Optimizing Mixed-Asset Portfolio With Real Estate: Why Price Predictions?

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***Abstract*—The main purpose of portfolio optimization is to reduce the risk, and/or maximize the return of a group of investments. Most of the works that have been done on portfolio optimization are based on the Modern Portfolio Theory introduced by Markowitz in 1959. Some of them have employed price predictions to compute optimal asset weights. It has been demonstrated that using price predictions, instead of historical data, might improve portfolio performance under a risk-adjusted perspective. However, contributions in the field mainly focused on stocks, while little attention has been given on multi-asset portfolios including real estate. In this paper, we fill this gap by running a genetic algorithm on 456 portfolios to demonstrate the added value of including price predictions in our asset allocation problem. To investigate this, we compare the theoretical case of having a perfect foresight, where the predicted price p_t is exactly the same as the expected price p_t; under this case, the portfolio optimization task takes place in the test set (since we have assumed a perfect price prediction). We compare the results under perfect foresight with results derived from portfolio optimization that only took place in the training set, and the weights were then directly applied to the test set. Our goal is to demonstrate the theoretical advantages of using price predictions on mixed-asset portfolios that include real estate. Our results show that there can be significant improvements (up to 45%) in sharpe ratio, rate of return, and risk, when using price predictions instead of a historical prices based portfolio.**

***Index Terms*—genetic algorithm, mixed-asset portfolio, perfect foresight, portfolio optimization, risk-adjusted return**

I. Introduction

Portfolio optimization involves selecting optimal weights for a given set of assets (i.e., that maximize return and/or minimize risk). One asset class that has been gaining popularity in mixed-asset portfolios is real estate and in particular Real Estate Investment Truests (REITs). A REIT is a company which owns and manages real estate assets. Buying shares in REITs provides investors with the same benefits of investing in real estate (i.e., steady income, diversification, etc.), and at the same time, it requires a relatively low initial expense without the need to engage in real estate management.

Many works in the literature perform portfolio optimization by calculating the optimal weights in a training set and then applying those weights to an unseen test set [1, 2, 3]. A potential disadvantage of this approach is that prices in the test set might be significantly different than prices in the training set. As a result, weights computed using the training set might not fit the test set very well and thus lead to worse portfolio performance (i.e., increased risk and/or reduced return).

To alleviate the above issue, an alternative approach is to try and predict prices in the test set, and then perform the portfolio optimization task (i.e. calculating the optimal weights) directly in the test set [4, 5, 6, 7, 8]. The advantage of this approach is that we focus only on the data period we're interested in (i.e., the test set); as a result, accurate predictions would closely reflect the prices in the test set, and thus lead to a more efficient portfolio selection. However, the quality of the results is very much dependent on the effectiveness of the price predictions.

While the above methodology has been used before in mixed-asset portfolios, it has never been used before in portfolios that include REITs. To the best of our knowledge, portfolio optimization with REITs has only taken place by calculating the optimal sets of weights in the training set [9, 10].

The novelty of this work lies in the consideration of multi-asset portfolios including (both domestic and international) REITs in optimization problems involving perfect foresight. Our goal is to investigate the potential added benefits of using price predictions in the test set for portfolios that include REITs, rather than historical data. To do this, we assume the theoretical case of having perfect foresight in the test set, i.e. our price predictions are 100% correct. We then perform the optimization task directly in the test set and compare the performance of financial metrics, such as Sharpe ratio; rate of return; and risk, to the results obtained by having estimated the optimal weights in the training set and then applying them to the test set. We will use a genetic algorithm to optimize the portfolio weights, as this a well-known state-of-the-art algorithm for this type of problems [11, 12].

Moreover, as the literature usually focuses on portfolios with REITs from a single country, they tend to miss potential opportunities that might arise from including REITs of different countries. This work considers mixed-asset portfolios composed of stocks, bonds, and REITs belonging to three different countries, and will demonstrate that such approach might offer potential diversification benefits. In addition, the current literature is referred to time periods ranging from the 1990s to the Global Financial Crisis (GFC) period. None of them analyzes the 2020-2021 period, which is of interest because of the Covid-19 pandemic, or other political events such as Brexit (in the case of the UK market).

