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# INSIDE MONEY, INVESTMENT, AND UNCONVENTIONAL MONETARY POLICY*

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I develop a model where banks play a central role in monetary policy transmission. By credibly committing to repayment, banks can perform liquidity transformation. Illiquid assets may pay a liquidity premium because they allow banks to create liquid assets. The policy analysis discusses how the monetary authority can affect nominal rates and inflation when the fiscal authority follows nominal or real debt targets. A main result is that under a nominal debt target, the monetary authority is only able to increase inflation at the zero-lower bound by issuing money via lump-sum transfers, while doing so via bond purchases is ineffective.

## 1. Introduction

After the financial crisis of 2007–9, central banks in many developed economies implemented a number of unconventional policies such as quantitative easing (QE; purchasing large amounts of assets), forward guidance (informing market participants in detail about future plans), and paying interest—or charging negative rates—on reserves. Other, arguably even more radical new policies have also been discussed, most notably helicopter money, that is, printing money and distributing it directly to households.[1] A main goal of these policies was to stimulate investment, which was perceived too low.[2] The reason that central banks tried novel policies was that their main tool—cuts in the policy rate—quickly came to a limit, as for example, the Federal Funds rate in the United States hit the zero-lower bound shortly after the onset of the financial crisis. The time after the financial crisis also made clear that the quantity theory of money fails at the zero-lower bound: This theory predicts that increases in the monetary base will lead to increases in inflation at least in the medium term. However, the monetary base grew at unprecedented rates from 2008 onward in the United States and many other developed economies, whereas inflation remained low.

Given these observations, the goal of this article is first to create a model where the economy can endogenously end up at the zero-lower bound, and that is able to capture the failure of the quantity theory of money at the zero-lower bound. The second goal is to use this

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[1] Amid the current crisis caused by COVID-19, helicopter money is again being discussed as a measure to stabilize the economy: See, for example, https://voxeu.org/article/helicopter-money-another-pandemic-recession-venice-1630and the references provided therein.

[2] Arguably, financial stability in terms of bank default was an even larger concern for central banks at the onset of the crisis, but it is clear that insufficient bank lending and thus a lack of investment was also a major issue for central banks. See, for example, this speech by Ben Bernanke from December 2008: https://www.federalreserve.gov/newsevents/speech/bernanke20081201a.htm. In this article, I focus on the effect of policies on bank lending and investment.

model to analyze whether and how various policies employed by central banks are able to affect investment and output, both at the zero-lower bound and away from it. To achieve this, I develop a model where banks play a crucial role in the transmission mechanism of monetary policy, and I take the separation between the monetary and the fiscal authority seriously. The model is able to replicate the failure of the quantity theory at the zero-lower bound if the fiscal authority follows a nominal debt target and the monetary authority increases the monetary base through asset purchases. If the monetary authority distributes newly issued money via lump-sum transfers instead, it can increase inflation even at the zero-lower bound. This policy can be interpreted as helicopter money, and it can be welfare-improving if entrepreneurs are borrowing-constrained. The model further shows that the monetary authority is also constrained at the zero-lower bound under a real debt target, unless it is able to use helicopter money. Under either target, QE can only be used to increase investment away from the zero-lower bound, and negative interest rates on reserves do not increase investment unless there is a large share of trades in the economy that require deposits while cash cannot be used.

1.1. *Model Summary.* The model is based on Lagos and Wright (2005), but adds entrepreneurs in need of funding and banks that can intermediate funds in order to make the role of the financial sector in the transmission of monetary policy explicit. Specifically, the model features both fiat (outside) money provided by the central bank, and inside money created by private banks. Buyers and sellers meet in a decentralized market (DM), and buyers need liquid assets to purchase goods from sellers. I assume that all sellers accept fiat money issued by the central bank, and most sellers accept inside money issued by banks. I further assume that government bonds and loans to entrepreneurs are illiquid, in the sense that sellers never accept them as payment. Banks differ from other agents in one main aspect: They are able to credibly commit to repayment, and thus their debt may be accepted by other agents as a means of payment. This allows banks to refinance themselves at lower rates, and makes them the natural lenders in the economy. In equilibrium, entrepreneurs prefer to borrow from banks, because they lend at lower rates. When creating inside money, banks need to take into account how many deposits agents are willing to hold at the equilibrium interest rate.[3] Buyers are willing to hold deposits because they need liquid assets to trade, and deposits weakly dominate cash due to interest rate payments. Thus, the ability of banks to issue liquid debt allows them to perform liquidity transformation and makes them essential: They are able to extend the set of liquid assets by investing in illiquid assets such as government bonds and loans to entrepreneurs.[4] This also implies that illiquid assets may attain a liquidity premium. The banking market is competitive, so the loan rate equals the deposit rate in equilibrium. Depending on demand for loans and deposits, this nominal rate can be either positive or zero. If it is zero, banks may hold excess reserves to satisfy the demand for deposits. Due to limited commitment, entrepreneurs can only pledge a fraction of their future output to receive credit. Therefore, their investment depends on both the interest rate and whether the borrowing constraint is binding—but in either case, they invest more at lower interest rates. The Friedman rule delivers first-best investment and deposit holdings if entrepreneurs are not constrained in their borrowing. With binding borrowing constraints, investment is below the first best at the Friedman rule. In this case, lowering the real interest rate can be welfare-improving.

I assume throughout the article that the fiscal authority and the monetary authority are independent. For the fiscal authority, I consider two targets: A real debt target where the fiscal authority keeps the value of its real debt constant, and a nominal debt target where nominal debt grows at a constant rate. For the monetary authority, I distinguish between two ways of

[3] This is true even though banks can create as many deposits as they want. In equilibrium, someone has to be willing to hold liquid assets—if banks create more deposits than agents want to hold, prices adjust, thereby reducing the real value of the deposits created.

[4] In turn, this (weakly) increases investment by entrepreneurs and quantities of goods traded against liquid assets, with at least one of those strictly increasing unless inflation is at the Friedman rule.

issuing fiat money: purchases of government bonds and lump-sum transfers to agents. If the fiscal authority follows a nominal debt target, the steady-state inflation rate cannot be controlled by the monetary authority, but the monetary authority can vary the real interest rate through open-market operations as long as the nominal rate is positive. At the zero-lower bound, increasing the money supply through lump-sum transfers (helicopter money) increases the inflation rate, which in turn allows to lower the real interest rate. If the monetary authority instead increases the money supply through bond purchases, inflation remains unchanged, which rationalizes the failure of the quantity theory at the zero-lower bound. If the fiscal authority follows a real debt target, the monetary authority can control steady-state inflation and nominal rates simultaneously if it can use helicopter money. Without helicopter money, the monetary authority's choice of steady-state inflation pins down nominal and real rates, and there is a lower bound on the real interest rate the monetary authority can implement. This lower bound is reached at the zero-lower bound. Thus, while the exact mechanism differs depending on the target the fiscal authority follows, the main result is the same: the monetary authority may not implement arbitrarily low real interest rates unless it can use helicopter money.

1.2. *Existing Literature.* Although this article is closely related to Williamson (2012), the model presented here is quite different. Importantly, banks in my model perform liquidity transformation, whereas banks in Williamson (2012) perform liquidity insurance. This can best be understood by comparing the main frictions in the two papers: In Williamson (2012), buyers can use bonds to make purchases in the DM with some probability, but they only learn about this after making their portfolio choice. Banks then insure them against this liquidity risk by holding a portfolio of bonds and cash. This is similar to Diamond and Dybvig (1983) or Berentsen et al. (2007). In my article, all agents know ex ante which assets they will be able to use in the next DM, so there is no role for liquidity insurance. However, by being able to credibly commit to repay their debt, banks are able to issue another liquid asset besides fiat money. Thus, the role of banks in my article builds on papers such as Gu et al. (2013).[5] Note that in Williamson (2012), banks do not issue deposits, and the role of insuring agents against liquidity risk can to some extent be performed by a financial market.[6] If instead banks are able to more credibly commit to repayment than other agents in the economy, a financial market cannot replace them. Despite these fundamental differences, the results on how monetary policy can affect interest rates and real outcomes in the economy are very similar, which shows that these results are robust regarding the particular role of banks in the economy. The new results I find on how helicopter money may increase investment and welfare at the zero-lower bound stem from two innovations compared to Williamson (2012): First, I assume that entrepreneurs are subject to a borrowing constraint, which implies that welfare may be increased through decreases in the real interest rate;[7] and second, I explicitly model how newly issued fiat money is introduced in the economy. While in Williamson (2012), the baseline assumption is that money is issued via lump-sum transfers, I assume that money is issued via bond purchases, while issuing money via lump-sum transfers instead is another policy tool for the monetary authority. The analysis shows that how money is issued makes a difference, particularly at the zero-lower bound.

Another related paper is Herrenbrueck (2019), which also studies helicopter money. However, the focus there lies on the distributional effects of this policy in a model with preference shocks, which is an aspect I abstract from. Furthermore, my article is related to the literature

[5] A similar way to introduce banks as here is used by Keister and Sanches (2019) and Chiu et al. (2019), which both study central bank digital currency.

[6] Zannini (2020) argues that welfare can be improved in an economy based on Williamson (2012) by introducing financial markets.

[7] Venkateswaran and Wright (2013) show numerically that increases in inflation can lead to increases in investment and output if there is a tax on capital. This is a different mechanism than in my article, but it has similar effects for welfare.

on indirect liquidity (w.g., Herrenbrueck and Geromichalos, 2017) which shows that assets can attain a liquidity premium even if they cannot directly be used as a means of payment, but can be traded on financial markets against cash. I show that a similar mechanism is at play if there are institutions such as banks that can issue liquidity by investing in illiquid assets. Furthermore, my article is related to a number of publications in the New Monetarist literature that make important contributions to our understanding of the zero-lower bound and related phenomena, such as: Williamson (2016), who studies QE, but focuses on different maturities of government debt, which I abstract from in this article; Andolfatto and Williamson (2015), who study how to escape from the zero-lower bound in an environment where increases in the real interest rate lead to increases in consumption and output—which is not generally the case in my model; Dai and He (2018), who study open-market operations, standing facilities, and lump-sum transfers in a model where preference shocks create borrowers and lenders, and where borrowing constraints lead to distributional effects of these policies; Rocheteau et al. (2018), who study the effect of one-time changes in the bonds-to-money ratio in New Monetarist models with a variety of different trading protocols and market structures; and Boel and Waller (2019), who show that there is a need for stabilization policy even at the Friedman rule if agents have heterogeneous discount rates.[8]

In the broader literature on monetary economics, the quantity theory of money and helicopter money are discussed in a few papers. Krugman et al. (1998) find that an expansion of the monetary base has no effect on broader monetary aggregates due to credibility problems, a mechanism which is not at play here. Kiyotaki and Moore (2012) study the effect of open-market operations and helicopter money after a liquidity shock, and find that open-market operations have real effects, whereas helicopter money does not. Their contrary findings to mine stem from the fact that there is no role for assets as investment opportunities for banks in their paper. Buiter (2014) argues that if three conditions are satisfied (i.e., fiat money is held for other reasons than its return, fiat money is irredeemable, and the price of money is positive), helicopter money can always be used to boost demand, which in turn increases inflation. All of these conditions are satisfied in my model, so my results support Buiter's claim. Gali (2014) shows that a money-financed fiscal stimulus (i.e., something like helicopter money) has strong effects on economic activity, but only relatively mild inflationary consequences, whereas in my article, helicopter money only has an effect on economic activity through the inflation rate. Open-market operations are discussed in Eggertsson and Woodford (2003, 2004); QE is discussed in Gertler and Karadi (2011) and Bacchetta et al. (2020); finally, negative interest on reserves are discussed in Demiralp et al. (2017), Dong and Wen (2017), and Rognlie (2016). Other papers discussing monetary and fiscal policy at the zero-lower bound are Werning (2012), Eggertsson and Krugman (2012), Christiano et al. (2011), Correia et al. (2013), Guerrieri and Lorenzoni (2017), and Cochrane (2017). The novel results found in my article relative to those cited here stem from the different modeling approaches of the New Monetarist literature relative to the broader literature on monetary economics. Whereas nominal rigidities play a central role in many of the papers cited, they are not at play in my article. Instead, liquidity is key: The demand for money, deposits, and bonds is microfounded and stems from their (direct or indirect) liquidity. Some of the papers above abstract from money entirely (e.g., Gertler and Karadi, 2011; Werning, 2012); if they include money, it is typically done through a money-in-the utility approach (e.g., Eggertsson and Woodford, 2003, 2004; Gali, 2014); and as far as I am aware, none of them have both money and deposits. However, only the coexistence of these two forms of money allows for a

[8] For the results I find on helicopter money, two model components are important: First, the monetary authority needs to be able to use both lump-sum transfers and bond purchases to issue new money; second, monetary and fiscal policy have to be separate, with fiscal policy not being completely passive. The only other paper mentioned in this paragraph that combines these aspects is Herrenbrueck (2019), but it differs in many other aspects; Rocheteau et al. (2018) and Dai and He (2018) allow for lump-sum transfers and bond purchases, but assume passive fiscal policy; Andolfatto and Williamson (2015) and Williamson (2016) have nonpassive fiscal policy, but only consider money issued via bond purchases; Boel and Waller (2019) has only lump-sum transfers.

zero-lower bound that arises endogenously, which in turn is necessary to assess whether different policies are able to overcome the constraint this puts on the set of allocations that are implementable by policymakers. This approach also allows to distinguish a zero-lower bound equilibrium from a Friedman rule equilibrium: In the former, the return on money and bonds is the same, but there is still a positive opportunity cost of holding either of them; in the latter, the opportunity cost of holding any liquid asset is zero, and thus agents are satiated in liquidity. This difference is key to understanding the welfare properties of a zero-lower bound equilibrium. Finally, explicitly modeling multiple assets also allows to explicitly model monetary policy implementation.

1.3. *Outline.* The rest of the article is organized as follows: In Section 2, the model is explained, and in Section 3, the equilibrium is defined and discussed. Section 4 discusses the welfare properties of the model and optimal monetary policy. Section 5 discusses how the monetary authority may vary inflation under a nominal debt target. Section 6 discusses some other policies that may stimulate investment at the zero-lower bound, and Section 7 concludes.

## 2. THE MODEL

Time is discrete and continues forever. Each period is divided into two subperiods. In the first subperiod, agents interact in a frictional, DM, and in the second subperiod, they interact in a Walrasian, centralized market (CM). There is a unit measure each of infinitely lived buyers and sellers, collectively called households. There is also a unit measure of infinitely lived banks, and in the CM of each period $t$, a unit measure of entrepreneurs is born and lives until the end of period $t+1$. There is also a monetary and a fiscal authority.

In the CM, a good $x$ can be produced by households on the spot. CM goods are nonstorable but can be transformed one to one into capital $k$, which can be invested by entrepreneurs in order to produce $x$ in the next period. In the DM, sellers can produce a nonstorable good $q$, and buyers gain utility from consuming it. Buyers' preferences are

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (u(q_t) - y_t), \tag{1}$$

with $\beta \in (0,1)$ denoting the discount factor, $u(0)=0$, $u'(q) > 0 > u''(q)$, $u'(0) = \infty$, and $-qu''(q)/u'(q) < 1$. $y_t$ denotes the net disutility from producing the CM good. Sellers' preferences are

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (-c(q_t) + x_t), \tag{2}$$

with $c(q)$ denoting the disutility of producing the DM good. Assume $c(q) = q$, that is, sellers produce DM goods at linear cost. $x_t$ denotes sellers' net utility from consuming the CM good. Define $q^*$ as $u'(q^*) = 1$; that is, the first-best quantity of consumption in the DM.

In the DM, buyers and sellers match bilaterally, and buyers get to make take it or leave it offers.[9] Buyers are anonymous during the DM, so they can only acquire goods from sellers via direct settlement.[10] Furthermore, sellers can be of two types: A fraction $1-\eta$ are outside sellers who are only able to accept fiat (outside) money, whereas a fraction $\eta$ are inside

[9] The effect of different bargaining protocols and powers has been analyzed by Lagos and Wright (2005), and the effect of different market structures in the DM has been analyzed by Rocheteau and Wright (2005). Their findings apply here as well.

