64-QAM Complementary Sets for High-Rate OFDM Transmissions

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***Abstract*—Existing 64-QAM Golay complementary pairs, each having impulse-like aperiodic auto-correlation sums, have been used to generate codebooks in code-keying orthogonal frequency-division multiplexing (OFDM) communications for the peak-to-mean power control. As a generalization of the 64-QAM Golay complementary pairs, in this paper, 64-QAM complementary sets, each of which contains two or more sequences, are proposed. It is shown that when the set size is not less than 4, they lead to significant code rate increases in code-keying OFDM communications.**

I. INTRODUCTION

A pair of sequences is called a Golay complementary pair (GCP) if their aperiodic auto-correlation sums are zero for all non-zero shifts [1]. In this case, either sequence is called a Golay sequence. The concept of GCP has been generalized to "complementary set", which consists of a set of two or more sequences [2].

A modern application of Golay sequences is for the peak-to-mean power control in code-keying orthogonal frequency-division multiplexing (OFDM) systems [3], [4]. For practical code-keying OFDM communications, however, the code rate is another important system parameter to be addressed. To improve the code rate, it is desirable to construct more Golay sequences for a codebook with larger size. Motivated by this goal, QAM Golay sequences, each having sequence elements over the QAM alphabet, are concerned by several researchers. In [5], 16-QAM Golay sequences were proposed by Rößing *et al*, and later extended by Chong *et al* [6]. In [7], 64-QAM Golay sequences were found by Lee *et al*, followed by the comments from Li *et al* for more Golay sequences [8], [9]. Generalized Case I-III 4^q-QAM ($q \geq 1$) Golay sequences are proposed in [10]. Very recently, Generalized Case IV-V 4^q-QAM ($q \geq 3$) Golay sequences are constructed by Liu, Li, and Guan using selected Gaussian integer pairs, each of which contains two distinct Gaussian integers with identical magnitude and which are not conjugate with each other [11].

Previous codebook constructions were all based on the polyphase and QAM Golay complementary pairs. To further improve the code rate, in this paper, we are concerned about the codebooks generating from the QAM complementary sets. We ask the following question: how much code rate improvement one can achieve when the size of complementary sets[1] is enlarged? Although this may mean a PEMPR increase, this research might be of interest for a code-keying OFDM system with higher priority for the code rate. We will answer this question by the proposed 64-QAM complementary sets in this paper.

The rest of this paper is organized as follows. In Section II, we present the definition of complementary set, and peak-to-mean power control problem in OFDM system. We then introduce generalized Boolean functions, and review existing constructions for polyphase complementary sets. In Section III, the proposed construction for 64-QAM complementary sets will be given. Enumeration on the number of 64-QAM complementary sequences and analysis on the resultant OFDM code-rate will be presented in section IV, followed by the conclusions in Section V.

II. PRELIMINARIES

A. Complementary set

Let $\mathbf{a} = [a_0, a_1, \cdots, a_{L-1}]$ and $\mathbf{b} = [b_0, b_1, \cdots, b_{L-1}]$ are two complex-valued sequences of length L. Define the aperiodic correlation function of $\mathbf{a}$ and $\mathbf{b}$ as

$$C(\mathbf{a},\mathbf{b})(\tau) = \begin{cases} \sum\limits_{t=0}^{L-1-\tau} a_t (b_{t+\tau})^*, & 0 \leq \tau \leq (L-1); \\ \sum\limits_{t=0}^{L-1+\tau} a_{t-\tau} (b_t)^*, & -(L-1) \leq \tau \leq -1; \\ 0, & |\tau| \geq L. \end{cases} \quad (1)$$

When $\mathbf{a} \neq \mathbf{b}$, $C(\mathbf{a},\mathbf{b})(\tau)$ is called the aperiodic cross-correlation function of $\mathbf{a}$ and $\mathbf{b}$; otherwise, aperiodic auto-correlation function. For simplicity, the aperiodic auto-correlation function of $\mathbf{a}$ will be sometimes written as $C(\mathbf{a})(\tau)$.