The rest of this paper is organized as follows. Section II presents a brief background on modern portfolio theory and a literature review on REITs and portfolio optimization, and Section III discusses the methodology of this paper. Our experimental setup is presented in Section IV. Section V provides a detailed discussion of the experimental results we obtained by both using historical data and perfect foresight. Finally, Section VI concludes the paper.

II. Background and Literature Review

Modern portfolio theory (MPT) is a mathematical framework that is largely used to solve asset allocation problems. The main assumption of MPT is that investors are risk averse in the sense that the less risky portfolio among those that portfolios that provide the same expected return. Consequently, one will choose a riskier portfolio only if compensated by a higher expected return. Different investors have different preferences over such tradeoffs based on their individual risk aversion levels.

According to MPT, a portfolio is considered efficient when its expected return is maximized for a given level of risk, or its expected risk is minimized for a given level of return. The expected return of the portfolio is expressed as a weighted average of the historical returns of the assets included in the portfolio, where the weighting factors are the proportions allocated to the different asset classes. The expected risk of the portfolio is expressed as the variance of the historical returns of the asset classes, and is a function of the correlations ρ_{ij}, for all pairs of asset (i, j). Given specific combinations of assets and standard deviations of asset returns, the highest possible standard deviation of portfolio returns is obtained when all correlations are equal to 1, which means that all asset pairs are perfectly correlated to each other. It is possible to reduce the portfolio's expected risk by selecting combinations of assets that are not perfectly positively correlated (i.e., $-1 < \rho_{ij} < 1$). This is known as *diversification*. If all asset pairs are perfectly uncorrelated ($\rho_{ij} = 0$ for all i, j), the variance of the portfolio returns is the sum of the squares of all asset weights times the asset's return variance. If all asset pairs are perfectly positively correlated ($\rho_{ij} = 1$ for all i, j), then the standard deviation of the portfolio returns is the sum of the standard deviations of the underlying asset returns, weighted by the proportion allocated to each asset class.

Including REITs in mixed-asset portfolios has been shown to have many advantages. Buying shares in REITs provides investors with the same benefits of investing in real estate (i.e., steady income, diversification, etc.), and at the same time, it requires a relatively low initial expense without the need to engage in real estate management. Several authors demonstrated the risk-adjusted performance and portfolio diversification benefits of REITs in mixed-asset portfolios [13, 14, 15, 16, 17, 18]. In particular, [13] used monthly total returns for stocks, bonds, and Japan REITs (J-REITs) to demonstrate that the risk-adjusted performance of J-REITs outperformed all of the other asset classes due to lower risk and higher average return. In the same way, [14] provided evidence that Malaysia REITs offer some diversification benefits when included in a mixed-asset portfolio due to their low correlation to the other asset classes. By using South Africa financial data, [15] demonstrated that while stocks are the best performing asset, REITs serve as return-enhancer when included in a mixed-asset portfolio, and tend to contribute to the portfolio risk reduction. With reference to the French financial market, [13] found out that the risk adjusted performance of REITs outperformed all of the other asset classes due to the lower risk and higher average return. This appeared to offer some diversification benefits to the mixed-asset portfolio. [17] examined the development of REITs in Thailand over the 2003-2010 period, and found out that pre-GFC and during the GFC, Thai REITs offered little diversification benefits to mixed-asset portfolios, while during the post-GFC period, Thai-REITs gained a significant role in the mixed-asset portfolios. [18] showed that Singapore REITs contributed to the risk reduction of mixed-asset portfolio during the 2003-2013 period.

All of the above-mentioned works focused on historical data: portfolio optimization problems were solved using historical average returns, instead of return predictions. Moreover, they focused on single countries, thus missing potential opportunities that might arise from including REITs of different countries. Moreover, the current literature is referred to time periods ranging from the 1990s to the post-GFC period. None of them analyzes the 2020-2021 period, which as mentioned earlier is of interest because of the Covid-19 pandemic and other political events such as the Brexit (in the case of the UK market).

