On the Performance of Single-Channel Source Separation of Two Co-Frequency PSK Signals with Carrier Frequency Offsets

Xiao-Bei Liu, Yong Liang Guan, Soo Ngee Koh, Teo Kok Ann Donny
School of Electrical and Electronic Engineering
Nanyang Technological University
Singapore 639798
E-mail: xpliu@ntu.edu.sg

Zilong Liu
Institute for Communication Systems
5G Innovation Centre (5GIC)
University of Surrey, UK
E-mail: zilong.liu@surrey.ac.uk

***Abstract*—Single-channel blind source separation (SCBSS) of uncoordinated, non-spread, co-frequency interfering signals with non-zero carrier frequency offset (CFO) and timing offset, and without training sequence for channel estimation, is a challenging task. To measure the impact of different parameters on the performance of the separated signal, most of the previous works rely on numerical evaluations. In this paper, the union bounds of the symbol error rates (SERs) of the separated symbols from the mixture of two PSK signals of different powers and initial phases, with and without carrier frequency offsets (CFOs), are derived. Simulation results show that the SER bounds are tight in high SNR region. Based on the SER bounds, the effects of power, phase and CFO on the SER of the stronger and weaker signals are discussed.**

I. Introduction

Single-channel blind source separation (SCBSS) is a problem that can be found in many practical applications, e.g., co-frequency overlapped signal separation [1], paired carrier multiple access (PCMA) signal monitoring [2], and space-based automatic identification system (AIS) signal detection [3]. Since in SCBSS, the number of observations is less than the number of sources, it is an ill-conditioned problem and very difficult to be solved.

Fortunately, SCBSS has been shown to be possible by taking advantage of certain properties of the signals. For example, in [4], a blind separation algorithm was proposed to separate two PSK signals which are filtered by different pulse shaping filters. In [5], a decorrelating filter approach was described for multi-user communications based on the assumption that the users have distinct delays. In [6], a turbo joint equalization technique was proposed to separate co-received signals by using frame and symbol timing offsets of different users. In [7], a single antenna interference cancellation scheme was proposed to separate co-channel signals by taking advantage of the independence of each user's multipath channel.

In all the above mentioned works on SCBSS, the performance of separated signals are measured based on simulation results. To the best of our knowledge, there has been no theoretical analysis on the performance of the signals separated from a mixture of PSK modulated signals, especially when carrier frequency offsets (CFOs) exist in the received signal. Furthermore, the performances of separated signals are affected by many parameters such as the transmission powers, the received phases, the channel fading parameters and relative delays between the signals, etc. To measure the impact of different parameters on the performance of separated signal, most of the previous works rely on numerical evaluations.

The contribution of this paper includes the following. Firstly, union bounds of the SERs of two PSK modulated signals separated from their mixture are derived. In the calculation, we assume that the two PSK modulated signals have the same symbol rate, transmitted in the same time-frequency plane and arrive the receiver at same time, which makes the separation of the two signals to be most difficult. Secondly, the effects of CFOs as well as other parameters, e.g., the powers and phases of the received signals, on the performance of the separated signal are discussed in detail.

This paper is organized as follows. In Section II, the basic system model for single-channel source separation of two co-frequency PSK signals is introduced. In Section III, the theoretical union bounds of the SERs of the symbols separated from a mixture of two PSK signals with and without CFOs are developed. Simulation results of the algorithm are shown in Section IV and conclusions are drawn in Section V.

