Single-Channel Blind Separation of Co-Frequency PSK Signals with Unknown Carrier Frequency Offsets

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***Abstract*—In this paper, the problem of single-channel blind source separation (SCBSS) of a mixture of two co-frequency phase-shift keying (PSK) signals with unknown carrier frequency offsets (CFOs) is investigated. Two SCBSS algorithms which are robust to CFOs are proposed to perform separation of the mixture signals. In the first algorithm, the phase changes of the received signals caused by CFOs are tracked when performing separation of the signals. In the second algorithm, the CFOs are estimated directly before separation by using the cyclostationary feature of the signal, and they are used as known values during the separation of the signals. Simulation results show that both of the two proposed algorithms lead to significant performance improvement as compared with that achieved by using the conventional SCBSS algorithm. When CFOs are small, the phase tracking based algorithm is preferred as it can achieve same performance as that achieved by using the cyclic-feature based algorithm, but with much lower complexity. When CFOs are big, the cyclic-feature based algorithm is a better choice because it can achieve much better performance than that achieved by using the phase tracking based algorithm.**

I. INTRODUCTION

Single-channel blind source separation (SCBSS) is a problem that can be found in many practical applications, e.g., co-frequency overlapped signal separation [1], Paired Carrier Multiple Access (PCMA) signal monitoring [2], and space-based Automatic Identification System (AIS) signal detection [3]. Since in SCBSS, the number of observations is less than the number of sources, it is an ill-conditioned problem and very difficult to be solved, especially when difference between the individual signals is small. In the past decade, researchers have made attempts to solve this problem, and works in this area include the joint maximum likelihood sequence estimation (JMLSE) and joint maximum a posteriori symbol detector (JMAPSD) algorithms of [4][5], and the per-survivor processing (PSP) algorithm of [6][7]. Among these proposed solutions, it is found in [7] that the PSP algorithm is most attractive in terms of the complexity involved and the performance achieved.

Most of the above mentioned works on SCBSS assume that there is no carrier frequency offsets (CFOs) or the CFOs existing in the received signals are perfectly known. However, in an actual situation, CFOs exist and they are unknown to the blind receiver. On the other hand, although a lot of CFO estimation algorithms exist in the literature [8]-[10], most of them are based on the assumption that the received signal is from a single user and they cannot be used to estimate CFOs in a mixed signals from more than one users.

In this paper, two SCBSS algorithms which are robust to CFOs are proposed to perform separation of the mixture signals. The two proposed algorithms use the conventional PSP algorithm to perform separation of the mixture signals, while the phase rotations caused by CFOs are estimated by different methods to make the PSP algorithm more robust to CFOs. In the first algorithm, the phase rotations of the received signals are tracked by estimating the phase changes in each symbol interval. In the second proposed algorithm, the phase rotations are calculated by using the estimated CFOs obtained from the cyclic-feature of the received signals. The symbol error ratio (SER) performance achieved by the two proposed CFO-robust algorithms are compared under different CFO scenarios. The complexity of the two proposed algorithms are compared in the end of the paper.

This paper is organized as follows. In Section II, the basic system model and PSP algorithm for SCBSS is introduced. In Section III, two CFO-robust PSP algorithms are proposed. Simulation results are shown in Section IV and conclusions are drawn in Section V.

II. BASIC SYSTEM MODEL AND PSP ALGORITHM

In SCBSS, for simplicity, the two-user case is usually considered. In Fig. 1, a block diagram of the single channel received mixture of two PSK signals is shown.

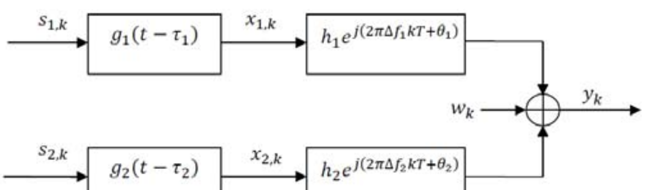

Fig. 1: Model for single-channel received mixture signals.

As shown in Fig. 1, the received signal can be expressed as

$$y_k = h_1 e^{j(2\pi\Delta f_1 kT+\theta_1)} x_{1,k} + h_2 e^{j(2\pi\Delta f_2 kT+\theta_2)} x_{2,k} + w_k, \tag{1}$$

where h_i $(i = 1, 2)$ are the magnitudes of the two modulated signals, Δf_i $(i = 1, 2)$ are their CFOs, θ_i $(i = 1, 2)$ are the initial phases, and w_k is the additive white Gaussian noise (AWGN) with mean 0 and per dimension variance σ^2. Moreover, in (1), the source signals $x_{i,k}$ at the output of the pulse shaping filter can be expressed as

$$x_{i,k} = \sum_{m=-L}^{L} s_{i,m+k} g_i(-mT - \tau_i), i = 1, 2 \tag{2}$$

where $s_{i,m}$ is the mth symbol transmitted by source i, $g_i(t)$ is the impulse response function which has a raised-cosine spectrum and finite duration from $-LT$ to LT (L is an integer constant). $0 \leq \tau_i < T$ is the time invariant relative delay between the ith received signal and the local clock reference, and T is the symbol period.

