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Essays on Incentives and Promotions

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Declaration of Authorship

I, Kai ZHANG, declare that this thesis titled, “Essays on Incentives and Promotions” and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
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Abstract

In this thesis, I study the problem of setting right incentives using promotion opportunities in organizations and its implications.

In the first chapter, I consider an environment assuming that a firm can commit to different promotion policies and highlight the role of commitment in promotion policy. In the presence of external hiring, committing to promotions can sharpen the incentive provisions. There is a trade-off between incentive provisions and job assignment. I show that a full commitment promotion policy is optimal if and only if the incentive role in promotion dominates the assignment role. In addition, I provide a strategic reason for not committing to promotions. Keeping external hiring open has a higher expected output for high-level positions.

In the second chapter, I explore the commitment role of training by considering an environment in which a firm cannot commit to a promotion policy but can commit to a given training level. I show that training can sharpen incentive provisions by increasing the promotion rate. This is twofold. First, training can increase profit by saving on wage costs or increasing outputs provided that promotions can provide incentives. Second, training increases the range over which promotion can provide incentives. From this perspective, I argue that training can serve as a commitment device for promotion policies.

The third chapter provides empirical evidence to support the findings in chapter 1. I examine the relationship between promotion rate and firm rank level profit of white-collar workers employed in Finnish manufacturing from 2002 to 2019. I show that promotion and firm rank level profit have a concave relationship. The result is robust in different settings and consistent with the theoretical model in chapter 1. This indicates that promotion policy is a firm's strategy to maximize its profit, which depends on firm rank level profitability and other characteristics.

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1 The Role of Commitment in Promotion Policy

1.1 Introduction

Promotions are widespread in most modern organizations (Jensen, 1986), and promotion opportunities play an important role in a dynamic principal-agent model. The first role of promotions is to provide incentives. In a long-term agency relationship, a principal can reward an agent through current compensation and future compensation. The principal can commit to paying high wages for a certain numbert of workers via promotion opportunities. However, if promotions are not contractible, it results in a dual moral hazard problem that both the principal and the agent face a moral hazard problem (Kahn and Huberman, 1988). The principal would have a commitment problem.

Promotions also serve the role of reassigning workers to the positions with which they are best matched. One of the reasons firms favour hiring internally for higher-level jobs is that the firm can learn information about workers' characteristics and productivity by observing them in entry-level jobs. The more information on workers firms have, the less uncertainty they are likely to face. The incentive role of promotions can increase organizational production, and the job assignment role can go further in this regard. Higher levels of the hierarchy, especially the managerial level, have more impact on organizational production than the lower-levels. However, in the presence of external hiring, a promotion policy still has a commitment problem, primarily when internal workers are found not to match with open manager positions.

From the perspective of workers, there are also career concerns around the effects of current performance on future compensation (Gibbons and Murphy, 1992). Even though firms commit to specific promotion rules, they also have incentives to avoid promoting people by giving poor performance evaluations to good workers. The current literature on promotions shows the commitment problem that a firm may face and how to commit to implementing a promotion policy (Waldman, 2003). For example, firms can build a reputation of only hiring internally, but that fails to explain the role of commitment in promotion policy. In fact, there is evidence that firms favour promotions but rarely commit to them. In this paper, I highlight the role of commitment in promotion policy. I argue that even if a firm has the ability to commit to promotions, it may not choose to do so.

A key issue in implementing promotion policy is how a firm commits to a promotion rule. To study the role of commitment in promotion policy, I assume that a firm's commitment is credible and that it can commit to different promotion policies. In the presence of external hiring, committing to promotions can sharpen incentives among low-level workers and could reduce productivity in high-level positions. There is a trade-off between incentive provisions and job assignment. I analyse the optimal promotion policy in light of this trade-off. Moreover, the optimal promotion policy might entail no commitment at all.

I explore this trade-off by using the efficiency wage model (Shapiro and Stiglitz, 1984) but assuming the firm has two types of job, job A and job B. In period 1, the firm hires two workers with unknown ability for job A. The contract specifies a wage policy for both periods and a promotion policy for period 2. I assume that a long-term contract is feasible and that the commitment to the promotion policy is credible. In period 2, the firm observes workers' ability and decides to promote the best worker or hire externally to fill job B according to the promotion policy. Job B is a managerial job, and there is a minimum wage restriction μ, which is high enough to make job B much more attractive for workers. The firm can provide incentives through promotion.

To eliminate promotion distortion concerns, I have a slot constraint on job B. I also assume that there is a minimum wage for job B that will retain the best worker and prevent poaching by another firm. In this case, there is no inefficiency concerning the number of promotions. However, there can be an inefficiency when lower-quality workers are promoted (Waldman and Zax, 2015). As long as a firm commits to promotions, there is a possibility that it will promote a less able worker and lose the the chance of obtaining the services of a better external worker. With this setting, the firm makes such a commitment if and only if it can increase the profit of job A such that this profit increase is larger than the profit loss of job B. Therefore, making such a commitment is optimal for the firm and is also socially desirable.

Once the wage and promotion policies are chosen, they remain fixed. The only uncertainty is the workers' ability. The firm has three different promotion policies: external hiring (no promotion), no commitment (promote internally or hire externally) and commitment (no external hiring). Among these three promotion policies, commitment has the highest promotion rate for the same type of worker with the same effort level. Common sense dictates that a firm can set the lower wage of job A at the same level of job B when the promotion rate is higher. The commitment can increase promotion opportunities and then reduce the wage paid of job A. More specifically, commitment can sharpen the incentive provisions of promotion.

On the other hand, commitment may reduce productivity in job B. There is a possibility that workers' abilities are limited. When the firm has committed to a promotion policy, the firm's hands are tied. The firm gives up the chance to obtain a better worker for job B through external hiring. The firm has information on workers' abilities, but it cannot fully use it. As a result, the firm would have low expected output from the worker in job B. Commitment weakens the job assignment role of promotion.

I show that commitment and no-commitment promotion policies are always better than external hiring for the firm. The optimal promotion policy depends on the relative size of the expected revenue in the two jobs. In the presence of external hiring, committing to promotion increases promotion opportunities and reduces productivity in job B. Finally, the trade-off exists if and only if the minimum wage μ is moderate.

The intuition for these findings is as follows. There is no role for promotions when the firm adopts an external hiring policy. Job A and job B are two independent jobs. Promotions cannot provide any incentives to low-level job workers. Moreover, the firm can gain information on workers' abilities through first-period outputs, but the firm only hires externally and thus cannot use this information. Even if promotions cannot provide incentives under no-commitment or commitment promotion policies, promotions can always play an assignment role, whereas under an external hiring policy, there is no room for promotions.

Therefore, the optimal promotion policy is either no commitment or commitment. If the minimum wage μ is relatively low, even with a high promotion rate under a commitment policy, promotion cannot provide incentives. Of course, the firm can raise the wage of job B to make the incentive constraints hold, but it may be too costly for the firm to do so. If the minimum wage μ is high enough, even a low promotion rate under a no-commitment policy can provide incentives, and there is no space to sharpen the incentive role of promotions. Under these two schemes (i.e., a low or high minimum wage μ), commitment and no-commitment policies play the same incentive role. However, no-commitment policy has another advantage. If one of the workers is high type, both promotion policies are the same, and the firm would promote the best internal worker. Nevertheless, if both workers are low type, under a commitment policy, the firm has to promote the best of the two low type workers, while no-commitment policy has the advantage of hiring a better external worker. In other words, if the minimum wage μ is either low enough or high enough, the role of commitment in the promotion policy only affects job B. Because a no-commitment policy could lead to a higher expected output of job B, it is always better than a commitment policy.

By contrast, if the minimum wage μ is moderate, there is room for commitment to sharpen the incentive role of promotions. A commitment policy can increase the promotion rate. With a higher promotion rate, the firm can either reduce the wage paid for either job A or job B. In other words, commitment increases the revenue attributable to job A by increasing its output or or lowering its wage or both. At the same time, it reduces the revenue of job B. There is a trade-off between job assignment efficiency and incentive provisions. When the increasing revenue of job A is the dominant effect, the optimal promotion policy is a commitment policy; otherwise, the no-commitment policy is optimal.

The findings on the role of commitment in promotion policy have certain important implications. First, I highlight the role of commitment in promotion policy. To the best of my knowledge, it has not been explored in the literature, which only shows the commitment problem a firm may face and how to address it. An interesting implication of the result is that even if a firm has the ability to implement commitment, it may not be optimal for that firm to commit. Empirical findings show that modern organizations favour internal workers for promotion and even use promotion-based incentives overwhelmingly. However, it is rare to find a firm that only promotes internally. In addition, not many firms take solely hire externally. Promotions have many advantages that do not increase firms' costs. Thus, a policy of only hiring externally is always inferior.

The contributions are twofold. First, I find that even when a firm can make a full commitment to internal promotion, it may not be optimal to do so. I argue that implementing commitment may not be a problem; the actual problem for the firm is the kind of commitment to make. Second, the optimal commitment to promotion policy for the firm may be partial commitment. Firms do not need to adopt a promotion-only policy, but they will benefit from ensuring that workers believe that they favour promotions.

1.2 Related Literature

There are two strands of literature to help understand the role of promotion (for an excellent review, see Waldman, 2013). One strand treats promotion as a tournament, following the seminal work of Lazear and Rosen (1981). From the perspective of promotion tournaments, firms commit to awarding a large prize (promotion) in the future to increase workers' current effort level. Promotion plays a role as an incentive provision, it can induce extra effort under certain wage policies. Another strand treats promotion as a signal of worker ability, following the seminal worker of Waldman (1984). Under asymmetric learning about workers' ability, promotion is a signal of higher ability, and outside firms can poach promoted workers. Given this concern, firms might provide fewer promotions (known as promotion distortion). There are many extensions of asymmetric information to study the conditions under which promotion distortion exists (Golan, 2005; Gibbons and Waldman, 2006; Waldman and Zax, 2015) or how to mitigate such inefficiency (Mukherjee and Vasconcelos, 2018). This research seeks to combine the two theories. In my model, promotion has two roles: incentives provisions and job reassignment.

This paper is related to the literature on the choice between external recruitment and internal promotion. The advantage of internal promotion has been well explored. Because of disparities in information learning, firms are more uncertain about external hires' abilities (Bidwell, 2011) and engage in external hiring if and only if outside workers are significantly superior to incumbent workers (Chan, 1996). In addition, there is a strategic reason for firms to consider external hiring, because it can increase incentive provisions by reducing the marginal return of negative effort, and avoid shirking equilibrium or workers' collusion (Chen, 2005). The negative effort is workers' effort to undercut the opponents' performances. The contribution is to provide another strategic reason for external hiring. When a firm learns that insider workers are less able, external hiring is the only way to use that information and information is useful if and only if a firm can use it. External hiring is one example of implementing such information.

It is also worth noting that this paper is closely related to Waldman (2003). He studies the role of time inconsistency in determining promotion policy. A time inconsistency problem is that the promotion rule is optimal at the time of promotion decision may differ from that is optimal before performance is determined. It is still a commitment problem that the firm may face, and this paper focuses on the role of commitment. I show that the capacity of commitment cannot solve the time inconsistency problem and that an optimal promotion policy may not require the firm to commit to promotion. In other words, firms do not need to adopt a consistent promotion policy.

Another closely related article is Krakel and Schottner (2012), who compare combined contracts (the no-commitment policy in the model of this paper) and separate contracts (the external-only hiring policy in the model of this paper) in different settings. In the model of this paper, I also consider another extreme promotion policy: the promotion-only policy. Under these settings, I can better understand the role of commitment in promotion policy. Krakel and Schottner (2012) find a combined contract strictly dominates external recruitment. I come to the same conclusion, but also I show that the promotion-only policy strictly dominates external recruitment. Thus, the topic of interest is to compare the combined contract and promotion-only policies.

The rest of the paper is organized as follows. Section 1.3 presents the model, while section 1.4 describes the optimal wage policy for a given promotion policy. The optimal promotion policy is provided in section 1.5, where I highlight the role of commitment in promotion policy. A final section draws a conclusion. All proofs are provided in the Appendix.

1.3 The Model

I consider a two-period principal-agent model that is described below in terms of its four key components: players, technology, contract and payoffs.

Players: A firm, F, has a fixed number (2) of positions for job A and one position for job B. Job A is an entry-level job, and job B is a managerial position. The labour pool consists a large number workers who are qualified for job A. Once hired, a worker may leave the firm, and the firm may fire a worker.

Technology: The technology of the firm is similar to Shapiro and Stiglitz's (1984) efficiency wage model. However, this model allows two types of jobs (job A and job B) within a single firm.

Workers privately choose their effort level for job A. Workers performing job A in period t choose effort level $e_t \in \{L, H\}$, $t = 1, 2$, $L < H$ and $L, H \in (0, 1)$. Exerting effort e implies a disutility for the worker that is equal to $C(e_t)$, with the normalizations $C(L) = 0$ and $C(H) = c$. An effort level e_t generates output y_A with probability e_t and 0 with $1 - e_t$. I impose the following restriction on the parameters.

Assumption 1.1 $\frac{c}{\triangle} < y_A$***, where*** $\triangle = H - L$.

This assumption makes sure that y_A is high enough for the firm to find it optimal to induce a high effort level.

However, job B is a high-level job. It only exists in period 2. The output of a worker with an ability of $\widehat{y_B}$ who performs job B is $\widehat{y_B}$. Without loss of generality, I assume the cost of performing job B for workers is zero; $\widehat{y_B}$ is unknown to all players and is assumed to follow a uniform distribution on $[\underline{y}, \overline{y}]$, where $\underline{y} > y_A$. In addition, the firm can learn workers' ability through their first-period output. At the beginning of period 2, if a worker worked for the firm in period 1 and still works for the firm in period 2, the firm has observed the worker's ability $\widehat{y_B}$.

Contract: I assume that long-term contracts on wages are feasible. As a worker's ability and effort level are not observable at the beginning of the game, the firm cannot offer a contract contingent on ability and effort level. Hence, I restrict attention to the following contract. At the beginning of the game, the firm announces the promotion policy and offers a contract w_i contingent on y_i to each worker (where $i = A, B$). Moreover, if a worker's output is zero, the worker receives zero and be fired. There is only one position for job B, and the firm can hire externally; therefore, there are three kinds of promotion policy P_J for job B: (i) external-only hiring policy P_E; (ii) commitment policy P_P (only internal hiring policy); and (iii) no-commitment policy P_N (hire internally or externally). To understand the role of commitment in promotion policy, I assume that as long as the firm announces the promotion policy, the promotion rule is credible.

At the beginning of period 1, the firm announces the promotion policy P_J and makes two take-it-or-leave-it offers (w_A, w_B) to the workers. At the end of period 1, after observing the output, the firm would fire workers whose output is zero without paying w_A. Then, the firm offers the same w_A to hire new workers to fill vacancies in job A. At the beginning of period 2, the firm observes existing workers' abilities $\widehat{y_B}$, and according to promotion policy P_J, the firm decides to promote the best incumbent worker or hire a worker externally for job B. The following timeline summarizes the game described above.

Period 1.0. The firm publicly announces the promotion policy P_J and offers a contract (w_A, w_B) to all the workers. If accepted by two, the game proceeds but ends otherwise. If more than two workers accept the offer, the firm would randomly choose two workers to fill the two positions.

Period 1.1. Workers choose their own effort levels. Production occurs, and output is realized. Workers whose output is zero are fired"; in other cases, wages are paid. The firm hires new workers with the same w_A to fill vacancies in job A.

Period 2.0. $\widehat{y_B}$ for all insider workers are observed. According to the promotion policy P_J, the firm decides to promote the best internal worker or hire a worker externally for job B.

Period 2.1. Workers choose their effort levels. Production occurs, and output is realized. Workers whose output is zero are fired; in other cases, wages are paid.

Payoffs: The firm and workers are risk neutral. Workers are protected by limited liability. The workers' transfer must always be nonnegative. There is no discounting. I assume that all players' outside options are 0. Upon successfully having two workers for job A in both periods, the firm's aggregate payoff is

$$\varPi = \pi_1 + \pi_2,$$

where π_t is the firm's payoff at time t, $\pi_1 = e_1^{\alpha}(y_A - w_A) + e_1^{\beta}(y_A - w_A)$, $\pi_2 = e_2^{\alpha}(y_A - w_A) + e_2^{\beta}(y_A - w_A) + E(\widehat{y_B}) - w_B$, e_t^s is the effort level of worker s at period t, $s = \alpha, \beta$, and $E(\widehat{y_B})$ is the expected output of job B.

Similarly, the workers' expected payoff of job A in period t is $u_t(e_t) = e_t w_A - c(e_t)$. The workers' expected payoff of job B in period 2 is w_B. Let $U_t(e_t)$ be the expected value of job A in period t with effort level e_t; then,

$$\begin{aligned} U_2(e_2) &= u_2(e_2) \\ U_1(e_1) &= u_1(e_1) + e_1 [p w_B + (1-p) U_2(e_2)], \end{aligned}$$

where p is the expected promotion rate. Since there is no moral hazard problem for job B, I impose a minimum wage μ on w_B, where $\mu > \frac{c}{\triangle}$. The restrictions on the lower bound of $\widehat{y_B}$ and the minimum wage μ guarantee job B is a better job than job A. μ is the minimum wage needed to retain the best worker and to prevent outside firms from poaching.

Strategies and equilibrium concept: The firm's strategy, σ_F, has two components : (i) at the beginning of period 1, it chooses the promotion policy P_J; (ii) at the beginning of period 1, it chooses the wage policy (w_A, w_B). The worker's strategy, σ_W, has four components: (i) at the beginning of period 1, he or she chooses accepts or rejects the firm's contract; (ii) if the worker gets the job, he or she chooses the effort level e_1; (iii) if he or she does not get the job at the beginning of period 1, at the end of period 1, he or she chooses to accept or reject the firm's w_A offer; and (iv) at the beginning of period 2, if he or she does not get promoted, he or she chooses the effort level e_2. I use subgame perfect equilibrium (SPE) as a solution concept.

1.4 Optimal Wage Policy for a Given Promotion Policy

In order to derive the optimal promotion policy contract for the firm, I first need to analyse the equilibrium wage policy (w_A, w_B) for a given promotion policy P_J. I use backward induction to solve the model. In what follows, I characterize the firm's equilibrium wage policy and workers' effort level for each promotion policy.

1.4.1 Period 2 Profits

Since there is no moral hazard problem in job B, the worker in job B would receive $w_B \geq \mu$, and the firm would obtain $E\left(\widehat{y_B}\right) - w_B$ for job B. The promotion policy would affect the $E\left(\widehat{y_B}\right)$ and promotion rate. For now, I do not need to compare the total payoffs for the firm, so I use $E\left(\widehat{y_B}\right)$. When I analyse the promotion rate for a given promotion policy, I provide the full results of $E\left(\widehat{y_B}\right)$ for different promotion policies.

For workers in job A, since workers are protected by limited liability and outside options are zero, I can ignore the individual rationality constraints (IR) and only consider the incentive compatibility constraints (IC). There are two different choices for workers that depend on w_A. When $w_A \geq \frac{c}{\triangle}$ - that is, the IC_2 constraint holds ($U_2(H) \geq U_2(L)$) - then all workers choose high effort H. I have $U_2(H) = Hw_A - c$, and the expected payoff of job A for the firm is $H(y_A - w_A)$ for each worker. When $w_A < \frac{c}{\triangle}$ - that is the IC_2 constraint does not hold - then all workers choose low effort L. I have $U_2(L) = Lw_A$, and the expected payoff of job A for the firm is $L(y_A - w_A)$ for each worker. The payoff for the firm in period 2 is

$$\pi_2 = \begin{cases} 2H(y_A - w_A) + E(\widehat{y_B}) - w_B & if \;\; w_A \geq \frac{c}{\triangle} \\ 2L(y_A - w_A) + E(\widehat{y_B}) - w_B & if \;\; w_A < \frac{c}{\triangle}. \end{cases}$$

1.4.2 Period 1 profits with $w_A \geq \frac{c}{\triangle}$

In this case, the IC_1 constraint ($U_1(H) > U_1(L)$) is

$$Hw_A - c + H[p_H w_B + (1-p_H)U_2(H)] > Lw_A + L[p_L w_B + (1-p_L)U_2(H)],$$

where p_H and p_L are the expected promotion rates when the worker chooses effort level H or L in period 1, respectively. These promotion rates depend on external and internal workers' abilities, rivals' effort level in period 1 and training level. I must have $p_H \geq p_L$ while holding all other factors fixed.

For this case, workers in job A would always choose high effort H. Promotion does not provide any incentives. Promotion policy only affects the expected output of job B. The payoff for the firm in period 1 is $\pi_1 = 2H(y_A - w_A)$. Then, the firm's problem is

$$\max_{\{w_A, w_B\}} \; \Pi = 4H(y_A - w_A) + E(\widehat{y_B})$$
$$s.t \;\; w_A \geq \frac{c}{\triangle} \;\; and \;\; w_B \geq 0.$$

To maximize profits, I must have $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, and

$$\Pi^*(4H) = 4H\left(y_A - \frac{c}{\triangle}\right) + E(\widehat{y_B}) - \mu.$$

1.4.3 Period 1 profits with $w_A < \frac{c}{\triangle}$

In this case, w_A is not enough to let the IC_1 constraint ($U_1(H) > U_1(L)$) hold. Promotion may play a role in providing incentives. Both the wage policy (w_A, w_B) and the promotion policy would affect workers' effort choice in period 1 because workers compete for the only managerial position. Workers' effort choices affect each other's expected promotion rate and the IC_1 constraint. The following is the total payoff matrix for the two workers:

Table 1.1: Total Payoffs Matrix

		Worker β	
		L	H
Worker α	L	A_{LL},A_{LL}	A_{LH},A_{HL}
	H	A_{HL},A_{LH}	A_{HH},A_{HH}

If both workers choose low effort, there are four different outcomes in period 1.

Table 1.2: Expected Payoff and Its Probability for (L, L)

Probability	Output	Payoff in period 1	Expected payoff in period 2
$(1-L)^2$	0,0	0,0	0,0
$(1-L)L$	0, y_A	0, w_A	0, $p^{one}w_B + (1-p^{one})Lw_A$
$(1-L)L$	y_A, 0	w_A, 0	$p^{one}w_B + (1-p^{one})Lw_A$, 0
L^2	y_A, y_A	w_A, w_A	$p^{two}w_B + (1-p^{two})Lw_A$, $p^{two}w_B + (1-p^{two})Lw_A$

In Table 1.2, p^{one} is the expected promotion rate when only one worker succeeds in period 1, and p^{two} is the expected promotion rate when two workers succeed in period 1. Thus, p^{one} and p^{two} are affected by the promotion policy and external and internal workers' abilities, but they are independent of their effort level. I provide full results in the next subsection. Workers' total payoffs are given by

$$\begin{aligned} A_{LL} &= L\left(1-L\right)\left[w_A + p^{one}w_B + \left(1-p^{one}\right)Lw_A\right] \\ &+ L^2\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right], \end{aligned} \tag{1.1}$$

Since both workers choose low effort in both periods, the total payoffs for the firm are given by

$$\Pi\left(4L\right) = 4L\left(y_A - w_A\right) + E\left(\widehat{y_B}\right) - w_B, \tag{1.2}$$

Similarly, if both workers choose high effort or one worker chooses low effort, and the other worker chooses high effort, there are also four different outcomes in period 1. All players' total payoffs for different cases are given by the following equations (more details are in Appendix 1.7.1):

$$\begin{aligned} A_{LH} &= L\left(1-H\right)\left[w_A + p^{one}w_B + \left(1-p^{one}\right)Lw_A\right] \\ &+ LH\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right], \end{aligned} \tag{1.3}$$

$$\begin{aligned} A_{HL} &= \left(1-L\right)H\left[w_A + p^{one}w_B + \left(1-p^{one}\right)Lw_A\right] \\ &+ LH\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] - c, \end{aligned} \tag{1.4}$$

$$\Pi\left(H\&3L\right) = \left(3L+H\right)\left(y_A - w_A\right) + E\left(\widehat{y_B}\right) - w_B,$$

$$\begin{aligned} A_{HH} &= H\left(1-H\right)\left[w_A + p^{one}w_B + \left(1-p^{one}\right)Lw_A\right] \\ &+ H^2\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] - c, \end{aligned} \tag{1.5}$$

$$\Pi\left(2H\&2L\right) = \left(2L+2H\right)\left(y_A - w_A\right) + E\left(\widehat{y_B}\right) - w_B.$$

1.4.4 Conditional Promotion Rate and $E\left(\widehat{y_B} \mid p^K\right)$

Only a worker whose output in period 1 is positive will remain with the firm and have the chance to get promoted in period 2. Her effort choice and her rival's effort choice would also affect the expected promotion rate. However, to understand the effect of different promotion policies on promotion rate and $E\left(\widehat{y_B} \mid p^K\right)$, where $K = one, two$ I want to exclude the impact of effort level. I calculate the conditional promotion rate and $E\left(\widehat{y_B} \mid p^K\right)$ at the beginning of period 2; the promotion rate and $E\left(\widehat{y_B} \mid p^K\right)$ are affected by the number of successful workers in period 1 and the promotion policy. The following table gives the different promotion rates for a given promotion policy (more details are in Appendix 1.7.2).

Table 1.3: Expected Promotion Rates for a Given Promotion Policy

	One worker succeeds		Two workers succeed	
	Promotion rate p^{one}	$E(\widehat{y_B} \mid p^{one})$	Promotion rate p^{two}	$E(\widehat{y_B} \mid p^{two})$
Commitment	1	$\frac{\underline{y}+\overline{y}}{2}$	$\frac{1}{2}$	$\frac{\underline{y}+2\overline{y}}{3}$
No commitment	$\frac{1}{2}$	$\frac{3\underline{y}+5\overline{y}}{8}$	$\frac{3}{8}$	$\frac{7\underline{y}+17\overline{y}}{24}$
External hiring	0	$\frac{\underline{y}+\overline{y}}{2}$	0	$\frac{\underline{y}+\overline{y}}{2}$

Proposition 1.1. **For the same situation, external hiring has the lowest conditional promotion rate and $E\left(\widehat{y_B} \mid p^K\right)$. For a given w_B, commitment increases promotion opportunities but reduces the expected productivity of job B.**

With an external hiring policy, the promotion rate is always zero ($p^{one} = p^{two} = 0$) since there are no promotion opportunities. For the commitment and no-commitment policies, the promotion rate depends on the number of successful workers in period 1. When only one worker succeeds in period 1, for the commitment policy, this worker would certainly be promoted ($p^{one} = 1$), while under the no-commitment policy, this worker would be promoted if and only if her ability is exceeded the average outsider worker ($\frac{\underline{y}+\overline{y}}{2}$); in that case, the conditional promotion rate p^{one} is $\frac{1}{2}$. When two workers succeed in period 1, under the commitment policy, one of them would certainly be promoted. Without any information on their abilities, they are identical and their chances are equal, meaning that the conditional promotion rate p^{two} is $\frac{1}{2}$. Meanwhile, under the no-commitment, the worker would be promoted if and only if her ability is better than her rival's ability and the average outsider worker ($\frac{\underline{y}+\overline{y}}{2}$); then, the conditional promotion rate ($p^{two} = \frac{3}{8}$) is lower than is the case under the commitment policy.

As to the expected output of job B ($E\left(\widehat{y_B} \mid p^K\right)$), for the external hiring policy, the firm can only obtain the average output ($\frac{\underline{y}+\overline{y}}{2}$), while under the commitment and no-commitment policies, the firm can obtain at least the average output, and it could obtain more than the average if it promotes a worker whose ability is better than the average. Therefore, the external hiring policy has the lowest conditional expected output. Comparing the no-commitment policy to the commitment policy, the probability of having or not having better insider workers is the same, and is irrelevant to promotion policies. When there are better inside workers, these two policies are the same; they would promote the best inside worker and obtain the same expected output. While when all inside workers' abilities are worse than average, under the commitment policy the firm has no choice but promote an inside worker and obtain below-average output. But under the no-commitment policy, the firm can hire externally and obtain average output. Therefore, the conditional expected output under the no-commitment policy is higher than under the commitment policy.

The key implication of Proposition 1.1 is internal hiring (commitment and no commitment) has two advantages over external hiring. First, internal hiring makes two jobs related, and it can provide incentives to low-level workers through promotion opportunities. Second, internal hiring could use the information learned about workers' abilities to in determining assignment. Only the external hiring policy cannot use this information at all. In contrast, the commitment and no-commitment policy can use this information by promoting the best inside worker. While this information is useless for commitment when both workers are lower than average, the no-commitment policy can still use this information by hiring externally.

On the other hand, commitment restricts the ability to hire externally. It has the highest promotion rate conditional on one worker or two workers succeeding. In other words, for a given effort level, workers always have a higher promotion rate in commitment policy. The effort level also affects the expected promotion rate; for example, if both workers choose high effort, the expected promotion rate for a worker is $H\left(1-H\right)p^{one}+H^2p^{two}$. If both workers choose low effort, the expected promotion rate for a worker is $L\left(1-L\right)p^{one}+L^2p^{two}$. The promotion policies also affect workers' effort choice. However, for a given w_B, because commitment has a higher p^{one} and p^{two}, if no commitment can induce high effort, a commitment must induce high effort. In contrast, if commitment can induce high effort, no commitment may not induce high effort. Therefore, commitment has a higher expected promotion rate than no commitment for a given w_B. The only concern is that different promotion policies may have different equilibrium wage policies and different equilibrium effort levels.

In subsection 1.4.4, I found the conditional promotion rate and $E\left(\widehat{y_B}\mid p^K\right)$. In the next three subsections, to find the optimal wage policy for a given promotion policy, I assume one effort level pair is the equilibrium effort choice, so I obtain the optimal wage policy and the firm's total payoffs under this effort level pair. Then comparing all the firm's total payoffs, I obtain the optimal wage policy. In addition, if $w_A \geq \frac{c}{\triangle}$, all workers would always choose high effort in both periods under all three promotion policies.

1.4.5 Optimal Wage Policy for the External-Only Hiring Policy

For the external-only hiring policy, the effort level in period 1 would never affect $E\left(\widehat{y_B}\right)$.

$$E\left(\widehat{y_B}\right) = \frac{\underline{y} + \overline{y}}{2}. \tag{1.6}$$

Therefore, in the case of $w_A \geq \frac{c}{\triangle}$, the optimal wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\Pi^*\left(4H\right) = 4H\left(y_A - \frac{c}{\triangle}\right) + \frac{\underline{y} + \overline{y}}{2} - \mu. \tag{1.7}$$

I now consider the case of $w_A < \frac{c}{\triangle}$. When $A_{LL} > A_{HL}$ and $A_{LH} > A_{HH}$, (L, L) is the unique NE in period 1. The wage policy is $w_A^* = 0$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\Pi^*\left(4L\right) = 4Ly_A + \frac{\underline{y} + \overline{y}}{2} - \mu. \tag{1.8}$$

When $A_{LL} < A_{HL}$ and $A_{LH} < A_{HH}$, (H, H) is the unique NE in period 1, the wage policy is $w_A^* = \frac{c}{\triangle(1+L)}$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\Pi\left(2H\&2L\right) = \left(2L + 2H\right)\left(y_A - \frac{c}{\triangle\left(1 + L\right)}\right) + \frac{\underline{y} + \overline{y}}{2} - \mu. \tag{1.9}$$

Since there is no promotion, workers are independent, and workers are identical for job A, they should have the same response to the same wage offer. There is no asymmetric equilibrium ((H, L) or (L, H)); more details are in Appendix 1.7.3. Comparing these three outcomes, I obtain Lemma 1.1, the proof for which is is in Appendix 1.7.4.

Lemma 1.1. **For external-only hiring policy,**

(a) If $y_A \geq \frac{Hc}{\triangle^2}$**, the optimal wage policy is** $w_A^* = \frac{c}{\triangle}$ **and** $w_B^* = \mu$**, and the total payoff for the firm is** (1.7)**;**

(b) If $y_A < \frac{Hc}{\triangle^2}$**, the optimal wage policy is** $w_A^* = 0$ **and** $w_B^* = \mu$**, and the total payoff for the firm is** (1.8)**.**

Under the external-only hiring policy, there is no promotion, and the two jobs are independent. The only problem for the firm is the moral hazard problem in job A. Workers are risk-neutral and protected by limited liability. As long as y_A is high enough, there is no efficiency loss because of the moral hazard, and it always achieves the first-best effort level. Meanwhile, if y_A is low, there is inefficiency, and the first-best effort cannot be achieved. In addition, I assume that a long-term contract is feasible; as long as the firm set the wage policy, there is no chance to revise it. The firm can set a slightly higher $w_A = \frac{c}{\triangle(1+L)}$, which gives enough incentive to induce high effort in period 1 but not in period 2. The problem with this wage policy is that the firm has to pay the same wage to the workers in job A, but this wage is not enough to incentivize workers to exert high effort in period 2. It gives workers too much surplus in period 2, so it is never the optimal wage policy.

1.4.6 Optimal Wage Policy for the Commitment Policy

Again, if $w_A \geq \frac{c}{\triangle}$, promotion has no incentive role, but it has a job assignment role and affects $E(\widehat{y_B})$. Note that only the effort level in period 1 has an effect on $E(\widehat{y_B})$, which is different from the external-only hiring policy. Promotion opportunities link two jobs and give the firm the ability to use information learned about worker types. In this case, both workers choose high effort in period 1:

$$\begin{aligned} E(\widehat{y_B}) &= 2H(1-H)E(\widehat{y_B} \mid p^{one}) + H^2 E\left(\widehat{y_B} \mid p^{two}\right) + (1-H)^2 E(outsider) \\ &= 2H(1-H)\left(\frac{\underline{y}+\overline{y}}{2}\right) + H^2\left(\frac{\underline{y}+2\overline{y}}{3}\right) + (1-H)^2\left(\frac{\underline{y}+\overline{y}}{2}\right) \\ &= \frac{\underline{y}+\overline{y}}{2} + H^2\left(\frac{\overline{y}-\underline{y}}{6}\right). \end{aligned} \tag{1.10}$$

Note that with the probability of $(1-H)^2$, both workers' outputs are zero in period 1 and would be fired, so the firm must hire externally in period 2. It is consistent with the commitment policy. In addition, with the same promotion policy, the firm obtains the same $E(\widehat{y_B})$ for the same level of effort. The wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\Pi^*(4H) = 4H\left(y_A - \frac{c}{\triangle}\right) + \frac{\underline{y}+\overline{y}}{2} + H^2\left(\frac{\overline{y}-\underline{y}}{6}\right) - \mu. \tag{1.11}$$

For the case of $w_A < \frac{c}{\triangle}$, when $A_{LL} > A_{HL}$ and $A_{LH} > A_{HH}$, (L, L) is the unique NE in period 1. The wage policy is $w_A^* = 0$ and $w_B^* = \mu$, the total payoff for the firm is

$$\Pi^*(4L) = 4Ly_A + \frac{\underline{y}+\overline{y}}{2} + L^2\left(\frac{\overline{y}-\underline{y}}{6}\right) - \mu. \tag{1.12}$$

I also need $\mu < \frac{c}{\triangle\left(1-\frac{L}{2}\right)}$; otherwise, promotion can provide enough incentives to make the IC_1 constraint hold and can induce workers to exert high effort in period 1.

When $A_{LL} < A_{HL}$ and $A_{LH} < A_{HH}$, (H, H) is the unique NE in period 1. There are three different wage policies for different μ. For $\mu \geq \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$, the wage policy is $w_A^* = 0$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\Pi^*\left(2H\&2L\right) = \left(2L+2H\right) y_A + \frac{\underline{y}+\overline{y}}{2} + H^2\left(\frac{\overline{y}-\underline{y}}{6}\right) - \mu. \tag{1.13}$$

For $\mu < \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$ and $2L+2H-\frac{3HL}{2}-H^2 < 1$, the wage policy is $w_A^* = \frac{\frac{c}{\triangle}-\mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\begin{aligned}\Pi^*\left(2H\&2L\right) = \left(2L+2H\right)&\left(y_A - \frac{\frac{c}{\triangle}-\mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}\right)\\ &+\frac{\underline{y}+\overline{y}}{2} + H^2\left(\frac{\overline{y}-\underline{y}}{6}\right) - \mu.\end{aligned} \tag{1.14}$$

For $\mu < \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$ and $2L+2H-\frac{3HL}{2}-H^2 > 1$, the wage policy is $w_A^* = 0$ and $w_B^* = \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$, and the total payoff for the firm is

$$\Pi^*\left(2H\&2L\right) = \left(2L+2H\right) y_A + \frac{\underline{y}+\overline{y}}{2} + H^2\left(\frac{\overline{y}-\underline{y}}{6}\right) - \frac{c}{\triangle\left(1-\frac{H}{2}\right)}. \tag{1.15}$$

When $A_{LL} < A_{HL}$ and $A_{LH} > A_{HH}$, (H, L) and (L, H) are the NEs in period 1, the wage policy is $w_A^* = 0$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\Pi^*\left(H\&3L\right) = \left(3L+H\right) y_A + \frac{\underline{y}+\overline{y}}{2} + HL\left(\frac{\overline{y}-\underline{y}}{6}\right) - \mu. \tag{1.16}$$

I need $\frac{c}{\triangle\left(1-\frac{L}{2}\right)} < \mu < \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$, or two inequations would not hold. More details are in Appendix 1.7.5. By comparing these outcomes, I obtain

Lemma 1.2, the proof of which is in Appendix 1.7.6.

***Lemma 1.2.* Under a commitment policy,**

- **(a) If $\mu < \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$, there are two cut-offs θ_1 and θ_2 for y_A, where $\theta_1 < \theta_2$; only under $\theta_1 < y_A < \theta_2$, promotion can provide incentives and induce both workers to choose high effort in period 1;**

- **(b) If $\frac{c}{\triangle\left(1-\frac{L}{2}\right)} < \mu < \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$, there are two cut-offs θ_3 and θ_2 for y_A; where $\theta_3 < \theta_2$, under $y_A < \theta_2$, promotion can provide incentives, while when $y_A < \theta_3$, promotion only induces one worker to choose high effort in period 1. When $\theta_3 < y_A < \theta_2$, promotion induces both workers to choose high effort in period 1;**

- **(c) If $\mu \geq \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$, there is one cut-off θ_4, under $y_A < \theta_4$; promotion can provide incentives and induce both workers to choose high effort in period 1.**

Since promotion exists in period 2, it only affects workers' effort choices in period 1. When μ is small, the interval $[\theta_1, \theta_2]$ for promotion to provide incentives is also small. When μ is moderate, promotion can always provide incentives (if it is not optimal to induce high effort in period 2). Moreover, the lowest effort level in period 1 is that one worker chooses low effort, and another chooses high effort this is an asymmetric equilibrium. This is different from Lemma 1.1. In the commitment policy, workers are competing with each other. A worker's own effort choice in period 1 affects her and her rival's expected promotion rate. When μ is large, there is no room for low effort in period 1. The firm can provide enough incentives to workers via promotion, without incurring extra cost.

