The S&P 500 Index inclusion effect: Evidence from the options market*

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Abstract

This paper employs forward-looking information from the options market to shed light on the comovement implications of S&P 500 index inclusion events over the 1997-2020 period. To this end, we test if forward–looking implied betas impound significant pre-inclusion information that is not embedded in historical betas. The empirical results show that the increase in post-inclusion implied betas is significantly smaller than the corresponding increase in historical or hybrid betas. In most cases, changes in implied betas show no evidence of excess comovement after S&P 500 index inclusion. Our findings suggest that added stocks experience changes prior to the index addition announcement and this leads to the "index inclusion effect" reported to the literature.

Key words: index inclusion effect, historical beta, implied beta

JEL classification: G12, G13, G14

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I. Introduction

The impact of the inclusion of new stocks in a stock market index has been the subject of considerable investigation and debate over recent decades. One issue investigated is return comovement or whether the post-inclusion returns on added stocks subsequently exhibit increased comovement with the index to which they are added. Empirical studies have shown that added stock returns do indeed commove more with the index than they did prior to inclusion. Barberis, Shleifer and Wurgler (2005) (BSW, hereafter), building on Vijh (1994), find that when a stock is added to the Standard & Poor's (S&P) 500 index, its beta with that index increases significantly. The increase is statistically significant over horizons of up to one year but not beyond due to mean reversion in investor sentiment. This "index-inclusion effect" is not confined to the US stock market. Similar results are also reported by Greenwood and Sosner (2007) for stocks added to the Nikkei 225 in Japan. Claessens and Yafeh (2013) establish widespread evidence for the index inclusion effect using a sample of some forty developed and emerging stock markets over a 10-year window. They find an increase both in the beta coefficient and in the explanatory power of market returns (R^2) .

Several explanations have been proposed for the index-inclusion effect which can be divided into two broad groups. On the one hand, the classical approach supports the view that the index-inclusion effect reflects changes in fundamentals. It argues that index inclusion is either an indirect signal that fundamentals have changed before the inclusion (as indicated for instance by pre-addition momentum) or that index inclusion leads to increased comovement, for example, through closer monitoring of management. Recent proponents of this approach include Kasch and Sarkar (2014) and Chen, Singal and Whitelaw (2016). Both studies stress

that many of the added stocks exhibit strong returns prior to index inclusion and that this is what largely drives the post-inclusion increase in comovement.¹

On the other hand, BSW (2005) propose the category or asset class approach as an explanation for increased comovement following index inclusion. Theoretical underpinnings consistent with their approach can be found in the style investment model of Barberis and Shleifer (2003) and in the Basak and Pavlova (2013) model where institutional investors focus on an index as a benchmark and thus tilt their portfolios toward index stocks.² BSW argue that some investors restrict themselves to trading within categories of stocks (funds tied to the performance of the S&P 500 index versus non-index funds, small-cap stocks versus large-cap stocks, etc.) in order to simplify portfolio decisions. Thus, when a stock is added to the index, its returns also reflect the common time-varying sentiment of investors trading those index stocks which in turn may induce excess comovement with other index stocks.³ The two contrasting approaches to explaining comovement raise the challenge of finding a context within which one can begin to distinguish between them.

This paper aims to shed light on the drivers of the index-inclusion effect using optionimplied beta estimates for stocks added to the S&P 500 index in the period 1997-2020. To the best of our knowledge, this is the first time in the literature that information from the option market is employed to study the impact of index additions on the comovement between the index and the added stock. ⁴ The main advantage that option-implied betas offer is that they

¹ Similarly, the "friction approach" suggests that the betas of stocks added to a market index increase because after inclusion they become more liquid with lower trading costs and incorporate market-wide news simultaneously with other index stocks.

² See, Agyei-Ampomah and Mazouz (2011), Wahal and Yavuz (2013) for empirical findings in favor of the style investing approach.

³ Note that both approaches focus on net beta changes following index additions. Liao, Coakley and Kellard (2022) show that net beta changes embody large offsetting increases and decreases in individual betas which make comovement even more perplexing.

⁴ Dash and Liu (2008) use data from the option markets to explore how option trading and prices change around an index inclusion announcement. Concurrent to this work, Holstein and Wese-Simen (2021) use option data to study the effect of index additions and removals on the implied volatility of the affected firm. We differentiate from these papers in that we investigate how implied betas respond to index composition changes as the focus of our work is comovement.

are forward-looking, i.e., they better reflect market expectations, and they are estimated using option data from a single day. This enables them to capture sudden changes in the fundamentals of stocks and in investor preferences, which can be hard to detect using OLS betas. As such, implied betas can be particularly insightful for our study and allow us to directly focus on any market effects caused by the index inclusion event.

Given the sensitivity of implied betas to unexpected changes in fundamentals and market expectations as well as their strong ability to predict realised betas (as discussed in Chang et al., 2012), we study the change in implied betas after a stock is included in the S&P 500 index. We hypothesize that if stocks experience changes before index addition and the relevant information is imbedded into option prices, implied betas should not change significantly after index addition. To test our hypothesis, we use the method of Chang, Christofersen, Jacobs, and Vainberg [CCJV] (2012) to calculate implied betas. This method is fully forward-looking as it expresses implied betas as a function of the implied volatility and implied skewness of the stock and the index and does not rely on historical data other than option prices on a particular day. In addition, we report OLS betas computed as in BSW and "hybrid betas" computed using implied volatilities and historical correlations as proposed by French, Groth, and Kolari [hereafter FGK] (1984). These two alternatives enable us to test if the change in implied betas is significantly different from the corresponding change in historical betas or hybrid betas.

