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Liquidity, the Mundell–Tobin Effect, and the Friedman Rule

We investigate how a positive relation between inflation and capital investment (the Mundell–Tobin effect, MT-E) affects optimal monetary policy in a framework that combines overlapping generations and new Monetarist models. We find that inflation rates above the Friedman rule are optimal if and only if there is an MT-E. In the absence of the MT-E, the Friedman rule is optimal. With an MT-E, increasing inflation above the Friedman rule leads to a first-order welfare gain from increasing capital investment, and only to a second-order welfare loss from reducing consumption in markets where liquidity matters.

JEL codes: E4, E5
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The Friedman rule for the optimal conduct of monetary policy is the most significant doctrine in monetary theory. Friedman (1969) argues that optimal monetary policy should equate the private opportunity costs of holding money (the nominal interest rate) to its social costs (which are zero). By this logic, the optimal monetary policy should be deflationary. This result is remarkably robust. It has been found by Friedman himself in a model with money in the utility (Friedman 1969), but also in a variety of other monetary models such as cash-in-advance

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(Grandmont and Younes 1973, Lucas and Stokey 1987), spatial separation (Townsend 1980), and New Monetarism (Lagos and Wright 2005). However, most central banks in developed economies follow an annual inflation target of around 2% and try to avoid deflation. Thus, the debate about the optimality of the Friedman rule has continued ever since Friedman proposed it, with several explanations for optimal inflation rates above the Friedman rule being brought forward.[1]

In this paper, we study optimal monetary policy in the presence of a Mundell–Tobin effect (MT-E; Mundell (1963) and Tobin (1965)). The MT-E postulates that inflation increases investment, the idea being that since inflation reduces the return on nominal assets, investment in capital becomes relatively more attractive and agents substitute away from nominal assets into capital. Thus if agents underinvest in capital at the Friedman rule, higher inflation might increase welfare through an increase in investment. We study this argument in an overlapping generations (OLG) framework that also incorporates a market where money is useful to trade in the spirit of Lagos and Wright (2005) (LW). The model features a basic liquidity-return trade-off: Agents can hold nominal (money) or real (capital) assets, with money being more liquid than capital, but capital being productive and thus more efficient to provide for old-age consumption. The model combines two usually separated roles of money. It is both used in intergenerational trade between young and old agents but also in intragenerational trade between agents of the same generation. In this environment, there may or may not be an MT-E, depending on the preferences of agents, the implementation of monetary policy, and the liquidity of capital. This allows us to study more carefully how the presence of an MT-E affects the optimality of the Friedman rule.[2]

We find that inflation rates above the Friedman rule are optimal if and only if there is an MT-E at the Friedman rule. In this case, increasing inflation above the Friedman rule leads to a first-order welfare gain from increasing capital investment, and only to a second-order welfare loss from reducing consumption resulting from intragenerational trade, and the optimal inflation rate lies between the Friedman rule and zero. Without an MT-E, this mechanism is not at play and the Friedman rule is optimal. We also show that an MT-E is more likely to occur at the Friedman rule if agents' demand for money is elastic and if capital is liquid.

From the policymaker's point of view, the fundamental trade-off in our model is that the Friedman rule delivers efficiency in intragenerational trade, but a constant price level is optimal regarding intergenerational trade. The reason for the latter point

1. Explanations for optimal inflation rates above the Friedman rule include: incomplete taxation (Aruoba and Chugh 2010, Finocchiaro et al. 2018); theft or socially undesirable activities financed by cash (Sanches and Williamson 2010, Williamson 2012); labor market frictions (Carlsson and Westermark 2016); or pecuniary exernalities (Brunnermeier and Sannikov 2016). In New Keynesian models with sticky prices, a constant price level is typically optimal as this eliminates inefficiencies from price adjustment costs. See Schmitt-Grohé and Uribe (2010) or Fuerst (2010) for an overview on the literature about optimal inflation rates.

2. We explain the relation of our approach to the existing literature in more detail in the literature review below.

is that there are two ways to provide for old-age consumption: accumulation of capital when young, or transfers from young to old agents. The inherent return of such transfers is equal to the population growth rate, which we normalize to one, while the return on capital is larger than that if capital investment is socially efficient; thus, the planner prefers capital accumulation for old-age consumption. However, if some intragenerational trade can only be settled with money, some old-age consumption must be financed through transfers from young to old agents as the money stock must be passed from old to young agents. Setting the gross inflation rate equal to one reflects the social return of using money (and thus intergenerational transfers) to acquire old-age consumption, but it inefficiently lowers intragenerational trade. Thus, there is no single inflation rate that allows for efficiency regarding both roles of money, and instead the inflation rate depends on how monetary policy is implemented, the liquidity of capital, and how elastic the agent's demand for money is.

Model summary

In our model, each period is divided into two subperiods, called the centralized market (CM) and the decentralized market (DM). Agents are born at the beginning of the CM and live until the end of the CM of the following period; that is, they are alive for three subperiods.[3] There are two assets in the economy, productive capital and fiat money. In the main body of the paper, we assume a linear return to capital that is high enough to make capital investment socially efficient.[4] When agents are born, they immediately learn whether they will be a buyer or a seller during the DM. During the first CM of their lives, all agents can work at linear disutility and accumulate capital and fiat money. In the DM, sellers can work at linear disutility and produce a DM good. Buyers cannot work during the DM, but they get concave utility from consuming the DM good. With some probability, buyers are relocated during the DM. If they are relocated, they can only use fiat money to settle trades, because we assume that capital is immobile and cannot be moved to different locations. If buyers are not relocated, they can use money and capital to purchase goods from sellers. Sellers are never relocated. During the final CM of their lives, buyers return to their original location and have access to all their remaining assets. Both buyers and sellers receive concave utility from consuming during the final CM of their lives. Monetary policy is implemented by paying transfers to either young or old agents. The relocation shock creates a basic liquidity/return trade-off between money and capital. Since capital pays a weakly higher return than money, it is better suited as a store of value. However, because buyers can use money in the DM even if they are

3. It is not crucial for our results that agents live for only three subperiods, but this is the easiest way of integrating OLG aspects into LW while keeping the model tractable.

4. In Appendix A, we also consider a case with linear return but socially inefficient capital investment, and in online Appendix D, we consider a case with concave returns to capital. We show that if capital is socially inefficient, a constant money stock is optimal, and that our main results go through if returns to capital are concave.

relocated, money is more liquid than capital, and the liquidity of capital decreases in the probability of relocation. As mentioned above, this setup blends standard OLG and LW frameworks—as in standard LW models, buyers and sellers trade with each other during the DM, and as in standard OLG models, young and old agents trade with each other during the CM. Using this setup allows us to identify the different roles of money in inter- and intragenerational trade and to study how this affects capital accumulation and optimal monetary policy.

We first study a benchmark case where no buyers are relocated, implying that capital is perfectly liquid. In this case, there is no trade-off between money and capital, so money is not valued and households only hold capital. Then, hours worked, DM and CM consumption are all at their first-best levels. Our main analysis then covers the case where all buyers are relocated, meaning that capital is perfectly illiquid.[5] In this case, CM consumption levels are independent of monetary policy and at the first-best level, and running the Friedman rule allows to implement the first-best consumption levels in the DM, but keeps the level of capital accumulation strictly below first best. We identify two channels through which inflation affects capital accumulation: on the one hand, sellers are aware that they can sell less goods in the DM at higher inflation rates, so they accumulate more capital at higher inflation rates to provide for their CM consumption (*seller channel*); on the other hand, buyers' CM consumption is financed through returns on capital minus the tax. Since buyers always consume the first-best quantity in the CM, their capital accumulation varies with the tax. In particular, buyers hold less capital for CM consumption and tax payments if the real tax payment decreases, which it does when inflation increases (*transfer channel*). The transfer channel is only active when old agents are taxed, so there is always an MT-E when monetary policy is implemented by taxing the young agents, and it turns out that a constant money stock is optimal in this case. For any deflationary policy, the welfare loss from the reduction in capital accumulation is larger than the gains from increasing the consumption levels in the DM. If old agents are taxed instead, whether or not there is an MT-E depends on the relative strength of the two channels, which is determined by the agents' preferences. If the elasticity of DM consumption is above one, there is an MT-E, while if it is below one, there is a reverse MT-E. From this, it follows that the Friedman rule is optimal if the old are taxed and the elasticity of DM consumption is below one; if the elasticity is above one, the optimal money growth rate lies somewhere between the Friedman rule and one, and it is an increasing function of the elasticity of DM consumption in that interval. We also show that for any deflationary policy, welfare is higher if monetary policy is implemented over old buyers only. This is because a deflationary policy implies levying a tax on buyers; if the tax can be paid by old buyers, parts of it can be financed through returns on capital, which is welfare-improving relative to financing it through labor income.

