Essays on Financial Markets with Asymmetric Information

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Summary

Chapter 1 analyzes a model of multiple overconfident traders submitting market orders where traders' private information is subject to correlated errors. We consider two standard types of overconfidence: overestimation of the trader's own information (κ-overconfidence) and underestimation of other traders' information (η-overconfidence). The analyses on the effects of overconfidence suggest that trading volume increases with κ-overconfidence and decreases with η-overconfidence. In addition, whereas κ-overconfidence causes trading volume to rise at a steeper pace with the number of traders so that large trading volume can occur with a small extent of κ-overconfidence, η-overconfidence may cause trading volume and price informativeness to decrease with the number of traders.

Chapter 2 presents an extension of the model considered in Chapter 1 to endogenous information. The analyses show that κ-overconfidence and the endogeneity of information lead to two qualitatively distinct equilibria in information acquisition. One of which features information aggregation with a sufficient number of traders, whereas the other one does not.

Chapter 3 analyzes a model of financial markets featuring heterogeneities in asset valuations and private information where a fraction of informed investors trading on the asset value incur transaction costs, as well as its variant with market power. We consider how reducing such costs affects the welfare of uninformed investors, which changes through their trading opportunities and learning from the price. In the basic model with price-taking investors, reducing the transaction costs makes uninformed investors better off if and only if these uninformed investors have a sufficiently large liquidity shock. Given any positive size of liquidity shock of uninformed investors, this is the case with sufficiently

large trading volume of the cost-bearing investors. In the variant with a large investor who is subject to his own liquidity shock, reducing the transaction costs may increase the welfare of uninformed investors even without their liquidity shock due to improved market liquidity.

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Contents

Chapter 1

Overconfidence and Correlated Information Structures: Exogenous Information

1.1 Introduction

A lot of evidence supports the argument that traders in financial markets tend to be overconfident about their valuation and trading skills (e.g., Barber and Odean, 2001; Statman, Thorley and Vorkink, 2006), and even financial professionals are not the exception (Gloede and Menkhoff, 2014). This naturally generates a disagreement over the asset value, thereby motivating speculative trade. Such argument is consistent with the fact that investors who trade the most appear to lose from trade (e.g., Odean, 1999). This has been proposed as an explanation for the fact that the world-wide volume of trades in financial markets is too large to be justified only by rational investors' risk-hedging motive.[1] A line of analytic studies develop various equilibrium results regarding the effects of overconfidence on trading volume and price informativeness, largely supporting these observations (Benos, 1998; Garcia, Sangiorgi and Urosevic, 2007; Ko and Huang, 2007; Kyle and Wang, 1997; Odean, 1998).

[1] See Daniel and Hirshleifer (2015) for this fact in detail, as well as a comprehensive overview of the literature. Also, note that overconfidence is not the only explanation as there are other forms of disagreement (e.g., Eyster, Rabin and Vayanos, 2019).

Building on these well-established results, this study analyzes further implications of overconfidence in financial markets by deviating from the literature in the following ways. First, the presence of correlated signal errors is considered for the analyses of overconfident traders. While the established results are based on a particular class of information structures involving the independence of errors of private signals, it is unknown whether these results extend to information structures with correlated signal errors. On the practical side, the presence of correlated signal errors is not exceptional in the real world. For example, the presence of public sources of information makes it more likely that investors are subject to a common error.[2] That is, investors recognize that they all are subject to such common error but they all believe that they are above average in processing their information. Then what is the role of overconfidence in affecting their behavior and forming the price? On the theoretical side, the complexity of information structures generates a different mechanism whereby overconfidence affects the market equilibrium. In particular, in contrast to information structures with independent errors where traders' private information is solely associated with the asset value, the presence of correlated errors makes it possible that each trader forms an expectation on others' private information, as well as their behavior. Naturally, such expectation is influenced by a biased belief about the structure of traders' private information caused by overconfidence. This illustrates the way in which overconfidence interacts with complex information structures to affect traders' behavior and the formation of the price, even apart from their own valuation. Overall, these suggest that consideration of correlated information structures could be more than a technical footnote put on the established results in the literature.

Second, this study examines whether overconfidence changes the properties of trading volume and price informativeness. While the established results are mostly quantitative in the sense that they tell us how overconfidence affects the market equilibrium "other things being equal," it is still unclear whether its effects are strong enough to generate testable differences in the qualitative properties of trading volume and price informativeness. In particular, their properties with respect to market size are relevant for the trends of

[2]See Han and Sangiorgi (2018) for a microfoundation for information structures with positively correlated errors.

globalization and financial technology, which tend to lower the entry barrier and thereby increase competition in financial markets. For example, the notion of price informativeness allows us to address the question of whether these trades affect the informational role of markets, which is connected with the conventional wisdom that the price aggregates information as markets grow large. Therefore, of particular interest here is whether and how overconfidence influences these properties regarding trading volume and price informativeness.

In this regard, this study considers a market with overconfident traders whose private information involves correlated errors. The market is oligopolistic in the sense that there are finitely many strategic traders, each of whom is large enough to affect the market price through his trading activity. As in previous studies in the literature, joint normality of relevant random variables is assumed to make the analyses tractable. I consider a class of symmetric information structures regarding traders' private signals: It is common knowledge that their signal errors are correlated each other's with a common correlation coefficient, but traders agree to disagree about the overall precision of their private signals. This study considers two types of overconfidence: Under the first type of overconfidence (κ-overconfidence), each trader believes that his own private signal is more precise than its true precision, whereas, under the second type of overconfidence (η-overconfidence), each trader believes that others' private signals are noisier than their true precision.

The trading mechanism of the model is a variant of Kyle's (1985) static model of strategic traders. At the trading stage, multiple strategic traders and noise traders simultaneously submit market orders and then the asset price is determined by market makers based on the total demand. This framework captures the situation where market participants' trading activities affects the market price. As in Kyle (1985) and many other studies in the literature, the main trade-off facing these traders is between an incentive to take profit opportunities by trading the asset and a disincentive to reveal private information through their demands. This tradeoff determines each trader's optimal demand for the asset.

The main focus of the analyses is the effects of the aforementioned types of overconfidence on trading volume and price informativeness. After providing a characterization of equilibrium for three cases of rational, κ-overconfident and η-overconfident traders (Proposition 1.3), we proceed to examine how overconfidence affects trading volume, price informativeness and their qualitative properties with respect to the number of traders, depending on the type of overconfidence.

The first set of main results answers the question of how overconfidence affects trading volume in the presence of correlated signal errors. Proposition 1.5 offers an answer to this question: On the one hand, κ-overconfidence leads to an increase in trading volume, and the positive effect of κ-overconfidence on trading volume is even higher when signal errors are correlated. On the other hand, η-overconfidence causes a decrease in trading volume. This stems from the fact that η-overconfidence and the correlation of signal errors interact with each other to weaken each trader's incentive to trade the asset following his private signal. These suggest that the relationship between overconfidence and trading volume qualitatively depends on the type of overconfidence and information structures. Overall, overconfidence does not necessarily increase trading volume in the presence of noise traders, in contrast to the intuitive idea that disagreement leads to more trade in general.[3] On the empirical side, these results can explain the mixed empirical and experimental findings regarding the relationship between overconfidence and trading volume (e.g., Biais, Hilton, Mazurier and Pouget, 2005; Fellner-Rohling and Krugel, 2014; Glaser and Weber, 2007; Merkle, 2017).

The second set of main results concerns whether and how the qualitative properties of trading volume are changed with overconfidence. In particular, the properties of trading volume with regard to the number of traders are considered in Proposition 1.6 for all three cases. In the benchmark case, trading volume increases with the number of traders and it goes to infinity at the rate of square roots. In contrast, in the case of κ-overconfident traders with correlated errors, trading volume increases with the number of traders as well, but at a faster rate of convergence toward infinity than that of square roots as in the benchmark case. As a result, it goes to infinity as the number of traders goes to a

[3] In the absence of noise traders, no-trade theorem ensures that this argument holds in a wide variety of information structures (e.g., Milgrom and Stokey, 1982).

finite value. This illustrates that, in the presence of correlated errors, large trading volume can occur with an arbitrary extent of κ-overconfidence and a sufficient number of traders. This stands in contrast to the benchmark case where large trading volume occurs only for a large number of traders or a strong extent of κ-overconfidence. Thus, large trading volume, which is observed occasionally in the real world, can be more easily explained with the combination of κ-overconfidence and correlated information structures even without a strong extent of κ-overconfidence. In the case of η-overconfident traders with correlated errors, neither of these properties of trading volume generally hold: Trading volume does not approach infinity as the number of traders grows large, and it even decreases with the number of traders when the extent of η-overconfidence is sufficiently high. These suggest that the aforementioned negative effect of η-overconfidence on trading volume can be large enough to change the qualitative properties of trading volume with respect to the number of traders.

The third set of main results considers the quantitative and qualitative effects of overconfidence on price informativeness, which measures the quality of information contained in the price. As price informativeness increases with the proportion of trades carried out by strategic traders relative to those by noise traders, which is positively associated with trading volume, the effects of overconfidence on price informativeness immediately follow from the above results regarding trading volume. As expected, Proposition 1.9 shows that κ-overconfidence increases price informativeness, whereas η-overconfidence decreases price informativeness. Regarding the qualitatitve side, Proposition 1.10 considers two well-known properties of price informativenessness: (i) Price informativeness rises in more competitive environments, and (ii) Price informativeness converges to the level at which all available information is aggregated as the market approaches perfect competition. This indicates that η-overconfidence not only decreases price informativeness but also changes the properties regarding price informativeness and undermines the conventional wisdom that promoting competition leads to a better functioning of the price.

Combined with the next chapter, whose analyses naturally follow from the current one, the main contribution of this work is to show that correlation in signal errors implies that two different types of overconfidence provide radically different implications regarding trading volume, price informativeness, and the incentive to acquire information. In particular, underestimation of others' private information leads to lower trading volume and price informativeness, whereas overestimation of the trader's own private information leads to higher trading volume and price informativeness. These implications stem from the formation of higher-order belief about each other's behavior caused by the presence of correlation in signal errors. Also, such difference turns out to be rather "large" in terms of the qualitative pace of changes in trading volume and price informativeness following an increase in the number of traders as well as the extent of information acquisition in large markets. Overall, these results stress the importance of the interplay between overconfidence and information structures in explaining various observable patterns of trading volume and price informativeness.

1.2 Related literature

The first line of literature related to this study is the theoretical literature on overconfidence and other related biases in financial markets. The literature focuses on how overconfidence affects market variables, such as trading volume and price informativeness, other things being equal. To address this question, Odean (1998) considers three settings of trading mechanism: (i) infinitely many traders taking the price as given, (ii) one strategic insider who submits market order to the market maker together with noise traders, and (iii) infinitely many traders identical to the first setting but involving costly information acquisition before the trading stage. The most consistent result here is on the relationship between overconfidence and trading volume: an increase in traders' overconfidence unambiguously leads to an increase in trading volume in the first and second models. Even though there is a possibility that this result is reversed in the third model with costly information acquisition, it is unclear whether this counterexample involves some general intuition or comes from idiosyncracies of the details of the model. On the other hand, the relationship between overconfidence and price informativeness depends on the details of trading mechanism: In the first model with price takers, overconfidence reduces price informativeness, whereas in the second model with a strategic insider, overconfidence increases price informativeness. The second model with a single insider is extended by Benos (1998) and Kyle and Wang (1997) to the case of multiple strategic traders, and they both give rise to similar results regarding comparative statics with respect to overconfidence. Other previous studies address whether these conclusions continue to hold with endogenous information acquisition, and they generally indicate that the effects of overconfidence on price informativeness vary across competitive and informational environments, whereas the positive effect of overconfidence on trading volume is still robust (Garcia, Sangiorgi and Urosevic, 2007; Ko and Huang, 2007). More recently, Eyster, Rabin and Vayanos (2019) consider "cursedness", which refers to traders not fully appreciating what prices convey about others' price information, and compare it with other biases including κ- and η-overconfidence considered here.[4] As in this study, they analyze whether these biases possibly generate large trading volume as the number of traders grows large, and show that the answer is positive only on cursedness. Despite the differences in many aspects of the model, this study is largely complementrary to their main results, as explained in Subsection 1.4.3 in detail. While this paper borrows much of its analytic framework from this literature, two takeaway points from the literature are (i) consideration of correlated signal errors and (ii) the analyses of its qualitative implications on price informativeness.

Another line of relevant literature is the literature on the informational role of market prices. It dates back to Hayek (1945), who argues that the superiority of market mechanism comes from its ability to aggregate the dispersed information of agents in the economy. Motivated by his informal argument, many previous studies formally examine whether price informativeness increases as markets grow large, and it converges towards the level at which the price aggregates all traders' private information available in the market. There is a substantial disagreement in the literature. On the one hand, many previous studies ask whether how price informativeness changes with market size when traders' private information is exogenously given. Rostek and Weretka (2012) and

[4] In their terminology, "overconfidence" corresponds to κ-overconfidence and "dismissiveness" refers to a set of different types of biases including η-overconfidence.

Rostek and Weretka (2015) consider a variant of Kyle's (1989) model which accounts for heterogeneity in correlation of trader values. The former characterizes the range of information structures where information aggregation is guaranteed, while the latter examines the condition under which price informativeness monotonically increases with market size. Lambert, Ostrovsky and Panov (2018) is more closely related to this study in that it generalizes Kyle's (1985) model to consider essentially arbitrarily correlations among the random variables involved in the model, such as the asset value, traders' and market makers' signals, and noise traders' demand. Their main result shows that information aggregation occurs in large markets under the mild condition that liquidity demand is positively correlated or uncorrelated with the asset value. In addition, the same question has also been addressed in the context of multi-unit auctions (e.g., Atakan and Ekmekci, 2014; Pesendorfer and Swinkels, 1997; Wilson, 1977). These studies generally suggest that information aggregation occurs in a wide variety of information structures, and the failure of information aggregation observed in reality could rather be attributed to other economic circumstances, such as the endogeneity of asset value (Atakan and Ekmekci, 2014). In contrast to these studies based on the assumption of exogenous information, if information is costly and endogenous, a well-known argument makes it difficult to interpret the price as information aggregators (e.g., Grossman and Stiglitz, 1980). If traders are more informed about the true value of the asset, the price then reflects their private information so that it becomes closer to the true value of the asset. But then traders now have lower incentives to acquire their information, essentially imposing an upper bound on price informativeness in any possible equilibrium. This argument appears to hold in a wide variety of trading mechanism, including Kyle (1985), and thus, most previous studies abstract from this issue by adopting the assumption of exogenous information. Compared to these previous studies, this paper considers both types of models with and without information acquisition and shows that these two conclusions can be altered with two types of overconfidence considered here. In particular, η-overconfidence can lead to a failure of information aggregation even with exogenous information, whereas κ-overconfidence raises the possibility that information aggregation occurs with endogenous information.

This study is also related to the literature on strategic complementarities in information acquisition. They have been a subject of long-standing interest in the literature, raising the possibility of multiple equilibria, as in this study's extended model. In the context of linear-quadratic global games, players' information acquisition activities are strategic complements when their actions are strategic complements (e.g., Hellwig and Veldkamp, 2009; Myatt and Wallace, 2012) and consideration of these activities yields a number of implications regarding the externalities across players and the social value of public information (e.g., Colombo, Femminis and Pavan, 2014). More closely related to this study are previous studies on information acquisition in financial markets. Multiple potential channels have been identified as the causes of complementarities in information acquisition: information about supply (Ganguli and Yang, 2009), multiple-dimensional uncertainty (Goldstein and Yang, 2015), decision makers' learning from prices (Dow, Goldstein and Guembel, 2017), and heterogeneous valuation (Rahi and Zigrand, 2018). The idea of complementarities in information acquisition is associated with the extreme amplification of shocks to asset prices, as information production may cease with small shocks to fundamentals. In line with these previous studies, this study offers a different mechanism whereby complementarities in information acquisition arise, possibly explaining fragility in financial markets, and this mechanism is distinct from previous ones in that it connects strategic complementarities in information with κ-overconfidence, which is one of the most widely-accepted types of possible deviation from the common prior assumption.

1.3 Basic model

A security is traded in a market whose value θ is not initially known to market participants. It is common knowledge that the asset value θ follows a normal distribution with mean θ_0 and variance σ_0^2, i.e., $\theta \sim N(\theta_0, \sigma_0^2)$. There are N strategic traders, noise traders, and competitive market makers in the market.

The trading game proceeds as follows: At the first stage, strategic and noise traders submit their demand to the market. Each strategic trader i's demand is denoted by x_i, and noise traders' demand is exogenously given by ω, where ω is normally distributed with mean zero and variance σ_ω^2, i.e., $\omega \sim N(0, \sigma_\omega^2)$. At the second stage, competitive market makers observe the total demand $X = \sum_{i=1}^{N} x_i + \omega$ and subsequently set the price P. At the final stage, the true value θ of the security is realized, and accordingly, each strategic trader i obtains profit $\pi_i = x_i(\theta - P)$.

At the first stage, each strategic trader i observes a private signal given by $s_i = \theta + \varepsilon_i$. The true distribution of these signal errors is given by $Var(\varepsilon_i) = \sigma_\varepsilon^2$ and $Corr(\varepsilon_i, \varepsilon_j) = \rho$ for every i and j where $i \neq j$. While the correlation coefficient ρ is common knowledge, strategic traders may agree to disagree about the precision of their signals. In particular, we consider three cases of strategic traders: The first case refers to "rational" traders who agree on the precision of their signals. In the second case, traders are "κ-overconfident" in the sense that each trader i perceives the variance of his own signal s_i to be smaller than the true variance by factor $\frac{1}{\kappa}$, i.e., $\frac{\sigma_\varepsilon^2}{\kappa}$, where $\kappa > 1$ represents the extent to which each trader overestimates his own signal's precision. Traders are unbiased about other traders' signal variances in that other traders' signal variances are perceived as the true variance σ_ε^2. In the third case, traders are "η-overconfident" in the sense that each trader i perceives the variance of all other traders' signals to be larger than the true variance by factor $\frac{1}{\eta}$, i.e., $\frac{\sigma_\varepsilon^2}{\eta}$, where $\eta \in (0,1)$ represents the extent to which each trader underestimates the precision of other traders' signals. In all these cases, market makers have the true belief on the distribution of these signals.

Definition 1.1. An equilibrium consists of demand functions $\{x_i(s_i)\}_{i=1,\cdots,N}$ and pricing rule $P(X)$ where the following two conditions hold:

1. At the second stage, the price set by market makers is equal to the expected value of the security conditional on X, i.e., $P = E[\theta|X]$;

2. For every trader i, for every realization of signal s_i, demand $x_i(s_i)$ maximizes his expected profit conditional on his beliefs on the distribution of signals, the pricing rule and the profile of other traders' strategies.

As in Kyle (1985) and many other previous studies in the literature, this study focuses on the class of linear equilibria.[5]

[5]It is largely an open question whether a unique linear equilibrium of the model is also unique in a wider

Definition 1.2. Equilibrium $(\{x_i(\cdot)\}_{i=1,\cdots,N}, P)$ is linear if demands x_i are linear in s_i, and pricing rule P is also linear in total demand X.

1.4 Analyses of the basic model

Throughout the section, we let the prior, each trader's signal precision, and noise trading (i.e., σ_0^2, σ_ε^2, and σ_ω^2) be fixed, and they turn out to be not critical in the analyses. In addition, we restrict attention to the case where $\sigma_\varepsilon^2 < \sigma_0^2$. This assumption rules out the possibility of the uninteresting no-equilibrium result due to infinite trading of strategic traders who are highly κ-overconfident, as noted by Kyle and Wang (1997) and Odean (1998).

1.4.1 Equilibrium

The following proposition proves the existence of a unique equilibrium of the game and characterizes its trading coefficient for each of three cases in the model (i.e., rational, κ-overconfident, and η-overconfident traders):

Proposition 1.3. *Fix $\kappa > 1$, $\eta \in (0,1)$, $\rho \in [0,1]$ and $N \geq 1$. Then the following statements hold true:*

(1) *In the benchmark case of rational traders, there is a unique equilibrium of the game. In the equilibrium, trader i submits $x_i^* = \beta_R^*(s_i - \theta_0)$, where his trading coefficient is given by*

$$\beta_R^* = \sqrt{\frac{\sigma_\omega^2}{N\left(\sigma_0^2 + \sigma_\varepsilon^2\right)}}.$$

(2) *In the case of κ-overconfident traders, there exists a unique equilibrium if and only if $N \in [1, \bar{N}_K)$, where $\bar{N}_K$ is defined as*

$$\bar{N}_K := 1 + \frac{\sigma_0^2 + \left(\frac{2}{\kappa} - 1\right)\sigma_\varepsilon^2}{\rho\left(1 - \frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2}$$

class of demand functions. See Mclennan, Monteiro and Tourky (2017) and Rochet and Vila (1994) for the case of a single strategic trader.

for $\rho > 0$, and it takes infinity for $\rho = 0$.

Whenever such an equilibrium exists, trader i submits $x_i^* = \beta_K^*(s_i - \theta_0)$, where his trading coefficient is given by

$$\beta_K^* = \sqrt{\frac{\sigma_\omega^2}{N\left\{\sigma_0^2 + \left(\frac{2}{\kappa} - 1\right)\sigma_\varepsilon^2 - \rho(N-1)\left(1 - \frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2\right\}}}.$$

(3) *In the case of η-overconfident traders, there is a unique equilibrium of the game. In the equilibrium, trader i submits $x_i^* = \beta_E^*(s_i - \theta_0)$, where his trading coefficient is given by*

$$\beta_E^* = \sqrt{\frac{\sigma_\omega^2}{N\left\{\sigma_0^2 + \sigma_\varepsilon^2 + \rho(N-1)\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\varepsilon^2\right\}}}.$$

The first case of rational traders serves as the benchmark case throughout the analyses in the following subsections. This case can be regarded as a special case of Lambert, Ostrovsky and Panov (2018). In contrast, in the second case of κ-overconfident traders, the existence of equilibrium is guaranteed only in the absence of signal error correlation (i.e., $\rho = 0$). If traders' signal errors are correlated (i.e., $\rho > 0$), the proposition implies that a unique equilibrium exists if and only if the number of traders is not too large (i.e., $N < \bar{N}_K$). Finally, in the third case of η-overconfident traders, there exists a unique equilibrium for every $N \geq 1$, whether traders' signal errors are correlated or uncorrelated.

Following Definitions 1.1 and 1.2, an equilibrium is determined by the following steps, as standard in the literature following Kyle (1985): First, given a conjecture on traders' behavior (i.e., $x_i^* = \beta(s_i - \theta_0)$), the resulting price is linear in total demand X, i.e.,

$$P = \theta_0 + \lambda X,$$

where its updating coefficient λ is given by[6]

$$\lambda = \frac{1}{N\beta}\frac{\sigma_0^2}{\sigma_0^2 + \frac{1+\rho(N-1)}{N}\sigma_\varepsilon^2 + \frac{\sigma_\omega^2}{N^2\beta^2}}. \tag{1.1}$$

[6]Equation (1.1) is derived in the proof of Proposition 1.3.

Second, correctly recognizing that the price depends on his submitted demand, i.e.,

$$P_i(x_i) = \theta_0 + \lambda(x_i + T_i + \omega),$$

where $T_i := \sum_{j \neq i} \beta(s_j - \theta_0)$ is the sum of other strategic traders' demands, each trader i chooses demand x_i so as to maximize his expected profit given by

$$E[\pi_i | \mathscr{I}_i] = E[x_i \{\theta - P_i(x_i)\} | \mathscr{I}_i].$$

In particular, plugging the price function $P_i(x_i)$ into the maximand in the above condition, and then solving its first-order condition, we obtain trader i's best response as follows:

$$B_i(\beta) = \frac{1}{2\lambda} E[\theta - \theta_0 | \mathscr{I}_i] - \frac{1}{2} E[T_i | \mathscr{I}_i].$$

Note that, as each trader's signal is decomposed into the asset value and his signal error, T_i is decomposed into two terms, i.e.,

$$\begin{aligned} T_i &= \sum_{j \neq i} \beta(s_j - \theta_0) \\ &= \sum_{j \neq i} \beta(\theta - \theta_0) + \sum_{j \neq i} \beta \varepsilon_j := T_{i\theta} + T_{i\varepsilon}, \end{aligned}$$

where $T_{i\theta}$ and $T_{i\varepsilon}$ represent the true-value component of other traders' signals and the error component of these signals, respectively.. Plugging this into trader i's best response as above, we can see that an equilibrium is formed at the fixed point of each trader i's best response function given by

$$B_i(\beta) = \frac{1}{2\lambda} E[\theta - \theta_0 | \mathscr{I}_i] - \frac{1}{2} \beta(N-1) E[\theta - \theta_0 | \mathscr{I}_i] - \frac{1}{2} \beta(N-1) E[\varepsilon_j | \mathscr{I}_i], \quad (1.2)$$

where trader i's expectation terms are obtained as follows: In the benchmark case, we have

$$\begin{aligned} E[\theta-\theta_0|\mathscr{I}_i] &= \frac{Cov(s_i,\theta-\theta_0)}{Var(s_i)}(s_i-\theta_0) = \frac{\sigma_0^2}{\sigma_0^2+\sigma_\varepsilon^2}(s_i-\theta_0)\,; \\ E[\varepsilon_j|\mathscr{I}_i] &= \frac{Cov(s_i,\varepsilon_j)}{Var(s_i)}(s_i-\theta_0) = \frac{\rho\sigma_\varepsilon^2}{\sigma_0^2+\sigma_\varepsilon^2}(s_i-\theta_0)\,. \end{aligned}$$

In the case of κ-overconfident traders, these terms are given by

$$\begin{aligned} E[\theta-\theta_0|\mathscr{I}_i] &= \frac{Cov(s_i,\theta-\theta_0)}{Var(s_i)}(s_i-\theta_0) = \frac{\sigma_0^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}(s_i-\theta_0)\,; \\ E[\varepsilon_j|\mathscr{I}_i] &= \frac{Cov(s_i,\varepsilon_j)}{Var(s_i)}(s_i-\theta_0) = \frac{\rho\frac{1}{\sqrt{\kappa}}\sigma_\varepsilon^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}(s_i-\theta_0)\,, \end{aligned}$$

and, in the case of η-overconfident traders, they are given by

$$\begin{aligned} E[\theta-\theta_0|\mathscr{I}_i] &= \frac{Cov(s_i,\theta-\theta_0)}{Var(s_i)}(s_i-\theta_0) = \frac{\sigma_0^2}{\sigma_0^2+\sigma_\varepsilon^2}(s_i-\theta_0)\,; \\ E[\varepsilon_j|\mathscr{I}_i] &= \frac{Cov(s_i,\varepsilon_j)}{Var(s_i)}(s_i-\theta_0) = \frac{\rho\frac{1}{\sqrt{\eta}}\sigma_\varepsilon^2}{\sigma_0^2+\sigma_\varepsilon^2}(s_i-\theta_0)\,. \end{aligned}$$

In Equation (1.2), the first term represents how trader i's optimal demand is affected by a tradeoff between the marginal cost from raising the price (i.e., λ) and the marginal benefit of increasing demand. On the other hand, the second and third terms represent trader i's expectation of other traders' demands, which is divided into the true-value component in the second term and the error component in the third term.

It is noteworthy that an equilibrium is non-existent if traders are κ-overconfident and the number of traders is large enough. Such a non-existence outcome occurs due to the fact that each trader has an infinite incentive to trade the asset regardless of other traders' trading strategies, so that there is no fixed point where each trader's best response equals the supposed trading coefficient. This is in line with Kyle and Wang (1997) and Odean (1998). However, while the non-existence outcome is attributed to highly overconfident traders in these previous papers, the proposition indicates that it is attributed to a large number of traders in the market. Indeed, even when traders have an arbitrarily small extent of overconfidence (i.e., κ just above one), the non-existence outcome possibly occurs with a sufficiently large number of traders. The intuition behind this outcome is as follows: As traders are κ-overconfident, each existing trader believes that the price is subject to a systematic error which is involved in other traders' private signals but not in his own signal, and thus, he (incorrectly) expects to make a further profit from the systematic error, in addition to that from noise traders, by trading the asset following his private signal. When a new trader enters the market, each existing trader believes that the price is now even more strongly correlated with the systematic error, rather than by the true value of the asset. This generates a further incentive for these traders to trade the asset. As the number of traders is large enough (i.e., $\bar{N}_K$), such complementarities eventually cause these traders to trade infinitely.

1.4.2 Effect of overconfidence on trading volume

In this subsection, we examine how overconfidence affects trading volume in equilibrium. Formally, trading volume is defined as follows:

Definition 1.4. Let $\rho \in [0,1]$ and $N \geq 1$ be given. Trading volume is defined the sum of the expected absolute values of strategic and noise traders' demands as follows:

$$TV(\rho, N) = N \cdot E\left[|x_i^*|\right] + E\left[|\omega|\right].$$

The below proposition compares trading volume in the case of overconfident traders with that in the case of rational traders:

Proposition 1.5. *Fix $\kappa \in (1,\infty)$, $\eta \in (0,1)$, $\bar{\rho} \in (0,1]$ and $N \geq 2$. Also, $\bar{N}_K$ is defined in Proposition* 1.3 *as the upper bound of N under which a unique equilibrium exists under κ-overconfidence. Then the following statements hold true:*

(1) *If $N < \bar{N}_K$, κ-overconfidence always increases trading volume, whether $\rho = 0$ or $\rho = \bar{\rho}$. Also, such effect is higher for $\rho = \bar{\rho}$ than for $\rho = 0$.*

(2) *If $\rho = 0$, η-overconfidence does not affect trading volume. In contrast, if $\rho = \bar{\rho}$, η-overconfidence decreases trading volume.*

Note that Kyle and Wang's (1997) model of two insiders corresponds to a special case of $\rho = 0$ and $N = 2$ in the model. As in their main result, when traders' signal errors are independent (i.e., $\rho = 0$), κ-overconfidence increases trading volume, whereas η-overconfidence doesn't affect trading volume. Thus, one could argue that the overall effect of overconfidence on trading volume is positive when both types of overconfidence coexist, leading to the prevailing argument in the literature that overconfidence increases trading volume.

In contrast, the proposition tells us that in the presence of correlated signal errors (i.e., $\rho = \bar{\rho}$), the effect of overconfidence on trading volume depends on the type of overconfidence: the positive effect of κ-overconfidence on trading volume is even higher than that for $\rho = 0$, whereas η-overconfidence strictly decreases trading volume. In order to get the intuition of these results, note first that in all three cases, each trader's best response is given by Equation (1.1) in the previous subsection. Without loss of generality, consider a high signal (i.e., $s_i > \theta_0$). With regard to the effect of κ-overconfidence, we can see that κ-overconfidence moves the best response function upward by (i) decreasing $Var(s_i) = \sigma_0^2 + \frac{\sigma_\varepsilon^2}{\kappa}$, which increases all three terms in Equation (1.2) through scaling up $E[\theta - \theta_0|\mathscr{I}_i]$ and $E[\varepsilon_j|\mathscr{I}_i]$,[7] and (ii) decreasing $Cov(s_i, \varepsilon_j) = \rho \frac{1}{\sqrt{\kappa}}\sigma_\varepsilon^2$, which decreases the third term in Equation (1.2). The first effect moves the best response upward (assuming that the best response is positive). The interpretation is that a negative bias in the unconditional signal variance causes an increase in the sensitivity of updating other variables (i.e., $\theta - \theta_0$ and ε_j) in response to a unit change in the signal, and this scales up the trader's best response. The second effect moves the best response upward as well. The interpretation is that a trader's overestimation of his signal precision leads to a negative bias in the covariance between his signal and others' errors, and such bias causes him to underestimate others' errors, which increases the best response. Given that an equilibrium is formed at the fixed point of the best response function, an upward movement of the best response

[7]This effect leads to opposite signs of term-by-term changes to the best response in Equation (1.2), making it difficult to interpret how changes in $Var(s_i)$ affect the best response. In particular, while it increases the first term, which increases the best response, its effects on the second and third terms decrease the best response. However, since all three terms are scaled up by the same proportion, the overall effect through these terms is to increase the best response.

caused by κ-overconfidence leads to an increase in each trader's trading coefficient β^* (i.e., from β_R^* to β_K^* in Proposition 1.3) in equilibrium, which increases trading volume.

With regard to the effect of η-overconfidence, we can see from Equation (1.2) that η-overconfidence moves the best response function downward by increasing $Cov(s_i, \varepsilon_j) = \rho \frac{1}{\sqrt{\eta}} \sigma_\varepsilon^2$, which decreases the third term in Equation (1.2). The interpretation is that a trader's underestimation of others' signal precision leads to a positive bias in the covariance between his signal and others' errors, and such bias causes him to overestimate others' errors, which decreases the best response. Given that an equilibrium is formed at the fixed point of the best response function, a downward movement in the best response caused by η-overconfidence leads to a decrease in each trader's trading coefficient β^* in equilibrium, which decreases trading volume.

In sum, the opposite signs of the effects of two types of overconfidence on trading volume stem from traders' inference about others' errors (i.e., the third term in Equation (1.2)). The definition of κ-overconfidence ("knowing the value better") means that each trader believes that his own signal is less correlated with others' errors. This implies that his signal is also less correlated with the price, which leads him to trade more. On the contrary, the definition of η-overconfidence ("others knowing the value less") means that each trader believes that his own signal is more correlated with others' signal errors. This implies that his signal is also more correlated with the price, which leads him to trade less.

With regard to empirical relevance, the proposition predicts the presence of heterogeneity across markets regarding trading volume and its relationship with overconfidence. Indeed, it indicates that excessive trade is not the only possible outcome from overconfidence. In the case of η-overconfident traders, inefficiencies do not arise from excessive trade, but rather from the lack of trade. This is interpreted as illiquidity or market breakdown, and thus stands in contrast to disagreement about the asset valuation, whose underlying intuition on its positive effect on trading volume appears to be unambiguous by no-trade theorem (Milgrom and Stokey, 1982). On the other hand, trading volume increases with κ-overconfidence even more as traders' signal errors are correlated with each other. Therefore, it could be argued that the relationship between overconfidence and trading volume greatly differs across information structures and the types of overconfidence. Even if empirical evidence supports the positive effect of overconfidence on trading volume on average, this could be due to the prevalence of a particular form of overconfidence, such as κ-overconfidence, and information structures.

Further, the proposition sheds light on empirical and experimental literature on the link between overconfidence and trading volume. Even though early strong results support the argument that overconfidence is a reason for high trading volume, these results largely hinge on the use of proxies for overconfidence, such as gender (e.g., Barber and Odean, 2001) and past returns (e.g., Statman, Thorley and Vorkink, 2006). In contrast, recent studies measure overconfidence more directly using survey data to capture individuals' beliefs and appear to provide mixed evidence on the relationship between overconfidence and trading volume. These studies employ so-called miscalibration scores, which are obtained by asking individuals to state confidence intervals for a number of general knowledge questions requiring numerical answers and then measuring the extent to which their confidence intervals are too narrow. Even though miscalibration scores seem to be the closest to the types of overconfidence considered in analytic studies in the literature, including "κ-overconfidence" in this study, these studies consistently find that such miscalibration scores do not increase trading volume even though they decrease trading profits (e.g., Biais, Hilton, Mazurier, Pouget, 2005; Fellner-Rohling and Krugel, 2014; Glaser and Weber, 2007; Merkle, 2017). On the other hand, a different type of overconfidence focusing on positive self-illusions, which is called the better-than-average effect, appears to be related to trading volume (e.g., Glaser and Weber, 2007; Merkle, 2017). Also, Fellner-Rohling and Krugel (2014) propose an alternative measure to capture misconception of signal reliability based on the past observation of signals and actual outcomes and show that the proposed measure increases trading volume. We can see that the mixed findings on miscalibration scores can be explained with the proposition above. Even if miscalibration scores correspond to κ-overconfidence, which increases trading volume, it is expected that these scores are also strongly associated with η-overconfidence, which decreases trading volume. Therefore, we conclude that the link between overconfidence and trading volume is unlikely to be clear without identifying the form of overconfident beliefs about ability.

1.4.3 Effect of overconfidence on properties of trading volume

Now we study the qualitative side, focusing on how overconfidence affects the properties of trading volume with regard to the number of traders.

Proposition 1.6. *Fix $\kappa \in (1,\infty)$, $\eta \in (0,1)$ and $\bar{\rho} \in (0,1]$. Also, $\bar{N}_K$ is defined in Proposition* 1.3 *as the upper bound of N under which a unique equilibrium exists under κ-overconfidence. Then the following statements hold true:*

(1) *In the benchmark case and the cases of κ- and η-overconfident traders with $\rho = 0$, trading volume increases with N, and goes to infinity at the rate of $\sqrt{N}$ as $N \to \infty$.*

(2) *In the case of κ-overconfident trader with $\rho = \bar{\rho}$, trading volume increases with N, and goes to infinity as $N \to \bar{N}_K$.*

(3a) *In the case of η-overconfident traders with $\rho = \bar{\rho}$ with $\eta \in \left(\left(\frac{\bar{\rho}\sigma_\varepsilon^2}{\sigma_0^2+\sigma_\varepsilon^2+\bar{\rho}\sigma_\varepsilon^2}\right)^2, 1\right)$, trading volume increases with N, and converges to a finite value as $N \to \infty$.*

(3b) *In the case of η-overconfident traders with $\rho = \bar{\rho}$ with $\eta \in \left(0, \left(\frac{\bar{\rho}\sigma_\varepsilon^2}{\sigma_0^2+\sigma_\varepsilon^2+\bar{\rho}\sigma_\varepsilon^2}\right)^2\right)$, trading volume decreases with N, and converges to a finite value as $N \to \infty$.*

Two properties of trading volume are considered: A new trader's entry increases trading volume, and it approaches infinity in large markets. In the benchmark case, these properties hold true. Note first that the best response function in Equation (1.2) changes with N. In particular, a new trader's entry has a positive effect on the best response by the first term but also has negative effects on it by the second and third terms in Equation

(1.2).[8] Canceling out three terms in Equation (1.2), we have

$$\begin{aligned} BR_i(\beta) &= \frac{N\beta}{2}\left\{\sigma_0^2 + \frac{1+\rho(N-1)}{N}\sigma_\varepsilon^2 + \frac{\sigma_\omega^2}{N^2\beta^2}\right\}\frac{1}{\sigma_0^2+\sigma_\varepsilon^2}(s_i-\theta_0) - \frac{(N-1)\beta}{2}\frac{\sigma_0^2+\rho\sigma_\varepsilon^2}{\sigma_0^2+\sigma_\varepsilon^2}(s_i-\theta_0) \\ &= \frac{\beta}{2}(s_i-\theta_0) + \frac{1}{2}\frac{1}{N\beta}\frac{\sigma_\omega^2}{\sigma_0^2+\sigma_\varepsilon^2}(s_i-\theta_0). \end{aligned}$$

This implies that trader i submits demand $x_i = \beta_i(s_i-\theta_0)$ where

$$\beta_i = \frac{\beta}{2} + \frac{1}{2}\frac{1}{\sigma_0^2+\sigma_\varepsilon^2}\frac{\sigma_\omega^2}{N\beta}. \tag{1.3}$$

Equation (1.3) represents the optimal level of demand equalizing the marginal cost of raising the price and the marginal benefit of increasing demand. Without noise (i.e., $\sigma_\omega^2 \to 0$) or with infinite trading volume (i.e., $N\beta \to \infty$), the marginal cost of raising the price is strong enough to make sure that each trader i individually has an incentive to reduce trade, as demonstrated by the fact that the first term in Equation (1.3) is less than β. However, the presence of noise weakens such incentive, thereby moving up the best response. As trading volume (i.e., $N\beta$) is small enough, the price is noisy enough to cause each trader i to trade as much as others (i.e., β). An equilibrium given by coefficient β_R^* is formed at the fixed point of Equation (1.3), i.e., $\beta_i = \beta$. Now we can see that the fixed point β_R^* goes to zero at the rate of $\frac{1}{\sqrt{N}}$. As N grows large, β_R^* goes to zero as the proportion of noise becomes marginal, as reflected in the second term in Equation (1.3). However, if the pace of its convergence toward zero is too fast, the proportion of noise (reflected in $\frac{\sigma_\omega^2}{N\beta}$ in the second term in Equation (1.3)) becomes significant enough to cause trade. As a result, β_R^* goes to zero at the rate of $\frac{1}{\sqrt{N}}$. This immediately implies that trading volume (i.e., $N\beta_R^*$) increases toward infinity at the rate of $\sqrt{N}$.

These properties have a general intuition, rather than being specific to the details of the model. Intuitively, as the number of traders increases, each trader's market power, which is denoted by λ in Equation (1.2), decreases, given other traders' demand, which is

[8] The interpretation is as follows: Given that different traders' signals are partially substitutes, the entry decreases the existing traders' contribution to market makers' inferences, causing a decrease in the slope of price function (i.e., λ) and thus increasing the first term in Equation (1.2). On the other hand, the entry negatively affects the best response by the second and third terms in Equation (1.2) due to an increase in the expectation of other traders' demands.

represented by the second and third terms in Equation (1.2). It follows that each trader's incentive to trade increases given trading volume. This leads to an increase in trading volume. Further, trading volume eventually goes to infinity. Otherwise, the price would be still noisy due to a significant proportion of noise traders in trading volume, causing a significant incentive to trade for each trader. This leads to a contradiction.

Now we consider how κ-overconfidence changes the properties of trading volume. Applying κ-overconfidence to each trader i's best response in Equation (1.2) and then arranging the terms, we have

$$BR_i(\beta) = \frac{\sigma_0^2+\sigma_\varepsilon^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}\left(\frac{\beta}{2}+\frac{1}{2}\frac{1}{N\beta}\frac{\sigma_\omega^2}{\sigma_0^2+\sigma_\varepsilon^2}\right)(s_i-\theta_0)+\frac{1}{2}\beta(N-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\frac{\rho\sigma_\varepsilon^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}(s_i-\theta_0).$$

This yields trader i's optimal demand as follows:

$$\beta_i = \frac{\sigma_0^2+\sigma_\varepsilon^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}\left(\frac{\beta}{2}+\frac{1}{2}\frac{1}{N\beta}\frac{\sigma_\omega^2}{\sigma_0^2+\sigma_\varepsilon^2}\right)+\frac{1}{2}\beta(N-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\frac{\rho\sigma_\varepsilon^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}. \tag{1.4}$$

As noted in Subsubsection 1.4.2, there are two effects of κ-overconfidence on the best response: (i) scaling up the best response, and (ii) an additional term coming from traders' biased belief on $Cov(s_i, \varepsilon_j)$. An equilibrium given by β_K^* is formed at the fixed point of Equation (1.4), i.e., $\beta_i = \beta$. Note that Equation (1.4) is qualitatively different from Equation (1.3) for the benchmark case due to the second term in Equation (1.4), which represents traders' biased belief on $Cov(s_i, \varepsilon_j)$. As this term grows large proportionally as N grows large, it qualitatively changes the limiting property of β_K^* with regard to N. Indeed, if β_K^* were proportional to $\frac{1}{\sqrt{N}}$ as in the benchmark case, the first term in Equation (1.4) would be also proportional to $\frac{1}{\sqrt{N}}$, but the second term in Equation (1.4) explodes to infinity, causing the best response to explode as well. As a result, under κ-overconfidence, the second term in Equation (1.4) causes trading volume (i.e., $N\beta_K^*$) to go up toward infinity even faster than the rate of $\sqrt{N}$.

