Asset Operation Detection Based on Fuzzy Logic and Phase Portrait

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***Abstract*—This article proposes a novel asset operation detection (AOD) solution by applying the fuzzy logic reasoning concept to the phase portraits (PPs) of time series data. Around the benefits of business insight and climate impact, we firstly provide relevant context to highlight the importance of asset operation features and necessity for efficient operation detection algorithms in the facility management industry. Then we will review several existing approaches for detecting asset operations and discuss their advantages and disadvantages. With these concerns in mind, we come to the operation detection solution proposed in this research, explaining the technical idea and mentioning two approaches regarding the algorithm inputs: physical phases and derivative phases. All the proposed analysis will be based on a real-case industrial dishwasher. Finally, we will come back to the benefits in terms of business insight and climate impact to showcase the application of detected operational features.**

***Index Terms*—asset operation detection, fuzzy logic, phase portrait, artificial intelligence (AI), internet of things (IoT)**

I. Introduction

The facility management (FM) industry is growing rapidly with the emergence of Internet of Things (IoT), reliable artificial intelligence (AI) technology and evolving IT infrastructure [1]. As this growing industry becomes more integral to the ways modern society conducts business, entertainment and lifestyles, there has been higher expectation on the FM industry in terms of business efficiency and environmental impact.

As a major player in the UK market, Cloudfm Group Ltd continually evolves its strategy to meet growing customer expectations and environmental demands, providing more intelligent asset monitoring services and developing predictive and prescriptive maintenance [2]. Behind this strategy, the technology of IoT and AI will be the engine of change. This investment in innovation underpins all services offered by Cloudfm Group, and the company has also created a subsidiary brand to focus wholly on IoT. Called Mindsett, this sub-brand is focused on using AI and machine learning (ML) to detect asset operational features and potential failures, enabling the automation of many FM and compliance processes.

An asset in the context of FM industry generally refers to an operational equipment or building. The asset operation features [5] are important aspects to describe the working condition of an asset, for example, the number of washing cycles of an industrial dishwasher or the heating/cooling working mode of an air conditioning unit. If we can build asset models in combination with their operation features, we can understand the asset performance much better.

In terms of commercial values, these features can be combined to solve problems in the areas of abnormality detection [12], energy saving, predictive maintenance [14] etc. For example, by combining the washing cycle feature with its actual electricity consumption, we can get the energy consumption per cycle. Consequently we can find abnormal cases with exceptionally high/low consumption, or figure out the energy efficient cases by bench-marking all cases together, comparing daily or weekly operations of the same type of asset across multiple sites. The we can provide suggestions to clients to determine where savings can be.

In this project, wireless sensors are designed to monitor asset data via the IoT platform. They can be fitted on the assets and collect data in real-time. To acquire the asset operation features, a simple solution would be to deploy the ideal sensors on assets for acquiring the desired data/signals. This makes the process of data analysis simple and is generally reliable since the data comes directly from the operation action we are concerned about monitoring. But in the actual applications, there should be a balance between the number of sensors we can deploy and the information to measure. To get the desired information, theoretically, we can have as many sensors as needed. While in the actual cases, on hardware level, we need to consider the costs of installation and maintenance of the deployed sensors. On the platform level, we need to consider the volume of data storage and the bandwidth for data transmission/streaming channels. With these considerations in mind, we find that there is a need for the operation detection based on data aggregation methods.

In terms of the operation detection algorithms [12], we can generally group them into two categories. The first category can be identified as methods related to machine learning algorithms, e.g. long short-term memory (LSTM) [6], dynamic time wrapping (DTW) [7], etc. These approaches are easily scalable and flexible, as we can handle the operation changes as the language from assets and consider it as a way of natural

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language processing (NLP) [9]. The main drawbacks with these approaches are that, firstly before training the learning models, we need to have the labelled data ready, which is itself an important but difficult task. Secondly, we can only get the detection result after reading the whole time-range of sensor data/signal, which might be the source of detection delay/lag.

On the other hand, we can design the detection algorithms by using signal processing methods. The main idea behind these is aggregating/transforming the data into new features which may directly/indirectly enable us to detect the operation features we want. These aggregation/transformations can be done in either frequency domain [10] or time domain. The methods in frequency domain is more suitable for the offline analysis since they require more computation power and longer time-range of data to get the wider spectrum of features. While the methods in time-domain can be efficient in both offline and real-time analysis.

With the above consideration, in this paper, we will consider the asset operation detection algorithms in the time-domain. The transformation will be done using a joint space called phase portrait (PP) [3]. This provides clear correlation between different signal channels and also gives direct interpretation for the dynamics of different asset operations. In combination of the transformation analysis, we will use fuzzy logic (FL) [11] to define the flexible boundary for different asset operation features extracted by using PP. To the authors' knowledge, this is the first work which outlines a methodology combining FL with PP for the topic of asset operation detection.