II. Basic system model

To make the problem tractable, consider a two-user SCBSS system in which the received signal at time index k can be expressed as

$$y_k = h_1 e^{j(2\pi\Delta f_1 kT+\theta_1)} x_{1,k} + h_2 e^{j(2\pi\Delta f_2 kT+\theta_2)} x_{2,k} + w_k, \quad (1)$$

where h_i $(i = 1, 2)$ are the amplitude of the received signals, Δf_i $(i = 1, 2)$ are their carrier frequency offsets (CFOs), θ_i $(i = 1, 2)$ are the initial phases, and w_k is the additive white Gaussian noise (AWGN) with zero mean and per dimension variance σ^2. Each source signals $x_{i,k}$ is the output of a pulse

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shaping filter which can be expressed as

$$x_{i,k} = \sum_{m=-L}^{L} s_{i,m+k} g_i(-mT - \tau_i), i = 1, 2 \qquad (2)$$

where $s_{i,m}$ is the mth symbol transmitted by source i, $g_i(t)$ is the filter impulse response which has a raised-cosine spectrum and finite duration from $-LT$ to LT (L is an integer constant). $0 \leq \tau_i < T$ is the relative delay of the ith received signal relative to the transmit signal, and T is the symbol period.

For compactness, the discrete received mixture signal y_k can also be expressed in a matrix form as

$$y_k = \boldsymbol{s}_k \boldsymbol{f}_k + w_k, \qquad (3)$$

where $\boldsymbol{s}_k = \begin{bmatrix} \boldsymbol{s}_{1,k-L:k+L} & \boldsymbol{s}_{2,k-L:k+L} \end{bmatrix}$ and $\boldsymbol{f}_k = \begin{bmatrix} \boldsymbol{f}_{1,k-L:k+L} & \boldsymbol{f}_{2,k-L:k+L} \end{bmatrix}^{\mathrm{T}}$, with $(\cdot)^{\mathrm{T}}$ denotes matrix transpose. $\boldsymbol{s}_{i,k-L:k+L} = [s_{i,k-L}, s_{i,k-L+1}, \ldots, s_{i,k+L}]$ is the ith source symbol vector at time index k, and $\boldsymbol{f}_{i,k-L:k+L}$ is the CIR vector which is defined as

$$\boldsymbol{f}_{i,k-L:k+L} = h_i e^{j(2\pi\Delta f_i kT + \theta_i)} [g_i(LT - \tau_i), \ldots, g_i(-LT - \tau_i)]^{\mathrm{T}}. \qquad (4)$$

The goal of the receiver is to accurately detect all the source symbol sequences $\boldsymbol{s}_{i,1:N} = [s_{i,1}, s_{i,2}, \ldots, s_{i,N}]$ by using the received sequence $\boldsymbol{y}_{1:N} = [y_1, y_2, \ldots, y_N]$. More specifically, the detector for the symbol sequences of users 1 and 2 is given by

$$(\hat{\boldsymbol{s}}_{1,1:N}, \hat{\boldsymbol{s}}_{2,1:N}) = \arg \max_{\substack{\tilde{\boldsymbol{s}}_{1,1:N} \\ \tilde{\boldsymbol{s}}_{2,1:N}}} \Pr(\tilde{\boldsymbol{s}}_{1,1:N}, \tilde{\boldsymbol{s}}_{2,1:N} | \boldsymbol{y}_{1:N}) \qquad (5)$$

where $\tilde{\boldsymbol{s}}_{i,1:N}$ is a trial of $\boldsymbol{s}_{i,1:N}$.

In this paper, we consider an extreme case, i.e., the two signals arrive at the receiver at the same time, so that they interfere with each other to the most extent. Obviously, the SERs obtained in such a condition will be the worst. Since $\Delta\tau = |\tau_1 - \tau_2| = 0$, Eq. (1) can be simplified to

$$y_k = h_1 s_{1,k} e^{j(2\pi\Delta f_1 kT + \theta_1)} + h_2 s_{2,k} e^{j(2\pi\Delta f_2 kT + \theta_2)} + w_k, \qquad (6)$$

and the detector for the kth symbol in the sequence is simplified to

$$(\hat{s}_{1,k}, \hat{s}_{2,k}) = \arg \max_{s_{1,k}, s_{2,k}} p(s_{1,k}, s_{2,k} | y_k). \qquad (7)$$