[10] Anonymity rules out credit because a buyer cannot credibly promise to repay later if the seller is unable to identify the buyer in the future.

sellers who are able to accept fiat money and bank deposits (inside money).[11] Importantly, buyers already learn during the CM of period $t$ which type of seller they will contact in period $t+1$. Therefore, in any given DM, the measure of buyers with an inside meeting is $\eta$, whereas the remaining measure $1-\eta$ have an outside meeting and thus need cash. Any value $\eta \in (0,1)$ can be assumed, but empirically $\eta$ close to, but strictly smaller than 1 is realistic—that is, only a few trades can be made only with cash. Other assets besides bank deposits and fiat money are not accepted by any seller in the DM.[12]

During the CM, buyers need to acquire liquid funds in order to purchase goods in the next DM, which they can do by producing CM goods and selling them against money. Sellers typically enter the CM with money, which they can use to purchase CM goods from buyers or old entrepreneurs.

Entrepreneurs born in period $t$ get linear utility from consuming CM goods during period $t+1$. Each entrepreneur has access to an individual investment opportunity that, upon investing $k$ units of capital in period $t$, yields $f(k)$ units of CM goods during the CM of period $t+1$, with $f(0)=0$, $f'(k)>0$, $f''(k)<0$, and $f'(0)=\infty$. During the CM, $x$ can be transformed into $k$ one for one. Capital fully depreciates after production. The first-best quantity of capital $k^*$ is given by $f'(k^*)=1/\beta$. Because entrepreneurs have no funds of their own, they need external funding, that is, loans. Suppose the nominal market clearing loan interest rate due in the following CM is denoted by $i^\ell$. Assume that entrepreneurs can run away without punishment with a share $1-\chi$ of their output, with $\chi \in (0,1]$. This creates a limited commitment friction, as it restricts the amount entrepreneurs can borrow by $\chi f(k)$.[13] Assume further that loans outstanding are public information.

Banks are agents that do not participate in the DM; in the CM, they cannot produce and get linear utility from consumption. They also discount future periods at rate $\beta$. Banks differ from other agents because they are not anonymous in the CM and are under full commitment. This allows them to credibly commit to repay their debt. As explained above, inside sellers have the necessary technology to accept bank deposits as a means of payment during the DM, which makes bank deposits liquid and distinguishes them from other assets in the economy, particularly loans to entrepreneurs and government bonds. Deposits are nominal claims that can be redeemed during the CM against fiat money. The market clearing deposit interest rate is denoted by $i^d$. Banks (as well as households) can invest either by making loans to entrepreneurs, by purchasing government bonds, or by acquiring fiat money. Note that banks do not need to acquire fiat money first before making loans or purchasing bonds: If the entrepreneur seeking the loan or the agent selling the bond is willing to accept inside money, the bank can simply create a liability (i.e., a bank deposit) to create the loan or pay for the bond. Banks take prices as given and compete for deposits and loans.

The monetary authority issues fiat money $M_t$, which it can produce without cost. If fiat money is held by a bank, it can be considered reserves.[14] Denote the value of fiat money in terms of $x_t$ as $\phi_t$, which implies the inflation rate is $\pi_{t+1} \equiv \phi_t/\phi_{t+1} - 1$. Furthermore, denote the growth rate of fiat money as $\gamma_t^M \equiv M_t/M_{t-1}$. The monetary authority may issue fiat money either by purchasing bonds, or by making lump-sum transfers to households. Denote bonds held by the monetary authority as $b_t^M$. I assume that the monetary authority rolls over bonds

[11] One way to motivate this assumption is that $\eta$ sellers own the technology required to accept debit card transactions, whereas it is infinitely costly for the remaining sellers to acquire the technology. Appendix B.1 additionally considers virtual meetings where only bank deposits are accepted.

[12] To motivate this, assume it is impossible for sellers to verify the validity of these assets during the DM. All results also go through qualitatively if other assets are assumed to be liquid, but less so than bank deposits.

[13] Along the lines of Kehoe and Levine (1993), $\chi$ could be made endogenous by assuming that entrepreneurs live forever, but get excluded from receiving credit in the future if they do not repay. $\chi$ then has to be set such that the discounted future profits of acting as an entrepreneur are higher than the immediate profits of consuming the full payoff of the investment opportunity. Empirically, $\chi$ can be interpreted as a covenant based on future earnings.

[14] Furthermore, all reserves held by banks can be considered excess reserves, because there is no reserve requirement. In reality, there are subtle but important differences between fiat money and reserves, but I abstract from these here.

it held in previous periods.[15] Assume that the monetary authority chooses $b_t^M/M_t$, that is, the fraction of money issued through bond purchases and lump-sum transfers. Note that $b_t^M/M_t = 0$ ($b_t^M/M_t = 1$) implies only lump-sum transfers (bond purchases). $b_t^M/M_t > 1$ is also possible; it implies the simultaneous use of bond purchases and lump-sum taxes to withdraw some of the money issued. Note also that the monetary authority may make profits if the bonds on its portfolio pay interest. This central bank profit (in real terms) is denoted by $\Pi_t$. I assume that $\Pi_t$ is transferred to the fiscal authority.

Finally, suppose the fiscal authority is the only entity in the economy able to levy taxes. It has to finance some spending $g_t$, and can do so by levying lump-sum taxes $\tau_t$ on households, issuing nominal, one-period bonds $B_t$, or using the profits transferred from the monetary authority. This gives rise to the following government budget constraint:

$$\phi_t B_t + 2\tau_t + \Pi_t = \phi_t(1 + i_t^B)B_{t-1} + g_t, \tag{3}$$

where $i_t^B$ denotes the nominal, market clearing interest rate on bonds. Assume that the government starts with an initial amount of nominal debt $B_0$. I consider two different targets for the fiscal authority. First, assume that nominal debt grows at constant rate $\gamma^B = B_t/B_{t-1}$. I will call this a nominal debt target. Alternatively, suppose the fiscal authority targets a constant level of real debt $\mathcal{B} = \phi_t B_t$ by issuing the amount of nominal debt required to hit the target, given the price level. I will call this a real debt target. In either case, the fiscal authority then raises the amount of lump-sum taxes needed to balance its budget, given $\Pi_t$, $g_t$, and either $\mathcal{B}$ or $\gamma^B$.

2.1. *The Household's Problem.* Start with the problem of buyers who will be in an outside meeting in the following DM. Denote the quantity of DM goods that these buyers consume as $q^o$. In the DM, buyers in outside meetings solve

$$\begin{aligned} &\max_{q_t^o} \quad u(q_t^o) - p(q_t^o) \\ &\text{s.t.} \quad \phi_t m_{t-1}^o \geq p(q_t^o) = c(q_t^o) = q_t^o, \end{aligned}$$

where $p(q)$ denotes the real payment required to purchase quantity $q$ of the DM good. Buyers choose $q_t^o$ such that they maximize the surplus from trade, which is given by the utility they get from consuming $q_t^o$ units of DM goods minus $p(q_t^o)$. Since buyers get to make take it or leave it offers in the DM, $p(q) = c(q)$. Finally, outside buyers cannot pay more than the real value of the fiat money they hold, which is given by $\phi_t m_{t-1}^o$. The solution to this problem is $q_t^o = \min\{\phi_t m_{t-1}^o, q^*\}$.

Next, turn to the CM problem of outside buyers. As usual in models based on Lagos and Wright (2005), the portfolio choice during the CM is independent of current wealth of a household. Thus, the CM problem is

$$\begin{aligned} \max_{m_t^o\geq 0, b_t^o\geq 0, \ell_t^o\geq 0, d_t^o\geq 0} \Bigg[ &-\left(\frac{1+\pi_{t+1}}{\beta} - (1+i_{t+1}^d)\right)\phi_{t+1}d_t^o - \left(\frac{1+\pi_{t+1}}{\beta} - 1\right)\phi_{t+1}m_t^o \\ &-\left(\frac{1+\pi_{t+1}}{\beta} - (1+i_{t+1}^B)\right)\phi_{t+1}b_t^o - \left(\frac{1+\pi_{t+1}}{\beta} - (1+i_{t+1}^\ell)\right)\phi_{t+1}\ell_t^o \\ &+u(\phi_{t+1}m_t^o) - \phi_{t+1}m_t^o \Bigg]. \end{aligned}$$

[15] Rolling over bonds allows the monetary authority to contract the money stock in the future by selling some of the bonds against fiat money.

The first four terms in this problem state the cost of acquiring deposits $d_t^o$, fiat money $m_t^o$, government bonds $b_t^o$, and loans to entrepreneurs $\ell_t^o$, in period $t$ and holding them to period $t+1$. The final two terms capture the surplus from trading in the DM, which depends on the liquid assets (fiat money in the case of buyers in outside meetings) an agent brings to the DM. It is easy to see that buyers in outside meetings only hold loans to entrepreneurs, bonds, and bank deposits if the respective interest rate is at least compensating them for discounting and inflation, that is, if $1+i_{t+1} \geq (1+\pi_{t+1})/\beta$. Following Geromichalos and Herrenbrueck (2022), I call $1+i_{t+1} = (1+\pi_{t+1})/\beta$ the Fisher interest rate—that is, the nominal interest rate that exactly compensates for inflation and discounting.[16] This is not true for fiat money, because fiat money allows buyers in outside meetings to purchase DM goods and thus has a liquidity value. Solving the above problem for money, we get:

$$u'(\phi_{t+1}m_t^o) = 1 + \frac{1+\pi_{t+1}-\beta}{\beta}. \tag{4}$$

It can be verified that the demand for real balances is decreasing in $\pi_{t+1}$.

Now turn to inside buyers. Denote their DM good consumption as $q^i$. Their problem is similar, except that they can also use bank deposits to purchase DM goods. Thus, their DM problem is

$$\max_{q_t^i} \quad u(q_t^i) - p(q_t^i)$$
$$\text{s.t.} \quad (1+i_t^d)\phi_t d_{t-1}^i + \phi_t m_{t-1}^i \geq p(q_t^i) = c(q_t^i) = q_t^i,$$

and the solution to this problem is $q_t^i = \min\{(1+i_t^d)\phi_t d_{t-1}^i + \phi_t m_{t-1}^i, q^*\}$.

In the CM, inside buyers solve

$$\begin{aligned}\max_{m_t^i\geq0, b_t^i\geq0, \ell_t^i\geq0, d_t^i\geq0} \Bigg[ &-\left(\frac{1+\pi_{t+1}}{\beta} - (1+i_{t+1}^d)\right)\phi_{t+1}d_t^i - \left(\frac{1+\pi_{t+1}}{\beta} - 1\right)\phi_{t+1}m_t^i \\ &- \left(\frac{1+\pi_{t+1}}{\beta} - (1+i_{t+1}^B)\right)\phi_{t+1}b_t^i - \left(\frac{1+\pi_{t+1}}{\beta} - (1+i_{t+1}^\ell)\right)\phi_{t+1}\ell_t^i \\ &+ u((1+i_{t+1})\phi_{t+1}d_t^i + \phi_{t+1}m_t^i)) - ((1+i_t)\phi_{t+1}d_t^i + \phi_{t+1}m_t^i)\Bigg].\end{aligned}$$

Just as outside buyers, inside buyers only hold bonds and loans to entrepreneurs if they pay the Fisher rate, but they are willing to hold deposits at lower rates due to their liquidity services. Furthermore, bank deposits (weakly) dominate fiat money for any $i_{t+1}^d \geq 0$. This implies that buyers in inside meetings do not hold $m$ if $i_{t+1}^d > 0$, and that they are indifferent to do so if $i_{t+1}^d = 0$.[17] For $0 \leq i_{t+1}^d \leq (1+\pi_{t+1})/\beta - 1$, the first-order condition (FOC) for $d$ is

$$u'\big((1+i_{t+1}^d)\phi_{t+1}d_t^i\big) = 1 + \frac{1+\pi_{t+1}-\beta(1+i_{t+1}^d)}{\beta(1+i_{t+1}^d)}. \tag{5}$$

Appendix C.1 shows that demand for real deposits $\phi_t d_t^i$ is decreasing in $\pi_{t+1}$ and that $d_t^i$ is increasing in $i_{t+1}^d$.[18] Define $\hat{z_d}$ as the real demand for deposits at the Fisher interest rate, and $\bar{z_d}$

[16] By defining $1+r = \frac{1}{\beta}$ as the natural real interest rate, the Fisher rate is the nominal interest rate at which the Fisher equation (Fisher, 1930) holds for the natural real interest rate.

[17] For $i_{t+1}^d < 0$, inside buyers do not hold any deposits, and their demand for money is identical to that of buyers in outside meetings, given by Equation (4).

[18] As shown in the proof, $-q\frac{u''(q)}{u'(q)} < 1$ is required for these results.

as the real demand for deposits at $i^d_{t+1} = 0$. Note that $\hat{z_d} > \bar{z_d}$, and that $\hat{z_d}$ is the amount of deposits required to purchase $q^*$, whereas $q^i_t < q^*$ for $\phi_t d^i_t < \hat{z_d}$. The complete demand schedule for deposits is given by

$$
d^i_t = \begin{cases} \infty \quad \text{if} \quad 1 + i^d_{t+1} > \frac{1+\pi_{t+1}}{\beta}, \\ \in (\frac{\hat{z_d}}{\phi_t}, \infty) \quad \text{if} \quad 1 + i^d_{t+1} = \frac{1+\pi_{t+1}}{\beta}, \\ \text{solution to Equation (5) if} \quad 0 < i^d_{t+1} < \frac{1+\pi_{t+1}}{\beta} - 1, \\ \in (0, \frac{\bar{z_d}}{\phi_t}) \quad \text{if} \quad i^d_{t+1} = 0, \\ 0 \quad \text{if} \quad i^d_{t+1} < 0. \end{cases} \tag{6}
$$

In words, for interest rates above the Fisher rate, buyers hold an infinity of deposits; at the Fisher rate, buyers are indifferent about any real amount of deposits at least equal to $\hat{z_d}$; for interest rates strictly between the Fisher rate and zero, the demand for deposits is given by Equation (5); at a deposit rate of zero, buyers are indifferent about holding any amount of deposits up to $\bar{z_d}$; and for negative deposit rates, buyers do not hold any deposits. Demand for fiat money by inside buyers is then given by

$$
m^i_t = \begin{cases} 0 \quad \text{if} \quad i^d_{t+1} > 0, \\ \frac{\bar{z_d}}{\phi_t} - d^i_t \quad \text{if} \quad i^d_{t+1} = 0, \\ m^o_t \quad \text{if} \quad i^d_{t+1} < 0, \end{cases} \tag{7}
$$

that is, inside buyers hold no $m$ if $i^d_{t+1} > 0$, and they hold $m^o_t$ if $i^d_{t+1} < 0$. If $i^d_{t+1} = 0$, inside buyers are indifferent between deposits and money, but the total real value of liquid assets they hold is pinned down by $\bar{z_d}$. Assume from here on that inside buyers weakly prefer to hold $d$ at $i^d_{t+1} = 0$, so they will only hold $m$ at the zero-lower bound if the supply of real deposits from banks is less than $\bar{z_d}$.

Since sellers have no need for liquid assets, they only hold an asset if it pays the Fisher rate, and they only hold fiat money if $1 + \pi_{t+1} = \beta$.

