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# Rate Splitting in the Presence of Untrusted Users: Outage and Secrecy Outage Performances

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**ABSTRACT** In this contribution, a thorough investigation of the performance of rate splitting is conducted in terms of outage and secrecy outage for the simultaneous service to a near user and far user, where the latter attempts to overhear the message of the former. The source transmits a linear combination of the users' common stream and private streams. Once the common stream is retrieved, two decoding strategies can be adopted by each user. In the first strategy, the nodes (near or far) treat the far user's private stream as noise to retrieve the private stream of the near user, then the far user decodes its own stream. In the second strategy, the nodes decode the far user's private stream by treating the one of the near user as noise, then the near user retrieves its private stream while the far user decodes the stream of the near user in its attempt to overhear it. Considering the four decoding combinations, we obtain exact closed-form expressions for the outage probability, and provide tight approximations for the secrecy outage probability. Comparative results are also provided. In particular, it is shown that to achieve better outage probability, with no concern about secrecy, once the decoding of the common stream is completed, each user should first retrieve the private stream with lower target data rate by treating the other private stream as noise. To improve the secrecy outage probability, once the common stream is decoded, the near user must first decode the far user's private stream, and the far user should first retrieve the private stream with lower target data rate.

**INDEX TERMS** Rate splitting, physical-layer security, outage probability, secrecy outage probability.

## I. INTRODUCTION

### A. CONTEXT

INCREASING the capacity of wireless communication systems in terms of the number of users that can be served within a limited spectrum band has always been a fundamental design goal. A promising approach to serve multiple users simultaneously in a shared bandwidth is rate splitting (RS), which is based on the concept of Han-Kobayashi (H-K) signaling [1]. In RS systems,[1] a user's message is split at the source into two parts, known as common and private messages. The common parts of the message are combined and encoded to form a single common stream to be decoded by all users. The private messages are separately encoded into private streams. Then, the source broadcasts a linear combination of the users' private streams and the common stream, with a predefined power allocation. After decoding the common stream, each user can obtain its private stream by treating those of the other users as noise [2].

For a system comprised of two-user single-input single-output interference channels, implementing H-K signalling is known to yield the best achievable rate region [1]. Also, [3] and [4] demonstrate that the achievable rate region

1. In this paper, an RS system refers to a system implementing the RS mechanism.

of networks with multiple-input multiple-output interference channels can be improved by utilizing H-K signaling. In particular, applying RS mechanism in multi-user networks can enhance the quality of service [2], [5]. For instance, RS multiple access (RSMA) can achieve equal or larger rate region compared to NOMA (non-orthogonal multiple access) and conventional access schemes such as space-division multiple access, and also reduce the computational complexity as compared to NOMA for instance [6]. NOMA and OMA in broadcast channels can be seen as special cases of RS [6], [7]. However, RSMA is superior over the aforementioned conventional multiple access schemes, as has been demonstrated from various aspects, e.g., in terms of spectral efficiency [8]–[11], max-min achievable rates [6], [12], [13], and ergodic sum-rate [14]. In [15], the performance of two-user uplink RSMA was investigated in terms of outage probability (OP) and throughput. Also, the OP of cooperative H-K signalling has been studied in [16], where the near user relays the common message or the far user to improve the corresponding received signal-to-noise ratio (SNR). This is a particular case of RSMA to serve near and far users, which we seek to improve in this paper by considering all possible strategies to decode the common and private messages of the RSMA users.

Several studies have also looked at the system performance when RS, or H-K signaling in a more general sense, operates in the presence of eavesdroppers. Indeed, since the signals are transmitted on free medium, eavesdroppers, whether external or internal to the communication system, have the opportunity to overhear the messages of the system's users, [17], [18]. Generally speaking, to ensure secure communication, alongside cryptography protocols, solutions based on physical-layer security (PLS) can leverage the randomness of the fading channels to guarantee message secrecy [19], [20]. A suitable metric to evaluate the secrecy performance of PLS-based schemes is the secrecy outage probability (SOP) [19]. In this context, several works have investigated the SOP of RSMA systems. The secrecy performance of H-K based communication assisted with unmanned aerial vehicle in the presence of external eavesdropper was investigated in [21]. Therein, it was shown that the H-K access outperforms OMA and NOMA in terms of secrecy throughput. In [19], considering a two-user RS system with external eavesdropper, robust beamforming was devised to maximize the secrecy achievable rate, where the common stream was modeled as artificial noise for the eavesdropper. Besides, by enabling cooperation, improvement in the secrecy sum-rate of RSMA in the presence of external eavesdropping was demonstrated in [22], [23]. Cooperative RS was also considered at an aerial base station to protect the secrecy of a two-user system against an external eavesdropper [24]. Deploying RSMA in a system with two legitimate users and a potential eavesdropper was also studied in [25], which addressed the maximization of the minimum achievable secrecy rate.

### B. CONTRIBUTION

In this paper, the focus is on the investigation of the robustness of RS technique in terms of SOP and OP. Specifically, the RS system under study is comprised of three nodes: the source, a near user, and a far user; the users being assumed to be scheduled for service. Each user has to first decode the common stream. In addition to retrieving its own private stream, the far user attempts to wiretap the message of the near user. After decoding the common stream, two decoding strategies can be implemented at each node, termed Strategy-1 and Strategy-2. When adopting Strategy-1, the users (near and/or far) first decode the near user's private stream while treating the stream of the far user as noise. In this case, the far user also retrieves its own private stream after decoding the one of the near user. When adopting Strategy-2, the users (near and/or far) retrieve the private stream of the far user while considering the stream of the near user as noise. Here, the near user decodes its own private stream after having access to the far user's private stream. The far user is capable of overhearing the near user's private stream after obtaining its own private stream via this decoding scheme. As each user in the system can individually apply either Strategy-1 or Strategy-2, a total of four decoding strategies are possible. Specifically, Strategy-*ij*, $i, j \in \{1, 2\}$, describes the RS system in which the near user and the far user adopt Strategy-*i* and Strategy-*j*, respectively. Aiming at the evaluation of the OP and SOP of the RS system in the different decoding scenarios, the focus of this work and its ensuing contributions can be summarized as follows.

We obtain the exact closed-form expressions for the OP in the four decoding strategies for the entire system, and for the near user separately. Monte-Carlo simulations are also presented and confirm the analysis. To compare the decoding strategies, the impacts of the power allocation, power budget, target data rates, and the far user's distance, on the outage performance without security constraints, are investigated.

Further, using the Gauss-Chebyshev quadrature method, we find tractable approximations for the SOP corresponding to the use of each one of the four decoding strategies, and characterize the exact SOP in the system operation with Strategy-22. We also investigate the effects of the power allocation, the source power budget, the target data rates, and the far user's distance, on the SOP of the four RS scenarios. The agreement between the Monte-Carlo simulations and analytical results confirms the accuracy of the analysis.

We also compare the system's OP and SOP in different operation scenarios, and provide guidelines on the decoding mechanisms that lead to enhanced performance. Important findings are revealed. In particular, it is shown that when the allocated powers and the streams' targets rates are equal, then all decoding schemes will yield the same outage performance. Also, when the powers allocated to the private streams of the users are the same, the near user must follow Strategy-2 to achieve the least SOP.

In detailing the above-highlighted contributions and findings, the following content of the paper is organized as follows. Section II elaborates on the RS system and the decoding strategies. The OP and SOP are studied in Sections III and IV, respectively. Numerical results are discussed in Section V, and Section VI concludes the paper.

*Notation:* For event $A$, $\mathbb{P}(A)$ denotes the probability of occurrence of $A$, and $\overline{A}$ is its complementary. $F_X(x)$ and $f_X(x)$ respectively denote the cumulative distribution function (CDF) and the probability density function (PDF) of random variable $X$. Operator $[y]^+$ returns $\max(0, y)$, $U(.)$ is the unit step function, and $\mathbb{1}_B$ is the indicative function corresponding to event $B$, where $\mathbb{1}_B = 1$ if $B$ occurs and $\mathbb{1}_B = 0$ otherwise.

## II. THE RATE SPLITTING SYSTEM UNDER EAVESDROPPING

The communication takes place from the source to a near user, $U_1$, and a far user, $U_2$, using the RS mechanism. Denote the messages intended to $U_1$ and $U_2$ by $W_1$ and $W_2$, respectively. As per the RS mechanism, the messages $W_i$, $i \in \{1, 2\}$, are split into a common part $W_{i,\mathrm{c}}$ and private parts $W_{i,\mathrm{p}}$. The common parts are combined together and encoded into a common stream $S_\mathrm{c}$. The private parts, $W_{1,\mathrm{p}}$ and $W_{2,\mathrm{p}}$, are encoded into the private streams $S_1$ and $S_2$, respectively. Here, it is assumed that $S_\mathrm{c}$, $S_1$ and $S_2$ are i.i.d. zero-mean circularly-symmetric complex Gaussian random variables of unit variance. The source transmits a linear combination of $S_\mathrm{c}$, $S_1$ and $S_2$, with a power allocation such that the private stream $S_1$ is to be kept confidential against the eavesdropping by $U_2$. Let the power allocation coefficients pertaining to $S_\mathrm{c}$, $S_1$ and $S_2$, be denoted by $a_\mathrm{c}$, $a_1$ and $a_2$, respectively, with $a_1 + a_2 + a_\mathrm{c} = 1$ and $0 < a_1, a_2, a_\mathrm{c} < 1$. Using a power budget $P$, the source broadcasts the superimposed streams, as

$$X = \sqrt{P}\left(\sqrt{a_1}S_1 + \sqrt{a_2}S_2 + \sqrt{a_\mathrm{c}}S_\mathrm{c}\right). \tag{1}$$

Denote the Rayleigh fading channel gains between the source transmitter and the users by $h_1$ and $h_2$, with distribution parameters $\lambda_1$ and $\lambda_2$, respectively. The source and node $U_t$, $t \in \{1, 2\}$, are separated by distance $d_t$, and the path-loss coefficient is $\alpha$. The signal at node $U_t$, $t \in \{1, 2\}$, can then be expressed as

$$Y_t = h_t d_t^{-\frac{\alpha}{2}} \sqrt{P}\left(\sqrt{a_1}S_1 + \sqrt{a_2}S_2 + \sqrt{a_\mathrm{c}}S_\mathrm{c}\right) + n_t, \tag{2}$$

where $n_t$ is the additive white Gaussian noise, with zero mean and variance $\sigma^2$.

To decode its private stream, a user first decodes $S_\mathrm{c}$ while treating $S_1$ and $S_2$ as noise. Define $g_t = |h_t|^2 d_t^{-\alpha}$, with CDF $F_{g_t}(x) = 1 - e^{-\lambda_t d_t^\alpha x}$. For any node $U_t$, the achievable rate to decode the common stream $S_\mathrm{c}$ is given by,[2]

$$R_{\mathrm{c}\to t} = \log(1 + \gamma_{\mathrm{c}\to t}), \tag{3}$$

where $\gamma_{\mathrm{c}\to t} = \frac{a_\mathrm{c} g_t}{(a_1+a_2)g_t+\rho}$ and $\rho = \frac{\sigma^2}{P}$.

The four decoding strategies are depicted in Fig. 1. For $i, j \in \{1, 2\}$, Strategy-$ij$ indicates that $U_1$ and $U_2$ follow Strategy-$i$ and Strategy-$j$, respectively. Strategy-12 corresponds to conventional RS system. As aforementioned, we consider three other decoding mechanisms, and investigate their performance in comparison to the conventional scheme.

2. In this paper, the unit of rates is bit per channel use (BPCU).

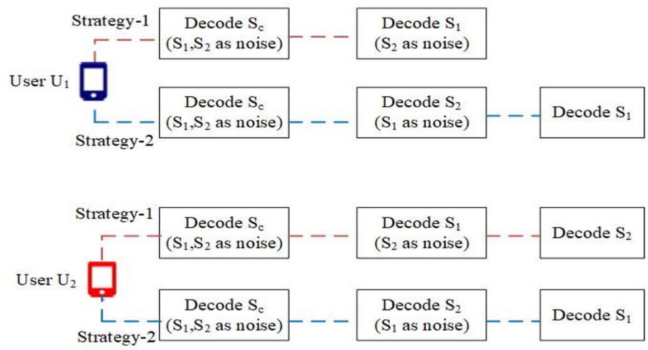



**FIGURE 1.** The four decoding strategies in the RS system. The nodes first decode the common stream $S_c$, then each can follow two strategies for decoding its own private stream, $S_1$ or $S_2$.

### 1) STRATEGY-11

In this decoding strategy, both users follow Strategy-1. After decoding $S_\mathrm{c}$, node $U_t$, $t \in \{1, 2\}$, treats $S_2$ as noise and retrieves $S_1$ with achievable rate

$$R^{(1)}_{1\to t} = \log\left(1 + \gamma^{(1)}_{1\to t}\right), \tag{4}$$

where superscript $^{(1)}$ refers to Strategy-1, and $\gamma^{(1)}_{1\to t} = \frac{a_1 g_t}{a_2 g_t+\rho}$. In this case, the achievable rate of $U_2$ to retrieve $S_2$ is given by (5), where $\gamma^{(1)}_{2\to 2} = \frac{a_2 g_2}{\rho}$.

$$R^{(1)}_{2\to 2} = \log\left(1 + \gamma^{(1)}_{2\to 2}\right). \tag{5}$$

### 2) STRATEGY-12

In this case, $U_1$ and $U_2$ use Strategy-1 and Strategy-2, respectively. After decoding $S_\mathrm{c}$, $U_1$ decodes $S_1$ by assuming $S_2$ as noise. Therefore, the achievable rate of $U_1$ for accessing $S_1$ is as per (4). At the $U_2$'s side, after having access to $S_\mathrm{c}$, $U_2$ retrieves $S_2$ with rate

$$R^{(2)}_{2\to 2} = \log\left(1 + \gamma^{(2)}_{2\to 2}\right), \tag{6}$$

where $\gamma^{(2)}_{2\to 2} = \frac{a_2 g_2}{a_1 g_2+\rho}$ and $S_1$ is considered as noise. Since $U_2$ also attempts to overhear $S_1$, the achievable rate of $U_2$ for decoding $S_1$ is given by (7), where $\gamma^{(2)}_{1\to 2} = \frac{a_1 g_2}{\rho}$.

$$R^{(2)}_{1\to 2} = \log\left(1 + \gamma^{(2)}_{1\to 2}\right). \tag{7}$$

### 3) STRATEGY-21

Here, nodes $U_1$ and $U_2$ respectively adopt Strategy-2 and Strategy-1. Hence, once $U_1$ obtains $S_\mathrm{c}$, it first decodes $S_2$ and then retrieves its own private stream $S_1$. The achievable rate of $U_1$ to obtain $S_2$ is

$$R^{(2)}_{2\to 1} = \log\left(1 + \gamma^{(2)}_{2\to 1}\right), \tag{8}$$

where superscript $^{(2)}$ refers to Strategy-2, and $\gamma^{(2)}_{2\to 1} = \frac{a_2 g_1}{a_1 g_1 + \rho}$. In decoding $S_1$, node $U_1$ achieves the rate

$$R^{(2)}_{1\to 1} = \log\left(1 + \gamma^{(2)}_{1\to 1}\right), \tag{9}$$

where $\gamma^{(2)}_{1\to 1} = \frac{a_1 g_1}{\rho}$. Besides, as node $U_2$ follows Strategy-1, the achievable rates for $U_2$ to decode $S_1$ and $S_2$ correspond to (4) and (5), respectively.