In the case of η-overconfident traders, applying η-overconfidence to Equation (1.2) yields

$$BR_i(\beta) = \left(\frac{\beta}{2}+\frac{1}{2}\frac{1}{N\beta}\frac{\sigma_\omega^2}{\sigma_0^2+\sigma_\varepsilon^2}\right)(s_i-\theta_0)-\frac{1}{2}\beta(N-1)\left(\frac{1}{\sqrt{\eta}}-1\right)\frac{\rho\sigma_\varepsilon^2}{\sigma_0^2+\sigma_\varepsilon^2}(s_i-\theta_0).$$

This leads to

$$\beta_i = \frac{\beta}{2}+\frac{1}{2}\frac{1}{N\beta}\frac{\sigma_\omega^2}{\sigma_0^2+\sigma_\varepsilon^2}-\frac{1}{2}\beta(N-1)\left(\frac{1}{\sqrt{\eta}}-1\right)\frac{\rho\sigma_\varepsilon^2}{\sigma_0^2+\sigma_\varepsilon^2}. \tag{1.5}$$

As noted in Subsubsection 1.4.2, η-overconfidence affects the best response with an additional term coming from traders' biased belief on $Cov(s_i,\varepsilon_j)$. An equilibrium given by β_E^* is formed at the fixed point of Equation (1.5), i.e., $\beta_i=\beta$. Similar to the case of κ-overconfident traders, the last term in Equation (1.5), which represents traders' biased belief on $Cov(s_i,\varepsilon_j)$, grows large proportionally with N and thus qualitatively changes the limiting property of β_E^* with regard to N. If β_E^* were proportional to $\frac{1}{\sqrt{N}}$ as in the benchmark case, the first and second terms in Equation (1.5) would be also proportional to $\frac{1}{\sqrt{N}}$, but the last term explodes to (negative) infinity, causing the best response to explode negatively as well. This implies that a new η-overconfident trader's entry increases trading volume (i.e., $N\beta_E^*$) at a slower rate compared to the benchmark case. If parameter η is sufficiently low, such entry even decreases trading volume.

In sum, these results generally indicate that overconfidence leads to a wide range of predictions regarding the properties of trading volume. In the benchmark case, the prevailing argument in the literature holds true in that trading volume increases with market size and becomes large in large markets. In contrast, in the case of η-overconfident traders, trading volume cannot be arbitrarily large in large markets and may even decrease with a new trader's entry. This stands in contrast to the conventional wisdom that promoting competition leads to more trading activities and thus is generally desirable for market efficiencies. On the other hand, κ-overconfidence makes it even more likely that large trading volume occurs as markets grow large. Overall, these imply that the properties of trading volume can vary across different markets in reality, depending on the type of overconfidence and information structures.

In order to stress the qualitative side of the positive effect of κ-overconfidence on trading volume, it is useful to compare between its effects with correlated errors (i.e., $\rho = \bar{\rho}$) and independent errors (i.e., $\rho = 0$), the latter of which has long been addressed in the literature (e.g., Kyle and Wang, 1997; Odean, 1998). The results on κ-overconfidence with independent errors indicate that the rate of convergence of trading volume toward infinity is of square roots, as in the benchmark case. That is, κ-overconfidence only increases trading volume roughly by multiplying constant. This implies that a moderate extent of κ-overconfidence does not cause such large trading volume by itself. This appears to cause a difficulty in explaining large trading volume in the real world given the intuitive observation that market participants show overconfidence on average but to a moderate degree and working experience indeed reduces overconfidence (e.g., Gloede and Menkhoff, 2014). In contrast, the results on κ-overconfidence with correlated errors suggest that κ-overconfidence increases the rate of convergence of trading volume toward infinity, and that inexplicably large trading volume occurs even with a moderate extent of κ-overconfidence and a sufficient number of traders. This makes large trading volume more likely to occur in equilibrium. Overall, large trading volume in the real world can be more easily explained with these results on κ-overconfidence with correlated errors.

It is worth comparing these results with Eyster, Rabin and Vayanos' (2019) results on the pattern of trading volume with respect to the number of traders. As mentioned in Subsection 1.2, they consider the case where traders are cursed in the sense that they do not fully appreciate the informational content of the price. Using a different trading mechanism where a finite number of price-taking traders submit price-contingent demands and the standard assumption of independent errors, they show that cursedness generates infinite trading volume in large markets, whereas κ- and η-overconfidence do not. The intuition behind their results is as follows: As the number of traders grows large, despite their biased beliefs, κ- and η-overconfident traders recognize that the price fully reveals the average signal of all other traders. In large markets, therefore, these traders base their expectations of the asset payoff almost exclusively on the price, and as a result, the difference between any two traders' expectations converges to zero and so does their per-trader volume. In contrast, cursed traders give their signal nonnegligible weight even when the number of traders grows large so that the price reveals the average signal of other traders. As a result, cursed traders' per-trader volume does not converge to zero in large markets. Of course, these appear to be rather different from Proposition 1.6 in that infinite trading volume occurs in large markets in this study even in the benchmark case of rational traders, mainly due to the differences in the framework of modeling trade in markets.[9] Nevertheless, Proposition 1.6 appears to be complementary to their results based on the assumption of independent errors in the sense that even κ-overconfidence could easily generate large trading volume which would be difficult to be explained with the assumption of rational traders if it is combined with correlated errors.

1.4.4 Effect of overconfidence on price informativeness and its properties

This subsection addresses the question of how overconfidence affects price informativeness, which measures the quality of information contained in the price. Formally, we define price informativeness as follows:

Definition 1.7. Let $\rho \in [0,1]$ and $N \geq 1$ be given. Price informativeness is defined as the precision (or the inverse of variance) of the asset value conditional on the price as follows:

$$PI(\rho, N) = \{Var(\theta|P)\}^{-1}.$$

Note that most previous studies (e.g., Rahi and Zigrand, 2018; Rostek and Weretka, 2012) define it slightly differently by normalizing and then reversing the sign of the conditional variance of the asset value (i.e., $1 - \frac{Var(\theta|P)}{\sigma_\theta^2}$). However, it is easy to see that these definitions are equivalent to each other once we fix the prior σ_0^2.

[9] For example, Kyle's (1985) framework is more likely to generate infinite trading volume in large markets due to the fact that traders are risk-neutral and the only factor that limits infinite trading volume is market power, which fades away as the number of traders grows large. In contrast, Eyster, Rabin and Vayanos adopt the assumption of constant absolute risk aversion to ensure that traders shy away from infinite trade to avoid a large amount of risk. Also, in their model, traders are price takers, making it possible that traders' beliefs converge toward the price so that these traders do not trade based on their own private information.

The best scenario for price informativeness is that the price fully aggregates all available signals held by traders in the market. This corresponds to the maximum of price informativeness defined as follows:

Definition 1.8. Let $\rho \in [0,1]$ and $N \geq 1$ be given. The maximum level of price informativeness is the precision of the asset value conditional on all traders' private signals:

$$PI^*(\rho,N) = \{Var(\theta|s_1,\cdots,s_N)\}^{-1}.$$

The following corollary concerns the effect of overconfidence on price informativeness other things being equal.

Proposition 1.9. *Fix $\kappa > 1$, $\eta \in (0,1)$, $\bar{\rho} \in (0,1]$ and $N \geq 2$. Also, $\bar{N}_K$ is defined in Proposition* 1.1 *as the upper bound of N under which a unique equilibrium exists under κ-overconfidence. Then the following statements hold true:*

(1) *If $N \in [2,\bar{N}_K)$, κ-overconfidence always increase price informativeness, whether $\rho = 0$ or $\rho = \bar{\rho}$.*

(2) *If $\rho = 0$, η-overconfidence does not affect price informativeness. In contrast, if $\rho = \bar{\rho}$, η-overconfidence decreases price informativeness.*

The only observation needed here is that the "normalized" price is thought of as an unbiased signal about θ. In particular, knowing the price is equivalent to knowing total demand, which consists of strategic traders' demands and noises. Since strategic traders' demands reflect their private signals, which in turn consist of the asset value and their errors, we have

$$\frac{X}{N\beta} = \theta - \theta_0 + \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i + \frac{\omega}{N\beta}, \tag{1.6}$$

which is regarded as an unbiased signal about θ. Thus, price informativeness is positively associated with the precision of such variable extracted from the price. By Proposition 1.6, trading volume from strategic traders increases with κ-overconfidence. This implies that κ-overconfidence causes a reduction in the relative proportion of noises, as demonstrated by a reduction in the last term in Equation (1.6). It increases the precision of total demand given by Equation (1.6), and and, as a result, increases price informativeness. Proposition 1.6 also implies that trading volume from strategic traders decreases with η-overconfidence. By symmetric argument, η-overconfidence decreases price informativeness.

The following proposition considers how overconfidence affects the qualitative properties of price informativeness.

Proposition 1.10. *Fix $\kappa \in (1,\infty)$, $\eta \in (0,1)$ and $\bar{\rho} \in (0,1]$. Also, $\bar{N}_K$ is defined in Proposition* 1.1 *as the upper bound of N under which a unique equilibrium exists under κ-overconfidence. Then the following statements hold true:*

(1) *In the benchmark case and the cases of κ- and η-overconfident traders with $\rho = 0$, price informativeness increases with N and converges to its maximum as $N \to \infty$.*

(2) *In the case of κ-overconfident traders with $\rho = \bar{\rho}$, price informativeness increases with N and converges to its maximum as $N \to \bar{N}_K$.*

(3a) *In the case of η-overconfident traders with $\rho = \bar{\rho}$ with $\eta \in \left(\left(\frac{\bar{\rho}\sigma_\varepsilon^2}{\sigma_0^2+2\sigma_\varepsilon^2}\right)^2, 1\right)$, price informativeness increases with N and converges to a value strictly below its maximum as $N \to \infty$.*

(3b) *In the case of η-overconfident traders with $\rho = \bar{\rho}$ with $\eta \in \left(0, \left(\frac{\bar{\rho}\sigma_\varepsilon^2}{\sigma_0^2+2\sigma_\varepsilon^2}\right)^2\right)$, price informativeness decreases with N and converges to a value strictly below its maximum as $N \to \infty$.*

In the benchmark case, a new trader's entry affects price informativeness in two ways: First, the new trader brings his own information into the price by submitting his demand to the market. Second, the new trader's entry increases trading volume, as formally shown by Proposition 1.9, thereby lowering the relative proportion of noise. Both of these effects lead to an increase in price informativeness. With regard to information aggregation in large markets, the benchmark case is a special case of the main result of Lambert, Ostrovsky and Panov (2018). As argued in their paper, the intuition of this result is as follows: Suppose that a strategic trader's private information is still not fully incorporated into the price even as N goes to infinity. Then each trader makes a nonnegligible profit in equilibrium, which implies that the sum of their expected profits goes to infinity. However, this cannot happen in any equilibrium because these profits must come from noise traders' random demand. This leads to a contradiction.

However, the proposition indicates that both of these properties can be changed with overconfidence combined with correlated errors. In the case of κ-overconfident traders, these properties hold with a faster rate of convergence in that price informativeness reaches its maximum even as the number of traders approaches $\bar{N}_K$. On the other hand, in the case of η-overconfident traders, price informativeness fails to converge toward its maximum in the limit, and it even decreases with the number of traders when parameter η is low. Intuitively, all these results immediately follow from Proposition 1.9. A new trader's entry amplifies the aforementioned effects of κ- and η-overconfidence on price informativeness, thereby increasing and decreasing the pace of converging toward its maximum in the cases of κ- and η-overconfident traders, respectively. When parameter η is low, the latter even reverses the direction of the effect of new trader's entry, as in statement (3b).

These results imply that overconfidence significantly impacts the properties of price informativeness. It is commonly believed that the promotion of competition, which possibly arises from the trends of globalization and financial technology, improves the functioning of markets, and that it eventually leads to the best scenario that prices accurately summarize the dispersed information held by market participants. These properties hold in the benchmark case, as in a lot of dfferent contexts of markets summarized in Subsection 1.2. However, in the world with overconfident traders, the proposition illustrates the possibility that market prices still involve a noise in environments arbitrarily close to perfect competition, and if the extent of overconfidence is severe, they even become noisier in these competitive environments. Even though it could be argued that every economic reasoning trivially breaks down with arbitrarily irrational players, overconfidence is thought of as a common form of irrationality which is empirically relevant and imposes some reasonably strong conditions on how players disagree on the state of nature. The sensitivity of these properties to this particular deviation from the common prior assumption undermines the possibility that they persist in various economic environments in reality.

1.5 Concluding remarks

The main objective of this study is to analyze the implications of overconfidence in financial markets. Two important features of the model are that overconfident traders are oligopolistic in the manner of Kyle (1985) and that their imperfect signals are subject to a common error in valuation. The analyses suggest that the direction of implications of overconfidence on trading volume and price informativeness differs radically across two different types of overconfidence. Given that these two types of overconfidence are likely to coexist in reality, one could argue that the implications of overconfidence depend on which type of overconfidence prevails in markets. As noted in Subsubsection 1.4.2, this is indeed consistent with the mixed empirical and experimental results on the relationship between overconfidence and trading volume (e.g., Biais, Hilton, Mazurier, and Pouget, 2005; Fellner-Rohling and Krugel, 2014; Glaser and Weber, 2007: Merkle, 2017).

It is noteworthy that the implications of overconfidence are complicated further by the observation that the degree of overconfidence is only moderate on average and even negative in many cases. Indeed, the degree of overconfidence varies across a lot of individual-specific and time-specific factors, such as working experience and ambiguity (e.g., Gloede and Menkhoff, 2014; Yang and Zhu, 2016). Though we do not formally analyze the effects of underconfidence here, the effects of underconfidence appear to be symmetric to those of overconfidence of the corresponding type. For example, the results in Subsubsection 1.4.2 immediately imply that underconfidence does not necessarily decrease trading volume, depending on the type of underconfidence. The results in Subsubsection 1.4.3 indicate that underconfidence of the type corresponding to κ-overconfidence resembles η-overconfidence in the sense that it prevents information aggregation and may cause a negative relationship between the number of traders and price informativeness. This raises another possibility of the price becoming noisier with the number of traders. On the theoretical side, the results presented in this study could be regarded as a rich set of possible outcomes arising from a reasonable range of forms of disagreements over the structure of information possibly observed in reality, rather than being specific to overconfidence. On the empirical viewpoint, these illustrate a difficulty in explaining the pattern of trading volume and price informativeness with overconfidence, as it is rather hard to observe the form of investors' beliefs about information structures.

Finally, we discuss the robustness of the main results to alternative trading mechanisms. In the basic model and its variant considered in this study, traders submit market orders conditional only on their own private information. This particular trading mechanism abstracts from learning from the price, thereby allowing for tractable analyses even with correlated errors. Otherwise, each trader would infer other traders' behavior from both the price and his own private information, whose weights are expected to be intractable due to the complexity of standard Bayesian updating. However, it is still noteworthy that learning from the price is likely to significantly change the interpretation of private information held by each trader, thereby affecting the implications of overconfidence in the presence of correlated errors. Nevertheless, the core idea of non-obvious relationship between overconfidence and trading volume still appears to persist as in the following example:

Example. The value of a risky asset is θ, which follows an improper prior. The asset is traded by a continuum of competitive risk-neutral traders $i \in [0,2]$. They all simultaneously submit price-contingent orders so as to maximize their profits and then the market clears as in Grossman and Stiglitz (1980).

Trade occurs due to a liquidity shock $\omega \sim N(0, \sigma_\omega^2)$ affecting the second half of traders (i.e., those with $i \in [1,2]$) privately. All traders incur quadratic transaction costs proportional to the square of their demands. In particular, for $i \in [0,1]$, trader i's valuation is simply θ in the sense that his profit is $\pi_i = x_i(\theta - p) - \frac{1}{2}x_i^2$, whereas, for $i \in [1,2]$, trader i's valuation is $\theta + \omega$ in the sense that his profit is $\pi_i = x_i(\theta + \omega - p) - \frac{1}{2}x_i^2$.

Traders $i \in [0,1]$ have a common signal $s = \theta + \varepsilon$, where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. These traders $i \in [0,1]$ are κ-overconfident in that he believes that his own signal is given by $s_i = \theta + \frac{\varepsilon}{\sqrt{\kappa}}$, where $\kappa \in (1, \infty)$, and is unbiased about others' signal s.

In contrast, traders $i \in [1,2]$ do not have such private information. However, they learn from the price. This implies that they are effectively informed about signal s because the price must be a function of s and ω, the latter of which is known to them. Also, they are "rational" in the sense that they have the true belief on the structure of information.

We can show that there is a unique equilibrium where the price is given by $p = s + \beta\omega$ with $\beta \in (0,1)$.[10] To solve the equilibrium, we first note that traders' optimal demands are given by $x_i^* = E[\theta|\mathscr{I}_i] - p$ for $i \in [0,1]$ and $x_i^* = E[\theta|\mathscr{I}_i] + \omega - p$ for $i \in [1,2]$, where $\mathscr{I}_i$ is trader i's information set incorporating his possibly biased belief on the structure of information. Given the conjectured price $p = s + \beta\omega$, each κ-overconfident trader $i \in [0,1]$'s expectation about the asset value is given by

$$E[\theta|\mathscr{I}_i] = \frac{1}{\left(\sqrt{\kappa}-1\right)^2 \beta^{-2}\sigma_\omega^{-2} + \sigma_\varepsilon^{-2}} \left\{ -\frac{\sqrt{\kappa}-1}{\beta^2\sigma_\omega^2} p + \left(\sigma_\varepsilon^{-2} + \frac{\sqrt{\kappa}\left(\sqrt{\kappa}-1\right)}{\beta^2\sigma_\omega^2} \right) s \right\},$$

whereas each rational trader $i \in [1,2]$'s expectation about the asset value is given by $E[\theta|\mathscr{I}_i] = s$. Note that κ-overconfidence makes traders learn more from the price p and κ-overconfident traders' expectation about the asset value decreases with the price p. This is in contrast to our typical understanding of overconfidence telling us that overconfident traders learn less from the price compared with rational ones. In the presence of correlated errors, such seemingly-unintuitive effect of κ-overconfidence arises from the fact that the realization of a higher price increases his expectation about the common error ε under κ-overconfidence and it is even more than that about the asset value θ. Plugging these traders' optimal demands obtained from the above expression of $E[\theta|\mathscr{I}_i]$ into the market clearing condition (i.e., $\int_{i\in[0,1]} x_i di + \int_{i\in[1,2]} x_i di = 0$), we get the clearing price as follows:

$$p = s + \frac{\sigma_\varepsilon^{-2} + \left(\sqrt{\kappa}-1\right)^2 \beta^{-2}\sigma_\omega^{-2}}{\sigma_\varepsilon^{-2} + \sqrt{\kappa}\left(\sqrt{\kappa}-1\right)\beta^{-2}\sigma_\omega^{-2} + \sigma_\varepsilon^{-2} + \left(\sqrt{\kappa}-1\right)^2 \beta^{-2}\sigma_\omega^{-2}} \omega.$$

The resulting price constitutes an equilibrium if it concides with the conjectured price $p = s + \beta\omega$. We can then show that coefficient β is uniquely determined and $\beta \in (0,1)$ in equilibrium.

[10]Here the price fully reflects the common signal $s$because traders $i \in [1,2]$ perfectly infer that from the price p. This point is intended to simplify the analyses compared with those in the basic model with risk-neutral market makers. Nevertheless, the core intuition behind the below analyses, which concerns the inference of overconfident traders $i \in [0,1]$ about the asset value from their signal s and the price p, would not change in case of risk-neutral market makers.

The main point here is that trading volume and price informativeness decrease with the degree of overconfidence $\kappa > 1$. Recall that trade occurs only between traders with $i \in [0,1]$ and those with $i \in [1,2]$. Therefore, trading volume is simply equal to the volume of traders with $i \in [1,2]$ whose optimal demand is given by

$$x_i^* = s + \omega - p = (1-\beta)\omega.$$

Given the optimal demand x_i^* for $i \in [1,2]$, trading volume, which corresponds to $E[|x_i^*|]$, is proportional to $(1-\beta)^2$. We can then show that coefficient β increases with the degree of overconfidence κ by using the expression for the equilibrium price. This immediately implies that trading volume decreases with the degree of overconfidence κ. Further, price informativeness decreases with the degree of overconfidence κ as well because it decreases with coefficient β in the price.

Combined with the fact that such negative relationship between overconfidence and trading volume would not generally hold if traders had independent signal errors,[11] the example tells us that the possibility of a negative relationship between overconfidence and trading volume, which arises from correlated information structures, is not necessarily an artifact from the specific trading mechanism considered in this study. Here, traders on the first half (i.e., $i \in [0,1]$ are κ-overconfident, whereas those on the second half (i.e., $i \in [1,2]$) correspond to noise traders in the basic model except that these noise-creating ones are price-elastic. Though the intuition underlying this example, which involves changes in learning from the price in response to κ-overconfidence, appears to be rather different from that in the main text due to differences in the trading mechanism, the example suggests that overconfidence and correlation in errors are combined together to qualitatively affect trading volume and price informativeness, regardless of whether traders observe the price or not.

[11] Though the formal analysis is omitted, this fact is intuitive in the limiting cases (i.e., comparing $\kappa \to \infty$ with $\kappa = 1$) because extremely large trades occur among highly overconfident traders $i \in [0,1]$ with different signal realizations, which arise from independent signal errors. This is in line with previous studies on overconfidence (e.g., Benos, 1998; Kyle and Wang, 1997; Odean, 1998).

Chapter 2

Overconfidence and Correlated Information Structures: Endogenous Information

2.1 Introduction

This chapter analyzes an extension of the basic model considered in Chapter 1 where strategic traders simultaneously choose whether to costly acquire their signals before the trading stage identical to the basic model. In the extended model, overconfidence affects the market equilibrium through traders' information choices at the first stage and their trading activities at the second stage. The main focus here is whether and how the endogeneity of information changes the aforementioned main results regarding the effects of overconfidence. Lemma 2.3 solves for subgames following the first stage in all three cases. Whereas subgames in the benchmark case and the case of η-overconfident traders reiterate strategic substitutability of costly information acquisition in line with Grossman and Stiglitz (1980), κ-overconfidence with correlated errors leads to a U-shaped curve of informed traders' expected profit with respect to the number of informed traders. Proposition 2.4 and 2.5 solve for endogenous information choices of traders at the first stage in the benchmark case and the case of κ-overconfident traders with correlated errors, respectively. In the benchmark case, there is a unique equilibrium, which involves an upper bound in the number of traders who choose to be informed. In contrast, in the case of κ-overconfident traders with correlated errors, two equilibria possibly exist: One of them exists throughout the entire range of the number of traders and involves an upper bound in the number of informed traders, as in the benchmark case, whereas the other equilibrium exists with a sufficient number of traders and all traders choose to be informed in this equilibrium. In both equilibria, the number of informed traders is higher than that in the benchmark case, implying that information acquisition increases with κ-overconfidence.

In parallel with the main results in the basic model, the analyses of the extended model examine the effects of overconfidence on trading volume and price informativeness. Given that Lemma 2.3 implies that κ-overconfidence qualitatively changes the pattern of endogenous information acquisition and η-overconfidence does not, we focus on the case of κ-overconfident traders here. On the quantitative side, Corollary 2.6 implies that trading volume and price informativeness increase with κ-overconfidence, as in the basic model. On the qualitative side, Corollary 2.7 compares the properties of trading volume and price informativeness between the benchmark case and the case of κ-overconfident traders with correlated errors. In the benchmark case, it naturally follows from Proposition 2.4 that trading volume and price informativeness increase in the beginning but then stay constant with the number of traders. However, in the case of κ-overconfident traders with correlated errors, among two equilibria, one of them qualitatively resembles the benchmark case in terms of the properties of trading volume and price informativeness, whereas the other one is qualitatively distinct from the benchmark case. In the "new" equilibrium where all traders choose to be informed, trading volume increases toward infinity as the number of traders goes to a finite number, and price informativeness also increases toward the level of full information aggregation in the limit. These raise the possibility that κ-overconfidence causes information aggregation in large markets even if information is costly and endogenous.

2.2 Model

This extension is in parallel with Grossman and Stiglitz (1980), in the sense that strategic traders choose to costly acquire their private signals before the trading stage which corresponds to the basic model. As in the basic model, there are N strategic traders, noise traders, and competitive market makers in the market, and they trade a security whose value θ is not initially known and follows $N(\theta_0, \sigma_0^2)$. At the first stage of the model, each strategic trader $i \in \{1, \cdots, N\}$ simultaneously decides whether to acquire a private signal s_i with exogenous cost c. The following stages are identical to the basic model: At the second stage, strategic and noise traders submit their demand to the market. Following the notation of the basic model, we denote by x_i each strategic trader i's demand, and by ω noise traders' demand, where ω follows $N(0, \sigma_\omega^2)$. At the third stage, the price is set by competitive market makers based on the total demand $X = \sum_{i=1}^{N} x_i + \omega$. At the final stage, the true asset value θ is realized, and each strategic trader i's profit is $\pi_i = x_i(\theta - P)$.

If a strategic trader i decides not to acquire his private signal at the first stage, he enters the second stage without any private information. In contrast, if he decides to acquire his signal, he observes $s_i = \theta + \varepsilon_i$ at the second stage. All other assumptions are identical to the basic model, regarding the distribution of signals and how traders perceive their own signal and others' signals. As in the basic model, market makers have the true belief on the distribution of these signals, but strategic traders' beliefs are possibly biased, depending on three cases of rational, κ-overconfident and η-overconfident traders.

Definition 2.1. An equilibrium of the extended model consists of the number of strategic traders who choose to be informed (i.e., M), informed and uninformed traders' demands (i.e., $(x_M^I(s), x_M^U(s))_{M=1}^N$), and the pricing rule (i.e., $(P_M(X))_{M=1}^N$) where the following three conditions hold:

1. At the third stage, the price set by market makers is equal to the expected value of the security conditional on X and M, i.e., $P_M = E[\theta | X, M]$;

2. At the second stage, for every realization of signal s, each strategic trader's demand $(x_M^I(s), x_M^U(s))_{M=1}^N$ maximizes his expected profit conditional on whether or not he

is informed, M, and his beliefs on the distribution of signals, the pricing rule and the profile of other traders' strategies.

3. At the first stage, each strategic trader pursues to maximize his expected profit net of the cost conditional on his own beliefs on the distribution of signals, the pricing rule and the profile of other traders' strategies. In particular, each strategic trader acquires a private signal if and only if his expected net profit from obtaining the signal is higher than that from not doing so;

Though symmetry is imposed on the concept of equilibrium, it does not hurt generality because the class of information structures is symmetric. It allows us to prevent redundancy arising from the presence of many equilibria which are symmetric to each other at the first stage.

Finally, as in the basic model, we restrict attention to the class of linear equilibria, as defined in Definition 1.2.

2.3 Solving for subgames

Following backward induction, we first solve for subgames starting from the second stage, and then, proceed to analyze how many strategic traders choose to be informed at the second stage. Note that a subgame is represented by the number of informed traders (i.e., M) where $M \in [1,N]$. The following definition will be useful in what follows:

Definition 2.2. For a subgame with M informed traders, each trader's expected profit from the subgame is denoted by $U_R(\rho,N,M)$, $U_K(\rho,N,M)$, and $U_E(\rho,N,M)$, for the cases of rational traders, κ-overconfident traders, and η-overconfident traders, respectively.

The lemma provides a characterization of equilibrium of subgames in all three cases.

Lemma 2.3. *Fix $\kappa \in (1,\infty)$, $\eta \in (0,1)$ and $\bar{\rho} \in (0,1]$. Also, $\bar{N}_K$ is defined in Proposition 1.3 as the upper bound of N under which a unique equilibrium exists for κ-overconfident subgames. Consider subgames with M informed traders in all three cases (i.e., benchmark, κ-overconfidence, and η-overconfidence). Then the following statements hold true:*

(a) *There is a unique equilibrium except for* κ*-overconfidence with* $M > \bar{N}_K$*. In this equilibrium, uninformed traders submit zero demand (i.e.,* $x_M^U = 0$*), while informed traders submit the same demand as they would submit in the basic model with M traders in the same case, as stated in Proposition* 1.3*;*

(b) *In the benchmark case, the case of* κ*-overconfident traders with* $\rho = 0$ *and the case of* η*-overconfident traders, each trader's expected trading profit (i.e.,* $U_R(\rho,N,M)$*,* $U_K(0,N,M)$ *or* $U_E(\rho,N,M)$*) does not depend on N, decreases with M, and converges to zero as* $M \to \infty$*;*

(c) *In the case of* κ*-overconfident traders with* $\rho = \bar{\rho}$ *and* $M \in [1,\bar{N}_K)$*, each trader's expected trading profit (i.e.,* $U_K(\bar{\rho},N,M)$*) does not depend on N, is U-shaped with M, and goes to infinity as* $M \to \bar{N}_K$*;*

(d) *Whether* $\rho = 0$ *or* $\rho = \bar{\rho}$*, it holds that* $U_R(\rho,N,M) < U_K(\rho,N,M)$ *for every* $M \in [1,\bar{N}_K)$.

The above lemma shows that a subgame with M informed traders is equivalent to the basic model with the same number of traders in the same case, as uninformed traders in the subgame behave as if they are "inactive". Then it naturally follows that informed traders' expected profit does not hinge on the number of uninformed traders, but rather depend on the number of these informed traders.

It is useful to compare the properties of informed traders' expected profit with respect to the number of informed traders. In the benchmark case and the case of η-overconfident traders, it decreases to zero as the number of informed traders increases to infinity. This is consistent with the argument that incentives to acquire information are strategic substitutes across traders, which holds in standard modeling frameworks such as Grossman and Stiglitz (1980). In contrast, in the case of κ-overconfident traders with correlated signal errors, informed traders' expected profit is non-monotonic with respect to the number of informed traders, and it eventually goes to infinity with a sufficient number of informed traders. Above this limit, there is no equilibrium due to infinite trading, as mentioned in Proposition 1.3. Such non-monotonicity is in sharp contrast with the benchmark case. Intuitively, as more κ-overconfident traders choose to be informed, two conflicting effects are at work: First, an increase in trading volume leads to a decrease in the proportion of noise traders, causing the price to be less noisier and thus lowering profit opportunities for existing informed traders, as expected even without overconfidence. Second, each existing informed trader believes that the price is now more strongly affected by a systematic error which is involved in others' signals. This increases his perceived profit opportunities from the asset, in contrast to the first effect. The proposition shows that as the number of informed traders is large enough, the second effect is dominates the first one. This implies that traders' information choices are strategic complements. Eventually, this leads to a non-equilibrium outcome due to infinite trading, as noted in Proposition 1.3.

Note that the above second effect arises only from a biased belief on others' signal errors, which comes down to the interplay between κ-overconfidence and correlated signal errors. It does not exist in the absence of correlation of signal errors because there is no systematic error involved in traders' signals. Also, in the case of η-overconfident traders, an inverse effect is at work: As more η-overconfident traders choose to be informed, each existing informed trader believes that the price is now even more correlated with his own signal. This effect further decreases his expected profit from the subgame, reinforcing the first effect as above.

Finally, statement (d) shows that κ-overconfidence increases each informed trader's expected profit given the number of informed traders. This result is intuitive, as his expected profit increases with what he believes is the precision of his own signal, which in turn increases with κ-overconfidence. The formal proof in the Appendix is complicated by strategic interaction among informed traders, which turns out not to reverse the aforementioned intuition. This statement is the main driving force leading to the analyses on the effects of overconfidence in what follows.

2.4 Information acquisition in equilibrium

Now we proceed to analyze information acquisition at the first stage. At the first stage, each trader chooses to be informed if and only if his expected profit from the following trading stage is higher than the cost of information c. As is implied by Lemma 2.3, only the case of κ-overconfident traders is qualitatively different from the benchmark case in terms of informed traders' expected profit. In other cases, the standard properties of information acquisition that hold in the benchmark case continue to hold. Therefore, we hereafter focus on the comparison between the benchmark case and the case of κ-overconfident traders with correlated errors.

In the benchmark case, the following proposition provides a characterization of strategic traders' behavior and price informativeness in equilibrium:

Proposition 2.4. *Consider the benchmark case of rational traders. Fix* $\rho = \bar{\rho} \in [0, 1]$.

If $c \in (0, U_R(\bar{\rho}, 1, 1))$, *then there exists a unique equilibrium. In this equilibrium, there exists* $N_R^*(c) \in (1, \infty)$ *such that the following statements hold true:* (a) *If* $N \in [1, N_R^*(c))$, *then all strategic traders choose to be informed at the first stage;* (b) *If* $N > N_R^*(c)$, *then only* $N_R^*(c)$ *strategic traders choose to be informed at the first stage.*

If $c > U_R(\bar{\rho}, 1, 1)$, *then there exists a unique equilibrium where all strategic traders choose not to be informed at the first stage.*

In equilibrium, the number of informed traders is uniquely determined at the first stage. If the number of traders is small, all traders choose to be informed at the first stage. However, as it exceeds the limit denoted by $N_R^*(c)$, the number of informed traders remains constant at $N_R^*(c)$ and the rest of them choose to be uninformed. Even though price informativeness would increase toward its maximum *if all traders were informed*, the endogeneity of information prevents the outcome of all traders choosing to be informed, thereby undermining information aggregation. Overall, the above proposition reiterates the idea that endogeneity of information prevents information aggregation.[1]

In contrast, in the case of κ-overconfident traders with correlated signal errors, the

[1] Note that this point is related to but is still distinguished from the Grossman-Stiglitz paradox that perfectly informationally efficient markets would collapse as traders would not incur the costs of acquiring information (Grossman and Stiglitz, 1980). It refers to the limiting case where small noise leads to nonexistence of equilibrium due to the lack of incentive to acquire information, which leads to the lack of trade. The driving force behind this paradox is similar to the failure of information aggregation in large markets, which holds in their model as well, as increasing the number of traders is effectively the same as reducing noise. Also, though Grossman and Stiglitz (1980) consider a different trading mechanism allowing for the observation of prices, strategic substitutability of information acquisition, which causes the failure of information acquisition, appears to hold more broadly than the particular mechanism considered by Grossman and Stiglitz.

following proposition suggests that their behavior and the properties of the price are qualitatively distinct:

Proposition 2.5. *Consider the case of κ-overconfident traders with correlated signal errors. Fix $\kappa \in (1,\infty)$ and $\rho = \bar{\rho} \in (0,1]$. Also, consider $N < \bar{N}_K$, where $\bar{N}_K$ is defined in Proposition* 1.3 *to rule out infinite trade under κ-overconfidence.*

If $c < U_K(\bar{\rho}, M_K^*, M_K^*)$, where M_K^* is defined in Lemma 2.3*, there is a unique equilibrium where all N traders choose to be informed at the first stage.*

If $c \in (U_K(\bar{\rho}, M_K^*, M_K^*), U_K(\bar{\rho}, 1, 1))$, then there exist $N_K^*(c)$ and $N_K^{**}(c)$ such that $1 < N_K^*(c) < N_K^{**}(c) < \bar{N}_K$ and the following statements hold true: (a) *If $N \in [1, N_K^*(c))$, there is a unique equilibrium where all N strategic traders choose to be informed at the first stage;* (b) *If $N \in (N_K^*(c), N_K^{**}(c))$, there is a unique equilibrium where only $N_K^*(c)$ strategic traders choose to be informed at the first stage;* (c) *If $N \in (N_K^{**}(c), \bar{N}_K)$, there are two equilibria: In one equilibrium, only $N_K^*(c)$ strategic traders choose to be informed at the first stage, whereas, in the other equilibrium, all N strategic traders choose to be informed at the first stage.*

Finally, it holds that $N_K^*(c) > N_R^*(c)$ for every $c \in (U_K(\bar{\rho}, M_K^*, M_K^*), U_K(\bar{\rho}, 1, 1))$. This implies that κ-overconfidence weakly increases the number of informed traders in both possible equilibria described above.

Throughout the range considered in the proposition (i.e., $N \in [1, \bar{N}_K)$), there are two different equilibria in information acquisition: One equilibrium exists for all N in this range, where no more than $N_K^*(c)$ traders choose to be informed. This feature arises from the left side of the U-shaped curve of κ-overconfident informed traders' expected profit in Lemma 2.3. While this equilibrium is similar to the benchmark case at this point, its upper bound on the number of informed traders (i.e., $N_K^*(c)$) is higher than that in the benchmark case (i.e., $N_R^*(c)$). This implies that κ-overconfidence weakly increases the number of informed traders in this equilibrium. In contrast, the other equilibrium exists only with a sufficient number of traders (i.e., $N \in (N_K^{**}(c), \bar{N}_K)$), where all traders choose to be informed. This feature arises from the right side of the U-shaped curve of κ-overconfident informed traders' expected profit in Lemma 2.3, and it is qualitatively distinct from the unique equilibrium in the case of rational traders. The number of informed traders equals the number of all traders, which is trivially higher than that in the benchmark case in this equilibrium.

It is worth commenting on the range of the number of traders which is not considered in the proposition. If the number of traders is so large that it exceeds the range (i.e., $N \geq \bar{N}_K$), the absence of equilibrium in subgames complicates the interpretation of the formal analyses. In particular, Lemma 2.3 implies that there is no equilibrium for subgames with more than $\bar{N}_K$ informed traders, and thus, there cannot be any equilibrium where these subgames are realized. Accordingly, the "new" equilibrium does not survive, whereas the other equilibrium still persists. However, note that such no-equilibrium outcome is due to infinite trading by κ-overconfident traders, not due to the absence of trade. Indeed, traders a priori expect infinite profits from these subgames with many κ-overconfident traders. Therefore, the new equilibrium occurs again with a slight perturbation guaranteeing the existence of equilibrium in all subgames. For example, this could be done by a perturbation of subgames by adopting a quadratic utility with a very slightly negative quadratic coefficient. This observation suggests that the absence of the new equilibrium for $N \geq \bar{N}_K$ appears to be an artifact of risk neutrality, supporting the argument that the new equilibrium is likely to be observed even in large markets in reality.

This sort of equilibrium multiplicity reminds us of Mondria, Vives and Yang (2020). They consider a model in which investors cannot costlessly process information from asset prices. It then naturally follows that investors optimally choose sophistication levels by balancing the benefit of beating the market against the cost of acquiring sophistication. They show that there can exist strategic complementarities in the choice of sophistication levels, leading to multiple equilibria. Compared with their results, the proposition identifies another potential channel whereby equilibrium multiplicity is caused by a deviation from the common prior assumption. This channel is distinguished from that of Mondria, Vives and Yang in that it does not require the notion of sophistication in processing information and instead combines two notions of correlation in errors and overconfidence, which are more or less established and, to the best of my knowledge, addressed separately in the literature as reviewed in Subsection 1.2, to raise the possibility of multiple equilibria.

2.5 Effects of overconfidence on trading volume and price informativeness

In parallel with Propositions 1.5 and 1.9 in the basic model, the following corollary considers how κ-overconfidence affects trading volume and price informativeness.

Corollary 2.6. *Fix* $\kappa \in (1,\infty)$, $\rho = \bar{\rho} \in (0,1]$ *and* $c \in (0, U_K(\bar{\rho},1,1))$. *If* $N < \bar{N}_K$, *where* $\bar{N}_K$ *is defined in Proposition* 1.3 *to rule out infinite trade under* κ*-overconfidence,* κ*-overconfidence increases trading volume and price informativeness in both possible equilibria.*

Note that κ-overconfidence has two effects on trading volume and price informativeness: (i) increasing trading volume and price informativeness given the number of informed traders, as implied by Proposition 1.5 and 1.9, respectively; (ii) (weakly) increasing the number of informed traders, as implied by Proposition 2.5. Both of these effects lead to increases in trading volume and price informativeness in both equilibria which possibly occur with κ-overconfidence. Overall, the endogeneity of information causes additional indirect effects of κ-overconfidence on both trading volume and price informativeness through an increase in the number of informed traders, but such effects do not reverse the results that hold in the basic model because they operate in the same direction with its direct effects on trading volume and price informativeness given the number of informed traders.

We now turn to the qualitative side of the analyses. In parallel with Propositions 1.6 and 1.10 in the basic model, the following corollary concerns how the qualitative properties of trading volume and price informativeness are changed with κ-overconfidence.

Corollary 2.7. *Fix* $\kappa \in (1,\infty)$, $\bar{\rho} \in (0,1]$ *and* $c \in (U_K(\bar{\rho}, M_K^*, M_K^*), U_K(\bar{\rho},1,1))$, *where* M_K^* *is defined in Proposition* 2.5. *Also, consider* $N < \bar{N}_K$, *where* $\bar{N}_K$ *is defined in Proposition*

1.3 *to rule out infinite trade under κ-overconfidence. With $N_R^*(c)$, $N_K^*(c)$ and $N_K^{**}(c)$ defined in Propositions* 2.4 *and* 2.5*, the following statements hold true:*

(1) *In the benchmark case, trading volume and price informativeness increase with N for $N \in [1, N_R^*(c))$, but then stay constant for $N > N_R^*(c)$.*

(2a) *In the case of κ-overconfident traders with $\rho = \bar{\rho}$, one equilibrium exists for $N \in [1, \bar{N}_K)$, and in this equilibrium, trading volume and price informativeness increase with N for $N \in [1, N_K^*(c))$, but then stay constant for $N \in (N_K^*(c), \bar{N}_K)$.*

(2b) *In the other equilibrium which exists for $N \in (N_K^{**}, \bar{N}_K)$ in the case of κ-overconfident traders with $\rho = \bar{\rho}$, trading volume and price informativeness increase with N throughout the range of N. As $N \to \bar{N}_K$, trading volume goes to infinity and price informativeness approaches its maximum.*

Statement (1) regarding the benchmark case follows from combining Proposition 1.6 (for trading volume) and Proposition 1.10 (for price informativeness) with Proposition 2.4. In particular, if all traders were informed, Propositions 1.6 and 1.10 would apply. However, Proposition 2.4 implies that as the number of traders increases above N_R^*, only N_R^* traders choose to be informed, thus imposing an upper bound in trading volume and price informativeness in response an increase in the number of traders. This is consistent with the idea that the endogeneity of information prevents information aggregation, as noted in Subsection 1.2.

