# Optimizing Mixed-Asset Portfolios Involving REITs

Fatim Z. Habbab, Michael Kampouridis, Alexandros A. Voudouris
*School of Computer Science and Electronic Engineering*
*University of Essex*
Wivenhoe Park, United Kingdom
{fh20175, mkampo, alexandros.voudouris}@essex.ac.uk

***Abstract*—Real Estate Investment Trusts (REITs) is a popular investment choice as it allows investors to hold shares in real estate rather than investing large sums of money to purchase real estate by themselves. Previous work studied the effectiveness of multi-asset portfolios that include REITs via an efficient frontier analysis. However, the advantages of including (both domestic and international) REITs in multi-asset portfolios, as well as analyzing all the possible combinations of asset classes, has not been investigated before. In this paper, we fill in this gap by performing a thorough investigation across 456 different portfolios to demonstrate the added value of including REITs in mixed-asset portfolios in terms of different important financial metrics. To this end, we use a genetic algorithm approach to maximize the Sharpe ratio of the portfolios. Our results show that optimization via a genetic algorithm outperforms the results obtained from a global minimum variance portfolio. More importantly, our results also show that there can be significant improvements in average returns, risk and Sharpe ratio when including REITs.**

***Index Terms*—mixed-asset portfolio, genetic algorithm, minimum variance, portfolio optimization, risk-adjusted return**

## I. Introduction

One of the main objectives of financial investors is to reduce the risk associated to their portfolios, which can be done through an effective asset allocation strategy. An investor could choose among various investment opportunities and rank them according to preference. Portfolio Theory describes how investors should allocate their wealth to maximize their investment return or minimize the associated risk.

Among the different asset classes, *real estate* has attracted the attention of billions of investors worldwide for the diversification potential that it could offer when included in investment portfolios. However, due to the high unit value that characterizes most real estate assets (such as offices, malls, hotels, etc), investors have been looking for more affordable alternatives, in particular, REIT shares. A *Real Estate Investment Trust* (REIT) is a company that owns, operates or finances income-producing real estate. On the other hand, direct investment in real estate (i.e., purchasing real estate assets) requires a large sum to be paid, and an effective management of the property. One of the biggest advantages of investing through REITs is that an investor can access the benefits that the real estate asset class offers without being required to spend a large amount of money and without the need to engage in the management of the underlying properties. In this paper we focus on REITs because of their simplicity and affordability over their direct investment counterpart.

The added value that REITs could offer to a mixed-asset portfolio in terms of risk-adjusted performance enhancement has been demonstrated in several previous works. For example, [1] examined the diversification effects of including REITs in a portfolio already composed by common stocks. [2] examined the effects of including U.S. commercial real estate in a portfolio composed of U.S. common stocks, U.S. bonds, and international common equity on the efficient frontier. [3] showed that both private and public REITs have positive effects on the efficient frontier. [4] showed that the added value of REITs in a mixed-asset portfolio increases with the time horizon. [5] studied the role of REITs in a mixed-asset portfolio for the Malaysian market by adopting a minimum-variance and downside deviation approach. [6] analyzed the diversification benefits of adding REITs in a mixed-asset portfolio by comparing four portfolios (equity shares and bonds, REITs and bonds, REITs and equity shares, equity shares, bonds, and REITs) for the 2009–2019 period. Other works confirmed the positive effects on the risk-adjusted performance of including REITs in a mixed-asset portfolios for different countries through an efficient frontier and asset allocation diagram analysis [7, 8, 9].

The aforementioned works followed mainly a *global minimum variance* (GMV) portfolio methodology, which aims at optimizing the weights of the assets included in the portfolio by searching for the combination that minimizes the portfolio risk. In other words, for a given number of $N$ assets, the GMV method computes the single, global optimal solution consisting of a specific weight allocated to each asset. In contrast, in this paper, we follow a *genetic algorithm* (GA) approach to search over the infinite number of possible asset weight combinations for those that maximize the Sharpe ratio, and therefore maximize return and minimize risk. We compare the two approaches and discuss their differences.