Because the demand for bonds and loans to entrepreneurs is the same for all types of households, denote the demand for these as $b^h = \eta b^i + (1 - \eta) b^o + b^s$ and $\ell^h = \eta \ell^i + (1 - \eta)\ell^o + \ell^s$. $b^h_t$ and $\ell^h_t$ are

$$
b^h_t = \begin{cases} 0 \quad \text{if} \quad 1 + i^B_{t+1} < \frac{1+\pi_{t+1}}{\beta}, \\ \in (0, \infty) \quad \text{if} \quad 1 + i^B_{t+1} = \frac{1+\pi_{t+1}}{\beta}, \\ \infty \quad \text{if} \quad 1 + i^B_{t+1} > \frac{1+\pi_{t+1}}{\beta}, \end{cases} \tag{8}
$$

$$
\ell^h_t = \begin{cases} 0 \quad \text{if} \quad 1 + i^\ell_{t+1} < \frac{1+\pi_{t+1}}{\beta}, \\ \in (0, \infty) \quad \text{if} \quad 1 + i^\ell_{t+1} = \frac{1+\pi_{t+1}}{\beta}, \\ \infty \quad \text{if} \quad 1 + i^\ell_{t+1} > \frac{1+\pi_{t+1}}{\beta}. \end{cases} \tag{9}
$$

Finally, assume without loss of generality that sellers and outside buyers never hold $d$, and that sellers never hold $m$.[19]

[19] Although these households want to hold these assets for some interest rates, these rates cannot occur in equilibrium.

2.2. *The Entrepreneur's Problem.* Entrepreneurs decide how many loans to demand, given the loan rate and the pledgeability constraint. Entrepreneurs use loans to purchase capital. Since loans are nominal, the amount of capital that can be purchased with a loan $\ell_t^e$ is given by $k_t = \phi_t \ell_t^e$. The entrepreneur's problem is

$$\max_{\ell_t^e} \quad f(\phi_t \ell_t^e) - (1 + i_{t+1}^\ell)\phi_{t+1}\ell_t^e$$
$$\text{s.t.} \quad \chi f(\phi_t \ell_t^e) \geq (1 + i_{t+1}^\ell)\phi_{t+1}\ell_t^e,$$

with the solution given by

$$f'(\phi_t \ell_t^e) = \frac{1 + i_{t+1}^\ell}{1 + \pi_{t+1}} \quad \text{if} \quad \chi f(\phi_t \ell_t^e) > (1 + i_{t+1}^\ell)\phi_{t+1}\ell_t^e$$
$$\chi f(\phi_t \ell_t^e) = (1 + i_{t+1}^\ell)\phi_{t+1}\ell_t^e \quad \text{otherwise.} \tag{10}$$

In both cases, the real amount of loans and the capital invested is increasing in $\pi$ and decreasing in $i_\ell$. This can directly be seen in the unconstrained case. For the constrained case, the proof can be found in Appendix C.2. Define the real demand for loans at the Fisher rate as $\hat{z}_\ell$, and the real demand for loans at $i^\ell = 0$ as $\bar{z}_\ell$. Note that $\hat{z}_\ell < \bar{z}_\ell$. Furthermore, assume throughout the article that $\eta \hat{z}_d > \hat{z}_\ell$, that is, that demand for deposits exceeds demand for loans at the Fisher rate.

2.3. *The Banks' Problem.* Banks decide on the amount of deposits they want to attract and how to invest in the different assets such that their profits are maximized:

$$\max_{d_t, \alpha_t^M, \alpha_t^B} \quad (1 - \alpha_t^M - \alpha_t^B)\phi_{t+1}(1 + i_{t+1}^\ell)d_t^b + \alpha_t^B \phi_{t+1}(1 + i_{t+1}^B)d_t^b + \alpha_t^M \phi_{t+1} d_t^b - \phi_{t+1}(1 + i_{t+1}^d)d_t^b$$
$$\text{s.t.} \quad \alpha_t^M \geq 0$$
$$\alpha_t^B \geq 0$$
$$\alpha_t^B + \alpha_t^M \leq 1,$$

where $\alpha_t^M$ denotes the share of assets held as fiat money, $\alpha_t^B$ denotes the share of assets held as bonds, and thus $(1 - \alpha_t^M - \alpha_t^B)$ denotes the share of assets held as loans to entrepreneurs. The first three terms in the maximization problem capture the return on loans, bonds, and fiat money, respectively, whereas the third term captures the cost of deposits. The constraints ensure that investment in all types of assets is nonnegative. FOCs are given by

$$i_{t+1}^\ell \geq 0,$$
$$i_{t+1}^\ell = i_{t+1}^B,$$
$$1 + i_{t+1}^d = (1 - \alpha_t^M - \alpha_t^B)(1 + i_{t+1}^\ell) + \alpha_t^B(1 + i_{t+1}^B) + \alpha_t^M.$$

The first condition holds with equality for $\alpha_t^M > 0$, that is, if banks hold positive quantities of fiat money.[20] Focusing on interior solutions for deposits, combining the FOCs yields

$$i_{t+1}^\ell = i_{t+1}^B = i_{t+1}^d \geq 0 \quad \text{if} \quad \alpha_t^M = 0; \quad i_{t+1}^\ell = i_{t+1}^B = i_{t+1}^d = 0 \quad \text{if} \quad \alpha_t^M > 0. \tag{11}$$

[20] In principle, the same is true for the second condition: $i_{t+1}^\ell \geq i_{t+1}^B$, with equality if $\alpha_t^B > 0$. However, since households only hold $b$ and $\ell$ at the Fisher rate, the condition holds with equality in equilibrium even if $\alpha_t^B = 0$.

At these interest rates, banks are willing to take any amount of deposits, so

$$d_t^b \in (0, \infty). \tag{12}$$

For $i_{t+1}^\ell > i_{t+1}^d$ ($i_{t+1}^\ell < i_{t+1}^d$), banks take an infinite amount of deposits (zero deposits), but this cannot be an equilibrium outcome. Thus, Equation (11) shows that (i) all interest rates have to be equal in this economy, even though the assets have different liquidity properties; (ii) unlike households, banks are willing to hold illiquid assets if they pay less than the Fisher rate; (iii) there is an endogenous zero-lower bound in this economy on interest rates, which arises because of the existence of fiat money; (iv) banks are only willing to hold fiat money (excess reserves) at the zero-lower bound. Equation (11) also shows that unsurprisingly, banks earn zero profits in equilibrium.

## 3. EQUILIBRIUM

DEFINITION 1. An equilibrium is a sequence of prices $i_t^\ell, i_t^B, i_t^d, \phi_t$, quantities $m_t^o, d_t^b, d_t^i, \ell_t^e, \ell_t^h, b_t^h$, and ratios $\alpha_t^M$, and $\alpha_t^B$ that simultaneously solve the Equations (4), (6), (7), (8), (9), (10), (11), (12) and the market clearing conditions

$$\phi_t M_t = \eta \phi_t m_t^i + (1-\eta)\phi_t m_t^o + \alpha_t^M \phi_t d_t^b, \tag{13}$$

$$\phi_t d_t \equiv \phi_t d_t^b = \eta \phi_t d_t^i, \tag{14}$$

$$\phi_t \ell_t \equiv \phi_t \ell_t^e = \phi_t \ell_t^h + (1 - \alpha^M - \alpha_t^B)\phi_t d_t^b, \tag{15}$$

$$\phi_t B_t - \phi_t b_t^M = \phi_t b_t^h + \alpha_t^B \phi_t d_t^b; \tag{16}$$

$\forall t$, given initial values.

Definition 1 gives general conditions for an equilibrium in this economy, but much of what follows is going to be focused on steady-state equilibria. In steady state, additional to the definition above, all real quantities are constant, so $\phi_t a_t = \phi_{t+1} a_{t+1}$ for $a = \{m, d, \ell, b\}$ $\forall t$. To simplify notation, I denote real quantities of asset $a^j$ as $z_a^j$; that is, $\phi_t d_t^b \equiv z_d^b$, $\phi_t \ell_t \equiv z_\ell$, and so on. Furthermore, making use of Equation (11), I use $i \equiv i^\ell = i^B = i^d$ to denote the equilibrium nominal interest rate.

PROPOSITION 1. *There is a unique monetary steady-state equilibrium in this economy. Steady-state inflation is $1 + \pi = \gamma^B = \gamma^M \geq \beta$. Under a nominal debt target, the fiscal authority's target $\gamma^B$ determines steady-state inflation; under a real debt target, the monetary authority's choice of $\gamma^M$ determines steady-state inflation.*

The proof of Proposition 1 can be found in Appendix C.3. Since all nominal variables need to grow at the same, constant rate for a steady-state equilibrium to exist, $1 + \pi = \gamma^B = \gamma^M$ must hold. Furthermore, for a steady-state equilibrium to exist, inflation must be weakly higher than the discount factor $\beta$. Under a nominal debt target, $\gamma^B$ thus determines steady-state inflation. Under a real debt target however, the monetary authority can control the inflation rate by choosing $\gamma^M$, and the fiscal authority will then set $\gamma^B = \gamma^M$ in order to keep real debt constant at $\mathcal{B}$.

Although Proposition 1 shows that a unique monetary steady-state exists, due to the bounds on interest rates at zero and at the Fisher interest rate, three steady-state equilibrium cases with differing properties can occur in this economy.[21]

PROPOSITION 2. *If* $\eta \hat{z}_d - \hat{z}_\ell \leq \phi_t(B_t - b_t^M)$, $i = \frac{1+\pi}{\beta} - 1$*; denote this as equilibrium case I. If* $\eta \hat{z}_d - \hat{z}_\ell > \phi_t(B_t - b_t^M) > \eta \bar{z}_d - \bar{z}_\ell$, $i \in (0, \frac{1+\pi}{\beta} - 1)$*; denote this as equilibrium case II. If* $\phi_t(B_t - b_t^M) \leq \eta \bar{z}_d - \bar{z}_\ell$, $i = 0$*, denote this as equilibrium case III. Under a nominal debt target,* $\phi_t(B_t - b_t^M) = (1-\eta) z_m^o \frac{B_t - b_t^M}{M_t}$*; under a real debt target,* $\phi_t(B_t - b_t^M) = \mathcal{B} - (1-\eta) z_m^o \frac{b_t^M}{M_t}$.

Proposition 2 defines the equilibrium cases according to the equilibrium nominal rate $i$ and discusses the conditions for each case to occur.[22] Proposition 2 follows from the proof of Proposition 1. The proposition shows that under a nominal debt target, the policy variable $(B_t - b_t^M)/M_t$ determines the equilibrium case the economy falls into, whereas under a real debt target, the policy variables $b_t^M/M_t$ and $\pi$ (through its effect on $z_m^o$, $\bar{z}_d$, and $\bar{z}_\ell$) determine the equilibrium case.[23] For given levels of $(B_t - b_t^M)/M_t$ or $\pi$ and $b_t^M/M_t$, respectively, the equilibrium case is determined by: the functional form of $u(q)$, which determines $\hat{z}_d$ and $\bar{z}_d$ through Equation (5); the functional form of $f(k)$ and $\chi$, since they determine $\hat{z}_\ell$ and $\bar{z}_\ell$ through Equation (10); and the fiscal authority's target $\gamma^B$ or $\mathcal{B}$, respectively. The remainder of this section discusses the intuition behind the equilibrium cases; the remainder of the article then discusses how exactly the monetary authority can affect the policy variables under the two targets of the fiscal authority.

3.1. *Equilibrium Case I.* In equilibrium case I, interest rates equal the Fisher rate, so $i = (1+\pi)/\beta - 1$. This implies that it is costless to hold assets from one period to the next, so households are willing to hold any amount of them. This also applies to deposits, so buyers in inside meetings bring enough funds to the DM to purchase $q^*$. This equilibrium case occurs if $\eta \hat{z}_d - \hat{z}_\ell \leq \phi_t(B_t - b_t^M)$, that is, if the difference between the demand for deposits and the demand for loans at the Fisher rate is less or equal than the supply of real government bonds. This implies that some bonds and/or loans to entrepreneurs might be held by households. Banks are essential in this case because their ability to create liquid assets by investing in illiquid assets reduces the cost of holding liquid assets for buyers in inside meetings and thereby increases the quantities consumed in the DM.

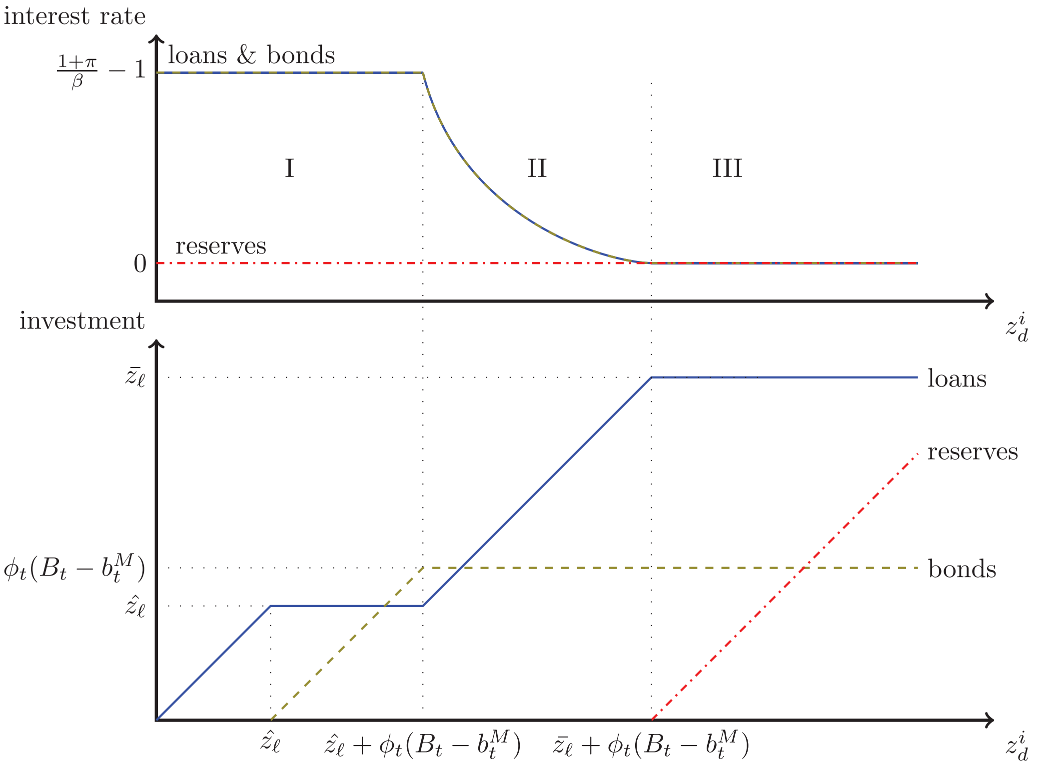
3.2. *Equilibrium Case II.* In equilibrium case II, interest rates are strictly between zero and the Fisher rate, so $0 < i < (1+\pi)/\beta - 1$. This case occurs if $\eta \bar{z}_d - \bar{z}_\ell < \phi_t(B_t - b_t^M) < \eta \hat{z}_d - \hat{z}_\ell$, that is, if the real bond supply is larger than the difference between demand for deposits and demand for loans at the zero-lower bound, but smaller than the difference between demand for deposits and demand for loans at the Fisher rate. This implies that there are not enough investment opportunities at the Fisher rate to satisfy the demand for deposits at that rate. Furthermore, $b^H = \ell^H = 0$: Illiquid assets are scarce and pay less than the Fisher rate, so households are not willing to hold them. In this equilibrium case, banks are essential because their existence increases the amount of DM goods traded and lowers the funding cost of entrepreneurs, which increases investment.

3.3. *Equilibrium Case III.* In equilibrium case III, interest rates are zero, so $i = 0$. This case occurs if $\eta \bar{z}_d - \bar{z}_\ell \geq \phi_t(B_t - b_t^M)$, that is, if the difference between the demand for de-

[21] Note that there also exists a nonmonetary steady-state equilibrium with $\phi_t = 0\ \forall t$. Note also that uniqueness here is defined in terms of real allocations, as there is some indeterminacy in nominal variables in some equilibrium cases (e.g., between $d_t^i$ and $m_t^i$ if $i = 0$), but those lead to the same real allocations.

[22] For $1 + \pi = \beta$, the three equilibrium cases coincide, that is, the Fisher rate equals a nominal rate of zero, and thus $\hat{z}_d = \bar{z}_d$ whereas $\hat{z}_\ell = \bar{z}_\ell$. For the remainder of the article, assume that $1 + \pi > \beta$ unless otherwise stated.

