Carbon Prices Forecasting in Quantiles

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Abstract

This paper proposes two new methods (the Quantile Group LASSO and the Quantile Group SCAD models) to evaluate the predictability of a large group of factors on carbon futures returns. The most powerful predictors are selected through the dimension-reduction mechanism of the two models, while potential differences of the statistically significant predictors for different quantiles of carbon returns are carefully considered. First, we find that the proposed models outperform a series of competing ones with respect to prediction accuracy. Second, impacts of the selected predictors over the carbon price distribution are estimated through a quantile approach, which outperforms the mean shrinkage model in our case with data featured by a non-normal distribution. Specifically, the Brent spot price, the crude oil closing stock in the UK, and the growth of natural gas production in the UK are found to impact carbon futures returns only in extreme conditions with a strong asymmetric feature. Importantly, our estimators remain robust against the extreme event caused by the Covid-19. Our findings reveal that the identification of appropriate carbon return predictors and their impacts hinge on the carbon market conditions, and should be of interest to various stakeholders.

Keywords: Carbon return predictability, Dimension reduction techniques, Out-of-sample forecasting, Quantile regression, LASSO penalty, SCAD penalty, Variable selection

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1. Introduction

The rising concentration of greenhouse gases (GHGs) results in adverse consequences of global warming and climate change whereby the sustainability of human activities and development could be potentially weakened. In response to global climate change, the carbon market has been specifically developed as an effective mechanism of the carbon emissions reduction, while considering the carbon market dynamics has become an integral part of the worldwide policymaking (Zhu et al., 2018). Operating on the principle of 'cap-and-trade', the European Union Emission Trading System (EU ETS) initialized on January 2005 is the largest multinational carbon market worldwide so far to constrain CO_2 emissions by carbon-consumed industries in Europe.[1]

An accurate prediction of carbon price dynamics and an in-depth investigation of its determination are of great importance for various stakeholders involving academic researchers, policymakers, carbon-consumed installations, and financial investors (Zhu and Chevallier, 2017), whereas existing efforts are surprisingly sparse. Specifically, carbon price fluctuations directly impact the performance of carbon emissions reduction in carbon market (Zhu et al., 2018). Carbon price dynamics also affect the cost of most human activities and economic development (such as power production, modern transportation, land-use changes, etc.). The latter is known to be largely driven by the carbon-consumed energy (i.e., oil, natural gas, and coal), which are major sources of carbon emissions (Balcılar et al., 2016; Kara et al., 2008). As an emerging financial product, futures contracts of carbon allowances provide investors with an important instrument for the risk diversification in their investment portfolios (Paolella and Tas-

[1] According to European Commission (https://ec.europa.eu/clima/policies/ets_en), the EU ETS covers around 50% of total CO_2 emissions in EU and controls for the emissions from more than 11,000 carbon-intensive installations in 31 European countries.

chini, 2008). Importantly, it is known that the carbon price formation is characterized with asymmetry (Duan et al., 2021), extant literature that moves beyond the mean-based predictions is nevertheless scant. Thus, thorough forecasting for future carbon price movements while considering the impact of extreme events worldwide, e.g., the ongoing Covid-19 epidemic, is of paramount importance and still left for research.

We propose two innovative dimension-reduction and quantile forecasting methods, i.e. the quantile group least absolute shrinkage and selection operator (Quantile Group LASSO) and the Quantile Group SCAD models, to identify statistically significant predictors of the dynamics of carbon futures returns in the EU ETS over the carbon price distribution. Unlike the existing literature that usually learns predictors of carbon futures returns via a small number of variables in a narrowed field, this study includes a large number of predictors, which may possibly determine the dynamics of carbon futures returns. Our massive data enable us to include the related information as much as possible, however, traditional statistical models which are widely used in return forecasting could not incorporate massive amount of variables.

Therefore, we advocate these two novel methods that are able to identify 'key factors' among a large number of variables to improve the predictive efficiency for carbon returns. The high predictive accuracy and feasibility of these methods is demonstrated in our empirical analysis. In our study, the predictors are selected from a comprehensive pool related to the carbon market dynamics including 44 market fundamental variables and 18 technical variables.[2] Impacts of the most powerful carbon-return predictors, which are allowed to be different at various carbon quantiles, on carbon return dynamics are estimated through a quantile regression. Performance of our employed

[2]'Carbon price' and 'Carbon return' are used interchangeably in the paper, as like the literature we transform the carbon price into returns to avoid nonstationarity. Detailed descriptions are in the data section.

estimators remains robust when facing extreme events associated with the ongoing Covid-19 epidemic worldwide.

Our research contributes to the literature in the following ways. First, via a large set of candidate models we account for a large set of predictive sources regarding the carbon return dynamics from aspects of energy demand-supply fluctuations, energy price dynamics, stock price indicators, aggregate credit provisions, macroeconomic conditions, and technical indicators, respectively. Based on two sterling properties of our proposed methods, i.e., the 'interpretability of the final estimator' and the 'fast computation', we are able to identify the most powerful predictors among all potential ones for future dynamics of carbon returns, while allowing for potential differences of significant predictors at various carbon return quantiles.

Second, to evaluate the forecasting performance, through comparisons of the mean-squared prediction error ($MSPE$) and the mean absolute value of prediction error ($MAPE$), we demonstrate that the Quantile Group LASSO model and the Quantile Group SCAD model have superior out-of-sample predictability compared to the currently-popular methods. Meanwhile, regarding the predictor selection, these two methods consider and allow for the heterogeneity of significant predictors at different quantiles of carbon returns.

Third, in contrast to mean-based approaches, we further employ a quantile regression model to estimate distinct impacts of the selected forecasting factors on carbon futures returns across all market conditions in the data set. Applying the quantile approach to examining the tail behavior of carbon futures prices could better capture the true interdependence between the carbon return and its predictors. We find that the Brent oil price, the crude oil closing stock in the UK, and the growth of natural gas production in the UK statistically significantly affect carbon returns during extreme events (i.e., at low and high quantile levels). In addition, it is worth noting that our estimators are also shown to be robust against extreme events in the ongoing Covid-19 epidemic.

Overall, our empirical research possesses important implications to a wide group of entities, involving policymakers, carbon-consumed industrial productions, and investors, for an accurate cost assessment of carbon-consumed productions and activities, a sensible risk diversification of the investment portfolio, and an effective reduction of carbon market risks.

The rest of the paper is organized as follows: Section 2 summarizes the extant related literature in carbon price forecasting; Section 3 proposes our methodology. Section 4 introduces our data set as well as the main variables used in this study. Section 5 discusses the empirical results. Section 6 concludes.

2. Literature review

2.1. Carbon price prediction

How is our research connected with the extant literature? Previous studies employ various methods for the carbon price/return prediction.[3] Early research mainly uses a qualitative research approach to discuss carbon price prediction. In recent studies, considering changes in carbon price over time as a time series, popular time-series forecasting methods are extensively applied to the carbon price prediction. For example, Paolella and Taschini (2008) model the conditional dynamics of CO_2 and SO_2 price returns in the US and EU markets using a novel Generalized Autoregressive Conditional Heteroskedasticity (GARCH)-structure approach, and find that a mixed-normal

[3] See a recent review of related studies in Zhu et al. (2018).

GARCH model outperforms standard GARCH and other GARCH models in terms of the forecastability. Benz and Trück (2009) apply the Markov switching and AR-GARCH model to capture distinct behaviors of carbon return volatility in different regimes in the EU ETS and examine the improvement of its forecasting performance compared with conventional prediction methods without considering switching regimes.

Focusing on the EU carbon markets, Chevallier (2011b) applies a nonparametric approach for the carbon price prediction and investigates that the approach outperforms conventional linear autoregression models, where forecasting errors could be reduced by almost 15% through the nonparametric modeling. Byun and Cho (2013) focus on the European Climate Exchange market and apply the GARCH-structured models to forecast carbon price dynamics. They observe a more effective predictive power of GJR-GARCH model against TGARCH and standard GARCH models. Koop and Tole (2013) forecast carbon price dynamics in the EU ETS using the dynamic averaging method and examine its forecast accuracy compared to conventional methods. They further discuss the forecastability of market fundamental and institutional factors for the carbon price dynamics. Sanin et al. (2015) find that the Autoregressive Moving Average X (ARMAX)-GARCH approach with an additive stochastic jump process outperforms the standard ARMAX-GARCH approach regarding the carbon price prediction in the EU ETS. Overall, while there is a growing literature in the carbon price forecasting, the methodology is mostly based on an assumption of linear movements of carbon prices, and the potentially-existing nonlinearity is nevertheless neglected.

2.2. Nonlinear carbon-price pattern

To model the nonlinear carbon-price changing patterns, existing research mainly relies on the techniques of artificial intelligence and ensemble (hybrid), respectively. For example, Fan et al. (2015) forecast carbon price movements in the EU ETS using a multi-layered perception (MLP)-artificial neural networks (ANN) approach and find a better predictive performance than the single and variant models. At the same time, the ensemble (hybrid) method is developed to further improve the weakness of single models and enhance forecasting accuracy. For example, Zhu et al. (2016) conduct the carbon and energy price prediction using an ensemble empirical mode decomposition (EEMD)-based least square support vector machines (LSSVM) and examine more accurate forecasting performance of the EEMD-LSSVM compared to conventional methods. Sun et al. (2016) confirm the improvement of forecasting accuracy when combining variational mode decomposition (VDM) and spiking neural networks (SNN) in contrast to conventional methods. Zhu et al. (2018) propose a multiscale nonlinear ensemble learning framework, including EMD and LSSVM with a kernel function prototype for the prediction of carbon prices in the EU. They find high levels of predictive accuracy and robustness of their proposed methods compared to standard forecasting methods.

Although more sophisticated methods have been developed to account for the nonlinearity of carbon price dynamics, potentially heterogeneous change patterns of carbon prices at different price quantiles are neglected. Moreover, most of the extant literature conducts the carbon price prediction merely based on historical information of carbon price changes, whereas the predictive power of its forecasting factors is still nevertheless ignored. Indeed, it has been well-established that carbon price changes are determined by a large number of factors mainly involving energy market dynamics, financial market performance, technical indicators, weather and macroeconomic conditions (See, e.g., Zhang and Wei, 2010). Specifically, Alberola et al. (2008) conduct an econometric analysis to find carbon price drivers by identifying the potential structural breaks in the EU ETS. They point out that energy prices and weather conditions can explain changes in carbon price levels in EU ETS. Chevallier (2009) uses a series of

GARCH-structured models to analyze the relationship between carbon futures returns and macroeconomic-financial factors involving stock, bond, commodity markets, and macroeconomic factors based on the EU ETS. Chevallier (2011a) applies a Markov-switching VAR approach to identify the 'boom-bust' cycle in the EU carbon market and measures the determination of carbon pricing by macroeconomic factors and energy prices.