In our empirical analysis, we find that the change in the implied betas after the addition to the index is significantly smaller than the respective change in historical and hybrid betas. This finding is consistent across three option maturities we assume for the estimation of implied and hybrid betas. Moreover, in most cases considered in the empirical analysis the change in implied betas is not even statistically significant. For instance, using options with a 30-day maturity, we find that historical (hybrid) betas significantly rise to

1.172 (1.063) from 1.056 (0.941) on average. However, the average post-inclusion implied beta is 1.096 compared to an average pre-inclusion beta of 1.080 and the difference is not statistically significant. Our results are consistent with Kasch and Sarkar (2014) and Chen, Singal and Whitelaw (2016) in that we do not find evidence of excess comovement following index inclusion in most cases considered. Our findings suggest that the reported indexinclusion effect is mainly driven by changes in the added stocks prior to index addition.

We carry out three robustness tests to confirm the validity of our results. First, we investigate whether the average volatility risk premium changes after index addition, in order to ensure that our results are not driven by time-varying premia embedded in implied betas. We confirm that there is no significant change in the volatility risk premium due to index addition. Second, we split our sample into two subsamples and rerun our analysis. As in our main results, the change in fully implied betas is considerable smaller than the change in historical and hybrid betas. Finally, we carry out our tests on median changes in the beta around index inclusion instead of average changes and obtain qualitatively similar results.

The rest of the paper is organized as follows. Section II discusses the relevant literature and Section III describes the methodology and specifies the hypotheses to be tested while Section IV discusses our empirical results. Section V presents our robustness tests and a final section concludes.

II. Literature review

One critical issue in the comovement debate is whether or not index inclusion conveys news about the fundamentals of included stocks. Standard & Poor's relies upon publicly available information in making index changes and it repudiates the suggestion that index inclusion implies a judgment on the future investment prospects of the company. However, the assumption that inclusion in the index provides no new information has been challenged

mainly in the price impact literature.⁵ Harris and Gurel (1986) found that the initial price increase associated with index inclusion is reversed over the subsequent 30 days. Thus, there is no permanent price impact and their results are consistent with index inclusion being an information-free event. At the international level, several studies report that stock price tends to increase following incorporation in the local main market index while index removals are associated with mixed findings. For example, Masse et al. (2000) document positive market reaction to index inclusion for Canadian stocks while index exclusion is not associated with significant market impact. Papachristou, Papadamou and Spyromitros (2018) report significant price increase following index addition and significant decrease following exclusion for Greek stocks.

More recent studies focus on the long-term effects of index inclusion and removal and cast doubt on whether index inclusion leads to favourable outcomes for the firm. Kot, Leung and Tang (2015) examine the long-term performance of firms added to or deleted from the Hang Seng Index. They find that deleted stocks outperform the index over the long run while added stocks do not. Patel and Welch (2017) find that any change in the investor demand for the stock after the addition/inclusion is short-lived. Bennett and Wang (2020) further show that index inclusion can impact a stock negatively in the long-term in terms of corporate performance, informativeness and governance.

A strand in the literature advocates that index inclusion may convey some information about firm's fundamentals. For instance, Denis et al. (2003) start from the assumption that the inclusion of new firms in an index leads to an improvement in their future performance. This could stem from increased monitoring of index firms or an increase in the cost of managerial reputation for such firms. They find that new additions to the index experience significant

⁵ Schleifer (1986), Lynch and Mendenhall (1997), Wurgler and Zhuravskaya (2002) and Biktimirov, Cowan, and Jordan (2004) and Chen (2006) also investigate the price impact and find that a stock's inclusion in an index usually results in a share price increase. Biktimirov (2004) finds that index deletions lead to a permanent stock price decline. Kappou, Brooks and Ward (2010) find significant overnight price changes after the announcement of an S&P 500 index change.

increases in earnings-per-share forecasts and in realized earnings relative to benchmark companies. Likewise, Cai (2007) using a matching firm methodology finds that index addition conveys favourable information about both included firms and their industry. Their results challenge the assumption that index inclusion is an information-free event.

Alternatively, BSW support the view that excess comovement with the index is driven by a change in the trading patterns of investors due to the index inclusion. As some investors tend to invest within categories of stocks, investors basing their investment decisions on the index may incorporate the added stocks to their portfolios. As a result, the stock may commove more with other index stocks.

However, some recent studies advocate that the "index inclusion effect" is an artifact of pre-inclusion changes in the stock. Kasch and Sarkar (2014) challenge the information-free assumption in the context of both price impact and comovement studies on the basis of the strong pre-inclusion performance of included firms. They show that new firms in the S&P 500 index are characterized by large increases in earnings and market value and positive price momentum in the pre-inclusion period compared with a matching firm benchmark. Using the three-factor model of Fama and French (1993), they conclude that index inclusion has no permanent effect on market value (price) and systematic risk. Along similar lines, Chen, Singal and Whitelaw (2016) provide a critique of the index-inclusion effect based on univariate regressions. When they employ (non-included stock) control samples matched on past returns and estimate Dimson betas (to allow for frictions), they find no evidence of excess comovement after the inclusion.

Liao, Coakley and Kellard (2022) adopt a different approach to comovement. They develops a stylised model for S&P 500 index changes with two distinct types of investor. On one hand, index trackers buy added stocks and sell deleted stocks. On the other, beta arbitrageurs trade in both high and low beta stocks – including S&P event stocks - to exploit

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beta mean reversion towards one. The implication is that the overall comovement effect typically reported in the literature has two components. For added stocks, pre-event low beta stocks drive the overall beta increases due to demand from both index trackers and arbitrageurs. By contrast, arbitrageur shorting of high beta added stocks reduces or can even reverse the beta increases for these stocks driven by index trackers. Thus, a small or insignificant overall comovement effect can hide larger underlying effects.