The rest of this paper will be organised as follows. Firstly in Section II, we will provide brief preliminary for the PP method and concept of FL. Then in Section III, the PP and FL based detection methodology will be explained in details. Following the method, in Section IV we will verify the result in terms of accuracy, detection delay and complexity. Finally, we will go back to the concerns related to business insight and environmental impact, and apply the operation detection result to further analyse the asset performance.

II. Preliminary

A. Preliminary on Phase Portrait

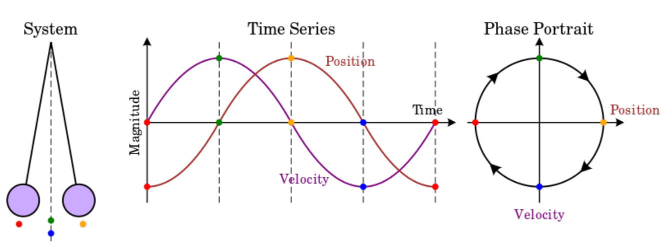


Fig. 1: Illustration of how a phase portrait would be constructed for the motion of a simple pendulum.

The concept of *phase portrait* (PP) is most commonly used to study the directional behaviour of a dynamic system. It gives the geometric representation of the time-domain trajectories of a dynamical system. From the data analysis prospective, we can consider it as the correlation plot of multiple data features. A special property of PP is that there is a clear directional connection among all the scattered points.

As we can see from Figure 1, for a simple pendulum system, we have the time-domain measures for the position and velocity of the ball. By plotting these two measures on different axes of a joint coordinate, we can get the PP representation of the pendulum dynamic. In the continuous-time case, the trajectory on PP will also be continuous. In the discrete-time case, the plot will be scattered points with directional connection.

It's worth mentioning that, if we define the position and velocity as x-y axes of the coordinate system, a point on PP will always move clockwise on its trajectory, see Figure 1. With this characteristic, it will be easier for us to predict the evolvement of time-series data or sensor signals.

B. Preliminary on Fuzzy Logic

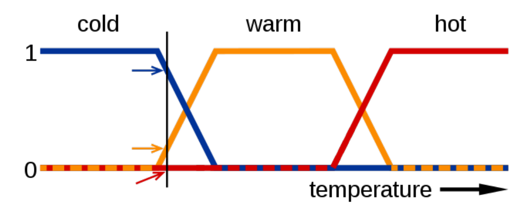


Fig. 2: Illustration describing the level of temperature with fuzzy logic

Contrary to boolean logic, where the degree of truth values of variables may only be the integer of 0 or 1, *fuzzy logic* (FL) is employed to handle the concept of partial truth, where the degree truth value may be any real number between 0 and 1, both inclusive [4].

At first it may seem similar to the concept of probability in mathematics, but fuzzy logic uses degrees of truth as a mathematical model of vagueness, while probability is a mathematical model of ignorance [8]. A basic application of fuzzy logic might be characterizing the various sub-ranges of a continuous variable.

For example, in Figure 2, the meanings of expressions "cold", "warm", and "hot" are fuzzy sets represented by *membership functions* mapping a temperature scale. A point on the temperature scale has three "truth values", one for each of the three fuzzy sets: "cold", "warm", and "hot". The vertical line in Figure 2 represents the outputs (truth values where the line intersects) of the three membership functions at a particular temperature. It is obvious that a low temperature has higher value for "cold", while a high temperature has higher value for "hot".

The membership functions $\mathcal{M}_i(x)$ ($i = 1, ..., n$, x is the concerned variable) for n sub-ranges expressions can be constructed based on their possibility distribution, but the following properties should always hold:

$$0 \leq \mathcal{M}_i(x) \leq 1, \quad \text{for } i = 1, ..., n \tag{1}$$

$$\sum_{i=1}^{n} \mathcal{M}_i(x) = 1. \tag{2}$$

C. Operation Detection for a Dishwasher

In this paper we are concerned about the operation for an industrial dishwasher, see Figure 3. In each of its washing cycles, it will firstly start with washing mode and then change to the heating mode (see Figure 5). From the deployed IoT sensing platform (see Figure 4), we now have the current values from the three phases of electricity power supply cables,

$$\begin{aligned} p_1(k) &:= \text{current value of phase 1, in } \textit{Amps} \\ p_2(k) &:= \text{current value of phase 2, in } \textit{Amps} \\ p_3(k) &:= \text{current value of phase 3, in } \textit{Amps} \end{aligned} \tag{3}$$

For simplicity, through out this paper, we may express $p_i(k)$ (for $i = 1, 2, 3$) as p_i. At a glance, a sample period of the average value can be represented as:

$$p_{\text{average}}(k) := (p_1 + p_2 + p_3)/3.$$

In Figure 5, it shows the signal output $p_{\text{average}}(k)$ of three washing cycles. We have partially labelled the signal by using a camera to record video footage of asset usage pattern. In each washing cycle in Figure 5, the blue colour means that the dishwasher is undertaking the washing operation, the yellow colour means that the dishwasher is undertaking the heating operation.