2.3. Carbon price determinants

In addition to macroeconomic factors, the impact of energy prices on carbon price determination has also been discussed. Kumar et al. (2012) conducts a VAR analysis and investigate the dynamic price linkage among carbon, fossil energy, and stock prices of clean energy and technology. Sadorsky (2012) applies a series of multivariate GARCH models and find strong correlations among oil prices and stock prices of clean energy and technology. Using a multivariate GARCH model, which can consider structural changes and the heterogeneity of price correlations between carbon market and market fundamentals in the economic upturn and downturn periods, Koch (2014) finds strong price linkages among carbon, energy, and financial markets. Ji et al. (2018) analyzes the information linkage and knowledge spillover between carbon and energy markets, viz. oil, natural gas, and coal, in the format of return and volatility, respectively. The close relationship between the oil price volatility and carbon prices have also been discussed in Gong and Lin (2017); Xu and Lin (2018); Gong and Lin (2021); Gong et al. (2021). While existing studies have discussed the determination of carbon prices considering different groups of forecasting factors, to the best of our knowledge, we are the first to investigate the predictability among possible forecasting factors of carbon prices in quantiles.

3. Methodology

In this section, we briefly introduce each of the candidate method we use to quantify the importance of potential Carbon price forecasting factors, as well as out-of-sample forecasting comparison method.

3.1. The candidate models

3.1.1. LASSO

The Least Absolute Shrinkage and Selection Operate (LASSO) proposed by Tibshirani (1996) is one of the most popular methods to solve the high dimensional estimation problem (See, e.g., Zhang et al., 2008). It penalizes the likelihood function and obtains a sparse solution.

The LASSO estimator is defined as

$$\hat{\beta}^{LASSO} = \arg\min_{\beta} \left\{ \frac{1}{N} \|Y - X\beta\|_2^2 + \lambda \sum_{i=1}^{N} |\beta_i| \right\},$$

where λ is the regularization parameter, and the ℓ_1 penalty $\sum_{i=1}^{N} |\beta_i|$ is employed to ensure sparsity.

With the increase of the regularization parameter λ, estimation parameters will be continuously shrunk towards zero by the LASSO. If the λ is large enough, some of them will be shrunk to exactly zero. According to this, the LASSO is often used in variable selection. Due to its high accuracy in prediction and variable selection, the LASSO is the most commonly used technique for solving high-dimensional estimation problems.

Our forecasts for the carbon price returns using LASSO are

$$\hat{p}_{t+1} = \hat{\beta}_0^{LASSO} + \sum_{i=1}^{N} \hat{\beta}_i^{LASSO} x_{i,t}.$$

Here

$$\hat{\beta}^{LASSO} = \arg\min_{\beta} \left\{ \frac{1}{t-1} \sum_{l=1}^{t-1} \left(p_{l+1} - \beta_0 - \sum_{i=1}^{N} \beta_i x_{i,l} \right)^2 + \lambda_{cv} \sum_{i=1}^{N} |\beta_i| \right\}, \quad (1)$$

where $\hat{\beta}^{LASSO}$ is the regression coefficients estimated by LASSO using the data up to month t, p_{t+1} is the log return of carbon prices at month $t+1$, $x_{i,t}$ is the ith predictor available at month t, and λ_{cv} is the non-negative regularization parameter selected by the cross-validation method.

3.1.2. Adaptive LASSO

The adaptive LASSO (Zou, 2006) is an advanced high-dimensional estimation method which is based on the LASSO. Unlike the LASSO which uses a standard ℓ_1 penalty, the adaptive LASSO employs a weighted ℓ_1 penalty, and therefore avoid the overestimation problem. Moreover, compared with LASSO, it holds consistent selection property with weaker conditions.

The adaptive LASSO estimator is

$$\hat{\beta}^{adapt} = \arg\min_{\beta} \left\{ \frac{1}{N} \|Y - X\beta\|_2^2 + \lambda \sum_{i=1}^{N} \frac{|\beta_i|}{|\hat{\beta}_{init,i}|} \right\}, \quad (2)$$

The adaptive LASSO estimator can be obtained in two steps. The first step is to obtain the weight value which is given by the formula (1) in LASSO, and the regularization parameter $\hat{\lambda}_{init,cv}$ in (1) is chosen by the cross-validation method, thus the weight value is $\hat{\beta}_{init,i} = \hat{\beta}(\hat{\lambda}_{init,cv})$. For the second step, we use the weight value in step 1, and chose the regularization parameter λ_{cv} in (2) by the cross-validation method again. In this way, we obtain the final estimator. The regularization parameters in adaptive LASSO are selected in step 1 and step 2 sequentially, and it is less computationally expensive than optimize them simultaneously.

Unlike the LASSO where the same regularization parameter λ are employed for all the parameters β_i $(i = 1, 2..., \ p)$ in the penalty term, the different parameter β_i in the adaptive LASSO has different penalty value which depends on the different weight value $\hat{\beta}_{init}$. Therefore, the adaptive LASSO has the following property:

(1) If $\hat{\beta}_{init,i} = 0$, then the estimator $\hat{\beta}_{adapt,i} = 0$, which ensures the sparsity of the solution.

(2) If $|\hat{\beta}_{init,i}|$ is large, then the value of penalty term for parameter β_i will be small. Similarly, if $|\hat{\beta}_{init,i}|$ is small, then the penalty value for parameter β_i will be large. Therefore, the adaptive LASSO not only has less biased estimators, but also avoid selecting undesired variables.

Our forecasts for the carbon price returns using Adaptive LASSO are

$$\hat{p}_{t+1} = \hat{\beta}_0^{adapt} + \sum_{i=1}^{N} \hat{\beta}_i^{adapt} x_{i,t}.$$

Here

$$\hat{\beta}^{adapt} = \arg\min_{\beta} \left\{ \frac{1}{t-1} \sum_{l=1}^{t-1} \left(p_{l+1} - \beta_0 - \sum_{i=1}^{N} \beta_i x_{i,l} \right)^2 + \lambda_{cv} \sum_{i=1}^{N} \frac{|\beta_i|}{|\hat{\beta}_{init,i}|} \right\},$$

where $\hat{\beta}^{adapt}$ is the regression coefficients estimated by the adaptive LASSO using the data up to month t, $\beta_{init,i}$ is an initial estimator, p_{t+1}, $x_{i,t}$ and λ_{cv} are defined the same as in (1).

3.1.3. Group LASSO

In some situations, the parametric vector β in a high-dimensional regression model has a group structure $\{g_1, g_2, ..., \; g_q\}$ which is essentially based on the index number $\{1, 2, ..., \; p\}$. That is, $\cup_{j=1}^{q} g_j = \{1, 2, ..., \; p\}$ and $g_j \cap g_k = \emptyset$.

Then the parametric vector β is

$$\beta = (\beta_{g_1}, \beta_{g_2}, ..., \; \beta_{g_q}), \; where \; \beta_{g_j} = \{\beta_r; \; r \in g_j\}.$$

The group LASSO estimator in a linear model (Yuan and Lin, 2006) is then defined as

$$\hat{\beta}^{group} = \arg\min_{\beta} \left\{ \frac{1}{N} \|Y - X\beta\|_2^2 + \lambda \sum_{j=1}^{q} m_j \|\beta_{g_j}\|_2 \right\}, \tag{3}$$

where $\|\beta_{g_j}\|_2$ denotes the standard Euclidean norm, that is $\|\beta_{g_j}\|_2 = \left(\sum_{l=1}^{k} \beta_{g_j,l}^2 \right)^{\frac{1}{2}}$. The multiplier m_j is used to balance cases where the groups are of very different sizes, usually we set

$$m_j = \sqrt{T_j}, \tag{4}$$

where T_j is the number of parameters in jth group. The advantages of the group LASSO estimator are two folds: First, it can deal with the data where features are organized into related groups. Second, it remains high prediction accuracy and estimation consistency as the LASSO.

Our forecasts for the carbon price returns using Group LASSO is

$$\hat{p}_{t+1} = \hat{\beta}_0^{group} + \sum_{i=1}^{N} \hat{\beta}_i^{group} x_{i,t}.$$

Here

$$\hat{\beta}^{group} = \arg\min_{\beta} \left\{ \frac{1}{t-1} \sum_{l=1}^{t-1} \left(p_{l+1} - \beta_0 - \sum_{i=1}^{N} \beta_i x_{i,l} \right)^2 + \lambda_{cv} \sum_{i=1}^{q} m_i \|\beta_{g_i}\|_2 \right\},$$

where $\hat{\beta}^{group}$ is the regression coefficients estimated by the group LASSO using the data up to month t, $\|\beta_{g_i}\|_2 = \left(\sum_{l=1}^{T_i} \beta_{g_i,l}^2\right)^{\frac{1}{2}}$, $m_i = \sqrt{T_i}$ with T_i being the number of parameters in ith group, p_{t+1}, $x_{i,t}$ and λ_{cv} are defined the same as in (1).

3.1.4. ARMA and ARMAX Models

Unlike the LASSO, the adaptive LASSO and the group LASSO which focus on high-dimensional regression problems, the AutoRegressive-Moving-Average (ARMA) model is one of the most famous methods in time-series analysis, which can understand and predict future value.

The ARMA model (Whittle, 1953, Box et al., 2015) is a combination of Autoregressive (AR) model and Moving-average (MA) model. The autoregressive model of order p which refers to AR(p) is written as

$$Y_t = c + \sum_{i=1}^{p} \alpha_i Y_{t-i} + \varepsilon_t$$

where Y_t is the observation at time t, $\alpha_1, ..., \alpha_p$ are parameters in AR(p) model, c is a constant, and the random variable ε_t is white noise which means they are independent and identically distributed with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = \sigma^2$.

The moving-average model of order q which refers to MA(q) is written as

$$Y_t = \mu + \varepsilon_t + \sum_{i=1}^{q} \beta_i \varepsilon_{t-i},$$

where Y_t is the observation at time t, $\beta_1, ..., \beta_q$ are parameters in MA(q) model, μ is the expectation of X_t, and ε_t, ε_{t-1} are white noise error terms.

Now the ARMA (p,q) model which refers to p autoregressive terms and q moving-average terms is written as

$$Y_t = c + \varepsilon_t + \sum_{i=1}^{p} \alpha_i Y_{t-i} + \sum_{i=1}^{q} \beta_i \varepsilon_{t-i},$$

where Y_t is the observation at time t, α_i and β_i are parameters in ARMA(p,q) model, c is a constant and ε_t, ε_{t-1} are white noise error terms.