Statements (2a) and (2b) consider two different possible equilibria in the case of κ-overconfident traders with correlated errors. As in the benchmark case, they follow from combining Propositions 1.6 and 1.10 with Proposition 2.4. The equilibrium in statement (2a) resembles the benchmark case in that the number of those who choose to be informed stays at N_K^* as the number of traders exceeds N_K^*, and thus, trading volume and price informativeness are bounded above as in the benchmark case. However, in the other equilibrium in statement (2b), which occurs with a sufficient number of traders (i.e., $N \in (N_K^{**}, \bar{N}_K)$), all traders choose to be informed. Therefore, Propositions 1.6 and 1.10 directly apply so that trading volume and price informativeness increase with the number of traders and they converge toward infinity and its maximum, respectively, as the number of traders approaches $\bar{N}_K$.

The equilibrium in statement (2b) which distinctly occurs in the case of κ-overconfident traders tells us that the standard argument that endogenous information prevents information aggregation hinges on the assumption of common prior, and it may not apply otherwise. It is notable that such equilibrium occurs even with an arbitrarily small extent of κ-overconfidence, indicating that the validity of the mechanism is more or less sensitive to a slight deviation from Bayesian rationality. Overall, information aggregation in markets can be explained by a large number of κ-overconfident traders with correlated signal errors, even when information is costly and endogenous.

Chapter 3

Taxing Speculative Trades in Financial Markets

3.1 Introduction

The policy debate on stock market liberalization and financial transaction taxes centers around speculative trading. For example, financial transaction taxes, which are called the "Robin Hood Tax" by some advocates, aim to curb short-term and high-frequency trading typically carried out by professional investors with relevant experience and expertise. As their nickname suggests, they appear to be seen as being "fair" in the sense that these taxes are disproportionately imposed on the financial sector and the revenues could be used to help recoup the costs of the financial crisis, including the financial-sector bailout, imposed on middle-class workers and retirees. Politicians and policymakers seemed to recognize this point in the aftermath of the global financial crisis in 2008 as well as the most recent proposal of the European Union financial transaction tax.[1] However, restrictions on trading may hurt the functioning of markets by reducing socially beneficial trades.

[1]At their Pittsburgh meeting in September 2009, the G-20 leaders tasked the IMF to explore "the range of options countries have adopted or are considering as to how the financial sector could make a fair and substantial contribution toward paying for any burdens associated with government interventions to repair the banking system." In response to their request, the IMF recommended the adoption of levies on financial institutions to pay for the resolution of troubled institutions in the event of future failures and crises and examined the possibility of raising revenue from the sector's activities more generally (IMF, 2010). In a similar vein, in 2011, the European commission stated that it has decided to propose a new tax on financial transactions to "ensure that the financial sector makes a fair contribution at a time of fiscal consolidation in the Member States." (European Commission, 2011).

In the context of cross-border investments, symmetric arguments arise regarding the advantages and disadvantages of promoting foreign institutional investments in emerging economies.[2]

The key issue in the above debate is how speculative trading of the financial sector affects investors outside of that. Its negative side is that it takes trading opportunities for others, and it is aggravated by its informational advantage. In particular, professional investors with relevant experience and expertise are likely to have superior information about the value of assets and they may trade with those without such information, resulting in a wealth transfer from the latter toward the former. Such wealth transfer, which could be viewed as rent-seeking at the expense of those without superior information, would be curbed by taxing trades carried out by those with superior information. This possibility is a straightforward concern to policymakers who are under pressure from voters whose majority are outside of the financial sector. Indeed, we can see this point from the fact that various rules and exemptions in financial transaction taxes are intended to identify speculation of the financial sector.[3] However, as seen in the above debate on financial transaction taxes and stock market liberalization, this point is valid only if speculative trading indeed hurts other investors through such wealth transfer even after taking into account its positive side in terms of informational and allocational efficiencies of financial markets. Despite its relevance for policymaking, this point has not been addressed well in the analytic literature on financial markets with asymmetric information (e.g., Subrahmanyam, 1998; Dow and Rahi, 2000; Vives, 2017), which appears to mainly focus on market efficiency in line with the original efficiency-based argument for

[2]Kacperczyk, Sundaresan and Wang (2021) document the positive effect of foreign institutional investors' stock ownership on price informativeness. With regard to its distributional impact across informed and uninformed investors, it appears to be less clear. In a broad perspective (i.e., assuming that differences in private information are associated with income differences), there is recent evidence that capital account liberalization tends to increase income inequality (Furceri and Loungani, 2018).

[3]There are a number of examples (Burman et al., 2016; Hemmelgarn, Nicodeme, Tasnadi, and Vermote, 2016): First, market making is exempt from many financial transaction taxes, including the national ones in France, Greece, Italy, and United Kingdom, in view of its perceived positive influence on market liquidity, whereas proprietary trading appears to be the main target of these taxes. Second, French taxes on credit default swaps and cancelled high-frequency trading orders are seen as ways to reduce speculation. Third, foreign exchange and short-term foreign loans and bonds are explicitly taxed in some emerging economies, including Brazil. Fourth, the EU financial transaction tax proposed in 2011 was wider in scope compared with others but is still mainly targeted at transactions carried out by financial institutions.

a Tobin tax (Tobin, 1978). This is in contrast to the large literature on the regulation of generic markets in public economics drawing on the premise that taxes and price regulations could be adopted as a response to inequality in markets (e.g., Dworczak, Kominers, and Akbarpour, 2021; Judd, 1985).

In this regard, this paper first identifies upsides of speculative trades arising from improved efficiencies in the viewpoint of uninformed investors. In particular, speculative trades give rise to more information from the price and lower price impact of uninformed investors' common liquidity trading. Also, they lower price impact of other large investors' own liquidity trading, thereby promoting such liquidity trading and thus generating trading opportunities for uninformed investors. These upsides stem from the informational and allocational roles of financial markets where prices provide useful information to investors and allocate the resources to those who value the most. Then this paper adds to the literature by comparing these upsides with the downside of the aforementioned wealth transfer in the viewpoint of uninformed investors. As a result, we show that the upsides may dominate the downside and this works through uninformed investors' trading on liquidity shocks, learning from the price, and the presence of market power. In the context of financial transaction taxes, these suggest that, despite the direct advantage of preventing speculators from taking profits from uninformed investors, a tax on speculative trades may hurt uninformed investors by discouraging liquidity trades of uninformed investors as well as those of investors with market power.

To capture financial markets' dual role of generating information and allocating the resources, we consider the situation where investors trade a risky asset whose per-unit valuations consist of a common value and their liquidity shocks, the latter of which reflect the possibility that investors differ with respect to their liquidity needs, investment opportunities, and the regulatory constraints facing them. Among these investors, some of them are informed in the sense that they have superior information about the common value. Trades are motivated by the average difference in liquidity shocks between informed and uninformed investors, which could arise from various different trading motives of professional and retail investors such as portfolio risk and life cycle. We model the regulation of trading activities, such as financial transaction taxes and trading barriers against cross-border investments, by assuming that transaction costs depending on the volume of trades are imposed on a fraction of informed investors who trade solely on the asset value with their informational advantage. These transaction costs reflect the fact that financial transaction taxes are usually targeted at certain types of trades based on superior information which are deemed speculative, such as short-term and high-frequency ones and foreign ones, rather than equally discouraging all trades including mutually beneficial ones, such as those motivated by liquidity and hedging. Without such transaction costs, these speculative trades take profits from uninformed investors by making the price track the asset value more closely, leading to the motivating argument that these speculative trades need to be taxed in order to prevent rent-seeking of the financial sector. Nevertheless, it is unclear a priori whether such negative side dominates potential improvements in informational and allocational efficiencies resulting from these speculative trades despite its key role in the debate on stock market liberalization and financial transaction taxes.

Drawing on the concept of competitive rational expectations equilibrium à la Grossman and Stiglitz (1980), the basic model of the paper quantifies the overall effect of informed investors' trading based on their superior information, which is caused by a reduction in the transaction costs, on the welfare of uninformed investors. In the model, we use terminology in the context of cross-border investments and thus call the cost-exempt and cost-bearing informed investors "(domestic) informed" and "foreign" investors, respectively. After proving the existence of unique equilibrium, we examine how the welfare of uninformed investors changes in equilibrium as cross-border costs decrease (increase) so that foreign investors' trades push the price toward (away from) the asset value. In order to clarify the intuition behind a rich set of possible equilibrium results, we note that uninformed investors' welfare consists of learning from the price and their gains from trade. The former reflects the efficiency of decision making and trading intensity, which hinges on the quality of information obtained from the price, whereas the latter incorporates trading opportunities for each unit of the asset apart from their lack of information. As cross-border costs decrease so that foreign investors trade more, learning from the price always increases due to increased price informativeness, whereas its effect on the gains from trade is ambiguous: Foreign investors' trades make trading on the asset value less profitable by making the price closer to the asset value, but, at the same time, can make trading on uninformed investors' liquidity shock more profitable through mutual gains arising from differential valuations.

The analyses of the basic model yield the following results depending on the structure of liquidity shocks across uninformed and domestic informed investors. First, when uninformed investors have no liquidity shock, Proposition 3.7 shows that reducing cross-border costs unambiguously makes uninformed investors worse off. In this case, despite a tradeoff between an increase in learning from the price and a decrease in the gains from trade, the former is dominated by the latter in the absence of mutual gains between uninformed and foreign investors. Second, as uninformed investors have a liquidity shock, foreign investors' trades lead to two additional effects as follows: (i) foreign investors' trades increase liquidity trades of uninformed investors directly through mutual gains arising from differential valuations. (ii) foreign investors' trades further promote these liquidity trades of uninformed investors through learning from the price resulting from increased price informativeness. The question of interest here is whether these effects can indeed cause foreign investors' trades to increase the welfare of uninformed investors. According to Propositions 3.8 and 3.9, the answer is yes either when uninformed investors have a sufficiently large liquidity shock compared with that of informed ones or when foreign investors' volume is sufficiently large. This provides the conditions under which a tax on speculative trades hurts uninformed investors, which correspond to a high proportion of uninformed investors' liquidity trading and large volume of foreign investors. While the former naturally follows from the presence of large trading opportunities in general, the latter holds for any positive size of uninformed investors' liquidity shock.

Next, the extended model is considered to take into account market power and the resulting role of liquidity as another plausible economic force behind cross-border investments and financial transaction taxes. For the sake of comparison with the basic model, the extended model is generally identical to the basic model except that a large investor exists with his own liquidity shock. As the presence of market power causes inefficiencies by allowing for monopolistic demand-reducing behaviors and the lack of liquidity is indeed an oft-cited disadvantage of taxing trades, this setting provides an interesting comparison with the basic model by incorporating the influence of informed investors' trading on market liquidity. Proposition 3.12 shows that reducing cross-border costs can increase the welfare of uninformed investors even without their liquidity shock, in contrast to what occurs in the basic model. This is generally driven by the well-known fact that the presence of market power increases the potential benefit of promoting trades in terms of the overall efficiency. In this particular context, reducing cross-border costs leads to lower price impact of the large investor, which in turn promotes his trades on liquidity shock and thereby increases uninformed investors' gains from trade. What is still not obvious in this result is two-fold: First, combined with learning from the price, such potential benefit from liquidity is set against increased competition and the resulting decrease in the trading opportunities on the asset value in the viewpoint of uninformed investors. This leads us to identify the condition under which reducing cross-border costs makes uninformed investors better off, which turns out to be associated with a smaller proportion of uninformed investors' trades. This point potentially undermines (supports) the argument for promoting (restricting) cross-border investments in emerging economies where the lack of liquidity is a significant concern and most domestic investors appear to be unsophisticated. Second, it highlights the importance of the learning effect in the sense that reducing cross-border costs would not otherwise improve the welfare of uninformed investors. In other words, speculative trading can benefit uninformed investors only through the interplay between liquidity and learning from the price.

In addition, recognizing the difficulty of identifying speculative trades in practice, we consider the case of uniform taxation in two models considered above (i.e., the basic model and the extended model with market power). Propositon 3.13 shows that all main results presented above concerning a tax on speculative trades generally continue to hold with uniform taxation under the condition that foreign investors' risk aversion is sufficiently small. This arises from the fact that less risk averse investors' trades are more sensitive to the tax compared with others, and suggests that a uniform transaction tax on all market participants can proxy for a (perhaps hypothetical) transaction tax selectively imposed on speculative trades considered in the main analyses.

Finally, we compare the basic model with the case where foreign investors have a liquidity shock so that their trading creates noises. Intuitively, such noise-creating trades go in the opposite direction compared with the case of speculative trades in terms of a tradeoff between the gains from trade and learning from the price. In particular, they generate more trading opportunities for uninformed investors, whereas they reduce learning from the price. Proposition 3.15 indicates the possibility that reducing cross-border costs decreases the welfare of uninformed investors. This can be the case when uninformed investors have a sufficiently large liquidity shock and arises due to the dominance of the loss in learning from the price over the increase in the gains from trade, thereby justifying the argument for a Tobin tax (Tobin, 1978). This case provides a contrast to the basic model in that a tax on noise-creating trades can be desirable in markets with large liquidity trades of uninformed investors, whereas a tax on speculative trades is desirable in markets with large liquidity trades of informed investors. Further, in contrast to the basic model where large volume of (speculative) foreign investors leads to suboptimality of a small tax, this case leads to the possibility that there is an optimal level of foreign investors' (noise-creating) trades, possibly justifying the policy of countercyclical capital control.

To summarize, the analyses in this paper identify the conditions under which speculative trades benefit uninformed investors through mutual gains, learning from the price, and improved liquidity even more than taking profits from uninformed investors. Stressing the importance of the learning channel whereby investors benefit from the information content of prices, these conditions hold when uninformed investors carry out large liquidity trades or when the volume of speculative trades is already large. Even without liquidity shock of uninformed investors, they may hold when there is a large investor who takes into account his price impact, and this is more likely the case with lower proportion of trades carried out by uninformed investors. Overall, these cases appear to be more likely to hold in developed financial markets with large volume of (cross-border) institutional traders, a high fraction of whom have a speculative motive. In these cases, a tax on speculative trades hurts uninformed investors through efficiency losses, despite the absence of direct tax burden. This point makes it challenging to justify such tax on speculative trades, as it is then neither a Pareto improvement nor desired by uninformed investors. The modeling framework for these analyses builds on many previous studies on markets with investors having correlated private valuations (see Subsubsection 3.1.1) but it is tailored in some ways, including the distinction between a common value and investors' liquidity shock and their knowledge about their own liquidity shock, to clarify the role of speculative trades in the provision of information and the allocation of resources in markets. Compared with Sorensen's (2017) similarly motivated analyses on uniform taxes with the redistribution of the tax revenues, our analyses shed light on the potential benefits of speculative trades based on superior information. While it is unclear in Sorensen (2017) whether uninformed investors still benefit from uniform taxes before the redistribution of the tax revenues, our analyses identify the conditions under which taxing speculative trades, which could be proxied by uniform taxation, makes uninformed investors better or worse off before the redistribution of the tax revenues. The mixed results presented in this study come from the fact that a tax on speculative trades reduces not only these speculative trades but also liquidity trades carried out by uninformed investors and those with market power. This is in contrast to Sorensen (2017) where such a selective tax would unambiguously promote liquidity trades of uninformed investors by reducing adverse selection.

More broadly, aside from taxing speculative trades, the analyses in the paper provide an insight about the welfare implications of expanding financial markets. These analyses generally suggest that its effect on the welfare of uninformed participants depends on whether additional trading activities are speculative or noise-creating, uninformed participants' trading motive, and market concentration, as summarized in Subsection 3.7. On the empirical side, these analyses clarify the relationship between price informativeness and welfare. In particular, many recent empirical studies measure price informativeness to investigate how it changes over time and its cross-sectional variations (e.g., Bai, Philippon, and Savov, 2016; Davila and Parlatore, 2021a; Farboodi et al., 2021; Kacperczyk, Sundaresan, and Wang, 2021). These appear to be attributed to an increase in market size via various recent trends such as technological advances, financial globalization, and institutional ownership. However, it is still unclear whether such change in price informativeness indeed makes market participants better or worse off. The analyses in the paper address this point in the viewpoint of uninformed participants, leading to mixed conclusions mainly depending on investors' trading motives as well as market concentration.

3.1.1 Related literature

In terms of substance, this paper is related to the literature on financial transaction taxes. The idea of taxing financial transactions dates long back to Keynes (1936), and, interestingly, similar arguments reappeared and gained the popularity following the collapse of the Bretton Woods system (Tobin, 1978), Black Monday (Stiglitz, 1989), and the global financial crisis in 2008. The last one then spawned the most recent proposal of the European Union financial transaction tax, as reviewed in Hemmelgarn, Nicodeme, Tasnadi, and Vermote (2016). On the empirical side, there is a large literature. See Burman et al. (2016) and Matheson (2011) for a comprehensive review with policy discussions. One of the existing analytic studies is Subrahmanyam (1998), who uses a strategic trading model of informed agents submitting market orders to analyze a financial transaction tax imposed on informed agents. Notably, he shows that the tax can increase market liquidity depending on the number of informed agents, but, even if it increases market liquidity, informed agents are unambiguously worse off. Nevertheless, if private information is endogenous and costly, a tax may be beneficial by preventing a race to obtain private information earlier than others. The mechanism at work in Subrahmanyam is different from this study because there is no learning from the price due to the nature of his trading mechanism. Rather, it depends on the idea that the use of short-term information causes negative externalities. Gümbel (2005) presents similar analyses and suggests the possibility of positive externalities of short-term speculation. Dow and Rahi (2000) appear to be closer to the basic model in this study in that they consider the situation where informed traders and uninformed hedgers trade with risk-neutral market makers by submitting price-contingent demands to these market makers. Therefore, they learn from prices. However, in their framework, risk-neutral market makers trade infinitely on the asset value so that uninformed investors earn zero from that. As a result, informed investors' trading on the asset value does not result in any wealth transfer across informed and uninformed investors due to the absence of uninformed investors' gains possibly taken by informed ones. Another difference from this paper is that price learning is not necessarily beneficial for these uninformed hedgers who trade for risk sharing due to the Hirshleifer effect, whereby public information kills the opportunity for risk sharing. Dow and Rahi derive various equilibrium results which feature the Hirshleifer effect and the comparison of theirs with those of this study provides some interesting points, as detailed in Subsubsection 3.3.2. More recently, Vives (2017) proposes a Tobin-style tax to correct an externality of the information choice facing informed speculators between private and public signals. In all these papers, taxes are Pigouvian in that they are intended to enhance the overall efficiency of markets by addressing externalities of trading activities, in line with the efficiency-based motivation for a Tobin tax.

Biais and Rochet (2020) and Sorensen (2017) provide a justification for financial transaction taxes as a policy instrument for redistribution. Sorensen (2017) considers a tax imposed on market makers who determine the bid and ask prices to trade with informed speculators and uninformed traders having idiosyncratic shocks. Here the tax reduces informed speculators' trades, thereby reducing adverse selection, whereas it also reduces uninformed traders' trades, thereby increasing adverse selection. Despite such tradeoff, Sorensen shows that the tax always increases adverse selection proxied by the bid-ask spreads. As a result, it decreases trading volume and total welfare. Nevertheless, if the revenues from the tax are redistributed according to trading volume, then uninformed investors can gain from the tax. Their benefit essentially stems from the fact that the tax reduces adverse selection by curbing informed speculators' trades. This point matches with the benchmark case in this study. However, his framework focuses on the downside of speculative trades and abstracts from their upsides arising from learning from the price and consideration of market power.[4] Biais and Rochet (2020) offer an alternative approach without information asymmetries to justify financial transaction taxes as a policy instrument for wealth redistribution. They consider the problem of optimizing wealth-weighted utilitarian social welfare through redistributive taxation on capital income and financial transactions. Under the condition that richer people are more inclined to trade in the financial market than poorer people, they show that the financial transaction tax helps to identify investors' unobservable wealth, implying that such a tax is always part of the optimal tax mix along with a tax on the capital income.

A separate line of research investigates the impact of financial transaction taxes under belief disagreements among investors. These investors engage in fundamental and non-fundamental trading in that they trade for different risk-adjusted valuations and different priors, respectively. While the former gives rise to welfare-enhancing allocations of resources and risks and improves price informativeness, the latter can generate welfare losses and reduce price informativeness. This sort of distinction between fundamental and non-fundamental trading is considered by Cipriani, Guarino, and Uthemann (2021), Davila (2021), and Davila and Parlatore (2021b). Such nonfundamental trading stemming from belief disagreements, which is not considered in this study, appears to provide additional gains (costs) of imposing transaction taxes (reducing cross-border costs) in the context of financial transaction taxes (cross-border investments).

The modeling framework of this study draws on the premise that agents have correlated private valuations for a risky asset and hold private information about their own valuation. The relevant literature starts from Vives (2011), who builds and analyzes a model of oligopolistic competition in strategic supply function. Vives deviates from the standard assumption of noise traders in order to study the welfare consequence of large

[4]In his modeling framework, uninformed investors observe predetermined ask and bid prices from market makers who are uninformed as well. Accordingly, these ask and bid prices do not have any information content which could arise from informed investors' trading. Further, the presence of risk-neutral market makers in his framework makes sure that these market makers trade infinitely on the asset value so that additional (informed) investors' trading on the asset value does not further reduce the price impact of other investors' liquidity trading. Instead, their trading merely causes adverse selection, thereby widening the bid-ask spreads.

markets. Rostek and Weretka (2012) and Rostek and Weretka (2015) introduce double auctions with linear-quadratic preferences under asymmetric structures of valuations, which feature tractability allowing for deriving closed-form equilibrium results. More recently, Lou and Rahi (2021) consider a similar framework with coexisting informed and uninformed investors to analyze the entry of these investors and show that the distinction between informed and uninformed investors gives rise to different implications of large market size on market liquidity. Compared with theirs, the current study derives its implications on the welfare of uninformed investors, whose relationship with market liquidity is not obvious as seen in the model with market power in Subsection 3.4.

The closest to this study in terms of the modeling framework are Rahi and Zigrand (2018) and Rahi (2021). Rahi and Zigrand (2018) build a model of competitive markets with asymmetric information and heterogeneous valuations to analyze strategic interaction of information acquisition as well as its externalities across agents. Whereas Rahi and Zigrand (2018) restrict attention to binary actions of acquiring information, Rahi (2021) allows for continuous actions of choosing the precision of private information, which turn out to be useful for the full characterization of information acquisition in the case of two different types of agents. Above all, the basic model in Subsection 3.2 is largely contained in their framework in terms of payoff structures, and thus, a tradeoff between learning from the price and the gains from trade naturally arises in their analyses as well. However, they restrict attention to the analyses of information acquisition, rather than the restriction of trading activities. In addition, the extended model with market power in Subsection 3.4 significantly deviates from their models.

An alternative modeling choice for heterogeneous valuations is to consider risk-sharing activities of traders explicitly. While investors' equilibrium behavior would not be qualitatively different from that in this study, the welfare analyses can be richer, notably involving the Hirshleifer effect, whereby public information kills the opportunity for risk sharing. It is named after Hirshleifer (1971) and applies to public disclosures of information (e.g., Gottardi and Rahi, 2014) and prices (e.g., Dow and Rahi, 2000; Kawakami, 2017; Marin and Rahi, 2000). Drawing on this idea, Kawakami (2017) analyzes the welfare consequence of changing market size and finds that information aggregation causes welfare costs by generating the Hirshleifer effect and thus large markets are not necessarily optimal in terms of total welfare. In comparison to this study, Kawakami elaborates more on the source of heterogeneous valuations, but he focuses on a change in the number of symmetrically informed agents, which makes it challenging to see the role of information held by these agents, and abstracts from consideration of market power.

The extended model with market power in this study features the coexistence of large and small investors. This feature is present in Glebkin and Kuong (2021), who analyze the coexistence of multiple informed large traders and a continuum of small partially informed traders with heterogeneous valuations and linear-quadratic preferences, as well as the core-fringe example in Manzano and Vives (2021). Glebkin and Kuong (2021) focus on competition among large traders and the quality of private signals held by small traders, showing that increased competition among large traders may make small traders worse off and an increase in the quality of small investors' private signals may render the price less informative. Most notably, these findings essentially arise from a complementarity between large and small traders' trades. This is the case as well in the extended model with market power, as discussed in Subsection 3.4.3. In fact, it appears to be a common feature arising from the presence of heterogeneity across market participants (e.g., Manzano and Vives, 2021).

3.2 Basic model

There is one risky asset whose value is intially unknown to investors. The true value of the asset is denoted by $\theta+\varepsilon$, where θ and ε are normally and independently distributed with mean zero and prior variance σ_θ^2 and σ_ε^2, respectively, i.e., $\theta \sim N(0,\sigma_\theta^2)$, $\varepsilon \sim N(0,\sigma_\varepsilon^2)$, and $\theta \perp \varepsilon$. As further explained below, θ is a forecastable component which is known to informed investors, whereas ε is a non-forecastable component, which is needed to rule out the unrealistic outcome of infinite trading among investors with different valuations. For the sake of brevity, we call the forecastable component θ the "asset value" throughout the paper.

The trading game proceeds as follows: At the first stage, all investors simultaneously submit their demand conditional on the price to the market. At the second stage, the price is set to clear the market. At the third stage, the true value of the asset $\theta+\varepsilon$ is realized, and accordingly, each investor obtains profit.

As mentioned in Subsection 3.1, there are price-taking informed and uninformed investors in the market and a fraction of informed investors incur transaction costs. Here, informed investors are "informed" in the sense that they perfectly know the asset value θ, whereas uninformed investors do not know the asset value θ at all. Uninformed investors have mass λ_U, whereas informed investors consist of mass λ_{DI} of cost-exempt informed investors and mass λ_F of cost-bearing ones, as detailed below. We hereafter call these cost-exempt and cost-bearing informed investors "domestic informed" and "foreign" ones, respectively, in the context of cross-border investments.

Trades occur among these investors as they take into account their type-dependent liquidity shocks affecting per-unit valuations for the asset in addition to the asset value θ. Uninformed investors are subject to a liquidity shock ω_U, whereas domestic informed investors are subject to a liquidity shock ω. These liquidity shocks are driven by various common factors affecting their liquidity demands, such as correlated flow shocks to mutual and pension funds studied by Koch, Ruenzi and Starks (2016) and Da, Larrain, Sialm and Tessada (2018), respectively. In case of uninformed investors, they may also have liquidity demands depending on life cycles and needs for cash.[5] Formally, uninformed and domestic informed investors have CARA utilities with coefficients φ_U and φ_{DI}, respectively, as follows:

$$u_U=-\exp\left\{-\varphi_U x_U\left(\theta+\varepsilon+\omega_U-p\right)\right\} \text{ and } u_{DI}=-\exp\left\{-\varphi_{DI} x_{DI}\left(\theta+\varepsilon+\omega-p\right)\right\},$$

where x_U is uninformed investors' asset holding, x_{DI} is that of domestic informed investors, and p is the price of the asset. We assume that these liquidity shocks ω_U and ω are independent with each other, as well as with θ and ε. All these uninformed and

[5]Even if their liquidity shock is idiosyncratic, the main insight of Propositions 1-3 in the analyses of the current model still persists. The formal analyses are available upon request.

domestic informed investors know their own liquidity shock. That is, domestic informed investors know ω and uninformed investors know ω_U. Such knowledge about liquidity shocks helps to extract information about the asset value θ from the price. This assumption is intended to rule out the channel where uninformed investors learn about their own liquidity shock from the price, and, thereby, clarify the informational role of the price as a source of information about the asset value θ. At this point, the current model is distinguished from the standard framework in previous studies (e.g., Rahi and Zigrand, 2018; Rostek and Weretka, 2012), where each agent observes a noisy private signal which consists of a common value plus the agent's own liquidity shock as well as a random error independent of all other variables.

Foreign investors, who are the cost-bearing informed investors, do not have any liquidity shock so that they solely trade on the asset value θ. They incur cross-border transaction costs which are quadratic with respect to their asset holding x_F, i.e., $-\frac{1}{2}cx_F^2$, where $c \geq 0$ is constant. The quadratic form of transaction costs conveniently parametrizes the degree of restriction on trading activities in the model and is common in previous studies (e.g., Dow and Rahi, 2000; Subrahmanyam, 1998). In particular, their CARA utility with coefficient φ_F is given as follows:

$$u_F = -\exp\left[-\varphi_F \left\{x_F\left(\theta + \varepsilon - p\right) - \frac{1}{2}cx_F^2\right\}\right],$$

where x_F is the asset holding of foreign investors.

These reflect the idea discussed in Subsection 3.1 that various trading restrictions, such as financial transaction taxes and capital control, are usually targeted at certain types of trades which are deemed speculative, such as cross-border investments and short-term and high-frequency trading. These transaction costs could be interpreted as the extent to which foreign investors' trades are affected by trading restrictions, rather than the actual tax rate. Even without explicitly taxing trades, implicit barriers such as the limits on foreign portfolio holdings could be regarded as the cross-border costs in the model as these barriers naturally reduce trading. Outside of the context of cross-border investments, when it is difficult to identify speculative trades, uniform taxation can be used as a proxy for taxing these speculative trades, as analyzed in Subsection 3.5.

Another issue is whether such cost-bearing investors in the contexts of financial transaction taxes and cross-border investments mainly carry out speculative trades rather than liquidity trades. This is an empirical question. In the context of cross-border investments, empirical evidence appears to be in favor of the current model. In particular, Kacperczyk, Sundaresan and Wang (2021) show that foreign institutional ownership increases price informativeness, which is measured by the predicted variation of cash flows using contemporaneous market prices, in 40 countries including both developed and emerging ones. In a similar vein, Bae, Ozoguz, Tan and Wirjanto (2012) and Ng, Wu, Yu and Zhang (2016) document the positive effects of foreign ownership on price discovery and market liquidity, respectively. Nonetheless, it can be reasonably argued that large heterogeneity across markets is likely to be present regarding foreign investors' trading motives and thus liquidity trading is crucial in explaining foreign institutional investors' behavior such as "flight to quality", leading to the well-known argument for a Tobin tax (Tobin, 1978). In the context of financial transaction taxes, empirical evidence is rather mixed. Colliard and Hoffmann (2017) find that the adoption of financial transaction tax in France decreased price efficiency and market liquidity, whereas Cipriani, Guarino, and Uthemann (2021) employ a structural estimation approach using transaction data on a sample of NYSE firms to show that a financial transaction tax would improve the informational efficiency of prices despite a loss of liquidity measured by bid-ask spreads. Given the well-known argument for a Tobin tax and the empirical nature of investors' trading motives, the case of liquidity shock of foreign investors in the model is analyzed in Subsection 3.6, focusing on the comparison with the basic model where foreign investors do not have a liquidity shock.

Definition 3.1. An equilibrium consists of demands of all types of investors and price function $p = \alpha\theta + \beta\omega + \beta_U \omega_U$ with coefficients α, β and β_U representing its sensitivities to θ, ω and ω_U, respectively, where the following two conditions hold:

1. Every investor of three different types submits demand for the asset (i.e., x_U, x_{DI}, and x_F) maximizing the expected utility conditional on his available information,

other investors' strategies and the price.

2. The market clears for the asset. In particular, it holds that $\lambda_U x_U + \lambda_{DI} x_{DI} + \lambda_F x_F = 0$.

3.2.1 Solving the model

Following Definition 3.1, the steps for solving the basic model are standard. First, recognizing a conjecture on the price $p = \alpha\theta + \beta\omega + \beta_U \omega_U$, each type of investors chooses demand so as to maximize the conditional expected utility. Due to investors' CARA preferences and foreign investors' quadratic cross-border costs, each type of investors' optimization problem turns out to be tractable, corresponding to maximizing a concave quadratic function with respect to their asset holding. As a result, we have

$$x_U = \frac{E[\theta|\mathscr{I}_U] + \omega_U - p}{\varphi_U Var[\theta + \varepsilon|\mathscr{I}_U]}, x_{DI} = \frac{\theta + \omega - p}{\varphi_{DI}\sigma_\varepsilon^2} \text{ and } x_F = \frac{\theta - p}{\varphi_F \sigma_\varepsilon^2 + c},$$

where $\mathscr{I}_U := \{p, \omega_U\}$ is the information set of uninformed investors. In order to obtain uninformed investors' expectation terms about the asset value θ (i.e., $E[\theta|\mathscr{I}_U]$ and $Var[\theta|\mathscr{I}_U]$), we use the fact that the (unique) unbiased sufficient statistic for the asset value θ based on uninformed investors' information set $\mathscr{I}_U = \{p, \omega_U\}$ is given by $\tilde{p} = \frac{p - \beta_U \omega_U}{\alpha}$. Then we get

$$E[\theta|\mathscr{I}_U] = E[\theta|\tilde{p}] = \frac{\sigma_p^{-2}}{\sigma_\theta^{-2} + \sigma_p^{-2}}\tilde{p} \text{ and } Var[\theta|\mathscr{I}_U] = Var[\theta|\tilde{p}] = \left(\sigma_\theta^{-2} + \sigma_p^{-2}\right)^{-1},$$

where $\sigma_p^{-2} := Var(\tilde{p}|\theta)^{-1}$ represents the precision of $\tilde{p}$ as an unbiased signal about the asset value θ. Next, we determine the market-clearing price which ensures that the integrated sum of these optimal demands is equal to zero. Finally, if this market-clearing price coincides with the conjectured price $p = \alpha\theta + \beta\omega + \beta_U\omega_U$, the conjectured price p constitutes an equilibrium.

In addition, it is useful to define V_U, V_{DI} and V_F, which represent the (information-adjusted) volume of uninformed, domestic informed and foreign investors' trades, respectively, as follows:

$$V_U := \frac{\lambda_U}{\varphi_U \sigma_\varepsilon^2},\ V_{DI} := \frac{\lambda_{DI}}{\varphi_{DI} \sigma_\varepsilon^2},\ \text{and } V_F := \frac{\lambda_F}{\varphi_F \sigma_\varepsilon^2 + c}.$$

The following lemma proves the existence of a unique equilibrium in the basic model and characterizes the equilibrium price.

Lemma 3.2. *There is a unique equilibrium with price $p = \alpha\theta + \beta\omega + \beta_U\omega_U$, whether $\sigma_{\omega U}^2 = 0$ or $\sigma_{\omega U}^2 > 0$. In this equilibrium, the relative ratios of coefficients $\frac{\beta}{\alpha}$ and $\frac{\beta_U}{\alpha}$ are given by*

$$\frac{\beta}{\alpha} = \frac{V_{DI}}{V_{DI} + V_F} \text{ and } \frac{\beta_U}{\alpha} = \frac{\frac{V_U}{V_{DI}+V_F}}{1 + \frac{V_U}{V_{DI}+V_F}\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}} \frac{\sigma_\theta^{-2} + \sigma_p^{-2}}{\sigma_\theta^{-2} + \sigma_p^{-2} + \sigma_\varepsilon^{-2}},$$

where $\sigma_p^{-2} := Var(\tilde{p}|\theta)^{-1} = \left(\frac{\beta}{\alpha}\right)^{-2} \sigma_\omega^{-2}$. Given these ratios of coefficients $\frac{\beta}{\alpha}$ and $\frac{\beta_U}{\alpha}$, coefficient α is given by

$$\alpha = \frac{V_U \frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + V_{DI} + V_F}{V_U \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}} + V_{DI} + V_F}.$$

Overall, these determine all coefficients α, β and β_U in the equilibrium price.

3.3 Analyses of the basic model

In this section, we start by defining price informativeness and the welfare of uninformed investors and then proceed to examine the effect of reducing cross-border costs on the welfare of uninformed investors. Price informativeness measures the quality of information from the price summarized in the aforementioned sufficient statistic $\tilde{p}$ in the viewpoint of uninformed investors. In particular, it is defined as follows:

Definition 3.3. Price informativeness is $\sigma_p^{-2} := Var(\tilde{p}|\theta)^{-1}$, where $\tilde{p}$ is an unbiased sufficient statistic for the asset value θ in the viewpoint of uninformed investors, as defined in the previous section.

Note that this definition is seemingly different from the standard one used in Rahi (2021), Rahi and Zigrand (2018), and Rostek and Weretka (2012). In these previous studies, price informativeness is defined as $PI := 1 - \sigma_\theta^{-2} Var(\theta | \mathscr{I}_U)$ in the viewpoint of uninformed investors in the current model. Using the fact that $Var(\theta | \mathscr{I}_U) = \left(\sigma_\theta^{-2} + \sigma_p^{-2}\right)^{-1}$, we get $PI = 1 - \sigma_\theta^{-2} \left(\sigma_\theta^{-2} + \sigma_p^{-2}\right)^{-1} = \sigma_p^{-2} \left(\sigma_\theta^{-2} + \sigma_p^{-2}\right)^{-1}$. Having the prior σ_θ^{-2} fixed, we can see that their definition of price informativeness PI is equivalent to the current definition σ_p^{-2}.

In equlibrium, Lemma 3.2 implies that price informativeness is given by $\sigma_p^{-2} = \left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2} \sigma_\omega^{-2}$. This in turn implies that reducing cross-border costs always increases price informativeness σ_p^{-2} by increasing the volume of foreign investors' trades V_F. As foreign investors trade more, the price incorporates more information from their trades.

Definition 3.4. The welfare of uninformed investors W is the ex ante expectation of the utility of uninformed investors. In particular, it is given by

$$W := E[u_U] = E\left[-\varphi_U x_U (\theta + \varepsilon + \omega_U - p)\right],$$

where x_U is the optimal demand of uninformed investors.

As mentioned in Subsection 3.1, our focus on the welfare of uninformed investors is motivated by the policy debate on financial transaction taxes and foreign institutional investments. In practice, policymakers may also take into account other types of agents, including informed investors in the current model. Even so, a negative impact on the welfare of uninformed investors provides a strong counterargument to the policy under consideration, possibly due to pressure from voters. An alternative justification is that the welfare of uninformed investors is likely to proxy for the utilitarian welfare when the mass of informed investors is small but these informed investors still take a large proportion in terms of trading volume as they are more informed and less risk averse compared with uninformed ones. Intuitively, the utilitarian welfare disproportionately reflects those who take large mass and those whose utilities are sensitive to changes in wealth, which correspond to uninformed investors in the current model. Note that risk aversion is crucial at this point. In case of risk-neutral agents, the utilitarian welfare would reflect their utilities proportionally.

In order to get a clear intuition on the welfare of uninformed investors W, we closely follow Rahi (2021) to define the following two variables:

Definition 3.5. Closely following Rahi (2021), we define uninformed investors' gains from trade G and learning from the price L as

$$G := \sigma_\theta^{-2} Var(\theta + \omega_U - p) \text{ and } L := \sigma_\theta^{-2} \left\{ \sigma_\theta^2 - Var(\theta | \mathscr{I}_U) \right\}.$$

The gains from trade G represent uninformed investors' trading opportunities for each unit of the asset apart from their lack of information. In particular, uninformed investors have more profitable trading opportunities the greater the distance betwen their own valuation $\theta + \omega_U$ and the price p. As their trading opportunities arise from the differences in valuations, the gains from trade G incorporate mutual gains between uninformed investors and others. In the absence of foreign investors' trades, the gains from trade G represent mutual gains between uninformed and domestic informed investors. In the presence of foreign investors' trades, the gains from trade G additionally reflect mutual gains between uninformed and foreign investors.

Learning from the price L reflects the quality of information obtained from the price, which affects uninformed investors' welfare through their decision making and trading intensity. In equilibrium, it is given by $L = \sigma_p^{-2} \left(\sigma_\theta^{-2} + \sigma_p^{-2} \right)^{-1}$, which is merely an increasing transformation of price informativeness σ_p^{-2} given the prior σ_θ^{-2}. The presence of foreign investors' trades improves learning from the price L by improving price informativeness σ_p^{-2}. This in turn increases the welfare of uninformed investors W by allowing them to face lower ex post uncertainties and trade better, given their trading opportunities for each unit of the asset.

The following lemma represents the welfare of uninformed investors W in terms of their gains from trade G and learning from the price L.

Lemma 3.6. *In equilibrium, given the prior σ_θ^{-2} and the unforecastable error σ_ε^{-2}, the* *welfare of uninformed investors W is divided into the gains from trade G and learning from the price L as follows:*

$$W=-\left(1+\frac{Var(\theta+\omega_U-p)-Var(\theta|\mathscr{I}_U)}{\sigma_\varepsilon^2+Var(\theta|\mathscr{I}_U)}\right)^{-\frac{1}{2}}=-\left(\frac{\sigma_\theta^{-2}\sigma_\varepsilon^2+G}{\sigma_\theta^{-2}\sigma_\varepsilon^2+1-L}\right)^{-\frac{1}{2}}.$$

For $\sigma_\varepsilon^2=0$, this expression matches with the decomposition of the welfare effect in Rahi (2021).[6] In line with Rahi (2021), the expression suggests that a change in the price affects uninformed investors' welfare through changes in the gains from trade G and learning from the price L. In the context of the welfare effects from information acquisition of investors, Rahi termed these two welfare effects "the gains-from-trade effect" and "the learning effect".

Then how does the model work as foreign investors trade more due to a reduction in cross-border costs? The resulting change in the welfare of uninformed investors is divided into the corresponding changes in learning from the price L and the gains from trade G. First, foreign investors' trades improve price informativeness σ_p^{-2}, thereby increasing learning from the price L. Second, their trades decrease uninformed investors' gains from trade G by making the price track the asset value more closely, thereby making trading on the asset value less profitable. This is because some of mutual gains between uninformed and domestic informed investors, which stem from their liquidity shocks ω and ω_U, are taken by foreign investors' trades. Third, their trades bring uninformed investors mutual gains arising from differential valuations of uninformed and foreign investors, thereby making trading on uninformed investors' liquidity shock ω_U more profitable. This works through a reduction in the price impact of uninformed investors' liquidity shock ω_U. At this point, it is not obvious whether the resulting change in the gains from trade G is positive or negative.

[6]To be specific, if $\sigma_\varepsilon^2=0$, we have $W=-\left(\frac{G}{1-L}\right)^{-\frac{1}{2}}$, which implies that the log-transformed utility is given by $\ln\left(W^{-2}\right)=\log G-\log(1-L)$. This is identical to a special case of agents without private information in Lemma 5.1 of Rahi (2021).

3.3.1 Benchmark case

In order to gain a clear insight, we consider the benchmark case where $\sigma_{\omega U}^2 = 0$. That is, uninformed investors do not have any liquidity shock and thus only domestic informed investors have liquidity shock ω. By shutting down mutual gains between uninformed and foreign investors leading to uninformed investors' liquidity trading, the benchmark case concerns the case where uninformed investors' gains from trade G only come from their trading on the asset value θ. In this case, foreign investors' trades unambiguously decrease uninformed investors' gains from trade G by making the price closer to the asset value θ. The following proposition presents the effect of reducing cross-border costs on the welfare of uninformed investors.

