# Multi-Stage Optimization Filter for Trend Based Short-Term Forecasting

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**Abstract**

A new method is proposed to estimate the long term seasonal component by a Multi-stage Optimization filter with a leading Phase Shift (MOPS). It can be utilized to provide better predictions in case of the seasonal component auto-regressive (SCAR) model, where data is filtered/decomposed into trend and remainder components, then forecasts for constituent components generated separately and later combined. This reinforces the importance of trend estimation filtering/decomposition methods, which are scarce and only few methods, primarily wavelet decomposition, have improved upon the forecasts generated by statistical linear models. We contribute to the literature by introducing a new trend estimation method and the forecast results are compared to the most popular trend estimation methods, such as frequency filters, wavelet decomposition, empirical mode decomposition and HP filter, through their performance in generating short term forecasts for day-ahead electricity prices. Our method for trend estimation performs better in terms of providing short-term forecasts as compared to some well-known methods and the best forecast, according to the Diebold and Mariano (1995) test, is obtained by using our MOPS filter with annual trend period length.

*Keywords:* Electricity Price Forecasting, Filtering, SCAR, Phase Shift, Trend, Multi-stage optimization filter

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## 1. Introduction

The trend or long-term seasonal component provides the future direction of the data series (Hyndman et al., 2008). This characteristic of the trend is often utilized in long-term predictions, generally for policy making and strategic planning. Conversely, in short-term forecasting the trend is often ignored; indeed, modelling the trend has been believed to make the short-term forecasts overly complex (Weron, 2014). However, a series of recent papers on electricity price forecasting (EPF), such as Uniejewski et al. (2019), Marcjasz et al. (2019), Nowotarski and Weron (2016), Weron and Zator (2015), Lisi and Nan (2014) highlight the importance of the trend in short-term forecasting. The forecasts generated by filtering/decomposing the trend and forecasting series components separately in a seasonal component auto-regressive (SCAR) model, referred to as trend-based forecasting in this paper, outperformed forecasts in which the trend was not predicted separately, in the cases of statistical (Nowotarski and Weron, 2016) and neural network models (Marcjasz et al., 2019).

The success of these trend-based forecasting procedures varies with selected trend period lengths, typically ranging from monthly to annual trend period lengths, and the specific filtering/decomposition procedures used for trend estimation. In the case of statistical linear forecasting models, only wavelet decomposition based trends have currently been shown to improve the forecasts for certain trend period lengths (Nowotarski and Weron, 2016). However, the difficulty with wavelet decomposition lies both with the complexity of choosing suitable wavelets and the lengthy computational run-time (Weron and Zator, 2015). The popular Hodrick-Prescott (HP) filter performs well only in neural network non-linear model based forecasts (Marcjasz et al, 2019) but not in linear statistical model based contexts (Nowotarski and Weron, 2016). Also, the HP filter is often criticised for end-point bias and causing the appearance of spurious dynamic relations (Hamilton, 2017). This all poses two crucial challenges for the trend based forecasting scheme; firstly, finding the most suitable trend estimation method that overcomes the potential disadvantages of wavelets and HP filters, which are commonly the only methods used for trend-based forecasting in the EPF literature. Secondly, the choice of period length to be used for trend or long-term seasonal component estimation also has a large impact on forecast performance (Nowotarski and Weron, 2016).

In order to assess the optimal trend estimation method for a trend-based forecasting scheme, we explore a number of the existing popular trend estimation methods in literature; additionally, we introduce a new multi-stage optimization filter with inherent phase shift (MOPS). The phase shifts normally occur after the filtration/decomposition, in shape of a lagging or leading time shift between the original data and the filtered output, see figure 1. These phase shifts are removed usually through a convolution function to bring the filtered output to the same level as original data. Contrary to this practice of fixing the phase shift, we aim to induce a leading phase shift in our filter; the intuition behind our proposed model is that a trend estimation method that closely follows the mean and contains a leading phase shift provides a natural opportunity for trend forecasting.

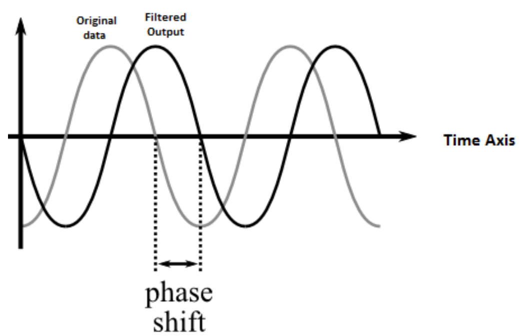


Figure 1: Phase shift occurring in the filtered output plotted against the original data with amplitude on vertical axis and time on horizontal axis

We focus specifically on forecasting the UK electricity day-ahead price data. This will be comparable to much of the earlier literature on trend-based forecasting which used similar datasets from other electricity day-ahead markets. Additionally, the stylized features of the electricity prices are suitable for trend-based forecasting procedures (see Nowotarski and Weron, 2016; Marcjasz et al., 2019). In particular, the stylized features of electricity prices are high volatility, mean reversion, multiple seasonality and extreme spikes due to economically in-viable large scale storage, and also, demand or supply shifts (Higgs and Worthington, 2008). These features make EPF a complex task and prediction using one specific model that encompasses all such features does not provide precise results; instead a combination of different methods are used for best forecasts (Maciejowska et al., 2015). One way to tackle this complexity is to decompose prices, predict the resulting constituent series separately and later combine the predictions to obtain an overall forecast for the price series. Such an approach is practised for trend-based forecasting and followed in this paper.

The structure for rest of the paper is that existing trend estimation methods used in our analysis are discussed in section 2. The literature covering statistical linear forecasting models and filtering/decomposition, specifically for electricity prices, is reviewed in section 3. The details for our proposed multi-stage optimization filter with phase shift (MOPS) are provided in section 4. The data and methodology for trend estimation and the forecasting exercise are given in section 5. Finally, sections 6 and 7 contain the discussion of results and conclusion.

## 2. Existing trend estimation methods

### *2.1. Frequency filters*

Filters are commonly used in signal processing to remove undesirable frequencies before or after a specific cut-off point or within or outside a frequency band range of the original signal to reduce noise. Frequency filters are also popular in economic and financial time series applications to remove trend and seasonal components, including Baxter-King and Christiano-Fitzgerald filters that are based on the analogous filtering principles. Of course, low *($f_L$)* and high frequencies *($f_H$)* are determined by the wavelength of the sinusoids; the low frequency sinusoids have a high wavelength and, in particular, the trend or long-term seasonal component falls into this category. As shown in figure 2, there are four main types of filters based on different cut-off points - low pass, high pass, band pass and band stop filters - which are discussed in more detail below.