$\boldsymbol{G} = \left(\boldsymbol{G}^0, \boldsymbol{G}^1, \cdots, \boldsymbol{G}^{N-1}\right)$ is a complementary set of size N if

$$\sum_{i=0}^{N-1} C(\boldsymbol{G}^i)(\tau) = 0 \quad (2)$$

[1]Note that the size of a complementary set refers to the number of sequences, which is different from the codebook size.

for any $\tau \neq 0$, and in that case, either $\boldsymbol{G}^i$ $(0 \leq i \leq N-1)$ is called a complementary sequence. In particular, when $N=2$, $\boldsymbol{G}$ reduces to a Golay complementary pair (GCP), and each $\boldsymbol{G}^i$ $(0 \leq i \leq N-1)$ is called a Golay sequence.

B. Peak-to-mean power control problem in OFDM system

Consider an OFDM system with L subcarriers, with $\triangle f$ as the subcarrier spacing and f_c the carrier frequency. For any length L complex-valued codeword $\mathbf{a}$ in a codebook $\mathbf{S}$, the OFDM signal in the symbol duration $0 \leq t \leq 1/\triangle f$ can be written as

$$s_{\mathbf{a}}(t) = \sum_{k=0}^{L-1} a_k \exp\left(\sqrt{-1}2\pi(f_c + \triangle f k)t\right). \tag{3}$$

Moreover, let the instantaneous envelope power and mean envelope power (in the symbol duration) of $\mathbf{a}$ be $P_{\mathbf{a}}(t) = |s_{\mathbf{a}}(t)|^2$ and $|\mathbf{a}|^2 = \sum_{k=0}^{L-1}|a_k|^2$, respectively. Suppose that the frequency of use for each codeword in $\mathbf{S}$ is equally often, then the mean envelope power when averaging over all the codewords is given by

$$P_{av}(\mathbf{S}) = \frac{1}{|\mathbf{S}|}\sum_{\mathbf{a}\in\mathbf{S}}|\mathbf{a}|^2, \tag{4}$$

and the peak-to-mean envelope power ratio (PMEPR) for $\mathbf{S}$ is defined as

$$\text{PMEPR}(\mathbf{S}) = \frac{1}{P_{av}(\mathbf{S})}\max_{\mathbf{a}\in\mathbf{S}}\max_{0\leq t\leq 1/\triangle f} P_{\mathbf{a}}(t) \tag{5}$$

In the end, the code-rate of the OFDM system is defined as the ratio of the information word-length to the codeword length (which is equal to the the number of subcarriers), that is:

$$\boldsymbol{R}(\mathbf{S}) := \frac{\log_2|\mathbf{S}|}{L}. \tag{6}$$

III. Construction of 64-QAM Complementary Sets

Let $\mathbb{Z}_q = \{0,1,\cdots,q-1\}$ and $\xi = \exp\left(\sqrt{-1}2\pi/q\right)$. For $\mathbf{x} = [x_0, x_1, \cdots, x_{m-1}] \in \mathbb{Z}_2^m$, a generalized Boolean function $f(\mathbf{x})$ is defined as a mapping $f : \{0,1\}^m \to \mathbb{Z}_q$. Let $(i_0, i_1, \cdots, i_{m-1})$ be the binary representation of the integer $i = \sum_{\alpha=0}^{m-1} i_\alpha 2^\alpha$, with i_{m-1} denoting the most significant bit. Given $f(\mathbf{x})$ (or $f(x_0, x_1, \cdots, x_{m-1})$), define $f_i := f(i_0, i_1, \cdots, i_{m-1})$. Denote the complex-valued vector associated with f by

$$\psi(f) := \left(\xi^{f(0,0,\cdots,0)}, \xi^{f(1,0,\cdots,0)}, \cdots, \xi^{f(1,1,\cdots,1)}\right). \tag{7}$$

From now on, let $q=4$, and $\xi = \exp\left(\sqrt{-1}2\pi/4\right)$. For $0 \leq k \leq m-1$, let

$$\begin{aligned} I &= [i_0, i_1, \cdots, i_{m-k-1}], \\ J &= [j_0, j_1, \cdots, j_{k-1}] = \{0,1,\cdots,m-1\} \setminus I, \end{aligned}$$