The existing literature has largely ignored an obvious source of publicly available information on expected fundamentals. Data on options on stocks added to the S&P 500 provide a rich source of forward-looking information that may be able to shed light on the index-inclusion effect on comovement. On one hand, if there is excess comovement following index inclusion, we would expect betas constructed using option data to increase at a magnitude similar to historical betas. On the other hand, as the Kasch and Sarkar (2014) and Chen, Singal and Whitelaw (2016) studies imply, if added stocks exhibit strong performance prior to addition which would affect expected growth prospects post-addition, then it is reasonable to expect this information to be reflected in options prices prior to inclusion and option-implied betas should not change significantly.

Implied betas have several interesting implications (e.g., see Siegel, 1995 and McNulty, Yeh, Schulze, and Lubatinet, 2002). They are reported to reflect more accurately sudden changes to fundamentals or investment sentiment than historical betas (see, also, Buss and Vilkov 2012). Moreover, implied betas are not affected by the choice of sampling frequency. This is important as BSW find a significant increase in the betas of stocks added to S&P 500 at both daily and weekly frequencies. However, they find that changes in betas are not significant at monthly frequencies for the three-year post-event estimation window and suggest that comovement disappears in the long run because of reversion in noise trader sentiment.

III. Methodology

As in CCJV (2012), we assume that the log return R_i of stock *i* is well described by a onefactor market model of the form:⁶

$$
R_i = a + \beta_i R_M + \varepsilon_i \tag{1}
$$

where R_M is the log return of the market. The standard OLS estimates of beta (β_i^{hist}) of stock *i* derived from historical market returns and individual equity returns can be written as

$$
\beta_i^{hist} = \rho_{i,M}^{hist} \left(\sigma_i^{hist} / \sigma_M^{hist} \right)
$$
 (2)

where $\rho_{i,M}^{hist}$ is the correlation coefficient between the log returns of stock *i* and the log returns of the market, and σ_i^{hist} , σ_m^{hist} are the standard deviations of the log returns of stock *i* and the log returns of the market, respectively. All the above parameters are calculated using historical data.

We calculate option-based betas using the method proposed by FGK (1984), and that proposed by CCJV (2012). The FGK beta estimator is a hybrid approach that uses both historical and option implied information. Their approach relies on conventional correlation estimates from historical returns.

The CCJV implied beta estimator is based solely on option implied data. Buss and Vilkov (2012) and Baule, Korn and Sassning (2016) also propose alternative implied beta estimators, which are also based solely on option implied data. Unfortunately, the Buss and Vilkov (2012) and Baule, Korn and Sassning (2016) estimators require the use of index weights and therefore cannot be applied to obtain implied betas before additions to the S&P 500.

⁶ Alternatively, one could assume excess returns in the context of the Capital Asset Pricing Model to yield implied betas (e.g., see Buss and Vilkov, 2012).

The FGK beta estimator (β_i^{FGK}) and the CCJV implied beta estimator (β_i^{CCIV}) are given by the following formulae, respectively:

$$
\beta_i^{FGK} = \rho_{i,M}^{hist} \left(\frac{VAR_i^Q}{VAR_M^Q} \right)^{1/2} \tag{3}
$$

and

$$
\beta_i^{CCJV} = \left(\frac{SKEW_i^Q}{SKEW_M^Q}\right)^{1/3} \left(\frac{VAR_i^Q}{VAR_M^Q}\right)^{1/2} \tag{4}
$$

where $SKEW^Q$ and VAR^Q are the risk-neutral skewness and variance of stock *i*, respectively, and *SKEW*^{Q}_{*M*} and *VAR*^{Q}_{*M*} are the risk-neutral skewness and variance of the market, respectively.⁷ Comparing equations (2) and (4), it is evident that the skewness ratio $\varrho \setminus^{1/3}$ *i Q M SKEW* $\left(\frac{SKEW_i^Q}{SKEW_M^Q}\right)^{1/2}$ can be used as a proxy for correlation.

The risk neutral moments of the return distribution are calculated using options data following the method of Bakshi, Kapadia and Madan (2003). They show that the risk-neutral skewness and variance can be extracted in a model-free manner using out-of-the-money call and put option prices. A merit of this method is that it takes into account the full cross section of option prices and, for any given maturity, the risk-neutral skewness and volatility are equivalent to portfolios of call and put option prices. See the Appendix for the derivation of the risk-neutral moments of the return distribution.

We calculate OLS historical betas, CCJV implied betas, and FGK hybrid betas using daily data for each sample stock added to the S&P 500 index. A one-year window before and after the index announcement/inclusion event is employed and the inclusion month data are excluded from the estimation. The OLS beta estimates are computed from daily returns using

⁷ CCJV make the additional assumption that SKEW_{ε,i} = 0 to obtain a solution for β_i^{ccIV} that depends only on moments of R_i and R_m . A similar assumption is also considered by Duan and Wei (2009). Chang et al. (2012) show that even if this assumption is violated, option-implied moments have good forecasting performance. A potential extension would be to estimate the error skewness and then incorporate it into the computation in the implied beta. We leave such an extension for future research.

the pre- and post-event windows. Daily implied variance and skewness are employed for computing the CCJV betas to obtain for each stock a time series of implied betas. For the FGK hybrid betas we use the pre- and post-event historical data to estimate the correlation coefficient and the average pre- and post-event stock implied and market implied volatility. All implied betas are computed for different horizons using using options with maturities of 30, 60, and 91 calendar days for the 1-year estimation window before and after a stock's addition to the S&P 500.

The main hypothesis that we test in the empirical analysis is that pre-addition CCJV implied betas impound significant pre-inclusion information that is not embedded in historical and hybrid betas. We examine this hypothesis using two different tests. First, we examine whether implied betas also display a statistically significant increase after index addition. If stocks experience changes before index addition and the relevant information is embedded into option prices, implied betas may not change significantly after index addition.