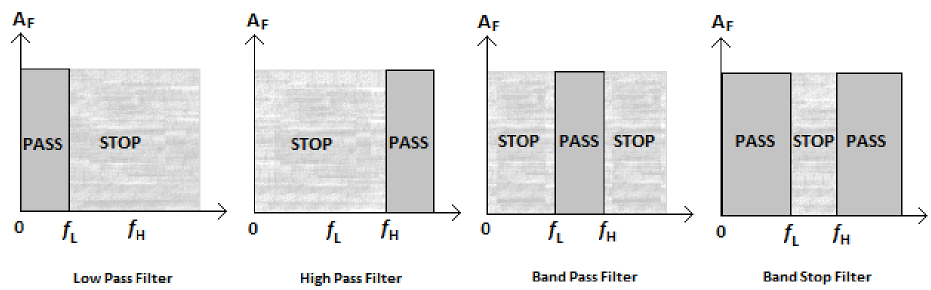


Figure 2: Categories of frequency filters

### *2.1.1. Low Pass filters*

In Low pass filters only low frequencies are allowed to pass, consequently removing all high frequencies in the data after the specified cut-off point. Low pass filters are useful for extracting short-term fluctuations and are commonly used to smooth time series. The Butterworth filter is an example of a low pass filter, in which the frequency response is particularly flat.

### *2.1.2. High Pass filters*

In high pass filters only high frequencies are allowed and they are often used to extract the trend of the time series by capturing the long term seasonal component. Of course, the frequencies retained by using a high pass filter can also be defined as those frequencies removed after applying a low pass filter.

### *2.1.3. Band Pass filters*

They let intermediate frequencies to pass, while removing the low and high frequencies and are often used to isolate business cycles in the given data series. The Christiano-Fitzgerald filter (see Christiano and Fitzgerald, 2003) is based on an ideal band pass filter that extracts the cyclical component $c$ at time $t$ for the time series $y$ through estimating coefficients $B$ at different intervals from beginning of time series $t$ till the end point in time $T$. The coefficient $B$ is estimated as $B = b - a/\pi$, where $a = 2\pi/p_u$ and $b = 2\pi/p_l$. Here, the upper and lower cut-off cycle lengths (user defined) are represented by $p_u$ and $p_l$, the filter output $c$ contains the cycle lengths shorter than $p_u$ and longer than $p_l$.

$$c_t = B_0 y_t + B_1 y_{t+1} + ... + B_{T-1-t} y_{T-1} + B_{T-t} y_t + B_1 y_{t-1} + ... B_{t-2} y_2 + B_{T-t} y_1 \quad (1)$$

### 2.2. Other decomposition techniques

#### 2.2.1. Wavelet decomposition

A wavelet decomposes a data series into a set of waves fluctuating around zero. A similar technique is the Fourier transformation, the standard Fourier transform provides decomposition based on frequency, whereas the wavelets take care of both the time and frequency, which is more useful for time series analysis. The wavelets have the property of functional bases and they can operate to extract information at different frequency domains along with recording the instance of wave oscillations.There are different families of wavelets that are normally chosen based on their merits for specific wavelet analysis. The wavelet used for estimating the daily trend in this paper belongs to Daubechies family order 24 as previously used by Nowotarski and Weron (2016).

#### 2.2.2. Empirical Mode decomposition

Based on the "Hilbert-Huang Transform", the empirical mode decomposition splits a given data series into different components called as Intrinsic Mode Functions. These Intrinsic mode functions produce decomposed series at different scales from low to high frequencies. It is being used extensively for decomposing economic data and identifying business cycles due to its flexibility and range of component series produced at different scales.

#### 2.2.3. HP Filter

This is one the most extensively utilized method for financial data filtering despite all the critism it has received. It was introduced by Hodrick and Prescott (1997)to analyse the business cycles in US labour market. The HP filter decomposes a time series into a growth $\tau_t$ and cyclical $c_t$ component.

$$y_t = \tau_t + c_t \tag{2}$$

The overall equation for smoothing time series is based on a constrained minimization problem for estimating the growth component $\tau_t$.

$$\min_{\tau_t} \sum_{t}^{T} c_t^2 + \lambda \sum_{t=1}^{T} [(\tau_t - \tau_{t-1}) - (\tau_{t-1} - \tau_{t-2})]^2 \tag{3}$$

Where, $c_t$ is the business cycle component, it shows the deviation from the trend. The parameter $\lambda$ is a constant which constraints the growth component and its larger values makes $\tau$ smoother, if $\lambda = 0$, then $\tau = y$.

The value for lambda is usually chosen based on the frequency of time series under observation, the most commonly used parameter to estimate the value of lambda is provided by Ravn  Uhlig (2002).

## 3. Literature review

The literature review section is discussed in two parts, providing the key focus points of this paper. The first part focuses on the linear statistical models for electricity price forecasting and the second part presents the filtering/decomposition schemes and methods. Before we explore these two streams of literature on electricity price forecasting (EPF), it is worthwhile to mention some key review or survey papers summarizing the different models used in the literature for forecasting electricity prices. Among these review papers, Weron (2014) stands out for its comprehensive and precise enquiry on the topic; it covers most of the methods and procedures suggested in the literature to forecast the electricity prices. Some other reviews of electricity price forecasting literature are more tailored towards providing in-depth knowledge of specific categories of the models, broadly statistical, economic or computer intelligence models (Amjady and Hemmati (2006);Ventosa et al. (2005); Chan et al. (2012); Garcia-Martos and Conejo (2013)) and the procedures applied such as variable segmentation, forecast combination and pre-processing (Aggarwal et al. (2009a); Bunn (2000)).

These review papers generally cover all procedures used in EPF but lack the in depth inquiry of filtering and decomposition of electricity prices which is a common practice in electricity price forecasting literature due to the price spikes and extreme volatility. However, one such review recently conducted by Shao et al. (2017) focuses particularly on decomposition methodologies but solely for electricity demand forecasting rather than the electricity price forecasting. An important point to take under consideration while looking at the surveys focused more on electricity demand forecasting is the difference in the characteristics of electricity price and demand as the former is much more difficult to forecast (Dominik Liebl, 2013). This is why we specifically focus only on the literature of electricity price forecasting instead of demand forecasting. Later in this section, we provide a general review of commonly used statistical methods followed by the price filtering and decomposition for electricity price forecasting.

### *3.1. Existing Models to Forecast Electricity Prices*

The electricity price forecasting models used in the literature are usually statistical or computer intelligence models that can be divided primarily into time series models and neural network (artificial intelligence) models that are equally famous in EPF literature, or a combination of both models is used for forecasts(Weron, 2014). However, a single most parsimonious model to predict spot electricity prices does not exist, instead a combination of different models are used for best forecasts(Maciejowska et al, 2015). The computer intelligence and neural networks are still equally popular in EPF, but we focus particularly on statistical methods as the aim of this paper is on decomposition of prices and forecasting through statistical models, hence the relevant literature is reviewed and researches using computational intelligence models are not reviewed.