This work thus considers mixed-asset portfolios composed of stocks, bonds, and REITs belonging to three different countries, and will demonstrate that such approach might offer potential diversification benefits. In the next section, we present our methodology of using perfect foresights and a genetic algorithm for the portfolio optimization task.

III. Methodology

A. Portfolio optimization under perfect foresight

As previously explained, our goal is to demonstrate that portfolio optimization in the test set under the assumption of perfect foresight will lead to better performance (in terms of financial metrics) when compared to optimization that has taken place on the training set.

The methodology used in this work follows two steps:

- The first step consists of optimizing asset weights using returns calculated on the test set.
- The second step consists of calculating the expected return, expected risk, and Sharpe ratio for all asset combinations.

As to the first step, we run a genetic algorithm (which we discuss in Section III-B) on all possible asset combinations. For example, if we have in total 3 assets (Asset A, B, and C), then we run the genetic algorithm for a total of 3 times (to optimize the weights if the portfolio consists only of Assets A and B; or if the portfolio consists only of Assets

B and C; of if the portfolio consists of Assets A, B, and C). The advantage of this approach (rather than performing the portfolio optimization task a single time on all available assets altogether) is that is allows us to investigate a much higher number of portfolio combinations and thus be able to better generalize our findings under perfect foresight. As mentioned above, the genetic algorithm is applied directly on the test set, as we have made the assumption of having a perfect foresight of the future prices.

In the second step, we use the optimal weights obtained from the first phase to compute the expected return, expected risk, and Sharpe ratio of the GA runs. The hypothesis behind our experiments is that this portfolio optimization strategy would result in better portfolio performance than in the case of optimal weights calculated on historical average returns.

B. *Portfolio optimization via a Genetic Algorithm*

Evolutionary algorithms have been widely used for financial applications, including portfolio optimization [19]. To tackle the portfolio optimization problem we consider in this paper, we use a particular type of evolutionary algorithm known as *genetic algorithm* (GA) [20, 21, 22]. Below we briefly discuss the GA we have used.

GA *chromosomes* (or, *individuals*) consist of N *genes* indicating the weights allocated to the N assets in the portfolio. The weight are real numbers in the interval $[0, 1]$, and their sum is equal to 1. For example, a GA individual that has the genotype [0.5 0.2 0.3] indicates that there are three assets, and the weight for those asset are 0.5, 0.2, and 0.3, respectively. Initially, all genes are assigned the same weight (in particular, $W_i = 1/N$ for each asset i), which are then evolved according to a set of operators.

We use *elitism*, *one-point crossover* and *one-point mutation*. Since we use market proxies in our experiments, the number of assets is small, and thus one-point crossover and mutation are sufficient (see Section IV for more details). After the application of crossover and mutation, we apply normalization to each GA individual, to ensure that the sum of weights remains equal to 1.

State-of-the-art methods for solving portfolio optimization problems have used many different metrics as fitness functions. In this paper, we use the *Sharpe ratio*, defined as the ratio of the difference between the average return and the risk-free rate, over the standard deviation of the returns, that is,

$$S = \frac{r - r_f}{\sigma_r}, \tag{1}$$

where r is the average return of the investment, r_f is the risk-free rate, and σ_r is the standard deviation of the returns.

IV. Experimental setup

Our experiments aim to provide evidence that optimizing asset weights under a hypothetically perfect foresight situation results in better portfolio performance than in the case of historical data.

A. *Data*

We use daily prices over the period between June 2017 and January 2021. The training period is from June 2017 to December 2019 (inclusive) and the test period is from January 2020 to January 2021 (inclusive). We adopt the perspective of an institutional investor from the US who wants to gain exposure to international markets (UK and Australia). The asset classes we consider are stocks, bonds, and listed real estate.