Proposition 3.7. *In the benchmark case, reducing cross-border costs decreases the welfare of uninformed investors W. The decrease in their welfare is divided into an increase in learning from the price L and a decrease in the gains from trade G.*

The negative sign of change in the gains from trade G is not surprising, meaning that uninformed investors lose from foreign investors' trades apart from learning from the price. What is non-trivial is that the decrease in the gains from trade G always dominates the corresponding increase in learning from the price L, thereby leading to a decrease in the welfare of uninformed investors W. Part of the decrease in uninformed investors' welfare is attributed to information about the asset value θ held by foreign investors. This part turns out to be unambiguously negative as well.[7] Apart from such part attributed to information of foreign investors, the rest of the decrease in uninformed investors' welfare arises from foreign investors' trading that would occur without information about the asset value θ, which simply takes away uninformed investors' gains from trade G. Overall, though it is difficult to say that such dominance of the gains-from-trade effect is general,

[7] Note that this statement is stronger than Proposition 1. The proof is available upon request. This exercise corresponds to increasing the proportion of informed traders in Grossman and Stiglitz's model where informed and uninformed traders coexist and the exogenous supply of the asset is normally distributed. However, Grossman and Stiglitz focus on the ratio between informed and uninformed traders' ex ante utilities and show that increasing the proportion of informed investors relatively disadvantages informed traders in terms of the ratio of their ex ante utilities. Though this point turns out to be useful to look at traders' incentives to acquire private information in their analyses, it sidesteps the explicit analyses of uninformed and informed traders' separate utilities. In fact, an increase in the proportion of informed traders leads to decreases in both of informed and uninformed traders' ex ante utilities.

it appears to hold broadly in the CARA-normal framework. This suggests that the learning channel may not be strong on the quantitative side compared with the corresponding change in the gains from trade in the absence of liquidity trading of uninformed investors.

3.3.2 General case

We turn to the general case with the presence of uninformed investors' liquidity shock ω_U. In line with the benchmark case, foreign investors' trades make trading on the asset value θ less profitable by making the price closer to the asset value, whereas their trades bring uninformed investors mutual gains arising from differential valuations.

Let us elaborate more on the role of uninformed investors' liquidity shock ω_U. In comparison to the benchmark case without their liquidity shock, the difference is that uninformed investors additionally trade on their liquidity shock ω_U and learning from the price further promotes their liquidity trades. The former corresponds to mutual gains between uninformed and foreign investors, whereas the latter strengthens the learning effect. Both of them make it more likely that foreign investors' trades increase the welfare of uninformed investors, thereby overturning the benchmark case. To see this in detail, note first that the equilibrium price $p = \alpha\theta + \beta\omega + \beta_U\omega_U$ yields uninformed investors' gains from trade G as follows:

$$
\begin{aligned}
G &:= Var(\theta + \omega_U - p) \\
&= Var[(1-\alpha)\theta - \beta\omega + (1-\beta_U)\omega_U] \\
&= Var[(1-\alpha)\theta - \beta\omega] + Var[(1-\beta_U)\omega_U] \\
&= Var[(1-\alpha)\theta - \beta\omega] + (1-\beta_U)^2\sigma_{\omega U}^2,
\end{aligned}
$$

where the first and other terms represent their trades on the asset value θ and liquidity shock ω_U, respectively. By Lemma 3.6, this implies that the welfare of uninformed investors W is an increasing transformation of the following expression:

$$\begin{aligned} W^{-2} &= \frac{\sigma_\varepsilon^2 + Var[(1-\alpha)\theta - \beta\omega] + (1-\beta_U)^2\sigma_{\omega U}^2}{\sigma_\varepsilon^2 + Var(\theta|\mathscr{I}_U)} \\ &= \frac{\sigma_\varepsilon^2 + Var[(1-\alpha)\theta - \beta\omega]}{\sigma_\varepsilon^2 + Var(\theta|\mathscr{I}_U)} + \frac{(1-\beta_U)^2\sigma_{\omega U}^2}{\sigma_\varepsilon^2 + Var(\theta|\mathscr{I}_U)} \equiv K_\theta + K_{\omega U}, \end{aligned}$$

where K_θ and $K_{\omega U}$ are the first and second terms on the second line, respectively. Here, K_θ represents the welfare of uninformed investors from trades they would carry out without liquidity shock ω_U, whereas $K_{\omega U}$ represents the change in their welfare arising from liquidity shock ω_U.[8] In line with Proposition 3.7 in the benchmark case, reducing cross-border costs always decreases uninformed investors' welfare apart from liquidity shock ω_U (i.e., K_θ). In contrast, provided that the presence of liquidity shock ω_U increases the welfare of uninformed investors (i.e., $K_{\omega U} > 0$) due to a sufficient liquidity shock of uninformed investors $\sigma_{\omega U}^2$, reducing cross-border costs can increase uninformed investors' welfare through their liquidity trades (i.e., an increase in $K_{\omega U}$) by bringing mutual gains to uninformed investors (i.e., a reduction in β_U) and improving learning from the price (i.e., a reduction in $Var(\theta|\mathscr{I}_U)$). At this point, the relevant questions answered by the model are whether these welfare-improving channels can dominate the negative effect through a reduction in K_θ, and, if so, which of them plays a crucial role in causing an improvement in the welfare of uninformed investors.

Proposition 3.8. *As uninformed investors' liquidity shock $\sigma_{\omega U}^2$ is large, reducing cross-border costs leads to an increase in learning from the price L, whereas it decreases the gains from trade G if and only if $\sigma_\varepsilon^{-2} - 8\sigma_\theta^{-2} > 0$ and*

$$\sigma_p^{-2} \in \left(\frac{1}{2}\sigma_\varepsilon^{-2} - \sigma_\theta^{-2} - \frac{1}{2}\sqrt{\sigma_\varepsilon^{-2}\left(\sigma_\varepsilon^{-2} - 8\sigma_\theta^{-2}\right)}, \frac{1}{2}\sigma_\varepsilon^{-2} - \sigma_\theta^{-2} + \frac{1}{2}\sqrt{\sigma_\varepsilon^{-2}\left(\sigma_\varepsilon^{-2} - 8\sigma_\theta^{-2}\right)}\right).$$

The above proposition indicates that uninformed investors' gains from trade G may increase or decrease as we reduce cross-border costs to encourage foreign investors' trades,

[8] An important technical fact is that a change in uninformed investors' liquidity shock $\sigma_{\omega U}^2$ does not affect any variable in K_θ, such as α, β, and $Var(\theta|\mathscr{I}_U)$. Depending on the CARA-normal framework, this allows us to algebraically decompose uninformed investors' welfare into K_θ and $K_{\omega U}$.

and this is the case even as uninformed investors' liquidity shock $\sigma^2_{\omega U}$ is large. On the one hand, if foreign investors' trades increase the gains from trade G, then it is obvious that these trades make uninformed investors better off through increases in the gains from trade and learning from the price. On the other hand, if foreign investors' trades decrease uninformed investors' gains from trade G, we can say that foreign investors' trades would make uninformed investors worse off setting aside the learning effect. This may occur because (i) foreign investors' trades still make uninformed investors' trading on the asset value less profitable, and (ii) foreign investors' trades may increase the price impact of uninformed investors' liquidity trading (i.e., coefficient β_U in the price) by promoting their liquidity trading through the learning channel. In the limit of large liquidity shock of uninformed investors, the first one is dwarfed but the second one still persists. This is more likely to occur when price informativeness σ_p^{-2} is intermediate, possibly due to intermediate size of domestic informed investors' liquidity shock σ^2_{ω}.

Proposition 3.9. *Reducing cross-border costs increases the welfare of uninformed investors W if and only if uninformed investors have a sufficiently large liquidity shock $\sigma^2_{\omega U}$. Further, given uninformed investors' liquidity shock $\sigma^2_{\omega U} > 0$, reducing cross-border costs increases the welfare of uninformed investors W with sufficiently large volume of foreign investors $V_F := \frac{\lambda_F}{\varphi_F \sigma^2_{\varepsilon} + c}$, which changes with their mass λ_F and cross-border costs c, or sufficiently small liquidity shock of domestic informed investors σ^2_{ω}.*

The above proposition shows that in the limit of large liquidity shock of uninformed investors, foreign investors' trades make uninformed investors better off regardless of the sign of change in the gains from trade G. Even in case where foreign investors' trades decrease the gains from trade G, such negative gains-from-trade effect is more than offset by an increase in learning from the price L. Though this appears to be the least obvious part of the proposition, it is explained with the complementarity between the learning channel and the proportion of uninformed investors' liquidity trades. In particular, as uninformed investors engage in more liquidity trades (i.e., large $K_{\omega U}$ defined in the expression above), increased price informativeness caused by foreign investors' trades leads to a larger increase in uninformed investors' liquidity trades, making it more likely that these uninformed investors are better off. Essentially, this arises from the complementarity between the gains from trade G and learning from the price L in the determination of uninformed investors' welfare W. Lemma 3.6 confirms this point in that a change in learning from the price L affects uninformed investors' welfare W even more with larger gains from trade G.

The below figure illustrates numerical results on the effects of a marginal reduction in cross-border cost c on uninformed investors' welfare W and the gains from trade G depending on their liquidity shock $\sigma^2_{\omega U}$. In two graphs, the numerical calculations are reported with reasonably given parameters as follows: On the left side, we have $V_U = 1\left(=\frac{\lambda_U}{\varphi_U \sigma^2_\varepsilon}\right)$, $V_{DI} = 1\left(=\frac{\lambda_{DI}}{\varphi_{DI}\sigma^2_\varepsilon}\right)$, $\sigma^2_\theta = \sigma^2_\omega = \sigma^2_\varepsilon = 1$, and $V_F = 1\left(=\frac{\lambda_F}{\varphi_F \sigma^2_\varepsilon + c}\right)$, whereas, on the right side, all parameters are identical except that $\sigma^2_\varepsilon = 0.1$. The graphs plot the partial derivatives of uninformed investors' welfare W with respect to the volume of foreign trades $V_F := \frac{\lambda_F}{\varphi_F \sigma^2_\varepsilon + c}$, which are equivalent to reducing cross-border cost c. Note that changes in uninformed investors' gains from trade G eventually become positive in the former case, whereas they are negative for every value of $\sigma^2_{\omega U}$ in the latter case. In both cases, the graphs show that changes in the welfare of uninformed investors (dW) are negative at $\sigma^2_{\omega U} = 0$ and then become positive for a sufficiently large liquidity shock of uninformed investors $\sigma^2_{\omega U}$. The large gaps between the changes in uninformed investors' gains from trade (dG) and their welfare (dW) demonstrate the significant size of the learning effect.

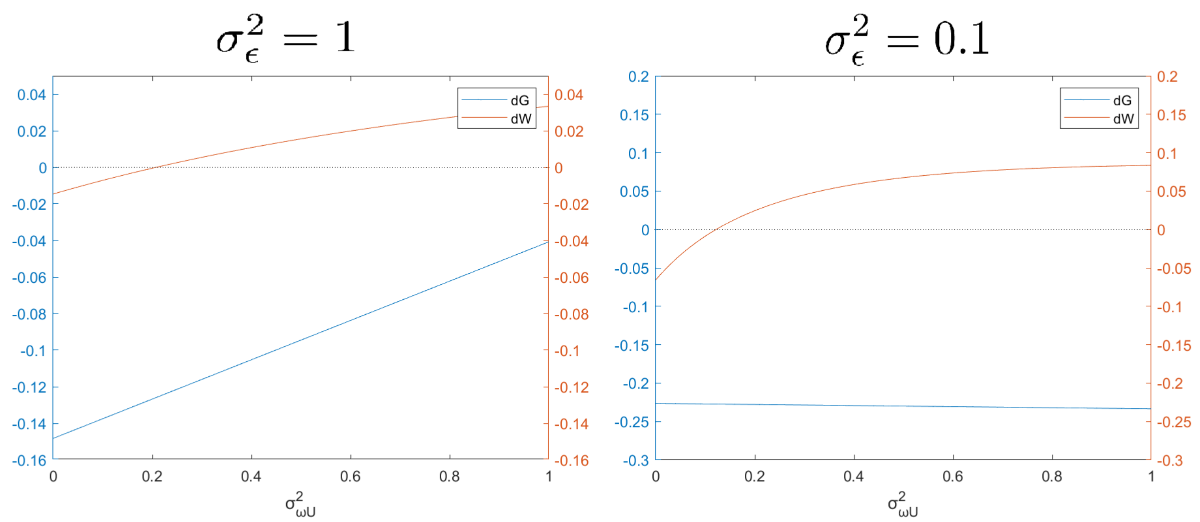

Figure 1: Numerical results on the effects of (marginally) reducing cross-border costs on uninformed investors' welfare W and the gains from trade G for different size of uninformed investors' liquidity shock $\sigma^2_{\omega U}$

A follow-up question is how large liquidity shock of uninformed investors $\sigma^2_{\omega U}$ is required for the positive effect of foreign investors' trades on the welfare of uninformed investors. The answer is that it can be arbitrarily small in the limiting cases of large volume of foreign investors V_F and small liquidity shock of domestic informed investors σ^2_{ω}. The intuition behind this result is as follows: Note first that uninformed investors' trades are motivated by domestic informed investors having liquidity shock ω and thus creating noise. With large volume of foreign investors V_F, which makes trading on the asset value θ less profitable, these uninformed investors trade less on the asset value θ. As these uninformed investors earn most of their utilities from liquidity trades, their welfare W is likely to increase with foreign investors' trades due to the dominance of their gains from liquidity trades. Similar arguments apply in the limit of small liquidity shock of domestic informed investors σ^2_{ω}.

It is worth comparing Proposition 3.9 with Dow and Rahi (2000), who consider a model where informed traders and uninformed hedgers trade a risky asset with competitive market makers and a financial transaction tax is imposed on informed traders. Here, uninformed hedgers have initial endowments correlated with the asset value, which generate their risk-hedging motives. These hedging activities match with uninformed investors' liquidity shock ω_U in the current model in that they generate trades and thereby make the price different from the asset value. In their model, the risk neutrality of market makers suppresses the possibility that uninformed hedgers trade on the asset value. As a result, the welfare of uninformed risk-hedgers solely reflects their risk-sharing opportunities. Dow and Rahi show that a tax on informed traders decreases price informativeness but may either increase or decrease the welfare of uninformed hedgers. The positive welfare effect of decreased price informativeness resulting from the tax is due to the Hirshleifer effect as reviewed in Subsubsection 3.1.1, whereas its negative sign is also possible due to the fact that more information allows these uninformed hedgers to hedge their endowment risk more accurately. In this respect, a learning effect is also at work in their model, though it is rather different from that in this study. The sign of the learning effect hinges on the distribution of the risk factor involved in the initial endowment of uninformed hedgers.[9] An alternative framework from this study is to explicitly model uninformed investors' risk-sharing activities stemming from their initial endowments instead of simply reducing these activities to liquidity shocks. This would generate the Hirshleifer effect as well as the aforementioned effect of hedging their endowment risk more accurately. It can move in any direction from the current model.

Overall, we conclude that informed investors' trading based on their superior information makes uninformed investors better off when trades on liquidity shocks are carried out by uninformed investors rather than informed ones. In this case, a tax on such speculative trades is not justifiable in terms of the welfare of uninformed investors. On the practical side, it is not straightforward to empirically observe who carries out liquidity trades in financial markets. Nevertheless, it is not inconceivable to measure the relative proportion of informed and uninformed investors' trades and then examine how the relative proportion of their trades is associated with fluctuations in the price which are unrelated to long-term fundamentals of the asset.

Moreover, a small tax is suboptimal with large volume of speculative trades, as (inversely) indicated by Proposition 3. In practice, such large volume of speculative trades is likely to hold in small open economies in the context of cross-border investments. In the context of financial transaction taxes, this is likely to be the case in financial markets where the financial sector takes a large proportion of trading activities and their trades are identifiable. Combined with the fact that most financial transaction taxes have a very small rate of less than 0.5 percent in practice, these suggest that such small taxes are unlikely to be the best policy in these cases. Nevertheless, there is still no guarantee that zero tax rate is the best policy as the proposition concerns only the limiting case of large

[9] To be specific, the distribution of this risk factor in Dow and Rahi (2000) reflects the covariance between the risk factor and the observable part of asset value, which corresponds to θ in the basic model, and that between the risk factor and the unobservable part of the asset valuation, which corresponds to ε in the basic model here. It is shown that the former is positively associated with the Hirshleifer effect, whereas the latter is positively associated with the extent to which learning from the price is valuable for uninformed hedgers.

volume of foreign investors.

Further, recall that the learning channel plays a significant role in explaining changes in the gains from trade G and uninformed investors' welfare W in the limiting case of large liquidity shock of uninformed investors. This generally suggests that reducing cross-border costs (increasing the tax rate) is less (more) likely to be justifiable in terms of uninformed investors' welfare unless the learning channel fully applies to all uninformed investors. Though this point goes beyond the current model, we can imagine that the information content of the price is not fully recognized for some uninformed investors in reality.[10] For example, if a fraction of uninformed investors are not sophisticated enough to learn from the price, then foreign investors' trades may hurt trading opportunities for these unsophisticated ones even with a large liquidity shock. At this point, despite their large difference in valuations from foreign investors, these unsophisticated investors are be hurt by foreign investors' speculative trading as such speculative trading increases the price impact of their liquidity trading, as indicated by Proposition 3.9. Still, the total welfare of uninformed investors, including the unsophisticated ones, is not clear. Nevertheless, the significance of the learning channel may complicate the argument for promoting speculative trading even in case of large liqudity shock of uninformed investors.

3.4 Model with market power

In this section, we build and analyze a variant of the basic model by adding to the benchmark case in the basic model one large investor who takes into account the impact of his market power on the price. The model could be viewed as reflecting the dominance of one or a few large investors in financial markets, which is a common feature around the world.[11] Even apart from the practical relevance for the real world, the presence of market

[10]This bias is known as "cursedness" in the literature and may arise from the fact that attention is costly. See Eyster, Rabin, and Vayanos (2019) for extensive evidence for and analyses of investors neglecting the information content of prices in the context of financial markets. Also, Mondria, Vives, and Yang (2022) offer alternative analyses of such bias based on costly attention.

[11]The significance of institutional investors and price impact has long been addressed in practice as well as in the literature. See Rostek and Yoon (2020) for a detailed literature review. This concern is well-known to be pronounced in emerging economies, and, even in US markets, Holderness (2009) finds that the majority of firms have blockholders and the ownership concentration of firms is similar to other countries.

power allows our analyses to accommodate the oft-cited concern for market liquidity as a positive (negative) side of promoting (taxing) trades. As consideration of market power is expected to add to the upsides of foreign investors' trades, our focus is on whether and how it indeed overturns the negative answer in the benchmark case without liquidity shock of uninformed investors in the basic model.

3.4.1 Model

Aside from market power, the model is in parallel with the benchmark case in the basic model in that we retain all assumptions regarding mass λ_U of uninformed investors, mass λ_{DI} of domestic informed investors, and mass λ_F of foreign investors. Until this point, these investors take the price as given. What is different from the basic model is that there is a large (domestic) investor who is risk-neutral and has a liquidity shock ω_L, where $\omega_L \sim N(0, \sigma^2_{\omega L})$. Accordingly, the utility of the large investor is represented by $u_L = x_L(\theta + \omega_L - p)$, where x_L is his asset holding. In addition, the large investor is informed in the sense that he knows the asset value θ and his own liquidity shock ω_L.

The large investor could be interpreted as an outside institutional investor, such as a national pension fund, or an insider. These types of traders often take a significant portion of asset ownership and thus have price impacts. Their large size gives rise to a variety of private elements of asset valuation which essentially arise from agency conerns, such as internal rules for management, sudden withdrawals of deposit, a personal need for cash, and control right. The presence of liquidity shock of the large investor matches with empirical evidence that large institutional investors' activities make price noisier (e.g., Ben-David, Franzoni, Moussawi, and Sedunov, 2021). Given that we are not interested in the welfare of the large investor, it is reasonable to view his liquidity shock as simply summarizing all private elements of asset valuation.

The timeline of the model is identical to the basic model: All investors simultaneously submit demands conditional on the price to the market, and then, the price is set to clear the market, and finally, the true value of the asset $\theta + \varepsilon$ is realized.

Definition 3.10. An equilibrium consists of demands of all types of investors and linear price function $p = \alpha\theta + \beta\omega + \gamma x_L$ with coefficients α, β and γ representing its sensitivities to θ, ω and x_L, respectively, where the following two conditions hold:

1. Every investor of four different types submits demand for the asset (i.e., x_U, x_L, x_{DI} and x_F) maximizing the expected utility conditional on his available information, other investors' strategies and the price.

2. The market clears for the asset. In particular, it holds that

$$\lambda_U x_U + x_L + \lambda_{DI} x_{DI} + \lambda_F x_F = 0.$$

The concept of equilibrium closely follows that of previous studies (e.g., Kyle, 1989; Rostek and Weretka, 2012; Vives, 2011), apart from the fact that agents with and without market power coexist. As in the benchmark case of the basic model, all uninformed investors are symmetric. It is worth noting that linearity of demand schedules is non-trivial in the presence of market power, though most previous studies in the literature restrict attention to linear equilibria. To my best knowledge, the only exception is Du and Zhu (2017), who find that non-linear equilibria exist for two-agent double auctions with linear-quadratic preferences. It is not known whether such non-linear equilibria exist in the current model.

3.4.2 Solving the model

The steps for deriving the equilibrium conditions are well-known in the literature (e.g., Kyle, 1989; Rostek and Weretka, 2012; Vives, 2011). Recognizing the conjectured price function $p(x_L) = \alpha\theta + \beta\omega + \gamma x_L$, the large investor solves

$$\max_{x_L} \left[x_L \left\{ \theta + \omega_L - p(x_L) \right\} \right].$$

This gives rise to the optimal demand $x_L^* = \frac{1}{2\gamma}\{(1-\alpha)\theta + \omega_L - \beta\omega\}$. Given the large investor's demand x_L^*, the equilibrium price p^* is represented by

$$p^* = \alpha\theta + \beta\omega + \gamma x_L^* = \frac{1+\alpha}{2}\theta + \frac{\beta}{2}\omega + \frac{1}{2}\omega_L.$$

Given the equilibrium price p^*, uninformed investors' learning from the price leads to the relevant expectation terms (i.e., $E[\theta|\mathscr{I}_U]$ and $Var[\theta|\mathscr{I}_U]$) in their optimal demand x_U, as detailed in the proof of Lemma 3.11 below. The remaining steps are identical to those in the basic model, plugging uninformed investors', domestic informed price-takers', and foreign investors' optimal demands x_U, x_{DI} and x_F into the market-clearing condition as in the basic model and then equalizing the resulting price with the conjectured price $p(x_L) = \alpha\theta + \beta\omega + \gamma x_L$ to obtain an equilibrium. However, what turns out to be more challenging is to show the existence of a unique solution satisfying the equilibrium conditions. Accordingly, Lemma 3 focuses on two tractable cases, i.e. Cases 1 and 2.

Lemma 3.11. *In Case 1 (i.e., $\sigma_{\omega L}^2 > 0$, $\sigma_\omega^2 = 0$), there is a unique equilibrium where the price is $p^* = \frac{1+\alpha}{2}\theta + \frac{1}{2}\omega_L$ with coefficient α determined by*

$$\frac{\left\{\sigma_{\omega L}^{-2}(1-\alpha^2) - \sigma_\theta^{-2}\right\} V_U}{\sigma_\varepsilon^{-2} + \sigma_\theta^{-2} + (1+\alpha)^2\sigma_{\omega L}^{-2}} + \frac{1-\alpha}{\alpha}(V_{DI} + V_F) = 0.$$

In Case 2 (i.e., $\sigma_{\omega L}^2 = 0$, $\sigma_\omega^2 > 0$), there is a unique equilibrium where the price is $p^* = \frac{1+\alpha}{2}\theta + \frac{\beta}{2}\omega$ with coefficients α and β determined by

$$\frac{\left\{\frac{1-\alpha^2}{\alpha^2}\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2}\sigma_\omega^{-2} - \sigma_\theta^{-2}\right\} V_U}{\sigma_\varepsilon^{-2} + \sigma_\theta^{-2} + \frac{(1+\alpha)^2}{\alpha^2}\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2}\sigma_\omega^{-2}} + \frac{1-\alpha}{\alpha}(V_{DI} + V_F) = 0 \text{ and } \beta = \frac{V_{DI}}{V_{DI}+V_F}\alpha.$$

The crucial difference between Cases 1 and 2 is who has a liquidity shock, thereby motivating others' trades. In Case 1, it is the large investor. In Case 2, domestic informed investors have a liquidity shock as in the basic model. In this case, the large investor does not create any noise and solely trades on the asset value θ. The distinction between these two cases allows us to understand the role of market power in generating trades.

3.4.3 Analyses

In parallel with the basic model, this subsection analyzes the effects of reducing cross-border costs on price informativeness and the welfare of uninformed investors. As Lemma 3.9 guarantees the existence and uniqueness of equilibrium only for Cases 1 and 2, we compare these tractable cases depending on which type of investors have liquidity shocks between the large investor and domestic informed price-takers.

Proposition 3.12. *In Case* 1 *where only the large investor has a liquidity shock* ω_L *(i.e.,* $\sigma^2_{\omega L} > 0$*,* $\sigma^2_{\omega} = 0$*), reducing cross-border costs may increase or decrease the welfare of uninformed investors. In particular, it depends on the relative proportion of uninformed investors' trades* $H := \frac{V_U}{V_{DI}+V_F}$ *as follows: For* $H \in \left[0, \frac{\sigma_\varepsilon^{-2}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}\right)}{2\left(\sigma_\theta^{-2}\right)^2}\right]$*, reducing cross-border costs always increases uninformed investors' welfare* W*. For* $H > \frac{\sigma_\varepsilon^{-2}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}\right)}{2\left(\sigma_\theta^{-2}\right)^2}$*, there is a cutoff* $\bar{\sigma}^2_{\omega L}(H)$ *such that reducing cross-border costs increases uninformed investors' welfare* W *if and only if the large investor's liquidity shock* $\sigma^2_{\omega L}$ *is smaller than the cutoff (i.e.,* $\sigma^2_{\omega L} < \bar{\sigma}^2_{\omega L}(H)$*). Also, the cutoff* $\bar{\sigma}^2_{\omega L}(H)$ *decreases with* H*.*

In Case 2 *where only domestic informed price-takers have a liquidity shock* ω *(i.e.,* $\sigma^2_{\omega L} = 0$*,* $\sigma^2_{\omega} > 0$*), reducing cross-border costs always decreases the welfare of uninformed investors* W*.*

In both Cases 1 *and* 2*, either positive or negative, the change in the welfare of uninformed investors* W *is divided into an increase in learning from the price* L *and a decrease in uninformed investors' gains from trade* G*.*

Reducing cross-border costs increases foreign investors' trades, which in turn decrease uninformed investors' gains from trades ($\Delta G < 0$) and increase learning from the price ($\Delta L > 0$). This is consistent with the benchmark case in the basic model. However, in contrast to the benchmark case, the above proposition tells us that the positive learning effect can dominate the negative gains-from-trade effect. In Case 1, where this can be the case, what is crucially different from the benchmark case is that even as foreign investors trade more, the impact of the large investor's liquidity shock ω_L on the price remains unchanged: Whether these foreign investors trade the asset or not, one half of the liquidity shock is added to the price so that coefficient α in the price function $p(x_L) = \alpha\theta + \gamma x_L$ pins down the equilibrium price p^*, i.e.,

$$p^* = \frac{1+\alpha}{2}\theta + \frac{1}{2}\omega_L.$$

The coefficient on liquidity shock ω_L is invariant because the large investor increases his trades on liquidity shock ω_L proportionally as foreign investors trade more. What occurs in Case 1 is that such monopolistic price-setting behavior of the large investor weakens the negative gains-from-trade effect compared with the benchmark case. As a result, it turns out to be able to upset the dominance of the negative gains-from-trade effect. Note also that uninformed investors always lose their gains from trade G apart from learning from the price L. That is, without learning from the price, the negative answer in the benchmark case would remain unchanged even in the presence of the large investor. This suggests that the role of learning from the price is still crucial here. In contrast, in Case 2 where the large investor does not have any liquidity shock, foreign investors' trades still make uninformed investors worse off by making the price closer to the asset value θ as in the benchmark case.

These results appear to be consistent with the conventional wisdom that the entry of informed participants improves market liquidity, thereby enhancing the overall efficiency of the market. Though the notion of market liquidity is broad and largely informal, a crucial feature of liquid markets is that participants do not face significant inefficiencies arising from the price impact of their trades. Thus, following Kyle (1989) and many others in the literature, market liquidity could be formalized as the reciprocal of the price impact of the large investor's demand in this framework. Indeed, using Claim 1 and Equations (37) and (39) in the proof, we can easily show that reducing cross-border costs in Case 1 decreases coefficient γ in the price function $p(x_L) = \alpha\theta + \gamma x_L$, which means that the large investor loses market power. This is consistent with Lou and Rahi (2021) in that the entry of informed agents eventually leads to deep liquidity.[12] Nevertheless, it is not obvi-

[12]Lou and Rahi (2021) stress the role of information held by investors entering the market (e.g., foreign investors in the current model). In particular, they show that deep liquidity results only from informed agents' entry but not from that of uninformed ones. The entry of uninformed agents may increase liquidity

ous whether deeper liquidity enhances the welfare of uninformed investors in the current model as these uninformed investors' trades are limited by their risk aversion rather than the lack of liquidity. Indeed, the negative sign of the gains-from-trade effect means that foreign investors still decrease uninformed investors' gains from trade by trading based on their superior information, as in the benchmark case in the basic model. However, combined with a positive learning effect through an improvement in price informativeness, foreign investors' trades and the resulting change in the large investor's behavior trades can increase the welfare of uninformed investors.

The large investor's liquidity shock (i.e., $\sigma_{\omega L}^2$) is the key to understanding these conditions throughout Cases 1 and 2. Let us fix a small liquidity shock of domestic informed investors $\sigma_\omega^2 > 0$ and the relative volume of uninformed investors $H > 0$. If the large investor's liquidity shock $\sigma_{\omega L}^2$ is much smaller than domestic informed investors' liquidity shock σ_ω^2 so that it is close to zero, then reducing cross-border costs decreases the welfare of uninformed investors as in Case 2. However, if the large investor's liquidity shock is dominant compared with domestic informed investors' liquidity shock as in Case 1 but it is still smaller than the cutoff $\bar{\sigma}_{\omega L}^2(H)$ defined in the proposition, then reducing cross-border costs increases the welfare of uninformed investors. Last, if the large investor's liquidity shock is even larger than the cutoff $\bar{\sigma}_{\omega L}^2(H)$, then reducing cross-border costs decreases the welfare of uninformed investors. These indicate that reducing cross-border costs makes uninformed investors better off if and only if the large investor's liquidity shock is not too large and domestic informed investors' one is even much smaller than that. The intuition behind the part of not-too-large shock of the large investor is that as the large investor has a smaller liquidity shock $\sigma_{\omega L}^2$, then uninformed investors' gains from trade G are smaller, making it more likely that the learning effect is dominant. This is in parallel with the basic model, except that uninformed investors' trades are motivated by the large investor instead of domestic informed ones.

In addition, the relative volume of uninformed investors H plays an important role in determining whether uninformed investors benefit from or are hurt by foreign investors'

but the market still remains illiquid to some extent.

trades through a reduction in cross-border costs. The proposition indicates that reducing cross-border costs is more likely to make uninformed investors better off with smaller relative volume of uninformed investors H. Though the underlying mechanism is complicated by the interaction among multiple economic forces including those present in the basic model as well as that of the large investor's liquidity trades, this is understood by noting that such positive welfare effect would not occur in the basic model, where the large investor is absent. Intuitively, the large investor's liquidity trades, which appear to cause a difference from the basic model, increase uninformed investors' gains from trade G and decrease learning from the price L. Given the presence of complementarity between the gains from trade and learning from the price,[13] these effects are most likely to benefit uninformed investors when their gains from trade G are small and learning from the price L is already large. This condition corresponds to small relative volume of uninformed investors H generally in parallel with the basic model.[14] On the practical side, this point explains heterogeneity across countries in addressing cross-border investments in financial markets. As the proportion of investors with relevant experience and expertise is associated with a distinction between developed and emerging economies, these conditions appear to suggest a complementary relationship between financial development and cross-border investments. Though it is challenging to empirically assess the welfare of investors, this might explain the sentiment against cross-border investments in emerging economies.

It is noteworthy that the there is a positive feedback between the large investor's trades, which involve both types of trades on the asset value and his liquidity shock, and foreign investors' trades reducing the large investor's market power. As mentioned in Subsection 3.2, this feature is present in Glebkin and Kuong (2021) and Manzano and Vives (2021). For example, Glebkin and Kuong (2021) consider the situation where multiple

[13]Note that learning from the price increases uninformed investors' welfare through their trading opportunities. Lemma 3.11 confirms this point in that a change in learning from the price L affects uninformed investors' welfare W even more with larger gains of trade G.

[14]One subtle difference from the basic model is that the "relative" volume of informed and uninformed investors matters in the current model with the large investor, in contrast to the basic model where the "absolute" volume of informed investors matters and changing the mass of uninformed investors does not affect price informativeness. This stems from the fact uninformed investors use up liquidity rather than providing it in the presence of the large investor, as noted in Lou and Rahi (2021).

large informed traders and a continuum of small partially-informed traders coexist with linear-quadratic preferences. In his sense, increased participation refers to an entry of large traders. It indeed reduces the existing large traders' market power in line with Case 1 considered here. What is notably different from Case 1 is that this may lead to a decrease in price informativeness, possibly hurting partially-informed small traders. This is not the case in Case 1, where the large investor fixes the weight of liquidity shock ω_L in the price, as seen by the equilibrium price p^* above, and thus his liquidity trades in response to foreign investors' trades are not sufficient to reduce price informativeness. Otherwise, the large investor would hurt his own trading margin through liquidity trades. In contrast, in Glebkin and Kuong's setting, there appears to be a strategic interaction among large informed traders leading to stronger incentives for them to engage in liquidity trades. Overall, the current model touches on a different consequence of the positive feedback between large and small investors' trades, complementing those in Glebkin and Kuong (2021) and Manzano and Vives (2021).

3.5 Uniform taxation

In the basic model, only foreign investors, whose trading is speculative, are taxed. In the context of cross-border investments, such assumption of selective taxation appears to be innocuous as these foreign investors are (mostly) identified by the government in practice. In contrast, in the context of financial transaction taxes, it is difficult to perfectly identify speculation as opposed to trading for other motives. Indeed, many financial transaction taxes in reality, including the proposed one in the European Union, are practically uniform, possibly due to the difficulty of identifying speculative trades.

One may ask whether uniform taxation has similar impacts on the formation of prices and the welfare of uninformed investors. This comes down to whether speculative trades are most sensitive to changes in the tax rate compared with other types of trades. First, it is intuitive that uninformed investors are less sensitive to the tax compared with domestic informed and foreign investors, assuming that they are more risk averse so that their overall trading scale is lower than that of foreign ones. This assumption is justified by the fact that domestic informed and foreign investors are likely to be professional investors with expertise and skills to manage their risks, whereas this is less likely to be the case for uninformed investors. Second, it can be reasonably argued that domestic informed investors tend to be more risk averse than foreign ones so that they are less sensitive to the tax. This may be due to the potential endogeneity of whether informed investors are speculative or have liquidity shocks, assuming that such liquidity shocks reflect their portfolio risk or hedging demands. In particular, more risk averse informed investors are likely to have more liquidity needs due to their concern about risk management. Then these investors with more liquidity needs are less sensitive to the same tax compared with those who are less risk averse and thus mostly engage in speculative trades. Overall, the answer can be yes depending on reasonable assumptions on the degrees of risk aversion of different types of investors.

One potentially interesting point is that the above argument for less risk aversion of foreign investors is further strengthened by consideration of heterogeneous risk preferences of investors and fixed costs of entry into the market. Intuitively, less risk aversion and large liquidity trading make it more likely that investors choose to enter the market. This implies that, given their entry into the market, foreign investors without any liquidity shock are likely to be less risk averse compared with domestic informed investors with liquidity shocks. This is because only those with small risk aversion choose to enter the market among foreign investors without any liquidity shock, whereas such selection effect is weaker for domestic informed investors with liquidity shocks. If uninformed investors also incur such fixed costs of entry, the same argument applies so that foreign investors are likely to be less risk averse than uninformed investors with liquidity shocks.

The following proposition formally verifies that the effects of uniform taxation are not so different from those of cross-border costs in case where speculative investors are least risk averse:

Proposition 3.13. *Consider a small uniform tax c instead of cross-border costs. In the basic model, reducing the uniform tax increases price informativeness σ_p^{-2} if and only if foreign investors are less risk averse than domestic informed ones (i.e., $\varphi_F < \varphi_{DI}$). Re-*

stricting attention to the case where this holds and uninformed investors are even more risk averse than domestic informed ones (i.e., $0 < \varphi_F < \varphi_{DI} < \varphi_U$*), the following statements hold true:*

(i) In the benchmark case (i.e., $\sigma^2_{\omega U} = 0$*) of the basic model, reducing the uniform tax decreases the welfare of uninformed investors* W *if foreign investors' risk aversion* φ_F *is sufficiently small.*

(ii) In the general case (i.e., $\sigma^2_{\omega U} > 0$*) of the basic model, reducing the uniform tax increases the welfare of uninformed investors* W *with their sufficiently large shock* $\sigma^2_{\omega U}$*. Further, given* $\sigma^2_{\omega U} > 0$*, it increases* W *with either sufficiently small shock of domestic informed investors* σ^2_{ω} *or sufficiently large volume of foreign investors' trades* V_F*.*

(iii) In Case 1 *(i.e.,* $\sigma^2_{\omega} = 0$ *and* $\sigma^2_{\omega L} > 0$*) of the extended model, reducing the uniform tax increases the welfare of uninformed investors* W *under the corresponding condition in the case of reducing cross-border costs in Proposition* 3.12.

Assuming that uninformed, domestic informed and foreign investors are the most risk averse, the second-most risk averse, and the least risk averse type of investors, respectively, the proposition concerns whether the main results concerning cross-border costs in the basic model and its extended model with market power (i.e., Propositions 3.7-3.9 and 3.12) continue to hold with a small uniform tax on all investors. First, the uniform tax still increases price informativeness, assuming that foreign investors' risk aversion is lower than that of domestic informed ones. Second, the main results on the welfare of uninformed investors generally continue to hold as well. The only non-trivial part among them is uniform taxation in the benchmark case, where reducing the uniform tax hurts uninformed investors *only when* foreign investors' risk aversion is sufficiently small and the tax is sufficiently small. This holds only conditionally, in contrast to the main case of taxing only foreign investors, due to the direct impact of the uniform tax on uninformed investors' trades. Despite the direct impact of reducing the uniform tax on uninformed investors' trades, which is beneficial for them, the proposition shows that the uniform tax still hurts the welfare of uninformed investors as long as foreign investors have sufficiently small risk aversion. Other main results concerning the possibilities of taxes hurting uninformed investors are intact to uniform taxation because reducing the uniform tax is likely to benefit uninformed investors even more compared with the main case of taxing only foreign investors due to its additional direct impact on these uninformed investors.

The above robustness results hold essentially due to the convexity of (foreign) investors' trades with respect to their trading frictions (i.e., risk aversion and trading costs). That is, when their trading costs are lower so that their trades have larger scale, these investors are more sensitive to a marginal change in the trading frictions. Though this point appears to hold generally under the CARA-normal framework, we can see this point in the basic model and the extended with market power by looking at the expression of foreign investors' volume $V_F := \frac{\lambda_F}{\varphi_F \sigma_\varepsilon^2 + c}$, which is more sensitive to any change in their costs c when the denominator is low. In the particular context of the proposition, as foreign investors' risk aversion is small enough, this means that their trading scale is large so that their trading volume is more sensitive to the uniform tax compared with domestic informed and uninformed ones. This leads to the fact that the (positive) impact of reducing the uniform tax on foreign investors' (speculative) trades is dominant over the corresponding impacts on domestic informed and uninformed investors. The former leads to an increase in price informativeness, whereas the latter leads to the robustness of Proposition 1 in terms of the welfare of uninformed investors in the benchmark case.

3.6 Comparison with taxing noise-creating trades

In this section, we consider a variant of the basic model where foreign investors have a liquidity shock. Apart from the obvious fact that investors generally carry out liquidity trades, this provides a useful comparison of the basic model with the long-held argument for a Tobin tax (Tobin, 1978), which is intended to prevent excessive volatility of prices possibly arising from the herd-like behavior of international investors referred to as "flight to quality". Compared with a tax on speculative trades considered in the analyses here, the idea of taxing noise-creating trades goes in the opposite direction in that foreign investors' trading is assumed to cause noises rather than making prices track the asset value more closely. Accordingly, the intuition behind its welfare effect appears to go in the opposite direction as well: On the one hand, foreign investors' liquidity shock adds to mutual gains between uninformed and foreign investors, possibly making it more likely that foreign investors' trades increase the welfare of uninformed investors. On the other hand, their liquidity shock makes the price noisier, thereby reversing the learning effect in the basic model and possibly making it less likely that foreign investors' trades increase the welfare of uninformed investors. At this point, the latter one appears to potentially provide a justification of the tax on noise-creating trades. However, as is seen in the main analyses of the basic model, it is still not obvious which side is dominant.

Motivated by the above argument for the Tobin tax as well as the empirical nature of investors' trading motives discussed in Subsection 3.2, we extend the basic model by assuming that foreign investors have a liquidity shock $\omega_F \sim N\left(0, \sigma_{\omega F}^2\right)$ with correlation coefficients $\rho_F := Corr\left(\omega_F, \omega\right) \in [0,1]$ and $\rho_{UF} := Corr\left(\omega_F, \omega_U\right) \in [0,1]$. All other assumptions are identical to those in the basic model. Now that foreign investors trade on their liquidity shock ω_F so that their trades are reflected on the price, the definition of equilibrium is extended naturally with price function $p = \alpha\theta + \beta\omega + \beta_U\omega_U + \beta_F\omega_F$. Price informativeness σ_p^{-2} and uninformed investors' welfare W are defined in the same manner as those of the basic model. The following lemma extends Lemma 3.2 in the basic model, thereby establishing the existence and uniqueness of equilibrium. The details for equilibrium coefficients are provided in the proof of the lemma in the Appendix.