5. In online Appendix C, we consider the case with random relocation, and show that all our main results go through.

Existing literature. Aruoba and Wright (2003) include capital in a LW framework and find that capital accumulation is independent of monetary policy if capital is fully illiquid. In our model, this is not true: because the OLG framework allows us to drop quasi-linear preferences, capital accumulation is affected by monetary policy even with fully illiquid capital. Lagos and Rocheteau (2008) show that the MT-E exists in LW models when capital is liquid. However, the Friedman rule still delivers the first-best outcome in their model. In Aruoba et al. (2011), capital is both a liquid asset and an input in DM production. They show that capital accumulation is affected more strongly by inflation if there is price-taking in the DM. Andolfatto et al. (2016) show that if taxes cannot be enforced and therefore the Friedman rule is not feasible, the first-best allocation can be implemented with a cleverly designed mechanism even if the capital stock is too small. In Wright et al. (2018), ,2020), the authors study models where capital is traded in frictional markets, and they show that if money is needed to purchase capital, a reverse MT-E may occur. In our model, a reverse MT-E may also occur for some parameters, but due to preferences, not frictional markets. Gomis-Porqueras et al. (2020) show that there is a hump-shaped relationship between inflation and aggregate capital, as inflation affects capital accumulation negatively on the extensive margin by reducing the number of firms, besides the usual positive effect on the intensive margin. Probably the paper most closely related to ours is Matsuoka and Watanabe (2019), who build on Williamson (2012). They also study an economy with relocation, money, and capital, but agents live forever and CM utility is quasi-linear. In contrast to our model, they find that the Friedman rule is always optimal even if there is an MT-E. In Matsuoka and Watanabe (2019), it is never efficient to finance CM consumption with capital. Thus if the monetary authority runs the Friedman rule it can always achieve efficiency without capital investments. In our model, it is socially efficient to finance CM consumption with capital investments. Thus, with an MT-E, there is a welfare cost of running the Friedman rule due to lower socially useful capital investments. This justifies optimal deviations from the Friedman rule in our model but not in theirs. A few papers in the LW literature find optimal deviations from the Friedman rule due to the MT-E—for example, Venkateswaran and Wright (2013), Geromichalos and Herrenbrueck (2022), Wright et al. (2018), or Altermatt (Forthcoming). These papers usually exhibit frictions that lead to underinvestment at the Friedman rule (e.g., limited pledgeability, taxes, or wage bargaining), but do not affect the presence of an MT-E itself. If the frictions are shut down, the MT-E still exists, but the Friedman rule becomes optimal.

In the OLG literature following Wallace (1980), Azariadis and Smith (1996) show that if there is private information about an agent's type, an MT-E exists for low levels of inflation, while a reverse MT-E exists for high levels of inflation. In models with relocation shocks, Smith (2002), ,2003) and Schreft and Smith (2002) have claimed to show that the Friedman rule is suboptimal because of the MT-E.[6] However, OLG

6. See also Schreft and Smith (1997), which focuses on positive inflation rates, but endogenizes the return on capital.

models typically find deviations from the Friedman rule to be optimal even without the MT-E, as in Weiss (1980), Abel (1987), or Freeman (1993).[7] Bhattacharya et al. (2005) and Haslag and Martin (2007) build on these results to show that for optimal deviations from the Friedman rule found in Smith (2002) and the other papers mentioned, the MT-E is not necessary. Instead, standard properties of OLG models, in particular the presence of intergenerational transfers, are sufficient to find these deviations to be optimal. The debate whether the MT-E itself can render deviations from the Friedman rule optimal in an OLG environment thus remained unsettled.[8] By transferring the basic OLG structure into an LW model, we can study these issues more carefully. The combined model gives us more flexibility by introducing additional heterogeneity with buyers and sellers (besides young and old generations) and an additional market (DM). As a result, we may or may not observe an MT-E at the Friedman rule (depending on preferences and the liquidity of capital) and we are able to investigate whether the MT-E is a necessary condition for the suboptimality of the Friedman rule. Within this framework, we can show that if an MT-E arises, deviations from the Friedman rule are always optimal. The richer model also allows us to investigate the different channels through which inflation affects capital investments, and we can separate liquidity and store of value properties of money and capital.

A combination of OLG and LW structures has first been studied by Zhu (2008). In his model, agents do not know their type during the first CM when they are able to accumulate assets. Therefore, the Friedman rule can be suboptimal for some parameters, as it makes saving relatively cheap and reduces the sellers' willingness to produce in the DM. In contrast to this, our model follows Altermatt (2019) by assuming that each agent knows their type. Hiraguchi (2017) extends the model of Zhu (2008) by including capital and shows that the Friedman rule remains suboptimal in this case. In another recent paper that combines OLG and LW, Huber and Kim (2020) show that the Friedman rule can be suboptimal if old agents face a higher disutility of labor than young agents. We replicate this result in the case of socially inefficient capital investment which we discuss in Appendix A.

We think that our paper contributes to the existing literature in a number of important ways. First, it is able to reconcile Smith (2002) with Bhattacharya et al. (2005) and Haslag and Martin (2007), by showing that even in a model with an OLG structure, the Friedman rule can be optimal for some parameters, but that it is never optimal when there is an MT-E. This supports the claims by Smith (2002), ,2003) that the MT-E can be the source of optimal deviations from the Friedman rule. Second, our paper shows that in an LW model with intergenerational aspects and strictly

7. Matsuoka (2011) shows that the Friedman rule becomes optimal again with a monopolistic banking sector.

8. There is a further complication in the welfare analysis of OLG models due to the absence of a representative agent. Freeman (1993) shows that the Friedman rule is typically Pareto optimal, but not maximizing steady-state utility in OLG models. In this paper, we are going to focus on steady-state optimality, but we also discuss Pareto optimality in online Appendix E.

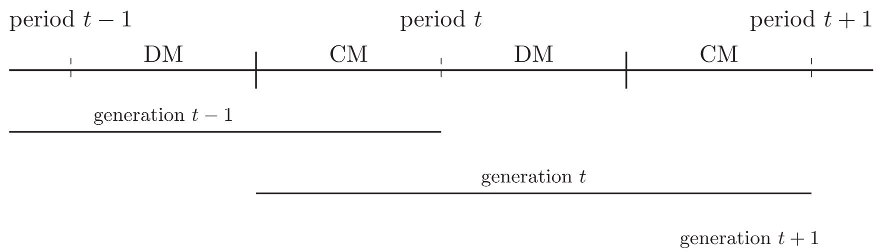

Fig 1. Timeline with Lifespans of Generations.

concave CM utility, capital accumulation is affected by monetary policy even if capital is illiquid, and capital accumulation can be inefficiently low at the Friedman rule due to the MT-E. Third, we made some advances in understanding the frictions that arise in a model that combines OLG and LW, most importantly by showing that the timing of monetary policy implementation matters for welfare in these models.