Second, we test if the change in implied betas is statistically smaller relative to the change in historical betas to determine whether new information after index addition has a stronger impact on historical relative to implied betas. To this end, we compute point estimates, $\sum_{i=1}^{n} (\Delta \beta_i^{CCJV} - \Delta \beta_i^{hist})/n$ and $\sum_{i=1}^{n} (\Delta \beta_i^{CCJV} - \Delta \beta_i^{FGK})/n$, where *n* is the number of the total inclusion events in the sample and examine the corresponding t-statistics.⁸

IV. Data and empirical results

IV.1 Characteristics of added stocks

The S&P 500 index addition events are downloaded from Professor Jeffrey Wurgler's website for the 1997-2000 period and they are extracted from Standard & Poor's annual

⁸ In our robustness tests in section V, we also consider the median changes in betas instead of average changes.

reviews for the period 2001 until December of 2020. Similar to BSW, inclusion events are excluded if the new firm is a spin-off or a restructured version of a firm already in the index, or if the firm is engaged in a merger or takeover around the inclusion event. We collect the corresponding options data from the Ivy Database of OptionMetrics for each stock included in the addition list. The estimation window considered in the main empirical analysis is one year before and one year after the addition of the stock, excluding the month of the announcement and inclusion implementation. As such, we require at least one year of option prices data before and after the addition event for a stock to be included in the analysis. Overall, our sample period yields a total of some 488 inclusion events.

The volatility surface file from OptionMetrics is employed to calculate the riskneutral skewness and variance following the approach of DeMiguel, Plyakha, Uppal, and Vilkov (2013). OptionMetrics provides historical prices of all US listed equity and index options based on the closing quotes at the Chicago Board of Option Exchange (CBOE). The volatility surface file contains the interpolated volatility surface for each security using a methodology based on a kernel smoothing algorithm. From the volatility surface, we consider volatilities of options with 30, 60, and 91 calendar days to expiration and select the volatilities of calls with deltas smaller or equal to 0.5 (out-of-the-money) and the implied volatilities of puts with deltas bigger than -0.5 (out-of-the-money). We use a cubic spline to interpolate-extrapolate the implied volatilities for each maturity and across moneyness levels and we construct a fine grid of 1001 implied volatilities. We apply the volatilities to calculate the corresponding option prices and use these to compute the risk-neutral moments of the return distribution in equation (4), as outlined in the Appendix.

Figure 1 plots the 30-day risk neutral moments of the S&P 500 and the 30-day riskneutral moments of the stocks employed in the empirical analysis for the 1997-2020 period.

[Figure 1 around here]

The top panel gives a plot of the implied variance of the S&P 500 and the average stock implied variance. At each point in time, the average stock implied variance is the equally-weighted, cross-sectional average of the implied variance of stocks added to the S&P 500 during the 1997-2020 period. Similarly, bottom panel plots the implied skewness of the S&P 500 and the average stock implied skewness for the same time period. As expected, the average stock implied variance is higher than the S&P's implied variance.

The S&P 500 is consistently more negatively skewed because out-of-the-money index put options are generally more expensive than the corresponding out-of-the money index call options. Rubinstein (1994) refers to this phenomenon as "crash-o-phobia" and attributes it to the strong demand for out-of-the-money put options to hedge against market crashes. Our findings are in line with Bakshi, Kapadia and Madan (2003) who also find that the riskneutral distribution of individual stocks is less negatively skewed and substantially more volatile than the risk-neutral distribution of the market index. Our time series average implied variance of the stocks added to the S&P 500 is 0.2 (45% volatility) and the time series average implied variance of the S&P 500 is 0.05 (22% volatility). The time series average of stocks' implied skewness is -0.31 and the time series average of the market's implied skewness is -1.23, and this difference is statistically significant at the 1% level.

Since the calculation of implied betas involves the implied second and third moments, we examine the time evolution of market-adjusted versions of these moments over the course of our sample period. Following DeFusco, Johnson, and Zorn (1990), the market-adjusted implied volatility is defined as the ratio of the implied volatility of the stock to the implied volatility of the market (VOL^Q _{*i}* / VOL^Q _{*u*}</sub>) and the market-adjusted implied skewness is defined as the ratio of the implied skewness of the stock to that of the market ($SKEW^Q$ / $SKEW^Q$). We examine changes in market-adjusted volatility and skewness using very short time windows.

Figure 2 plots the daily cross-sectional average of market-adjusted volatility and skewness.

[Figure 2 around here]

The time window is from 60 days before to 60 days after index addition announcement or implementation where these are different. On the announcement day, the market-adjusted implied volatility of stocks added to the S&P 500 spikes and then reverts to normal levels within a 10-day period. It is consistent with the findings of Dash and Liu (2008) who show that option trading volume surges after the announcement and option prices increase. However, they also show that it is not possible to profit from options trading since changes in option prices happen very shortly after the announcement. This is one justification for excluding the announcement month. The time evolution of market-adjusted implied skewness does not display any systematic change on announcement day.

Figure 3 finally presents the time series of the cross-sectional average of pre-addition CCJV implied betas, FGK implied betas and OLS historical betas of all stocks employed in the empirical analysis. The graph shows that there is substantial variation in the betas of the stocks added to the S&P 500 index and in most cases the FGK betas appear to be somewhat lower than the CCJV and OLS betas.

[Figure 3 around here]

IV.2 Changes in implied betas

Table 1 reports the average changes for the three beta estimators.

[Table 1 around here]

As expected, the historical betas of stocks added to the S&P 500 increase during the period following the inclusion. The OLS beta increases from 1.056 before the inclusion to 1.172 in the post-inclusion period. This increase of 0.116 is highly significant at the 1% level ($t =$ 5.997). The above results are broadly in line with those of BSW who found a 0.214 increase in beta for the earlier 1988-2000 period for a sample of 259 additions.