To the best of our knowledge, no work has been dedicated to studying the impact of international REITs on a mixed-asset portfolio composed of both domestic and international stocks and bonds. In most cases, they either focus on single countries, such as [7], or consider domestic REITs coupled together with international equity, such as [3]. To fill in this gap, we experiment with 9 different asset classes: 3 stock indices, 3 bonds, and 3 REITs from the UK, US, and Australian markets; this leads to 456 different portfolios. The advantage of our approach is that we are able to consider a large number of portfolios that include REITs and compare their performance to a large number of portfolios that do not contain REITs. We examine the performance of all these different portfolios and provide a thorough analysis in terms of sharpe ratio, returns, risk, as well as other metrics and distribution moments, such as maximum drawdown, skewness and kurtosis. In addition, the data we use include the 2020-2021 COVID-19 pandemic period, which is of great interest due to its global effect in the markets, making them more volatile and uncertain.

The rest of this paper is organized as follows. Section II presents a brief background on modern portfolio theory, and Section III discusses the methodology of this paper. Our experimental setup is presented in Section IV. Section V provides a detailed discussion of the experimental results we obtained by both the genetic algorithm we implemented and the GMV method. Finally, Section VI concludes the paper.

## II. Background

*Modern portfolio theory* (MPT) is a mathematical framework that is largely used to solve asset allocation problems. The main assumption of MPT is that investors are risk averse in the sense that the less risky portfolio among those that portfolios that provide the same expected return. Consequently, one will choose a riskier portfolio only if compensated by a higher expected return. Different investors have different preferences over such tradeoffs based on their individual risk aversion levels.

According to MPT, a portfolio is considered efficient when its expected return is maximized for a given level of risk, or its expected risk is minimized for a given level of return. The expected return of the portfolio is expressed as a weighted average of the historical returns of the assets included in the portfolio, where the weighting factors are the proportions allocated to the different asset classes. The expected risk of the portfolio is expressed as the variance of the historical returns of the asset classes, and is a function of the correlations $\rho_{ij}$, for all pairs of asset $(i, j)$. Given specific combinations of assets and standard deviations of asset returns, the highest possible standard deviation of portfolio returns is obtained when all correlations are equal to 1, which means that all asset pairs are perfectly correlated to each other. It is possible to reduce the portfolio's expected risk by selecting combinations of assets that are not perfectly positively correlated (i.e., $-1 < \rho_{ij} < 1$). This is known as *diversification*. If all asset pairs are perfectly uncorrelated ($\rho_{ij} = 0$ for all $i, j$), the variance of the portfolio returns is the sum of the squares of all asset weights times the asset's return variance. If all asset pairs are perfectly positively correlated ($\rho_{ij} = 1$ for all $i, j$), then the standard deviation of the portfolio returns is the sum of the standard deviations of the underlying asset returns, weighted by the proportion allocated to each asset class.

## III. Portfolio optimization via a Genetic Algorithm

Evolutionary algorithms have been widely used for financial applications, including portfolio optimization [10]. To tackle the portfolio optimization problem we consider in this paper, we use a particular type of evolutionary algorithm known as *genetic algorithm* (GA) [11, 12, 13]. Below we briefly discuss the GA we have used.

### A. Representation

GA *chromosomes* (or, *individuals*) consist of $N$ *genes* indicating the weights allocated to the $N$ assets in the portfolio. The weight are real numbers in the interval $[0, 1]$, and their sum is equal to 1. For example, a GA individual that has the genotype [0.5 0.2 0.3] indicates that there are three assets, and the weight for those asset are 0.5, 0.2, and 0.3, respectively. Initially, all genes are assigned the same weight (in particular, $W_i = 1/N$ for each asset $i$), which are then evolved according to a set of operators.

### B. Operators

We use *elitism*, *one-point crossover* and *one-point mutation*. Since we use market proxies in our experiments, the number of assets is small, and thus one-point crossover and mutation are sufficient (see Section IV for more details). After the application of crossover and mutation, we apply normalization to each GA individual, to ensure that the sum of weights remains equal to 1.

### C. Fitness function

State-of-the-art methods for solving portfolio optimization problems have used many different metrics as fitness functions. In this paper, we use the *Sharpe ratio*, defined as the ratio of the difference between the average return and the risk-free rate, over the standard deviation of the returns, that is,

$$S = \frac{r - r_f}{\sigma_r}, \tag{1}$$

where $r$ is the average return of the investment, $r_f$ is the risk-free rate, and $\sigma_r$ is the standard deviation of the returns.