As other authors did previously [23, 1], we use index prices as data for our experiments. Stocks are proxied by the S&P 500 index for the US marked, by the FTSE 100 index for the UK market, and by the S&P/ASX 200 index for the Australian market. For the bond asset class, we use the indices issued by Dow Jones for all the three markets considered. Finally, we use the FTSE/EPRA NAREIT indices to proxy the real estate markets. We thus have 9 asset classes, namely 3 stocks, 3 bonds, and 3 REITs.

Table I presents the Sharpe ratio for each asset class for the period between 2017 and 2019. The Sharpe ratio is calculated as the ratio between each asset's average return and its risk. From the values shown in Table I, we can observe that the real estate asset class generally presents a lower level of performance (that is, lower Sharpe ratio) compared to the other asset classes for the considered period.

Table II presents the Sharpe ratio for each asset class for the period between 2020 and 2021 (which corresponds to the Covid-19 pandemic period). From the values shown in Table II, we can observe that the risk-adjusted performance of the real estate asset class improves with respect to the previous period, and appears to be even better than that of the other asset classes. Such improvement in REIT performance is expected to result in better portfolio optimization performance when the testing set (rather than the training set) is used.

The reason we have chosen to include REITs in this mixed-asset portfolio is because of the diversification they bring in. From the correlation matrix shown in Table III, we can observe that the real estate asset class generally has relatively lower correlation with the other asset classes, thus justifying its diversification potential. More specifically, a low or zero correlation between two asset classes might reduce a portfolio's overall level of risk. For example, regarding the correlation between real estate and stocks, the US REIT index has a correlation of 0.267 with the FTSE 100, and of 0.217 with the S&P/ASX 200 index. Its correlation with the S&P 500 index (belonging to the same country) presents a higher value of 0.541 which is considered a low correlation value. Moreover, the UK REIT has a correlation of 0.34 with the S&P 500 index, of 0.728 with the FTSE 100 index, and of 0.329 with the S&P/ASX 200 index. As to the correlation between real estate and bonds, we observe that US REITs have a correlation of 0.547 with the US bonds, of 0.249 with the UK bonds, and of 0.17 with the Australian bonds. Moreover, the UK REITs have a correlation of 0.347 with the US bonds, of 0.726 with the UK bonds, and of 0.299 with the Australian

TABLE I: Sharpe ratio for each asset class from 2017 to 2019 (training set)

S&P 500	FTSE 100	S&P/ASX 200	US bond	UK bond	AU bond	US REIT	UK REIT	AU REIT
5.16%	0.22%	0.58%	5.06%	0.29%	0.69%	0.43%	-0.77%	1.02%

TABLE II: Sharpe ratio for each asset class from 2020 to 2021 (test set)

S&P 500	FTSE 100	S&P/ASX 200	US bond	UK bond	AU bond	US REIT	UK REIT	AU REIT
3.73%	1.27%	2.33%	3.53%	1.32%	2.49%	4.49%	4.26%	3.10%

TABLE III: Correlation coefficients between asset classes.

	S&P 500	FTSE 100	S&P/ASX 200	US bond	UK bond	AU bond	US REIT	UK REIT	AU REIT
S&P 500	1	0.515	0.326	0.999	0.48	0.259	0.541	0.34	0.249
FTSE 100	0.515	1	0.464	0.521	0.946	0.449	0.267	0.728	0.334
S&P/ASX 200	0.326	0.464	1	0.329	0.443	0.939	0.217	0.329	0.73
US bond	0.999	0.521	0.329	1	0.488	0.263	0.547	0.347	0.251
UK bond	0.480	0.946	0.443	0.488	1	0.489	0.249	0.726	0.31
AU bond	0.259	0.449	0.939	0.263	0.489	1	0.17	0.299	0.668
US REIT	0.541	0.267	0.212	0.547	0.249	0.17	1	0.272	0.234
UK REIT	0.34	0.728	0.329	0.347	0.726	0.299	0.27213	1	0.283
AU REIT	0.249	0.334	0.73	0.25	0.31	0.668	0.234	0.283	1

bonds.