Outline. The rest of this paper is organized as follows. In Section 1, the environment and the planner's solution is explained. In Section 2, we present the market outcome for perfectly liquid capital, and in Section 3, we discuss the market outcome for perfectly illiquid capital, monetary policy implementation, and the main results of the paper. Finally, Section 4 concludes.

1. THE MODEL

Our model combines the LW (2005) environment and an OLG structure with relocation shocks such as Townsend (1987) or Smith (2002). Time is discrete and continues forever. Each period is divided into two subperiods: First, the CM takes place, followed by the DM. There are two distinct locations, which we sometimes call islands. The two locations are completely symmetric, and everything described happens simultaneously on both islands.[9] A new generation of agents is born at the beginning of each period, consisting of a unit mass of buyers and a unit mass of sellers on each island. An agent born in period t lives until the end of the CM in period $t+1$. In the CM of period 0, there is also a unit mass of sellers called the "initial old" on each island. Figure 1 gives an overview of the sequence of subperiods and the lifespans of generations. There is also a single monetary authority in charge of monetary policy on both islands.

Both buyers and sellers are able to produce a general good x during the first CM of their life at linear disutility h, whereas incurring disutility h yields h units of x; during the second CM of their life, buyers and sellers both receive utility from consuming x.

9. To simplify notation, we state market-clearing and other conditions in terms of a single island throughout the paper, unless otherwise stated.

During the DM, sellers can produce special goods q at linear disutility; buyers receive utility from consuming q. A fraction π of buyers are relocated during the DM, meaning that they are transferred to the other island without the ability to communicate with their previous location. Sellers are not relocated. During the CM, relocated buyers return to their original location for the final CM of their life.[10] Relocation occurs randomly, so for an individual buyer, the probability of being relocated is π. Buyers learn at the beginning of the DM whether they are relocated. Buyers' preferences are

$$-h_t^b + \pi\big[u(q_t^m) + \beta U(x_{t+1}^m)\big] + (1-\pi)\big[u(q_t^b) + \beta U(x_{t+1}^b)\big], \tag{1}$$

where superscript m denotes consumption of relocated buyers (movers). In the main body of the paper, we focus on the corner cases $\pi = \{0, 1\}$, that is, either all buyers or nobody is relocated, so that there is no uncertainty about relocation. In online Appendix C, we discuss an economy with $\pi \in (0, 1)$. Buyers discount the second period of their life by $\beta \in (0, 1)$, get linear disutility h from producing x during the CM when young, and gain utility $u(q)$ from consuming q in the DM and $U(x)$ from consuming x in the CM when old, with $U(0) = u(0) = 0, u'(q) > 0 > u''(q), U'(x) > 0 > U''(x), U'(0) = u'(0) = \infty$, and $\bar{q} = u(\bar{q})$ for some $\bar{q} > 0$.[11] Sellers' preferences are

$$-h_t^s - q_t^s + \beta U(x_{t+1}^s). \tag{2}$$

Sellers also discount the second period of their life by β and get utility $U(x)$ from consuming in the CM. They get linear disutility from producing in the CM and the DM.

The monetary authority issues a stock of fiat money M_t per island, which it can produce without cost. ϕ_t denotes the value of M_t in terms of x_t, which implies gross inflation is ϕ_t/ϕ_{t+1}. The growth rate of M_t is $\gamma = M_t/M_{t-1}$. Newly printed fiat money is distributed to young or old buyers via lump-sum transfers (or lump-sum taxes if $\gamma < 1$).[12] We denote transfers to young buyers as τ^y, and transfers to old buyers as τ^o. We use the indicator variable $\mathcal{I}$ to denote which generation receives transfers. If $\mathcal{I} = 1$ ($\mathcal{I} = 0$), young buyers (old buyers) get transfers, which means the monetary authority sets τ^y (τ^o) such that the money growth rate γ can be implemented, while $\tau^o = 0$ ($\tau^y = 0$). In period 0, the monetary authority issues M_0 to initially old sellers.

10. In Smith (2002), agents live for two periods and relocation occurs during the second period, meaning that all assets they cannot spend during that period are wasted from their point of view. Our model crucially differs from Smith (2002) in that regard, as our agents have access to all their assets during the final period of their life.

11. In online Appendix C, we also assume strictly convex marginal utility in the CM, that is, $U'''(x) > 0$, to simplify some proofs. Most commonly used utility functions satisfy this assumption.

12. As we will show in this paper, the exact timing of the lump-sum taxes is irrelevant for consumption allocations, but not for welfare. Assuming that only buyers are taxed is without loss of generality.

During the CM, agents can also invest general goods into capital. We assume a linear return on capital, so one unit of k delivers R goods in the CM of the following period and fully depreciates after production.[13] In contrast to money, capital is immobile, that is, it is impossible to move it to other locations during the DM. Furthermore, agents cannot verify claims on capital.

1.1 Planner's Problem

We focus on maximizing expected steady-state welfare of a representative generation.[14] We denote total CM labor as H and total capital as K, and we impose that $H \geq K$, that is, that steady-state consumption cannot be financed through a preexisting capital stock. Then, the planner's problem is

$$\max_{H,K,q^b,q^m,q^s,x^b,x^m,x^s} -H - q^s + \pi\left[u(q^m) + \beta U(x^m)\right] + (1-\pi)\left[u(q^b) + \beta U(x^b)\right] + \beta U(x^s) \tag{3}$$

$$\text{s.t.} \qquad \pi q^m + (1-\pi)q^b = q^s, \tag{4}$$

$$H \geq K, \tag{5}$$

$$\pi x^m + (1-\pi)x^b + x^s + K = RK + H, \tag{6}$$

where the first and third constraints are the resource constraints in the DM and CM, respectively. It is easy to see that first-best level of DM consumption q^* is given by

$$q^b = q^m = q^s = q^* \quad \text{solving} \quad u'(q^*) = 1. \tag{7}$$

To finance CM consumption, the planner has two options: Either young agents work for the old and CM consumption is financed by transfers, or young agents work to invest in capital and consume the returns when old. Denoting λ as the Lagrange multiplier for (5), the first-order conditions (FOCs) for CM consumption and capital are

$$x^b = x^m = x^s = x^* \quad \text{solving} \quad \beta U'(x^*) = 1 - \lambda$$
$$R - 1 \leq R\lambda \text{ with equality if } \quad K > 0.$$

13. The capital in our model can also be interpreted as a storage technology. In online Appendix D, we consider a more general production function with endogenous R and show that our main results go through.

14. Steady-state welfare places an equal weight on the utility of each generation. Since there is an infinite number of future generations but only a single initial old generation, this welfare objective asymptotically ignores the welfare of the initial old. For this and other reasons, this welfare measure is commonly used in the OLG literature, including in the papers we want to compare our results to, such as Smith (2002) and Haslag and Martin (2007). We also believe that focusing on steady-state welfare allows us to compare our results more easily to papers with infinitely lived agents, such as Lagos and Rocheteau (2008). To complete our analysis, we discuss Pareto-welfare and the effects of policy changes on the agents who are old at the time of the policy change in online Appendix E.

Suppose first (5) binds, which happens if $R > 1$. Then, the planner wants to use capital, so agents only work to build up the capital stock. x^*, H^*, and K^* are

$$R\beta U'(x^*) = 1, \tag{8}$$

$$H^* = K^* = \frac{2x^*}{R}. \tag{9}$$

If instead $R \leq 1$, the planner only uses intergenerational transfers and capital investment is zero. H^* and x^* are then given by

$$\beta U'(x^*) = 1, \tag{10}$$

$$H^* = 2x^*. \tag{11}$$

To have a meaningful trade-off between money and capital we assume in the following that CM consumption should be financed with capital, that is, that investing in capital is socially efficient and more specifically that

$$R\beta = 1, \tag{12}$$

which ensures $R > 1$ and has the added benefit of simplifying the definition of the Friedman rule, as we will discuss in a moment.[15]

Before moving on, let us introduce two important concepts. First, we define the Friedman Rule (FR) as the inflation rate for which the opportunity cost of holding money is zero. This is true for $\phi/\phi_{+1} = 1/R$.[16] Second, we define an MT-E as a positive relation between capital and inflation, that is, as $\partial K/\partial(\phi/\phi_{+1}) > 0$, and a reverse MT-E as $\partial K/\partial(\phi/\phi_{+1}) < 0$.