### D. Asset combinations

To overcome the limitations of the current literature, where a single minimum variance portfolio is built, and to also better understand the added value of REITs in multi-asset portfolios, we apply the GA multiple times to each of the possible asset combinations. For example, for $N$ assets, we run the GA on all possible combinations of portfolios consisting of $x$ assets, for each $x \in \{2, \ldots, N\}$. This way, we create multiple portfolios that include REITs, and are able to compare their performance to all portfolios that do not include REITs. This allows for a more in-depth analysis of the benefits of using REITs, when compared to the state-of-the-art method of searching for one solution within the whole universe of asset classes considered together.

## IV. Experimental setup

Our experiments aim to (i) provide evidence that including real estate in a mixed-asset portfolio increases its diversification potential, and (ii) demonstrate that the genetic algorithm can lead to better results when compared to the global minimum variance portfolio approach (state-of-the-art) in terms of expected return and Sharpe ratio.

TABLE I
MEAN, STANDARD DEVIATION AND SHARPE RATIO FOR EACH ASSET CLASS.

| | S&P 500 | FTSE 100 | S&P/ASX 200 | US bond | UK bond | AU bond | US REIT | UK REIT | AU REIT |
|---|---|---|---|---|---|---|---|---|---|
| **Mean return** | 0.04% | 0.005% | 0.011% | 0.039% | 0.006% | 0.012% | 0.014% | 0.006% | 0.019% |
| **Standard deviation** | 0.86% | 1.05% | 1.07% | 0.86% | 1.06% | 1.10% | 0.96% | 1.37% | 1.16% |
| **Sharpe ratio** | 4.51% | 0.32% | 0.86% | 4.38% | 0.40% | 0.96% | 1.24% | 0.28% | 1.44% |

TABLE II
CORRELATION COEFFICIENTS BETWEEN ASSET CLASSES.

| | S&P 500 | FTSE 100 | S&P/ASX 200 | US bond | UK bond | AU bond | US REIT | UK REIT | AU REIT |
|---|---|---|---|---|---|---|---|---|---|
| **S&P 500** | 1 | 0.515 | 0.326 | 0.999 | 0.48 | 0.259 | 0.541 | 0.34 | 0.249 |
| **FTSE 100** | 0.515 | 1 | 0.464 | 0.521 | 0.946 | 0.449 | 0.267 | 0.728 | 0.334 |
| **S&P/ASX 200** | 0.326 | 0.464 | 1 | 0.329 | 0.443 | 0.939 | 0.217 | 0.329 | 0.73 |
| **US bond** | 0.999 | 0.521 | 0.329 | 1 | 0.488 | 0.263 | 0.547 | 0.347 | 0.251 |
| **UK bond** | 0.480 | 0.946 | 0.443 | 0.488 | 1 | 0.489 | 0.249 | 0.726 | 0.31 |
| **AU bond** | 0.259 | 0.449 | 0.939 | 0.263 | 0.489 | 1 | 0.17 | 0.299 | 0.668 |
| **US REIT** | 0.541 | 0.267 | 0.212 | 0.547 | 0.249 | 0.17 | 1 | 0.272 | 0.234 |
| **UK REIT** | 0.34 | 0.728 | 0.329 | 0.347 | 0.726 | 0.299 | 0.27213 | 1 | 0.283 |
| **AU REIT** | 0.249 | 0.334 | 0.73 | 0.25 | 0.31 | 0.668 | 0.234 | 0.283 | 1 |

### A. *Data*

We use daily prices over the period between June 2017 and January 2021. We adopt the perspective of an institutional investor from the US who wants to gain exposure to international markets (UK and Australia). The asset classes we consider are stocks, bonds, and listed real estate.

As other authors did previously [14, 15], we use index prices as data for our experiments. Stocks are proxied by the S&P 500 index for the US marked, by the FTSE 100 index for the UK market, and by the S&P/ASX 200 index for the Australian market. For the bond asset class, we use the indices issued by Dow Jones for all the three markets considered. Finally, we use the FTSE/EPRA NAREIT indices to proxy the real estate markets. We thus have 9 asset classes, namely 3 stocks, 3 bonds, and 3 REITs.

Table I presents the main statistics for each asset class: the mean of the returns of the assets (as a proxy of their expected return), their standard deviation (as a proxy of their volatility or risk), and the Sharpe ratio (as a measure of the asset's risk-adjusted return). In order to calculate the Sharpe ratio, we have considered a risk-free rate equal to $0.0019\%$ (corresponding to an average of the daily government bond rates in the three countries). From the results shown in Table I, we can observe that the real estate asset class generally presents a lower level of performance (that is, lower Sharpe ratio) compared to the other asset classes. This indicates that real estate investments could be less profitable than the other types of investments if considered individually. Our aim is to assess the *added value* that real estate could bring within a multi-asset portfolio.