Write $\mathbf{x}_I = [x_{i_0}, x_{i_1}, \cdots, x_{i_{m-k-1}}]$ and $\mathbf{x}_J = [x_{j_0}, x_{j_1}, \cdots, x_{j_{k-1}}]$. For every $0 \leq b \leq 2^k-1$, with its binary representation $(b_0, b_1, \cdots, b_{k-1})$, let $g_b \in \mathbb{Z}_4$ and $\boldsymbol{g}_b \in \mathbb{Z}_4^{m-k}$. Also, let $r = \exp(\sqrt{-1}\pi/4)$, $r_0 = 4/\sqrt{21}$, $r_1 = 2/\sqrt{21}$, $r_2 = 1/\sqrt{21}$.

We present below the proposed construction of 64-QAM complementary sets. Due to the space limitation, the proof is omitted.

Theorem 1: Let $f : \{0,1\}^m \to \mathbb{Z}_4$ be a generalized Boolean function as follows,

$$\begin{aligned} f =& 2\sum_{\alpha=0}^{m-k-2} x_{i_{\pi(\alpha)}} x_{i_{\pi(\alpha+1)}} \\ &+ \sum_{b=0}^{2^k-1} x_{j_0}^{b_0} x_{j_1}^{b_1} \cdots x_{j_{k-1}}^{b_{k-1}} \left(\boldsymbol{g}_b \cdot \mathbf{x}_I + g_b\right), \end{aligned} \tag{8}$$

where π is a permutation of $\{0,1,\cdots,m-k-1\}$. Also, let $0 \leq t = c\cdot 2^k + \sum_{\alpha=0}^{k-1} c_\alpha 2^\alpha \leq 2^{k+1}$. Then, for $\beta \in \{i_{\pi(0)}, i_{\pi(m-k-1)}\}$ and

$$\begin{aligned} f_0^{(t)}(\mathbf{x}) &= f(\mathbf{x}) + 2\left(\boldsymbol{c}\cdot\mathbf{x}_J + cx_\beta\right) \\ f_1^{(t)}(\mathbf{x}) &= f_0^{(t)}(\mathbf{x}) + s_1(\mathbf{x}) \\ f_2^{(t)}(\mathbf{x}) &= f_0^{(t)}(\mathbf{x}) + s_2(\mathbf{x}), \end{aligned} \tag{9}$$

a 64-QAM complementary set

$$\boldsymbol{G} = \left(\boldsymbol{G}^0, \boldsymbol{G}^1, \cdots, \boldsymbol{G}^{2^{k+1}-1}\right) \tag{10}$$

can be formed by

$$\boldsymbol{G}^t = r\left[r_0\psi\left(f_0^{(t)}\right) + r_1\psi\left(f_1^{(t)}\right) + r_2\psi\left(f_2^{(t)}\right)\right], \tag{11}$$

where $s_1(\mathbf{x})$ and $s_2(\mathbf{x})$ are in one of the following cases:

1) Case I & II[2]:

$$s_p(\mathbf{x}) = \sum_{b=0}^{2^k-1} x_{j_0}^{b_0} x_{j_1}^{b_1} \cdots x_{j_{k-1}}^{b_{k-1}} \left(e_b^{(p)} x_\lambda + d_b^{(p)}\right), \tag{12}$$

where $1 \leq p \leq 2$, $\lambda = \{i_{\pi(0)}, i_{\pi(m-k-1)}\}/\beta$, and $e_b^{(p)}, d_b^{(p)} \in \mathbb{Z}_4$.

2) Case III:

$$\begin{aligned} s_1(\mathbf{x}) &= d_0 + d_1 x_{i_{\pi(w)}} + d_2 x_{i_{\pi(w+1)}} \\ s_2(\mathbf{x}) &= d_0' + d_1' x_{i_{\pi(w)}} + d_2' x_{i_{\pi(w+1)}} \end{aligned} \tag{13}$$

with $0 \leq w \leq m-k-2$, and $2d_0 + d_1 + d_2 = 0$, $2d_0' + d_1' + d_2' = 0$.