Lemma 3.14. *In case where foreign investors have a liquidity shock $\omega_F \sim N(0, \sigma_{\omega F}^2)$, there exists a unique equilibrium with price $p = \alpha\theta + \beta\omega + \beta_U\omega_U + \beta_F\omega_F$.*

In what follows, we restrict attention to the "extreme" case where foreign investors have a liquidity shock ω_F independent of that of uninformed investors ω_U, whereas domestic informed investors do not. That is, $\sigma_\omega^2 = 0$ and $\sigma_{\omega F}^2 > 0$. This case is tractable and can be viewed as the opposite of the basic model where foreign investors' trades solely make the price track the asset value more closely. Indeed, foreign investors' trades always make the price noisier in this case. This captures the situation where a Tobin tax is justified as an instrument for reducing noises created by foreign investors' trades. We also assume the independence of liquidity shocks of uninformed and foreign investors ω_U

and ω_F in order to clearly distinguish between the effects of uninformed investors' and foreign investors' liquidity shocks ω_U and ω_F.

Proposition 3.15. *Suppose that domestic informed investors do not have a liquidity shock (i.e., $\sigma_\omega^2 = 0$). Then reducing cross-border costs always decreases price informativeness σ_p^{-2} given by $\sigma_p^{-2} = \left(\frac{V_{DI}}{V_F} + 1\right)^2 \sigma_{\omega F}^{-2}$. Also, the following statements hold true:*

(i) If uninformed investors do not have a liquidity shock (i.e., $\sigma_{\omega U}^2 = 0$), reducing cross-border costs always increases the welfare of uninformed investors W.

(ii) If uninformed investors have a liquidity shock ω_U (i.e., $\sigma_{\omega U}^2 > 0$) and it holds that

$$\left(\sigma_\theta^{-2} + \sigma_p^{-2}\right)^2 < \sigma_\varepsilon^{-2}\sigma_p^{-2}\left\{\frac{V_{DI}}{V_U}\left(\frac{V_{DI}}{V_F} + 1\right) - \frac{V_{DI}}{V_F}\frac{\sigma_\theta^{-2} + \sigma_p^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\theta^{-2} + \sigma_p^{-2}}\right\},$$

then reducing cross-border costs decreases the welfare of uninformed investors W if and only if their liquidity shock $\sigma_{\omega U}^2$ is sufficiently large.

(iii) If uninformed investors have a liquidity shock ω_U (i.e., $\sigma_{\omega U}^2 > 0$) and the above inequality does not hold, then reducing cross-border costs always increases the welfare of uninformed investors W.

As expected, foreign investors' trades decrease price informativeness as these trades only create noises. However, it is not obvious whether these trades benefit or hurt uninformed investors, generally depending on the relative weight of uninformed investors' speculative and liquidity motives. Intuitively, foreign investors' trades bring mutual gains to uninformed investors, thereby generating trading opportunities for uninformed investors. At the same time, these trades hurt learning from the price due to decreased price informativeness. According to the proposition, if uninformed investors do not have a liquidity shock so that they trade only on the asset value θ, then the positive gains-from-trade effect is dominant so that foreign investors' trades make uninformed investors better off. In a sense, this is symmetric to the dominance of the negative gains-from-trade effect in the benchmark case in the basic model. However, if uninformed investors have a liquidity shock ω_U, then the sign of the welfare effect is not obvious as these uninformed investors' welfare is even further affected by the negative learning effect due to the presence of their liquidity trades. Nevertheless, even as uninformed investors carry out large liquidity trades, they face a tradeoff between losing learning from the price and having mutual gains between uninformed and foreign investors. That is, uninformed investors with a large liquidity shock suffer from a loss in learning from the price but they also gain from different valuations between uninformed and foreign investors. Both of them appear to become larger simultaneously with a larger liquidity shock. Indeed, the proposition indicates a mixed result on the welfare of uninformed investors by identifying the condition under which uninformed investors are hurt by a reduction in cross-border costs as they carry out large liquidity trades.

The below figure illustrates numerical results on the effect of a marginal reduction in cross-border costs c on uninformed investors' welfare W depending on their liquidity shock $\sigma^2_{\omega U}$. In parallel with Figure 1 in the basic model where foreign investors do not have a liquidity shock, the numerical calculations in two graphs are reported with the following parameters: On the left side, we have $V_U = 1\left(= \frac{\lambda_U}{\varphi_U \sigma^2_\varepsilon}\right)$, $V_{DI} = 1\left(= \frac{\lambda_{DI}}{\varphi_{DI} \sigma^2_\varepsilon}\right)$, $\sigma^2_\theta = \sigma^2_\omega = \sigma^2_\varepsilon = 1$, and $V_F = 1\left(= \frac{\lambda_F}{\varphi_F \sigma^2_\varepsilon + c}\right)$, whereas, on the right side, all parameters are identical except that $\sigma^2_\varepsilon = 0.1$. The graphs plot the partial derivatives of uninformed investors' welfare W with respect to the volume of foreign trades $V_F := \frac{\lambda_F}{\varphi_F \sigma^2_\varepsilon + c}$, which are equivalent to reducing cross-border cost c. Note that changes in uninformed investors' welfare are always positive in the former case, whereas they eventually become negative in the latter case. At this point, the latter case suggests that a tax on foreign investors' trades increases the welfare of uninformed investors with a sufficiently large liquidity shock through an improvement in learning from the price.

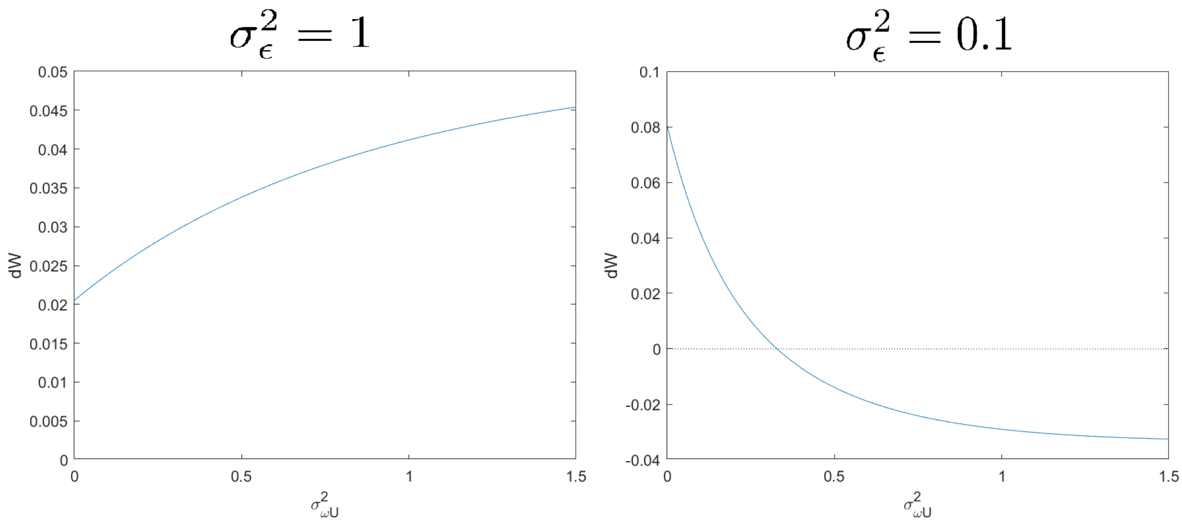

Figure 2: Numerical results on the effect of (marginally) reducing cross-border costs on uninformed investors' welfare W for different size of uninformed investors' liquidity shock $\sigma^2_{\omega U}$

It is noteworthy that the second part of Proposition 3.15 provides a justification for a Tobin tax imposed on foreign investors' trades. The inequality in the second part of the proposition identifies the condition under which taxing these foreign investors' trades improves the welfare of uninformed investors through the learning channel, whereas this would not be the case in other parts of the proposition. The economic interpretation of the inequality appears to be complicated by multiple economic forces. Generally, the condition is more likely to hold with higher trading intensity (i.e., higher σ_ε^{-2}) because it means a greater importance of the learning effect. In other words, higher trading intensity makes it more likely that a Tobin tax is justifiable in terms of the welfare of uninformed investors. This point is illustrated in Figure 2 and appears to be in line with the informal idea that a Tobin tax is effective when trading is "excessive".

In terms of motivation for taxation, such a tax on noise-creating trades is rather different from a tax on speculative trades considered in the basic model. Here, the tax on foreign investors' noise-creating trades may improve the welfare of uninformed investors through the learning channel which may be dominant over the loss in the gains from trade. In contrast, a tax on foreign investors' speculative trades considered in the basic model may improve the welfare of uninformed investors by preventing these foreign investors from taking away the gains from trade. Therefore, the same tax on foreign investors may be justifiable in different ways depending on trading motives of uninformed and foreign investors.

In the special case where uninformed investors have a large liquidity shock $\sigma_{\omega U}^2$ and the ex ante uncertainty is large (i.e., $\sigma_\theta^{-2} \to 0$), the aforementioned condition in Proposition 3.15 is simplified as follows:

$$\frac{V_{DI}}{V_U} > \frac{\left(1+\frac{V_{DI}}{V_F}\right)^2 \sigma_{\omega F}^{-2}\left\{\sigma_\varepsilon^{-2}+\left(1+\frac{V_{DI}}{V_F}\right)\sigma_{\omega F}^{-2}\right\}}{\sigma_\varepsilon^{-2}\left\{\sigma_\varepsilon^{-2}+\left(1+\frac{V_{DI}}{V_F}\right)^2\sigma_{\omega F}^{-2}\right\}}.$$

It is easy to see that the right-hand side is infinity as $V_F \to 0$ and it decreases with V_F toward $\frac{\sigma_{\omega F}^{-2}}{\sigma_\varepsilon^{-2}}$. Therefore, as long as the limiting value of the right-hand side (i.e., $\frac{\sigma_{\omega F}^{-2}}{\sigma_\varepsilon^{-2}}$) is smaller than the left-hand side (i.e., $\frac{V_{DI}}{V_U}$), the Intermediate Value Theorem implies that the above condition holds if and only if V_F is sufficiently large. This point leads to the following corollary, which provides a simple condition under which there is an optimal level of noise-creating trades of foreign investors $V_F := \frac{\lambda_F}{\varphi_F \sigma_\varepsilon^2 + c}$:

Corollary 3.16. *Suppose that uninformed investors have a large liquidity shock $\sigma_{\omega U}^2$ and the ex ante uncertainty is large (i.e., $\sigma_\theta^{-2} \to 0$). If $\frac{V_{DI}}{V_U} > \frac{\sigma_{\omega F}^{-2}}{\sigma_\varepsilon^{-2}}$, then the welfare of uninformed investors W is inverse-U-shaped in the volume of foreign investors V_F. Otherwise, it increases with the volume of foreign investors V_F.*

According to the corollary, as long as the overall extent of trading intensity is sufficiently high (i.e., large σ_ε^{-2}) or the size of foreign liquidity shock is sufficiently large (i.e., small $\sigma_{\omega F}^{-2}$), there is an optimal level of volume of foreign investors maximizing the welfare of uninformed investors W. Above this level, further noise-creating trades hurt uninformed investors as the loss in learning from the price dominates the corresponding increase in the gains from the price. Though the direct applicability of the corollary in practice is limited by its strong assumptions, it provides a simple illustration of the possibility that the policy of "countercyclically" controlling the level of noise-creating trades toward a certain level, whether through a transaction tax or through an explicit barrier to the market, can be effective in the viewpoint of the welfare of uninformed market participants. Overall, this point is another contrast to the basic model where a small tax on speculative trades is suboptimal with large volume of these speculative trades (Proposition 3.9).

3.7 Concluding remarks

The main objective of this study is to analyze the welfare consequence of promoting (restraining) trades through a reduction (an increase) in transaction costs imposed on speculative trades in financial markets. The basic model and the extended model with market power feature the coexistence of informed and uninformed investors subject to different liquidity shocks affecting their valuations, and these models differ from each other with regard to consideration of market power. Motivated by the policy debate on financial transaction taxes and stock market liberalization, the anayses focus on the welfare of uninformed investors, which incorporates their trading opportunities and learning from the price.

The main results in the basic model and the extended model with market power generally suggest that informed investors' speculative trades make uninformed investors better off when liquidity trades are mostly carried out by uninformed investors in the market or when the volume of the speculative trades is large. Even without liquidity trading of uninformed investors, if liquidity trades are carried out by a large investor, informed investors' speculative trades can make uninformed investors better off with a not-too-large proportion of uninformed investors. In these cases, the fact that a tax on speculative trades hurts even uninformed investors makes it challenging to justify such tax. If neither of them holds, trading restrictions targeted at speculative trading activities based on superior information, which are exemplfied as cross-border investments in the models in the paper, are justifiable in terms of uninformed investors' welfare even after taking into account the losses in price informativeness and market liquidity. This could be regarded as the condition under which the informally advocated argument for the "Robin Hood Tax" indeed works in favor of uninformed participants.

Admittedly, the policy debate on cross-border investments and financial transaction taxes is broader than their impacts on the welfare of market participants in the viewpoint of the entire economy. Let us discuss two potentially important points below.

First, in the context of financial transaction taxes, the possibility of redistribution of the tax revenues generally makes it more likely that uninformed investors can benefit from financial transaction taxes. Note that this point typically arises together with the case for uniform taxation considered in Section 3.5. Then we could regard it as counteracting the direct impact of the uniform tax on uninformed investors, thereby leading to the possibility that the qualitative main results in the main case of selectively taxing speculative trades without the redistribution of the tax revenues are not qualitatively overturned even after taking into account these two opposing forces of uniform taxation and the redistribution of the tax revenues. Further, it is unclear whether an arbitrary lump-sum transfer toward uninformed investors is practically executable, given the potential difficulty of identifying different types of investors. Otherwise, the tax revenues may be distributed according to the volume of trades, as considered in Sorensen (2017). In this case of volume-wise redistribution of the tax revenues combined with the typical scenario where highly risk-averse uninformed investors take a smaller proportion of trading volume compared with their large population, it is not obvious whether such redistribution would indeed generally overturn the qualitative main results in the paper.

Second, in the presence of the production sector of the economy, an increase in price informativeness following from speculative trades may enhance the real efficiency of firms and policymakers by providing agents, such as CEOs, with payoff-relevant information held by market participants.[15] Also, it may cause an improvement in the efficiency of managerial contracts in the presence of moral hazard (e.g., Holmstrom and Tirole, 1993). These concerns would add to the informational role of prices, which is covered only by the learning effect ΔL in the analyses of the paper. Generally, they appear to

[15]There is a large literature on this argument starting from Hayek (1945). The informational role of prices and the resulting feedback effect have been formally studied by Bond and Goldstein (2015), Dow and Gorton (1997), Dow and Rahi (2003), Goldstein and Guembel (2008), Goldstein, Ozdenoren and Yuan (2013), Goldstein and Yang (2019), and Siemroth (2019) among many others. See Bond, Edmans and Goldstein (2012) for a detailed review.

provide another reason for promoting speculative trades apart from those provided in the analyses of the paper. Nevertheless, the analyses of the paper, which abstract from the informational role of prices in the real sector, suggest that even the learning channel through uninformed participants' trading activities alone can provide a sufficient argument against taxing speculative trades.

Combining the main results with the case of noise-creating foreign investors, the following table summarizes the implications of foreign investors' trades on price informativeness and uninformed investors' welfare in the models, which turn out to depend on the presence of market power and foreign and uninformed investors' trading motives:

Market structure	Foreign investors' trading motive	Uninformed investors' trading motive	Change in price informativeness	Change in uninformed investors' welfare
Competitive market	Speculative	Speculative ($\sigma_{\omega U}^2 = 0$)	Increase	Decrease
	Speculative	Liquidity trading ($\sigma_{\omega U}^2$ is large)	Increase	Increase
	Liquidity trading	Speculative ($\sigma_{\omega U}^2 = 0$)	Decrease	Increase
	Liquidity trading	Liquidity trading ($\sigma_{\omega U}^2$ is large)	Decrease	Decrease if trading is excessive
Concentrated market	Speculative	Speculative ($\sigma_{\omega U}^2 = 0$)	Increase	Increase if uninformed investors' proportion is not too large

Appendix 1: Proofs of Propositions in Chapter 1

Proof of Proposition 1.3

Suppose that every strategic trader i follows a linear strategy given by $x_i = \alpha + \beta(s_i - \theta_0)$. As in the main text of the paper, this strategy profile constitutes an equilibrium if two conditions in Definitions 1.1 and 1.2 are satisfied. We first show that the first condition corresponds to a linear price with slope given by Equation (1.1), and then, proceed to show that the second condition corresponds to each trader i's best response given by Equation (1.2). Finally, we prove the proposition by solving the fixed point of the above best response.

We first show that the price is given by $P = \theta_0 + \lambda(X - N\alpha)$, where λ is given by Equation (1.1). Note that market makers believe that total demand is given by $X = N\alpha + \beta\sum_{j=1}^{N}(s_j - \theta_0) + \omega$, which implies

$$\frac{X - N\alpha}{N\beta} = \frac{1}{N}\sum_{j=1}^{N}(s_j - \theta_0) + \omega = \theta - \theta_0 + \frac{1}{N}\sum_{j=1}^{N}\varepsilon_j + \frac{\omega}{N\beta}.$$

Thus, market makers regard $\theta_0 + \frac{X-N\alpha}{N\beta}$ as an unbiased signal on the asset value with error variance given by

$$
\begin{aligned}
Var\left(\frac{X-N\alpha}{N\beta}|\theta\right)=Var\left[\frac{1}{N}\sum_{j=1}^{n}\varepsilon_i+\frac{\omega}{N\beta}\right] &= \frac{1}{N^2}Var\left[\sum_{j=1}^{N}\varepsilon_j\right]+\frac{\sigma_\omega^2}{N^2\beta^2} \\
&= \frac{1}{N^2}\left\{N\sigma_\varepsilon^2+N(N-1)\rho\sigma_\varepsilon^2\right\}+\frac{\sigma_\omega^2}{N^2\beta^2} \\
&= \frac{1+\rho(N-1)}{N}\sigma_\varepsilon^2+\frac{\sigma_\omega^2}{N^2\beta^2}. \qquad (1)
\end{aligned}
$$

The price is then Bayesian updated as follows:

$$
\begin{aligned}
P &= E[\theta|X] \\
&= \theta_0+\gamma\left(\frac{X-N\alpha}{N\beta}\right)
\end{aligned}
$$

where updating weight γ is obtained by the Projection Theorem as follows:

$$
\begin{aligned}
\gamma &= \frac{Cov\left(\theta,\theta_0+\frac{X-N\alpha}{N\beta}\right)}{Var\left(\theta_0+\frac{X-N\alpha}{N\beta}\right)} \\
&= \frac{Cov(\theta,\theta)}{Var(\theta)+Var\left(\frac{1}{N}\sum_{j=1}^{n}\varepsilon_i+\frac{\omega}{N\beta}\right)}=\frac{\sigma_0^2}{\sigma_0^2+\frac{1+\rho(N-1)}{N}\sigma_\varepsilon^2+\frac{\sigma_\omega^2}{N^2\beta^2}} \qquad (2)
\end{aligned}
$$

by Equation (1.4). This implies that slope $\lambda := \frac{\gamma}{N\beta}$ is given by Equation (1.1).

Now we proceed to show that the second condition in Definition 1.1 is equivalent to each trader i's best response given by Equation (1.2). With his correct belief on the price function $P_i(x_i)=\theta_0+\lambda(x_i+T_i+(N-1)\alpha+\omega-N\alpha)$, where $T_i:=\sum_{j\neq i}(x_j-\alpha)=\beta\sum_{j\neq i}(s_j-\theta_0)$ similar to main text, trader i's expected profit is

$$
\begin{aligned}
E[\pi_i|\mathscr{I}_i] &= E\left[x_i\left\{\theta-P_i(x_i)\right\}|\mathscr{I}_i\right] \\
&= x_iE\left[\theta-\theta_0|\mathscr{I}_i\right]-x_i\lambda E\left[T_i|\mathscr{I}_i\right]+x_i\lambda\alpha-\lambda x_i^2.
\end{aligned}
$$

By solving the first-order condition, trader i's optimal demand is given by

$$\begin{aligned} x_i^* &= \frac{1}{2\lambda}\left\{E\left[\theta-\theta_0|\mathscr{I}_i\right]-\lambda E\left[T_i|\mathscr{I}_i\right]+\lambda\alpha\right\}. \\ &\equiv \alpha_i+\beta_i(s_i-\theta_0). \end{aligned}$$

Note that the first two terms in the above equation are proportional to $s_i-\theta_0$, and only the last term is constant. Thus, we have $\alpha_i=\frac{\alpha}{2}$, which implies $\alpha=0$ in equilibrium. As in the main text, T_i is decomposed into two terms, and accordingly, trader i's best response is represented by his expectation terms, as in Equation (1.2).

Finally, we prove the proposition by calculating the expectation terms in Equation (1.2) and then solving the fixed point of Equation (1.2). To prevent redundancies, we consider the "combined" case nesting all three cases. In particular, suppose that each strategic trader believes that his own signal error has variance $\frac{\sigma_\varepsilon^2}{\kappa}$, and other traders' signal errors have variance $\frac{\sigma_\varepsilon^2}{\eta}$. By the Projection Theorem, the expectation terms in Equation (1.2) are given by

$$\begin{aligned} E[\theta-\theta_0|\mathscr{I}_i] &= \frac{Cov(\varepsilon_j,s_i)}{Var(s_i)}(s_i-\theta_0)=\frac{\sigma_0^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}(s_i-\theta_0); \\ E[\varepsilon_j|\mathscr{I}_i] &= \frac{Cov(\varepsilon_j,s_i)}{Var(s_i)}(s_i-\theta_0)=\frac{\frac{1}{\sqrt{\kappa\eta}}\rho\sigma_\varepsilon^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}(s_i-\theta_0). \end{aligned}$$

Applying these into Equation (1.2), we have

$$\begin{aligned} x_i^* &= \frac{1}{2\lambda}\left[\{1-\lambda\beta(N-1)\}E\left[\theta-\theta_0|\mathscr{I}_i\right]-\lambda\beta(N-1)E\left[\varepsilon_j|\mathscr{I}_i\right]\right] \\ &= \frac{1}{2\lambda}\left[\{1-\lambda\beta(N-1)\}\frac{\sigma_0^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}-\lambda\beta(N-1)\frac{\frac{1}{\sqrt{\kappa\eta}}\rho\sigma_\varepsilon^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}\right](s_i-\theta_0) \\ &\equiv \beta_i(s_i-\theta_0). \end{aligned}$$

This implies

$$\begin{aligned}
\beta_i &= \frac{1}{2\lambda}\left[\{1-\lambda\beta(N-1)\}\frac{\sigma_0^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}-\lambda\beta(N-1)\frac{\frac{1}{\sqrt{\kappa\eta}}\rho\sigma_\varepsilon^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}\right]\\
&= \left\{\frac{1}{2\lambda}-\frac{\beta(N-1)}{2}\right\}\frac{\sigma_0^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}-\frac{\beta(N-1)}{2}\frac{\frac{1}{\sqrt{\kappa\eta}}\rho\sigma_\varepsilon^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}\\
&= \left\{\frac{N\beta}{2}\frac{\sigma_0^2+\frac{1+\rho(N-1)}{N}\sigma_\varepsilon^2+\frac{\sigma_\omega^2}{N^2\beta^2}}{\sigma_0^2}-\frac{\beta(N-1)}{2}\right\}\frac{\sigma_0^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}-\frac{\beta(N-1)}{2}\frac{\frac{1}{\sqrt{\kappa\eta}}\rho\sigma_\varepsilon^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}\\
&= \left[\frac{\beta}{2}+\frac{\beta}{2}\{1+\rho(N-1)\}\frac{\sigma_\varepsilon^2}{\sigma_0^2}+\frac{1}{2}\frac{\sigma_\omega^2}{N\beta\sigma_0^2}\right]\frac{\sigma_0^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}-\frac{\beta(N-1)}{2}\frac{\frac{1}{\sqrt{\kappa\eta}}\rho\sigma_\varepsilon^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}\\
&= \frac{\beta}{2}\frac{\sigma_0^2+\sigma_\varepsilon^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}+\frac{\beta}{2}(N-1)\rho\frac{\sigma_\varepsilon^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}\left(1-\frac{1}{\sqrt{\kappa\eta}}\right)+\frac{1}{2N\beta}\frac{\sigma_\omega^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}}. \qquad (3)
\end{aligned}$$

At the fixed point, we have $\beta_i = \beta$. Applying this into the above equation and then arranging the terms with respect to β, we have

$$\beta^2 = \frac{\sigma_\omega^2}{N}\frac{1}{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2-\rho(N-1)\sigma_\varepsilon^2\left(1-\frac{1}{\sqrt{\kappa\eta}}\right)}.$$

We then determine the unique trading coefficient β^* satisfying the equilibrium conditions (if it exists) as follows:

$$\beta^* = \sqrt{\frac{\sigma_\omega^2}{N\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2-\rho(N-1)\left(1-\frac{1}{\sqrt{\kappa\eta}}\right)\sigma_\varepsilon^2\right\}}}. \qquad (4)$$

Note that the above equation nests all three cases considered in the proposition. The cases of rational, κ-overconfident, and η-overconfident traders correspond to (i) $\kappa = \eta = 1$, (ii) $\kappa > 1$ and $\eta = 1$, and (iii) $\kappa = 1$ and $\eta \in (0,1)$, respectively. The existence of equilibrium is non-trivial only in the second case of κ-overconfident traders with correlated signal errors: If $\rho = \bar{\rho} > 0$, Equation (4) implies that there exists such β^* satisfying the equilibrium conditions if and only if

$$N < \bar{N}_K := 1+\frac{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2}{\bar{\rho}\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2},$$

as stated in the proposition.

Proof of Proposition 1.5

Denote by $TV_R(\rho,N)$, $TV_K(\rho,N)$, and $TV_E(\rho,N)$ trading volume in the cases of rational, κ-overconfident, and η-overconfident traders, respectively. To prove the first statement of the proposition, we show that if $N \in [2,\bar{N}_K)$, we have

$$TV_R(0,N) = TV_R(\bar{\rho},N) < TV_K(0,N) < TV_K(\bar{\rho},N).$$

Also, the second statement corresponds to

$$TV_E(\bar{\rho},N) < TV_E(0,N) = TV_R(0,N) = TV_R(\bar{\rho},N),$$

for every $N \in [2,\infty)$.

Note first that given the number of traders N, $TV(\rho,N)$ is proportional to trading coefficient β^* in equilibrium, which is described in Proposition 1.3. Both statements in the proposition are easily shown by comparing such trading coefficient between different types of traders.

In the case of rational traders, β_R^* does not depend on ρ, which implies $TV_R(0,N) = TV_R(\bar{\rho},N)$. In the case of κ-overconfident traders with $\rho = 0$, we have

$$\beta_K^*(0,N) = \sqrt{\frac{\sigma_\omega^2}{N\left\{\sigma_0^2 + \left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2\right\}}},$$

which is larger than β_R^* as defined in Proposition 1.3. It follows that $TV_K(0,N) > TV_R(0,N)$. In the case of κ-overconfident traders with $\rho = \bar{\rho}$, we have

$$\beta_K^*(\bar{\rho},N) = \sqrt{\frac{\sigma_\omega^2}{N\left\{\sigma_0^2 + \left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2 - \bar{\rho}(N-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2\right\}}},$$

which is larger than $\beta_K^*(0,N)$ as above. This implies that $TV_K(\bar{\rho},N) > TV_K(0,N)$.

In the case of η-overconfident traders with $\rho = 0$, $\beta_E^*(0,N)$ is the same as $\beta_R^*(0,N)$

as above by Proposition 1.3. This yields $TV_E(0,N) = TV_R(0,N)$. Finally, in the case of η-overconfident traders with $\rho = \bar{\rho}$, Proposition 1.3 implies that

$$\beta_E^*(\bar{\rho},N) = \sqrt{\frac{\sigma_\omega^2}{N\left\{\sigma_0^2 + \sigma_\varepsilon^2 + \bar{\rho}(N-1)\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\varepsilon^2\right\}}},$$

which is smaller than $\beta_E^*(0,N)$. This implies $TV_E(\bar{\rho},N) < TV_E(0,N)$.

Proof of Propositions 1.6

Note that $TV(\rho,N)$ is proportional to the number of traders times their trading coefficient (i.e., $N\beta^*$) in equilibrium where $\beta^* = \beta_R^*$ in the benchmark case, $\beta^* = \beta_K^*$ in the case of κ-overconfident traders, and $\beta^* = \beta_E^*$ in the case of η-overconfident traders. All statements in the proposition follow from the properties of $N\beta^*$ with respect to N. The only non-trivial statement among them is that in the case of η-overconfident traders with $\rho = \bar{\rho}$, trading volume increases with N for $\eta \in \left(\left(\frac{\bar{\rho}\sigma_\varepsilon^2}{\sigma_0^2+\sigma_\varepsilon^2+\bar{\rho}\sigma_\varepsilon^2}\right)^2, 1\right)$, whereas it decreases with N for $\eta \in \left(0, \left(\frac{\bar{\rho}\sigma_\varepsilon^2}{\sigma_0^2+\sigma_\varepsilon^2+\bar{\rho}\sigma_\varepsilon^2}\right)^2\right)$. Applying Proposition 1.3, and then arranging the terms, we have

$$\begin{aligned} N\beta_E^* &= \sqrt{\frac{N\sigma_\omega^2}{\sigma_0^2 + \sigma_\varepsilon^2 + \bar{\rho}(N-1)\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\varepsilon^2}}, \\ &= \sqrt{\frac{\sigma_\omega^2}{\bar{\rho}\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\varepsilon^2 + \frac{1}{N}\left\{\sigma_0^2 + \sigma_\varepsilon^2 - \bar{\rho}\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\varepsilon^2\right\}}}. \end{aligned}$$

For $\eta \in \left(\left(\frac{\bar{\rho}\sigma_\varepsilon^2}{\sigma_0^2+\sigma_\varepsilon^2+\bar{\rho}\sigma_\varepsilon^2}\right)^2, 1\right)$, it is easy to see that $\sigma_0^2 + \sigma_\varepsilon^2 - \bar{\rho}\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\varepsilon^2 > 0$, which implies that trading volume, which is equivalent to $N\beta_E^*$, increases with N. Otherwise, we have $\sigma_0^2 + \sigma_\varepsilon^2 - \bar{\rho}\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\varepsilon^2 < 0$ so that trading volume decreases with N.

Proof of Proposition 1.9

Denote price informativeness in equilibrium by $PI_R(\rho,N)$, $PI_K(\rho,N)$, and $PI_E(\rho,N)$ for the cases of rational, κ-overconfident, and η-overconfident traders, respectively. Recall from the proof of Proposition 1.3 that the price function is represented by

$$P = \theta_0 + \lambda X,$$

where λ is given by Equation (1.1). Thus, knowing the price is equivalent to knowing total demand X. As noted in Equation (1.6) in the main text, $\frac{X}{N\beta} = \theta - \theta_0 + \frac{1}{N}\sum_{j=1}^{N}\varepsilon_j + \frac{\omega}{N\beta}$ is regarded as an unbiased signal about θ. Applying the standard Bayesian updating formula for normal distribution into the definition of price informativeness, we have

$$\begin{aligned} PI &= \{Var(\theta|P)\}^{-1} = \left\{Var\left(\theta|\frac{X}{N\beta}\right)\right\}^{-1}. \\ &= \left[\frac{1}{\sigma_0^{-2} + \left\{Var\left(\frac{X}{N\beta}|\theta\right)\right\}^{-1}}\right]^{-1} = \sigma_0^{-2} + \left\{Var\left(\frac{X}{N\beta}|\theta\right)\right\}^{-1} \\ &= \sigma_0^{-2} + \left\{\frac{1+\rho(N-1)}{N}\sigma_\varepsilon^2 + \frac{\sigma_\omega^2}{N^2\beta^2}\right\}^{-1}, \end{aligned} \tag{5}$$

where the last line follows from Equation (1) with $\alpha = 0$ (which is proven in Proposition 1.3). The proof of Proposition 1.5 shows that $\beta_R^*(0,N) = \beta_R^*(\bar{\rho},N) < \beta_K^*(0,N) < \beta_K^*(\bar{\rho},N)$ and $\beta_E^*(\bar{\rho},N) < \beta_E^*(0,N) = \beta_R^*(0,N) = \beta_R^*(\bar{\rho},N)$. The former implies the first statement of the proposition, as an increase in β caused by κ-overconfidence leads to an increase in *PI* by Equation (5), whereas the latter implies the second statement of the proposition, as a decrease in β caused by η-overconfidence leads to a decrease in *PI* by Equation (5).

Proof of Proposition 1.10

Denote price informativeness in equilibrium by $PI_R(\rho,N)$, $PI_K(\rho,N)$, and $PI_E(\rho,N)$ for the cases of rational, κ-overconfident, and η-overconfident traders, respectively. We first prove the following claim, and then proceed to prove the proposition.

Claim. [1] The maximum level of price informativeness (i.e., PI^*) is given by

$$PI^* = \sigma_0^{-2} + \left\{\frac{1+\rho(N-1)}{N}\sigma_\varepsilon^2\right\}^{-1}.$$

Proof. By the definition of $PI^* = \{Var(\theta|s_1, \cdots, s_N)\}^{-1}$, it suffices to show that

$$Var(\theta|s_1, \cdots, s_N) = \frac{\sigma_0^2 \frac{1+\rho(N-1)}{N} \sigma_\varepsilon^2}{\sigma_0^2 + \frac{1+\rho(N-1)}{N} \sigma_\varepsilon^2}.$$

To show this, define $s = (s_1, \cdots, s_N)'$. Its variance-covariance matrix is given by

$$[Var(s)]_{ii} = \sigma_0^2 + \sigma_\varepsilon^2; \ [Var(s)]_{ij} = \sigma_0^2 + \rho \sigma_\varepsilon^2 \text{ for every } i \neq j.$$

Applying the formula for conditional variance of multivariate normal distribution, we have

$$Var(\theta|s_1, \cdots, s_N) = Var(\theta) - Cov(\theta, s) Var(s)^{-1} Cov(s, \theta).$$

By straightforward calculation, it holds that $Cov(\theta, s) = (\sigma_0^2, \cdots, \sigma_0^2)$, $Cov(\theta, s) = (\sigma_0^2, \cdots, \sigma_0^2)'$, and $Var(s)^{-1}$ is given by

$$\left[Var(s)^{-1}\right]_{ii} = \frac{1}{(1-\rho)\sigma_\varepsilon^2} - \frac{1}{C(1-\rho)^2(\sigma_\varepsilon^2)^2}; \ \left[Var(s)^{-1}\right]_{ij} = -\frac{1}{C(1-\rho)^2(\sigma_\varepsilon^2)^2} \text{ for every } i \neq j,$$

where $C := (\sigma_0^2 + \sigma_\varepsilon^2)^{-1} + N\left\{(1-\rho)\sigma_\varepsilon^2\right\}^{-1}$. Plugging these into the above equation, we get the conditional variance $Var(\theta|s_1, \cdots, s_N)$ as in the claim.[16] □

To prove the proposition, first consider the benchmark case and the cases of κ- and η-overconfident traders with $\rho = 0$. In all these cases, Proposition 1.6 implies that $N\beta^*$ increases with N, and converges to infinity as $N \to \infty$. Applying the former to Equation (5), we can easily see that PI increases with N. Applying the latter to Equation (5), we have, as $N \to \infty$,

$$PI \to \sigma_0^{-2} + \left\{\frac{1+\rho(N-1)}{N} \sigma_\varepsilon^2\right\}^{-1},$$

which equals PI^* by Claim 1.

Next we consider the case of κ-overconfident traders with $\rho = \bar{\rho}$. By Proposition 1.6, $N\beta_K^*$ increases with N, and converges to infinity as $N \to \bar{N}_K$. Applying the former to Equation (1.5), PI_K increases with N. Applying the latter to Equation (1.5), we have, as

[16]The detailed proof of this part is available upon request. It uses basic linear algebra but is tedious.

$N \to \infty$,

$$PI_K \to \sigma_0^{-2} + \left\{ \frac{1+\rho(N-1)}{N} \sigma_\varepsilon^2 \right\}^{-1} = PI^*.$$

Finally, consider the case of η-overconfident traders with $\rho = \bar{\rho}$. By Proposition 1.3, we have

$$N\beta_E^* = \sqrt{\frac{N\sigma_\omega^2}{\sigma_0^2 + \sigma_\varepsilon^2 + \bar{\rho}(N-1)\left(\frac{1}{\sqrt{\eta}} - 1\right)\sigma_\varepsilon^2}}.$$

Plugging this into Equation (5), we have

$$\begin{aligned} PI_E &= \sigma_0^{-2} + \left\{ \frac{1+\bar{\rho}(N-1)}{N} \sigma_\varepsilon^2 + \frac{\sigma_\omega^2}{(N\beta_E^*)^2} \right\}^{-1} \\ &= \sigma_0^{-2} + \left\{ \bar{\rho}\frac{\sigma_\varepsilon^2}{\eta} + \frac{1}{N}\left(\sigma_0^2 + 2\sigma_\varepsilon^2 - \bar{\rho}\frac{\sigma_\varepsilon^2}{\eta}\right) \right\}^{-1}. \qquad (6) \end{aligned}$$

If $\eta \in \left(\left(\frac{\bar{\rho}\sigma_\varepsilon^2}{\sigma_0^2 + 2\sigma_\varepsilon^2} \right)^2, 1 \right)$, then $\sigma_0^2 + 2\sigma_\varepsilon^2 - \bar{\rho}\frac{\sigma_\varepsilon^2}{\eta} > 0$, which implies that PI_E increases with N, as stated in the proposition. Otherwise, we have $\sigma_0^2 + 2\sigma_\varepsilon^2 - \bar{\rho}\frac{\sigma_\varepsilon^2}{\eta} > 0$, which implies that PI_E decreases with N, as stated in the proposition. Finally, for every $\eta \in (0,1)$, Equation (6) implies that PI_E does not converge to PI^* as $N \to \infty$.

Appendix 2: Proofs of Propositions in Chapter 2

Proof of Lemma 2.3

This proof is mostly parallel with that of Proposition 1.3, except that uninformed traders' demand is shown to be zero. As in the proof of Proposition 1.3, we consider the combined case where each strategic trader believes that his own signal error has variance $\frac{\sigma_\varepsilon^2}{\kappa}$, and other traders' signal errors have variance $\frac{\sigma_\varepsilon^2}{\eta}$. Consider a subgame with M informed traders involved in the extended model with N strategic traders. Without loss of generality, we rearrange strategic traders' indices so that traders $i \in \{1, \cdots, M\}$ are informed and other traders are uninformed.

First consider the formation of price, which is given by $P_M = \lambda_M \left(X - M\alpha_M^I - (N-M)\alpha_M^U \right)$. The total demand of these strategic traders and noise traders is given by

$$X = M\alpha_M^I + \beta_M^I \sum_{i=1}^{M} (s_i - \theta_0) + (N-M)\alpha_M^U + \omega.$$

This is normalized to an unbiased signal about θ as follows:

$$\frac{X - M\alpha_M^I - (N-M)\alpha_M^U}{M\beta_M^I} = \theta - \theta_0 + \frac{1}{M}\sum_{i=1}^{M} \varepsilon_i + \frac{\omega}{M\beta_M^I}.$$

The price is then set by

$$\begin{aligned} P_M &= E[\theta | X, M] \\ &= \theta_0 + \gamma_M \left(\frac{X - M\alpha_M^I - (N-M)\alpha_M^U}{M\beta_M^I} \right), \end{aligned}$$

where updating coefficient γ_M is given by

$$\gamma_M = \frac{\sigma_0^2}{\sigma_0^2 + \frac{1+\rho(M-1)}{M}\sigma_\varepsilon^2 + \frac{\sigma_\omega^2}{M^2\left(\beta_M^I\right)^2}}, \tag{7}$$

which is parallel with Equation (1.2). Therefore, the price P_M is given as above with $\lambda_M = \frac{\gamma_M}{M\beta_M^I}$.

Next, consider uninformed traders' demand (i.e., α_M^U). Each uninformed trader's expected profit is given by

$$\begin{aligned} E[\pi_u] &= E\left[x_u \left\{ \theta - \theta_0 - \frac{\gamma}{M\beta_M^I}\left(X - M\alpha_M^I - (N-M)\alpha_M^U \right) \right\} \right] \\ &= x_u E\left[\theta - \theta_0 - \frac{\gamma}{M\beta_M^I}\left(-\alpha_M^U + \beta_M^I \sum_{i=1}^{M}(s_i - \theta_0) \right) \right] - \frac{\gamma}{M\beta_M^I}x_u^2 \\ &= \frac{\gamma}{M\beta_M^I}\alpha_M^U x_u - \frac{\gamma}{M\beta_M^I}x_u^2 \end{aligned}$$

where the second equality follows from

$$X = x_u + (N-M-1)\alpha_M^U + M\alpha_M^I + \beta_M^I \sum_{i=1}^{M}(s_i - \theta_0),$$

and the third equality holds because

$$E[\theta] = E[s_i] = \theta_0$$

for every informed trader $i \in \{1, \cdots, M\}$. Differentiating his expected profit with respected to x_u, we have the first-order condition as follows:

$$\frac{\gamma_M}{M\beta_M^I}\alpha_M^U - \frac{2\gamma_M}{M\beta_M^I}x_u = 0.$$

This yields

$$x_u^* \equiv \alpha_M^U = \frac{1}{2}\alpha_M^U,$$

which implies $\alpha_M^U = 0$. That is, uninformed traders demand zero in any equilibrium, as stated in the lemma.

Given that uninformed traders are "inactive" in equilibrium, this subgame is equivalent to the combined case of the basic model with M strategic traders, described in the proof of Proposition 1.3. Thus, $\alpha_M^I = 0$ holds, and β_M^I is given by Proposition 1.3, only replacing N by M. This holds for all three cases, as stated in the lemma.