1.4.7 Optimal Wage Policy for the No-Commitment Policy

For $w_A \geq \frac{c}{\triangle}$, the only difference between commitment and no commitment is the $E\left(\widehat{y_B}\right)$. In this case, both workers choose high effort in period 1:

$$\begin{aligned} E\left(\widehat{y_B}\right) &= 2H\left(1-H\right)E\left(\widehat{y_B} \mid p^{one}\right) + H^2 E\left(\widehat{y_B} \mid p^{two}\right) + \left(1-H\right)^2 E\left(outsider\right) \\ &= 2H\left(1-H\right)\left(\frac{3\underline{y}+5\overline{y}}{8}\right) + H^2\left(\frac{7\underline{y}+17\overline{y}}{24}\right) + \left(1-H\right)^2\left(\frac{\underline{y}+\overline{y}}{2}\right) \\ &= \frac{\underline{y}+\overline{y}}{2} + \left(H-\frac{H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right). \end{aligned} \tag{1.17}$$

Note that (1.17) $>$ (1.10); as Proposition 1.1 states, the no-commitment policy has a higher expected output in job B than the commitment policy. The wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\Pi^*\left(4H\right) = 4H\left(y_A - \frac{c}{\triangle}\right) + \frac{\underline{y}+\overline{y}}{2} + \left(H-\frac{H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right) - \mu. \tag{1.18}$$

For $w_A < \frac{c}{\triangle}$, when $A_{LL} > A_{HL}$ and $A_{LH} > A_{HH}$, (L, L) is the unique NE in period 1, the wage policy is $w_A^* = 0$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\Pi^*\left(4L\right) = 4Ly_A + \frac{\underline{y}+\overline{y}}{2} + \left(L-\frac{L^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right) - \mu. \tag{1.19}$$

I need $\mu < \frac{c}{\triangle\left(\frac{1}{2}-\frac{L}{8}\right)}$ to make sure the two inequations hold.

When $A_{LL} < A_{HL}$ and $A_{LH} < A_{HH}$, (H, H) is the unique NE in period 1. There are two different wage policies for different μ. For $\mu \geq \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)}$, the wage policy is $w_A^* = 0$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\Pi^*(2H\&2L) = (2L+2H)\,y_A + \frac{\underline{y}+\overline{y}}{2} + \left(H - \frac{H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right) - \mu. \quad (1.20)$$

For $\mu < \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)}$, the wage policy is $w_A^* = \frac{\frac{c}{\triangle}-\mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}}$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\begin{aligned}\Pi^*(2H\&2L) = {} & (2L+2H)\left(y_A - \frac{\frac{c}{\triangle}-\mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}}\right) \\ & + \frac{\underline{y}+\overline{y}}{2} + \left(H - \frac{H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right) - \mu. \quad (1.21)\end{aligned}$$

When $A_{LL} < A_{HL}$ and $A_{LH} > A_{HH}$, (H, L) and (L, H) are the NEs in period 1, the wage policy is $w_A^* = 0$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\begin{aligned}\Pi^*(H\&3L) = {} & (3L+H)\,y_A + \frac{\underline{y}+\overline{y}}{2} \\ & + \left(H + L - \frac{HL}{3}\right)\left(\frac{\overline{y}-\underline{y}}{8}\right) - \mu. \quad (1.22)\end{aligned}$$

I need $\frac{c}{\triangle\left(\frac{1}{2}-\frac{L}{8}\right)} < \mu < \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)}$ to make sure the two inequations hold. More details are in Appendix 1.7.7. Compared to these outcomes, I obtain Lemma 1.3, the proof of which is in Appendix 1.7.8.

Lemma 1.3. **Under a no-commitment policy,**

- **(a) If $\mu < \frac{c}{\triangle\left(\frac{1}{2}-\frac{L}{8}\right)}$, there are two cut-offs θ_5 and θ_6 for y_A, where $\theta_5 < \theta_6$, only under $\theta_5 < y_A < \theta_6$; promotion can provide incentives and induce both workers to choose high effort in period 1;**

- **(b) If $\frac{c}{\triangle\left(\frac{1}{2}-\frac{L}{8}\right)} < \mu < \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)}$, there are two cut-offs θ_7 and θ_6 for y_A, where $\theta_7 < \theta_6$; under $y_A < \theta_6$, promotion can provide incentives, while when $y_A < \theta_7$ promotion only induces**

one worker to choose high effort in period 1, and when $\theta_7 < y_A < \theta_6$, promotion induces both workers to choose high effort in period 1;

- **(c) If $\mu \geq \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)}$, there is one cut-off θ_4; under $y_A < \theta_4$, promotion can provide incentives and induce both workers to choose high effort in period 1.**

Lemma 1.3 is similar to Lemma 1.2. However, since no-commitment policy has a lower conditional promotion rate than the commitment policy, the critical value to induce high effort ($\frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)} > \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$) is higher. The advantage of a no-commitment policy is the higher $E\left(\widehat{y_B}\right)$. In Lemma 1.2, when μ is smaller than the critical value ($\frac{c}{\triangle\left(1-\frac{H}{2}\right)}$), the firm can increase w_A or w_B to induce high effort, which depends on the parameters L and H (or whichever costs less). In Lemma 1.3, when μ is not enough to induce high effort, the firm would always choose to raise w_A to incentivize workers. It has a low promotion rate for the same effort level and a higher critical value to make the IC constraint hold for a no-commitment policy. This shows that when the promotion rate is low or the w_B is high (but not high enough to induce high effort), increasing w_A to induce high effort costs less than increasing w_B.

1.5 The Optimal Promotion Policy

In section 1.4, I identify the optimal wage policies for three different promotion policies. By comparing these optimal wage policies, I determine the optimal promotion policy. However, I first highlight the roles of promotion by comparing Lemma 1.2 and Lemma 1.3 to Lemma 1.1.

Proposition 1.2. **Commitment and no-commitment promotion policies are always better for the firm than the external-only hiring policy.**

Proposition 1.1 shows external hiring has the lowest conditional promotion rate and $E\left(\widehat{y_B} \mid p^K\right)$. The key implication of proposition 1.1 is that promotion could increase the conditional promotion rate and $E\left(\widehat{y_B} \mid p^K\right)$. Proposition 1.2 is similar to the results of Krakel and Schottner (2012). In their setting, the combined contracts (no commitment) always dominate two separate contracts (external-only hiring). This is because promotion opportunities have two roles: incentives and job assignment. More specifically, the role of job assignment always exists, while the role of incentives only exists when the y_A is moderate. For the external-only hiring policy, the cut-off is $\frac{Hc}{\triangle^2}$. For the commitment and no-commitment policy, there are two cut-offs if μ is small, and there is one cut-off if μ is high. If y_A is small (smaller than θ_1, $\theta_1^{'}$, θ_3, $\theta_3^{'}$, θ_5, or θ_7 for different settings that are all smaller than $\frac{Hc}{\triangle^2}$), promotion cannot provide incentives but retains the role of job assignment. It is not worth inducing high effort because y_A is too small. If y_A is high (higher than θ_2, $\theta_2^{'}$, θ_6, or θ_4 for different settings that are all greater than $\frac{Hc}{\triangle^2}$), promotion also cannot provide incentives. It is worth inducing high effort in both periods because y_A is high, and promotion cannot affect the effort choice in period 2. The only way for the firm is to raise the wage of job A. If and only if y_A is moderate can promotion provide incentives.

For the external-only hiring policy, when y_A is greater than the cut-off, the moral hazard problem is not an issue as I can always get the first-best effort level. Promotion can sharpen incentive provisions and increase the firm's payoff, but it can also increase efficiency and reduce efficiency. For y_A is bigger than θ_1 ($\theta_1^{'}$, θ_3, $\theta_3^{'}$, θ_5, or θ_7 for different settings) but smaller than $\frac{Hc}{\triangle^2}$, promotion increases the expected output of job A in period 1 by inducing high effort. It increases efficiency and is thus socially desirable. However, when y_A is bigger than $\frac{Hc}{\triangle^2}$ but smaller than θ_2 ($\theta_2^{'}$, or θ_6, or θ_4 for different settings), promotion increases the firm's profit by saving the wage of job A (w_A drops form $\frac{c}{\triangle}$ to 0); it also reduces the expected output of job A in period 2. It reduces efficiency and is thus socially. This is different from the results of Krakel and Schottner (2012). They obtain efficiency wages in a more literal sense.

As proposition 1.2, the optimal promotion policy for the firm is either commitment or no commitment. The results of the optimal promotion policy are summarized in the following proposition; the proof of which is in Appendix 1.7.9.

Proposition 1.3. **For $\mu > \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)}$, the optimal promotion policy is a no-commitment policy. For $\mu < \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)}$, the optimal promotion policy depends on the role of promotion.**

Proposition 1.1 shows that commitment increases promotion opportunities but also reduces the expected output of job B. Promotion opportunities and $E\left(\widehat{y_B}\right)$ are always different in commitment and no-commitment policies. The former has a higher promotion rate but a lower $E\left(\widehat{y_B}\right)$. A lower $E\left(\widehat{y_B}\right)$ always makes commitment policy unpreferable, but a higher promotion rate does not always make commitment preferable. The trade-off between a higher promotion rate and a lower $E\left(\widehat{y_B}\right)$ does not always exist.

When μ is high enough ($\mu > \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)}$), even with a low promotion rate (under the no-commitment policy), it can induce high effort. There is no role for commitment. In this case, the commitment and no-commitment policy are the same (wage policy and effort levels) except that no commitment has a higher $E\left(\widehat{y_B}\right)$. There never is a trade-off. Therefore, the optimal promotion policy is always a no-commitment policy.

When μ is small ($\mu < \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)}$), if the incentive role dominates the assignment role, the optimal promotion policy is commitment; otherwise, the optimal promotion policy is a no-commitment policy. There are only two cases in which the assignment role dominates the incentive role. With a small μ ($\mu < \frac{c}{\triangle\left(\frac{1}{2}-\frac{L}{8}\right)}$) and a small y_A (i.e, $y_A < \theta_1$), it's not worth to induce high effort in period 1 for the firm, there is no incentive role for promotion. With a moderate μ and high y_A (i.e, $y_A > \theta_2$), it is worth inducing high effort in both periods for the firm which has to raise w_A to incentivize workers. In summary, Proposition 1.3 shows the relationship between profits (y_A) and promotion is inverse U-shape. This inverse U-shape relationship is tested in Chapter 3.

The key implication of Proposition 1.3 is that commitment is not always the optimal promotion policy for a firm. Common sense dictates that any firm that uses promotion-based incentives always faces a commitment problem. However, Proposition 1.3 shows that evenwhen the firm has the ability to commit, the firm would not commit since it is not optimal to commit. When a commitment policy is not the optimal promotion policy, then commitment is not a problem for the firm.

1.6 Conclusion

I analyse the role of commitment in promotion policy through an efficiency wage model where the firm has two types of job. In this setting, promotion (internal hiring) may play two roles: incentive and job assignment. I also assume that the firm can commit to three different promotion policies: an external-only hiring, a commitment and a no-commitment policy. Most importantly, these three policies are credible, which allows us to investigate the role of commitment and understand the trade-off between incentive provisions and job assignment.

The key finding is that a commitment policy is not always the optimal promotion policy. If promotions cannot provide incentives in a commitment policy, it is never optimal. Even if promotion can provide incentives, it is not optimal when the job assignment role of the promotion dominates the incentive role. This finding shows that even when the firm has the ability to commit, it may not commit to promotion because commitment is not optimal. In this setting, how to commit is not a problem for the firm. In reality, one can rarely find a firm that only hires internally. The results provide a simple answer. This is not because the firm does not have the ability to commit, but because committing is not optimal. Moreover, I find that when the promotion rate is low, and the firm uses promotion to provide incentives, increasing the wage in the low level of the hierarchy is better than increasing the wage at the higher level. Finally, promotion can sharpen the incentive provisions and boost the payoffs for the principal, sometimes even without extra cost. However, it could either increase efficiency or reduce efficiency. When a promotion reduces efficiency, it raises a firm's payoff by saving wages paid.

However, the firm still faces the problem of implementing the commitment policy when that commitment policy is optimal. One option is to establish a reputation that it only hires internally (Milgrom and Roberts, 1988). Moreover, specific human capital may also play an important role in the promotion policy. Other economic effects are interesting and related to the promotion. For the role of incentives, the promotion also plays a role in acquiring human capital skills. For the role of job assignment, what is the role of firm-sponsored training and external poaching? Training can increase workers' abilities while providing a way to learn about workers abilities. But external poaching reduces the firm's incentives for training and promotion. All issues raised above are worth investigating in further research.

1.7 Appendices

1.7.1 Equilibrium Outcomes for Different Effort Levels

- (L, H) and (H, L)

In these two cases, one worker chooses low effort, and another worker chooses high effort; there are also four different outcomes in period 1.

Table 1.4: Expected Payoff and Its Probability for (L, H) and (H, L)

Probability	Output	Payoff in period 1	Expected payoff in period 2
$(1-L)(1-H)$	$0, 0$	$0, -c$	$0, 0$
$(1-L)H$	$0, y_A$	$0, w_A - c$	$0, p^{one}w_B + (1-p^{one})Lw_A$
$L(1-H)$	$y_A, 0$	$w_A, -c$	$p^{one}w_B + (1-p^{one})Lw_A, 0$
LH	y_A, y_A	$w_A, w_A - c$	$p^{two}w_B + (1-p^{two})Lw_A, p^{two}w_B + (1-p^{two})Lw_A$

Then, workers' total payoffs are given by

$$A_{LH} = L(1-H)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] + LH\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right], \quad (1.3)$$

$$A_{HL} = (1-L)H\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] + LH\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] - c. \quad (1.4)$$

The total payoffs for the firm are given by

$$\begin{aligned}\Pi(H\&3L) &= \pi_1 + \pi_2 \\ &= (L+H)(y_A - w_A) + 2L(y_A - w_A) + E(\widehat{y_B}) - w_B \\ &= (3L+H)(y_A - w_A) + E(\widehat{y_B}) - w_B.\end{aligned}$$

- (H, H)

In this case, both workers choose high effort, and there are also four different outcomes in period 1.

Table 1.5: Expected Payoff and Its Probability for (H, H)

Probability	Output	Payoff in period 1	Expected payoff in period 2
$(1-H)^2$	$0, 0$	$-c, -c$	$0, 0$
$(1-H)H$	$0, y_A$	$-c, w_A - c$	$0, p^{one}w_B + (1-p^{one})Lw_A$
$(1-H)H$	$y_A, 0$	$w_A - c, -c$	$p^{one}w_B + (1-p^{one})Lw_A, 0$
H^2	y_A, y_A	$w_A - c, w_A - c$	$p^{two}w_B + (1-p^{two})Lw_A, p^{two}w_B + (1-p^{two})Lw_A$

Then, workers' total payoffs are given by

$$\begin{aligned} A_{HH} &= H(1-H)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] \\ &\quad + H^2\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] - c. \end{aligned} \tag{1.5}$$

The total payoffs for the firm are given by

$$\begin{aligned} \Pi\left(2H\&2L\right) &= \pi_1 + \pi_2 \\ &= 2H\left(y_A - w_A\right) + 2L\left(y_A - w_A\right) + E\left(\widehat{y_B}\right) - w_B \\ &= (2L+2H)\left(y_A - w_A\right) + E\left(\widehat{y_B}\right) - w_B. \end{aligned}$$

1.7.2 Conditional Promotion Rate and $E\left(\widehat{y_B} \mid p^K\right)$

x_1 and x_2 are two workers' ability, and both follow the uniform distribution on $\left[\underline{y}, \overline{y}\right]$. The probability density function is $f(x) = \frac{1}{\overline{y}-\underline{y}}$, and the cumulative distribution function is $F(x) = \frac{x-\underline{y}}{\overline{y}-\underline{y}}$, where $\underline{y} \leq x \leq \overline{y}$.

External Hiring

$$p_E^{one} = p_E^{two} = 0$$

$$E\left(\widehat{y_B}\right) = E\left(output \mid p^K\right) = \frac{\underline{y} + \overline{y}}{2}.$$

Commitment

- Only one worker succeeds:

$$p_{NC}^{one} = 1$$

$$E\left(output \mid p_{NC}^{one}\right) = \int_{\underline{y}}^{\overline{y}} x f\left(x\right) dx = \frac{\underline{y} + \overline{y}}{2}.$$

- Two workers succeed:

$$p_{NC}^{one} = \int_{\underline{y}}^{\overline{y}} f\left(x_1\right) P\left\{x_1 > x_2\right\} dx_2 = \int_{\underline{y}}^{\overline{y}} \frac{1}{\overline{y} - \underline{y}} F\left(x\right) dx = \frac{1}{2}.$$

For x_1 and x_2, i.i.d. continuous random variables with pdf $f\left(x\right)$ and cdf $F\left(x\right)$, the density of the maximum is $f_{(2)}\left(x\right) = 2f\left(x\right) F\left(x\right)$:

$$\begin{aligned}
P\left(X_{(n)} \in [x, x+\epsilon]\right) &= P\left(one\ of\ the\ X's\ \in [x, x+\epsilon]\ and\ all\ others < x\right) \\
&= \textstyle\sum_{i=1}^{n} P\left(X_i \in [x, x+\epsilon]\ and\ all\ others < x\right) \\
&= nP\left(X_i \in [x, x+\epsilon]\ and\ all\ others < x\right) \\
&= nP\left(X_i \in [x, x+\epsilon]\right) P\left(all\ others < x\right) \\
&= nP\left(X_i \in [x, x+\epsilon]\right) P\left(X_2 < x\right) \cdots P\left(X_n < x\right) \\
&= nf\left(x\right) \epsilon F\left(x\right)^{n-1}.
\end{aligned}$$

Then,

$$\begin{aligned}
E\left(output \mid p_{NC}^{one}\right) &= \int_{\underline{y}}^{\overline{y}} x f_{(2)}\left(x\right) dx \\
&= 2\int_{\underline{y}}^{\overline{y}} x f\left(x\right) F\left(x\right) dx \\
&= 2\int_{\underline{y}}^{\overline{y}} x \frac{1}{\overline{y} - \underline{y}} \frac{x - \underline{y}}{\overline{y} - \underline{y}} dx \\
&= \frac{\underline{y} + 2\overline{y}}{3}.
\end{aligned}$$

No Commitment

- Only one worker succeeds:

$$p_{NC}^{one} = P\left\{x_1 > \frac{\underline{y}+\overline{y}}{2}\right\} = 1 - F\left(\frac{\underline{y}+\overline{y}}{2}\right) = \frac{1}{2}$$

$$E\left(internal\ worker \mid p_{NC}^{one}\right) = \int_{\frac{\underline{y}+\overline{y}}{2}}^{\overline{y}} xf\left(x\right)dx = \frac{\underline{y}+3\overline{y}}{8}$$

$$\begin{aligned} E\left(output \mid p_{NC}^{one}\right) &= E\left(internal\ worker \mid p_{NC}^{one}\right) + P\left\{x_1\ or\ x_2 < \frac{\underline{y}+\overline{y}}{2}\right\} E\left(\frac{\underline{y}+\overline{y}}{2}\right) \\ &= \frac{3\underline{y}+5\overline{y}}{8}. \end{aligned}$$

Two workers succeed:

$$p_{NC}^{two} = P\left\{x_1 > x_2\ and\ x_1 > \frac{\underline{y}+\overline{y}}{2}\right\} = \int_{\frac{\underline{y}+\overline{y}}{2}}^{\overline{y}} f\left(x\right)F\left(x\right)dx = \frac{3}{8}$$

$$\begin{aligned} E\left(internal\ worker \mid p_{NC}^{two}\right) &= \int_{\frac{\underline{y}+\overline{y}}{2}}^{\overline{y}} xf\left(x\right)F\left(x\right)dx \\ &= \int_{\frac{\underline{y}+\overline{y}}{2}}^{\overline{y}} x\frac{1}{\overline{y}-\underline{y}}\frac{x-\underline{y}}{\overline{y}-\underline{y}}dx \\ &= \frac{2\underline{y}+7\overline{y}}{24} \end{aligned}$$

$$\begin{aligned} E\left(output \mid p_{NC}^{two}\right) &= P\left\{x_1 < \frac{\underline{y}+\overline{y}}{2}\right\} P\left\{x_2 < \frac{\underline{y}+\overline{y}}{2}\right\} E\left(\frac{\underline{y}+\overline{y}}{2}\right) \\ &+ 2E\left(internal\ worker \mid p_{NC}^{two}\right) \\ &= \frac{7\underline{y}+17\overline{y}}{24}. \end{aligned}$$

1.7.3 Nash Equilibrium with External Hiring

(1) (L, L) is the only NE.

The firm's problem is

$$\underset{\{w_A,w_B\}}{max} \mathit{\Pi}(4L) = 4L(y_A - w_A) + E(\widehat{y_B}) - w_B$$

s.t $w_A < \frac{c}{\triangle}$, $w_B \geq \mu$, $A_{LL} > A_{HL}$, and $A_{LH} > A_{HH}$

$$\begin{aligned} A_{LL} > A_{HL} &\iff A_{LL} - A_{HL} \geq 0 \\ &\iff -\triangle(1-L)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] - \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c > 0 \\ &\iff \triangle(1-L)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] + \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] < c \\ &\iff \triangle(w_A + Lw_A) < c \end{aligned}$$

$$\begin{aligned} A_{LH} > A_{HH} &\iff A_{LH} - A_{HH} \geq 0 \\ &\iff -\triangle(1-H)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] - \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c > 0 \\ &\iff D_{24} = \triangle(1-H)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] + \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] < c \\ &\iff \triangle(w_A + Lw_A) < c. \end{aligned}$$

To maximize profits, we must have $w_A^* = 0$ and $w_B^* = \mu$,

$$\mathit{\Pi}^*(4L) = 4Ly_A + \frac{\underline{y} + \overline{y}}{2} - \mu. \tag{1.8}$$

(2) (H, H) is the only NE.

The firm's problem is

$$\underset{\{w_A,w_B\}}{max} \mathit{\Pi}(2H\&2L) = (2L + 2H)(y_A - w_A) + E(\widehat{y_B}) - w_B$$

s.t $w_A < \frac{c}{\triangle}$, $w_B \geq \mu$, $A_{LL} < A_{HL}$, and $A_{LH} < A_{HH}$

$$\begin{aligned} A_{LL} < A_{HL} &\iff A_{LL} - A_{HL} < 0 \\ &\iff -\triangle(1-L)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] - \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c < 0 \\ &\iff \triangle(1-L)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] + \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] > c \\ &\iff \triangle(w_A + Lw_A) > c \end{aligned}$$

$$\begin{aligned} A_{LH} < A_{HH} &\iff A_{LH} - A_{HH} < 0 \\ &\iff -\triangle(1-H)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] - \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c < 0 \\ &\iff D_{24} = \triangle(1-H)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] + \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] > c \\ &\iff \triangle(w_A + Lw_A) > c. \end{aligned}$$

To maximize profits, we must have $w_A^* = \frac{c}{\triangle(1+L)}$ and $w_B^* = \mu$,

$$\Pi^* (2H\&2L) = (2L + 2H) \left(y_A - \frac{c}{\triangle (1 + L)} \right) + \frac{\underline{y} + \overline{y}}{2} - \mu. \quad (1.9)$$

(3) (H, L) and (L, H) are the NEs.

The firm's problem is

$$\max_{\{w_A, w_B\}} \Pi (H\&3L) = (3L + H) (y_A - w_A) + E (\widehat{y_B}) - w_B$$

s.t $w_A < \frac{c}{\triangle}$, $w_B \geq \mu$, $A_{LL} < A_{HL}$, and $A_{LH} > A_{HH}$

$$\begin{aligned} A_{LL} < A_{HL} \quad \Longleftrightarrow \quad & A_{LL} - A_{HL} < 0 \\ \Longleftrightarrow \quad & -\triangle (1 - L) [w_A + p^{one} w_B + (1 - p^{one}) Lw_A] - \triangle L \left[w_A + p^{two} w_B + \left(1 - p^{two}\right) Lw_A\right] + c < 0 \\ \Longleftrightarrow \quad & \triangle (1 - L) [w_A + p^{one} w_B + (1 - p^{one}) Lw_A] + \triangle L \left[w_A + p^{two} w_B + \left(1 - p^{two}\right) Lw_A\right] > c \\ \Longleftrightarrow \quad & \triangle (w_A + Lw_A) > c \end{aligned}$$

$$\begin{aligned} A_{LH} > A_{HH} \quad \Longleftrightarrow \quad & A_{LH} - A_{HH} > 0 \\ \Longleftrightarrow \quad & -\triangle (1 - H) [w_A + p^{one} w_B + (1 - p^{one}) Lw_A] - \triangle H \left[w_A + p^{two} w_B + \left(1 - p^{two}\right) Lw_A\right] + c > 0 \\ \Longleftrightarrow \quad & \triangle (1 - H) [w_A + p^{one} w_B + (1 - p^{one}) Lw_A] + \triangle H \left[w_A + p^{two} w_B + \left(1 - p^{two}\right) Lw_A\right] < c \\ \Longleftrightarrow \quad & \triangle (w_A + Lw_A) < c. \end{aligned}$$

There is no solution for this case.

1.7.4 Proof for Lemma 1.1

There are three kinds of outcomes as follows:

- $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$; $\Pi^* (4H) = 4H \left(y_A - \frac{c}{\triangle} \right) + \frac{\underline{y}+\overline{y}}{2} - \mu$.
- $w_A^* = \frac{c}{\triangle(1+L)}$ and $w_B^* = \mu$; $\Pi^* (2H\&2L) = (2L + 2H) \left(y_A - \frac{c}{\triangle(1+L)} \right) + \frac{\underline{y}+\overline{y}}{2} - \mu$.
- $w_A^* = 0$ and $w_B^* = \mu$; $\Pi^* (4L) = 4Ly_A + \frac{\underline{y}+\overline{y}}{2} - \mu$.

Note that $w_A^* = \frac{c}{\triangle(1+L)}$ and $w_B^* = \mu$; $\Pi^*(H\&3L) = (2L+2H)\left(y_A - \frac{c}{\triangle(1+L)}\right) + \frac{y+\overline{y}}{2} - \mu$ is not optimal:

$$\begin{aligned}
\Pi^*(H\&3L) = (2L+2H)\left(y_A - \frac{c}{\triangle(1+L)}\right) + \frac{y+\overline{y}}{2} - \mu &> \Pi^*(4L) = 4Ly_A + \frac{y+\overline{y}}{2} - \mu \\
y_A &> \frac{(L+H)c}{\triangle^2(1+L)} \\
\Pi^*(H\&3L) = (2L+2H)\left(y_A - \frac{c}{\triangle(1+L)}\right) + \frac{y+\overline{y}}{2} - \mu &> \Pi^*(4H) = 4H\left(y_A - \frac{c}{\triangle}\right) + \frac{y+\overline{y}}{2} - \mu \\
y_A &< \frac{2Hc}{\triangle^2} - \frac{(L+H)c}{\triangle^2(1+L)} \\
\frac{(L+H)c}{\triangle^2(1+L)} &> \frac{2Hc}{\triangle^2} - \frac{(L+H)c}{\triangle^2(1+L)} \\
L+H &> H(1+L) \\
L &> HL.
\end{aligned}$$

The optimal wage policy (w_A, w_B) for only external hiring policy is given by

- $y_A < \frac{Hc}{\triangle^2}$. $w_A^* = 0$ and $w_B^* = \mu$; $\Pi^*(4L) = 4Ly_A + \frac{y+\overline{y}}{2} - \mu$

$$\begin{aligned}
\Pi^*(4L) = 4Ly_A + \frac{y+\overline{y}}{2} - \mu &> \Pi^*(2H\&2L) = (2L+2H)\left(y_A - \frac{c}{\triangle(1+L)}\right) + \frac{y+\overline{y}}{2} - \mu \\
y_A &< \frac{(L+H)c}{\triangle^2(1+L)} \\
\Pi^*(4L) = 4Ly_A + \frac{y+\overline{y}}{2} - \mu &> \Pi^*(4H) = 4H\left(y_A - \frac{c}{\triangle}\right) + \frac{y+\overline{y}}{2} - \mu \\
y_A &< \frac{Hc}{\triangle^2} \\
\frac{(L+H)c}{\triangle^2(1+L)} &> \frac{Hc}{\triangle^2} \\
L+H &> H(1+L) \\
L &> HL.
\end{aligned}$$

- $y_A \geq \frac{Hc}{\triangle^2}$. $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$; $\Pi^*(4H) = 4H\left(y_A - \frac{c}{\triangle}\right) + \frac{y+\overline{y}}{2} - \mu$

$$\begin{aligned}
\Pi^*(4H) = 4H\left(y_A - \frac{c}{\triangle}\right) + \frac{y+\overline{y}}{2} - \mu &> \Pi^*(2H\&2L) = (2L+2H)\left(y_A - \frac{c}{\triangle(1+L)}\right) + \frac{y+\overline{y}}{2} - \mu \\
y_A &> \frac{2Hc}{\triangle^2} - \frac{(L+H)c}{\triangle^2(1+L)} \\
\Pi^*(4H) = 4H\left(y_A - \frac{c}{\triangle}\right) + \frac{y+\overline{y}}{2} - \mu &> \Pi^*(4L) = 4Ly_A + \frac{y+\overline{y}}{2} - \mu \\
y_A &> \frac{Hc}{\triangle^2} \\
\frac{Hc}{\triangle^2} &> \frac{2Hc}{\triangle^2} - \frac{(L+H)c}{\triangle^2(1+L)} \\
L+H &> H(1+L) \\
L &> HL.
\end{aligned}$$

1.7.5 Nash Equilibrium with Commitment

(1) (L, L) is the only NE.

The firm's problem is

$$\max_{\{w_A, w_B\}} \mathit{\Pi}(4L) = 4L(y_A - w_A) + E(\widehat{y_B}) - w_B$$

s.t $w_A < \frac{c}{\triangle}$, $w_B \geq \mu$, $A_{LL} > A_{HL}$, and $A_{LH} > A_{HH}$

$$\begin{aligned}
A_{LL} > A_{HL} \quad &\Longleftrightarrow \quad A_{LL} - A_{HL} > 0 \\
&\Longleftrightarrow \quad -\triangle(1-L)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] - \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c > 0 \\
&\Longleftrightarrow \quad D_1 = \triangle(1-L)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] + \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] < c \\
&\Longleftrightarrow \quad D_1 = \triangle(1-L)\left[w_A + w_B\right] + \triangle L\left[w_A + \tfrac{1}{2}w_B + \tfrac{1}{2}Lw_A\right] < c
\end{aligned}$$

$$\begin{aligned}
A_{LH} > A_{HH} \quad &\Longleftrightarrow \quad A_{LH} - A_{HH} > 0 \\
&\Longleftrightarrow \quad -\triangle(1-H)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] - \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c > 0 \\
&\Longleftrightarrow \quad D_2 = \triangle(1-H)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] + \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] < c \\
&\Longleftrightarrow \quad D_2 = \triangle(1-H)\left[w_A + w_B\right] + \triangle H\left[w_A + \tfrac{1}{2}w_B + \tfrac{1}{2}Lw_A\right] < c.
\end{aligned}$$

To maximize profits, we must have $w_A^* = 0$ and $w_B^* = \mu$, and $\mu < \frac{c}{\triangle\left(1-\frac{L}{2}\right)}$,

$$\begin{aligned}
\mathit{\Pi}^*(4L) &= 4Ly_A + 2L(1-L)\left(\frac{\underline{y}+\overline{y}}{2}\right) + L^2\left(\frac{\underline{y}+2\overline{y}}{3}\right) + (1-L)^2\left(\frac{\underline{y}+\overline{y}}{2}\right) - \mu \\
&= 4Ly_A + \frac{\underline{y}+\overline{y}}{2} + L^2\left(\frac{\overline{y}-\underline{y}}{6}\right) - \mu. \qquad (1.12)
\end{aligned}$$

(2) (H, H) is the only NE.

The firm's problem is

$$\max_{\{w_A, w_B\}} \mathit{\Pi}(2H\&2L) = (2L + 2H)(y_A - w_A) + E(\widehat{y_B}) - w_B$$

s.t $w_A < \frac{c}{\triangle}$, $w_B \geq \mu$, $A_{LL} < A_{HL}$, and $A_{LH} < A_{HH}$

$$\begin{aligned} A_{LL} < A_{HL} \quad & \Longleftrightarrow \quad A_{LL} - A_{HL} < 0 \\ & \Longleftrightarrow \quad -\triangle(1-L)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] - \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c < 0 \\ & \Longleftrightarrow \quad D_1 = \triangle(1-L)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] + \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] > c \\ & \Longleftrightarrow \quad D_1 = \triangle(1-L)\left[w_A + w_B\right] + \triangle L\left[w_A + \tfrac{1}{2}w_B + \tfrac{1}{2}Lw_A\right] > c \end{aligned}$$

$$\begin{aligned} A_{LH} < A_{HH} \quad & \Longleftrightarrow \quad A_{LH} - A_{HH} < 0 \\ & \Longleftrightarrow \quad -\triangle(1-H)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] - \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c < 0 \\ & \Longleftrightarrow \quad D_2 = \triangle(1-H)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] + \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] > c \\ & \Longleftrightarrow \quad D_2 = \triangle(1-H)\left[w_A + w_B\right] + \triangle H\left[w_A + \tfrac{1}{2}w_B + \tfrac{1}{2}Lw_A\right] > c. \end{aligned}$$

Since $D_1 - D_2 = \triangle^2\left(\frac{w_B}{2} - \frac{Lw_A}{2}\right) > 0 \left(w_B > \mu > \frac{c}{\triangle} > w_A\right)$, then $IC(A_{LL} < A_{HL})$ is slack, and $IC(A_{LH} < A_{HH})$ is binding. We only need consider $D_2 > c$. In fact, D_2 is the IC constraint given that the rival worker chooses high effort.

- If $\mu \geq \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$, the minimum wage can make the IC constraint hold. The wage policy is $w_A^* = 0$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\Pi^*(2H\&2L) = (2L+2H)\,y_A + \frac{\underline{y}+\overline{y}}{2} + H^2\left(\frac{\overline{y}-\underline{y}}{6}\right) - \mu. \quad \textbf{(1.13)}$$

- If $\mu < \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$, the minimum wage cannot make the IC constraint hold. The firm can increase either w_A or w_B to make the IC constraint hold. We need to check (1.14) and (1.15). We find that when $2L+2H-\frac{3HL}{2}-H^2 > 1$, that is $(2L+2H)\frac{\frac{c}{\triangle}-\mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}} > \frac{c}{\triangle\left(1-\frac{H}{2}\right)} - \mu$. Therefore,

  ✧ If $2L+2H-\frac{3HL}{2}-H^2 < 1$, the wage policy is $w_A^* = \frac{\frac{c}{\triangle}-\mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}$

and $w_B^* = \mu$,

$$\Pi^*\left(2H\&2L\right) = \left(2L+2H\right)\left(y_A - \frac{\frac{c}{\triangle} - \mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}\right) + \frac{\underline{y}+\overline{y}}{2} + H^2\left(\frac{\overline{y}-\underline{y}}{6}\right) - \mu. \tag{1.14}$$

✧ If $2L+2H-\frac{3HL}{2}-H^2 > 1$, the wage policy is $w_A^* = 0$ and $w_B^* = \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$, and the total payoff for the firm is

$$\Pi^*\left(2H\&2L\right) = \left(2L+2H\right)y_A + \frac{\underline{y}+\overline{y}}{2} + H^2\left(\frac{\overline{y}-\underline{y}}{6}\right) - \frac{c}{\triangle\left(1-\frac{H}{2}\right)}. \tag{1.15}$$

(3) (H,L) and (L,H) are the NEs.

The firm's problem is

$$\underset{\{w_A,w_B\}}{max}\ \Pi\left(H\&3L\right) = \left(3L+H\right)\left(y_A - w_A\right) + E\left(\widehat{y_B}\right) - w_B$$

s.t $w_A < \frac{c}{\triangle}$, $w_B \geq \mu$, $A_{LL} < A_{HL}$, and $A_{LH} > A_{HH}$

$$\begin{aligned} A_{LL} < A_{HL} &\iff A_{LL} - A_{HL} < 0 \\ &\iff -\triangle\left(1-L\right)\left[w_A + p^{one}w_B + \left(1-p^{one}\right)Lw_A\right] - \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c < 0 \\ &\iff D_1 = \triangle\left(1-L\right)\left[w_A + p^{one}w_B + \left(1-p^{one}\right)Lw_A\right] + \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] > c \\ &\iff D_1 = \triangle\left(1-L\right)\left[w_A + w_B\right] + \triangle L\left[w_A + \tfrac{1}{2}w_B + \tfrac{1}{2}Lw_A\right] > c \end{aligned}$$

$$\begin{aligned} A_{LH} > A_{HH} &\iff A_{LH} - A_{HH} > 0 \\ &\iff -\triangle\left(1-H\right)\left[w_A + p^{one}w_B + \left(1-p^{one}\right)Lw_A\right] - \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c > 0 \\ &\iff D_2 = \triangle\left(1-H\right)\left[w_A + p^{one}w_B + \left(1-p^{one}\right)Lw_A\right] + \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] < c \\ &\iff D_2 = \triangle\left(1-H\right)\left[w_A + w_B\right] + \triangle H\left[w_A + \tfrac{1}{2}w_B + \tfrac{1}{2}Lw_A\right] < c. \end{aligned}$$

To let $A_{LL} < A_{HL}$ and $A_{LH} > A_{HH}$, we must have $\frac{c}{\triangle\left(1-\frac{L}{2}\right)} < \mu < \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$, and the wage policy is $w_A^* = 0$ and $w_B^* = \mu$; the total payoff for the firm is

$$\Pi^*\left(H\&3L\right) = \left(3L+H\right)y_A + \frac{\underline{y}+\overline{y}}{2} + HL\left(\frac{\overline{y}-\underline{y}}{6}\right) - \mu. \tag{1.16}$$

1.7.6 Proof for Lemma 1.2

(1) $\mu < \frac{c}{\triangle\left(1-\frac{L}{2}\right)}$

There are three kinds of outcomes as follows:

- $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(4L) = (1.12)$.
- $w_A^* = \frac{\frac{c}{\triangle}-\mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}$ and $w_B^* = \mu\,(2L+2H-\frac{3HL}{2}-H^2 < 1)$, $\Pi^*(2H\&2L) = (1.14)$ or $w_A^* = 0$ and $w_B^* = \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$ $(2L+2H-\frac{3HL}{2}-H^2 > 1)$, $\Pi^*(2H\&2L) = (1.15)$.
- $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (1.11)$.