3) Case IV:

$$\left(s_1(\mathbf{x}), s_2(\mathbf{x})\right) = \left(d_0 + d_1 x_{i_{\pi(w)}}, d_0' + d_1' x_{i_{\pi(w)}}\right), \tag{14}$$

with $1 \leq w \leq m-k-2$ and $(d_0, d_1, d_0', d_1') \in \left\{(0,1,2,3), (0,3,2,1), (1,3,1,1), (3,1,3,3)\right\}$.

4) Case V:

$$\begin{aligned} s_1(\mathbf{x}) &= d_0 + d_1 x_{i_{\pi(w_1)}} + d_2 x_{i_{\pi(w_2)}} \\ s_2(\mathbf{x}) &= d_0' + d_1' x_{i_{\pi(w_1)}} + d_2' x_{i_{\pi(w_2)}} \end{aligned} \tag{15}$$

with $0 \leq w_1 \leq m-k-3, w_1 + 2 \leq w_2 \leq m-k-1$, and $(d_0, d_1, d_2, d_0', d_1', d_2') \in$

[2]Note that when $k=0$, Case I & II corresponds to the combined offset pairs belonging to Case I and Case II in [9]. We label in this way for simplicity.

$\{(0,1,3,2,3,1),(0,3,1,2,1,3),(1,3,3,1,1,1),$
$(3,1,1,3,3,3)\}$.

When $k=0$, the above complementary set is reduced to a 64-QAM GCP. For 64-QAM GCPs, Case I & II and Case III were originally obtained by Lee and Golomb [7], and later corrected by Li [8]. Case IV and Case V were found by Li as well by computer search [8], and then proved to be true in [9]. The total number of 64-QAM GCP offset pairs for Case I & II and Case III is $240(m+1)+4$, and for Case IV and Case V is $2(m-2)(m+1)$, respectively. The total number of 64-QAM complementary sequences is equal to $(m!/2)\,4^{m+1}$ times of the number of offset pairs [9]. Generalized Case I-III and Generalized Case IV-V 4^q-QAM Golay complementary sequences can be found in [10] and [11], respectively.

IV. Code Rate Analysis for the Proposed 64-QAM Complementary Sequences

Enumeration will be performed first in order to study the code-rate of the proposed 64-QAM complementary sequences.

Denote by $N_{\text{I \& II}}^{(k)}$ the number of offset pairs in Case I & II of *Theorem 1*. Also, denote by $N_{\text{III}}^{(k)}$ the number of offset pairs which are in Case III only, but not in Case I & II. When $k=0$, $N_{\text{I \& II}}^{(k=0)}=2\cdot 4^{2\times 2}-4^2=496$ and $N_{\text{III}}^{(k=0)}=240(m+1)+4-N_{\text{I \& II}}^{(k=0)}=240m-252$ [10]. In addition, denote by $N_{\text{IV}}^{(k)}$ and $N_{\text{V}}^{(k)}$ the numbers of offset pairs belonging to Case IV and Case V [3], respectively. According to [9], $N_{\text{IV}}^{(k=0)}=4(m-2)$ and $N_{\text{V}}^{(k=0)}=2(m-2)(m-1)$.

In general,

$$\begin{aligned} N_{\text{I \& II}}^{(k)} &= 2\cdot 4^{4\cdot 2^k}-4^{2\cdot 2^k},\\ N_{\text{III}}^{(k)} &= 240(m-k)-252,\\ N_{\text{IV}}^{(k)} &= 4(m-k-2),\\ N_{\text{V}}^{(k)} &= 2(m-k-2)(m-k-1). \end{aligned} \tag{16}$$