1.2 Market Outcomes

In the DM, q is sold in competitive manner.[17] As is standard in the LW literature, we assume that frictions such as anonymity and the lack of a record-keeping technology hinder credit in the DM, so all trades have to be settled immediately. Therefore, buyers have to transfer assets to sellers in order to purchase q. Furthermore, because k cannot be transported to other locations and claims on k are not verifiable, relocated buyers can only use m to settle trades. Nonrelocated buyers can use m and k to purchase q.

15. For completeness, Appendix A studies an economy with $R < 1$.

16. The remainder of the paper applies to any $R > 1$. However, by assuming $R\beta = 1$ the inflation rate at the FR also equals β, which is the standard definition of the FR in LW models.

17. Zhu (2008) studies an economy with bilateral meetings and ex ante uncertainty about an agent's type in a model that is otherwise similar to ours, and shows that these frictions can make deviations from the FR optimal under some conditions. By assuming fixed types and competitive markets, we want to highlight that our results stem from different frictions than those present in Zhu (2008).

The relocation shock makes k less liquid than m and thus introduces a basic liquidity-return trade-off between m and k, with $1 - \pi$ representing the liquidity of capital.[18] In the DM, all buyers face the same nominal price p_t regardless of their means of payment. In the CM, x is also sold competitively. Because the problem is symmetric, we focus on one location for the remainder of the analysis.

Buyer's lifetime problem. A buyer's value function at the beginning of his life is

$$
\begin{aligned}
V^b = \max_{h_t, q_t^m, q_t^b, x_{t+1}^m, x_{t+1}^b} \quad & -h_t + \pi\big(u(q_t^m) + \beta U(x_{t+1}^m)\big) + (1-\pi)\big(u(q_t^b) + \beta U(x_{t+1}^b)\big) \\
\text{s.t.} \quad & h_t + \mathcal{I}\tau_t^y = \phi m_t + k_t^b \\
& p_t q_t^m \leq m_t, \\
& p_t q_t^b \leq m_t + \frac{Rk_t^b}{\phi_{t+1}}, \\
& x_{t+1}^m = \phi_{t+1} m_t + Rk_t^b - \phi_{t+1} p_t q_t^m + (1-\mathcal{I})\tau_{t+1}^o, \\
& x_{t+1}^b = \phi_{t+1} m_t + Rk_t^b - \phi_{t+1} p_t q_t^b + (1-\mathcal{I})\tau_{t+1}^o.
\end{aligned}
$$

Variables with superscript b indicate decisions of buyers prior to learning about relocation, or those of nonrelocated buyers, depending on the context. The first constraint is the budget constraint for the portfolio choice when young. The second constraint denotes that relocated buyers cannot spend more than their money holdings during the DM, and the third constraint denotes that nonrelocated buyers cannot spend more than their total wealth for consumption during the DM.[19] The fourth and fifth constraints denote that buyers use all remaining resources for consumption when old.

We only consider inflation rates where k (weakly) dominates m in terms of return, that is, $\phi_{t+1}/\phi_t \leq R$. In this case, the second constraint always holds at equality, as there is no reason for buyers to save m for the CM if k pays a higher return. We assume throughout the paper that if agents are indifferent between m and k, they only hold k. Furthermore, we restrict our attention to cases where the third constraint does not

18. One could also use search frictions to microfound the liquidity of k. For example, we could assume DM trade is bilateral, and buyers meet a seller who accepts m and k with probability $1 - \pi$, while the probability of meeting a seller who accepts only m is π. Our results go through qualitatively under this alternative specification, but the analysis in online Appendix C is slightly more complicated. Furthermore, using a relocation shock allows us to compare our results more directly to Smith (2002) and Haslag and Martin (2007).

19. The purchasing power of capital is scaled by R/ϕ_{t+1} to ensure that buyers give up the same amount of CM consumption by paying with capital and money.

bind.[20] After simplification, the buyer's problem is

$$V^b = \max_{m_t, k_t^b, q_t^b} \quad \mathcal{I}\tau_t^y - \phi_t m_t - k_t^b + \pi\left(u\left(\frac{m_t}{p_t}\right) + \beta U(Rk_t^b + (1-\mathcal{I})\tau_{t+1}^o)\right)$$
$$+ (1-\pi)\big(u(q_t^b) + \beta U(\phi_{t+1} m_t + Rk_t^b - \phi_{t+1} p_t q_t^b + (1-\mathcal{I})\tau_{t+1}^o)\big). \quad (13)$$

Seller's lifetime problem. A seller's value function at the beginning of his life is

$$V^s = \max_{h_t, q_t^s, x_{t+1}^s} \quad -h_t^s - q_t^s + \beta U(x_{t+1}^s)$$
$$\text{s.t. } h_t^s = k_t^s,$$
$$x_{t+1}^s = Rk_t^s + \phi_{t+1} p_t q_t^s,$$

where we imposed that sellers do not accumulate m in the first CM, which is true for $\phi_{t+1}/\phi_t \leq R$. The first constraint denotes sellers work to accumulate k, and the second constraint denotes that a seller's CM consumption is equal to the return on k plus his revenue from selling q in the DM. After simplification, the seller's problem is

$$V^s = \max_{q_t^s, k_t^s} \quad -k_t^s - q_t^s + \beta U(Rk_t^s + \phi_{t+1} p_t q_t^s). \quad (14)$$

As mentioned earlier, we only discuss the corner cases of $\pi = \{0, 1\}$ in the main body, as these are sufficient to derive the main results. The model with $\pi \in (0, 1)$ is discussed in online Appendix C.

2. EQUILIBRIUM WITH PERFECTLY LIQUID CAPITAL

Suppose $\pi = 0$, which implies that no relocation occurs and k is perfectly liquid as all buyers can use k during the DM. As m and k are equally liquid and safe in this case, agents only hold the asset with the higher rate of return. For $\phi_{t+1}/\phi_t \leq R$, k has a weakly higher return, so we abstract from m and monetary policy in this section. Given this, the buyer's problem from equation (13) becomes

$$V^b = \max_{k_t^b, q_t^b} \quad -k_t^b + u(q_t^b) + \beta U((k_t^b - \rho_t q_t^b)R),$$

where ρ_t denotes the price of q_t in terms of k_t.[21] The FOCs are

20. This constraint may bind if τ^o is large enough to finance a buyer's optimal CM consumption, such that he optimally spends all his funds in the DM. Note that this constraint only becomes relevant for the analysis in online Appendix C, and one can show that it never binds for $\gamma \leq 2$.

21. Since p denotes the price of q in terms of m, we have to change notation here as p is undefined if m is not held in equilibrium.

$$q_t^b: \quad u'(q_t^b) = \rho_t \beta R U'((k_t^b - \rho_t q_t^b)R), \tag{15}$$

$$k_t^b: \quad 1 = \beta R U'((k_t^b - \rho_t q_t^b)R). \tag{16}$$

The seller's problem is only affected by the change in notation. Solving equation (14) yields

$$q_t^s: \quad 1 = \beta R \rho_t U'((k_t^s + \rho_t q_t^s)R), \tag{17}$$

$$k_t^s: \quad 1 = \beta R U'((k_t^s + \rho_t q_t^s)R). \tag{18}$$

We see from equations (16) and (18) that $x^b = x^s = x^*$. Combining equations (17) and (18) gives $\rho_t = 1$ assuming optimal capital holdings of sellers are interior, which means that DM prices are such that sellers are indifferent between working in the CM or the DM.[22] Then, combining this with equations (15) and (16) yields

$$u'(q_t^b) = 1 \forall t,$$

so $q^b = q^s = q^*$. Furthermore, it is easily confirmed that $H = H^*$ and $K = K^*$. Thus, perfectly liquid capital allows to implement the planner's solution.

