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CORRECTION

Correction to: Orientation preserving and orientation reversing mappings: a new description

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This corrigendum refers to Theorem 3 of [1], which characterises the sets of mappings on a finite cycle that preserve orientation, $\mathcal{OP}_n$, or that reverse orientation, $\mathcal{OR}_n$, in terms of actions of mappings on oriented triples. The statement of the theorem mistakenly omitted a necessary condition on the rank (the number of elements in the image) of the mappings concerned. Since the theorem clearly holds for constant mappings, a correct formulation is as follows.

Theorem *Let $\alpha \in \mathcal{T}_n$ be of rank other than 2. Then $\alpha \in \mathcal{OP}_n$ (resp. $\alpha \in \mathcal{OR}_n$) if and only if for every triple $S = (i, j, k)$ taken from $[n] := \{0, 1 \cdots, n-1\}$, the orientation of $S\alpha$ is the same as (resp. is opposite to) that of S.*

The argument given in [1] proves, without alteration, this amended theorem.

The approach to prove the converse direction is via the contrapositive in that we take a mapping $\alpha \notin \mathcal{OP}_n$ and identify a cyclic triple (r, s, t) such that $(r\alpha, s\alpha, t\alpha)$ is not cyclic.

Since $\alpha \notin \mathcal{OP}_n$, there exist distinct integers i, j such that $i\alpha > (i+1)\alpha$ and $j\alpha > (j+1)\alpha$ (addition modulo n). If $i\alpha \neq j\alpha$ or if $(i+1)\alpha \neq (j+1)\alpha$, then the rank of α is at least 3, as shown in [1, Theorem 3]. However in the case where $i\alpha = j\alpha$ and $(i+1)\alpha = (j+1)\alpha$, the argument needs to postulate the existence of a third member in the range of α, a condition explicitly assumed in Case (3) of the proof of [1, Theorem 3].

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In general, the exclusion of rank 2 mappings is necessary as, for example, the mapping $\alpha \in \mathcal{T}_4$ with $0\alpha = 2\alpha = 1$ and $1\alpha = 3\alpha = 0$ is not orientation-preserving although α preserves orientation for each triple.

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Reference

1. Higgins, P.M., Vernitski, A.: Orientation preserving and orientation reversing mappings: a new description. Semigroup Forum **104**(2), 509–514 (2022). https://doi.org/10.1007/s00233-022-10256-8