#### 4) STRATEGY-22

In this approach, $U_1$ and $U_2$ utilize Strategy-2 to decode their streams. Therefore, after decoding $S_c$, both users retrieve $S_2$ by treating $S_1$ as noise, and lastly obtain $S_1$. The achievable rates for $U_1$ to access $S_2$ and $S_1$ are given by (8) and (9), respectively. Also, $U_2$ achieves the rates (6) and (7) to decode $S_2$ and overhear $S_1$, respectively.

## III. OUTAGE PROBABILITY

First, we investigate the OP of each strategy without considering any security constraint. The target date rates associated with $S_c$, $S_1$ and $S_2$ are denoted by $R_c$, $R_1$ and $R_2$, respectively. For simplicity, define $C_x = 2^{R_x} - 1$, $x \in \{c, 1, 2\}$.

*Definition 1:* For $U_t$, $t \in \{1, 2\}$, define $E_{c\to t}$ as the event that node $U_t$ is not able to decode $S_c$ correctly, i.e.,

$$E_{c\to t} = \{R_{c\to t} < R_c\}, \tag{10}$$

and, for $i, j \in \{1, 2\}$, define $E^{(j)}_{i\to t}$ as the event that $U_t$ fails to retrieve $S_i$ via Strategy-$j$, i.e.,

$$E^{(j)}_{i\to t} = \left\{R^{(j)}_{i\to t} < R_i\right\}. \tag{11}$$

### A. PERFORMANCE WITH STRATEGY-11

Here, both users adopt Strategy-1. This strategy faces outage in the following cases: $U_t$, $t \in \{1, 2\}$, fail to decode $S_c$, i.e., $E_{c\to t}$; $U_1$ cannot retrieve $S_1$, i.e., $E^{(1)}_{1\to 1}$; $U_2$ is not able to decode $S_1$, i.e., $E^{(1)}_{1\to 2}$; $U_2$ is not able to decode $S_2$, i.e., $E^{(1)}_{2\to 2}$. Hence, the OP in Strategy-11 is given by

$$\mathrm{OP}^{(11)} = 1 - \mathbb{P}\left(\overline{E}_{c\to 1} \cap \overline{E}_{c\to 2} \cap \overline{E}^{(1)}_{1\to 1} \cap \overline{E}^{(1)}_{1\to 2} \cap \overline{E}^{(1)}_{2\to 2}\right). \tag{12}$$

*Theorem 1:* Let $\tau_c = \frac{\rho C_c}{a_c - C_c + a_c C_c}$, $\tau_1 = \frac{\rho C_1}{a_1 - a_2 C_1}$, $\mu_1 = \max(\tau_c, \tau_1)$, and $\beta_1 = \max(\mu_1, \frac{\rho C_2}{a_2})$. To avoid the occurrence of outage, the power allocation must satisfy the conditions $a_c > \frac{C_c}{1+C_c}$ and $\frac{a_1}{a_2} > C_1$, and $\mathrm{OP}^{(11)}$ is obtained as

$$\mathrm{OP}^{(11)} = 1 - e^{-\left(\lambda_1 d_1^{\alpha} \mu_1 + \lambda_2 d_2^{\alpha} \beta_1\right)}, \tag{13}$$

where superscript $^{(11)}$ refers to Strategy-11.

*Proof:* The proof is provided in Appendix A. ■

### B. PERFORMANCE WITH STRATEGY-12

In this case, outage occurs in four cases: $U_t$, $t \in \{1, 2\}$, fail to decode $S_c$, i.e., $E_{c\to t}$; $U_1$ is not able to decode $S_1$, i.e., $E^{(1)}_{1\to 1}$; $U_2$ cannot retrieve $S_2$, i.e., $E^{(2)}_{2\to 2}$. As such, the outage probability is given by

$$\mathrm{OP}^{(12)} = 1 - \mathbb{P}\left(\overline{E}_{c\to 1} \cap \overline{E}_{c\to 2} \cap \overline{E}^{(1)}_{1\to 1} \cap \overline{E}^{(2)}_{2\to 2}\right). \tag{14}$$

*Theorem 2:* As proven in [16], by defining $\tau_2 = \frac{C_2 \rho}{a_2 - C_2 a_1}$ and $\mu_2 = \max(\tau_c, \tau_2)$, the power allocation coefficients must satisfy $a_c > \frac{C_c}{1+C_c}$ and $C_1 < \frac{a_1}{a_2} < \frac{1}{C_2}$. Under these conditions, $\mathrm{OP}^{(12)}$ is obtained as

$$\mathrm{OP}^{(12)} = 1 - e^{-\left(\lambda_1 d_1^{\alpha} \mu_1 + \lambda_2 d_2^{\alpha} \mu_2\right)}, \tag{15}$$

in which the superscript $^{(12)}$ indicates Strategy-12.

### C. PERFORMANCE WITH STRATEGY-21

Outage here occurs in five events: $U_t$, $t \in \{1, 2\}$, are not capable of decoding $S_c$, i.e., $E_{c\to t}$; $U_1$ cannot decode $S_2$, i.e., $E^{(2)}_{2\to 1}$; $U_1$ fails to decode $S_1$, i.e., $E^{(2)}_{1\to 1}$; $U_2$ fails to retrieve $S_1$, i.e., $E^{(1)}_{1\to 2}$; $U_2$ is not able to decode $S_2$, i.e., $E^{(1)}_{2\to 2}$. Hence, the outage probability in this case is given by

$$\mathrm{OP}^{(21)} = 1 - \mathbb{P}\left(\overline{E}_{c\to 1} \cap \overline{E}_{c\to 2} \cap \overline{E}^{(2)}_{2\to 1} \cap \overline{E}^{(2)}_{1\to 1} \cap \overline{E}^{(1)}_{1\to 2} \cap \overline{E}^{(1)}_{2\to 2}\right). \tag{16}$$

*Theorem 3:* Let $\beta_2 = \max(\mu_2, \frac{\rho C_1}{a_1})$. In order to prevent outage, the power allocation coefficients must satisfy the conditions $a_c > \frac{C_c}{1+C_c}$ and $C_1 < \frac{a_1}{a_2} < \frac{1}{C_2}$, which yields

$$\mathrm{OP}^{(21)} = 1 - e^{-\left(\lambda_1 d_1^{\alpha} \beta_2 + \lambda_2 d_2^{\alpha} \beta_1\right)}, \tag{17}$$

where the superscript $^{(21)}$ indicates Strategy-21.

*Proof:* The proof is provided in Appendix B. ■

### D. PERFORMANCE WITH STRATEGY-22

In this system operation, outage is comprised of five events: $U_t$, $t \in \{1, 2\}$, are not able to decode $S_c$, i.e., $E_{c\to t}$; $U_1$ cannot decode $S_2$, i.e., $E^{(2)}_{2\to 1}$; $U_1$ cannot retrieve $S_1$, i.e., $E^{(2)}_{1\to 1}$; $U_2$ fails to decode $S_2$, i.e., $E^{(2)}_{2\to 2}$. As a result, $\mathrm{OP}^{(22)}$ is given by

$$\mathrm{OP}^{(22)} = 1 - \mathbb{P}\left(\overline{E}_{c\to 1} \cap \overline{E}_{c\to 2} \cap \overline{E}^{(2)}_{2\to 1} \cap \overline{E}^{(2)}_{1\to 1} \cap \overline{E}^{(2)}_{2\to 2}\right). \tag{18}$$

*Theorem 4:* For the system operation with Strategy-22, the power allocation coefficients must satisfy $a_c > \frac{C_c}{1+C_c}$ and $\frac{a_1}{a_2} < \frac{1}{C_2}$. With this power allocation, $\mathrm{OP}^{(22)}$ is obtained as

$$\mathrm{OP}^{(22)} = 1 - e^{-\left(\lambda_1 d_1^{\alpha} \beta_2 + \lambda_2 d_2^{\alpha} \mu_2\right)}, \tag{19}$$

where the superscript $^{(22)}$ refers to Strategy-22.

*Proof:* The proof is provided in Appendix C. ■

### E. OUTAGE PROBABILITY OF $U_1$

The OP in Strategy-11 and Strategy-12 can be individually evaluated at node $U_1$ by considering the cases when $U_t$, $t \in \{1, 2\}$ fail to decode $S_c$, and $U_1$ cannot retrieve $S_1$. Thus, it is obtained as

$$\begin{aligned} \mathrm{OP}_1^{(1j)} &= 1 - \mathbb{P}\Big(\overline{E}_{c\to 1} \cap \overline{E}_{c\to 2} \cap \overline{E}_{1\to 1}^{(1)}\Big) \\ &= 1 - e^{-\lambda_1 d_1^\alpha \mu_1}, \end{aligned} \tag{20}$$

where $j \in \{1, 2\}$ denotes the decoding strategy at $U_2$.

Similarly, the OP of $U_1$ in Strategy-21 and Strategy-22 can be evaluated by considering the cases when $U_t$, $t \in \{1, 2\}$, fail to decode $S_c$, and $U_1$ fails to decode $S_1$ and $S_2$. That is,

$$\begin{aligned} \mathrm{OP}_1^{(2j)} &= 1 - \mathbb{P}\Big(\overline{E}_{c\to 1} \cap \overline{E}_{c\to 2} \cap \overline{E}_{2\to 1}^{(2)} \cap \overline{E}_{1\to 1}^{(2)}\Big) \\ &= 1 - e^{-\lambda_1 d_1^\alpha \beta_2}, \end{aligned}$$

where $j \in \{1, 2\}$ denotes the decoding strategy at node $U_2$.

Calculation of the OP at $U_2$ can straightforwardly be obtained using the same approach and, hence, not shown here.

### F. COMPARISON OF THE DECODING STRATEGIES

The following Lemma compares the OP of the RS systems when the allocated powers to $S_1$ and $S_2$ are equal, and the target date rate of $S_2$ is higher than that of $S_1$.

*Lemma 1:* To avoid outage with Strategy-11, Strategy-12, Strategy-21 and Strategy-22, consider $a_c > \frac{C_c}{1+C_c}$ and $C_1 < \frac{a_1}{a_2} < \frac{1}{C_2}$. Assuming $R_2 > R_1$ and $a_2 = a_1$, and $\tau^* = (\beta_2 - \mu_1)/(\mu_2 - \beta_1)$, the OP performance is such that

$$\begin{cases} \mathrm{OP}^{(11)} < \mathrm{OP}^{(21)} < \mathrm{OP}^{(12)} < \mathrm{OP}^{(22)} & \text{if } \lambda_2 d_2^\alpha > \lambda_1 d_1^\alpha \tau^* \\ \mathrm{OP}^{(11)} < \mathrm{OP}^{(12)} < \mathrm{OP}^{(21)} < \mathrm{OP}^{(22)} & \text{otherwise.} \end{cases} \tag{21}$$

*Proof:* The proof is provided in Appendix D. ■

That is, when $a_1 = a_2$ and $R_2 > R_1$, the users must follow Strategy-11 to yield the least OP. Similar to Lemma 1, when $a_1 = a_2$ and $R_1 > R_2$, one can claim that better OP performance can be attained when the users adopt Strategy-22. Indeed, with $R_1 > R_2$, decoding $S_1$ first at $U_1$ or $U_2$ will lead to a higher OP. On the other hand, decoding $S_2$ first at both nodes, trying to achieve a lower data rate $R_2$, will lead to a lower OP, thereby justifying the fact that Strategy-22 is the best. Similar inference can also be drawn for the case with $R_2 > R_1$, where Strategy-11 performs the best.

## IV. SECRECY OUTAGE PROBABILITY

Node $U_1$ must be able to decode its private stream $S_1$ securely, while $U_2$ attempts to overhear $S_1$ in addition to retrieving its own stream $S_2$. Now, we develop closed-form approximations for the SOP when the RS system operates with either one of the four decoding strategies. Let $R_s$ be the secrecy target data rate of $S_1$, and $C_s = 2^{R_s} - 1$.

### A. PERFORMANCE WITH STRATEGY-11

A secrecy outage event occurs when: a node $U_t$, $t \in \{1, 2\}$, is not able to decode $S_c$, i.e., $E_{c\to t}$; $U_1$ is not able to decode its private stream securely, i.e.,[3]

$$E_s^{(11)} = \Big\{R_{1\to 1}^{(1)} - R_{1\to 2}^{(1)} < R_s\Big\}, \tag{22}$$

$U_2$ fails to decode $S_1$, i.e., $E_{1\to 2}^{(1)}$; $U_2$ cannot retrieve $S_2$, i.e., $E_{2\to 2}^{(1)}$. As such, the SOP of the system can be calculated using

$$\mathrm{SOP}^{(11)} = 1 - \mathbb{P}\Big(\overline{E}_{c\to 1} \cap \overline{E}_{c\to 2} \cap \overline{E}_s^{(11)} \cap \overline{E}_{1\to 2}^{(1)} \cap \overline{E}_{2\to 2}^{(1)}\Big). \tag{23}$$

Since $U_2$ decodes $S_1$ by treating $S_2$ as noise, perfect secrecy is not maintained for the private stream of $U_1$, i.e., $S_1$. In this paper, we evaluate the SOP w.r.t. the condition in which the secrecy achievable rate of $U_1$ to retrieve $S_1$, i.e., $R_{1\to 1}^{(1)} - R_{1\to 2}^{(1)}$, is higher than $R_s$. The goal is to make $R_{1\to 1}^{(1)} - R_{1\to 2}^{(1)} \geq R_s$ simultaneously with $R_{1\to 2}^{(1)} \geq R_1$.

