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A Theory of Strategic Diffusion

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Abstract

The important role of friends, neighbors and colleagues in shaping individual choices has been brought out in a number of studies over the years. The presence of significant 'local' influence in shaping individual behavior suggests that firms, governments and developmental agencies should explicitly incorporate it in the design of their marketing and developmental strategies. This paper develops a framework for the study of optimal strategies in the presence of *social interaction*.

We focus on the case of a single player who exerts costly effort to get a set of individuals – engaged in social interaction – to choose a certain action. Our formulation allows for different types of social interaction and also allows for the player to have incomplete information concerning the connections among individuals.

We first show that incorporating information on social interaction can have large effects on the profits of a player. Then, we establish that an increase in the level and dispersion of social interaction can raise or lower the optimal strategy and profits of the player, depending on the *content* of the interaction. Finally, we study the value of social network information for the player and find that it depends on the dispersion in social connections. The economic interest of these results is illustrated via a discussion of two economic applications: advertising in the presence of word of mouth communication and seeding a network.

Keywords: Social interaction, seeding the network, word of mouth communication, diffusion strategy.

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1 Introduction

In making their choices individuals use personally gathered information along with the information they get from their acquaintances. The important role of friends, neighbors and colleagues in shaping individual choices has been brought out in a number of studies over the years.[1] The presence of significant 'local' influence in shaping individual behavior suggests that firms, governments and developmental agencies should explicitly incorporate local network effects into their marketing strategy.

Indeed, the popularity of the expression *word of mouth communication* reflects the fact that both academics as well as practitioners are aware of the potential importance of local influence. The use of social networks in commercial marketing is increasingly popular and has given rise to the term *viral marketing*: a marketing strategy which takes advantages of existing networks of influence among consumers to generate a large product demand with limited advertising resources.[2] Similarly, governmental and developmental agencies has coined the expression *peer-leading* interventions to describe social programmes which attempt to reduce risky behavior by exploiting the presence of network effects.[3]

Local social influence is, however, a nebulous concept and it has been resistant to formal treatment. Two difficulties are worth noting. The first difficulty is a conceptual one. Networks of social relationships overlap in complicated ways and have a number of different dimensions. The need is for a way to define patterns of relationships that is simple and also general enough that it captures key intuitions we have about flows of influence. The second difficulty is a practical one. Firms and other players who are trying to get individuals to choose certain actions usually do not know a great

[1] For example, Coleman (1966) presents evidence on how a doctor's prescription of new drugs was influenced by his location in communication networks. Conley and Udry (2004) and Foster and Rosenzweig (1995) present evidence that farmers are influenced by their neighbors in the choice of crops and agricultural inputs. In the context of brand and product choice, Feick and Price (1987), Reingen et al. (1984), Godes and Mayzlin (2004) present evidence for word of mouth communication.

[2] The advent of new communication tools such as email, chat rooms, web sites, allow consumers to talk with others essentially at no costs. This has led marketers to re-evalute the impact of network-based marketing strategy as an effective way to market products. Perhaps the best known case of viral marketing is the rapid adoption of HOTMAIL: almost 12 million people signed up with Hotmail within eighteen months of its start, and the firm spent around 50,000 dollars in advertising the product. For a discussion on the empirical importance of viral marketing see Leskovec, Adamic and Huberman (2006) and Godes and Mayzlin (2004). See also Rosen (2000) for a variety of case studies in which commercial organizations target key individuals in social contexts to generate desirable outcomes.

[3] Organizations often incorporate peer effects in designing policies to promote behavior change, such as reducing smoking and reducing risk behavior that can lead to sexually transmitted diseases. See Rogers (2003) and Rosen (2000) for an illustration of different case studies. See Valente et al. (2003) and Kelly et al. (1991) for a discussion about the empirical importance of peer-leading interventions.

deal about the actual structures. Thus, we need a way to think about the design of marketing or development strategies in a context of incomplete information about ties that connect individuals.

This paper proposes a general framework which addresses these difficulties, and thereby opens up the study of optimal strategies in contexts characterized by 'local' influence. There are two groups of players, $\mathcal{M}$ and $\mathcal{N}$. Every member of group $\mathcal{M}$ chooses a strategy with a view to influencing members of group $\mathcal{N}$ to choose certain actions. The actions taken by members of group $\mathcal{M}$ lead to some information/resources reaching individuals in $\mathcal{N}$. This information is shared by individuals in $\mathcal{N}$ 'locally'. This local sharing leads to a new distribution of resources or information. Group $\mathcal{N}$ members make decisions based on this distribution, which in turn generates payoffs for members in group $\mathcal{M}$. In the present paper we will study the case in which there is only one player in group $\mathcal{M}$. In a companion paper Galeotti and Goyal (2007), we study the general case with $|\mathcal{M}| \geq 1$.

Two aspects of social interaction are important in our study. The *first* aspect is the *level* of social interaction. Do individuals have many friends or few friends, and is the distribution of friends even or is it characterized by inequality with some individuals having many friends while others have only a few? We will model levels of social interaction via the distribution of connections that individuals have. The number of connections of an individual will be termed her degree and we will study the effects of the degree distribution on optimal strategy and profits. The degree distribution summarizes a large amount of information about the network in a very simple and natural way. It also allows to formalize ideas about adding links or redistributing links in the network. The notion of adding links to a network is studied in terms of first order stochastic dominance relations, while the idea of redistributing links is formalized in terms of second order stochastic dominance relations.[4]

The *second* aspect is the *content* of social interaction. Inter-personal interaction is important due to two types of factors: information sharing and adoption externalities. Information sharing about prices and quality of products is important in shaping demand for them. In the case of products such as fax machines, softwares and e-mail, the rewards to an individual depend on how many others adopt the product. Our model allows for both these types of interaction.

The analysis examine two set of questions. We first address how the content of interaction and the distribution of connections jointly shape the optimal strategy of

[4]Many processes governing network formation exhibit stochastic dominance relationships as parameters describing the underlying process are altered. For example, in Jackson and Rogers (2006) links between nodes occur both at a random base and at a network base (local network search). The authors show that by changing the random-network ratio the resulting degree distributions can be ordered in the sense of stochastic dominance. Furthermore, for different networks, these parameters can be easily estimated.

player $\mathcal{M}$ and the level of surpluses that she can hope to earn. We then study how a monopolist incorporates additional information on the network in the design of marketing strategies and its effect on profits.

Our analysis brings out three general points. *First*, we show that incorporating information on social interaction can have large effects on the profits of a player $\mathcal{M}$. Thus it is important for firms and governments to take social interaction seriously. *Second*, we show that an increase in the level and dispersion of social interaction can increase or decrease the strategy and profits of player $\mathcal{M}$; the effect depends on the *content* of the interaction. Therefore players like firms and governments should pay attention to the type of interaction as well as the level of social interaction in designing their strategies. *Third*, we show that the dispersion of connections determines the value of additional information on connections. This means that player $\mathcal{M}$ should be willing to pay more for details of network information in contexts with greater dispersion in social connections.

The economic interest of these findings is illustrated via a detailed analysis of two prominent economic applications: advertising in the presence of word of mouth communication and product introduction in the presence of adoption externalities. In the information sharing example, there is a firm advertising to a group of consumers, who share product information among themselves.[5] Potential buyers are not aware of the existence of the product and the firm undertakes costly informative advertising. Consumers share the information they receive from the firm with their friends and neighbors.

In the adoption externalities example, we study how a firm can optimally induce the adoption of a new product in a context with local interaction.[6] Suppose individuals buy the product only if all their social contacts have already adopted the product. There are two periods, 1 and 2. In period 1 no individual will buy the product and to generate demand in period 2, the firm must seed the network: distribute some free samples of the product in period 1.[7]

We illustrate one set of implications of our general results: profits are increasing in level but decreasing in the dispersion of social interaction in the word of mouth communication application. The opposite pattern obtains in the model with adoption externalities. Profits are decreasing in the level and increasing in the dispersion of social interaction! Thus the content of interaction plays a key role in mediating the

[5]This example combines the formulation of advertising from Butters (1977) with the word of mouth communication formulation from Ellison and Fudenberg (1995).

[6]This example is based on the threshold model which has been studied in a variety of fields ranging from economics, sociology, epidemiology and statistical physics. See, e.g., Granovetter (1978), Lopez-Pintado (2007), Jackson and Yariv (2007) and, Pastor-Satorras and Vespigniani (2001).

[7]Active seeding strategies are widely used in practice. For a discussion of a number of case studies see Rosen (2000).

relation between social interaction and payoffs. Similarly, contrasting implications also arise with regard to the effects of social interaction on optimal strategy.

We now place our paper in perspective. There is a large literature on optimal firm strategies with regard to advertising and the adoption of goods with adoption externalities.[8] Similarly there is a large literature on local interaction both with regard to word of mouth communication and with regard to adoption in the presence of local externalities.[9] An important contribution of our paper is a simple model in which interaction can involve local information sharing or local adoption externalities and firms incorporate the local interaction explicitly in their choice of optimal influence strategies.[10] Our paper thus bridges these two literatures. The analysis shows that this model is tractable and that it yields a number of insights into how the content and level of social interaction jointly shape optimal strategies and profits.

Our paper is related to two recent papers which study optimal strategies in the face of local interaction, Ballester et al. (2006) and Banerji and Dutta (2006). Ballester et al. (2006) study a model in which individuals located in a network choose actions (criminal activities) which affect the payoffs of other individuals within the network. They examine the question: which individuals should be eliminated from the network if the objective is to minimize crime? This problem is related to the issue of targeting which we study. Banerji and Dutta (2006) study a setting where firms sell to consumers located on a network and there are local adoption externalities. Their interest is in characterizing networks which can sustain different technologies in equilibrium. Ballester et al (2006) and Banerji and Dutta (2006) both assume that the network is common knowledge among the players. By contrast, we study a model in which the network is imperfectly known and we are then led to a study of networks in terms of degree distributions, something which is quite distinct from these two papers.

Section 2 presents a basic model of strategic diffusion. Section 3 develops the main ideas of our paper via a detailed discussion of the information sharing example; in section 3.1 we analyze the general model. Section 4 presents two extensions which allow for targeted strategies and the presence of opinion leaders. Section 5 concludes. A number of extensions are proposed in the appendix. These extensions show that our method of analysis can be easily extended to accommodate a variety of phenomena.

[8] For early work on advertising see Butters (1977) and Grossman and Shapiro (1984). For early work on firms in markets with adoption externalities, see Farrell and Saloner (1986), Rolfs (1974), and Katz and Shapiro (1986); for a recent survey, see Shy (2001).