As we can observe, there appears to be a low correlation between asset classes belonging to different markets. This could open opportunities to an international diversification. In other words, an investor might find diversification opportunities in gaining exposure to foreign markets.

B. Experimental parameters

To decide the parameter values, we undertook a parameter tuning process using the I/F-Race package [24]. I/F-Race implements the iterated racing procedure, which is an extension of the Iterated F-Race process and builds upon the race package by [25]. Its main purpose is to automatically configure optimization algorithms by finding the most appropriate settings, given a set of instances of a problem.

In our case, I/F-Race was applied to data for the period from June 2017 to December 2018. The following twelve months (January-December 2019) were used only with the already tuned parameters, after I/F-Race was completed. In other words, the first period was used as a training dataset for parameter tuning, while the second period was used as a validation dataset for parameter testing. The period January-January 2021 was the test set, and remained unseen during the parameter tuning process. At the end of the tuning process, we picked the best parameters returned by I/F-Race, which constitute the experimental parameters used by our algorithms, and are presented in Table IV.

TABLE IV: I-Race Parameter Tuning Results.

Parameter	Value
Tournament size	3
Population size	300
Crossover rate	1.0
Mutation rate	0.01
Number of generations	10

C. Benchmark: Historical data approach

In order to demonstrate the potential improvement from the perfect foresight situation, we compare their results with results obtained from experiments under the historical method. In other words, we used the 2017-2019 period as the training set, where we ran the portfolio optimization task. After the weights were obtained in the training set, we then applied them to the test set (2020-2021 period), and then compared the financial performance (Sharpe ratio, rate of return, risk) against the perfect foresight results. We again used a genetic algorithm for the portfolio optimization task. The GA used the same parameters that were presented above in Table IV.

V. Results

A. Summary statistics

In this Section, we present results from our experiments conducted under a perfect foresight situation and historical data case, and discuss our conclusions (in Section V-C). Results are presented as averages over 20 individual GA runs. It should also be noted that all results are daily results. So when, for example, we present a seemingly “low” return of around 0.03%, its annual equivalent would be around 11.6%.

As previously explained, under the perfect foresight situation, we assume that the predicted price at t_i is exactly the same as the actual price at t_i. In other words, this hypothetical prediction model leads to a zero error rate. We compare the portfolio performance results obtained from such model with those obtained from the historical data approach. As mentioned in Section III, have we run the genetic algorithm multiple

TABLE V: Average GA expected return for the portfolios.

Metrics	Historical data approach	Perfect foresight approach	Relative difference	KS test results (p-values)
Overall	4.03e-04	4.60e-04	14.21%	9.08e-75
Two assets	3.63e-04	4.30e-04	18.36%	0.0059
Three assets	3.90e-04	4.50e-04	15.32%	7.59e-13
Four assets	4.08e-04	4.63e-04	13.38%	5.03e-28
Five assets	4.17e-04	4.73e-04	13.30%	1.66e-37

TABLE VI: Average GA expected risk for the portfolios.

Metrics	Historical data approach	Perfect foresight approach	Relative difference	KS test results (p-values)
Overall	9.98e-03	8.01e-03	-19.75%	1.21e-104
Two assets	9.85e-03	9.14e-03	-7.15%	1.31e-04
Three assets	9.97e-03	8.40e-03	-15.76%	1.46e-18
Four assets	1.00e-02	7.89e-03	-21.14%	3.14e-41
Five assets	1.00e-02	7.54e-03	-24.55%	9.53e-52

TABLE VII: Average GA expected Sharpe ratio for the portfolios.

Metrics	Historical data approach	Perfect foresight approach	Relative difference	KS test results (p-values)
Overall	4.02%	5.82%	44.86%	6.26e-138
Two assets	3.65%	4.75%	30.09%	3.76e-09
Three assets	3.89%	5.43%	39.56%	3.73e-29
Four assets	4.07%	5.92%	13.38%	1.32e-52
Five assets	4.17%	6.30%	13.30%	4.10e-56

times, on all possible asset combinations. Given we have 9 asset classes (3 stocks, 3 bonds, 3 REITs), this created 456 different asset combinations to which the GA was applied. The advantage of this approach (rather than performing the portfolio optimization task a single time on all available assets altogether) is that is allows us to investigate a much higher number of portfolio combinations and thus be able to better generalize our findings under perfect foresight. As a result of this approach, different portfolios were created; some included all 9 assets, while there were others that only created 2 assets, 3 assets, 4 assets, or 5 assets. We did not consider 6, 7, or 8-asset portfolios, as these did not include the REITs class.