[23] Under a real debt target, a necessary condition for equilibrium case I to exist is $\eta \hat{z}_d - \hat{z}_\ell < \mathcal{B}$. Suppose for the remainder of the article that this holds.




The first panel shows the nominal interest rate on the three different assets as a function of total investment, which equals deposits received. The second panel shows the amounts invested in the separate assets as a function of total investment.

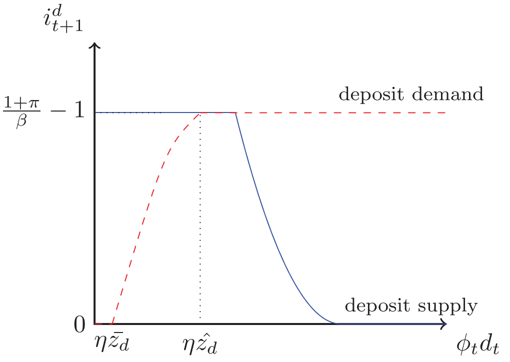
FIGURE 1

BANKS' INVESTMENT DECISION

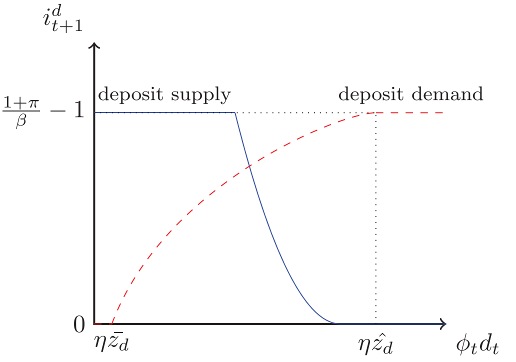
posits and the demand for loans is (weakly) larger than the supply of bonds at a nominal rate of zero. In this case, banks may hold reserves (i.e., fiat money) in order to satisfy deposit demand. At $i^d = 0$, $q^i = q^o$, so the existence of banks does not improve outcomes in the DM. However, banks are still essential because as in case II, they lend to entrepreneurs at lower rates than households, which increases capital investment.

3.4. *Discussion.* Figure 1 shows the banks' investment decision, taking total deposits as given. The upper panel shows the nominal rate on the three different assets as a function of total investment, whereas the lower panel shows the banks' investment in each asset. Note that no asset can pay a higher return than the Fisher rate. If banks receive few deposits, loans and bonds pay the Fisher rate, banks are indifferent between the two and strictly prefer holding these instead of reserves.[24] In this region, the economy is in equilibrium case I. Marginal investors in loans and bonds are households, so these assets have to be priced at the Fisher rate in this region. When total deposits exceed $\hat{z}_\ell + \phi_t(B_t - b_t^M)$, banks hold more assets than there are investment opportunities at the Fisher rate, so the economy moves into equilibrium case II. Since demand is higher than supply, the interest rate has to fall. At lower rates, entrepreneurs demand more loans, so banks can invest more in region II by making more loans—but the more loans they make, the lower is the interest rate they earn on loans. As banks must be indifferent between bonds and loans, $i^B$ also decreases with loans made.

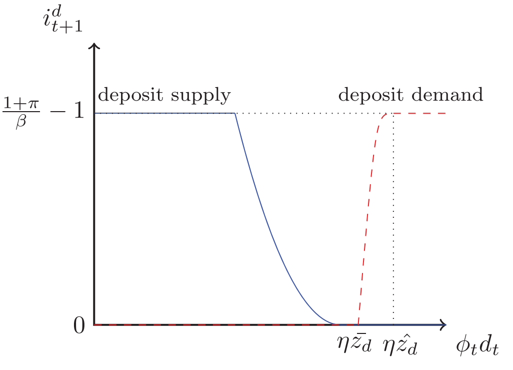
[24] In the figure, banks first invest only in loans before they start investing in bonds in region I. This is done for visual clarity, and banks might just as well invest in bonds first before investing in loans.




(a) Equilibrium case I.




(b) Equilibrium case II.




(c) Equilibrium case III.

FIGURE 2

THE THREE EQUILIBRIUM CASES

When total deposits exceed $\bar{z}_\ell + \phi_t(B_t - b_t^M)$, the economy is in equilibrium case III. Increasing loans further would push $i^\ell$ into negative territory, so instead banks hold fiat money if they invest more. In steady state, the return on fiat money is determined by the money growth rate $\gamma^M$, so the return on money is not affected by the amount of fiat money held by banks.

From this, the banks' deposit supply curve as a function of $i^d$ can be derived. Since $i^d = i^B = i^\ell$ in equilibrium, the upper envelope of the upper panel in Figure 1 gives the supply of deposits by banks. The demand for deposits by households is given by Equation (6).

Figure 2 shows examples of deposit demand and supply curves for the three cases. The dashed red line shows the demand for deposits by buyers, and the solid blue line captures the supply of deposits by banks. Depending on the segment of the supply curve on which the two curves intersect, the prevalent equilibrium case is determined.[25]

Before moving on, I want to highlight the important features of the model, which are (i) the banks' ability to credibly promise to repay loans allows them to perform liquidity transformation; (ii) due to the banks' ability to perform liquidity transformation, illiquid assets may pay a liquidity premium; (iii) banks are essential because they lend to entrepreneurs at weakly lower rates than households and issue liquid assets that pay weakly higher interest rates than

[25] Note that in equilibrium case III, the amount of deposits and fiat money held by households is not pinned down exactly, but their sum is. Deposits are given by $\phi_t d_t^i \in (\bar{z}_\ell + \phi_t(B_t - b_t^M), \bar{z}_d)$, and fiat money holdings are given by $\phi_t m_t^i = \bar{z}_d - \phi_t d_t^i$. Similarly, banks' fiat money holdings are given by $\phi_t \alpha_t^M d_t = \phi_t d_t^i - (\bar{z}_\ell + \phi_t(B_t - b_t^M))$. This indeterminacy has no effect on real allocations however, as the total amount of fiat money held in the economy, and more importantly the total amount of liquid assets held by buyers in inside meetings, is uniquely determined.

fiat money, with at least one of them being strict if $1+\pi_t > \beta$. In Appendix A.1, I also provide a brief discussion of the banks' ability to create money.

## 4. WELFARE AND OPTIMAL POLICY

To study welfare, three variables need to be assessed: $k$, $q^i$, and $q^o$; that is, steady-state capital investment and consumption in both types of DM meetings. DM consumption is optimal if $q^o = q^i = q^*$. Equation (4) shows that $q^o = q^*$ if and only if $1+\pi = \beta$. Thus, the quantity consumed in outside meetings is at the first best at the Friedman rule, and it is independent of the nominal interest rates on deposits, loans, or bonds. For $1+\pi > \beta$, $q^o$ and thus welfare is decreasing in inflation. Equation (6) shows $q^i = q^*$ if $1+i^d = (1+\pi)/\beta$, that is, if the interest rate on deposits equals the Fisher interest rate. For $1+i^d < (1+\pi)/\beta$, $q^i$ and thus welfare is increasing in the nominal deposit interest rate for a given inflation rate, and decreasing in the inflation rate for a given nominal deposit interest rate. Regarding capital, note that in all three equilibrium cases, the pledgeability constraint for entrepreneurs can be binding or slack, depending on $\chi$. Whether the constraint is binding affects steady-state $k$ and thus also welfare. With a nonbinding pledgeability constraint, Equation (10) shows that $k = k^*$ if $1+i^\ell = (1+\pi)/\beta$, that is, if the loan rate equals the Fisher interest rate. With a binding pledgeability constraint, $\hat{z_\ell} < k^*$, so $1+i^\ell < (1+\pi)/\beta$ is required to achieve $k^*$. If $\bar{z_\ell} > k^*$, some loan rate $i^\ell > 0$ delivers the first-best capital investment, whereas if $\bar{z_\ell} < k^*$, capital investment is below first best even at $i^\ell = 0$. This is more likely for low values of $\chi$ and $1+\pi$.[26]

From this, welfare is a function of $\pi$ and $i^d$. Since $k$ and $q^i$ jointly depend on $\pi$ and $i^d$, it is helpful to define the real interest rate in the economy as

$$1+r = \frac{1+i^d}{1+\pi} = \frac{1+i^\ell}{1+\pi} = \frac{1+i^B}{1+\pi}, \tag{17}$$

with $k$ decreasing and $q^i$ increasing in $r$. Next, I discuss the welfare-maximizing real interest rate in the economy with and without a binding pledgeability constraint for entrepreneurs. I then discuss whether the monetary authority is able to achieve the welfare-maximizing real interest rate under either target of the fiscal authority.

PROPOSITION 3. *If the pledgeability constraint is nonbinding, the Friedman rule* $(1+\pi = \beta)$ *allows to achieve the first best. For a given* $1+\pi > \beta$*, the Fisher interest rate* $(1+r = 1/\beta)$ *maximizes welfare. If the pledgeability constraint is binding, some real interest rate* $1+r^* < 1/\beta$ *(while* $i = 0$*) maximizes welfare.*

With a nonbinding pledgeability constraint, the Fisher interest rate simultaneously delivers $k = k^*$ and $q^i = q^*$. Furthermore, only the Friedman rule allows $q^o = q^*$. Since at the Friedman rule, the Fisher interest rate is satisfied automatically, it delivers first-best outcomes for all parameters.[27] Deviations from both the Friedman rule and the Fisher interest rate are required to maximize welfare with a binding pledgeability constraint.[28] To see this, first note that there is no equilibrium case which allows to achieve the first-best outcome in all markets when the pledgeability constraint is binding, as the loan rate that delivers $k^*$ is different from the deposit rate that delivers $q^*$, but loan and deposit rates are equal in equilibrium. At

[26] Note that this analysis focuses on steady-state welfare, but this should not be a concern because due to capital fully depreciating and the quasi-linear utility of agents, the economy can get to steady-state immediately after policy changes, so there is no trade-off between higher steady-state welfare and lower welfare during the transition.

[27] This is a standard result in many monetary models. For an example with investment, see Lagos and Rocheteau (2008).

[28] Since the Fisher interest rate is the only feasible interest rate at the Friedman rule, policymakers cannot deviate from the Fisher interest rate unless they also deviate from the Friedman rule.

the Fisher interest rate, $q^i = q^*$ is achieved, but $k = \hat{z}_\ell < k^*$. By the envelope theorem, a decrease in the real interest rate therefore strictly increases welfare. Since deviations from the Friedman rule are necessary to implement $1 + r < 1/\beta$, but increasing $\pi$ reduces $q^o$, welfare is maximized at $r^*$ with $i = 0$.[29] This implies that equilibrium case I is never optimal with a binding pledgeability constraint. Instead, the zero-lower bound (equilibrium case III) is optimal if steady-state inflation is set correctly. The exact value of $r^*$ depends on parameters, and on welfare weights on DM consumption and capital investment, with $r^*$ increasing with $\chi$ and decreasing with higher welfare weights on capital investment. Next, we turn to the monetary authority's ability to affect steady-state $i$, $r$, and $\pi$ under either target of the fiscal authority.

PROPOSITION 4. *Under a nominal debt target, the monetary authority may choose any $i \in [0, (1+\pi)/\beta - 1]$ by varying $(B_t - b_t^M)/M_t$ through open-market operations. In equilibrium cases I and III, a marginal change in $(B_t - b_t^M)/M_t$ has no effect on real allocations.*

As already established, steady-state inflation is $1 + \pi = \gamma^B$, and is thus fully determined by the fiscal authority's nominal debt target. However, as shown by Proposition 2 and the proof to Proposition 1, varying $(B_t - b_t^M)/M_t$ allows the monetary authority to choose any $i \in [0, (1+\pi)/\beta - 1]$.[30] By varying $i$, the monetary authority may in turn choose any $1 + r \in [1/(1+\pi), 1/\beta]$. Since $B_t$ is exogenously given under a nominal debt target through the initial debt level $B_0$ and the fiscal authority's target $\gamma^B$, the monetary authority can increase (decrease) $(B_t - b_t^M)/M_t$ through an open-market purchase (sale) of government bonds, as this varies $M_t$ and $b_t^M$. Note that this is possible even if $b_t^M = M_t$, that is, if the monetary authority issues all money through bond purchases. Thus, the monetary authority may vary the nominal rate through open-market operations while the economy is in equilibrium case II. Once $(B_t - b_t^M)/M_t$ becomes large (small) enough to get the economy into case I (case III), a further increase (decrease) has no effect on nominal interest rates, which, together with steady-state inflation resulting from the fiscal authority's target, implies that it also has no effect on real allocations. This is a standard result in related papers; see, for example, Williamson (2012) or Rocheteau et al. (2018). A short discussion of the intuition is provided in Appendix A.2.

Combined with the results from Proposition 3, Proposition 4 shows that the monetary authority should set $i = (1+\pi)/\beta - 1$ if the pledgeability constraint is loose, and it should set $1 + i = (1 + r^*)(1 + \pi)$ if the pledgeability constraint is binding. If $1 + r^* < 1/\gamma^B$, the monetary authority cannot implement $r^*$ through open-market operations under a nominal debt target.

PROPOSITION 5. *Under a real debt target, the monetary authority may choose any $\pi \geq \beta$ and any $i \in [0, (1+\pi)/\beta - 1]$ by varying $\gamma^M$ and $b_t^M/M_t$. If money can only be issued through bond purchases, which implies $b_t^M = M_t\ \forall t$, the monetary authority may only choose $\pi$ through varying $\gamma^M$, which then determines $i(\pi)$ and $r(\pi)$. In particular, the monetary authority may set $\pi = \beta - 1$, which implies $r = 1/\beta - 1$, or it may set $\pi \geq \underline{\pi} > \beta - 1$, which determines $r(\pi)$, where $r(\pi) = 1/\beta - 1$ for $\pi \geq \bar{\pi} > \underline{\pi}$, $r(\underline{\pi}) = 1/(1 + \underline{\pi}) - 1$, and $\partial r/\partial \pi > 0$ for $\pi \in (\underline{\pi}, \bar{\pi})$.*

The proof to this proposition can be found in Appendix C.4. The proposition shows that under a real debt target, the monetary authority may only independently choose steady-state inflation and the real interest rate if it can issue some fraction of newly printed money through lump-sum transfers, that is, as helicopter money, each period. In this case and given the results from Proposition 3, the monetary authority should run the Friedman rule if the pledgeability

[29] If $r^*$ were achieved with $i > 0$ instead, $k$ and $q^i$ would be exactly the same, but $q^o$ would be strictly lower.

[30] This is true as long as $\gamma^B$ is not too high. Remember that for $i = 0$ to prevail, $(1 - \eta) z_m^o (B_t - b_t^M)/M_t \leq \eta \bar{z}_d - \bar{z}_\ell$ is required. Since the left-hand side (LHS) of this condition is bounded below at zero, implementing $i = 0$ is possible only if $\eta \bar{z}_d \geq \bar{z}_\ell$ holds. Since $\eta \bar{z}_d - \bar{z}_\ell$ is decreasing in steady-state inflation, this condition will be violated at high enough $\gamma^B$. However, since the zero-lower bound has been an empirical reality in recent years, assume for the rest of the article that the fiscal authority's target is such that $\eta \bar{z}_d \geq \bar{z}_\ell$ holds.

TABLE 1
WELFARE-MAXIMIZING POLICIES IN ALL CASES

| | Pledgeability Constraint Loose | Pledgeability Constraint Binding |
|---|---|---|
| Nominal debt target | Set $1+i=(1+\pi)/\beta$ | Set $1+i=(1+r^*)(1+\pi)$ |
| Real debt target, hel. money | Set $1+\pi=\beta$ and $i=0$ | Set $1+\pi=1/(1+r^*)$ and $i=0$ |
| Real debt target, no hel. money | Set $1+\pi=\beta$ | Set $\pi$ s.t. $r(\pi)=r^*$ |

constraint is loose, while it should set inflation such that $1+\pi=1/(1+r^*)$ holds and simultaneously set the nominal interest rate to zero if the pledgeability constraint is binding.