#### *3.1.1. Parameter Rich models*

Generally, using a parameter rich model with substantial number of explanatory variables and their past values is often a trend observed in EPF researches and up to 100 parameters have been used in a single model for electricity price forecasting (Ziel, 2016). Accordingly, selecting the right variable is a key issue in predicting the future prices and load. A common way adopted for choosing the right variables by earlier researches is to eliminate statistically insignificant variables for (AR) models (Uniejewski et al, 2016).

Another way to deal with large number of variables recently applied by Alonso et al (2016) is to choose few components or factors to explain the major changes without using all the parameters. They adopted the dynamic factor model based on extracting principle components that provides dimensionality reduction and avoids multicollinearity and it is a substitute to parameter rich models that is based on regression for the entire explanatory variables. As the dynamic factor models extract the information before the forecasts and the forecast combination works after the forecasts, they combined both techniques to obtain a single prediction. Forecasts generated for ARIMA models and factors are later converted to original variable values to obtain individual predictions for each hour and forecast combination is applied by averaging the forecasts of alternative models for improved forecasts. Log prices were used to avoid heteroscedasticity but it did not make difference to forecasting performance. A rolling window of 1.5 years was used with 36 alternative seasonal ARIMA for each factor in a window. In terms of further research, they suggest using different weights for combination of forecasts rather than fixed weights, seaDFA and GARCH- seaDFA, other variables like demand, fuel, weather etc, and use of bootstrap procedures to obtain confidence intervals of the predictions.

*3.1.2. Long memory models*

In terms of model building, different specifications have been used for electricity price forecasting autoregressive structures perform well due to the lag dependence and mean reverting properties of electricity prices. Autoregressive (AR, ARX) models are most widely used for electricity price forecasting for time series models. These models express the future price in terms of combination of their past prices and they are useful in exploring the mean reverting properties of the electricity prices, the electricity prices for some countries do exhibit non-stationarity and deviation from mean, for which GARCH models are more suitable. To cope with this, researcher sometimes use both the models, as Liu Shi (2013) used the combination of ARMA and different GARCH models, in ARMA-GARCH framework to forecast hour ahead electricity prices. In their case of New England market, electricity prices show time-varying volatility and daily, weekly and monthly periodicities.

In a comparison by Weron and Misiorek (2008) of 12 short term electricity price forecasting models including regime switching, stochastic mean reverting jump diffusion model and different autoregressive models, the smoothed nonparametric calibrations of ARX and AR models performed the best. Furthermore, the use of demand as an exogenous variable improved the forecasts in most of the cases.

*3.1.3. Structural/Equilibrium models*

Equilibrium models are often used to study the dynamics of electricity market, based on the balance between demand and supply, and they are referred as structural models in electricity pricing literature (Carmona and Coulon, 2014). These models are used to study the behaviour of electricity wholesale prices on the industrial side through the marginal costs, and supply /demand curves. However, such models have not yet been extensively explored to forecast the electricity prices in wholesale markets.

Electricity prices are inelastic to demand, and lack of storage usually causes the prices to spike massively due to supply and demand shocks. Subsequently, earlier researches used the relation between the price with demand and with supply (or available capacity/margins) for pricing electricity derivatives. Boogert and Dupont (2008) used a supply-demand framework to model day-ahead electricity spot prices for each of the 24 hours and forecast the price spikes. Kanamura and Ōhashi (2007) proposed a model for pricing electricity based on demand and supply, providing a better fit for spikes than jump diffusion and Box cox transformation model.

#### *3.1.4. Quantitative/Mathematical models*

The quantitative models, also referred as reduced form models, often use stochastic differential equations and thus they are more useful in pricing derivatives rather than forecasting(Weron, 2014). However, the quantitative models that encompass correctly the properties of electricity prices, provide good forecasts. As the electricity prices are mean reverting and have huge spikes, mean reverting jump diffusion models are often used to capture these properties. Ornstein-Uhlenbeck processes are often used to explore these mean reverting properties, as by Barlow (2002), Benth et al (2007) and more recently for power demand by Verdejo et al (2016). Moreover, Geometric Brownian Motion is often used to depict the stochastic component of electricity prices that at times does not respond to the extreme spikes and fails in the reversion to mean(Weron, 2014).

### *3.2. Filtering and decomposition*

The filtering processes involve retaining a specific component of the data and discarding the rest of the components, whereas decomposition processes breaks up the data series based on specific features but then retain all the constituent components. Due to the seasonality and spikes in the electricity prices, filtering and decomposition of prices before building a model is popular in EPF literature. Filtering and decomposing the original prices before applying the statistical models can provide at least better short term day-ahead forecasts as compared to forecasts obtained without prior filtering (Conejo et al, 2005; Janczura et al, 2013; Nowotarski and Weron, 2016; Weron, 2014). The figure below illustrates the filtering and decomposition into resulting constituent series that is often performed in EPF literature(Weron, 2014).

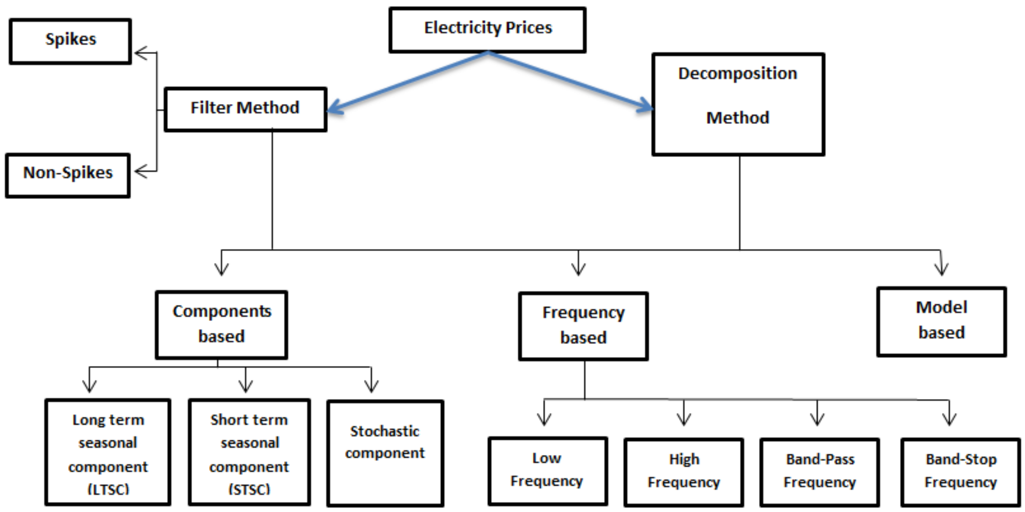


Figure 3: Filtering and decomposition procedures for electricity price forecasting (own elaboration)

*3.2.1. Filtering spikes*

The filtering price spikes process generally involves removing or substituting the spikes from the data. After applying a filter, the resulting series of spikes and non-spikes are modelled separately and the later the forecasts of each series are combined. This is done because the statistical methods for both price-only and fundamental variable models do not provide good forecasts in the presence of spikes (Weron, 2014). And so the extreme spikes in electricity prices are suggested to be pre-filtered before modelling the electricity prices. However, there is no single best method to identify or filter the price spikes but regardless which method is chosen, the forecast results are observed to be better when spikes are removed(Janczura et al, 2013).