On the other hand, denote by $N_f^{(k)}$ the number of generalized Boolean function f in (8). Then,

$$N_f^{(k)}=\binom{m}{k}\frac{(m-k)!}{2}4^{(m-k+1)2^k}=\frac{1}{2}\frac{m!}{k!}4^{(m-k+1)2^k}, \tag{17}$$

with $N_f^{(k=0)}=(m!/2)4^{m+1}$. By (16) and (17), we have

Theorem 2: There are

$$\begin{aligned} &N_{\text{total}}^{(k)}\\ =&N_f^{(k)}\cdot\left[N_{\text{I \& II}}^{(k)}+N_{\text{III}}^{(k)}+N_{\text{IV}}^{(k)}+N_{\text{V}}^{(k)}\right]\\ =&\frac{1}{2}\frac{m!}{k!}4^{(m-k+1)2^k}\cdot\Big[2\cdot 4^{4\cdot 2^k}-4^{2\cdot 2^k}\\ &\qquad +2(m-k)^2+238(m-k)-256\Big] \end{aligned}$$

64-QAM complementary sequences of length 2^m identified in *Theorem 1*, where $0\le k\le m-1$. In particular, when $k=0$, $N_{\text{total}}^{(k=0)}$ agrees with the number of the existing 64-QAM Golay complementary sequences.

[3]Note that for Case IV and Case V, each of them doesn't intersect with any other case.

Note that when all 64-QAM complementary sequences generated in *Theorem 1* are applied to an OFDM system, the PMEPR analysis is challenging. This is because the binary variables in $\mathbf{x}_J$ in the offset pairs in Case I & II gives rise to very complicated sequence element distributions over the 64-QAM constellation. We, however, will show next that there exists a group of 64-QAM complementary sequences whose elements have identical magnitude and whose PMEPRs therefore are at most 2^{k+1}.

For the offset pair coefficients $\{d_i\}$ and $\{d'_i\}$ $(0\le i\le 2)$ in Case III, Case IV, and Case V, define

$$\begin{aligned} M_0 &= 4+2\xi^{d_0}+\xi^{d'_0},\\ M_1 &= 4+2\xi^{d_0+d_1}+\xi^{d'_0+d'_1},\\ M_2 &= 4+2\xi^{d_0+d_2}+\xi^{d'_0+d'_2},\\ M_3 &= 4+2\xi^{d_0+d_1+d_2}+\xi^{d'_0+d'_1+d'_2}. \end{aligned} \tag{18}$$

Since the magnitude of ith element of $\boldsymbol{G}^t$ $(0\le t\le 2^{k+1}-1)$ can be written as

$$\left|\boldsymbol{G}_i^t\right|=rr_2\left|4+2\xi^{s_1(i_0,i_1,\cdots,i_{m-1})}+\xi^{s_2(i_0,i_1,\cdots,i_{m-1})}\right|,$$

where $0\le i\le 2^m-1$, with its binary representation $(i_0,i_1,\cdots,i_{m-1})$. Therefore, $\boldsymbol{G}^t$ is a polyphase complementary sequence if and only if

$$\begin{aligned} &\left|4+2\xi^{s_1(i_0,i_1,\cdots,i_{m-1})}+\xi^{s_2(i_0,i_1,\cdots,i_{m-1})}\right|\\ =&\left|4+2\xi^{s_1(i'_0,i'_1,\cdots,i'_{m-1})}+\xi^{s_2(i'_0,i'_1,\cdots,i'_{m-1})}\right| \end{aligned} \tag{19}$$

holds for any $0\le i\ne i'\le 2^m-1$. Considering the offset pairs in Case III, Case IV, and Case V, the condition specified in (19) is equivalent to

$$\left|M_0\right|=\left|M_1\right|=\left|M_2\right|=\left|M_3\right|. \tag{20}$$

It is interesting to note that all the offset pair coefficients in Case IV and Case V satisfy the condition in (20). Moreover, all the elements of the resultant complementary sequences have identical magnitude of $5\sqrt{2}/\sqrt{21}$ and are distributed over a PSK circle in the 64-QAM constellation, as shown by the star symbols in Fig. 1.