Fig. 3: An industrial dishwasher at the client restaurant

Our objective is to design the efficient processing algorithm so that the different operation modes can be detected in real-time from the three phases current values in (3).

III. Main Methodology

In this part, we will analyse the dishwasher sensing signals in details and propose two different approaches for the detection algorithm design. For the first approach, we will directly use the multi-phase signals in equation (3) which we get from the electricity cables. As we are encouraged to achieve detection functionality with reduced sensor installation cost, in the second approach, we will propose an alternative

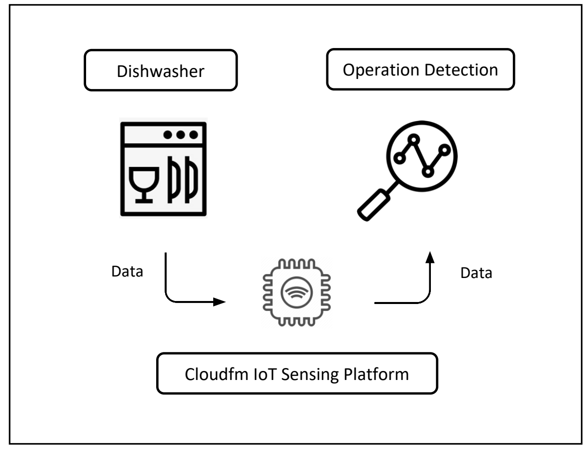


Fig. 4: Diagram of the IoT sensing platform at Cloudfm

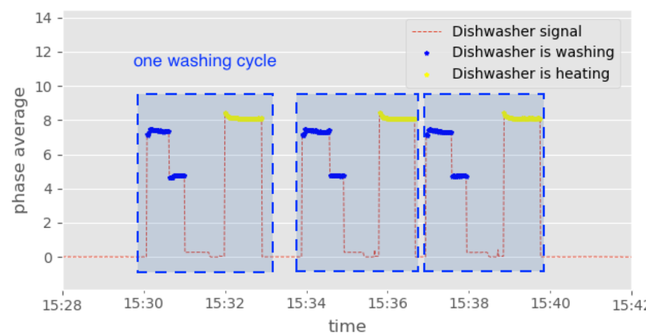


Fig. 5: Dishwasher signal with labelled heating and washing operation fractions

detection algorithm based on the value of only one phase of the electricity cables. Both approaches will combine the concept of FF to define the boundaries for different operation patterns.

A. Approach One: Detection Based on Physical Phases

From Figure 5, we can get the general impression of different operation features. For heating operation, the average value tends to be high at a static value. While for the washing operation, the average value tends to be varying among a certain range of values. To get better insight, firstly let us visualise all the three phases of signals behind the operations within a certain time range, see Figure 6.

By looking into the values of individual phases, we may find that, for phases 1 and 2, their values for different operation modes tend to be static with less variation, which makes it easier for us to capture the pattern differences. For phase 2, both the heating and washing operations will result in a high static value. While for phase 1, both operations will result in none-zero static values, but the value for heating operation will be relatively higher. Intuitively we may simply regard the high value in phase 1 as heating operation, and the the medium high values as washing operation. But in the practical case, it is actually not that easy to separate the time-ranges with different

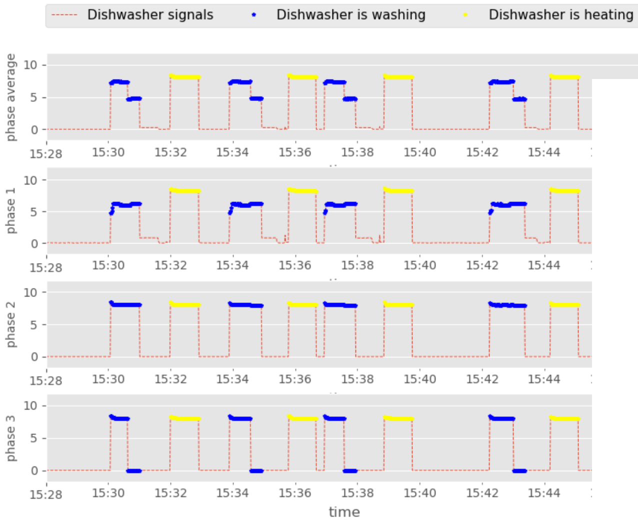


Fig. 6: Dishwasher signals of the three phases of elect cables with labelled operations

sensor values. Since the sensor signal is changing continuous, a low value will firstly go through the medium value before reaching its highest values.