One of the earliest research conducted in this regard was Lu et al (2005), in which, electricity prices were filtered mainly using fixed threshold of two standard deviations. The resulting two series were named normal price and spikes, the spikes were further filtered to obtain the abnormal high price, which was forecasted using statistical method based on demand and supply of electricity. Whereas, the normal price was decomposed using wavelet analysis and forecasts were produced using neural network method. Finally, the forecasts of spikes and normal price (non-spikes) were reconstructed to obtain the comprehensive forecasts of the electricity price.

One of the key issues in filtering the spikes is choosing the filtering method and cut-off price threshold, as a price spike in one market may be considered a normal price in other electricity markets. The best way is to study all the procedures applied in the literature and choose the one that performs better for a given market or look at the studies that provide a comparison for these filtering procedures for electricity prices. In this regard, Janczura et al (2013) is the most detailed and comprehensive study on the topic, it provides a detailed comparison of the filtering techniques namely fixed price threshold, variable price threshold, fixed price change threshold, variable price change threshold , Gaussian threshold, wavelet filtering, Markov regime switching model classification and recursive seasonal model. Out of these methods, the Markov regime switching model classification and the variable price thresholds provided the best forecast results.

*3.2.2. Components based Decomposition/Filtering*

The electricity price decomposition procedures can be divided into broadly three inherent components based categories; firstly the deterministic and stochastic components, secondly the time-frequency components and finally model based components, as illustrated in figure 3. Once the decomposition is performed, the constituent series are forecasted individually using a suitable method and then these forecasts are combined in either a multiplicative or additive manner, as explained in the introduction section of the paper.

The traditional approach in business and economics is to decompose a series in three constituent components including long term seasonal component (LTSC), short term seasonal component (STSC) and stochastic component; then model them separately and combine the forecasts(Hyndman et al, 2008). In EPF literature, the decomposition of electricity price ($P_t$) is often focused on one of the three components; either the long term seasonal component ($T_t$), short term seasonal component ($s_t$) or remainder ($X_t$) component, shown in equation (4).

$$P_t = T_t + X_t \tag{4}$$

$$P_t = s_t + X_t$$

Conejo et al (2005) was one of the first papers to decompose the original prices into short term seasonal component ($s_t$) and remainder component

($X_t$) using wavelet transformation before applying the ARIMA model for day-ahead electricity price forecasting. Since then, more focus has been on modelling and forecasting of STSC in electricity price forecasting literature, which was based on decomposing the prices into only stochastic and short term seasonal component, whereas the LTSC merged with the STSC was often modelled and forecasted together. The LTSC alone was considered to add complexity in forecasting electricity prices, however, this approach has changed recently with evidence of better forecasts from the trend based forecasting scheme by Nowotarski and Weron (2016) , Marcjasz et al (2019) and Uniejewski et al (2019).

The long term seasonal component(LTSC), also referred as trend provides the long term direction to the series (Hyndman et al, 2008). The LTSC gradually changes so it is often considered to be same as previous day ($LTSC = LTSC_{(t-1)}$). Thus, persistent forecasts are used for the trend in short-term electricity price forecasting as suggested by Lisi Nan (2014) and later implemented by Nowotarski and Weron (2016) to study the impact of long term seasonal component in day-ahead electricity price forecasting. Nowotarski and Weron (2016) used different decomposition levels of wavelet transformations and the HP filter in a seasonal component autoregressive models (SCAR) and the benchmark Autoregressive models (ARX). The variables chosen were one day, two day and weekly lags of spot price (hourly Nordpool) along with day dummies, hourly load and last day minimum price. The methods used for forecast evaluation/comparison were WMAE (weekly mean absolute error) and the DM (Diebold Mariano) test. The SCAR model with different trend decompositions and AR without decomposition was used and the SCAR-type models with selective wavelet transformations with higher decomposition levels for the long term seasonal component (LTSC) performed the best. Although the approach for identifying these components has changed over time but still similar methods are being used for component estimation, primarily based on wavelet methods for more than a decade.

*3.2.3. Frequency based Decomposition/Filtering*

The frequency based filtering and decomposition methods are borrowed from the signal processing field of engineering and they have gain much popularity in business and economics literature, especially to study the business cycles using low pass, band pass and high pass frequency filters. The most commonly used method in EPF to decompose the prices based on frequency is again wavelet decomposition (specifically Discrete Wavelet Transformation), for example see Xu and Niimura (2004); Mandal et al (2012); Osório et al (2014); Yang et al (2017). The discrete wavelet transformation by Mallat (1989) is unique among the wavelet family as it decomposes prices based on low and high frequency rather than the mother and father wavelet approach in rest of the wavelet methods. Only few papers have explored other frequency based methods due to excessive reliance on the traditional wavelet families.

Lisi and Nan (2014) made an attempt to identify the best trend estimation methods for electricity prices. Out of the 11 methods used to estimate the trend, two frequency based methods were used namely, Kolmogorov–Zurbenko (KZ) filter, which is a low pass frequency filter and Christiano–Fitzgerald (CF) filter, which belongs to the band pass frequency filter category. Although, the focus of their research was not forecasting electricity prices, however, one of the criteria used for ranking the trend estimation method was the predictive performance of electricity spot prices, which is close to our aim of this paper. They suggest filtering based on smoothing splines performs best with regard to the overall set of criteria but they did not specifically rank based solely on the forecasting abilities of the trend estimation methods. We have also incorporated these smoothing splines as the second stage of our multi-stage optimization filter.

### *3.3. Model based decomposition/Filtering*

There are certain methods that decompose or filter the prices based on inherent characteristics of the models, which are not necessarily based on time-frequency or component estimation. One such model, used in EPF is the empirical mode decomposition (EMD), which is based on Hilbert–Huang transform that decomposes data into intrinsic mode functions by restricting the maxima and minima points in the data. Lisi and Nan (2014) used empirical mode decomposition in their comparison study of the trend estimation methods and the results for EMD trend were not as promising as the trends estimated by other methods. He et al (2015) also used empirical mode decomposition for de-noising the electricity prices before producing forecasts, however, it has been more often used in electricity demand forecasting rather than electricity price forecasting due to its compatibility with non-stationary and non-linear demand data, see An et al (2013); Fan et al (2016); Ghelardoni et al (2013); Hong et al (2013).

Another category of filtering/decomposition technique is based on optimization, in order to minimize certain features of the data; some popular methods are Kalman Filter and Hodrick-Prescott (HP) filter. Weron and Zator (2015) compared the performance of HP filter against the wavelet decomposition, which is best performing method and most commonly used in EPF literature, to estimate the trend in electricity prices. They emphasized on the ease of use and computational efficiency of HP filter and in this regard ranked it superior to the wavelet method. Since then, HP filter has received some attention in EPF to estimate the trend of electricity prices, see Lisi Nan (2014); Nowotarski and Weron (2016); Marcjasz et al (2019).