Further, when $(s_1(\mathbf{x}),s_2(\mathbf{x}))=(d_0,d'_0)$, with $(d_0,d'_0)\in\{(1,1),(3,3),(0,2)\}$, or $(s_1(\mathbf{x}),s_2(\mathbf{x}))=\left(d_0+d_1x_{i_{\pi(w)}},d'_0+d'_1x_{i_{\pi(w)}}\right)$, with $0\le w\le m-k-1$, and $(d_0,d_1,d'_0,d'_1)\in\{(1,2,1,2),(3,2,3,2)\}$, the condition in (20) will be satisfied too, and all the resultant sequence elements are over the same PSK circle which is characterized by the star-symbols in Fig. 1.

Taking account of the offset pairs in Case IV, Case V, and the additional two special cases, we arrive at the following theorem.

Theorem 3: Out of the $N_{\text{total}}^{(k)}$ (where $0\le k\le m-1$) complementary sequences in *Theorem 1*, there exists a subset

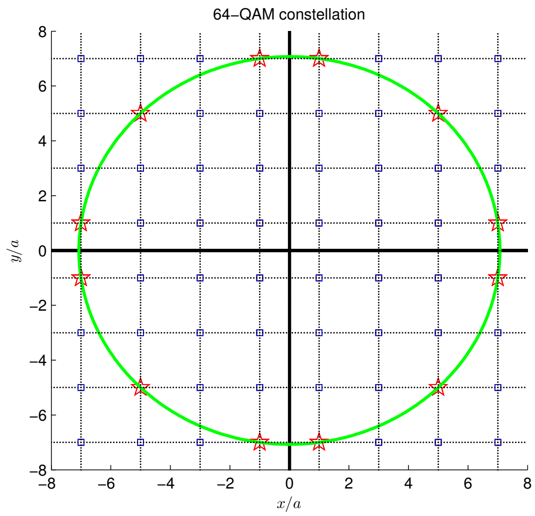

Fig. 1: Element distribution of Case IV-V 64-QAM complementary sequences.

of

$$\begin{aligned} N_{\text{subset}}^{(k)} &= N_f^{(k)} \cdot \left[3 + 2(m-k) + N_{\text{IV}}^{(k)} + N_{\text{V}}^{(k)}\right] \\ &= \frac{1}{2}\frac{m!}{k!}4^{(m-k+1)2^k} \cdot \left[2(m-k)^2 - 1\right] \end{aligned}$$

64-QAM complementary sequences (which have set size of 2^{k+1}) whose elements all are over the PSK circle in Fig. 1, and therefore the subset code-rate

$$\begin{aligned} \boldsymbol{R}_{\text{subset}}^{(k)} &= \frac{\log_2 N_{\text{subset}}^{(k)}}{2^m} \\ &= 2^{-m} \cdot \left[2(m-k+1)2^k + \log_2 m! - \log_2 k! \right. \\ &\qquad \left. + \log_2(2(m-k)^2 - 1) - 1\right] \end{aligned}$$

and $\text{PMEPR}_{\text{subset}}^{(k)} \leq 2^{k+1}$.

It is interesting to note that

$$\begin{array}{rcl} \boldsymbol{R}_{\text{subset}}^{(k=0)} & \approx & \frac{2m + \log_2 m! + 2\log_2 m}{2^m} \\ \boldsymbol{R}_{\text{subset}}^{(k=m-1)} & = & 2 + (\log_2 m - 1)2^{-m} \approx 2. \end{array} \tag{21}$$

V. CONCLUSIONS

This paper presents an algebraic construction of 64-QAM complementary sets (each of set size 2^{k+1}, where k is a non-negative integer) for high-rate code-keying OFDM communications. It is interesting to note that a subset of the proposed 64-QAM complementary sequences have identical element magnitude and are all over the same PSK circle (see Fig. 1). Therefore, the PMEPR upper bound of this subset is equal to the set size value of 2^{k+1}, where $0 \leq k \leq m-1$, and 2^m is the sequence length. Note that the code rates in code-keying OFDM systems using existing polyphase or QAM Golay sequences, in general are much less than 1 and decrease quickly for increasing sequence length. The critical observation in this paper is that, by using the subset of the proposed 64-QAM complementary sequences, the resultant code rate can be about 2, as shown in (21). In practice, by choosing a modest k (for a tolerable PMEPR), our proposed construction may be useful for a code-keying OFDM system with higher priority for the code rate.

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