For simplicity, the discrete received mixture signal y_k can also be expressed in a matrix form as

$$y_k = \boldsymbol{s}_k \boldsymbol{f}_k + w_k, \tag{3}$$

where $\boldsymbol{s}_k = \begin{bmatrix} \boldsymbol{s}_{1,k} & \boldsymbol{s}_{2,k} \end{bmatrix}$ and $\boldsymbol{f}_k = \begin{bmatrix} \boldsymbol{f}_{1,k} & \boldsymbol{f}_{2,k} \end{bmatrix}'$, with $(')$ denotes matrix transpose. $\boldsymbol{s}_{i,k}$ is the ith source symbol vector at time index k which is defined as $\boldsymbol{s}_{i,k} = [s_{i,k-L}, s_{i,k-L+1}, \ldots, s_{i,k+L}]$, and $\boldsymbol{f}_{i,k}$ is the channel impulse response (CIR) vector which is defined as

$$\boldsymbol{f}_{i,k} = h_i e^{j(2\pi\Delta f_i kT+\theta_i)} [g_i(LT - \tau_i), \ldots, g_i(-LT - \tau_i)]'. \tag{4}$$

The goal of the receiver is to accurately estimate the source symbol sequences $\boldsymbol{s}_{i,1:N} = [s_{i,1}, s_{i,2}, \ldots, s_{i,N}]$ by using the received sequence $\boldsymbol{y}_{1:N} = [y_1, y_2, \ldots, y_N]$. More specifically, the detector for the symbol sequences of users 1 and 2 is given by

$$\{\hat{\boldsymbol{s}}_{1,1:N}, \hat{\boldsymbol{s}}_{2,1:N}\} = \arg \max_{\substack{\boldsymbol{s}_{1,1:N} \\ \boldsymbol{s}_{2,1:N}}} \Pr(\boldsymbol{s}_{1,1:N}, \boldsymbol{s}_{2,1:N} | \boldsymbol{y}_{1:N}). \tag{5}$$

At the receiver side, if the knowledge of the CIR $\boldsymbol{f}_k$ is available, the conventional Viterbi algorithm can be directly employed [11]. However, unlike the conventional Viterbi algorithm, in the blind detector, the CIR $\boldsymbol{f}_k$ is unknown. Therefore, a joint detection of $\boldsymbol{s}_k$ and $\boldsymbol{f}_k$ is needed. This can be achieved by using the PSP algorithm proposed in [6][7]. Define the state at time index k in the PSP algorithm as $\boldsymbol{\phi}_k = [\bar{\boldsymbol{s}}_{k-L+1}, \bar{\boldsymbol{s}}_{k-L+2}, \ldots, \bar{\boldsymbol{s}}_{k+L}]$, where $\bar{\boldsymbol{s}}_k = [s_{1,k}, s_{2,k}]$ is the pair of symbols transmitted at time index k from sources 1 and 2 respectively. The PSP algorithm includes the following steps. We call this algorithm "Conventional PSP algorithm".

Conventional PSP algorithm

1) Initialize the CIR ($\boldsymbol{f}_0$). For each state $\boldsymbol{\phi}_0$, initialize the metric as $M_0(\boldsymbol{\phi}_0) = 0$.
2) Calculate the branch metrics at time index k by

$$m(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k) = |e(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k)|^2. \tag{6}$$

where

$$e(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k) = y_k - \boldsymbol{s}_k(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k)\boldsymbol{f}_k. \tag{7}$$

In (7), $\boldsymbol{s}_k(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k) = [\boldsymbol{s}_{1,k}, \boldsymbol{s}_{2,k}]$ is the symbol vector associated with the state transition $\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k$.

3) Calculate the accumulated metrics at time index k as

$$M_k(\boldsymbol{\phi}_k) = \min_{\boldsymbol{\phi}_{k-1}} (M_{k-1}(\boldsymbol{\phi}_{k-1}) + m(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k)). \tag{8}$$

4) Update the CIR vector $\boldsymbol{f}_k$ by the least mean squares (LMS) algorithm [12], which is given by

$$\boldsymbol{f}_k = \boldsymbol{f}_{k-1} + \eta \cdot e(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k) \cdot \boldsymbol{s}_k(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k)^H, \tag{9}$$

where η is the step size and $(\cdot)^H$ denotes Hermitian transposition.

5) Set $k = k + 1$ and go back to step 2 or stop if reaching the end of the sequence.