III. Performance analysis

Let the transmitted PSK modulated symbols from sources 1 and 2 at time index k be $s_{1,k} = e^{j(\frac{2\pi n_k}{M_1})}$ and $s_{2,k} = e^{j(\frac{2\pi m_k}{M_2})}$ respectively, where $n_k = 0, 1, \ldots, M_1 - 1$ and $m_k = 0, 1, \ldots, M_2 - 1$. M_1 and M_2 are the alphabet sizes of sources 1 and 2 respectively. Firstly, we assume that $\Delta f_1 = \Delta f_2 = 0$ and the received signal y_k can be expressed as

$$y_k = h_1 e^{j(\frac{2\pi n_k}{M_1} + \theta_1)} + h_2 e^{j(\frac{2\pi m_k}{M_2} + \theta_2)} + w_k. \qquad (8)$$

A wrong detection of the signal pair $(s_{1,k}, s_{2,k})$ will occur if the detected symbols are $\hat{s}_{1,k} \neq s_{1,k}$ or $\hat{s}_{2,k} \neq s_{2,k}$. The wrong detection probabilities for symbols from sources 1 and 2 can be expressed as

$$P_1 = \sum_{\substack{\hat{s}_{1,k}, \hat{s}_{2,k} \\ (\hat{s}_{1,k} \neq s_{1,k})}} p(\hat{s}_{1,k}, \hat{s}_{2,k} | y_k, s_{1,k}, s_{2,k}) \qquad (9)$$

and

$$P_2 = \sum_{\substack{\hat{s}_{1,k}, \hat{s}_{2,k} \\ (\hat{s}_{2,k} \neq s_{2,k})}} p(\hat{s}_{1,k}, \hat{s}_{2,k} | y_k, s_{1,k}, s_{2,k}) \qquad (10)$$

respectively. In the following, for simplicity, we omit the time index k and use $s(m,n)$ and $\hat{s}(m,n)$ to denote the constellation points which correspond to the signal pairs $(s_{1,k}, s_{2,k})$ and $(\hat{s}_{1,k}, \hat{s}_{2,k})$. $s(m,n)$ and $\hat{s}(m,n)$ are expressed as

$$s(m,n) = h_1 e^{j(\frac{2\pi n}{M_1} + \theta_1)} + h_2 e^{j(\frac{2\pi m}{M_2} + \theta_2)} \qquad (11)$$

and

$$\begin{aligned} \hat{s}(m,n) &= h_1 e^{j(\frac{2\pi n'}{M_1} + \theta_1)} + h_2 e^{j(\frac{2\pi m'}{M_2} + \theta_2)} \\ &= h_1 e^{j(\frac{2\pi(n+\Delta n)}{M_1} + \theta_1)} + h_2 e^{j(\frac{2\pi(m+\Delta m)}{M_2} + \theta_2)}, \\ &\Delta n = 0, 1, \ldots, M_1 - 1, \Delta m = 0, 1, \ldots, M_2 - 1 \end{aligned} \qquad (12)$$

respectively. It should be noted that the values of Δn and Δm cannot be equal to 0 at the same time as $\hat{s}(m,n)$ is a wrong detection. Letting N_o be the noise power spectral density, we have

$$p(\hat{s}(m,n) | y_k, s(m,n)) \propto Q\left(\frac{d(n,m,\Delta n,\Delta m)}{\sqrt{2N_o}}\right), \qquad (13)$$

where the Q-function is defined by

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(\frac{-u^2}{2}\right) du, \qquad (14)$$

and the received distance metric is given by

$$\begin{aligned} d(n,m,\Delta n,\Delta m) = &|h_1 e^{j(\frac{2\pi n}{M_1} + \theta_1)} + h_2 e^{j(\frac{2\pi m}{M_2} + \theta_2)} \\ &- (h_1 e^{j(\frac{2\pi(n+\Delta n)}{M_1} + \theta_1)} + h_2 e^{j(\frac{2\pi(m+\Delta m)}{M_2} + \theta_2)})|. \end{aligned} \qquad (15)$$