When $2L + 2H - \frac{3HL}{2} - H^2 > 1$, we have

$$\begin{aligned}
(1.12) < (1.15) \quad &\Longleftrightarrow \quad y_A > \frac{1}{2\triangle}\left[\frac{c}{\triangle\left(1-\frac{H}{2}\right)} - \mu\right] - \frac{H+L}{2}\left(\frac{\overline{y}-\underline{y}}{6}\right) = \theta_1 \\
(1.12) < (1.11) \quad &\Longleftrightarrow \quad y_A > \frac{Hc}{\triangle^2} - \frac{H+L}{4}\left(\frac{\overline{y}-\underline{y}}{6}\right) \\
(1.11) > (1.15) \quad &\Longleftrightarrow \quad y_A > \frac{2Hc}{\triangle^2} - \frac{1}{2\triangle}\left[\frac{c}{\triangle\left(1-\frac{H}{2}\right)} - \mu\right] = \theta_2.
\end{aligned}$$

I assume $\left(\overline{y}-\underline{y}\right)$ is not high enough; this means the difference between the highest type of worker and the lowest type of worker is small, which makes $\theta_1 > 0$ and $\frac{Hc}{\triangle^2} - \frac{H+L}{4}\left(\frac{\overline{y}-\underline{y}}{6}\right) > 0$. Otherwise, $\theta_1 < 0$ and $\frac{Hc}{\triangle^2} - \frac{H+L}{4}\left(\frac{\overline{y}-\underline{y}}{6}\right)$ <0, (1.11) and (1.15) are always greater than (1.12), and $w_A^* = 0$ and $w_B^* = \mu$ would never be the optimal wage policy. For this case, the firm prefers a high effort level in period 1 because of higher expected outputs in job B, not because of higher expected outputs in job A. Also, note that I assume $\mu > \frac{c}{\triangle}$; then, we have

$$\frac{Hc}{\triangle^2} > \frac{1}{2\triangle}\left[\frac{c}{\triangle\left(1-\frac{H}{2}\right)} - \frac{c}{\triangle}\right] > \frac{1}{2\triangle}\left[\frac{c}{\triangle\left(1-\frac{H}{2}\right)} - \mu\right].$$

Therefore,

$$\theta_1 < \frac{Hc}{\triangle^2} - \frac{H+L}{4}\left(\frac{\overline{y}-\underline{y}}{6}\right) < \theta_2.$$

When $2L + 2H - \frac{3HL}{2} - H^2 < 1$, we have

$$
\begin{aligned}
(1.12) < (1.14) &\iff y_A > \frac{L+H}{\triangle}\left[\frac{\frac{c}{\triangle}-\mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}\right] - \frac{H+L}{2}\left(\frac{\overline{y}-\underline{y}}{6}\right) = \theta_1^{'} \\
(1.12) < (1.11) &\iff y_A > \frac{Hc}{\triangle^2} - \frac{H+L}{4}\left(\frac{\overline{y}-\underline{y}}{6}\right) \\
(1.11) > (1.14) &\iff y_A > \frac{2Hc}{\triangle^2} - \frac{L+H}{\triangle}\left[\frac{\frac{c}{\triangle}-\mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}\right] = \theta_2^{'}.
\end{aligned}
$$

Similarly, we have $\theta_1^{'} < \frac{Hc}{\triangle^2} - \frac{H+L}{4}\left(\frac{\overline{y}-\underline{y}}{6}\right) < \theta_2^{'}$.

The optimal wage policy (w_A, w_B) for commitment policy is given by,

- If $y_A < \theta_1$ or $\theta_1^{'}$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \mu$, $\mathit{\Pi}^*(4L) = (1.12)$.
- If $\theta_1 < y_A < \theta_2$ or $\theta_1^{'} < y_A < \theta_2^{'}$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$ $(2L + 2H - \frac{3HL}{2} - H^2 > 1)$, $\mathit{\Pi}^*(2H\&2L) = (1.15)$ (or $w_A^* = \frac{\frac{c}{\triangle}-\mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}$ and $w_B^* = \mu$ $(2L + 2H - \frac{3HL}{2} - H^2 < 1)$, $\mathit{\Pi}^*(2H\&2L) = (1.14)$).
- If $\theta_2 < y_A$ or $\theta_2^{'}$, the optimal wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\mathit{\Pi}^*(4H) = (1.11)$.

(2) $\frac{c}{\triangle\left(1-\frac{L}{2}\right)} < \mu < \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$

There are three kinds of outcomes as follows:

- $w_A^* = 0$ and $w_B^* = \mu$, $\mathit{\Pi}^*(H\&3L) = (1.16)$.
- $w_A^* = \frac{\frac{c}{\triangle}-\mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}$ and $w_B^* = \mu$ $(2L+2H-\frac{3HL}{2}-H^2 < 1)$, $\mathit{\Pi}^*(2H\&2L) = (1.14)$ or $w_A^* = 0$ and $w_B^* = \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$ $(2L + 2H - \frac{3HL}{2} - H^2 > 1)$, $\mathit{\Pi}^*(2H\&2L) = (1.15)$.
- $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\mathit{\Pi}^*(4H) = (1.11)$.

When $2L + 2H - \frac{3HL}{2} - H^2 > 1$, we have

$$
\begin{aligned}
(1.16) < (1.15) &\iff y_A > \frac{1}{\triangle}\left[\frac{c}{\triangle\left(1-\frac{H}{2}\right)} - \mu\right] - H\left(\frac{\overline{y}-\underline{y}}{6}\right) = \theta_3 \\
(1.16) < (1.11) &\iff y_A > \frac{4Hc}{3\triangle^2} - \frac{H}{3}\left(\frac{\overline{y}-\underline{y}}{6}\right) \\
(1.11) > (1.15) &\iff y_A > \frac{2Hc}{\triangle^2} - \frac{1}{2\triangle}\left[\frac{c}{\triangle\left(1-\frac{H}{2}\right)} - \mu\right] = \theta_2.
\end{aligned}
$$

Similarly, we have $\theta_3 < \frac{4Hc}{3\triangle^2} - \frac{H}{3}\left(\frac{\overline{y}-\underline{y}}{6}\right) < \theta_2$.

When $2L + 2H - \frac{3HL}{2} - H^2 < 1$, we have

$$
\begin{aligned}
(1.16) < (1.14) &\iff y_A > \frac{2L+2H}{\triangle}\left[\frac{\frac{c}{\triangle} - \mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}\right] - H\left(\frac{\overline{y}-\underline{y}}{6}\right) = \theta_3' \\
(1.16) < (1.11) &\iff y_A > \frac{4Hc}{3\triangle^2} - \frac{H}{3}\left(\frac{\overline{y}-\underline{y}}{6}\right) \\
(1.11) > (1.14) &\iff y_A > \frac{2Hc}{\triangle^2} - \frac{L+H}{\triangle}\left[\frac{\frac{c}{\triangle} - \mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}\right] = \theta_2'.
\end{aligned}
$$

Similarly, we have $\theta_3' < \frac{4Hc}{3\triangle^2} - \frac{H}{3}\left(\frac{\overline{y}-\underline{y}}{6}\right) < \theta_2'$.

The optimal wage policy (w_A, w_B) for commitment policy is given by,

- If $y_A < \theta_3$ or θ_3', the optimal wage policy is $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(H\&3L) = (1.16)$.
- If $\theta_3 < y_A < \theta_2$ or $\theta_3' < y_A < \theta_2'$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$ ($2L + 2H - \frac{3HL}{2} - H^2 > 1$), $\Pi^*(2H\&2L) = (1.15)$ (or $w_A^* = \frac{\frac{c}{\triangle} - \mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}$ and $w_B^* = \mu$ ($2L + 2H - \frac{3HL}{2} - H^2 < 1$), $\Pi^*(2H\&2L) = (1.14)$).
- If $\theta_2 < y_A$ or θ_2', the optimal wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (1.11)$.

(3) $\mu \geq \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$

There are two kinds of outcomes as follows:

- $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(2H\&2L) = (1.13)$.

- $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (1.11)$.

We have $(1.11) > (1.13) \iff y_A > \frac{2Hc}{\triangle^2} = \theta_4$.

The optimal wage policy (w_A, w_B) for commitment policy is given by,

- If $y_A < \theta_4$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(2H\&2L) = (1.13)$.

- If $\theta_4 < y_A$, the optimal wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (1.11)$.

1.7.7 Nash Equilibrium with No Commitment

(1) (L, L) is the only NE.

The firm's problem is

$$\max_{\{w_A, w_B\}} \Pi(4L) = 4L(y_A - w_A) + E(\widehat{y_B}) - w_B$$

s.t $w_A < \frac{c}{\triangle}$, $w_B \geq \mu$, $A_{LL} > A_{HL}$, and $A_{LH} > A_{HH}$

$$\begin{aligned}
A_{LL} > A_{HL} \iff & A_{LL} - A_{HL} > 0 \\
\iff & -\triangle(1-L)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] - \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c > 0 \\
\iff & D_3 = \triangle(1-L)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] + \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] < c \\
\iff & D_3 = \triangle(1-L)\left[w_A + \tfrac{1}{2}w_B + \tfrac{1}{2}Lw_A\right] + \triangle L\left[w_A + \tfrac{3}{8}w_B + \tfrac{5}{8}Lw_A\right] < c
\end{aligned}$$

$$\begin{aligned}
A_{LH} > A_{HH} \iff & A_{LH} - A_{HH} > 0 \\
\iff & -\triangle(1-H)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] - \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c > 0 \\
\iff & D_4 = \triangle(1-H)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] + \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] < c \\
\iff & D_4 = \triangle(1-H)\left[w_A + \tfrac{1}{2}w_B + \tfrac{1}{2}Lw_A\right] + \triangle H\left[w_A + \tfrac{3}{8}w_B + \tfrac{5}{8}Lw_A\right] < c.
\end{aligned}$$

To maximize profits, we must have $w_A^* = 0$ and $w_B^* = \mu$, and $\mu < \frac{c}{\triangle\left(\frac{1}{2}-\frac{L}{8}\right)}$,

$$\begin{aligned} \Pi^*(4L) &= 4Ly_A + 2L(1-L)\left(\frac{3\underline{y}+5\overline{y}}{8}\right) + L^2\left(\frac{7\underline{y}+17\overline{y}}{24}\right) + (1-L)^2\left(\frac{\underline{y}+\overline{y}}{2}\right) - \mu \\ &= 4Ly_A + \frac{\underline{y}+\overline{y}}{2} + \left(L - \frac{L^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right) - \mu. \end{aligned} \tag{1.19}$$

(2) (H, H) is the only NE.

The firm's problem is

$$\max_{\{w_A, w_B\}} \Pi(2H\&2L) = (2L+2H)(y_A - w_A) + E\left(\widehat{y_B}\right) - w_B$$

s.t $w_A < \frac{c}{\triangle}$, $w_B \geq \mu$, $A_{LL} < A_{HL}$, and $A_{LH} < A_{HH}$

$$\begin{aligned} A_{LL} < A_{HL} &\iff A_{LH} - A_{HL} < 0 \\ &\iff -\triangle(1-L)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] - \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c < 0 \\ &\iff D_3 = \triangle(1-L)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] + \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] > c \\ &\iff D_3 = \triangle(1-L)\left[w_A + \tfrac{1}{2}w_B + \tfrac{1}{2}Lw_A\right] + \triangle L\left[w_A + \tfrac{3}{8}w_B + \tfrac{5}{8}Lw_A\right] > c \end{aligned}$$

$$\begin{aligned} A_{LH} < A_{HH} &\iff A_{LH} - A_{HH} < 0 \\ &\iff -\triangle(1-H)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] - \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c < 0 \\ &\iff D_4 = \triangle(1-H)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] + \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] > c \\ &\iff D_4 = \triangle(1-H)\left[w_A + \tfrac{1}{2}w_B + \tfrac{1}{2}Lw_A\right] + \triangle H\left[w_A + \tfrac{3}{8}w_B + \tfrac{5}{8}Lw_A\right] > c. \end{aligned}$$

Since $D_3 - D_4 = \triangle^2\left(\frac{w_B}{8} - \frac{Lw_A}{8}\right) > 0$ $\left(w_B > \mu > \frac{c}{\triangle} > w_A\right)$, $IC(A_{LL} < A_{HL})$ is slack. and $IC(A_{LH} < A_{HH})$ is binding. We only need consider $D_4 > c$. In fact, D_4 is the IC constraint given that the rival worker chooses high effort.

- If $\mu \geq \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)}$, the minimum wage can make the IC constraint hold. The wage policy is $w_A^* = 0$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\Pi^*\left(2H\&2L\right) = \left(2L+2H\right)y_A + \frac{\underline{y}+\overline{y}}{2} + \left(H-\frac{H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right) - \mu .1.20$$

- If $\mu < \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)}$, the minimum wage cannot make the IC constraint hold. The firm can either increase w_A or w_B to make the IC constraint hold. I need to check (1.21) and (1.23). I find that $\left(2L+2H\right)\frac{\frac{c}{\triangle}-\mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}} < \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)} - \mu$. Therefore, the wage policy is $w_A^* = \frac{\frac{c}{\triangle}-\mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}}$ and $w_B^* = \mu$, and the total payoff for the firm is (1.21).

  ✧ If the wage policy is $w_A^* = \frac{\frac{c}{\triangle}-\mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}}$ and $w_B^* = \mu$, the total payoff for the firm is

$$\Pi^*\left(2H\&2L\right) = \left(2L+2H\right)\left(y_A - \frac{\frac{c}{\triangle}-\mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}}\right) + \frac{\underline{y}+\overline{y}}{2} + \left(H-\frac{H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right) - \mu. \quad (1.21)$$

  ✧ If the wage policy is $w_A^* = 0$ and $w_B^* = \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)}$, the total payoff for the firm is

$$\Pi^*\left(2H\&2L\right) = \left(2L+2H\right)y_A + \frac{\underline{y}+\overline{y}}{2} + \left(H-\frac{H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right) - \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)}. \quad (1.23)$$

(3) (H,L) and (L,H) are the NEs.

The firm's problem is

$$\underset{\{w_A,w_B\}}{max}\ \Pi\left(H3\&L\right) = \left(3L+H\right)\left(y_A-w_A\right) + E\left(\widehat{y_B}\right) - w_B$$

s.t $w_A < \frac{c}{\triangle}$, $w_B \geq \mu$, $A_{LL} < A_{HL}$, and $A_{LH} > A_{HH}$

$$\begin{aligned} A_{LL} < A_{HL} &\iff A_{LL} - A_{HL} < 0 \\ &\iff -\triangle(1-L)\left[w_A + p^{one} w_B + (1-p^{one}) L w_A\right] - \triangle L\left[w_A + p^{two} w_B + \left(1-p^{two}\right) L w_A\right] + c < 0 \\ &\iff D_3 = \triangle(1-L)\left[w_A + p^{one} w_B + (1-p^{one}) L w_A\right] + \triangle L\left[w_A + p^{two} w_B + \left(1-p^{two}\right) L w_A\right] > c \\ &\iff D_3 = \triangle(1-L)\left[w_A + \tfrac{1}{2} w_B + \tfrac{1}{2} L w_A\right] + \triangle L\left[w_A + \tfrac{3}{8} w_B + \tfrac{5}{8} L w_A\right] > c \end{aligned}$$

$$\begin{aligned} A_{LH} > A_{HH} &\iff A_{LH} - A_{HH} > 0 \\ &\iff -\triangle(1-H)\left[w_A + p^{one} w_B + (1-p^{one}) L w_A\right] - \triangle H\left[w_A + p^{two} w_B + \left(1-p^{two}\right) L w_A\right] + c > 0 \\ &\iff D_4 = \triangle(1-H)\left[w_A + p^{one} w_B + (1-p^{one}) L w_A\right] + \triangle H\left[w_A + p^{two} w_B + \left(1-p^{two}\right) L w_A\right] < c \\ &\iff D_4 = \triangle(1-H)\left[w_A + \tfrac{1}{2} w_B + \tfrac{1}{2} L w_A\right] + \triangle H\left[w_A + \tfrac{3}{8} w_B + \tfrac{5}{8} L w_A\right] < c. \end{aligned}$$

To let $A_{LL} < A_{HL}$, and $A_{LH} > A_{HH}$, We must have $\frac{c}{\triangle\left(\frac{1}{2}-\frac{L}{8}\right)} < \mu < \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)}$, and the wage policy is $w_A^* = 0$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\begin{aligned} \Pi^*(H\&3L) &= (3L+H)\,y_A + H(1-L)\left(\frac{3\underline{y}+5\overline{y}}{8}\right) \\ &+ L(1-H)\left(\frac{3\underline{y}+5\overline{y}}{8}\right) + HL\left(\frac{7\underline{y}+17\overline{y}}{24}\right) \\ &+ (1-H)(1-L)\left(\frac{\underline{y}+\overline{y}}{2}\right) - \mu \\ &= (3L+H)\,y_A + \frac{\underline{y}+\overline{y}}{2} + \left(H+L-\frac{HL}{3}\right)\left(\frac{\overline{y}-\underline{y}}{8}\right) - \mu. \end{aligned} \tag{1.24}$$

1.7.8 Proof for Lemma 1.3

(1) $\mu < \frac{c}{\triangle\left(\frac{1}{2}-\frac{L}{8}\right)}$

There are three kinds of outcomes as follows:

- $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(4L) = (1.19)$.
- $w_A^* = \frac{\frac{c}{\triangle} - \mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}}$ and $w_B^* = \mu$, $\Pi^*(2H\&2L) = (1.21)$.
- $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (1.18)$.

$$\begin{aligned}
(1.19) < (1.21) &\iff y_A > \tfrac{L+H}{\triangle}\left[\frac{\frac{c}{\triangle}-\mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}}\right] - \left(\tfrac{1}{2}-\tfrac{H+L}{12}\right)\left(\tfrac{\overline{y}-\underline{y}}{4}\right) = \theta_5 \\
(1.19) < (1.18) &\iff y_A > \tfrac{Hc}{\triangle^2} - \left(\tfrac{1}{4}-\tfrac{H+L}{24}\right)\left(\tfrac{\overline{y}-\underline{y}}{4}\right) \\
(1.18) > (1.21) &\iff y_A > \tfrac{2Hc}{\triangle^2} - \tfrac{L+H}{\triangle}\left[\frac{\frac{c}{\triangle}-\mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}}\right] = \theta_6.
\end{aligned}$$

Similar to Lemma 1.2, I assume $(\overline{y}-\underline{y})$ is not high enough which means that the difference between the highest type of worker and the lowest type of worker is small, which makes $\theta_5 > 0$ and $\frac{Hc}{\triangle^2} - \left(\frac{1}{4}-\frac{H+L}{24}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right) > 0$. Otherwise, $\theta_5 < 0$ and $\frac{Hc}{\triangle^2} - \left(\frac{1}{4}-\frac{H+L}{24}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right)$<0, (1.18) and (1.21) are always greater than (1.19), and $w_A^* = 0$ and $w_B^* = \mu$ would never be the optimal wage policy. For this case, the firm prefers a high effort level in period 1 because of higher expected outputs in job B, not because of higher expected outputs in job A. Then,

$$\theta_5 < \frac{Hc}{\triangle^2} - \left(\frac{1}{4}-\frac{H+L}{24}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right) < \theta_6.$$

The optimal wage policy (w_A, w_B) for the commitment policy is given by,

- If $y_A < \theta_5$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(4L) = (1.19)$.
- If $\theta_5 < y_A < \theta_6$, the optimal wage policy is $w_A^* = \frac{\frac{c}{\triangle}-\mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}}$ and $w_B^* = \mu$, $\Pi^*(2H\&2L) = (1.21)$.
- If $\theta_6 < y_A$, the optimal wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (1.18)$.

(2) $\frac{c}{\triangle\left(\frac{1}{2}-\frac{L}{8}\right)} < \mu < \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)}$

There are three kinds of outcomes as follows:

- $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(H\&3L) = (1.22)$.

- $w_A^* = \frac{\frac{c}{\triangle}-\mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}}$ and $w_B^* = \mu$, $\Pi^*(2H\&2L) = (1.21)$.
- $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (1.18)$.

We have

$$\begin{aligned}
(1.22) < (1.21) &\iff y_A > \frac{2L+2H}{\triangle}\left[\frac{\frac{c}{\triangle}-\mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}}\right] - \left(\frac{1}{2}-\frac{H}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right) = \theta_7 \\
(1.22) < (1.18) &\iff y_A > \frac{4Hc}{3\triangle^2} - \left(\frac{1}{6}-\frac{H}{18}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right) \\
(1.18) > (1.21) &\iff y_A > \frac{2Hc}{\triangle^2} - \frac{L+H}{\triangle}\left[\frac{\frac{c}{\triangle}-\mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}}\right] = \theta_6.
\end{aligned}$$

Similarly, we have $\theta_7 < \frac{4Hc}{3\triangle^2} - \left(\frac{1}{6}-\frac{H}{18}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right) < \theta_6$.

The optimal wage policy (w_A, w_B) for the commitment policy is given by,

- If $y_A < \theta_7$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(H\&3L) = (1.22)$.
- If $\theta_7 < y_A < \theta_6$, the optimal wage policy is $w_A^* = \frac{\frac{c}{\triangle}-\mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}}$ and $w_B^* = \mu$, $\Pi^*(2H\&2L) = (1.21)$.
- If $\theta_6 < y_A$, the optimal wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (1.18)$.

(3) $\mu \geq \frac{c}{\triangle\left(\frac{1}{2}-\frac{H}{8}\right)}$

There are two kinds of outcomes as follows:

- $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(2H\&2L) = (1.20)$.
- $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (1.18)$.

We have $(1.18) > (1.20) \iff y_A > \frac{2Hc}{\triangle^2} = \theta_4$.

The optimal wage policy (w_A, w_B) for the commitment policy is given by,

- If $y_A < \theta_4$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(2H\&2L) = (1.20)$.

- If $\theta_4 < y_A$, the optimal wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_A^* = \mu$, $\Pi^*(4H) = (1.18)$.

1.7.9 Proof for Proposition 1.3

(1) $\mu < \frac{c}{\triangle\left(\frac{1}{2}-\frac{L}{8}\right)}$ and $2L + 2H - \frac{3HL}{2} - H^2 > 1$:

Table 1.6: Outcome Comparison Under Case (1)

Promotion policy	Commitment		No commitment	
	(w_A^*, w_B^*)	Π^*	(w_A^*, w_B^*)	Π^*
$y_A < \theta_1$	$(0, \mu)$	(1.12)	$(0, \mu)$	(1.19)
$\theta_1 < y_A < \theta_5$	$(0, \frac{c}{\triangle\left(1-\frac{H}{2}\right)})$	(1.15)	$(0, \mu)$	(1.19)
$\theta_5 < y_A < \theta_6$	$(0, \frac{c}{\triangle\left(1-\frac{H}{2}\right)})$	(1.15)	$(\frac{\frac{c}{\triangle}-\mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}}, \mu)$	(1.21)
$\theta_6 < y_A < \theta_2$	$(0, \frac{c}{\triangle\left(1-\frac{H}{2}\right)})$	(1.15)	$(\frac{c}{\triangle}, \mu)$	(1.18)
$\theta_2 < y_A$	$(\frac{c}{\triangle}, \mu)$	(1.11)	$(\frac{c}{\triangle}, \mu)$	(1.18)

$$(1.12) < (1.19).$$

$$(1.15) > (1.19) \iff y_A > \frac{1}{2\triangle}\left[\frac{c}{\triangle\left(1-\frac{H}{2}\right)} - \mu\right] + \frac{1}{2\triangle}\left(L - \frac{L^2}{6} - \frac{4H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right).$$

$$(1.15) > (1.21) \iff \frac{\frac{c}{\triangle} - \mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}} > \left(\frac{1}{2L+2H}\right)\left(\frac{c}{\triangle\left(1-\frac{H}{2}\right)} - \mu\right) + \left(H - \frac{5H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right).$$

$$(1.15) > (1.18) \iff y_A < \frac{2Hc}{\triangle^2} - \frac{1}{2\triangle}\left[\frac{c}{\triangle\left(1-\frac{H}{2}\right)} - \mu\right] - \frac{1}{2\triangle}\left(H - \frac{5H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right).$$

$$(1.11) < (1.18).$$

(2) $\mu < \frac{c}{\triangle\left(\frac{1}{2}-\frac{L}{8}\right)}$ and $2L + 2H - \frac{3HL}{2} - H^2 < 1$:

$$(1.12) < (1.19).$$

Table 1.7: Outcome Comparison Under Case (2)

Promotion policy	Commitment		No commitment	
	(w_A^*, w_B^*)	Π^*	(w_A^*, w_B^*)	Π^*
$y_A < \theta_1^{'}$	$(0, \mu)$	**(1.12)**	$(0, \mu)$	**(1.19)**
$\theta_1^{'} < y_A < \theta_5$	$(\frac{\frac{c}{\triangle}-\mu(1-\frac{H}{2})}{1+\frac{HL}{2}}, \mu)$	**(1.14)**	$(0, \mu)$	**(1.19)**
$\theta_5 < y_A < \theta_6$	$(\frac{\frac{c}{\triangle}-\mu(1-\frac{H}{2})}{1+\frac{HL}{2}}, \mu)$	**(1.14)**	$(\frac{\frac{c}{\triangle}-\mu(\frac{1}{2}-\frac{H}{8})}{1+\frac{L}{2}+\frac{HL}{8}}, \mu)$	**(1.21)**
$\theta_6 < y_A < \theta_2^{'}$	$(\frac{\frac{c}{\triangle}-\mu(1-\frac{H}{2})}{1+\frac{HL}{2}}, \mu)$	**(1.14)**	$(\frac{c}{\triangle}, \mu)$	**(1.18)**
$\theta_2^{'} < y_A$	$(\frac{c}{\triangle}, \mu)$	**(1.11)**	$(\frac{c}{\triangle}, \mu)$	**(1.18)**

$$(1.14) > (1.19) \iff y_A > \frac{L+H}{\triangle}\left[\frac{\frac{c}{\triangle}-\mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}\right]+\frac{1}{2\triangle}\left(L-\frac{L^2}{6}-\frac{4H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right).$$

$$(1.14) > (1.21) \iff \left(\frac{\frac{c}{\triangle}-\mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}}-\frac{\frac{c}{\triangle}-\mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}\right) > \left(\frac{H-\frac{5H^2}{6}}{2L+2H}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right).$$

$$(1.14) > (1.18) \iff y_A < \frac{2Hc}{\triangle^2}-\frac{L+H}{\triangle}\left[\frac{\frac{c}{\triangle}-\mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}\right]-\frac{1}{2\triangle}\left(H-\frac{5H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right).$$

$$(1.11) < (1.18).$$

(3) $\frac{c}{\triangle\left(1-\frac{L}{2}\right)} < \mu < \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$ **and** $2L+2H-\frac{3HL}{2}-H^2 > 1$:

Table 1.8: Outcome Comparison Under Case (3)

Promotion policy	Commitment		No commitment	
	(w_A^*, w_B^*)	Π^*	(w_A^*, w_B^*)	Π^*
$y_A < \theta_3$	$(0, \mu)$	**(1.16)**	$(0, \mu)$	**(1.19)**
$\theta_3 < y_A < \theta_5$	$(0, \frac{c}{\triangle(1-\frac{H}{2})})$	**(1.15)**	$(0, \mu)$	**(1.19)**
$\theta_5 < y_A < \theta_6$	$(0, \frac{c}{\triangle(1-\frac{H}{2})})$	**(1.15)**	$(\frac{\frac{c}{\triangle}-\mu(\frac{1}{2}-\frac{H}{8})}{1+\frac{L}{2}+\frac{HL}{8}}, \mu)$	**(1.21)**
$\theta_6 < y_A < \theta_2$	$(0, \frac{c}{\triangle(1-\frac{H}{2})})$	**(1.15)**	$(\frac{c}{\triangle}, \mu)$	**(1.18)**
$\theta_2 < y_A$	$(\frac{c}{\triangle}, \mu)$	**(1.11)**	$(\frac{c}{\triangle}, \mu)$	**(1.18)**

$$(1.16) > (1.19) \iff y_A > \frac{1}{\triangle}\left(L-\frac{L^2}{6}-\frac{4HL}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right).$$

$$(1.15) > (1.19) \iff y_A > \frac{1}{2\triangle}\left[\frac{c}{\triangle\left(1-\frac{H}{2}\right)}-\mu\right]+\frac{1}{2\triangle}\left(L-\frac{L^2}{6}-\frac{4H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right).$$

$$(1.15) > (1.21) \iff \frac{\frac{c}{\triangle} - \mu\left(\frac{1}{2} - \frac{H}{8}\right)}{1 + \frac{L}{2} + \frac{HL}{8}} > \left(\frac{1}{2L+2H}\right)\left(\frac{c}{\triangle\left(1-\frac{H}{2}\right)} - \mu\right) + \left(H - \frac{5H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right).$$

$$(1.15) > (1.18) \iff y_A < \frac{2Hc}{\triangle^2} - \frac{1}{2\triangle}\left[\frac{c}{\triangle\left(1-\frac{H}{2}\right)} - \mu\right] - \frac{1}{2\triangle}\left(H - \frac{5H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right).$$

$$(1.11) < (1.18).$$

(4) $\frac{c}{\triangle\left(1-\frac{L}{2}\right)} < \mu < \frac{c}{\triangle\left(1-\frac{H}{2}\right)}$ **and** $2L + 2H - \frac{3HL}{2} - H^2 < 1$:

Table 1.9: Outcome Comparison Under Case (4)

Promotion policy	Commitment		No commitment	
	(w_A^*, w_B^*)	Π^*	(w_A^*, w_B^*)	Π^*
$y_A < \theta_3'$	$(0, \mu)$	**(1.16)**	$(0, \mu)$	**(1.19)**
$\theta_3' < y_A < \theta_5$	$\left(\frac{\frac{c}{\triangle}-\mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}, \mu\right)$	**(1.14)**	$(0, \mu)$	**(1.19)**
$\theta_5 < y_A < \theta_6$	$\left(\frac{\frac{c}{\triangle}-\mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}, \mu\right)$	**(1.14)**	$\left(\frac{\frac{c}{\triangle}-\mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}}, \mu\right)$	**(1.21)**
$\theta_6 < y_A < \theta_2'$	$\left(\frac{\frac{c}{\triangle}-\mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}, \mu\right)$	**(1.14)**	$\left(\frac{c}{\triangle}, \mu\right)$	**(1.18)**
$\theta_2' < y_A$	$\left(\frac{c}{\triangle}, \mu\right)$	**(1.11)**	$\left(\frac{c}{\triangle}, \mu\right)$	**(1.18)**

$$(1.16) > (1.19) \iff y_A > \frac{1}{\triangle}\left(L - \frac{L^2}{6} - \frac{4HL}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right).$$

$$(1.14) > (1.19) \iff y_A > \frac{L+H}{\triangle}\left[\frac{\frac{c}{\triangle} - \mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}\right] + \frac{1}{2\triangle}\left(L - \frac{L^2}{6} - \frac{4H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right).$$

$$(1.14) > (1.21) \iff \left(\frac{\frac{c}{\triangle} - \mu\left(\frac{1}{2}-\frac{H}{8}\right)}{1+\frac{L}{2}+\frac{HL}{8}} - \frac{\frac{c}{\triangle} - \mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}\right) > \left(\frac{H - \frac{5H^2}{6}}{2L+2H}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right).$$

$$(1.14) > (1.18) \iff y_A < \frac{2Hc}{\triangle^2} - \frac{L+H}{\triangle}\left[\frac{\frac{c}{\triangle} - \mu\left(1-\frac{H}{2}\right)}{1+\frac{HL}{2}}\right] - \frac{1}{2\triangle}\left(H - \frac{5H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right).$$

$$(1.11) < (1.18).$$

(5) $\frac{c}{\triangle\left(1-\frac{H}{2}\right)} < \mu < \frac{c}{\triangle\left(\frac{1}{2}-\frac{L}{8}\right)}$:

$$(1.13) > (1.19) \iff y_A > \frac{1}{\triangle}\left(L - \frac{L^2}{6} - \frac{4H^2}{6}\right)\left(\frac{\overline{y}-\underline{y}}{4}\right).$$

Table 1.10: Outcome Comparison Under Case (5)

Promotion policy	Commitment		No commitment	
	(w_A^*, w_B^*)	Π^*	(w_A^*, w_B^*)	Π^*
$y_A < \theta_5$	$(0, \mu)$	(1.13)	$(0, \mu)$	(1.19)
$\theta_5 < y_A < \theta_6$	$(0, \mu)$	(1.13)	$(\frac{\frac{c}{\triangle} - \mu\left(\frac{1}{2} - \frac{H}{8}\right)}{1 + \frac{L}{2} + \frac{HL}{8}}, \mu)$	(1.21)
$\theta_6 < y_A < \theta_4$	$(0, \mu)$	(1.13)	$(\frac{c}{\triangle}, \mu)$	(1.18)
$\theta_4 < y_A$	$(\frac{c}{\triangle}, \mu)$	(1.11)	$(\frac{c}{\triangle}, \mu)$	(1.18)

$$(1.13) > (1.21) \iff \frac{\frac{c}{\triangle} - \mu\left(\frac{1}{2} - \frac{H}{8}\right)}{1 + \frac{L}{2} + \frac{HL}{8}} > \frac{1}{2L + 2H}\left(H - \frac{5H^2}{6}\right)\left(\frac{\overline{y} - \underline{y}}{4}\right).$$

$$(1.13) > (1.18) \iff y_A < \frac{2Hc}{\triangle^2} - \frac{1}{2\triangle}\left(H - \frac{5H^2}{6}\right)\left(\frac{\overline{y} - \underline{y}}{4}\right).$$

$$(1.11) < (1.18).$$

(6) $\frac{c}{\triangle\left(\frac{1}{2} - \frac{L}{8}\right)} < \mu < \frac{c}{\triangle\left(\frac{1}{2} - \frac{H}{8}\right)}$:

Table 1.11: Outcome Comparison Under Case (6)

Promotion policy	Commitment		No commitment	
	(w_A^*, w_B^*)	Π^*	(w_A^*, w_B^*)	Π^*
$y_A < \theta_7$	$(0, \mu)$	(1.13)	$(0, \mu)$	(1.22)
$\theta_7 < y_A < \theta_6$	$(0, \mu)$	(1.13)	$(\frac{\frac{c}{\triangle} - \mu\left(\frac{1}{2} - \frac{H}{8}\right)}{1 + \frac{L}{2} + \frac{HL}{8}}, \mu)$	(1.21)
$\theta_6 < y_A < \theta_4$	$(0, \mu)$	(1.13)	$(\frac{c}{\triangle}, \mu)$	(1.18)
$\theta_4 < y_A$	$(\frac{c}{\triangle}, \mu)$	(1.11)	$(\frac{c}{\triangle}, \mu)$	(1.18)

$$(1.13) > (1.22) \iff y_A > \frac{1}{\triangle}\left(\frac{H + L}{2} - \frac{HL}{6} - \frac{4H^2}{6}\right)\left(\frac{\overline{y} - \underline{y}}{4}\right).$$

$$(1.13) > (1.21) \iff \frac{\frac{c}{\triangle} - \mu\left(\frac{1}{2} - \frac{H}{8}\right)}{1 + \frac{L}{2} + \frac{HL}{8}} > \frac{1}{2L + 2H}\left(H - \frac{5H^2}{6}\right)\left(\frac{\overline{y} - \underline{y}}{4}\right).$$

$$(1.13) > (1.18) \iff y_A < \frac{2Hc}{\triangle^2} - \frac{1}{2\triangle}\left(H - \frac{5H^2}{6}\right)\left(\frac{\overline{y} - \underline{y}}{4}\right).$$

$$(1.11) < (1.18).$$

(7) $\frac{c}{\triangle\left(\frac{1}{2} - \frac{H}{8}\right)} < \mu$:

Table 1.12: Outcome Comparison Under Case (7)

Promotion policy	Commitment		No commitment	
	(w_A^*, w_B^*)	Π^*	(w_A^*, w_B^*)	Π^*
$y_A < \theta_4$	$(0, \mu)$	**(1.13)**	$(0, \mu)$	**(1.20)**
$\theta_4 < y_A$	$(\frac{c}{\triangle}, \mu)$	**(1.11)**	$(\frac{c}{\triangle}, \mu)$	**(1.18)**

$$(1.13) < (1.20).$$

$$(1.11) < (1.18).$$

2 Training as a Commitment Device

2.1 Introduction

Firms can fill managerial vacancies through internal hiring (promotions) and external recruitment. There is evidence that shows firms favour internal hiring for high-level positions (Parrino, 1997; Lauterbach et al., 1999; Agrawal et al., 2006) and prefer external hiring for entry-level jobs (Bidwell, 2011). Because firms have more information about incumbent workers than external workers, internal hiring involves less risk (Bidwell, 2011). Promotions also provide incentives to induce effort from existing employees and incentives to acquire firm-specific skills (Kahn and Huberman, 1988; Prendergast, 1993). In addition, promotions help firms reallocate the workforce so as to best match firm needs (Baker et al., 1988, 1994a; Gibbons and Waldman, 1999b) and create new vacancies which can incentivize low-level workers (DeVaro et al., 2019). In contrast, external workers might have high potential. Evidence shows that external workers have better backgrounds, such as education and working histories (Bidwell, 2011). External hiring is a threat to insider workers, keeping them working hard (Chen, 2005). When firms hire both internally and externally, they may face a commitment problem in terms of incentives for promotion. Firms can always hire externally if outside workers are better than incumbents.

Modern organizations frequently use promotions as incentives (Jensen, 1986), and some incentive systems are promotion-based. A successful promotion-based incentive system relies on other policies (Demougin and Siow, 1994), such as firm-sponsored training. Evidence shows that firms that rely heavily on promotions also provide more training (DeVaro and Morita, 2013). Training helps workers accumulate both firm-specific and general skills and increases the productivity of insider workers. Training also helps workers adapt to new high-level jobs. The better insider workers are, the greater their promotion opportunities. Promotion is non-contractable, while training is contractable. Firms cannot commit to a promotion policy but can commit to a given training level. In this chapter, I highlight the commitment role of training and provide a strategic reason for firm-sponsored training. I argue that training can serve as a commitment device for promotion policies.

Haidilao, the highly successful Chinese hotpot company, serves as the primary example for this chapter, specifically for its distinctive compensation and promotion system. It incentivizes workers by providing a highly mobile promotion system. Since 2006, it very rarely hire externally for high-level jobs, including service staff and floor managers, especially restaurant managers. Before it went public in mid-2018, Haidilao had 363 existing restaurant managers, one for each restaurant and approximately 400 restaurant manager candidates. It has recently expanded extremely rapidly. Since the end of 2017, when it owned 273 restaurants, it opened 200 and 308 new restaurants in 2018 and 2019, respectively. Even in the pandemicchallenged first half of 2020, it opened 173 new restaurants. Most importantly, it was able to maintain the same table turnover rate and achieve the same sales levels at new restaurants as at existing restaurants. It also operates a distinctive apprenticeship system that is the core of their bottom-up driven expansion strategy, which enables continuous replicative growth. Haidilao provides significant amounts of training to ensure a large number of manager candidates and enable growth without sacrificing quality. It even has a Haidilao University. The key to Hadilaos's high-quality growth is "aligned interests and disciplined management", which entails a promotion-based incentive system and apprenticeship training system. This chapter seeks to investigate the relationship between promotion and training.