3. EQUILIBRIUM WITH PERFECTLY ILLIQUID CAPITAL

Consider now $\pi = 1$, which implies k is perfectly illiquid as all buyers are relocated. Then, for $\phi_{t+1}/\phi_t \leq R$, buyers face no trade-off between holding m and k, as only m allows them to acquire q, while k (weakly) dominates in terms of providing x.

With $\pi = 1$, the buyer's lifetime value function (13) simplifies to

$$V^b = \max_{m_t^b, k_t^b} \quad \mathcal{I}\tau_t^y - \phi_t m_t^b - k_t^b + u\left(\frac{m_t^b}{p_t}\right) + \beta U(Rk_t^b + (1-\mathcal{I})\tau_{t+1}^o).$$

Solving this problem yields two FOCs:

$$m_t^b: \quad p_t \phi_t = u'\left(\frac{m_t^b}{p_t}\right), \tag{19}$$

$$k_t^b: \quad \frac{1}{\beta R} = U'(Rk_t^b + (1-\mathcal{I})\tau_{t+1}^o), \tag{20}$$

22. To guarantee that sellers are willing to work in both markets at the first best, utility functions have to be such that sellers want to consume at least as much in the CM as they receive from selling q^* at $\rho = 1$. Thus, we need $U'(q^* R) \geq 1/(\beta R)$. We assume this holds for the remainder of the paper.

while solving the seller's problem (14) yields the following FOCs:

$$q_t^s: \quad 1 = \phi_{t+1} p_t \beta_t U'(Rk_t^s + p_t q_t^s \phi_{t+1}), \tag{21}$$

$$k_t^s: \quad 1 = \beta R U'(Rk_t^s + p_t q_t^s \phi_{t+1}). \tag{22}$$

We first derive the stationary equilibrium when $\mathcal{I} = 1$. In a stationary equilibrium we must have: $q_t^m = q_t^s$ (DM clearing), $m_t^b = M_t$ (money market clearing), and $\phi_t/\phi_{t+1} = \gamma$, that is, the inflation rate must equal the growth rate of the money supply since the real value of money is constant over time, implying $\phi_t M_t = \phi_{t+1} M_{t+1}$. Furthermore, the real value of transfers to young buyers is $\tau_t^y = \phi_t(M_t - M_{t-1}) = \frac{\gamma-1}{\gamma}\phi_t M_t$. Using this, we can then define a stationary equilibrium with $\pi = 1$ as a list of eight variables $\{h^b, h^s, k^b, k^s, \phi_{+1}M, q^m, x^b, x^s\}$ solving:

$$u'(q^m) = \gamma R, \tag{23}$$

$$x^m = x^s = x^*, \tag{24}$$

$$\phi_{+1}M = q^m R, \tag{25}$$

$$k^{b,\mathcal{I}=1} = \frac{x^*}{R}, \tag{26}$$

$$h^{b,\mathcal{I}=1} = q^m R + \frac{x^*}{R}, \tag{27}$$

$$k^s = \frac{x^*}{R} - q^m, \tag{28}$$

$$h^s = k^s, \tag{29}$$

and total capital investment and labor supply with $\mathcal{I} = 1$ are

$$K^{\mathcal{I}=1} = k^{b,\mathcal{I}=1} + k^s = \frac{2x^*}{R} - q^m, \tag{30}$$

$$H^{\mathcal{I}=1} = h^{b,\mathcal{I}=1} + h^s = \frac{2x^*}{R} + q^m(R-1). \tag{31}$$

Next, we derive the stationary equilibrium when $\mathcal{I} = 0$. The real value of transfers to old buyers is $\tau_{t+1}^o = \phi_{t+1} M_t(\gamma - 1)$. Thus under stationarity $\tau^y = \tau^o = \tau$ for a given γ, and equilibrium consumption levels are unaffected by the different monetary policy regimes. The only changes in the equilibrium allocation affect the labor supply (equation (27)) and capital accumulation of buyers (equation (26)). These now are

$$h^{b,\mathcal{I}=0} = \gamma q^m R + \frac{x^*}{R} - q^m(\gamma - 1), \tag{32}$$

$$k^{b,\mathcal{I}=0} = \frac{x^*}{R} - q^m(\gamma - 1), \tag{33}$$

and thus aggregate capital and labor supply are

$$K^{\mathcal{I}=0} = \frac{2x^*}{R} - \gamma q^m, \tag{34}$$

$$H^{\mathcal{I}=0} = \frac{2x^*}{R} + \gamma q^m (R-1). \tag{35}$$

Comparing these with equations (30) and (31) leads us to our first proposition.

PROPOSITION 1. *If $\gamma > 1$, $K^{\mathcal{I}=0} < K^{\mathcal{I}=1}$, and $H^{\mathcal{I}=0} > H^{\mathcal{I}=1}$, while if $\gamma < 1$, $K^{\mathcal{I}=0} > K^{\mathcal{I}=1}$, and $H^{\mathcal{I}=0} < H^{\mathcal{I}=1}$. For $\gamma = 1$, the two equilibria coincide.*

To put this in words, if monetary policy is inflationary ($\gamma > 1$) and buyers receive newly printed money as a transfer ($\tau > 0$), capital investment is higher and total labor is lower if monetary policy is implemented over young buyers. The opposite is true if monetary policy is deflationary ($\gamma < 1$) and buyers are taxed to withdraw money from circulation ($\tau < 0$). In this case, capital investment is higher and total labor is lower if monetary policy is implemented over old buyers. The timing of monetary policy matters because it is strictly more efficient for buyers to finance old age consumption with k due to $R > 1$. The timing of monetary policy affects the incentives for capital investment in the following way: If buyers receive a transfer ($\tau > 0$), it is better to receive it when young and invest it in k instead of receiving it when old and work more when young. Whereas if buyers pay a tax ($\tau < 0$), it is better to use the return on k to pay it when old instead of paying it by working more when young.[23] Because higher capital means less work, higher K and lower H always go hand in hand.

Next, we characterize an FR equilibrium where $\gamma = 1/R$:

PROPOSITION 2. *For $\pi = 1$, $q^m = q^*$, and $x^s = x^m = x^*$ at the FR. However, $H^{\mathcal{I}=1}|_{FR} > H^{\mathcal{I}=0}|_{FR} > H^*$.*

It can easily be seen from equation (23) that $q^m = q^*$ for $\gamma = 1/R$, and equation (24) shows that $x^s = x^b = x^*$ always holds. Thus, consumption is efficient at the FR. K and H however are not efficient, so the FR equilibrium does not allow to implement the first-best allocation. From (30) and (34), we see that $K^{\mathcal{I}=1}|_{FR} = 2x^*/R - q^*$ and $K^{\mathcal{I}=0}|_{FR} = 2x^*/R - q^*/R$. Thus, capital investment is too low in both monetary policy regimes compared to the first-best level. This is not surprising, as sellers can only be compensated with money, which implies that their CM consumption is partially financed through transfers from young to old agents—old sellers enter the CM with money and use it to purchase consumption goods from young buyers. At the first best, all CM consumption is financed with capital investment, so these intergenerational transfers imply an inefficiency. The inefficiency shows up in H, which is too

23. One could also consider giving a transfer to all buyers, both young and old. But with a linear return on capital, it is strictly better to make transfers to either only young or old buyers. As discussed here, which of those is better depends on the sign of $\gamma - 1$.

high compared to the first best:

$$H^{\mathcal{I}=1}|_{FR} = \frac{2x^*}{R} + q^*(R-1) > H^{\mathcal{I}=0}|_{FR} = \frac{2x^*}{R} + \frac{q^*(R-1)}{R} > H^*.$$

Since $R > 1$, it can easily be seen that implementing the FR by taxing old buyers is more efficient—it allows to achieve the same consumption levels at strictly lower hours worked.[24] Proposition 2 shows that even though consumption is at the first-best level at the FR, there is still a welfare loss from hours worked, so it is not obvious that the FR is welfare-maximizing.