The average pre- addition CCJV implied beta is 1.08 for the 30-day options, 1.279 for the 60-day options and 1.266 for the 91-day options. Across the various maturities the implied beta change is not statistically significant. The average pre-addition FGK beta is 0.941, 0.922 and 0.887 for the 30-day, 60-day and 91-day options, respectively. The change in FGK betas is highly statistically significant across all maturities, with an average beta change of about 0.12 and t-statistics in excess of 6. The change in FGK betas is similar in magnitude with the change in OLS betas and as shown in the ensuing analysis their changes are the same from a statistical point of view. This is to be expected since both OLS and FGK estimators use historical correlation as an input. The Table 1 results suggest that implied betas do not display any statistically significant increase after index addition.

The above results are consistent with our hypotheses that pre-addition fully implied betas impound pre-inclusion information that is not captured by historical and hybrid betas. This motivates us to further explore the information content of OLS and FGK betas with regard to CCJV betas in two ways. First, we run cross-sectional regressions of the pre-inclusion CCJV betas against the OLS and FGK betas and of the FGK betas against the OLS betas, respectively. We present these results in Table 2.

[Table 2 around here]

The adjusted $R²$ s indicate that, while historical betas can explain up to 69% of the variation in hybrid betas, they can only explain up to 26% of the variation in implied betas. This finding indicates that implied betas contain significant information about the fundamentals of the firm which historical betas do not fully capture. Second, we study the extent that preinclusion OLS and FGK betas predict pre-inclusion CCJV betas, respectively, by examining the corresponding mean-squared errors (MSE) and mean absolute errors (MAE). We report these errors in Table 2. We observe that the errors for the pair OLS-FGK betas pair are considerably smaller than the errors for the OLS-CCJV and FGK-CCJV pairs that are similar in size. This finding is in line with our regressions results indicating that FGK and OLS betas do not fully embed the information in implied betas.

Table 3 reports the results with respect to the difference in beta changes using the pairs CCJV/OLS betas, CCJV/FGK betas and FGK/OLS betas, respectively.

[Table 3 around here]

The change in CCJV implied betas is statistically smaller compared to the change in both the OLS and FGK betas. The point estimate of the difference across the various maturities is around -0.1 with highly significant t-statistics. By contrast, the difference between changes in FGK and OLS betas is insignificant across all maturities with very low t-statistics.

The results reported in Table 3 highlight the differential impact of stock additions on historical and implied betas, respectively. The difference in beta changes is likely more informative, from an economic and statistical point of view, for testing whether stock additions per se are the underlying cause of the excess comovement reported in the literature. The results from this analysis unambiguously point to a systematic divergence between changes in implied betas and historical or hybrid betas. In particular, pre-inclusion implied betas appear to embed additional information which is not incorporated in pre-inclusion historical or hybrid betas. These results are consistent with the hypothesis that, if added stocks experience changes prior to index addition, as Kasch and Sarkar (2014) and Chen, Singal and Whitelaw (2016) suggest, then this information should be better reflected in implied betas.

V. Robustness Tests

V.1 Do stock volatility risk premiums change relative to the market premiums?

A potential criticism of our approach is that the empirical results may be driven by riskpremiums embedded in implied betas that capture volatility risk premiums. However, since our baseline results are based on differences and not on the levels of implied betas, the impact of risk premiums is expected to be washed out, unless the risk premiums are strongly time varying and change significantly after index addition. To investigate this, we calculate the pre- and post-event equity volatility risk premium corrected for the market volatility risk premium for the sample of stocks under consideration using options with maturities 30, 60 and 91 days, respectively. ⁹

We calculate the pre- and post-event average relative volatility risk premium of each stock using the relationship $ARVR$ _{*i*} = $\sqrt{VAR_i^Q} / \sqrt{VAR_i^P}$, where $\sqrt{VAR_i^Q}$ is the average riskneutral volatility and $\sqrt{VAR_i^P}$ is the historical volatility.¹⁰ The relative pre- and post-event market volatility risk premium is calculated using the relationship $ARVR_M = \sqrt{VAR_M^Q} / \sqrt{VAR_M^P}$, where $\sqrt{VAR_M^Q}$ is the average risk-neutral volatility of the S&P 500 and $\sqrt{VAR_M^P}$ is the historical volatility. To examine if stock volatility risk premiums change relative to the market we test the average change in the ratio $ARVRP_i/ARVRP_M$ across all stocks. The results are reported in Table 4.

[Table 4 around here]

The results are highly insignificant with very low t-statistics for all options considered. The conclusion here is that the empirical results are not driven by risk-premiums embedded in implied betas that capture volatility risk premiums.

V.2 Sub-sample analysis

Table 5 reports changes in implied, historical and hybrids betas in two different subsamples that approximately capture two different decades. The first subsample covers the period from

⁹ See Hollstein and Prokopczuk (2016) for a similar approach.

 10 The calculation of the volatility risk premium is based on the assumption that the magnitude of the volatility risk premium of a stock is proportional to the level of its historical volatility. This is a common assumption in the literature (see, for example, DeMiguel et al., 2013).

1997 until the end of 2009 and the second subsample covers the period from 2010 until the end of 2020. The first subsample includes the global financial crisis of 2008 and the second subsample includes the significant market turmoil due to the outbreak of the COVID-19 pandemic.

[Table 5 around here]

The results suggest that the increases in historical and hybrid betas are statistically significant in the first part of the sample period (1997-2009). Also, the implied betas with maturities 30 and 60 days display significant positive changes which are however smaller than the respective changes in historical and hybrid betas. By contrast, in the second part of the sample changes in all the implied and hybrid betas are not statistically significant. The increase in the historical betas is significant in both periods but the results are somewhat weaker in the second period given that the change in the point estimate drops from 0.15 to 0.04 and the t-statistic is reduced from 5.919 to 1.759.