[9] For local learning and word of mouth communication, see Bala and Goyal (1998), Ellison and Fudenberg (1993,1995); for a survey of this work see Goyal (2005). For local interaction and adoption externalities see Ellison (2003), Morris (2000), Jackson and Yariv (2007), Sundararajan (2006), Lopez-Pintado (2007). For a general treatment of games with local externalities, see Galeotti et. al. (2006).

[10] As we have mentioned in the introduction above the companion paper Galeotti and Goyal (2006) studies the case of competition among firms.

2 Basic Model

We study the problem of a player $\mathcal{M}$ who exerts costly effort with a view to getting a group of individuals to choose an action. Individual behavior is influenced by *social interaction*. There are two dimensions of social interaction which will be relevant in our paper. The *first* dimension concerns the content of the interaction. Broadly speaking social interaction may involve sharing of valuable information and adoption externalities. Our model accommodates both these aspects; see example 2.1-2.2 below. The *second* dimension is about who meets whom, i.e., the distribution of personal connections. The analysis will examine how these two dimensions, the content of interaction and the distribution of connections, shape the optimal strategy of player $\mathcal{M}$ and the level of surpluses that she can hope to earn. We now get into the details of the model.[11]

There is a unit measure of individuals $\mathcal{N} = [0, 1]$. Individuals are located in a social network and in principle the structure of the network can be complex and take on a variety of forms. However, $\mathcal{M}$ has limited knowledge about this network. We will model the beliefs of $\mathcal{M}$ about this network as follows: she knows the proportions of individuals having different levels of social interaction. We now elaborate on this formulation.

For an individual $i \in \mathcal{N}$, the level of social interaction is parameterized by a number k, where k is termed the degree. We will suppose that each individual draws k others with probability $P(k) \geq 0$, $k \in \{1, 2, ..., \bar{k}\} = O$ and $\sum_{k \in O} P(k) = 1$. She uses an (atomless) uniform distribution on the unit interval to pick her sample. So, if she has a k sized sample, she makes k draws, and each draw is independent.[12] Now suppose that the draw of the sample size is independent across individuals. We can then say that there is fraction $P(k)$ of individuals who choose a k sized sample.[13] We will refer to P as the degree distribution. Define $\hat{k} = \sum_{k \in O} P(k)k$ as the average degree of social interaction.

As a benchmark case, we will suppose that $\mathcal{M}$ knows P and chooses an action $x \in [0, 1]$.[14] Let the profits from an individual influenced by k others be given by $\phi_k(x)$, where $\phi_k(\cdot) : [0, 1] \to \mathcal{R}$; assume that $\phi_k(\cdot)$ is twice continuously differentiable, for every $k \in O$. The expected net payoffs to player $\mathcal{M}$ under P and effort x are:

[11] It is important to emphasize that heterogeneity in the level of social interaction across individuals has a very different effect on individual behavior and on optimal strategies of the external player as compared to individual heterogeneity with regard to willingness to play; see the appendix for a detailed discussion on this point.

[12] The probability of drawing the same person two or more times is zero, given that there is a continuum of individuals.

[13] Note that we are invoking a variant of the law of large numbers here.

[14] In section 4.1 we relax this assumption and we allow player $\mathcal{M}$ to know more about the pattern of personal connections.

$$\Pi(x|P) = \sum_{k \in O} P(k)\phi_k(x) - C(\alpha, x), \tag{1}$$

where $C(\alpha, \cdot) : [0,1] \to \mathcal{R}$ is the cost of effort and the parameter $\alpha \geq 0$ indicates the efficiency in generating efforts.

We now make a number of remarks on the scope of this framework. First we discuss the continuum of individuals formulation. The continuum formulation allows us to move from the distribution of sample sizes at the individual level to the fraction of individuals with a certain degree easily. It also simplifies the exposition of the arguments in some places. Second we note that the atomless uniform distribution assumption is convenient, but the framework allows for a non-uniform draw of samples as well. Indeed, section 4.2 extends the model to allow for heterogeneities in the probability of being drawn. It is also worth noting an implication of the atomless distribution: the probability of two individuals picking a common partner is zero. In other words, it rules out any clustering in the network. Thus, in our framework, we would need probability distributions with atoms if we want to study clustering. Third, we have written the payoffs in terms of individual degree, but the framework allows for indirect social interaction effects: for instance, it is easy to accommodate indirect flow of information from i to j to k (on this issue also see the discussion in word of mouth application below).

We are now ready to introduce our two leading economic examples.

Example 2.1 *Word of mouth communication*

Consider a firm advertising to a group of consumers, who share product information among themselves. The model here combines the formulation of advertising from Butters (1977) with the word of mouth communication formulation from Ellison and Fudenberg (1995).[15] Player $\mathcal{M}$ is a monopolist selling a good at price 1; the cost of producing the good is zero. The set of buyers is $\mathcal{N} = [0, 1]$; each buyer has inelastic demand and her reservation value for the object is $v = 1$. Suppose first that potential buyers are not aware of the existence of the product and the monopolist undertakes costly informative advertising.

The monopolist chooses $x \in [0, 1]$; this is the fraction of individuals in $\mathcal{N}$ who receive advertisements. Let the cost of effort x be $\alpha x^2/2$, where $\alpha > 0$. A buyer buys either if she receives the advertisement from the monopolist or if she receives information via word of mouth communication from her cohort. In this example, a buyer with degree k contacts k other consumers, from whom she obtains information about the

[15]For recent empirical work on the importance of word of mouth communication in the diffusion of products, see Mobius, Niehaus and Rosenblat (2006) and Godes and Mayzlin (2004)

product, if any. It then follows that if the monopolist chooses x, her expected profits from a k degree buyer are:

$$\phi_k(x) = 1 - (1-x)^{k+1}. \tag{2}$$

This is the probability that a consumer with k friends becomes aware of the product. For a given distribution, P, we can write the expected profits under strategy x as:

$$\Pi(x|P) = \sum_{k \in O} P(k)[1 - (1-x)^{k+1}] - \frac{\alpha}{2}x^2.$$

We would like to emphasize three aspects of this example. First, note that we are assuming that individuals obtain information only from their direct neighbors. This assumption is valid when advertisement is about information which is only valuable for a short length of time, such as discounts, sales and last-minute offers. However, when advertisement is about information which is more stable, it is appropriate to allow for indirect information transmission across neighbor or neighbors' neighbor etc. In the appendix we show that our framework and the methods of analysis can be extended in a natural way to cover richer patters of information diffusion. Second, in this example the function of word of mouth communication is to create product awareness among consumers. The case where consumers are aware of the product but do not know the quality of the product can be modeled similarly, and leads to very similar payoffs and incentives for the firm. This formulation is developed in detail in the appendix. Finally, in the appendix we extend this example to allow for competition between firms. This extension shows that most of the results obtained in the single firm case extends to an oligopoly advertising model. ■

Example 2.2 *Seeding the network*[16]

In some interesting contexts, the returns from adopting a product depend on whether others do likewise. Well known examples of products which display adoption externalities include fax machines, software programmes, telephones and e-mail accounts. This example considers the case of a new product and asks how a firm can optimally induce its adoption in a context with local interaction. The model introduces optimal marketing strategies in a standard threshold model.[17]

Suppose individuals use the following simple decision rule. If a consumer with degree k observes that s of her social contacts have already the product then she buys the object with probability $\psi(k, s)$. Hence, $\psi(k, s)$ indicates the probability of adoption of a k-degree consumer in the event that s of her neighbors have the product. We

[16] This example is inspired by discussions with Arun Sundararajan. We thank him for his comments.

[17] See footnote 6 for detailed references on previous existing work on the threshold model in economics, sociology and statistical physic.

assume that a consumer adopts the product only if at least one of her neighbors has the product, i.e., $\psi(k,0)=0$ for all $k \in O$.

Suppose there are two periods, 1 and 2. Under our assumptions, in period 1 no individual will buy the product. To generate demand in period 2, the firm can seed the network: it can distribute some free samples of the product in period 1. Let $x \in [0,1]$ be the fraction of individuals who are sent free samples and let the price of the object be 1. The expected returns to the monopolist from a k degree individual are then given by:

$$\phi_k(x) = (1-x)\sum_{s=1}^{k}\binom{k}{s}x^s(1-x)^{k-s}\psi(k,s).$$

This is simply the probability that a k degree consumer does not receive the product for free and that she will adopt the object. Clearly, the behavior of the function $\phi_k(\cdot)$ ultimately depends on the assumptions of $\psi(k,s)$.[18] For simplicity, here we focus on the case in which for all $k \in O$, $\psi(k,s)=1$ if $s=k$, otherwise it is zero. That is, consumers adopt the product if all their social contacts have already adopted the product.[19] In this case, the expected returns to the monopolist from a k degree individual are then given by:

$$\phi_k(x) = (1-x)x^k.$$

It can be checked that $\phi_k(x)$ is increasing and convex for low x, and decreasing and concave for large x, and that it is decreasing and convex in k. The expected profits to a monopolist who chooses x are:

$$\Pi(x|P) = \sum_{k\in O} P(k)(1-x)x^k.$$

The direct cost of production and dissemination of samples is zero, but a free sample has an implicit cost for the monopolist since a consumer who gets a free product does not buy at a positive price later. It is easy to see that expected profits at $x=0$ and at $x=1$ equal 0. Further, the expected profits are positive for every $x \in (0,1)$. ■

The examples illustrate how different aspects of social interaction can be accommodated within our framework. We now introduce a few concepts which help us in studying changes in the levels of social interaction.

[18] In the appendix we show that our methods of analysis can accommodate a variety of different processes of adoption, i.e., different functional forms of $\psi(k,s)$.

[19] This decision rule is reasonable if there is a status quo technology which everyone uses and a switch to a new technology is justified only if communication with the new technology is possible with everyone. Consider a school which sends messages to parents via post, and is considering switching to e-mail. It will switch to e-mail only if every parent can be reached via e-mail. We also emphasize that our findings can be generalized to a situation in which individuals adopt the product if a sufficiently large fraction of their neighbors adopt the new technology.

An increase in the level of social interaction is modeled in terms of first order stochastic shifts in the degree distribution (FOSD). We are also interested in the role of the dispersion of social interaction. Changes in dispersion are studied in terms of second order stochastic shifts in the degree distribution. When studying increasing dispersion, we will focus on mean preserving spreads of distributions (MPS). While these concepts are standard, we present them here for easy reference. In what follows, P and P' are distinct degree distributions defined on O. Given a degree distribution P, let the cumulative distribution function be denoted by $\mathcal{P} : \{1, 2, ..., \bar{k}\} \to [0, 1]$ i.e. $\mathcal{P}(y) = \sum_{k=1}^{y} P(k)$.