I consider an environment in which a firm cannot commit to a promotion policy but can commit to a given training level. Training not only increases the productivity of workers but also increases the expected promotion rate, which can provide additional incentives to insider workers. As training increases the expected promotion rate, providing training can sharpen incentive provisions. Therefore, the training trade-off is not just between productivity increases and training costs it should also take incentive provisions into account. In other words, firms can commit to a promotion policy through its training levels, even if it is not fully committed.

I explore the commitment role of training by using the two-period efficiency wage model (Shapiro and Stiglitz, 1984) but assuming the firm has two types of jobs, job A and job B. Job A is an entry-level job, and job B is a managerial job which only exists in period 2. In period 1, the firm hires two workers for job A by offering each a contract. These two workers are identically qualified for job A but have unknown ability for job B. The contracts specify a wage policy for both jobs and both periods and a training level θ. I assume that long-term contract is feasible and the commitment of training level is credible, but the commitment of promotion policy is non-credible. In period 1, the firm provides the θ level training to both workers, according to the contract. In period 2, the firm has observed the workers' ability and decides to promote internally or hire externally to fill the vacancy in job B. There is a concern that promotion is a signal and outside firms may poach the promoted worker. I impose a minimum wage restriction μ for job B that is also the minimum wage to prevent poaching by other firms. I also assume μ is high enough to make job B much more attractive to workers than job A, which means that the firm can provide incentives through promotion.

Once the wage policy and training level are set, only workers' effort level and ability can affect the expected promotion rate. The firm can promote the insider worker or hire externally to fill job B, depending on the ability of the best inside worker. Comparing no training to a positive level training, training increases the ability of inside workers; therefore, for the same level of effort, training has a high promotion rate. If a promotion provides enough incentives, a firm can use a high promotion rate to set lower wages for job A and job B, which makes the incentive constraints still hold and induces high effort in period 1. A high effort level would also increase the expected promotion rate. That is, training can amplify the incentive provision of promotion by increasing the expected promotion rate.

The only concern is training costs. In the model of this paper, the firm would always provide positive training because of the specific training cost function. The optimal training level may be zero. However, common sense dictates that the firm provide either general or firm-specific skill training, as long as the profits the firm reaps from the training can cover the training cost. This chapter highlights the incentive provisions of training. Firms should also consider the gains in wage savings from sharpening incentives in designing their training policies. Firms should provide training even when the profits gained from training are less than the training cost when the wage-saving effect dominates.

I show that training can sharpen incentives provisions by increasing the promotion rate. This is twofold. First, training could increase firms' profit by saving wages or increasing outputs as long as promotions can provide incentives. Second, training increases the range by which promotion can provide incentives. Under the same wage policy, without training the promotion rate is low, and promotion cannot provide incentives with the incentive constraints do not hold, while because training increases the promotion rate, the incentive constraints might hold and promotion can provide incentives.

The findings regarding the commitment role of training have some important implications. First, I provide a strategic reason for providing firm-sponsored training. To the best of my knowledge, this is the first theoretical paper to investigate the impact of training on promotion rates. I argue that training serves as a commitment device for promotion and can sharpen incentive provisions, a view that is supported by the empirical finding (Melero, 2010). Second, I expand the interaction between human capital and promotion. The existing literature on the interaction between human capital and promotion shows that promotion can incentivize workers to acquire firm-specific training (Carmichael, 1983; Kahn and Huberman, 1988). That is, promotion has an effect on human capital acquisition. This chapter shows that the opposite effect also exists. Acquiring human capital also helps implement a promotion-based reward system.

2.2 Related Literature

This analysis contributes to the theoretical literature on human capital theory following the seminal work of Becker (1962). The main concern of the theoretical literature is on-the-job training, particularly, why firms provide general training and why workers acquire firm-specific training. Becker argued that if training is general and labour markets are competitive, firms will not finance training, because firms cannot reap any benefits from the training, and the possibility of external poaching exists (Acemoglu and Pischke, 1999). For a similar reason, because of the hold-up problem (Williamson, 1985), if training is firm-specific, workers have no incentives to acquire those skills. Subsequent research has helped explain this phenomenon. The main answers are imperfect competition and information asymmetries, such as costs of changing jobs, training not observable by outside firms (Katz and Ziderman, 1990; Chang and Wang, 1996) and ability not observable by outside firms (Greenwald, 1986; Chun and Wang, 1995). The contribution is to provide a strategic reason for firms to provide training as a commitment device to help firms to implement their promotion policy.

This study is also closely related to research on the interaction between training and promotion. There are contractual solutions to solve the firm's hold-up problem. Promotion is one of the remedies. Up-or-out practices (Kahn and Huberman, 1988) and up-or-stay promotion rules (Prendergast, 1993) can induce workers to acquire firm-specific skills. The finding complements this insight. On the one hand, promotion has a positive effect on training. On the other, I show that training also has a positive effect on promotion, which is consistent with empirical findings (DeVaro and Morita, 2013). It's worth noting that research shows that promotion rules help firms to induce workers to acquire firm-specific skills, while both firm-specific training and general training can increase the potential promotion rate as long as the training is firm-sponsored.

This paper combines the tournament theory (Lazear and Rosen, 1981) and the signal theory (Waldman, 1984) of promotion. Promotion is a tool of incentive provisions and a tool of job reassignment. Promotion tournaments exclude outside poaching and face a commitment problem. The model in this paper uses a minimum wage as an exogenous variable to explore the effect of outside poaching. The result is similar to Waldman (2013) and DeVaro and Kauhanen (2016); because there is a slot constraint for the managerial potion, promotion distortion does not exist. As training increases the expected promotion rate, it can partly solve the commitment problem. As long as the training cost is not too high, there is always a positive level of training, which means there is always a partial commitment to promotion. However, promotion does not always provide incentives; outside that range, the only role of promotion is job reassignment. Training also increases the range for certain incentive provisions of promotion

This paper also contributes to the literature on internal promotion versus external recruitment. The pros and cons of internal promotion and external recruitment have been well explored (Chan, 1996; Chen, 2005; Bidwell, 2011). My models have an additional feature that allows the firm to choose training. Training decision helps the implementing of promotion policy. Most cases cannot rule out the possibility of external recruitment. The strategic reason for external hiring is efficiency. Because the firm learns information about inside workers, external hiring is one way to use this information if outside workers are better than inside workers. One article closely related to ours is DeVaro and Morita (2013), who also introduce training into the model. While their training is about firm-specific skills, in the model of this paper, training is not firm-specific but general skills. I also highlight the commitment role of the training. In addition, they focus on the consequences for the whole market, whereas I focus on one firm and its strategy when choosing between an external hiring or internal promotion policy.

The rest of the chapter is organized as follows. Section 2.3 presents the model, while section 2.4 gives the optimal wage policy for a given training level. The optimal training level is discussed in section 2.5. A final section draws a conclusion; all proofs are given in the Appendix.

2.3 The Model

I consider a two-period principal-agent model that is described below in terms of its four key components: players, technology, contract and payoffs.

Players: A firm, F, has a fixed number (2) of positions for job A and one position for job B. Job A is an entry-level job, and job B is a managerial job. The labour pool consists of a large number of workers who are qualified for job A. Workers who have been hired may leave the firm or be fired.

Technology: The technology of the firm is similar to Shapiro and Stiglitz's (1984) efficiency wage model. However, this model allows for two types of jobs (job A and job B) within a single firm. In addition, the firm can offer on-the-job training.

Workers privately choose their effort level for job A. Workers performing job A in period t choose an effort level $e_t \in \{L, H\}$, $t = 1, 2$, $L < H$ and $L, H \in (0, 1)$. Exerting effort e implies a disutility for the worker that is equal to $C(e_t)$ with the normalizations $C(L) = 0$ and $C(H) = c$. An effort level e_t generates output y_A with probability e_t and 0 with $1 - e_t$. Similar to chapter 1, I assume that $\frac{c}{\triangle} < y_A$, *where* $\triangle = H - L$. This assumption ensures that y_A is high enough for the firm to induce a high effort level.

However, job B is a high-level job with and only exists in period 2. The output of a worker with an ability of $\widehat{y_B}$ who performs job B is $\widehat{y_B}$. Without loss of generality, I assume that the cost of performing job B for workers is zero. $\widehat{y_B}$ is unknown to all players and is assumed to follow a uniform distribution on $\left[\underline{y}, \overline{y}\right]$, where $\underline{y} > y_A$. The firm can also provide on-the-job training in period 1, which only affects the output of job B. The firm chooses the training level of θ at cost $h(\theta) = \frac{\theta^2}{2}$. With θ level of training, the output of a worker with an ability of $\widehat{y_B}$ is $\widehat{y_B} + \theta$. That is the ability of a trained worker following a uniform distribution on $\left[\underline{y} + \theta, \overline{y} + \theta\right]$. I impose the following restriction on θ.

Assumption 2.1. $\overline{y} - \underline{y} > 4$**.**

Assumption 2.1 ensures that the firm would not choose the training level of $\frac{\overline{y} - \underline{y}}{2}$, as $\frac{\overline{y} - \underline{y}}{2} < h\left(\frac{\overline{y} - \underline{y}}{2}\right)$, because it would rule out of the possibility of external hiring. In addition, the firm can learn workers' ability through their first-period output and on-the-job training. If a worker was employed for the firm for both periods, at the beginning of period 2, her ability $\widehat{y_B}$ is observed by the firm and the worker.

Contract: I assume that long-term contracts on wages are feasible. As a worker's ability and effort level are not observable at the beginning of the game, the firm cannot offer a contract contingent on ability and effort level. Moreover, the firm does not have the ability to commit to a promotion policy. Hence, I restrict attention to the following contract. At the beginning of the game, the firm announces the training level of θ and offers a contract w_i contingent on y_i to each worker, where $i = A, B$. Moreover, if the output of a worker is zero, the worker would receive zero and be fired. There is only one position for job B, and the firm can hire externally.

At the beginning of period 1, the firm announces the training level θ and makes two take-it-or-leave-it offers (w_A, w_B) to the workers. At the end of period 1, after observing the output, the firm would fire workers whose output is zero without paying w_A. Then, the firm offers the same w_A to hire new workers to fill vacancies in job A. At the beginning of period 2, after the firm has observed existing workers' abilities $\widehat{y_B}$, the firm decides to promote the best incumbent worker or hire a worker externally for job B. The following timeline summarizes the game described above.

Period 1.0. The firm publicly announces training policy θ and offers a contract (w_A, w_B) to all workers. If only two workers accept the offer, the game proceeds. If no workers or just one worker accept it, the game ends. If more than two workers accept it, the firm would randomly choose two workers to fill the two positions.

Period 1.1. Workers choose their effort level. Production and training occur, and output is realized. Workers whose output is zero are fired. Otherwise, wages are paid. The firm hires new workers with the same w_A to fill vacancies in job A.

Period 2.0. $\widehat{y_B}$ for workers whose output is positive in period 1 are observed. Based to $\widehat{y_B}$, the firm decides to promote the best insider worker or hire a worker externally for job B.

Period 2.1. Workers choose their effort level. Production occurs, and output is realized. Workers whose output is zero are fired. Otherwise, wages are paid.

Payoffs: The firm and workers are risk neutral. Workers are protected by limited liability. The workers' transfer must always be non-negative. There is no discounting. I assume that all players' outside options are zero. Upon successfully having two workers for job A in both periods, the firm's aggregate payoff is

$$\Pi = \pi_1 + \pi_2,$$

where π_t is the firm's payoff at time t, $\pi_1 = e_1^{\alpha}(y_A - w_A) + e_1^{\beta}(y_A - w_A) - h(\theta)$, and $\pi_2 = e_2^{\alpha}(y_A - w_A) + e_2^{\beta}(y_A - w_A) + E(\widehat{y_B}) - w_B$. e_t^s is the effort level of worker s at period t, $s = \alpha, \beta$. $E(\widehat{y_B})$ is the expected output of job B.

Similarly, the workers' expected payoff for job A in period t is $u_t(e_t) = e_t w_A - c(e_t)$, and workers' expected payoff for job B in period 2 is w_B. Let $U_t(e_t)$ be the expected value of job A in period t with effort level e_t; then I have

$$\begin{aligned} U_2(e_2) &= u_2(e_2) \\ U_1(e_1) &= u_1(e_1) + e_1\left[p w_B + (1-p) U_2(e_2)\right], \end{aligned}$$

where p is the expected promotion rate. Since there is no moral hazard problem with job B, I impose a minimum wage μ on w_B, where

$\mu > \frac{c}{\triangle}$. The restrictions on the lower bound of $\widehat{y_B}$ and the minimum wage μ guarantee that job B is a better job than job A; μ is the minimum wage to retain the best worker and to prevent poaching by other firms.

Strategies and equilibrium concept: The firm's strategy, σ_F, has two components: (i) at the beginning of period 1, it chooses training level θ; (ii) at the beginning of period 1, it chooses wage policy (w_A, w_B). The worker's strategy, σ_W, has four components: (i) at the beginning of period 1, the worker accepts or rejects the firm's contract in period 1; (ii) if the worker gets the job, the worker chooses effort level e_1; (iii) at the end of period 1, if the worker does not get the job at the beginning, the worker accepts or rejects the firm's w_A offer for period 2; and (iv) at the beginning of period 2, if the worker does not get promoted, the worker chooses effort level e_2. I use subgame perfect equilibrium (SPE) as a solution concept.

2.4 The Optimal Wage Policy for a Given Training Level

In order to derive the optimal training level for the firm, I first need to analyse the equilibrium wage policy (w_A, w_B) for a given training level θ. I use backward induction to solve the model. In what follows, I characterize the firm's equilibrium wage policy and workers' effort level for a given training level θ.

2.4.1 Period 2 Profits

Since there is no moral hazard problem in job B, the worker in job B would receive $w_B \geq \mu$, and the firm would receive $E(\widehat{y_B}) - w_B$ in outputs from job B. The training level would affect the $E(\widehat{y_B})$ and promotion rate. For now, I do not need to obtain the total payoffs for the firm, so I simply use the term $E(\widehat{y_B})$. When I anaylze the promotion rate for a given training level, I give the full results of $E(\widehat{y_B})$ for training level θ.

For workers in job A, who are protected by limited liability and whose outside options are zero, I can ignore the individual rationality (IR) constraints and only consider the incentive compatibility (IC) constraints. There are two choices for workers that depend on w_A. When $w_A \geq \frac{c}{\triangle}$ (that is the IC_2 constraint $U_2(H) \geq U_2(L)$ holds), then all workers choose high effort H. I have $U_2(H) = Hw_A - c$, and the expected payoff of job A for the firm is $H(y_A - w_A)$ for each worker. When $w_A < \frac{c}{\triangle}$ (that is the IC_2 constraint does not hold), then all workers choose low effort L. I have $U_2(L) = Lw_A$, and the expected payoff of job A for the firm is $L(y_A - w_A)$ for each worker. The payoff for the firm in period 2 is

$$\pi_2 = \begin{cases} 2H(y_A - w_A) + E(\widehat{y_B}) - w_B & if \;\; w_A \geq \frac{c}{\triangle} \\ 2L(y_A - w_A) + E(\widehat{y_B}) - w_B & if \;\; w_A < \frac{c}{\triangle}. \end{cases}$$

2.4.2 Period 1 Profits with $w_A \geq \frac{c}{\triangle}$

In this case, the IC_1 constraint is

$$Hw_A - c + H\left[p_H w_B + (1-p_H)\, U_2(H)\right] > Lw_A + L\left[p_L w_B + (1-p_L)\, U_2(H)\right],$$

where p_H and p_L are the expected promotion rates, respectively, when the worker chooses effort levels H or L in period 1. These promotion rates depend on external and internal workers' abilities, rivals' effort levels in period 1 and training levels. I must have $p_H \geq p_L$ holding all other factors fixed.

For this case, workers in job A would always choose high effort H. Promotion does not provide any incentives, and training level only affects the expected output of job B. The payoff for the firm in period 1 is $\pi_1 = 2H\,(y_A - w_A)$. Then, the firm's problem is

$$\max_{\{w_A, w_B\}} \ \Pi = 4H\,(y_A - w_A) + E\,(\widehat{y_B}) - \frac{\theta^2}{2}$$
$$s.t\ \ w_A \geq \frac{c}{\triangle}\ \ and\ \ w_B \geq \mu.$$

To maximize the profit, $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, and

$$\Pi^*\,(4H) = 4H\left(y_A - \frac{c}{\triangle}\right) + E\,(\widehat{y_B}) - \mu - \frac{\theta^2}{2}.$$

2.4.3 Period 1 Profits with $w_A < \frac{c}{\triangle}$

In this case, w_A is not enough to let the IC_1 constraint ($U_1\,(H) > U_1\,(L)$) hold. Promotion may have a role in providing incentives. Both the wage policy (w_A, w_B) and the training level would affect workers' effort choice in period 1, because workers are competing for the only one managerial position. Workers' effort choices affect each other's expected promotion rates and the IC_1 constraint. The following is the total payoffs matrix for the two workers.

Table 2.13: Total Payoffs Matrix

		Worker β	
		L	H
Worker α	L	A_{LL},A_{LL}	A_{LH},A_{HL}
	H	A_{HL},A_{LH}	A_{HH},A_{HH}

If both workers exert low effort, there are four different outcomes in period 1.

Table 2.14: Expected Payoff and Its Probability for (L, L)

Probability	Output	Payoff in period 1	Expected payoff in period 2
$(1-L)^2$	0,0	0,0	0,0
$(1-L)L$	0, y_A	0, w_A	0, $p^{one}w_B + (1-p^{one})Lw_A$
$(1-L)L$	y_A, 0	w_A, 0	$p^{one}w_B + (1-p^{one})Lw_A$, 0
L^2	y_A, y_A	w_A, w_A	$p^{two}w_B + (1-p^{two})Lw_A, p^{two}w_B + (1-p^{two})Lw_A$

In the Table 2.14, p^{one} is the expected promotion rate conditional on only one worker succeeding in period 1, and p^{two} is the expected promotion rate conditional on two workers succeeding in period 1; p^{one} and p^{two} are affected by the training level and external and internal workers' abilities, but they are independent of workers' effort level. I present the full results in the next subsection. Workers' total payoffs are given by

$$\begin{aligned} A_{LL} &= L\left(1-L\right)\left[w_A + p^{one}w_B + \left(1-p^{one}\right)Lw_A\right] \\ &\quad + L^2\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right]. \end{aligned} \tag{2.25}$$

Since both workers exert low effort in both periods, the total payoffs for the firm are given by

$$\Pi\left(4L\right) = 4L\left(y_A - w_A\right) + E\left(\widehat{y_B}\right) - w_B - \frac{\theta^2}{2}. \tag{2.26}$$

Similarly, if both workers choose high effort or one worker chooses low effort and the other worker chooses high effort, there are four different outcomes in period 1. All players' total payoffs for different cases are given by

$$\begin{aligned} A_{LH} &= L(1-H)\left[w_A + p^{one} w_B + (1-p^{one}) L w_A\right] \\ &\quad + LH\left[w_A + p^{two} w_B + \left(1-p^{two}\right) L w_A\right], \end{aligned} \tag{2.27}$$

$$\begin{aligned} A_{HL} &= (1-L)H\left[w_A + p^{one} w_B + (1-p^{one}) L w_A\right] \\ &\quad + LH\left[w_A + p^{two} w_B + \left(1-p^{two}\right) L w_A\right] - c, \end{aligned} \tag{2.28}$$

$$\Pi\left(H\&3L\right) = (3L+H)(y_A - w_A) + E\left(\widehat{y_B}\right) - w_B - \frac{\theta^2}{2},$$

$$\begin{aligned} A_{HH} &= H(1-H)\left[w_A + p^{one} w_B + (1-p^{one}) L w_A\right] \\ &\quad + H^2\left[w_A + p^{two} w_B + \left(1-p^{two}\right) L w_A\right] - c, \end{aligned} \tag{2.29}$$

$$\Pi\left(2H\&2L\right) = (2L+2H)(y_A - w_A) + E\left(\widehat{y_B}\right) - w_B - \frac{\theta^2}{2}.$$

2.4.4 Conditional Promotion Rate and $E\left(\widehat{y_B} \mid p^K\right)$

Only workers whose output in period 1 are positive will stay with the firm and have the chance to be promoted in period 2. A worker's effort choice and her rival's effort choice would also affect the expected promotion rate. However, to understand the effect of different training levels on promotion rate and $E\left(\widehat{y_B} \mid p^K\right)$, I want to exclude the impact of effort level. I calculate the conditional promotion rate and $E\left(\widehat{y_B} \mid p^K\right)$ at the beginning of period 2; the promotion rate and $E\left(\widehat{y_B} \mid p^K\right)$ are affected by the number of workers who succeed and the training level.

The following table gives different conditional promotion rates for a given training level (more details are in Appendix 2.7.1).

Table 2.15: Expected Promotion Rates for a Given Training Level

One worker succeeds	
Conditional promotion rate p^{one}	$E\left(\widehat{y_B} \mid p^{one}\right) = E^{one}$
$\frac{1}{2} + \frac{\theta}{\overline{y}-\underline{y}}$	$\frac{3\underline{y}+5\overline{y}}{8} + \frac{\theta^2}{2\left(\overline{y}-\underline{y}\right)} + \frac{\theta}{2}$
Two workers succeed	
Conditional promotion rate p^{two}	$E\left(\widehat{y_B} \mid p^{two}\right) = E^{two}$
$\frac{3}{8} + \frac{\theta\left(\overline{y}-\underline{y}-\theta\right)}{2\left(\overline{y}-\underline{y}\right)^2}$	$\left(\frac{1}{2} - \frac{\theta}{\overline{y}-\underline{y}}\right)^2 \left(\frac{\underline{y}+\overline{y}}{2}\right) + \frac{2\underline{y}+7\overline{y}+5\theta}{12} + \frac{\theta\left(\underline{y}+5\overline{y}\right)}{6\left(\overline{y}-\underline{y}\right)} - \frac{\theta^2\left(3\underline{y}+\theta\right)}{3\left(\overline{y}-\underline{y}\right)^2}$

Proposition 2.1. **Training increases the conditional promotion rate and conditional expected output** $E\left(\widehat{y_B} \mid p^K\right)$**.**

Comparing these results to conditional promotion rates and $E\left(\widehat{y_B} \mid p^K\right)$ in Chapter 1, we can easily obtain Proposition 2.1. The key implication of Proposition 2.1 is that training has two advantages over no training (which is the case of the no-commitment policy in Chapter 1). This is because training increases the ability of incumbent workers. A high-ability insider worker would have a high promotion rate and expected productivity in job B; similarly, the firm can benefit from a high promotion rate and expected productivity in job B. A high promotion rate benefits firms in two ways. First, a high promotion rate could help the firm save wages by inducing a high effort level in either in job A or job B, as long as the wage level can make the IC constraints hold. On the other hand, a high promotion rate has a high probability of inducing high effort, and a high effort has a high $E\left(\widehat{y_B}\right)$. However, training incurs cost $\frac{\theta^2}{2}$. Thus, the overall effect of training is unclear.

Comparing a commitment policy to the no-commitment policy in Chapter 1, commitment increases the conditional promotion rate p^{one} from

$\frac{1}{2}$ to 1 and the conditional promotion rate p^{two} from $\frac{3}{8}$ to $\frac{1}{2}$. Training increases the conditional promotion rate p^{one} from $\frac{1}{2}$ to $\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}$ and the conditional promotion rate p^{two} from $\frac{3}{8}$ to $\frac{3}{8}+\frac{\theta\left(\overline{y}-\underline{y}-\theta\right)}{2\left(\overline{y}-\underline{y}\right)^{2}}$. Both conditional promotion rates are increasing in θ, while $\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}\leq 1$ and $\frac{3}{8}+\frac{\theta\left(\overline{y}-\underline{y}-\theta\right)}{2\left(\overline{y}-\underline{y}\right)^{2}}\leq\frac{1}{2}$; that is, a commitment policy has the highest conditional promotion rate. However, firms may not have the ability to commit, in which case the only choice is providing training. As long as the training cost is not too high, firms have incentives to provide training, and training serves as a commitment device.

2.4.5 The Optimal Wage Policy for a Given Training Level θ

From subsection 2.4.4, we know the conditional promotion rate and $E\left(\widehat{y_B}\mid p^{K}\right)$. In this subsection, to find the optimal wage policy for a given training level, I assume one effort level pair is the equilibrium effort choices. I obtain the equilibrium wage policy and the firm's total payoffs under this effort level pair. Then, by comparing all the firm's total payoffs, I obtain the optimal wage policy.

Note that if $w_A\geq\frac{c}{\triangle}$, all workers would always choose high effort in both periods. In addition, I have $E\left(\widehat{y_B}\mid p^{one}\right)$ and $E\left(\widehat{y_B}\mid p^{two}\right)$, so I obtain $E\left(\widehat{y_B}\right)$ with different effort levels. Hence, for $w_A^{*}=\frac{c}{\triangle}$ and $w_B^{*}=\mu$, the total profit for the firm is

$$\Pi^{*}\left(4H\right)=4H\left(y_A-\frac{c}{\triangle}\right)+2H\left(1-H\right)E^{one}+ \\ +H^{2}E^{two}+\left(1-H\right)^{2}\left(\frac{\underline{y}+\overline{y}}{2}\right)-\mu-\frac{\theta^{2}}{2}, \tag{2.30}$$

I now consider the case of $w_A<\frac{c}{\triangle}$. When $A_{LL}>A_{HL}$ and $A_{LH}>A_{HH}$, (L,L) is the only NE in period 1. The wage policy is $w_A^{*}=0$ and

$w_B^* = \mu$, and the total payoff for the firm is

$$\begin{aligned} \Pi^* (4L) &= 4Ly_A + 2L(1-L)E^{one} + L^2 E^{two} \\ &+ (1-L)^2 \left(\frac{\underline{y} + \overline{y}}{2}\right) - \mu - \frac{\theta^2}{2}. \end{aligned} \tag{2.31}$$

I also need $\mu < \frac{c}{\triangle(1-L)p^{one} + \triangle Lp^{two}} = \frac{c}{\triangle p^{one} - \triangle L(p^{one} - p^{two})}$; otherwise, promotion can provide enough incentives to make the IC_1 constraint hold and can induce workers to exert high effort in period 1.

When $A_{LL} < A_{HL}$ and $A_{LH} < A_{HH}$, (H, H) is the unique NE in period 1. There are three different wage policies for different μ. For $\mu \geq \frac{c}{\triangle(1-H)p^{one} + \triangle Hp^{two}}$, the wage policy is $w_A^* = 0$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\begin{aligned} \Pi^* (2H\&2L) &= (2L + 2H) y_A + 2H(1-H)E^{one} \\ &+ H^2 E^{two} + (1-H)^2 \left(\frac{\underline{y} + \overline{y}}{2}\right) - \mu - \frac{\theta^2}{2}. \end{aligned} \tag{2.32}$$

For $\mu < \frac{c}{\triangle(1-H)p^{one} + \triangle Hp^{two}}$ and $4L + 4H - H(3L + 2H) - 2 < 0$, the wage policy is $w_A^* = \frac{c - \triangle(1-H)p^{one}\mu - \triangle Hp^{two}\mu}{\triangle(1+L) - \triangle(1-H)p^{one}L - \triangle Hp^{two}L}$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\begin{aligned} \Pi^* (2H\&2L) = (2L + 2H) &\left(y_A - \frac{c - \triangle(1-H)p^{one}\mu - \triangle Hp^{two}\mu}{\triangle(1+L) - \triangle(1-H)p^{one}L - \triangle Hp^{two}L}\right) \\ &+ 2H(1-H)E^{one} \\ &+ H^2 E^{two} + (1-H)^2 \left(\frac{\underline{y} + \overline{y}}{2}\right) - \mu - \frac{\theta^2}{2}. \end{aligned} \tag{2.33}$$

For $\mu < \frac{c}{\triangle(1-H)p^{one} + \triangle Hp^{two}}$ and $4L + 4H - H(3L + 2H) - 2 > 0$, there is a cut-off $\alpha = \left[\frac{1 - \sqrt{1 - \frac{2H(1+L)}{3L+2H}}}{H} - \frac{1}{2}\right] \in \left(0, \frac{1}{2}\right)$, where $J\left(\frac{1}{2} + \alpha\right) = 0$. If $\frac{\theta}{\overline{y} - \underline{y}} > \alpha$, $J\left(\frac{1}{2} + \frac{\theta}{\overline{y} - \underline{y}}\right) > 0$, the wage policy is $w_A^* = 0$ and $w_B^* =$

$\frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}$, the total payoff for the firm is

$$\begin{aligned}\mathit{\Pi}^*\left(2H\&2L\right) &= \left(2L+2H\right)y_A+2H\left(1-H\right)E^{one}\\&+H^2E^{two}+\left(1-H\right)^2\left(\frac{\underline{y}+\overline{y}}{2}\right)\\&-\frac{c}{\triangle\left(1-H\right)p^{one}+\triangle Hp^{two}}-\frac{\theta^2}{2}.\end{aligned} \tag{2.34}$$

If $\frac{\theta}{\overline{y}-\underline{y}}<\alpha$, $J\left(\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}\right)<0$, the wage policy is $w_A^*=\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L}$ and $w_B^*=\mu$, and the total payoff for the firm is (2.33).

When $A_{LL}<A_{HL}$ and $A_{LH}>A_{HH}$, (H,L) and (L,H) are the NEs in period 1. The wage policy is $w_A^*=0$ and $w_B^*=\mu$, and the total payoff for the firm is

$$\begin{aligned}\mathit{\Pi}^*\left(H\&3L\right) &= \left(3L+H\right)y_A+H\left(1-L\right)E^{one}+L\left(1-H\right)E^{one}\\&+HLE^{two}+\left(1-H\right)\left(1-L\right)\left(\frac{\underline{y}+\overline{y}}{2}\right)-\mu-\frac{\theta^2}{2}.\end{aligned} \tag{2.35}$$

I need $\frac{c}{\triangle(1-L)p^{one}+\triangle Lp^{two}}<\mu<\frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}$; otherwise, the two inequations would not hold. More details of equilibrium wage policies under different effort level pairs are presented in Appendix 2.7.2. By comparing these outcomes, I obtain Proposition 2.2; the proof is in Appendix 2.7.3.

Proposition 2.2. **The optimal wage policy for a given training level and μ depends on y_A. If y_A is small, the optimal wage policy is $w_A^*=0$ and $w_B^*=\mu$. If y_A is moderate, the optimal wage policy is $w_A^*>0$ or $w_B^*>\mu$. If y_A is large, the optimal wage policy is $w_A^*=\frac{c}{\triangle}$ and $w_B^*=\mu$. Different wage policies would induce different effort levels and payoffs.**

Proposition 2.2 is similar to Lemma 1.3 in Chapter 1 (the optimal wage policy for the no-commitment policy) but with different ranges of μ and cut-offs. However, training has a higher conditional promotion rate than the no-commitment policy and a lower conditional promotion rate than commitment. The critical value to induce high effort is lower than the no-commitment policy but higher than a commitment policy. The role of training is similar to commitment; it increases the conditional promotion rate, which allows the firm to lower w_A or w_B while the ICs still hold. Meanwhile, promotion can only provide incentives in certain ranges, which depend on y_A. The reason is quite simple; if y_A is low enough, there is no need to induce high effort. If y_A is high enough, it is worth inducing high effort in both periods. Promotion cannot provide incentives in period 2. The only choice for the firm is to increase w_A.

2.5 The Optimal Training Level

From section 2.4, I have the optimal wage policy (w_A, w_B) for a given training level θ with different ranges of μ. The payoff function for the firm is a function of θ. In this section, I seek the optimal training level θ^*, conditional promotion rates ($p^{one}(\theta^*)$ and $p^{two}(\theta^*)$), ranges of μ and cut-offs (φ is also a function of θ) for y_A. More details and proofs are in Appendix 2.7.4 and Appendix 2.7.5. I have Proposition 2.3.

Proposition 2.3. **The firm always provides a positive level of training. The optimal training level θ^* is a strictly positive and increasing function of effort level.**

From Proposition 2.1, we know that training can increase the conditional promotion rates and $E\left(\widehat{y_B} \mid p^K\right)$. The only question is the cost of training, whereas with this setting (the training cost function is $h(\theta) = \frac{\theta^2}{2}$), we always have an inner solution. Training can increase promotion rates and the expected output of job B. Even if the promotion does not provide incentives, the firm still provides positive training, because training can increase the expected output of job B. In addition, the optimal training level is a function of effort level. θ^* increases with effort level. A high effort level has a high training level. For example, when y_A is low enough or y_A is high enough, promotion does not provide incentives, and the effort level pair in period 1 is (L, L) and (H, H). While the optimal training levels for each case are $\theta^*(L)$ and $\theta^*(H)$, where $\theta^*(H) > \theta^*(L)$. This is because the $E\left(\widehat{y_B} \mid p^K\right)$ and the conditional promotion rates depend on the training level θ, while the total expected output of job B ($E\left(\widehat{y_B}\right)$) and promotion rates also depend on the effort level pair in period 1. A high effort level pair in period 1 means a high probability of positive output in period 1. Only positive-output workers can continue to work for the firm and be promoted, and only with positive output can the obtain information about workers' abilities and apply this information.

When y_A is moderate, promotion can provide incentives. Either w_A or w_B is also a function of L and H. The optimal training level θ^* is much more complicated but is still a function of L and H. The optimal training level θ^* is also moderate; it is below $\theta^*(H)$ and greater than $\theta^*(L)$. Note that the firm payoffs are increasing in θ at interval $[0, \theta^*]$. In this case, training serves as a commitment device that sharpens the incentive provisions. This is a strategic reason for the firm to provide training. The total effect also relies on the cost function of training. If the cost of training is extremely high, the optimal training would be zero. Under this condition, there is no inner solution. Note that as long as a promotion can provide incentives, even if the cost of training is higher than the increase of the output of job B, the firm might provide positive training. As long as that strategic reason exists, training could sharpen the incentive provisions and increase total profits by saving on total wages.

Corollary 2.1. **Training increases the ranges of cut-offs (φs) for the provision of the incentive of promotion. Training decreases the cut-offs of the minimum wage (μ) for the firm to provide incentives.**

Corollary 2.1 is because training increases conditional promotion rates. For the ranges of cut-offs (φs) to provide incentives of promotion, a high conditional promotion rate means a low wage to induce high effort. Even with a low y_A, it is still worth inducing high effort. Training decreases the low bound of the range to induce high effort. For a high y_A, without training, it is worth it for the firm to choose $w_A = \frac{c}{\triangle}$, and promotion does provide incentives. Since training increases conditional promotion rates, it could save on total wages. As long as this effect dominates the loss of expected output from job A in period 2, it is worth it for the firm to set $w_A = 0$. Training increases the high bound of the range to induce high effort. In a word, training sharpens the provision of the incentive of promotion.

For the cut-offs of the minimum wage (μ) for the firm to provide incentives, the decisive factor is the IC constraints in period 1. Since training increases conditional promotion rates, a low μ alone can induce high effort.This is similar to the comparison of the commitment and the no-commitment policies in Chapter 1. The commitment policy increases conditional promotion rates and thus has low cut-offs of μ.

2.6 Conclusion

In this chapter, I analysed the commitment role of training in hiring policy through a two-period efficiency wage model and assuming the firm has two types of job. I assume the firm cannot commit to a promotion policy. In the presence of external hiring, the firm faces a commitment problem when implementing promotion policy. However, I do assume that the firm can commit to a certain training level. This allows us to investigate the effect of training provision on promotion policy. The key finding is that training can increase the expected promotion rate. Promotion does not always provide incentives. However, as long as it does, training can sharpen incentive provisions and boost payoffs for the firm. In addition, training increases the range over which promotion can provide incentives. This result indicates that training can serve as a commitment device for promotion. In most cases, training does not rule out the possibility of external hiring. In the model of this paper, this is due to efficiency. As long as outside workers are "better" than incumbents, the firm would hire externally.

In my model, I assume a specific training cost function and that the firm always provides a positive level of training. The specific training cost function is not the key assumption of the results, but in reality, the training cost would affect the firm's decision on hiring policy. It would be worthwhile to analyse the effect of training cost on internal promotion. In addition, few empirical studies on the interaction between training and promotion have been carried out. These are all promising avenues for future work.

2.7 Appendices

2.7.1 Conditional Promotion Rate and $E\left(\widehat{y_B} \mid p^K\right)$

As the firm provides a θ training level, then x_1 and x_2 are two workers' ability; both follow the uniform distribution on $\left[\underline{y}+\theta, \overline{y}+\theta\right]$. The probability density function is $f(x) = \frac{1}{\overline{y}-\underline{y}}$, and the cumulative distribution function is $F(x) = \frac{x-\underline{y}-\theta}{\overline{y}-\underline{y}}$, where $\underline{y}+\theta \leq x \leq \overline{y}+\theta$.