III. Proposed SCBSS algorithms robust to CFOs

Although the PSP algorithm can be used to separate two signals existing in a single channel, it is required that Δf_1 and Δf_2 in (1) be small enough. This is because according to (4), the change of the CIR is caused by the change in h_i, $\Delta f_i kT$ and τ_i. h_i and τ_i are usually constant or vary slowly with time. $\Delta f_i kT$ changes with the time index k. The bigger the value of Δf_i, the larger the difference between $\Delta f_i kT$ and $\Delta f_i(k-1)T$, and hence the larger the change in CIR between consecutive symbols. If the change in CIR is too big, the LMS algorithm will fail to track the change and the PSP algorithm will not converge.

To overcome the drawback of the existing PSP algorithm, two CFO-robust PSP algorithms are proposed in this section. In the following, the two proposed algorithms are described in detail.

A. *CFO-robust PSP algorithm based on phase tracking*

According to (3), the received signal y_k can be expressed as

$$\begin{aligned} y_k &= \boldsymbol{s}_{1,k} \tilde{\boldsymbol{f}}_{1,k} e^{j2\pi\Delta f_1 kT} + \boldsymbol{s}_{2,k} \tilde{\boldsymbol{f}}_{2,k} e^{j2\pi\Delta f_2 kT} + w_k \\ &= y_{1,k} e^{j\tilde{\theta}_{1,k}} + y_{2,k} e^{j\tilde{\theta}_{2,k}} + w_k \end{aligned} \tag{10}$$

In (10), $y_{1,k}$ and $y_{2,k}$ are the received signals from users 1 and 2 respectively if CFOs do not exist. $\tilde{\theta}_{1,k}$ and $\tilde{\theta}_{2,k}$ are the phase rotations caused by CFOs at time k. $\tilde{\boldsymbol{f}}_{\boldsymbol{1,k}}$ and $\tilde{\boldsymbol{f}}_{\boldsymbol{2,k}}$ are the CIRs excluding the phase rotations caused by the CFOs, i.e.,

$$\tilde{\boldsymbol{f}}_{\boldsymbol{i,k}} = h_i e^{j\theta_i} [g_i(LT - \tau_i), \ldots, g_i(-LT - \tau_i)]'. \tag{11}$$

In the following, we will describe how to track the phase rotation caused by CFO for user 1. The tracking of the phase rotation caused by CFO for user 2 can be similarly done. Since $y_{1,k}$ is unknown, it can only be approximated from the separated symbol vector, i.e.

$$y_{1,k} \approx \hat{\boldsymbol{s}}_{1,k} \tilde{\boldsymbol{f}}_{1,k} \tag{12}$$

where $\hat{\boldsymbol{s}}_{1,k} = [\hat{s}_{1,k-L}, \hat{s}_{i,k-L+1}, \ldots, \hat{s}_{i,k+L}]$ is the estimated symbol vector at time k from user 1, which can be obtained from the survivor path. Suppose the estimated phase rotation caused by CFO for user 1 at time k is $\hat{\theta}_{1,k}$, the difference between the actual and estimated phases at time k is then

$$\begin{aligned} \Delta\theta_{1,k} &= \tilde{\theta}_{1,k} - \hat{\theta}_{1,k} \\ &= \angle[y_{1,k} e^{j\tilde{\theta}_{1,k}} y_{1,k}^{*} e^{-j\hat{\theta}_{1,k}}] \\ &\approx \angle[(y_k - y_{2,k} e^{j\tilde{\theta}_{2,k}})[\hat{\boldsymbol{s}}_{1,k}\tilde{\boldsymbol{f}}_{1,k}]^{*} e^{-j\hat{\theta}_{1,k}}] \\ &\approx \angle[(y_k - \hat{\boldsymbol{s}}_{2,k}\tilde{\boldsymbol{f}}_{2,k} e^{j\hat{\theta}_{2,k}})[\hat{\boldsymbol{s}}_{1,k}\tilde{\boldsymbol{f}}_{1,k}]^{*} e^{-j\hat{\theta}_{1,k}}] \end{aligned} \tag{13}$$

$\Delta\theta_{1,k}$ is the phase rotation caused by CFO for user 1 at time k. When $\Delta\theta_{1,k}$ is obtained, we can update the estimated phase rotation by

$$\hat{\theta}_{1,k+1} = \hat{\theta}_{1,k} + \Delta\theta_{1,k} \tag{14}$$

In reality, due to the existence of noise and estimation errors, the estimated $\Delta\theta_{1,k}$ might not be very accurate. To overcome this problem, we propose to pass the estimated $\Delta\theta_{1,k}$ through a very simple infinite-impulse-response (IIR) low pass filter. The transfer function of the low pass filter is

$$H(Z) = \frac{\beta}{1-(1-\beta)Z^{-1}}, \tag{15}$$

where $\beta \ll 1$. The output of the low pass filter is $\Delta\hat{\theta}_{1,k}$ and finally, the estimated phase rotation is updated by

$$\hat{\theta}_{1,k+1} = \hat{\theta}_{1,k} + \Delta\hat{\theta}_{1,k} \tag{16}$$

The above proposed phase tracking algorithm can be applied to the PSP algorithm to make the PSP algorithm more robust to the change of CIR caused by CFOs. In the following, the PSP with the phase tracking algorithm is described. For simplicity, we call this algorithm "PSP with phase tracking".