Table 6 reports changes in implied correlation and changes in historical correlations for the period 1997-2009 and for the period 2010-2020, respectively. The implied correlation is calculated using the implied skewness ratio in equation (4). Changes in implied correlations are significant in the first part of the sample (1997-2009) and insignificant in the second part of the sample (2010-2020). Changes in the historical correlation are significant in both samples. Note also that, in the first sub-sample, the changes in historical correlations are much higher compared to the changes in implied correlations. Given the results for the first sub-sample, one cannot completely rule out the possibility that in certain time periods addition announcements have an impact on both implied and historical betas. However, as shown below, the increase in historical betas in the first subsample is still significantly higher than the increase in the implied betas.

[Table 6 around here]

We appreciate that further testing through statistically comparing changes in betas is required to accept or reject our hypothesis in this subsample setting. The results for the two sub-samples are given in Table 7.

[Table 7 around here]

The results for the two sub-samples are quite similar to those obtained for the full sample (Table 3). The change in CCJV implied betas is significantly smaller than the change in historical and FGK hybrid betas in all cases. This finding further confirms the validity of our main hypothesis that pre-addition implied betas incorporate information about stock fundamentals or other factors that is not captured by historical betas. The results also show that the difference between changes in FGK and OLS betas is insignificant across all maturities.

V.3 Median Changes

Table 8 reports median changes in implied, historical and hybrids betas of S&P 500 added stocks during the period 1997-2020.

[Table 8 around here]

From a statistical significance viewpoint, the results are similar to those obtained using mean changes in Table 1. All implied beta changes are insignificant, while the changes in OLS and FGK betas remain significant but are now smaller. The OLS beta increase is now 0.079 compared to 0.116 in Table 1 and the FGK increase is 0.08 compared to 0.121 in Table 1.

Table 9 compares the median changes across the beta estimators. These median results are virtually identical to the corresponding mean results in Table 2.

[Table 9 around here]

Table 10 reports median changes in implied, historical and hybrids betas for the 1997- 2009 period and for the 2010-2020 period, respectively. Using the median, the CCJV implied betas changes are now insignificant in the first subsample, while changes in hybrid and historical beta changes remain statistically significant. In the second subsample all median beta changes (implied, historical, hybrid) are statistically insignificant.

[Table 10 around here]

Table 11 compares the median changes across the beta estimators for the 1997-2009 and 2010-2020 periods, respectively. The median results for the 1997-2009 period are virtually identical those for the full sample median results in Table 7. The CCJV/OLS median differences are statistically significant like those in Table 7 but these differences are larger in absolute terms. Finally, all the median differences for the 2010-2020 period in Table 11 are statistically insignificant.

[Table 11 around here]

VI. Conclusions

The comovement literature provides evidence that the betas of stocks added to a stock index exhibit a significant increase after inclusion in the index. This paper examines the S&P 500 index inclusion effect from a different angle using information from the option market for a sample of 488 addition events over the 1997-2020 period. We compute the pre- and postinclusion betas using historical data and/or option implied data for each firm in the sample. More specifically, we compute OLS beta estimates based on historical data, the CCJV implied beta estimate which is based solely on option data and the FGK beta estimate which is a hybrid approach that uses both historical and option implied information.

We test the hypothesis that pre-inclusion options of stocks added to the S&P index potentially impound significant pre-inclusion information not embedded in historical betas. The empirical analysis shows that CCJV implied betas changes are either insignificant or significantly smaller than historical or hybrid beta changes. Our main conclusion from the

empirical results is that there is no strong evidence of excess comovement when optionimplied information is used to measure comovement.

Our results could help to shed light on the debate regarding the justification of the empirically observed increase in comovement by using the hitherto ignored publicly available information from options markets. Since the options of stocks added to the S&P500 index impound forward-looking information on fundamentals or other factors, our analysis is not subject to the endogeneity bias that Kasch and Sarkar (2014) attribute to extant comovement studies. Overall, our empirical results are consistent with the results of Kasch and Sarkar (2014) and Chen, Singal and Whitelaw (2016) in suggesting that added stocks experience changes before index addition and these lead to the "index-inclusion effect" reported in the literature.

Our findings have important implications for investors, firms and researchers. We provide evidence that the systematic risk of the firm as reflected in the implied beta is not affected by index inclusions in most cases. Likewise, our results indicate that the cost of capital of the firm does not considerably change due to index inclusion in contrast to evidence provided in earlier studies. Finally, our findings highlight the importance of also accounting for information from the option markets in financial research. As our results show, such information can significantly alter the conclusions based on historical stock market data alone.

Data Availability Statement

The S&P 500 index addition events are downloaded from Professor Jeffrey Wurgler's website [\(http://people.stern.nyu.edu/jwurgler/\)](https://eur01.safelinks.protection.outlook.com/?url=http%3A%2F%2Fpeople.stern.nyu.edu%2Fjwurgler%2F&data=04%7C01%7CA.Kourtis%40uea.ac.uk%7Cba010bbe70b54683d4df08d8c13b3658%7Cc65f8795ba3d43518a070865e5d8f090%7C0%7C0%7C637471807368405661%7CUnknown%7CTWFpbGZsb3d8eyJWIjoiMC4wLjAwMDAiLCJQIjoiV2luMzIiLCJBTiI6Ik1haWwiLCJXVCI6Mn0%3D%7C1000&sdata=a%2FOVS%2FC5yu9bUDVfbdC3MGVDYhkRlcDqgBpqB1SjaAw%3D&reserved=0) for the 1997-2000 period and they are extracted from Standard & Poor's annual reviews for the period 2001 until December of 2020. Stock and index option data used in the paper are available on the Ivy Database of OptionMetrics.

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Figure 1: Implied Variance and Skewness

The top panel plots the implied variance of the S&P 500 and the average stock implied variance. The average stock implied variance at each point in time is the cross-sectional average of the implied variance of stocks added to the S&P 500 during the period 1997-2020. The lower panel plots the implied skewness of the S&P 500 and the average stock implied skewness. The time series data in the graph include stock implied variance and skewness one year before and one year after addition and cover the period 1996 to 2021.