To clearly describe the differences of heating operation and washing operation, we may view the values of phases 1 and 2 in the proposed PP in Figure 7, which shows both phases 1 and 2 as dimensions on the same plot. With the additional dimension from phase 1, we can now easily differentiate the clustering areas for washing operation and heating operation, see Figure 7.

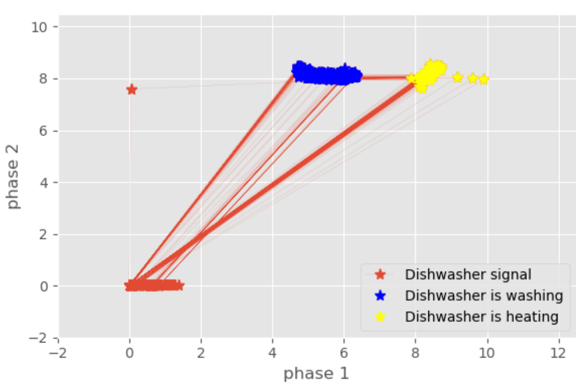


Fig. 7: Dishwasher signal trajectory in a 2-dimensional PP with phase 1 and phase 2 as the coordinate axes

To double check whether the data from phase 3 will further differentiate different operations, we can also view all the three phases in a 3-dimensional PP. From Figure 8, we can see that phase 3 is not necessary as it only separates the data from the same operations. Instead of making it easier for the detection analysis, phase 3 will only add unnecessary complexity. Thus we will conduct the following analysis based the two-dimensional PP in Figure 7, which involves just phases 1 and 2.

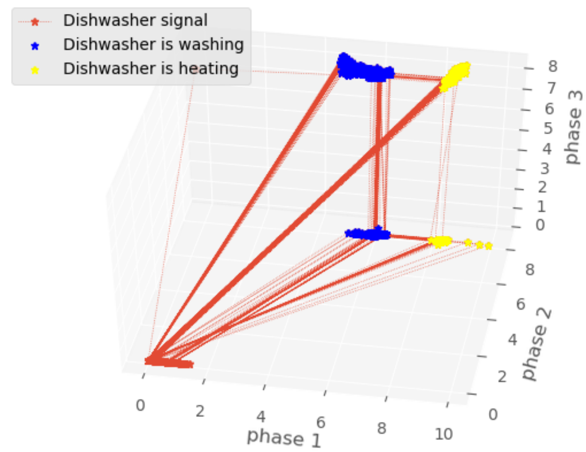


Fig. 8: Dishwasher signal trajectory in a 3-dimensional PP with phases 1, 2, and 3 as the coordiate axes

To further clearly define the boundaries for these concerned operation areas, we will embed the concept of *fuzzy sets* and *fuzzy interface system (FIS)* into our analysis.

In this research, we will firstly design the FIS and then identify the corresponding fuzzy set membership functions. Following the above reasoning process, the FIS will be directly described using If-Then rules as follows:

IF phase 1 is high and phase 2 is high
  THEN the dishwasher is under heating operation (4)
IF phase 1 is medium high and phase 2 is high
  THEN the dishwasher is under washing operation (5)
IF other cases
  THEN the dishwasher is under other operations (6)

We define the fuzzy sets based on the actual statistical distribution of the dishwasher's operation data in the three months period between July and September 2019. Firstly, we generate the kernel density estimation (KDE) [13] for the washing and heating operations on phases 1 and 2, see Figures 9.

For simplicity and efficient processing, we define the membership functions

$$\mathcal{M}_{1h}(p_1) : \text{phase 1 is high} \quad (7)$$
$$\mathcal{M}_{1mh}(p_1) : \text{phase 1 is medium high} \quad (8)$$
$$\mathcal{M}_{2h}(p_2) : \text{phase 2 is high} \quad (9)$$

as trapezoidal functions and trapezoidal shoulder functions, see Figure 10, where p_1 (or $p_1(k)$) and p_2 (or $p_2(k)$) are the values for signals phase 1 and phase 2 (at sample point k).