Without helicopter money, the monetary authority is constrained under a real debt target. While it has (some) control over $\pi$, it cannot set $i$ independently of $\pi$, implying that the choice of $\pi$ also determines $r(\pi)$. Since $r(\pi)$ is increasing in $\pi$ and since the choice over $\pi$ is bounded from below at $\underline{\pi}$, there is a lower bar on the real interest rate implementable. This lower bar is reached at the zero-lower bound. The intuition behind this lower bar on $\pi$ is that if money is only issued via bond purchases under a real debt target, an increase in the money supply has two effects: On the one hand, it makes money less attractive through a usual quantity theory of money argument, but on the other hand, it also reduces the quantity of bonds available for banks to hold. At the zero-lower bound, these two effects exactly offset each other. The reason for this is that money is a perfect substitute for bonds at $i=0$, so if the monetary authority increases the money supply, banks react by holding all the additional money as reserves. Thus, the monetary authority loses control over the inflation rate at the zero-lower bound. This in turn implies that—just as under a nominal debt target—there is a lower bar on the real interest rate which the monetary authority may implement.[31]

Without helicopter money, the monetary authority is still able to implement the Friedman rule, so it should do so if the pledgeability constraint is loose. If the pledgeability constraint is binding, the welfare-maximizing policy is to set $r(\pi)=r^*$. If $r^*<r(\underline{\pi})$, it is not possible for the monetary authority to set the real rate low enough to maximize welfare without using helicopter money.

Table 1 summarizes how policy should be set to maximize welfare with or without a binding pledgeability constraint, and under the two targets of the fiscal authority.

This section showed that it may be welfare-maximizing to set $1+r<1/\beta$ if the entrepreneur's pledgeability constraint is binding. However, both under a real and a nominal debt target, the monetary authority is constrained by a lower bound on the real interest rate when it uses conventional monetary policy. The analysis also showed that under a real debt target, the constraint can be relaxed by using helicopter money.

## 5. CONTROLLING INFLATION UNDER A NOMINAL DEBT TARGET

The previous section showed that under a nominal debt target, the monetary authority is unable to lower the real interest rate at the zero-lower bound, even though it may be optimal to do so. While the steady-state inflation rate is determined by $\gamma^B$ under a nominal debt target, the question analyzed in this section is whether the monetary authority is able to increase inflation at the zero-lower bound at least temporarily by setting $\gamma^M>\gamma^B$.

To answer this question, I will first analyze what happens if the monetary authority sets $\gamma_t^M>\gamma_t^B$, and issues the newly printed fiat money by purchasing bonds. Second, I will analyze what happens if the monetary authority instead does so by issuing the newly printed

[31] It may seem counterintuitive that a lower bound on $\pi$ implies a lower bound on $1+r=(1+i)/(1+\pi)$, but this is the case because $i$ is also an (increasing) function of $\pi$ under a real debt target, and in fact increases more than one to one with $\pi$. Since the lowest $r$ is attained at the zero-lower bound, to reduce $r$ further the monetary authority would want to increase $\pi$ while staying at the zero-lower bound—but since $i$ and $r$ are both increasing in $\pi$, this is not possible.

money through lump-sum transfers to agents. Note that with bond purchases, $\gamma_t^M > \gamma_t^B$ cannot be held up $\forall t$, as at some point $b_t^M = B_t$, and from then on the monetary authority has to revert back to $\gamma_t^M = \gamma_t^B$. Therefore, I assume $\gamma_t^M > \gamma_t^B$ temporarily, for $n \geq 1$ periods. To be specific, starting in a steady-state equilibrium in period $t$, the monetary authority announces that it will set $\gamma_{t+j}^M > \gamma^B$ for $j \in \{1, n\}$, and $\gamma_{t+j}^M = \gamma^B$ for $j > n$, with $\gamma^B$ being the steady-state bond growth rate.[32] Furthermore, I restrict attention to the economy being in equilibrium case III in steady state, as this is the only situation where an increase in $\pi_{t+1}$ can be welfare-improving.

5.1. *Setting $\gamma^M > \gamma^B$ through Bond Purchases.* Assume the monetary authority sets $\gamma_{t+j}^M > \gamma^B$ for $j \in \{1, n\}$, and issues the additional newly printed currency through purchases of government bonds.

PROPOSITION 6. *At the zero-lower bound, setting $\gamma_{t+j}^M > \gamma^B$ for $j \in \{1, n\}$ through bond purchases has no effect on $\pi_{t+j}$. Instead, inflation remains at the steady-state level, that is, $1 + \pi_{t+j} = \gamma^B$.*

The proof to Proposition 6 can be found in Appendix C.5. The intuition behind it is as follows: At the zero-lower bound, fiat money and bonds are perfect substitutes for banks, and all available bonds are held by banks. Using bond purchases to increase the fiat money growth rate implies that $M_{t+j}$ increases by exactly the same amount as $b_{t+j}^M$ increases, which in turn means that the amount of publicly available bonds $B_{t+j} - b_{t+j}^M$ decreases by the same amount. Thus, the sum of fiat money and publicly available bonds remains constant. The banks absorb all the newly issued money to replace the bonds purchased by the central bank, and therefore the amount of money in the goods market remains unaltered, and, consequently, the newly issued fiat money has no inflationary effect. To put it differently, even though the supply of fiat money increases, the demand for fiat money increases by exactly the same amount, thus offsetting any potential real effects of this policy.

This result mirrors the one from Proposition 5 under a real debt target. Together, these results show that under both targets of the fiscal authority, the quantity theory of money breaks down at the zero-lower bound if the monetary authority varies the money growth rate through bond purchases: Although the money growth rate increases, only the banks' reserve holdings increase, but the inflation rate remains constant. The result rationalizes the empirical observation that after the financial crisis, central banks increased the monetary base by large amounts, but inflation remained constant. Instead, the growth in the monetary base was mirrored by a growth in excess reserves held by banks. The result can also be seen as a version of Wallace (1981)'s irrelevance result for open-market operations: Although open-market operations are generally not irrelevant here, they are at the zero-lower bound.

5.2. *Setting $\gamma^M > \gamma^B$ through Lump-Sum Transfers.* Now, assume the monetary authority sets $\gamma_{t+j}^M > \gamma^B$ for $j \in \{1, n\}$, and issues the additional newly printed currency through lump-sum transfers to households, that is, as helicopter money.

PROPOSITION 7. *At the zero-lower bound, setting $\gamma_{t+j}^M > \gamma^B$ for $j \in \{1, n\}$ through lump-sum transfers to households leads to an increase in $\pi_{t+j}$. The increase in inflation leads to an increase in $k_{t+j}$ and decreases in $q_{t+j}^o$ and $q_{t+j}^i$.*

[32] It is important that the monetary authority announces the policy change prior to implementing it, since for investment it is expected inflation that matters, not past inflation. This is similar to the effects in Gu et al. (2019) or Berentsen and Waller (2011, 2015).

The proof to this proposition can be found in Appendix C.6. Proposition 7 states that helicopter money enables the monetary authority to increase inflation at the zero-lower bound.[33] The intuition behind this result is that with lump-sum transfers to agents, the sum of fiat money and publicly available bonds increases with the increase in the fiat money growth rate. Since in this case, the banks have no need to hold additional fiat money, the newly printed fiat money reaches the goods market, and this leads to an increase in prices and inflation. Once inflation increases, households want to hold less deposits and thus consume less in the DM. Meanwhile, banks are still willing to lend to entrepreneurs at a nominal interest rate of zero, but due to the increase in inflation this translates to a lower real interest rate, so more loans are made in equilibrium and capital investment increases. This shows that helicopter money can be used to increase investment at the zero-lower bound under a nominal debt target.[34]

## 6. OTHER POLICIES

The analysis so far has shown that the zero-lower bound constrains the monetary authority, but that this constraint can be relaxed through the use of helicopter money. This section considers some other approaches that have been discussed to get around the constraint imposed by the zero-lower bound. The approaches considered here are: QE, both in the form of the central bank buying loans to entrepreneurs from banks, or by lending directly to entrepreneurs; paying interest on reserves, both positive and negative; and finally, eliminating cash altogether, as suggested by Rogoff (2017).

6.1. *Quantitative Easing.* Instead of purchasing government bonds, the monetary authority may also purchase loans to entrepreneurs. As with bonds, assume that the monetary authority rolls over all assets it purchases. Denote the amount of loans held by the monetary authority at $t$ as $\ell_t^M$. A purchase of private loans by the central bank is sometimes called QE.[35] With QE, the market clearing condition for loans is $\ell_t = \ell_t^M + \ell_t^h + (1 - \alpha^M - \alpha^B)d_t^b$.

PROPOSITION 8. *With QE, the policy choices of the monetary authority remain exactly the same as with conventional monetary policy (i.e., open-market operations and variations in the money growth rate $\gamma^M$) under either target of the fiscal authority.*

The proof to this proposition can be found in Appendix C.7. The intuition behind this result is as follows: Under a nominal debt target, the monetary authority is already able to affect the real quantity of assets available for banks to invest in directly through open-market operations; while QE increases the universe of assets the monetary authority may purchase, this does not address the issue that the nominal rate cannot be lowered further at the zero-lower

[33] The proposition discusses a temporary increase in inflation, but since it is possible to set $\gamma^M > \gamma^B$ forever by distributing the newly printed money via lump-sum transfers, the monetary authority can also use this policy to permanently increase the inflation rate.

[34] It is possible that a central bank is restricted by law from making lump-sum transfers to agents. Whereas lump-sum transfers are the most straightforward way to implement helicopter money in this model, there are other methods that have the same effect. What is needed in general is that the fiat money reaches the CM goods market, and that the quantity of outstanding bonds $B_t - b_t^M$ is unaffected. Specifically, the following methods have the same effect as a lump-sum transfer to agents: The monetary authority could buy CM goods with the newly printed fiat money and then consume these goods. The monetary authority could also transfer either the newly printed fiat money or goods acquired with that fiat money to the fiscal authority. Then, if the fiscal authority either increases spending $g_t$ or lowers taxes $\tau_t$ as a reaction to this transfer from the monetary authority, the policy tool still has the same effect as a lump-sum transfer to agents. The tool does not work, however, if the fiscal authority instead reduces its debt as a reaction to the transfer, because then helicopter money essentially becomes equivalent to a purchase of government bonds by the monetary authority.

[35] Note that in practice, the term QE captures a variety of policies that involve the purchase of large amounts of assets by the central bank, and in particular assets that the central bank does not purchase in "normal times." Besides private debt purchases as discussed here, the purchase of long-term government securities is another common form of QE. Williamson (2016) discusses such a policy.

bound. Under a real debt target and without helicopter money, the monetary authority is constrained because if it purchases assets by issuing more money, banks increase the demand for money and thus the inflation rate remains constant. This issue remains even if the monetary authority purchases loans to entrepreneurs. Thus, QE, at least in the form modeled here, is unable to increase bank lending at the zero-lower bound.[36]

6.2. *Direct Loans to Entrepreneurs.* Suppose the economy is at the zero-lower bound, with a binding borrowing constraint and $k < k^*$. There are multiple ways how the government can offer direct lending to entrepreneurs: Consider first a policy of offering loans at $i^{\ell,g} < 0$. In this case, entrepreneurs strictly prefer borrowing from the government, and by setting $i^{\ell,g}$ according to $\chi f(k^*) = k^*(1 + i^{\ell,g})/(1 + \pi)$, the government can achieve $k^*$. The loans will be repaid in full, since the borrowing constraint is respected. With this policy, the government will take over the entire lending market. Banks replace the loans on their balance sheet with additional reserves, and DM consumption remains unchanged. Alternatively, the government can offer additional loans $\ell^g$ at the market rate, so $i^{\ell,g} = 0$. Entrepreneurs are willing to take additional loans at this rate, but banks are not willing to extend these due to the binding borrowing constraint. The effect of this policy depends on whether government loans are considered senior to bank loans in the case of default. Suppose first that this is the case. Then, total real lending will remain unchanged at $\bar{z}_\ell$. Remember that we assumed loans received by an entrepreneur are public knowledge. Then, bank loans will be $\phi_t \ell_t^b = \bar{z}_\ell - \phi_t \ell_t^g$, as banks understand that entrepreneurs will default on any debt beyond $\bar{z}_\ell$, and given that government loans are senior, the banks will be suffering the losses. Thus, this policy simply crowds out bank lending and does not affect capital investment. Now, suppose instead that bank loans are senior. Then, banks are willing to extend $\phi_t \ell_t^b = \bar{z}_\ell$, so any additional loans made by the government will increase capital investment. Thus, by choosing $\phi_t \ell_t^g = k^* - \bar{z}_\ell$, the government can achieve $k^*$ with this policy, while DM consumption remains unchanged. However, entrepreneurs will default on all loans made by the government. This shows that direct lending allows to achieve $k^*$, but the question is how to finance it, as both policies essentially require a subsidy to entrepreneurs: Even though entrepreneurs do not default in the first case, setting $i^{\ell,g} < 0$ also leads to losses for the government, as it is making loans at lower rates than its own refinancing rate. The fiscal authority can finance these losses through adjustments in the lump-sum tax, so if it is willing to run these policies, it can do so. In fact, these policies are a more efficient way to increase welfare than increasing the inflation rate, as the latter lowers DM consumption. If instead the monetary authority runs either of these policies, it will make a loss, since central bank profits are zero at the zero-lower bound. To recoup the loss, the monetary authority is reliant on transfers from the fiscal authority, so in essence, the monetary authority cannot run such policies without fiscal backing. To conclude, direct loans are an efficient way to increase capital investment, but they require fiscal backing. In addition, they lead to redistribution from households to entrepreneurs. While this has no welfare effect in the model, it might be an issue in reality.

6.3. *Interest Rates on Reserves.* Next, consider interest on reserves, both positive and negative.[37]

To model interest on reserves, assume the monetary authority pays $i^R$ on banks' fiat money holdings, whereas households earn no interest on fiat money even if $i^R \neq 0$.[38] With interest on

[36] Williamson (2012) also shows that QE is ineffective at the zero-lower bound. In his article, QE is also not welfare-improving in other cases, whereas it may improve welfare in equilibrium case II here.

[37] In the United States, the Federal Reserve started to pay interest on excess reserves (IOER) in 2008. In 2016, the Fed started to increase the IOER, which was constant at 0.25% before for a long time. Some central banks in Europe raised negative rates on reserves after the financial crisis of 2007–9.

[38] While reserves are not modeled as a separate asset, this approach captures an essential property of reserves—that they can only be held by banks. It is also not unrealistic to assume the monetary authority is able to observe fiat money holdings of banks, whereas it cannot do so for households. This implies that banks cannot avoid to pay nega-

reserves, the banks' problem is

$$\max_{d_t,\alpha_t^M,\alpha_t^B} \quad (1-\alpha_t^M-\alpha_t^B)\phi_{t+1}(1+i_{t+1}^{\ell})d_t^b + \alpha_t^B\phi_{t+1}(1+i_{t+1}^B)d_t^b + \alpha_t^M\phi_{t+1}(1+i_{t+1}^R)d_t^b$$
$$-\phi_{t+1}(1+i_{t+1}^d)d_t^b$$
$$\text{s.t.} \quad \alpha_t^M \geq 0$$
$$\alpha_t^B \geq 0$$
$$\alpha_t^B + \alpha_t^M \leq 1,$$

and the resulting equilibrium conditions are

$$i_{t+1}^{\ell} = i_{t+1}^B = i_{t+1}^d \geq i_{t+1}^R \quad \text{if} \quad \alpha_t^M = 0; \quad i_{t+1}^{\ell} = i_{t+1}^B = i_{t+1}^d = i_{t+1}^R \quad \text{if} \quad \alpha_t^M > 0. \tag{18}$$

Furthermore, denote the steady-state nominal interest rate in the economy in the absence of interest on reserves as $\tilde{i}$.