From Eqs. (9), (10) and (13), it can be derived that the union bounds for the SER which can be achieved by the receiver for symbols from sources 1 and 2 are

$$P_1 \leq \frac{1}{M_1 M_2} \sum_{n=0}^{M_1-1} \sum_{m=0}^{M_2-1} \sum_{\Delta n=1}^{M_1-1} \sum_{\Delta m=0}^{M_2-1} Q\left(\frac{d(n,m,\Delta n,\Delta m)}{\sqrt{2N_o}}\right), \qquad (16)$$

and

$$P_2 \leq \frac{1}{M_1 M_2} \sum_{n=0}^{M_1-1} \sum_{m=0}^{M_2-1} \sum_{\Delta n=0}^{M_1-1} \sum_{\Delta m=1}^{M_2-1} Q\left(\frac{d(n,m,\Delta n,\Delta m)}{\sqrt{2N_o}}\right), \qquad (17)$$

respectively. After some derivation, $d(n,m,\Delta n,\Delta m)$ which is given by (15) can be finally re-expressed by the phase factor $\Delta\theta = \theta_2 - \theta_1$, as given in Eq. (18).

When CFOs exist, i.e., $\Delta f_1 \neq 0$ and $\Delta f_2 \neq 0$, the position of $s(m,n)$ will vary with the time index k. We use $s(m,n,k)$

$$d(n,m,\Delta n,\Delta m)=2\sqrt{\left(h_1\sin\frac{\pi\Delta n}{M_1}\right)^2+\left(h_2\sin\frac{\pi\Delta m}{M_2}\right)^2+2h_1h_2\sin\frac{\pi\Delta n}{M_1}\sin\frac{\pi\Delta m}{M_2}\cos\left(\Delta\theta+\frac{\pi(2n+\Delta n)}{M_1}-\frac{\pi(2m+\Delta m)}{M_2}\right)}. \quad (18)$$

to represent the constellation points which change with time index k. $s(m,n,k)$ can be expressed as

$$\begin{aligned} s(m,n,k) &= h_1 e^{j(\frac{2\pi n}{M_1}+2\pi\Delta f_1 kT+\theta_1)}+h_2 e^{j(\frac{2\pi m}{M_2}+2\pi\Delta f_2 kT+\theta_2)} \\ & n=0,1,\ldots,M_1-1, m=0,1,\ldots,M_2-1, \end{aligned} \quad (19)$$

and the distance between $s(m,n,k)$ and $\hat{s}(m,n,k)$ is given by Eq. (20) given in the next page. Let $\Delta\theta' = \Delta\theta + 2\pi(\Delta f_1-\Delta f_2)kT+\frac{\pi(2n+\Delta n)}{M_1}-\frac{\pi(2m+\Delta m)}{M_2}$. Suppose the received sequence is long enough and $\Delta f=|\Delta f_1-\Delta f_2|>0$. We can see that $\Delta\theta'$ varies periodically in the range $[0,2\pi]$ regardless of the values of m and n. Therefore, when CFOs exist, the average SERs which can be achieved by the receiver for symbols from sources 1 and 2 do not depend on m and n. The union bound for the SER of symbol from source 1 is

$$P_1' \le \frac{1}{2\pi}\sum_{\Delta n=1}^{M_1-1}\sum_{\Delta m=0}^{M_2-1}\int_{\Delta\theta=0}^{2\pi} Q\left(\frac{d(\Delta n,\Delta m,\Delta\theta)}{\sqrt{2N_o}}\right)d(\Delta\theta), \quad (21)$$

and the union bound for the SER of symbols from source 2 is

$$P_2' \le \frac{1}{2\pi}\sum_{\Delta n=0}^{M_1-1}\sum_{\Delta m=1}^{M_2-1}\int_{\Delta\theta=0}^{2\pi} Q\left(\frac{d(\Delta n,\Delta m,\Delta\theta)}{\sqrt{2N_o}}\right)d(\Delta\theta), \quad (22)$$

where $d(\Delta n,\Delta m,\Delta\theta)$ is given by Eq. (23) given in the next page.