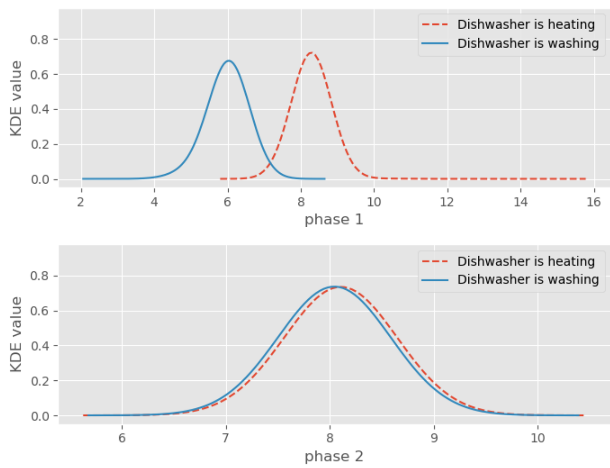


Fig. 9: Kernel density estimation for the washing and heating operations on phase 1 and phase 2

$$\mathcal{M}_{1h}(p_1) = \begin{cases} 1 & p_1 > 7.7 \\ p_1 - 6.7 & 7.7 \geq p_1 > 6.7 \\ 0 & 6.7 \geq p_1 \end{cases} \tag{10}$$

$$\mathcal{M}_{1mh}(p_1) = \begin{cases} 0 & p_1 > 7.7 \\ 7.7 - p_1 & 7.7 \geq p_1 > 6.7 \\ 1 & 6.7 \geq p_1 > 5.1 \\ (p_1 - 3.9)/1.2 & 5.1 \geq p_1 > 3.9 \\ 0 & 3.9 \geq p_1 \end{cases} \tag{11}$$

$$\mathcal{M}_{2h}(p_2) = \begin{cases} 1 & p_2 > 7.5 \\ p_2 - 6.5 & 7.5 \geq p_2 > 6.5 \\ 0 & 6.5 \geq p_2 \end{cases} \tag{12}$$

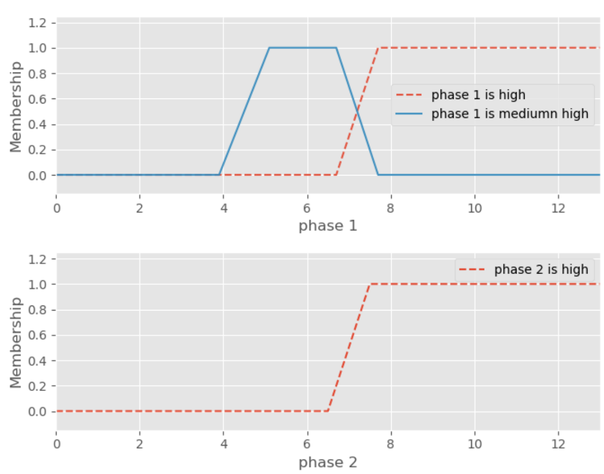
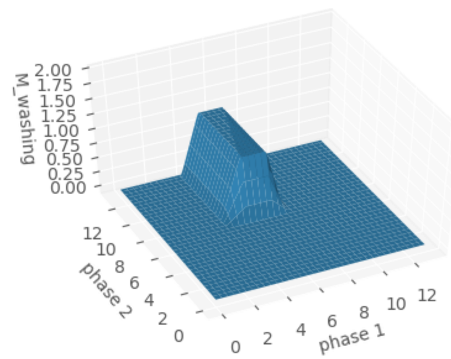


Fig. 10: Membership functions for fuzzy sets $\mathcal{M}_{1h}(p1)$, $\mathcal{M}_{1mh}(p1)$ and $\mathcal{M}_{2h}(p2)$

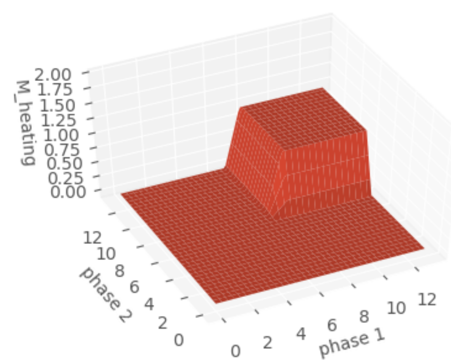
Following the fuzzy logic in rules (4) and (5), we can get the output membership functions for the washing operation and heating operation, see the surface plots in Figure 11.

$$\mathcal{M}_{\text{washing}}(p_1, p_2) = \mathcal{M}_{1h}(p_1) \cdot \mathcal{M}_{2h}(p_2) \tag{13}$$

$$\mathcal{M}_{\text{heating}}(p_1, p_2) = \mathcal{M}_{1mh}(p_1) \cdot \mathcal{M}_{2h}(p_2) \tag{14}$$



(a) Output membership function for washing: $\mathcal{M}_{\text{washing}}(p_1, p_2)$



(b) Output membership function for heating: $\mathcal{M}_{\text{heating}}(p_1, p_2)$

