha_e L'(z_e)]\left[N_4 + \frac{H(z_h)\tilde{U}_2}{z_h^2} - \alpha_h L'(z_h)\right] - \alpha_m \alpha_e L'(z_m) L'(z_e).$$

Now, the effects of changes in i are ambiguous. There is still an effect that an increase in i reduces z_m and increases z_h, but then the increase in z_h affects $H(z_h)$. As shown above, the effects of changes in H are ambiguous, and hence the effects of i are ambiguous when we endogenize the supply of housing.

6 Conclusion

To conclude, we summarize what we consider the main lessons from the above exercises. First, it is not difficult to build a tractable GE model that incorporates sufficient frictions to generate an endogenous role for money as a medium of exchange. Second, once one sees how fiat currency gets priced above its fundamental value, because it provides liquidity, it is not difficult to see how other assets can get priced above their fundamental values, because they can also provide liquidity. Third, an asset in general can provide liquidity by acting as a medium of exchange, for immediate settlement, or by serving as collateral, in deferred settlement, and the equations are very similar even if one or the other interpretation can be more natural in particular applications (e.g., cash is naturally used as a medium of exchange while home equity is more naturally used as collateral). Fourth, by parameterizing both acceptability and pledgeability it is possible to capture the notion of liquidity on the extensive margin and the intensive margin.

Fifth, many interesting special cases can be understood as various extreme versions of the framework, as summarized in Table 1. Sixth, one can derive bounds on parameters that allow the existence of equilibria where liquidity matters and on the prices or rates of return that can obtain for different assets. Seventh, the methods can be adapted quite easily to understand various types of assets, including equity in productive technologies, simple bonds, neoclassical capital, and housing. And finally, the tractability of the approach is apparent from the various comparative static exercises performed above. Indeed, classic ideas such as the Mundell-Tobin effect emerge readily with recourse to ad hoc assumptions, like putting money in the utility function. We think all of this suggests people in monetary, macro and financial economics should be pursuing further research in this class of models, and hope the reader agrees.

Appendix A

Figure 3: The M and A curves in (z_m, z_a) space.

Here we show uniqueness of the monetary steady state. The M curve defined by $\Delta z_m = 0$, i.e., by $z_{m,t} = z_{m,t+1}$, solves

$$i = \alpha_m \chi_m L(z_m) + \alpha_e \chi_m L(z_m + z_a).$$

Hence,

$$\frac{\partial z_a}{\partial z_m}_{|\Delta z_m = 0} = -\frac{\alpha_m L'(z_m) + \alpha_e L'(z_m + z_a)}{\alpha_e L'(z_m + z_a)} < 0.$$

Similarly, the A curve defined by $\Delta z_a = 0$ solves

$$r = \frac{\chi_a \rho A(1+r)}{z_a} + \alpha_a \chi_a L(z_a) + \alpha_e \chi_a L(z_m + z_a).$$

Hence,

$$\frac{\partial z_a}{\partial z_m}_{|\Delta z_a = 0} = \frac{\alpha_e L'(z_m + z_a)}{\rho A(1+r)/z_a^2 - \alpha_a L'(z_a) - \alpha_e L'(z_m + z_a)} < 0.$$

Figure 3 shows the M and A curves as functions of z_m and z_a. Both curves are downward sloping, so uniqueness is not automatic. However, the difference in slopes is

$$\frac{-[\alpha_m L'(z_m) + \alpha_e L'(z_m + z_a)][\rho A(1+r)/z_a^2 - \alpha_a L'(z_a)] + \alpha_m \alpha_e L'(z_m) L'(z_m + z_a)}{\alpha_e L'(z_m + z_a)[\rho A(1+r)/z_a^2 - \alpha_a L'(z_a) - \alpha_e L'(z_m + z_a)]},$$

which is negative. Thus, in (z_m, z_a) space, as shown in figure 3, the M curve is steeper than the A curve, and steady state is unique.

References

Lukas Altermatt (2019) "Savings, Asset Scarcity, and Monetary Policy," *Journal of Economic Theory* 182, 329-59.

S. Boragan Aruoba, Christopher Waller and Randall Wright (2011) "Money and Capital," *Journal of Monetary Economics* 58, 98-116.

S. Boragan Aruoba and Randall Wright (2003) "Search, Money and Capital," *Journal of Money, Credit, and Banking* 35, 1085-05.

Yu Awaya, Kohei Iwasaki and Makoto Watanabe (2019) "Rational Bubbles and Middlemen," mimeo.

Aleksander Berentsen and Christopher Waller (2011) "Outside versus Inside Bonds: A Modigliani-Miller Type Result for Liquidity Constrained Economies," *Journal of Economic Theory* 146, 1852-87.

Paola Boel and Gabriele Camera (2006) "Efficient Monetary Allocations and the Illiquidity of Bonds," *Journal of Monetary Economics* 53, 1693-715.

Athanasios Geromichalos and Lucas Herrenbrueck (2016) "Monetary Policy, Asset Prices, and Liquidity in Over-the-Counter Markets," *Journal of Money, Credit, and Banking* 48,35-79.

Athanasios Geromichalos and Lucas Herrenbrueck (2017) "A Tractable Model of Indirect Asset Liquidity," *Journal of Economic Theory* 48,252-60.

Athanasios Geromichalos, Lucas Herrenbrueck and Kevin Salyer (2016) "A search-theoretic model of the term premium," *Theoretical Economics* 11, 897-935.