PSP with phase tracking

1) Set $k = 0$; Initialize the CIR ($\tilde{\boldsymbol{f}}_0$), the metrics $M_0(\boldsymbol{\phi}_0) = 0$, and the estimates of the phase rotations, i.e., $\hat{\theta}_{1,0} = \hat{\theta}_{2,0} = 0$, $\Delta\hat{\theta}_{1,0} = \Delta\hat{\theta}_{2,0} = 0$.
2) Calculate the branch metrics at time index k by

$$m(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k) = |e(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k)|^2. \tag{17}$$

where

$$\begin{aligned} e(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k) =& y_k - \boldsymbol{s}_{1,k}(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k)\tilde{\boldsymbol{f}}_{\boldsymbol{1},\boldsymbol{k}} e^{j\hat{\theta}_{1,k}} \\ & - \boldsymbol{s}_{2,k}(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k)\tilde{\boldsymbol{f}}_{\boldsymbol{2},\boldsymbol{k}} e^{j\hat{\theta}_{2,k}} \end{aligned} \tag{18}$$

where $\boldsymbol{s}_{i,k}(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k)$ is the ith transmit symbol vector associated with the state transition $\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k$.
3) Same as Step 3 in the **Conventional PSP algorithm**.
4) Same as Step 4 in the **Conventional PSP algorithm**.
5) Estimate $\Delta\theta_{1,k}$ and $\Delta\theta_{2,k}$ based on (13) as

$$\Delta\theta_{1,k} \approx \angle[(y_k - y_{2,k} e^{j\hat{\theta}_{2,k}}) y_{1,k}^{*} e^{-j\hat{\theta}_{1,k}}] \tag{19}$$

and

$$\Delta\theta_{2,k} \approx \angle[(y_k - y_{1,k} e^{j\hat{\theta}_{1,k}}) y_{2,k}^{*} e^{-j\hat{\theta}_{2,k}}] \tag{20}$$

6) Pass $\Delta\theta_{1,k}$ and $\Delta\theta_{1,k}$ through the low pass filter. The outputs of the low pass filter can be expressed as

$$\Delta\hat{\theta}_{1,k} = (1-\beta)\Delta\hat{\theta}_{1,k-1} + \beta\Delta\theta_{1,k} \tag{21}$$

and

$$\Delta\hat{\theta}_{2,k} = (1-\beta)\Delta\hat{\theta}_{2,k-1} + \beta\Delta\theta_{2,k} \tag{22}$$

7) Update the phase rotations caused by CFOs by

$$\hat{\theta}_{1,k+1} = \hat{\theta}_{1,k} + \Delta\hat{\theta}_{1,k} \tag{23}$$

and

$$\hat{\theta}_{2,k+1} = \hat{\theta}_{2,k} + \Delta\hat{\theta}_{2,k} \tag{24}$$

respectively.
8) Set $k = k+1$ and go back to step 2 or stop if reaching the end of the sequence.

B. CFO-robust PSP algorithm based on cyclic-feature

Although the algorithm proposed in Section III-A is simple and fast, it does not work well if large values of CFOs exist. This is because in (13), the estimated phase $\hat{\theta}_{i,k}$ is used to approximate the actual phase $\tilde{\theta}_{i,k}$ caused by CFO. This approximation only holds when the CFO is small enough. In this section, another CFO-robust PSP algorithm is proposed. In this algorithm, the phase rotations caused by CFOs are calculated directly by using CFOs estimated from cyclic-feature of the received signals. As long as the cyclic-feature based CFO estimation algorithm is accurate enough, the phase rotation can be tracked even when the CFOs are very large.

Assuming $x(t)$ is a cyclostationary signal, its correlation function is defined as $R_x(t+\frac{\tau}{2}, t-\frac{\tau}{2}) = E[x(t+\frac{\tau}{2})x^{*}(t-\frac{\tau}{2})]$. Since the autocorrelation function is periodic, it can be represented by the Fourier coefficients

$$R_x(t+\frac{\tau}{2}, t-\frac{\tau}{2}) = \sum_{\alpha} R_x^{\alpha}(\tau) e^{j2\pi\alpha t}. \tag{25}$$

The Fourier coefficients can be represented by

$$\begin{aligned} R_x^{\alpha}(\tau) &= \frac{1}{T}\int_{-T/2}^{T/2} R_x(t+\frac{\tau}{2}, t-\frac{\tau}{2}) e^{-j2\pi\alpha t} dt \\ &= \lim_{T\to\infty} \frac{1}{T}\int_{T} x(t+\frac{\tau}{2})x^{*}(t-\frac{\tau}{2}) e^{-j2\pi\alpha t} dt, \end{aligned} \tag{26}$$

where $\alpha = \frac{m}{T}, m = \pm1, \pm2, \ldots$ and $R_x^{\alpha}(\tau)$ is called cyclic autocorrelation function. The Fourier transform of the cyclic autocorrelation function is

$$S_x^{\alpha}(f) = F\{R_x^{\alpha}(\tau)\} = \int_{-\infty}^{\infty} R_x^{\alpha}(\tau) e^{-j2\pi f\tau} d\tau, \tag{27}$$

and $S_x^{\alpha}(f)$ is the spectral correlation function (SCF) [13][14].