PROPOSITION 9. *For $(1+\pi)/\beta - 1 > i^R > \tilde{i}$, an increase in $i_{t+1}^R$ increases $i_{t+1}^{\ell}$, $i_{t+1}^B$, $i_{t+1}^d$, $q_{t+1}^i$, and banks' reserve holdings, and reduces $k_t$. Any $i^R < \tilde{i}$ has no effect on the economy.*

The intuition is as follows: Paying $i^R > \tilde{i}$ creates an outside option for banks' investment, and thus equilibrium rates increase to that level. If $\tilde{i} > i^R > 0$, it has no effect since other assets pay higher returns. With $i^R < 0$ things are more complex, but the result that $i^R$ has no effect on the economy still holds. The intuition for this is that the zero-lower bound effectively binds twice for banks: Once through their reserve holdings, and once through deposit demand. Whereas negative interest rates remove the zero-lower bound on reserves, it does not remove it on deposit demand. Appendix A.3 discusses this in more detail. However, this result is not completely general: Appendix B.1 shows that negative interest rates can increase investment if there is a large enough measure of virtual meetings where only deposits are accepted. In summary, interest on reserves are an effective tool for the monetary authority if $(1+\pi)/\beta - 1 > i^* > \tilde{i}$. This increases the policy options somewhat for a monetary authority operating under a real debt target of the fiscal authority, as it gives them a policy tool to vary $i$ while keeping $\pi$ unchanged. However, since it is only possible to set $i \geq \tilde{i}$, this does not allow the monetary authority to implement any $r$ that it could not already implement without interest on reserves. Furthermore, since $\tilde{i} \geq 0$, interest on reserves only work in a subset of the parameter space in which open-market operations and QE are also effective under a nominal debt target.[39]

6.4. *Eliminating Cash.* Suppose there is a ban on cash, in the sense that households are not allowed to hold $M$. First, note that without cash, it is not obvious how the monetary authority can affect the economy at all through open-market operations. To get around this, suppose the monetary authority sets a reserve requirement $\delta$, so banks need to hold $m_t^b \geq \delta d_t^b$, which implies that the reserve constraint in the banks' problem becomes $\alpha_t^M \geq \delta$. This ensures that there is a positive demand for reserves at any interest rate, so banks are willing to

tive interest rates by substituting reserves for cash. In reality, central banks have taken measures to prevent this: for example, the Swiss National Bank has measured cash holdings by banks before introducing negative interest rates on reserves, and increases in cash holdings over the level prior to the introduction of negative interest rates are interpreted as reserves, and thus subject to negative interest rates. For details, see the SNB (Swiss National Bank) press release on the introduction of negative interest rates from December 18, 2014: https://www.snb.ch/en/mmr/reference/pre_20141218/source/pre_20141218.en.pdf.

[39] Somewhat similar results on positive interest on reserves are shown in Berentsen et al. (2014); Berentsen et al. (2020) finds that negative rates do have real effects, but are bad for welfare.

sell bonds against reserves. The effect on outside buyers of eliminating cash is obvious: Without cash, they cannot consume in the DM, so $q_t^o = 0\ \forall t$. Inside buyers are less affected. For them, Equation (5) now determines deposit demand for any $i_{t+1}^d < (1 + \pi_{t+1})/\beta - 1$. Without the zero-lower bound on $i^d$, negative interest rates become effective, so the monetary authority can increase capital investment by setting $i^R < 0$.[40] Whether this increases welfare of course depends on whether the benefit from increasing capital investment outweighs the cost of shutting down outside meetings. Even though the share of these meetings should be considered small, the welfare cost for households can be substantial if these meetings are completely eliminated. In a similar model, Altermatt et al. (2021) calibrate the cost of eliminating currency and find a welfare cost of almost 7% of GDP when trade in the DM is characterized by take-it-or-leave-it offers as assumed here, and the measure of outside meetings is 0.05. This shows that eliminating cash can only increase overall welfare if underinvestment due to pledgeability constraints is a very severe problem.

## 7. CONCLUSION

This article shows that microfounding the financial system's role in liquidity creation and embedding it in a general equilibrium model creates a rich framework with three different equilibrium regimes. The equilibrium regimes are defined by interest rates and investment decisions by banks, and the effects of policies differ across the three regimes. Banks are essential in this framework because they are able to create liquid assets that pay a higher interest rate than fiat money, and they simultaneously provide funding for entrepreneurs at lower interest rates than households. Because of the banks' ability to create liquid assets by investing in illiquid assets such as government bonds and loans to entrepreneurs, these illiquid assets also attain a liquidity premium. If entrepreneurs are not constrained in their borrowing, running the Friedman rule allows to achieve the first-best outcome. If entrepreneurs are constrained, the first best is not achievable, and deviating from the Friedman rule increases welfare. If the fiscal authority follows a nominal debt target, the steady-state inflation rate is out of the monetary authority's control. In this case, the monetary authority can unilaterally increase inflation and investment at the zero-lower bound by using helicopter money. If the monetary authority increases the money supply through bond purchases instead, it only leads to growth in excess reserves, while the inflation rate and capital investment remain unchanged. If the fiscal authority follows a real debt target, the monetary authority may only independently choose inflation and nominal rates simultaneously if it is able to use helicopter money.

## APPENDIX A: ADDITIONAL DISCUSSION

A.1. *Discussion of banks' ability to create inside money.* Here I want to briefly discuss how banks can create inside money in the model, and why the banks are still constrained in the amount of loans they extend even though they can create money out of nothing. Trade in the centralized market (CM) is Walrasian, meaning that it is not specified who trades with whom, and which goods are exchanged between individuals—instead, agents are able to trade against their wealth denoted in numeraire (i.e., CM goods), and markets have to clear. Therefore, it does not matter whether banks first receive deposits from households, and then proceed to lend those to entrepreneurs so that they can invest, or whether banks lend to entrepreneurs by creating inside money, which entrepreneurs then use to purchase CM goods to invest from households, and households then deposit the inside money they receive as payment at the bank. Either story works, and the equations are exactly the same in both cases. Importantly, even if banks have the ability to create inside money, as they do in this article,

[40] Appendix B.1 discusses how negative interest rates work when only deposits can be used in some virtual meetings. Without cash, all inside meetings become virtual meetings, and otherwise the same analysis applies.

they need to take into account market forces—most importantly, how many deposits households are willing to hold at a given interest rate. Suppose instead banks were ignoring this and the amount of loans created at the equilibrium interest rate were higher than the amount of deposits households are willing to hold. In this case, households would consume more CM goods, which increases the CM price level—that is, it creates inflation, which increases the price of capital and lowers the entrepreneurs' return—which then means that banks make losses and go out of business. Instead, as in any Walrasian market, agents take into account equilibrium prices when making decisions—most importantly, banks take into account the equilibrium deposit interest rate when creating loans. So while banks are allowed to create an unlimited amount of loans in this economy, they still choose a finite quantity in equilibrium due to market forces. Importantly, this model captures the banks' ability to create liquid assets, and also the relevant constraints that are restricting banks in reality.

A.2. *Discussion of Proposition 4.* A decrease in $(B_t - b_t)/M_t$ reduces the real amount of publicly available bonds. It can easily be seen that this does not affect equilibrium outcomes at the margin in equilibrium case I, as the marginal bond is held by a household in that case, and thus the bond rate has to remain at the Fisher interest rate. However, if the decrease in $(B_t - b_t)/M_t$ is larger than $z_b^h + z_\ell^h$, that is, the amount of illiquid held by households, it can move the economy into equilibrium case II. Similarly, in equilibrium case III, the economy is at the zero-lower bound, and banks are already holding excess reserves. Thus, a further decrease in the amount of publicly available bonds has no effect on interest rates, but it further increases banks' excess reserve holdings, as these pay the same interest rate (zero) as the bonds. In equilibrium case II, open-market operations matter: A purchase of bonds by the monetary authority reduces the amount of bonds available for banks, which means that bond demand is larger than supply at the prevailing interest rate before the open-market operation. As a result, the interest rate is reduced. Lower interest rates induce households to hold less deposits and thus consume less goods in inside meetings. Meanwhile, banks make more loans to entrepreneurs, because entrepreneurs want to invest more at lower rates. All of this can easily be seen from Figure 2(b), where a decrease in $(B_t - b_t)/M_t$ shortens the upper flat segment of the solid blue curve and thus shifts the remaining segments of that curve to the left. An increase in the amount of publicly available bonds brought by an open-market sale of bonds by the monetary authority has the reverse effect.

A.3. *Discussion of negative interest rates.* Consider an economy in equilibrium case III with $i^R = 0$. Then, the equilibrium amount of real deposits is $\bar{z}_d$, real loans are $\bar{z}_\ell$, and $m_t^i = 0$. Furthermore, banks' fiat money holdings in real terms are $\bar{z}_d - (\bar{z}_\ell + \phi_t(B_t - b_t^M))$. With $i^R < 0$, banks are only willing to offer $\bar{z}_\ell + \phi_t(B_t - b_t^M)$ deposits at $i^d = 0$. To offer more deposits, banks would need to set $i^d < 0$. However, Equation (6) shows that households do not hold any deposits at negative rates. So with $i^R < 0$, banks ration deposit supply, and households hold $\phi_t d_t^i = \bar{z}_\ell + \phi_t(B_t - b_t^M)$ real deposits, plus real fiat money balances equal to $\phi_t m_t^i = \bar{z}_d - \phi_t d_t^i$. This allows them to purchase the same quantities in the decentralized market (DM) as with $i^R = 0$. Furthermore, since $i^d = 0$ even with $i^R < 0$, $i^\ell = 0$ still holds and thus $k_t$ remains unchanged. Figure A.1 depicts this equilibrium graphically.

## APPENDIX B: EXTENSIONS

B.1. *Virtual meetings.* As an extension, suppose a measure $\psi$ of buyers meet sellers in the DM who only accept deposits, but no fiat money. Call these *virtual meetings*. Thus, the measure of inside meetings remains $\eta$, but the measure of outside meetings is now $1 - \eta - \psi$. As before, suppose buyers learn in the CM what type their next DM meeting will be. The prob-

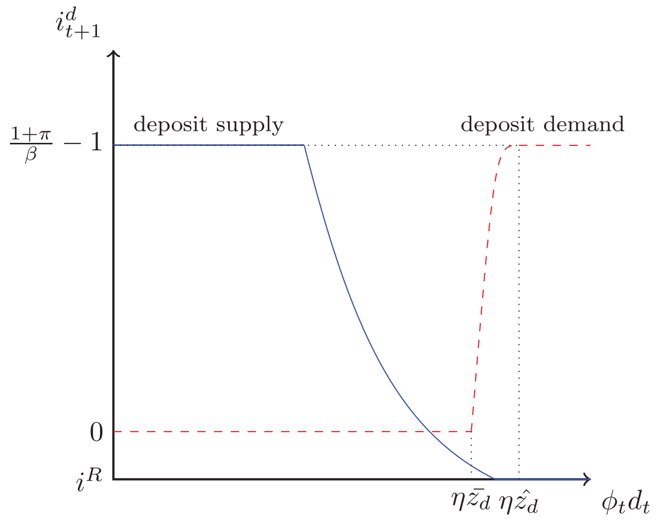



FIGURE A.1

DEPOSIT MARKET CLEARING WITH NEGATIVE INTEREST RATES

lem of inside and outside buyers remains unchanged. The DM problem of virtual buyers is

$$\max_{q_t^v} \quad u(q_t^v) - p(q_t^v)$$
$$\text{s.t.} \quad (1+i_t)\phi_t d_t^v \geq p(q_t^v) = c(q_t^v) = q_t^v,$$

with the solution $q_t^v = \min\{(1+i_t)\phi_t d_t^v, q^*\}$.

In the CM, virtual buyers solve

$$\max_{m_t^v \geq 0, b_t^v \geq 0, \ell_t^v \geq 0, d_t^v \geq 0} \Bigg[ -\left(\frac{1+\pi_{t+1}}{\beta} - (1+i^d_{t+1})\right)\phi_{t+1} d_t^v - \left(\frac{1+\pi_{t+1}}{\beta} - 1\right)\phi_{t+1} m_t^v$$
$$-\left(\frac{1+\pi_{t+1}}{\beta} - (1+i^B_{t+1})\right)\phi_{t+1} b_t^v - \left(\frac{1+\pi_{t+1}}{\beta} - (1+i^\ell_{t+1})\right)\phi_{t+1} \ell_t^v$$
$$+ u((1+i_{t+1})\phi_{t+1} d_t^v)) - (1+i_t)\phi_{t+1} d_t^v \Bigg].$$

The problem is similar, except that fiat money has no liquidity value for virtual buyers and thus they only hold it at the Friedman rule. In turn, virtual buyers are willing to hold deposits even at negative interest rates. For any $i^d_{t+1} \leq (1+\pi_{t+1})/\beta - 1$, their demand for deposits is given by

$$u'\big((1+i^d_{t+1})\phi_{t+1} d_t^v\big) = 1 + \frac{1+\pi_{t+1} - \beta(1+i^d_{t+1})}{\beta(1+i^d_{t+1})}, \tag{B.1}$$

so virtual buyers' demand schedule for deposits is

$$d_t^v = \begin{cases} \infty \quad \text{if} \quad 1+i^d_{t+1} > \frac{1+\pi_{t+1}}{\beta}, \\ \in (\frac{\hat{z}_d}{\phi_t}, \infty) \quad \text{if} \quad 1+i^d_{t+1} = \frac{1+\pi_{t+1}}{\beta}, \\ \text{solution to Equation (A.1) if} \quad i^d_{t+1} < \frac{1+\pi_{t+1}}{\beta} - 1. \end{cases} \tag{B.2}$$

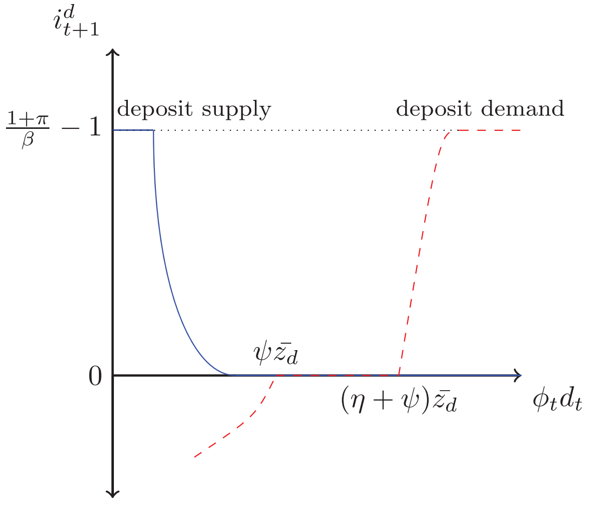



FIGURE B.2

EQUILIBRIUM CASE III WITH VIRTUAL MEETINGS

With virtual buyers, $b^h = \eta b^i + \psi b^v + (1-\eta-\psi)b^o + b^s$, whereas $\ell^h = \eta\ell^i + \psi\ell^v + (1-\eta-\psi)\ell^o + \ell^s$. Furthermore, assume without loss of generality that virtual buyers never hold $m$. The problem of banks and entrepreneurs is unchanged; so is the definition of equilibrium, except that market clearing conditions for money and deposits are now given by

$$M_t = \eta m^i_t + (1-\eta-\psi)m^o_t + \alpha^M_t d^b_t;\; d_t \equiv d^b_t = \eta d^i_t + \psi d^v_t. \tag{B.3}$$

In the baseline model, the same three equilibrium cases may exist. To see why, Figure B.2 depicts deposit demand and supply in a case III equilibrium with virtual meetings. While there is positive demand for deposits at negative interest rates, perfect competition among banks and their ability to hold reserves at nominal rates of zero prevent $i^d$ from becoming negative in equilibrium.