For particular values of M_1 and M_2, the Eqs. (16), (17), (21) and (22) can be simplified. For example, when both sources 1 and 2 are BPSK modulated, we have $M_1=2$ and $M_2=2$. Eq. (16) is reduced to

$$P_1 \le \frac{1}{4}\sum_{n=0}^{1}\sum_{m=0}^{1}\sum_{\Delta m=0}^{1} Q\left(\frac{d(n,m,\Delta m)}{\sqrt{2N_o}}\right), \quad (24)$$

where $d(n,m,\Delta m)$ can be expressed by (25).

By putting the values of $d(n,m,\Delta m)$ in Eq. (24), we have

$$\begin{aligned} P_1 \le\; & Q\left(\frac{2h_1}{\sqrt{2N_o}}\right) \\ & +\frac{1}{2}Q\left(2\sqrt{\frac{{h_1}^2+{h_2}^2-2h_1h_2\cos\Delta\theta}{2N_o}}\right) \\ & +\frac{1}{2}Q\left(2\sqrt{\frac{{h_1}^2+{h_2}^2+2h_1h_2\cos\Delta\theta}{2N_o}}\right). \end{aligned} \quad (26)$$

Suppose the power of the signals from source 1 is larger than or equal to that of the signals from source 2, i.e., $h_1 \ge h_2$. Letting $E_s=h_1^2+h_2^2$ and $g=\frac{h_2}{h_1}, 0<g\le 1$, (26) becomes

$$\begin{aligned} P_1 \le\; & Q\left(\sqrt{\frac{2E_s}{N_o}\left(\frac{1}{1+g^2}\right)}\right) \\ & +\frac{1}{2}Q\left(\sqrt{\frac{2E_s}{N_o}\left(1-\frac{2g}{1+g^2}\cos(\Delta\theta)\right)}\right) \\ & +\frac{1}{2}Q\left(\sqrt{\frac{2E_s}{N_o}\left(1+\frac{2g}{1+g^2}\cos(\Delta\theta)\right)}\right). \end{aligned} \quad (27)$$

Similarly, the union bound for the SER of symbols from source 2, i.e., P_2, is given by

$$\begin{aligned} P_2 \le\; & Q\left(\sqrt{\frac{2E_s}{N_o}\left(\frac{g^2}{1+g^2}\right)}\right) \\ & +\frac{1}{2}Q\left(\sqrt{\frac{2E_s}{N_o}\left(1-\frac{2g}{1+g^2}\cos(\Delta\theta)\right)}\right) \\ & +\frac{1}{2}Q\left(\sqrt{\frac{2E_s}{N_o}\left(1+\frac{2g}{1+g^2}\cos(\Delta\theta)\right)}\right). \end{aligned} \quad (28)$$

When CFOs exist, according to (21) and (22), the SER bounds of sources 1 and 2 can be expressed as

$$\begin{aligned} P_1' \le\; & \frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{\frac{2E_s}{N_o}\left(1-\frac{2g}{1+g^2}\cos(\Delta\theta)\right)}\right)d(\Delta\theta) \\ & +Q\left(\sqrt{\frac{2E_s}{N_o}\left(\frac{1}{1+g^2}\right)}\right), \end{aligned} \quad (29)$$

and

$$\begin{aligned} P_2' \le\; & \frac{1}{2\pi}\int_0^{2\pi} Q\left(\sqrt{\frac{2E_s}{N_o}\left(1-\frac{2g}{1+g^2}\cos(\Delta\theta)\right)}\right)d(\Delta\theta) \\ & +Q\left(\sqrt{\frac{2E_s}{N_o}\left(\frac{g^2}{1+g^2}\right)}\right). \end{aligned} \quad (30)$$