Next, we investigate the effects of inflation on H and K for $\mathcal{I} = \{0, 1\}$. For this, denote the coefficient of relative risk aversion as $\eta(q) = -qu''(q)/u'(q)$, and the elasticity of DM consumption with respect to inflation as $|\varepsilon_{q^m}|$.

PROPOSITION 3. $|\varepsilon_{q^m}| = \frac{1}{\eta(q^m)}$ *with* $\pi = 1$*. For* $\mathcal{I} = 0$*, we have*

(1.) If $|\varepsilon_{q^m}| < 1$*:* $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} > 0$ *and* $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} < 0$*: Reverse MT-E.*
(2.) If $|\varepsilon_{q^m}| > 1$*:* $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} < 0$ *and* $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} > 0$*: MT-E.*
(3.) If $|\varepsilon_{q^m}| = 1$*:* $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} = 0$ *and* $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} = 0$*: No MT-E.*

With $\mathcal{I} = 1$*,* $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} < 0$ *and* $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} > 0$ *and thus there is an MT-E* $\forall |\varepsilon_{q^m}|$.

The proof to this proposition can be found in Appendix B. There are two channels through which γ affects K when $\pi = 1$. First, equation (28) shows that k^s is decreasing in q^m—and since q^m is decreasing in γ from equation (23), k^s is increasing in γ. The intuition is that sellers accumulate less k if they expect to sell more q. This is the first channel through which capital accumulation is affected by the inflation rate, and it is active independent of the tax regime. Since it affects the sellers' capital accumulation, we call it the *seller channel*. Second, equation (33) shows that when $\mathcal{I} = 0$, k^b depends on γ and on q^m, which is decreasing in γ. Since equation (26) shows that k^b is independent of γ for $\mathcal{I} = 1$, this channel is only active when $\mathcal{I} = 0$, so we call this the *transfer channel*. The effect of γ on k^b through the transfer channel has two components: on the one hand, higher γ decreases ϕ. On the other hand, higher γ increases the nominal value of the transfer. For $\gamma < 1$, both effects go in the same direction, as higher γ decreases ϕ, but increases the nominal value of the (negative) transfer. Thus, $\tau^o < 0$ increases and k^b decreases in γ. For $\gamma > 1$, either effect can dominate, depending on the elasticity of DM consumption.[25]

The effect on aggregate capital accumulation is given by the net effect of the two channels. With $\mathcal{I} = 1$, only the seller channel is active, and since the effect of γ

24. Remember we assume that if agents are indifferent between m and k, they accumulate k. This implies that agents only use k to save for old age at the FR, not m.

25. Specifically, if $|\varepsilon_{q^m}| > \frac{\gamma}{\gamma-1}$, the effect through the transfer channel of γ on k^b is positive, so a positive correlation is more likely for high $|\varepsilon_{q^m}|$ and high γ.

on K through the seller channel is positive, there is always an MT-E in this case. For $\mathcal{I} = 0$, the aggregate effect depends on $|\varepsilon_{q^m}|$, as this governs both the sign of the transfer channel at higher γ and the relative strength of the two channels. If $|\varepsilon_{q^m}| = 1$, the increase in k^s with γ through the seller channel is exactly offset by a decrease in k^b from the transfer channel, so K remains constant and there is no MT-E. With $|\varepsilon_{q^m}| > 1$, the (positive) effect through the seller channel is strong while the (negative) effect from the transfer channel is weak, so there is an MT-E on aggregate. The reason that this happens for high $|\varepsilon_{q^m}|$ is that in this case, buyers reduce q^m by a lot if γ increases, so in turn k^s is reacting strongly to changes in γ. Strong changes in q^m also imply that ϕ decreases strongly as γ increases, which weakens the negative effect on capital accumulation from the transfer channel. The contrary is true for $|\varepsilon_{q^m}| < 1$: q^m changes very little as γ varies, implying that k^s also varies very little with inflation. On the other hand, higher γ leaves ϕ almost unchanged, so k^b reacts strongly to changes in γ through the transfer channel. On aggregate, the negative effect from the transfer channel dominates, such that there is a reverse MT-E.

Proposition 3 also shows that an MT-E is always correlated with a negative effect on aggregate labor supply. Independent of $\mathcal{I}$, CM clearing implies

$$H = K + 2x^* - KR = 2x^* - K(R - 1). \tag{36}$$

Since CM consumption is independent of inflation, H and K are negatively related for $R > 1$, so if there is an MT-E (reverse MT-E), H decreases in γ (increases in γ).

Understanding this, we can derive the optimal monetary policy.

Proposition 4. *With $\pi = 1$, optimal inflation γ^* is*

- $\gamma^* = 1/R$ *under* $\mathcal{I} = 0$ *if* $|\varepsilon_{q^m}| \leq 1$;
- $\gamma^* = \frac{|\varepsilon_{q^m}|}{|\varepsilon_{q^m}|+R-1} \in (1/R, 1)$ *under* $\mathcal{I} = 0$ *if* $|\varepsilon_{q^m}| > 1$;
- $\gamma^* = 1$ *under* $\mathcal{I} = 1$.

The optimal monetary policy consists of setting $\mathcal{I}^* = 0$ and $\gamma^* = 1/R$ for $|\varepsilon_{q^m}| \leq 1$, and $\gamma^* = \frac{|\varepsilon_{q^m}|}{|\varepsilon_{q^m}|+R-1}$ for $|\varepsilon_{q^m}| > 1$. The first-best allocation is not achievable with $\pi = 1$.

Proposition 4 states our main result: Inflation rates above the FR are optimal if and only if there is an MT-E. The proof to this proposition can be found in Appendix B. The intuition is as follows: We know from Proposition 2 that the FR allows to achieve q^*. Thus if $\partial H/\partial\gamma \geq 0$ (Cases 1 and 3 from Proposition 3) the FR must be optimal. Higher inflation would decrease q^m while weakly increasing H, so higher inflation rates clearly decrease welfare. If $\partial H/\partial\gamma < 0$, there is a policy trade-off: Increasing γ reduces q^m, but also reduces H. Thus, by an envelope argument, the optimal γ must be above the FR: The marginal costs of decreasing q^m are zero at the FR, but the benefits of decreasing H are positive. This is what happens in Case 2 from Proposition 3 and with $\mathcal{I} = 1$. By how much γ can be increased above the FR to further increase welfare then depends on the strength of the MT-E, that is, how much K increases and H decreases with γ. If the MT-E is strong and H strongly decreases with γ, γ^*, which balances the marginal benefits of lower H with the marginal cost of lower q^m, is relatively far from the FR. If the MT-E is weak, γ^* is close to the FR. As discussed after Proposition 3, the strength of the MT-E also depends on how monetary policy is implemented. With $\mathcal{I} = 1$, there is always an MT-E because only the seller channel is active. With $\mathcal{I} = 0$, the transfer channel is also active, and for $\gamma < 1$, the effect of γ on K through the transfer channel is positive. So with $\mathcal{I} = 0$, the seller channel and the transfer channel go into opposite directions, implying that the MT-E is always weaker with $\mathcal{I} = 0$ than with $\mathcal{I} = 1$. This explains why optimal monetary policy depends on $\mathcal{I}$, and why γ^* is higher under $\mathcal{I} = 1$ than under $\mathcal{I} = 0$.