### B. PERFORMANCE WITH STRATEGY-12

When operating with this decoding strategy, a secrecy outage event occurs if $U_t$, $t \in \{1, 2\}$, are not capable of decoding $S_c$, i.e., $E_{c\to t}$, or that $U_1$ is not able to decode $S_1$ securely, i.e.,

$$E_s^{(12)} = \Big\{R_{1\to 1}^{(1)} - R_{1\to 2}^{(2)} < R_s\Big\}, \tag{24}$$

or $U_2$ fails to decode $S_2$, i.e., $E_{2\to 2}^{(2)}$. Accordingly,

$$\mathrm{SOP}^{(12)} = 1 - \mathbb{P}\Big(\overline{E}_{c\to 1} \cap \overline{E}_{c\to 2} \cap \overline{E}_s^{(12)} \cap \overline{E}_{2\to 2}^{(2)}\Big). \tag{25}$$

*Theorem 5:* The SOP of the RS system under the decoding strategy $i \in \{11, 12\}$ is approximated in tractable form as

$$\begin{aligned} \mathrm{SOP}^{(i)} \approx 1 - \mathbb{1}_{M^{(i)} \cap \{\delta_2^{(i)} > \beta^{(i)}\} \cap \{a_c > \frac{C_c}{1+C_c}\}} \\ \times \Bigg[\mathbb{1}_{\{\delta_2^{(i)} > \Delta^{(i)}\}} \sum_{l=1}^{N_1} \omega_l^{(i)} A_l^{(i)} B_l^{(i)} \\ + \mathbb{1}_{\{\phi^{(i)} > \beta^{(i)}\} \cap \{a_1 > C_s a_2 + \frac{C_s \rho}{\tau_c}\}} \\ \times e^{-\lambda_1 d_1^\alpha \tau_c}\Big(e^{-\lambda_2 d_2^\alpha \beta^{(i)}} - e^{-\lambda_2 d_2^\alpha \phi^{(i)}}\Big)\Bigg], \end{aligned} \tag{26}$$

where $N_1$ is the complexity-vs-accuracy parameter for the Gauss-Chebyshev quadrature rule, $\theta_l = \cos(\frac{2l-1}{2N_1}\pi)$, and the other parameters are as follows:

For $i = 11$: $M^{(11)} = \{\frac{a_1}{a_2} > \max(C_s, C_1)\}$, $\delta_2^{(11)} = \frac{a_1\rho - C_s a_2 \rho}{C_s a_1 a_2 + C_s a_2^2}$, $\beta^{(11)} = \beta_1$, $\eta^{(11)} = \frac{\tau_c a_1 \rho - \tau_c C_s a_2 \rho - C_s \rho^2}{2^{R_s} a_1 \rho + \tau_c C_s a_1 a_2 + \tau_c C_s a_2^2 + C_s a_2 \rho}$, $\Delta^{(11)} = \max(\beta^{(11)}, \eta^{(11)})$, $k_1^{(11)} = \frac{\delta_2^{(11)} - \Delta^{(11)}}{2}$, $k_2^{(11)} = \frac{\delta_2^{(11)} + \Delta^{(11)}}{2}$, $\Theta_l^{(11)} = k_1^{(11)}\theta_l + k_2^{(11)}$, $\omega_l^{(11)} = \frac{\lambda_2 d_2^\alpha k_1^{(11)} \pi \sqrt{1-\theta_l^2}}{N_1}$, $A_l^{(11)} = e^{-\lambda_2 d_2^\alpha \Theta_l^{(11)}}$, $B_l^{(11)}$

3. In general, for $i, j \in \{1, 2\}$, $E_s^{(ij)} = \{[R_{1\to 1}^{(i)} - R_{1\to 2}^{(j)}]^+ < R_s\}$. In this paper, we find the parameters such that $[R_{1\to 1}^{(i)} - R_{1\to 2}^{(j)}]^+ = R_{1\to 1}^{(i)} - R_{1\to 2}^{(j)}$.

$= e^{-\lambda_1 d_1^{\alpha} \frac{C_s\rho(a_2\Theta_l^{(11)}+\rho)+2^{R_s}a_1\rho\Theta_l^{(11)}}{-C_s a_1 a_2\Theta_l^{(11)}+a_1\rho-C_s a_2(a_2\Theta_l^{(11)}+\rho)}}$, and $\phi^{(11)} = \min(\delta_2^{(11)}, \eta^{(11)})$;

For $i = 12$: $M^{(12)} = \{C_s < \frac{a_1}{a_2} < \frac{1}{C_2}\}$, $\delta_2^{(12)} = \frac{\rho(a_1-C_s a_2)}{2^{R_s}a_1 a_2}$, $\beta^{(12)} = \mu_2$, $\eta^{(12)} = \frac{\rho\tau_c a_1-\rho\tau_c C_s a_2-C_s\rho^2}{2^{R_s}a_1(\tau_c a_2+\rho)}$, $\Delta^{(12)} = \max(\beta^{(12)}, \eta^{(12)})$, $k_1^{(12)} = \frac{\delta_2^{(12)}-\Delta^{(12)}}{2}$, $k_2^{(12)} = \frac{\delta_2^{(12)}+\Delta^{(12)}}{2}$, $\Theta_l^{(12)} = k_1^{(12)}\theta_l + k_2^{(12)}$, $\omega_l^{(12)} = \frac{\lambda_2 d_2^{\alpha} k_1^{(12)}\pi\sqrt{1-\theta_l^2}}{N_1}$, $A_l^{(12)} = e^{-\lambda_2 d_2^{\alpha}\Theta_l^{(12)}}$, $B_l^{(12)} = e^{-\frac{\lambda_1 d_1^{\alpha}\rho(C_s\rho+2^{R_s}a_1\Theta_l^{(12)})}{a_1\rho-C_s a_2\rho-2^{R_s}a_1 a_2\Theta_l^{(12)}}}$, and $\phi^{(12)} = \min(\delta_2^{(12)}, \eta^{(12)})$.

*Proof:* The proof is provided in Appendix E. ■

### C. PERFORMANCE WITH STRATEGY-21

In this case, a secrecy outage event is the result of the following events: $U_t$, $t \in \{1, 2\}$, are not capable of retrieving $S_c$, i.e., $E_{c\to t}$; $U_1$ cannot decode $S_2$, i.e., $E_{2\to 1}^{(2)}$; $U_1$ fails to retrieve $S_1$ securely, i.e.,

$$E_s^{(21)} = \left\{R_{1\to 1}^{(2)} - R_{1\to 2}^{(1)} < R_s\right\}, \tag{27}$$

$U_2$ cannot decode $S_1$, i.e., $E_{1\to 2}^{(1)}$; or $U_2$ fails to retrieve $S_2$, i.e., $E_{2\to 2}^{(1)}$. As a result, $\text{SOP}^{(21)}$ is given by

$$\text{SOP}^{(21)} = 1 - \mathbb{P}\left(\overline{E}_{c\to 1} \cap \overline{E}_{c\to 2} \cap \overline{E}_{2\to 1}^{(2)} \cap \overline{E}_s^{(21)} \cap \overline{E}_{1\to 2}^{(1)} \cap \overline{E}_{2\to 2}^{(1)}\right). \tag{28}$$

Similar to Section IV-A, perfect secrecy at $S_1$ cannot be provided with this strategy. In this paper, we investigate the secrecy of $S_1$ when the secrecy achievable rate of $U_1$ to decode $S_1$, i.e., $R_{1\to 1}^{(2)} - R_{1\to 2}^{(1)}$, is higher than $R_s$. Next, we provide an approximation for the SOP in Strategy-21.

*Theorem 6:* The SOP for the system operation with Strategy-21 is obtained as

$$\begin{aligned}\text{SOP}^{(21)} \approx 1 - &\mathbb{1}_{\{a_c > \frac{C_c}{1+C_c}\}\cap\{C_1 < \frac{a_1}{a_2} < \frac{1}{C_2}\}} \\ &\times \Bigg[\sum_{l=1}^{N_1}\omega_l^{(21)}A_l^{(21)}B_l^{(21)} \\ &\times M_{1R}^{(21)} + e^{-(\lambda_1 d_1^{\alpha}\mu_2+\lambda_2 d_2^{\alpha}\beta_1)}M_2^{(21)} \\ &+ e^{-\lambda_1 d_1^{\alpha}\mu_2}\left(e^{-\lambda_2 d_2^{\alpha}\beta_1} - e^{-\lambda_2 d_2^{\alpha}\eta^{(21)}}\right)M_3^{(21)}\Bigg],\end{aligned} \tag{29}$$

where $\eta^{(21)} = \frac{a_1\rho\mu_2-\rho^2 C_s}{a_2\rho C_s+2^{R_s}a_1\rho-a_1 a_2\mu_2}$, $M_1^{(21)} = \{a_1\mu_2 \geq \rho C_s\} \cap \{a_2\rho C_s + 2^{R_s}a_1\rho \geq a_1 a_2\mu_2\}$, $M_{1R}^{(21)} = \mathbb{1}_{\{a_1\mu_2<\rho C_s\}} + \mathbb{1}_{M_1^{(21)}}$, $\Delta^{(21)} = \max(\beta_1, \eta^{(21)})$, $k_1^{(21)} = 1/2(0.01 + \Delta^{(21)})$, $\Theta_l^{(21)} = k_1^{(21)}(\theta_l + 1)$, $\omega_l^{(21)} = \lambda_2 d_2^{\alpha}\pi k_1^{(21)}\sqrt{1-\theta_l^2}/N_1$, $A_l^{(21)} = \frac{1}{\Theta_l^{(21)2}}e^{-\lambda_2 d_2^{\alpha}(\frac{1}{\Theta_l^{(21)}}-0.01)}$, $B_l^{(21)} = e^{-\lambda_1 d_1^{\alpha}\frac{(1-0.01\Theta_l^{(21)})(a_2\rho C_s+2^{R_s}a_1\rho)+\rho^2 C_s\Theta_l^{(21)}}{a_1 a_2(1-0.01\Theta_l^{(21)})+a_1\rho\Theta_l^{(21)}}}$, $M_2^{(21)} = \mathbb{1}_{\{a_1\mu_2\geq\rho C_s\}\cap\{a_2\rho C_s+2^{R_s}a_1\rho<a_1 a_2\mu_2\}}$, and $M_3^{(21)} = \mathbb{1}_{\{\eta^{(21)}>\beta_1\}\cap M_1^{(21)}}$.

*Proof:* The proof is provided in Appendix F. ■

### D. PERFORMANCE WITH STRATEGY-22

Here, the secrecy outage event occurs in four cases: when users are not able to decode $S_c$, i.e., $E_{c\to t}$, $t \in \{1, 2\}$; if $U_1$ fails to decode $S_2$, i.e., $E_{2\to 1}^{(2)}$; when the near user $U_1$ is not able to decode $S_1$ securely, i.e.,

$$E_s^{(22)} = \left\{R_{1\to 1}^{(2)} - R_{1\to 2}^{(2)} < R_s\right\}, \tag{30}$$

or if $U_2$ fails to retrieve $S_2$, i.e., $E_{2\to 2}^{(2)}$. As such, we have

$$\text{SOP}^{(22)} = 1 - \mathbb{P}\left(\overline{E}_{c\to 1} \cap \overline{E}_{c\to 2} \cap \overline{E}_{2\to 1}^{(2)} \cap \overline{E}_s^{(22)} \cap \overline{E}_{2\to 2}^{(2)}\right). \tag{31}$$

The next theorem characterizes the exact SOP in closed-from.

*Theorem 7:* The SOP for the system operation with Strategy-22 is obtained as

$$\begin{aligned}\text{SOP}^{(22)} = 1 - &\mathbb{1}_{\{a_c > \frac{C_c}{1+C_c}\}\cap\{\frac{a_1}{a_2} < \frac{1}{C_2}\}}\frac{\lambda_2 d_2^{\alpha}}{\lambda_2 d_2^{\alpha} + 2^{R_s}\lambda_1 d_1^{\alpha}} \\ &\times e^{-\frac{\lambda_1 d_1^{\alpha}\rho C_s}{a_1}}e^{-\mu_2(\lambda_2 d_2^{\alpha}+2^{R_s}\lambda_1 d_1^{\alpha})}.\end{aligned} \tag{32}$$

*Proof:* The proof is provided in Appendix G. ■

*Remark:* When $U_2$ adopts Strategy-1, it first retrieves $S_1$ to obtain $S_2$. This decoding scheme does not provide perfect secrecy at node $U_1$. By setting $R_s$ as the secrecy target rate of $S_1$, some sort of secrecy can be maintained for $U_1$. In fact, $U_2$ is permitted to retrieve $S_1$ with the achievable rate $R_{1\to 2}^{(1)}$ higher than $R_1$, while the secrecy achievable rate of $U_1$ to decode $S_1$ must be higher than $R_s$. Clearly, when node $U_2$ uses Strategy-1, there exists a tradeoff between the ability of $U_2$ to retrieve $S_2$ and the secrecy of the private stream of node $U_1$. Indeed, on one hand, since $U_2$ retrieves $S_1$ before decoding $S_2$, then $R_{1\to 2}^{(1)}$ must be sufficiently high to be greater than $R_1$. On the other hand, $R_{1\to 2}^{(1)}$ must be as small as possible to make $R_{1\to 1}^{(i)} - R_{1\to 2}^{(1)}$, $i \in \{1, 2\}$, i.e., the secrecy achievable rate of $U_1$ to decode $S_1$ with Strategy-$i$1, greater than $R_s$.