**Negative interest rates on reserves**

Now, suppose the monetary authority sets $i^R < 0$ in an economy with virtual meetings. Whether this policy allows to increase investment depends on whether banks are willing to set negative interest rates on deposits. Figure B.2 illustrates that there is a discontinuity in deposit demand at $i^d = 0$. At negative rates, inside buyers will stop using deposits and rely on fiat money instead, so the bank can only retain virtual buyers as depositors. Doing so is optimal for banks if $\psi\bar{z}_d > \bar{z}_\ell + \phi_t(B_t - b^M_t)$, that is, if deposit demand of virtual buyers exceeds available investment opportunities at the zero-lower bound. Then, banks still want to hold reserves, so $i^\ell = i^B = i^d = i^R$ must hold in equilibrium. Thus, for $i^R < 0$, loan rates are also negative, and capital investment increases relative to an equilibrium with $i^R = 0$. If instead $\psi\bar{z}_d < \bar{z}_\ell + \phi_t(B_t - b^M_t) < (\psi+\eta)\bar{z}_d$, setting $i^R < 0$ will have similar effects to an economy without virtual meetings: Banks get rid of their reserves and ration deposits, and inside buyers use a mix of fiat money and deposits to pay in DM meetings. Virtual buyers continue holding $\bar{z}_d$, so the amount of deposits held by virtual and inside buyers differs, but not the total amount of liquidity. This analysis shows that negative interest rates can be effective if the measure of virtual meetings is large enough. In that case, $i^d < 0$, and banks reduce reserves relative to the zero-lower bound, but do not completely get rid of them.

## APPENDIX C: PROOFS

C.1. *Proof for* $\frac{\partial \phi_t d_t^i}{\partial(1+\pi_{t+1})} < 0$ *and* $\frac{\partial d_t^i}{\partial i_{t+1}^d} > 0$. PROOF. Start by rewriting Equation (5) as

$$u'\left(\frac{1+i_{t+1}^d}{1+\pi_{t+1}}\phi_t d_t^i\right) = \frac{1+\pi_{t+1}}{\beta(1+i_{t+1}^d)}.$$

Totally differentiating this with respect to $1+\pi_{t+1}$ yields

$$u''(q_{t+1}^i)\left(\frac{\partial \phi_t d_t^i}{\partial(1+\pi_{t+1})}\frac{1+i_{t+1}^d}{1+\pi_{t+1}} - \frac{q_{t+1}^i}{1+\pi_{t+1}}\right) = \frac{1}{\beta(1+i_{t+1}^d)},$$

where I made use of $q_{t+1}^i = \frac{1+i_{t+1}^d}{1+\pi_{t+1}}\phi_t d_t^i$. Rearrange to get

$$\frac{\partial \phi_t d_t^i}{\partial(1+\pi_{t+1})} = \frac{1}{1+i_{t+1}^d}\left(\frac{u'(q_{t+1}^i)}{u''(q_{t+1}^i)} + q_{t+1}^i\right),$$

where I used $u'(q_{t+1}^i) = \frac{1+\pi_{t+1}}{\beta(1+i_{t+1}^d)}$. Thus, $\frac{\partial \phi_t d_t^i}{\partial(1+\pi_{t+1})} < 0$ if $\frac{u'(q_{t+1}^i)}{u''(q_{t+1}^i)} + q_{t+1}^i < 0$, which can be written as

$$1 > -q_{t+1}^i\frac{u''(q_{t+1}^i)}{u'(q_{t+1}^i)}.$$

Since I assumed $u(q)$ satisfies $1 > -q\frac{u''(q)}{u'(q)}$, this condition holds, so $\frac{\partial \phi_t d_t^i}{\partial(1+\pi_{t+1})} < 0$.

Now turn to $\frac{\partial d_t^i}{\partial i_{t+1}^d}$. Totally differentiating Equation (5) with respect to $i_{t+1}^d$ yields

$$u''(q_{t+1}^i)\left(q_{t+1}^i + \phi_{t+1}(1+i_{t+1}^d)^2\frac{\partial d_t}{\partial i_{t+1}^d}\right) + u'(q_{t+1}^i) = 0,$$

where I made use of $q_{t+1}^i = (1+i_{t+1}^d)\phi_{t+1}d_t^i$. Rearrange this to get

$$\frac{\partial d_t}{\partial i_{t+1}^d} = -\frac{1}{\phi_{t+1}(1+i_{t+1}^d)^2}\left(\frac{u'(q_{t+1}^i)}{u''(q_{t+1}^i)} + q_{t+1}^i\right).$$

Thus, $\frac{\partial d_t}{\partial i_{t+1}^d} > 0$ if $\frac{u'(q_{t+1}^i)}{u''(q_{t+1}^i)} + q_{t+1}^i < 0$. This is the same condition as above, so $\frac{\partial d_t}{\partial i_{t+1}^d} > 0$. □

C.2. *Proof for* $\frac{\partial k_t}{\partial(1+\pi_{t+1})} > 0$ *and* $\frac{\partial k_t}{\partial(1+i_{t+1}^\ell)} < 0$ *if pledgeability is binding.* PROOF. With the pledgeability constraint binding, the amount of loans borrowed by entrepreneurs is given by

$$\chi f(\phi_t \ell_t) = (1+i_{t+1}^\ell)\phi_{t+1}\ell_t.$$

We can rewrite this in terms of capital to get

$$\chi f(k_t) = \frac{1+i_{t+1}^\ell}{1+\pi_{t+1}}k_t. \tag{C.1}$$

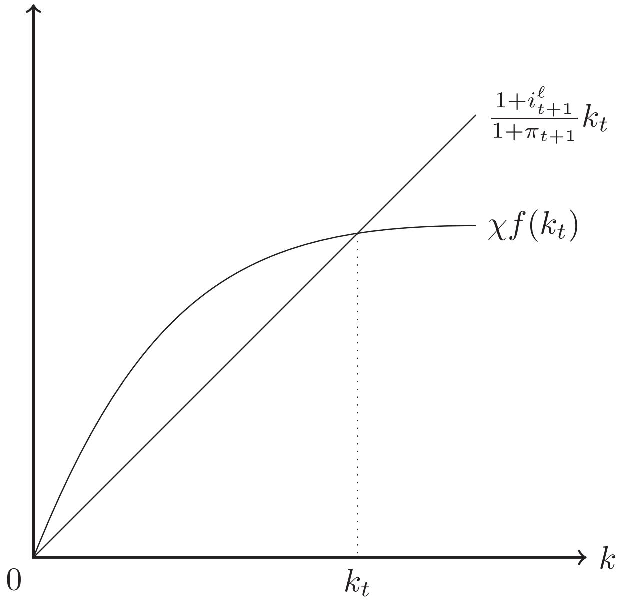



FIGURE C.3

SOLUTION FOR $k_t$ IF THE PLEDGEABILITY CONSTRAINT BINDS

Deriving Equation (C.1) after inflation $1+\pi_{t+1}$ yields

$$\frac{\partial k_t}{\partial(1+\pi_{t+1})} = \frac{-k_t \frac{1+i^{\ell}_{t+1}}{(1+\pi_{t+1})^2}}{\chi f'(k_t) - \frac{1+i^{\ell}_{t+1}}{1+\pi_{t+1}}},$$

so $\frac{\partial k_t}{\partial(1+\pi_{t+1})} > 0$ if $\chi f'(k_t) < \frac{1+i^{\ell}_{t+1}}{1+\pi_{t+1}}$.

Deriving Equation (C.1) after the loan rate $1+i^{\ell}_{t+1}$ yields

$$\frac{\partial k_t}{\partial(1+i^{\ell}_{t+1})} = \frac{\frac{k_t}{1+\pi_{t+1}}}{\chi f'(k_t) - \frac{1+i^{\ell}_{t+1}}{1+\pi_{t+1}}},$$

so $\frac{\partial k_t}{\partial(1+i^{\ell}_{t+1})} < 0$ if $\chi f'(k_t) < \frac{1+i^{\ell}_{t+1}}{1+\pi_{t+1}}$. What is left now is to show that $\chi f'(k_t) < \frac{1+i^{\ell}_{t+1}}{1+\pi_{t+1}}$ at the solution, and this can best be done graphically.

In Figure C.3, it can easily be seen that the slope of the right-hand side (RHS) of Equation (C.1) has to be less than the slope of the RHS at the solution $k_t$. Since the slope of the left-hand side (LHS) is given by $\chi f'(k_t)$ and the slope of the RHS is given by $\frac{1+i^{\ell}_{t+1}}{1+\pi_{t+1}}$, this shows that $\frac{\partial k_t}{\partial(1+\pi_{t+1})} > 0$ and $\frac{\partial k_t}{\partial(1+i^{\ell}_{t+1})} < 0$. □

C.3. *Proof of Proposition 1.* PROOF. In steady state, all nominal variables need to grow at the same, constant rate. Thus, $\gamma^M = \gamma^B$ is required. Then, from Equation (13), $1+\pi = \gamma^M = \gamma^B$ follows. $1+\pi \geq \beta$ is required as otherwise holding on to money strictly dominates spending it. I prove the remainder of the proposition separately for the two targets of the fiscal authority.

**Nominal debt target**

Under a nominal debt target, $\gamma^B$ is exogenously given, so the monetary authority has no choice but to set $\gamma^M = \gamma^B$ in steady state. In turn, steady-state inflation is exogenously determined through $\gamma^B$. For a given $\pi$, Equation (4) pins down $z_m^o$, while $z_\ell = z_\ell(i)$ according to Equation (10) and $z_d = z_d(i)$ according to Equation (6). Then, Equation (15) can be rewritten as

$$z_\ell(i) = z_\ell^h(i) + (1 - \alpha^M(i) - \alpha_t^B)\eta z_d^i(i), \tag{C.2}$$

where $z_\ell^h(i) > 0$ iff $i = \frac{1+\pi}{\beta} - 1$ according to Equation (9), while $\alpha^M(i) > 0$ iff $i = 0$ according to Equation (11). Rearranging Equation (16), we get

$$\alpha^B \eta z_d^i(i) = \phi_t M_t \frac{B_t - b_t^M}{M_t} - z_b^h(i),$$

with $z_b^h(i) > 0$ iff $i = \frac{1+\pi}{\beta} - 1$ from Equation (8). Using Equation (13) to replace $\phi_t M_t$, assuming without loss of generality that $m_t^i = 0 \; \forall i \geq 0$ as discussed in Subsection 2.1, and plugging this into Equation (C.2), the deposit market clearing condition becomes

$$z_\ell(i) = z_\ell^h(i) + z_b^h(i) + \eta z_d^i(i)\left(1 - \alpha^M(i)\left[1 + \frac{B_t - b_t^M}{M_t}\right]\right) - (1-\eta) z_m^o \frac{B_t - b_t^M}{M_t}. \tag{C.3}$$

To see that there is a unique $i$ solving this equation, it is helpful to go through three cases. First, suppose that $i = \frac{1+\pi}{\beta} - 1$. Then, Equation (C.3) becomes

$$\hat{z}_\ell = z_\ell^h + z_b^h + \eta \hat{z}_d - (1-\eta) z_m^o \frac{B_t - b_t^M}{M_t}. \tag{C.4}$$

Thus, an equilibrium with $i = \frac{1+\pi}{\beta} - 1$ exists if Equation (C.4) holds for $z_\ell^h + z_b^h \geq 0$. Next, suppose that $i = 0$, in which case (C.3) can be written as

$$\bar{z}_\ell = \eta \bar{z}_d \left(1 - \alpha^M\left[1 + \frac{B_t - b_t^M}{M_t}\right]\right) - (1-\eta) z_m^o \frac{B_t - b_t^M}{M_t}. \tag{C.5}$$

This shows that an equilibrium with $i = 0$ exists if (C.5) holds for $\alpha^M \in [0, 1)$.[41] Finally, suppose $i \in (0, \frac{1+\pi}{\beta} - 1)$. Then, Equation (C.3) reduces to

$$\eta z_d^i(i) - z_\ell(i) = (1-\eta) z_m^o \frac{B_t - b_t^M}{M_t}. \tag{C.6}$$

An equilibrium with $i \in (0, \frac{1+\pi}{\beta} - 1)$ exists if there is an $i$ satisfying this condition for which Equation (C.6) holds. Furthermore, since the LHS of (C.6) is strictly increasing in $i$, whereas the RHS is independent of $i$, if such an $i$ exists it must be unique. Finally, note that at $i = \frac{1+\pi}{\beta} - 1$, Equation (C.6) is equivalent to equation (C.4) with $z_\ell^h + z_b^h = 0$, while at $i = 0$, equation (C.6) is equivalent to Equation (C.5) with $\alpha^M = 0$, showing that only one equation out of (C.4)–(C.6) can be satisfied simultaneously.

[41] To see that $\alpha^M \geq 1$ can never be an equilibrium, rewrite Equation (C.5) as $\eta \bar{z}_d(1 - \alpha^M[1 + \frac{B_t - b_t^M}{M_t}]) - \bar{z}_\ell = (1 - \eta) z_m^o \frac{B_t - b_t^M}{M_t}$ and notice that the LHS becomes negative at $\alpha^M \geq 1$, whereas the RHS cannot be negative.

**Real debt target**

Under a nominal debt target, the monetary authority controls steady-state inflation through its choice of $\gamma^M$. Since $1+\pi=\gamma^M$, the fiscal authority will then set $\gamma^B=\gamma^M$ in order to keep real debt constant at $\mathcal{B}$. Then, $z_m^o(\pi)$ is determined by Equation (4), whereas real loans and real deposits are functions of $i$ and $\pi$ according to Equations (10) and (6), respectively. Given this, Equation (15) can be rewritten as

$$z_\ell(i,\pi)=z_\ell^h(i)+\big(1-\alpha^M(i)-\alpha_t^B\big)\eta z_d^i(i,\pi). \tag{C.7}$$

Rearranging Equation (16), we get

$$\alpha^B\eta z_d^i(i,\pi)=\mathcal{B}-\phi_t b_t^M-z_b^h(i).$$

Using Equation (13) to replace $\phi_t$, assuming again that $m_t^i=0$, and plugging this into Equation (C.7), the deposit market clearing condition becomes

$$z_\ell(i,\pi)=z_\ell^h(i)+z_b^h(i)+\eta z_d^i(i,\pi)\left(1-\alpha^M(i)\left[1-\frac{b_t^M}{M_t}\right]\right)+(1-\eta)z_m^o(\pi)\frac{b_t^M}{M_t}-\mathcal{B}. \tag{C.8}$$

To show uniqueness, it is again helpful to go through three cases. First, suppose that $i=\frac{1+\pi}{\beta}-1$. Then, Equation (C.8) becomes

$$\hat{z}_\ell=z_\ell^h+z_b^h+\eta\hat{z}_d+(1-\eta)z_m^o(\pi)\frac{b_t^M}{M_t}-\mathcal{B}. \tag{C.9}$$

Thus, an equilibrium with $i=\frac{1+\pi}{\beta}-1$ exists if, for a given $1+\pi=\gamma^M$, Equation (C.9) holds for $z_\ell^h+z_b^h\geq 0$. Next, suppose that $i=0$, in which case (C.8) can be written as

$$\bar{z}_\ell(\pi)=\eta\bar{z}_d(\pi)\left(1-\alpha^M\left[1-\frac{b_t^M}{M_t}\right]\right)+(1-\eta)z_m^o(\pi)\frac{b_t^M}{M_t}-\mathcal{B}. \tag{C.10}$$

This shows that an equilibrium with $i=0$ exists if, for a given $1+\pi=\gamma^M$, Equation (C.10) holds for $\alpha^M\in[0,1)$.[42] Finally, suppose $i\in(0,\frac{1+\pi}{\beta}-1)$. Then, Equation (C.8) reduces to

$$\eta z_d^i(i,\pi)-z_\ell(i,\pi)=\mathcal{B}-(1-\eta)z_m^o(\pi)\frac{b_t^M}{M_t}. \tag{C.11}$$

An equilibrium with $i\in(0,\frac{1+\pi}{\beta}-1)$ exists if, for a given $1+\pi=\gamma^M$, there is an $i$ satisfying this condition for which Equation (C.11) holds. Furthermore, since the LHS of (C.11) is strictly increasing in $i$, whereas the RHS is independent of $i$, if such an $i$ exists it must be unique. Finally, note that at $i=\frac{1+\pi}{\beta}-1$, Equation (C.11) is equivalent to Equation (C.9) with $z_\ell^h+z_b^h=0$, while at $i=0$, Equation (C.11) is equivalent to Equation (C.10) with $\alpha^M=0$, showing that only one equation out of (C.9)–(C.11) can be satisfied simultaneously. □

C.4. *Proof of Proposition 5.* PROOF. Suppose first that $\frac{b_t^M}{M_t}=1$ must hold. From Appendix C.3, choosing $\gamma^M$ determines steady-state inflation $\pi$, and Equation (C.8) reduces to

$$\eta z_d^i(i,\pi)-z_\ell(i,\pi)-(z_\ell^h(i)+z_b^h(i))=\mathcal{B}-(1-\eta)z_m^o(\pi).$$

[42] One can rule out $\alpha^M\geq 1$ by a similar reasoning as in the case of a nominal debt target.