Consider the received signal as expressed in (1). Its corresponding cyclic autocorrelation function can be derived based on Equation (26) as follows:

$$
\begin{aligned}
R^{\alpha}_{yy^*}(l) &= \langle y_{k+\frac{l}{2}} y_{k-\frac{l}{2}} e^{-j2\pi\alpha k} \rangle \\
&= \langle y_k y_{k-l} e^{-j2\pi\alpha k} \rangle e^{j\pi\alpha l} \\
&= h_1^2 \langle x_{1,k} x_{1,k-l} e^{j2\pi(2\Delta f_1-\alpha)k} \rangle e^{-j[\pi(2\Delta f_1-\alpha)l-2\theta_1]} \\
&+ h_2^2 \langle x_{2,k} x_{2,k-l} e^{j2\pi(2\Delta f_2-\alpha)k} \rangle e^{-j[\pi(2\Delta f_2-\alpha)l-2\theta_2]} \\
&+ \langle w_k w_{k-l} e^{-j2\pi\alpha k} \rangle e^{j\pi\alpha l} \\
&= \begin{cases} h_1^2 R_{x_1}(l) e^{j2\theta_1}, & \text{for } \alpha = 2\Delta f_1 \\ h_2^2 R_{x_2}(l) e^{j2\theta_2}, & \text{for } \alpha = 2\Delta f_2 \\ \sigma^2 \delta(l), & \text{for } \alpha = 0 \\ 0, & \text{otherwise,} \end{cases}
\end{aligned}
\tag{28}
$$

where $\langle\cdot\rangle \triangleq \lim_{Z\to\infty} \frac{1}{2Z+1} \sum_{k=-Z}^{Z}(\cdot)$. $R_{x_1}(l)$ and $R_{x_2}(l)$ are the autocorrelation functions of symbols from users 1 and 2, respectively. The SCF of the received signals is

$$
\begin{aligned}
S^{\alpha}_{yy^*}(f) &= \sum_{l=-\infty}^{\infty} R^{\alpha}_{yy^*}(l) e^{-j2\pi f l} \\
&= \begin{cases} h_1^2 S^{\alpha}_{x_1}(f) e^{j2\theta_1}, & \text{for } \alpha = 2\Delta f_1 \\ h_2^2 S^{\alpha}_{x_2}(f) e^{j2\theta_2}, & \text{for } \alpha = 2\Delta f_2 \\ \sigma^2, & \text{for } \alpha = 0 \\ 0, & \text{otherwise.} \end{cases}
\end{aligned}
\tag{29}
$$

From (29), it can be observed that $S^{\alpha}_{yy^*}(f)$ exhibits impulses at $\alpha = 2\Delta f_1$, $\alpha = 2\Delta f_2$ and $\alpha = 0$. This property can be used to detect Δf_1 and Δf_2. For simplicity, in this paper, we only consider $S^{\alpha}_{yy^*}(0)$. Suppose $S^{\alpha}_{yy^*}(0)$ has impulse when $\alpha = \alpha_1$ and $\alpha = \alpha_2$, $\alpha_1 \neq 0, \alpha_2 \neq 0$. Consequently, the estimated CFOs are $\tilde{\Delta f_1} = \frac{\alpha_1}{2}$ and $\tilde{\Delta f_2} = \frac{\alpha_2}{2}$ respectively. Based on the property of the SCF of the received signals as shown in (29), the proposed PSP algorithm which uses cyclic-feature for CFO estimation is described as follows. For simplicity, we call this algorithm "PSP with CFO estimation".

PSP with CFO estimation

1) For $l = 0, 1, \ldots, l_{max}$, calculate $R_{yy^*}(l)$ by $R_{yy^*}(l) = E[y(k)y(k-l)] = \frac{1}{N-l}\sum_{k=l+1}^{N} y(k)y(k-l)$. l_{max} is the maximum value of l and we have $R_{yy^*}(l) = 0$ when $l > l_{max}$.
2) For $\alpha = 0, \Delta\alpha, 2\Delta\alpha, \ldots, \alpha_{max}$ and $l = 0, 1, \ldots, l_{max}$, calculate $R^{\alpha}_{yy^*}(l)$ by (28). $\Delta\alpha$ is the resolution of α and α_{max} is the maximum CFO in the search.
3) For $\alpha = 0, \Delta\alpha, 2\Delta\alpha, \ldots, \alpha_{max}$, calculate $S^{\alpha}_{yy^*}(f)$ by (29).
4) Find α_1 and α_2 where $S^{\alpha_1}_{yy^*}(0)$ and $S^{\alpha_2}_{yy^*}(0)$ are two biggest impulses in $S^{\alpha}_{yy^*}(0)$ in the region that $\alpha \neq 0$, and calculate $\Delta\hat{f}_1$ and $\Delta\hat{f}_2$ by $\Delta\hat{f}_1 = \frac{\alpha_1}{2}$ and $\Delta\hat{f}_2 = \frac{\alpha_2}{2}$.
5) Set $k = 0$. Initialize the CIR ($\tilde{\boldsymbol{f}}_0$) and the metrics $M_0(\boldsymbol{\phi}_0) = 0$.
6) Calculate the branch metrics at time index k by