Now we want to obtain each informed trader i's expected profit from a subgame. By applying P_M which is given by Equation (7) to informed trader i's expected profit, we have

$$\begin{aligned}
E[\pi_i] &= E\left[\beta_M^I(s_i-\theta_0)\left\{\theta-\theta_0-\frac{\gamma_M}{M\beta_M^I}\beta_M^I(s_i-\theta_0)-\frac{\gamma_M}{M\beta_M^I}\sum_{j\in\{1,\cdots,M\}\setminus\{i\}}\beta_M^I(s_j-\theta_0)\right\}\right] \\
&= E\left[\beta_M^I(s_i-\theta_0)\left\{\theta-\theta_0-\frac{\gamma_M}{M}(s_i-\theta_0)-\frac{\gamma_M}{M}\sum_{j\in\{1,\cdots,M\}\setminus\{i\}}\left((\theta-\theta_0)+\varepsilon_j\right)\right\}\right] \\
&= \beta_M^I\sigma_0^2-\frac{\gamma_M\beta_M^I}{M}\left(\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}\right)-\frac{\gamma_M\beta_M^I}{M}\sigma_0^2(M-1)-\frac{\gamma_M\beta_M^I}{M}\frac{\rho}{\sqrt{\kappa\eta}}\sigma_\varepsilon^2(M-1) \\
&= \sqrt{\frac{\sigma_\omega^2}{M\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2-\rho(M-1)\left(1-\frac{1}{\sqrt{\kappa\eta}}\right)\sigma_\varepsilon^2\right\}}}\left\{\sigma_0^2-\frac{\gamma_M}{M}\left(M\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}+(M-1)\frac{\rho}{\sqrt{\kappa\eta}}\sigma_\varepsilon^2\right.\right. \\
&= \sqrt{\frac{\sigma_\omega^2}{M\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2-\rho(M-1)\left(1-\frac{1}{\sqrt{\kappa\eta}}\right)\sigma_\varepsilon^2\right\}}} \\
&\quad\times\left[\sigma_0^2-\frac{\sigma_0^2}{2\left(\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}\right)+(M-1)\left(\sigma_0^2+\frac{\rho}{\sqrt{\kappa\eta}}\sigma_\varepsilon^2\right)}\left(M\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}+(M-1)\frac{\rho}{\sqrt{\kappa\eta}}\sigma_\varepsilon^2\right)\right] \\
&= \sqrt{\frac{\sigma_\omega^2}{M\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2-\rho(M-1)\left(1-\frac{1}{\sqrt{\kappa\eta}}\right)\sigma_\varepsilon^2\right\}}}\frac{\sigma_0^2\left(\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}\right)}{2\left(\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}\right)+(M-1)\left(\sigma_0^2+\frac{\rho}{\sqrt{\kappa\eta}}\sigma_\varepsilon^2\right)},
\end{aligned}$$

where the third equality follows from

$$E[(s_i-\theta_0)^2]=\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}, E[(s_i-\theta_0)(\theta-\theta_0)]=\sigma_0^2, \text{ and } E[(s_i-\theta_0)\varepsilon_j]=\frac{\rho}{\sqrt{\kappa\eta}}\sigma_\varepsilon^2,$$

and the fifth equality holds by Equation (7).

Consider the case of rational traders (i.e., $\kappa=\eta=1$). Plugging these parameters into Equation (8), we have

$$U_R(\rho,N,M)=\sqrt{\frac{\sigma_\omega^2}{M\left(\sigma_0^2+\sigma_\varepsilon^2\right)}}\frac{\sigma_0^2\left(\sigma_0^2+\sigma_\varepsilon^2\right)}{2\left(\sigma_0^2+\sigma_\varepsilon^2\right)+(M-1)\left(\sigma_0^2+\rho\sigma_\varepsilon^2\right)}. \tag{9}$$

It is easy to see that it decreases with M, and approaches zero as M goes to infinity, as stated in the lemma.

In the case of κ-overconfident traders (i.e., $\kappa>1$ and $\eta=1$), Equation (8) implies that

$$U_K(\rho,N,M)=\sqrt{\frac{\sigma_\omega^2}{M\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2-\rho(M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2\right\}}}\frac{\sigma_0^2\left(\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}\right)}{2\left(\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}\right)+(M-1)\left(\sigma_0^2+\frac{\rho}{\sqrt{\kappa}}\sigma_\varepsilon^2\right)}. \tag{10}$$

If $\rho=0$, it is easy to see that $U_K(0,N,M)$ decreases with M, and converges to zero as M goes to infinity, as stated in the proposition. If $\rho=\bar\rho$, $U_K(\bar\rho,N,M)$ is represented by

$$U_K(\bar\rho,N,M)=\frac{\sqrt{\sigma_\omega^2}\sigma_0^2\left(\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}\right)}{g(\bar\rho,M)},$$

where $g(\bar\rho,M)$ is defined as

$$g(\bar\rho,M)=\sqrt{M\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2-\bar\rho(M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2\right\}}\left\{2\left(\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}\right)+(M-1)\left(\sigma_0^2+\frac{\bar\rho}{\sqrt{\kappa}}\sigma_\varepsilon^2\right.\right.$$

By noting that

$$
\begin{aligned}
\frac{dg(\bar{\rho},M)}{dM} &= \frac{1}{2}\frac{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2-\rho(2M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2}{\sqrt{M\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2-\rho(M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2\right\}}}\left\{2\left(\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}\right)+(M-1)\left(\sigma_0^2+\frac{\bar{\rho}}{\sqrt{\kappa}}\right.\right. \\
&\quad +\sqrt{M\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2-\rho(M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2\right\}}\left(\sigma_0^2+\frac{\bar{\rho}}{\sqrt{\kappa}}\sigma_\varepsilon^2\right),
\end{aligned}
$$

we also define $h(\bar{\rho},M)$ so that it holds $\frac{dg(\bar{\rho},M)}{dM}=\frac{h(\bar{\rho},M)}{\sqrt{M\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2-\rho(M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2\right\}}}$ as follows:

$$
\begin{aligned}
h(\bar{\rho},M) &= \frac{1}{2}\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2-\bar{\rho}(2M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2\right\}\left\{2\left(\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}\right)+(M-1)\left(\sigma_0^2+\frac{\bar{\rho}}{\sqrt{\kappa}}\sigma\right.\right. \\
&\quad +M\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2-\bar{\rho}(M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2\right\}\left(\sigma_0^2+\frac{\bar{\rho}}{\sqrt{\kappa}}\sigma_\varepsilon^2\right).
\end{aligned}
$$

Note that $h(\bar{\rho},M)$ is a quadratic equation with respect to M with a negative quadratic coefficient, and that $h(\bar{\rho},1)>0$ and $h(\bar{\rho},M)$ converges to a negative value as $M\to\bar{N}_K$. These imply that $h(\bar{\rho},M)$ crosses zero once and only once as M increases from one to $\bar{N}_K$. Define $M_K^*\in(1,\bar{N}_K)$ as such M at the crossing point. By definition, $h(\bar{\rho},M)$ is positive for $M\in(1,M_K^*)$, whereas it is negative for $M\in(M_K^*,\bar{N}_K)$. Given that $\frac{dg(\bar{\rho},M)}{dM}>0$ if and only if $h(\bar{\rho},M)>0$, the same conclusion on whether it is positive or negative is drawn for $\frac{dg(\bar{\rho},M)}{dM}$. This implies that $g(\bar{\rho},M)$ is inverse-U-shaped, which in turn implies that $U_K(\bar{\rho},N,M)$ is U-shaped, as in the lemma, because $U_K(\bar{\rho},N,M)$ is inversely proportional to $g(\bar{\rho},M)$. Last, by the definition of $g(\bar{\rho},M)$, we can easily see that $g(\bar{\rho},M)\to 0$ as $M\to\bar{N}_K$, implying that $U_K(\bar{\rho},N,M)\to\infty$ in this limit, as stated in the lemma.

In the case of η-overconfident traders, plugging $\kappa=1$ and $\eta\in(0,1)$ into Equation (14), we have

$$
U_E(\rho,N,M)=\sqrt{\frac{\sigma_\omega^2}{M\left\{\sigma_0^2+\sigma_\varepsilon^2+\rho(M-1)\left(\frac{1}{\sqrt{\eta}}-1\right)\sigma_\varepsilon^2\right\}}}\frac{\sigma_0^2\left(\sigma_0^2+\sigma_\varepsilon^2\right)}{2\left(\sigma_0^2+\sigma_\varepsilon^2\right)+(M-1)\left(\sigma_0^2+\frac{\rho}{\sqrt{\eta}}\sigma_\varepsilon^2\right)}. \tag{11}
$$

for every $\rho\in[0,1]$. Then it is easy to see that it decreases with M, and converges to zero as $M\to\infty$, as stated in the lemma.

Finally, we want to prove that $U_R(\rho,N,M) < U_K(\rho,N,M)$ for every $\rho \in [0,1]$ and $M \geq 1$. Given that the benchmark case corresponds to $\kappa \to 1$ in the case of κ-overconfident traders, it suffices to show that $U_K(\rho,N,M)$ increases with $\kappa$for every $\rho \in [0,1]$ and $M \geq 1$. We take the logarithm of $g(\rho,M)$ and then differentiate it with respect to κ as follows:

$$\begin{aligned}\frac{\partial}{\partial\kappa}[U_K(\rho,N,M)] &= \frac{-\kappa^{-2}\sigma_\varepsilon^2}{\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}} - \frac{1}{2}\frac{M\left\{-2\kappa^{-2}\sigma_\varepsilon^2 - \frac{1}{2}(M-1)\rho\sigma_\varepsilon^2\kappa^{-\frac{3}{2}}\right\}}{M\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2 - \rho(M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2\right\}} \\ &\quad - \frac{-2\sigma_\varepsilon^2\kappa^{-2} - \frac{1}{2}(M-1)\rho\sigma_\varepsilon^2\kappa^{-\frac{3}{2}}}{2\left(\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}\right)+(M-1)\left(\sigma_0^2+\frac{\rho}{\sqrt{\kappa}}\sigma_\varepsilon^2\right)} \\ &= -\frac{\sigma_\varepsilon^2}{\kappa^2\sigma_0^2+\kappa\sigma_\varepsilon^2} + \left\{2\kappa^{-2}\sigma_\varepsilon^2 + \frac{1}{2}(M-1)\rho\sigma_\varepsilon^2\kappa^{-\frac{3}{2}}\right\}\left(\frac{1}{L_1}+\frac{1}{L_2}\right),\end{aligned}$$

where

$$\begin{aligned}L_1 &= 2\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2 - \rho(M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2\right\}; \\ L_2 &= 2\left(\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}\right)+(M-1)\left(\sigma_0^2+\frac{\rho}{\sqrt{\kappa}}\sigma_\varepsilon^2\right).\end{aligned}$$

The above derivative is positive if and only if

$$\frac{\sigma_\varepsilon^2}{\kappa^2\sigma_0^2+\kappa\sigma_\varepsilon^2}L_1L_2 < \left\{\frac{2\sigma_\varepsilon^2}{\kappa^2}+\rho(M-1)\sigma_\varepsilon^2\kappa^{-\frac{3}{2}}\right\}(L_1+L_2),$$

which is equivalent to

$$L_1L_2 < \left(\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}\right)\left\{2+\rho(M-1)\sqrt{\kappa}\right\}(L_1+L_2). \tag{12}$$

Claim. [2] $L_1 < \left(\sigma_0^2+\frac{\sigma_\varepsilon^2}{\kappa}\right)\left\{2+\rho(M-1)\sqrt{\kappa}\right\}$ holds true.

Proof. Noting that $L_1 = 2\left\{\sigma_0^2+\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2 - \rho(M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2\right\}$ by definition, the claim is equivalent to

$$2\left(\frac{2}{\kappa}-1\right)\sigma_\varepsilon^2 - 2\rho(M-1)\left(1-\frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2 < \sqrt{\kappa}\rho(M-1)\sigma_0^2 + \frac{\sigma_\varepsilon^2}{\kappa}\left\{2+\rho(M-1)\sqrt{\kappa}\right\},$$

which is in turn equivalent to

$$\frac{2}{\kappa}\sigma_\varepsilon^2 - 2\sigma_\varepsilon^2 < \sqrt{\kappa}\rho(M-1)\sigma_0^2 + 2\rho(M-1)\left(1 - \frac{1}{\sqrt{\kappa}}\right)\sigma_\varepsilon^2 + \frac{\rho}{\sqrt{\kappa}}(M-1)\sigma_\varepsilon^2.$$

It is easy to see that the above inequality holds true, since the left-hand side is negative but the right-hand side is positive. □

By Claim 2 and $L_2 < L_1 + L_2$, it immediately follows that Equation (12) holds true, completing the proof that $U_K(\rho, N, M)$ increases with $\kappa$for every $\rho \in [0,1]$ and $M \geq 1$.

Proof of Proposition 2.4

Recall from Lemma 2.3 that U_R decreases with M and converges to zero as M goes to infinity. If $c \in (0, U_R(\bar{\rho}, 1, 1))$, this implies that there exists a unique crossing point $N_R^*(c)$ where $U_R(\rho, N_R^*(c), N_R^*(c)) = c$ by the Intermediate Value Theorem.

If $N \in [1, N_R^*(c))$, it holds that $U_R(\rho, N, M) > c$ for every $M \in [1, N]$ by the definition of $N_R^*(c)$. This implies that all traders choose to be informed. The resulting subgame is equivalent to the basic model with N strategic traders by Lemma 2.3.

If $N > N_R^*(c)$, it holds that $U_R(\rho, N, M) > c$ for $M \in [1, N_R^*(c))$, whereas $U_R(\rho, N, M) < c$ for $M \in (N_R^*(c), N]$. This implies that only $N_R^*(c)$ traders choose to be informed in equilibrium. The resulting subgame with $N_R^*(c)$ informed traders is equivalent to the basic model with $N_R^*(c)$ strategic traders by Lemma 2.3.

Finally, if $c > U_R(\bar{\rho}, 1, 1))$, then it holds that $c > U_R(\bar{\rho}, M, M)$ for every $M \geq 1$. This implies that all traders choose not to be informed.

Proof of Proposition 2.5

Recalling from Lemma 2.3 that $U_K(\bar{\rho}, M, M)$ is U-shaped in M, denote its minimum by $M_K^* \in (1, \bar{N}_K)$.

If $c \in (0, U_K(\bar{\rho}, M_K^*, M_K^*))$, then it holds that $c < U_K(\bar{\rho}, M, M)$ for every $M \in [1, \bar{N}_K)$. This implies that all traders choose to be informed.

Now consider $c \in (U_K(\bar{\rho}, M_K^*, M_K^*), U_K(\bar{\rho}, 1, 1))$, there exists two crossing points $N_K^*(c) \in (1, M_K^*)$ and $N_K^{**}(c) \in (M_K^*, \bar{N}_K)$ such that $U_K(\bar{\rho}, N_K^*, N_K^*) = U_K(\bar{\rho}, N_K^{**}, N_K^{**}) = c$ by applying the Intermediate Value Theorem separately to $U_K(\bar{\rho}, M, M)$ defined on $(1, M_K^*)$ and $(M_K^*, \bar{N}_K)$. The existence of the crossing point for the former interval is guaranteed by $c < U_K(\bar{\rho}, 1, 1)$, and that for the latter interval is guaranteed by $U_K(\bar{\rho}, M, M) \to \infty$ as $M \to \bar{N}_K$ by Lemma 2.3.

If $N \in [1, N_K^*(c))$, it holds that $U_K(\bar{\rho}, N, M) > c$ for every $M \in [1, N]$ by the definition of $N_K^*(c)$. Thus, all traders choose to be informed. The resulting subgame is equivalent to the basic model with N strategic traders by Lemma 2.3.

If $N \in (N_K^*(c), N_K^{**}(c))$, it holds that $U_K(\bar{\rho}, N, M) > c$ for $M \in [1, N_K^*(c))$, whereas $U_K(\bar{\rho}, N, M) < c$ for $M \in (N_K^*(c), N_K^{**}(c))$ by the definitions of $N_K^*(c)$ and $N_K^{**}(c)$. Thus, only $N_K^*(c)$ traders choose to be informed in equilibrium. The resulting subgame with $N_K^*(c)$ informed traders is equivalent to the basic model with $N_K^*(c)$ strategic traders by Lemma 2.3.

If $N \in (N_K^{**}(c), \bar{N}_K)$, it holds that $U_K(\bar{\rho}, N, M) > c$ for $M \in [1, N_K^*(c))$, $U_K(\bar{\rho}, N, M) < c$ for $M \in (N_K^*(c), N_K^{**}(c))$, and $U_K(\bar{\rho}, N, M) > c$ for $M \in (N_K^{**}(c), \bar{N}_K)$. As a result, there are two equilibria at $M = N^*(c)$ and $M = N$, respectively. The former equilibrium results in a subgame with $N_K^*(c)$ informed traders, which is equivalent to the basic model with $N_K^*(c)$ strategic traders by Lemma 2.3. On the other hand, the latter equilibrium results in a subgame with N informed traders, which is equivalent to the basic model with N strategic traders.

Finally, we want to prove the last statement that $N_K^*(c) > N_R^*(c)$ for every $c \in (U_K(\bar{\rho}, M_K^*, M_K^*), U_K(\bar{\rho}, 1, 1$
Note that two functions $U_R(\bar{\rho}, M, M)$ and $U_K(\bar{\rho}, M, M)$ defined on $[1, M_K^*)$ are decreasing in M, and it holds that $U_R(\bar{\rho}, M, M) < U_K(\bar{\rho}, M, M)$. Given that $N_R^*(c)$ and $N_K^*(c)$ are their crossing points with c as defined above, and that $N_K^*(c) > 0$ by $c < U_K(\bar{\rho}, 1, 1)$, we have $N_K^*(c) > N_R^*(c)$.

The above inequality implies that κ-overconfidence weakly increases the number of informed traders even in the equilibrium with a lower number of informed traders. Denote by M_R and M_K the number of informed traders in the benchmark case and the equilibrium with a lower number of traders under κ-overconfidence, respectively. The observation that $M_K \geq M_R$ appears to be intuitive by comparing the curves of $U_K(\bar{\rho},M,M)$ and $U_R(\bar{\rho},M,M)$ on the (M,U)-space and then noting that M_R and M_K are determined at these curves' crossing points with $U=c$ subject to $M_R \leq N$ and $M_K \leq N$. We formally prove this by considering two cases: (1) $U_R(\bar{\rho},1,1) > U_K(\bar{\rho},M_K^*,M_K^*)$ and (2) $U_R(\bar{\rho},1,1) < U_K(\bar{\rho},M_K^*,M_K^*)$. In the first case, the range of $c \in (0,U_K(\bar{\rho},1,1))$ is divided into three subcases: (1a) $c \in (0,U_K(\bar{\rho},M_K^*,M_K^*))$, (1b) $c \in (U_K(\bar{\rho},M_K^*,M_K^*),U_R(\bar{\rho},1,1))$, and (1c) $c \in (U_R(\bar{\rho},1,1),U_K(\bar{\rho},1,1))$. Similarly, in the second case, the range of $c \in (0,U_K(\bar{\rho},1,1))$ is divided into three subcases: (2a) $c \in (0,U_R(\bar{\rho},1,1))$, (2b) $c \in (U_K(\bar{\rho},M_K^*,M_K^*),U_K(\bar{\rho},M_K^*,M_K^*))$, and (2c) $c \in (U_K(\bar{\rho},M_K^*,M_K^*),U_K(\bar{\rho},1,1))$. The following table describes M in equilibrium in all these possible subcases to verify that M weakly increases with κ-overconfidence:

(1) $U_R(\bar{\rho},1,1) > U_K(\bar{\rho},M_K^*,M_K^*)$	Benchmark (M_R)	κ-overconfidence (M_K)
(1a) $c \in (0,U_K(\bar{\rho},M_K^*,M_K^*))$	$\max\{N_R^*(c),N\}$	N
(1b) $c \in (U_K(\bar{\rho},M_K^*,M_K^*),U_R(\bar{\rho},1,1))$	$\max\{N_R^*(c),N\}$	$\max\{N_K^*(c),N\}$
(1c) $c \in (U_R(\bar{\rho},1,1),U_K(\bar{\rho},1,1))$	0	$\max\{N_K^*(c),N\}$
(2) $U_R(\bar{\rho},1,1) < U_K(\bar{\rho},M_K^*,M_K^*)$	Benchmark (M_R)	κ-overconfidence (M_K)
(2a) $c \in (0,U_R(\bar{\rho},1,1))$	$\max\{N_R^*(c),N\}$	N
(2b) $c \in (U_K(\bar{\rho},M_K^*,M_K^*),U_K(\bar{\rho},M_K^*,M_K^*))$	0	$\max\{N_K^*(c),N\}$
(2c) $c \in (U_K(\bar{\rho},M_K^*,M_K^*),U_K(\bar{\rho},1,1))$	0	$\max\{N_K^*(c),N\}$

Proof of Corollary 2.6

Denote by $TV_R(\rho,M)$ and $TV_K(\rho,M)$ trading volume in subgame (ρ,N,M) in the benchmark case and the case of κ-overconfident traders, respectively, and by $PI_R(\rho,M)$ and $PI_K(\rho,M)$ price informativeness in subgame (ρ,N,M) in the benchmark case and the case of κ-overconfident traders, respectively. Also, as in the proof of Proposition 2.5, denote by M_R and M_K the number of informed traders in the benchmark case and the equilibrium with a lower number of traders under κ-overconfidence, respectively.

Under κ-overconfidence, trading volume equals $TV_K(\bar{\rho},M_K)$. Given that $TV_K(\bar{\rho},M_K) \geq TV_K(\bar{\rho},M_R)$ by Propositions 1.10 and 2.5, and that $TV_K(\bar{\rho},M_R) > TV_R(\bar{\rho},M_R)$ by Proposition 1.5, we have $TV_K(\bar{\rho}, M_K) > TV_R(\bar{\rho}, M_R)$, which equals trading volume in the benchmark case.

Also, price informativeness under κ-overconfidence corresponds to $PI_K(\bar{\rho}, M_K)$. Given that $PI_K(\bar{\rho}, M_K) \geq PI_K(\bar{\rho}, M_R)$ by Propositions 1.10 and 2.5, and that $PI_K(\bar{\rho}, M_R) > PI_R(\bar{\rho}, M_R)$ by Proposition 1.5, we have $PI_K(\bar{\rho}, M_K) > PI_R(\bar{\rho}, M_R)$, which equals price informativeness in the benchmark case.

Proof of Corollary 2.7

Denote by $TV_R(\rho, M)$ and $TV_K(\rho, M)$ trading volume in subgame (ρ, N, M) in the benchmark case and the case of κ-overconfident traders, respectively, and by $PI_R(\rho, M)$ and $PI_K(\rho, M)$ price informativeness in subgame (ρ, N, M) in the benchmark case and the case of κ-overconfident traders, respectively. Also, as in the proof of Proposition 2.5, denote by M_R and M_K the number of informed traders in the benchmark case and the equilibrium with a lower number of traders under κ-overconfidence, respectively.

In the benchmark case, trading volume and price informativeness correspond to $TV_R(\bar{\rho}, M_R)$ and $PI_R(\bar{\rho}, M_R)$, respectively. Note that $M_R = \max\{N_R^*(c), N\}$ where $N_R^*(c)$ takes zero for $c \in (U_R(\bar{\rho}, 1, 1), U_K(\bar{\rho}, 1, 1))$ by Proposition 2.4. For $N < N_R^*(c)$, M_R increases with N, and thus, trading volume and price informativeness also increase with N as well. For $N > N_R^*(c)$, M_R stays constant with respect to N, and thus, trading volume and price informativeness also stay constant as well.

In the case of κ-overconfident traders, we have two possible equilibria by Proposition 2.5. In the equilibrium with a lower number of informed traders, note that trading volume and price informativeness correspond to $TV_R(\bar{\rho}, M_K)$ and $PI_R(\bar{\rho}, M_K)$, respectively, and $M_K = \max\{N_K^*(c), N\}$. For $N < N_K^*(c)$, M_K increases with N, and thus, trading volume and price informativeness also increase with N as well. For $N > N_K^*(c)$, M_K stays constant with respect to N, and thus, trading volume and price informativeness also stay constant as well. In the other equilibrium that exists for $N \in (N_K^*, \bar{N}_K)$, note first that all strategic traders choose to be informed. Thus, trading volume and price informativeness correspond to $TV_K(\bar{\rho}, N)$ and $PI_K(\bar{\rho}, N)$, respectively. It follows that $TV_K(\bar{\rho}, N)$ and

$PI_K(\bar{\rho},N)$ increase with N by Propositions 1.6 and 1.10, respectively, and that as $N \to \bar{N}_K$, $TV_K(\bar{\rho},N) \to \infty$ and $PI_K(\bar{\rho},N) \to PI^*$ by Propositions 1.6 and 1.10, respectively.

Proofs of Propositions in Chapter 3

The following facts are standard and will be useful in the proof:

Fact. [1] *Let u_U be the utility of uninformed investors and suppose that an uninformed investor $i \in [0, \lambda_U]$ submits demand $x_U = \frac{E[\theta|\mathscr{I}_U]+\omega_U-p}{\varphi_U Var[\theta|\mathscr{I}_U]}$. Then the expected utility of the uninformed investor conditional on his information set $\mathscr{I}_U = \{p, \omega_U\}$ is given by*

$$\begin{aligned}
E[u_U|\mathscr{I}_U] &= E\left[-\exp\{-\varphi_U x_U (\theta + \omega_U + \varepsilon - p)\} | \mathscr{I}_U\right] \\
&= -\exp\left\{-\varphi_U x_U (E[\theta|\mathscr{I}_U] + \omega_U - p) + \frac{1}{2}\varphi_U^2 x_U^2 Var[\theta + \varepsilon|\mathscr{I}_U]\right\} \\
&= -\exp\left\{-\frac{(E[\theta|\mathscr{I}_U] + \omega_U - p)^2}{2Var[\theta + \varepsilon|\mathscr{I}_U]}\right\}.
\end{aligned}$$

Fact. [2] *Let X be a random variable with $N(0,1)$. Then X^2 follows $\chi^2(1)$ and $E\left[e^{tX^2}\right] = (1-2t)^{-\frac{1}{2}}$ for every $t \in \mathbb{R}$.*

Proof of Lemma 3.2

As mentioned in the main text, we start by noting that the following $\tilde{p}$ is the unbiased sufficient statistic for θ:

$$\tilde{p} := \frac{p}{\alpha} - \frac{\beta_U}{\alpha}\omega_U.$$

Also, its variance conditional on θ is given by

$$\sigma_p^2 := Var(\tilde{p}|\theta) = \left(\frac{\beta}{\alpha}\right)^2 \sigma_\omega^2. \tag{13}$$

By the standard Bayesian updating formula, this yields

$$E[\theta|\mathscr{I}_U] = \frac{\sigma_p^{-2}}{\sigma_\theta^{-2} + \sigma_p^{-2}} \tilde{p} \text{ and } Var[\theta|\mathscr{I}_U] = \left(\sigma_\theta^{-2} + \sigma_p^{-2}\right)^{-1}, \tag{14}$$

where σ_p^{-2} is defined in Equation (19).

It is easy to see that $x_U = \frac{E[\theta|\mathscr{I}_U] + \omega_U - p}{\varphi_U Var[\theta+\varepsilon|\mathscr{I}_U]}$, $x_{DI} = \frac{\theta+\omega-p}{\varphi_{DI}\sigma_\varepsilon^2}$, and $x_F = \frac{\theta-p}{\varphi_F\sigma_\varepsilon^2+c}$ by the first-order conditions of these three types of investors' log-transformed expected utilities. Applying these to the market clearing condition together with Equation (20), we have

$$\frac{\lambda_U}{\varphi_U} \frac{\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}\left(\frac{p-\beta_U\omega_U}{\alpha}\right) + \omega_U - p}{\sigma_\varepsilon^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}\left(\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}\right)} + \frac{\lambda_{DI}}{\varphi_{DI}\sigma_\varepsilon^2}(\theta+\omega-p) + \frac{\lambda_F}{\varphi_F\sigma_\varepsilon^2+c}(\theta-p) = 0.$$

Rearranging the terms, we have

$$\begin{aligned} &\frac{\lambda_U}{\varphi_U} \frac{\left(\frac{\sigma_p^{-2}}{\alpha} - \sigma_\theta^{-2} - \sigma_p^{-2}\right)p - \frac{\sigma_p^{-2}}{\alpha}\beta_U\omega_U + \left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\omega_U}{\sigma_\varepsilon^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}\right)} \\ &+ \frac{\lambda_{DI}}{\varphi_{DI}\sigma_\varepsilon^2}(\theta+\omega-p) + \frac{\lambda_F}{\varphi_F\sigma_\varepsilon^2+c}(\theta-p) = 0. \end{aligned} \tag{15}$$

Setting aside the terms multiplied by p, other terms must be proportional to p as follows:

$$\frac{\lambda_U}{\varphi_U} \frac{-\frac{\sigma_p^{-2}}{\alpha}\beta_U\omega_U + \left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\omega_U}{\sigma_\varepsilon^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}\right)} + \frac{\lambda_{DI}}{\varphi_{DI}\sigma_\varepsilon^2}(\theta+\omega) + \frac{\lambda_F}{\varphi_F\sigma_\varepsilon^2+c}\theta = Cp,$$

where C is a constant. Equalizing coefficients on both sides of the above equality yields

$$C = \frac{V_{DI}+V_F}{\alpha}, \ \frac{\beta}{\alpha} = \frac{V_{DI}}{V_{DI}+V_F} \text{ and } \frac{\beta_U}{\alpha} = \frac{\frac{V_U}{V_{DI}+V_F}\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}}{1+\frac{V_U}{V_{DI}+V_F}\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}},$$

as stated in the lemma for $\sigma_{\omega U}^2 > 0$. In case where $\sigma_{\omega U}^2 = 0$, we can effectively take $\rho = 0$ because uninformed investors do not know ω at all. Then the β_U-term in the price p is zero, as stated in the lemma. Whether $\sigma_{\omega U}^2 = 0$ or $\sigma_{\omega U}^2 > 0$, note that all non-p terms in Equation (21) are equal to $Cp = \frac{V_{DI}+V_F}{\alpha}p$. Thus, plugging Cp into all non-p terms in

Equation (21), we have

$$\frac{V_U\left(\frac{\sigma_p^{-2}}{\alpha}-\sigma_\theta^{-2}-\sigma_p^{-2}\right)}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}p-(V_{DI}+V_F)\,p+\frac{V_{DI}+V_F}{\alpha}p=0,$$

which yields

$$\alpha=\frac{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}{V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F},$$

as stated in the lemma.

Proof of Lemma 2

As stated in Fact 1 in the Appendix, we first apply the optimal demand of uninformed investors $x_U=\frac{E[\theta|\mathscr{I}_U]+\omega_U-p}{\varphi_U Var[\theta+\varepsilon|\mathscr{I}_U]}$ to get the expectation of their utility u_U conditional on the information set of uninformed investors $\mathscr{I}_U$ (i.e. $E[u_U|\mathscr{I}_U]$). Then, further taking its expectation over the realization of the information set $\mathscr{I}_U=\{p,\omega_U\}$, we obtain the welfare of uninformed investors W. In particular, using Fact 1 yields

$$E\left[u_U|\mathscr{I}_{Ui}\right]=-\exp\left\{-\frac{(E[\theta|\mathscr{I}_U]+\omega_U-p)^2}{2Var[\theta+\varepsilon|\mathscr{I}_U]}\right\}.$$

Noting that the fraction term within the curly bracket above is normally distributed, we define

$$X^2:=\frac{(E[\theta|\mathscr{I}_U]+\omega_U-p)^2}{Var\,(E[\theta|\mathscr{I}_U]+\omega_U-p)}$$

so that X (X^2) follows $N(0,1)$ ($\chi^2(1)$). Then the numerator $(E[\theta|\mathscr{I}_U]+\omega_U-p)^2$ can be written as

$$(E[\theta|\mathscr{I}_U]+\omega_U-p)^2=Var\,(E[\theta|\mathscr{I}_U]+\omega_U-p)\,X^2,$$

yielding the following expression of $E\left[u_U|\mathscr{I}_U\right]$:

$$E\left[u_U|\mathscr{I}_U\right]=-\exp\left\{-\frac{Var\,(E[\theta|\mathscr{I}_U]+\omega_U-p)}{2Var[\theta+\varepsilon|\mathscr{I}_U]}X^2\right\}.$$

Given that all relevant variables (i.e., θ, ε, and p) are normally distributed, $Var\left(E[\theta|\mathscr{I}_U]+\omega_U-p\right)$ and $Var[\theta+\varepsilon|\mathscr{I}_U]$ are independent of the realization of the price p and idiosyncratic shocks ω_U and thus are treated as constants when integrating over the information set $\mathscr{I}_U$. Recalling that $X^2 \sim \chi^2(1)$ and taking the expectation of $E[u_U|\mathscr{I}_U]$ over the realization of $\mathscr{I}_U$, we get the welfare of uninformed investors as follows:

$$\begin{aligned} W &= E\left[-\exp\left\{-\frac{Var\left(E[\theta|\mathscr{I}_U]+\omega_U-p\right)}{2Var[\theta+\varepsilon|\mathscr{I}_U]}X^2\right\}\right] \\ &= -\left(1+\frac{Var\left(E[\theta|\mathscr{I}_U]+\omega_U-p\right)}{Var[\theta+\varepsilon|\mathscr{I}_U]}\right)^{-\frac{1}{2}}, \end{aligned} \tag{16}$$

where the last line is obtained by using the moment-generating function of chi-square distribution. Note that we have

$$\begin{aligned} Var\left(E[\theta|\mathscr{I}_U]+\omega_U-p\right) &= Var\left(E\left[\theta+\omega_U-p|\mathscr{I}_U\right]\right) \\ &= Var\left(\theta+\omega_U-p\right)-E\left[Var\left(\theta+\omega_U-p|\mathscr{I}_U\right)\right] \\ &= Var\left(\theta+\omega_U-p\right)-Var\left(\theta+\omega_U-p|\mathscr{I}_U\right) \\ &= Var\left(\theta+\omega_U-p\right)-Var\left(\theta|\mathscr{I}_U\right), \end{aligned}$$

where the second line follows from the Law of Total Variance, and the third line is a special property following from the fact that θ and p are normally distributed. Plugging this into Equation (22), we have

$$W = -\left(1+\frac{Var\left(\theta+\omega_U-p\right)-Var\left(\theta|\mathscr{I}_U\right)}{\sigma_\varepsilon^2+Var\left(\theta|\mathscr{I}_U\right)}\right)^{-\frac{1}{2}}.$$

We then decompose the welfare of uninformed investors W into their gains from trade G and learning from the price L as follows:

$$W = -\left(1+\frac{G+L-1}{\sigma_\theta^{-2}\sigma_\varepsilon^2-L+1}\right)^{-\frac{1}{2}} = -\left(\frac{\sigma_\theta^{-2}\sigma_\varepsilon^2+G}{\sigma_\theta^{-2}\sigma_\varepsilon^2+1-L}\right)^{-\frac{1}{2}}.$$

Proof of Propositions 3.7, 3.8 and 3.9

Take the general case at first. We have $\tilde{p} = \frac{p-\beta_U\omega_U}{\alpha} = \theta + \frac{\beta}{\alpha}\omega$. By Equation (20), we have

$$\begin{aligned}
E[\theta|\mathscr{I}_U] + \omega_U - p &= \frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}\tilde{p} - p + \omega_U \\
&= \frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}\left(\theta + \frac{\beta}{\alpha}\omega\right) - \alpha\theta - \beta\omega - \beta_U\omega_U + \omega_U \\
&= \left(\frac{1}{\alpha}\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}} - 1\right)(\alpha\theta+\beta\omega) + (1-\beta_U)\omega_U \\
&= \left(\frac{1}{\alpha}\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}} - 1\right)(\alpha\theta+\beta\omega) + \left(1 - \frac{\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\alpha}{1+\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\frac{1}{V_{DI}+V_F}}\right)\omega_U
\end{aligned} \tag{17}$$

where the last line is obtained by using the ratio of coefficients $\frac{\beta_U}{\alpha}$ from Lemma 3.2. Using coefficient α obtained from Lemma 3.2, we have

$$\begin{aligned}
\frac{1}{\alpha}\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}} - 1 &= \frac{V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + V_{DI} + V_F}{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + V_{DI} + V_F}\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}} - 1 \\
&= \frac{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + \frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}(V_{DI}+V_F)}{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + V_{DI} + V_F} - 1 = -\frac{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}(V_{DI}+V_F)}{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + V_{DI} + V_F}
\end{aligned} \tag{18}$$

We first prove Proposition 3.7 by taking the benchmark case, i.e. $\sigma_{\omega U}^2 = 0$. Plugging Equation (24) into Equation (23) yields

$$E[\theta|\mathscr{I}_U] + \omega_U - p = -\frac{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}(V_{DI}+V_F)}{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + V_{DI} + V_F}(\alpha\theta+\beta\omega).$$

Noting that uninformed investors' welfare W increases with $K := \frac{Var(E[\theta|\mathscr{I}_U]+\omega_U-p)}{Var[\theta+\varepsilon|\mathscr{I}_U]}$ by

Equation (22), we use the above expression to get

$$
\begin{aligned}
K &= \frac{Var\left(E[\theta|\mathscr{I}_U]+\omega_U-p\right)}{Var[\theta+\varepsilon|\mathscr{I}_U]} = \frac{Var\left(-\frac{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}(V_{DI}+V_F)}{\frac{\lambda_U}{\varphi_U\sigma_\varepsilon^2}\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}(\alpha\theta+\beta\omega)\right)}{\sigma_\varepsilon^2+\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}} \\
&= \left(\frac{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}(V_{DI}+V_F)}{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\right)^2 \frac{\alpha^2 Var\left(\theta+\frac{\beta}{\alpha}\omega\right)}{\sigma_\varepsilon^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)} \\
&= \left(\frac{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}(V_{DI}+V_F)}{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\right)^2 \left(\frac{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}{V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\right)^2 \frac{\sigma_\theta^2\sigma_p^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)}{\sigma_\varepsilon^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}\right.} \\
&= \left(\frac{1}{V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\right)^2 \frac{\left(V_{DI}+V_F\right)^2\sigma_\theta^{-2}\sigma_p^2}{\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)} \\
&= \left(\frac{1}{V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\right)^2 \frac{V_{DI}^2\sigma_\theta^{-2}\sigma_\omega^2}{\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)},
\end{aligned}
$$

where the last line is obtained by using $\sigma_p^2 = \left(\frac{\beta}{\alpha}\right)^2\sigma_\omega^2$ and $\frac{\beta}{\alpha} = \frac{V_{DI}}{V_{DI}+V_F}$ from Lemma 3.2. We can easily see that K decreases with σ_p^{-2} and V_F, both of which increase with a reduction in c. Therefore, reducing c decreases W, as stated in Proposition 3.7. This immediately leads to other statements in Proposition 3.7. It is easy to see that reducing c increases L. Combined with Lemma 3.5, this implies that reducing c decreases G.

Now we consider the general case where $\sigma_{\omega U}^2 > 0$ to prove Propositions 3.8 and 3.9. Plugging Equation (24) into Equation (23) yields

$$
E[\theta|\mathscr{I}_U]+\omega_U-p = -\frac{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}(V_{DI}+V_F)}{\frac{\lambda_U}{\varphi_U\sigma_\varepsilon^2}\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}(\alpha\theta+\beta\omega)+\left(1-\frac{\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\alpha}{1+\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\frac{1}{V_{DI}+V_F}}\right)\omega_U.
$$

Using the fact that ω_U is independent of θ and ω, we have

$$
\begin{aligned}
Var\left(E[\theta|\mathscr{I}_U]+\omega_U-p\right) &= Var\left\{\frac{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}\left(V_{DI}+V_F\right)\left(\alpha\theta+\beta\omega\right)}{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}+\left(1-\frac{\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\alpha}{1+\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\frac{1}{V_{DI}+V_F}}\right)\omega_U\right. \\
&= Var\left\{\frac{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}\left(V_{DI}+V_F\right)\left(\alpha\theta+\beta\omega\right)}{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\right\}+\left(1-\frac{\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\alpha}{1+\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\frac{1}{V_{DI}+V_F}}\right.
\end{aligned}
$$

Then it follows

$$
\begin{aligned}
K &:= \frac{Var\left(E[\theta|\mathscr{I}_U]+\omega_U-p\right)}{Var[\theta+\varepsilon|\mathscr{I}_U]} \\
&= \frac{1}{Var[\theta+\varepsilon|\mathscr{I}_U]}\left[Var\left\{\frac{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}\left(V_{DI}+V_F\right)\left(\alpha\theta+\beta\omega\right)}{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\right\}+\left(1-\frac{\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\alpha}{1+\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\frac{1}{V_{DI}+V_F}}\right)^2\sigma_{\omega U}^2\right.
\end{aligned}
$$

Note that the first term in Equation (26) corresponds to Equation (25) since $\sigma_p^2=\left(\frac{\beta}{\alpha}\right)^2\sigma_\omega^2$ and $\frac{\beta}{\alpha}=\frac{V_{DI}}{V_{DI}+V_F}$ still holds by Lemma 1 even when $\sigma_{\omega U}^2>0$, which implies that the first term in Equation (26) decreases with a reduction in c. On the other hand, the second term in Equation (26) is given by

$$
\begin{aligned}
K_2 &:= \frac{1}{\sigma_\varepsilon^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}\left(1-\frac{\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\alpha}{1+\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\frac{1}{V_{DI}+V_F}}\right)^2\sigma_{\omega U}^2 \\
&= \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}\left(1-\frac{\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}}{1+\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\frac{1}{V_{DI}+V_F}}\frac{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}{V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\right)^2\sigma_{\omega U}^2 \\
&= \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}\left\{\frac{V_{DI}+V_F+\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}}{\left(1+\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\frac{1}{V_{DI}+V_F}\right)\left(V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F\right)}\right\}^2\sigma_{\omega U}^2 \\
&= \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}\left(\frac{V_{DI}+V_F}{V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\right)^2\sigma_{\omega U}^2, \qquad (21)
\end{aligned}
$$

where the second line is obtained by using coefficient α from Lemma 3.2. Here, K_2 given by Equation (27) is a function of V_F as $\sigma_p^{-2} = \left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2} \sigma_\omega^{-2}$. If K_2 increases with V_F, which increases with a reduction in c, then a reduction in c increases the second term in Equation (26), which corresponds to K_2, whereas it decreases the first term in Equation (26). Combined with the fact that the second term is multiplied by $\sigma_{\omega U}^2$, this implies that a reduction in c increases K given by Equation (26) and thus increases W with sufficiently large $\sigma_{\omega U}^2$. Otherwise (i.e., if K_2 decreases with V_F), a reduction in c decreases both of the first and second terms in Equation (26). This implies that it decreases K given by Equation (26), and thus, decreases W regardless of $\sigma_{\omega U}^2$.