Table V shows the results obtained in terms of expected returns. We can notice an increase in all the results, both at overall level and for single combinations. In particular, at overall level (i.e., considering all the portfolios together), we observe an increase of around 14.21% from the historical data approach to the perfect foresight method. We also observe an improvement of around 18.36% in the case of two asset combinations, of around 15.32% in the case of three asset combinations, of around 13.38% in the case of four asset combinations, and of around 13.30% in the case of five asset combinations. Under a financial perspective, an investor sees the portfolio profitability increasing under a perfect foresight situation.

We also performed five Kolmogorov-Smirnov (KS) tests at the 5% significance level, one per distribution pair that we wanted to compare: returns from perfect foresight method and returns from historical data approach at global level, and for sub-portfolios. The null hypothesis for each test was that the two distributions come from the same probability distribution. Given that we were making multiple comparisons, we adjusted the tests' p-value according to the Bonferroni correction to $0.05/5 = 0.01$. The p-value for the first test was 9.08e-75; the p-value for the second test was 0.0059; the p-value for the third test was 7.59e-13; the p-value for the fourth test was 5.03e-28; and the p-value for the fifth test was 1.66e-37. As we can observe, all five values are well below the adjusted p-value of 0.01, thus making the differences statistically significant at the 5% level.

Table VI shows results for the average expected risks. In this case, under a perfect foresight situation, we observe a decrease in the average risk levels both for the whole sets of assets and for their subsets. In particular, average expected risks show a decrease of around 19.75% from the historical data approach to the perfect foresight method at overall level, of around 7.15% in the case of two asset combinations, of around 15.76% in the case of three asset combinations, of around 21.14% in the case of four asset combinations, and of 24.55% in the case of five asset combinations. These results can be interpreted as an improvement in portfolio performance under a perfect foresight situation.

In order to compare the risk distribution pairs (risks from perfect foresight method and risks from historical data approach at global level, and for sub-portfolios), we performed five Kolmogorov-Smirnov (KS) tests at the 5% significance level. As we have seen before, the null hypothesis for each test was that the two distributions come from the same probability distribution. The-adjusted p-value is again equal to 0.01, as we have again applied the Bonferroni correction. The p-value for the first test was 1.21e-104; the p-value for the second test was 1.31e-04; the p-value for the third test was 1.46e-18; the p-value for the fourth test was 3.14e-41; and the p-value for the fifth test was 9.53e-52. As we can observe, again all five values are well below the adjusted p-value of 0.01, thus making the differences statistically significant at the 5% level.

Table VII shows results obtained for the average expected Sharpe ratios. As we can observe, the perfect foresight method leads to an improvement in the risk-adjusted portfolio performance with respect to the historical data approach in all cases. In particular, we can observe an increase of around 44.86% at overall level, of around 30.09% in the case of two asset combinations, of around 39.56% in the case of three asset combinations, of around 45.52% in the case of four asset combinations, and of around 51.22% in the case of five asset combinations.

We performed five Kolmogorov-Smirnov (KS) tests at the 5% significance level, one per distribution pair that we wanted to compare: Sharpe ratios from perfect foresight method and Sharpe ratios from historical data approach at global level, and for sub-portfolios. The null hypothesis is again that the two distributions come from the same probability distribution. The adjusted p-value is equal to 0.01, according to the Bonferroni correction. The p-value for the first test was 6.26e-138; the p-value for the second test was 3.76e-09; the p-value for the third test was 3.73e-29; the p-value for the fourth test was 1.32e-52; and the p-value for the fifth test was 4.10e-56. As we can observe, all five values are well below the adjusted p-value of 0.01, thus making the differences statistically significant at the 5% level.