- Only one worker succeeds:

$$p^{one} = Pr\left\{x_1 > \frac{\underline{y}+\overline{y}}{2}\right\} = 1-F\left(\frac{\underline{y}+\overline{y}}{2}\right) = 1-\left(\frac{1}{2}-\frac{\theta}{\overline{y}-\underline{y}}\right) = \frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}$$

$$\begin{aligned} E\left(internal\ worker \mid p^{one}\right) &= \int_{\frac{\underline{y}+\overline{y}}{2}}^{\overline{y}+\theta} xf(x)\,dx \\ &= \frac{1}{2\left(\overline{y}-\underline{y}\right)}\left[\left(\overline{y}+\theta\right)^2 - \left(\frac{\underline{y}+\overline{y}}{2}\right)^2\right] \\ &= \frac{\underline{y}+3\overline{y}}{8} + \frac{\theta^2+2\overline{y}\theta}{2\left(\overline{y}-\underline{y}\right)} \end{aligned}$$

$$\begin{aligned} E^{one} = E\left(output \mid p^{one}\right) =& E\left(internal\ worker \mid p^{one}\right) + Pr\left\{x_1 < \frac{\underline{y}+\overline{y}}{2}\right\} E\left(\frac{\underline{y}+\overline{y}}{2}\right) \\ =& \frac{\underline{y}+3\overline{y}}{8} + \frac{\theta^2+2\overline{y}\theta}{2\left(\overline{y}-\underline{y}\right)} + \left(\frac{1}{2}-\frac{\theta}{\overline{y}-\underline{y}}\right)\frac{\underline{y}+\overline{y}}{2} \\ =& \frac{3\underline{y}+5\overline{y}+4\theta}{8} + \frac{\theta^2}{2\left(\overline{y}-\underline{y}\right)}. \end{aligned}$$

- Two workers succeed:

$$\begin{aligned} p^{two} &= Pr\left\{x_1 > x_2 \, and \, x_1 > \frac{\underline{y}+\overline{y}}{2}\right\} \\ &= \int_{\frac{\underline{y}+\overline{y}}{2}}^{\overline{y}+\theta} f(x)\, F(x)\, dx \\ &= \frac{3}{8} + \frac{\theta}{2\left(\overline{y}-\underline{y}\right)} - \frac{\theta^2}{2\left(\overline{y}-\underline{y}\right)^2} = \frac{3}{8} + \frac{\theta\left(\overline{y}-\underline{y}-\theta\right)}{2\left(\overline{y}-\underline{y}\right)^2} \end{aligned}$$

$$\begin{aligned} E\left(internal\,worker \mid p^{two}\right) &= \int_{\frac{\underline{y}+\overline{y}}{2}}^{\overline{y}+\theta} xf(x)\, F(x)\, dx \\ &= \int_{\frac{\underline{y}+\overline{y}}{2}}^{\overline{y}+\theta} x\frac{1}{\overline{y}-\underline{y}}\frac{x-\underline{y}-\theta}{\overline{y}-\underline{y}} dx \\ &= \frac{1}{\left(\overline{y}-\underline{y}\right)^2}\left[\frac{1}{3}x^3 - \frac{1}{2}\left(\underline{y}+\theta\right)x^2\right] \Big|_{\frac{\underline{y}+\overline{y}}{2}}^{\overline{y}+\theta} \\ &= \frac{2\underline{y}+7\overline{y}+5\theta}{24} + \frac{\theta\left(\underline{y}+5\overline{y}\right)}{12\left(\overline{y}-\underline{y}\right)} - \frac{\theta^2\left(3\underline{y}+\theta\right)}{6\left(\overline{y}-\underline{y}\right)^2} \end{aligned}$$

$$\begin{aligned} E^{two} = E\left(output \mid p^{two}\right) &= P\left\{x_1 < \frac{\underline{y}+\overline{y}}{2}\right\} P\left\{x_2 < \frac{\underline{y}+\overline{y}}{2}\right\} E\left(\frac{\underline{y}+\overline{y}}{2}\right) + 2E\left(internal\,worker \mid p^{two}\right) \\ &= \left(\frac{1}{2} - \frac{\theta}{\overline{y}-\underline{y}}\right)^2 \left(\frac{\underline{y}+\overline{y}}{2}\right) + \frac{2\underline{y}+7\overline{y}+5\theta}{12} + \frac{\theta\left(\underline{y}+5\overline{y}\right)}{6\left(\overline{y}-\underline{y}\right)} - \frac{\theta^2\left(3\underline{y}+\theta\right)}{3\left(\overline{y}-\underline{y}\right)^2} \\ &= \frac{7\underline{y}+17\overline{y}+18\theta}{24} + \frac{\theta^2}{2\left(\overline{y}-\underline{y}\right)} - \frac{\theta^3}{3\left(\overline{y}-\underline{y}\right)^2}. \end{aligned}$$

2.7.2 NE with a Given Training Level θ

(1) (L, L) is the unique NE.

The firm's problem is

$$\max_{\{w_A, w_B\}} \Pi(4L) = 4L\left(y_A - w_A\right) + E\left(\widehat{y_B}\right) - w_B - \frac{\theta^2}{2}$$

s.t $w_A < \frac{c}{\triangle}$, $w_B \geq \mu$, $A_{LL} > A_{HL}$, and $A_{LH} > A_{HH}$

$$\begin{aligned} A_{LL} > A_{HL} &\iff A_{LL} - A_{HL} > 0 \\ &\iff -\triangle(1-L)\left[w_A + p^{one}w_B + \left(1-p^{one}\right)Lw_A\right] - \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c > 0 \\ &\iff D_3 = \triangle(1-L)\left[w_A + p^{one}w_B + \left(1-p^{one}\right)Lw_A\right] + \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] < c \end{aligned}$$

$$\begin{aligned} A_{LH} > A_{HH} &\iff A_{LH} - A_{HH} > 0 \\ &\iff -\triangle(1-H)\left[w_A + p^{one}w_B + \left(1-p^{one}\right)Lw_A\right] - \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c > 0 \\ &\iff D_4 = \triangle(1-H)\left[w_A + p^{one}w_B + \left(1-p^{one}\right)Lw_A\right] + \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] < c. \end{aligned}$$

To maximize profits, $w_A^* = 0$ and $w_B^* = \mu$, and $\mu < \frac{c}{\triangle(1-L)p^{one}+\triangle Lp^{two}} = \frac{c}{\triangle p^{one}-\triangle L(p^{one}-p^{two})}$,

$$\Pi^*(4L) = 4Ly_A + 2L(1-L)E^{one} + L^2E^{two} + (1-L)^2\left(\frac{\underline{y}+\overline{y}}{2}\right) - \mu - \frac{\theta^2}{2}.$$

(2) (H, H) is the unique NE.

The firm's problem is

$$\max_{\{w_A,w_B\}} \Pi(2H\&2L) = (2L+2H)(y_A - w_A) + E(\widehat{y_B}) - w_B - \frac{\theta^2}{2}$$

s.t $w_A < \frac{c}{\triangle}$, $w_B \geq \mu$, $A_{LL} < A_{HL}$, and $A_{LH} < A_{HH}$

$$\begin{aligned} A_{LL} < A_{HL} &\iff A_{LL} - A_{HL} < 0 \\ &\iff -\triangle(1-L)[w_A + p^{one}w_B + (1-p^{one})Lw_A] - \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c < 0 \\ &\iff D_3 = \triangle(1-L)[w_A + p^{one}w_B + (1-p^{one})Lw_A] + \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] > c \end{aligned}$$

$$\begin{aligned} A_{LH} < A_{HH} &\iff A_{LH} - A_{HH} < 0 \\ &\iff -\triangle(1-H)[w_A + p^{one}w_B + (1-p^{one})Lw_A] - \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c < 0 \\ &\iff D_4 = \triangle(1-H)[w_A + p^{one}w_B + (1-p^{one})Lw_A] + \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] > c. \end{aligned}$$

Since $D_3 - D_4 > 0$ $\left(w_B > \mu > \frac{c}{\triangle} > w_A\right)$, then $IC(A_{LL} < A_{HL})$ is slack, and $IC(A_{LH} < A_{HH})$ is binding. We only need consider $D_4 > c$. In fact, D_4 is the IC constraint if the rival worker chooses the high effort.

- If $\mu \geq \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}$, the minimum wage can make the IC constraint hold. The wage policy is $w_A^* = 0$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\begin{aligned} \Pi^*(2H\&2L) = {} & (2L+2H)y_A + 2H(1-H)E^{one} \\ & + H^2E^{two} + (1-H)^2\left(\frac{\underline{y}+\overline{y}}{2}\right) - \mu - \frac{\theta^2}{2}. \end{aligned}$$

  If $\mu < \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}$, the minimum wage cannot make the IC constraint hold. The firm can either increase w_A or w_B to make the IC constraint hold. I need to check (2.33) and (2.34). I find that

$$
\begin{aligned}
(2L+2H)\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L} &< \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}-\mu \\
(2L+2H)\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L} &< \frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1-H)p^{one}+\triangle Hp^{two}} \\
(2L+2H)\frac{1}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L} &< \frac{1}{\triangle(1-H)p^{one}+\triangle Hp^{two}} \\
(2L+2H)\left[(1-H)p^{one}+Hp^{two}\right] &< (1+L)-(1-H)p^{one}L-Hp^{two}L \\
(3L+2H)\left[(1-H)p^{one}+Hp^{two}\right] &< 1+L \\
(1-H)\left(\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}\right)+H\left(\frac{3}{8}+\frac{\theta}{2(\overline{y}-\underline{y})}-\frac{\theta^2}{2(\overline{y}-\underline{y})^2}\right) &< \frac{1+L}{3L+2H} \\
\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}-H\left(\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}-\frac{3}{8}-\frac{\theta}{2(\overline{y}-\underline{y})}+\frac{\theta^2}{2(\overline{y}-\underline{y})^2}\right) &< \frac{1+L}{3L+2H} \\
\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}-H\left(\frac{1}{8}+\frac{\theta}{2(\overline{y}-\underline{y})}+\frac{\theta^2}{2(\overline{y}-\underline{y})^2}\right) &< \frac{1+L}{3L+2H} \\
\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}-\frac{H}{2}\left(\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}\right)^2-\frac{1+L}{3L+2H} &< 0.
\end{aligned}
$$

Set $x=\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}$ ($\theta<2$ and $\overline{y}-\underline{y}>4$) and $J(x)=x-\frac{H}{2}x^2-\frac{1+L}{3L+2H}$, where $x\in\left(\frac{1}{2},1\right)$. $J'(x)=1-Hx>0$ and $J''(x)=-H<0$. Then $J(x)$ is increasing at $x\in\left(\frac{1}{2},1\right)$. $J\left(\frac{1}{2}\right)<0$ and $J(1)=1-\frac{H}{2}-\frac{1+L}{3L+2H}=\frac{4L+4H-H(3L+2H)-2}{3L+2H}>0$ if $4L+4H-H(3L+2H)-2>0$ and $J(1)=1-\frac{H}{2}-\frac{1+L}{3L+2H}=\frac{4L+4H-H(3L+2H)-2}{3L+2H}<0$ if $4L+4H-H(3L+2H)-2<0$. Therefore,

- ✧ When $4L+4H-H(3L+2H)-2<0$, the wage policy is $w_A^*=\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L}$ and $w_B^*=\mu$, and the total payoff for the firm is (2.33).

- ✧ When $4L+4H-H(3L+2H)-2>0$, there is a cut-off $\alpha=\left[\frac{1-\sqrt{1-\frac{2H(1+L)}{3L+2H}}}{H}-\frac{1}{2}\right]\in\left(0,\frac{1}{2}\right)$, where $J\left(\frac{1}{2}+\alpha\right)=0$. If $\frac{\theta}{\overline{y}-\underline{y}}>\alpha$, $J\left(\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}\right)>0$, the wage policy is $w_A^*=0$ and $w_B^*=\frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}$, and the total payoff for the firm is (2.34). If $\frac{\theta}{\overline{y}-\underline{y}}<\alpha$, $J\left(\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}\right)<0$, the wage policy is $w_A^*=\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L}$ and $w_B^*=\mu$, and the total payoff for the firm is (2.33).

1. If the wage policy is $w_A^* = \frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L}$ and $w_B^* = \mu$, the total payoff for the firm is

$$\Pi^*(2H\&2L) = (2L+2H)\left(y_A - \frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L}\right) + 2H(1-H)E^{one}$$
$$+H^2E^{two} + (1-H)^2\left(\frac{\underline{y}+\overline{y}}{2}\right) - \mu - \frac{\theta^2}{2}.$$

2. If the wage policy is $w_A^* = 0$ and $w_B^* = \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}$, the total payoff for the firm is

$$\Pi^*(2H\&2L) = (2L+2H)y_A + 2H(1-H)E^{one}$$
$$+H^2E^{two} + (1-H)^2\left(\frac{\underline{y}+\overline{y}}{2}\right) - \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}} - \frac{\theta^2}{2}.$$

(3) (H, L) and (L, H) are the NEs.

The firm's problem is

$$\underset{\{w_A, w_B\}}{max}\ \Pi(H3\&L) = (3L+H)(y_A - w_A) + E(\widehat{y_B}) - w_B - \frac{\theta^2}{2}$$

s.t $w_A < \frac{c}{\triangle}$, $w_B \geq \mu$, $A_{LL} < A_{HL}$, and $A_{LH} > A_{HH}$

$$\begin{aligned} A_{LL} < A_{HL} \quad &\Longleftrightarrow \quad A_{LL} - A_{HL} < 0 \\ &\Longleftrightarrow \quad -\triangle(1-L)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] - \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c < 0 \\ &\Longleftrightarrow \quad D_3 = \triangle(1-L)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] + \triangle L\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] > c \end{aligned}$$

$$\begin{aligned} A_{LH} > A_{HH} \quad &\Longleftrightarrow \quad A_{LH} - A_{HH} > 0 \\ &\Longleftrightarrow \quad -\triangle(1-H)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] - \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] + c > 0 \\ &\Longleftrightarrow \quad D_4 = \triangle(1-H)\left[w_A + p^{one}w_B + (1-p^{one})Lw_A\right] + \triangle H\left[w_A + p^{two}w_B + \left(1-p^{two}\right)Lw_A\right] < c. \end{aligned}$$

To let $A_{LL} < A_{HL}$, and $A_{LH} > A_{HH}$, we must have $\frac{c}{\triangle(1-L)p^{one}+\triangle Lp^{two}} < \mu < \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}$, and the wage policy is $w_A^* = 0$ and $w_B^* = \mu$, and the total payoff for the firm is

$$\Pi^*(H\&3L) = (3L+H)y_A + H(1-L)E^{one} + L(1-H)E^{one}$$
$$+HLE^{two} + (1-H)(1-L)\left(\frac{\underline{y}+\overline{y}}{2}\right) - \mu - \frac{\theta^2}{2}.$$

2.7.3 The Optimal Wage Policy for a Given Training Level θ

(1a) $\mu < \frac{c}{\triangle(1-L)p^{one}+\triangle Lp^{two}}$ and $(2L+2H)\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L} < \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}} - \mu$.

There are three kinds of outcome.

- $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(4L) = (2.31)$;
- $w_A^* = \frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L}$ and $w_B^* = \mu$, $\Pi^*(2H\&2L) = (2.33)$;
- $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (2.30)$.

$$
\begin{aligned}
(2.31) < (2.33) &\iff y_A > \varphi_1 \\
(2.31) < (2.30) &\iff y_A > \varphi_2 \\
(2.30) > (2.33) &\iff y_A > \varphi_3.
\end{aligned}
$$

$$(2.31) < (2.33)$$

$$
\begin{aligned}
2\triangle y_A - (2L+2H)\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L} &> -E^{one}[2H(1-H)-2L(1-L)] - E^{two}\left(H^2-L^2\right) \\
&\quad - \left(\frac{\underline{y}+\overline{y}}{2}\right)\left[(1-H)^2-(1-L)^2\right] \\
2\triangle y_A - (2L+2H)\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L} &> -2E^{one}[\triangle-\triangle(H+L)] - E^{two}\triangle(H+L) + \left(\frac{\underline{y}+\overline{y}}{2}\right)\triangle(2-H-L) \\
2y_A - \left(\frac{2L+2H}{\triangle}\right)\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L} &> -2E^{one}[1-(H+L)] - E^{two}(H+L) + \left(\frac{\underline{y}+\overline{y}}{2}\right)(2-H-L) \\
2y_A - \left(\frac{2L+2H}{\triangle}\right)\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L} &> (H+L)\left(2E^{one}-E^{two}-\frac{\underline{y}+\overline{y}}{2}\right) - \frac{\overline{y}-\underline{y}}{4} - \frac{\theta^2}{\overline{y}-\underline{y}} - \theta \\
y_A &> \varphi_1.
\end{aligned}
$$

$$\varphi_1 = \left(\frac{L+H}{\triangle}\right)\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L} + \left(\frac{L+H}{2}\right)\left[\frac{\overline{y}-\underline{y}+6\theta}{24} + \frac{\theta^2}{2\left(\overline{y}-\underline{y}\right)} + \frac{\theta^3}{3\left(\overline{y}-\underline{y}\right)^2}\right] - \frac{\overline{y}-\underline{y}}{8} - \frac{\theta^2}{2\left(\overline{y}-\underline{y}\right)} - \frac{\theta}{2}.$$

$$(2.31) < (2.30)$$

$$
\begin{aligned}
4\triangle y_A &> \frac{4Hc}{\triangle} - E^{one}\left[2H(1-H) - 2L(1-L)\right] - E^{two}\left(H^2 - L^2\right) - \left(\frac{\underline{y}+\overline{y}}{2}\right)\left[(1-H)^2 - (1-L)^2\right] \\
4\triangle y_A &> \frac{4Hc}{\triangle} - 2E^{one}\left[\triangle - \triangle(H+L)\right] - E^{two}\triangle(H+L) + \left(\frac{\underline{y}+\overline{y}}{2}\right)\triangle(2-H-L) \\
4y_A &> \frac{4Hc}{\triangle^2} - 2E^{one}\left[1-(H+L)\right] - E^{two}(H+L) + \left(\frac{\underline{y}+\overline{y}}{2}\right)(2-H-L) \\
4y_A &> \frac{4Hc}{\triangle^2} + (L+H)\left[\frac{\overline{y}-\underline{y}+6\theta}{24} + \frac{\theta^2}{2\left(\overline{y}-\underline{y}\right)} + \frac{\theta^3}{3\left(\overline{y}-\underline{y}\right)^2}\right] - \frac{\overline{y}-\underline{y}}{4} - \frac{\theta^2}{\overline{y}-\underline{y}} - \theta \\
y_A &> \frac{Hc}{\triangle^2} + \left(\frac{L+H}{4}\right)\left[\frac{\overline{y}-\underline{y}+6\theta}{24} + \frac{\theta^2}{2\left(\overline{y}-\underline{y}\right)} + \frac{\theta^3}{3\left(\overline{y}-\underline{y}\right)^2}\right] - \frac{\overline{y}-\underline{y}}{16} - \frac{\theta^2}{4\left(\overline{y}-\underline{y}\right)} - \frac{\theta}{4} = \varphi_2.
\end{aligned}
$$

$$(2.30) < (2.33)$$

$$
\begin{aligned}
4H\left(y_A - \frac{c}{\triangle}\right) &> (2L+2H)\left(y_A - \frac{c - \triangle(1-H)p^{one}\mu - \triangle Hp^{two}\mu}{\triangle(1+L) - \triangle(1-H)p^{one}L - \triangle Hp^{two}L}\right) \\
2\triangle y_A &> \frac{4Hc}{\triangle} - (2L+2H)\frac{c - \triangle(1-H)p^{one}\mu - \triangle Hp^{two}\mu}{\triangle(1+L) - \triangle(1-H)p^{one}L - \triangle Hp^{two}L} \\
y_A &> \frac{2Hc}{\triangle^2} - \left(\frac{L+H}{\triangle}\right)\frac{c - \triangle(1-H)p^{one}\mu - \triangle Hp^{two}\mu}{\triangle(1+L) - \triangle(1-H)p^{one}L - \triangle Hp^{two}L} = \varphi_3.
\end{aligned}
$$

Let $\frac{Hc}{\triangle^2} = a > 0$, $\left(\frac{L+H}{\triangle}\right)\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L} = b > 0$ and $\left(\frac{L+H}{2}\right)\left[\frac{\overline{y}-\underline{y}+6\theta}{24} + \frac{\theta^2}{2\left(\overline{y}-\underline{y}\right)} + \frac{\theta^3}{3\left(\overline{y}-\underline{y}\right)^2}\right] - \frac{\overline{y}-\underline{y}}{8} - \frac{\theta^2}{2\left(\overline{y}-\underline{y}\right)} - \frac{\theta}{2} = -c < 0$. We have $a > b > c$, $\varphi_1 = b - c$, $\varphi_2 = a - \frac{c}{2}$ and $\varphi_3 = 2a - b$. We easily obtain $\varphi_1 < \varphi_2 < \varphi_3$.

The optimal wage policy (w_A, w_B) is given by the following:

- If $y_A < \varphi_1$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(4L) = (2.31)$;
- If $\varphi_1 < y_A < \varphi_3$, the optimal wage policy is $w_A^* = \frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L}$ and $w_B^* = \mu$, $\Pi^*(2H\&2L) = (2.33)$;
- If $\varphi_3 < y_A$, the optimal wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (2.30)$.

(1b) $\mu < \frac{c}{\triangle(1-L)p^{one}+\triangle Lp^{two}}$ and $(2L+2H)\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L} > \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}} - \mu$.

There are three kinds of outcome.

- $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(4L) = (2.31)$;
- $w_A^* = 0$ and $w_B^* = \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}$, $\Pi^*(2H\&2L) = (2.34)$;
- $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (2.30)$.

$$
\begin{aligned}
(2.31) < (2.34) &\iff y_A > \varphi_1' \\
(2.31) < (2.30) &\iff y_A > \varphi_2 \\
(2.30) > (2.34) &\iff y_A > \varphi_3'.
\end{aligned}
$$

$$(2.31) < (2.34)$$

$$2\triangle y_A - \frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1-H)p^{one}+\triangle Hp^{two}} > -2E^{one}\left[\triangle-\triangle(H+L)\right] - E^{two}\triangle(H+L) + \left(\frac{\underline{y}+\overline{y}}{2}\right)\triangle(2-H-L)$$

$$y_A > \varphi_1'.$$

$$\varphi_1' = \frac{1}{2\triangle}\cdot\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1-H)p^{one}+\triangle Hp^{two}} + \left(\frac{H+L}{2}\right)\left[\frac{\overline{y}-\underline{y}+6\theta}{24}+\frac{\theta^2}{2\left(\overline{y}-\underline{y}\right)}+\frac{\theta^3}{3\left(\overline{y}-\underline{y}\right)^2}\right] - \frac{\overline{y}-\underline{y}}{8} - \frac{\theta^2}{2\left(\overline{y}-\underline{y}\right)} - \frac{\theta}{2}.$$

$$(2.30) < (2.34)$$

$$
\begin{aligned}
2\triangle y_A - \frac{4Hc}{\triangle} &> -\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1-H)p^{one}+\triangle Hp^{two}} \\
y_A &> \frac{2Hc}{\triangle^2} - \frac{1}{2\triangle}\cdot\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1-H)p^{one}+\triangle Hp^{two}} = \varphi_3'.
\end{aligned}
$$

Similarly, we have $\varphi_1' < \varphi_2 < \varphi_3'$

The optimal wage policy (w_A, w_B) is given by the following:

- If $y_A < \varphi_1'$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(4L) = (2.31)$;

- **If** $\varphi_1' < y_A < \varphi_3'$, **the optimal wage policy is** $w_A^* = 0$ **and** $w_B^* = \frac{c}{\triangle(1-H)p^{one}+\triangle H p^{two}}$, $\Pi^*(2H\&2L) = (2.34)$;
- **If** $\varphi_3' < y_A$, **the optimal wage policy is** $w_A^* = \frac{c}{\triangle}$ **and** $w_B^* = \mu$, $\Pi^*(4H) = (2.30)$.

(2a) $\frac{c}{\triangle(1-L)p^{one}+\triangle L p^{two}} < \mu < \frac{c}{\triangle(1-H)p^{one}+\triangle H p^{two}}$ and $(2L+2H)\frac{c-\triangle(1-H)p^{one}\mu-\triangle H p^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle H p^{two}L} < \frac{c}{\triangle(1-H)p^{one}+\triangle H p^{two}} - \mu.$

There are three kinds of outcome.

- $w_A^* = 0$ **and** $w_B^* = \mu$, $\Pi^*(H\&3L) = (2.35)$;
- $w_A^* = \frac{c-\triangle(1-H)p^{one}\mu-\triangle H p^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle H p^{two}L}$ **and** $w_B^* = \mu$, $\Pi^*(2H\&2L) = (2.33)$;
- $w_A^* = \frac{c}{\triangle}$ **and** $w_B^* = \mu$, $\Pi^*(4H) = (2.30)$.

We have

$$(2.35) < (2.33) \iff y_A > \varphi_4$$
$$(2.35) < (2.30) \iff y_A > \varphi_5$$
$$(2.30) > (2.33) \iff y_A > \varphi_3.$$

$$(2.35) < (2.33)$$

$$\triangle y_A - (2L+2H)\frac{c-\triangle(1-H)p^{one}\mu-\triangle H p^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle H p^{two}L} > \triangle H E^{one} - \triangle(1-H)E^{one} - \triangle H E^{two} + \triangle(1-H)\frac{\underline{y}+\overline{y}}{2}$$
$$y_A > \frac{2L+2H}{\triangle}\cdot\frac{c-\triangle(1-H)p^{one}\mu-\triangle H p^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle H p^{two}L} + H\left(2E^{one} - E^{two} - \frac{\underline{y}+\overline{y}}{2}\right) - E_1 + \frac{\underline{y}+\overline{y}}{2}$$
$$y_A > \varphi_4.$$

$$\varphi_4 = \frac{2L+2H}{\triangle}\cdot\frac{c-\triangle(1-H)p^{one}\mu-\triangle H p^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle H p^{two}L} + H\left(\frac{\overline{y}-\underline{y}+6\theta}{24} + \frac{\theta^2}{2\left(\overline{y}-\underline{y}\right)} + \frac{\theta^3}{3\left(\overline{y}-\underline{y}\right)^2}\right) - \frac{\overline{y}-\underline{y}}{8} - \frac{\theta^2}{2\left(\overline{y}-\underline{y}\right)} - \frac{\theta}{2}.$$

$$(2.35) < (2.30)$$

$$3\triangle y_A - \frac{4Hc}{\triangle} > \triangle HE^{one} - \triangle(1-H)E^{one} - \triangle HE^{two} + \triangle(1-H)\frac{\underline{y}+\overline{y}}{2}$$

$$y_A > \frac{4Hc}{3\triangle^2} + \frac{H}{3}\left(\frac{\overline{y}-\underline{y}+6\theta}{24} + \frac{\theta^2}{2(\overline{y}-\underline{y})} + \frac{\theta^3}{3(\overline{y}-\underline{y})^2}\right) - \frac{\overline{y}-\underline{y}}{24} - \frac{\theta^2}{6(\overline{y}-\underline{y})} - \frac{\theta}{6} = \varphi_5.$$

Similarly, we have $\varphi_4 < \varphi_5 < \varphi_3$.

The optimal wage policy (w_A, w_B) **is given by the following:**

- If $y_A < \varphi_4$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(H\&3L) = (2.35);$
- If $\varphi_4 < y_A < \varphi_3$, the optimal wage policy is $w_A^* = \frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L}$ and $w_B^* = \mu$, $\Pi^*(2H\&2L) = (2.33);$
- If $\varphi_3 < y_A$, the optimal wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (2.30).$

(2b) $\frac{c}{\triangle(1-L)p^{one}+\triangle Lp^{two}} < \mu < \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}$ and $(2L+2H)$ $\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L} > \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}} - \mu.$

There are three kinds of outcome.

- $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(H\&3L) = (2.35);$
- $w_A^* = 0$ and $w_B^* = \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}$, $\Pi^*(2H\&2L) = (2.34);$
- $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (2.30).$

We have

$$(2.35) < (2.34) \iff y_A > \varphi_4'$$
$$(2.35) < (2.30) \iff y_A > \varphi_5$$
$$(2.30) > (2.34) \iff y_A > \varphi_3'.$$

$$(2.35) < (2.34)$$

$$\triangle y_A - \frac{c - \triangle(1-H)p^{one}\mu - \triangle Hp^{two}\mu}{\triangle(1-H)p^{one} + \triangle Hp^{two}} > \triangle HE^{one} - \triangle(1-H)E^{one} - \triangle HE^{two} + \triangle(1-H)\frac{\underline{y}+\overline{y}}{2}$$

$$y_A > \frac{1}{\triangle} \cdot \frac{c - \triangle(1-H)p^{one}\mu - \triangle Hp^{two}\mu}{\triangle(1-H)p^{one} + \triangle Hp^{two}} + H\left(2E^{one} - E^{two} - \frac{\underline{y}+\overline{y}}{2}\right) - E^{one} + \frac{\underline{y}+\overline{y}}{2}$$

$$y_A > \varphi_4'.$$

$$\varphi_4' = \frac{1}{\triangle} \cdot \frac{c - \triangle(1-H)p^{one}\mu - \triangle Hp^{two}\mu}{\triangle(1-H)p^{one} + \triangle Hp^{two}} + H\left(\frac{\overline{y} - \underline{y} + 6\theta}{24} + \frac{\theta^2}{2(\overline{y}-\underline{y})} + \frac{\theta^3}{3(\overline{y}-\underline{y})^2}\right) - \frac{\overline{y}-\underline{y}}{8} - \frac{\theta^2}{2(\overline{y}-\underline{y})} - \frac{\theta}{2}.$$

Similarly, we have $\varphi_4' < \varphi_5 < \varphi_3'$.

The optimal wage policy (w_A, w_B) **is given by the following:**

- If $y_A < \varphi_4'$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \mu$, $\varPi^*(H\&3L) = (2.35)$;
- If $\varphi_4' < y_A < \varphi_3'$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}$, $\varPi^*(2H\&2L) = (2.34)$;
- If $\varphi_3' < y_A$, the optimal wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\varPi^*(4H) = (2.32)$.

(3) $\mu \geq \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}$.

There are two kinds of outcome.

- $w_A^* = 0$ and $w_B^* = \mu$, $\varPi^*(2H\&2L) = (2.32)$;
- $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\varPi^*(4H) = (2.30)$;

I have $(2.30) > (2.32) \Longleftrightarrow y_A > \frac{2Hc}{\triangle^2} = \varphi_6$.

The optimal wage policy (w_A, w_B) **is given by the following:**

- If $y_A < \varphi_6$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \mu$, $\varPi^*(2H\&2L) = (2.32)$;

- If $\varphi_6 < y_A$, the optimal wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (2.30)$.

2.7.4 The Optimal Training Level - Preliminary Analysis

$E^{one}(\theta) = \frac{3\underline{y}+5\overline{y}+4\theta}{8} + \frac{\theta^2}{2(\overline{y}-\underline{y})}$, $E^{two}(\theta) = \frac{7\underline{y}+17\overline{y}+18\theta}{24} + \frac{\theta^2}{2(\overline{y}-\underline{y})} - \frac{\theta^3}{3(\overline{y}-\underline{y})^2}$,
$p^{one}(\theta) = \frac{1}{2} + \frac{\theta}{\overline{y}-\underline{y}}$ and $p^{two}(\theta) = \frac{3}{8} + \frac{\theta}{2(\overline{y}-\underline{y})} - \frac{\theta^2}{2(\overline{y}-\underline{y})^2}$

$E^{one\prime} = \frac{1}{2} + \frac{\theta}{\overline{y}-\underline{y}} > 0$, $E^{one\prime\prime} = \frac{1}{\overline{y}-\underline{y}} > 0$ and $E^{one\prime\prime\prime} = 0$

$E^{two\prime} = \frac{3}{4} + \frac{\theta}{\overline{y}-\underline{y}} - \frac{\theta^2}{(\overline{y}-\underline{y})^2} > 0$, $E^{two\prime\prime} = \frac{1}{\overline{y}-\underline{y}} - \frac{2\theta}{(\overline{y}-\underline{y})^2} > 0$ and $E^{two\prime\prime\prime} = -\frac{2}{(\overline{y}-\underline{y})^2} < 0$

$p^{one\prime} = \frac{1}{\overline{y}-\underline{y}}$, $p^{two\prime} = \frac{1}{2(\overline{y}-\underline{y})} - \frac{\theta}{(\overline{y}-\underline{y})^2} > 0$,

$$
\begin{aligned}
w_A^* &= \frac{c - \triangle(1-H)p^{one}\mu - \triangle H p^{two}\mu}{\triangle(1+L) - \triangle(1-H)p^{one}L - \triangle H p^{two}L} \\
&= \frac{c}{\triangle(1+L) - \triangle(1-H)p^{one}L - \triangle H p^{two}L} + \frac{-\triangle(1-H)p^{one}\mu - \triangle H p^{two}\mu}{\triangle(1+L) - \triangle(1-H)p^{one}L - \triangle H p^{two}L} \\
&= \frac{c}{\triangle}\cdot\frac{1}{(1+L)-(1-H)p^{one}L - Hp^{two}L} + \frac{-(1-H)p^{one}\mu - Hp^{two}\mu}{(1+L)-(1-H)p^{one}L - Hp^{two}L} \\
&= \frac{c}{\triangle}\cdot\frac{1}{(1+L)-(1-H)p^{one}L - Hp^{two}L} + \frac{\mu}{L}\cdot\frac{-(1-H)p^{one} - Hp^{two}}{\left(\frac{1+L}{L}\right)-(1-H)p^{one} - Hp^{two}} \\
&= \frac{c}{\triangle}\cdot\frac{1}{(1+L)-(1-H)p^{one}L - Hp^{two}L} + \frac{\mu}{L}\cdot\left[\frac{\frac{1+L}{L}-(1-H)p^{one} - Hp^{two}}{\left(\frac{1+L}{L}\right)-(1-H)p^{one} - Hp^{two}} - \frac{\frac{1+L}{L}}{\left(\frac{1+L}{L}\right)-(1-H)p^{one} - Hp^{two}}\right] \\
&= \frac{c}{\triangle}\cdot\frac{1}{(1+L)-(1-H)p^{one}L - Hp^{two}L} + \frac{\mu}{L}\cdot\left[1 - \frac{\frac{1+L}{L}}{\left(\frac{1+L}{L}\right)-(1-H)p^{one} - Hp^{two}}\right] \\
&= \frac{c}{\triangle}\cdot\frac{1}{(1+L)-(1-H)p^{one}L - Hp^{two}L} + \frac{\mu}{L} - \left(\frac{1+L}{L}\right)\mu\frac{1}{(1+L)-(1-H)p^{one}L - Hp^{two}L} \\
&= \left[\frac{c}{\triangle} - \left(\frac{1+L}{L}\right)\mu\right]\frac{1}{(1+L)-(1-H)p^{one}L - Hp^{two}L} + \frac{\mu}{L}.
\end{aligned}
$$

$$
\begin{aligned}
w_A^{*\prime} &= \left[\frac{c}{\triangle} - \left(\frac{1+L}{L}\right)\mu\right]\frac{\frac{(1-H)L}{\overline{y}-\underline{y}} + HL\left[\frac{1}{2(\overline{y}-\underline{y})} - \frac{\theta}{(\overline{y}-\underline{y})^2}\right]}{[(1+L)-(1-H)p^{one}L - Hp^{two}L]^2} \\
&= \left[\frac{c}{\triangle} - \left(\frac{1+L}{L}\right)\mu\right]\frac{\frac{2L-HL}{2(\overline{y}-\underline{y})} - \frac{HL\theta}{(\overline{y}-\underline{y})^2}}{[(1+L)-(1-H)p^{one}L - Hp^{two}L]^2} \\
&= \frac{2L-HL}{2(\overline{y}-\underline{y})}\left[\frac{c}{\triangle} - \left(\frac{1+L}{L}\right)\mu\right]\frac{1}{[(1+L)-(1-H)p^{one}L - Hp^{two}L]^2} - \frac{1}{(\overline{y}-\underline{y})^2}\left[\frac{c}{\triangle} - \left(\frac{1+L}{L}\right)\mu\right]\frac{HL\theta}{[(1+L)-(1-H)p^{one}L - Hp^{two}L]^2} \\
&< 0.
\end{aligned}
$$

$\frac{c}{\triangle} - \left(\frac{1+L}{L}\right)\mu < 0$ as $\frac{c}{\triangle} < \mu$.