### *3.4. Summary of literature*

The key takeaway from the literature for electricity price forecasting is that linear statistical models with long memory dynamics and properly calibrated for seasonality work better than most of the complex or parameter rich models (Weron, 2014). This is why; we aim for a simple linear autoregressive model with more focus on short term and long term seasonality. In terms of filtering/decomposition schemes, literature suggest that newly developed trend based forecasting is best, in terms of convenience and minimum forecast error. Lastly, there is a scarcity of well performing trend estimation methods, which is a hurdle for trend based forecasting scheme and is addressed in this paper. The best method suggested by the literature for trend estimation is wavelet decomposition (Nowotarski et al, 2013), but due to the complexity of the wavelets, as discussed in the introduction section, new methods need to be explored for trend estimation that could improve the forecasts.

## 4. MOPS Filter

We are particularly interested in designing a filter to obtain the trend of the series that could be later used for forecasting electricity prices. Our aim is to take the input time series vector of electricity prices $y$ and transform using filter function $T$ to generate the trend output $c[n]$ over the time period $n$ as

$$c[n] = T\{y\} \tag{5}$$

In order to record the abrupt changes over different time intervals, $y$ is split based on regular intervals of time, such that these sub-intervals can be concatenated to give the original time series y

$$y = \{y_{[n]} | y_{[1,d]} +\!\!+ y_{[d+1,2d]} +\!\!+ ... +\!\!+ y_{[id+1,nd]}\} \forall i = (1, 2...n-1) \quad (6)$$

Where, $i$ represents the number of sub-intervals and $d$ is the length of each sub-interval in number of days ($d$ stands for one day, $2d$ stands for 2 days and $nd$ shows $n$ number of days), for tri-weekly sub-intervals $|d|$=17, Quarterly $|d|$=73, semi-annually $|d|$=183 and annually $|d|$=365; which are chosen based on the trend cycles in the electricity market and approximate comparison with other filter lengths. These sub-intervals are then transformed using our filter to obtain the trend.

In terms of building the filter, the first and foremost important part is the filter designing method that has to be chosen from the different filter design and estimation methods that can be broadly categorized as finite impulse response (FIR) filters, infinite impulse response (IIR) filters, Nyquist filters, Multi-rate Filter Design, Multistage Filter Design and Special Multi-rate Filters (Losada, 2008). Out of these filters, multistage filters can provide ten times more efficiency than the rest of single stage filters (Zhu et al, 2016). To benefit from these efficiency gains, we base our method on the Multistage Filter Design, in which, the first stage applies optimization and second stage performs interpolation. In the first stage, the optimization is performed by specifying an optimal design filter, which are normally used to minimize certain deviations using the optimization method; specifically between the ideal filter and the proposed filter (Losada, 2008). However, we are more interested in minimizing the deviation from mean of the series through choosing to minimize $f(y)$ by the input choice of $y_{[n]} \in y$, where $y$ is a vector of electricity prices.

$$\min_{y_{[n]} \in y} f(y) \quad (7)$$

As a measure of deviation from mean for the function $f$ , the root mean square error is used as it cumulates the error magnitude and it is scale-dependant that penalises abrupt deviations.

$$\min_{y_{[n]} \in y} f(\sqrt{\sum_{t=1}^{n} (x_{[n]} - y_{[n]})^2}) \quad (8)$$

The minimization function, given in equation (8), minimizes $y_{[n]}$ to obtain minimum root mean square error closer to zero for a given length of data by using the mean $x_{[n]}$, resulting in the trend to be on the same mean level as the original data series. This minimization function is applied over a rolling window of approximately 3 weeks, quarter, and semi-annual and annual time intervals to be used for forecasting and comparison with other filtering methods. In the second stage, the minimized solutions obtained at regular intervals are used for interpolation splines to produce trend curve of the same length as the original data series.

The requirements that are considered essential for decomposition/filtering method to work well and give desired results are mainly filter response, phase shift, stability and consistency of the filter. The phase shift is usually expected once a filter is applied on the data, this makes the filtered output data to either lead or lag the original data points. The phase shifts are normally resolved using convolution methods to remove the delay between filtered data and the original data points. However, this delay could be useful in terms of forecasting the future prices if the filtered data leads the original data series, as this brings a natural forecast for the future. For making use of this natural forecast opportunity, we keep the lead phase shift (that naturally occurs in the filtered output) in estimating trend from our MOPS (Multi-stage optimization with phase shift) filter.

## 5. Data and Methodology

The electricity day-ahead market prices for UK are obtained from Nordpool (data is available on request from the authors). The day-ahead market is a blind auction, in which, participants have to submit their prices for each of the 24 hours of next day before the gate closure at 12:00 CET (Central European Time). The final price is calculated by matching the demand and supply and publically announced at 12:42 CET or later. The data for intraday market consists of 24 hourly prices of each day for the periods between 01/01/2015-31/12/2017 (3 years). This time has been chosen because of the rapid changes in electricity generation mix and integration of renewable energy that makes the demand and supply more uncertain, which leads to more volatility and difficulty in forecasting the electricity prices

The descriptive statistics for the data are provided in table 1, the electricity day-ahead prices and load both have only positive values. The maximum and minimum values for both price and load have large margins; also the volatility in terms of standard deviation is quite high, pointing towards the

(Insert Table 1 here)

presence of extreme spikes. The skewness and kurtosis are both positive and provide evidence of huge jumps particularly for day-ahead electricity prices and non-normality, also confirmed by the Jarque-Bera. As stationarity is a pre-requisite for time series regression analysis, both data series are stationary and contain no unit roots, the results are not included here.

The electricity load data is taken from ELEXON website, which is also in hourly frequency and the same time period as electricity prices (01/01/2015-31/12/2017). Both electricity prices and load are converted into natural logarithmic values as it would be convenient to convert back the forecasted output through exponents for comparison against the real prices, as it is a norm in electricity price forecasting literature. The plots for electricity day-ahead prices in GBP/MWh and electricity load in GWh are presented below, in figure 4.

### *5.1. Forecast Scheme and trend estimation*

Electricity price forecasting is carried out for short term horizon, as the short-term forecasts are important for smooth daily operations to keep the system in balance. The short-term forecasts are generated for 24 hours-ahead for day ahead prices using a rolling window as displayed in the figure 4. The estimation period ends on 31/12/2016 and forecast is generated for 24 hours of next day (01/01/2017). In the next step, the estimation period is extended by 24 hours to 01/01/2017 and then forecasts are generated for next 24 hours of 02/01/2017.