### E. SECRECY OUTAGE PROBABILITY OF $U_1$

Now, we focus on the secrecy of the near user $U_1$ only and not on the entire decoding process at the near and far users. The SOP of $U_1$ with Strategy-$ij$, $i, j \in \{1, 2\}$, is

$$\text{SOP}_1^{(ij)} = \mathbb{P}\left(E_s^{(ij)}\right) = \mathbb{P}\left(R_{1\to 1}^{(i)} - R_{1\to 2}^{(j)} < R_s\right). \tag{33}$$

Next, we approximate the SOP of $U_1$ for Strategy-$i$, $i \in \{11, 12, 21\}$, and obtain the exact SOP of $U_1$ for Strategy-22.

*Theorem 8:* The SOP of $U_1$ under Strategy-$i$, $i \in \{11, 12, 21\}$, is approximated as

$$\text{SOP}_1^{(i)} \approx 1 - \sum_{l=1}^{N_2}\Omega_l^{(i)}H_l^{(i)}J_l^{(i)}M_s^{(i)}, \tag{34}$$

where $N_2$ is the complexity-vs-accuracy parameter in the Gauss-Chebyshev quadrature rule, $\psi_l = \cos(\frac{2l-1}{2N_2}\pi)$, and the other parameters are as follows:

*Strategy-11:* $M_{\rm s}^{(11)} = \mathbb{1}_{\{\frac{a_1}{a_2} > C_{\rm s}\}}$, $k_0 = (a_1 - a_2 C_{\rm s})/2^{R_{\rm s}}$, $\epsilon^{(11)} = \rho k_0 / 2(a_1 a_2 - a_2 k_0)$, $\Psi_l^{(11)} = \epsilon^{(11)}(\psi_l + 1)$, $\Omega_l^{(11)} = \lambda_2 d_2^{\alpha} \pi \epsilon^{(11)} \sqrt{1-\psi_l^2}/N_2$, $H_l^{(11)} = e^{-\lambda_2 d_2^{\alpha} \Psi_l^{(11)}}$, and $J_l^{(11)} = \exp(-\lambda_1 d_1^{\alpha} \frac{\rho C_{\rm s}(a_2 \Psi_l^{(11)} + \rho) + 2^{R_{\rm s}} a_1 \rho \Psi_l^{(11)}}{(a_1 - a_2 C_{\rm s})(a_2 \Psi_l^{(11)} + \rho) - 2^{R_{\rm s}} a_1 a_2 \Psi_l^{(11)}})$;

*Strategy-12:* $M_{\rm s}^{(12)} = \mathbb{1}_{\{\frac{a_1}{a_2} > C_{\rm s}\}}$, $\epsilon^{(12)} = \rho 2^{-R_{\rm s}}(a_1 - a_2 C_{\rm s})/2a_1 a_2$, $\Psi_l^{(12)} = \epsilon^{(12)}(\psi_l + 1)$, $\Omega_l^{(12)} = \lambda_2 d_2^{\alpha} \epsilon^{(12)} \pi \sqrt{1-\psi_l^{(12)}}/N_2$, $H_l^{(12)} = e^{-\lambda_2 d_2^{\alpha} \Psi_l^{(12)}}$, and $J_l^{(12)} = e^{-\lambda_1 d_1^{\alpha} \frac{\rho^2 C_{\rm s} + 2^{R_{\rm s}} a_1 \rho \Psi_l^{(12)}}{(a_1 - a_2 C_{\rm s})\rho - 2^{R_{\rm s}} a_1 a_2 \Psi_l^{(12)}}}$;

*Strategy-21:* $M_{\rm s}^{(21)} = 1$, $\Psi_l^{(21)} = 10(\psi_l + 1)$, $\Omega_l^{(21)} = \lambda_2 d_2^{\alpha} 10\pi \sqrt{1-\psi_l^2}/N_2$, $H_l^{(21)} = \exp(-\lambda_2 d_2^{\alpha}(\frac{1}{\Psi_l^{(21)}} - 0.05))/\Psi_l^{(21)2}$, and $J_l^{(21)} = e^{-\lambda_1 d_1^{\alpha}(\frac{\rho C_{\rm s}}{a_1} + \frac{2^{R_{\rm s}} \rho (1 - 0.05\Psi_l^{(21)})}{a_2(1-0.05\Psi_l^{(21)}) + \rho \Psi_l^{(21)}})}$;

In the above SOP evaluations pertaining to node $U_1$, Strategy-11 involves the situation when $U_1$ is unable to decode $S_c$ and fails to retrieve $S_1$ securely with $U_2$ performing Strategy-1, i.e., $\bar{E}_s^{11}$, Strategy-12 involves the situation when $U_1$ is unable to decode $S_c$ and fails to retrieve $S_1$ securely with the user $U_2$ performing Strategy-2, i.e., $\bar{E}_s^{12}$, and Strategy-21 involves the situation when $U_1$ is unable to decode $S_c$ and $S_2$ correctly and fails to retrieve $S_1$ securely, i.e., $\bar{E}_s^{21}$.

In the case of Strategy-22, the SOP of $U_1$ is given by

$$\mathrm{SOP}_1^{(22)} = 1 - \frac{\lambda_2 d_2^{\alpha}}{\lambda_2 d_2^{\alpha} + 2^{R_{\rm s}} \lambda_1 d_1^{\alpha}} e^{-\frac{\lambda_1 d_1^{\alpha} \rho C_{\rm s}}{a_1}}. \tag{35}$$

*Proof:* The proof is provided in Appendix H. ■

### F. COMPARISON OF THE DECODING STRATEGIES

Now, we compare the behavior of the decoding strategies in high-SNR regimes. We assume that the source transmit power, $P$, is sufficiently large to make the users decode the common stream, and $U_2$ is able to retrieve its private stream perfectly, i.e., the power allocation satisfy $a_c >> a_1$ and $a_c >> a_2$. In these cases, the secrecy of the private stream of $U_1$ has the major impact on the SOP, and the system SOP is almost equal to that of $U_1$. This will be confirmed in Section V. Table 1 compares the strategies for high transmit power budget at the source transmitter. For instance, comparing Strategy-11 and Strategy-12 (cf. case 1 in Table 1), it is observed that the former decoding strategy yields better SOP.[4]

4. In Table 1, for a fair comparison between the SOP of the decoding strategies, we assume that the power allocation coefficients are such that the users are able to decode the common stream, and that $U_2$ retrieves $S_2$ correctly.

**TABLE 1.** Comparison between the decoding strategies for high-SNR regimes.

| | Comparison | Less SOP | Proof |
|---|---|---|---|
| **Case 1** | Strategy-11 vs. Strategy-12 | Strategy-11 | Appendix I |
| **Case 2** | Strategy-21 vs. Strategy-22 | Strategy-21 | Appendix J |
| **Case 3** | Strategy-11 vs. Strategy-21 | Strategy-21 | Appendix K |
| **Case 4** | Strategy-12 vs. Strategy-22 | Strategy-22 | Appendix L |

*Corollary 1:* By adopting the results of Table 1, conventional RS, i.e., when the system operates with Strategy-12, yields the worst SOP, while Strategy-21 excels in SOP compared to the other schemes.

Finally, the next two lemmas investigate the SOP performance for extreme values of $P$.

*Lemma 2:* When $P \to \infty$, i.e., $\rho \to 0$, the system SOP under Strategy-11 and Strategy-12 tend to unity.

*Proof:* The proof is provided in Appendix M. ■

*Lemma 3:* When $P \to \infty$, the SOP under Strategy-21 tends to zero, and the SOP under Strategy-22 converges to $1 - \frac{\lambda_2 d_2^{\alpha}}{\lambda_2 d_2^{\alpha} + 2^{R_{\rm s}} \lambda_1 d_1^{\alpha}}$.

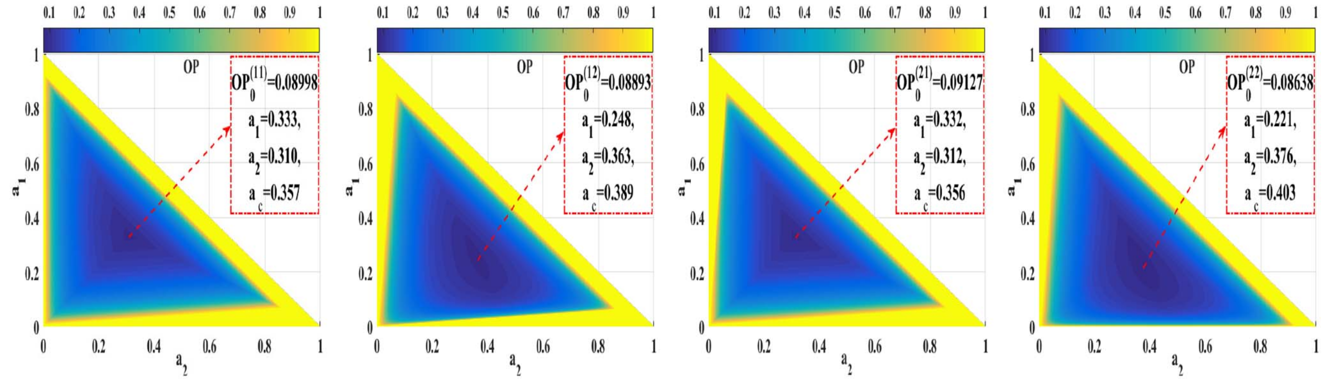
*Proof:* The proof is provided in Appendix N. ■

## V. NUMERICAL RESULTS AND DISCUSSIONS

In this part, numerical results corresponding to the obtained OP, SOP and $\mathrm{SOP}_1$, are presented. These are in full agreement with the Monte-Carlo simulations obtained from $10^5$ iterations. The goal is to investigate the effects of the power allocation, the target data rates, the power budget of the source transmitter, and the far user's distance, on performance. Here, the path-loss exponent is set as $\alpha = 2.2$, the distribution parameters of the channel power gains are such that $\lambda_1 = \lambda_2 = 1$, the noise variance is $\sigma^2 = 1$, and the complexity-vs-accuracy parameters of the Gauss-Chebyshev quadrature rule are $N_1 = N_2 = 150$.

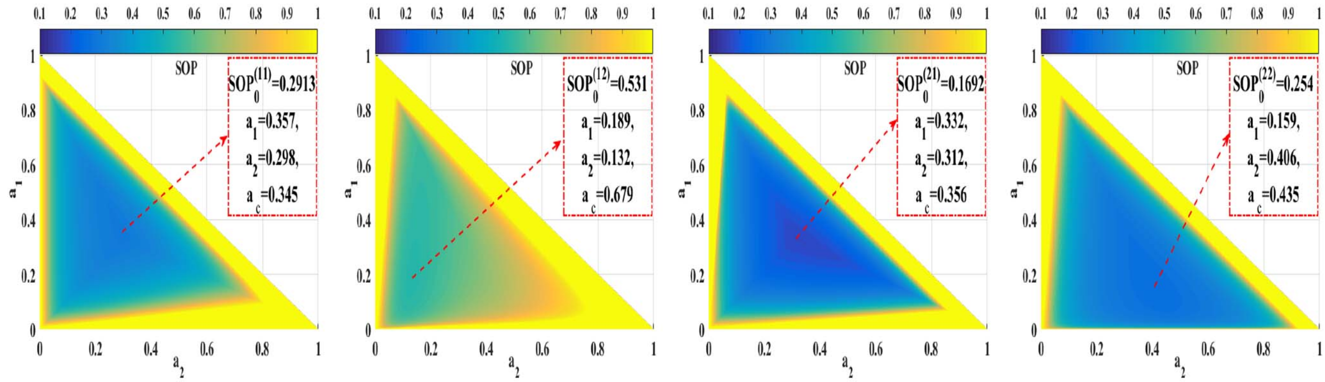
### A. EFFECTS OF THE POWER ALLOCATION

Figure 2 illustrates the OP of the four strategies w.r.t. the power allocation coefficients, for constant $P = 40$dBW and target rates $R_1 = R_2 = R_{\rm c} = 0.1$BPCU. For different sets of $(a_1, a_2, a_{\rm c})$, the least OP with Strategy-$i$, $i \in \{11, 12, 21, 22\}$, is denoted as $\mathrm{OP}_0^{(i)}$, while these sets are not necessarily the same for all strategies. When a sufficient portion of $P$ is allocated to each stream such that the users are able to correctly decode the streams, it is demonstrated that $\mathrm{OP}_0^{(22)} < \mathrm{OP}_0^{(12)} < \mathrm{OP}_0^{(11)} < \mathrm{OP}_0^{(21)}$, which confirms that Strategy-22 is superior. This validates Lemma 1, which illustrates the fact that for equal target rates, by assigning appropriate power allocation coefficients, the users can achieve lower OP when following Strategy-2. The users retrieve the far user's private stream $S_2$ by treating $S_1$ as noise. When $a_2$ is significantly small, Strategy-11 outperforms the other strategies. In fact, with Strategy-2, the users should decode $S_2$ after obtaining $S_{\rm c}$, where small values of $a_2$ indicating low levels of $P$ cannot achieve a high enough rate for the retrieving of $S_2$. Hence, when the power allocated to $S_2$ is much smaller than those allocated to $S_1$ and $S_{\rm c}$, then Strategy-11 leads users




(a) Strategy-11 (b) Strategy-12 (c) Strategy-21 (d) Strategy-22

**FIGURE 2.** The range of OP vs. $a_1$ and $a_2$ ($a_c = 1 - (a_1 + a_2)$) is determined by the color for: (a) $U_1$ and $U_2$ adopt Strategy-1; (b) $U_1$ uses Strategy-1, and $U_2$ uses Strategy-2; (c) $U_1$ and $U_2$ respectively utilize Strategy-2 and Strategy-1; (d) $U_1$ and $U_2$ use Strategy-2. Here, $P$ = 40dBW, $R_1 = R_2 = R_c$ = 0.1BPCU, $d_1$ = 20m and $d_2$ = 40m. $\text{OP}_0^{(i)}$ is the least achievable OP with Strategy-$i$, $i \in \{11, 12, 21, 22\}$.