From the correlation matrix shown in Table II, we can observe that the real estate asset class generally has relatively lower correlation with the other asset classes, thus justifying its diversification potential. More specifically, a low or zero correlation between two asset classes might reduce a portfolio's overall level of risk. Moreover, we observe a low correlation between asset classes belonging to different markets (e.g., S&P 500 and UK REITs). This could open opportunities to an international diversification. In other words, an investor might find diversification opportunities in gaining exposure to foreign markets.

### B. *Experimental parameters*

To decide the parameter values, we undertook a parameter tuning process using the I/F-Race package [16]. I/F-Race implements the iterated racing procedure, which is an extension of the Iterated F-Race process and builds upon the race package by [17]. Its main purpose is to automatically configure optimization algorithms by finding the most appropriate settings, given a set of instances of a problem.

In our case, I/F-Race was applied to data for the period from June 2017 to December 2018. The following twelve months (January-December 2019) were used only with the already tuned parameters, after I/F-Race was completed. In other words, the first period was used as a training dataset for parameter tuning, while the second period was used as a validation dataset for parameter testing. The period January-December 2020 was the test set, and remained unseen during the parameter tuning process. At the end of the tuning process, we picked the best parameters returned by I/F-Race, which constitute the experimental parameters used by our algorithms, and are presented in Table III.

### C. *Benchmark: The global minimum-variance approach*

In addition to our genetic algorithm, we also adopt the *global minimum variance* (GMV) portfolio method that has been extensively used to estimate the optimal portfolio weights [18, 19, 14]. The global minimum variance portfolio $w =$

TABLE III
I-RACE PARAMETER TUNING RESULTS.

| Parameter | Value |
|---|---|
| Tournament size | 3 |
| Population size | 300 |
| Crossover rate | 1.0 |
| Mutation rate | 0.01 |
| Number of generations | 10 |

$(w_1, \ldots, w_N)$ consisting of $N$ asset weights is the solution of the following constrained minimization problem:

$$\min_w \sigma_w^2 = w^T \Sigma w \qquad (2)$$
$$\text{subject to } w^T \mathbf{1} = 1$$

where $\sigma_w^2$ is the variance of the portfolio returns, $w^T$ indicates the transposed vector of asset weights, $\Sigma$ is the variance-covariance matrix. According to the math program (2), the optimal weights are selected so that the variance of the portfolio returns is minimized, under the constraint that the sum of all asset weights is 1. We used Microsoft Excel's build-in Generalized Reduced Gradient Nonlinear algorithm to perform this optimization task. As mentioned earlier, this approach admits a global optimal solution, rather than multiple solutions. Thus, given the initial universe of 9 asset classes that we use in our experiments, the GMV approach leads to one global optimal portfolio.

## V. RESULTS

In this section we present our experimental results for the genetic algorithm (Section V-A) and the minimum-variance approach (Section V-B), and discuss our conclusions (in Section V-C). Results are presented as averages over 20 individual GA runs. It should also be noted that all results are daily results. So when, for example, we present a seemingly "low" return of around 0.03%, its annual equivalent would be around 11.6%. [1]

### A. Genetic algorithm

We start by presenting our results for the genetic algorithm. In the following three subsections, we discuss these results in terms of expected return (Section V-A1), expected risk (Section V-A2), and expected Sharpe ratio (Section V-A3). To show the added value of investing in REITs, we compare the performance of all the portfolios that included REITs against all the portfolios that did not include REITs, but only stocks and bonds. The total number of portfolios not including REITs is 57, while the total number of portfolios using REITs is 399. Due to this imbalance, we also present results from undersamples of the REITs sample (both with and without replacement), to make the comparison fairer. To assess the diversification potential of real estate, we compare different risk measures (standard deviation, downside deviation, maximum drawdown, minimum-maximum range, and average expected risk) obtained for portfolios that include stocks, bonds and real estate, and portfolios that include only stocks and bonds.

*1) Expected portfolio returns:* Table IV summarizes the differences between the two types of portfolios (all combinations of portfolios that include REITs and all combinations that do not) in terms of average returns. As we can observe, the average expected return is higher for portfolios that include REITs than for those that include only stocks and bonds, regardless of whether we perform a sampling of the return distribution (for the portfolios including real estate) or not. Specifically, the increase is 5.15% (if we do not perform sampling), 4.94% (if we perform sampling with replacement), and 5.25% (if we perform sampling without replacement).