The filtering and forecasting is performed in the following steps:

1. In the first step the log Prices are filtered/decomposed using six most popular trend estimation methods in the literature e.g. HP filter, CF filter, Butterworth filter, Wavelet decomposition and Empirical mode decomposition along with our MOPS filter. Persistent forecasts for the

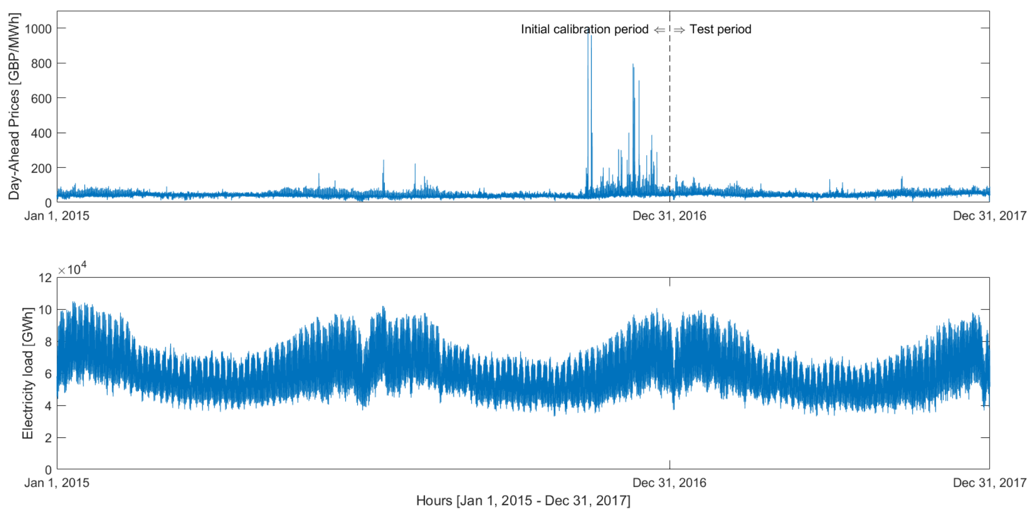


Figure 4: Plots of electricity day-ahead market price and load data with estimation period and the Forecast Window

next 24 hours are used (assuming the price in the trend estimated for yesterday to be the same as today, following the Nowotarski and Weron (2016) ). Out of these methods, only wavelet and HP filter have been previously used for trend based forecasting, and only the forecasts of trend from wavelet decomposition performed well for linear statistical models.

2. The out of sample 24 hours ahead forecasts for daily trend are obtained using persistent forecasts of last 24 hour trend as suggested by Nowotarski and Weron (2016).The remainder part after subtracting the trend is forecasted using autoregressive lags structure (ARX) proposed by Misiorek et al. (2006) and used extensively in electricity price forecasting literature, provided in equation (9).

$$S_{h,t} = \theta_1 S_{h,t-1} + \theta_2 S_{h,t-2} + \theta_3 S_{h,t-7} + \beta mp_t + \gamma ld_{t-1} + \sum_{l=1}^{3} d_l D_l + \epsilon_t \quad (9)$$

Where, $S$ is the spot price, $h$ denotes the hour and $t$ is the day, $mp_t$ is the minimum price from last day, $ld$ is the electricity load from yesterday and three dummy for Monday, Saturday and Sunday.

3. In the third step, the forecasts obtained for the trend and remainder parts are combined.

4. Fourth step, the forecast results from this Trend-ARX scheme are then compared with the ARX forecasts without decomposition and Naive method, in which the forecast for each hour $h$ of monday, saturday and sunday is the price for each hour $h$ of previous week's monday, saturday and sunday. While, the forecast of each hour $h$ of tuesday, wednesday, thursday and friday is the previous days price for that hour $h$. This naive method was originally proposed by Nogales et al. (2002) and it is often used in EPF literature as a benchmark method[1].
5. In the last step, the exponents are used to convert the log prices and then evaluated against the original electricity prices using Weekly-weighted Mean Absolute Error (WMSE) and Diebold Mariano test.

*5.2. Forecast evaluation*

The Diebold and Mariano (1995) test is used to examine if the forecasts from different models have equal predictive accuracy. The test statistic is computed as: extensively in electricity price forecasting literature, provided in equation (9).

$$M = \frac{\bar{d}}{s_d} \tag{10}$$

Where, $\bar{d}$ and $s_d$ are the mean and sample standard deviation of $d$.

$$d = L_1 - L_2$$

And $L_i, (i = 1, 2)$ is absolute difference between the forecast and the actual,

$$L_i = \mid \hat{y} - y \mid \tag{11}$$

The null hypothesis suggests similar forecast accuracy for the different models and it is represented as $H0 : E(d_t) = 0$, while the alternative hypothesis would suggest the superiority of forecasts out of the given models

[1]As this paper focuses on the performance of different filters in forecasting the long-term seasonal component (LTSC) and compares these with our MOPS filter, exogenous variables have not been included. However, practitioners can include exogenous variables (relating to their data set and particular market) to improve their forecasts. Moreover, the setup for the forecasting exercise is the same as earlier papers on trend forecasting in electricity prices for consistency and comparison, (see, for example, Nowotarski and Weron, 2016). Finally, the setup is formulated to be used in the electricity auction market where historic load and prices only up till 12:00 CET are used to forecast the next 24 hours electricity prices, in a rolling window scheme to prevent looking ahead.

and it is represented as $H1 : E(d_t) \neq 0$. In order to avoid auto-correlation that multi-period forecast errors usually exhibit, the DM test is conducted separately for the 24-hours of each day, following Bordignon et al. (2013).

## 6. Forecast Results

In this section, the results for the UK day-ahead electricity price forecast are presented. As the trend forecasts alone are not comparable to the real electricity prices for the next day, we combine the forecast of next 24 hours of the day $(t + 1)$ for the trend $(T_{t+1})$ and the remainder part $(X_{t+1})$ and then evaluate it against the real prices for the 24 hours of the next day $P_{t+1} = forecasted(T_{t+1} + X_{t+1})$. Out of all three models compared, Naive, ARX and Trend –ARX model, the Naive forecast is the worst while Trend-ARX is better than ARX with only selective trend estimation methods and period lengths, with respect to WMAE in table 2, confirming previous findings of Nowotarski and Weron (2016) and Marcjasz et al (2019), they used a different day-ahead electricity prices dataset from Global energy forecasting competition(2014) and Nordic countries (Denmark, Finland, Norway and Sweden).

Wavelet based trend performs the best while our proposed MOPS based trend performs better than other filters after wavelets. Also, our proposed trend procedure provides better forecast than estimating the ARX model without decomposition and the Naive method for all four trend periods.

(Insert Table 2 here)

In terms of WMAE, the best forecasts among the different trend estimation methods used in Trend-ARX model is given by wavelet, similar to Nowotarski and Weron (2016) and Marcjasz et al (2019), as it has outperformed all other methods in three out of the four trend length periods. Our proposed optimization filter performs equally well with only slightly worse

WMAE but still ranks in second, out of all six trend estimation methods considered.

Butterworth filter, which is a low pass frequency filter and the HP filter also performed better than the ARX benchmark model for all four trend length periods and could be a good second tier choice for trend estimation after wavelet and MOPS filter. Both these methods provided best forecasts for the semi-annual trend period lengths out of all six trend estimation methods.