$$
\begin{aligned}
w_A^{*\prime\prime} &= \frac{2L-HL}{\overline{y}-\underline{y}}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{\frac{(1-H)L}{\overline{y}-\underline{y}}+HL\left[\frac{1}{2\left(\overline{y}-\underline{y}\right)}-\frac{\theta}{\left(\overline{y}-\underline{y}\right)^2}\right]}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
&-(2HL\theta)\left[\frac{\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu}{\left(\overline{y}-\underline{y}\right)^2}\right]\frac{\frac{(1-H)L}{\overline{y}-\underline{y}}+HL\left[\frac{1}{2\left(\overline{y}-\underline{y}\right)}-\frac{\theta}{\left(\overline{y}-\underline{y}\right)^2}\right]}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
&-\left[\frac{\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu}{\left(\overline{y}-\underline{y}\right)^2}\right]\frac{HL}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^2} \\
&=\frac{1}{2}\left(\frac{2L-HL}{\overline{y}-\underline{y}}\right)^2\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
&-HL\theta\left[\frac{2L-HL}{\left(\overline{y}-\underline{y}\right)^3}\right]\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
&-(HL\theta)\left[\frac{2L-HL}{\left(\overline{y}-\underline{y}\right)^3}\right]\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
&+2\,(HL\theta)^2\left[\frac{\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu}{\left(\overline{y}-\underline{y}\right)^4}\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
&-HL\left[\frac{\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu}{\left(\overline{y}-\underline{y}\right)^2}\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^2} \\
&=(2L-HL)^2\,\frac{1}{2\left(\overline{y}-\underline{y}\right)^2}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
&-2HL\theta\,(2L-HL)\,\frac{1}{\left(\overline{y}-\underline{y}\right)^3}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
&-2\,(HL\theta)^2\,\frac{1}{\left(\overline{y}-\underline{y}\right)^4}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
&-HL\frac{1}{2\left(\overline{y}-\underline{y}\right)^2}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^2}>0.
\end{aligned}
$$

$$
\begin{aligned}
w_A^{*'''} = {} & \frac{3}{2}\left(\frac{2L-HL}{\overline{y}-\underline{y}}\right)^2\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{\frac{(1-H)L}{\overline{y}-\underline{y}}+HL\left[\frac{1}{2\left(\overline{y}-\underline{y}\right)}-\frac{\theta}{\left(\overline{y}-\underline{y}\right)^2}\right]}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^4} \\
& -2HL\left[\frac{2L-HL}{\left(\overline{y}-\underline{y}\right)^3}\right]\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
& -6HL\theta\left[\frac{2L-HL}{\left(\overline{y}-\underline{y}\right)^3}\right]\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{\frac{(1-H)L}{\overline{y}-\underline{y}}+HL\left[\frac{1}{2\left(\overline{y}-\underline{y}\right)}-\frac{\theta}{\left(\overline{y}-\underline{y}\right)^2}\right]}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^4} \\
& -6\left(HL\theta\right)^2\left[\frac{\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu}{\left(\overline{y}-\underline{y}\right)^4}\right]\frac{\frac{(1-H)L}{\overline{y}-\underline{y}}+HL\left[\frac{1}{2\left(\overline{y}-\underline{y}\right)}-\frac{\theta}{\left(\overline{y}-\underline{y}\right)^2}\right]}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^4} \\
& -4\theta\left(HL\right)^2\left[\frac{\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu}{\left(\overline{y}-\underline{y}\right)^4}\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
& -2HL\left[\frac{\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu}{\left(\overline{y}-\underline{y}\right)^2}\right]\frac{\frac{(1-H)L}{\overline{y}-\underline{y}}+HL\left[\frac{1}{2\left(\overline{y}-\underline{y}\right)}-\frac{\theta}{\left(\overline{y}-\underline{y}\right)^2}\right]}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
= {} & \frac{3}{4}\left(2L-HL\right)^3\frac{1}{\left(\overline{y}-\underline{y}\right)^3}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^4} \\
& -\frac{3}{2}HL\theta\left(2L-HL\right)^2\frac{1}{\left(\overline{y}-\underline{y}\right)^4}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^4} \\
& -2HL\left(2L-HL\right)\frac{1}{\left(\overline{y}-\underline{y}\right)^3}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
& -3HL\theta\left(2L-HL\right)^2\frac{1}{\left(\overline{y}-\underline{y}\right)^4}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^4} \\
& +6\left(HL\theta\right)^2\left(2L-HL\right)\frac{1}{\left(\overline{y}-\underline{y}\right)^5}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^4} \\
& -3\left(HL\theta\right)^2\left(2L-HL\right)\frac{1}{\left(\overline{y}-\underline{y}\right)^5}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^4} \\
& +6\left(HL\theta\right)^3\frac{1}{\left(\overline{y}-\underline{y}\right)^6}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^4} \\
& -4\theta\left(HL\right)^2\frac{1}{\left(\overline{y}-\underline{y}\right)^4}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
& -HL\left(2L-HL\right)\frac{1}{\left(\overline{y}-\underline{y}\right)^3}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
& +2\theta\left(HL\right)^2\frac{1}{\left(\overline{y}-\underline{y}\right)^4}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
= {} & \frac{3}{4}\left(2L-HL\right)^3\frac{1}{\left(\overline{y}-\underline{y}\right)^3}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^4} \\
& -\frac{9}{2}HL\theta\left(2L-HL\right)^2\frac{1}{\left(\overline{y}-\underline{y}\right)^4}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^4} \\
& -3HL\left(2L-HL\right)\frac{1}{\left(\overline{y}-\underline{y}\right)^3}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} \\
& +3\left(HL\theta\right)^2\left(2L-HL\right)\frac{1}{\left(\overline{y}-\underline{y}\right)^5}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^4} \\
& +6\left(HL\theta\right)^3\frac{1}{\left(\overline{y}-\underline{y}\right)^6}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^4} \\
& -2\theta\left(HL\right)^2\frac{1}{\left(\overline{y}-\underline{y}\right)^4}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{\left[(1+L)-(1-H)\,p^{one}L-Hp^{two}L\right]^3} > 0.
\end{aligned}
$$

$$
w_B^{*} = \frac{c}{\triangle\left(1-H\right)p^{one}+\triangle Hp^{two}} = \frac{c}{\triangle}\frac{1}{\left(1-H\right)p^{one}+Hp^{two}}.
$$

$$
\begin{aligned}
w_B^{*\prime} &= -\frac{c}{\triangle}\frac{\frac{1-H}{\overline{y}-\underline{y}} + H\left[\frac{1}{2\left(\overline{y}-\underline{y}\right)} - \frac{\theta}{\left(\overline{y}-\underline{y}\right)^2}\right]}{\left[(1-H)\,p^{one} + Hp^{two}\right]^2} \\
&= -\frac{c}{\triangle}\left\{\frac{\frac{2-H}{2\left(\overline{y}-\underline{y}\right)}}{\left[(1-H)\,p^{one} + Hp^{two}\right]^2} - \frac{\frac{H\theta}{\left(\overline{y}-\underline{y}\right)^2}}{\left[(1-H)\,p^{one} + Hp^{two}\right]^2}\right\} < 0.
\end{aligned}
$$

$$
\begin{aligned}
w_B^{*\prime\prime} =& \frac{c}{\triangle}\frac{2-H}{\overline{y}-\underline{y}}\frac{\frac{2-H}{2\left(\overline{y}-\underline{y}\right)} - \frac{H\theta}{\left(\overline{y}-\underline{y}\right)^2}}{\left[(1-H)\,p^{one} + Hp^{two}\right]^3} \\
&+ \frac{c}{\triangle}\frac{H}{\left(\overline{y}-\underline{y}\right)^2}\frac{1}{\left[(1-H)\,p^{one} + Hp^{two}\right]^2} \\
&- \frac{3c}{\triangle}\frac{H\theta}{\left(\overline{y}-\underline{y}\right)^2}\frac{\frac{2-H}{2\left(\overline{y}-\underline{y}\right)} - \frac{H\theta}{\left(\overline{y}-\underline{y}\right)^2}}{\left[(1-H)\,p^{one} + Hp^{two}\right]^3} \\
=& \frac{2c}{\triangle}\frac{(2-H)^2}{\left(\overline{y}-\underline{y}\right)^2}\frac{1}{\left[(1-H)\,p^{one} + Hp^{two}\right]^3} - \frac{c}{\triangle}\frac{H\,(2-H)\,\theta}{\left(\overline{y}-\underline{y}\right)^3}\frac{1}{\left[(1-H)\,p^{one} + Hp^{two}\right]^3} \\
&+ \frac{c}{\triangle}\frac{H}{\left(\overline{y}-\underline{y}\right)^2}\frac{1}{\left[(1-H)\,p^{one} + Hp^{two}\right]^2} \\
&- \frac{3c}{\triangle}\frac{H\,(2-H)\,\theta}{2\left(\overline{y}-\underline{y}\right)^3}\frac{1}{\left[(1-H)\,p^{one} + Hp^{two}\right]^3} + \frac{3c}{\triangle}\frac{(H\theta)^2}{\left(\overline{y}-\underline{y}\right)^4}\frac{1}{\left[(1-H)\,p^{one} + Hp^{two}\right]^3} \\
=& \frac{2c}{\triangle}\frac{(2-H)^2}{\left(\overline{y}-\underline{y}\right)^2}\frac{1}{\left[(1-H)\,p^{one} + Hp^{two}\right]^3} - \frac{4c}{\triangle}\frac{H\,(2-H)\,\theta}{\left(\overline{y}-\underline{y}\right)^3}\frac{1}{\left[(1-H)\,p^{one} + Hp^{two}\right]^3} \\
&+ \frac{c}{\triangle}\frac{H}{\left(\overline{y}-\underline{y}\right)^2}\frac{1}{\left[(1-H)\,p^{one} + Hp^{two}\right]^2} + \frac{3c}{\triangle}\frac{(H\theta)^2}{\left(\overline{y}-\underline{y}\right)^4}\frac{1}{\left[(1-H)\,p^{one} + Hp^{two}\right]^3} > 0.
\end{aligned}
$$

$$
\begin{aligned}
w_B^{*'''} = & -\frac{6c}{\triangle}\frac{(2-H)^2}{\left(\overline{y}-\underline{y}\right)^2}\frac{\frac{2-H}{2\left(\overline{y}-\underline{y}\right)}-\frac{H\theta}{\left(\overline{y}-\underline{y}\right)^2}}{\left[(1-H)\,p^{one}+Hp^{two}\right]^4}-\frac{2c}{\triangle}\frac{H}{\left(\overline{y}-\underline{y}\right)^2}\frac{\frac{2-H}{2\left(\overline{y}-\underline{y}\right)}-\frac{H\theta}{\left(\overline{y}-\underline{y}\right)^2}}{\left[(1-H)\,p^{one}+Hp^{two}\right]^3} \\
& -\frac{4c}{\triangle}\frac{H\,(2-H)}{\left(\overline{y}-\underline{y}\right)^3}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^3}+\frac{12c}{\triangle}\frac{H\,(2-H)\,\theta}{\left(\overline{y}-\underline{y}\right)^3}\frac{\frac{2-H}{2\left(\overline{y}-\underline{y}\right)}-\frac{H\theta}{\left(\overline{y}-\underline{y}\right)^2}}{\left[(1-H)\,p^{one}+Hp^{two}\right]^4} \\
& +\frac{6c}{\triangle}\frac{H^2\theta}{\left(\overline{y}-\underline{y}\right)^4}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^3}-\frac{9c}{\triangle}\frac{(H\theta)^2}{\left(\overline{y}-\underline{y}\right)^4}\frac{\frac{2-H}{2\left(\overline{y}-\underline{y}\right)}-\frac{H\theta}{\left(\overline{y}-\underline{y}\right)^2}}{\left[(1-H)\,p^{one}+Hp^{two}\right]^4} \\
= & -\frac{3c}{\triangle}\frac{(2-H)^3}{\left(\overline{y}-\underline{y}\right)^3}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^4}+\frac{6c}{\triangle}\frac{(2-H)^2H\theta}{\left(\overline{y}-\underline{y}\right)^4}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^4} \\
& -\frac{c}{\triangle}\frac{H\,(2-H)}{\left(\overline{y}-\underline{y}\right)^3}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^3}+\frac{2c}{\triangle}\frac{H^2\theta}{\left(\overline{y}-\underline{y}\right)^4}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^3} \\
& -\frac{4c}{\triangle}\frac{H\,(2-H)}{\left(\overline{y}-\underline{y}\right)^3}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^3}+\frac{6c}{\triangle}\frac{H^2\theta}{\left(\overline{y}-\underline{y}\right)^4}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^3} \\
& +\frac{6c}{\triangle}\frac{(2-H)^2H\theta}{\left(\overline{y}-\underline{y}\right)^4}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^4}-\frac{12c}{\triangle}\frac{(H\theta)^2(2-H)}{\left(\overline{y}-\underline{y}\right)^5}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^4} \\
& -\frac{9c}{2\triangle}\frac{(H\theta)^2(2-H)}{\left(\overline{y}-\underline{y}\right)^5}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^4}+\frac{9c}{\triangle}\frac{(H\theta)^3}{\left(\overline{y}-\underline{y}\right)^6}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^4} \\
= & -\frac{3c}{\triangle}\frac{(2-H)^3}{\left(\overline{y}-\underline{y}\right)^3}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^4}+\frac{12c}{\triangle}\frac{(2-H)^2H\theta}{\left(\overline{y}-\underline{y}\right)^4}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^4} \\
& -\frac{5c}{\triangle}\frac{H\,(2-H)}{\left(\overline{y}-\underline{y}\right)^3}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^3}+\frac{8c}{\triangle}\frac{H^2\theta}{\left(\overline{y}-\underline{y}\right)^4}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^3} \\
& -\frac{33c}{2\triangle}\frac{(H\theta)^2(2-H)}{\left(\overline{y}-\underline{y}\right)^5}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^4}+\frac{9c}{\triangle}\frac{(H\theta)^3}{\left(\overline{y}-\underline{y}\right)^6}\frac{1}{\left[(1-H)\,p^{one}+Hp^{two}\right]^4}>0.
\end{aligned}
$$

2.7.5 The Optimal Training Level

(1a) $\mu<\frac{c}{\triangle(1-L)p^{one}+\triangle Lp_2}$ and $(2L+2H)\,\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L}<\frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}-\mu.$

The optimal wage policy (w_A, w_B) for a given training level θ is given by the following:

- (i) If $y_A<\varphi_1$, the optimal wage policy is $w_A^*=0$ and $w_B^*=\mu$, $\mathit{\Pi}^*(4L)=(2.31)\,;$
- (ii) If $\varphi_1<y_A<\varphi_3$, the optimal wage policy is $w_A^*=\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L}$ and $w_B^*=\mu$, $\mathit{\Pi}^*(2H\&2L)=(2.33)\,;$

- (iii) If $\varphi_3 < y_A$, the optimal wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (2.30)$.

(i) $\Pi^*(4L) = 4Ly_A + 2L(1-L)E^{one}(\theta) + L^2 E^{two}(\theta) + (1-L)^2\left(\frac{\underline{y}+\overline{y}}{2}\right) - \mu - \frac{\theta^2}{2}$.

The first order derivative is given by (more details of the derivatives of $E^{one}(\theta)$ and $E^{two}(\theta)$ appear in Appendix 2.7.4),

$$\Pi^*(4L)' = 2L(1-L)\left(\frac{1}{2} + \frac{\theta}{\overline{y}-\underline{y}}\right) + L^2\left(\frac{3}{4} + \frac{\theta}{\overline{y}-\underline{y}} - \frac{\theta^2}{\left(\overline{y}-\underline{y}\right)^2}\right) - \theta.$$

At $\theta = 0$, $\Pi^*(4L)' = L(1-L) + \frac{3}{4}L^2 > 0$, while at $\theta = 2$, $\Pi^*(4L)' = 2L(1-L)\left(\frac{1}{2} + \frac{2}{\overline{y}-\underline{y}}\right) + L^2\left(\frac{3}{4} + \frac{2}{\overline{y}-\underline{y}} - \frac{4}{\left(\overline{y}-\underline{y}\right)^2}\right) - 2 < 0$. Since $\Pi^*(4L)$ is a continuation function at $\theta \in [0,2]$, the optimal training level θ^* must satisfy the first order condition; that is,

$$\Pi^*(4L)' = 2L(1-L)\left(\frac{1}{2} + \frac{\theta}{\overline{y}-\underline{y}}\right) + L^2\left(\frac{3}{4} + \frac{\theta}{\overline{y}-\underline{y}} - \frac{\theta^2}{\left(\overline{y}-\underline{y}\right)^2}\right) - \theta = 0$$

$$-\frac{L^2}{\left(\overline{y}-\underline{y}\right)^2}\theta^2 - \left(1 - \frac{2L-L^2}{\overline{y}-\underline{y}}\right)\theta + L - \frac{L^2}{4} = 0.$$

According to the roots of a quadratic equation, we have (the other root is negative)

$$\begin{aligned}\theta^*(L) &= \frac{1 - \frac{2L-L^2}{\overline{y}-\underline{y}} - \sqrt{\left(1 - \frac{2L-L^2}{\overline{y}-\underline{y}}\right)^2 + 4\frac{L^2}{\left(\overline{y}-\underline{y}\right)^2}\left(L - \frac{L^2}{4}\right)}}{-\frac{2L^2}{\left(\overline{y}-\underline{y}\right)^2}} \\ &= \frac{\overline{y}-\underline{y} - 2L + L^2 - \sqrt{\left(\overline{y}-\underline{y}-2L+L^2\right)^2 + 4L^2\left(L - \frac{L^2}{4}\right)}}{-2L^2}\left(\overline{y}-\underline{y}\right).\end{aligned}$$

Therefore, $p^{one}(\theta^*(L)) = \frac{1}{2} + \frac{\theta^*(L)}{\overline{y}-\underline{y}}$ and $p^{two}(\theta^*(L)) = \frac{3}{8} + \frac{\theta^*(L)}{2\left(\overline{y}-\underline{y}\right)} - \frac{\theta^*(L)^2}{2\left(\overline{y}-\underline{y}\right)^2}$, and $\mu \le \frac{c}{\triangle(1-L)p^{one}(\theta^*(L)) + \triangle L p^{two}(\theta^*(L))}$.

(ii) $\Pi^*(2H\&2L) = (2L+2H)\left(y_A - \frac{c - \triangle(1-H)p^{one}\mu - \triangle H p^{two}\mu}{\triangle(1+L) - \triangle(1-H)p^{one}L - \triangle H p^{two}L}\right) + 2H(1-H)E^{one} + H^2E^{two} + (1-H)^2\left(\frac{\underline{y}+\overline{y}}{2}\right) - \mu - \frac{\theta^2}{2}$.

The first order derivative is given by (more details of derivatives of w_A^* appear in Appendix 2.7.4),

$$\mathit{\Pi}^*(2H\&2L)' = -(2L+2H)\,w_A^{*\prime} + 2H(1-H)\left(\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}\right) + H^2\left(\frac{3}{4}+\frac{\theta}{\overline{y}-\underline{y}}-\frac{\theta^2}{\left(\overline{y}-\underline{y}\right)^2}\right) - \theta.$$

At $\theta = 0$, $\mathit{\Pi}^*(2H\&2L)' = -(2L+2H)\,w_A^{*\prime} + H(1-H) + \frac{3}{4}H^2 > 0$ as $w_A^{*\prime} < 0$, while at $\theta = 2$, $\mathit{\Pi}^*(2H\&2L)' < 0$ (proofs appear in Appendix 2.7.6).

The second order derivative is given by,

$$\mathit{\Pi}^*(2H\&2L)'' = -(2L+2H)\,w_A^{*\prime\prime} + 2H(1-H)\left(\frac{1}{\overline{y}-\underline{y}}\right) + H^2\left(\frac{1}{\overline{y}-\underline{y}}-\frac{2\theta}{\left(\overline{y}-\underline{y}\right)^2}\right) - 1.$$

At $\theta = 0$, $\mathit{\Pi}^*(2H\&2L)'' = -(2L+2H)\,w_A^{*\prime\prime} + \frac{2H-H^2}{\overline{y}-\underline{y}} - 1 < 0$ as $\frac{2H-H^2}{\overline{y}-\underline{y}} - 1 < 0$ and $w_A^{*\prime\prime} > 0$. The value of $\mathit{\Pi}^*(2H\&2L)''$ at $\theta = 2$ is ambiguous, while, because the third order derivative $\mathit{\Pi}^*(2H\&2L)''' = -(2L+2H)\,w_A^{*\prime\prime\prime} + H^2\left(-\frac{2}{\left(\overline{y}-\underline{y}\right)^2}\right) < 0$ (as $w_A^{*\prime\prime\prime} > 0$), $\mathit{\Pi}^*(2H\&2L)''$ decreases with θ. Therefore, $\mathit{\Pi}^*(2H\&2L)'' < 0$ at $\theta \in [0,2]$, which means $\mathit{\Pi}^*(2H\&2L)'$ is decreasing at $\theta \in [0,2]$. Since $\mathit{\Pi}^*(2H\&2L)$ is a continuation function at $\theta \in [0,2]$, the optimal training level θ^* must satisfy the first order condition; that is,

$$-(2L+2H)\,w_A^{*\prime} + 2H(1-H)\left(\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}\right) + H^2\left(\frac{3}{4}+\frac{\theta}{\overline{y}-\underline{y}}-\frac{\theta^2}{\left(\overline{y}-\underline{y}\right)^2}\right) - \theta = 0,$$

where $w_A^{*\prime} = \frac{2L-HL}{2\left(\overline{y}-\underline{y}\right)}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{1}{[(1+L)-(1-H)p^{one}L-Hp^{two}L]^2} - \frac{1}{\left(\overline{y}-\underline{y}\right)^2}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\frac{HL\theta}{[(1+L)-(1-H)p^{one}L-Hp^{two}L]^2}.$

Rearranging, we have (more details appear in Appendix 2.7.6)

$$\frac{H^4L^2}{4\left(\overline{y}-\underline{y}\right)^6}\theta^6+\frac{H^2L^2\left[\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)\right]-4\left(2-H\right)H^3L^2}{4\left(\overline{y}-\underline{y}\right)^5}\theta^5+$$
$$\frac{\left(2-H\right)^2H^2L^2+\left(1+\frac{L}{2}+\frac{HL}{8}\right)H^3L-H^2L^2\left(H-\frac{H^2}{4}\right)-\left(2-H\right)HL^2\left[\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)\right]}{4\left(\overline{y}-\underline{y}\right)^4}\theta^4+$$
$$\frac{\left(2-H\right)HL^2\left(H-\frac{H^2}{4}\right)+\left[\left(2-H\right)^2L^2+\left(1+\frac{L}{2}+\frac{HL}{8}\right)HL\right]\left[\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)\right]-2\left(1+\frac{L}{2}+\frac{HL}{8}\right)\left(2-H\right)H^2L}{\left(\overline{y}-\underline{y}\right)^3}\theta^3+$$
$$\frac{\left(1+\frac{L}{2}+\frac{HL}{8}\right)^2H^2-\left[2\left(1+\frac{L}{2}+\frac{HL}{8}\right)\left(2-H\right)L\right]\left[\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)\right]-\left[\left(2-H\right)^2L^2+\left(1+\frac{L}{2}+\frac{HL}{8}\right)HL\right]\left(H-\frac{H^2}{4}\right)}{\left(\overline{y}-\underline{y}\right)^2}\theta^2+$$
$$\frac{\left[2\left(1+\frac{L}{2}+\frac{HL}{8}\right)\left(2-H\right)L\right]\left(H-\frac{H^2}{4}\right)+\left(1+\frac{L}{2}+\frac{HL}{8}\right)^2\left[\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)\right]}{\overline{y}-\underline{y}}\theta-\frac{\left(2L+2H\right)HL}{\left(\overline{y}-\underline{y}\right)^2}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\theta+$$
$$\frac{\left(L+H\right)\left(2L-HL\right)}{\left(\overline{y}-\underline{y}\right)}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]-\left(1+\frac{L}{2}+\frac{HL}{8}\right)^2\left(H-\frac{H^2}{4}\right)=0. \tag{2.36}$$

Equation (2.36) is a sextic equation; let

$$a_1\theta^6+a_2\theta^5+a_3\theta^4+a_4\theta^3+a_5\theta^2+a_6\theta+a_7=0, \tag{2.37}$$

where $a_1=\frac{H^4L^2}{4\left(\overline{y}-\underline{y}\right)^6}$, $a_2=\frac{H^2L^2\left[\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)\right]-4(2-H)H^3L^2}{4\left(\overline{y}-\underline{y}\right)^5}$,

$$a_3=\frac{\left(2-H\right)^2H^2L^2+\left(1+\frac{L}{2}+\frac{HL}{8}\right)H^3L-H^2L^2\left(H-\frac{H^2}{4}\right)-\left(2-H\right)HL^2\left[\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)\right]}{4\left(\overline{y}-\underline{y}\right)^4}$$

$$a_4=\frac{\left(2-H\right)HL^2\left(H-\frac{H^2}{4}\right)+\left[\left(2-H\right)^2L^2+\left(1+\frac{L}{2}+\frac{HL}{8}\right)HL\right]\left[\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)\right]-2\left(1+\frac{L}{2}+\frac{HL}{8}\right)\left(2-H\right)H^2L}{\left(\overline{y}-\underline{y}\right)^3}$$

$$a_5=\frac{\left(1+\frac{L}{2}+\frac{HL}{8}\right)^2H^2-\left[2\left(1+\frac{L}{2}+\frac{HL}{8}\right)\left(2-H\right)L\right]\left[\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)\right]-\left[\left(2-H\right)^2L^2+\left(1+\frac{L}{2}+\frac{HL}{8}\right)HL\right]\left(H-\frac{H^2}{4}\right)}{\left(\overline{y}-\underline{y}\right)^2}$$

$$a_6=\frac{\left[2\left(1+\frac{L}{2}+\frac{HL}{8}\right)\left(2-H\right)L\right]\left(H-\frac{H^2}{4}\right)+\left(1+\frac{L}{2}+\frac{HL}{8}\right)^2\left[\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)\right]}{\overline{y}-\underline{y}}-\frac{\left(2L+2H\right)HL}{\left(\overline{y}-\underline{y}\right)^2}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]$$

$$a_7=\frac{\left(L+H\right)\left(2L-HL\right)}{\left(\overline{y}-\underline{y}\right)}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]-\left(1+\frac{L}{2}+\frac{HL}{8}\right)^2\left(H-\frac{H^2}{4}\right).$$

The works of Abel (1826) and Galois (1832) have shown that the general polynomial equations of degree higher than the fourth cannot be solved in radicals, but the general sextic can be solved in terms of Kampe de Feriet functions.

$\theta^*\left(H,w_A^*\right)$ is the only solution at $\theta\in[0,2]$. Therefore, $p^{one}\left(\theta^*\left(H,w_A^*\right)\right)=$

$\frac{1}{2} + \frac{\theta^*\left(H,w_A^*\right)}{\overline{y}-\underline{y}}$ and $p^{two}\left(\theta^*\left(H,w_A^*\right)\right) = \frac{3}{8} + \frac{\theta^*\left(H,w_A^*\right)}{2\left(\overline{y}-\underline{y}\right)} - \frac{\theta^*\left(H,w_A^*\right)^2}{2\left(\overline{y}-\underline{y}\right)^2}$.

(iii) $\mathit{\Pi}^*\left(4H\right) = 4H\left(y_A - \frac{c}{\triangle}\right)+2H\left(1-H\right)E^{one}++H^2E^{two}+\left(1-H\right)^2\left(\frac{\underline{y}+\overline{y}}{2}\right)-\mu-\frac{\theta^2}{2}$.

The first order derivative is given by (more details of derivatives of $E^{one}\left(\theta\right)$ and $E^{two}\left(\theta\right)$ appear in Appendix 2.7.4),

$$\mathit{\Pi}^*\left(4H\right)' = 2H\left(1-H\right)\left(\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}\right)+H^2\left(\frac{3}{4}+\frac{\theta}{\overline{y}-\underline{y}}-\frac{\theta^2}{\left(\overline{y}-\underline{y}\right)^2}\right)-\theta,$$

At $\theta = 0$, $\mathit{\Pi}^*\left(4H\right)' = H\left(1-H\right)+\frac{3}{4}H^2 > 0$, while, at $\theta = 2$, $\mathit{\Pi}^*\left(4H\right)' = 2H\left(1-H\right)\left(\frac{1}{2}+\frac{2}{\overline{y}-\underline{y}}\right)+H^2\left(\frac{3}{4}+\frac{2}{\overline{y}-\underline{y}}-\frac{4}{\left(\overline{y}-\underline{y}\right)^2}\right)-2 < 0$. Since $\mathit{\Pi}^*\left(4H\right)$ is a continuation function at $\theta \in [0,2]$, the optimal training level θ^* must satisfy the first order condition; that is,

$$\mathit{\Pi}^*\left(4H\right)' = 2H\left(1-H\right)\left(\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}\right)+H^2\left(\frac{3}{4}+\frac{\theta}{\overline{y}-\underline{y}}-\frac{\theta^2}{\left(\overline{y}-\underline{y}\right)^2}\right)-\theta = 0$$

$$-\frac{H^2}{\left(\overline{y}-\underline{y}\right)^2}\theta^2-\left(1-\frac{2H-H^2}{\overline{y}-\underline{y}}\right)\theta+H-\frac{H^2}{4} = 0.$$

According to the roots of a quadratic equation, we have (the other root is negative)

$$\begin{aligned}\theta^*\left(H\right) &= \frac{1-\frac{2H-H^2}{\overline{y}-\underline{y}}-\sqrt{\left(1-\frac{2H-H^2}{\overline{y}-\underline{y}}\right)^2+4\frac{H^2}{\left(\overline{y}-\underline{y}\right)^2}\left(H-\frac{H^2}{4}\right)}}{-\frac{2H^2}{\left(\overline{y}-\underline{y}\right)^2}} \\ &= \frac{\overline{y}-\underline{y}-2H+H^2-\sqrt{\left(\overline{y}-\underline{y}-2H+H^2\right)^2+4H^2\left(H-\frac{H^2}{4}\right)}}{-2H^2}\left(\overline{y}-\underline{y}\right).\end{aligned}$$

Therefore, $p^{one}\left(\theta^*\left(H\right)\right) = \frac{1}{2}+\frac{\theta^*(H)}{\overline{y}-\underline{y}}$ and $p^{two}\left(\theta^*\left(H\right)\right) = \frac{3}{8}+\frac{\theta^*(H)}{2\left(\overline{y}-\underline{y}\right)}-\frac{\theta^*(H)^2}{2\left(\overline{y}-\underline{y}\right)^2}$.

Comparing these three levels of θ^*, it is obvious that

$$\theta^*\left(H\right) > \theta^*\left(H,w_A^*\right) > \theta^*\left(L\right)$$

$$p^{one}\left(\theta^*\left(H\right)\right) > p^{one}\left(\theta^*\left(H,w_A^*\right)\right) > p^{one}\left(\theta^*\left(L\right)\right)$$

$$p^{two}\left(\theta^*\left(H\right)\right) > p^{two}\left(\theta^*\left(H, w_A^*\right)\right) > p^{two}\left(\theta^*\left(L\right)\right).$$

(1b) $\mu < \frac{c}{\triangle(1-L)p^{one}+\triangle L p^{two}}$ and $(2L+2H)\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L} > \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}} - \mu$.

The optimal wage policy (w_A, w_B) is given by the following:

- If $y_A < \varphi_1'$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(4L) = (2.31)$;
- If $\varphi_1' < y_A < \varphi_3'$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}$, $\Pi^*(2H\&2L) = (2.34)$;
- If $\varphi_3' < y_A$, the optimal wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (2.30)$.

(i) $\Pi^*(4L) = 4Ly_A + 2L(1-L)E^{one} + L^2E^{two} + (1-L)^2\left(\frac{\underline{y}+\overline{y}}{2}\right) - \mu - \frac{\theta^2}{2}$.

As with with (1a) (i)

(ii) $\Pi^*(2H\&2L) = (2L+2H)y_A + 2H(1-H)E^{one} + H^2E^{two} + (1-H)^2\left(\frac{\underline{y}+\overline{y}}{2}\right) - \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}} - \frac{\theta^2}{2}$.

The first order derivative is given by (more details of derivatives of $E^{one}(\theta)$ and $E^{two}(\theta)$ appear in Appendix 2.7.4),

$$\Pi^*(2H\&2L)' = -w_B^{*\prime} + 2H(1-H)\left(\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}\right) + H^2\left(\frac{3}{4}+\frac{\theta}{\overline{y}-\underline{y}}-\frac{\theta^2}{(\overline{y}-\underline{y})^2}\right) - \theta.$$

At $\theta = 0$, $\Pi^*(2H\&2L)' = -w_B^{*\prime} + H(1-H) + \frac{3}{4}H^2 > 0$, while, at $\theta = 2$, $\Pi^*(2H\&2L)' = -w_B^{*\prime} + 2H(1-H)\left(\frac{1}{2}+\frac{2}{\overline{y}-\underline{y}}\right) + H^2\left(\frac{3}{4}+\frac{2}{\overline{y}-\underline{y}}-\frac{4}{(\overline{y}-\underline{y})^2}\right) - 2 < 0$ (proofs appear in Appendix 2.7.7). Since $\Pi^*(2H\&2L)$ is a continuation function at $\theta \in [0, 2]$, the optimal training level θ^* must satisfy the first order condition; that is,

$$-w_B^{*\prime} + 2H(1-H)\left(\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}\right) + H^2\left(\frac{3}{4}+\frac{\theta}{\overline{y}-\underline{y}}-\frac{\theta^2}{(\overline{y}-\underline{y})^2}\right) - \theta = 0,$$

where $w_B^{*\prime} = -\frac{c}{\triangle}\left\{\frac{\frac{2-H}{2(\overline{y}-\underline{y})}}{[(1-H)p^{one}+Hp^{two}]^2} - \frac{\frac{H\theta}{(\overline{y}-\underline{y})^2}}{[(1-H)p^{one}+Hp^{two}]^2}\right\}$. Rearranging, we have also obtain a sextic equation, and $\theta^*(H, w_B^*)$ is the only solution at $\theta \in [0,2]$. Therefore, $p^{one}(\theta^*(H, w_B^*)) = \frac{1}{2} + \frac{\theta^*(H,w_B^*)}{\overline{y}-\underline{y}}$ and $p^{two}(\theta^*(H, w_B^*)) = \frac{3}{8} + \frac{\theta^*(H,w_B^*)}{2(\overline{y}-\underline{y})} - \frac{\theta^*(H,w_B^*)^2}{2(\overline{y}-\underline{y})^2}$.

(iii) $\Pi^*(4H) = 4H\left(y_A - \frac{c}{\triangle}\right) + 2H(1-H)E^{one} + +H^2E^{two} + (1-H)^2\left(\frac{\underline{y}+\overline{y}}{2}\right) - \mu - \frac{\theta^2}{2}$.

As with (1a) (iii)

Comparing these three levels of θ^*, it is obvious that

$$\theta^*(H) > \theta^*(H, w_B^*) > \theta^*(L)$$

$$p^{one}(\theta^*(H)) > p^{one}(\theta^*(H, w_B^*)) > p^{one}(\theta^*(L))$$

$$p^{two}(\theta^*(H)) > p^{two}(\theta^*(H, w_B^*)) > p^{two}(\theta^*(L)).$$

(2a) $\frac{c}{\triangle(1-L)p^{one}+\triangle Lp^{two}} < \mu < \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}$ and $(2L+2H)\ \frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L} < \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}} - \mu$.

There are three kinds of outcome.

The optimal wage policy (w_A, w_B) is given by the following:

- If $y_A < \varphi_4$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(H\&3L) = (2.35)$;
- If $\varphi_4 < y_A < \varphi_3$, the optimal wage policy is $w_A^* = \frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L}$ and $w_B^* = \mu$, $\Pi^*(2H\&2L) = (2.33)$;
- If $\varphi_3 < y_A$, the optimal wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (2.30)$.

(i) $\Pi^*(H\&3L) = (3L+H)y_A + H(1-L)E^{one} + L(1-H)E^{one} + HLE^{two} + (1-H)(1-L)\left(\frac{\underline{y}+\overline{y}}{2}\right) - \mu - \frac{\theta^2}{2}$.

The first order derivative is given by (more details of derivatives of $E^{one}(\theta)$ and $E^{two}(\theta)$ appear in Appendix 2.7.4),

$$\varPi^*(H\&3L)' = [H(1-L)+L(1-H)]\left(\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}\right)+HL\left(\frac{3}{4}+\frac{\theta}{\overline{y}-\underline{y}}-\frac{\theta^2}{\left(\overline{y}-\underline{y}\right)^2}\right)-\theta.$$

At $\theta=0$, $\varPi^*(H\&3L)' = \frac{H+L}{2}-\frac{HL}{4}>0$, while, at $\theta=2$,$\varPi^*(H\&3L)' = (H+L-2HL)\left(\frac{1}{2}+\frac{2}{\overline{y}-\underline{y}}\right)+HL\left(\frac{3}{4}+\frac{2}{\overline{y}-\underline{y}}-\frac{4}{\left(\overline{y}-\underline{y}\right)^2}\right)-2<0.$ Since $\varPi^*(H\&3L)$ is a continuation function at $\theta\in[0,2]$, the optimal training level θ^* must satisfy the first order condition; that is,

$$(H+L-2HL)\left(\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}\right)+HL\left(\frac{3}{4}+\frac{\theta}{\overline{y}-\underline{y}}-\frac{\theta^2}{\left(\overline{y}-\underline{y}\right)^2}\right)-\theta=0$$

$$-\frac{HL}{\left(\overline{y}-\underline{y}\right)^2}\theta^2-\left(1-\frac{H+L-HL}{\overline{y}-\underline{y}}\right)\theta+\frac{H+L}{2}-\frac{3HL}{4}=0.$$

According to the roots of a quadratic equation, we have (the other root is negative)

$$\theta^*(H,L) = \frac{1-\frac{H+L-HL}{\overline{y}-\underline{y}}-\sqrt{\left(1-\frac{H+L-HL}{\overline{y}-\underline{y}}\right)^2+4\frac{HL}{\left(\overline{y}-\underline{y}\right)^2}\left(\frac{H+L}{2}-\frac{3HL}{4}\right)}}{-\frac{2HL}{\left(\overline{y}-\underline{y}\right)^2}}$$
$$= \frac{\overline{y}-\underline{y}-H-L+HL-\sqrt{\left(\overline{y}-\underline{y}-H-L+HL\right)^2+HL(2H+2L-3HL)}}{2HL}(\overline{y}-\underline{y})$$

Therefore, $p^{one}(\theta^*(H,L)) = \frac{1}{2}+\frac{\theta^*(H,L)}{\overline{y}-\underline{y}}$ and $p^{two}(\theta^*(H,L)) = \frac{3}{8}+\frac{\theta^*(H,L)}{2\left(\overline{y}-\underline{y}\right)}-\frac{\theta^*(H,L)^2}{2\left(\overline{y}-\underline{y}\right)^2}$.

(ii) $\varPi^*(2H\&2L) = (2L+2H)\left(y_A-\frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L}\right)+2H(1-H)E^{one}+H^2E^{two}+(1-H)^2\left(\frac{\underline{y}+\overline{y}}{2}\right)-\mu-\frac{\theta^2}{2}.$

As with (1a) (ii)

(iii) $\varPi^*(4H) = 4H\left(y_A-\frac{c}{\triangle}\right)+2H(1-H)E^{one}++H^2E^{two}+(1-H)^2\left(\frac{\underline{y}+\overline{y}}{2}\right)-\mu-\frac{\theta^2}{2}.$

As with (1a) (iii)

Comparing these three levels of θ^*, it is obvious that

$$\theta^* (H) > \theta^* (H, w_A^*) > \theta^* (H, L)$$

$$p^{one} (\theta^* (H)) > p^{one} (\theta^* (H, w_A^*)) > p^{one} (\theta^* (H, L))$$
$$p^{two} (\theta^* (H)) > p^{two} (\theta^* (H, w_A^*)) > p^{two} (\theta^* (H, L)) .$$

(2b) $\frac{c}{\triangle(1-L)p^{one}+\triangle Lp^{two}} < \mu < \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}$ and $(2L+2H) \frac{c-\triangle(1-H)p^{one}\mu-\triangle Hp^{two}\mu}{\triangle(1+L)-\triangle(1-H)p^{one}L-\triangle Hp^{two}L} > \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}} - \mu$.

The optimal wage policy (w_A, w_B) is given by the following:

- **If $y_A < \varphi_4'$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \mu$,** $\mathit{\Pi}^* (H\&3L) = (2.35)$;
- **If $\varphi_4' < y_A < \varphi_3'$, the optimal wage policy is $w_A^* = 0$ and $w_B^* =$** $\frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}}$, $\mathit{\Pi}^* (2H\&2L) = (2.34)$;
- **If $\varphi_3' < y_A$, the optimal wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$,** $\mathit{\Pi}^* (4H) = (2.30)$.

(i) $\mathit{\Pi}^* (H\&3L) = (3L+H) y_A + H(1-L) E^{one} + L(1-H) E^{one} + HLE^{two} + (1-H)(1-L) \left(\frac{\underline{y}+\overline{y}}{2}\right) - \mu - \frac{\theta^2}{2}$.

As with (2a) (i)

(ii) $\mathit{\Pi}^* (2H\&2L) = (2L+2H) y_A + 2H(1-H) E^{one} + H^2 E^{two} + (1-H)^2 \left(\frac{\underline{y}+\overline{y}}{2}\right) - \frac{c}{\triangle(1-H)p^{one}+\triangle Hp^{two}} - \frac{\theta^2}{2}$.

As with (1b) (ii)

(iii) $\mathit{\Pi}^* (4H) = 4H \left(y_A - \frac{c}{\triangle}\right) + 2H(1-H) E^{one} + +H^2 E^{two} + (1-H)^2 \left(\frac{\underline{y}+\overline{y}}{2}\right) - \mu - \frac{\theta^2}{2}$.

As with (1a) (iii)

Comparing these three levels of θ^*, it is obvious that

$$\theta^*(H) > \theta^*(H, w_B^*) > \theta^*(H, L)$$
$$p^{one}(\theta^*(H)) > p^{one}(\theta^*(H, w_B^*)) > p^{one}(\theta^*(H, L))$$
$$p^{two}(\theta^*(H)) > p^{two}(\theta^*(H, w_B^*)) > p^{two}(\theta^*(H, L)).$$

(3) $\mu \geq \frac{c}{\triangle(1-H)p^{one}+\triangle H p^{two}}$.