In addition to the mean, we have analyzed other distribution moments as measures of the volatility of the distributions, in particular, the standard deviation, downside deviation, maximum drawdown, and minimum-maximum range. According to the results obtained, there is a decrease for the return distributions of all the combinations including real estate of around 40% in the case of the standard deviation and the downside deviation. Specifically, we could observe a decrease in the standard deviation of 37.21% (if we do not perform sampling), of 37.03% (if we perform sampling with replacement), and of 39.60% (if we perform sampling without replacement) for the combinations including real estate. In the case of downside deviation, we observe a decrease of 36.41% (if we do not perform sampling), of 36.75% (if we perform sampling with replacement), and of 39.24% (if we perform sampling without replacement). For the maximum drawdown values, we observe an increase of 7.32% (if we do not perform sampling), which might be due to the larger range of observations for the combinations including real estate, a reduction of 17.25% (if we perform sampling with replacement), and of 18.51% (if we perform sampling without replacement). If we have a look at the minimum-maximum ranges, we could notice an increase of 24.80% (if we do not perform sampling), which might be due to the higher range of observations for the combinations including real estate, a reduction of 11.67% (if we perform sampling with replacement), and of 12.95% (if we perform sampling without replacement). These results indicate a reduction in the observed volatility for the returns of combinations including real estate, which might indicate some level of diversification occurring when including real estate in a mixed-asset portfolio.

Regarding the Skewness and Kurtosis values, we observe an increase in the Skewness value of 82.10% (if we do not perform sampling), of 56.37% (if we perform sampling with replacement), and of 60.69% (if we peform sampling without replacement) for the combinations including real estate. We also observe an increase of 236.20% (if we do not perform sampling), of 168.20% (if we perform sampling with replacement), and of 179.43% (if we perform sampling without replacement). These results indicate that the return distribution for the combinations including real estate tends to be more skewed to the left (which indicates a higher likelihood of observing higher returns than the mean), and to have fatter

[1] $\text{AnnualizedReturn} = [(\text{DailyReturn} + 1)^{365} - 1] \times 100 = 11.6\%$.

TABLE IV
AVERAGE GA RETURNS OF THE PORTFOLIOS.

| Metrics | Not including real estate | Including real estate (no sampling) | Relative difference | Including real estate (with replacement) | Relative difference | Including real estate (w/o replacement) | Relative difference |
|---|---|---|---|---|---|---|---|
| **Mean** | 0.0336% | 0.0354% | 5.15% | 0.0353% | 4.94% | 0.0354% | 5.25% |
| **Standard deviation** | 0.006% | 0.0037% | -37.21% | 0.0038% | -37.03% | 0.0036% | -39.6% |
| **Downside deviation** | 0.0053% | 0.0034% | -36.41% | 0.0034% | -36.75% | 0.0032% | -39.2% |
| **Max drawdown** | 0.6051 | 0.6494 | 7.32% | 0.5007 | -17.25% | 0.4931 | -18.51% |
| **Min-max range** | 0.023% | 0.0287% | 24.8% | 0.0203% | -11.67% | 0.02% | -12.95% |
| **Skewness** | -1.5834 | -2.8833 | 82.1% | -2.4759 | 56.37% | -2.5444 | 60.69% |
| **Kurtosis** | 3.9957 | 13.4336 | 236.2% | 10.7165 | 168.2% | 11.1653 | 179.43% |

TABLE V
AVERAGE GA RISK FOR THE PORTFOLIOS.

| Metrics | Not including real estate | Including real estate (no sampling) | Relative difference | Including real estate (with replacement) | Relative difference | Including real estate (w/o replacement) | Relative difference |
|---|---|---|---|---|---|---|---|
| **Mean** | 0.93% | 0.8903% | -4.27% | 0.8889% | -4.42% | 0.8887% | -4.44% |
| **Standard deviation** | 0.0703% | 0.1011% | 43.97% | 0.1016% | 44.56% | 0.1021% | 45.38% |
| **Downside deviation** | 0.0589% | 0.0856% | 45.48% | 0.0855% | 45.35% | 0.0859% | 46.01% |
| **Max drawdown** | 0.2434 | 0.4249 | 74.57% | 0.3671 | 50.80% | 0.3683 | 51.30% |
| **Min-max range** | 0.2505% | 0.4713% | 88.11% | 0.3870% | 54.49% | 0.3861% | 54.11% |
| **Skewness** | -1.1555 | -1.1738 | 1.59% | -1.1649 | 0.82% | -1.1560 | 0.05% |
| **Kurtosis** | 3.4051 | 3.3498 | -1.62% | 3.4161 | 0.32% | 3.3745 | -0.90% |