The AutoRegressive-Moving-Average model with exogenous inputs (ARMAX) is a generalization of the ARMA model. The ARMAX(p,q,g) model adds external covariates to an ARMA (p,q) model, which is given by

$$Y_t = c + \varepsilon_t + \sum_{i=1}^{p} \alpha_i Y_{t-i} + \sum_{i=1}^{q} \beta_i \varepsilon_{t-i} + \sum_{i=1}^{g} \gamma_i X_{t-i},$$

where γ_i is the parameter of the exogenous covariate X.

By comparing the autocorrelation (ACF) function which gives correlations between p_t and $p_t - h$ for $h = 1, 2, 3...$, we use the following ARMA (1,1) and ARMAX(1,1,1) models to understand and predict our carbon price return data

- ARMA (1,1):

$$p_t = c + \alpha p_{t-1} + \varepsilon_t + \beta \varepsilon_{t-1},$$

- ARMAX(1,1,1):

$$p_t = c + \alpha p_{t-1} + \varepsilon_t + \beta \varepsilon_{t-1} + \gamma X_{t-1},$$

where p_t is the observation of the carbon price return at time t, X_{t-1} is the external covariate at time $t-1$, α, β, and γ are parameters, c is a constant, and ε_t is the white noise.

Then the forecasts of the carbon price returns using ARMA (1,1) model is

- one-month ahead: $\hat{p}_{t+1} = \hat{c} + \hat{\alpha} p_t + \hat{\beta}\hat{\varepsilon}_t$
- two-months ahead: $\hat{p}_{t+2} = \hat{c} + \hat{\alpha}\hat{p}_{t+1}$
- three-months ahead: $\hat{p}_{t+3} = \hat{c} + \hat{\alpha}\hat{p}_{t+2}$

  $\vdots$

- i-months ahead: $\hat{p}_{t+i} = \hat{c} + \hat{\alpha}\hat{p}_{t+i-1}$

where $\hat{p}_{t+i}$ is the predicted value of the carbon price return at time $t+i$, p_t is the true value of the carbon price return at time t, $\hat{\varepsilon}_t = p_t - \hat{p}_t$, and $\hat{c}$, $\hat{\alpha}$, $\hat{\beta}$ are parametric estimators of ARMA(1,1) model. Here we omit the prediction procedure of the ARMAX(1,1,1) model which can be constructed similarly.

3.1.5. GARCH and GARCHX Models

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model (Engle, 2001) is the most commonly used financial time-series model and has inspired a family of sophisticated models in econometrics (i.e., GARCH-family). The GARCH model is a generalized version of the Autoregressive conditional heteroskedasticity (ARCH) model, which describes the variance of the current error term as a function of the actual sizes of the previous periods' error terms. When the variance of the error is assumed to follow the autoregressive moving average (ARMA) model, this model is called the GARCH model.

The GARCH(p,q) regression model is defined by

$$y_t = \mu_t + \varepsilon_t,\ \ \varepsilon_t|\Psi_{t-1} \sim N(0, \sigma_t^2)$$
$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2,$$

where Ψ_{t-1} denotes the information set at time $t-1$, μ_t is the expected value of y_t at time t, ε_t is the error term at time t, σ_t^2 is the variance of the current error term conditioned on all the information up to time $t-1$, ω, α_i, β_j are parameters in GARCH(p,q) model, $\omega > 0$, $\alpha_i \geq 0$, $\beta \geq 0$.

The main idea of GARCH model is that the conditional variance σ_t of current error term ε_t given information up to time $t-1$ is correlated to its own past values σ_{t-j}^2 $(j = 1, 2, ..., q)$ and the recent values of squared errors ε_{t-i}^2 $(i = 1, 2, ..., p)$. This model can be augmented with exogenous variables, which is the so-called GARCHX model.

By comparing the autocorrelation (ACF) function which gives correlations between p_t and $p_t - h$ for $h = 1, 2, 3...$, we use the following GARCH (1,1) and GARCHX (1,1,1) models to understand and predict our carbon price return data

$$p_t = \mu_t + \varepsilon_t,\ \ \varepsilon_t|\Psi_{t-1} \sim N(0, \sigma_t^2)$$

- GARCH (1,1):

$$\sigma_t^2 = \omega + \alpha p_{t-1}^2 + \beta \sigma_{t-1}^2,$$

- GARCHX (1,1,1):

$$\sigma_t^2 = \omega + \alpha p_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma X_{t-1},$$

where p_t is the observation of carbon price return at time t, X_{t-1} is the exogenous covariate at time $t-1$, μ_t is the expected value of p_t, ω, α, β, and γ are parameters.

3.2. Our models

3.2.1. Quantile Group LASSO

The quantile regression (Koenker and Hallock, 2001), which focuses on obtaining the information of conditional median or conditional quantiles of the response, is an important analysis method in econometrics and statistics (e.g., Koenker, 2004; Machado and Mata, 2005; Buchinsky, 1994; Yu et al., 2003). Compared with the standard linear regression, which is only able to capture the relationship between the predictors and the mean response, the quantile regression can provide more information about different conditional quantiles of the response, and therefore outliers have fewer effects in the analysis.

Let Y be a random variable and the cumulative distribution function is

$$F_Y(y) = P(Y \leq y),$$

then the τth quantile of Y is defined by

$$Q_Y(\tau) = F_Y^{-1}(\tau) = inf\{y : \; F_Y(y) \geq \tau\}.$$

where inf is the infimum. Suppose the τth quantile function is

$$Q_{Y|X} = X^T\beta_\tau,$$

then the parametric estimator $\hat{\beta}_\tau$ is given by

$$\hat{\beta}_\tau = \arg\min_\beta \sum_{i=1}^{n} (\rho_\tau(y_i - x_i^T \beta)),$$

where the check function $\rho_\tau(u) = u\{\tau - I(u \leq 0)\}$ and I is an indicator function.

During the last decade, the analysis for high-dimensional data has drawn much attention. The key feature of the high-dimensional problem is the number of predictors is larger than the sample size, and the most common way for solving this problem is to introduce a penalty term in the estimation function.

The parametric estimator in the penalized quantile regression model is

$$\hat{\beta}_\tau = \arg\min_\beta \left\{ \frac{1}{N} \sum_{i=1}^{N} (\rho_\tau(y_i - x_i^T \beta)) + \sum_{j=1}^{p} p_\lambda(|\beta_j|) \right\},$$

where $\rho_\tau(u)$ is the check function and $p_\lambda(\cdot)$ is a penalty function with a tuning parameter λ.

In this study, we employ two most popular and commonly used penalties: The LASSO penalty (Tibshirani, 1996) and the SCAD penalty (Fan and Li, 2001). Moreover, due to the structure of potential forecasting factors, we use more proper and advanced versions which are constructed on the LASSO and the SCAD, respectively: The Group LASSO penalty (Yuan and Lin, 2006), and the Group SCAD penalty (Wang et al., 2007). Both are wildly used in statistical and economic analysis (See, e.g., Meier et al., 2008).

Suppose the parametric vector β has a group structure $\{g_1, g_2, ..., \ g_q\}$ which is a combination of the index number $\{1, 2, ..., \ p\}$. That is, $\cup_{j=1}^{q} g_j = \{1, 2, ..., \ p\}$ and

$g_j \cap g_k = \emptyset$, then the parametric vector β is

$$\beta = (\beta_{g_1}, \beta_{g_2}, ..., \beta_{g_q}),\ where\ \beta_{g_j} = \{\beta_r;\ r \in g_j\}.$$

The parametric estimator in the penalized quantile regression with Group LASSO penalty is defined as

$$\hat{\beta}_{\tau}^{qgLASSO} = \arg\min_{\beta} \left\{ \frac{1}{N} \sum_{i=1}^{N} (\rho_\tau (y_i - x_i^T \beta)) + \lambda \sum_{j=1}^{q} m_j \|\beta_{g_j}\|_2 \right\}, \tag{5}$$

where β_{g_j} are the parameters in g_jth group, $\|\beta_{g_j}\|_2$ denotes the standard Euclidean norm $\|\beta_{g_j}\|_2 = \left(\sum_{l=1}^{k} \beta_{g_j,l}^2 \right)^{\frac{1}{2}}$. The multiplier m_j is used to balance cases where the groups are of very different sizes, usually we set $m_j = \sqrt{T_j}$, where T_j is the number of parameters in jth group.

Compared with the classic penalization method LASSO (Tibshirani, 1996), which intends to select explanatory variables individually, the group LASSO penalty proposed by Yuan and Lin (2006) considers a common scenario that features can be organized into related groups. In this case, there is indeed information contained in the grouping structure, thus ignoring it and using standard methods will lead inaccurate estimators. The quantile group LASSO estimator in (5) employs the group LASSO penalty in the classic quantile regression, which makes it not only be able to capture the information in the feature groups, but also can discover useful predictive relationships between variables under different quantile levels.

Our forecasts of the carbon price returns at the median case using the Quantile Group LASSO are

$$\hat{p}_{t+1} = \hat{\beta}_{\tau_t,0}^{qgLASSO} + \sum_{i=1}^{N} \hat{\beta}_{\tau_t,i}^{qgLASSO} x_{i,t}.$$

Here

$$\hat{\beta}_{\tau_t}^{qgLASSO} = \arg\min_{\beta} \left\{ \frac{1}{t-1} \sum_{l=1}^{t-1} (\rho_{\tau_t}(p_{l+1} - \beta_0 - \sum_{i=1}^{N} \beta_i x_{i,l})) + \lambda \sum_{i=1}^{q} m_i \|\beta_{g_i}\|_2 \right\},$$

where $\hat{\beta}_{\tau_t}^{qgLASSO}$ is the regression coefficients estimated by the Quantile Group LASSO using the data up to month t with $\tau = 0.5$, β_{gi} are the parameters in g_ith group and $\|\beta_{g_i}\|_2 = \left(\sum_{l=1}^{T_i} \beta_{g_i,l}^2\right)^{\frac{1}{2}}$, p_{t+1} is the log return of carbon price at month $t+1$, $x_{i,t}$ is the ith predictor available at month t, $m_i = \sqrt{T_i}$ and T_i is the number of parameters in ith group.