Definition 2.1 *P' first-order stochastically dominates (FOSD) P if and only if $\mathcal{P}'(k) \leq \mathcal{P}(k)$ for every $k \in \{1, 2, .., \bar{k}\}$.*

Definition 2.2 *P' is a mean preserving spread of P if and only if P and P' have the same mean and*

$$\sum_{k=1}^{y} \mathcal{P}(k) \leq \sum_{k=1}^{y} \mathcal{P}'(k),$$

for every $y \in \{1, 2, ...\bar{k}\}$.

This completes the description of the model we now turn to a study of the effects of the content of interaction, which is captured by the function ϕ, and the distribution of connections P, on the optimal strategy, x^*, and profits, $\Pi(x^*|P)$.

3 Optimal advertising with word of mouth communication

In this section we develop some of the main ideas of our paper via a detailed discussion of optimal advertising in the presence of word of mouth communication among consumers.

We start with a preliminary enquiry into the potential advantages of using word of mouth communication. Perhaps the simplest way to do this is to compare the profits under a strategy which ignores word of mouth communication with profits under a strategy that optimally responds to word of mouth distribution. To fix ideas suppose that $\alpha = 1$ and that everyone draws the same sample size k. Then the optimal strategy of a firm which ignores word of mouth advertising is to set $x = 1$. The profits under this strategy are $\Pi(1) = 1/2$. On the other hand, the optimal strategy of a firm which incorporates the word of mouth communication is given by x_k^*, where x_k^* solves:

$$(k + 1)(1 - x^*)^k - x^* = 0.$$

We can substitute this strategy in the profits to get the profits under optimal strategy. Denote this profit by $\Pi(x_k^*)$. Figure 1 plots the percentage advantages, i.e., $[\Pi(x_k^*) - \Pi(1)]/\Pi(1)$, from incorporating word of mouth communication in the design of advertising strategy. The figure shows that if $k \geq 10$ then optimally responding to word of mouth communication can lead to a 80% increase in profits!

Having established that a firm can make substantial gains in profits by incorporating word of mouth communication in its marketing strategy, we now examine the ways in which optimal strategy and profits vary with the level and dispersion in word of mouth communication.

We start with the effects of an increase in word of mouth communication. As individuals talk to more people two forces are at work. On the one hand, a potential buyer with more connections is more likely to hear about the product from neighbors. In this sense, advertising and word of mouth communication may be viewed as substitutes. On the other hand, a higher degree of social communication means that for any advertisement there are now more people who hear about it through word of mouth communication. In this sense social communication and advertising are complements. The effects of an increase in the level of word of mouth communication will therefore depend on which of these two effects will prevail.

If advertising technology is inefficient (i.e., α is large), then optimal x is small and few consumers receive advertisements. Consequently it is unlikely that a person will hear about it from others even if she has more contacts. This suggests that the second positive effect dominates. In contrast, when the advertising technology is efficient (i.e., α is low), the monopolist chooses high x. Since many consumers hear directly about the product, an increase in word of mouth makes it much more likely that an uninformed buyer will hear about the product via word of mouth and this lowers the incentives for the firm to advertise. The following result summarizes these ideas.

Proposition 3.1 *Suppose that P' FOSD P. There exist $\underline{\alpha}_P$ and $\bar{\alpha}_P$, with $0 < \underline{\alpha}_P \leq \bar{\alpha}_P$, such that if $\alpha < \underline{\alpha}_P$ then $x_P^* \geq x_{P'}^*$, while if $\alpha > \bar{\alpha}_P$ then $x_P^* \leq x_{P'}^*$.*

Proof: The derivative of the marginal returns with respect to degree k at x_P^* is:

$$\frac{\partial^2 \phi_k(x_P^*)}{\partial x \partial k} = (1 - x_P^*)^k \left[1 + (k+1)\ln(1 - x_P^*)\right].$$

For sufficiently low x_P^* the marginal returns are increasing in k, while for sufficiently high x_P^* the converse holds. Next, note that at the optimum the following holds

$$\frac{\partial \Pi(x_P^*|P)}{\partial x} = \sum_{k \in O} P(k)(k+1)(1 - x_P^*)^k - \alpha x_P^* = 0,$$

and it is easy to see that x_P^* is decreasing in α and that $x_P^* \to 0$ when $\alpha \to \infty$, while $x_P^* \to 1$ when $\alpha \to 0$. The proof is completed by noting that P' FOSD P and that $\Pi(x|P)$ is concave in x. ∎

The effects of greater word of mouth communication on firm's profits are straightforward. Suppose the firm keeps the advertisement constant. Then the costs remain the same, but an increase in word of mouth communication means that more potential buyers will hear about the product and so revenue will increase. Clearly this will also hold when the firm reacts optimally to the new word of mouth communication regime. The following result summarizes this argument.

Proposition 3.2 *Firm's profits are increasing with the level of word of mouth communication.*

Proof: Start with some $x_P^* \in (0,1)$. Then,

$$\Pi(x_P^*|P) \leq \Pi(x_P^*|P') \leq \Pi(x_{P'}^*|P'),$$

where the first inequality follows because $\phi_k(\cdot)$ is increasing in k and P' FOSD P, while the second inequality follows from the optimality of $x_{P'}^*$, under P'. ∎

We now turn to the effects of greater dispersion in word of mouth communication. The simplest way to study this is to consider the case of a mean preserving spread in word of mouth communication. The optimal response depends on whether the marginal returns to advertisements are concave or convex in the degrees. If they are concave then greater dispersion implies a fall in expected marginal returns and under the concavity of returns with respect to advertisement this means that optimal advertisement goes down. The converse holds if the marginal returns are convex in degrees. The following result summarizes these ideas.

Proposition 3.3 *Given a P, there exists $\tilde{\alpha}_P$ and $\tilde{\tilde{\alpha}}_P$, with $0 < \tilde{\alpha}_P \leq \tilde{\tilde{\alpha}}_P$, such that if $\alpha < \tilde{\alpha}_P$ then marginal returns are convex in k, while if $\alpha > \tilde{\tilde{\alpha}}_P$ then marginal returns are concave in k. Consequently, optimal efforts increase (decline) with mean preserving spread of word of mouth communication if $\alpha < \tilde{\alpha}_P$ and decline if $\alpha > \tilde{\tilde{\alpha}}_P$.*

Proof: We can write the second derivative of the marginal returns with respect to k at the optimal strategy x_P^* as follows:

$$\frac{\partial^3 \phi_k(x_P^*)}{\partial x \partial k^2} = (1 - x_P^*)^k \ln(1-x) \left[2 + (k+1)\ln(1-x_P^*)\right]$$

For sufficiently low x_P^* this expression is negative while for sufficiently high x_P^* it is positive. This means that marginal returns are concave in k for small x_P^* and convex in k for large x_P^*. The proof now follows from the observation that x_P^* is decreasing in α, and that $x_P^* \to 0$ when $\alpha \to \hat{k} + 1$, while $x_P^* \to 1$ when $\alpha \to 0$. ∎

Finally, we study the effects of greater dispersion in word of mouth communication on firm's profits. Here the intuition is quite simple: the value of sampling others is increasing but concave in the size of the sample. Consequently, greater dispersion has the effect of lowering the potential for profits. The following result summarizes this intuition.

Proposition 3.4 *Firm profits are falling with greater dispersion in word of mouth communication.*

Proof: Suppose that P' is a mean preserving spread of P. Note that,

$$\Pi(x^*_{P'}|P') \leq \Pi(x^*_{P'}|P) \ \leq \Pi(x^*_P|P)$$

where the first inequality follows because $\phi_k(\cdot)$ is concave in k and P' is a mean preserving spread of P, while the second inequality follows by optimality of x^*_P, under P. ■

3.1 General results in the basic model

The analysis of the word of mouth communication example yields a number of interesting results on how optimal strategy and profits depend on word of mouth communication. We now use the intuitions gained to develop general results on how social interaction affects optimal strategies. The relevance of these results is to highlight the important point that the effects of increasing social interaction or greater dispersion in social interaction on optimal strategy depend very much on the *content* of the interaction. In particular, we will show that the effects of social interaction on optimal strategies and profits are very different in the seeding the network application.

We start by formally stating the key properties of the returns function $\phi_k(\cdot)$ and the cost function $C(\alpha, x)$:

$$\text{Concavity of returns in effort :}\forall x \in [0,1], \ \ \forall k \in O, \ \frac{\partial^2 \phi_k(x)}{\partial x^2} \leq 0 \qquad (R.1)$$

$$\forall x \in (0,1], \forall \alpha > 0, \ \ \frac{\partial C(\alpha, x)}{\partial \alpha} > 0; \ \ \frac{\partial C(\alpha, x)}{\partial x} > 0; \ \ \frac{\partial^2 C(\alpha, x)}{\partial x^2} > 0. \qquad (C.1)$$

Denote the optimal strategy under a degree distribution P by x^*_P. Throughout this section, we will assume that for any P, there exists an interior optimal strategy $x^*_P \in (0,1)$. It is worth noting that in the word of mouth communication model and in the seeding the network model, the optimal strategy is always interior.

Recall that we measure an increase in social interaction in terms of a first order stochastic shift in the distribution P. The following result shows that the effects of increasing social interaction depend on whether the function ϕ displays increasing or decreasing marginal returns with respect to degree. It is useful to define these properties of $\phi_k(\cdot)$ formally.

Definition 3.1 *The function ϕ exhibits increasing marginal returns in degree (IMRD) if for all $x > x'$ and $k < \bar{k}$: $\phi_{k+1}(x) - \phi_{k+1}(x') \geq \phi_k(x) - \phi_k(x')$. Analogously, the function ϕ exhibits decreasing marginal returns in degree (DMRD) if for all $x > x^{'}$ and $k < \bar{k}$: $\phi_{k+1}(x) - \phi_{k+1}(x') \leq \phi_k(x) - \phi_k(x')$.*

Denote the optimal strategy under P and P' by x^*_P and $x^*_{P'}$, respectively. We can now state our first result on effects of social interaction.