The optimal wage policy (w_A, w_B) is given by the following:

- If $y_A < \varphi_6 = \frac{2Hc}{\triangle^2}$, the optimal wage policy is $w_A^* = 0$ and $w_B^* = \mu$, $\Pi^*(2H\&2L) = (2.32)$;
- If $\varphi_6 < y_A$, the optimal wage policy is $w_A^* = \frac{c}{\triangle}$ and $w_B^* = \mu$, $\Pi^*(4H) = (2.30)$;

(i) $\Pi^*(2H\&2L) = (2L + 2H)y_A + 2H(1-H)E^{one} + H^2 E^{two} + (1-H)^2\left(\frac{\underline{y}+\overline{y}}{2}\right) - \mu - \frac{\theta^2}{2}$.

As with (1a) (iii)

(ii) $\Pi^*(4H) = 4H\left(y_A - \frac{c}{\triangle}\right) + 2H(1-H)E^{one} + +H^2 E^{two} + (1-H)^2\left(\frac{\underline{y}+\overline{y}}{2}\right) - \mu - \frac{\theta^2}{2}$.

As with (1a) (iii)

Comparing these three levels of θ^*, it is obvious that

$$\theta^*(H) > \theta^*(L)$$
$$p^{one}(\theta^*(H)) > p^{one}(\theta^*(L))$$
$$p^{two}(\theta^*(H)) > p^{two}(\theta^*(L)).$$

2.7.6 The sextic equation

$$-\left(2L+2H\right)w_A^{*\prime}+2H\left(1-H\right)\left(\frac{1}{2}+\frac{\theta}{\overline{y}-\underline{y}}\right)+H^2\left(\frac{3}{4}+\frac{\theta}{\overline{y}-\underline{y}}-\frac{\theta^2}{\left(\overline{y}-\underline{y}\right)^2}\right)-\theta=0$$

$$\left(2L+2H\right)w_A^{*\prime}+\frac{H^2}{\left(\overline{y}-\underline{y}\right)^2}\theta^2+\frac{\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)}{\overline{y}-\underline{y}}\theta-\left(H-\frac{H^2}{4}\right)=0$$

$$\left[\frac{\left(L+H\right)\left(2L-HL\right)}{\left(\overline{y}-\underline{y}\right)}-\frac{\left(2L+2H\right)HL\theta}{\left(\overline{y}-\underline{y}\right)^2}\right]\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]+$$
$$\left[\left(1+L\right)-\left(1-H\right)p^{one}L-Hp^{two}L\right]^2\left[\frac{H^2}{\left(\overline{y}-\underline{y}\right)^2}\theta^2+\frac{\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)}{\overline{y}-\underline{y}}\theta-\left(H-\frac{H^2}{4}\right)\right]=0$$

$$\left[\frac{\left(L+H\right)\left(2L-HL\right)}{\left(\overline{y}-\underline{y}\right)}-\frac{\left(2L+2H\right)HL\theta}{\left(\overline{y}-\underline{y}\right)^2}\right]\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]+$$
$$\left[\left(1+\frac{L}{2}+\frac{HL}{8}\right)-\frac{\left(2-H\right)L}{\overline{y}-\underline{y}}\theta+\frac{HL}{2\left(\overline{y}-\underline{y}\right)^2}\theta^2\right]^2\left[\frac{H^2}{\left(\overline{y}-\underline{y}\right)^2}\theta^2+\frac{\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)}{\overline{y}-\underline{y}}\theta-\left(H-\frac{H^2}{4}\right)\right]=0$$

$$\left[\frac{\left(L+H\right)\left(2L-HL\right)}{\left(\overline{y}-\underline{y}\right)}-\frac{\left(2L+2H\right)HL\theta}{\left(\overline{y}-\underline{y}\right)^2}\right]\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]+$$
$$\left\{\left(1+\frac{L}{2}+\frac{HL}{8}\right)^2+\left[\frac{\left(2-H\right)L}{\overline{y}-\underline{y}}\theta-\frac{HL}{2\left(\overline{y}-\underline{y}\right)^2}\theta^2\right]^2-2\left(1+\frac{L}{2}+\frac{HL}{8}\right)\left[\frac{\left(2-H\right)L}{\overline{y}-\underline{y}}\theta-\frac{HL}{2\left(\overline{y}-\underline{y}\right)^2}\theta^2\right]\right\}*$$
$$\left[\frac{H^2}{\left(\overline{y}-\underline{y}\right)^2}\theta^2+\frac{\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)}{\overline{y}-\underline{y}}\theta-\left(H-\frac{H^2}{4}\right)\right]=0$$

$$\left[\frac{\left(L+H\right)\left(2L-HL\right)}{\left(\overline{y}-\underline{y}\right)}-\frac{\left(2L+2H\right)HL\theta}{\left(\overline{y}-\underline{y}\right)^2}\right]\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]+$$
$$\left\{\left(1+\frac{L}{2}+\frac{HL}{8}\right)^2+\left[\left[\frac{\left(2-H\right)L}{\overline{y}-\underline{y}}\right]^2\theta^2+\left[\frac{HL}{2\left(\overline{y}-\underline{y}\right)^2}\right]^2\theta^4-\frac{\left(2-H\right)HL^2}{\left(\overline{y}-\underline{y}\right)^3}\theta^3\right]-2\left(1+\frac{L}{2}+\frac{HL}{8}\right)\left[\frac{\left(2-H\right)L}{\overline{y}-\underline{y}}\theta-\frac{HL}{2\left(\overline{y}-\underline{y}\right)^2}\theta^2\right]\right\}*$$
$$\left[\frac{H^2}{\left(\overline{y}-\underline{y}\right)^2}\theta^2+\frac{\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)}{\overline{y}-\underline{y}}\theta-\left(H-\frac{H^2}{4}\right)\right]=0$$

$$\left[\frac{\left(L+H\right)\left(2L-HL\right)}{\left(\overline{y}-\underline{y}\right)}-\frac{\left(2L+2H\right)HL\theta}{\left(\overline{y}-\underline{y}\right)^2}\right]\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]+$$
$$\left\{\frac{H^2L^2}{4\left(\overline{y}-\underline{y}\right)^4}\theta^4-\frac{\left(2-H\right)HL^2}{\left(\overline{y}-\underline{y}\right)^3}\theta^3+\frac{\left(2-H\right)^2L^2+\left(1+\frac{L}{2}+\frac{HL}{8}\right)HL}{\left(\overline{y}-\underline{y}\right)^2}\theta^2-\frac{2\left(1+\frac{L}{2}+\frac{HL}{8}\right)\left(2-H\right)L}{\overline{y}-\underline{y}}\theta+\left(1+\frac{L}{2}+\frac{HL}{8}\right)^2\right\}*$$
$$\left[\frac{H^2}{\left(\overline{y}-\underline{y}\right)^2}\theta^2+\frac{\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)}{\overline{y}-\underline{y}}\theta-\left(H-\frac{H^2}{4}\right)\right]=0$$

$$\frac{H^4L^2}{4\left(\overline{y}-\underline{y}\right)^6}\theta^6+\frac{H^2L^2\left[\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)\right]-4\left(2-H\right)H^3L^2}{4\left(\overline{y}-\underline{y}\right)^5}\theta^5+$$
$$\frac{\left(2-H\right)^2H^2L^2+\left(1+\frac{L}{2}+\frac{HL}{8}\right)H^3L-H^2L^2\left(H-\frac{H^2}{4}\right)-\left(2-H\right)HL^2\left[\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)\right]}{4\left(\overline{y}-\underline{y}\right)^4}\theta^4+$$
$$\frac{\left(2-H\right)HL^2\left(H-\frac{H^2}{4}\right)+\left[\left(2-H\right)^2L^2+\left(1+\frac{L}{2}+\frac{HL}{8}\right)HL\right]\left[\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)\right]-2\left(1+\frac{L}{2}+\frac{HL}{8}\right)\left(2-H\right)H^2L}{\left(\overline{y}-\underline{y}\right)^3}\theta^3+$$
$$\frac{\left(1+\frac{L}{2}+\frac{HL}{8}\right)^2H^2-\left[2\left(1+\frac{L}{2}+\frac{HL}{8}\right)\left(2-H\right)L\right]\left[\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)\right]-\left[\left(2-H\right)^2L^2+\left(1+\frac{L}{2}+\frac{HL}{8}\right)HL\right]\left(H-\frac{H^2}{4}\right)}{\left(\overline{y}-\underline{y}\right)^2}\theta^2+$$
$$\frac{\left[2\left(1+\frac{L}{2}+\frac{HL}{8}\right)\left(2-H\right)L\right]\left(H-\frac{H^2}{4}\right)+\left(1+\frac{L}{2}+\frac{HL}{8}\right)^2\left[\left(\overline{y}-\underline{y}\right)-\left(2H-H^2\right)\right]}{\overline{y}-\underline{y}}\theta-\frac{\left(2L+2H\right)HL}{\left(\overline{y}-\underline{y}\right)^2}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]\theta+$$
$$\frac{\left(L+H\right)\left(2L-HL\right)}{\left(\overline{y}-\underline{y}\right)}\left[\frac{c}{\triangle}-\left(\frac{1+L}{L}\right)\mu\right]-\left(1+\frac{L}{2}+\frac{HL}{8}\right)^2\left(H-\frac{H^2}{4}\right)=0.$$

2.7.7 $\Pi^*(2H\&2L)' < 0$ at $\theta = 2$

At $\theta = 2$,

$$\begin{aligned}
\Pi^*(2H\&2L)' =& -(2L+2H)\,w_A^{*\prime} + 2H(1-H)\left(\frac{1}{2} + \frac{2}{\overline{y}-\underline{y}}\right) + H^2\left(\frac{3}{4} + \frac{2}{\overline{y}-\underline{y}} - \frac{4}{\left(\overline{y}-\underline{y}\right)^2}\right) - 2 \\
=& (2L+2H)\left[\frac{2L-HL}{2\left(\overline{y}-\underline{y}\right)} - \frac{2HL}{\left(\overline{y}-\underline{y}\right)^2}\right]\left[\left(\frac{1+L}{L}\right)\mu - \frac{c}{\triangle}\right]\frac{1}{[(1+L)-(1-H)p^{one}L - Hp^{two}L]^2} \\
& + H - \frac{H^2}{4} + \frac{4H-2H^2}{\overline{y}-\underline{y}} - \frac{4H^2}{\left(\overline{y}-\underline{y}\right)^2} - 2 \\
=& \frac{1}{\overline{y}-\underline{y}}\left\{(L+H)(2L-HL)\left[\left(\frac{1+L}{L}\right)\mu - \frac{c}{\triangle}\right]\frac{1}{[(1+L)-(1-H)p^{one}L - Hp^{two}L]^2} + 4H - 2H^2\right\} \\
& - \frac{1}{\left(\overline{y}-\underline{y}\right)^2}\left\{2(2L+2H)HL\left[\left(\frac{1+L}{L}\right)\mu - \frac{c}{\triangle}\right]\frac{1}{[(1+L)-(1-H)p^{one}L - Hp^{two}L]^2} + 4H^2\right\} \\
& + H - \frac{H^2}{4} - 2 \\
=& \frac{L(L+H)}{\left(\overline{y}-\underline{y}\right)^2}\left[\left(\frac{1+L}{L}\right)\mu - \frac{c}{\triangle}\right]\frac{1}{[(1+L)-(1-H)p^{one}L - Hp^{two}L]^2}\left[(2-H)\left(\overline{y}-\underline{y}\right) - 4H\right] \\
& + \frac{2H}{\left(\overline{y}-\underline{y}\right)^2}\left[(2-H)\left(\overline{y}-\underline{y}\right) - 2H\right] + H - \frac{H^2}{4} - 2.
\end{aligned}$$

$\Pi^*(2H\&2L)'$ at $\theta = 2$ is decreasing on $\left(\overline{y}-\underline{y}\right)$. Therefore, the maximum of $\Pi^*(2H\&2L)'$ at $\theta = 2$ is when $\overline{y}-\underline{y} = 4$; that is,

$$\begin{aligned}
\Pi^*\left(2H\&2L|\theta=2,\overline{y}-\underline{y}=4\right)' =& \frac{L(L+H)}{16}\left[\left(\frac{1+L}{L}\right)\mu - \frac{c}{\triangle}\right]\frac{1}{\left(1+\frac{HL}{2}\right)^2}[4(2-H)-4H] \\
& + \frac{2H}{16}[4(2-H)-2H] + H - \frac{H^2}{4} - 2 \\
=& \frac{L(L+H)}{2}\left[\left(\frac{1+L}{L}\right)\mu - \frac{c}{\triangle}\right]\frac{1}{\left(1+\frac{HL}{2}\right)^2}(1-H) + 2H - H^2 - 2 \\
=& \frac{L(L+H)}{2}\left[\left(\frac{1+L}{L}\right)\mu - \frac{c}{\triangle}\right]\frac{1}{\left(1+\frac{HL}{2}\right)^2}(1-H) - (1-H)^2 - 1.
\end{aligned}$$

Since $\mu < \frac{c}{\triangle(1-L)p^{one}+\triangle Lp^{two}}$, as long as $\frac{c}{\triangle}$ is small, $\Pi^*\left(2H\&2L|\theta=2,\overline{y}-\underline{y}=4\right)' < 0$; if $\frac{c}{\triangle}$ is big enough, $\Pi^*\left(2H\&2L|\theta=2,\overline{y}-\underline{y}=4\right)' > 0$. If $\Pi^*\left(2H\&2L|\theta=2,\overline{y}-\underline{y}=4\right)' > 0$, the optimal training level is $\theta^* = 2$. I am more interested in the case of $\Pi^*\left(2H\&2L|\theta=2,\overline{y}-\underline{y}=4\right)' < 0$. Therefore, let us assume that $\frac{c}{\triangle}$ is small.

3 Promotion and Profit: Evidence from Linked Employer-Employee Data

3.1 Introduction

Haidilao, a Chinese hotpot company, is a cultural phenomenon. As early as 2006, Haidilao had already established a subsidiary responsible for recruitment; it rarely or ever hires externally for higher-level jobs, including servers and floor managers, and especially restaurant managers. Substantially all of its restaurant managers and headquarters coaches are homegrown. Haidilao has a well-designed incentive system which relies heavily on its promotion policy and its reputation for never recruiting externally to fill vacancies in the managerial hierarchies.

A lot of Chinese restaurants have studied and copied the Haidilao customer service style; they also enjoy great success. However, no one has tried to build a reputation or rely so heavily on its promotion policy as Haidilao. Promotions are general in most modern organisations (Jensen, 1986), and firms have a preference for internal promotion rather than external hiring (Baker et al., 1994a; DeVaro and Morita, 2013), especially for the top of the job hierarchy (Lazear and Oyer, 2004; Kauhanen and Napari, 2012a). By contrast, certain sectors, like academia, strongly prefer external hiring. It's challenging to find a firm which has a reputation and only hires internally. Using promotions can lead to tremendous achievements, so why do firms rarely highly rely on internal promotions?

Promotion is a tool for firms providing incentives and serves the role of talent assignment, but promotion faces a commitment problem in the presence of external hiring. When external workers are better than the hardest-working internal employees, firms have the incentive to deviate from their promotion rules. Therefore, there is a trade-off between these two roles.

However, this is not the reason that firms do not rely heavily on promotions. In chapter 1, I assume that firms have the ability to fully commit to promotions and find that it is not always optimal for a firm to commit to promotions. I find a concave relationship between promotion rates and firms' rank-level profits. When the profits at high ranks are low, there is no need to use promotions to provide incentives; when the profits at high ranks are high, it is better to use wages to create incentives. In both cases, the efficiency allocation role of promotions dominates. Firms use promotions only when internal workers are better than external workers. While when the profits at high ranks are moderate, the incentive provision role of promotions dominates. In this case, firms would use promotions even if internal workers are worse than external workers. Therefore, in this paper, I try to find empirical evidence to support this principle and practice.

I examine the determinants of promotions from the perspective of firms. To do so, I run regressions with different settings at the rank level, not the individual or firm level, using a large linked Finnish employer-employee dataset (EK data). The novel feature of that dataset is that it includes four job levels; as the same levels are used for all firms, comparison across firms is facilitated. The details of the dataset and certain descriptive results are provided in section 3.3.

Promotions happen within a firm and between different hierarchies. In chapter 1, I show that promotions in their incentive provision role can increase firm profits by either saving wages or inducing extra effort or both and promotions in their efficiency allocation role can increase firm profits by improving productivity. The trade-off between incentive provision and efficiency allocation depends onrank-level profits. Therefore, I look at different ranks of firms and focus on the determinants of promotions at the rank level.

Another benefit of rank-level regressions is that I use actual promotion rates rather than dummy promotion variables. Promotions provide incentives through future compensation. There is uncertainty if workers who work hard could not be promoted and be certain of being rewarded. The magnitude of promotion rates should also matter; different promotion rates should have different effects. Therefore, I use actual promotion rates as the dependent variable. To examine the nonlinear relationship between promotion rate and firm rank-level profit, the main independent variables are firm rank-level profit and its square. The control variables are size, performance, years of education and age. All these variables are at the rank level, so they measure competition between co-workers (Kauhanen and Napari, 2012a).

I begin by investigating the determinants of promotion in different settings at the rank level. The main finding shows a concave relationship between promotion rate and firm rank-level profit. This evidence supports the results of the theoretical model in chapter 1. All internal promotion competition variables have a negative impact on promotions, and all of these factors are less important than firm rank-level profit. This indicates that the primary determinant of promotion is rank-level profit.

In section 3.5, I examine the determinants of promotions from the perspective of workers. I run regressions on the dummy promotion variable at the individual level. Again, I find a concave relationship between promotion rate and firm rank level profit, with rank-level profit identified as the primary determinant of promotion, although wages also play an important role in promotion. Individuals' and co-workers' characteristics matter and have a similar magnitude but opposite effect. For instance, if a worker obtains an additional year of education, the worker's expected promotion rate should increase by 0.1%. If co-workers obtains an additional year of education, the worker's expected promotion rate should decrease by 0.09%. Moreover, rank level has an impact on promotion. In organizations with pyramidal hierarchies, getting promoted is more difficult when one is already in the top tier.

Then I move to the determinants of wages. Promotion and wage dynamics is the central question of the internal labour market. It turns out that promotion (higher rank level) contributes most to wage growth. This is consistent with evidence that promotion is often associated with large wage growth. There is a long-standing argument in the literature between classic and market-based tournaments of promotion. In the classic tournament model of Lazear and Rosen (1981), promotion isprimarily used for incentive provision. In the market-based tournament model of Waldman (1984), promotion is a signal of worker ability, so high wages are implemented to prevent the worker from being poached by other firms. If the classic tournament is accurate, wages and individuals' and co-workers characteristics should contribute most to promotion, whereas the findings indicate that rank-level profit is the most important factor in promotion. Therefore, this paper is more supportive of the market-based tournament of promotion. High wage growth associated with promotion is used to keep the best workers.

Finally, I explore the robustness of the key findings using a set of alternative modelling assumptions. First, the overall promotion rate is around 5%, so they are many zeros on the dependent variable. I use a zero-inflated Poisson model to eliminate the potential zero-inflation problem and biased results. Second, I am also concerned about fluctuations in the size of ranks over time. Therefore, I run the same regressions by using data without rank 1. Moreover, if a promotion policy is a firm's strategic choice, the opposite of promotion, demotion, should have the opposite effect. The results showing a concave relationship between promotion rate and firm-rank level profit are robust in different settings, whereas I find inconsistent results for the determinants of demotion in different settings, indicating that demotion is much more complicated than promotion.

The findings on the concave relationship between promotion rate and firm rank-level profit have some important implications. First, to the best of my knowledge, this is the first paper to investigate determinants of promotion at the rank level. Promotions happen inside firms, and I should explore promotion to account for its rank-level characteristics. Second, this is also the first paper to use actual promotion rates rather than dummy promotion variables to analyse the factors involved in promotion. Because not all workers get promoted, the magnitude of promotion rates matters. Finally, I explore the determinants of demotion, which need further investigation.

The structure of the paper is as follows. The relevance of the paper in the context of the related literature is assessed in Section 3.2, while Section 3.3 provides details about the EK data and some descriptive results. Section 3.4 presents the regression results at the rank level, while Section 3.5 shows the regression results at the individual level. Section 3.6 studies the determinants of wages. Section 3.7 provides robustness checks and extends the models to alternative settings. A final section draws a conclusion.

3.2 Related Literature

This paper contributes to the literature on internal labour markets developed by Baker et al. (1994a; 1994b) by providing empirical evidence on wage and promotion dynamics. Baker et al. (1994a) explore personnel data from one firm over 20 years to identify the mechanisms that underlie internal labour markets. Some important empirical evidence on wage and promotion dynamics has been found, including the fact that real wages decreases are not rare, but demotions are (Baker et al. 1994a; 1994b; Seltzer and Merrett 2000; Treble et al. 2001); there is a fast track (Podolny and Baron, 1997; Belzil and Bognanno, 2010), and promotions are associated with large wage increases (Lazear, 1992; Main et al., 1993; Gibbons and Waldman, 1999a, 2006). Follow-up studies have examined the roles of schooling and performance in wage and promotion dynamics. The findings include the fact that schooling positively affects wage and promotion rates (McCue, 1996; Lluis, 2005; Lin, 2006). First, unlike some studies (Baker et al., 1994a; Seltzer and Merrett 2000; Treble et al. 2001) that find that demotions to be rare, my data show that demotion does not prevail over promotion, but it is by no means rare. This adds some evidence of a significant rate of demotions (Dohmen et al., 2004; Lin, 2005; Kauhanen and Napari, 2012a). In addition, I provide serial correlation in demotion rates. Demotions also depend heavily on firm rank-level profit. Second, I find strong support that promotions are associated with large wage increases, which is in line with numerous studies (e.g., Lazear 1992; McCue 1996). Third, the findings support the view that schooling is positively related to promotion probabilities. This is in line with Baker et al. (1994a), McCue (1996) and Lin (2006).

Several studies have explored internal economics using a single firm's data. Baker et al. (1994a) use personnel records of a medium-sized U.S. financial services firm over 20 years to show career movements, the structure of the hierarchy and the internal labour market. Using the same dataset, Ekinci et al. (2019) combine the classic tournament approach and the market-based approach to test predictions on bonuses rather than wages. They find that bonus size increases with job level, holding job level tenure and worker age fixed, and is negatively related to the size of the expected promotion prizes. Dohmen (2004) shows that better performance ratings increase a worker's chance of climbing faster, using personnel data from the Dutch aircraft manufacturer Fokker for white-collar employees. Lin (2005; 2006) finds that career mobility is different among different workers, using data from an auto dealer in Taiwan. There are also several studies that explore internal economics using a large linked employer-employee dataset. Lazear and Oyer (2004) examine internal and external labour markets using a linked Sweden employer-employee dataset. Jones and Kato (2011) find that individuals who get promoted receive more training, using Finnish bank-level data. Jin and Waldman (2020) show that lateral mobility positively affects future promotions using a linked panel dataset on senior managers in a sample of large U.S. firms between 1981 and 1985.

While in this paper, I use a large linked employer-employee dataset covering the period from 2002 to 2019. The specific feature of this dataset enables us to compare job levels across all firms and distinguish between many types of career moves. Some other papers have used the same dataset. For example, Cassidy et al. (2016) examine the markert-based approach of promotion theory using the same dataset over the 2000–2012 period. Kauhanen and Napari (2012a) study career and wage dynamics using the same dataset over the period between 1981 to 2006. Ekinci et al. (2019) test bonuses and promotion tournaments using the dataset over the 2003–2012 period. My dataset is updated, and there was a change in the classification of job titles in 2002 that makes job titles and hierarchies more comparable across firms. Finally, the main focus of this paper is not on the features of the internal labour market but on the nonlinear relationship between rank profit and promotion rates.

This paper also contributes to the literature on determinants of promotions in the internal labour market. Previous empirical research on promotions has primarily used household data, for example, using British Household Panel Survey data Francesconi (2001) and Booth et al. (2003) investigate the determinants of promotion. Some studies use firm-level datasets; for example, Pfeifer (2010) analyses the determinants of promotion by combing the tournament and job assignment theories using personnel records from a large German company, but there little research using a large linked employer-employee dataset. The most important advantage of my dataset is that the identification of hierarchies is reliable, consistent and comparable across all firms.

There exists a large body of empirical evidence concerning the determinants of wage growth. For instance, McCue (1996) shows that position changes within a firm account for approximately 15% of wage growth for males. Smeets and Warzynski (2008) find that higher spans of control are associated with higher wages. Green et al. (2021) demonstrate a size effect on wages; that is, larger employers pay higher wages. Belzil and Bognanno (2008) report a convexity of pay structures and show that past promotion has a positive impact on wage growth but not on bonus growth. Their paper also provides evidence that internal mobility is an important source of wage growth. The results show that wages are mainly determined by firms' profit and then by workers' hierarchy level. One of the novelties of this paper is that I include characteristics of co-workers. For example, a higher average performance (i.e., competition) leads to a lower wage paid. Regarding gender differences in wages (Pinkston, 2003), I do observe that females earn less than males, but the results are statistically insignificant when accounting for the unobserved heterogeneities.

It is also worth noting that this paper is closely related to Kauhanen and Napari (2012a). I use the same dataset but cover different periods. Some results are consistent with theirs, but others are different. Most importantly, this paper focuses on the determinants of promotion; more specifically, I show a concave relationship between promotion rate and firm rank-level profit. In addition, this is the first paper to try running regression at the rank level and using actual promotion rates. The findings and the way I use them provide new insights into the factors involved in promotion.

3.3 Data

3.3.1 The EK Data and the Hierarchy

This paper uses a large linked employer-employee dataset from 2002 to 2019. The data is collected by the Confederation of Finnish Industries (EK), which is the leading business organization in Finland; see Kauhanen and Napar (2012a; 2012b) and DeVaro and Kauhanen (2016) for a detailed description of the data. EK represents the entire private sector and companies of all sizes, including 20 member associations, 15,300 member companies across all business sectors (96% of which are small- and medium-sized enterprises) and 900,000 employees. The member companies are collectively responsible for over 70% of Finland's GDP and over 70% of Finland's exports, employing around 980,000 people.

The EK collects data by sending mandatory annual surveys to its member companies (the response rate is nearly 100%). The data are based on the administrative records of the member companies. Therefore, all information on the dataset is high quality in terms of accuracy. Most importantly, the dataset allows us to follow individuals' career moves over the long term (as much as 38 years). I am able to distinguish between many different types of career moves, including promotions and demotions, within firm mobility and employer changes. Another advantage of the dataset is that it includes 56 job titles and four job levels, and the same job titles and levels are in use by all member companies. This feature enables us to compare job levels across all companies. The dataset also includes information on wages, bonuses and demographic variables such as age, gender and education.

I restrict the analysis to full-time, white-collar employees in manufacturing and to the time period between 2002 to 2019. In this sector, all firms use the same job levels. In addition, there was a change in the classification of job titles in 2002. These restrictions make job levels comparable across firms. The sample consists of 2,367,355 total observations, 367,868 individual persons and 4,702 firms.

I apply the task level of difficulty (based on job title) and sort them into four rank levels. This is consistent with DeVaro and Kauhanen (2016) but different from Kauhanen and Napari (2012a), who use six levels. The top of the rank consists of managerial staff with financial responsibility and administrative duties. The second rank includes senior specialists. The specialists are allocated to the third level. The bottom rank consists of auxiliary staff with simple tasks. In this paper, promotion is defined as a transition from a lower rank level to a higher position within a firm. Note that the rank levels are based on independent competence classifications. It does not invite obvious endogeneity problems related to the analysis of the wage effects of level changes (Kauhanen and Napari, 2012a).

3.3.2 Some Descriptive Results

Figures 3.1 and 3.2 present the relative size distribution of rank levels from 2002 to 2019. In most cases, the relative size increases as I move from the top rank, except the lowest level, rank 1. This is consistent with Kauhanen and Napari (2012a). One possible explanation is that, on average, white-collar employees are well or even highly educated, and relatively few of them are in the lowest rank of entry-level jobs (Kauhanen and Napari, 2012a). The structure of the

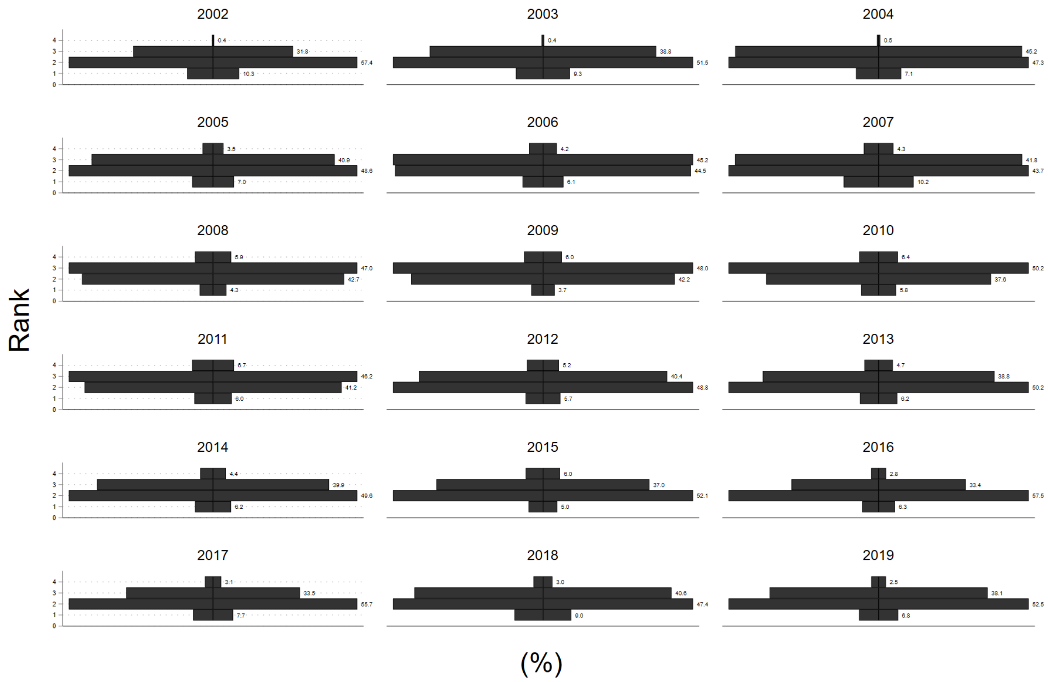

Figure 3.1: Size Distribution of Hierarchical Levels

hierarchy fluctuates but is relative stable over time. I do not observe the large increase in the middle of the hierarchy that Kauhanen and Napari (2012a) found in their dataset. The main reasons are that an increase in the average level of schooling and structural changes in the Finnish economy took place during the last two decades of the 20th century, a period that is not covered in my dataset. The lack of structural change in the hierarchy improves the reliability of the analysis.

Table 3.16 presents all transitions between hierarchical levels both within and between firms with at least two observations. In line with Kauhanen and Napari (2012a), most employees (more than 90%) remain at the same rank, and promotions are more prevalent than demotions. However, demotions are not rare, especially for high-rank employees. In addition, when an employee is promoted or demoted, she is most likely to move up or down only one rank at a time.

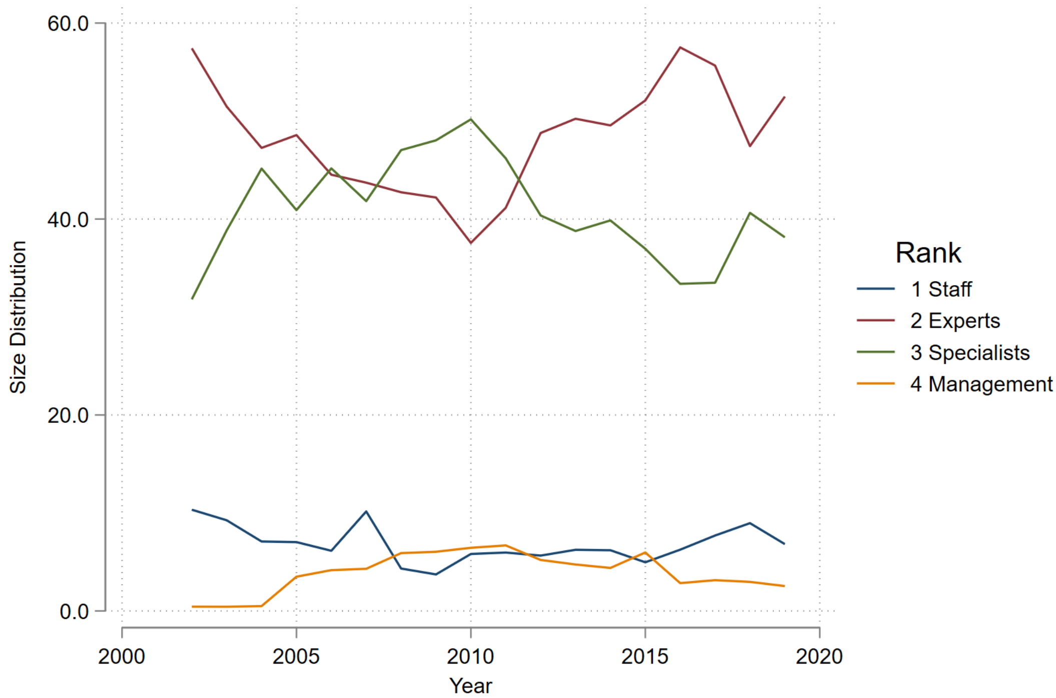

Figure 3.2: Size Distribution over Years by Rank

Table 3.16: Transitions between Hierarchical Levels

	Level t+1				
Level t	1 Staff	2 Experts	3 Specialists	4 Management	N
1 Staff	90.53	8.19	1.17	0.11	360,968
2 Experts	1.58	93.30	4.75	0.36	964,499
3 Specialists	0.31	3.84	93.40	2.45	581,451
4 Management	0.20	1.66	6.32	91.81	128,983

3.4 Promotion Rate and Regressions at the Rank Level

3.4.1 The Methodology and Key Hypothesis

The theoretical starting point is that promotion is a firm's strategy to maximise its profit, no matter whether the role of promotion is with incentive provision or efficiency allocation. Promotion can increase a firm's profit by saving wage costs, inducing extra effort, increasing productivity, or a combination thereof. Firms' promotion strategies must be related to their profitability and other characteristics. That is, different hierarchical level profitabilities have an impact on promotion strategies. More specifically, the theoretical model shows that when the profits of a rank are low, it is not worthwhile to provide extra incentives by indicating a high possibility of promotion; while it is better to use wages and bonuses when the profits of a rank are high. Only when the profits of a rank are moderate is it worthwhile to use high promotion possibilities to provide incentives. Therefore, the main hypothesis of this paper is as follows:

Hypothesis: The profits of a rank have a nonlinear impact on the possibility of promotion to a higher rank.

While the theoretical motivation of the research is briefly outlined above, I further examine the testable predictions. For the purpose of empirical investigation, I identify the target dependent variable as the possibility of promotion and the main explanatory variable as profits at the rank level. Therefore, this study provides a clear causal relation between promotion strategies and profits at the rank level. The empirical model, tentatively presented below, takes the form

$$p_{ijt} = \beta_1 Profit_{ijt} + \beta_2 Profit_{ijt}^2 + \gamma X_{ijt}, \tag{3.38}$$

where i is the rank, j is the firm, t is the year, p_{ijt} is the promotion possibility of rank i of firm j in year t, $Profit_{ijt}$ is the profit, and X_{ijt} are the control variables: size, performance, years of education and age.

3.4.2 Variables and Summary Statistics

Table 3.17 presents the definitions of all variables employed in the above model at rank i at firm j in year t. Promotion rate (%) is the actual possibility of promotion, as calculated by the following formula:

$$p_{ijt} = \frac{the\ number\ of\ internal\ hiring\ workers\ at\ rank\ i+1/2/3\ at\ firm\ j\ at\ year\ t+1}{the\ number\ of\ workers\ at\ rank\ i\ at\ firm\ j\ at\ year\ t} * 100(\%),$$

Table 3.17: Definition of Variables

Name	Variable	Definition
p_{ijt}	Promotion rate	the number of internal promotion at rank $i+$ in year $t+1$/size
$Profit_{ijt}$	Profit	the average wages of workers in terms of logs
$Profit^2_{ijt}$	Profit squared	log profit squared
X_{ijt}	Size	the number of workers in terms of logs
	Average performance	the average performance wages of workers in terms of logs
	Average education	the average years of education of workers
	Average age	the average ages of workers

Table 3.18: Summary Statistics

	N	mean	sd	p25	p50	p75
Promotion rate	79,170	2.52	13.61	0.00	0.00	0.00
Profit	79,170	10.36	0.26	10.17	10.35	10.54
Profit squared	79,170	107.40	5.31	103.53	107.05	111.09
Size	79,170	2.07	1.44	1.10	1.95	3.00
Average performance	79,170	2.93	3.48	0.00	0.00	6.80
Average education	79,170	16.99	1.57	16.00	17.00	18.09
Average age	79,170	43.94	6.48	40.14	44.00	47.87

By definition, the promotion rate of the highest rank, rank 4, is always zero. Therefore, I drop rank 4. Due to limitations in the dataset, I use the average wages of workers in terms of logs as a proxy for the rank profit. There is extensive empirical evidence that suggests a positive relationship between firm performance and average worker wages, based on efficiency wage models, especially for manufacturing industries: workers seek rent-sharing (Holzer et al., 1991; Blanchflower et al., 1996; Ouimet and Simintzi, 2021); higher pay lead to less shirking (Cappelli and Chauvin, 1991); higher pay makes it easier to attract and retain higher-quality workers (Nickell et al., 1994; Propper and Van Reenen, 2010); higher pay results in higher productivity (Raff and Summers, 1987; Mas, 2006). Bell and Van Reenen (2011)replicated the rent-sharing study of Blanchflower et al. (1996) and found that workers may be able to share in the rents from firms when unions were stronger or markets in manufacturing were less competitive. While the primary purpose of the EK data is to provide wage statistics to central wage negotiations and lobbying. EK wage statistics are part of Finland's official statistics and are used to monitor income development and wage levels and structure at EK and to assess the cost impact of wage increases in different sectors. Some studies also use wages as a proxy for firm performance and wages as a proxy for productivity (Medoff and Abraham, 1980; Holzer, 1990; Korenman and Neumark, 1991); for example, Maliranta and Asplund (2007) use wages plus social security payments paid to measure firm profitability. Therefore, with the EK data, the average wage of workers is a good proxy for firm profit. I also introduce profit squared to test the nonlinear relationship between promotion rate and firm rank profit. The number of workers in a rank in terms of logs measures the size of that rank.

For a similar reason using average wages as a proxy for profit, I use the average performance wages (performance-related compensation) of workers in terms of logs as a proxy for average performance. I also have average years of education of workers and their average ages as the average years of education variable and average ages variable. All these control variables measure internal promotion competition. If these competition factors matter, these variables should affect the promotion rate (DeVaro, 2006). All variables are measured at the rank level in year t; Table 3.18 provides the summary statistics of the regressions. In the Appendix, I also provide the summary statistics of the regressions when I exclude rank 1 in the Table 3.27.