Proposition 4 also states that there is an ordering of the two ways of implementing monetary policy. We know that for $\gamma = 1$, both regimes are equivalent in terms of allocations and we also know that if $\mathcal{I} = 1$, $\gamma^* = 1$ is optimal. This allocation is always feasible, but not optimal, if $\mathcal{I} = 0$. Thus, we can conclude that the optimal monetary policy is to set $\mathcal{I}^* = 0$, and to set $\gamma^* = 1/R$ for $|\varepsilon_{q^m}| \leq 1$ and to set $\gamma^* = \frac{|\varepsilon_{q^m}|}{|\varepsilon_{q^m}|+R-1} \in (1/R, 1)$ for $|\varepsilon_{q^m}| > 1$. This also follows from the weaker MT-E with $\mathcal{I} = 0$. It implies that a more efficient DM allocation, which requires a lower γ, is less costly in terms of how much capital accumulation is reduced with $\mathcal{I} = 0$. This reiterates our result from Proposition 1 that implementing any $\gamma < 1$ is less costly with $\mathcal{I} = 0$ than with $\mathcal{I} = 1$, and thus if optimal policy rates are chosen, welfare is higher with $\mathcal{I} = 0$ than with $\mathcal{I} = 1$.

Note that $\gamma = 1$ implies the return on money equals the return on intergenerational transfers which the social planner faces; note also that financing x with m implies intergenerational transfers, as only young agents are willing to sell x against m. With $\pi = 1$, buyers are only able to compensate sellers with m for q, so unless the DM is completely shut down, sellers inevitably end up with m when they enter the second CM of their life. Setting $\gamma = 1$ leads to the correct price for providing x with intergenerational transfers. On the other hand, setting $\gamma = 1/R$ is the only way to reach q^*. This analysis points out the fundamental policy trade-off in our model: Efficiency in the DM (i.e., in intragenerational trade) requires a different γ than efficiency in the CM (i.e., in intergenerational trade), and the optimal inflation rate depends on the monetary policy regime and on how elastic DM consumption is.

Haslag and Martin (2007) have shown that $\gamma = 1$ is typically optimal in an OLG model, independent of the MT-E. We can confirm that $\gamma = 1$ is the optimal monetary policy for $\mathcal{I} = 1$, independent of all other parameters. However, we have also shown that an MT-E still exists in this case. More generally, we can show that implementing monetary policy in a less costly way, namely, by taxing agents only once they are old, allows to shut down the MT-E completely for certain parameters, and that in these cases, the FR is the optimal monetary policy.[26]

26. While this works nicely in our model, it would not do the trick in pure OLG models. The difference is that relocation occurs during the final stage of an agent's life in models such as Smith (2002) or Haslag and Martin (2007). The reason that taxing the old is strictly cheaper in our model is that all agents know they have access to their capital when they have to pay the tax, and can thus fully pay the tax via capital

Before concluding, we want to discuss the main results from online Appendix C, which considers the case of partially liquid capital, that is, $\pi \in (0, 1)$. With partially liquid capital, the monetary authority is able to perfectly insure agents against the relocation shock by running the Friedman rule, but then all DM trades are made with money, even though capital would be accepted in some of them. This adds a third channel through which inflation affects capital accumulation, which we call the *liquidity channel*. The higher the liquidity of capital, the more willingly buyers switch to accumulating capital instead of real balances if γ increases. We show that our main result, that is, that the Friedman rule is optimal if and only if there is no MT-E, still prevails with partially liquid capital. However, due to the liquidity channel, the MT-E is more likely to occur at the Friedman rule if capital is liquid, which in turn makes it less likely that the Friedman rule is the optimal monetary policy, even if $\mathcal{I} = 0$. In the limit when capital is fully liquid, the Friedman rule is never optimal.

Finally, we want to highlight again that this analysis focuses on steady-state welfare. This means γ^* maximizes welfare for all generations born in or after the period when γ^* is implemented. To complete our analysis, in online Appendix E, we derive how changes in γ affect welfare of all generations, including the welfare of buyers and sellers who are old in the period when γ^* is implemented. Specifically, we discuss how central banks can implement γ^* in a Pareto-efficient way, given that the current γ is different from γ^*. We show there that if monetary policy is implemented efficiently, that is, if $\mathcal{I} = 0$ for $\gamma \leq 1$ and $\mathcal{I} = 1$ for $\gamma > 1$, any $\gamma \in [\frac{1}{R}, 1]$ is Pareto-optimal for $|\varepsilon_{q^m}| < 1$, while any $\gamma \in [\frac{1}{R}, \gamma^*]$ is Pareto-optimal for $|\varepsilon_{q^m}| \geq 1$. Since most central banks are currently targeting positive inflation rates, this result is encouraging: A decrease from a positive inflation target to γ^* leads to a Pareto-improvement in welfare if $|\varepsilon_{q^m}| \geq 1$. With $|\varepsilon_{q^m}| < 1$, the central bank can lower the inflation target from a positive rate to $\gamma = 1$, but not to γ^* without hurting old agents. The analysis also shows that the FR is always Pareto-optimal.

4. CONCLUSION

We have added a market which requires liquid assets to trade to an OLG model with relocation shocks in order to study whether the MT-E can make deviations from the Friedman rule optimal in a model where money serves a role in both intra- and intergenerational trade. We have shown that the Friedman rule is optimal if and only if there is no MT-E at the Friedman rule, and that an MT-E is more likely to occur if capital is relatively liquid, risk aversion of buyers is low, and if monetary policy is implemented by taxing the young. If the Friedman rule is not optimal, the optimal money growth rate lies somewhere between the Friedman rule and a constant money stock. While the Friedman rule allows for first-best consumption levels in the DM,

investment. In pure OLG models, only nonrelocated agents have access to their capital during the final stage of their life.

it misrepresents the cost of using intergenerational transfers to provide for CM consumption during old age. These costs are correctly represented by a constant money stock. We have also shown that for any deflationary policy, taxing old agents is strictly better than taxing young agents when there is a productive investment opportunity in the economy.

APPENDIX A: AN ECONOMY WITH $R < 1$

Suppose now that $R < 1$. From the planner's problem, we know this makes capital accumulation socially inefficient. This implies that any $\gamma > 1/R$ cannot be optimal as it makes k more attractive than m to provide for the centralized market (CM) consumption. Thus, we restrict the analysis to $\gamma < 1/R$, which implies agents do not accumulate k and the relocation shock becomes irrelevant. To simplify notation, we set $\pi = 0$ here, but the results are equivalent for any $\pi \in [0, 1]$. Then, the buyer's problem is

$$V^b = \max_{m_t^b, q_t^b} \quad \mathcal{I}\tau_t^y - \phi_t m_t^b + u(q_t^b) + \beta U\big[\phi_{t+1}(m_t^b - p_t q_t^b) + (1 - \mathcal{I})\tau_{t+1}^o\big],$$

with the first-order conditions (FOCs)

$$U'(x_t^b) = \frac{\phi_t}{\phi_{t+1}} \frac{1}{\beta},$$
$$u'(q_t^b) = \phi_t p_t.$$

Meanwhile, the seller's problem is

$$V^s = \max_{m_t^s, q_t^s} \quad -\phi_t m_t^s - q_t^s + \beta U\big[\phi_{t+1}(m_t^s + p_t q_t^s)\big].$$

Solving this yields

$$U'(x_t^s) = \frac{1}{p_t \phi_{t+1}} \frac{1}{\beta},$$
$$U'(x_t^s) = \frac{\phi_t}{\phi_{t+1}} \frac{1}{\beta}, \tag{A.1}$$

which implies $p_t = 1/\phi_t$.[27] In a stationary equilibrium, we have $\phi_t/\phi_{t+1} = \gamma$ as always, $q_t^s = q_t^b$, $m_t^b + m_t^s = M_t$, and

$$u'(q^b) = 1,$$

27. As in the main body of the paper, we focus on cases where sellers always work in both markets. Otherwise, $m_t^s \geq 0$ may bind and thus (A.1) does not hold. Here, this holds if $U'(\frac{q^s}{\gamma}) \geq \frac{\gamma}{\beta}$.