TABLE VI
AVERAGE GA SHARPE RATIO FOR THE PORTFOLIOS.

| Metrics | Not including real estate | Including real estate (no sampling) | Relative difference | Including real estate (with replacement) | Relative difference | Including real estate (w/o replacement) | Relative difference |
|---|---|---|---|---|---|---|---|
| **Overall** | 3.5957% | 4.0171% | 11.72% | 4.015% | 11.66% | 4.0165% | 11.70% |
| **Two assets** | 3.2835% | 4.0171% | 22.34% | 3.7207% | 13.31% | 3.7451% | 14.06% |
| **Three assets** | 3.6090% | 3.9653% | 9.87% | 3.9608% | 9.75% | 3.9701% | 10% |
| **Four assets** | 3.7748% | 4.05% | 7.29% | 4.0324% | 6.82% | 4.0274% | 6.69% |
| **Five assets** | 3.8843% | 4.0641% | 4.63% | 4.0405% | 4.02% | 4.0636% | 4.62% |

tails (which indicates a higher probability for the investor of experiencing occasional extreme returns).

We also performed three Kolmogorov-Smirnov (KS) tests at the 5% significance level, one per distribution pair that we wanted to compare: not including real estate vs including real estate (no sampling); not including real estate vs including real estate (with replacement); and not including real estate vs including real estate (without replacement). The null hypothesis for each test was that the two distributions come from the same probability distribution. Given that we were making multiple comparisons, we adjusted the tests' p-value according to the Bonferroni correction to $0.05/3 = 0.0166$. The p-value for the first test was 3.5376e-04; the p-value for the second test was 5.5466e-10; and the p-value for the third test was 5.5466e-10. As we can observe, all three values are well below the adjusted p-value of 0.0166, thus making the differences statistically significant at the 5% level.

In conclusion, the results we obtained from our genetic algorithm confirm that including real estate in a mixed-asset portfolio leads to an increase in the overall return level, and at the same time to a reduction of the volatility level of the return distribution.

*2) Expected portfolio risk:* This part aims at making a comparison between the expected risk distributions for the combinations including real estate and those not including real estate, both in the case of no sampling and of sampling (with and without replacement). As we can observe from Table V, there is a decrease in the average expected risk of 4.27% (if we do not perform sampling), of 4.42% (if we perform sampling with replacement), and of 4.44% (if we perform sampling without replacement) for the combinations including real estate.

The standard deviation, downside deviation, maximum drawdown, and minimum-maximum range appear to increase from the combinations not including real estate to those including real estate, indicating that there are risk values that might be far from each other for combinations including real estate. In particular, we observe an increase in the standard deviation values of 43.97% (if we do not perform sampling), of 44.56% (if we perform sampling with replacement), and of 45.38% (if we perform sampling without replacement) for the combinations including real estate. For the downside deviation, we observe an increase of 45.48% (if we do not perform sampling), of 45.35% (if we perform sampling with replacement), and of 46.01% (if we perform sampling without replacement).

In the case of maximum drawdown, we observe an increase of 74.57% (if we do not perform sampling), of 50.80% (if we perform sampling with replacement), and of 51.30% (if we perform sampling without replacement). Regarding the minimum-maximum range, we observe an increase of 88.11% (if we do not perform sampling), of 54.49% (if we perform sampling with replacement), and of 54.11% (if we perform sampling without replacement).

There are small differences between the two risk distributions in terms of the Skewness and Kurtosis, indicating similarities between the shapes of the two distributions. In particular, we observe an increase in the Skewness values of 1.59% (if we do not perform sampling), of 0.82% (if we perform sampling with replacement), and of 0.05% (if we perform sampling without replacement). On the other hand, we observe a decrease in the Kurtosis value of 1.62% (if we do not perform sampling), an increase of 0.32% (if we perform sampling with replacement), and a decrease of 0.90% (if we perform sampling without replacement). These results indicate a return distribution with a slightly higher negative asymmetry, and lighter tails for combination including real estate, which might mean a higher likelihood of observing extreme values.