Proposition 3.5 *Suppose the payoffs are given by (1) and satisfy (R.1) and (C.1). Let P' FOSD P. If ϕ satisfies IMRD (DMRD), then $x^*_{P'} \geq x^*_P$ ($x^*_{P'} \leq x^*_P$).*

Proof: Suppose $x^*_P \in (0,1)$. Then,

$$\frac{\partial \Pi(x^*_P|P)}{\partial x} = \sum_{k \in O} P(k) \frac{\partial \phi_k(x^*_P)}{\partial x} - \frac{\partial C(\alpha, x^*_P)}{\partial x} = 0. \tag{3}$$

Since P' FOSD P and IMRD holds, it follows that $\frac{\partial \Pi(x^*_P|P')}{\partial x} \geq 0$. Now (R.1) and (C.1) together imply that the expected returns function is strictly concave in x for $x \in (0,1]$, and so $x^*_{P'} \geq x^*_P$. The proof for the DMRD case is analogous and omitted. ■

Remark 1: It is easy to see that this result can be strengthened and holds so long as IMRD (DMRD) holds at the optimum x^*_P.

Recall that in the word of mouth communication model we showed that IMRD and DMRD obtain depending on whether the costs of advertising are large or small. Thus optimal strategy will increase with increase in social interaction if costs are high but fall if they are low. The value of the result lies in identifying a simple property of the returns function as being key to understanding how the level of social interaction affects optimal strategy. So as an example, consider the seeding the network model. For the case of equal samples k, the marginal returns are increasing in degree, i.e., IMRD obtains. The above result then immediately implies that optimal seeding increases with social interaction.[20]

We now turn to the effects of increasing social interaction on the profits of player $\mathcal{M}$. The following result shows that the effects of a first order stochastic shift in social interaction depend on whether the function ϕ is increasing or decreasing in degree.

[20]Specifically, it is easy to check that the optimal strategy of the monopolist is given by $x^* = k/(k+1)$, which is increasing in k.

Proposition 3.6 *Suppose the payoffs are given by (1) and P' FOSD P.*

1. *If $\phi_k(x) \geq \phi_{k-1}(x)$, $\forall x \in (0,1)$, $\forall k \in O$, then $\Pi(x^*_{P'}|P') \geq \Pi(x^*_P|P)$;*
2. *If $\phi_k(x) \leq \phi_{k-1}(x)$, $\forall x \in (0,1)$, $\forall k \in O$, then $\Pi(x^*_{P'}|P') \leq \Pi(x^*_P|P)$.*

Proof: Start with some $x^*_P \in (0,1)$. In case 1,

$$\Pi(x^*_P|P) \leq \Pi(x^*_P|P') \leq \Pi(x^*_{P'}|P'),$$

where the first inequality follows because $\phi_k(\cdot)$ is increasing in k and P' FOSD P, while the second inequality follows from the optimality of $x^*_{P'}$, under P'. The proof for the second case follows from analogous arguments and is omitted. ■

We have already seen that in the word of mouth communication model the payoff is increasing in degree. Proposition 3.6 then implies that profits increase with social interaction. By contrast, in the seeding network model returns are decreasing in degree, and so Proposition 3.6 immediately implies that profits are falling in the level of social interaction! This result is relatively easy to obtain in our framework, but it is worth emphasizing its substantive interest: an increase in social interaction is not always beneficial for the outside player. The impact on payoffs depend very much on the content of the social interaction.

We next turn to the effects of an increase in dispersion of social interaction. An examination of the first order condition, (3), suggests that the effect of such changes depend on the curvature of the marginal returns with respect to k.

Proposition 3.7 *Suppose the payoffs are given by (1), satisfy (R.1) and (C.1), and that P' is a mean preserving spread of P. If the marginal returns are concave (convex) in k then $x^*_{P'} \leq x^*_P$ ($x^*_P \geq x^*_{P'}$).*

Proof: Let us start with $x^*_P \in (0,1)$. Since P' is a mean preserving spread of P and since the marginal returns are concave in degree, it follows that

$$\frac{\partial \Pi(x^*_P|P')}{\partial x} \leq \sum_{k \in O} P(k) \frac{\partial \phi_k(x^*_P)}{\partial x} - \frac{\partial C(\alpha, x^*_P)}{\partial x} = \frac{\partial \Pi(x^*_P|P)}{\partial x} = 0.$$

Next note that (R.1) and (C.1) together imply that expected payoffs are strictly concave in x for every $x \in (0,1)$. This in turn implies $x^*_{P'} \leq x^*_P$. The proof for the case where marginal returns are convex in k follows from analogous arguments and is omitted.

■

Remark 2: It is easy to see that this result can be strengthened and holds so long as concavity (convexity) of the marginal returns holds at the optimum x^*_P.

We now ask how profits of the monopolist vary with a mean preserving spread in social interaction.

Proposition 3.8 *Suppose payoffs are given by (1) and that P' is a mean preserving spread of P.*

1. *If $\phi_k(x)$ is concave in k, for all $k \in O$, then $\Pi(x^*_{P'}|P') \leq \Pi(x^*_P|P)$.*

2. *If $\phi_k(x)$ is convex in k for all $k \in O$, then $\Pi(x^*_{P'}|P') \geq \Pi(x^*_P|P)$.*

Proof: We prove 1.

$$\Pi(x^*_{P'}|P') \leq \Pi(x^*_{P'}|P) \leq \Pi(x^*_P|P)$$

where the first inequality follows because $\phi_k(.)$ is concave in k and P' is a mean preserving spread of P, while the second inequality follows by optimality of x^*_P, under P. The proof for case 2 is analogous and is omitted. ■

Recall that in the word of mouth communication model payoffs are concave in degrees, and so Proposition 3.8 implies that profits are falling with greater dispersion in social interaction. By contrast, in the seeding the network model payoffs are convex in k; Proposition 3.8 then tells us that profits are increasing with dispersion in social interaction. Thus the implications of increasing dispersion in profits goes in opposite directions in the two models. These differences reiterate the point we made above: the impact of changes in the dispersion of social interaction on firm's profits is sensitive to the content of social interaction.

4 Extensions

In this section we present two extensions. The first extends the benchmark model to a setting in which player $\mathcal{M}$ has additional information on the distribution of personal connections. The second extension studies optimal advertising when some individuals in group $\mathcal{N}$ are drawn more than others. These individuals are the opinion leaders or influencers.

4.1 Targeted Strategies

We now consider a situation in which the player $\mathcal{M}$ knows the degree of each individual, and is therefore able to target her effort.[21] We would like to understand the circumstances under which $\mathcal{M}$ should target the high or the low degree individuals.

[21]There are some well known examples in which organizations have tried to target key individuals in a social context to generate desirable outcomes. For instance, in 2001 the firm Hasbro, a worldwide leader in manufacture and marketing of games and toy, collected data on the social network of boys aged eight to thirteen in Chicago. This data was used to identify 1600 influential children who were then the focus of personal advertising and training. For further details on this case, see Tierney (2001). See Rosen (2000) for detailed discussions on a variety of similar case studies.

We also examine the relative value of using a targeted strategy as compared to an un-targeted strategy and how this relates to the nature of social interaction.[22]

Formally, the monopolist knows the distribution of degrees P and is able to partition the set $\mathcal{N}$ in $\bar{k}$ groups. $P(k)$ is the fraction of individuals in group k and individuals in group k have degree k. Hence, the strategy of the monopolist is a vector $\mathbf{x} = (x_1, ..., x_{\bar{k}})$, where $x_k \in [0,1]$ indicates the effort that player $\mathcal{M}$ targets to group $k \in O$. It follows that $\mathbf{x} \in [0,1]^{\bar{k}}$. A strategy $\mathbf{x}$ generates a total effort $\theta(\mathbf{x}) = \sum_{k \in O} P(k) x_k$. We will consider models in which expected returns from a k degree individual can be represented as $\phi_k(x_k, \theta(\mathbf{x}))$. The expected net profits to the monopolist from a strategy $\mathbf{x}$ are:

$$\Pi(\mathbf{x}|P) = \sum_{k \in O} P(k)\phi_k(x_k, \theta(\mathbf{x})) - C(\alpha, \theta(\mathbf{x})) \tag{4}$$

We now discuss our two examples, word of mouth communication and seeding the network, in a context where $\mathcal{M}$ knows the degree of every individual.

Example 4.1 *Word of mouth communication, continued*

Given $\mathbf{x}$ the expected returns from a k degree consumer are:

$$\phi_k(x_k, \theta(\mathbf{x})) = 1 - (1 - x_k)(1 - \theta(\mathbf{x}))^k,$$

which is the probability that an individual with degree k will be informed either from direct advertisements or from word of mouth communication.[23] Note that $\phi_k(x_k, \theta(\mathbf{x}))$ is concave in the first argument. The expected profits of the monopolist are,

$$\Pi(\mathbf{x}|P) = \sum_{k \in O} P(k)[1 - (1 - x_k)(1 - \theta(\mathbf{x}))^k] - \frac{\alpha}{2}(\theta(\mathbf{x}))^2.$$

■

Example 4.2 *Seeding the network, continued.*

In this case the expected returns from an k degree individual are:

$$\phi_k(x_k, \theta(\mathbf{x})) = (1 - x_k)(\theta(\mathbf{x}))^k.$$

[22] The massive quantities of data available on social networks, such as chat rooms, social networking web-sites, newsgroups, has led to a booming literature on social network data mining in computer science. Its main objective is to construct algorithms which allow to analyse social network data in order to determine measures of what is called the *network value of customers*: the expected increase in sales to others that results from marketing to that customer. Not surprisingly, the degree of a node is an important determinant of the network value of a consumer. See Domingos and Richardson (2001), Kempe et al (2003).

[23] Note that here we are assuming that consumers sample other consumers randomly from $\mathcal{N}$.

This is the probability that an individual with degree k will buy the product in period 2. It may be checked that $\phi_k(x_k, \theta(\mathbf{x}))$ is concave in the first argument. The expected profits of player $\mathcal{M}$ under strategy $\mathbf{x}$ are

$$\Pi(\mathbf{x}|P) = \sum_{k \in O} P(k)(1 - x_k)(\theta(\mathbf{x}))^k$$

■

We say that a strategy $\mathbf{x}$ is monotonically increasing in degree if $x_{k+1} \geq x_k$ for all $k \in O \setminus \{\bar{k}\}$. A monotonically decreasing strategy in degree is defined similarly. We now develop simple conditions on the returns function $\phi_k(\cdot, \cdot)$ under which the optimal strategy is monotonically increasing and decreasing in degree, respectively.

Definition 4.1 *The returns function $\phi_k(x_k, \theta(\mathbf{x}))$ exhibits IMRD if for all $x > x'$ and $k < \bar{k}$, $\phi_{k+1}(x, \theta(\mathbf{x})) - \phi_{k+1}(x', \theta(\mathbf{x})) \geq \phi_k(x, \theta(\mathbf{x})) - \phi_k(x', \theta(\mathbf{x}))$. Analogously, the monopolist faces decreasing marginal returns in degree (DMRD) if for all $x > x'$ and $k < \bar{k}$, $\phi_{k+1}(x, \theta(\mathbf{x})) - \phi_{k+1}(x', \theta(\mathbf{x})) \leq \phi_k(x, \theta(\mathbf{x})) - \phi_k(x', \theta(\mathbf{x}))$.*

The following result can now be stated.