3.2.2. Quantile Group SCAD

The parametric estimator in the penalized quantile regression with group SCAD penalty is

$$\hat{\beta}_{\tau}^{qgscad} = \arg\min_{\beta} \left\{ \frac{1}{N} \sum_{i=1}^{N} (\rho_{\tau}(y_i - x_i^T \beta)) + \sum_{j=1}^{q} P_{\lambda} \left(\|\beta_{g_j}\|_2 \right) \right\},$$

where β_{g_j} are the parameters in g_jth group, and $P_{\lambda}(\cdot)$ is the group SCAD penalty which is defined as

$$P_{\lambda}(|x|) = \begin{cases} \lambda|x|, & \text{if } |x| \leq \lambda. \\ -\frac{(|x|^2 - 2a\lambda|x| + \lambda^2)}{2(a-1)}, & \text{if } \lambda < |x| < a\lambda. \\ \frac{(a+1)\lambda^2}{2}, & \text{if } |x| > a\lambda. \end{cases}$$

Our forecasts of the carbon price returns at the median case using Quantile Group SCAD is

$$\hat{p}_{t+1} = \hat{\beta}_{\tau_t,0}^{qgscad} + \sum_{i=1}^{N} \hat{\beta}_{\tau_t,i}^{qgscad} x_{i,t} \tag{6}$$

Here

$$\hat{\beta}^{qgscad}_{\tau_t} = \arg\min_{\beta} \left\{ \frac{1}{t-1} \sum_{l=1}^{t-1} (\rho_{\tau_t}(p_{l+1} - \beta_0 - \sum_{i=1}^{N} \beta_i x_{i,l})) + \sum_{j=1}^{q} P_\lambda \left(\|\beta_{g_j}\|_2 \right) \right\},$$

where $\hat{\beta}^{qgscad}_{\tau_t}$ is the regression coefficients estimated by Quantile Group SCAD using the data up to month t with $\tau = 0.5$, p_{t+1}, $x_{i,t}$ and β_{gi} are defined the same as in (6).

3.3. Out-of-sample Comparisons

The out-of-sample performance test, which can avoid the over-fitting problem of using the whole data set, is commonly used to test statistical models' prediction ability in many areas, such as statistics, econometrics, envirometrics, computer science and so on (See, e.g., Welch and Goyal, 2007; Rapach et al., 2010; Clark and West, 2006). It is conducted by dividing the original data set into two parts: the in-sample data set and the out-of-sample data set. We train the statistical model in the in-sample data set, and then compare the forecasting result of the obtained statistical model with the original data in the out-of-sample data set.

Inspired by Campbell and Thompson (2007) and jointly considering the sterling performance and wide applications of the following criteria in the extant literature, we employ the mean-squared prediction error ($MSPE$), the mean absolute prediction error ($MAPE$), the R^2 statistic of mean-squared prediction error (R^2_{MSPE}) and the R^2 statistic of the absolute value of prediction error (R^2_{MAPE}) to compare the out-of-sample prediction accuracy of the candidate forecast model (Quantile Group LASSO model and the Quantile Group SCAD model) with the benchmark model (the LASSO model, the adaptive LASSO model, the group LASSO model, the ARMA model, and the GARCH model).

The out-of-sample R^2 statistic of mean-squared prediction error (R^2_{MSPE}) is

$$R^2_{MSPE} = 1 - \frac{MSPE_C}{MSPE_B},$$

and

$$MSPE_C = \frac{1}{q}\sum_{i=1}^{q}(r_{m+i} - \hat{r}^C_{m+i})^2,$$

$$MSPE_B = \frac{1}{q}\sum_{i=1}^{q}(r_{m+i} - \hat{r}^B_{m+i})^2,$$

where $MSPE_C$ is the mean-squared prediction error of the candidate model and $MSPE_B$ is the mean-squared prediction error of the benchmark model, r_{m+i} is the actual carbon price return at time $m+i$, $\hat{r}^C_{m+i}$ and $\hat{r}^B_{m+i}$ are the predicted carbon price returns of the candidate model and the benchmark model at time $m+i$ respectively, m is the length of the in-sample estimation data set, and q is the length of the out-of-sample prediction data set.

Similarly, the out-of-sample R^2 statistic of the absolute value of prediction error (R^2_{MAPE}) is

$$R^2_{MAPE} = 1 - \frac{MAPE_C}{MAPE_B},$$

and

$$MAPE_C = \frac{1}{q}\sum_{i=1}^{q}|r_{m+i} - \hat{r}^C_{m+i}|,$$

$$MAPE_B = \frac{1}{q}\sum_{i=1}^{q}|r_{m+i} - \hat{r}^B_{m+i}|,$$

where $MAPE_C$ is the average absolute value of the prediction error of the candidate model and $MAPE_B$ is the average absolute value of the prediction error of the benchmark model. Here r_{m+i}, $\hat{r}^C_{m+i}$ and $\hat{r}^B_{m+i}$ are defined the same as before.

The R^2_{MSPE} statistic and the R^2_{MAPE} statistic can evaluate the proportional reduction of the prediction errors $MSPE$ and $MAPE$ for the candidate forecast model to the benchmark model respectively. To know whether the accurate predictability of the candidate forecast model is better than the benchmark model, people usually test whether the MSPE of the candidate forecast model is smaller than the MSPE of the benchmark model, which means the candidate forecast model has more accurate out-of-sample performance, or the MSPE of candidate forecast model is larger than or equals to the MSPE of the benchmark model, which means the candidate forecast model is not that competitive (See, e.g., Campbell and Thompson, 2007; Baumeister and Kilian, 2015; Wang et al., 2017). With the same spirit, here a positive value of R^2_{MSPE} indicates that compared with the benchmark forecast model, the candidate forecast model is more accurate and has less prediction error in terms of the $MESP$ criterion in the out-of-sample data set. Similarly, a positive value of R^2_{MAPE} means the candidate model has better forecast ability than the benchmark model in terms of the $MAPE$ criterion in the out-of-sample data set. Moreover, the larger values of R^2_{MSPE} and R^2_{MAPE} mean the candidate model has higher forecast accuracy than the benchmark model.

4. Data

4.1. Carbon price

Although the underlying asset of the Carbon futures is an annual product[4] , the prices of Carbon futures are constantly fluctuating during trading hours. To construct our dependent variable, we collect the monthly observations of carbon futures closing price

[4]We thank an anonymous referee for pointing it out.

at the end of each month from the ICE ECX EUA futures continuous contract #1[5]. These data enable us to construct monthly observations for our key dependent variable of interest: the carbon futures returns, which is defined as logarithmic monthly changes in carbon futures closing prices. Other data are collected from DataStream covering the period from March 2009 to December 2020. Table 1 reports the descriptive statistics for our response variable —the carbon futures price return.

[Table 1 about here.]

The monthly carbon price return is skewed left as the negative skewness value in Table 1, and the kurtosis value means the data is platykurtic. The Jarque-Bera test shows the carbon price return series is not normally distributed. This information drives us to use the Quantile Group LASSO and Quantile Group SCAD models, which can capture different information under different quantiles when the data doesn't hold the normality assumption. Based on the ADF test statistic, the data is stationary as the null hypothesis of non-stationary is rejected at the 5% significance level.

As in Inoue and Kilian (2005), we divide the whole sample set into a in-sample training data set and a out-of-sample forecast data set. First, we obtain the parametric estimators of each model in the in-sample training data set, and then use them to get forecasting results in the out-of-sample period. Second, we compare the forecasting results with the true value in the out-of-sample data set. The in-sample training period spans from March 2009 to December 2019, and then we make the six-month and twelve-month forecasts of the carbon price return.

[5]https://www.theice.com/index

4.2. Predictors

In this paper, we use a large number of predictors, including 18 technical indicators and 44 macroeconomic variables. To ensure the stationarity, the macroeconomic variables have been preprocessed by taking logarithm and differencing transformations, i.e., the log returns. The technical indicators could provide more information where the importance and the predictive ability of technical analysis have been found (See, e.g., Gehrig and Menkhoff, 2006; Neely et al., 2014; Tan et al., 2021). In this paper, we employ 18 technical indicators suggested by Yin and Yang (2016). These technical indicators are constructed on the following three technical rules: the moving-average (MA) rule, the momentum (MOM) rule, and the on-balance volume averages (VOL).

1. The moving-average rule, which is a mechanical trading rule and aims to capture trends, is constructed by generating a buy signal ($S_{i,t} = 1$) or sell signal ($S_{i,t} = 0$) at the end of time t by comparing two moving averages:

$$S_{i,t} = \begin{cases} 1, \ if \ MA_{s,t} \geq MA_{l,t} \\ 0, \ if \ MA_{s,t} < MA_{l,t} \end{cases}$$

   and

$$MA_{j,t} = \frac{1}{j} \sum_{i=0}^{j-1} P_{t-i} \ for \ j = s, l$$

   where P_t is the level of carbon price at time t, s is the length of the short MA, l is the long-term MA and $s < l$. Based on the above formula, the MA rule is sensitive about the changes in price trends. In this paper, we use six moving-average indicators with $s = 1, 2, 3$ and $l = 9, 12$.

2. The momentum rule is constructed by generating a buy signal ($S_{i,t} = 1$) or sell

signal ($S_{i,t} = 0$) at the end of time t by comparing the current carbon price and its level at m periods ago:

$$S_{i,t} = \begin{cases} 1, \; if \; P_t \geq P_{t-m} \\ 0, \; if \; P_t < P_{t-m} \end{cases}$$

where P_t denotes the carbon price at time t, and P_{t-m} denotes the level of carbon price at m periods ago. We use six momentum indicators with $m = 1, 2, 3, 6, 9, 12$.

3. The on-balance volume averages rule, which aims to capture market trend using past prices, is constructed by generating a buy signal ($S_{i,t} = 1$) or sell signal ($S_{i,t} = 0$) at the end of time t by comparing two moving averages based on OBV_t:

$$S_{i,t} = \begin{cases} 1, \; if \; MA^{OBV}_{s,t} \geq MA^{OBV}_{l,t} \\ 0, \; if \; MA^{OBV}_{s,t} < MA^{OBV}_{l,t} \end{cases}$$

and

$$MA^{OBV}_{j,t} = \frac{1}{j} \sum_{i=0}^{j-1} OBV_{t-i} \; for \; j = s, l,$$

$$OBV_t = \sum_{k=1}^{t} VOL_k, D_k$$

where VOL_k is the trading volume during month k, D_k is a binary variable which equals 1 if $P_k - P_{k-1} \geq 0$ and takes a value of -1 if $P_k - P_{k-1} < 0$. Similar to the moving-average rule, we employ six on-balance volume average indicators for $s = 1, 2, 3$ and $l = 9, 12$.

To construct the above 18 technical indicators, we use the data of carbon price from Datastream and trading volumes of ICE ECX EUA Futures Contract 1.

Besides these technical variables, by referring to Neely et al. (2014) and Tan et al.

(2021), we also consider the 44 macroeconomic variables covering information of energy commodities, financial markets and economic activities, which may have the predictive power for the carbon price. These data are available in DataStream, EIA[6], and ICE[7]. Moreover, the 44 macroeconomic variables can be divided into the following groups: the energy source group, the energy price group, the stock market index group, the monetary policy group, and the economic information group, where each group usually contains similar information from different countries. The full group information is provided in List 1 of the Appendix.