Figure 2: Market-adjusted implied volatility and skewness before and after index addition announcements

The market-adjusted implied volatility is defined as the ratio of the implied volatility of the stock to the implied volatility of the market (VOL^Q _{*i}* VOL^Q _{*M*}) and market-adjusted implied skewness is defined</sub> as the ratio of the implied skewness of the stock to the implied skewness of the market ($SKEW_i^Q$ / $SKEW_M^Q$). The figure plots the daily stock cross sectional average of market-adjusted volatility (top panel) and skewness (lower panel). The time window is from 60 days before to 60 days after the index addition announcement.

Market-adjusted implied volatility

Market-adjusted implied skewness

Figure 3: CCJV implied beta, FGK beta and historical OLS beta before index-inclusion

The graph plots the pre-inclusion CCJV implied betas, FGK implied betas and historical OLS betas of all S&P 500 added stocks during the period 1997-2020.

Table 1: Changes in implied and historical betas

This table presents the results for changes in the implied and historical betas for 488 additions to the S&P 500 index during the period 1997-2020. The CCJV implied beta is calculated using equation (4) and the FGK implied beta is calculated using equation (3). The historical beta is the OLS estimate from regression (1). For each stock in our sample we calculate the daily implied betas using daily data one year before and one year after the addition of the stock to the S&P 500 excluding the month of the announcement/ implementation. Then for each stock we calculate the average implied beta before and after the inclusion event and we average across all stocks to obtain the pre- and post-event implied beta estimates. All implied betas are computed for different horizons using options with maturities of 30, 60 and 91 calendar days.

Table 2: Information content of beta estimates

This table presents estimates of the intercept (a) , the slope (β) and the adjusted R-squared (R^2) from the cross-sectional regression of the pre-inclusion CCJV betas on the pre-inclusion OLS betas and the pre-inclusion FGK betas, respectively, for the period 1997-2020. It also presents these coefficients for the regression of the pre-inclusion FGK betas on the pre-inclusion OLS betas. Finally, it reports Mean Squared Errors (MSE) and Mean Absolute Errors (MAE) for the three pairs of pre-inclusion beta estimates. These results are computed for three different horizons corresponding to option maturities of 30, 60 and 91 calendar days, respectively.

Table 3. Difference in changes of implied and historical betas

This table presents the difference in beta changes of three pairs of betas: CCJV/OLS, CCJV/FGK, and FGK/OLS. The results for these pairs are given in separate columns for options with 30, 60 and 91 day to maturity. The betas are computed for all S&P 500 added stocks during the period 1997-2020.

Table 4: Changes in Volatility Risk Premiums

This table presents the results for the pre- and post-addition event equity volatility risk premium corrected for the market volatility risk premium. We calculate the pre- and post-event average relative volatility risk premium of each stock using the relationship $ARVRP_i = \sqrt{VAR_i^Q} / \sqrt{VAR_i^P}$, where VAR_i^Q is the average risk-neutral volatility and $\sqrt{VAR_i^P}$ is the historical volatility. The relative preand post-event market risk premium is calculated using the relationship ARVR $P_M = \sqrt{VAR_M^Q} / \sqrt{VAR_M^P}$, where $\sqrt{VAR_M^Q}$ is the average risk-neutral volatility of the S&P 500 and $\sqrt{VAR_M^P}$ is the historical volatility. We investigate the stock volatility risk premiums change relative to the market by testing the change in the ratio $ARVRP_i/ARVRM$ and averaging across all stocks. The changes in volatility risk premiums are computed for all S&P 500 added stocks during the period 1997-2020.

Table 5: Changes in implied and historical betas – Subsample analysis

This table presents separate results for changes in the implied and historical betas for stock additions to the S&P 500 index over the 1997-2009 and 2010-2020 periods. The implied beta CCJV is calculated using equation (4) and the implied beta FGK is calculated using equation (3). The historical beta is the OLS estimate from regression (1). For each stock in our sample we calculate the daily implied betas using daily data from one year before to one year after the addition of the stock to the S&P 500 excluding the month of the announcement/implementation. Then for each stock we calculate the average implied beta before and after the inclusion event and we average across all stocks to obtain the pre- and post-event implied beta estimates. All implied betas are computed for different horizons using options with maturities of 30, 60 and 91 calendar days.

Table 6: Changes in implied and historical correlation – Subsample analysis

This table presents separate results for changes in the implied and historical correlations for stock additions to the S&P 500 index over the 1997-2009 and 2010-2020 periods. The implied correlation CCJV is calculated using the implied skewness ratio in equation (4) and the historical correlation is estimated using the Pearson correlation coefficient. For each stock in our sample we calculate the daily implied correlation using daily data from one year before to one year after the addition of the stock to the S&P 500 excluding the month of the announcement/implementation. Then for each stock we calculate the average implied correlation before and after the inclusion event and we average across all stocks to obtain the pre- and post-event implied correlation estimates. All implied correlations are computed for different horizons using options with maturities of 30, 60 and 91 calendar days.

Table 7. Difference in changes of implied and historical betas– Subsample analysis

This table presents the difference in beta changes of three pairs of betas: CCJV/OLS, CCJV/FGK, and FGK/OLS. The results for these pairs are given in separate columns for options with 30, 60 and 91 days to maturity. Top panel reports the results over the 1997 to 2009 period and the bottom panel the results over the 2010 to 2020 period.