(a) Strategy-11 (b) Strategy-12 (c) Strategy-21 (d) Strategy-22

**FIGURE 3.** The range of SOP vs. $a_1$ and $a_2$ ($a_c = 1 - (a_1 + a_2)$) is shown by the color for: (a) Strategy-11; (b) Strategy-12; (c) Strategy-21; (d) Strategy-22. Here, $P$ = 40dBW, $R_1 = R_2 = R_c$ = 0.1BPCU, $R_s$ = 0.05BPCU, $d_1$ = 20m and $d_2$ = 40m. $\text{SOP}_0^{(i)}$ is the least achievable SOP with Strategy-$i$, $i \in \{11, 12, 21, 22\}$.

to achieve better outage performance. Similarly, when $a_1$ is close to zero, Strategy-22 excels in OP, since small levels of $a_1$ do not allow the users to correctly decode $S_1$ via Strategy-1.

Figure 3 shows the range of SOP of all strategies when $R_1 = R_2 = R_c = 0.1$BPCU, $R_s = 0.05$BPCU, and $P = 40$dBW. Here, $\text{SOP}_0^{(i)}$, $i \in \{11, 12, 21, 22\}$, represents the least achievable SOP with Strategy-$i$. As illustrated, $\text{SOP}_0^{(21)} < \text{SOP}_0^{(22)} < \text{SOP}_0^{(11)} < \text{SOP}_0^{(12)}$, which indicates that by adopting suitable power allocation, Strategy-21 has the potential to provide the least SOP. Unlike the OP performance without security constraints in which $\text{OP}_0^{(12)} < \text{OP}_0^{(11)}$ and $\text{OP}_0^{(21)} > \text{OP}_0^{(11)}$, for the SOP it is found that $\text{SOP}_0^{(12)} > \text{SOP}_0^{(11)}$ and $\text{SOP}_0^{(21)} < \text{SOP}_0^{(11)}$. Therefore, with suitable power allocation, the achievable rate of $U_1$ and $U_2$ to decode $S_1$ gets improved with Strategy-2, while the increment in the rate of $U_1$ to retrieve $S_1$ reduces the SOP, and the increment in the rate of $U_2$ to decode $S_1$ raises it. Similar to the OP performance, when $a_1$ is significantly smaller than $a_2$ and $a_c$, then Strategy-22 yields better SOP and, for small $a_2$, Strategy-11 outperforms the other strategies. If $a_1 + a_2$ is close to one, the SOP would be close to unity, since with

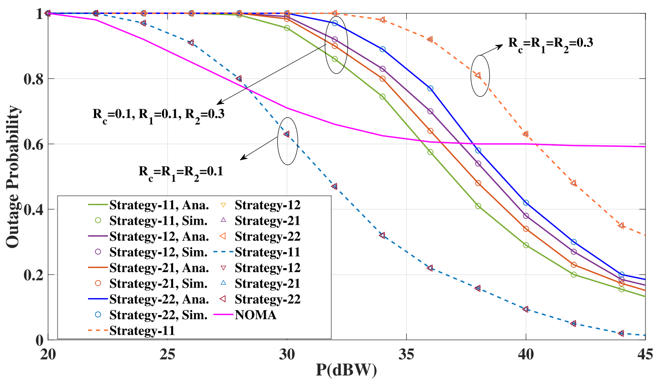



**FIGURE 4.** OP vs. $P$ for different target data rates when $a_1 = a_2 = a_c = \frac{1}{3}$, $d_1$ = 20m and $d_2$ = 40m.

a low value of $a_c$ the users' SINRs are not sufficient for decoding the common stream.

### *B. IMPACTS OF THE POWER BUDGET AND THE TARGET RATES*

Figure 4 compares the OP of the strategies w.r.t. $P$, for different target rates and $a_1 = a_2 = a_c = \frac{1}{3}$. When the rates associated with $S_1$ and $S_2$ are equal, e.g., $R_1 = R_2 = 0.1$BPCU, and $R_c = 0.1$ or 0.3BPCU, the plots of all strategies coincide. While all strategies yield the same performance in these cases, Strategy-12 reduces the decoding complexity at the receiving sides. When $R_1 > R_2$ or $R_2 > R_1$, according to Lemma 1, after decoding $S_c$, all users must decode the private stream for which the target rate is less. For instance, when $R_1 = R_c = 0.1$BPCU and $R_2 = 0.3$BPCU, $\mathrm{OP}^{(11)} < \mathrm{OP}^{(21)} < \mathrm{OP}^{(12)} < \mathrm{OP}^{(22)}$, i.e., Strategy-11 yields the least OP for the RS system, where each user after retrieving $S_c$ decodes $S_1$ by treating $S_2$ as noise. An increase in the target rates raises the OP, according to the fact that the decoding ability of the users gets reduced by increasing the rates. For example, the RS system with $R_1 = R_2 = 0.1$BPCU and $R_c = 0.3$BPCU, and the one with $R_1 = R_c = 0.1$BPCU and $R_2 = 0.3$BPCU, have higher OP than the system with $a_1 = a_2 = a_c = 0.1$BPCU. Changing the power allocation from $a_1 = a_2 = a_c = 0.1$BPCU to $a_1 = a_2 = 0.1$BPCU and $a_c = 0.3$BPCU affects the OP more than changing the allocation from $a_1 = a_2 = a_c = 0.1$BPCU to $a_1 = a_c = 0.1$BPCU and $a_2 = 0.3$BPCU. Therefore, the target rate $R_c$ has the major impact on the OP as compared to $R_1$ and $R_2$, since each user must decode $S_c$ before having access to its own private stream.

The OP performance of the proposed system has also been compared with a benchmark system, namely, a NOMA system where $R_1 = R_2 = 0.1$BPCU and $a_1 = a_2 = 1/2$. As observed from Fig. 4, at low transmit power values, the performance with NOMA is slightly better than RSMA because the power budget in the latter is divided into three parts for the common and private streams of the two users, thereby resulting in lower power levels for each stream, whereas the power allocation is only for the messages of the two users in NOMA. On the other hand, for high values of the power budget $P$, the proposed decoding strategies are able to overcome the allocation of lesser power for the private and common streams as compared to NOMA, thereby yielding improved performance for RSMA as compared to NOMA.

Figure 5 illustrates the SOP and $\mathrm{SOP}_1$ w.r.t. the source power, $P$, when $a_1 = a_2 = a_c = \frac{1}{3}$, $R_1 = R_2 = R_c = 0.1$BPCU, and $R_s = 0.05$BPCU. As discussed in Table 1, when the RS strategies operate with high power, e.g., $P = 50$dBW, $\mathrm{SOP}_1 \approx \mathrm{SOP}$, which confirms the major impact of the $U_1$'s private stream secrecy on the SOP at high SNRs. If the signals are transmitted with $P < 50$dBW, then the SOP of Strategy-21 and Strategy-22 decrease with $P$, due to the increment in the received SNR at $U_1$ to retrieve $S_1$. For $P \geq 50$dBW, as per Lemma 3, the SOP of Strategy-21 goes to zero, and $\mathrm{SOP}^{(12)}$ tends to the constant value $1 - \frac{\lambda_2 d_2^{\alpha}}{\lambda_2 d_2^{\alpha} + 2^{R_s}\lambda_1 d_1^{\alpha}} = 0.1839$.

Now, let $P_{\min}^{(i)}$ be the transmit power that achieves the least SOP, denoted $\mathrm{SOP}_{\min}^{(i)}$, when the RS system operates with Strategy-$i$, $i \in \{11, 12\}$. When $P \leq P_{\min}^{(i)}$, increasing $P$ enhances the SINR at the receivers and reduces the SOP of

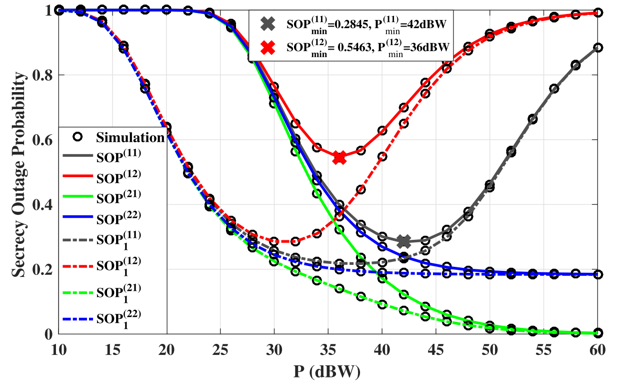



**FIGURE 5.** SOP and $\mathrm{SOP}_1$ of the RS decoding schemes vs. $P$. $\mathrm{SOP}_{\min}^{(i)}$, $i \in \{11, 12\}$ is the minimum SOP of Strategy-$i$ that is achievable with power $P_{\min}^{(i)}$. Here, $R_1 = R_2 = R_c = 0.1$BPCU, $R_s = 0.05$BPCU, $a_1 = a_2 = a_c = \frac{1}{3}$, $d_1 = 20$m and $d_2 = 40$m.

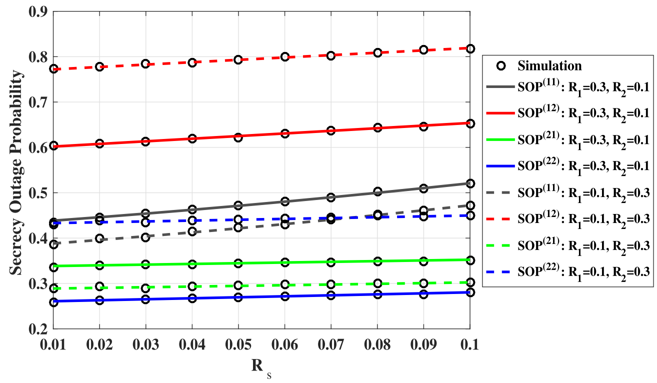



**FIGURE 6.** SOP vs. $R_s$ for different values of $R_1$ and $R_2$. Here, $R_c = 0.1$BPCU, $P = 40$dBW, $a_1 = a_2 = a_c = \frac{1}{3}$, $d_1 = 20$m and $d_2 = 40$m.

Strategy-$i$. When $P > P_{\min}^{(i)}$, an increase in $P$ raises the SOP. Indeed, in addition to improving the SINR at $U_1$ to decode $S_1$, an increase in $P$ enhances the achievable rate of $U_2$ to decode $S_1$, which degrades the SOP. As a result, the SOP of Strategy-$i$, $i \in \{11, 12\}$, converges to unity in high-SNR regimes, as stated in Lemma 2. Comparing the results for large values of $P$ reveals that $\mathrm{SOP}^{(21)} < \mathrm{SOP}^{(11)} < \mathrm{SOP}^{(12)}$ and $\mathrm{SOP}^{(21)} < \mathrm{SOP}^{(22)} < \mathrm{SOP}^{(12)}$, which is in agreement with Table 1. Hence, although $U_1$ is not aware of the strategy adopted by $U_2$, following Strategy-2 leads to the RS system performing more securely.

The SOP of the RS strategies w.r.t. $R_s$ is shown in Fig. 6, for $P = 40$dBW and $R_c = 0.1$BPCU, and different $R_1$ and $R_2$. The SOP increases with $R_s$, due to the fact that the ability of $U_1$ to decode $S_1$ securely gets reduced when $R_s$ increases. When $R_1 > R_2$, e.g., $R_1 = 0.3$BPCU and $R_2 = 0.1$BPCU, Strategy-22 leads to better SOP, while for $R_2 > R_1$, e.g., $R_2 = 0.3$BPCU and $R_1 = 0.1$BPCU, Strategy-21 performs more securely than the other decoding mechanisms. Therefore, to achieve the best performance, $U_1$ must follow Strategy-2 by decoding $S_2$ after having access to $S_c$. In order for $U_2$ to assist the RS system in achieving the least SOP, it must follow a similar approach to Lemma 1. In

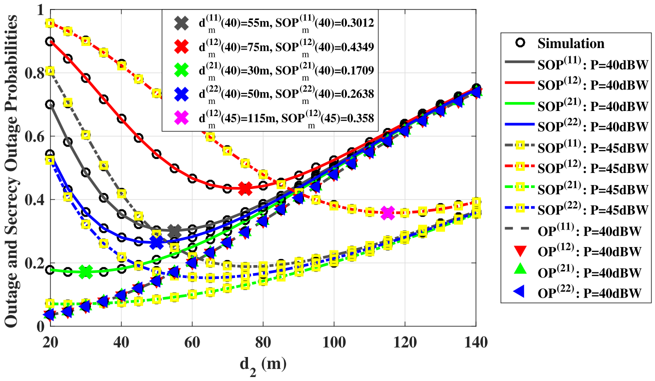



**FIGURE 7.** SOP and OP vs. $d_2$ for different values of $P$. Here, $R_1 = R_2 = R_c = 0.1$BPCU, $R_s = 0.05$BPCU, $a_1 = a_2 = a_c = \frac{1}{3}$ and $d_1 = 20$m. For Strategy-$i$, $i \in \{11, 12, 21, 22\}$, the minimum SOP with fixed power $P$, i.e., $\mathrm{SOP}_m^{(i)}(P)$, is achievable with the distance $d_2 = d_m^{(i)}(P)$.

this regard, if $R_1 > R_2$ then $U_2$ must adopt Strategy-2, while if $R_2 > R_1$ then $U_2$ should decode its own private stream via Strategy-1. When the system operates with $R_1 = 0.1$BPCU, $R_2 = 0.3$BPCU and $R_s \in [0.01, 0.07]$, then Strategy-11 yields better performance compared to Strategy-22, due to the fact that $U_2$ by following Strategy-2 improves the outage. For $R_s \in [0.07, 0.1]$, an increase in $R_s$ degrades the secrecy achievable rate of $U_1$ to decode $S_1$, significantly, and makes Strategy-11 yield higher SOP than Strategy-22.