5. Empirical results

This section presents and discusses our empirical results. We start with a horse race of a set of forecasting models, and move to analysis of forecasting factors for the dynamics of carbon prices. A quantile analysis is then conducted to investigate the impacts of the selected predictors for different quantiles of carbon returns over the carbon price distribution. Robustness of our estimators and the corresponding findings in the face of extreme events associated with the ongoing Covid-19 epidemic is further examined.

5.1. A horse race of forecasting models

5.1.1. Baseline Out-of-sample Forecasting Results

First, we report the results of the MSPE and MAPE of the six-month and twelve-month out-of-sample forecast tests in Table 2[8]. Table 2(a) shows the result in the six-month

[6]Source: The US Energy Information Administration https://www.eia.gov

[7]Source: The Intercontinental Exchange https://www.theice.com/

[8]We also make the three-month out-of-sample forecast, which are consistent with our six-month and twelve-month out-of-sample forecast tests. While the three-month results are not reported due to limited space in the paper, they are available from the authors upon request.

forecast, and Table 2(b) shows the result in the twelve-month forecast. The list of candidate forecast models is in the first column of each table[9]. Here the forecasting abilities of the Quantile Group LASSO and the Quantile Group SCAD models are evaluated at the median quantile level, i.e., the 50% quantile, for fair comparisons with the competing models which do not take quantiles into consideration.

[Table 2 about here.]

The smaller MSPE and MAPE values indicate higher prediction accuracy of the forecast model. Therefore, the most important finding in Table 2 is that in both the six and twelve months forecast tests, the Quantile Group LASSO and the Quantile Group SCAD models have much smaller MSPE and MAPE values than all the other models. This means these two models have better prediction performances than the competing models in our test.

Although the ARMA, GARCH, ARMAX(S) and GARCHX(S) models are the classical and widely used time-series models in economics and financial analysis, here all of them have statistically significantly larger MSPE and MAPE values, which means lower prediction accuracy, compared with the rest high-dimensional forecast model group (the Quantile Group LASSO model, the Quantile Group SCAD model, the LASSO model, the adaptive LASSO model, the group LASSO model).

Among the high-dimensional forecast model group, the Quantile Group LASSO and the Quantile Group SCAD models have much better performance than others. In the six-month forecast, the Quantile Group LASSO model has the lowest MAPE value and the second-lowest MSPE value, and the Quantile Group SCAD model has the lowest MSPE value and the same lowest MAPE value. In the twelve-month forecast, the

[9]The ARMAXS and GARCHXS in the list are the ARMAX and GARCHX models with selected variables described in Section 5.2.1.

Quantile Group LASSO model has the smallest values for both MSPE and MAPE, and the Quantile Group SCAD model has the second smallest MSPE and MAPE values. In other words, no matter for MSPE or MAPE criterion, in the six-month and twelve-month forecast tests, the Quantile Group LASSO and Quantile Group SCAD models are the top two models in terms of prediction accuracy.

5.1.2. Comparisons among high-dimensional models using time-series models as benchmarks

From Table 2, we know that the time-series forecast group (the ARMA (1,1), GARCH (1,1), ARMAX (1,1,1), GARCHX (1,1,1), ARMAXS (1,1,1), and GARCHXS (1,1,1) models) has obvious lower prediction accuracy than the high-dimensional forecast group (the Quantile Group LASSO, the Quantile Group SCAD, the LASSO, the adaptive LASSO, and the group LASSO) in both the six-month and twelve-month forecasts. To quantitatively compare the forecasting ability of these two groups, we set the ARMA (1,1) and GARCH (1,1) models, which have relatively better forecasting performances in the time-series group, as the benchmark models and the rest models as the candidate models. The R^2_{MSPE} and R^2_{MAPE} values are employed to evaluate the predictive ability of each model. It is worth mentioning that the positive (negative) R^2_{MSPE} and R^2_{MAPE} values indicate that the candidate model has better (lower) forecasting accuracy than the benchmark model. The larger positive value of R^2_{MSPE} and R^2_{MAPE}, the better prediction ability of the candidate forecast model compared with the corresponding benchmark model.

The following Table 3 contains the results of R^2_{MSPE} and R^2_{MAPE} in the six-month and twelve-month forecasts. Table 3(a) and 3(b) show the results when the benchmark model is the ARMA (1,1) model and the GARCH (1,1) model, respectively. The upper part (i.e., Panel A) in each table is the results in the six-month forecast, and the bottom part (i.e., Panel B) is the results in the twelve-month forecast. The list of candidate forecast models is in the first column of each table.

[Table 3 about here.]

The most important finding in Table 3 is that the Quantile Group LASSO and the Quantile Group SCAD have larger R^2_{MSPE} and R^2_{MAPE} values than all the other models in both the six-month and twelve-month forecasts, no matter the benchmark model is the ARMA (1,1) model or the GARCH(1,1) model. This means compared with these two well-known time-series models, the Quantile Group LASSO and the Quantile Group SCAD have much better prediction performances in our out-of-sample tests. Besides, all the R^2_{MSPE} and R^2_{MAPE} values of the high-dimensional forecast group in Table 3 are positive, except the Adaptive Lasso which has been found unstable in our experiments. This indicates that most of our high-dimensional forecasting models have higher forecast accuracy than the time-series forecasting models.

It is also worth noticing that in the six-month forecast, all the time-series candidate forecasting models (the ARMAX (1,1,1), GARCHX (1,1,1), ARMAXS (1,1,1), and GARCHXS (1,1,1)) have negative R^2_{MSPE} and R^2_{MAPE} values. For the twelve-month forecast, the ARMAX (1,1,1) and ARMAXS (1,1,1) models also have negative R^2_{MSPE} and R^2_{MAPE} values, while the GARCHX (1,1,1) and GARCHXS (1,1,1) models have small positive R^2_{MSPE} values, but part of the R^2_{MAPE} values are still negative. This means brute-force introduction of a large number of variables into the time-series models cannot provide better prediction accuracy than the vanilla ARMA (1,1) and GARCH (1,1) models.

Although compared with the ARMAX (1,1,1) and the GARCHX (1,1,1) models which consider all possible variables, the ARMAXS (1,1,1) and the GARCHXS (1,1,1) with carefully selected variables have better forecasting performances, they are still not comparable to the ARMA (1,1) and the GARCH(1,1) models, let alone the aforementioned high-dimensional forecast group. This indicates that information of relative variables should be smartly wrapped into the forecasting models, just as our high-dimensional group, rather than naively combining them.

To conclude, the high-dimensional model, which can predict future prices by capturing important information from a large number of variables, is more accurate than the time-series models, no matter these time-series models consider the variables or not. Among the high-dimensional forecast group, the Quantile Group LASSO model and the Quantile Group SCAD model have better prediction performances than all the other methods, for the reason that they can obtain more information of forecasting factors at different quantiles, thus are more accurate and proper in the situation where the response series is not normally distributed.

5.1.3. Comparisons between Quantile Group LASSO/SCAD and other high-dimensional methods

From Table 3, we can know that the Quantile Group LASSO and the Quantile Group SCAD have better forecasting results than others. To quantitatively compare the prediction ability of these two models with the competing ones, we set the Quantile Group LASSO and the Quantile Group SCAD as the candidate models respectively, and all the other models as the benchmark models.

The following Table 4 reports the results of R^2_{MSPE} and R^2_{MAPE} in both the six-month and twelve-month forecast tests. Table 4(a) and 4(b) show the results when the candidate models are the Quantile Group LASSO and the Quantile Group SCAD, respectively. The upper part in each table is the results in the six-month forecast, and the bottom part is the results in the twelve-month forecast. The list of benchmark forecast models is in the first column of each table.

[Table 4 about here.]

According to Table 4, there are larger R^2_{MSPE} and R^2_{MAPE} values when the time-series models as the benchmark in both the six-month and twelve-month forecasts. It indicates that the Quantile Group LASSO and the Quantile Group SCAD have greatly improved the prediction accuracy over the time series model group, as R^2_{MSPE} and R^2_{MAPE} values show how much accuracy of the candidate model has improved over the benchmark model. This finding is consistent with the results in Table 2 and 3 as well. Besides, all the values in Table 4 are positive, which implies the Quantile Group LASSO and the Quantile Group SCAD have better prediction results than all the other models in our experiments. This finding is exciting since these competing models are widely used in many areas (See, e.g., Engle, 2001; McLeod and Li, 1983; Varian, 2014).

In summary, the Quantile Group LASSO and the Quantile Group SCAD models have the best out-of-sample prediction performances, and the time-series group has the worst prediction results in our six-month and twelve-month forecast tests. In addition, the high-dimensional forecast group (the quantile group LASSO, the quantile group SCAD, the LASSO, and the group LASSO) have higher forecast accuracy than the time-series group, even if the time-series models take into account the relative variables as well. This indicates that the high-dimensional models are better at handling information from a large number of important variables, thus are more accurate than the traditional time-series models in prediction. More importantly, among these high-dimensional models, the Quantile Group LASSO and the Quantile Group SCAD, which have more flexibility and fewer model restrictions, are useful in the case where the response series has a complex distribution.

5.2. Analysis of forecasting factors

5.2.1. General information

As we mentioned before, the Quantile Group LASSO and the Quantile Group SCAD models can select the most important variables and use them to implement forecasting. In the last section, we focus on the forecasting results. Now we continue to analyze the potential forecasting factors.

Based on the related literature (See, e.g., Fezzi and Bunn, 2009; da Silva et al., 2016; Hammoudeh et al., 2014; Tan and Wang, 2017), we consider a large set of 44 macroeconomic variables, and they are divided into the following five groups: the energy source group, the energy price group, the stock market index group, the monetary policy group, and the economic information group. One of the most important findings in the variable selection is that both the Quantile Group LASSO and the Quantile Group SCAD only select the variables in the energy source group and the energy price group. In other words, among all the macroeconomic variables, the monthly carbon futures price is affected by the crude oil and natural gas only. This information is beneficial since it sheds light on the most important factors affecting the carbon futures price. The common variables selected by the Quantile Group LASSO and the Quantile Group SCAD are as follows: the Europe Brent spot price, the growth of crude oil import in the United Kingdom, the growth of crude oil import in Germany, the growth of crude oil stock in the United Kingdom, the growth of natural gas import in France, the growth of natural gas import in the United Kingdom, and the growth of natural gas import in Italy.

These forecasting factors suggest that, among a large number of factors that consists of the traditional energy (oil, gas) price and demand, the economic factors and financial market index, only the Brent spot price and the demand for crude oil and natural gas are the determinants of the carbon price.

The finding that the Brent price has a link to the carbon price in the EU is consistent with other studies. For example, Bachmeier and Griffin (2006) showed the Brent price is the key factor of the carbon price. Mansanet-Bataller et al. (2007) found that the Brent price is the most important variable affecting the carbon price return, Fezzi and Bunn (2009) and Alberola et al. (2008) showed that the energy price highly influences the carbon price.