$$
m(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k) = |e(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k)|^2. \tag{30}
$$

where

$$
\begin{aligned}
e(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k) =& y_k - \boldsymbol{s}_{1,k}(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k) \tilde{\boldsymbol{f}}_{\mathbf{1},\boldsymbol{k}} e^{j2\pi\Delta\hat{f}_1 T} \\
&- \boldsymbol{s}_{2,k}(\boldsymbol{\phi}_{k-1} \to \boldsymbol{\phi}_k) \tilde{\boldsymbol{f}}_{\mathbf{2},\boldsymbol{k}} e^{j2\pi\Delta\hat{f}_2 T}
\end{aligned}
\tag{31}
$$

7) Same as Step 3 in the **Conventional PSP algorithm**.
8) Same as Step 4 in the **Conventional PSP algorithm**.
9) Set $k = k + 1$ and go back to step 6 or stop if reaching the end of the sequence.

IV. Simulation Results

In our simulations, a mixture of two BPSK signals is generated with $h_1 = h_2$. It is assumed that $\tau_1 = 0.2T$, $\tau_2 = 0.4T$ and $L = 2$. $g_1(t)$ and $g_2(t)$ are raised-cosine pulses with a roll-off factor of 0.33. The initial phases of signals from users 1 and 2 are 0.1 and 0.8 respectively. The symbol rate of each user is assumed to be 100,000 symbols per second. The oversampling rate is 4. The performance of the proposed algorithms are tested under different CFO scenarios. In the first scenario, we have $\Delta f_1 = 200$Hz and $\Delta f_2 = 600$Hz. In the second scenario, we have $\Delta f_1 = 400$Hz and $\Delta f_2 = 1200$Hz.

In Figure 2, the phases caused by CFOs estimated by using the PSP with the phase tracking algorithm are compared with the actual phases at each time instance. The SNR in the estimation is set to 20dB, and we use $\beta = 0.001$ for the low pass filter. It can be observed that the phase tracking algorithm works well for small CFOs ($\Delta f_i T < 0.01$). When CFO increases, the gap between the estimated and actual phases becomes bigger. When $\Delta f_i T = 0.012$, the gap becomes non-negligible.

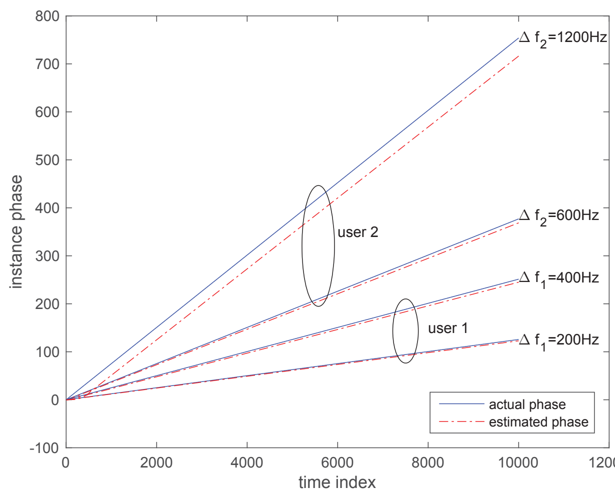

Fig. 2: Comparison of actual and estimated phases by using PSP with phase tracking algorithm.

In Figure 3, the SCFs, $S^{\alpha}_{yy^*}(0)$, of the received signals for the above mentioned two scenarios are plotted. It can be observed that when $\Delta f_1 = 200$Hz and $\Delta f_2 = 600$Hz, $S^{\alpha}_{yy^*}(0)$ exhibits two impulses at $\alpha = 400$Hz and $\alpha = 1200$Hz. When

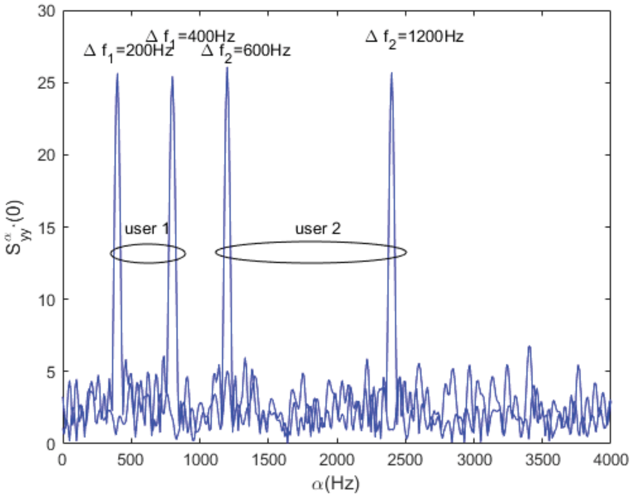

Fig. 3: The SCFs $S^{\alpha}_{yy^*}(0)$ of the received signals ($E_s/N_o = 20$dB, $\beta = 0.001$).