Proposition 4.1 *Suppose the payoffs are given by (4). In addition suppose that the returns function $\phi(\cdot, \cdot)$ satisfies concavity in its first argument, and that $P(k) > 0$ for all $k \in O$.*

1. *If the returns function $\phi_k(.,.)$ satisfies IMRD then the optimal targeted strategy is monotonically increasing in k.*

2. *If the returns function $\phi_k(.,.)$ satisfies DMRD then the optimal targeted strategy is monotonically decreasing in k.*

Proof: We prove part 1. Note that for all $s \in O$ we have that

$$\frac{d\Pi(\mathbf{x}|P)}{dx_s} = P(s)\left[\frac{\partial \phi_s(x_s, \theta(\mathbf{x}))}{\partial x_s} + \sum_{k \in O} P(k)\frac{\partial \phi_k(x_k, \theta(\mathbf{x}))}{\partial \theta(\mathbf{x})} - \frac{\partial C(\alpha, \theta(\mathbf{x}))}{\partial \theta}\right]$$

Within the brackets only the first term differs across the s. Let $\mathbf{x}^*$ be the optimal targeted strategy. Suppose there is some s and some $x_s^* > 0$ such that $\frac{d\Pi(\mathbf{x}^*|P)}{dx_s} \geq 0$. It then follows from IMRD that the total derivative will be positive at $x_{s'} = x_s$ for all $s' \geq s$. The result then follows from concavity of the payoff function with respect to its first argument x_s for each s. We note that if there is no s with this property then clearly the result is true. The proof for the DMRD case is analogous and omitted. ■

The two conditions on payoffs in Proposition 4.1 are interesting as they are simple to check and they are satisfied by our two examples. In particular, the word of mouth example satisfies DMRD while the seeding the network satisfies IMRD. In fact the implications of IMRD and DMRD are a little stronger in these examples, and yield cut-off optimal strategies.

An increasing cut-off strategy $\mathbf{x}$ has a $\tilde{k} \in O$ such that $x_{\tilde{k}} \in [0,1]$, while for all $k > \tilde{k}$, $x_k = 1$ and for all $k < \tilde{k}$, $x_k = 0$. A decreasing cut-off strategy is defined similarly. The following result covers the two examples.

Corollary 4.1 *In the word of mouth communication example the optimal targeted strategy is a decreasing cut-off strategy, while in the seeding the network the optimal targeted strategy is an increasing cut-off strategy.*

Proof: We prove this result for the word of mouth communication model. Similar arguments can be used to prove the result for the seeding the network model. First note that the function $\phi_k(\cdot,\cdot)$ in the word of mouth communication example satisfies DMRD and it is concave in its first argument. So, we can apply Proposition 4.1 to infer that an optimal strategy is monotonically decreasing in degree. Second, the marginal returns for a degree s individual are:

$$\frac{d\Pi(\mathbf{x}|P)}{dx_s} = P(s)\left[(1-\theta(\mathbf{x}))^s + \sum_{k\in O} P(k)k(1-x_k)(1-\theta(\mathbf{x}))^{k-1} - \alpha\theta(\mathbf{x})\right]$$

Suppose $\mathbf{x}^*$ is optimal and that $1 > x^*_{s'} \geq x^*_s > 0$, for some $s' < s$. Since $x^*_s > 0$, it follows that $\frac{d\Pi(\mathbf{x}^*|P)}{dx_s} \geq 0$. However, from DMRD and the fact that the first term $(1-\theta(\mathbf{x}^*))$ only depends on $\theta(\mathbf{x}^*)$, it follows that $\frac{d\Pi(\mathbf{x}^*|P)}{dx_{s'}} > 0$. So aggregate payoffs can be strictly increased relative to $\mathbf{x}^*$. This contradicts the hypothesis that $\mathbf{x}^*$ is optimal, and concludes the argument. ■

Corollary 4.1 tells us that when the content of social interaction is about information sharing, the optimal strategy involves targeting the low degree consumers, as they are less likely to be informed by their cohort. In contrast, in the adoption externalities example, high degree consumers are the natural target, as they are unlikely to buy the product via social influence and so the marginal cost of sending them free samples is smaller. Hence, the structure of optimal targeted strategies is inherently linked to the content of social interaction.

We next examine how the value of using targeted strategies as against un-targeted strategies depends on social interaction. We will focus on the word of mouth communication example, hereafter. We start with a simple remark.

Let $\Delta_P = \Pi(\mathbf{x}^*|P) - \Pi(x^*|P)$, where $\Pi(\mathbf{x}^*|P)$ and $\Pi(x^*|P)$ indicate the maximum profits under targeted and un-targeted strategies, respectively.

Proposition 4.2 *Consider the word of mouth communication example and fix some $P(\cdot)$. If $P(\cdot)$ assigns positive probability to two or more degrees in O, then $\Delta_P > 0$.*

The proof of Proposition 4.2 is provided in the appendix. Proposition 4.2 shows that targeted strategies strictly increase firm's profits relative to un-targeted strategies. This result obtains whenever P assigns positive probability to two or more degrees; for otherwise, there is not difference between a targeted and an un-targeted strategy. More generally, this suggests that a greater dispersion in the distribution of connections favors the use of targeted strategies. This is addressed in our next result.

Proposition 4.3 *Consider the word of mouth communication with targeted strategies. If P' is a mean preserving spread of P, then there exists a $\alpha^* > 0$ such that for all $\alpha < \alpha^*$, profits under P' are (weakly) higher than profits under P and $\Delta_{P'} > \Delta_P$.*

Proof. Suppose that P' is a mean preserving spread of P. We first show that this implies that there exists a $k^* \in O \setminus \{1, \bar{k}\}$ such that $\mathcal{P}'(s) \leq \mathcal{P}(s)$ for all $s \geq k^*$. Suppose not; then there must exist $k^* \in O \setminus \{1, \bar{k}\}$ such that $\mathcal{P}'(s) \geq \mathcal{P}(s)$ for all $s \geq k^*$ and the inequality holds strictly for some of these values of s. We now show that this contradicts that P' is a MPS of P. Note that since P' is MPS of P, then $\sum_{s=1}^{\bar{k}} [\mathcal{P}'(s) - \mathcal{P}(s)] = \sum_{s=1}^{k^*-1} [\mathcal{P}'(s) - \mathcal{P}(s)] + \sum_{s=k^*}^{\bar{k}} [\mathcal{P}'(s) - \mathcal{P}(s)] = 0$. But since $\mathcal{P}'(s) \geq \mathcal{P}(s)$ for all $s \geq k^*$ and the inequality holds strictly for some of these values of s it follows that $\sum_{s=k^*}^{\bar{k}} [\mathcal{P}'(s) - \mathcal{P}(s)] > 0$, which then implies that $\sum_{s=1}^{k^*-1} [\mathcal{P}'(s) - \mathcal{P}(s)] < 0$, which contradicts that P' is a MPS of P. This proves the claim.

Next, let $\mathbf{x}^*$ be the optimal targeted strategy under P and let $\tilde{k}$ be the cut-off of this strategy. Lemma 6.1 implies that $\theta(\mathbf{x}^*) \in (0, 1)$ for every α and that there exists a $\alpha^* > 0$ such that if $\alpha < \alpha^*$ then $\tilde{k} > k^*$. Suppose then that $\alpha < \alpha^*$. Next, under P' construct a targeted strategy, $\mathbf{x}'$, such that $\sum_{k \in O} P'(k) x'_k = \theta(\mathbf{x}^*)$. Let $\tilde{k}'$ be the cut-off under $\mathbf{x}'$. Since $\tilde{k} > k^*$ and since P' is a mean preserving spread of P it follows that either $\tilde{k}' > \tilde{k}$ or $\tilde{k}' = \tilde{k}$ and $x^*_{\tilde{k}} < x'_{\tilde{k}'}$.

Now note that $\Pi(\mathbf{x}'|P') - \Pi(\mathbf{x}^*|P) \geq 0$ if and only if the following condition holds:

$$\sum_{k \in O} P'(k) \frac{(1 - x'_k)}{1 - \theta(\mathbf{x}^*)} [1 - \theta(\mathbf{x}^*)]^k \leq \sum_{k \in O} P(k) \frac{(1 - x_k)}{1 - \theta(\mathbf{x}^*)} [1 - \theta(\mathbf{x}^*)]^k. \tag{5}$$

To show that this holds, define for each $k \in O$, $g(k) = \frac{P(k)(1-x_k)}{1-\theta(\mathbf{x}^*)}$ and $g'(k) = \frac{P'(k)(1-x'_k)}{1-\theta(\mathbf{x}^*)}$; by construction, g and g' are two probability distributions. Note that for condition 5 to hold it is sufficient that g' first order stochastic dominates g, which is true if and only if,

$$\sum_{k=1}^{s} P'(k)(1 - x'_k) \leq \sum_{k=1}^{s} P(k)(1 - x_k). \tag{6}$$

Note that for all $s < \tilde{k}'$ the LHS of that inequality is equal to zero, while the RHS is non-negative, so the inequality is clearly satisfied. For $s \geq \tilde{k}'$, $\sum_{k=1}^{s} P'(k)x'_k = \theta(\mathbf{x}^*) = \sum_{k=1}^{s} P(k)x^*_k$. Thus, for all $s \geq \tilde{k}'$, condition (6) holds if and only if $\mathcal{P}'(s) \leq \mathcal{P}(s)$, which follows by the hypothesis that P' is a mean preserving spread of P together with the fact that $k^* < \tilde{k}$. Since g' FOSD g and since $(1 - \theta(\mathbf{x}^*))^k$ is strictly falling in k, the claim follows. Since $\Pi(\mathbf{x}'|P') \geq \Pi(\mathbf{x}^*|P)$ it follows that the monopolist will obtain (weakly) higher profits with the optimal strategy under P' as well. Finally, since under un-targeted strategies optimal profits fall with mean preserving shifts (c.f. Proposition 3.4) it immediately follows that $\Delta_{P'} > \Delta_P$. This concludes the proof. ■

We now elaborate on why greater dispersion in connections has contrasting effects on profits under targeted and un-targeted strategies.[24] A mean preserving spread shift augments the fraction of consumers with low degree, while it increases the fraction of consumers with high degree. Under un-targeted strategies the first effect decreases profits, while the latter increases profits. Since marginal returns are decreasing in degree, the former effect dominates and consequently profits go down. In contrast, under targeted strategies the monopolist informs most of the low degree consumers and this alleviates the negative effect induced by the increase of dispersion in connections. Specifically, if the monopolist targets sufficiently enough low degree consumers, which will be the case when advertising is relatively inexpensive, greater dispersion increases profits.