3.4.3 Regressions Results at the Rank Level

Table 3.19 shows the determinants of promotion in different settings using equation (3.38) at the rank level. I use actual promotion rates as the dependent variable rather than a dummy promotion variable because I believe that the difficulty of being promoted matters. The main explanatory variables are profit and profit squared. The difference between model 1 and model 2 is whether the profit squared variable is included. The first two columns in Table 3.19 provide the linear regression results. Because firms may apply different standards and rules for promotions, the third and fourth columns provide the fixed effect regression results that account for unobserved firm heterogeneity. Moreover, I use the average wages of workers in terms of logs as a proxy for rank profit and the average performance wages of workers in terms of logs as a proxy for average performance; firms' performance and promotion rates are endogenous, and current promotion rates could be affected by the past promotion rates because of slot constraints. Therefore, I also use a system GMM approach to overcome potential endogeneity and autocorrelation issues.

Table 3.19 shows that profit is positively associated and profit squared is negatively associated with promotion rates. Therefore, the positive coefficient of the profit and the negative coefficient of the profit squared indicate a concave relationship between promotion rate and profit. This is consistent with the theoretical model. The economic logic is that 1) when the profit of a rank is low, there is no need to provide incentives by using promotion; when the profit of a rank is high, it is better to use wage compensation to provide incentives than to use promotion. For these two cases, promotions play only an assignment role. 2) Only when the profit is moderate would firms use promotions to provide incentives. For instance, the fixed effect model predicts that when the profit in terms of logs is below zero, the promotion rate is negatively associated with rank profit. That is, firms do not use promotion opportunities to provide incentives. Meanwhile, when the profit is above zero but below 24.83, promotion rate is positively associated with rank profit. Firms use promotion opportunities to provide incentives, and the promotion rate reaches its peak at 12.41%, when the log profit is at 5.4. Once log profit is above 24.83, the promotion rate is negatively associated with rank profit. According to the theoretical model, firms use increasing wages to provide incentives instead of increasing promotion opportunities. The only concern is that the first column shows a negative but insignificant relationship between profit and promotion rates, whereas all other models reverse this conclusion. Given the time-invariant heterogeneity, potential endogeneity and autocorrelation issues, a concave relationship between promotion rate and profit is more reliable. Empirically, this means that time-invariant firm rank heterogeneity is correlated with firm rank profit.

The results also show that the size of a rank has a negative impact on promotion rates. This also indicates that the difficulty of being promoted matters. For a given promotion slot, a larger size means a lower promotion rate. It is more difficult for a firm with a large size rank to use promotions to provide incentives. As expected, internal promotion competition matters. The signs of competition variables, average performance and average age are negative, showing that the promotion rate is low when the competition is high. On one hand, this

Table 3.19: Regression Results for Promotion Rate at the Rank Level

Dependent variable: Promotion rate (%)	(1) OLS-1	(2) OLS-2	(3) FE-1	(4) FE-2	(5) GMM-1	(6) GMM-2
Profit	-0.16	32.80***	14.27***	86.89***	17.54***	295.37***
	(0.28)	(12.29)	(0.88)	(29.78)	(2.88)	(93.17)
Profit squared		-1.59***		-3.50**		-13.72***
		(0.59)		(1.43)		(4.45)
Size	-1.13***	-1.15***	-0.78***	-0.81***	-1.04***	-1.05***
	(0.04)	(0.04)	(0.13)	(0.13)	(0.08)	(0.08)
Average performance	-0.06***	-0.05***	-0.04*	-0.04*	-0.11***	-0.07***
	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
Average education	-0.02	-0.02	0.59***	0.58***	-1.19***	-0.60***
	(0.05)	(0.05)	(0.11)	(0.11)	(0.24)	(0.15)
Average age	-0.15***	-0.15***	-0.10***	-0.10***	-0.27***	-0.20***
	(0.01)	(0.01)	(0.02)	(0.02)	(0.03)	(0.02)
Promotion lag					-0.01	-0.01
					(0.01)	(0.01)
_cons	12.15***	-158.88**	-148.53***	-524.86***	-143.93***	-1562.72***
	(2.27)	(63.85)	(8.76)	(154.94)	(24.48)	(485.97)
N	79,170	79,170	79,170	79,170	63,041	63,041

FE models control for firm rank effect.GMM modles are system GMM and control for size, average performance, average education,average, age and promotion lag.
Robust standard errors in parentheses. All columns contain year dummies.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

result indicates that an individual has a better chance of being promoted if her co-workers have lower performance or are younger. On the other, better performance and more experiences mean a higher profit for the firm. Due to the concave relationship between promotion rate and profit, firms with a higher profit most likely do not heavily use promotion to provide incentives.

The OLS model shows a negative but insignificant relationship between promotion rates and average education, and the GMM model supports this negative correlation. In contrast, the fixed effect model reverses this conclusion. This indicates a potential multicollinearity issue between wages and education, as I use average wages as a proxy for profit. Average education measures the competition between co-workers, with higher average education indicating better co-workers and a greater difficulty in being promoted. Therefore, average education should have a negative impact on promotion rates. But average education is also correlated with firm rank profit, and profit is most likely positively correlated with promotion rates. Excluding the endogeneity issue, the results of the GMM model are more reliable.

Comparing the effect of profit on promotion to the effect of all other control variables, the magnitude of the effect of profit is notably higher than other variables. For instance, the fixed effect model predicts that when profits increase by 1% (I assume that log profit is below 24.83), promotion rates increase by 0.87%, holding all other factors constant. Meanwhile, when size increases by 1%, promotion rates only decrease by 0.0081%, holding all other factors constant. This may indicate that promotion rate depends heavily on profit.

Promotion lag has a negative but insignificant impact on promotion rates. Unlike Kauhanen and Napari (2012a), who observed a large increase in the middle of the hierarchy, in my dataset, the structure of the hierarchy fluctuates but is relatively stable over time. Therefore, firms' promotion rules are more consistent over time. I take this as evidence that the results in this paper are reliable.

Finally, the fluctuations of the size of rank 2 and rank 3 over time are addressed. I run regressions by using data without rank 1. The summary statistics and regression results are provided in the Appendix in Table 3.27 and Table 3.28, respectively. As with Table 3.19, the difference in Table 13 between model 1 and model 2 is whether to include the profit squared variable. The first two columns of Table 3.28 provide the linear regression results. The third and fourth columns provide the fixed effect regression results to account for unobserved firm heterogeneity. The last two columns provide the results from the system GMM approach. There is no large difference between these and the above results. Therefore, I believe that the concave relationship between promotion rate and profit is robust.

3.5 Dummy Promotion Variable and Regressions at the Individual Level

3.5.1 The Methodology and Key Hypothesis

In this section, I run regressions at the individual level. The model and key hypothesis are similar to the equation (3.38) (regressions at rank level) but take place at the individual level, which allows me to control for gender, province, industry, tenure and rank dummies. The dependent variable is a dummy promotion variable that also differs from equation (3.38).

The empirical model tentatively presented below takes the form

$$p_{nt} = \beta_1 Profit_{ijt} + \beta_2 Profit_{ijt}^2 + \gamma X_{ijt} + \delta Y_{nt} + \eta Z, \tag{3.39}$$

where n is an individual at rank i at firm j in year t, p_{nt} is the dummy variable of promotion for individual n in year t, $Profit_{ijt}$ is the profit of rank i at firm j in year t, X_{ijt} are the variables controlling for firm-rank characteristics: size, performance, years of education and ages, Y_{nt} measures individuals' characteristics: wage, performance, years of education and ages, and Z represents other dummy control variables: gender, rank and tenure of the individual, and province and industry of the firm.

3.5.2 Variables and Summary Statistics

Table 3.20 presents the definitions of all variables employed in the above model. For a promotion of individual n in year t, $p_{nt} = 1$ if individual n is promoted in year $t+1$; otherwise $p_{nt} = 0$. Again, promotions of individuals at the top rank are always zero, and I drop all individuals at rank 4. Profit and firm-rank characteristics variables X_{ijt} are the same as in the above model. Firm-rank characteristic variables measure the internal promotion competition faced by an individual with co-workers at the same rank and firm. Y_{nt} are individual characteristics including log wages and log performance wages as a proxy for performance, years of education and ages. Z represents other dummy control variables: gender, rank and tenure of the individual, and province and industry of the firm. Table 3.21 provides a summary of the statistics of the data used in equation (3.39).

Table 3.20: Definition of Variables

Name	Variable	Definition
p_{nt}	Promotion	$p_{nt} = 1$ if n gets promoted at year $t+1$, otherwise 0
$Profit_{ijt}$	Profit	the average wages of workers in terms of logs
$Profit^2_{ijt}$	Profit squared	log profit squared
	Size	the number of workers in terms of logs
X_{ijt}	Average performance	the average performance wages of workers in terms of logs
	Average education	the average years of education of workers
	Average age	the average ages of workers
	Wage	the wages of n at year t in terms of logs
Y_{nt}	Performance	the performance wages of n at year t in terms of logs
	Education	the years of education of n at year t
	Age	the age of n at year t
	Gender	the gender of n
	Tenure	the type of the contract for n
Z	Rank	the rank of n at year t
	Province	the province of the firm j
	Industry	the industry of the firm j

Table 3.21: Summary Statistics

	N	mean	sd	p25	p50	p75
Promotion	2,367,355	0.04	0.20	0.00	0.00	0.00
Profit	2,367,355	10.41	0.22	10.27	10.40	10.58
Profit squared	2,367,355	108.48	4.60	105.48	108.22	111.96
Size	2,367,355	5.01	1.91	3.64	4.94	6.35
Average performance	2,367,355	5.40	3.11	3.81	6.76	7.68
Average education	2,367,355	17.43	1.22	16.54	17.61	18.42
Average age	2,367,355	42.57	4.13	40.00	42.84	45.32
Wages	2,367,355	10.41	0.29	10.22	10.40	10.60
Performance	2,367,355	3.89	3.80	0.00	5.35	7.59
Education	2,367,355	17.43	2.38	15.00	17.00	19.00
Age	2,367,355	42.57	10.26	42.00	34.00	51.00
Gender	2,367,355	1.35	0.48	1.00	1.00	2.00
Province	2,367,355	1.66	0.95	1.00	1.00	2.00
Industry	2,367,355	50.47	27.13	40.00	40.00	66.00
Tenure	2,367,355	0.96	0.32	1.00	1.00	1.00
Rank	2,367,355	2.11	0.70	2.00	2.00	3.00

3.5.3 Regression Results at the Individual Level

Table 3.22 examines determinants of promotion in different settings using equation (3.39) at the individual level. The first setting is a linear probability model, the second is a fixed effects model that accounts for time-invariant unobserved individual heterogeneity, and the third one is a fixed effects logit model. Most of the promotion variables are zeros, so I am concerned about rare events bias. Therefore, the fourth model is a complementary log-log model using an extreme value distribution.

Table 3.22 shows a concave relationship between promotion and profit, which is in line with the above results, using equation (3.38) at the rank level and the theoretical model. For instance, the fixed effects logit model predicts that when the profit in terms of logs is below zero or above 10, promotion is negatively associated with rank profit. When profit in in terms of logs is between zero or 10, promotion is positively associated with rank profit. The reason is explained above

Table 3.22: Regression Results For Promotion at the Individual Level

Dependent variable: Promotion dummy	(1) LPM	(2) FE	(3) XTLOGIT	(4) CLOGLOG	(5) CLOGLOG-AME
Profit	1.550***	2.474***	35.686***	18.487***	0.763***
	(0.07)	(0.13)	(3.04)	(1.88)	(0.00)
Profit squared	-0.076***	-0.121***	-1.794***	-0.914***	-0.038***
	(0.00)	(0.01)	(0.15)	(0.09)	(0.00)
Size	0.001***	-0.007***	-0.023***	0.056***	0.002***
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)
Average performance	-0.001***	-0.001***	-0.071***	-0.038***	-0.002***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Average education	-0.002***	0.000	-0.095***	-0.052***	-0.002***
	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)
Average age	-0.002***	-0.001***	-0.020***	-0.043***	-0.002***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Wage	0.134***	0.124***	3.448***	3.051***	0.126***
	(0.00)	(0.00)	(0.06)	(0.02)	(0.00)
Performance	0.001***	0.001***	0.058***	0.028***	0.001***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Education	0.002***	0.001***	0.096***	0.047***	0.002***
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)
Age	-0.001***	-0.001***	-0.072***	-0.038***	-0.002***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Female	-0.004***	0.025	-0.216	-0.098***	-0.004***
	(0.00)	(0.06)	(1.00)	(0.01)	(0.00)
Rank 2	-0.068***	-0.204***	-3.710***	-1.429***	-0.123***
	(0.00)	(0.00)	(0.03)	(0.01)	(0.00)
Rank 3	-0.109***	-0.367***	-7.690***	-2.663***	-0.155***
	(0.00)	(0.00)	(0.05)	(0.02)	(0.00)
_cons	-9.071***	-13.594***		-124.048***	
	(0.36)	(0.66)		(9.72)	
N	2,367,355	2,367,355	813,853	2,220,219	

Robust standard errors in parentheses. All columns contain gender, tenure, rank, province, industry and year dummies.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

in subsection 3.4.3.

The results also show that rank size has a positive impact on promotion, but when accounting for the time-invariant unobserved individual heterogeneity, the fixed effect LPM and logit models reverse this conclusion. A negative relationship between promotion and size aligns with the above results using equation (3.39). Kauhanen and

Napari (2012a) found that internal promotions are more common in larger firms. This might be because larger firms have well-defined career paths and are more likely to rely on internal promotions (Chan, 1996). A potential explanation is that a large firm has more promotion slots, which has a positive impact on promotions. While a large firm also has a large size, when accounting for unobserved individual heterogeneity or the difficulty of being promoted, the overall effect on promotion is negative. On one hand, more workers mean a higher level of promotion competition for a given position. On the other, larger firms prefer internal promotions, and more workers also mean more promotion positions. And the first effect dominates the second one in my dataset. This result also suggests that the difficulty of being promoted matters.

Regarding competition variables, average performance, average years of education and average age all have a negative impact on promotion, which is also in line with the above results using equation (3.38) at the rank level. Kauhanen and Napari (2012a) also showed that an individual has a higher chance of getting promoted within a firm if her co-workers are less educated.

Regarding individual characteristics, wage is positively related to promotion, which is in line with Kauhanen and Napari (2012a) but in contrast to Acosta (2010), who found that individuals at lower wage deciles are more likely to be promoted than individuals at higher wage deciles using data from a single U.S. firm. Years of education are also positively related to promotion, which is in line with both Acosta (2010) and Kauhanen and Napari (2012a) . When it comes to the relationship between performance and promotion, the results show a positive relationship. Ages are negatively related to promotion, which is also in line with Kauhanen and Napari (2012a). Kauhanen and

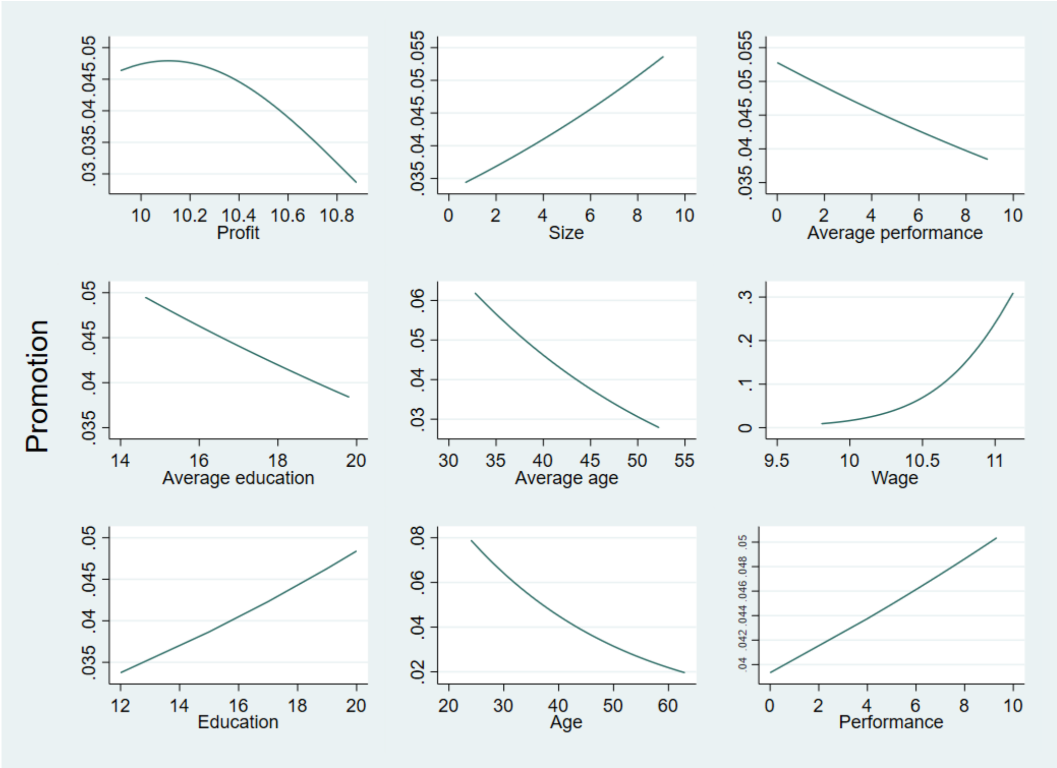

Figure 3.3: Average Marginal Effect Using Cloglog Model

Napari (2012a) believe that this supports the existence of fast tracks.

The results show that females are less likely to be promoted than men, as has been found in many studies (e.g., Ransom and Oaxaca 2005; Blau and DeVaro 2007; Kauhanen and Napari 2012a). But this is not true when considering unobserved individual or firm heterogeneities. Kauhanen and Napari (2012a) show a similar result: a positive but insignificant effect on promotion in the fixed effects model.

Both rank 2 and rank 3 are negatively related to promotion, and the marginal effect of rank 3 on promotion is higher than the marginal effect of rank 2. This indicates that promotions are more likely at the bottom of a rank and reflect that there is more room for promotions at lower ranks (Kauhanen and Napari, 2012a).

Finally, the partial effect in panel logit regression is not meaning-

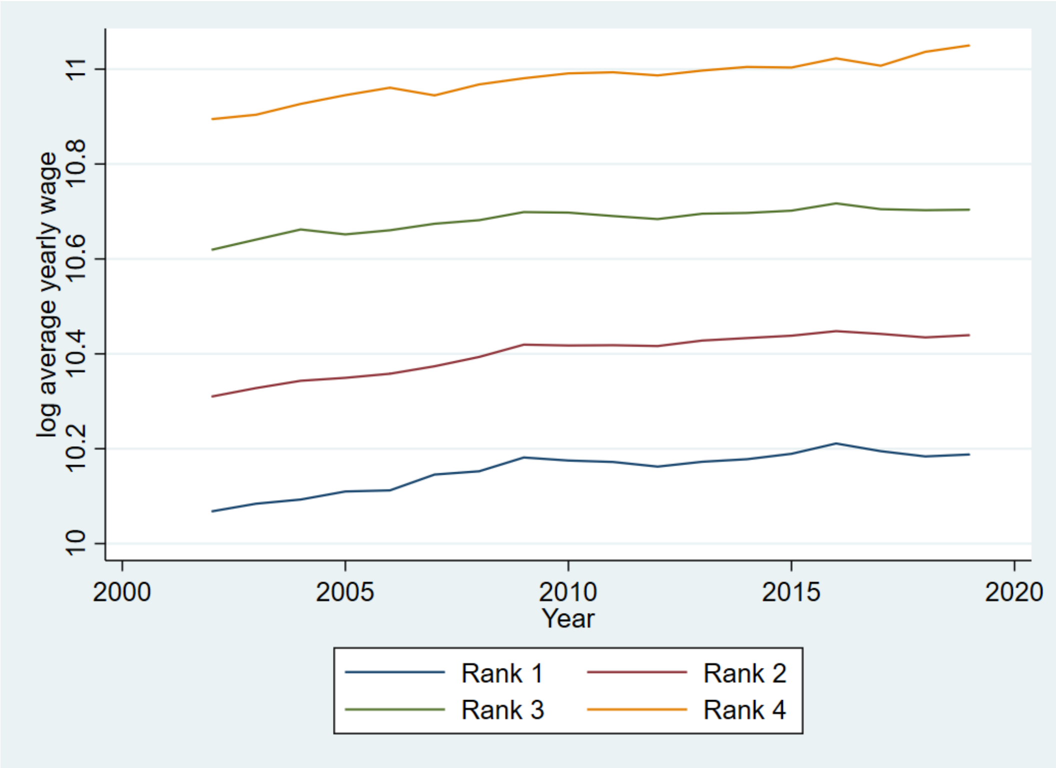

Figure 3.4: Average Annual Wages by Rank Level, 2002–2019

ful because of the choice of where to evaluate the individual effect is completely arbitrary (Kemp and Silva, 2016). Therefore, column (5) provides the average marginal effect using the cloglog model. Similar to the results in the rank level regressions, comparing the effect of profit on promotion to the effect of all other control variables shows that the magnitude of the effect of profit is substantially higher than other variables. For instance, the clogclog model predicts that an extra unit of log profit increases the probability of promotion by 76.3%. The second important group of variables for promotion are wage and rank levels because promotion normally comes with a substantial salary increase. Figure 3.3 shows the average marginal effect on promotion.

3.6 The Determinants of Wages

The impact of rank levels on wages and promotion on wage growth are the main questions in the literature on careers in organisations. Therefore, in this section, I investigate the determinants of wages. Figure 3.4 presents the relative wage structure in Finnish manufacturing over the 2002–2019 period by showing the development of average annual wages by rank level over time. Like Kauhanen and Napari (2012a), I find that average annual wages increase with level, and wages have grown in a very similar fashion across rank levels, indicating that the relative wage structure is practically unchanged between 2002 and 2019. A rigidity in wage structure implies that rank has a meaningful impact on wages (Baker et al., 1994a). In addition, unlike Kauhanen and Napari (2012a), I do not observe a small difference, and there is no crossing between the top two levels. This suggests that the four scales of rank levels are more meaningful and informative than their six scales of rank levels.

Following Baker et al. (1994a) and Kauhanen and Napari (2012a), Figure 3.5 shows the variation in wages within rank levels by providing wage ranges by rank level. In line with them, I find log wages increase linearly with rank level. Unlike Kauhanen and Napari (2012a), I find no kink between the top two levels, which again indicates that the four scales of rank levels are more reliable. Like those authors, I find substantial wage overlap between rank levels. For example, workers in the upper quartile of the wage distribution at rank 1 have higher wages than workers in the lower quartile at rank 4. The variation in wages within rank levels implies that wages are not only determined by rank levels but also by other factors, such as education (Gibbons and Waldman, 2006).

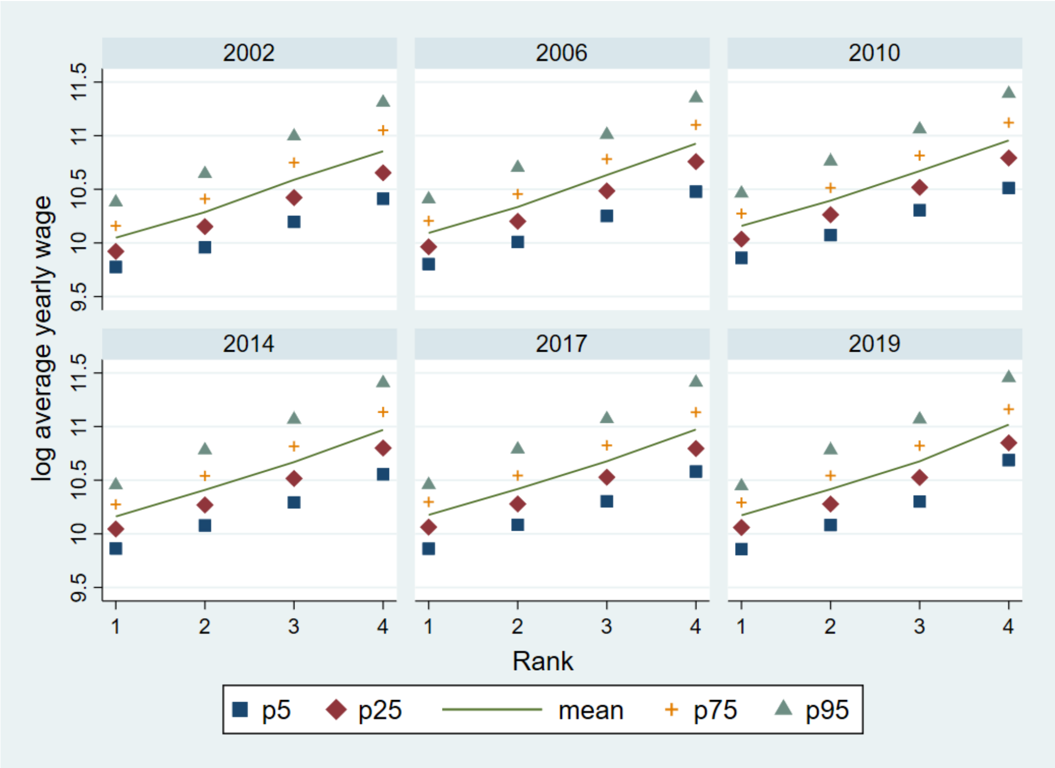

Figure 3.5: Variation in Wages within Rank Levels

In Table 3.23, I investigate the determinants of wages. Model (1) is the OLS regression; model (2) considers the time and individual effects; model (3) considers the time, individual and firm effects; and model (4) considers the individual and firm effects. The difference between my model and the one in Kauhanen and Napari (2012a) is that I include internal competition variables (size, average performance, average education and average age). If these competition factors matter, these variables should affect wages. Performance, education, age and rank level make positive contributions to wages. This is consistent with Kauhanen and Napari's model (2012a). Average performance has a negative impact on wages, while average education and age each has a positive impact on wages.

Finally, all the results are consistent in different settings, except for gender. The OLS model shows that, on average, females have lower wages than males, as in Kauhanen and Napari (2012a). When I ac-

Table 3.23: Regression Results For Wages at the Individual Level

Dependent variable: Log wages	(1) OLS	(2) FE-1	(3) FE-2	(4) FE-3	(5) FE-4
Size	0.008***	0.004***	0.004***	0.004***	0.003***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Average performance	-0.004***	-0.001***	-0.002***	-0.002***	-0.001***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Average education	0.041***	0.005***	0.005***	0.004***	0.004***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Average age	0.001***	0.000***	0.000***	0.000***	0.000***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Performance	0.007***	0.002***	0.002***	0.002***	0.002***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Education	0.024***	0.021***	0.021***	0.021***	0.021***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Age	0.006***	0.016***	0.016***	0.000	0.015***
	(0.00)	(0.00)	(0.00)	(.)	(0.00)
Female	-0.099***	0.016	0.018	0.031	0.029
	(0.00)	(0.02)	(0.02)	(0.02)	(0.02)
Rank2	0.126***	0.040***	0.040***	0.042***	0.043***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Rank3	0.304***	0.100***	0.100***	0.102***	0.103***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
_cons	8.811***	9.237***	9.178***	9.839***	9.213***
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
N	2,367,355	2,367,355	2,367,355	2,367,355	2,301,794

Standard errors in parentheses. All columns contain gender, tenure, rank, province and industry dummies.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

count for unobserved individual heterogeneity, the opposite turns out to be true, but those results are insignificant.

3.7 Discussion and Extensions

In this section, I highlight the implications of promotion in my model and explore the robustness of the key findings to a set of alternative modelling assumptions.

3.7.1 Many Zeros and Rare Events Bias

Table 3.24: ZIP Regression Results at the Rank Level

Dependent variable: Promotion rate	(1) ZIP-1	(2) ZIP-2
Profit	0.08	25.50***
	(0.08)	(5.57)
Profit squared		-1.23***
		(0.27)
Size	-0.55***	-0.56***
	(0.02)	(0.02)
Average performance	-0.03***	-0.03***
	(0.01)	(0.01)
Average education	-0.01	-0.01
	(0.01)	(0.01)
Average age	-0.04***	-0.04***
	(0.00)	(0.00)
_cons	-1.89**	-133.31***
	(0.75)	(28.75)
inflate		
_cons	-25.27***	-25.27***
	(0.02)	(0.02)
N	79,170	79,170

Standard errors in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Most employees (more than 90%) remain at the same rank, and the overall promotion rate is around 5%. Therefore, most of the promotion rate variables are zeros, which leads to a potential zero inflation problem and biased results. Table 3.24 shows the alternative regression results using zero-inflated Poisson models at the rank level (equation 3.38). Zero-inflated Poisson models generate counts governed by a Poisson distribution, which are suitable for analysing non-negative count outcomes that have many zeros and areright-skewed (note that the promotion rate variables are continuous outcomes).

Most of the ZIP model results are similar to the above results using equation (3.38) at the rank level. Therefore, I believe that the concave relationship between rank profit and promotion rates is reliable.

The only exception is the effect of average education on promotion. It shows a negative but insignificant impact on promotion rates. I have already identified and reported on this issue in Section 3.4. The effect of average education is not consistent in different settings. One potential explanation is that education significantly impacts wages (see results in Section 3.5), and I use average wages as a proxy for profit.

Finally, for regressions using equation (3.39) at the individual level, I use a complementary log-log model for the potential zero inflation problem and biased results. The results are shown in Table 3.22.

3.8 Conclusion

I have examined the relationship between promotion rate and firm rank-level profit of white-collar workers employed in Finnish manufacturing. This study adds to Kauhanen and Napari's (2012a) paper in five important ways. First, I use the same linked employer-employee dataset but cover different periods. Their data covers periods that saw increases in the average level of schooling and structural changes in the Finnish economy, while no structural transformation of the hierarchy occurred during the period which fluctuates but is relatively stable over time in my data, making the analysis more reliable. Second, the promotion variables include not only dummy promotion variables but also the actual possibility of promotion. I believe that the difficulty of being promoted matters. Third, the analysis of determinants of promotion extends to both rank-level regression and individual-level regression. Fourth, the analysis of determinants of wages includes internal competition variables. If these competition factors matter, these variables should affect wages. Fifth, I also analyse the determinants of demotion. The results weakly support the findings of determinants of promotion.

The key finding is that promotion and firm rank-level profit have a concave relationship and that demotion and firm rank-level profit might have a convex relationship. The results are robust in different settings and consistent with the theoretical model in chapter 1. This indicates that promotion and demotion are firms' strategies to maximize profits, which depend on firm rank-level profitability and other characteristics. Furthermore, in line with Kauhanen and Napari (2012a) , the characteristics of co-workers (internal competition issues like size, average performance, average education and average age) also matter.

However, the evidence shows that firms that heavily use promotions also provide more training (DeVaro and Morita, 2013), and the theoretical model in chapter 2 also indicates that training affects promotion policies. Due to data limitations, I am not able to analyse the effect of training on promotions. Future empirical work should consider these aspects of firm-sponsored training.

3.9 Appendices

3.9.1 Summary Statistics and Regression Results - No Rank 1

Table 3.25: Summary Statistics – No Rank 1

	N	mean	sd	p25	p50	p75
Promotion rate	55,325	1.63	10.74	0.00	0.00	0.00
Profit	55,325	10.46	0.22	10.31	10.45	10.62
Profit squared	55,325	109.52	4.60	106.32	109.13	112.68
Size	55,325	2.21	1.45	2.08	1.10	3.14
Average performance	55,325	3.11	3.57	0.00	0.00	7.04
Average education	55,325	17.40	1.45	17.46	16.55	18.43
Average age	55,325	44.32	6.00	40.77	44.33	47.93

3.9.2 Demotion

As shown in Table 3.16, demotions are less typical than promotions, but demotions are still not rare. I have shown that firms' promotion strategies depend on hierarchical level profitabilities and other characteristics. Hierarchical-level profitabilities should also have an impact on demotions. Therefore, I examine determinants of demotion using both equations (3.38) and (3.39) with different settings. Table 3.25 provides the regression results for demotion rates using equation (3.38) at the rank level. Similar to Table 3.19, the difference between model 1 and model 2 is whether to include the profit squared variable. The first two columns in Table 3.25 provide the linear regression results. The third and fourth columns provide the

Table 3.26: Regression Results for Promotion Rate at the Rank Level – No Rank 1

Dependent variable: Promotion rate (%)	(1) OLS-1	(2) OLS-2	(3) FE-1	(4) FE-2	(5) GMM-1	(6) GMM-2
Profit	1.41***	14.70	11.30***	31.43	15.52***	155.15*
	(0.30)	(16.17)	(0.92)	(37.57)	(2.63)	(87.80)
Profit squared		-0.63		-0.96		-6.95*
		(0.77)		(1.80)		(4.17)
Size	-0.84***	-0.84***	-0.79***	-0.80***	-0.66***	-0.69***
	(0.04)	(0.04)	(0.13)	(0.13)	(0.06)	(0.06)
Average performance	-0.06***	-0.06***	-0.02	-0.02	-0.12***	-0.08***
	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
Average education	-0.05	-0.05	0.41***	0.41***	-0.75***	-0.37***
	(0.05)	(0.05)	(0.11)	(0.11)	(0.17)	(0.13)
Average age	-0.09***	-0.09***	-0.06**	-0.06**	-0.18***	-0.12***
	(0.01)	(0.01)	(0.02)	(0.02)	(0.03)	(0.02)
Promotion lag					0.00	-0.01
					(0.01)	(0.01)
_cons	-6.82***	-76.45	-118.56***	-223.81	0.00	0.00
	(2.60)	(84.53)	(9.18)	(196.44)	(.)	(.)
N	55,325	55,325	55,325	55,325	44,265	44,265

Cluster robust standard errors in parentheses. All columns contain year dummies.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

fixed effect regression results to account for unobserved firm heterogeneity. The last two columns provide the results from the system GMM approach.

Table 3.25 provides inconsistent results regarding the relationship between rank profit and demotion rates. The OLS and fixed effect models indicate a convex relationship between demotion and rank profit, but the GMM results do not support this. The size of rank has a negative impact on demotion rates, which is the same as its impact on promotion rates. This also indicates that the difficulty of being demoted matters. For a given demotion slot, a larger size means a lower demotion rate.

As to the competition variables, the OLS and fixed effect models provide similar results. All the competition variables have a negative

Table 3.27: Regression Results for Demotion Rate at the Rank Level

Dependant variable: Demotion rate (%)	(1) OLS-1	(2) OLS-2	(3) FE-1	(4) FE-2	(5) GMM-1	(6) GMM-2
Profit	1.82***	42.60***	-6.68***	54.92***	-8.92***	-62.43
	(0.19)	(7.83)	(0.67)	(19.56)	(1.79)	(42.59)
Profit squared		-1.96***		-2.97***		2.75
		(0.38)		(0.95)		(2.03)
Size	-0.54***	-0.56***	-0.27***	-0.30***	-0.11**	-0.14***
	(0.03)	(0.03)	(0.08)	(0.08)	(0.04)	(0.04)
Average performance	-0.04***	-0.04***	-0.02*	-0.02	0.01	-0.01
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Average education	-0.03	-0.03	-0.31***	-0.31***	0.71***	0.41***
	(0.03)	(0.03)	(0.08)	(0.08)	(0.15)	(0.10)
Average age	-0.02**	-0.02**	-0.02	-0.02	0.08***	0.04**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)
Demotion lag					-0.00	-0.01
					(0.01)	(0.01)
_cons	-15.54***	-227.15***	75.75***	-243.53**	0.00	343.54
	(1.49)	(40.50)	(6.75)	(100.41)	(.)	(222.38)
N	79,170	79,170	79,170	79,170	63,041	63,041

FE models control for firm rank effect.GMM modles are system GMM and control for size, average performance, average education,average, age and demotion lag.
Cluster robust standard errors in parentheses. All columns contain year dummies.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

impact on demotion rates. For instance, average performance measures how good one's competitors are, and a higher average performance tends to result in lower rates of both promotion and demotion. Again, the GMM results do not support this. Similar to the lag effect of promotion rates, the demotion lag has a negative but insignificant impact on demotion rates. This again indicates that the rank structure of my dataset is stable and that the demotion rules are also consistent over time.

Table 3.26 presents the determinants of demotion in different settings using equation (3.39) at the individual level. Similar to Table 3.22, the first setting is a linear probability model, the second is a fixed effect that accounts for time-invariant unobserved individual

Table 3.28: Regression Results For Demotion at the Individual Level

Dependent variable: Demotion dummy	(1) LPM	(2) FE	(3) XTLOGIT
Profit	0.48804***	0.07984	-14.98538***
	(0.04)	(0.08)	(5.37)
Profit squared	-0.02125***	-0.00228	0.78596***
	(0.00)	(0.00)	(0.25)
Size	-0.00184***	-0.00318***	-0.04914***
	(0.00)	(0.00)	(0.01)
Average performance	0.00012***	0.00012*	-0.03356***
	(0.00)	(0.00)	(0.00)
Average education	-0.00352***	-0.00403***	-0.05087**
	(0.00)	(0.00)	(0.02)
Average age	-0.00046***	-0.00098***	0.01097***
	(0.00)	(0.00)	(0.00)
Wage	-0.05739***	-0.09619***	-3.79831***
	(0.00)	(0.00)	(0.09)
Performance	-0.00066***	-0.00060***	-0.00404
	(0.00)	(0.00)	(0.00)
Education	-0.00051***	-0.00163***	0.00277
	(0.00)	(0.00)	(0.01)
Age	-0.00000	-0.00158***	0.32104***
	(0.00)	(0.00)	(0.00)
Female	0.00301***	0.07474	1.64931
	(0.00)	(0.07)	(1.34)
Rank2	0.02303***	0.09734***	8.56201***
	(0.00)	(0.00)	(0.41)
Rank3	0.05611***	0.19716***	12.02158***
	(0.00)	(0.00)	(0.41)
_cons	-2.10088***	0.50423	
	(0.19)	(0.40)	
N	2,367,355	2,367,355	383,193

Standard errors in parentheses. All columns contain gender, tenure, rank, province, and industry dummies.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

heterogeneity, and the third is a fixed effects logit model.

The LPM and fixed effects models provide similar results, while the results of the panel logit model with a fixed effects are quite different. The LPM and fixed effects models indicate a convex relationship between demotion and rank profit, while the results of the panel logit model do not support this. Size, average education, wage and performance have a negative impact on demotion; the results are consistent in all three models. The rank level has a positive impact on demotion, which is also consistent.

In the LPM and fixed effects models, average performance has a positive impact on demotion, and average age, education and age all have a negative impact on demotion, whereas the panel logit model finds the opposite results. Females are most likely to get demoted than males in the LPM model. When considering the unobserved individual heterogeneity, this result is insignificant.

Regarding the magnitude of the effect of different variables on demotion, I find that they are similar to promotion; rank profits have the most impact, followed by wages and rank levels. In summary, the determinants of demotion are much more complicated than the determinants of promotion. This difference should be investigated in further research.

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