We again performed KS tests for the three distribution pairs. The KS test results denote statistically significant differences at the 5% level (we again adjusted the p-value according to the Bonferroni correction), which implies the null hypothesis (no statistically significant differences exist between the two analyzed distributions) being rejected. The p-value for the first test is 3.2473e-09. The p-value for the second test was 5.5466e-10, as well as the p-value for the third test.

In conclusion, including real estate in a multi-asset portfolio tends to reduce the overall risk level of that portfolio.

*3) Expected Sharpe ratio:* This part shows the results obtained in terms of expected Sharpe ratios resulting from the expected return and risk distributions analyzed in the previous parts (see Table VI). Since the Sharpe ratio is essentially given by the ratio of returns and risk, to avoid repetition, we do not present the standard deviation, downside deviation, max drawdown, min-max range, skewness, and kurtosis metrics. Instead, we only present the mean Sharpe ratio values. In addition, we present the different asset combinations in terms of the number of assets included in a given portfolio, i.e., two assets, three assets, four assets, and five assets.

First of all, we can observe an increase in the Sharpe ratio values in all cases where real estate is included. In particular, we observe an increase of 11.72% (if we do not perform sampling), of 11.66% (if we perform sampling with replacement), and of 11.70% (if we perform sampling without replacement) for the combinations including real estate.

In terms of the different number of asset combinations, we can notice that for two asset combinations there is an increase in the Sharpe ratio of 22.34% by including real estate (without sampling), of 13.31% (in the case of sampling with replacement), and of 14.06% (in the case of sampling without replacement). Similarly, for three asset combinations, there is an increase in the Sharpe ratio of 9.87% by including real estate (without sampling), of 9.75% (in the case of sampling with replacement), and of 10% (in the case of sampling without replacement). For four asset combinations, we can observe an increase in the Sharpe ratio of 7.29% by including real estate (without sampling), of 6.82% (in the case of sampling with replacement), and of 6.69% (in the case of sampling without replacement). Finally, for five asset combinations, there is an increase in the Sharpe ratio of 4.63% by including real estate (without sampling), of 4.02% (in the case of sampling with replacement), and of 4.62% (in the case of sampling without replacement).

We again performed a Kolmogorov-Smirnov test for each distribution pair: all combinations not including real estate vs all combination including real estate (no sampling); all combinations not including real estate vs all combinations including real estate (with replacement); and all combinations not including real estate vs combinations including real estate. Table V-A3 presents the p-values resulting from the KS tests performed. As we can observe, the p-value was the same across all tests - very close to 0 (5.5466E-10). Those results denote statistically significant differences at the 5% level (we again adjusted the p-value according to the Bonferroni correction; thus a 5% statistical significance has a p-value of 0.01666).

In conclusion, our results show that including real estate in a mixed-asset portfolio tends to increase the risk-adjusted return level related to that portfolio, and such results appears to be more evident when a lower number of assets is included in the portfolio – the improvements appear to be larger for two asset combinations.

TABLE VII
P-VALUES FOR THE KS TEST ON SHARPE RATIO DISTRIBUTIONS

| | No sampling | With replacement | w/o replacement |
|---|---|---|---|
| **Overall** | 5.5466E-10 | 5.5466E-10 | 5.5466E-10 |
| **Two assets** | 5.5466E-10 | 5.5466E-10 | 5.5466E-10 |
| **Three assets** | 5.5466E-10 | 5.5466E-10 | 5.5466E-10 |
| **Four assets** | 5.5466E-10 | 5.5466E-10 | 5.5466E-10 |
| **Five assets** | 5.5466E-10 | 5.5466E-10 | 5.5466E-10 |

## B. Comparison with GMV results

In addition to running a GA to optimize the portfolio thresholds from multiple portfolios, we also used the global minimum variance method to find the weights of the less risky portfolio. Due to the process being deterministic, there is only a single value for return, risk and Sharpe ratio. In order to compare the GMV results to the GA ones, we obtain the best GA results across all portfolios' combinations over the 20 independent runs and present them in Table VIII. The reported financial metrics were calculated on the testing dataset (2020-2021) once the optimization algorithms have been run on the training dataset.

As we can observe, the improvements in terms of return and Sharpe ratio are of 7.28% for return, and 6.99% for Sharpe ratio. This is due to the fact that our GA seeks to maximize the Sharpe ratio, thus minimizing the risk and maximizing the return at the same time.