IV. Simulation Results

In our simulations, we consider the scenario described in section III, i.e., both sources employing BPSK modulation. $g_1(t)$ and $g_2(t)$ are raised-cosine pulses with a roll-off factor 0.33. The symbol rates of both sources are 100,000 symbols per second. The CIRs are assumed to be unknown at the receiver and the per-surviving processing algorithm [8] is used to perform joint detection of the symbols from the two sources. When CFOs exist, we assume that the CFOs of the two signals are 100 and 200Hz respectively and they are perfectly known at the receiver [9].

$$d(n,m,\Delta n,\Delta m,k) = 2\sqrt{\left(h_1 \sin\frac{\pi\Delta n}{M_1}\right)^2 + \left(h_2 \sin\frac{\pi\Delta m}{M_2}\right)^2 + 2h_1h_2 \sin\frac{\pi\Delta n}{M_1}\sin\frac{\pi\Delta m}{M_2}\cos\left(\Delta\theta + 2\pi(\Delta f_1 - \Delta f_2)kT + \frac{\pi(2n+\Delta n)}{M_1} - \frac{\pi(2m+\Delta m)}{M_2}\right)}. \quad (20)$$

$$d(\Delta n,\Delta m,\Delta\theta) = 2\sqrt{\left(h_1 \sin\frac{\pi\Delta n}{M_1}\right)^2 + \left(h_2 \sin\frac{\pi\Delta m}{M_2}\right)^2 + 2h_1h_2 \sin\frac{\pi\Delta n}{M_1}\sin\frac{\pi\Delta m}{M_2}\cos\Delta\theta}. \quad (23)$$

$$d(n,m,\Delta m) = 2\sqrt{(h_1)^2 + \left(h_2 \sin\frac{\pi\Delta m}{2}\right)^2 + 2h_1h_2 \sin\frac{\pi\Delta m}{2}\cos\left(\Delta\theta + \frac{\pi(2n+1)}{2} - \frac{\pi(2m+\Delta m)}{2}\right)}. \quad (25)$$

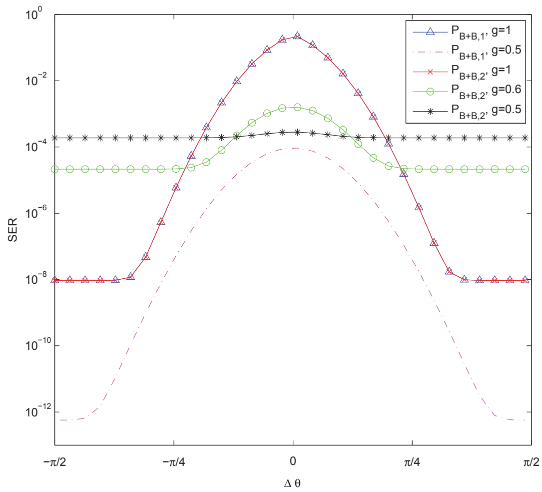
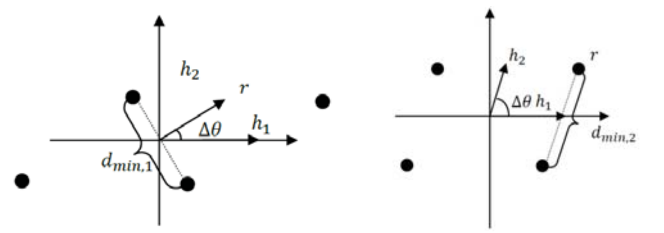


Fig. 1: The variation of SERs of separated signals versus $\Delta\theta$ (BPSK+BPSK, no CFOs, $E_s/N_o = 15$dB).