The most disappointing forecast performance was given by the Christiano Fitzgerald filter and the empirical mode decomposition, as the Trend-ARX forecasts with both these methods were worse than the benchmark ARX model in almost all trend period lengths, except for the semi-annual trend period length of EMD, also observed by Lisi and Nan (2014). The reason for this unsatisfactory performance could be higher sensitivity of both these methods to subtle changes in electricity prices, which Is a demerit for trend prediction, as electricity prices revert to the mean quickly.

(Insert Table 3 here)

Hence, any trend estimation method that is immune to sudden changes would be much useful for at least forecasting the trend of day-ahead electricity prices.

The four different trend length periods chosen at tri-weekly, quarterly, semi-annually and annually period lengths are representative of the long term seasonality that exists in electricity prices. Our choice of these trend lengths comes from the existing literature on the subject and it is backed empirically by our results, as the Trend-ARX gives superior forecasts in most of the cases for the given trend period lengths over the ARX model. However, in order to suggest best trend length period and estimation method, we refer to Diebold and Mariano (1995) test results for predictive superiority, presented in table 3 and figure 5.

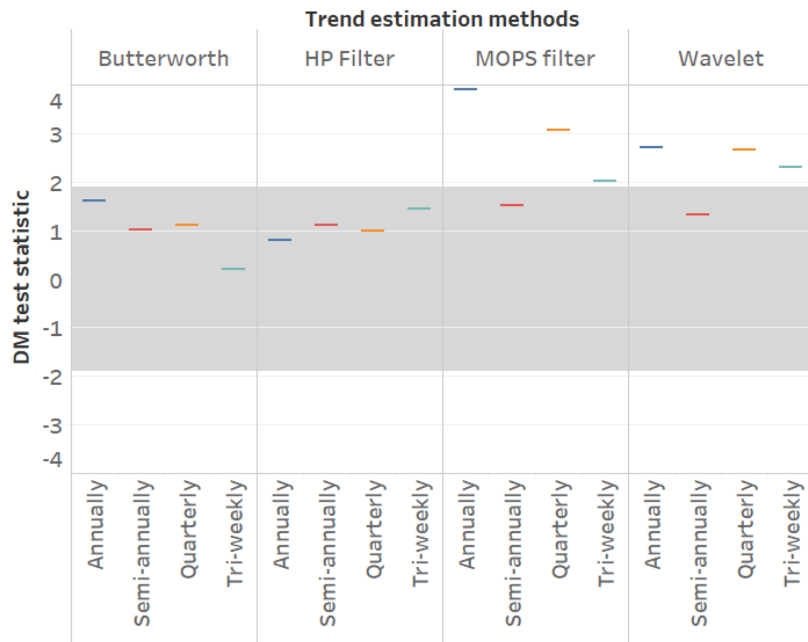


Figure 5: Diebold and Mariano (1995) test static calculated for each Trend-ARX model against the benchmark ARX model at different Trend period lengths.The test statistics above the shaded area (-1.96< $z$ <1.96) provide better predictive accuracy than the benchmark ARX model.

For the Diebold and Mariano (1995) test, only four trend estimation methods are considered, namely, wavelet decomposition, MOPS filter, HP filter and Butterworth filter. Both CF filter and EMD filter are eliminated from the test, as they performed worse than the benchmark ARX model. The DM test is conducted for individual trend method for each trend length period to evaluate which Trend-ARX forecast is better against benchmark ARX forecasts. The results from DM test show that only wavelet and our optimization based trend-ARX models perform better than the benchmark model, except for the semi-annual trend-ARX model. The best forecasts are given by our MOPS filter with annual and quarterly trend, see figure 5.

Plots of the trend estimated by different filters used in forecasting are presented in figure 6.

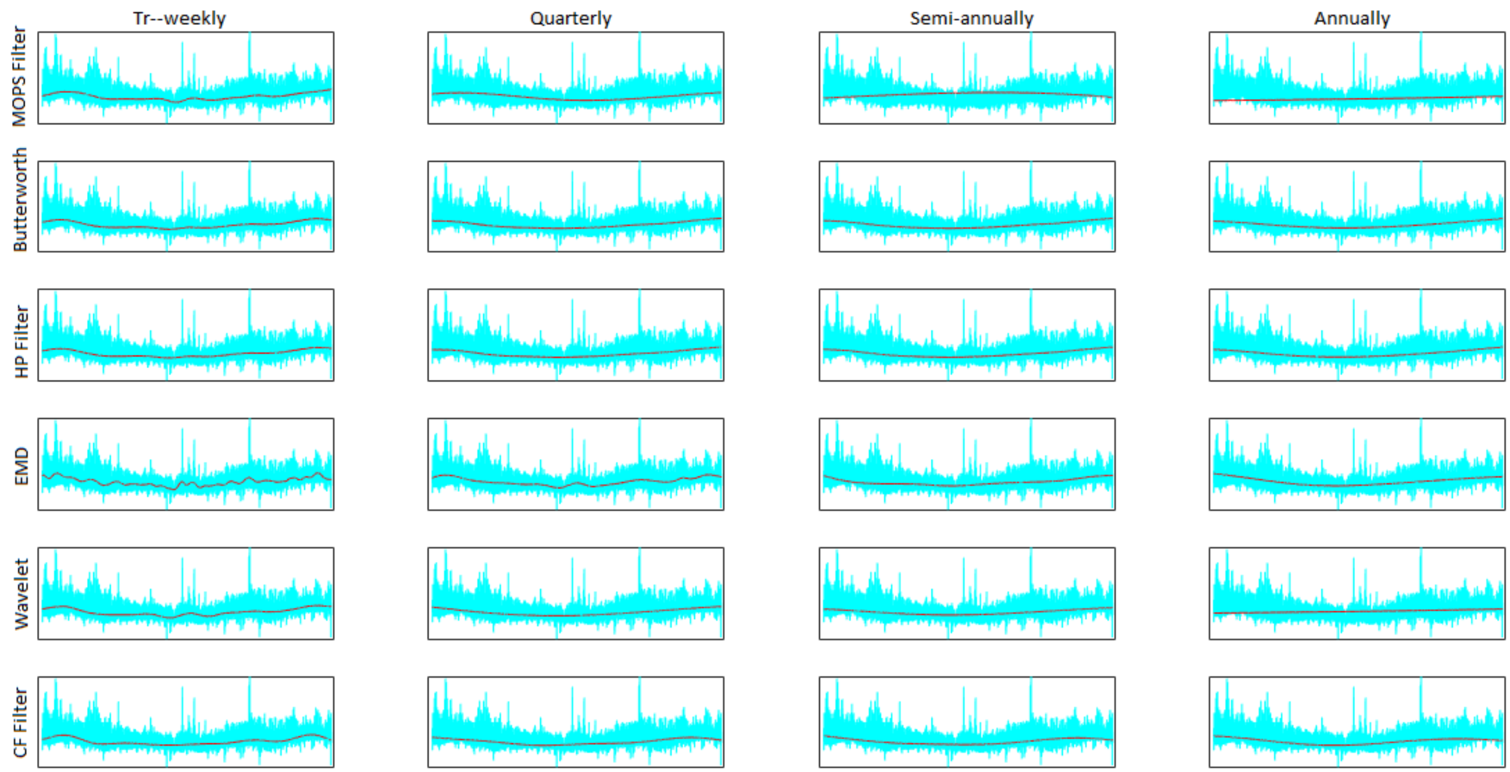


Figure 6: Plots of the trend estimated by different filters for tri-weekly, quarterly, semi-annual and annual intervals used in forecasting.