### *C. EFFECTS OF THE POSITION OF THE FAR USER*

Figure 7 illustrates the OP and SOP of the RS strategies w.r.t. $d_2$ for different values of $P$ when $a_1 = a_2 = a_c = \frac{1}{3}$, $R_1 = R_2 = R_c = 0.1$BPCU and $R_s = 0.05$BPCU. As discussed, when $a_1 = a_2$ and $R_1 = R_2$, all strategies yield the same OP. The OP increases with $d_2$. Indeed, such increments reduce the received SINR/SNR at $U_2$ to decode the streams. When it comes to secrecy, if $d_2$ is such that $U_2$ receives sufficient SINR/SNR to decode its private stream, an increase in $d_2$ improves the secrecy and reduces the SOP. In these cases, said increase reduces the rate of $U_2$ to retrieve $S_1$, and raises the secrecy achievable rate of $U_1$ to decode $S_1$, i.e., $R_{1\to 1}^{(i)} - R_{1\to 2}^{(j)} (i, j \in \{1, 2\})$. When $d_2$ is large enough to significantly reduce the rate of $U_2$ to decode $S_1$, $U_1$ would be able to retrieve $S_1$ securely. In these cases, the achievable rates of $U_2$ to decode the streams $S_c$ and $S_2$ have the main effects on the SOP. Hence, increasing $d_2$ reduces the ability of $U_2$ to decode $S_c$ and $S_2$, which raises the SOP.

Now, let $d_m^{(i)}(P)$ be the distance $d_2$ for which the SOP is minimum, denoted $\mathrm{SOP}_m^{(i)}(P)$, when Strategy-$i$, $i \in \{11, 12, 21, 22\}$, operates with the source power $P$. An increase in $P$ decreases the SOP of Strategy-$i$, while increasing $d_2$ enhances the SOP when $d_2 > d_m^{(i)}(P)$. As observed from Fig. 7, $d_m^{(21)}(40) < d_m^{(22)}(40) < d_m^{(11)}(40) < d_m^{(12)}(40)$. On the other hand, $\mathrm{SOP}_m^{(21)}(40) < \mathrm{SOP}_m^{(22)}(40) < \mathrm{SOP}_m^{(11)}(40) < \mathrm{SOP}_m^{(12)}(40)$. Hence, by comparing the SOP of Strategy-$i$ and Strategy-$j$, $i, j \in \{11, 12, 21, 22\}$, we can say that $d_m^{(i)}(40) < d_m^{(j)}(40)$ when $\mathrm{SOP}^{(i)} < \mathrm{SOP}^{(j)}$. In fact, the strategy with a higher SOP provides $U_1$ with a lower secrecy achievable rate to decode $S_1$. To improve the SOP with less secrecy achievable rate, it is necessary to degrade the achievable rate of $U_2$ to decode $S_1$ more by increasing $d_2$. Furthermore, comparing $d_m^{(12)}(40)$ and $d_m^{(12)}(45)$ shows that an increase in $P$ raises the value of $d_2$ for which the SOP is minimum. As a matter of fact, in addition to enhancing the received SINR/SNR at $U_1$ to decode $S_1$, increasing $P$ raises the received SINR/SNR at $U_2$ to retrieve $S_1$. Hence, to improve the secrecy outage and reduce the increment in the achievable rate of $U_2$ to access $S_1$, which is caused by increasing $P$, it is required that $d_2$ be increased.

## VI. CONCLUSION

We characterized the OP and SOP of a RS communication system, where the service to a near user and far user is such that the latter attempts to overhear the message of the former. The impacts of the power budget, power allocation, target data rates, and far user's distance, on the OP and SOP performance, were thoroughly investigated. In particular, without concerns about secrecy, it was shown that if the allocated power to one user's private stream is significantly smaller than the other, both users, after decoding the common stream, must first decode the private stream for which the power allocation coefficient is higher. Also, it was demonstrated that for high power budget, the system SOP is almost equal to the SOP of the near user. Besides, it was shown that when the powers allocated to the users' private streams are equal, to achieve the best OP and SOP performances, the far user must decode the private stream for which the target rate is less by treating the other private stream as noise. Furthermore, the near user by decoding the private stream whose rate is less than the other makes the RS system attain a better OP, whereas the least SOP is achievable when the near user first retrieves the far user's private stream by considering its own private stream as noise.

## APPENDIX A

*Proof of Theorem 1:* To simplify the calculations, we present this lemma (cf. proof in [16]).

*Lemma 4:* Consider the event $E_T = \{\frac{a_n Z}{a_m Z + a_p} \geq x\}$, where $Z$ is a non-negative random variable, and where $a_n$, $a_m$, $a_p$ and $x$ are positive deterministic parameters. For $E_T$ to occur, it is required that $\frac{a_n}{a_m} > x$. Under this condition, $E_T$ can be simplified to $E_T = \{Z \geq \frac{a_p x}{a_n - a_m x}\}$.

Now, we characterize the OP of Strategy-11. Using (3)-(5), (12), and adopting Lemma 4, it is ensured that $a_c > \frac{C_c}{1+C_c}$ and $\frac{a_1}{a_2} > C_1$. Therefore, $\mathrm{OP}^{(11)}$ is obtained as $\mathrm{OP}^{(11)} = 1 - \mathbb{P}(\{g_1 \geq \mu_1\} \cap \{g_2 \geq \beta_1\})$, and then as shown in (13).

## APPENDIX B

*Proof of Theorem 3:* Using (3)-(5), (8), (9), (16) and Lemma 4 urges the power allocation to satisfy $a_c > \frac{C_c}{1+C_c}$ and $C_1 < \frac{a_1}{a_2} < \frac{1}{C_2}$. Hence, $\mathrm{OP}^{(21)} = 1 - \mathbb{P}(\{g_1 \geq \beta_2\} \cap \{g_2 \geq \beta_1\})$, which yields (17).

## APPENDIX C

*Proof of Theorem 4:* Using (3), (6)-(9), (19) and Lemma 4 makes the power allocation coefficients satisfy $a_c > \frac{C_c}{1+C_c}$ and $\frac{a_1}{a_2} < \frac{1}{C_2}$ and, therefore, we have $\mathrm{OP}^{(22)} = 1 - \mathbb{P}(\{g_1 \geq \beta_2\} \cap . \{g_2 \geq \mu_2\})$, which leads to (19).

## APPENDIX D

*Proof of Lemma 1:* Assume $R_2 > R_1$ and $a_2 = a_1$, which results in $C_2 > C_1$, $\tau_2 > \tau_1$, $\beta_1 < \mu_2$ and $\beta_2 > \mu_1$. According to $\beta_1 < \mu_2$, comparing $\mathrm{OP}^{(11)}$ (13) with $\mathrm{OP}^{(12)}$ (15), and $\mathrm{OP}^{(21)}$ (17) with $\mathrm{OP}^{(22)}$ (19), leads to

$$\mathrm{OP}^{(11)} < \mathrm{OP}^{(12)}, \mathrm{OP}^{(21)} < \mathrm{OP}^{(22)}. \tag{36}$$

With $\beta_2 > \mu_1$, comparing $\mathrm{OP}^{(11)}$ (13) with $\mathrm{OP}^{(21)}$ (17) and $\mathrm{OP}^{(12)}$ (15) with $\mathrm{OP}^{(22)}$ (19) yields

$$\mathrm{OP}^{(11)} < \mathrm{OP}^{(21)}, \mathrm{OP}^{(12)} < \mathrm{OP}^{(22)}. \tag{37}$$

The last step indicates comparing Strategy-12 and Strategy-21. Using (15) and (17), we get

$$\begin{cases} \mathrm{OP}^{(12)} > \mathrm{OP}^{(21)} & \text{if } \lambda_2 d_2^{\alpha} > \lambda_1 d_1^{\alpha} \tau^* \\ \mathrm{OP}^{(12)} < \mathrm{OP}^{(21)} & \text{otherwise.} \end{cases} \tag{38}$$

Finally, according to (36)-(38), the outage performance is obtained as per (21).

## APPENDIX E

*Proof of Theorem 5:* We use $i \in \{11, 12\}$ to differentiate between the decoding strategies. First, define $f^{(11)}(g_2) = \frac{C_s\rho(a_2 g_2+\rho)+2^{R_s}a_1\rho g_2}{-C_s a_1 a_2 g_2 + a_1\rho - C_s a_2(a_2 g_2+\rho)}$ and $f^{(12)}(g_2) = \frac{\rho(C_s\rho+2^{R_s}a_1 g_2)}{a_1\rho - C_s a_2\rho - 2^{R_s}a_1 a_2 g_2}$. With regard to the events shown in (22) and (24), considering non-negative $g_1$ and $g_2$, and following a similar approach as in Lemma 4 along with simple algebraic manipulations, shows that the inequality $\frac{a_1}{a_2} > C_s$ must hold. As such, $\overline{E}_s^{(i)}$ is reformulated as

$$\overline{E}_s^{(i)} = \left\{g_2 < \delta_2^{(i)}\right\} \cap \left\{g_1 \geq f^{(i)}(g_2)\right\}. \tag{39}$$

Using Lemma 4, (23), (25), (39), and some calculations, we can simplify $\mathrm{SOP}^{(i)}$ as

$$\mathrm{SOP}^{(i)} = 1 - \mathbb{P}\left(F^{(i)}\right) \times \mathbb{1}_{M^{(i)} \cap \left\{\delta_2^{(i)} > \beta^{(i)}\right\} \cap \left\{a_c > \frac{C_c}{1+C_c}\right\}}, \tag{40}$$

where $F^{(i)} = \{g_1 \geq \max(\tau_c, f^{(i)}(g_2))\} \cap \{\beta^{(i)} \leq g_2 < \delta_2^{(i)}\}$. Define $E_d^{(i)} = \{\tau_c \leq f^{(i)}(g_2)\}$. $F^{(i)}$ is expanded as the union of two events satisfying $F_1^{(i)} \cap F_2^{(i)} = \varnothing$ and $F_1^{(i)} \cup F_2^{(i)} = F^{(i)}$, where

$$F_1^{(i)} = F^{(i)} \cap E_d^{(i)}, \tag{41}$$

$$F_2^{(i)} = F^{(i)} \cap \overline{E}_d^{(i)}, \tag{42}$$

while it is clear that

$$\mathbb{P}\left(F^{(i)}\right) = \mathbb{P}\left(F_1^{(i)}\right) + \mathbb{P}\left(F_2^{(i)}\right). \tag{43}$$

The first step for obtaining the SOP consists in deriving $\mathbb{P}(F_1^{(i)})$. Now, we rewrite $E_d^{(i)}$ as $E_d^{(i)} = \{g_2 \geq \eta^{(i)}\}$. Applying $E_d^{(i)}$ to (41) leads to $\mathbb{P}(F_1^{(i)}) = \mathbb{P}(\{g_1 \geq f^{(i)}(g_2)\} \cap \{\Delta^{(i)} \leq g_2 < \delta_2^{(i)}\})$. Note that although $g_2 > 0$, since $\Delta^{(i)} = \max(\beta^{(i)}, \eta^{(i)})$ and $\beta^{(i)}$ is supposed to be greater than zero, the sign of $\eta^{(i)}$ is not a concern here. If $\Delta^{(i)} \geq \delta_2^{(i)}$, $\mathbb{P}(F_1^{(i)}) = 0$; otherwise, since $g_1$ and $g_2$ are independent, and by using the Gauss-Chebyshev quadrature rule, $\mathbb{P}(F_1^{(i)})$ is approximated as

$$\mathbb{P}\left(F_1^{(i)}\right) = \int_{\Delta^{(i)}}^{\delta_2^{(i)}} f_{g_2}(x) \int_{f^{(i)}(x)}^{\infty} f_{g_1}(y) \mathrm{d}y\, \mathrm{d}x \approx \sum_{l=1}^{N_1} \omega_l^{(i)} A_l^{(i)} B_l^{(i)}. \tag{44}$$

Now, we derive $\mathbb{P}(F_2^{(i)})$. First, we have $\overline{E}_d^{(i)} = \{g_2 < \eta^{(i)}\}$. Since $g_2$ is non-negative, to make the event $\overline{E}_d^{(i)}$ occur, $\eta^{(i)}$ must be greater than zero, which results in $a_1 > C_s a_2 + \frac{C_s\rho}{\tau_c}$. Using (42), then when $\beta^{(i)} > \phi^{(i)}$ is satisfied we have $\mathbb{P}(F_2^{(i)}) = 0$; otherwise, $\mathbb{P}(F_2^{(i)})$ is given by

$$\begin{aligned} \mathbb{P}\left(F_2^{(i)}\right) &= \mathbb{P}\left(\{g_1 \geq \tau_c\} \cap \left\{\beta^{(i)} \leq g_2 < \phi^{(i)}\right\}\right) \\ &= e^{-\lambda_1 d_1^{\alpha}\tau_c}\left(e^{-\lambda_2 d_2^{\alpha}\beta^{(i)}} - e^{-\lambda_2 d_2^{\alpha}\phi^{(i)}}\right). \end{aligned} \tag{45}$$

Substituting (43)-(45) into (40), the SOP under Strategy-11 and Strategy-12 is as shown in (26).

## APPENDIX F

*Proof of Theorem 6:* First of all, define $f^{(21)}(g_2) = \frac{g_2(a_2\rho C_s + 2^{R_s}a_1\rho) + \rho^2 C_s}{a_1 a_2 g_2 + a_1\rho}$. Using Lemma 4 and (28), $\mathrm{SOP}^{(21)}$ is rewritten as

$$\mathrm{SOP}^{(21)} = 1 - \mathbb{P}\left(F^{(21)}\right)\mathbb{1}_{\left\{a_c > \frac{C_c}{1+C_c}\right\} \cap \left\{C_1 < \frac{a_1}{a_2} < \frac{1}{C_2}\right\}}, \tag{46}$$

where $F^{(21)} = \{g_1 \geq \mu_2\} \cap \{g_2 \geq \beta_1\} \cap \{g_1 \geq f^{(21)}(g_2)\}$. Now, we write $\mathbb{P}(F^{(21)}) = \mathbb{P}(F_1^{(21)}) + \mathbb{P}(F_2^{(21)})$, where $F_1^{(21)} = F^{(21)} \cap \{f^{(21)}(g_2) \geq \mu_2\}$ and $F_2^{(21)} = F^{(21)} \cap \{f^{(21)}(g_2) < \mu_2\}$.