Figure 1a represents the expected return distributions obtained from the historical data approach and the perfect foresight method. As we can observe, the return distribution obtained from the historical data approach presents a higher peak. On the other hand, the distribution of the perfect foresight has its mass concentrated on the right of the figure, indicating higher returns. Values for the Kurtosis of return distributions from the historical data approach and the perfect foresight method are 6.3368 and 5.9943 respectively, while those for the Skewness are -1.9697 and -1.3295 respectively. In financial terms, this means that an investor could expect similar returns to the average under the perfect foresight situation.

Figure 1b shows the expected risk distributions obtained from the historical data approach and the perfect foresight method. As we can observe, the risk distribution obtained from the historical data approach presents a higher peak, and is more skewed to the right than the risk distribution obtained from the perfect foresight method. In particular, values for the Kurtosis of return distributions from the historical data approach and the perfect foresight method are 3.3836 and 5.2604 respectively, while those for the Skewness are -0.2192 and 1.3701 respectively. In financial terms, this means that an investor could expect closer risk levels to the average under the perfect foresight hypothesis, which translates into lower volatility.

B. Computational times

A single run of the GA did not take longer than 30 seconds, under the parameter values presented in Table IV. As the portfolio optimization task is an offline approach, this duration is relatively fast and does not constitute a problem. Besides, speedups can be obtained by parallelizing the evolutionary process, as it has previously been shown in the literature (e.g. [26]).

C. Discussion

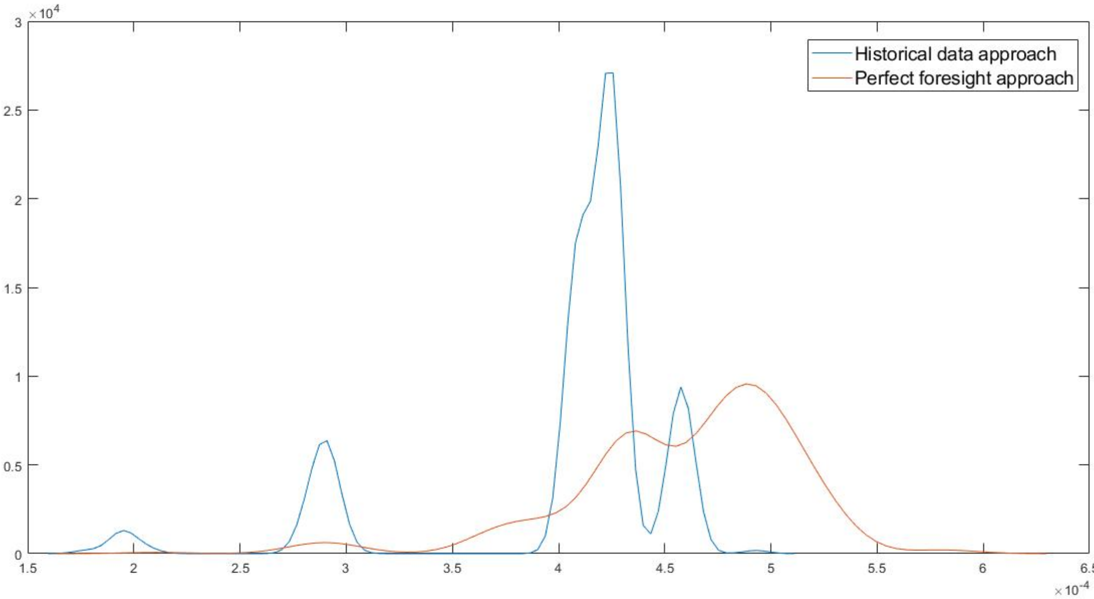
The main aim of our experiments was to demonstrate the potential improvement in mixed-asset portfolio performance that can be obtained from hypothetically perfect price predictions. As we have observed, the average portfolio returns appear to increase under a perfect foresight situation, and given the KS test results, such increases appear to be statistically significant. At the same time, the average portfolio risks appear to decrease when the perfect foresight case is applied, and based on the KS test results, such differences can be considered statistically significant. Such results lead to an improvement in the risk-adjusted portfolio performance.