## 7. Conclusion

We have analysed different methods for trend estimation and introduced a new procedure based on optimization principles that could be used for forecasting day-ahead electricity prices. Recently, in EPF literature a new procedure of filtering/decomposing the electricity prices into LTSC and remainder part and then forecasting them independently has proved to improve the prediction of day-ahead electricity prices. However, this improvement in predictions previously could only be achieved using the trend estimated through the wavelet decomposition for statistical models as observed by Nowotarski and Weron (2016) and HP filter only performed well for neural network model (Marcjasz et al, 2019).

The aim of this paper was to enhance this pool of trend estimation methods, in case of statistical methods, providing the practitioners with more choice of models rather than relying just on wavelet decomposition. In our quest of finding trend estimation methods that can be used for day-ahead electricity price forecasting, we compared five existing well known trend estimation methods including, Butterworth low pass frequency filter, Christiano Fitzgerald band pass frequency filter, wavelet decomposition, Empirical mode decomposition and HP filter. Moreover, we introduced a new procedure for estimation of the trend, which is a multi-stage optimization filter with phase shift (MOPS). Out of all these trend estimation methods, only the wavelet decomposition and our proposed MOPS filter were effective in improving the predictions to the benchmark ARX model. In short, we have successfully enhanced the pool of trend estimation methods for day-ahead electricity price forecasting by introducing the MOPS filter, which has even provided better forecasts than the wavelet decomposition for the annual and quarterly trend period lengths in Diebold and Mariano (1995) test. We believe that a key reason for the success of our filter is the phase shift that leads the original prices and creates natural opportunity to look into the future and provide better forecasts. For the future research on this subject, we suggest to include the lead phase shift in other trend estimation methods instead of removing it through convolution, which is the normally practiced for filters.

In terms of trend length periods, we used tri-weekly, quarterly, semi-annually and annual lengths following the literature (Nowotarski and Weron, 2016) and taking into account the seasonal periods for UK electricity market. These trend length periods might vary with different electricity markets in the world, but for British electricity market, we suggest using the annual trend length would be most suitable based on the results of Diebold and Mariano (1995) test.For future research, more markets and recent data could be used to extend the scope of this forecasting scheme and trend estimation methods.

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| | **Day-ahead prices (£/MWh)** | **Load (GWh)** |
|---|---|---|
| **Mean** | 42.0602 | 62621.37 |
| **Median** | 39.3900 | 62015.50 |
| **Maximum** | 999 | 104854 |
| **Minimum** | 1.5700 | 33040 |
| **Std. Dev.** | 20.6975 | 14115.20 |
| **Skewness** | 19.8927 | 0.3071 |
| **Kurtosis** | 709.9900 | 2.4324 |
| **Jarque-Bera** | 5.5000 | 766.5909 |
| **Probability** | 0.0000 | 0.0000 |
| **Observations** | 26304 | 26304 |

Table 1: Descriptive statistics for electricity day-ahead price (£/MWh) and Load (GWh)

| FORECASTING MODELS | | WMAE | | | |
|---|---|---|---|---|---|
| **Naive** | | 12.2538 | | | |
| **ARX** | | 10.4111 | | | |
| **Trend-ARX** | | **Tri-weekly** | **Quarterly** | **Semi-annually** | **Annually** |
| | **Wavelet** | **<u>10.2968</u>** ($S9$) | **<u>10.2798</u>** ($S11$) | 10.4138 ($S12$) | **<u>10.3432</u>** ($S13$) |
| | **MOPS Filter** | **<u>10.3008</u>** (D=17) | **<u>10.3064</u>** (D=73) | **10.4099** (D=183) | **<u>10.3444</u>** (D=365) |
| | **HP Filter** | **10.3320** (5e9) | **10.3300** (5e10) | **<u>10.3243</u>** (1e11) | **10.3699** (5e11) |
| | **Butterworth** | **10.3295** (S=1500) | **10.3296** (S=3000) | **<u>10.3244</u>** (S=3500) | 10.4355 (S=6500) |
| | **CF Filter** | 10.5766 (S=1500) | 10.5483 (S=3000) | 10.5039 (S=3500) | 10.4265 (S=6000) |
| | **EMD** | 10.6747 (IMF=6) | 10.4682 (IMF=7) | **10.3882** (IMF=8) | 10.6238 (IMF=9) |

Table 2: The forecast results in terms of average Weekly-weighted Mean Absolute Error (WMAE) are provided in this table. The filtering is done for four different time interval lengths to obtain 3weekly, quarterly, semi-annual and annual trends that are comparable to different decomposition methods. The forecasts better than the benchmark ARX are in bold and the best two forecasts for each trend period length, considering all trend estimation methods, are underlined. *Note: The approximations used in trend estimation for each of the methods are provided in* ()

| **ARX compared to Trend-ARX** | | **DM test statistics** | | |
|---|---|---|---|---|
| | **Tri-weekly** | **Quarterly** | **Semi-annually** | **Annually** |
| **Wavelet** | **<u>2.3382</u>**<br>($S$9) | **2.6974**<br>($S$11) | 1.3377<br>($S$12) | **2.7238**<br>($S$13) |
| **MOPS Filter** | **2.0320**<br>(D=17) | **<u>3.0839</u>**<br>(D=73) | 1.5402<br>(D=183) | **<u>3.9245</u>**<br>(D=365) |
| **HP Filter** | 1.4692<br>(5e9) | 1.0232<br>(5e10) | 1.1319<br>(1e11) | 0.8289<br>(5e11) |
| **Butterworth** | 0.2231<br>(S=1500) | 1.1265<br>(S=3000) | 1.0283<br>(S=3500) | 1.6308<br>(S=6500) |

Table 3: The test statistics from Diebold and Mariano (1995) test for the equality of forecast accuracy are presented in this table. The test static is calculated for each Trend-ARX model against the benchmark ARX model to compare if the forecast accuracy from Trend-ARX models is same or different than ARX model. Test statistic value greater than z=1.96 means we can reject the null hypothesis and conclude the Trend-ARX models to provide better predictive accuracy. The forecasts better than the benchmark ARX are in bold and the best forecast for each trend length, considering all trend estimation methods, are underlined.*Note: The approximations used in trend estimation for each of the methods are provided in* ()