Indeed, the result in Proposition 4.3 is tight in the sense that if the costs of advertising are sufficiently high it is easy to construct examples where mean preserving shifts in degree distributions decrease the profits under targeted strategies as well as the relative value of targeting. To see this suppose that $\alpha = 100$, and that $P(1) = P(3) = \beta$ and $P(2) = 1 - 2\beta$, $\beta \in [0, 0.5]$. Note that an increase in β is equivalent to a mean preserving spread shifts in degree distribution. Figure 2 plots the relative value of targeting, $\Pi(\mathbf{x}^*|\beta) - \Pi(x^*|\beta)$, for different values of β. As said, in contrast with the case of low advertising costs, when costs of advertising are high, the value of targeting may be lower in more dispersed networks.

4.2 Influencers

We have so far studied a setting in which heterogeneity in social interaction is reflected in the idea that some individuals are more susceptible to social influence as compared to others. In the marketing and social communication literatures a great

[24] To fix ideas, suppose that $\alpha = 1$ and consider a shift from a distribution in which all individuals have degree 2 to a new distribution in which individuals have either degree 1 or degree 3 with equal probability. Such a shift increases the value of targeting from zero to 10%.

deal of attention has focussed on opinion leaders or influencers.[25] In our setting a natural way to model influencers is to suppose that some individuals are sampled more often as compared to others. In this section, we will study a model in which every individual draws a sample of the same size, but some individuals are drawn more than others.[26] We will first study the case of un-targeted strategies and then turn to targeted strategies. The discussion will be carried out within the framework of the word of mouth communication example.

Let $I = \{1, ..., \bar{l}\}$ and let $H : O \to [0, 1]$, be a probability distribution, where $H(l)$ indicates the fraction of individuals in $\mathcal{N}$ who are sampled by l others. The mean of H will be denoted by $\hat{l} = \sum_{l \in I} H(l)l$. If an individual is sampled by l other individuals this means that there are l "links" pointing to individual i. We will refer to this as the in-degree and, in this framework, it represents how much individual i influences others.

We note that P and H satisfy the condition $\hat{l} = \hat{k}$. For simplicity we focus on the case where $P(\hat{k}) = 1$, in other words everyone draws a sample of the same size.

Example 4.3 *Word of mouth communication, continued*

For a given untargeted strategy $x \in [0, 1]$, the expected net profits of the monopolist are:

$$\Pi(x|P) = [1 - (1 - x)^{\hat{k}+1}] - \frac{\alpha}{2}x^2, \tag{7}$$

An inspection of the payoffs above reveals that when comparing two distributions H and H' the only factor that matters is the average in-degree. For example if H' first order stochastic dominates H, then $\hat{l}' = \hat{k}' \geq \hat{l} = \hat{k}$. This immediately implies from equation (7) that in the word of mouth communication example profits under H' are higher than profits under H. This finding is consistent with Proposition 3.6.

[25]Generally speaking, influencers may be said to be the people that friends, family and acquaintances turn to for information about what to buy, what to read, how to invest and how to vote. The ideas of influencers, opinion leaders and mavens figure prominently in the popular press. For instance, in a survey taken in the year 2003, the newspaper Washington Post reported that influencers constitute only 10% of the population and that they shape the attitudes and behavior of the other 90%. The classic work on opinion leaders is Katz and Lazersfeld (1955); see also Rogers (2003) and Kotler and Armstrong (2004) for a general discussion on the importance of opinion leaders and influencers.

[26]A major issue in the design of peer-leader network intervention policies is to identify the influencers. A general practice is to submit questionnaires to members of the targeted group. Subjects are asked, among other things, to answer questions about their social network such as to nominate their best friends, to nominate other individuals with whom they talk about specific issues, etc. Individuals who receive more nominations from others are identified as network leaders. In turn, network leaders are asked to attend training section and then they are asked to spread what they have learnt to their acquaintances. See Kelly et al.(1991) and Valente et al. (2003) for a detailed discussion of the implementation of these policies.

We next observe that since only the average in-degree is relevant, a mean preserving spread in the in-degree distribution H has no effect on profits; this finding is in sharp contrast to Proposition 3.8. Recall that in the basic model, greater dispersion in social interaction lowers profits in the word of mouth communication example!

We conclude by studying the optimal targeted strategy in this setting. Let a targeted strategy be denoted by $\mathbf{x} = \{x_1, x_2, ..., x_{\bar{l}}\}$, where x_l is the effort that the monopolist targets to consumers that are sampled by l other consumers.

Let us denote by $\tilde{H}(l)$, the probability that a consumer i samples a consumer who has in-degree l. Using Bayes' rule, we can express $\tilde{H}(l)$ as follows

$$\tilde{H}(l) = \frac{H(l)l}{\hat{l}}.$$

The following result characterizes the optimal strategy of $\mathcal{M}$ when she can target consumers with different in-degree.

Proposition 4.4 *In the word of mouth communication model with in-degree heterogeneity, the optimal targeted strategy is an increasing cut-off strategy.*

Proof: Given a targeted strategy $\mathbf{x}$, let $\omega(\mathbf{x}) = \sum_{l \in I} \tilde{H}(l) x_l$. The expected net payoffs to the monopolist from strategy $\mathbf{x}$ are:

$$\Pi(\mathbf{x}|H) = 1 - \sum_{l \in I} H(l)(1 - x_l)(1 - \omega(\mathbf{x}))^{\hat{l}} - \frac{\alpha}{2}(\theta(\mathbf{x}))^2,$$

and for any $s \in I$ we have that

$$\frac{d\Pi(\mathbf{x}|H)}{dx_s} = H(s)\left[(1 - \omega(\mathbf{x}))^{\hat{l}} + s \sum_{l \in I} H(l)(1 - x_l)(1 - \omega(\mathbf{x}))^{\hat{l}-1} - \alpha\theta(\mathbf{x})\right].$$

Notice that within the brackets only the second term differs across the different in-degree consumers. In particular the second term is increasing in the in-degree of the consumer. Now consider the case that there is some s and some $x_s > 0$ such that $\frac{d\Pi(\mathbf{x},|H)}{dx_s} \geq 0$. It then follows directly from the above observation that the total derivative will be strictly positive at $x_{s'} = x_s$ for all $s' \geq s$, and therefore $x_{s'} = 1$. ■

In this example note that, by assumption, the size of the sample that each consumer draws is the same, and so the probability that a consumer is informed by word of mouth communication is constant across consumers. Hence, more influential consumers are the natural target for the monopolist. More generally, one could allow that consumers are heterogeneous in their susceptibility to social influence as well as their degree of influencing others. Our findings point out that these two dimensions might have different implications for the design of effective strategies.

5 Conclusion

A broad range of work in economics, as well as in other disciplines, such as marketing and social psychology, suggests that friends, neighbors and acquaintances, play an important role is shaping individual behavior. In recent years, firms, governments and political parties have increasingly tried to incorporate such social effects in the design of their marketing and development strategies. To the best of our knowledge there is no theoretical model which examines the effects of social influence on the design of optimal strategies.

The main contribution of the paper has been to introduce a framework within which this question can be systematically studied. Our analysis brings out three general points. *First*, we show that incorporating information on social interaction can have large effects on the profits of a player. Thus it is important for firms and governments to take social interaction seriously. *Second*, we show that an increase in the level and dispersion of social interaction can increase or decrease the strategy and profits of the player; the effect depends on the *content* of the interaction. Therefore players should pay attention to the type of interaction as well as the level of social interaction in designing their strategies. *Third*, we show that the dispersion of connections determines the value of additional information on connections. This means that the player should be willing to pay more to learn the details of the network in contexts with greater dispersion in social connections. We have also illustrated the economic content of these results via a detailed discussion of two economic applications: optimal advertising in the presence of word of mouth communication and optimal seeding strategies in the presence of adoption externalities.

This paper has focussed on the case in which there is only one player external to the network. In many situations, it is more natural to consider multiple players. For example, these players could be firms competing for consumers or developmental agencies conducting complementary social programmes. The presence of multiple players introduces a new key element into the problem, namely the nature of payoff externalities between external players. In the appendix we have discussed the role of competition in the example of word of mouth communication. For general preliminary results with two or more players, see our companion paper Galeotti and Goyal (2007). Moreover, the model we study is essentially static. Generally, different content of social interaction as well as a different distribution of connections will lead to different dynamics of diffusion and therefore to different dynamically optimal diffusion strategies. In on-going work we are exploring the dynamics of diffusion.

6 Appendix

Proof Proposition 4.2**.** Start with the optimal untargeted strategy under P, say x; we know that $x \in (0,1)$. Construct a decreasing cut-off strategy $\mathbf{x}'$, such that $\theta(\mathbf{x}') = \sum_{k \in O} P(k) x'_k = x$. Note that $\Pi(\mathbf{x}'|P) - \Pi(x|P) > 0$ if and only if

$$\sum_{k \in O} P(k)(1 - x'_k)[1-x]^k < \sum_{k \in O} P(k)(1-x)[1-x]^k,$$

or

$$\sum_{k \in O} P(k) \frac{1 - x'_k}{1-x} [1-x]^k < \sum_{k \in O} P(k)[1-x]^k,$$

Let us define $g'(k) = \frac{P(k)(1-x'_k)}{1-x}$, for each $k \in O$. By construction of $\mathbf{x}'$ it follows that $g' : O \to [0,1]$ is a probability distribution. Denote by $G'(s) = \sum_{k=1}^{s} g'(k)$ its cumulative distribution. It is easy to check that $G'(s) \leq \mathcal{P}(s)$ for all $s \in O$ and that for some $s \in O$ the inequality is strict. We use the hypothesis that P assigns positive probability to two or more degrees to ensure the strict inequality for some s. That is, g' FOSD P; since $(1-x)^k$ is strictly decreasing in k, the claim follows. Since the monopolist obtains strictly higher profits by using $\mathbf{x}'$, she will also obtain strictly higher profits under an optimal targeted strategy. This completes the proof. ■

Lemma 6.1 *Consider the word of mouth communication example with targeted strategies. For every finite $\alpha > 0$ the optimal strategy $\mathbf{x}^*_P$ is such that $\theta(\mathbf{x}^*_P) \in (0,1)$. Furthermore, if α increases then $\theta(\mathbf{x}^*_P)$ (weakly) decreases. Finally, if α tends to zero then $\theta(\mathbf{x}^*_P)$ tends to one, if α tends to infinity then $\theta(\mathbf{x}^*_P)$ tends to zero.*

Proof. Fix $\alpha > 0$. We first show that $\theta(\mathbf{x}^*_P) \in (0,1)$. Suppose that $\theta(\mathbf{x}^*_P) = 0$, then $x^*_s = 0$ for all s. However, $\frac{d\Pi(\mathbf{x}^*_P|P)}{dx_s} = P(s)\left(1 + \hat{k}\right) > 0$, for all $s \in O$, and therefore aggregate profits can be strictly increased relative to $\mathbf{x}^*_P$. This contradicts the hypothesis that $\mathbf{x}^*_P$ is optimal. Similarly, suppose now that $\theta(\mathbf{x}^*_P) = 1$, then $x^*_s = 1$ for all s. Notice that $\frac{d\Pi(\mathbf{x}^*_P|P)}{dx_s} = -P(s)\alpha < 0$, for all $s \in O$, which contradicts the hypothesis that $\mathbf{x}^*_P$ is optimal.