Overall, it suffices to identify the condition under which K_2 as a function of V_F (given by Equation (27) and $\sigma_p^{-2} = \left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2} \sigma_\omega^{-2}$) increases with V_F. This provides a necessary and sufficient condition for K to increase with a reduction in c for a sufficiently large liquidity shock of uninformed investors $\sigma_{\omega U}^2$. Differentiating the logarithm of K_2 given by Equation (27) and $\sigma_p^{-2} = \left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2} \sigma_\omega^{-2}$ with respect to V_F, we have

$$\begin{aligned}
\frac{d}{dV_F}\ln K_2 &= \left(\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\right)^{-1} \frac{d}{dV_F}\left(\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\right) + \frac{2}{V_{DI}+V_F} \\
&\quad -2\frac{V_U \frac{d}{dV_F}\left(\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\right)+1}{V_U \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + V_{DI}+V_F} \\
&= \frac{d}{dV_F}\left(\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\right)\left\{\left(\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\right)^{-1} - \frac{2V_U}{V_U \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + V_{DI}+V_F}\right\} \\
&\quad + \frac{2}{V_{DI}+V_F} - \frac{2}{V_U \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + V_{DI}+V_F} \\
&= \frac{d}{dV_F}\left(\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\right) \frac{V_{DI}+V_F - V_U \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}}{\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\left(V_U \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + V_{DI}+V_F\right)} \\
&\quad + \frac{2V_U \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}}{(V_{DI}+V_F)\left(V_U \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + V_{DI}+V_F\right)}. \qquad (22)
\end{aligned}$$

Note that

$$\frac{d}{dV_F}\left(\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\right) = \frac{d}{dV_F}\left(1-\frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\right)$$
$$= \sigma_\varepsilon^{-2}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-2}\frac{2\left(V_{DI}+V_F\right)}{V_{DI}^2}\sigma_\omega^{-2}, \quad (23)$$

where the second line is obtained by differentiating $\sigma_p^{-2}=\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2}\sigma_\omega^{-2}$ with respect to V_F. Plugging Equation (29) into Equation (28) and then arranging the terms, we have $\frac{d}{dV_F}\ln K_2>0$ if and only if

$$\sigma_\varepsilon^{-2}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-2}\frac{2\left(V_{DI}+V_F\right)}{V_{DI}^2}\sigma_\omega^{-2}\left(V_{DI}+V_F-V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\right)$$
$$+\frac{2V_U}{V_{DI}+V_F}\left(\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\right)^2>0,$$

which is equivalent to

$$\sigma_\varepsilon^{-2}\sigma_p^{-2}\left(\frac{V_{DI}+V_F}{V_U}-\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\right)+\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^2>0, \quad (24)$$

which is in turn equivalent to

$$\frac{V_{DI}+V_F}{V_U} > \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}-\frac{\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^2}{\sigma_\varepsilon^{-2}\sigma_p^{-2}}$$
$$= \left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\frac{\sigma_\varepsilon^{-2}\sigma_p^{-2}-\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)}{\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)\sigma_\varepsilon^{-2}\sigma_p^{-2}}.$$

Note that the right-hand side of the above inequality is always negative, implying that the above inequality always holds. This in turn implies that $\frac{d}{dV_F}\ln K_2>0$. As mentioned above, this proves the statement about W in Proposition 3.9.

Now we proceed to prove the statement about G in Proposition 3.8. Using coefficients

α and β_U given by Lemma 3.2, we have

$$
\begin{aligned}
G &= Var\left(\theta+\omega_U-p\right) \\
&= Var\left[(1-\alpha)\,\theta-\beta\,\omega\right]+(1-\beta_U)^2\,\sigma_{\omega U}^2,
\end{aligned}
$$

where the first term is given by

$$
\begin{aligned}
Var\left[(1-\alpha)\,\theta-\beta\,\omega\right] &= (1-\alpha)^2\,\sigma_\theta^2+\alpha^2\left(\frac{\beta}{\alpha}\right)^2\sigma_\omega^2 \\
&= \frac{\left(V_U\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\right)^2\sigma_\theta^2+\left(V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F\right)^2\sigma_p^2}{\left(V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F\right)^2}
\end{aligned}
$$

and the second term is given by

$$
\begin{aligned}
(1-\beta_U)^2\,\sigma_{\omega U}^2 &= \left(1-\frac{V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}}{V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}+V_{DI}+V_F}\right)^2\sigma_{\omega U}^2 \\
&= \left(\frac{V_{DI}+V_F}{V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}+V_{DI}+V_F}\right)^2\sigma_{\omega U}^2 \\
&= \left(\frac{1}{\frac{V_U}{V_{DI}}\sqrt{\sigma_\omega^{-2}}\frac{V_{DI}}{V_{DI}+V_F}\sqrt{\sigma_\omega^2}\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}+1}\right)^2\sigma_{\omega U}^2 \\
&= \left(\frac{1}{\frac{V_U}{V_{DI}}\sqrt{\sigma_\omega^{-2}}\frac{1}{\sqrt{\sigma_p^{-2}}}\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}+1}\right)^2\sigma_{\omega U}^2. \qquad (25)
\end{aligned}
$$

Plugging Equation (31) into the expression of G, we get

$$
G = \frac{\left(V_U\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\right)^2\sigma_\theta^2+\left(V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F\right)^2\sigma_p^2}{\left(V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F\right)^2}+\frac{\sigma_{\omega U}^2}{\left(\frac{V_U}{V_{DI}}\sqrt{\sigma_\omega^{-2}}\frac{1}{\sqrt{\sigma_p^{-2}}}\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}+\right.}
$$

In fact, we can show that the first term, which is unrelated to $\sigma_{\omega U}^2$ and corresponds to

G in the benchmark case, always decreases with V_F, as consistent with Proposition 3.7. However, this first term is dwarfed by the second term multiplied by $\sigma_{\omega U}^2$ as $\sigma_{\omega U}^2$ is large. Therefore, as $\sigma_{\omega U}^2$ is large, G decreases with V_F if and only if the second term above multiplied by $\sigma_{\omega U}^2$ decreases with V_F. Noting that σ_p^{-2} increases with V_F, this is equivalent to

$$\frac{d}{d\sigma_p^{-2}} \log\left[\frac{1}{\sqrt{\sigma_p^{-2}}} \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\right] > 0.$$

This holds if and only if

$$-\frac{1}{2}\frac{1}{\sigma_p^{-2}} + \frac{1}{\sigma_\theta^{-2}+\sigma_p^{-2}} - \frac{1}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}} = -\frac{1}{2}\frac{1}{\sigma_p^{-2}} + \frac{\sigma_\varepsilon^{-2}}{\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)} > 0,$$

which is equivalent to

$$\begin{aligned} 2\sigma_p^{-2}\sigma_\varepsilon^{-2} &> \left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right) \\ &= \sigma_\theta^{-2}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}\right)+\sigma_p^{-2}\left(2\sigma_\theta^{-2}+\sigma_\varepsilon^{-2}\right)+\left(\sigma_p^{-2}\right)^2, \end{aligned}$$

which is in turn equivalent to

$$\left(\sigma_p^{-2}\right)^2+\sigma_p^{-2}\left(2\sigma_\theta^{-2}-\sigma_\varepsilon^{-2}\right)+\sigma_\theta^{-2}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}\right) < 0.$$

As a quadratic equation with respect to σ_p^{-2}, this holds for the intermediate interval stated in Proposition 3.8, provided that the determinant is positive as follows:

$$\left(2\sigma_\theta^{-2}-\sigma_\varepsilon^{-2}\right)^2-4\sigma_\theta^{-2}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}\right) = \left(\sigma_\varepsilon^{-2}\right)^2-8\sigma_\varepsilon^{-2}\sigma_\theta^{-2} > 0.$$

Now we prove the statement in Proposition 3.9 in the limiting cases of $V_F \to \infty$ and

$\sigma_\omega^2 \to 0$. Differentiating Equation (26) with respect to V_F, we have

$$
\begin{aligned}
\frac{d\ln K}{dV_F} &= \frac{d}{dV_F}\left[\ln\left\{\frac{V_{DI}^2\sigma_\theta^{-2}\sigma_\omega^2+\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\left(V_{DI}+V_F\right)^2\sigma_{\omega U}^2}{\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)\left(V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F\right)^2}\right\}\right] \\
&= \frac{d}{dV_F}\ln\left\{V_{DI}^2\sigma_\theta^{-2}\sigma_\omega^2+\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\left(V_{DI}+V_F\right)^2\sigma_{\omega U}^2\right\} \\
&\quad -\frac{d}{dV_F}\left\{\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)\left(V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F\right)^2\right\} \\
&= \frac{2\sigma_\theta^{-2}\left(V_{DI}+V_F\right)\sigma_{\omega U}^2+4\left(V_{DI}+V_F\right)^3V_{DI}^{-2}\sigma_\omega^{-2}\sigma_{\omega U}^2}{V_{DI}^2\sigma_\theta^{-2}\sigma_\omega^2+\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\left(V_{DI}+V_F\right)^2\sigma_{\omega U}^2}-\frac{2\left(V_{DI}+V_F\right)V_{DI}^{-2}\sigma_\omega^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}} \\
&\quad -2\frac{V_U}{V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\frac{2\sigma_\varepsilon^{-2}\left(V_{DI}+V_F\right)V_{DI}^{-2}\sigma_\omega^{-2}}{\left(\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}\right)^2}.
\end{aligned}
$$

As $V_F \to \infty$, we have $\sigma_p^{-2} \to \infty$, which implies

$$
\begin{aligned}
\frac{d\ln K}{dV_F} &\to \frac{4\left(V_{DI}+V_F\right)^3V_{DI}^{-2}\sigma_\omega^{-2}\sigma_{\omega U}^2}{\left(V_{DI}+V_F\right)^4V_{DI}^{-2}\sigma_\omega^{-2}\sigma_{\omega U}^2}-\frac{2\left(V_{DI}+V_F\right)V_{DI}^{-2}\sigma_\omega^{-2}}{\left(V_{DI}+V_F\right)^2V_{DI}^{-2}\sigma_\omega^{-2}} \\
&\quad -2\frac{V_U}{V_U+V_{DI}+V_F}\frac{2\sigma_\varepsilon^{-2}\left(V_{DI}+V_F\right)V_{DI}^{-2}\sigma_\omega^{-2}}{\left(\left(V_{DI}+V_F\right)^2V_{DI}^{-2}\sigma_\omega^{-2}\right)^2} \\
&= \frac{4}{V_{DI}+V_F}-\frac{2}{V_{DI}+V_F}-4\frac{V_U}{V_U+V_{DI}+V_F}\frac{\sigma_\varepsilon^{-2}}{\left(V_{DI}+V_F\right)^3V_{DI}^{-2}\sigma_\omega^{-2}} \\
&\approx \frac{2}{V_{DI}+V_F}>0,
\end{aligned}
$$

where the last line is obtained by using the fact that the last term decreases at the pace of V_F^{-4} while other terms decrease at the pace of V_F^{-1}. Therefore, as V_F is large, K increases with V_F, and thus, it does so with a reduction in c. Likewise, as $\sigma_\omega^2 \to 0$, we have $\sigma_\omega^{-2} \to \infty$

and $\sigma_p^{-2} \to \infty$, which imply

$$\begin{aligned}
\frac{d\ln K}{dV_F} &\to \frac{4\left(V_{DI}+V_F\right)^3 V_{DI}^{-2}\sigma_\omega^{-2}\sigma_{\omega U}^2}{\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\left(V_{DI}+V_F\right)^2\sigma_{\omega U}^2} - \frac{2\left(V_{DI}+V_F\right)V_{DI}^{-2}\sigma_\omega^{-2}}{\left(V_{DI}+V_F\right)^2 V_{DI}^{-2}\sigma_\omega^{-2}} \\
&\quad -2\frac{V_U}{V_U+V_{DI}+V_F}\frac{2\sigma_\varepsilon^{-2}\left(V_{DI}+V_F\right)V_{DI}^{-2}\sigma_\omega^{-2}}{\left(\left(V_{DI}+V_F\right)^2 V_{DI}^{-2}\sigma_\omega^{-2}\right)^2} \\
&= \frac{4}{V_{DI}+V_F} - \frac{2}{V_{DI}+V_F} - 2\frac{V_U}{V_U+V_{DI}+V_F}\frac{\sigma_\varepsilon^{-2}}{\left(V_{DI}+V_F\right)^3 V_{DI}^{-2}\sigma_\omega^{-2}} \\
&\approx \frac{2}{V_{DI}+V_F} > 0.
\end{aligned}$$

Therefore, as σ_ω^2 is small, K increases with V_F, and thus, it does so with a reduction in c.

Proof of Lemma 3.11

We first derive the equilibrium conditions which generally hold for every value of $\sigma_{\omega L}^2$ and σ_ω^2, and then, concentrate on Cases 1 and 2 to prove the existence and uniqueness of equilibrium in each of these cases.

We start by considering the general case with $\sigma_{\omega L}^2$ and σ_ω^2. Given the conjectured price $p = \alpha\theta + \beta\omega + \gamma x_L$, the large investor's profit is given by

$$u_L(x_L) = x_L\left(\theta + \omega_L + \varepsilon - p\right) = x_L\left(\theta + \omega_L + \varepsilon - \alpha\theta - \beta\omega - \gamma x_L\right).$$

Using the first-order condition of his expected profit given θ, ω_L and p, the large investor's optimal demand is represented by coefficients α, β and γ as follows:

$$x_L^* = \frac{1}{2\gamma}\left\{(1-\alpha)\theta + \omega_L - \beta\omega\right\}.$$

As a result, the equilibrium price is given by

$$p^* = \alpha\theta + \beta\omega + \gamma x_L^* = \frac{1+\alpha}{2}\theta + \frac{1}{2}\beta\omega + \frac{1}{2}\omega_L, \tag{26}$$

Other investors' optimal demands follow from their optimization problems. As in the basic model, define

$$\sigma_p^2 := \left(\frac{\beta}{1+\alpha}\right)^2 \sigma_\omega^2 + \left(\frac{1}{1+\alpha}\right)^2 \sigma_{\omega L}^2$$

as the error variance of the price which is normalized to be an unbiased signal about θ, i.e.,

$$\tilde{p} := \frac{2}{1+\alpha} p = \theta + \frac{1}{1+\alpha}\beta\omega + \frac{1}{1+\alpha}\omega_L.$$

Uninformed investors' optimal demand is given by

$$\begin{aligned} x_U &= \frac{E[\theta|\mathscr{I}_U]-p}{\varphi_U\left\{\sigma_\varepsilon^2 + Var[\theta|\mathscr{I}_U]\right\}} \\ &= \frac{\sigma_p^{-2}\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}\tilde{p}-p}{\varphi_U\sigma_\varepsilon^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}, \end{aligned}$$

where the second line follows from using the normalized price mentioned above (i.e., $\tilde{p}$) to obtain $E[\theta|\mathscr{I}_U]$ and $Var[\theta|\mathscr{I}_U]$ in line with Equation (19) in the benchmark case. Arranging the terms further, we have

$$x_U = \frac{\left(\frac{1-\alpha^2}{\beta^2\sigma_\omega^2+\sigma_{\omega L}^2} - \sigma_\theta^{-2}\right)p}{\varphi_U\sigma_\varepsilon^2\left\{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\frac{(1+\alpha)^2}{\beta^2\sigma_\omega^2+\sigma_{\omega L}^2}\right\}}. \tag{27}$$

Also, domestic informed price-takers and foreign investors form their optimal demands as

$$x_{DI} = \frac{\theta+\omega-p}{\varphi_{DI}\sigma_\varepsilon^2} \text{ and } x_F = \frac{\theta-p}{\varphi_F\sigma_\varepsilon^2+c}. \tag{28}$$

by the first-order conditions of their expected profits, as in the benchmark case.

By Equations (33) and (34), the market-clearing condition yields

$$\begin{aligned} &\lambda_U x_U + x_L + \lambda_{DI}x_{DI} + \lambda_F x_F \\ = \;& \frac{\lambda_U\left(\frac{1-\alpha^2}{\beta^2\sigma_\omega^2+\sigma_{\omega B}^2} - \sigma_\theta^{-2}\right)p}{\varphi_U\sigma_\varepsilon^2\left\{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\frac{(1+\alpha)^2}{\beta^2\sigma_\omega^2+\sigma_{\omega L}^2}\right\}} + x_L + \lambda_{DI}\frac{\theta+\omega-p}{\varphi_{DI}\sigma_\varepsilon^2} + \lambda_F\frac{\theta-p}{\varphi_F\sigma_\varepsilon^2+c} = 0 \end{aligned} \tag{29}$$

As the first term is proportional to p, other terms must be proportional to p as well. This implies

$$x_L + \lambda_{DI}\frac{\theta+\omega-p}{\varphi_{DI}\sigma_\varepsilon^2} + \lambda_F\frac{\theta}{\varphi_F\sigma_\varepsilon^2+c} = C^{'}z = C^{'}\left(\alpha\theta+\beta\omega+\gamma x_L\right). \tag{30}$$

By equalizing the coefficients for ω on the left- and right-hand sides of Equation (36), we get $C^{'} = \frac{\lambda_{DI}}{\beta}$. Then, by equalizing the ratios between the coefficients for θ and ω in both sides of Equation (36) and using the definitions of V_{DI} and V_F, we obtain

$$\frac{\beta}{\alpha} = \frac{V_{DI}}{V_{DI}+V_F} \text{ and } \frac{\gamma}{\alpha} = \frac{1}{V_{DI}+V_F}, \tag{31}$$

as stated in the lemma. By applying Equation (36) together with $C^{'} = \frac{\lambda_{DI}}{\beta}$ to Equation (35) and then arranging the terms using Equation (37) and the definition of H, we have

$$\frac{\left(\frac{1-\alpha^2}{\beta^2\sigma_\omega^2+\sigma_{\omega L}^2} - \sigma_\theta^{-2}\right)H}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\frac{(1+\alpha)^2}{\beta^2\sigma_\omega^2+\sigma_{\omega L}^2}}p + \frac{1-\alpha}{\alpha}p = 0. \tag{32}$$

Existence and uniqueness of equilibrium in Case 1

Applying $\sigma_\omega^2 = 0$ to Equation (32), we have $p = \frac{1+\alpha}{2}\theta + \frac{1}{2}\omega_L$. Also, applying $\sigma_\omega^2 = 0$ to Equation (38), we get

$$\left\{(1-\alpha^2)\sigma_{\omega L}^{-2} - \sigma_\theta^{-2}\right\}H + \frac{1-\alpha}{\alpha}\left\{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+(1+\alpha)^2\sigma_{\omega L}^{-2}\right\} = 0, \tag{33}$$

which is equivalent to the equilibrium condition stated in the proposition. We then want to prove the following claim, which implies the proposition in Case 1 and will also be useful in the proof of Proposition 3.12:

Claim. [1] There is a unique solution of α satisfying Equation (39) for every $H \in (0,\infty)$. If $\sigma_{\omega L}^2 < \sigma_\theta^2$, then it decreases from one to $\sqrt{\frac{\sigma_{\omega L}^{-2}-\sigma_\theta^{-2}}{\sigma_{\omega L}^{-2}}} \in (0,1)$ as H increases from zero to infinity. If $\sigma_{\omega L}^2 > \sigma_\theta^2$, then it decreases from one to zero as H increases from zero to infinity.

Proof. Note first that the left-hand side of Equation (39) approaches infinity as $\alpha \to 0^+$, whereas it goes to a negative value, i.e. $-V_U\sigma_\theta^{-2}$, as $\alpha \to 1$. Also, the first term of the left-hand side of the equation is clearly decreasing in α, and its second term is also decreasing in α for $0 < \alpha < 1$, since $\frac{1-\alpha}{\alpha}(1+\alpha)^2 = (1-\alpha^2)\left(\frac{1}{\alpha}+1\right)$ is decreasing in α. By the Intermediate Value Theorem, these imply that there exists a unique value $\alpha^* \in (0,1)$ such that Equation (41) holds.

As H increases, the left-hand side of Equation (39) goes downward pointwisely for each $\alpha \in (0,1)$. This implies that the unique solution α^* of the equation is also decreasing in H.

As $H \to 0$, whether $\sigma_{\omega L}^2 < \sigma_\theta^2$ or $\sigma_{\omega L}^2 > \sigma_\theta^2$, the left-hand side of Equation (39) goes upward toward infinity pointwisely for each $\alpha \in (0,1)$, thereby moving the solution α^* toward one.

As $H \to \infty$, the left-hand side of Equation (39) goes downward toward $V_U\left\{(1-\alpha^2)\sigma_{\omega L}^{-2} - \sigma_\theta^{-2}\right\}$ pointwisely for each $\alpha \in (0,1)$. If $\sigma_{\omega L}^2 < \sigma_\theta^2$, then this pointwise limit equals zero at $\alpha = \sqrt{\frac{\sigma_{\omega L}^{-2}-\sigma_\theta^{-2}}{\sigma_{\omega L}^{-2}}} \in (0,1)$, and thus, the solution α^* of the left-hand side of Equation (39) also approaches $\sqrt{\frac{\sigma_{\omega L}^{-2}-\sigma_\theta^{-2}}{\sigma_{\omega L}^{-2}}}$, as stated in the claim. If $\sigma_{\omega L}^2 > \sigma_\theta^2$, the pointwise limit described above is negative for every $\alpha \in (0,1)$. Thus, the solution α^* of the left-hand side of Equation (39) approaches zero. $\square$

Existence and uniqueness of equilibrium in Case 2

Applying $\sigma_{\omega L}^2 = 0$ to Equations (32) and (38), respectively, we get

$$p = \frac{1+\alpha}{2}\theta + \frac{\beta}{2}\omega \tag{34}$$

and

$$V_U\left\{\frac{1-\alpha^2}{\alpha^2}\left(\frac{V_{DI}+V_F}{V_{DI}}\right)^2\sigma_\omega^{-2} - \sigma_\theta^{-2}\right\} + \frac{1-\alpha}{\alpha}(V_{DI}+V_F)\left\{\sigma_\varepsilon^{-2} + \sigma_\theta^{-2} + \left(\frac{1+\alpha}{\alpha}\right)^2\left(\frac{V_{DI}+V_F}{V_{DI}}\right)^2\sigma_\omega^{-2}\right\} \tag{35}$$

which is equivalent to the equilibrium conditions stated in the proposition. We then want to prove the following claim, which implies the proposition in Case 2 and will also be useful in the proof of Proposition 3.11:

Claim. [2] There is a unique solution of α satisfying Equation (41). It decreases from one to $\alpha_0^* \in (0,1)$, which is determined by Equation (42) below, as V_F increases from zero to infinity. Further, for such unique solution α satisfying Equation (41), it holds that $\frac{1-\alpha}{\alpha}(V_{DI}+V_F)$ decreases with V_F and $\frac{1+\alpha}{\alpha}(V_{DI}+V_F)$ increases with V_F for every $V_F \in (0,\infty)$.

Proof. Given other variables, it is easy to see that the first term of the left-hand side of Equation (41) is decreasing in α for $\alpha \in (0,1)$, and its second term is also decreasing in α for $\alpha \in (0,1)$. Thus, the left-hand side of Equation (41) is decreasing in α for $\alpha \in (0,1)$. Also, it approaches infinity as $\alpha \to 0^+$ and goes to a negative value, i.e. $-V_U \sigma_\theta^{-2}$, as $\alpha \to 1$. By the Intermediate Value Theorem, these imply that there exists a unique solution $\alpha \in (0,1)$ such that Equation (41) holds.

Given $\alpha \in (0,1)$, the left-hand side of Equation (41) is decreasing in V_F. This implies that the unique solution α is also decreasing in V_F.

As $V_F \to 0$, for each $\alpha \in (0,1)$, Equation (41) leads to

$$V_U\left(\frac{1-\alpha^2}{\alpha^2}\sigma_\omega^{-2}-\sigma_\theta^{-2}\right)+\frac{1-\alpha}{\alpha}V_{DI}\left\{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\left(\frac{1+\alpha}{\alpha}\right)^2\sigma_\omega^{-2}\right\}=0. \tag{36}$$

As shown above, there exists a unique solution $\alpha_0^* \in (0,1)$ satisfying Equation (42), which corresponds to the limit of α as $V_F \to 0$.

As $V_F \to \infty$, the left-hand side of Equation (41) explodes to infinity pointwisely for each $\alpha \in (0,1)$, thereby pushing the solution α of Equation (41) toward one.

Finally, we want to prove the last statement of the claim. Suppose that $\frac{1-\alpha}{\alpha}(V_{DI}+V_F)$ weakly increases with V_F. Arranging the terms in Equation (41) further, we get

$$V_U\sigma_\theta^{-2}=\frac{1-\alpha}{\alpha}(V_{DI}+V_F)\left\{\sigma_\theta^{-2}+\sigma_\varepsilon^{-2}+\left(\frac{1+\alpha}{\alpha}\right)^2\left(\frac{V_{DI}+V_F}{V_{DI}}\right)^2\sigma_\omega^{-2}+V_U\frac{1+\alpha}{\alpha}\left(\frac{V_{DI}+V_F}{V_{DI}}\right)^2\sigma_\omega^{-2}\right\}.$$

By the fact that the right-hand side must be constant, the term in the curly bracket on the right-hand side weakly decreases with V_F. This implies that $\frac{1+\alpha}{\alpha}(V_{DI}+V_F)$ weakly decreases with V_F as well. However, this leads to a contradiction because we have $\frac{1+\alpha}{\alpha}(V_{DI}+V_F)=\frac{1-\alpha}{\alpha}(V_{DI}+V_F)+2(V_{DI}+V_F)$, where the first term $\frac{1-\alpha}{\alpha}(V_{DI}+V_F)$ weakly increases with V_F, by the assumption we have made, and the second term $2(V_{DI}+V_F)$ strictly increases with V_F. Therefore, it holds that $\frac{1-\alpha}{\alpha}(V_{DI}+V_F)$ decreases with V_F. Further, this implies that the curly bracket on the left-hand side of the above equation must increase with V_F, which in turn implies that $\frac{1+\alpha}{\alpha}(V_{DI}+V_F)$ increases with V_F. $\square$

Proof of Proposition 3.12

Case 1: $\sigma_{\omega L}^2 > 0$ and $\sigma_{\omega}^2 = 0$

Claim 1 immediately implies that price informativeness $\sigma_p^{-2} = (1+\alpha)^2\sigma_{\omega L}^{-2}$ always increases with a reduction in c, as stated in the proposition.

Consider uninformed investors' welfare W. Recall that $\sigma_p^2 = \frac{\sigma_{\omega L}^2}{(1+\alpha)^2}$ is the error variance of the normalized price which is unbiased about θ, which corresponds to $\tilde{p} := \frac{2}{1+\alpha}p$. Combined with the standard Bayesian updating formula, this leads to

$$\begin{aligned}
(E[\theta|\mathscr{I}_U]-p)^2 &= \left\{\sigma_p^{-2}\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}\frac{2}{1+\alpha}-1\right\}^2 p^2 \\
&= \left(\sigma_p^{-2}\frac{2}{1+\alpha}-\sigma_\theta^{-2}-\sigma_p^{-2}\right)^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}\frac{1}{4}\sigma_\theta^2\sigma_{\omega L}^2X^2 \\
&= \left\{\left(1-\alpha^2\right)\sigma_{\omega L}^{-2}-\sigma_\theta^{-2}\right\}^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}\frac{1}{4}\sigma_\theta^2\sigma_{\omega L}^2X^2 \qquad (37)
\end{aligned}$$

where the second line is obtained by defining $X := \frac{1}{\sqrt{\sigma_\theta^2+\sigma_p^2}}\frac{2}{1+\alpha}p \sim N(0,1)$ and then using

$$\begin{aligned}
p^2 &= \left(\frac{1+\alpha}{2}\right)^2\left(\theta+\frac{\omega_L}{1+\alpha}\right)^2 = \left(\frac{1+\alpha}{2}\right)^2\left(\sigma_\theta^2+\sigma_p^2\right)X^2 \\
&= \left(\frac{1+\alpha}{2}\right)^2\sigma_\theta^2\sigma_p^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)X^2 = \frac{1}{4}\sigma_\theta^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)X^2.
\end{aligned}$$

Using Fact 1 and Equation (43), we have

$$\begin{aligned}
E[u_U|\mathscr{I}_U] &= -\exp\left(-\frac{(E[\theta|\mathscr{I}_U]-p)^2}{2Var[\theta+\varepsilon|\mathscr{I}_U]}\right) \\
&= -\exp\left(-\frac{\left\{\left(1-\alpha^2\right)\sigma_{\omega L}^{-2}-\sigma_\theta^{-2}\right\}^2\frac{1}{4}\sigma_\theta^2\sigma_{\omega L}^2X^2}{2\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}\right).
\end{aligned}$$

By $X \sim N(0,1)$ and Fact 2, this implies

$$W = E[E[u_U|\mathscr{I}_U]] = -(1+K)^{-\frac{1}{2}},$$

where

$$K = \frac{\frac{1}{4}\sigma_\theta^2\sigma_{\omega L}^2\left\{(1-\alpha^2)\sigma_{\omega L}^{-2} - \sigma_\theta^{-2}\right\}^2}{\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}. \tag{38}$$

Noting that $(1-\alpha^2)\sigma_{\omega L}^{-2} - \sigma_\theta^{-2} < 0$ by Equation (39), by Claim 1, K increases with α and thus decreases with H if and only if

$$\begin{aligned}
\frac{\partial}{\partial\alpha}[\log K] &= 2\frac{2\alpha\sigma_{\omega L}^{-2}}{\sigma_\theta^{-2}-(1-\alpha^2)\sigma_{\omega L}^{-2}} - \frac{2(1+\alpha)\sigma_{\omega L}^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+(1+\alpha)^2\sigma_{\omega L}^{-2}} \\
&\propto 2\sigma_{\omega L}^{-2}\left[2\alpha\left\{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+(1+\alpha)^2\sigma_{\omega L}^{-2}\right\} - (1+\alpha)\left\{\sigma_\theta^{-2}-(1-\alpha^2)\sigma_{\omega L}^{-2}\right\}\right] \\
&= 2\sigma_{\omega L}^{-2}\left[2\alpha\sigma_\varepsilon^{-2} - (1-\alpha)\sigma_\theta^{-2} + (1+\alpha)^3\sigma_{\omega L}^{-2}\right] > 0, \qquad (39)
\end{aligned}$$

where the positive denominator is omitted in the second and third lines.

If $\sigma_{\omega L}^2 < \sigma_\theta^2$, then we can see that a reduction in cross-border cost c decreases H, which in turn increases α by Claim 1. This leads to an increase in K since Equation (45) is positive. This implies that W increases as well, as stated in the proposition.

If $\sigma_{\omega L}^2 > \sigma_\theta^2$, then Equation (45) is negative at $\alpha = 0$ but it is increasing in α and is positive at $\alpha = 1$. By the Intermediate Value Theorem, these imply that Equation (45) is positive (i.e., $\frac{\partial K}{\partial\alpha} > 0$) if and only if $\alpha > \hat{\alpha}$, where $\hat{\alpha} \in (0,1)$ is uniquely determined by

$$\hat{\alpha}\left(2\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}\right) + (1+\hat{\alpha})^3\sigma_{\omega L}^{-2} = \sigma_\theta^{-2}. \tag{40}$$

By $\sigma_{\omega L}^2 > \sigma_\theta^2$ and Claim 1, α decreases from one to zero as H increases from zero to infinity. This implies that there exists a unique cutoff $\hat{H} \in (0,\infty)$ such that Equation (45) is positive (i.e., $\frac{\partial K}{\partial\alpha} > 0$) if and only if $H < \hat{H}$. By Equation (45) and $\hat{\alpha}$ defined by Equation (46), $\hat{H}$ is given by

$$\hat{H} = \frac{1-\hat{\alpha}}{\hat{\alpha}}\frac{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+(1+\hat{\alpha})^2\sigma_{\omega L}^{-2}}{\sigma_\theta^{-2}-(1-\hat{\alpha})\sigma_{\omega L}^{-2}}, \tag{41}$$

where the denominator is positive by Equation (39). We can see that $\hat{H}$ is independent of λ_{DI} and λ_F. Given that α decreases with H by Claim 1 and H decreases with c by definition, this implies that a reduction in c increases K given by Equation (38), and thus, increases W as well if and only if $H < \hat{H}$, where $\hat{\alpha}$ is given by Equation (39). Here we can see that as $\sigma_{\omega L}^2$ increases from σ_θ^2 toward infinity, $\hat{H}$ decreases from infinity toward $\frac{\left(\sigma_\theta^{-2}\right)^2}{2\sigma_\varepsilon^{-2}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}\right)}$.[17] At this point, we can define $\bar{\sigma}_{\omega L}^2(H) > 0$ for $H \in \left[\frac{\left(\sigma_\theta^{-2}\right)^2}{2\sigma_\varepsilon^{-2}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}\right)}, \infty\right]$ as stated in the proposition. The properties of $\bar{\sigma}_{\omega L}^2(H)$ immediately follow from the properties of $\hat{H}$ shown above.

Finally, we want to show that whether $\sigma_\theta^2 > \sigma_{\omega L}^2$ or $\sigma_\theta^2 < \sigma_{\omega L}^2$, reducing c decreases the gains from trade G and increases learning from the price L. By $p = \frac{1+\alpha}{2}\theta + \frac{1}{2}\omega_L$, the gains from trade G are given by

$$\begin{aligned} G &= \sigma_\theta^{-2} Var(\theta - p) = \sigma_\theta^{-2} Var\left(\frac{1-\alpha}{2}\theta - \frac{1}{2}\omega_L\right) \\ &= \sigma_\theta^{-2}\left\{\left(\frac{1-\alpha}{2}\right)^2 \sigma_\theta^2 + \frac{1}{4}\sigma_{\omega L}^2\right\}. \end{aligned} \tag{42}$$

Applying Claim 1, which implies that reducing c increases coefficient α, to Equation (48), we conclude that reducing c decreases the gains from trade G. Also, given that reducing c increases price informativeness σ_p^{-2}, reducing c increases learning from the price L. These imply a negative gains-from-trade effect ($\Delta G < 0$) and a positive learning effect ($\Delta L > 0$), as stated in the proposition.

Case 2: $\sigma_{\omega L}^2 = 0$ and $\sigma_\omega^2 > 0$

The last statement of Claim 2 implies that price informativeness σ_p^{-2} increases with V_F because applying Equation (40) to the definition of σ_p^2 and using Equation (41) we have

$$\sigma_p^2 = \left(\frac{\beta}{1+\alpha}\right)^2 \sigma_\omega^2 = \left(\frac{\alpha}{1+\alpha}\frac{V_{DI}}{V_{DI}+V_F}\right)^2 \sigma_\omega^2 = \left(\frac{1+\alpha}{\alpha}\right)^{-2}\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^2 \sigma_\omega^2,$$

[17] At $\sigma_{\omega L}^2 = \sigma_\theta^2$, Equation (46) implies $\hat{\alpha} = 0$. Plugging this to Equation (47), we have $\hat{H} = 0$. As $\sigma_{\omega L}^2$ increases toward infinity, Equation (46) implies that $\hat{\alpha}$ increases toward $\frac{\sigma_\theta^{-2}}{2\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}}$. By Equation (47), this in turn implies that $\hat{H}$ decreases toward $\frac{\left(\sigma_\theta^{-2}\right)^2}{2\sigma_\varepsilon^{-2}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}\right)}$.

which decreases with V_F by the last statement of Claim 2. This in turn implies that a reduction in c increases price informativeness σ_p^{-2}, as stated in the proposition.

Now consider uninformed investors' welfare W. Recall that $\sigma_p^2 = \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{V_{DI}}{V_{DI}+V_F}\right)^2 \sigma_\omega^2$ is the error variance of the normalized price which is unbiased about θ, which corresponds to $\tilde{p} := \frac{2}{1+\alpha} p$. Using Fact 2 and the standard Bayesian updating formula to obtain $E[\theta|\mathscr{I}_U]$ and $Var[\theta+\varepsilon|\mathscr{I}_U]$, we have

$$\begin{aligned}
E[u_U|\mathscr{I}_U] &= -\exp\left[-\frac{\left\{\sigma_p^{-2}\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}\frac{2}{1+\alpha}-1\right\}^2 p^2}{2\sigma_\varepsilon^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}\right] \\
&= -\exp\left[-\frac{\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}\left(\sigma_p^{-2}\frac{2}{1+\alpha}-\sigma_\theta^{-2}-\sigma_p^{-2}\right)^2}{2\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}\frac{(1+\alpha)^2}{4}\sigma_\theta^2\sigma_p^2 X^2\right] \\
&= -\exp\left[-\frac{\left\{\frac{1-\alpha^2}{\alpha^2}\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2}\sigma_\omega^{-2}-\sigma_\theta^{-2}\right\}^2}{2\sigma_\varepsilon^2\left\{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\left(\frac{1+\alpha}{\alpha}\right)^2\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2}\sigma_\omega^{-2}\right\}}\frac{1}{4}\sigma_\theta^2\alpha^2\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^2\sigma_\omega^2 X^2\right],
\end{aligned}$$

where the second line is obtained by defining $X := \frac{1}{\sqrt{\sigma_\theta^2+\sigma_p^2}}\frac{2}{1+\alpha}p \sim N(0,1)$, similarly to Case 1, and then using

$$\begin{aligned}
p^2 &= \left(\frac{1+\alpha}{2}\right)^2\left(\theta+\frac{\beta}{1+\alpha}\omega\right)^2 \\
&= \left(\frac{1+\alpha}{2}\right)^2\left(\sigma_\theta^2+\sigma_p^2\right)X^2 = \left(\frac{1+\alpha}{2}\right)^2\sigma_\theta^2\sigma_p^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)X^2 \\
&= \frac{1}{4}\sigma_\theta^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)X^2,
\end{aligned}$$

and the third line follows from the definition of σ_p^2. Given that $X \sim N(0,1)$, by Fact 2, we have

$$W = E[E[u_U|\mathscr{I}_U]] = -(1+K)^{-\frac{1}{2}},$$

where

$$K := \frac{\left\{\frac{1-\alpha^2}{\alpha^2}\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2}\sigma_\omega^{-2}-\sigma_\theta^{-2}\right\}^2}{\sigma_\varepsilon^2\left\{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\left(\frac{1+\alpha}{\alpha}\right)^2\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2}\sigma_\omega^{-2}\right\}}\frac{1}{4}\sigma_\theta^2\alpha^2\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^2\sigma_\omega^2. \tag{43}$$

Note that Equation (41) in the proof of Lemma 3.11 yields

$$\frac{\frac{1-\alpha^2}{\alpha^2}\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2}\sigma_\omega^{-2}-\sigma_\theta^{-2}}{\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\left(\frac{1+\alpha}{\alpha}\right)^2\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2}\sigma_\omega^{-2}\right)}=-\frac{\varphi_U}{\lambda_U}\left(\frac{1}{\alpha}-1\right)(V_{DI}+V_F).$$

Applying the square of the above expression to Equation (43), we get

$$\begin{aligned}
K &= \left\{\frac{\varphi_U}{\lambda_U}\left(\frac{1}{\alpha}-1\right)(V_{DI}+V_F)\right\}^2\frac{1}{4}\sigma_\theta^2\alpha^2\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^2\sigma_\omega^2\sigma_\varepsilon^2\left\{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\left(\frac{1+\alpha}{\alpha}\right)^2\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-}\right. \\
&= \left\{\frac{\varphi_U}{\lambda_U}(1-\alpha)\right\}^2\frac{1}{4}\sigma_\theta^2V_{DI}^2\sigma_\omega^2\sigma_\varepsilon^2\left\{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\left(\frac{1+\alpha}{\alpha}\right)^2\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2}\sigma_\omega^{-2}\right\} \\
&= \left(\frac{\varphi_U}{\lambda_U}\right)^2\frac{1}{4}\sigma_\theta^2V_{DI}^2\sigma_\omega^2\sigma_\varepsilon^2\left\{(1-\alpha)^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}\right)+\left(\frac{1-\alpha^2}{\alpha}\right)^2\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2}\sigma_\omega^{-2}\right\}.
\end{aligned}$$

Here, with a slight abuse of notation, define $q:=\frac{1-\alpha}{\alpha}(V_{DI}+V_F)$ and then K can be viewed as a function of α and q as follows:

$$K(\alpha,q)=\left(\frac{\varphi_U}{\lambda_U}\right)^2\frac{1}{4}\sigma_\theta^2V_{DI}^2\sigma_\omega^2\sigma_\varepsilon^2\left\{(1-\alpha)^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}\right)+(1+\alpha)^2q^2V_{DI}^{-2}\sigma_\omega^{-2}\right\}. \quad (44)$$

By applying the Chain Rule to $K(\alpha,q)$ given by Equation (50), we have

$$\frac{dK(\alpha,q)}{dV_F}=\frac{d\alpha}{dV_F}\frac{\partial K(\alpha,q)}{\partial\alpha}+\frac{dq}{dV_F}\frac{\partial K(\alpha,q)}{\partial q}.$$

Note that $\frac{dq}{dV_F}<0$ and $\frac{d\alpha}{dV_F}>0$ by Claim 2, and that $\frac{\partial K(\alpha,q)}{\partial q}>0$ by Equation (50). Therefore, it suffices to show that $\frac{\partial K(\alpha,q)}{\partial\alpha}<0$ to ensure that K decreases with V_F. Noting that we can focus on the term in the curly bracket of Equation (50) because other terms are constant over α, we have

$$\begin{aligned}
\frac{\partial K(\alpha,q)}{\partial\alpha} &\propto -2(1-\alpha)\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}\right)+2(1+\alpha)q^2V_{DI}^{-2}\sigma_\omega^{-2} \\
&= 2(1-\alpha)\left\{\frac{1-\alpha^2}{\alpha^2}\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2}\sigma_\omega^{-2}-\sigma_\varepsilon^{-2}-\sigma_\theta^{-2}\right\}, \quad (45)
\end{aligned}$$

where the second line is obtained by the definition of q, i.e. $q:=\frac{1-\alpha}{\alpha}(V_{DI}+V_F)$. Note that applying $\alpha \in (0,1)$ to Equation (41) in the proof of Lemma 3.11 yields

$$\frac{1-\alpha^2}{\alpha^2}\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2}\sigma_\omega^{-2} < \sigma_\theta^{-2}.$$

Applying this to Equation (51), we have

$$\begin{aligned}\frac{\partial K(\alpha,q)}{\partial \alpha} &\propto 2(1-\alpha)\left\{\frac{1-\alpha^2}{\alpha^2}\left(\frac{V_{DI}}{V_{DI}+V_F}\right)^{-2}\sigma_\omega^{-2}-\sigma_\varepsilon^{-2}-\sigma_\theta^{-2}\right\}\\ &< -2(1-\alpha)\sigma_\varepsilon^{-2} < 0,\end{aligned}$$

which, by the aforementioned argument, establishes that K decreases with V_F, and thus, the same argument holds for W. Combined with the fact that a reduction in c increases V_F, this implies that a reduction in c decreases W, as stated in the proposition.