Moreover, we compared the risk and return distributions under the historical data situation and the perfect foresight case. We observed a concentration of returns on the right side for the perfect foresight approach, which indicates higher returns on average. At the same time, we observed a concentration of risk values on the left side for the perfect foresight approach, which indicates lower risks on average. In addition, both the return and risk distributions appear to have a greater concentration around the mean under the perfect foresight situation, which translates into lower volatility. Under a financial perspective, such results are promising for an investor who wishes to increase the return, and at the same time to reduce the risk associated to an investment.

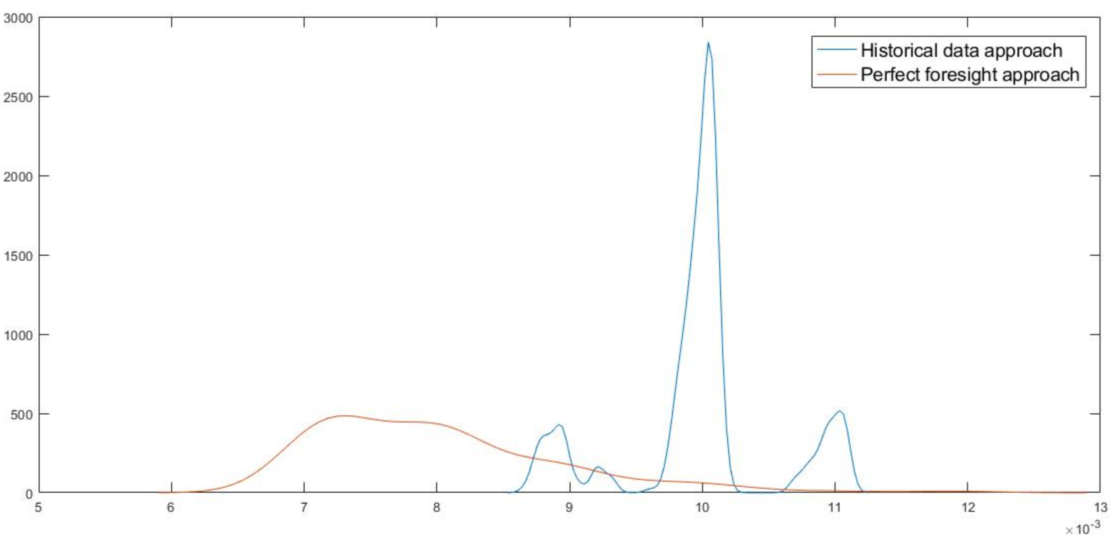
VI. Conclusion

We focused on the problem of optimizing portfolios made of stocks, bonds, and REITs for three different countries by using a genetic algorithm. The aim of this work was to demonstrate the potential improvement that can be obtained from predicting asset prices in the testing set with respect to prices of the training set. Our experimental analysis indicates that involving hypothetically perfect price predictions in the portfolio allocation process increases the overall return level and decreases the risk level, thus enhancing the risk-adjusted return. Portfolios obtained under a perfect foresight situation showcase a better average Sharpe ratio than those obtained from historical data.

Our results show that using price predictions can lead to better risk-adjusted performance than when using historical data. This is mainly explained by the fact that prices in the training set might be significantly different than those in the testing set (as we demonstrated through the KS tests), thus leading to under-performing portfolios. The results that we obtained



(a) Expected return distributions



(b) Expected risk distributions

motivate us to engage in price prediction tasks in order to solve mixed-asset portfolio optimization problems involving REITs. Future work will thus focus on finding appropriate machine learning algorithms to predict future prices of stocks, bonds, and REITs, which are as close as possible to the real values that appear in the test set. Succeeding in this task will allow us to observe similarly good performance in returns and risk, as we have observed under the theoretical case of perfect foresight.

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