Fig. 11: Output membership functions surface plots presented on the phase portrait

Considering the membership function property in (2), we can get the output membership function for other undefined operations as

$$\mathcal{M}_{\text{others}}(p_1, p_2) = 1 - \mathcal{M}_{\text{washing}}(p_1, p_2) - \mathcal{M}_{\text{heating}}(p_1, p_2) \tag{15}$$

For the final detection method, we use the $\arg\max$ function in the defuzzification process and choose the operation with the highest membership values as the output. The final detection output will be

$$\textbf{Operation}: \quad \arg\max_{x \in \mathcal{S}} \mathcal{M}_x(p_1, p_2) \tag{16}$$

$$\textbf{Possibility}: \quad \max_{x \in \mathcal{S}} \mathcal{M}_x(p_1, p_2) \tag{17}$$

for the given values (p_1, p_2), where

$$\mathcal{S} := \{\text{washing}, \ \text{heating}, \ \text{others}\}$$

B. Approach Two: Detection Based on Derivative Phases

As we have mentioned in the introduction, one motivation for the investigation of data/algorithm based detection techniques is to reduce the number of deployed sensors. In this sense, we may consider whether we can further reduce the data source needed for the operation detection algorithm. Here we will analyse the detection algorithm based only on one phase of the dishwasher signals.

In the previous sub-section III-A, it is clear that the signal from phase 1 plays more important role in the detection algorithm. Thus we choose phase 1 as the raw data source. To construct a conceptual phase portrait similar to that in sub-section III-A, we follow the idea in Figure 1 and create a virtual phase $\dot{p}_1$ as the derivative of phase 1. In discrete-time case, it can be obtained as:

$$\dot{p}_1(k) := p_1(k) - p_1(k-1).$$

Then the layout of dishwasher signal and labelled operations can be represented as shown in Figure 12.

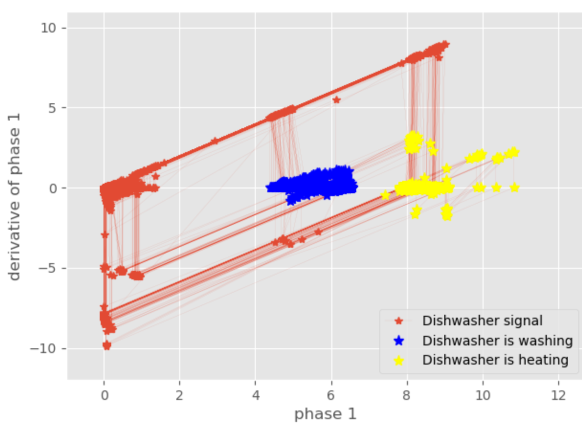


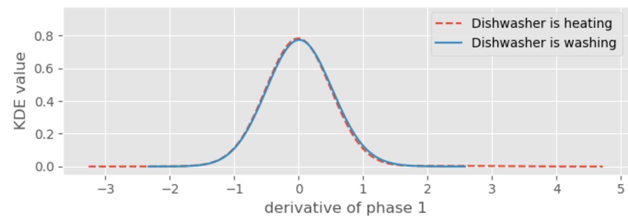
Fig. 12: Dishwasher signal trajectory in a 2-dimensional PP with phase 1 and its derivative as the coordinate axes

Correspondingly, the fuzzy interface system for washing operation and heathing operation can be designed as

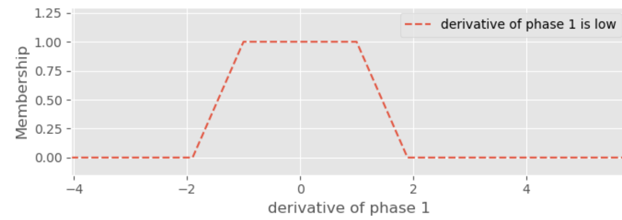
IF phase 1 is high and its derivative is low
THEN the dishwasher is under heating operation (18)
IF phase 1 is medium high and its derivative is low
THEN the dishwasher is under washing operation (19)
IF other cases
THEN the dishwasher is under other operations (20)

As for the membership functions, $\mathcal{M}_{1h}(p_1)$ and $\mathcal{M}_{1mh}(p_1)$ can be designed the same as that in (10) and (11). The construction of the additional membership function

$$\mathcal{M}_{d1l}(\dot{p}_1) : \text{derivative of phase 1 is low} \tag{21}$$



(a) KDE for the washing & heating operations on $\dot{p}_1$



(b) Membership function for $\mathcal{M}_{d1l}(\dot{p}_1)$

Fig. 13: Kernel density estimation and membership function on $\dot{p}_1$

will be based on the KDE distribution of washing and heating operations on the dimension: derivative of phase 1 $\dot{p}_1(k)$, See Figure 13.