Proof of Proposition 3.13

We first show that Lemma 1 continues to hold, thereby determining all coefficients α, β and β_U in the price, by redefining V_U and V_{DI}. This follows from noting that

$$\begin{aligned}&\lambda_U\frac{\left(\frac{\sigma_p^{-2}}{\alpha}-\sigma_\theta^{-2}-\sigma_p^{-2}\right)p-\frac{\sigma_p^{-2}}{\alpha}\left(\beta_U+\beta\rho\sqrt{\frac{\sigma_\omega^2}{\sigma_{\omega U}^2}}\right)\omega_U+\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\omega_U}{\varphi_U\sigma_\varepsilon^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}\right)+c\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)}\\&+\frac{\lambda_{DI}}{\varphi_{DI}\sigma_\varepsilon^2+c}(\theta+\omega-p)+\frac{\lambda_F}{\varphi_F\sigma_\varepsilon^2+c}(\theta-p)=0,\end{aligned}$$

which corresponds to Equation (21) in case of taxing only foreign investors' trades. Here, the three terms correspond to demands of uninformed, domestic informed, and foreign investors, respectively. Arranging the first term further, we have

$$\begin{aligned}&\frac{\lambda_U}{\varphi_U\sigma_\varepsilon^2+\frac{c\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}}\frac{\left(\frac{\sigma_p^{-2}}{\alpha}-\sigma_\theta^{-2}-\sigma_p^{-2}\right)p-\frac{\sigma_p^{-2}}{\alpha}\left(\beta_U+\beta\rho\sqrt{\frac{\sigma_\omega^2}{\sigma_{\omega U}^2}}\right)\omega_U+\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\omega_U}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\\&+\frac{\lambda_{DI}}{\varphi_{DI}\sigma_\varepsilon^2+c}(\theta+\omega-p)+\frac{\lambda_F}{\varphi_F\sigma_\varepsilon^2+c}(\theta-p)=0.\end{aligned}$$

This is equivalent to Equation (21) after redefining V_U and V_{DI} as follows:

$$V_U := \frac{\lambda_U}{\varphi_U \sigma_\varepsilon^2 + \frac{c\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}} \text{ and } V_{DI} := \frac{\lambda_{DI}}{\varphi_{DI}\sigma_\varepsilon^2 + c}.$$

We then proceed to prove that reducing the uniform tax c increases σ_p^{-2}, increases $\frac{V_F}{V_{DI}}$, and decreases $\frac{V_U}{V_{DI}}$. We have

$$\frac{V_F}{V_{DI}} = \frac{\lambda_F\left(\varphi_F\sigma_\varepsilon^2 + c\right)^{-1}}{\lambda_{DI}\left(\varphi_{DI}\sigma_\varepsilon^2 + c\right)^{-1}} = \frac{\lambda_F}{\lambda_{DI}}\frac{\varphi_{DI}\sigma_\varepsilon^2 + c}{\varphi_F\sigma_\varepsilon^2 + c} = \frac{\lambda_F}{\lambda_{DI}}\left\{1 + \frac{\left(\varphi_{DI} - \varphi_F\right)\sigma_\varepsilon^2}{\varphi_F\sigma_\varepsilon^2 + c}\right\}, \quad (46)$$

which decreases with c given the assumption that $\varphi_{DI} > \varphi_F$. Also, by definition, we have

$$\sigma_p^{-2} = \left(\frac{V_{DI} + V_F}{V_{DI}}\right)^2 \sigma_\omega^{-2},$$

which also decreases with c as $\frac{V_F}{V_{DI}}$ does so. Further, we have

$$\frac{V_U}{V_{DI}} = \frac{\lambda_U\left\{\varphi_U\sigma_\varepsilon^2 + \frac{c\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\right\}^{-1}}{\lambda_{DI}\left(\varphi_{DI}\sigma_\varepsilon^2 + c\right)^{-1}} = \frac{\lambda_U}{\lambda_{DI}}\frac{\varphi_{DI}\sigma_\varepsilon^2 + c}{\varphi_U\sigma_\varepsilon^2 + \frac{c\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}}.$$

Differentiating its logarithm with respect to c and then using $\frac{d\sigma_p^{-2}}{dc} < 0$, we have

$$\begin{aligned}
\frac{d}{dc}\ln\left(\frac{V_U}{V_{DI}}\right) &= \frac{1}{\varphi_{DI}\sigma_\varepsilon^2 + c} - \frac{\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + c\sigma_\varepsilon^{-2}\left(\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}\right)^{-2}\frac{d\sigma_p^{-2}}{dc}}{\varphi_U\sigma_\varepsilon^2 + \frac{c\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}} \\
&= \frac{\varphi_U\sigma_\varepsilon^2 + \frac{c\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} - \left(\varphi_{DI}\sigma_\varepsilon^2 + c\right)\left\{\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + c\sigma_\varepsilon^{-2}\left(\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}\right)^{-2}\frac{d\sigma_p^{-2}}{dc}\right\}}{\left(\varphi_{DI}\sigma_\varepsilon^2 + c\right)\left\{\varphi_U\sigma_\varepsilon^2 + \frac{c\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\right\}} \\
&\geq \frac{\varphi_U\sigma_\varepsilon^2 - \left(\varphi_{DI}\sigma_\varepsilon^2 + c\right)\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}}{\left(\varphi_{DI}\sigma_\varepsilon^2 + c\right)\left\{\varphi_U\sigma_\varepsilon^2 + \frac{c\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\right\}},
\end{aligned}$$

which is always positive as long as $\varphi_U > \varphi_{DI}$. This implies that $\frac{V_U}{V_{DI}}$ increases with c.

We first consider the benchmark case of the basic model where $\sigma_{\omega U}^2 = 0$. Note that

Equation (25) still holds with V_U and V_{DI} defined differently as above. This leads to

$$K = \left(\frac{1}{\frac{V_U}{V_{DI}} \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + 1 + \frac{V_F}{V_{DI}}} \right)^2 \frac{\sigma_\theta^{-2}\sigma_\omega^2}{\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}.$$

As σ_p^{-2} decrease with c, it suffices to show that $T := \frac{V_U}{V_{DI}} \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + 1 + \frac{V_F}{V_{DI}}$ decreases with c as this would imply that K increases with c. By (new) definitions of V_U, V_{DI}, and V_F, we have

$$\begin{aligned} T &= \frac{\lambda_U}{\lambda_{DI}} \frac{\varphi_{DI}\sigma_\varepsilon^2 + c}{\varphi_U\sigma_\varepsilon^2 + \frac{c\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}} \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + 1 + \frac{V_F}{V_{DI}} \\ &= \frac{\lambda_U}{\lambda_{DI}} \frac{\varphi_{DI}\sigma_\varepsilon^2 + c}{\varphi_U\sigma_\varepsilon^2 \frac{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}} + c} + 1 + \frac{V_F}{V_{DI}}. \end{aligned}$$

Differentiating this with respect to c, we have

$$\begin{aligned} &\frac{d}{dc}\left(\frac{V_U}{V_{DI}} \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + 1 + \frac{V_F}{V_{DI}} \right) \\ = \;& \frac{\lambda_U}{\lambda_{DI}} \frac{d}{dc}\left(\frac{\varphi_{DI}\sigma_\varepsilon^2 + c}{\varphi_U\sigma_\varepsilon^2 \frac{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}} + c} \right) + \frac{d}{dc}\left(\frac{V_F}{V_{DI}}\right) \\ = \;& \frac{\lambda_U}{\lambda_{DI}} \frac{\varphi_U\sigma_\varepsilon^2 \frac{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}} + c - \left(\varphi_{DI}\sigma_\varepsilon^2 + c\right)\left(\varphi_U\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-2}\frac{d\sigma_p^{-2}}{dc} + 1\right)}{\left(\varphi_U\sigma_\varepsilon^2 \frac{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}} + c\right)^2} + \frac{d}{dc}\left(\frac{V_F}{V_{DI}}\right) \\ = \;& \frac{\lambda_U}{\lambda_{DI}} \frac{\left(\varphi_U - \varphi_{DI}\right)\sigma_\varepsilon^2 + \varphi_U\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1} - \left(\varphi_{DI}\sigma_\varepsilon^2 + c\right)\varphi_U\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-2}\frac{d\sigma_p^{-2}}{dc}}{\left(\varphi_U\sigma_\varepsilon^2 \frac{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}} + c\right)^2} + \frac{d}{dc}\left(\frac{V_F}{V_{DI}}\right). \end{aligned}$$

Note that

$$\frac{V_F}{V_{DI}} = \frac{\lambda_F}{\lambda_{DI}} \frac{\varphi_{DI}\sigma_\varepsilon^2 + c}{\varphi_F\sigma_\varepsilon^2 + c} = \frac{\lambda_F}{\lambda_{DI}} \left\{ 1 + \frac{\left(\varphi_{DI} - \varphi_F\right)\sigma_\varepsilon^2}{\varphi_F\sigma_\varepsilon^2 + c} \right\},$$

whose derivative with respect to c is given by

$$\frac{d}{dc}\left(\frac{V_F}{V_{DI}}\right) = -\frac{\lambda_F}{\lambda_{DI}} \frac{\left(\varphi_{DI} - \varphi_F\right)\sigma_\varepsilon^2}{\left(\varphi_F\sigma_\varepsilon^2 + c\right)^2},$$

and that

$$\begin{aligned}
\frac{d\sigma_p^{-2}}{dc} &= \frac{d}{dc}\left\{\left(1+\frac{V_F}{V_{DI}}\right)^2\sigma_\omega^{-2}\right\} = 2\sigma_\omega^{-2}\left(1+\frac{V_F}{V_{DI}}\right)\frac{d}{dc}\left(\frac{V_F}{V_{DI}}\right) \\
&= -2\sigma_\omega^{-2}\left(1+\frac{V_F}{V_{DI}}\right)\frac{\lambda_F}{\lambda_{DI}}\frac{(\varphi_{DI}-\varphi_F)\,\sigma_\varepsilon^2}{\left(\varphi_F\sigma_\varepsilon^2+c\right)^2}.
\end{aligned}$$

Applying these into the above derivative of $T := \frac{V_U}{V_{DI}}\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}$, we get

$$\begin{aligned}
\frac{d}{dc}T &= \frac{\lambda_U}{\lambda_{DI}}\frac{(\varphi_U-\varphi_{DI})\,\sigma_\varepsilon^2+\frac{\varphi_U}{\sigma_\theta^{-2}+\sigma_p^{-2}}+2\sigma_\omega^{-2}\frac{(\varphi_{DI}\sigma_\varepsilon^2+c)\varphi_U}{\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^2}\left\{1+\frac{\lambda_F}{\lambda_{DI}}+\frac{\lambda_F}{\lambda_{DI}}\frac{(\varphi_{DI}-\varphi_F)\sigma_\varepsilon^2}{\varphi_F\sigma_\varepsilon^2+c}\right\}\frac{\lambda_F}{\lambda_{DI}}\frac{(\varphi_{DI}-\varphi_F)\sigma_\varepsilon^2}{\left(\varphi_F\sigma_\varepsilon^2+c\right)^2}}{\left(\varphi_U\sigma_\varepsilon^2\frac{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}+c\right)^2} \\
&\quad -\frac{\lambda_F}{\lambda_{DI}}\frac{(\varphi_{DI}-\varphi_F)\,\sigma_\varepsilon^2}{\left(\varphi_F\sigma_\varepsilon^2+c\right)^2} \\
&= \frac{\lambda_U}{\lambda_{DI}}\frac{(\varphi_U-\varphi_{DI})\,\sigma_\varepsilon^2+\frac{\varphi_U}{\sigma_\theta^{-2}+\sigma_p^{-2}}}{\left(\varphi_U\sigma_\varepsilon^2\frac{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}+c\right)^2}+2\sigma_\omega^{-2}\frac{\lambda_U}{\lambda_{DI}}\frac{\left(\varphi_{DI}\sigma_\varepsilon^2+c\right)\varphi_U\left\{1+\frac{\lambda_F}{\lambda_{DI}}+\frac{\lambda_F}{\lambda_{DI}}\frac{(\varphi_{DI}-\varphi_F)\sigma_\varepsilon^2}{\varphi_F\sigma_\varepsilon^2+c}\right\}\frac{\lambda_F}{\lambda_{DI}}\,(\varphi_{DI}}{\left\{\varphi_U\sigma_\varepsilon^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}\right)+c\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\right\}^2(\varphi} \\
&\quad -\frac{\lambda_F}{\lambda_{DI}}\frac{(\varphi_{DI}-\varphi_F)\,\sigma_\varepsilon^2}{\left(\varphi_F\sigma_\varepsilon^2+c\right)^2}.
\end{aligned}$$

Given that $\sigma_p^{-2} \propto \left(\varphi_F\sigma_\varepsilon^2+c\right)^{-1}$ and that it goes to infinity as φ_F and c approach zero, the first term on the second line goes to a finite value as φ_F and c approach zero. Also, in the same limit, the second term is proportional to $\left(\varphi_F\sigma_\varepsilon^2+c\right)^{-1}$, whereas the third term is proportional to $\left(\varphi_F\sigma_\varepsilon^2+c\right)^{-2}$. Therefore, the third term dominates in the limit, which leads to $\frac{d}{dc}T<0$ as φ_F and c approach zero. As mentioned above, this implies that K increases with c with sufficiently small φ_F and c, as stated in the proposition.

Next, consider the general case of the basic model where $\sigma_{\omega U}^2>0$. Note that Equations (26) and (27) in the proof of Propositions 3.8 and 3.9 continue to hold with new definitions of V_U and V_{DI} as above, thereby determining the log-transformed welfare K as in Equation (26). The second term in Equation (26) is given by Equation (27), which yields

$$K_2 = \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}\left(\frac{V_{DI}+V_F}{V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\right)^2\sigma_{\omega U}^2$$

$$= \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}\left(\frac{1+\frac{V_F}{V_{DI}}}{\frac{V_U}{V_{DI}}\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+1+\frac{V_F}{V_{DI}}}\right)^2\sigma_{\omega U}^2,$$

where price informativeness is given by $\sigma_p^{-2}=\left(1+\frac{V_F}{V_{DI}}\right)^2\sigma_\omega^{-2}$. Note that σ_p^{-2} is a function of $\frac{V_F}{V_{DI}}$ and K_2 changes with $\frac{V_F}{V_{DI}}$ and $\frac{V_U}{V_{DI}}$, the latter of which works through changes in σ_p^{-2} as well. Throughout the proof of Proposition 3.9, given $\frac{V_U}{V_{DI}}$, we have shown that K_2 increases with $\frac{V_F}{V_{DI}}$ for every $\sigma_{\omega U}^2>0$ and that K in Equation (26) does so as well for sufficiently large $\sigma_{\omega U}^2$. On the other hand, both K in Equation (26) and its second term K_2 given by Equation (27) decrease with $\frac{V_U}{V_{DI}}$, given other variables. Combining these together, we establish that an increase in c leads to an increase in $\frac{V_U}{V_{DI}}$ and a decrease in $\frac{V_F}{V_{DI}}$, both of which lead to a decrease in K in Equation (26) for sufficiently large $\sigma_{\omega U}^2$. This leads to the statement regarding the limit of large $\sigma_{\omega U}^2$ in the proposition. Further, the same argument holds for large V_F, starting from Proposition 3 implying that K_2 in Equation (27) increases with $\frac{V_F}{V_{DI}}$ for every $\sigma_{\omega U}^2>0$ and that K in Equation (26) does so as well for sufficiently large V_F. This leads to the statement regarding the limit of large V_F in the proposition.

Last, we consider Case 1 of the extended model with market power. Denote by c_L the tax imposed on the large investor and by c that imposed on all other investors. We want to show that small values of c and/or c_L always decrease W compared with $c=c_L=0$. For $c>0$, we can see that Equation (39) continues to hold with new definitions of V_U and V_{DI} as above. Also, $H:=\frac{V_U}{V_{DI}+V_F}=\frac{\frac{V_U}{V_{DI}}}{1+\frac{V_F}{V_{DI}}}$ increases with small c as Equations (52) and (53) still hold in the extended model. By Claim 1 in the proof of Lemma 3.11, this implies that W decreases with small c. Now it remains to show that W decreases with small c_L. In

Case 1, given the conjectured price $p = \alpha\theta + \gamma x_L$, the large investor's profit is given by

$$\begin{aligned} u_L(x_L) &= x_L(\theta + \omega_L + \varepsilon - p) - \frac{1}{2}c_L x_L^2 \\ &= x_L(\theta + \omega_L + \varepsilon - \alpha\theta - \gamma x_L) - \frac{1}{2}c x_L^2 \\ &= x_L\{(1-\alpha)\theta + \omega_L + \varepsilon\} - \frac{1}{2}(c_L + 2\gamma)x_L^2. \end{aligned}$$

Using the first-order condition of his expected profit given θ, ω_L, and p, the large investor's optimal demand is given by

$$x_L^* = \frac{1}{2\gamma + c_L}\{(1-\alpha)\theta + \omega_L\}.$$

As a result, the equilibrium price is given by

$$\begin{aligned} p^* &= \alpha\theta + \gamma x_L^* = \alpha\theta + \frac{\gamma}{2\gamma + c_L}\{(1-\alpha)\theta + \omega_L\} \\ &= \frac{\alpha(c_L + \gamma) + \gamma}{c_L + 2\gamma}\theta + \frac{\gamma}{c_L + 2\gamma}\omega_L = \frac{\alpha(1+t)+1}{2+t}\left(\theta + \frac{\omega_L}{\alpha(1+t)+1}\right), \end{aligned}$$

where $t := \frac{c_L}{\gamma}$. In parallel with the proof of Lemma 3.11, we define $\sigma_p^2 := \left(\frac{1}{\alpha(1+t)+1}\right)^2 \sigma_{\omega L}^2$. Applying this into the standard Bayesian updating formula, we have

$$\begin{aligned} (E[\theta|\mathscr{I}_U] - p)^2 &= \left\{\sigma_p^{-2}\left(\sigma_\theta^{-2} + \sigma_p^{-2}\right)^{-1}\frac{2+t}{\alpha(1+t)+1} - 1\right\}^2 p^2 \\ &= \left(\sigma_p^{-2}\left(\sigma_\theta^{-2} + \sigma_p^{-2}\right)^{-1} - \frac{\alpha(1+t)+1}{2+t}\right)^2 \left(\sigma_\theta^{-2} + \sigma_p^{-2}\right)\sigma_\theta^2\sigma_p^2 X^2, \end{aligned}$$

where the second line is obtained by defining $X := \frac{1}{\sqrt{\sigma_\theta^2 + \sigma_p^2}}\frac{2}{1+\alpha}p \sim N(0,1)$ and then using

$$p^2 = \left(\frac{\alpha(1+t)+1}{2+t}\right)^2 \sigma_\theta^2\sigma_p^2\left(\sigma_\theta^{-2} + \sigma_p^{-2}\right)X^2.$$

Using Fact 1, this leads to

$$
\begin{aligned}
E[u_U|\mathscr{I}_U] &= -\exp\left(-\frac{(E[\theta|\mathscr{I}_U]-p)^2}{2Var[\theta+\varepsilon|\mathscr{I}_U]}\right) \\
&= -\exp\left(-\frac{\left(\sigma_p^{-2}\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}-\frac{\alpha(1+t)+1}{2+t}\right)^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\sigma_\theta^2\sigma_p^2X^2}{2\sigma_\varepsilon^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}\right) \\
&= -\exp\left(-\frac{\left(\sigma_p^{-2}-\frac{\alpha(1+t)+1}{2+t}\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\right)^2\sigma_\theta^2\sigma_p^2X^2}{2\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}\right).
\end{aligned}
$$

By $X\sim N(0,1)$ and Fact 2, this implies

$$
W = E[E[u_U|\mathscr{I}_U]] = -(1+K)^{-\frac{1}{2}},
$$

where

$$
\begin{aligned}
K &= \frac{\left(\sigma_p^{-2}-\frac{\alpha(1+t)+1}{2+t}\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\right)^2\sigma_\theta^2\sigma_p^2}{2\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)} \\
&= \frac{\left(\sigma_p^{-2}\frac{(1+t)(1-\alpha)}{2+t}-\frac{\alpha(1+t)+1}{2+t}\sigma_\theta^{-2}\right)^2\sigma_\theta^2\sigma_p^2}{2\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)} \\
&= \frac{\left((\alpha(1+t)+1)^2(1+t)(1-\alpha)\sigma_{\omega L}^{-2}-(\alpha(1+t)+1)\sigma_\theta^{-2}\right)^2\sigma_\theta^2(\alpha(1+t)+1)^{-2}\sigma_{\omega L}^2}{2\sigma_\varepsilon^2\left\{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+(\alpha(1+t)+1)^2\sigma_{\omega L}^{-2}\right\}(2+t)^2} \\
&= \frac{\left\{(\alpha(1+t)+1)(1+t)(1-\alpha)\sigma_{\omega L}^{-2}-\sigma_\theta^{-2}\right\}^2\sigma_\theta^2\sigma_{\omega L}^2}{2\sigma_\varepsilon^2\left\{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+(\alpha(1+t)+1)^2\sigma_{\omega L}^{-2}\right\}(2+t)^2}.
\end{aligned}
$$

Here, given that $(\alpha(1+t)+1)(1+t)(1-\alpha)\sigma_{\omega L}^{-2}-\sigma_\theta^{-2}<0$ for small $t\geq 0$ by Equation (39), we can see that small t leads to a decrease in K compared with $t=0$. This implies that small c_L leads to a decrease in W.

Proof of Lemma 3.14

The proof is generally in parallel with Lemma 3.2, which corresponds to a special case of the current lemma taking $\sigma_{\omega F}^2=0$, except for the first step to find an unbiased sufficient statistic $\tilde{p}$ for the asset value θ given uninformed investors' information set $\mathscr{I}_U = \{p, \omega_U\}$. We first take $\tilde{p} = \kappa_1 p + \kappa_2 \omega_U$ with coefficients κ_1 and κ_2. It is straightforward that unbiasedness implies $\kappa_1 = \frac{1}{\alpha}$. According to the Factorization Theorem, the main task here is to find κ_2 such that the conditional distribution of p and ω_U given the realization of $\tilde{p}$ and θ is independent of θ. Then such κ_2 leads to an unbiased sufficient statistic $\tilde{p} = \frac{1}{\alpha} p + \kappa_2 p$ for the asset value θ. Noting that given $\tilde{p}$, the conditional distribution of p is equivalent to that of ω_U, it suffices to find κ_2 such that the conditional distribution of ω_U given the realization of $\tilde{p}$ and θ is independent of θ. Given that ω_U is independent of θ and that the realization of $\tilde{p} = \theta + \frac{\beta}{\alpha}\omega + \left(\frac{\beta_U}{\alpha} + \kappa_2\right)\omega_U$ is equivalent to that of $\tilde{p} - \theta$ given the realization of θ, this is equivalent to κ_2 such that the conditional distribution of ω_U given the realization of $\tilde{p} - \theta$ is independent of θ. This is the case when ω_U is independent of $\tilde{p} - \theta$. To find such κ_2, we have

$$\begin{aligned}
Cov(\tilde{p} - \theta, \omega_U) &= Cov\left(\frac{\beta}{\alpha}\omega + \left(\frac{\beta_U}{\alpha} + \kappa_2\right)\omega_U + \frac{\beta_F}{\alpha}\omega_F, \omega_U\right) \\
&= \frac{\beta}{\alpha} Cov(\omega, \omega_U) + \left(\frac{\beta_U}{\alpha} + \kappa_2\right) Var(\omega_U) + \frac{\beta_F}{\alpha} Cov(\omega_F, \omega_U) \\
&= \left(\frac{\beta_U}{\alpha} + \kappa_2\right)\sigma^2_{\omega U} + \frac{\beta_F}{\alpha}\rho_{UF}\sqrt{\sigma^2_{\omega F}\sigma^2_{\omega U}}.
\end{aligned}$$

Taking the above covariance to be zero to use the Factorization Theorem, we get

$$\kappa_2 = -\frac{\beta_U}{\alpha} - \rho_{UF}\frac{\beta_F}{\alpha}\sqrt{\frac{\sigma^2_{\omega F}}{\sigma^2_{\omega U}}}.$$

Therefore, we get the following unbiased sufficient statistic for θ:

$$\tilde{p} := \frac{p}{\alpha} - \left(\frac{\beta_U}{\alpha} + \frac{\beta_F}{\alpha}\rho_{UF}\sqrt{\frac{\sigma^2_{\omega F}}{\sigma^2_{\omega U}}}\right)\omega_U.$$

Also, its variance conditional on θ is given by

$$\begin{aligned}
\sigma_p^2 &:= Var(\tilde{p}|\theta) \\
&= Var\left[\frac{p}{\alpha}-\left(\frac{\beta_U}{\alpha}+\frac{\beta_F}{\alpha}\rho_{UF}\sqrt{\frac{\sigma_{\omega F}^2}{\sigma_{\omega U}^2}}\right)\omega_U|\theta\right] \\
&= Var\left[\frac{\beta}{\alpha}\omega+\frac{\beta_F}{\alpha}\omega_F-\frac{\beta_F}{\alpha}\rho_{UF}\sqrt{\frac{\sigma_{\omega F}^2}{\sigma_{\omega U}^2}}\omega_U\right] \\
&= Var\left[\frac{\beta}{\alpha}\omega\right]+Var\left[\frac{\beta_F}{\alpha}\omega_F-\frac{\beta_F}{\alpha}\rho_{UF}\sqrt{\frac{\sigma_{\omega F}^2}{\sigma_{\omega U}^2}}\omega_U\right]+2\frac{\beta}{\alpha}\frac{\beta_F}{\alpha}Cov\left[\omega,\omega_F-\rho_{UF}\sqrt{\frac{\sigma_{\omega F}^2}{\sigma_{\omega U}^2}}\omega_U\right] \\
&= \left(\frac{\beta}{\alpha}\right)^2\sigma_\omega^2+\left(\frac{\beta_F}{\alpha}\right)^2(1-\rho_{UF})^2\sigma_{\omega F}^2+2\frac{\beta}{\alpha}\frac{\beta_F}{\alpha}\rho_F\sqrt{\sigma_\omega^2\sigma_{\omega F}^2},
\end{aligned}$$

where the third term on the last line is obtained by

$$Cov\left[\omega,\omega_F-\rho_{UF}\sqrt{\frac{\sigma_{\omega F}^2}{\sigma_{\omega U}^2}}\omega_U\right]=Cov(\omega,\omega_F)-Cov\left(\omega,\rho_{UF}\sqrt{\frac{\sigma_{\omega F}^2}{\sigma_{\omega U}^2}}\omega_U\right)=\rho_F\sqrt{\sigma_\omega^2\sigma_{\omega F}^2}.$$

By the standard Bayesian updating formula, this yields the below equations identical to Equation (20) in the proof of Lemma 1 except that $\tilde{p}$ and σ_p^{-2} are defined differently as above:

$$E[\theta|\mathscr{I}_U]=\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}\tilde{p} \text{ and } Var[\theta|\mathscr{I}_U]=\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}.$$

It is easy to see that $x_U=\frac{E[\theta|\mathscr{I}_U]+\omega_U-p}{\varphi_U Var[\theta+\varepsilon|\mathscr{I}_U]}$, $x_{DI}=\frac{\theta+\omega-p}{\varphi_{DI}\sigma_\varepsilon^2}$, and $x_F=\frac{\theta+\omega_F-p}{\varphi_F\sigma_\varepsilon^2+c}$ by the first-order conditions of these three types of investors' log-transformed expected utilities. Applying these to the market clearing condition together with Equation (20), we have

$$\frac{\lambda_U}{\varphi_U}\frac{\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}\left(\frac{p-\beta_U\omega_U}{\alpha}-\frac{\beta_F}{\alpha}\rho_{UF}\sqrt{\frac{\sigma_{\omega F}^2}{\sigma_{\omega U}^2}}\omega_U\right)+\omega_U-p}{\sigma_\varepsilon^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}\left(\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}\right)}+\frac{\lambda_{DI}}{\varphi_{DI}\sigma_\varepsilon^2}(\theta+\omega-p)+\frac{\lambda_F}{\varphi_F\sigma_\varepsilon^2+c}(\theta+\omega_F-p)=0.$$

Rearranging the terms, we have

$$\frac{\lambda_U}{\varphi_U}\frac{\left(\frac{\sigma_p^{-2}}{\alpha}-\sigma_\theta^{-2}-\sigma_p^{-2}\right)p-\frac{\sigma_p^{-2}}{\alpha}\left(\beta_U+\frac{\beta_F}{\alpha}\rho_{UF}\sqrt{\frac{\sigma_{\omega F}^2}{\sigma_{\omega U}^2}}\right)\omega_U+\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\omega_U}{\sigma_\varepsilon^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}\right)}$$
$$+\frac{\lambda_{DI}}{\varphi_{DI}\sigma_\varepsilon^2}(\theta+\omega-p)+\frac{\lambda_F}{\varphi_F\sigma_\varepsilon^2+c}(\theta+\omega_F-p)=0,$$

which is in parallel with Equation (21) in the proof of Lemma 3.2. Setting aside the terms multiplied by p, other terms must be proportional to p as follows:

$$\frac{\lambda_U}{\varphi_U}\frac{-\frac{\sigma_p^{-2}}{\alpha}\left(\beta_U+\beta\rho\sqrt{\frac{\sigma_\omega^2}{\sigma_{\omega U}^2}}+\frac{\beta_F}{\alpha}\rho_{UF}\sqrt{\frac{\sigma_{\omega F}^2}{\sigma_{\omega U}^2}}\right)\omega_U+\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)\omega_U}{\sigma_\varepsilon^2\left(\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}\right)}+\frac{\lambda_{DI}}{\varphi_{DI}\sigma_\varepsilon^2}(\theta+\omega)+\frac{\lambda_F}{\varphi_F\sigma_\varepsilon^2+c}(\theta+\omega_F$$

where C is a constant. Equalizing coefficients on both sides of the above equality yields

$$C=\frac{V_{DI}+V_F}{\alpha},\ \frac{\beta}{\alpha}=\frac{V_{DI}}{V_{DI}+V_F},\ \frac{\beta_F}{\alpha}=\frac{V_F}{V_{DI}+V_F},$$
$$\frac{\beta_U}{\alpha}=\frac{\frac{V_U}{V_{DI}+V_F}\frac{\sigma_\theta^{-2}+\sigma_p^{-2}-\frac{V_F}{V_{DI}+V_F}\sigma_p^{-2}\rho_{UF}\sqrt{\frac{\sigma_{\omega F}^2}{\sigma_{\omega U}^2}}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}}{1+\frac{V_U}{V_{DI}+V_F}\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}},$$

as stated in the lemma for $\sigma_{\omega U}^2>0$. In case where $\sigma_{\omega U}^2=0$, the β_U-term in the price p is zero, as stated in the lemma. Whether $\sigma_{\omega U}^2=0$ or $\sigma_{\omega U}^2>0$, note that all non-p terms in the above equation in parallel with Equation (21) are equal to $Cp=\frac{V_{DI}+V_F}{\alpha}p$. Thus, plugging Cp into all non-p terms in that equation, we have

$$\frac{V_U\left(\frac{\sigma_p^{-2}}{\alpha}-\sigma_\theta^{-2}-\sigma_p^{-2}\right)}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}p-(V_{DI}+V_F)p+\frac{V_{DI}+V_F}{\alpha}p=0,$$

which yields the same expression for α as in Lemma 3.2.

Overall, the relative ratios of coefficients $\frac{\beta}{\alpha}$, $\frac{\beta_U}{\alpha}$, and $\frac{\beta_F}{\alpha}$ are given by

$$\begin{aligned}
\frac{\beta}{\alpha} &= \frac{V_{DI}}{V_{DI}+V_F};\ \frac{\beta_F}{\alpha} = \frac{V_F}{V_{DI}+V_F};\\
\frac{\beta_U}{\alpha} &= \begin{cases} 0 & \text{if } \sigma_{\omega U}^2 = 0;\\ \frac{\frac{V_U}{V_{DI}+V_F}}{1+\frac{V_U}{V_{DI}+V_F}\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}} \frac{\sigma_\theta^{-2}+\sigma_p^{-2}-\frac{V_F}{V_{DI}+V_F}\sigma_p^{-2}\rho_{UF}\sqrt{\frac{\sigma_{\omega F}^2}{\sigma_{\omega U}^2}}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} & \text{if } \sigma_{\omega U}^2 > 0,\end{cases}
\end{aligned}$$

where $\sigma_p^{-2} := Var(\tilde{p}|\theta)^{-1}$ is given by

$$\sigma_p^{-2} = \left\{ \left(\frac{V_{DI}}{V_{DI}+V_F}\right)^2 \sigma_\omega^2 + \left(\frac{V_F}{V_{DI}+V_F}\right)^2 (1-\rho_{UF})^2 \sigma_{\omega F}^2 + \frac{2V_{DI}V_F\rho_F\sqrt{\sigma_\omega^2\sigma_{\omega F}^2}}{(V_{DI}+V_F)^2} \right\}^{-1}.$$

Given these ratios of coefficients $\frac{\beta}{\alpha}$, $\frac{\beta_U}{\alpha}$, and $\frac{\beta_F}{\alpha}$, coefficient α is given by

$$\alpha = \frac{V_U \frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}} + V_{DI} + V_F}{V_U \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}} + V_{DI} + V_F}.$$

These determine all coefficients α, β, β_U, and β_F in the equilibrium price.

Proof of Proposition 3.15

Note that

$$\tilde{p} := \frac{p-\beta_U\omega_U}{\alpha} - \frac{\beta_F}{\alpha}\rho_{UF}\sqrt{\frac{\sigma_{\omega F}^2}{\sigma_{\omega U}^2}}\omega_U = \frac{p-\beta_U\omega_U}{\alpha},$$

when $\sigma_\omega^2 = 0$ and $\rho_{UF} = 0$. Also, applying $\sigma_\omega^2 = 0$ and $\rho_{UF} = 0$ to Lemma 3.14 yields

$$\sigma_p^2 = \left(\frac{V_F}{V_{DI}+V_F}\right)^2 \sigma_{\omega F}^2. \tag{48}$$

In parallel with Equation (41), we have

$$
\begin{aligned}
E[\theta|\mathscr{I}_U]+\omega_U-p &= \frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}\tilde{p}-p+\omega_U \\
&= \frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}\left(\theta+\frac{\beta_F}{\alpha}\omega_F\right)-\alpha\theta-\beta\omega-\beta_U\omega_U-\beta_F\omega_F+\omega_U \\
&= \left(\frac{1}{\alpha}\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}-1\right)(\alpha\theta+\beta_F\omega_F)+(1-\beta_U)\omega_U \\
&= \left(\frac{1}{\alpha}\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}-1\right)(\alpha\theta+\beta_F\omega_F)+\left(1-\frac{\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\alpha}{1+\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\frac{1}{V_{DI}+V_F}}\right)\omega_U,
\end{aligned}
$$

where the last line is obtained by applying $\sigma_\omega^2=0$ and $\rho_{UF}=0$ to the ratio of coefficients $\frac{\beta_U}{\alpha}$ from Lemma 3.14. Using coefficient α obtained from Lemma 3.14, we have the same equation as in Equation (24), except that σ_p^{-2} is defined differently:

$$
\frac{1}{\alpha}\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}-1=-\frac{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}(V_{DI}+V_F)}{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}.
$$

This leads to

$$
\begin{aligned}
& E[\theta|\mathscr{I}_U]+\omega_U-p \\
= \; & -\frac{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}(V_{DI}+V_F)}{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}(\alpha\theta+\beta_F\omega_F)+\left(1-\frac{\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\alpha}{1+\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\frac{1}{V_{DI}+V_F}}\right)\omega_U \qquad (49)
\end{aligned}
$$

where σ_p^{-2} is given by Equation (54).

We first consider the case where $\sigma_{\omega U}^2=0$. In this case, Equation (55) yields

$$
E[\theta|\mathscr{I}_U]+\omega_U-p=-\frac{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}(V_{DI}+V_F)}{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}(\alpha\theta+\beta_F\omega_F).
$$

Noting that uninformed investors' welfare W increases with $K:=\frac{Var(E[\theta|\mathscr{I}_U]+\omega_U-p)}{Var[\theta+\varepsilon|\mathscr{I}_U]}$ by

Equation (22), we use the above expression to get

$$
\begin{aligned}
K &= \frac{Var\left(E[\theta|\mathscr{I}_U]+\omega_U-p\right)}{Var[\theta+\varepsilon|\mathscr{I}_U]} = \frac{Var\left(-\frac{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}\left(V_{DI}+V_F\right)}{\frac{\lambda_U}{\varphi_U\sigma_\varepsilon^2}\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\left(\alpha\theta+\beta_F\omega_F\right)\right)}{\sigma_\varepsilon^2+\left(\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-1}} \\
&= \left(\frac{1}{V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\right)^2 \frac{\left(V_{DI}+V_F\right)^2\sigma_\theta^{-2}\sigma_p^2}{\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}, \qquad (50)
\end{aligned}
$$

which corresponds to Equation (25) in the proof of Propositions 3.7. 3.8, and 3.9, except that σ_p^2 is defined differently by Equation (54) here. Combined with Equation (54), this leads to

$$
K = \left(\frac{1}{V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\right)^2 \frac{V_F^2\sigma_\theta^{-2}\sigma_{\omega F}^2}{\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}.
$$

We can easily see that K increases with V_F, implying that reducing c increases W, as stated in the proposition.

Next, we consider the case where $\sigma_{\omega U}^2$ is large. Using Equation (55) and the fact that ω_U is independent of θ and ω_F, we have

$$
\begin{aligned}
Var\left(E[\theta|\mathscr{I}_U]+\omega_U-p\right) &= Var\left\{\frac{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}\left(V_{DI}+V_F\right)\left(\alpha\theta+\beta_F\omega_F\right)}{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F} + \left(1-\frac{\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\alpha}{1+\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\frac{1}{V_{DI}+V_F}}\right) \right. \\
&= Var\left\{\frac{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}\left(V_{DI}+V_F\right)\left(\alpha\theta+\beta_F\omega_F\right)}{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\right\} + \left(1-\frac{\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\alpha}{1+\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\frac{1}{V_{DI}+V_F}} \right.
\end{aligned}
$$

Then it follows

$$
\begin{aligned}
K &:= \frac{Var\left(E[\theta|\mathscr{I}_U]+\omega_U-p\right)}{Var[\theta+\varepsilon|\mathscr{I}_U]} \\
&= \frac{1}{Var[\theta+\varepsilon|\mathscr{I}_U]}\left[Var\left\{\frac{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}}\left(V_{DI}+V_F\right)\left(\alpha\theta+\beta_F\omega_F\right)}{V_U\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\right\} + \left(1-\frac{\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\alpha}{1+\frac{\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}\frac{1}{V_{DI}+V_F}}\right)^2 \sigma \right.
\end{aligned}
$$

Note that the first term corresponds to Equation (56) in the case where $\sigma_{\omega U}^2 = 0$ and it increases with V_F, which is equivalent to a reduction in c. On the other hand, the second term in Equation (56) is identical to Equation (27), except that σ_p^{-2} is defined differently by Equation (36), as below:

$$K_2 \quad := \quad \frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^2\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)}\left(\frac{V_{DI}+V_F}{V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F}\right)^2\sigma_{\omega U}^2. \tag{52}$$

The symmetric argument as in the proof of Proposition 3.9 holds as follows: (i) If K_2 above decreases with V_F, then K in Equation (57) decreases with V_F, which is equivalent to a reduction in c, if and only if $\sigma_{\omega U}^2$ is sufficiently large. Therefore, W does so as well if and only if $\sigma_{\omega U}^2$ is sufficiently large. (ii) If K_2 above increases with V_F, then K in Equation (57) increases with V_F for every $\sigma_{\omega U}^2 > 0$. Therefore, W does so as well for every $\sigma_{\omega U}^2 > 0$.

Overall, it suffices to identify the condition under which K_2 above as a function of V_F (given by Equation (58) and $\sigma_p^{-2} = \left(\frac{V_F}{V_{DI}+V_F}\right)^2\sigma_{\omega F}^2$) decreases with V_F. This provides a necessary and sufficient condition for K in Equation (57) to decrease with a reduction in c for sufficiently large $\sigma_{\omega U}^2$. Differentiating K_2 mentioned above with respect to V_F, we have the same equation as in Equation (28), except that σ_p^{-2} is defined differently by Equation (54) as below:

$$\begin{aligned}
\frac{d}{dV_F}\ln K_2 \quad &= \quad \frac{d}{dV_F}\left(\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\right)\frac{V_{DI}+V_F-V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}}{\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\left(V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F\right)} \\
&\quad +\frac{2V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}}{(V_{DI}+V_F)\left(V_U\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\theta^{-2}+\sigma_p^{-2}+\sigma_\varepsilon^{-2}}+V_{DI}+V_F\right)}.
\end{aligned}$$

Note that

$$\begin{aligned}
\frac{d}{dV_F}\left(\frac{\sigma_\theta^{-2}+\sigma_p^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\right) \quad &= \quad \frac{d}{dV_F}\left(1-\frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}}\right) \\
&= \quad -\sigma_\varepsilon^{-2}\left(\sigma_\varepsilon^{-2}+\sigma_\theta^{-2}+\sigma_p^{-2}\right)^{-2}\frac{2\left(V_{DI}+V_F\right)V_{DI}}{V_F^3}\sigma_{\omega F}^{-2}
\end{aligned} \tag{53}$$

where the second line is obtained by differentiating $\sigma_p^{-2} = \left(\frac{V_F}{V_{DI}+V_F}\right)^{-2} \sigma_{\omega F}^{-2}$ with respect to V_F. Plugging Equation (59) into the above equation identical to Equation (28) and then arranging the terms, we have $\frac{d}{dV_F} \ln K_2 < 0$ if and only if

$$-\sigma_\varepsilon^{-2} \left(\sigma_\varepsilon^{-2} + \sigma_\theta^{-2} + \sigma_p^{-2}\right)^{-2} \frac{2\left(V_{DI} + V_F\right) V_{DI}}{V_F^3} \sigma_{\omega F}^{-2} \left(V_{DI} + V_F - V_U \frac{\sigma_\theta^{-2} + \sigma_p^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\theta^{-2} + \sigma_p^{-2}}\right)$$
$$+ \frac{2V_U}{V_{DI} + V_F} \left(\frac{\sigma_\theta^{-2} + \sigma_p^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\theta^{-2} + \sigma_p^{-2}}\right)^2 < 0,$$

which is equivalent to

$$\left(\sigma_\theta^{-2} + \sigma_p^{-2}\right)^2 < \sigma_\varepsilon^{-2} \sigma_p^{-2} \left\{ \frac{V_{DI}}{V_U} \left(\frac{V_{DI}}{V_F} + 1\right) - \frac{V_{DI}}{V_F} \frac{\sigma_\theta^{-2} + \sigma_p^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\theta^{-2} + \sigma_p^{-2}} \right\},$$

as stated in the proposition.

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