Athanasios Geromichalos and Ina Simonovska (2014) "Asset liquidity and International Portfolio Choice," *Journal of Economic Theory* 151, 342-380.

Athanasios Geromichalos, Juan Manuel Licari and José Suárez-Lledó (2007) "Asset Prices and Monetary Policy," *Review of Economic Dynamics* 10, 761-79.

Pedro Gomis-Porqueras, Timothy Kam and Christopher Waller (2017) "Nominal exchange rate determinacy under the threat of currency counterfeiting," *American Economic Journal: Macroeconomics* 9, 256-73.

Chao Gu, Fabrizzio Mattesini and Randall Wright (2015) "Money and Credit Redux," *Econometrica*.

Chao Gu and Randall Wright (2016) "Monetary Mechanisms," *Journal of Economic Theory* 163, 644-57.

John Hicks (1935) "A Suggestion for Simplifying the Theory of Money," *Economic*a 2, 1-19.

Chao He, Randall Wright and Yu Zhu (2015) "Housing and Liquidity," *Review of Economic Dynamics* 18, 435-455.

Tai-Wei Hu, John Kennan and Neil Wallace (2009) "Coalition-Proof Trade and the Friedman Rule in the Lagos-Wright Model," *Journal of Political Economy* 117, 116-37.

Kukmo Jung and Seungduck Lee (2015) "A liquidity-based resolution of the uncovered interest parity puzzle," *Journal of Money, Credit and Banking* forthcoming.

John Kareken and Neil Wallace (1980) "Introduction," in *Models of Monetary Economies*, John Kareken and Neil Wallace, eds. FRB Minneapolis.

John Kareken and Neil Wallace (1981) "On the Indeterminacy of Equilibrium Exchange Rates," *The Quarterly Journal of Economics* 96, 207-22.

Timothy Kehoe and David Levine (1993) "Debt-Constrained Asset Markets," *Review of Economic Studies* 60, 865-88.

Narayana Kocherlakota (1998) "Money is Memory," *Journal of Economic Theory* 81, 232-51.

Arvind Krishnamurthy and Annette Vissing-Jorgensen (2012) "The Aggregate Demand for Treasury Debt," *Journal of Political Economy* 120, 233-67.

Ricardo Lagos (2010) "Asset Prices and Liquidity in an Exchange Economy," *Journal of Monetary Economics* 57, 913-30.

Ricardo Lagos (2011) "Asset Prices, Liquidity, and Monetary Policy in an Exchange Economy," *Journal of Money, Credit, and Banking* 43, 521-52.

Ricardo Lagos and Guillaume Rocheteau (2008) "Money and Capital as Competing Media of Exchange," *Journal of Economic Theory* 142, 247-258.

Ricardo Lagos, Guillaume Rocheteau and Randall Wright (2017) "Liquidity: A new monetarist perspective," *Journal of Economic Literature* 55, 371-440.

Ricardo Lagos and Randall Wright (2005) "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy* 113, 463-44.

Ricardo Lagos and Shengxing Zhang (2019) "A Monetary Model of Bilateral Over-the-Counter Markets," *Review of Economic Dynamics* 33, 205-27.

Benjamin Lester, Andrew Postlewaite and Randall Wright (2012) "Liquidity, Information, Asset Prices and Monetary Policy," *Review of Economic Studies* 79, 1209-38.

Yiting Li, Guillaume Rocheteau and Pierre-Olivier Weill (2012) "Liquidity and the Threat of Fraudulent Assets," *Journal of Political Economy* 120, 815-46.

Michele Loberto (2018) "Safety Traps, Liquidity and Information-Sensitive Assets," mimeo.

Robert Lucas (1978) "Asset Prices in an Exchange Economy," *Econometrica* 46, 1426-45.

Florian Madison (2018) "Asymmetric Information in Frictional Markets for Liquidity – Collateralized Credit vs Asset Sale," mimeo.

Ed Nosal and Guillaume Rocheteau (2017) *Money, Payments, and Liquidity*, MIT Press.

Guillaume Rocheteau (2011) "Payments and Liquidity under Adverse Selection," *Journal of Monetary Economics* 58, 191-205.

Guillaume Rocheteau and Randall Wright (2005) "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium," *Econometrica* 73, 175-202.

Guillaume Rocheteau, Randall Wright and Sylvia Xiao (2018) "Open Market Operations," *Journal of Monetary Economics* 98, 114-28.

Venky Venkateswaran and Randall Wright (2014) "Pledgability and liquidity: A new monetarist model of financial and macroeconomic activity," *NBER Macroeconomics Annual* 28, 227-70.

Neil Wallace (1981) "A Modigliani-Miller Theorem for Open-Market Operations," *American Economic Review* 71, 267-74.

Steven Williamson and Randall Wright (2010*a*) "New Monetarist Economics: Methods," *FRB St. Louis Review*.

Steven Williamson and Randall Wright (2010*b*) "New Monetarist Economics: Models," *Handbook of Monetary Economics*, vol. 2, Benjamin M. Friedman and Michael Woodford , eds. North-Holland.

Tsz-Nga Wong (2016) "A Tractable Monetary Model under General Preferences," *Review of Economic Studies* 83, 402-420.

Randall Wright, Sylvia Xiao, and Yu Zhu (2018) "Frictional Capital Reallocation I: Ex ante Heterogeneity," *Journal of Economic Dynamics and Control* 71, 100-16.

Randall Wright, Sylvia Xiao, and Yu Zhu (2019) "Frictional Capital Reallocation II: Ex post Heterogeneity," mimeo.