For the relationship between the demanding of traditional energy (oil and nature gas) and the carbon price, the impact is quite intuitive: the more imports of crude oil and natural gas, the more likely to have higher energy consumption, and hence the more likely to have increased CO_2 emission, and therefore the more likely larger CO_2 allowances are needed which affects the carbon price. This finding coincides with some literature. For instance, Chevallier (2011a) showed that economic activities influences the carbon price. Declercq et al. (2011) investigated the relationship between the economic recession and the CO_2 emission. Bredin and Muckley (2011) also highlighted the impact of economic activities and the industrial production on the carbon price.

However, for the rest forecasting factors, there are some debates in the literature, and our findings provide some new perspectives. For example, Chevallier (2009) showed that the interest rate and treasury bill yields are not robust in the carbon price forecast. However, there are some factors found by other studies but unselected here. For instance, Oberndorfer (2009) found that there is a relationship between the stock market index of the EU and the carbon price. Chevallier (2009) showed that the stock and bond markets in the EU affect the carbon price.

Except for the macroeconomic variables, our analysis also sheds light on the role of the technical indicators. Some literature stated they have advantages over the standard fundamental variables in terms of forecasting. See, e.g., Neely et al. (2014); Lin (2018); Yin and Yang (2016). However, these technical indicators are not shown to be important in our study.

In summary, among a large set of potential forecasting factors, the Brent price and the demands for crude oil and natural gas in the EU are the main drivers of the carbon price. The Quantile Group LASSO and Quantile Group SCAD models can select these important variables and use them to make accurate forecasting.

5.2.2. Does the importance of each forecasting factor vary across quantiles?

The previous subsection shows that the Quantile Group LASSO and the Quantile Group SCAD methods can select important factors and use them to implement forecasting. Does the importance of each forecasting factor vary across quantiles? We address this question by analyzing these forecasting factors using quantile regressions. Quantile regression is an extension of the basic and standard linear regression in which researchers use the values of several variables to explain or predict the mean values of the response variable. Compared with the ordinary least squares, the quantile regression has three main advantages: First, it makes no assumption about the distribution of the target variable; Second, it can model the relationship between the predictor variables and specific quantiles of the response variable; Third, it tends to resist the influence of outliers. Thus, it is highly suitable for our case.

[Table 5 about here.]

Table 5 displays the estimated coefficients of quantile regressions under the low, medium, and high quantile levels ($\tau = 0.1, 0.5, 0.9$), respectively. At each quantile level, all the forecasting factors by the Quantile Group LASSO method are taken into consideration[10]. Overall, the most powerful factors/predictors for carbon futures returns and their corresponding impacts hinge on carbon market conditions (i.e., whether normal or extreme scenarios).

At the low quantile level ($\tau = 0.1$), the Quantile Group LASSO method selects nine important variables. Four of them are statistically significant: the Europe Brent spot price return, the crude oil closing stock return in the UK, the growth of natural gas production in the UK, and the growth of natural gas import in Italy. The negative estimated coefficients of these three variables, i.e., the Europe Brent spot price return, the growth of natural gas production in the UK, and the growth of natural gas import in Italy, indicate that the increase (decrease) of them will decrease (increase) the carbon futures price return in the EU. Meanwhile, the positive estimated coefficient of the crude oil closing stock return in the UK implies that, the increase (decrease) of it will increase (decrease) the carbon futures price return in the EU, at the low quantile level.

At the median quantile level ($\tau = 0.5$), there are seven variables selected by the Quantile Group LASSO method, but only the growth of natural gas import in Italy is statistically significant. It has a negative estimated coefficient as well, which means at the median quantile level, the increase (decrease) of it will decrease (increase) the carbon futures price return in the EU. This is consistent with the previous findings in the low quantile case.

At the high quantile level ($\tau = 0.9$), among all the variables selected by the Quantile Group LASSO, the following factors are statistically significant: the Europe Brent spot price return, the crude oil closing stock return in the UK, the growth of natural gas production in the UK, and the FTSE 100 index. Moreover, the increase (decrease)

[10]Due to the limited space, here we omit the results of the Quantile Group SCAD which has very similar performances as the Quantile Group Lasso.

of the crude oil closing stock return in the UK and the FTSE 100 index will increase (decrease) the carbon futures price return in the EU. The increase (decrease) of the Brent price return and the growth of natural gas production in the UK will decrease (increase) the carbon futures price return in the EU, at the high quantile level.

Now we further analyze the similarities and differences between the results at different quantile levels. The Brent price return, the crude oil closing stock return in the UK, and the growth of natural gas production in the UK have been shown " statistically significant" and have important relationships with the carbon futures price return at both the low and high quantile levels. This finding is quite intuitive and straightforward. It means that they are "key factors", and highly influence the carbon futures price during extreme events (i.e., at the high quantile level)

However, these factors are not statistically significant at the median quantile level, where the growth of natural gas import in Italy has been found to be statistically significant. This is also the only factor that is statistically significant at both the low and median quantile levels, which means the growth of natural gas import in Italy has an important impact on the carbon futures price return in the EU, at the low to median quantile levels. The high quantile level case has one additional statistically significant variable: the FTSE 100 index. It means that, at the high quantile level, the FTSE index has an impact on the carbon futures price in the EU, but not at the low and median quantile level cases.

In summary, we analyze the variables selected by Quantile Group LASSO at different quantile levels. The Brent price, the crude oil closing stock return in the UK, and the growth of natural gas production in the UK are important factors in the carbon futures price prediction during extreme events (i.e., at the high quantile level). This finding is consistent with other studies (See, e.g., Fezzi and Bunn, 2009; Alberola et al., 2008).

The growth of natural gas import in Italy is an important factor at the median quantile level, and the FTSE index has a statistically significant impact on the carbon futures price during extreme events (i.e., at the high quantile level), which is novel.

5.3. Extreme event due to the Covid-19 & Robustness

In the last section, we have seen many results of factor analysis under different quantile levels, where the time spans from 2009 to 2020. However, as we all know, 2020 is a quite different year due to the worldwide pandemic. Coronavirus has impacted everyone and every area of people's life, e.g., people have been asked to work from home to keep social distancing, many factories have been temporarily closed, a huge number of flights have been canceled, there are very few cars on the street in most cities, and so on. Thus, a natural question arises: does the extreme event in 2020 have an impact on our previous findings of the carbon price? A straightforward approach to answer this question is constructing an additional index associated with the happening/absence of the extreme event, and then testing the significance of this new index variable. Based on this idea, first we design a dummy variable in which the element is 0 when the samples are collected in the time period before 2020, and the element is 1 when the data are collected in 2020. Then we have an augmented set of variables which includes this dummy variable and the variables considered in Table 5. Finally we conduct a similar quantile regression as in the last section to see whether the extreme event has a significant impact on the carbon futures price in the EU or not. The following Table 6 shows the estimated coefficients of quantile regressions under the low, medium, and high quantile levels ($\tau = 0.1, 0.5, 0.9$), respectively, and the results of the dummy variable are displayed on the last row in the table.

[Table 6 about here.]

From Table 6, we can see that the dummy variable is statistically significant at the highest level (1%) under all quantile levels, which means the extreme event in 2020 has a huge impact on the carbon futures price in the EU, no matter what quantile levels we care about. This finding, however, is not so surprising, since it is quite intuitive that the "pause" of human activities in the whole world results in drastically reduced energy consumption and CO_2 emission, thus the carbon price fluctuates.

Now we know the Covid-19 significantly influences the carbon futures price in the EU, but can we have a more in-depth understanding of the difference introduced by the extreme event? We tackle this problem by conducting further analysis on the data collected in 2020 only. A similar quantile regression with variables shown in Table 5 is taken into consideration to make a reasonable comparison. The following Table 7 displays the estimated coefficients of quantile regressions under different quantile levels using the data in 2020 only.

[Table 7 about here.]

Comparing the results in Table 7 with Table 5, there are indeed some differences. At the low quantile level ($\tau = 0.1$), the Europe Brent spot price return, the crude oil closing stock return in the UK, and the growth of natural gas import in Italy are significant in both 5 and Table 7, which means they are key factors at the quantile level ($\tau = 0.1$) regardless of extreme conditions. However, the growth of natural gas import in France and the natural gas futures return in the US are significant in Table 7, but not in Table 5, which means these two factors have an impact on the carbon price when the extreme event happens.

At the median quantile level ($\tau = 0.5$), similar to Table 5, there is only one significant variable in Table 7, i.e., the growth of crude oil import in Germany. It means, during the extreme event, this factor is important to the carbon price at the median quantile level.

At the high quantile level ($\tau = 0.9$), the Europe Brent spot price return and the crude oil closing stock return in the UK are also significant in both tables, as well as the growth of natural gas production in the UK and the FTSE index, which again demonstrates the importance of these factors in both the long term period and the extreme event. Besides, the growth of crude oil import in Germany, the growth of natural gas import in the UK, and the growth of natural gas production in France are shown to be significant in Table 7, but not in Table 5. This means during the extreme event, the high quantile level of the carbon price has more determinants than the normal situation.

Generally speaking, compared with Table 5, there are more factors shown to have significant relationships with the carbon price during the extreme event, especially at the extreme quantile levels (the low/high quantile level). This finding somehow coincides with the reality, as in extreme cases, price fluctuations are usually quite different from normal periods, and it is often caused by more factors in different areas (Ren et al., 2019; Duan et al., 2021).

Thus far, we have analyzed impacts and differences caused by the Covid-19. Now here is another question: are our estimators robust to the extreme event? To answer this question, we use the data before 2020, i.e., the time spans from 2009 to 2019, and conduct a similar quantile regression as in Table 5 to obtain comparable results. The following Table 8 displays the estimated coefficients of quantile regressions under different quantile levels using the data before 2020.

[Table 8 about here.]

Comparing the results in Table 8 and Table 5 where the time spans from 2009 to

2020, we can know that most of the significant predictors in Table 5 are also shown to be significant in Table 8, which indicates the robustness of our estimators against the extreme event 2020. At the low quantile level, there are three factors: the Europe Brent spot price return, the growth of natural gas production in the UK, and the growth of natural gas import in Italy, are significant in both Table 8 and Table 5. Here is only one predictor, the crude oil stock return in the UK, which has been found significant in Table 5, but not in Table 8. This may be caused by the extreme event, as it is also significant in Table 7 which considers the data in the extreme event only.

At the median quantile level, the only significant predictor in Table 5, the growth of natural gas import in Italy, is found to be significant in Table 8 as well. At the high quantile level, these two factors: the Europe Brent spot price return and the growth of natural gas production in the UK, are significant in both Table 8 and Table 5. In contrast, the crude oil closing stock return in the UK and the FTSE index are significant in Table 5, but not in Table 8. This may be due to the extreme event, since these two factors are also found to be significant when we analyze the data in 2020 only. In short, most of the significant factors with the data from 2009 to 2020 are found to be also significant with the data before 2020, which shows the robustness of our estimators against the extreme event due to the Covid-19.