Given $\pi$, this equation determines $i$, showing that the monetary authority may only choose a combination $(\pi, i(\pi))$ by varying $\gamma^M$. Suppose first that the monetary authority runs the Friedman rule, that is, $\gamma^M = \beta$. Then, $i = 0 = \frac{1+\pi}{\beta} - 1$ must hold, and Equation (C.8) becomes

$$\eta\hat{z}_d - \hat{z}_\ell - (z_\ell^h + z_b^h) = \mathcal{B} - (1-\eta)z_m^o(\beta).$$

Since any $(z_\ell^h + z_b^h) > 0$ is an equilibrium outcome for $i = \frac{1+\pi}{\beta} - 1$, this shows that a Friedman rule equilibrium can be implemented by the monetary authority. Now, suppose $\gamma^M > \beta$. To understand how $i(\pi)$ varies in $\pi$ in this case, it is helpful to go through the three equilibrium cases. With $\frac{b_t^M}{M_t} = 1$, the economy is in case I if

$$\eta\hat{z}_d - \hat{z}_\ell \leq \mathcal{B} - (1-\eta)z_m^o(\pi).$$

Since $z_m^o(\pi)$ is decreasing in $\pi$ and $z_m^o(\pi) \to 0$ as $\pi \to \infty$, the monetary authority is able to get the economy into case I by choosing some $\bar{\pi}$ for which the above condition holds at equality. Setting $\pi > \bar{\pi}$ further reduces $z_m^o(\pi)$, but leaves $i = \frac{1+\pi}{\beta} - 1$ unchanged. Next, turn to case III with $i = 0$. Under a real debt target, Equation (C.10) must hold for this equilibrium to exist. With $\frac{b_t^M}{M_t} = 1$, Equation (C.10) reduces to

$$\eta\bar{z}_d(\underline{\pi}) - \bar{z}_\ell(\underline{\pi}) = \mathcal{B} - (1-\eta)z_m^o(\underline{\pi}),$$

where $\underline{\pi}$ is the only $\pi$ consistent with a zero-lower bound equilibrium. Since $\eta\hat{z}_d - \hat{z}_\ell > \eta\bar{z}_d(\pi) - \bar{z}_\ell(\pi)\ \forall\pi > \beta$, it follows that $z_m^o(\bar{\pi}) < z_m^o(\underline{\pi})$, so in turn $\bar{\pi} > \underline{\pi}$. Note that real excess reserve holdings $\alpha^M\bar{z}_d(\underline{\pi})$ at the zero-lower bound are indeterminate under a real debt target with $\frac{b_t^M}{M_t} = 1$. Thus, if the monetary authority sets $\gamma^M < \bar{\pi}$, the economy remains at the zero-lower bound and inflation stays constant at $\underline{\pi}$, as the lower money growth rate simultaneously reduces the nominal quantity of bonds available for banks to invest, but increases the real value of these bonds as they become more scarce, and the two effects exactly offset each other. Finally, for $\gamma^M \in (\underline{\pi}, \bar{\pi})$, $i \in (0, \frac{1+\pi}{\beta} - 1)$ is determined by Equation (C.11), which reduces to

$$\eta z_d^i(i,\pi) - z_\ell(i,\pi) = \mathcal{B} - (1-\eta)z_m^o(\pi),$$

with $\frac{b_t^M}{M_t} = 1$. Since the LHS of this equation is increasing in $i$ and decreasing in $\pi$, whereas the RHS is increasing in $\pi$, this shows that $i(\pi)$ is increasing in $\pi$. To see how the choice of $\pi \in (\underline{\pi}, \bar{\pi})$ affects the real interest rate $r = \frac{1+i}{1+\pi} - 1$, rewrite the above equation as

$$\eta z_d^i(r) - z_\ell(r) = \mathcal{B} - (1-\eta)z_m^o(\pi).$$

Since the LHS is increasing in $r$, this shows that $r(\pi)$ is also increasing in $\pi$. $r = \frac{1}{\beta} - 1$ is the real rate consistent with $\bar{\pi}$, whereas $r = \frac{1}{1+\underline{\pi}} - 1$ is the real rate consistent with $\underline{\pi}$. Since $r$ is increasing in $\pi$ and $\bar{\pi} > \underline{\pi}$, this also shows that $1 + \underline{\pi} > \beta$.

Now, suppose instead that the monetary authority may choose any $\frac{b_t^M}{M_t}$. Choosing $\gamma^M$ still determines $\pi$, but, according to (C.8), the monetary authority may now implement any $i \in [0, \frac{1+\pi}{\beta} - 1]$ for a given $\pi$ by varying $\frac{b_t^M}{M_t}$. To understand how $i$ changes in $\frac{b_t^M}{M_t}$, it is easiest to look at the intermediate case given by Equation (C.11). Since the RHS of that equation is decreasing in $\frac{b_t^M}{M_t}$ and unaffected by $i$, whereas the LHS is increasing in $i$ and unaffected by $\frac{b_t^M}{M_t}$, this shows that $i$ must be decreasing in $\frac{b_t^M}{M_t}$ for a given $\pi$. This in turn implies that $r$ is also decreasing in $\frac{b_t^M}{M_t}$. □

C.5. *Proof of Proposition 6.* PROOF. To prove this proposition, I will show that there is an equilibrium with a constant inflation rate even if the monetary authority increases the money growth rate by purchasing bonds. I will assume $n = 1$ and $\gamma^B = 1$ in order to keep notation simple, but the proof generalizes.

Assume that the economy is in an equilibrium case III steady state in period $t$, when the monetary authority announces to set $\gamma^M_{t+1} > 1$, and $\gamma^M_{t+j} = 1\ \forall j > 1$. For this proof, I will posit that as a reaction to this policy, only $\alpha^M_{t+1}$ and $\alpha^B_{t+1}$ will differ from their steady-state values, but the real variables do not; Then, I will show that this indeed constitutes an equilibrium.

Two equilibrium conditions are directly affected by an increase in the money growth rate: the money market clearing condition $\phi_t M_t = \eta z^i_m + (1-\eta)z^o_m + \alpha^M_t z^d_m$. Under the assumption that only $\alpha^M$ and $\alpha^B$ change as a reaction to the policy change, the money market clearing condition in period $t+1$ becomes

$$\gamma^M_{t+1}\phi_t M_t = \eta z^i_m + (1-\eta)z^o_m + \alpha^M_{t+1} z^b_d.$$

Rearranging this for $\alpha^M_{t+1}$, the fraction of assets banks hold as fiat money during period $t+1$, yields

$$\alpha^M_{t+1} = \frac{\gamma^M_{t+1}\phi_t M_t - \eta z^i_m - (1-\eta)z^o_m}{z^b_d}.$$

Meanwhile, the bond market clearing condition $\phi_t(B_t - b^M_t) = \alpha^B_t z^b_d$, becomes

$$\alpha^B_{t+1} z^b_d = \phi_t(\gamma^B B_t - b^M_{t+1}) = \phi_t(B_t - b^M_t - (\gamma^M_{t+1} - 1)M_t)$$

in period $t+1$. Rearranging for $\alpha^B_{t+1}$ yields

$$\alpha^B_{t+1} = \frac{\phi_t(B_t - b^M_t - (\gamma^M_{t+1} - 1)M_t)}{z^b_d}.$$

Now, adding $\alpha^M_{t+1}$ and $\alpha^B_{t+1}$ yields:

$$\begin{aligned}
\alpha^B_{t+1} + \alpha^M_{t+1} &= \frac{\phi_t(B_t - b^M_t - (\gamma^M_{t+1} - 1)M_t) + \gamma^M_{t+1}\phi_t M_t - \eta z^i_m - (1-\eta)z^o_m}{z^b_d} \\
&= \frac{\phi_t(B_t - b^M_t - M_t) - -\eta z^i_m - (1-\eta)z^o_m}{z^b_d} z^b_d \\
&= \alpha^B_t + \alpha^M_t.
\end{aligned}$$

This shows that if I posit that the changes in the money growth rates are all absorbed by changes in $\alpha^B_{t+1}$ and $\alpha^M_{t+1}$, the sum of these two variables does not change. But since bonds and reserves are perfect substitutes for banks at the zero-lower bound, this implies that none of the real variables change as a reaction to the policy change, and this is indeed an equilibrium. In this equilibrium, the increase in the fiat money supply is exactly offset by an increase in the demand for fiat money by banks, which implies that the price of money, and thus the inflation rate, remain unchanged. □

C.6. *Proof of Proposition 7.* PROOF. To prove this proposition, I will proceed in two steps. I will first show that there cannot be an equilibrium with a constant inflation rate if the monetary authority increases the money growth rate through lump-sum transfers to households. Then, I will show that there is an equilibrium where inflation increases while the monetary authority increases the money growth rate, and that inflation returns to the steady-state level once the monetary authority reduces the money growth rate. I again assume $n = 1$ and $\gamma^B = 1$ in order to keep notation simple.

For the first part of this proof, I will try to replicate the proof for Proposition 6 and show that this leads to a contradiction. Consider an announcement by the monetary authority at time $t$ that $\gamma_{t+1}^M > 1$, with the newly issued money distributed via lump-sum transfers to households. Suppose again that in the money market clearing condition $\phi_t M_t = \eta z_m^i + (1-\eta) z_m^o + \alpha_t^M z_m^d$, only $\alpha_{t+1}^M$ changes. Since the bond market clearing condition $\phi_t(B_t - b_t^M) = \alpha_t^B z_d^b$ is unaffected by the increase in $\gamma_{t+1}^M$ through lump-sum transfers, the increase in $\alpha_{t+1}^M$ is not countered by a decrease in $\alpha_{t+1}^B$, so the sum $\alpha_{t+1}^M + \alpha_{t+1}^B$ has to increase. If loans from banks are kept constant, this is only possible if overall deposits increase—but from Equation (6), we know that households are only willing to hold more deposits at higher interest rates. Thus, the increased growth rate cannot lead to only a change in $\alpha_{t+1}^M$ and $\alpha_{t+1}^B$. But since the LHS of the money-market clearing condition is changing, we know that some other variables in this equation have to change too, and since all variables apart from $\alpha_t^M$ in the equation are real, it is clear that real variables have to change. Thus, we have ruled out that inflation remains constant after the policy change.

Next, to show that inflation indeed increases in period $t+1$, let us rewrite the money market clearing condition without assuming any of the variables remains in steady state:

$$\phi_{t+1}\gamma_{t+1}^M M_t = \eta\phi_{t+1} m_{t+1}^i + (1-\eta)\phi_{t+1} m_{t+1}^o + \alpha_{t+1}^M \phi_{t+1} d_{t+1}^b.$$

From Equations (4) and (6), we know that $\phi_{t+1} m_{t+1}^o$ and $\phi_{t+1} d_{t+1}^b$ are decreasing functions of $\pi_{t+2}$.[43] In period $t+2$, the economy can move back to the steady state, so $\pi_{t+2} = 0$ and thus $\phi_{t+1} m_{t+1}^o$ and $\phi_{t+1} d_{t+1}^b$ are equal to their steady-state value.[44] But since $\gamma_{t+1}^M > 1$, $\phi_{t+1}$ has to be lower than the previous steady-state value of money $\phi$, so $\phi_{t+1} < \phi$ is required for the money market to clear in period $t+1$. In period $t$, the money market clearing condition is:

$$\phi_t M_t = \eta\phi_t m_t^i + (1-\eta)\phi_t m_t^o + \alpha^M \phi_t d_t^b.$$

Suppose that $\phi_t$ is equal to its steady-state value $\phi$. This requires $\pi_{t+1} > 0$, since $1 + \pi_{t+1} = \frac{\phi_t}{\phi_{t+1}}$. But since $\phi_t m_t^o$ and $\phi_t d_t^b$ are decreasing in $\pi_{t+1}$, they have to be lower than their steady-state values if $\pi_{t+1} > 0$—but that in turn means that $\phi_t$ also has to be below its steady-state value, as otherwise the money market in period $t$ cannot clear. Thus, assuming that $\phi_t$ remains at the steady-state value leads to a contradiction. Now suppose instead that $\phi_t = \phi_{t+1} = \phi'$; that i, the value of money immediately drops to the new steady-state value when the policy is announced. However, this implies $\pi_{t+1} = 0$, which in turn means that $\phi_t m_t^o$ and $\phi_t d_t^b$ remain at their steady-state values. But this again leads to a contradiction, as this would require $\phi_t = \phi$. This leaves as a solution only $\phi > \phi_t > \phi_{t+1}$ and $\gamma_{t+1}^M > 1 + \pi_{t+1} > 1$. This shows that inflation is increasing as a reaction to the increase in $\gamma_{t+1}^M$ brought by lump-sum transfers, but not one-to-one (the reason that the increase in inflation is smaller than the increase in money growth is that the increase is only temporary, and agents expect the return to the steady-state level). □

C.7. *Proof of Proposition 8.* PROOF. With quantitative easing (QE), Equation (6) becomes

$$z_\ell = z_\ell^h + (1 - \alpha^M - \alpha_t^B)\eta z_d^i + \phi_t \ell_t^M.$$

[43] To be more precise, the sum $\eta\phi_{t+1} m_{t+1}^i + \phi_{t+1} d_{t+1}^b$ is decreasing in the inflation rate, since households in inside meetings are indifferent between fiat money and bank deposits at the zero-lower bound. For the sake of the argument, suppose households in inside meetings hold only bank deposits. $\alpha_{t+1}^M$ is also decreasing in inflation, because banks make more loans at higher inflation rates. Since real deposits go down with inflation and loans increase, loans clearly make up a larger share of all assets, while the share of bonds and fiat money holdings goes down.

[44] It can easily be shown that inflation indeed returns to steady state in period $t+2$ by making similar arguments as in this proof, but using the money market clearing condition in period $t+2$.

Combined with Equation (16), this becomes

$$z_\ell = z_\ell^h + z_\ell^h + (1-\alpha^M)\eta z_d^i - \phi_t(B_t - b_t^M - \ell_t^M).$$

Now, note that with QE and without helicopter money, $b_t^M + \ell_t^M = M_t$ must hold; that is, the nominal money supply must equal the sum of bonds and loans held by the monetary authority. Imposing this in the above condition, we get

$$z_\ell = z_\ell^h + z_\ell^h + (1-\alpha^M)\eta z_d^i - \phi_t(B_t - M_t).$$

Then, using Equation (13), we can rewrite this as

$$z_\ell = z_\ell^h + z_\ell^h + \eta z_d^i\left[1 - \alpha^M \frac{B_t}{M_t}\right] - (1-\eta) z_m^o \frac{B_t - M_t}{M_t}$$

under a nominal debt target, and as

$$z_\ell = z_\ell^h + z_\ell^h + \eta z_d^i - (1-\eta) z_m^o - \mathcal{B}$$

under a real debt target. Since these are identical to Equations (C.3) and (C.8) with $b_t = M_t$ imposed, respectively, this shows that QE does not give the monetary authority additional policy choices under either target of the fiscal authority. □

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