The exact expression of $\mathcal{M}_{d1l}(\dot{p}_1)$ will be

$$\mathcal{M}_{d1l}(\dot{p}_1) = \begin{cases} 0 & \dot{p}_1 > 1.9 \\ (1.9 - \dot{p}_1)/0.9 & 1.9 \geq \dot{p}_1 > 1 \\ 1 & 1 \geq \dot{p}_1 > -1 \\ (\dot{p}_1 + 1.9)/0.9 & -1 \geq \dot{p}_1 > -1.9 \\ 0 & -1.9 \geq \dot{p}_1 \end{cases} \tag{22}$$

Based on the FIS rules in (18) and (19), the output membership functions for washing operation and heating operation can be obtained as

$$\mathcal{M}_{\text{washing}}(p_1, \dot{p}_1) = \mathcal{M}_{1h}(p_1) \cdot \mathcal{M}_{d1l}(\dot{p}_1) \tag{23}$$
$$\mathcal{M}_{\text{heating}}(p_1, \dot{p}_1) = \mathcal{M}_{1mh}(p_1) \cdot \mathcal{M}_{d1l}(\dot{p}_1) \tag{24}$$

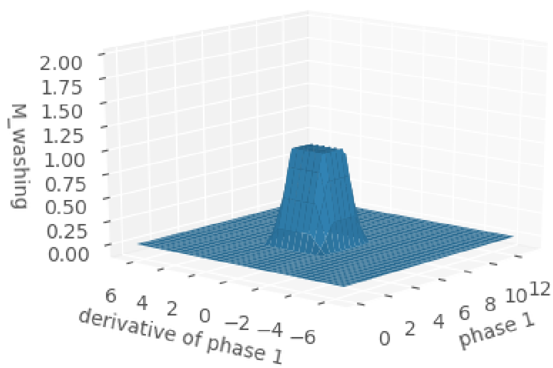
The output membership function for other undefined operations can be derived from (2) as:

$$\mathcal{M}_{\text{others}}(p_1, \dot{p}_1) = 1 - \mathcal{M}_{\text{washing}}(p_1, \dot{p}_1) - \mathcal{M}_{\text{heating}}(p_1, \dot{p}_1). \tag{25}$$

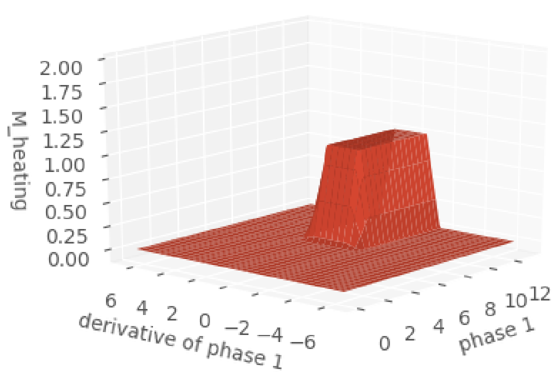
For the final detection method, we use the $\arg\max$ function in the defuzzification process and choose the operation with the highest membership values as the output:

$$\textbf{Operation}: \quad \arg\max_{x \in \mathcal{S}} \mathcal{M}_x(p_1, \dot{p}_1) \tag{26}$$
$$\textbf{Possibility}: \quad \max_{x \in \mathcal{S}} \mathcal{M}_x(p_1, \dot{p}_1) \tag{27}$$

for the given values $(p_1, \dot{p}_1)$, where $\dot{p}_1(k) := p_1(k) - p_1(k-1)$ in discrete-time case.



(a) Membership function for washing: $\mathcal{M}_{\text{washing}}(p_1, \dot{p}_1)$



(b) Membership function for heating: $\mathcal{M}_{\text{heating}}(p_1, \dot{p}_1)$

Fig. 14: Membership functions presented on the phase portrait

IV. Verifying the results

As we can see, the result mentioned in the previous section will be based only on the sensor signal values $(p_1(k), p_2(k))$ at time-step k, or two-step values $(p_1(k), p_1(k-1))$ at time-steps k and $k-1$. In this sense, there will be less delay in the detection process, as there is no need to go through the whole operation cycles before recognizing the operation pattern, (as in LSTM and DTW). Also, there will be less computation load in the detection process as less points are involved to calculate the distance/similarity. In our future work, the precise comparison test will be considered.

In terms of the accuracy, the result will be highly reliable (100% except for the transitional points) if there is no unexpected disturbance involved in the signals. Additional filters [15] can be combined to further improve the detection results in the case of signal uncertainty or disturbance. Since this is out of our research focus in this article, we will not elaborate on the details here.