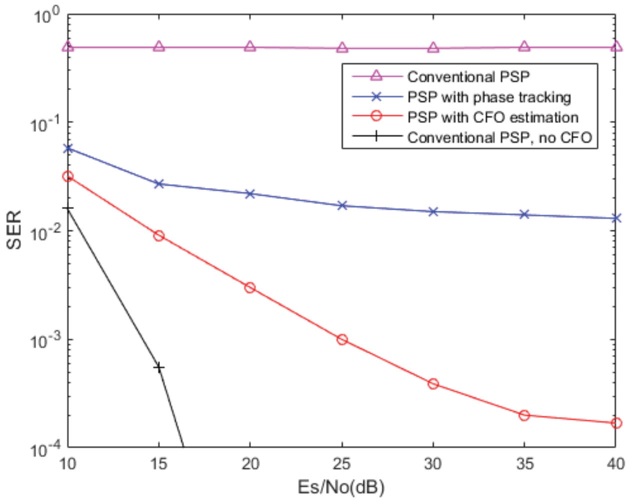

Fig. 5: SERs obtained by conventional and proposed algorithms ($\Delta f_1 = 400$Hz, $\Delta f_2 = 1200$Hz).

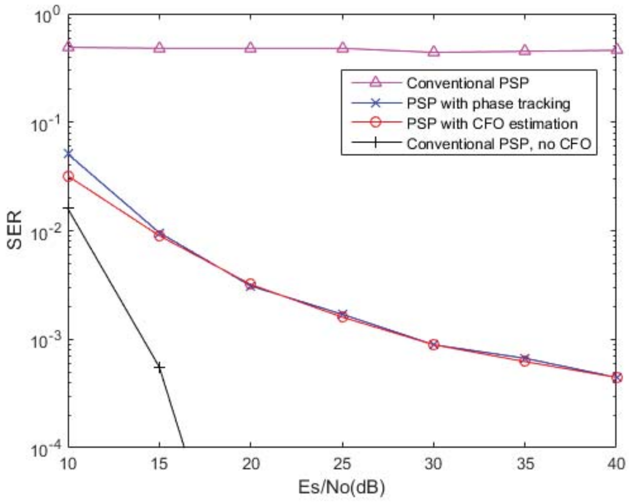

Fig. 4: SERs obtained by conventional and proposed algorithms ($\Delta f_1 = 200$Hz, $\Delta f_2 = 600$Hz).

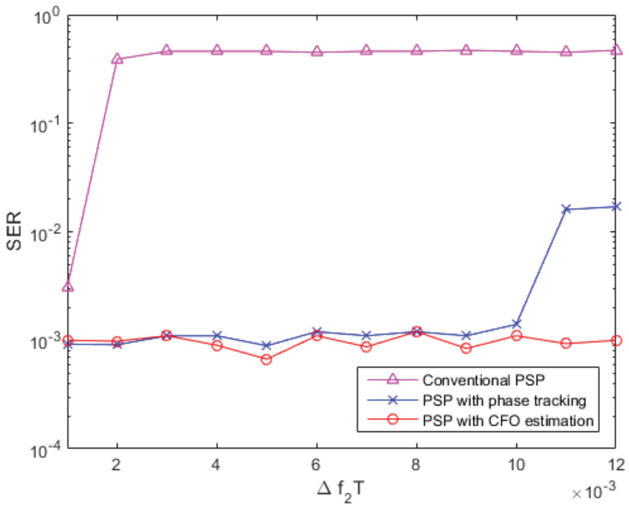

Fig. 6: SERs obtained by conventional and proposed algorithms for different normalized CFO values.

$\Delta f_1 = 400$Hz and $\Delta f_2 = 1200$Hz, $S^{\alpha}_{yy^*}(0)$ exhibits impulses at $\alpha = 800$Hz and $\alpha = 2400$Hz. This corresponds well to the results shown in Eq. (29).

In Figs. 4 and 5, the SERs obtained by the two proposed algorithms for the above mentioned two scenarios are compared. Since in each scenario, the SERs of the separated symbols from users 1 and 2 are almost the same, we only show results for one user. The SERs obtained by using the conventional PSP with unknown CFOs, and conventional PSP when there are no CFOs existing are also shown in Figs. 4 and 5 for comparison purpose.