Next, consider $\alpha' > \alpha > 0$ and let $\mathbf{x}'_P$ and $\mathbf{x}_P$ be the optimal targeted strategies under α' and α, respectively. Let $\tilde{k}$ the cut-off under strategy $\mathbf{x}_P$. We show that $\theta(\mathbf{x}'_P) \leq \theta(\mathbf{x}_P)$. Suppose that $\theta(\mathbf{x}'_P) > \theta(\mathbf{x}_P)$; this together with the fact that $\mathbf{x}'_P$ and $\mathbf{x}_P$ are decreasing cut-off strategies imply that $x'_{\tilde{k}} > x_{\tilde{k}}$. Notice that $\frac{d\Pi(\mathbf{x}|P)}{dx_s}$ is decreasing in α and $\theta(\mathbf{x})$ for all $s \in O$ and for all $\theta(\mathbf{x}) \in (0,1)$. This contradicts that under α' the optimal targeted strategy is such that $x'_{\tilde{k}} > x_{\tilde{k}}$. It is easy to verify the remaining part of the lemma. ■

Appendix A: Richer Patters of Word of mouth communication.

An important element in *viral marketing* is the idea that information can be passed on from person to person via social connections. However, the model in Section 3 assumes that word of mouth communication decays just after one step. Our model can be extended in a natural way to cover richer patterns of information diffusion, as we now illustrate.

For convenience, suppose that every buyer has the same degree, say k, and suppose that information flows r steps; $r \geq 1$ is an integer and it indicates the radius of information diffusion. For example, $r = 2$ means that a buyer receives information from her friends and, indirectly, from the friends of her friends. Given information radius r and strategy x, it is easy to check that the probability that a consumer with k friends becomes aware of the product, is $\phi_k(x|r) = 1 - (1-x)^{\sum_{s=0}^{r} k^s}$. Hence, the expected profits to firm $\mathcal{M}$ are

$$\Pi(x|k, r) = 1 - (1-x)^{\sum_{s=0}^{r} k^s} - \frac{\alpha}{2}x^2. \tag{8}$$

It is immediate to check that: $(I.)$ $\phi_k(x|r)$ is increasing and concave in x, in k and in r, (II.) the function $\phi_k(\cdot)$ exhibits IMRD for low values of x, otherwise it exhibits DMRD and (III.) $\frac{\partial^2 \phi_k(x,r)}{\partial x \partial r}$ is positive for low values of x and negative otherwise.

Using properties I-III and the results developed in section 3.1, we immediately obtain the following results. First, the effects of an increase in the level of word of mouth communication on optimal advertising strategy and profits presented in Proposition 3.1 and 3.2 extend to richer patterns of information diffusion, i.e. $r \geq 1$. Second, an increase in the radius of information flow is analogous to an increase in the level of word of mouth communication. Thus, Proposition 3.1 and Proposition 3.2 also address the issue of expanding the radius of information.

Appendix B. Word of mouth communication about product quality

Here we present a model in which there is asymmetric information between firms and consumers about quality and consumers share their experience about product quality via word of mouth communication.[27] As before, suppose that there is one firm which is selling to a set of consumers. The set of buyers is $\mathcal{N} = [0, 1]$; each buyer has inelastic demand and her reservation value for the object is $v = 1$ if the quality is HIGH, but the reservation utility $v = 0$ if the quality is LOW. At the start all consumers are pessimistic about the quality so that no one is willing to pay a positive price. Hence, the only way the firm can generate sales is to give away free samples

[27] In a recent paper, Navarro (2006) also studies the role of word of mouth communication in a model of asymmetric information about product quality. Her interest is in the affects of word of mouth communication in mitigating the inefficiencies generated by asymmetric information. She focuses on the use of prices by firms and free riding in experimentation by firms. By contrast, our interest is in the impact of word of mouth communication on optimal advertising and the profits of the firm.

of the product, and hope that the consumers will pass on good information about the product. Consider a two period model, where in period 1 the firm chooses the number of samples to give away for free $x \in [0, 1]$, and in period 2 it chooses the price to charge, $p \geq 0$. Moreover, to simplify matters, suppose that there are no direct costs of producing the good, which implies that the only cost is an indirect one, via the loss of potential sales. Given that consumers only buy if they are informed that the product is HIGH, it is optimal for the firm to set price $p = 1$ in the second period. We assume this in what follows.

The payoffs to a firm from a consumer with degree k, are then given by

$$\phi_k(x) = (1 - x)[1 - (1 - x)^k].$$

Notice that $(1 - x)$ refers to the probability that a consumers has not been given the product for free in period 1. It is easy to check that $\phi_k(x)$ is concave in x and it is increasing and concave in k. These properties correspond to the properties obtained in the optimal advertising with word of mouth communication about product existence. For a given distribution, P, we can write the expected profits under strategy x as:

$$\Pi(x|P) = (1 - x) \sum_{k \in O} P(k)[1 - (1 - x)^k]$$

The monopolist chooses x to maximize profits. It follows that there is a unique solution to this optimization problem and that this solution is interior. The effects of changes in P on the optimal strategy depend on how marginal returns change with respect to k. It is easy to check that these relations are as in the case when word of mouth communication is with regard to existence of product. ■

Appendix C. Distinction between individual preference heterogeneity and heterogeneity in social influence.

This section shows how heterogenous social influence is different from a standard form of individual heterogeneity with regard to willingness to pay. For illustration suppose there is a unit measure of consumers and that they demand a single unit of a good; they differ only in their willingness to pay for it. This heterogeneity is reflected in a distribution $P(.)$, where $P(k)$ is the fraction of consumers who are willing to pay k for the good. Suppose that the firm knows this distribution and sets a price $p \in \{1, 2....\bar{k}\}$. It then follows that the expected profits for the firm with price p and advertising $x \in [0, 1]$ is given by:

$$\Pi(x|P) = p \sum_{k=p}^{\bar{k}} P(k)x - C(\alpha, x).$$

At an interior optimal x^*, the following must be true:

$$p\sum_{k=p}^{\bar{k}} P(k) - \frac{\partial C(\alpha, x^*)}{\partial x} = 0.$$

Now consider a distribution P' which first order dominates P. It follows that

$$\sum_{k=p}^{\bar{k}} P'(k) \geq \sum_{k=p}^{\bar{k}} P(k) \tag{9}$$

and if (C.1) holds then the optimal strategy is (weakly) increasing with first order shifts in willingness to pay. This is in contrast to the result in the word of mouth communication example; recall from Proposition 3.1 that the effects of increasing social interaction depend on the costs of advertising. For low costs of advertising a first order shift in social interaction actually implies a lowering in the level of advertising. This difference in results arises out of the substitutes relation between social connections and advertising at low costs of advertising.

Appendix D. Firms' competition in the presence of word of mouth communication

We now analyse how competition between firms affects advertising strategies and firms profits in the presence of word of mouth communication. We consider the case of a duopoly. In this context a strategy profile is $s = (x_i, x_j)$, $x_i, x_j \in [0, 1]$. We focus on symmetric Nash equilibria.[28] For a given strategy s, the expected profits to firm i from a consumer with degree k are:

$$\begin{aligned}\phi_k(x_i, x_j) &= \left[1 - (1 - x_i)^{k+1}\right]\left[(1 - x_j)^{k+1} + \frac{1}{2}(1 - (1 - x_j)^{k+1})\right] \\ &= \frac{1}{2}\left[1 - (1 - x_i)^{k+1}\right]\left[1 + (1 - x_j)^{k+1}\right]. \end{aligned} \tag{10}$$

A buyer buys the object from firm i whenever she is only aware of firm i or (by assumption, with 1/2 probability) when she is aware of both firms. The expected profits to firm i can be then written as follows:

$$\Pi_i(x_i, x_j|P) = \sum_{k\in O} \frac{P(k)}{2}[1 - (1 - x_i)^{k+1}][1 + (1 - x_j)^{k+1}] - \frac{\alpha}{2}x_i^2. \tag{11}$$

It is easy to check that firms' advertising intensities are strict strategic substitutes, i.e. $\frac{\partial^2\Pi_i(x_i,x_j|P)}{\partial x_i,\partial x_j} < 0$, and that the game is of negative externalities, i.e., $\frac{\partial\Pi_i(x_i,x_j|P)}{\partial x_j} < 0$.

[28] For simplicity we are assuming that firms charge a price $p = 1$, so that firms do not compete in price. Price competition can be accommodated in our setting in a standard way.

Furthermore, there exists a unique interior symmetric Nash equilibrium, x_P^*, and it solves:

$$\frac{\partial \Pi_i(x_P^*, x_P^*|P)}{\partial x_i} = \sum_{k \in O} \frac{P(k)}{2}(k+1)(1-x_P^*)^k[1+(1-x_P^*)^{k+1}] - \alpha x^* = 0. \qquad (12)$$

In what follows we study the effect on equilibrium outcomes of an increase in the level of word of mouth communication. The following proposition summarizes the result.