TABLE VIII
BEST GA RUN VS GMV RESULTS.

| Metrics | Best GA run | GMV | Relative difference |
|---|---|---|---|
| **Return** | 0.0383% | 0.0357% | 7.28% |
| **Risk** | 0.72% | 0.72% | 0 |
| **Sharpe ratio** | 5.05% | 4.72% | 6.99% |

Furthermore, Table IX shows the portfolio weights obtained from the best GA run, and the GMV approach. The best GA portfolio suggests an allocation of: 0.07% to the S&P/ASX 200 index, 15.09% to the Australian aggregate bond index, 39.29% to FTSE/EPRA NAREIT US index, 0.22% to the FTSE/EPRA NAREIT UK index, 44.55% to the FTSE/EPRA NAREIT Australia index. On the other hand, the GMV approach returned the S&P 500 index with a proportion of 47.59%, the UK aggregate bond index with an allocation of 5.91%, the Australian aggregate bond index with an allocation of 16.52%, the FTSE/EPRA NAREIT US index with an allocation of 22.48%, and the FTSE/EPRA NAREIT US index with an allocation of 7.50%. As we can see, the best GA portfolio has taken full advantage of the REITs classes. Such a portfolio allowed to obtain the same minimum risk as that resulting from the GMV approach, and at the same time, a higher expected return and Sharpe ratio, as we saw earlier in Table VIII. This shows the advantage of considering different combinations of the asset classes in the portfolio optimization solution, rather than admitting one global 9-asset solution.

### C. Discussion

The first aim of our experiments was to demonstrate that including REITs in a mixed-asset portfolio increases its diversification potential. From the results we obtained using our GA approach, the return distribution for the combinations including REITs indeed appears to be better than that for the combinations not including REITs under multiple perspectives: the average value appears to increase after REITs are included, while the volatility of the return distribution decreases (not only in terms of standard deviation, but also of maximum drawdown, downside deviation, and minimum-maximum range); the expected risk distribution for combinations including REITs presents a lower average value and a similar shape with respect to the expected risk distribution for combinations not including REITs; the Sharpe ratio appears to increase after REITs are included.

The second aim of our experiments was to show the potential improvement that could be achieved when using a GA with respect to the state-of-the-art GMV-based portfolio optimization. We have demonstrated that our GA could lead to a maximum Sharpe ratio of 5.05%, which is 6.99% higher than the 4.72% Sharpe ratio obtained through a GMV method. This is explained by a higher GA return (7.28% more compared to the GMV-based return). Moreover, the GA optimization results suggest a higher allocation to real estate than the benchmark method.

TABLE IX
BEST GA RUN VS GMV WEIGHTS.

| Asset | Best GA run | GMV |
|---|---|---|
| S&P 500 | 0 | 47.59% |
| FTSE 100 | 0 | 0 |
| S&P/ASX 200 | 0.07% | 0 |
| Dow Jones aggregate bond UK | 0 | 0 |
| Dow Jones aggregate bond UK | 0 | 5.91% |
| Dow Jones aggregate bond AU | 15.09% | 16.52% |
| FTSE/EPRA NAREIT US | 39.29% | 22.48% |
| FTSE/EPRA NAREIT UK | 0.22% | 0 |
| FTSE/EPRA NAREIT AU | 44.55% | 7.50% |

## VI. CONCLUSION

We focused on the problem of optimizing portfolios that involve REITs by using a genetic algorithm. Our experimental analysis indicates that investing in real estate increases the overall return level and decreases the risk level, thus enhancing the risk-adjusted return. This is mainly explained by the fact that the class of real estate generally presents low correlation to the other asset classes. Portfolios that include real estate showcase a better average Sharpe ratio than those not including real estate. Moreover, the expected returns of combinations including real estate are more concentrated around the mean (due to the lower volatility values, and a higher Kurtosis value), and more negatively skewed with respect to the expected returns for combinations including real estate, indicating a higher likelihood of observing returns that are higher than the mean.

While our results show that adding real estates to investment portfolios can have positive effect under the diversification perspective, further research is required to fully explore the potential of using genetic algorithms for optimizing portfolios including real estate. For example, in this paper we mainly used historical data to find the optimal weights. An alternative would be to use perspective data (i.e., price predictions), which might lead to better portfolio performance levels.

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