In Figs. 6, the SERs obtained by the conventional PSP and the two proposed algorithms are compared under the condition that $\Delta f_1 T = 0$ and $\Delta f_2 T$ varies from 0.001 to 0.012. The E_s/N_o is set to 25dB in the comparison.

From Figs. 4 to 6, it can be observed that the conventional PSP algorithm only works when the normalized CFO is very small ($\Delta f_i T \leq 0.001$), and it fails badly when the normalized CFO is larger than 0.001. Both the two proposed CFO-robust PSP algorithms lead to significant performance improvement, as compared with that obtained by using the conventional PSP algorithm. The SERs obtained by the two proposed algorithms are almost the same when $\Delta f_i T \leq 0.01$. When one of the normalized CFO becomes larger than 0.01, the SERs obtained by the PSP with phase tracking algorithm is higher than those obtained by using the PSP with phase estimation. This is expected since from Fig. 2, it can be observed that when $\Delta f_i T$ is larger than 0.01, the gap between the actual and estimated

TABLE I: Comparison of the complexity of the conventional and proposed algorithms

Algorithms	Number of complex additions	Number of complex multiplications
Conventional PSP	$(4L+3)(M_1M_2)^{4L-2}N+(4L+2)(M_1M_2)^{2L}N$	$(4L+3)(M_1M_2)^{4L-2}N+(4L+3)(M_1M_2)^{2L}N$
Proposed PSP with phase tracking	$(4L+3)(M_1M_2)^{4L-2}N+(4L+2)(M_1M_2)^{2L}N$ $+(4L+4)N$	$(4L+3)(M_1M_2)^{4L-2}N+(4L+3)(M_1M_2)^{2L}N$ $+(8L+7)N$
Proposed PSP with CFO estimation	$(4L+3)(M_1M_2)^{4L-2}N+(4L+2)(M_1M_2)^{2L}N$ $+\frac{\alpha_{max}}{\Delta\alpha}(l_{max}+1)N$	$(4L+3)(M_1M_2)^{4L-2}N+(4L+3)(M_1M_2)^{2L}N$ $+\frac{\alpha_{max}}{\Delta\alpha}(l_{max}+1)(2N+2)$

phases is non-negligible. From Figs. 5 and 6, it can be seen that the PSP with CFO estimation still work well when $\Delta f_i T$ is larger than 0.01, since the CFOs can be accurately estimated by using the cyclostationary features in the received signal.

From Figs. 4 and 5, it can also be observed that when CFOs exist (suppose $\Delta f_1 \neq \Delta f_2$), the SER performance of the separated signal is much worse that that of the separated signals when CFOs do not exist. This is because when CFOs exist, the relative phase between the two received signal is changing with time. Since the power of the two signals are the same, when the two signals have opposite phases, they will cancel each other and lead to very high SER for the separated signals. This high SER cannot be reduced by using more accurate CFO estimation algorithm or higher power in the transmission, and it makes the overall SERs of the separated signal decrease very slowly with the increase of the E_s/N_o.

Finally, the complexities of the two proposed algorithms are calculated and compared with that of the conventional algorithm. Results are shown in Table I. The number of symbols in the received signal is assumed to be N. In the PSP with CFO estimation, we use $\alpha_{max} = 4000$Hz and $\Delta\alpha = 10$Hz. l_{max} is set to 100. From Table I, it can be seen that the proposed PSP with CFO estimation is more complex than the PSP with phase tracking algorithm. With consideration of a trade-off between the performance and complexity, we suggest that if the ranges of CFOs are known already (e.g., from the specification of the oscillators) and they are small, i.e., $\Delta f_i T \leq 0.01$, the proposed PSP with phase tracking algorithm should be used; Otherwise, the proposed PSP with CFO estimation is a better choice.

V. Conclusion

In this paper, the problem of blind separation of a mixture of two co-frequency PSK signals with unknown CFOs are investigated. Two CFO-robust algorithms based on the conventional PSP algorithm are proposed to perform separation of the co-frequency PSK signals. One of the algorithm tracks the phase changes of the received signals caused by CFOs during the separation, and the other estimates the CFOs directly by using cyclostationary feature of the received signals before the separation. Simulation results show that both proposed algorithms lead to significant performance improvement over that obtained by using the conventional PSP algorithm. When the normalized CFO is small ($\Delta f_i T \leq 0.01$), the SER performance obtained by the two proposed algorithms are almost the same. When one of the normalized CFOs becomes larger than 0.01, the SERs obtained by the PSP with CFO estimation is better than that achieved by the PSP with phase tracking. Considering that the complexity of the second proposed algorithm is higher than that of the first one, we suggest that the PSP with phase tracking be used when CFOs are small, and PSP with CFO estimation be used when at least one of the normalized CFO is larger than 0.01. It should be noted that the second proposed algorithm can also be used to estimate CFOs for more than two co-frequency signals and the accuracy of the estimation will not be affected. However, for the first proposed algorithm, further investigation is needed to see if the accuracy of the phase tracking will be affected due to the existence of more than two users in a same frequency band.

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