V. Application of the detection results

By observing the dishwasher signals labelled with washing and heating operations and recorded operational footage, we noticed that for each washing cycle, there was a continuous period of washing operation. As a result, the detection result for washing operation can be used to count the number of washing cycles each day, see Figure 15 for the washing cycle detection results by a physical-phases approach. With this type of operation information we can further analyse the business operational statistics and even asset performance in terms of carbon footprint.

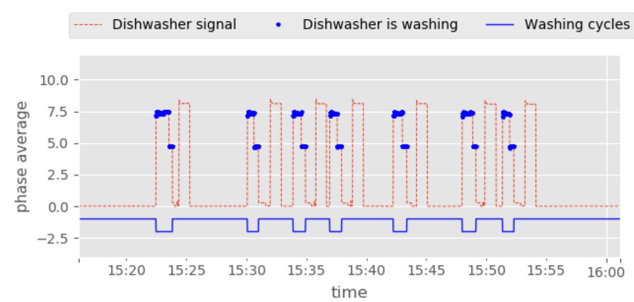


Fig. 15: Dishwasher phase average signal with labelled washing operation and washing cycles (lower value on the line indicate the washing cycles)

For example, we can categorise the daily number of washing cycles based on the weekdays to see which weekday has relatively more customers dining in the restaurant and thus make suggestions to optimize the stock storage and staff service, see Figure 16.

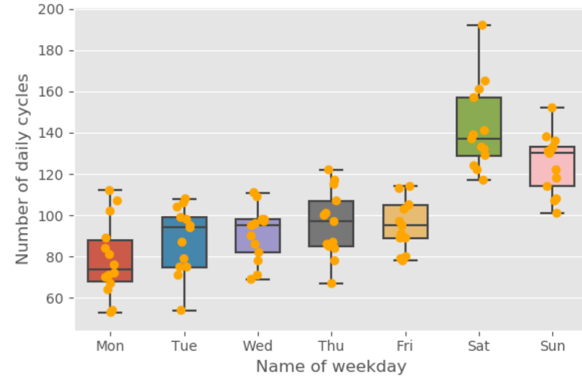


Fig. 16: Dishwasher daily washing cycles statistics by weekdays, where weekend shows higher number than weekdays, during the period July 2019 to September 2019

On the other hand we can also analyse the dishwasher performance based on the correlation between daily energy consumption and daily number of washing cycles. For example, in Figure 17 we plot the correlation between number of cycles and daily consumption on a joint plane. From the scattered points we can obtain a regression model as

$$Y = 214.99 + 7.66X \tag{28}$$

Generally we can determine that, the minimum energy to keep the dishwasher on for a day will be about 215 kWh. In addition to that, each washing cycle will consume about 7.66 kWh on average. By these key performance factors, we can compare the efficiency of different dishwashers over multiple sites, and make procurement suggestions based on the size of client restaurants. Meanwhile, if a scattered point on Figure 17 is far away from the regression model in (28), it might be a potential abnormal case due to a fault in the equipment or incorrect usage behaviour. In addition, we can also analyse the energy saving strategy by focusing on the scattered points underneath the regression line. By identifying the common usage behaviour behind the low consumption cases, we can optimise the daily operations and make bigger impact on reducing the carbon footprints.

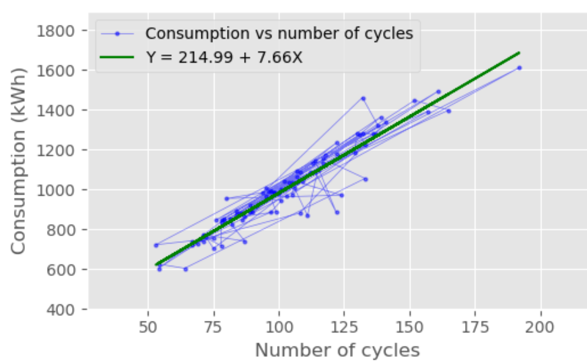


Fig. 17: Correlation and regression model between dishwasher daily energy consumption and number of washing cycles, during the period July 2019 to September 2019

VI. Conclusions

In this paper, around the context of FM industry, we have provided an effective asset operation detection solution for the time series data from business assets of Cloudfm clients. Based on the concepts of phase portrait (PP) and fuzzy logic (FL), the algorithm can instantly detect asset operation status based on multiple or single physical sensor signal(s) input. The algorithm will be helpful in terms of reducing sensors deployment costs. Compared to alternative machine learning approaches like LSTM and DTW, this solution will be more computationally efficient since less steps of sample points is involved in the calculation. In addition, the combined FL concept will provided reasonable possibility estimation for the detection result. Finally, brief explanation has been provided to showcase the application of operation detection results in terms of business insight and environmental impact.

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