We firstly analyse the effect of $\Delta\theta$ to the separate BERs. The variations of P_1 and P_2 versus $\Delta\theta$ are shown in Fig. 1.

From Fig. 1, it can be seen that when $\Delta\theta = 0$, P_1 reaches the maximum. This is because under such conditions, signals from sources 1 and 2 will cancel each other if they have opposite phases. Separation of the two signals in such a situation is very difficult, especially when the powers of the two signals are similar. When $\Delta\theta$ increases from 0 to $\frac{\pi}{2}$, P_1 becomes smaller, and when $\Delta\theta = \frac{\pi}{2}$, P_1 reaches minimum as the two signals become orthogonal to each other. For P_2, it can be seen that when $|\Delta\theta|$ is less than a certain value, the variation of P_2 versus $|\Delta\theta|$ is similar to the variation of P_1; however, beyond that value, P_2 does not change with $|\Delta\theta|$ anymore. This phenomenon can be explained by Figs. 2(a) and 2(b).

Figs. 2(a) and 2(b) show two typical constellation diagrams for the received signal based on different values of $\Delta\theta$ and g. If the minimum distance of the constellation points is between diagonal points, as shown in Fig. 2(a), P_2 can be approximately expressed as



(a) d_{min} is between diagonal points. (b) d_{min} is between neighbouring points.

Fig. 2: Two typical constellation diagrams for the received mixture of two BPSK signals.

$$P_2 \approx \frac{1}{2}Q\left(\sqrt{\frac{2E_s}{N_o}\left(1 - \frac{2g}{1+g^2}\cos(\Delta\theta)\right)}\right). \quad (31)$$

If the minimum distance of the constellation points is between neighboring points, as shown in Fig. 2(b), P_2 can be approximately expressed as

$$P_2 \approx Q\left(\sqrt{\frac{2E_s}{N_o}\left(\frac{g^2}{1+g^2}\right)}\right). \quad (32)$$

Comparing (31) and (32), it can be seen that only (31) is affected by $\Delta\theta$. Between $d_{min,1}$ and $d_{min,2}$, the smaller one will determine P_2. When $d_{min,1} < d_{min,2}$, we have

$$\frac{1}{1+g^2} > \frac{2g}{1+g^2}\cos(\Delta\theta) \Rightarrow |\Delta\theta| < \arccos\left(\frac{1}{2g}\right). \quad (33)$$

Let $\psi = \arccos(\frac{1}{2g})$, according to (33), when $|\Delta\theta| < \psi$, $d_{min,1} < d_{min,2}$; otherwise, $d_{min,1} \geq d_{min,2}$. In other words, when $|\Delta\theta| < \psi$, P_2 is affected by parameters $\Delta\theta$; otherwise, P_2 is not affected by $\Delta\theta$.

Next, we analyse the performance of the separated signals under different E_s/N_o. The initial phases of sources 1 and 2 are randomly chosen to be 0.1 and 0.8, respectively. In Figs. 3 and 4, the SERs obtained by simulation for a mixture of two BPSK signals with and without CFOs are compared with the SER bounds obtained by Eqs. (26), (28), (29) and (30). The

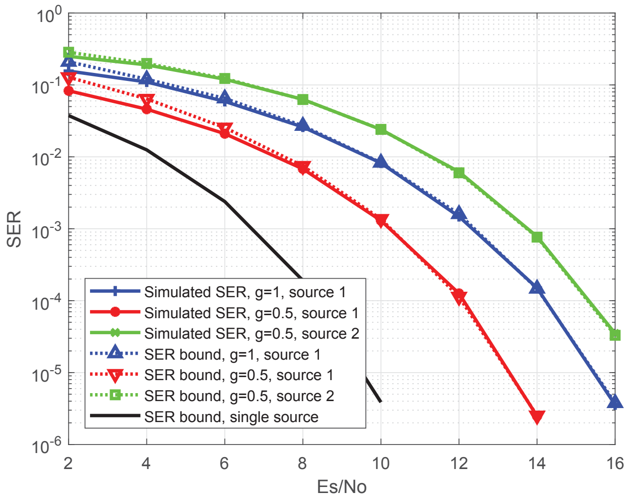


Fig. 3: Simulated and theoretical SERs of the separated signals. (BPSK+BPSK, no CFOs, $\Delta\theta = 0.7$).