Table 8: Median changes in implied and historical betas

This table presents the results for changes in the implied and historical betas for 488 additions to the S&P 500 index during the period 1997-2020. The CCJV implied beta is calculated using equation (4) and the FGK implied beta is calculated using equation (3). The historical beta is the OLS estimate from regression (1). For each stock in our sample we calculate the daily implied betas using daily data one year before and one year after the addition of the stock to the S&P 500 excluding the month of the announcement/ implementation. Then for each stock we calculate the average implied beta before and after the inclusion event and compute the median change. All implied betas are computed for different horizons using options with maturities of 30, 60 and 91 calendar days.

Table 9. Difference in median changes of implied and historical betas

This table presents the difference in median beta changes of three pairs of betas: CCJV/OLS, CCJV/FGK, and FGK/OLS. The results for these pairs are given in separate columns for options with 30, 60 and 91 days to maturity. The betas are computed for all S&P 500 added stocks during the period 1997-2020.

Table 10: Median changes in implied and historical betas – Subsample analysis

This table presents separate results for the median changes in the implied and historical betas for stock additions to the S&P 500 index over the 1997-2009 and 2010-2020 periods. The implied beta CCJV is calculated using equation (4) and the implied beta FGK is calculated using equation (3). The historical beta is the OLS estimate from regression (1). For each stock in our sample we calculate the daily implied betas using daily data from one year before to one year after the addition of the stock to the S&P 500 excluding the month of the announcement/implementation. Then for each stock we calculate the average implied beta before and after the inclusion event and compute the median change. All implied betas are computed for different horizons using options with maturities of 30, 60 and 91 calendar days.

Table 11. Difference in median changes of implied and historical betas – Subsample analysis

This table presents the difference in median changes of three pairs of betas: CCJV/OLS, CCJV/FGK, and FGK/OLS. The results for these pairs are given in separate columns for options with 30, 60 and 91 days to maturity. Top panel reports the results over the 1997 to 2009 period and the bottom panel the results over the 2010 to 2020 period.

Appendix

Let Q denote the probability distribution function under the risk-neutral measure. At date *t* and for a given horizon *τ* we define the "quadratic", "cubic", and "quartic" contracts as following, respectively:

$$
V(t,\tau) \equiv E^{\mathcal{Q}}\left(e^{-r\tau}R_{t,\tau}^{2}\right)
$$
 (5)

$$
W(t,\tau) \equiv E^{\mathcal{Q}}\left(e^{-r\tau}R^3_{t,\tau}\right) \tag{6}
$$

$$
X(t,\tau) \equiv E^{\mathcal{Q}}\left(e^{-r\tau}R_{t,\tau}^{4}\right) \tag{7}
$$

Following Bakshi, Kapadia and Madan (2003), the prices of the above contracts are given by:

$$
V(t,\tau) = \int_{S_t}^{\infty} \frac{2\left(1 - \ln\frac{K}{S_t}\right)}{K^2} C(t,\tau;K) dK
$$

+
$$
\int_{0}^{S_t} \frac{2\left(1 + \ln\frac{S_t}{K}\right)}{K^2} P(t,\tau;K) dK,
$$
 (8)

$$
W(t,\tau) = \int_{s_i}^{\infty} \frac{6 \ln \frac{K}{S_t} - 3 \left(\ln \frac{K}{S_t} \right)^2}{K^2} C(t,\tau;K) dK
$$

$$
- \int_{0}^{S_t} \frac{6 \ln \frac{S_t}{K} + 3 \left(\ln \frac{S_t}{K} \right)^2}{K^2} P(t,\tau;K) dK
$$
 (9)

$$
X(t,\tau) = \int_{S_t}^{\infty} \frac{12\left(\ln\frac{K}{S_t}\right)^2 - 4\left(\ln\frac{K}{S_t}\right)^3}{K^2} C(t,\tau;K) dK + \int_{0}^{S_t} \frac{12\left(\ln\frac{S_t}{K}\right)^2 + 4\left(\ln\frac{S_t}{K}\right)^3}{K^2} P(t,\tau;K) dK,
$$
\n(10)

where S_t is the price of the underlying asset and $C(t, \tau; K)$ and $P(t, \tau; K)$ are OTM call and put option prices, respectively, with strike price K and τ time to maturity.

Finally, the risk-neutral variance and skewness are defined as:

$$
VARQ ≡ EQ \left[\left(R_{t,\tau} - Eq \left(R_{t,\tau} \right) \right)2 \right] = ert V \left(t, \tau \right) - \mu \left(t, \tau \right)2
$$
\n(11)

$$
SKEW^{Q} = \frac{E^{Q}\left[\left(R_{t,\tau} - E^{q}\left(R_{t,\tau}\right)\right)^{2}\right]}{\left(VAR^{Q}\right)^{\frac{3}{2}}} = \frac{e^{r\tau}W\left(t,\tau\right) - 3e^{r\tau}\mu\left(t,\tau\right)V\left(t,\tau\right) + 2\mu\left(t,\tau\right)^{3}}{\left[e^{r\tau}V\left(t,\tau\right) - \mu\left(t,\tau\right)^{2}\right]^{\frac{3}{2}}}
$$
(12)

for $\mu(t,\tau) = e^{rt} - 1 - \frac{e^{rt}}{2}V(t,\tau) - \frac{e^{rt}}{6}W(t,\tau) - \frac{e^{rt}}{24}X(t,\tau)$ $\frac{e^{r\tau}}{2}V(t,\tau)-\frac{e^{r\tau}}{6}W(t,\tau)-\frac{e^{r\tau}}{24}$ $f(\tau) = e^{r\tau} - 1 - \frac{e^{r\tau}}{2} V(t, \tau) - \frac{e^{r\tau}}{6} W(t, \tau) - \frac{e^{r\tau}}{24} X(t)$ $\mu(t,\tau) = e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V(t,\tau) - \frac{e^{r\tau}}{6}W(t,\tau) - \frac{e^{r\tau}}{24}X(t,\tau)$. We calculate equations and (11)

and (12) using the methodology outlined in DeMiguel et al. (2013).