First, we derive $\mathbb{P}(F_1^{(21)})$, which is simplified to

$$\mathbb{P}\left(F_1^{(21)}\right) = \mathbb{P}\left(\left\{g_1 \geq f^{(21)}(g_2)\right\} \cap \{g_2 \geq \beta_1\} \cap \{g_2\delta_3 \geq \delta_4\}\right), \tag{47}$$

where $\delta_3 = a_2\rho C_s + 2^{R_s}a_1\rho - a_1 a_2\mu_2$ and $\delta_4 = a_1\rho\mu_2 - \rho^2 C_s$. Here, there exist two possibilities for $\delta_4$: $\delta_4 < 0$ and $\delta_4 \geq 0$. If $\delta_4 < 0$, i.e., $a_1\mu_2 < \rho C_s$, then one can simply prove that $\delta_3 > 0$. Therefore, when $\delta_4 < 0$, since $\eta^{(21)} < 0$ and $g_2$ is non-negative, (47) is given by

$$\mathbb{P}\left(F_1^{(21)}\right) = \mathbb{P}\left(\left\{g_1 \geq f^{(21)}(g_2)\right\} \cap \{g_2 \geq \beta_1\}\right)\mathbb{1}_{\{a_1\mu_2 < \rho C_s\}}. \tag{48}$$

For $\delta_4 \geq 0$, i.e., $a_1\mu_2 \geq \rho C_s$, since $g_2$ is greater than zero, it is necessary that $\delta_3 \geq 0$, which leads the power allocation satisfying $a_2\rho C_s + 2^{R_s}a_1\rho \geq a_1 a_2\mu_2$. Then, (47) is reformulated as

$$\mathbb{P}\left(F_1^{(21)}\right) = \mathbb{P}\left(\left\{g_1 \geq f^{(21)}(g_2)\right\} \cap \left\{g_2 \geq \max\left(\beta_1, \eta^{(21)}\right)\right\}\right)\mathbb{1}_{M_1^{(21)}}, \tag{49}$$

Since $\beta_1 \geq 0$, using (48) and (49), we can rearrange (47) as

$$\mathbb{P}\left(F_1^{(21)}\right) = \mathbb{P}\left(F_{1\mathrm{R}}^{(21)}\right) M_{1\mathrm{R}}^{(21)}, \tag{50}$$

where $F_{1\mathrm{R}}^{(21)} = \{g_1 \geq f^{(21)}(g_2)\} \cap \{g_2 \geq \Delta^{(21)}\}$. The next step is to obtain $\mathbb{P}(F_{1\mathrm{R}}^{(21)})$:

$$\mathbb{P}\left(F_{1\mathrm{R}}^{(21)}\right) = \int_{\Delta^{(21)}}^{\infty} f_{g_2}(x) \int_{f^{(21)}(x)}^{\infty} f_{g_1}(y) \mathrm{d}y \, \mathrm{d}x. \tag{51}$$

Now, setting $t = \frac{1}{0.01+x}$ and using some simple mathematical calculations, we get

$$\mathbb{P}\left(F_{1\mathrm{R}}^{(21)}\right) = \lambda_2 d_2^{\alpha} \int_0^{\frac{1}{0.01+\Delta^{(21)}}} \frac{e^{-\lambda_2 d_2^{\alpha}\left(\frac{1}{t}-0.01\right)}}{t^2} \times e^{-\lambda_1 d_1^{\alpha} \frac{\left(\frac{1}{t}-0.01\right)\left(a_2 \rho C_\mathrm{s} + 2^{R_\mathrm{s}} a_1 \rho\right) + \rho^2 C_\mathrm{s}}{a_1 a_2 \left(\frac{1}{t}-0.01\right) + a_1 \rho}} \mathrm{d}t, \tag{52}$$

which by Gauss-Chebyshev quadrature can be approximated as $\mathbb{P}(F_{1\mathrm{R}}^{(21)}) \approx \sum_{l=1}^{N_1} \omega_l^{(21)} A_l^{(21)} B_l^{(21)}$. Finally, using (50), $\mathbb{P}(F_1^{(21)})$ is obtained as

$$\mathbb{P}\left(F_1^{(21)}\right) \approx \sum_{l=1}^{N_1} \omega_l^{(21)} A_l^{(21)} B_l^{(21)} M_{1\mathrm{R}}^{(21)}. \tag{53}$$

The last step for obtaining $\mathbb{P}(F^{(21)})$ consists in deriving $\mathbb{P}(F_2^{(21)})$. Using Lemma 4 and similarly to (47), we get $F_2^{(21)} = \{g_1 \geq \mu_2\} \cap \{g_2 \geq \beta_1\} \cap \{g_2 \delta_3 < \delta_4\}$. When $\delta_4 < 0$, then $\delta_3 > 0$, which, due to the non-negative $g_2$, results in $\mathbb{P}(F_2^{(21)}) = 0$. For $\delta_4 \geq 0$, we rewrite $\mathbb{P}(F_2^{(21)})$ as

$$\mathbb{P}\left(F_2^{(21)}\right) = \mathbb{P}(\{g_1 \geq \mu_2\} \cap \{g_2 > \beta_1\}) M_2^{(21)} + \mathbb{P}\left(\{g_1 \geq \mu_2\} \cap \left\{\beta_1 \leq g_2 < \eta^{(21)}\right\}\right) M_3^{(21)}, \tag{54}$$

and then obtain

$$\mathbb{P}\left(F_2^{(21)}\right) = e^{-\left(\lambda_1 d_1^{\alpha} \mu_2 + \lambda_2 d_2^{\alpha} \beta_1\right)} M_2^{(21)} + e^{-\lambda_1 d_1^{\alpha} \mu_2}\left(e^{-\lambda_2 d_2^{\alpha} \beta_1} - e^{-\lambda_2 d_2^{\alpha} \eta^{(21)}}\right) M_3^{(21)}. \tag{55}$$

Then, using (46), (53) and (55), $\mathrm{SOP}^{(21)}$ is found as per (29).

## APPENDIX G

*Proof of Theorem 7:* Using Lemma 4 and (31), the power allocation must satisfy $a_\mathrm{c} > \frac{C_\mathrm{c}}{1+C_\mathrm{c}}$ and $\frac{a_1}{a_2} < \frac{1}{C_2}$. Under these conditions, we can write $\mathrm{SOP}^{(22)} = 1 - \mathbb{P}(\{g_1 \geq \mu_2\} \cap \{g_2 \geq \mu_2\} \cap \{g_1 \geq \frac{\rho C_\mathrm{s}}{a_1} + 2^{R_\mathrm{s}} g_2\})$, which is equal to

$$\mathrm{SOP}^{(22)} = 1 - \int_{\mu_2}^{\infty} f_{g_2}(x) \int_{\frac{\rho C_\mathrm{s}}{a_1} + 2^{R_\mathrm{s}} x}^{\infty} f_{g_1}(y) \mathrm{d}y \, \mathrm{d}x. \tag{56}$$

As a result, $\mathrm{SOP}^{(22)}$ is obtained as shown in (32).

## APPENDIX H

*Proof of Theorem 8:* Using (33), we present the proof for each strategy, separately.

*1) Strategy-11:* According to (33), $\mathrm{SOP}_1^{(11)}$ is given by

$$\mathrm{SOP}_1^{(11)} = 1 - \mathbb{P}\left(g_1\left(a_1 - a_2 C_\mathrm{s} - \frac{2^{R_\mathrm{s}} a_1 a_2 g_2}{a_2 g_2 + \rho}\right) \geq \rho C_\mathrm{s} + \frac{2^{R_\mathrm{s}} a_1 \rho g_2}{a_2 g_2 + \rho}\right). \tag{57}$$

Since $g_1$ and $g_2$ are non-negative random variables, it is necessary to have $a_1 - a_2 C_\mathrm{s} - \frac{2^{R_\mathrm{s}} a_1 a_2 g_2}{a_2 g_2 + \rho} > 0$, which results in $\frac{a_1}{a_2} > C_\mathrm{s}$ and $g_2 < 2\epsilon^{(11)}$. Then, (57) is simplified to

$$\mathrm{SOP}_1^{(11)} = 1 - \mathbb{1}_{\left\{\frac{a_1}{a_2} > C_\mathrm{s}\right\}} \mathbb{P}\left(\left\{g_1 \geq \frac{\rho C_\mathrm{s} + \frac{2^{R_\mathrm{s}} a_1 \rho g_2}{a_2 g_2 + \rho}}{a_1 - a_2 C_\mathrm{s} - \frac{2^{R_\mathrm{s}} a_1 a_2 g_2}{a_2 g_2 + \rho}}\right\} \cap \left\{g_2 < 2\epsilon^{(11)}\right\}\right). \tag{58}$$

Using the Gauss-Chebyshev quadrature method and following a similar approach as in (44), (58) is found as shown in (34).

*2) Strategy-12:* With the aid of (33), $\mathrm{SOP}_1^{(12)}$ is found as

$$\mathrm{SOP}_1^{(12)} = 1 - \mathbb{P}\left(g_1\left(a_1 - a_2 C_\mathrm{s} - \frac{2^{R_\mathrm{s}} a_1 a_2 g_2}{\rho}\right) \geq \rho C_\mathrm{s} + 2^{R_\mathrm{s}} a_1 g_2\right). \tag{59}$$

Since $g_1$ and $g_2$ are non-negative, it is sure that $a_1 - a_2 C_\mathrm{s} - \frac{2^{R_\mathrm{s}} a_1 a_2 g_2}{\rho} > 0$, which results in $\frac{a_1}{a_2} > C_\mathrm{s}$ and $g_2 < 2\epsilon^{(12)}$. Hence, (59) is written as

$$\mathrm{SOP}_1^{(12)} = 1 - \mathbb{1}_{\left\{\frac{a_1}{a_2} > C_\mathrm{s}\right\}} \mathbb{P}\left(\left\{g_1 \geq \frac{\rho C_\mathrm{s} + 2^{R_\mathrm{s}} a_1 g_2}{a_1 - a_2 C_\mathrm{s} - \frac{2^{R_\mathrm{s}} a_1 a_2 g_2}{\rho}}\right\} \cap \left\{g_2 < 2\epsilon^{(12)}\right\}\right), \tag{60}$$

which, by using the Gauss-Chebyshev quadrature and following a similar way as (44), can be approximated as per (34).

*3) Strategy-21:* According to (33), $\mathrm{SOP}_1^{(21)}$ is obtained as

$$\mathrm{SOP}_1^{(21)} = 1 - \mathbb{P}\left(g_1 \geq \frac{\rho C_\mathrm{s}}{a_1} + \frac{2^{R_\mathrm{s}} \rho g_2}{a_2 g_2 + \rho}\right) = 1 - \int_0^{\infty} \lambda_2 d_2^{\alpha} e^{-\lambda_2 d_2^{\alpha} x} e^{-\lambda_1 d_1^{\alpha}\left(\frac{\rho C_\mathrm{s}}{a_1} + \frac{2^{R_\mathrm{s}} \rho x}{a_2 x + \rho}\right)} \mathrm{d}x. \tag{61}$$

First, we replace the variable $t = \frac{1}{0.05+x}$ with $x$ in (61). Thus,

$$\mathrm{SOP}_1^{(21)} = 1 - \int_0^{20} \frac{\lambda_2 d_2^{\alpha}}{t^2} \times e^{-\lambda_2 d_2^{\alpha}\left(\frac{1}{t}-0.05\right)} e^{-\lambda_1 d_1^{\alpha}\left(\frac{\rho C_\mathrm{s}}{a_1} + \frac{2^{R_\mathrm{s}} \rho\left(\frac{1}{t}-0.05\right)}{a_2\left(\frac{1}{t}-0.05\right) + \rho}\right)} \mathrm{d}t. \tag{62}$$

Finally, adopting the Gauss-Chebyshev method, $\mathrm{SOP}_1^{(21)}$ is approximated as shown in (34).

*4) Strategy-22:* As per (33), $\text{SOP}_1^{(22)}$ is given by $\text{SOP}_1^{(22)} = 1 - \mathbb{P}(g_1 \geq \frac{\rho C_s}{a_1} + 2^{R_s} g_2)$, which after some mathematical calculations is obtained as shown in (35).