Proposition 6.1 *Consider two degree distributions P and P' and suppose that P' FOSD P. There exists $\underline{\alpha}$ and $\bar{\alpha}$ such that if $\alpha > \bar{\alpha}$ then $x_{P'}^* > x_P^*$, while if $\alpha < \underline{\alpha}$ then $x_{P'}^* < x_P^*$. Furthermore, If $\alpha < \underline{\alpha}$ then firms' equilibrium profits are increasing with the level of word of mouth communication.*

Proof. We first prove part I. The derivative of the marginal returns with respect to degree in a symmetric equilibrium x_P^* is:

$$\frac{\partial^2 \phi_k(x_P^*, x_P^*)}{\partial x_i \partial k} = \frac{(1-x_P^*)^k}{2}\left[1+(1-x_P^*)^{k+1} + (k+1)\left[1+2(1-x_P^*)^{k+1}\right]\ln(1-x_P^*)\right]$$

Note that $\frac{\partial^2 \phi_k(x_P^*, x_P^*)}{\partial x_i \partial k} > 0$ for sufficiently low x_P^*, while $\frac{\partial^2 \phi_k(x_P^*, x_P^*)}{\partial x_i \partial k} < 0$ for sufficiently high x_P^*. Next, by inspection of the equilibrium condition 12, it follows that x_P^* is decreasing in α, x_P^* goes to zero when $\alpha \to \infty$, while x_P^* goes to 1 when $\alpha \to 0$. The proof now follows by using the same arugment adopted in Proposition 3.1.

Finally, to see that if $\alpha < \underline{\alpha}$ equilibrium profits are increasing in the level of word of mouth communication note that

$$\Pi_i(x_P^*, x_P^*|P) < \Pi_i(x_P^*, x_P^*|P') < \Pi_i(x_P^*, x_{P'}^*|P') < \Pi_i(x_{P'}^*, x_{P'}^*|P'),$$

where the first inequality follows because $\phi_k(\cdot, \cdot)$ is increasing in k and P' FOSD P, the second inequality follows because $x_{P'}^* < x_P^*$ and the game is of negative externalities, while the third inequality follows by optimality of firm i. ■

We first note that the effects of an increase in the level of word of mouth communication on advertising are similar to the ones we find in the monopoly model (see Proposition 3.1). Second, when the costs of advertising are sufficiently low, firms' equilibrium profits are increasing with the level of word of mouth communication. This is consistent with our findings in the monopoly case (see Proposition 3.2). However, this does not have to hold for all advertising costs. The reason is that when α is high, greater word of mouth communication increases the intensities under which firms advertise. In turn, this effect changes the level of competition between firms and it may result in lower profits.

We conclude with a result on the effects on advertising and profits of MPS shifts in degree distributions.

Proposition 6.2 *Suppose P' is a MPS of P. There exists $\underline{\underline{\alpha}}$ and $\bar{\bar{\alpha}}$ such that if $\alpha > \bar{\bar{\alpha}}$ then $x^*_{P'} < x^*_P$, while if $\alpha < \underline{\underline{\alpha}}$ then $x^*_{P'} > x^*_P$. Furthermore, if $\alpha < \underline{\underline{\alpha}}$ then firms' equilibrium profits are falling with greater dispersion in word of mouth communication.*

Proof. Note that:

$$\frac{\partial^3 \phi_k(x^*_P, x^*_P)}{\partial x_i \partial k^2} = (1-x^*_P)^k \ln(1-x^*_P)[2+4(1-x^*_P)^{k+1}+(k+1)(1+4(1-x^*_P)^{k+1})\ln(1-x^*_P)].$$

It is readily seen that for sufficiently low x^*_P the marginal returns are concave in k, while they are convex for sufficiently large x^*_P. The proof follows by replicating the arguments used in Proposition 3.3.

To show that if $\alpha < \underline{\underline{\alpha}}$ then equilibrium profits are falling with MPS shifts, note that,

$$\Pi(x^*_{P'}, x^*_{P'}|P') \leq \Pi(x^*_{P'}, x^*_{P'}|P) \leq \Pi(x^*_{P'}, x^*_P|P') \leq \Pi(x^*_P, x^*_P|P),$$

where the first inequality follows because P' is a MPS of P and the returns from a buyer with degree k are concave in her degree, the second inequality follows because $x^*_P \leq x^*_{P'}$ and the game is of negative externalities, while the last inequality follows from optimality. ■

Appendix E. Seeding the network under different processes of adoption.

This section shows how different assumptions on the process of adoption can be accommodated in the seeding the network application. Recall that in the general seeding the network model the probability that a potential buyer eventually adopts the product is

$$\phi_k(x) = (1-x)\sum_{s=1}^{k} x^s (1-x)^{k-s}\psi(k,s),$$

where the function $\psi(k, s)$ indicates the probability that a k-degree consumer adopts the product in the event that s of her neighbors have already the product. Recall that $\psi(k, 0) = 0$ for all $k \in O$.

Different assumptions on the behavior of the probability of adoption with respect to k and s should be interpreted as indicating different processes of adoption. In what follows we consider two distinct cases. The first one is suitable for contexts in which the probability that a consumer adopts the product is only influenced by the absolute number of her neighbors who have already the product. In contrast, the second case is suitable in situations in which the probability of adoption depends on the proportion of consumers' neighbors who have already the product.

Number of neighbors' adopters. Suppose that $\psi(k, s) = s/\bar{k}$, for all $s, k \in O$. That is, the probability that a consumer adopts the product is increasing in the number of her neighbors who have already the product, but it is independent of the consumer's neighbor size. Under this assumption, it is easy to very that the expected returns from a k-degree buyer are:

$$\phi_k(x) = (1-x)x\frac{k}{\bar{k}}.$$

It is easy to see that the optimal seeding strategy is to seed, in period 1, 1/2 of the consumers, i.e. $x = 1/2$. Moreover, ϕ_k is increasing and linear in k. Therefore, Proposition 3.6 implies that a FOSD shifts in the degree distribution increases profits. Finally, profits are not affected by mean preserving shifts.

We now consider the targeted case. For a given $\mathbf{x}$, the expected returns from a k-degree consumer are

$$\phi_k(x_k, \theta(\mathbf{x}) = \frac{k}{\bar{k}}(1-x_k)\theta(\mathbf{x}).$$

This function is concave in the first argument and it exhibits DMRD (see definition 4.1.). Proposition 4.1 immediately implies that the optimal targeted strategy is monotonically decreasing in k.

Proportion of neighbors' adopters. Suppose now that $\psi(k,s) = \frac{s}{k^\beta}$, where $\beta > 1$. That is, the probability that a consumer adopts the product depends on the proportion of individuals in her neighbors who have already adopted the product. As an illustration, let $\beta = 2$ and consider a 2 degree consumer and a 3 degree consumer. In the event that both consumers observe that two of their neighbors have the product, then the 2 degree consumer will adopt with probability 1/2, while the 3 degree consumer will adopt with probability 2/9. It is easy to check that the returns from a k degree consumer are:

$$\phi_k(x) = (1-x)xk^{1-\beta}.$$

First, the optimal seeding strategy is $x = 1/2$, and therefore optimal strategies are not affected by changes in the degree distribution. Second, it is easy to verify that the expected returns from a k-degree consumers are decreasing and convex in k. Thus, Proposition 3.6 implies that FOSD shifts in the degree distribution decreases profits, while Proposition 3.8 implies that MPS shifts in the degree distribution increase profits.

Finally, suppose the monopolist uses targeted strategies. In this case, for a given strategy $\mathbf{x}$ the returns from a k degree consumer are,

$$\phi_k(x_k, \theta(\mathbf{x}) = (1-x_k)k^{1-\beta}\theta(\mathbf{x}).$$

This function is concave in the first argument and it exhibits IMRD (see definition 4.1.). Thus, Proposition 4.1 implies that the optimal targeted strategy is monotonically increasing in k.

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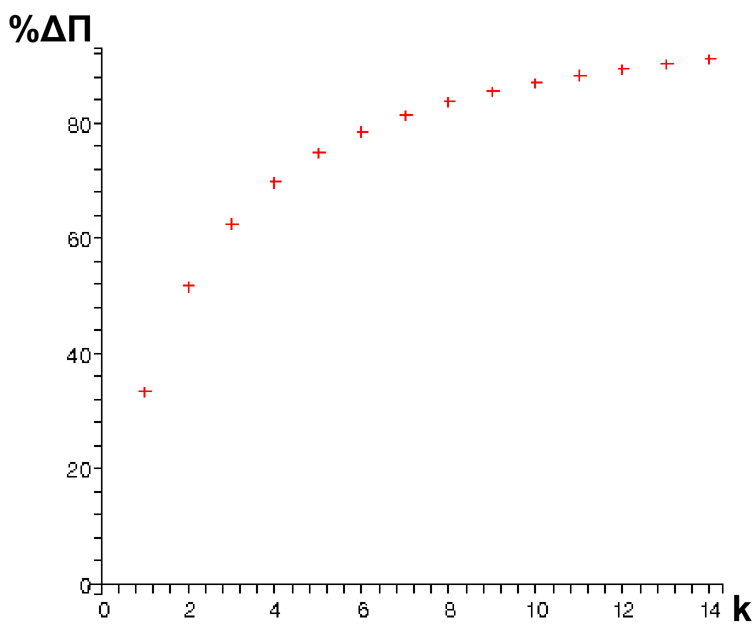
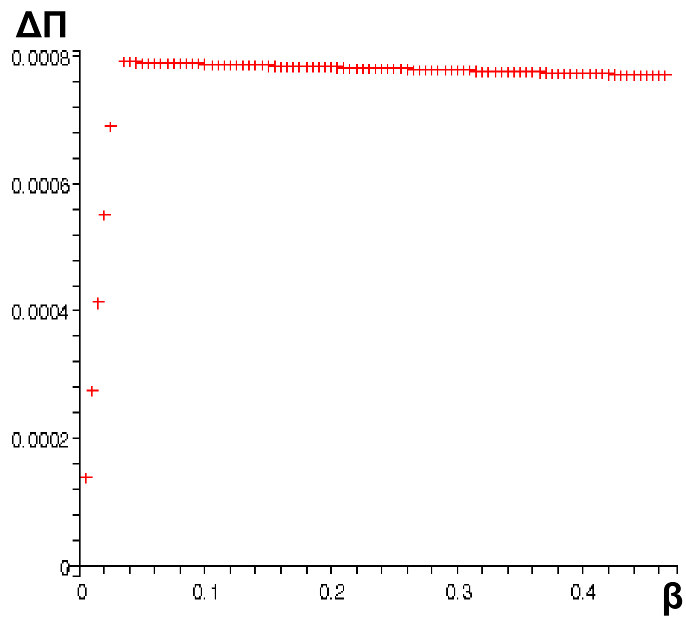
Figure 1. Value of incorporating WOM:
Percentage difference in profits.



P(k)=1, k = 1,2... and α=1.

Figure 2. Value of Targeting and MPS shifts: Difference in profits.



P(1)=P(3)= β, P(2)=1-2 β and α=100.