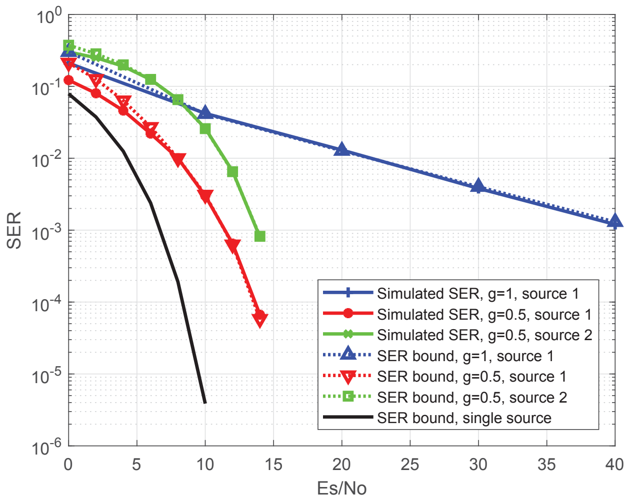


Fig. 4: Simulated and theoretical SERs of the separated signals. (BPSK+BPSK, CFOs exist).

SER bound for the situation that only one BPSK signal exists is also shown as a benchmark case. When $h_1 = h_2$, since the SERs of sources 1 and 2 are the same, only SER of source 1 is plotted.

From Figs. 3 and 4, it can be seen that for the mixture of two BPSK signals, the union bounds derived are tight in high SNR condition (or when the SER is lower than 10^{-2}). Comparing Fig. 3 and Fig. 4, it can be seen that when CFOs exist, the SERs of both the separated sources increase significantly when the two sources have identical power, i.e., $g = 1$. In this case, even at very high SNR (40dB) and with perfectly known CFOs, the SERs of sources 1 and 2 are still larger than 10^{-3}. This is unlike the single user case, in which the SERs obtained under the conditions that CFO exists or not are the same, as long as the CFO can be tracked or estimated accurately. The increase of the SERs is due to the fact that when CFOs exist, the SERs will be affected most by the SERs obtained in the worst case when CFOs do not exist, i.e., when the two signals have opposite phases and cancel or partially cancel with each other.

V. Conclusion

In this paper, we have conducted a theoretical analysis of the SERs of the symbols separated from a mixture of two PSK modulated signals occupying the same frequency band. Firstly, union bounds of the SERs of signals separated from the mixture of two arbitrary PSK signals with and without CFOs have been derived. Then, they are used to calculate SERs of the symbols separated from a particular mixture case, i.e., $M_1 = M_2 = 2$. Comparing with SERs obtained by computer simulations, it has been found that the union bounds derived are tight in high SNR condition.

The effect of different paramters to the SERs of the separated signals has also been analyzed in this paper. From both the theoretical and simulation results, it has been found that the effect of CFOs on the two-user case is unlike the single user case, i.e., the SERs of the separated signals is very high if the powers of the two signals are similar, even when channel SNR is very high and CFOs are perfectly known to the receiver. It has also been found that the SERs of the separated signals vary significantly with $\Delta\theta$, and they reach maximum when $\Delta\theta = 0$. In our future work, we will extend our theoretical analysis of the SERs of the separated signals to other conditions, e.g., $\Delta\tau > 0$, and we will also consider other channel models instead of AWGN channel, and the cases where more than two users exist in the mixture signals.

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