## APPENDIX I

*Proof of Case 1 of Table 1 (Comparing Strategy-11 and Strategy-12):* For large enough transmit power at the source, the SOP is almost equal to the SOP of the near user, i.e., $\text{SOP}^{(i)} \approx \text{SOP}_1^{(i)}$, $i \in \{11, 12\}$. Hence, using (33), the SOPs of Strategy-11 and Strategy-12 are approximately equal to

$$\text{SOP}^{(11)} \approx 1 - \mathbb{P}\left(1 + \frac{a_1 g_2}{a_2 g_2 + \rho} \leq 2^{-R_s}\left(1 + \frac{a_1 g_1}{a_2 g_1 + \rho}\right)\right), \tag{63}$$

$$\text{SOP}^{(12)} \approx 1 - \mathbb{P}\left(1 + \frac{a_1 g_2}{\rho} \leq 2^{-R_s}\left(1 + \frac{a_1 g_1}{a_2 g_1 + \rho}\right)\right), \tag{64}$$

respectively. First, we need to derive $F_{\frac{a_1 g_t}{a_2 g_t + \rho}}(x)$ for $t \in \{1, 2\}$. According to $F_{g_t}(x) = 1 - e^{-\lambda_t d_t^\alpha x}$ and using a similar approach as Lemma 4, we have

$$\begin{aligned} F_{\frac{a_1 g_t}{a_2 g_t + \rho}}(x) &= F_{g_t}\left(\frac{\rho x}{a_1 - a_2 x}\right) U(a_1 - a_2 x) + U(-a_1 + a_2 x) \\ &= \left(1 - e^{-\lambda_t d_t^\alpha \frac{\rho x}{a_1 - a_2 x}}\right) U(a_1 - a_2 x) + U(-a_1 + a_2 x). \end{aligned} \tag{65}$$

Then, taking the derivative of (65) w.r.t. $x$, we obtain

$$f_{\frac{a_1 g_t}{a_2 g_t + \rho}}(x) = \frac{\lambda_t d_t^\alpha \rho a_1}{(a_1 - a_2 x)^2} e^{-\lambda_t d_t^\alpha \frac{\rho x}{a_1 - a_2 x}} U(a_1 - a_2 x). \tag{66}$$

Next, we derive $F_{\frac{a_1 g_t}{\rho}}(x)$. To do so, the CDF $F_{\frac{a_1 g_t}{\rho}}(x)$ is obtained as

$$F_{\frac{a_1 g_t}{\rho}}(x) = \mathbb{P}\left(g_t \leq \frac{\rho x}{a_1}\right) = F_{g_t}\left(\frac{\rho x}{a_1}\right) = 1 - e^{-\lambda_t d_t^\alpha \frac{\rho x}{a_1}}. \tag{67}$$

Let $Y = \frac{a_1 g_1}{a_2 g_1 + \rho}$, where $f_Y(x)$ is obtained in (66). According to (63), (64) and (66), we get

$$1 - \text{SOP}^{(11)} \approx \int_{C_s}^{\frac{a_1}{a_2}} f_Y(x) F_{\frac{a_1 g_2}{a_2 g_2 + \rho}}\left(2^{-R_s}(1 + x) - 1\right) \mathrm{d}x, \tag{68}$$

$$1 - \text{SOP}^{(12)} \approx \int_{C_s}^{\frac{a_1}{a_2}} f_Y(x) F_{\frac{a_1 g_2}{\rho}}\left(2^{-R_s}(1 + x) - 1\right) \mathrm{d}x. \tag{69}$$

As mentioned in Theorem 8, to avoid the occurrence of a secrecy outage in Strategy-11 and Strategy-12, the power allocation condition $\frac{a_1}{a_2} > C_s$ must hold. Accordingly, for $x \in [C_s, \frac{a_1}{a_2}]$, $\lambda_2 d_2^\alpha \frac{\rho(2^{-R_s}(1+x)-1)}{a_1} < \lambda_2 d_2^\alpha \frac{\rho(2^{-R_s}(1+x)-1)}{a_1 - a_2(2^{-R_s}(1+x)-1)}$. Therefore, using (65)-(67), and comparing (68) and (69), it is guaranteed that $(1 - \text{SOP}^{(12)}) < (1 - \text{SOP}^{(11)})$. As a result, $\text{SOP}^{(12)} > \text{SOP}^{(11)}$, and Strategy-11 outperforms Strategy-12 in terms of SOP for large values of $P$.

## APPENDIX J

*Proof of Case 2 of Table 1 (Comparing Strategy-21 and Strategy-22):* As mentioned, for high values of $P$, the system SOP and the SOP of the near user are almost equal. Hence, using (33), the SOP of Strategy-21 and Strategy-22 in high-SNR regimes are approximately equal to

$$\begin{aligned} \text{SOP}^{(21)} &\approx \mathbb{P}\left(g_1 < \frac{\rho C_s}{a_1} + \frac{2^{R_s} \rho g_2}{a_2 g_2 + \rho}\right) \\ &= \int_0^\infty f_{g_2}(x) F_{g_1}\left(\frac{\rho C_s}{a_1} + \frac{2^{R_s} \rho x}{a_2 x + \rho}\right) \mathrm{d}x, \end{aligned} \tag{70}$$

$$\begin{aligned} \text{SOP}^{(22)} &\approx \mathbb{P}\left(g_1 < \frac{\rho C_s}{a_1} + 2^{R_s} g_2\right) \\ &= \int_0^\infty f_{g_2}(x) F_{g_1}\left(\frac{\rho C_s}{a_1} + 2^{R_s} x\right) \mathrm{d}x. \end{aligned} \tag{71}$$

Since $\lambda_1 d_1^\alpha (\frac{\rho C_s}{a_1} + 2^{R_s} x) > \lambda_1 d_1^\alpha (\frac{\rho C_s}{a_1} + \frac{2^{R_s} \rho x}{a_2 x + \rho})$, comparing (70) and (71) demonstrates that $\text{SOP}^{(22)} > \text{SOP}^{(21)}$ for high values of $P$.

## APPENDIX K

*Proof of Case 3 of Table 1 (Comparing Strategy-11 and Strategy-21):* According to Theorem 8, it is sure that $\frac{a_1}{a_2} > C_s$ must be satisfied for Strategy-11. Let $Z = \frac{a_1 g_2}{a_2 g_2 + \rho}$, where $f_Z(x)$ is as per (66). For high transmit powers, by adopting (33) and (66), defining $C^* = (\frac{a_1}{a_2} - C_s) 2^{-R_s}$, and following a similar approach as Lemma 4, we can write

$$\begin{aligned} &1 - \text{SOP}^{(11)} \\ &\approx \mathbb{P}\left(\left\{g_1 \geq \frac{\rho(C_s + 2^{R_s} Z)}{a_1 - a_2(C_s + 2^{R_s} Z)}\right\} \cap \{Z < C^*\}\right) \\ &= \int_0^{C^*} f_Z(x)\left(1 - F_{g_1}\left(\frac{\rho(C_s + 2^{R_s} x)}{a_1 - a_2(C_s + 2^{R_s} x)}\right)\right) \mathrm{d}x. \end{aligned} \tag{72}$$

Since $Z$ and $g_1$ are non-negative, $\frac{\rho(C_s + 2^{R_s} x)}{a_1 - a_2(C_s + 2^{R_s} x)}$ must be greater than zero, which leads the variable $x$ in (72) to satisfy $0 < x < C^*$. Further, with the aid of (71) and (66), we can write

$$\begin{aligned} 1 - \text{SOP}^{(21)} &\approx \mathbb{P}\left(g_1 \geq \frac{\rho}{a_1}\left(C_s + 2^{R_s} Z\right)\right) \\ &= \int_0^{\frac{a_1}{a_2}} f_Z(x)\left(1 - F_{g_1}\left(\frac{\rho}{a_1}\left(C_s + 2^{R_s} x\right)\right)\right) \mathrm{d}x. \end{aligned} \tag{73}$$

For $x \in [0, C^*]$, $\lambda_1 d_1^\alpha \frac{\rho(C_s + 2^{R_s} x)}{a_1 - a_2(C_s + 2^{R_s} x)} > \lambda_1 d_1^\alpha \frac{\rho(C_s + 2^{R_s} x)}{a_1}$. On the other hand, $C^* < \frac{a_1}{a_2}$, which according to (73) gives

$$\int_0^{C^*} f_Z(x)\left(1 - F_{g_1}\left(\frac{\rho}{a_1}\left(C_s + 2^{R_s} x\right)\right)\right) \mathrm{d}x \leq 1 - \text{SOP}^{(21)}. \tag{74}$$

Comparing (72)-(74) for high SNRs confirms that $\text{SOP}^{(11)} > \text{SOP}^{(21)}$.

## APPENDIX L

*Proof of Case 4 of Table 1 (Comparing Strategy-12 and Strategy-22):* Given (33), and since $g_1$ and $g_2$ are non-negative, $(1-\mathrm{SOP}^{(12)})$ and $(1-\mathrm{SOP}^{(22)})$ for high power budget at the source are given by

$$1-\mathrm{SOP}^{(12)} \approx \mathbb{P}\left(g_2 \le \left(2^{-R_s}-1\right)\frac{\rho}{a_1}+\frac{2^{-R_s}\rho g_1}{a_2 g_1+\rho}\right) = \int_{\frac{\rho C_s}{a_1-a_2C_s}}^{\infty} f_{g_1}(x) F_{g_2}\left(\left(2^{-R_s}-1\right)\frac{\rho}{a_1}+\frac{2^{-R_s}\rho x}{a_2 x+\rho}\right)\mathrm{d}x, \tag{75}$$

$$1-\mathrm{SOP}^{(22)} \approx \mathbb{P}\left(g_2 \le \left(2^{-R_s}-1\right)\frac{\rho}{a_1}+\frac{g_1}{2^{R_s}}\right) = \int_{\frac{\rho C_s}{a_1}}^{\infty} f_{g_1}(x) F_{g_2}\left(\left(2^{-R_s}-1\right)\frac{\rho}{a_1}+\frac{x}{2^{R_s}}\right)\mathrm{d}x. \tag{76}$$

As per Theorem 8, it is necessary for the power allocation coefficients to satisfy $\frac{a_1}{a_2} > C_s$ so as to prevent secrecy outage with Strategy-12. Since $g_1$ and $g_2$ are non-negative, (75) satisfies $(2^{-R_s}-1)\frac{\rho}{a_1}+\frac{2^{-R_s}\rho x}{a_2x+\rho} > 0$, which leads $x$ to be in the interval $(\frac{\rho C_s}{a_1-a_2C_s}, \infty)$. Also, $g_1, g_2 > 0$ makes (76) satisfy $(2^{-R_s}-1)\frac{\rho}{a_1}+2^{-R_s}x > 0$, which indicates that $\frac{\rho C_s}{a_1} < x < \infty$ in (76). Define $F_s(x) = F_{g_2}((2^{-R_s}-1)\frac{\rho}{a_1}+2^{-R_s}x)$. For $x \in [\frac{\rho C_s}{a_1-a_2C_s}, \infty]$, it is sure that $\frac{\rho x}{a_2x+\rho} < x$. Furthermore, it is found that $\frac{\rho C_s}{a_1-a_2C_s} > \frac{\rho C_s}{a_1}$. Hence, comparing (75) with (76), we get

$$1-\mathrm{SOP}^{(12)} < \int_{\frac{\rho C_s}{a_1-a_2C_s}}^{\infty} f_{g_1}(x)F_s(x)\mathrm{d}x < 1-\mathrm{SOP}^{(22)}, \tag{77}$$

which shows that $\mathrm{SOP}^{(12)} > \mathrm{SOP}^{(22)}$ in high-SNR regimes.

## APPENDIX M

*Proof of Lemma 2:* Clearly, when $P \to \infty$, then $\rho \to 0$. As stated earlier, for high values of $P$, $\mathrm{SOP}^{(11)}$ is almost equal to $\mathrm{SOP}_1^{(11)}$. Therefore, for $\rho \to 0$, by using (33), we have

$$\mathrm{SOP}_1^{(11)} \approx 1-\mathbb{P}\left(1+\frac{a_1}{a_2} \ge 2^{R_s}\left(1+\frac{a_1}{a_2}\right)\right). \tag{78}$$

When the transmit power $P \to \infty$, then $\mathrm{SOP}^{(12)} = \mathrm{SOP}_1^{(12)}$. Using (33), for $\rho \to \infty$ we obtain

$$\mathrm{SOP}_1^{(12)} \approx 1-\mathbb{P}\left(1+\frac{a_1}{a_2} \ge 2^{R_s}\left(1+\frac{a_1Pg_2}{\sigma^2}\right)\right) = 1-\int_0^{\frac{\left(2^{-R_s}\left(1+\frac{a_1}{a_2}\right)-1\right)\rho}{a_1}} f_{g_2}(x)\mathrm{d}x = e^{-\lambda_2 d_2^{\alpha}\frac{\left(2^{-R_s}\left(1+\frac{a_1}{a_2}\right)-1\right)\rho}{a_1}}. \tag{79}$$

Since $1+\frac{a_1}{a_2} < 2^{R_s}(1+\frac{a_1}{a_2})$, it is sure that $\mathbb{P}(1+\frac{a_1}{a_2} \ge 2^{R_s}(1+\frac{a_1}{a_2})) = 0$. Hence, according to (78), for $P \to \infty$ we have $\mathrm{SOP}^{(11)} \approx \mathrm{SOP}_1^{(11)} \approx 1$. When $\rho \to 0$, $e^{-\lambda_2 d_2^{\alpha}\frac{(2^{-R_s}(1+\frac{a_1}{a_2})-1)\rho}{a_1}} \to 1$, and $\mathrm{SOP}^{(12)}$ is approximated as $\mathrm{SOP}^{(12)} \approx \mathrm{SOP}_1^{(12)} \approx 1$.

## APPENDIX N

*Proof of Lemma 3:* We first provide the proof for Strategy-21. For $P \to \infty$, $U_2$ receives a sufficient rate to decode $S_c$ and $S_2$ with Strategy-1, and $U_1$ can decode $S_c$ correctly. As such, $\mathrm{SOP}^{(21)} \approx \mathrm{SOP}_1^{(21)}$. In this case, $\rho \to 0$ and, according to (33), $\mathrm{SOP}_1^{(21)}$ is approximated as follows

$$\mathrm{SOP}_1^{(21)} \approx 1-\mathbb{P}\left(g_1 \ge \frac{\rho}{a_1}\left(2^{R_s}\left(1+\frac{a_1}{a_2}\right)-1\right)\right) = 1-e^{-\lambda_1 d_1^{\alpha}\frac{\rho}{a_1}\left(2^{R_s}\left(1+\frac{a_1}{a_2}\right)-1\right)}. \tag{80}$$

Since $\rho \to 0$, then $e^{-\lambda_1 d_1^{\alpha}\frac{\rho}{a_1}(2^{R_s}(1+\frac{a_1}{a_2})-1)} \to 1$. Hence, by adopting (80), when $P \to \infty$, then $\mathrm{SOP}^{(21)}$ is obtained as $\mathrm{SOP}^{(21)} \approx \mathrm{SOP}_1^{(21)} \approx 0$.

Now, we present the Lemma proof for Strategy-22. For high values of $P$, $\mathrm{SOP}^{(22)}$ is very close to $\mathrm{SOP}_1^{(22)}$. According to $\rho \to 0$, $\mathrm{SOP}_1^{(22)}$ in (71) is written as

$$\mathrm{SOP}_1^{(22)} \approx 1-\mathbb{P}\left(g_1 \ge 2^{R_s}g_2\right) = 1-\int_0^{\infty} f_{g_2}(x)\int_{2^{R_s}x}^{\infty} f_{g_1}(y)\mathrm{d}y\,\mathrm{d}x. \tag{81}$$

Using (81), when $P \to \infty$, $\mathrm{SOP}^{(22)}$ is found as $\mathrm{SOP}^{(22)} \approx \mathrm{SOP}_1^{(22)} \approx 1-\frac{\lambda_2 d_2^{\alpha}}{\lambda_2 d_2^{\alpha}+2^{R_s}\lambda_1 d_1^{\alpha}}$.

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