6. Concluding remarks

This paper proposes the Quantile Group LASSO model and the Quantile Group SCAD model for the prediction of dynamics of carbon futures returns in the EU ETS. The predictive performance of the two models is examined to outperform popular and competing ones as demonstrated by smaller values of both $MSPE$ and $MAPE$ for the former two. Through a dimension-reduction mechanism, the most powerful carbon-return predictors are selected from a wide group of potential candidates, and the selected predictors are allowed to be different across various quantiles of carbon futures returns. Moreover, a quantile regression method is applied to identify possibly heterogeneous impacts of the predictors on carbon returns across the data distribution. The quantile method is documented to outperform the mean shrinkage models, especially when data like ours are featured by the abnormal price distribution, viz. non-normal distribution. Our results indicate that the Brent spot price, the crude oil closing stock in the UK, and the growth of natural gas production in the UK exert statistically significant impacts on carbon futures returns during extreme events (i.e., at low and high quantile levels). Importantly, our obtained estimators are shown to be robust against the extreme event due to the Covid-19 epidemic.

We demonstrate that the most powerful factors/predictors for carbon futures returns and their corresponding impacts hinge on carbon market conditions (i.e., whether normal or extreme scenarios). Policymakers and market practitioners should recognize such the variation, rather than simply assuming that the statistically significant carbon-return predictors are constant over the carbon price distribution, for a clearer interpretation of carbon return dynamics. Our findings possess statistically significant implications for various stakeholders. In a carbon-constrained environment, a clear comprehension of the significant carbon-return predictors and their impacts can help policymakers uncover the dynamics of carbon returns. Through this, the effectiveness of policy interventions towards carbon price stabilization, as well as the health and prosperity of the carbon market, is enhanced. At the same time, this study improves the assessment of production costs of carbon-intensive sectors and other carbon-consumed economic and human activities by revealing the future price dynamics of carbon emission allowances. This study also contributes to sensible risk diversifications of the investment portfolio, which underlying assets involve carbon futures contracts.

Inclusion and diversity

While citing references scientifically relevant for this work, we also activelyworked to promote gender balance in our reference list. The author list of this paper includes contributors from the location where the research was conducted who participated in the data collection, desig, analysis, and/or interpretation of the work.

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Appendix

- **Appendix A: Variable List**

  (1) Group 1: the energy source group

  1. Growth of crude oil production in France: calculated as the first difference of log the volume of crude oil primary production in France.
  2. Growth of crude oil production in the United Kingdom: calculated as the first difference of log the volume of crude oil primary production in the UK.
  3. Growth of crude oil production in Germany: calculated as the first difference of log the volume of crude oil primary production in Germany.
  4. Growth of crude oil import in France: calculated as the first difference of logging the crude oil imports in France.
  5. Growth of crude oil import in the United Kingdom: calculated as the first difference of log the crude oil imports in the United Kingdom.
  6. Growth of crude oil import in Germany: calculated as the first difference of log the crude oil imports in Germany.
  7. Growth of crude oil stock in France: calculated as the first difference of log the crude oil ending stocks in France.
  8. Growth of crude oil stock in the United Kingdom: calculated as the first difference of log the crude oil ending stocks in the United Kingdom.
  9. Growth of crude oil stock in Germany: calculated as the first difference of log the crude oil ending stocks in Germany.
  10. Growth of natural gas import in France: calculated as the first difference of log the natural gas imports in France.

    11. Growth of natural gas import in the United Kingdom: calculated as the first difference of log the natural gas imports in the United Kingdom.
    12. Growth of natural gas import in Italy: calculated as the first difference of log the natural gas imports in Italy.
    13. Growth of natural gas production in France: calculated as the first difference of log the natural gas production in France.
    14. Growth of natural gas production in the United Kingdom: calculated as the first difference of logging the natural gas production in the United Kingdom.
    15. Growth of natural gas production in Italy: calculated as the first difference of log the natural gas production in Italy.

(2) Group 2: the energy price group

    1. Europe Brent spot price: calculated as the first difference of log EU Brent spot price.
    2. US natural gas liquid composite price: calculated as the first difference of log US natural gas liquid composite price.
    3. US natural gas futures: calculated as the first difference of log US natural gas futures of contract 1.
    4. UK natural gas futures: calculated as the first difference of log UK natural gas futures.

(3) Group 3: the stock market index group

    1. Stock return in the US: calculated as the first difference of log Dow Jones industrial average index.
    2. Stock return in the United Kingdom: calculated as the first difference of log FTSE100 index.

3. Stock return in France.: calculated as the first difference of log CAC 40 index.
4. Stock return in Germany.: calculated as the first difference of log Dax performance index.

(4) Group 4: the monetary policy group

1. Money supply in France: calculated as the first difference of log France money supply M2.
2. Money supply in the United Kingdom: calculated as the first difference of log UK money supply M2.
3. Money supply in Germany: calculated as the first difference of log Germany money supply M2.
4. Money supply in Italy: calculated as the first difference of log Italy money supply M2.

(5) Group 5: the economic information group.

1. Unemployment rate in the UK: the total unemployment rate in the UK.
2. Unemployment rate in Germany: the registered unemployment rate in Germany.
3. Unemployment rate in France: the total unemployment rate in France.
4. Unemployment rate in Italy: the total unemployment rate in Italy.
5. Inflation in France: the monthly inflation rate in France.
6. Inflation in Germany: the monthly inflation rate in Germany.
7. Inflation in Italy: the monthly inflation rate in Italy.
8. Short-term interest rate in the US.
9. Short-term interest rate in the UK.

10. Short-term interest rate in the EU.
11. Long-term interest rate in the US.
12. Long-term interest rate in the UK.
13. Long-term interest rate in the EU.
14. Long-term yield in the UK: the long-term government bond yield in the UK.
15. Long-term yield in France: the long-term government bond yield in France.
16. Long-term yield in Germany: the long-term government bond yield in Germany.
17. Long-term yield in Italy: the long-term government bond yield in Italy.

- **Appendix B: Variable Name in Table 5**
  - BPRI: Europe Brent spot price
  - UKCS: Growth of crude oil stock in the United Kingdom
  - UKGF: UK natural gas futures
  - BDOI: Growth of crude oil import in Germany
  - UKGI: Growth of natural gas import in the United Kingdom
  - FRGP: Growth of natural gas production in France
  - UKGP: Growth of natural gas production in the United Kingdom
  - FTSE: Stock return in the United Kingdom
  - UKOP: Growth of crude oil production in the United Kingdom
  - FRGI: Growth of natural gas import in France

- ITGI: Growth of natural gas import in Italy
- GFC1: US natural gas futures
- UKOI: Growth of crude oil import in the United Kingdom

Table 1: *Descriptive statistics for the carbon price return data*

Mean	Stdev	Skewness	Kurtosis	ADF	Jarque-Bera
0.0078	0.1319	-0.6174	1.7039	-3.6736^{**}	0.000^{***}

Notes: This Table reports summary statistic for the response variable —the monthly carbon price returns, and the sample period runs from March 2009 to December 2020. The ADF shows the value of the Augmented Dickey-Fuller (ADF) test with the null hypothesis of nonstationarity. Jarque-Bera shows the p-values of the Jarque-Bera test with the null hypothesis of normality.
*** denotes statistically significance at 1% level
** denotes statistically significance at 5% level
* denotes statistically significance at 10% level

Table 2: *The MSPE and MAPE in the six-month and twelve-month forecasts*

(a) *six-month forecast* | **(b)** *twelve-month forecast*

forecasting model	MSPE	MAPE	forecasting model	MSPE	MAPE
LASSO	0.0121	0.0941	LASSO	0.0158	0.1087
Adaptive LASSO	0.0121	0.0942	Adaptive LASSO	0.0257	0.1272
Group LASSO	0.0101	0.0865	Group LASSO	0.0156	0.1078
ARMA (1,1)	0.0159	0.1087	ARMA (1,1)	0.0198	0.1184
GARCH (1,1)	0.0158	0.1087	GARCH (1,1)	0.0212	0.1245
ARMAX (1,1,1)	0.0324	0.1444	ARMAX (1,1,1)	0.1689	0.2599
GARCHX (1,1,1)	0.0331	0.1367	GARCHX (1,1,1)	0.0191	0.1208
ARMAXS (1,1,1)	0.0298	0.1359	ARMAXS (1,1,1)	0.0219	0.1255
GARCHXS (1,1,1)	0.0645	0.1692	GARCHXS (1,1,1)	0.0198	0.1190
Quantile Group LASSO	**0.0086**	**0.0445**	Quantile Group LASSO	**0.0098**	**0.0781**
Quantile Group SCAD	**0.0081**	**0.0445**	Quantile Group SCAD	**0.0111**	**0.0871**

Notes: Table 2 reports MSPE and MAPE values of the six-month and twelve-month forecasts. The smaller MSPE and MAPE values indicate higher prediction accuracy of the forecast model.

Table 3: *The R^2_{MSPE} and R^2_{MAPE} based on the time-series forecast group*

(a) *ARMA(1,1) as the benchmark model* | (b) *GARCH(1,1) as the benchmark model*

forecasting model	R^2_{MSPE}	R^2_{MAPE}	forecasting model	R^2_{MSPE}	R^2_{MAPE}
Panel A: Six-month forecast			Panel A: Six-month forecast		
ARMAX (1,1,1)	-1.0377	-0.3284	ARMAX (1,1,1)	-1.0506	-0.3284
GARCHX (1,1,1)	-1.0817	-0.2575	GARCHX (1,1,1)	-1.0949	-0.2575
ARMAXS (1,1,1)	-0.8742	-0.2502	ARMAXS (1,1,1)	-0.8861	-0.2502
GARCHXS (1,1,1)	-3.0566	-0.5565	GARCHXS (1,1,1)	-3.0822	-0.5565
LASSO	0.2389	0.1343	LASSO	0.2342	0.1343
Adaptive LASSO	0.2389	0.1334	Adaptive LASSO	0.2342	0.1334
Group LASSO	0.3648	0.2042	Group LASSO	0.3608	0.2042
Quantile Group LASSO	**0.4591**	**0.5906**	Quantile Group LASSO	**0.4557**	**0.5906**
Quantile Group SCAD	**0.4906**	**0.5906**	Quantile Group SCAD	**0.4873**	**0.5906**
Panel B: Twelve-month forecast			Panel B: Twelve-month forecast		
ARMAX (1,1,1)	-7.5303	-1.1951	ARMAX (1,1,1)	-6.9669	-1.087
GARCHX (1,1,1)	0.0353	-0.0202	GARCHX (1,1,1)	0.0991	0.0297
ARMAXS (1,1,1)	-0.1061	-0.0599	ARMAXS (1,1,1)	-0.0330	-0.0080
GARCHXS (1,1,1)	0.0000	-0.0051	GARCHXS (1,1,1)	0.0660	0.0441
LASSO	0.2020	0.0819	LASSO	0.2547	0.1269
Adaptive LASSO	-0.2979	-0.0743	Adaptive LASSO	-0.2123	-0.0217
Group LASSO	0.2121	0.0895	Group LASSO	0.2641	0.1341
Quantile Group LASSO	**0.5051**	**0.3404**	Quantile Group LASSO	**0.5377**	**0.3727**
Quantile Group SCAD	**0.4393**	**0.2644**	Quantile Group SCAD	**0.4764**	**0.3004**

Notes: Table 3 reports the results of R^2_{MSPE} and R^2_{MAPE} in the six-month and twelve-month forecasts. Table 3(a) and 3(b) show the results when the benchmark models are the ARMA (1,1) model and the GARCH (1,1) model respectively. The larger positive value of R^2_{MSPE} and R^2_{MAPE}, the better prediction ability of the candidate forecast model compared with the corresponding benchmark model.