$$U'(x^b) = U'(x^s) = \frac{\gamma}{\beta},$$
$$H = x^s + x^b,$$

where the third equation follows directly from the CM resource constraint with $K = 0$. This shows that contrary to the model with $R > 1$, the monetary policy regime is irrelevant here. Interestingly, $q^b = q^*$ independent of γ *as long as sellers work in both markets*. The reason is that in order to keep sellers indifferent between working in the decentralized market (DM) and the CM, p_t adjusts to changes in γ in a way that incentivizes buyers to always purchase q^*. This result resembles a finding in Huber and Kim (2020). Furthermore, it can easily be seen that for $\gamma = 1$, $x^b = x^s = x^*$, and $H = H^*$, showing that a constant money supply allows to implement the first-best allocation for $R < 1$.

APPENDIX B: PROOFS

B.1 Proof of Proposition 3

PROOF. We begin with the Mundell–Tobin effect (MT-E) in both regimes. When $\mathcal{I} = 1$,

$$K^{\mathcal{I}=1} = \frac{2x^*}{R} - q^m. \tag{30}$$

Since q^m is decreasing in inflation, we always get an MT-E with $\mathcal{I} = 1$:

$$\frac{\partial K^{\mathcal{I}=1}}{\partial \gamma} = -\frac{\partial q^m}{\partial \gamma} > 0. \tag{B.1}$$

When $\mathcal{I} = 0$,

$$K^{\mathcal{I}=0} = \frac{2x^*}{R} - q^m(\gamma - 1) - q^m = \frac{2x^*}{R} - q^m\gamma, \tag{34}$$

with the first derivative:

$$\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} = -(\gamma\frac{\partial q^m}{\partial \gamma} + q^m) = \frac{u'(q^m)}{-u''(q^m)} - q^m = q^m\left(\frac{1}{\eta(q^m)} - 1\right). \tag{B.2}$$

Furthermore, $\frac{1}{\eta(q^m)} = |\varepsilon_{q^m}|$, since

$$|\varepsilon_{q^m}| = -\frac{dq^m/q^m}{d\gamma/\gamma} = -\frac{\gamma}{q^m}\frac{\partial q^m}{\partial \gamma} = -\frac{u'(q^m)}{q^m u''(q^m)} \tag{B.3}$$

as $\frac{\partial q^m}{\partial \gamma} = \frac{R}{u''(q^m)}$ and $u'(q^m) = \gamma R$ from equation (23). Thus $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} > 0$ if $|\varepsilon_{q^m}| = \frac{1}{\eta(q^m)} > 1$.

Next we turn to the effects on total labor supply. In both monetary policy regimes, total labor supply is the sum of capital investments K and the work of buyers to acquire real balances. If $\mathcal{I} = 1$, total labor supply is

$$H^{\mathcal{I}=1} = \gamma q^m R + \frac{x^*}{R} - q^m R(\gamma - 1) + \frac{x^*}{R} - q^m = \frac{2x^*}{R} + q^m(R - 1). \tag{31}$$

Buyers acquire real balances $\phi M = \gamma q^m R$ and get a transfer of $\tau = q^m R(\gamma - 1)$. So in this case the wealth effects of inflation on the holdings of real balances are canceled out and the effect of inflation on total labor supply must be negative:

$$\frac{\partial H^{\mathcal{I}=1}}{\partial \gamma} = \frac{\partial q^m}{\partial \gamma}(R - 1) < 0. \tag{B.4}$$

With $\mathcal{I} = 0$ instead, total labor supply in the CM is

$$\begin{aligned} H^{\mathcal{I}=0} &= \gamma q^m R + \frac{x^*}{R} - q^m(\gamma - 1) + \frac{x^*}{R} - q^m \\ &= K^{\mathcal{I}=0} + \gamma q^m R = \frac{2x^*}{R} + (R - 1)\gamma q^m. \end{aligned} \tag{35}$$

Total labor supply is the sum of buyer real balances $\gamma q^m R$ and total capital investments. The effects of real balances on $K^{\mathcal{I}=0}$ and the real balance holdings $\gamma q^m R$ simplify to $(R - 1)\gamma q^m$. Thus the sign of the derivative of γq^m, which is determined by $\eta(q^m)$, will also determine the sign of the derivative of total labor supply:

$$\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} = (R - 1)(\frac{\partial q}{\partial \gamma}\gamma + q^m) = q^m(R - 1)\left(1 - \frac{1}{\eta(q^m)}\right). \tag{B.5}$$

Thus we must have $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} > 0$, $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} < 0$ for $|\varepsilon_{q^m}| = \frac{1}{\eta(q^m)} > 1$, and $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} < 0$, $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} > 0$ for $|\varepsilon_{q^m}| = \frac{1}{\eta(q^m)} < 1$. □

B.2 Proof of Proposition 4

PROOF. Welfare of a representative generation with $\pi = 1$ can be written as

$$V^g = -H + u(q^m) - q^m + 2\beta U(x^*). \tag{B.6}$$

Because CM consumption is independent of γ for $\pi = 1$, γ affects welfare through DM consumption and aggregate labor supply:

$$\frac{\partial V^g}{\partial \gamma} = -\frac{\partial H}{\partial \gamma} + \frac{\partial q^m}{\partial \gamma}(u'(q^m) - 1). \tag{B.7}$$

In cases 1 and 3 from Proposition 3, $\frac{\partial H}{\partial \gamma} \geq 0$. Thus, the FR must be the optimal monetary policy since it maximizes welfare in the DM ($q^m = q^*$) from (23). Higher γ would decrease q^m while weakly increasing H.[28]

In case 2 and with $\mathcal{I} = 1$, $\frac{\partial H}{\partial \gamma} < 0$. Thus, the optimal γ must lie above the FR due to the envelope theorem. $\frac{\partial H}{\partial \gamma}$ is given by (B.5) and the optimal inflation rate $\gamma^* < 1$ solves

$$-q^m(R-1) - \frac{\partial q^m}{\partial \gamma}\gamma^*(R-1) + \frac{\partial q^m}{\partial \gamma}(\gamma^* R - 1) = 0$$

$$\leftrightarrow$$

$$-\frac{\partial q^m}{\partial \gamma}\frac{\gamma^*}{q^m} = |\varepsilon_{q^m}| = \frac{(R-1)\gamma^*}{1-\gamma^*}. \tag{B.8}$$

The interior solution solves

$$\gamma^* = \frac{|\varepsilon_{q^m}|}{|\varepsilon_{q^m}| + R - 1} \quad \in (1/R, 1). \tag{B.9}$$

If $\mathcal{I} = 1$, $\frac{\partial H}{\partial \gamma}$ is given by (B.4) and γ^* solves

$$-\frac{\partial q}{\partial \gamma}(R-1) + \frac{\partial q}{\partial \gamma}(\gamma^* R - 1) = 0. \tag{B.10}$$

Thus $\gamma^* = 1$. □

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28. For inflation rates below the FR, our derivation of results is incorrect, because we assumed $\gamma \geq \frac{1}{R}$. It looks like further decreasing γ is welfare-increasing if $|\varepsilon_{q^m}| < 1$, but this is incorrect, as inflation below the FR leads to a regime switch where nobody accumulates capital. This clearly reduces aggregate welfare. Thus, $\gamma = \frac{1}{R}$ is a corner solution for $|\varepsilon_{q^m}| < 1$.

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SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of the article.

Supporting Data S1
Supporting Data S2