Table 4: *The R^2_{MSPE} and R^2_{MAPE} of the Quanitle Group LASSO (SCAD) models*

(a) *Quantile Group LASSO as the candidate model*

benchmark model	R^2_{MSPE}	R^2_{MAPE}
Panel A: Six-month forecast		
LASSO	0.2893	0.5271
Adaptive LASSO	0.2893	0.5276
Group LASSO	0.1485	0.4855
ARMA (1,1)	0.4591	0.5906
GARCH (1,1)	0.4557	0.5906
ARMAX (1,1,1)	0.7345	0.6918
GARCHX (1,1,1)	0.7401	0.6744
ARMAXS (1,1,1)	0.7114	0.6725
GARCHXS (1,1,1)	0.8666	0.7369
Panel B: Twelve-month forecast		
LASSO	0.3797	0.2815
Adaptive LASSO	0.6187	0.3860
Group LASSO	0.3718	0.2755
ARMA (1,1)	0.5051	0.3404
GARCH (1,1)	0.5377	0.3727
ARMAX (1,1,1)	0.9419	0.6994
GARCHX (1,1,1)	0.4869	0.3534
ARMAXS (1,1,1)	0.5525	0.3776
GARCHXS (1,1,1)	0.5050	0.3436

(b) *Quantile Group SCAD as the candidate model*

benchmark model	R^2_{MSPE}	R^2_{MAPE}
Panel A: Six-month forecast		
LASSO	0.3306	0.5271
Adaptive LASSO	0.3306	0.5276
Group LASSO	0.1980	0.4855
ARMA (1,1)	0.4906	0.5906
GARCH (1,1)	0.4873	0.5906
ARMAX (1,1,1)	0.7500	0.6918
GARCHX (1,1,1)	0.7552	0.6744
ARMAXS (1,1,1)	0.7281	0.6725
GARCHXS (1,1,1)	0.8744	0.7369
Panel B: Twelve-month forecast		
LASSO	0.2975	0.1987
Adaptive LASSO	0.5681	0.3152
Group LASSO	0.2885	0.1920
ARMA (1,1)	0.4394	0.2644
GARCH (1,1)	0.4764	0.3004
ARMAX (1,1,1)	0.9342	0.6648
GARCHX (1,1,1)	0.4188	0.2789
ARMAXS (1,1,1)	0.4931	0.3059
GARCHXS (1,1,1)	0.4393	0.2680

Notes: Table 4 reports the results of R^2_{MSPE} and R^2_{MAPE} in both the six-month and twelve-month forecast tests. Table 4(a) and 4(b) show the result when the candidate model is the Quantile Group LASSO and Quantile Group SCAD model, respectively. The larger value of R^2_{MSPE} and R^2_{MAPE} means a larger promotion of prediction ability of the candidate model over the corresponding benchmark model.

Table 5: *Regression results under different quantiles*

	Panel A: $\tau = 0.1$	Panel B: $\tau = 0.5$	Panel C: $\tau = 0.9$
BPRI	$\mathbf{-0.126}^{**}$	-0.139	$\mathbf{-0.123}^{**}$
	(0.058)	(0.078)	(0.057)
UKCS	$\mathbf{0.301}^{***}$	0.054	$\mathbf{0.261}^{***}$
	(0.078)	(0.061)	(0.066)
UKGF	-0.014		−0.014
	(0.063)		(0.055)
BDOI		-0.049	−0.076
		(0.062)	(0.052)
UKGI		-0.059	0.012
		(0.054)	(0.031)
FRGP	-0.006		−0.022
	(0.016)		(0.014)
UKGP	$\mathbf{-0.226}^{***}$		$\mathbf{-0.209}^{***}$
	(0.055)		(0.054)
FTSE			$\mathbf{0.361}^{**}$
			(0.171)
UKOP	-0.029		
	(0.074)		
FRGI	0.057	0.089	
	(0.064)	(0.131)	
ITGI	$\mathbf{-0.137}^{*}$	$\mathbf{-0.262}^{*}$	
	(0.075)	(0.148)	
GFC1	-0.006		
	(0.062)		
UKOI		-0.049	
		(0.062)	

Notes: (i) This table summarizes coefficient results of quantile regression under different quantile levels ($\tau = 0.1$, $\tau = 0.5$ and $\tau = 0.9$). (ii) At each quantile level, the model contains all the variables selected by the Quantile Group LASSO method. (iii) standard errors are in parentheses.

*** denotes statistically significance at 1% level

** denotes statistically significance at 5% level

* denotes statistically significance at 10% level

Table 6: *Regression results with an indicator of the extreme event in 2020*

	Panel A: $\tau = 0.1$	Panel B: $\tau = 0.5$	Panel C: $\tau = 0.9$
BPRI	0.026	-0.092	0.023
	(0.033)	(0.101)	(0.057)
UKCS	**0.141*****	0.052	**0.213*****
	(0.045)	(0.071)	(0.065)
UKGF	0.040		-0.087
	(0.036)		(0.055)
BDOI		-0.133	-0.054
		(0.105)	(0.052)
UKGI		-0.048	0.019
		(0.048)	(0.029)
FRGP	**−0.023****		-0.018
	(0.009)		(0.014)
UKGP	**−0.122*****		**−0.188*****
	(0.031)		(0.054)
FTSE			0.097
			(0.171)
UKOP	0.013		
	(0.042)		
FRGI	-0.021	0.173	
	(0.036)	(0.115)	
ITGI	**−0.180*****	**−0.340****	
	(0.043)	(0.130)	
GFC1	-0.031		
	(0.035)		
UKOI		-0.037	
		(0.054)	
Dummy	**−0.101*****	**0.130*****	**0.162*****
	(0.015)	(0.054)	(0.024)

Notes: (i) This table summarizes coefficient results of quantile regression under different quantile levels ($\tau = 0.1$, $\tau = 0.5$ and $\tau = 0.9$) with an indicator of the extreme event in 2020. (ii) The indicator (dummy variable) is displayed on the last row of the table. (iii) At each quantile level, the model contains all the variables selected by the Quantile Group LASSO method and the new dummy variable. (iv) standard errors are in parentheses.
*** denotes statistically significance at 1% level
** denotes statistically significance at 5% level
* denotes statistically significance at 10% level

Table 7: *Regression results at the extreme event during 2020*

	Panel A: $\tau = 0.1$	Panel B: $\tau = 0.5$	Panel C: $\tau = 0.9$
BPRI	**0.135****	-0.101	$\mathbf{-0.035}$***
	(0.030)	(0.224)	(0.038)
UKCS	**1.248****	-0.046	**0.439*****
	(0.345)	(0.348)	(0.087)
UKGF	0.056		0.134
	(0.026)		(0.065)
BDOI		$\mathbf{-0.320}$*	$\mathbf{-0.631}$***
		(0.164)	(0.088)
UKGI		-0.015	**0.215*****
		(0.209)	(0.037)
FRGP	0.037		$\mathbf{-0.056}$***
	(0.023)		(0.003)
UKGP	-0.868		$\mathbf{-0.540}$***
	(0.241)		(0.096)
FTSE			**0.784*****
			(0.104)
UKOP	-0.064		
	(0.139)		
FRGI	**0.549*****	-0.407	
	(0.053)	(0.596)	
ITGI	$\mathbf{-0.505}$**	0.278	
	(0.108)	(0.632)	
GFC1	$\mathbf{-0.337}$**		
	(0.084)		
UKOI		-0.239	
		(0.587)	

Notes: (i) This table summarizes coefficient results of quantile regression under different quantile levels ($\tau = 0.1$, $\tau = 0.5$ and $\tau = 0.9$) at the extreme event during 2020. (ii) At each quantile level, the model contains all the variables selected by the Quantile Group LASSO method. (iii) standard errors are in parentheses.
*** denotes statistically significance at 1% level
** denotes statistically significance at 5% level
* denotes statistically significance at 10% level

Table 8: *Regression results before the extreme event in 2020*

	Panel A: $\tau = 0.1$	Panel B: $\tau = 0.5$	Panel C: $\tau = 0.9$
BPRI	$\mathbf{-0.121}^{**}$	0.017	$\mathbf{0.138}^{***}$
	(0.065)	(0.109)	(0.099)
UKCS	-0.011	0.095	0.089
	(0.091)	(0.137)	(0.086)
UKGF	-0.030		-0.076
	(0.072)		(0.086)
BDOI		-0.034	0.001
		(0.080)	(0.059)
UKGI		-0.011	0.014
		(0.050)	(0.025)
FRGP	-0.018		-0.023
	(0.019)		(0.018)
UKGP	$\mathbf{-0.312}^{***}$		$\mathbf{-0.302}^{***}$
	(0.082)		(0.068)
FTSE			0.103
			(0.245)
UKOP	0.089		
	(0.095)		
FRGI	0.037	-0.005	
	(0.067)	(0.090)	
ITGI	$\mathbf{-0.194}^{***}$	$\mathbf{-0.173}^{*}$	
	(0.065)	(0.112)	
GFC1	0.023		
	(0.056)		
UKOI		-0.042	
		(0.045)	

Notes: (i) This table summarizes coefficient results of quantile regression under different quantile levels ($\tau = 0.1$, $\tau = 0.5$ and $\tau = 0.9$) before the extreme event in 2020. (ii) At each quantile level, the model contains all the variables selected by the Quantile Group LASSO method. (iii) standard errors are in parentheses.
*** denotes statistically significance at 1% level
** denotes statistically significance at 5% level
* denotes statistically significance at 10% level